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SUBJECT INDEX

PAPERS, DISCUSSIONS, AND REPORTS

Subject Classifications

American Institute of Electrical Engineers Affairs.........1172

Cables.........1172
Central Station Systems.........1172
Circuit Breakers and Switches.........1172
Communication.........1172
Control Devices.........1172
Conversion Methods and Equipment.........1172
Corona.........1173

Dielectric Phenomena.........1173
Distribution.........1173

Electric Circuit Theory.........1173
Electrochemistry.........1173
Electrophysics.........1173

Generators, Alternating Current.........1173

Industrial Application of Power.........1173
Insulators.........1173
Interconnected Systems.........1173

Lighting.........1173
Lightning Arresters.........1173
Loss and Efficiency Measurements.........1173
Loud Speakers.........1173

Magnetic Phenomena.........1173
Marine Applications.........1173
Measuring Instruments.........1173
Measuring Methods.........1173
Mining Applications.........1174
Motors, Alternating Current.........1174

Power Factor.........1174
Power Stations.........1174
Protective Devices.........1174

Radio Phenomena.........1174
Railways.........1174
Reactors.........1174
Rectifiers.........1174
Relays.........1174
Research.........1174

Standardization.........1174
Starting Currents.........1174
Substations.........1174
Switches.........1174

Technical Committee Reports.........1174
Telegraph.........1175
Telephone.........1175
Television.........1175
Transformer Connections.........1175
Transformers.........1175
Transients.........1170
Transmission Lines (Electrical Characteristics).........1175
Transmission Lines (General).........1175

Wave Form.........1175

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS AFFAIRS

See also Standardization, Technical Committee Reports

A. I. E. E. Outstanding Features of the Past Year, President's Address, *Chesney, C. C.*, (June)......... 1153
Report of the Board of Directors for the Fiscal Year Ending April 30, 1927......... 1159

CABLES

High-Voltage Measurements on Cables and Insulators, *Kasson, C. L.*, (May)......... 635
Joints in High-Voltage Multiple-Conductor Cable, *Peterson, Thos. F.*, (June)......... 963
New 132,000-Volt Cable Joint, A, *Simons, Donald M.*, (February)......... 240
Printing Telegraphs on Non-Loaded Ocean Cables, *Angel, Herbert*, (June)......... 884
Proximity Effect in a Seven-Strand Cable, *Tweeddale, J. E. L.*, (May)......... 1148

CENTRAL STATION SYSTEMS

See also Hydroelectric Power Stations

Auxiliary Power at Richmond Station, *Anderson, J. W., and Monteith, A. C.*, (June)......... 827
Characteristics of Interconnected Power Systems, *Blume, L. F.*, (May)......... 590
Combined Light and Power Systems for A-C. Secondary Networks, *Richter, H.*, (February)......... 216
Holtwood Steam Plant, The, *Allner, F. A.*, (June)......... 815
Power Generation Committee Report (June)......... 748
Standardization of Voltage Ratings for Power Systems and Equipment, *Silver, A. E., and Harding, A. L.*, (February)......... 175
Voltage Standardization from a Consulting Engineer's Point of View, *Argersinger, R. E.*, (February)......... 172
Voltage Standardization, Its Relation to the Interconnected Power Companies of the Southeast, *Scholz, H. J., Eberhardt, W. W., and Jones, S. Murray*, (February)......... 137
Voltage Standardization of A-C. Systems from the Viewpoint of the Electrical Manufacturer, *Hanker, F. C., and Summerhayes, H. R.*, (February)......... 161
Voltage Standards for Electrical Distribution, *Gear, H. B.*, (February)......... 185

CIRCUIT BREAKERS AND SWITCHES

Oil Circuit Breaker Development (A. I. E. E. JOURNAL, July, 1927, p. 707)
Tests on High- and Low-Voltage Oil Circuit Breakers, *Sporn, Philip, and St. Clair, Harry P.*, (February)......... 289

COMMUNICATION

See also Telephone, Telegraph, and Radio Phenomena

Communication Committee Report, (June)......... 713
Fifty Year's Progress in Electrical Communications, (A. I. E. E. JOURNAL, January, 1927, p. 59 and February, 1927, p. 171)
High-Frequency Measurement of Communication Apparatus, *Shackelton, W. J. and Ferguson, J. G.*, (May)......... 519
High-Frequency Measurements of Communication Lines, *Affel, H. A., and O'Leary, J. T.*, (May)......... 504

CONTROL DEVICES

See also Industrial Applications of Power, Motors

Carrier-Current Selector Supervisory Equipment, *Stewart, C. E., and Whitney, C. F.*, (March)......... 431
Development of Automatic Switching Equipments in the United States and Europe, *de Goede, A. H.*, (May)......... 660
Use of High-Frequency Currents for Control, The, *Boddie, C. A.*, (June)......... 975

CONVERSION METHODS AND EQUIPMENT

See Substations

Synchronous Converter Theory and Calculations, The, *Hambleton, T. T., and Bewley, L. V.*, (February)......... 60

CORONA

See also Dielectric Phenomena

Law of Corona and the Dielectric Strength of Air—IV, The, The Mechanism of Corona Formation and Loss, *Peek, F. W.*, (June) 1009

Methods Used in Investigating Corona Loss by Means of the Cathode Ray Oscillograph, *Lloyd, W. L., and Starr, E. C.*, (June) 997

Space Charge and Current in Alternating Corona, *Willis, C. H.*, (February) 271

DIELECTRIC PHENOMENA

See also Corona

Electric Strength of Solid and Liquid Dielectrics, *Del Mar, W. A., Davidson, W. F., and Marvin, R. H.*, (June) 1049

Electrical Resistivity of Insulating Materials, The, *Curtis, Harvey L.*, (June) 1039

Maxwell's Theory of the Layer Dielectric, *Murnaghan, Francis D.*, (February) 259

Oil Breakdown at Large Spacings, *Miner, Douglas F.*, (February) 248

Theory of Imperfect Solid Dielectrics, A, *Malti, Michel G.*, (May) 616

DISTRIBUTION

See also Substations, Transmission Lines

Voltage Standards for Electrical Distribution, *Gear, H. B.*, (February) 185

ELECTRIC CIRCUIT THEORY

See also Power Factor, Transients, Wave Form

Current Analysis in Circuits Containing a Resistance Modulator, *Grandy, L. S.*, (March) 421

Electric Arc, The, *Compton, K. T.*, (June) 868

Empirical Analysis of Complex Electric Waves, The, *Horton, J. W.*, (May) 535

Experimental Measurement of Mechanical Forces in Electric Circuits, *Roper, J. Walter*, (May) 574

Impedance of a Non-Linear Circuit Element, *Peterson, Eugene*, (May) 528

Mechanical Forces between Electric Currents and Saturated Magnetic Fields, *Karapetoff, Vladimir*, (May). 563

Non-Harmonic Alternating Currents, *Bedell, Frederick*, (May) 648

Studies on Sparking in Air, *Sah, A. Pen-Tung*, (May). 604

Substitution Method for the Determination of Resistance of Inductors and Capacitors at Radio Frequencies, *Burke, C. T.*, (May) 482

Two Cases of Calculation of Mechanical Forces in Electric Circuits, *Dwight, H. B.*, (May) 570

ELECTROCHEMISTRY

Electrochemistry and Electrometallurgy Committee Report, (June) 812

ELECTROPHYSICS

Electrophysics Committee Report (June) 700

GENERATORS, ALTERNATING

See also Loss and Efficiency Measurements, Temperature in Apparatus, Wave Form

Additional Losses of Synchronous Machines, *Laffoon, C. M., and Calvert, J. F.*, (February) 84

Study of Transverse Armature Reaction in Synchronous Machines, A, (A. I. E. E. JOURNAL, August, 1927, p. 811)

Synchronous Machines—III Torque-Angle Characteristics Under Transient Conditions, *Doherty, R. E., and Nickle, C. A.* (February) 1

Transverse Reaction in Synchronous Machines, *Douglas, J. F. H.*, (February) 30

INDUSTRIAL APPLICATIONS OF POWER

See also Control Devices, Furnaces and Electric Heating, Lamps, Mining Applications

Application of Electricity in Cement Mills, *North, W. E.*, (April) 462

Applications to Iron and Steel Production Committee Report, (June) 799

Developments in the Manufacture of Copper Wire, *Shea, John R., and McMullan, Samuel*, (February) 387

Electric Welding Committee Report, (June) 807

Electric Welding of Pipes (A. I. E. E. JOURNAL, June, 1927, p. 593)

Electricity in the Drilling of Oil Wells, *Murphy, L. J.*, (March) 455

Recent Developments in Electric Drives for Rolling Mills, *Umansky, L. A.*, (April) 469

INSULATORS

See Transmission Lines (General)

High-Voltage Measurements on Cables and Insulators, *Kasson, C. L.*, (May) 635

INTERCONNECTED SYSTEMS

See Central Station Systems

Characteristics of Interconnected Power Systems, *Blume, L. F.*, (May) 590

Voltage Standardization, Its Relation to the Interconnected Power Companies of the Southeast, *Scholz, H. J., Eberhardt, W. W., and Jones, S. Murray*, (February) 187

LIGHTING

Production and Application of Light Committee Report (June) 718

LIGHTNING ARRESTERS

Measurement of Surge Voltages on Transmission Lines Due to Lightning, The, *Lee, Everett S. and Foust, C. M.*, (February) 339

LOSS AND EFFICIENCY MEASUREMENTS

Additional Losses of Synchronous Machines, *Laffoon, C. M., and Calvert, J. F.*, (February) 84

Calculation of Stray Load Losses, *Rockwood, G. H.*, (May) 1139

Methods Used in Investigating Corona Loss by Means of the Cathode Ray Oscillograph, *Lloyd, W. L., and Starr, E. C.*, (June) 997

Reduction of Armature Copper Losses, *Summers, Ivan H.*, (February) 101

LOUD SPEAKERS

Electrical Reproduction from Phonograph Records, *Kellogg, Edward W.*, (June) 903

MAGNETIC PHENOMENA

Graphical Determination of Magnetic Fields, Comparison of Calculations and Tests, *Johnson, E. E., and Green, C. H.*, (February) 136

Graphical Determination of Magnetic Fields, Practical Applications to Salient-Pole Synchronous Machine Design, *Wieseman, Robert W.*, (February) 141

Graphical Determination of Magnetic Fields, Theoretical Considerations, *Stevenson, A. R., Jr., and Park, R. H.*, (February) 112

Mechanical Forces between Electric Currents and Saturated Magnetic Fields, *Karapetoff, Vladimir*, (May) 563

MARINE APPLICATIONS

Applications to Marine Work, Committee Report, (June) 803

MEASURING INSTRUMENTS

See also Loss and Efficiency Measurements, Measuring Methods, Temperature Measurements

Instrument for Measuring Short-Circuit Torque, An, *Penney, G. W.*, (May) 683

New Thermionic Instrument, A, *Hoare, S. C.*, (May). 541

Notes on the Use of a Radio Frequency Voltmeter, *Goodwin, W. N., Jr.*, (May) 479

Oscilloscope: a Stabilized Cathode-Ray Oscillograph with Linear Time-Axis, The, *Bedell, Frederick, and Reich, Herbert J.*, (May) 546

Stroboscopic Method of Testing Watthour Meters, *Sparkes, H. P.*, (February) 405

Theory of Action of the Induction Watthour Meter and Analysis of its Temperature Errors, *Canfield, D. T.*, (February) 411

MEASURING METHODS

See also Loss and Efficiency Measurements, Measuring Instruments, Temperature Measurements

Condenser Shunt for Measurement of High-Frequency Currents of Large Magnitude, *Nyman, Alexander*, (May) 488

High-Frequency Measurement of Communication Apparatus, *Shackelton, W. J., and Ferguson, J. G.*, (May) 519

High-Frequency Measurements of Communication Lines, *Affel, H. A., and O'Leary, J. T.*, (May) 504

High-Voltage Measurements on Cables and Insulators, *Kasson, C. L.*, (May) 635

Instruments and Measurements Committee Report, (June) 705
Klydonograph Surge Investigations, *Cox, J. H., McAuley, P. H., and Huggins, L. Gale,* (February) 315
Measurement of Surge Voltages on Transmission Lines Due to Lightning, The, *Lee, Everett S., and Foust, C. M.,* (February) 339
Measurement of Telegraph Transmission, *Nyquist, H., Shank, R. B., and Cory, S. I.,* (February) 367
Methods for the Measurement of Radio Field Strengths, *Englund, C. R., and Friis, H. T.,* (May) 492
Methods of Measuring the Insulation of Telephone Lines at High Frequencies, *Green, E. I.,* (May) 514
Methods Used in Investigating Corona Loss by Means of the Cathode Ray Oscillograph, *Lloyd, W. L., and Starr, E. C.,* (June) 997
Microammeter Indication of High-Frequency Bridge Balance, *Turner, H. M.,* (May) 559
Precision Measurement of Puncture Voltage, A, *Bush, V., and Moon, P. H.,* (June) 1025
Sensitivity Characteristics of a Low-Frequency Bridge Network for Locating Opens in Telephone Circuits, *Edwards, P. G., and Herrington, H. W.,* (May) 551
Stroboscopic Method of Testing Watthour Meters, *Sparkes, H. P.,* (February) 405

MINING APPLICATIONS

Applications to Mining, Committee Report, (June) 801

MOTORS, ALTERNATING CURRENT

See also Control Devices, Industrial Applications of Power, Loss and Efficiency Measurements, Transients

A-C. Elevator Motor Drive, *Thurston, E. B.,* (February) 397
Additional Losses of Synchronous Machines, *Laffoon, C. M., and Calvert, J. F.,* (February) 84
Starting Performances of Synchronous Motors, *Putman, H. V.,* (February) 39
Study of Transverse Armature Reaction in Synchronous Machines, A, (A. I. E. E. JOURNAL, August, 1927, p. 811)
Synchronous Machines—III Torque-Angle Characteristics under Transient Conditions, *Doherty, R. E., and Nickle, C. A.,* (February) 1
Transverse Reaction in Synchronous Machines, *Douglas, J. F. H.,* (February) 30

POWER FACTOR

Most Economical Power Factor, The, A Practical Design Formula for Distribution Circuits, *Litchfield, H. S.,* (May) 667

POWER STATIONS

See Central Station Systems, Hydroelectric, Power Factor

Auxiliary Power at Richmond Station, *Anderson, J. W., and Monteith, A. C.,* (June) 827
Holtwood Steam Plant, The, *Allner, F. A.,* (June) 815

PROTECTIVE DEVICES

See also Grounded Neutral, Lightning Arresters, Reactors

Directional Ground Relay Protection of High-Tension Isolated Neutral Systems, *Breisky, J. V., North, J. R., and King, G. W.,* (June) 853
Ground Relay Protection for Transmission Systems, *Jones, B. M., and Dodds, G. B.,* (June) 847
Protective Devices Committee Report, (June) 791

RADIO PHENOMENA

Carrier-Current Selector Supervisory Equipment, *Stewart, C. E., and Whitney, C. F.,* (March) 431
Electromagnetic Waves Guided by Parallel Wires, *Levin, S. A.,* (June) 983
Methods for the Measurement of Radio Field Strengths, *Englund, C. R., and Friis, H. T.,* (May) 492
Quantitative Determination of Radio Receiver Performance, *Oakley, H. D.,* (May) 498

RAILWAYS

Cab Signals for Railway Signaling, *Stevens, T. S.,* (March) 426
Catenary Construction for Chicago Terminal, *Thorp, J. S.,* (June) 1108
Catenary Design for Overhead Contact Systems, *Brown, H. F.,* (June) 1082
Collection of Current from Overhead Contact Wires, The, *Wade, R. E., and Linebaugh, J. J.,* (June) 1114
Current Collection from an Overhead Contact System Applied to Railroad Operation, *Viele, S. M.,* (June) 1072
Railway Inclined Catenary Standardized Design, *Jorstad, O. M.,* (June) 1125
Transportation Committee Report, (June) 804

REACTORS

See also Protective Devices

Design of Reactances and Transformers Which Carry Direct Current, *Hanna, C. R.,* (February) 155

RECTIFIERS

Application of Mercury Arc Power Rectifiers, (A. I. E. E. JOURNAL, May 1927, p. 446)
Mercury Arc Power Rectifiers, Their Applications and Characteristics, *Marti, O. K., and Winograd, H.,* (March) 437
Mercury Arc Rectifier Phenomena, *Prince, D.,* (June) 1064
New Electronic Rectifier, A, *Grondahl, L. O., and Geiger, P. H.,* (February) 357
Steel-Tank Mercury Arc Rectifiers (A. I. E. E. JOURNAL, June 1927, p. 597)

RELAYS

Directional Ground Relay Protection of High-Tension Isolated Neutral Systems, *Breisky, J. V., North, J. R., and King, G. W.,* (June) 853
Ground Relay Protection for Transmission Systems, *Jones, B. M., and Dodds, G. B.,* (June) 847

RESEARCH

Research Committee Report, (June) 697

STANDARDIZATION

Electrical Standardization (A. I. E. E. JOURNAL, March, 1927, opp. p. 214)
Present Status of the International Electrical Units, *Crittenden, E. C.,* (June) 990
Standardization of Voltage Ratings for Power Systems and Equipment, *Silver, A. E., and Harding, A. L.,* (February) 175
Standards Committee Report, (June) 703
Suggested Transformer Voltage Standards, Their Relationship to Pacific Coast Practise, Committee Report, *Minor, H. H.,* Chairman, (February) 195
Voltage Standardization, *Huber-Ruf, A.,* (February) 199
Voltage Standardization from a Consulting Engineer's Point of View, *Argersinger, R. E.,* (February) 172
Voltage Standardization, Its Relation to the Interconnected Power Companies of the Southeast, *Scholz, H. J., Eberhardt, W. W., and Jones, S. Murray,* (February) 187
Voltage Standardization of A-C. Systems from the Viewpoint of the Electrical Manufacturer, *Hanker, F. C., and Summerhayes, H. R.,* (February) 161
Voltage Standards for Electrical Distribution, *Gear, H. B.,* (February) 185

STARTING CURRENTS

See also Transients

Starting Performance of Synchronous Motors, *Putman, H. V.,* (February) 39

SUBSTATIONS

Inspection, Maintenance, and Test of Automatic Substation Equipment (A. I. E. E. JOURNAL, June, 1927, p. 603)
21,000 Kv-A. Automatic Substation, A, (A. I. E. E. JOURNAL, August, 1927, p. 802)

SWITCHES

See also Circuit Breakers

Development of Automatic Switching Equipments in the United States and Europe, *de Goede, A. H.,* (May) 660

TECHNICAL COMMITTEE REPORTS

Applications to Iron and Steel Production Committee Report (June) 799
Applications to Marine Work, Committee Report, (June) 803
Applications to Mining, Committee Report, (June) 801
Communication Committee Report, (June) 713
Electric Welding Committee Report, (June) 807
Electrical Machinery Committee Report, (June) 726
Electrochemistry and Electrometallurgy Committee Report, (June) 812
Electrophysics Committee Report, (June) 700
Instruments and Measurements Committee Report (June) 705
Power Generation Committee Report (June) 748

Power Transmission and Distribution Committee Report (June) 783
Production and Application of Light Committee Report, (June) 718
Protective Devices Committee Report, (June) 791
Research Committee Report, (June) 697
Standards Committee Report, (June) 703
Transportation Committee Report, (June) 804

TELEGRAPH
See also Telephone

Measurement of Telegraph Transmission, *Nyquist, H., Shank, R. B., and Cory, S. I.*, (February) 367
Non-Rotary Regenerative Telegraph Repeater, A, *Connery, A. F.*, (June) 897
Printing Telegraphs on Non-Loaded Ocean Cables, *Angel, Herbert*, (June) 884
Telegraph Traffic Engineering, *Mason, H., and Walbran, C. J.*, (February) 377

TELEPHONE
See also Telegraph

Methods of Measuring the Insulation of Telephone Lines at High Frequencies, *Green, E. I.*, (May) 514
Sensitivity Characteristics of a Low-Frequency Bridge Network for Locating Opens in Telephone Circuits, *Edwards, P. G., and Herrington, H. W.*, (May) 551

TELEVISION

Production and Utilization of Television Signals, The, *Gray, Frank, Horton, J. W., Mathes, R. C.*, (June) 918
Radio Transmission System for Television, *Nelson, Edward L.*, (June) 954
Synchronization of Television, *Stoller, H. M., and Morton, E. R.*, (June) 940
Television, *Ives, Herbert E.*, (June) 913
Wire Transmission System for Television, *Gannett, D. K., and Green, E. I.*, (June) 946

TRANSFORMER CONNECTIONS

Instability in Transformer Banks, *Gould, King E.*, (May) 676
Transformer Tap Changing under Load, *Hill, L. H.*, (May) 582

TRANSFORMERS
See also Transformer Connections

Constant-Current Regulating Transformer Characteristics, *Louis, H. C., and Albaugh, A.*, (February) 77
Design of Reactances and Transformers Which Carry Direct Current, *Hanna, C. R.*, (February) 155
Mechanical Forces in Transformers, *Clem, J. E.*, (May) 577
Reduction of Transformer Exciting Current to Sine-Wave Basis, *Camilli, G.*, (May) 692
Suggested Transformer Voltage Standards, Their Relationship to Pacific Coast Practise, Committee Report, *Minor, H. H.*, Chairman, (February) 195

TRANSIENTS
See also Electric Circuit Theory

Klydonograph Surge Investigations, *Cox, J. H., McAuley, P. H., and Huggins, L. Gale*, (February) 315
Measurement of Surge Voltages on Transmission Line Due to Lightning, The, *Lee, Everett S., and Foust, C. M.*, (February) 339
Synchronous Machines—III Torque-Angle Characteristics under Transient Conditions, *Doherty, R. E., and Nickle, C. A.*, (February) 1
Transmission Line Voltage Surges, *Cox, J. H.*, (February) 330

TRANSMISSION LINES (ELECTRICAL CHARACTERISTICS)

Recent Investigation of Transmission Line Operation, *Hemstreet, J. G.*, (June) 835
Transmission Line Voltage Surges, *Cox, J. H.*, (February) 330

TRANSMISSION LINES (GENERAL)

Power Transmission and Distribution Committee Report, (June) 783

WAVE FORM

Empirical Analysis of Complex Electric Waves, The, *Horton, J. W.*, (May) 535
M. M. F. Wave of Polyphase Windings with Special Reference to Sub-Synchronous Harmonics, The, *Graham, Quentin*, (February) 19
Reduction of Transformer Exciting Current to Sine-Wave Basis, *Camilli, G.*, (May) 692

INDEX OF AUTHORS

A

Adkerson, B. O., *Discussion* 365
Affel, H. A., and O'Leary, J. T., *Paper* 504
Albaugh, A., and Louis, H. C., *Paper* 77
Albaugh, A., and Louis, H. C., *Discussion* 83
Alexander, P. P., *Discussion* 881
Alger, P. L., *Discussion* 28, 58, 98, 109
Allner, F. A., *Paper* 815
Ames, N. B., *Discussion* 208
Anderson, J. W., and Monteith, A. C., *Paper* 827
Anderson, W. B., *Discussion* 450
Angel, Herbert, *Paper* 884
Angel, Herbert, *Discussion* 896, 902
Antoniono, Caesar, *Discussion* 448
Argersinger, R. E., *Paper* 172
Atherton, A. L., *Discussion* 358
Atkinson, R. W., *Discussion* 350
Austin, A. O., *Discussion* 844

B

Bedell, Frederick, *Paper* 648
Bedell, F., *Discussion* 550, 659
Bedell, Frederick, and Reich, Herbert J., *Paper* 546
Bell, J. H., *Discussion* 376, 386
Bewley, L. V., and Hambleton, T. T., *Paper* 60
Bewley, L. V., *Discussion* 76
Biesecker, A. S., *Discussion* 448
Blake, D. K., *Discussion* 234, 237
Blakeslee, H. J., *Discussion* 409
Blasser, B., *Discussion* 445
Blume, L. F., *Paper* 590
Blume, L. F., *Discussion* 602, 603
Boddie, C. A., *Paper* 975
Boddie, C. A., *Discussion* 982
Bonn, N. E., *Discussion* 527, 540, 544
Borden, P. A., *Discussion* 481, 550
Boyajian, A., *Discussion* 658, 696
Brand, F. F., *Discussion* 599
Bratt, Donald, *Discussion* 268
Breisky, J. V., North, J. R., and King, G. W., *Paper* 853
Breisky, J. V., *Discussion* 867
Brodsky, V. P., *Discussion* 866
Brown, B. J., *Discussion* 409, 419
Brown, H. F., *Paper* 1082
Brown, H. F., *Discussion* 1133, 1135, 1137
Brownell, F. A., *Discussion* 971
Bullard, W. R., *Discussion* 213
Burke, C. T., *Paper* 482
Burke, C. T., *Discussion* 487
Burnham, G. A., *Discussion* 313
Bush, V., and Moon, P. H., *Paper* 1025
Butcher, C. A., *Paper* (A. I. E. E. JOURNAL, May 1927, p. 446)
Butcher, C. A., *Discussion* 453

C

Caldwell, F. C., *Discussion* 466, 698
Calvert, J. F., *Discussion* 150
Calvert, J. F., and Laffoon, C. M., *Paper* 84
Calvert, J. F., and Laffoon, C. M., *Discussion* 99
Camilli, G., *Paper* 692
Canfield, D. T., *Paper* 411
Canfield, D. T., *Discussion* 420
Carroll, J. S., and Ryan, H. J., *Discussion* 288
Charlesworth, H. P., *Committee Report* 713
Chase, P. H., *Discussion* 210, 213, 235, 237
Chesney, C. C., *Paper* (A. I. E. E. JOURNAL, March, 1927, opp. p. 214)
Chesney, C. C., *President's Address* 1153
Clark, F. M., *Discussion* 257, 632, 1035, 1048, 1062
Clark, O. S., *Discussion* 448, 451, 468
Clem, J. E., *Paper* 577
Colburn, W. H., *Discussion* 98
Compton, K. T., *Paper* 868
Compton, K. T., *Discussion* 882
Connery, A. F., *Paper* 897
Connery, A. F., *Discussion* 895, 902
Cook, A. L., *Discussion* 503
Cooper, M. D., *Discussion* 205
Cory, S. I., Nyquist, H., and Shank, R. B., *Paper* 367
Cox, J. H., *Paper* 330
Cox, J. H., *Discussion* 352
Cox, J. H., McAuley, P. H., and Huggins, L. Gale, *Paper* 315
Cox, J. H., and McAuley, P. H., *Discussion* 355
Craighead, J. R., *Discussion* 527
Crawford, M. T., *Discussion* 215, 237
Crichton, L. N., *Discussion* 866
Crisson, George 364
Crittenden, E. C., *Paper* 990
Crittenden, E. C., *Discussion* 810
Curtis, Harvey L., *Paper* 1039
Curtis, R. E., *Discussion* 448

D

Damon, J. C., *Discussion* 1134
Dann, W. M., *Discussion* 600
Davidson, W. F., Marvin, R. H., and Del Mar, W. A., *Paper* 1049
Davidson, W. F., *Discussion* 972, 1022, 1063
Dawson, W. F., *Discussion* 97, 206
de Goede, A. H., *Paper* 660
de Goede, A. H., *Discussion* 666
Del Mar, W. A., Davidson, W. F., and Marvin, R. H., *Paper* 1049
Del Mar, W. A., *Discussion* 246, 288, 699, 790, 1036, 1063
de Mulinen, E., *Discussion* 665
Doan, G. H., *Discussion* 843, 866
Dodds, G. B., and Jones, B. M., *Paper* 847
Dodge, E. H., *Discussion* 573
Doggett, L. A., *Discussion* 451
Doherty, R. E., and Nickle, C. A., *Paper* 1
Doherty, R. E., *Discussion* 17, 37
Douglas, J. F. H., *Paper* 30
Douglas, J. F. H., *Discussion* 14, 38, 152, 159
Drake, H. W., *Discussion* 376
Dreyer, J. F., Jr., *Discussion* 366
Duer, J. V. B., *Committee Report* 804
Duer, J. V. B., *Discussion* 811
Dunbar, J. R., *Discussion* 109
DuVall, W. C., *Discussion* 450
Dwight, H. B., *Paper* 570
Dwight, H. B., *Discussion* 573, 581

E

Eberhardt, W. W., Jones, S. Murray, and Scholz, H. J., *Paper* 187
Eby, E. D., *Discussion* 246, 967
Edwards, P. G., and Herrington, H. W., *Paper* 551
Edwards, P. G., *Discussion* 558
Elden, L. L., *Discussion* 212, 237
Ellyson, D. W., *Paper* (A. I. E. E. JOURNAL, August, 1927, p. 802)
Englund, C. R., and Friis, H. T., *Paper* 492
Evans, R. D., *Discussion* 14

F

Fairman, F. E., *Discussion* 467
Fairman, J. F., *Discussion* 971
Ferguson, J. G., and Shackelton, W. J., *Paper* 519
Ferguson, J. G., *Discussion* 487, 527
Fischle, C. R., *Discussion* 558
Foote, J. H., *Discussion* 207
Foust, C. M., and Lee, Everett S., *Paper* 339
Foust, C. M., and Lee, E. S., *Discussion* 355
Freiburghouse, E. H., *Discussion* 98
French, B. V. K., *Discussion* 503
Friis, H. T., and Englund, C. R. *Paper* 492
Funk, F. W., *Discussion* 811

G

Gannett, D. K. and Green, E. I., *Paper* 946
Gealy, E. J., *Discussion* 802
Gear, H. B., *Paper* 185
Gehrkens, E. F., *Discussion* 599
Geiger, P. H., and Grondahl, L. O., *Paper* 357
George, E. E., *Discussion* 865
Gibbs, J. B., *Discussion* 82
Goodwin, V. E., *Discussion* 354
Goodwin, W. N. Jr., *Paper* 479
Goodwin, W. N. Jr., *Discussion* 481
Gorsuch, W. S., *Committee Report* 748
Gorsuch, W. S., *Discussion* 782
Gould, King E., *Paper* 676
Gould, K. E., *Discussion* 682

Graff, P. T., *Discussion* 646
Graham, Quentin, *Paper* 19
Graham, Quentin, *Discussion* 29, 58, 76
Grandy, L. S., *Paper* 421
Grandy, L. S., *Discussion* 425
Gray, Frank, Horton, J. W., and Mathes, R. C., *Paper* ... 918
Green, C. H., and Johnson, E. E., *Paper* 136
Green, E. I., and Gannett, D. K., *Paper* 946
Green, E. I., *Paper* 514
Grondahl, L. O., and Geiger, P. H., *Paper* 357
Grondahl, L. O., *Discussion* 366
Grotzinger, John, *Discussion* 451, 467
Grover, F. W., *Discussion* 581

H

Halperin, Herman, *Discussion* 633, 645, 790, 972, 1035
Hambleton, T. T., and Bewley, L. V., *Paper* 60
Hambleton, T. T., *Discussion* 76
Hamdi, A. F., *Discussion* 83
Hanker, F. C., and Summerhayes, H. R., *Paper* 161
Hanker, F. C., *Discussion* 211
Hanna, C. R., *Paper* 155
Hanna, C. R., *Discussion* 159, 160, 911
Harding, A. L. and Silver, A. E., *Paper* 175
Harding, C. F., *Discussion* 1022
Healy, K. T., *Discussion* 1134
Hellmund, R. E., *Discussion* 699
Hemstreet, J. G., *Paper* 835
Hemstreet, J. G., *Discussion* 846
Herrington, H. W., and Edwards, P. G., *Paper* 551
Herz, A., *Discussion* 430, 449, 789, 798
Hill, L. H., *Paper* 582
Hill, L. H., *Discussion* 601, 603
Hilliard, J. D., *Discussion* 312
Hoare, S. C., *Paper* 541
Hoare, S. C., *Discussion* 544
Hobart, H. M., *Committee Report* 726
Hooke, R. G., *Discussion* 351, 969
Horton, J. W., *Paper* 535
Horton, J. W., *Discussion* 540
Horton, J. W., Mathes, R. C., and Gray, Frank, *Paper* ... 918
Houghtaling, W. A., *Discussion* 895
Huber-Ruf, A., *Paper* 199
Huggins, L. Gale, Cox, J. H., and McAuley, P. H., *Paper*. 315
Hunt, F. L., *Committee Report* 791
Hunt, F. L., *Discussion* 208

I

Ives, Herbert E., *Paper* 913

J

Jamieson, B. G., *Discussion* 205, 209
Janson, G. W., *Discussion* 364
Johnson, E. E., and Green, C. H., *Paper* 136
Johnson, E. E., *Discussion* 97, 154
Johnson, J. Allen, *Discussion* 798, 866
Jones, B. M., and Dodds, G. B., *Paper* 847
Jones, B. M., *Discussion* 866
Jones, S. Murray, Scholz, H. J., and Eberhardt, W. W., *Paper* 187
Jorstad, O. M., *Paper* 1125
Junken, L. H., *Discussion* 981

K

Karapetoff, V., *Committee Report* 700
Karapetoff, Vladimir, *Paper* 563
Karapetoff, Vladimir, Discussion ... 38, 153, 266, 269, 573, 881, 1020
Kasson, C. L., *Paper* 635
Kasson, C. L., *Discussion* 646
Katzman, Jacob, *Discussion* 258
Kehoe, A. H., *Discussion* 206, 236, 972, 981
Kellogg, E. S., *Discussion* 911
Kellogg, Edward W., *Paper* 903
Kendall, B. W., *Discussion* 558
Kennedy, G. M., *Discussion* 451, 466
Kerchner, R. M., *Discussion* 425
Kierstead, F. H., *Discussion* 580
King, G. E., *Discussion* 448
King, G. W., Breisky, J. V., and North, J. R., *Paper* 853
Kinnard, I. F., *Discussion* 419
Kirke, W. B., *Discussion* 236, 237
Knowlton, A. E., *Committee Report* 705
Knowlton, A. E., *Discussion* 409
Knowlton, H. S., *Discussion* 665
Kolff, P. van Santen, *Discussion* 409
Kouwenhoven, W. B., *Discussion* 82, 269, 409, 615, 645

L

Laffoon, C. M. and Calvert, J. F., *Paper* 84
Laffoon, C. M. and Calvert, J. F., *Discussion* 99
Laffoon, C. M., *Discussion* 110, 150
Langstaff, H. A. P., *Discussion* 865
Lebovici, J., *Discussion* 404
Lee, E. S., and Foust, C. M., *Paper* 339
Lee, E. S. and Foust, C. M., *Discussion* 355
Lee, E. S., *Discussion* 632, 646
Leh, H. H., *Discussion* 467
Lehmann, Th., *Discussion* 152
Lenehan, B. E., *Discussion* 491
Lennox, J. S., *Discussion* 600
Lesser, W. H., *Committee Report* 801
Lesser, W. H., *Discussion* 451, 467, 802
Levin, S. A., *Paper* 983
Levin, S. A., *Discussion* 989
Lichtenberg, Chester, *Discussion* 665, 981
Lichtenberg, Chester, *Paper*, (A. I. E. E. Journal, June, 1927, p. 603)
Lincoln, J. C., *Committee Report* 807
Lincoln, J. C., *Discussion* 811, 880
Lincoln, J. F., *Paper* (A. I. E. E. Journal, June, 1927, p. 593)
Lindemann, P. A., *Discussion* 404
Linder, C. H., *Discussion* 148
Linebaugh, J. J., and Wade, R. E., *Paper* 1114
Linebaugh, J. J., and Wade, R. E., *Discussion* 1137
Litchfield, H. S., *Paper* 667
Litchfield, Norman, *Discussion* 1133
Lloyd, W. E., *Discussion* 467, 477
Lloyd, W. L. and Starr, E. C., *Paper* 997
Lloyd, W. L., *Discussion* 846
Louis, H. C., and Albaugh, A., *Paper* 77
Louis, H. C., and Albaugh, A., *Discussion* 83
Lyon, W. V., *Discussion* 15, 29, 110

M

MacCutcheon, A. M., *Discussion* 802
MacNeill, J. B., *Discussion* 313
Major, O. S., *Discussion* 429
Malti, Michel G., *Paper* 616
Malti, M. G., *Discussion* 633
Manz, M. W., *Discussion* 1133
Marti, O. K., and Winograd, H., *Paper* 437
Marti, O. K., and Winograd, H., *Discussion* 452
Marvin, R. H., Del Mar, W. A., and Davidson, W. F., *Paper* 1049
Marvin, R. H., *Discussion* 1036
Mason, H., and Walbran, C. J., *Paper* 377
Mason, H., *Discussion* 386
Mathes, R. C., Gray, Frank, and Horton, J. W., *Paper* ... 918
McAuley, P. H., Huggins, L. Gale, and Cox, J. H., *Paper*. 315
McAuley, P. H., and Cox, J. H., *Discussion* 355
McCormick, B. T., *Discussion* 461
McCoy, R. L., *Discussion* 843
McCurdy, R. G., *Discussion* 446, 448
McEachron, K. B., *Discussion* 348, 550
McMaster, R. K., *Discussion* 210
McMullan, Samuel, and Shea, John R., *Paper* 387
Meyer, J. Franklin, *Committee Report* 703
Millar, P. S., *Committee Report* 718
Miner, Douglas F., *Paper* 248
Miner, D. F., *Discussion* 258
Minor, H. H., Chairman, *Committee Report* 195
Monteith, A. C., and Anderson, J. W., *Paper* 827
Montsinger, V. M., *Discussion* 256, 682
Moody, W. S., *Discussion* 580
Moon, P. H. and Bush, V., *Paper* 1025
Moon, P. H., *Discussion* 1037
Morton, E. R., and Stoller, H. M., *Paper* 940
Murnaghan, Francis D., *Paper* 259
Murnaghan, F. D., *Discussion* 270
Murphy, L. J., *Paper* 455
Murphy, L. J., *Discussion* 461

N

Nelson, Edward L., *Paper* 954
Newbury, F. D., *Discussion* 98, 109
Nickle, C. A., and Doherty, R. E., *Paper* 1
Nickle, C. A., *Discussion* 16, 28, 37
North, J. R., King, G. W., and Breisky, J. V., *Paper* 853

North, W. E., *Paper* 462
North, W. E., *Discussion* 466, 467, 468
Noyes, J. D., *Discussion* 810
Nyman, Alexander, *Paper* 488
Nyman, Alexander, *Discussion* 287, 487, 491
Nyquist, H., Shank, R. B., and Cory, S. I., *Paper* 367
Nyquist, H., *Discussion* 376

O

Oakley, H. D., *Paper* 498
Oakley, H. D., *Discussion* 503
Oehler, A. G., *Discussion* 364, 1134
Oesterreicher, S. I., *Discussion* 245
O'Leary, J. T., and Affel, H. A., *Paper* 504
Oplinger, K. A., *Discussion* 600

P

Palueff, K. K., *Discussion* 665, 696
Park, R. H., and Stevenson, A. R., Jr., *Paper* 112
Park, R. H., *Discussion* 16, 58
Peek, F. W., *Paper* 1009
Peek, F. W., *Discussion* 257, 350, 1024
Penney, G. W., *Paper* 683
Peterman, W. C., *Discussion* 895, 902
Peterson, Eugene, *Paper* 528
Peterson, L. C., *Discussion* 989
Peterson, Thos. F., *Paper* 963
Peterson, T. F., *Discussion* 245, 973
Petty, D. M., *Discussion* 466, 476
Pierce, A. G., *Committee Report* 799
Pierce, G. A., *Committee Report* 803
Plumb, H. H., *Discussion* 355
Pragst, Ernest, *Discussion* 211
Pratt, W. H., *Discussion* 419
Prince, D., *Paper* 1064
Prince, D. C., *Discussion* 158, 159, 451
Puckstein, A. F., *Discussion* 269
Pupin, M. I., *Paper* (A. I. E. E. JOURNAL, January 1927, p. 59, and February, 1927, p. 171)
Putman, H. V., *Paper* 39
Putman, H. V., *Discussion* 15, 36, 59

R

Rakestraw, C. N., *Discussion* 246
Reich, Herbert J. and Bedell, Frederick, *Paper* 546
Richmond, H. S., *Discussion* 1134
Richter, H., *Paper* 216
Richter, H., *Discussion* 238
Rockwood, G. H., *Paper* 1139
Rockwood, G. H., Jr., *Discussion* 109
Roper, D. W., *Discussion* 245, 267, 789, 967
Roper, J. Walter, *Paper* 574
Rosch, S. J., *Discussion* 632, 646, 789
Rosenberger, H. J., *Discussion* 366
Rutter, A. R., *Discussion* 420
Ryan, H. J., and Carroll, J. S., *Discussion* 288
Ryan, J., *Discussion* 1023

S

Sah, A. Pen-Tung, *Paper* 604
Sah, A. P. T., *Discussion* 615
Sawin, G. A., *Discussion* 409
Schaefer, R. A., *Paper* (A. I. E. E. JOURNAL, August, 1927, p. 811)
Scheffler, F. A., *Discussion* 468, 782
Schirmer, A. H., *Discussion* 349
Scholz, H. J., Eberhardt, W. W., and Jones, S. Murray, *Paper* 187
Schurig, O. R., *Discussion* 255
Seelye, H. P., *Discussion* 238
Shackelton, S. P., *Discussion* 558
Shackelton, W. J. and Ferguson, J. G., *Paper* 519
Shackelton, W. J., *Discussion* 527
Shand, E. B., *Paper* (A. I. E. E. JOURNAL, June 1927, p. 597)
Shand, E. B., *Discussion* 75, 453
Shank, R. B., Cory, S. I., and Nyquist, H., *Paper* 367
Shaver, W. W., *Discussion* 1035
Shea, John R., and McMullan, Samuel, *Paper* 387
Shea, T. E., *Discussion* 486
Shoemaker, J. J., *Discussion* 811
Silver, A. E., and Harding, A. L., *Paper* 175
Simons, Donald M., *Paper* 240
Simons, D. M., *Discussion* 247, 968
Sipher, E. F., *Discussion* 450
Skinner, C. E., *Discussion* 206, 209
Sleeper, H. P., *Discussion* 864
Slepian, Joseph, *Discussion* 267, 287, 365, 878
Smith, Aubrey, *Discussion* 467
Snell, F. E., *Discussion* 429
Sorensen, R. W., *Discussion* 698, 702, 882
Sparkes, H. P., *Paper* 405
Sparkes, H. P., *Discussion* 410
Sporn, Philip, and St. Clair, Harry P., *Paper* 289
Sporn, Philip, *Discussion* 314
Spurck, R. M., *Paper* (A. I. E. E. JOURNAL, July 1927, p. 707)
Standing, A. J., *Discussion* 451, 466, 476
Starr, E. C., and Lloyd, W. L., *Paper* 997
Starr, E. C., *Discussion* 882, 1024
St. Clair, B. W., *Discussion* 544
St. Clair, Harry P., and Sporn, Philip, *Paper* 289
Stein, I. M., *Discussion* 557
Stephens, H. O., *Discussion* 600
Stevens, T. S., *Paper* 426
Stevens, T. S., *Discussion* 430
Stevenson, A. R., Jr., and Park, R. H., *Paper* 112
Stevenson, A. R., *Discussion* 154
Stewart, C. E., and Whitney, C. F., *Paper* 431
Stoller, H. M., and Morton, E. R., *Paper* 940
Stone, E. C., *Discussion* 209
Summerhayes, H. R., and Hanker, F. C., *Paper* 161
Summers, Ivan H., *Paper* 101
Summers, I. H., *Discussion* 97, 111
Sutton, H. C., *Discussion* 208

T

Taylor, H. W., *Discussion* 111
Taylor, W. G., *Discussion* 461
Thomas, P. H., *Discussion* 602
Thorp, J. S., *Paper* 1108
Thurston, E. B., *Paper* 397
Thurston, E. B., *Discussion* 404
Torchio, Philip, *Committee Report* 783
Torchio, Philip, *Discussion* 782, 790
Treanor, E. D., *Discussion* 82
Trucksess, D. E., *Discussion* 366
Trueblood, H. M., *Discussion* 864, 981
Turner, H. M., *Paper* 559
Turner, H. M., *Discussion* 481, 550
Tweeddale, J. E. L., *Paper* 1148
Tykocinski-Tykociner, J., *Discussion* 699

U

Umansky, L. A., *Paper* 469
Umansky, L. A., *Discussion* 477

V

Viele, S. M., *Paper* 1072
Viele, S. M., *Discussion* 1137
Vinal, G. W., *Committee Report* 812
Vincent, H. B., *Discussion* 351
Vinet, Eugene, *Discussion* 208

W

Wade, R. E., and Linebaugh, J. J., *Paper* 1114
Wade, R. E., and Linebaugh, J. J., *Discussion* 1137
Wagner, C. F., *Discussion* 16
Wagner, E. B., *Discussion* 466
Walbran, C. J. and Mason, H. 377
Wallau, H. L., *Discussion* 210, 235, 352
Wallis, C. T., *Discussion* 420
Waugh, J. T., *Discussion* 451, 468
Wensley, R. J., *Discussion* 435
Whitehead, John B., *Committee Report* 697
Whitney, C. F., and Stewart, C. E., *Paper* 431
Wieseman, Robert W., *Paper* 141
Wieseman, R. W., *Discussion* 153
Williford, E. A., *Discussion* 725
Willis, C. H., *Paper* 271
Willis, C. H., *Discussion* 288
Wilson, L. T., *Discussion* 481
Winograd, H., and Marti, O. K., *Paper* 437
Winograd, H., and Marti, O. K., *Discussion* 452
Wiseman, R. J., *Discussion* 246, 256
Withington, Sidney, *Discussion* 451, 1136
Woodward, J. S., *Discussion* 151
Wolf, H. Carl, *Discussion* 215
Wright, G. I., *Discussion* 1133
Wright, R. H., *Discussion* 477

Preface

This is the forty-sixth and last annual volume of the TRANSACTIONS of the American Institute of Electrical Engineers. It consists of the complete papers and accompanying discussions presented at the two conventions and three regional meetings of the Institute prior to September 1927. The January 1928 Quarterly TRANSACTIONS contains the papers from the two succeeding meetings in 1927. The annual report of the Board of Directors for the fiscal year ending April 30, 1927, and a list of the officers, committeemen, and Section and Branch officers for that period, are included in this volume. The index of subjects has been made as complete as possible, with ample cross references. The authors' index contains as well the names of all those contributing to the discussions. In addition to the papers published in this volume, the index lists certain articles of more transitory interest, which were printed only in the JOURNAL.

In 1928, with Volume 47, the Institute inaugurates its new policy under which the JOURNAL consists of abridged papers, while the Quarterly TRANSACTIONS will contain the papers and discussions in full.

9

Synchronous Machines—III

Torque-Angle Characteristics Under Transient Conditions

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Associate, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—This is the third part of a series of papers on the subject of synchronous machines. The first two were:*

I. An Extension of Blondel's Two-Reaction Theory,

II. Steady State Power-Angle Characteristics.

The present paper deals with the power-angle, or torque-angle, characteristics under transient conditions, namely,

A. Cyclic variation of impressed torque,

B. Sudden angular displacement,

C. Synchronizing out of phase.

It is shown, as in Fig. 6, that although the slope of the torque-angle characteristic (which is an important factor in the determination of the resonant frequency) under the oscillatory condition is greater over a large range of values of the average angle δ' than under steady operation, nevertheless in the range of normal operation, i. e., from $\delta' = 0$ to $\delta' = 25$ deg., the two slopes, in the case of salient-pole machines, are practically the same. Hence, it is only in rather rare, special cases that a correction in the slope for the oscillatory condition is necessary. For such cases, Equation 27 gives the correction.

Referring to condition B, Fig. 13 shows the steady state torque-angle characteristic and also the characteristics for the condition of sudden angular displacement, the latter occurring from various given points on the steady state curve. The slopes indicated by dotted line segments in Fig. 6 merely correspond to parts of the complete characteristics shown in Fig. 13. The latter are calculated from Equation 46.

It is fairly well known that synchronizing out of phase gives rise to much larger torque than would exist at the same angular displacement under steady operation. The difference between these two torques is shown in Fig. 17 for a steam turbine type generator. The steady state torque is calculated from Equation 26; the transient torque from Equation 61.

In certain applications of synchronous machines it is required to determine the relation between the torque and the displacement angle under transient conditions. For instance, when such a machine is direct-coupled to a reciprocating engine or compressor, there is, of course, a resultant alternating component of torque which causes an angular oscillation of the machine. This produces current pulsations in the line which are a function of the torque-angle characteristic,—that is, of the "synchronizing torque." Such an oscillation induces currents in the field winding, thus affecting the values of synchronizing torque, and therefore must properly be treated as a transient phenomenon. The present treatment, on this basis, however, shows that although there may be special cases where the effect of the oscillation on synchronizing torque is significant, there are nevertheless many practical applications where it is not.

Another instance is sudden angular displacement. If, when operating under a given load condition and angular displacement, the rotor is suddenly displaced to a different angle, the torque at the new angle will be different, under this transient condition, from the value at the same angle under steady operation.

Still another instance is synchronizing an incoming alternator out of phase. In this case, the resulting current and torque are much greater than at the same phase difference under steady operation. There may be danger of overstressing the shaft or coupling in such a case; hence it is important to have means for predetermining such forces.

The problem of steady state operation has already been treated by the authors[1]. It has also been treated more recently by Putman[2], giving the same results for those aspects of steady operation treated by him. His paper also investigated certain conditions of transient operation. The results for the latter, however, differ in certain important respects from the present treatment.[3]

It is the purpose of the present paper to establish mathematically, from what appear to be reasonable and practical premises, the relation between torque and the displacement angle of a synchronous machine under the following conditions:

A. Cyclic variation of impressed torque,

B. Sudden angular displacement,

C. Synchronizing out of phase.

PREMISES

For condition A it is assumed that:

1. *The machine is connected to a relatively large power system,*

2. *The effect of armature resistance is negligible.* This has been justified in a previous paper.[1]

3. *The currents are polyphase, balanced, sine waves (in time).* They can be resolved, therefore, into two complementary polyphase current systems, one in which the current in each individual phase reaches maximum at the instant the axis of the field pole coincides with the axis of magnetization of the phase under consideration—this is termed the *direct* component of current; and another in which the current in the same phase reaches maximum one quarter-cycle later, that is, in time quadrature. This is termed the *quadrature* component[1].

4. *The machine has salient poles.* Cylindrical rotor thus becomes a special case of salient poles in which the synchronous reactances in the *direct* and *quadrature* axes are equal. The transient reactances in the two axes, however, may or may not be equal.

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[1] All numbered references are to the Bibliography.

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5. *The machine has a short-circuited rotor winding in the quadrature axis, as well as the main field winding in the direct axis.* The effect of an amortisseur winding may thus be taken into account as a practical approximation.

6. *Saturation is negligible.* While the results apply strictly only to machines in which magnetic saturation is negligible, nevertheless this does not mean that practical calculations, within practical accuracy, can not be thus made when saturation is present. Indeed, they are made, just as many other similar calculations are made, by exercising engineering judgment in shading the constants of the machine with respect to the degree and distribution of the saturation.

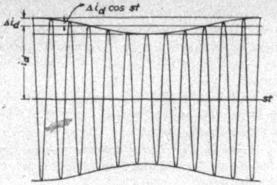

FIG. 1

Such a procedure should be recognized, however, as an approximation and should not be treated, as it sometimes is, as a correct method. The theory of superposition does not apply when saturation is present. Hence a linear relation between field current and nominal voltage can not be combined properly with other equations of armature voltage.

7. *The pulsation in the magnitude of the fundamental component of armature m. m. f. is harmonic.* The pulsation may comprise more than one harmonic, in which case each may be treated separately, and the results superposed.

8. *The frequency of this pulsation is low compared with the normal electrical frequency.* It thus becomes permissible, as a close approximation, to express the current as a vector of harmonically varying amplitude. The modulation frequency, *i. e.*, the frequency of the envelope of the current wave, must be low enough for the polyphase relation of the currents at any instant not to be appreciably disturbed. In still other words, the current wave throughout any cycle of normal frequency is not appreciably changed from a sine wave by the modulation, as indicated in Fig. 1.

For conditions B and C, it is assumed, in addition to the above and excepting 7 and 8, that

9. *The effect of the field circuit resistance is negligible in the first moment.* It is assumed that the time interval in which the displacement occurs is small enough to justify this.

10. *The direct component of the transient armature current is neglected.* This component gives rise to an alternating torque comprising normal frequency and higher harmonics. These rapidly alternating torques do not, as a rule, produce significant motional effects, on account of the relatively large inertia of the rotor with respect to the frequency of the torques. These will be treated in a future paper. The present treatment deals only with that component of torque which is unidirectional for a given displacement and is the component which may throw large stresses on the shaft if a machine is synchronized out of phase.

A. CYCLIC VARIATION OF IMPRESSED TORQUE

When a synchronous machine is direct-coupled to a reciprocating engine or compressor, the impressed torque comprises a steady component with superposed alternating components. The latter may be substantially a single harmonic, or it may comprise a number of significant torque harmonics. Each harmonic may be treated independently.

The problem may therefore be definitely stated as follows: A synchronous machine, operating on a relatively large power system, and carrying a given average mechanical load, experiences also an impressed cyclic, angular oscillation which produces an harmonic modulation of the armature current, as indicated in Fig. 1. This induces, by transformer action, a corresponding alternating component of current in the field winding, thus affecting the power-angle, or torque-angle, characteristic of the machine—that is, changing the slope, as indicated in Fig. 2. Thus, instead of an oscillation above and below the point *p*, from *a* to *b* on the steady state characteristic, as would occur at very low frequency of impressed oscillation, the actual oscillation, for the above reasons, would be from *a′* to *b′*, *i. e.*, at a different slope. In other words, if the frequency of oscillation is low enough in relation to the resistance of the field circuit, the adjustment of the field flux to the changing armature m. m. f. will be effected without appreciable induced currents in the field. On the other hand, for the same angular oscillations, if the frequency is so high that the field flux cannot change, the maximum induced field current will result—*i. e.*, that which is required to maintain constant flux linkages. Hence the induced field current will be between these limits and the slope of the torque-angle characteristic will also fall between corresponding limits.

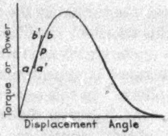

FIG. 2

The slope, designated by T_s, is an important factor in the *resonant* frequency of the machine. The relation which this bears to the frequency of the *impressed* oscillation is an important factor in determining the magnitude of the oscillation[4].

The present problem, therefore, is to determine the slope of the torque-angle characteristics at the point p corresponding to the average torque, Fig. 2.

The plan of attack is, first, to determine the rotor current induced by the oscillation. This, added to the average value supplied by the exciter, determines the total nominal voltage as a function of time. From the vector diagram of voltages, the displacement angle of the machine is also expressed as a function of time. Since the torque has been shown[1] to be a function of the nominal voltages and displacement angle, it may also be expressed, from the foregoing relations, as a function of time. The torque thus expressed contains a constant term and an alternating component. The plan is to obtain the complex expression for "motional impedance* by dividing the alternating component of torque by the alternating component of velocity, both expressed as vectors. The two components of the motional impedance give, respectively, the damping coefficient and the resilience coefficient, *i. e.*, synchronizing torque.

The use of motional impedance is convenient because of its analogy with electrical impedance. In the particular convention chosen[5], velocity corresponds to current, torque to voltage, damping constant to resistance, motional reactance to capacity reactance, etc.

Thus, the real term of the motional impedance consumes a torque which is in phase with the velocity (just as resistance consumes a voltage in phase with the current) and therefore represents damping. The imaginary term, the motional reactance, consumes a torque which is in time quadrature with the velocity (just as capacity reactance consumes a voltage in quadrature with the current) and hence in phase with, and proportional to, the displacement—just as the voltage across the condenser is in phase with and proportional to the charge. It therefore represents the synchronizing torque. That is, the change of angular displacement is accompanied by a proportional change in torque. This proportionality factor, *i. e.*, the slope of the torque-angle characteristic, is the objective of the investigation for condition A.

Percentage Representation of Quantities. As in the first two sections of this investigation, already published[1], the various quantities here will be expressed as a percentage (as a fraction) of some definite value, thus avoiding cumbersome conversion factors and other constants. For instance, all armature currents are expressed as fractions of normal current; all voltages, as fractions of normal voltage; etc.

Equations for Condition A. Since the modulation of the armature current produced by the angular oscillation of the machine is assumed to be harmonic, the current wave will be as shown in Fig. 1. The total direct component of current i_d at any time t and expressed in terms of the peak value, is

$$i_d = i_d' + \Delta\, i_d \cos s\, t \qquad (1)$$

where

$\Delta\, i_d$ = the peak value of the low-frequency alternating component which causes the modulation, as shown in Fig. 1,

i_d' = the steady, or average, value of the direct component expressed in terms of the peak value as a fraction of the peak value of normal current,

s = angular velocity corresponding to the frequency of modulation, expressed as a fraction of the angular velocity corresponding to normal electrical frequency,

t = time, expressed as a fraction of the time required, at normal frequency, to pass one electrical radian.

Thus the time angles corresponding to the two frequencies are expressed, respectively, at $s\, t$ and t.

Likewise, the quadrature component is

$$i_q = i_q' + \Delta\, i_q \cos(s\, t + \alpha) \qquad (2)$$

where α = the time phase difference between the low-frequency modulations of the direct and quadrature components of current.

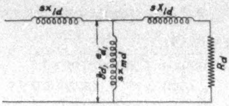

FIG. 3

Zero time can be chosen so that (1) is true, since the total current, and therefore each of the two components, is assumed to be modulated harmonically. But the phase difference α is not known.

The modulation of the polyphase armature currents produces a corresponding low-frequency variation in the otherwise constant magnitude of the armature m. m. f. (space sinusoid), *i. e.*, the armature reaction, which rotates at the same speed with the field poles. That particular component of armature reaction which is directly opposite the pole, that is, the direct component produced by i_d, is impressed on the same magnetic circuit with which the main field winding is linked. Thus, by transformer action, these harmonic variations in armature currents induce corresponding variations in the field winding, the latter being short-circuited through the exciter armature.

The variations in the quadrature component, however, are obviously not short-circuited by the main field winding. If there is an amortisseur winding, or some other short-circuited winding in the quadrature axis, the currents induced therein by the variations of i_q must also be taken into account.

The next step is to determine these induced field currents. Just as the armature reaction can be expressed in terms of the current of only one phase (because all of the currents are related in phase and magnitude so that this is possible) it is obviously also possible to express in the same way the variation of that armature reaction, within the limits of the premises. Hence the transformer relations may be determined from the well-known equivalent circuit. Thus in Fig. 3,

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*This term has been used by Dr. A. E. Kennelly and others.

x_l = armature leakage reactance at normal frequency (see notation). This is taken to be the same for i_d and i_q. See "Reactance," in Bibliography 1.

X_{lda} = field leakage reactance, in armature terms, for the winding in the direct axis—*i. e.*, main field winding.

R_{da} = resistance of main field winding, armature terms.

x_{md} = mutual reactance, direct axis.

x_{d1} = equivalent impedance. See Fig. 3.

e_{d1} = voltage across z_{d1}.

It is shown in Appendix A that the variation in the field current, direct axis, corresponding to Δi_d is

$$\Delta \underset{\cdot}{I}_d = \Delta \underset{\cdot}{i}_d (a + j b) \qquad (3)$$

or, as a function of time, from (**1**),

$$\Delta I_d = \Delta i_d (a \cos s t - b \sin s t) \qquad (3a)$$

Likewise

$$\Delta \underset{\cdot}{I}_q = \Delta \underset{\cdot}{i}_q (c + j d) \qquad (4)$$

and, from (**2**) and (**4**),

$$\Delta I_q = \Delta i_q (c \cos \alpha - d \sin \alpha) \cos s t - (c \sin \alpha + d \cos \alpha) \sin s t \qquad (4a)$$

where

$$a = \frac{s^2 x_{md}^2 (x_{md} + X_{lda})}{R_{da}^2 + s^2 (x_{md} + X_{lda})^2}$$

$$b = \frac{s\, x_{md}^2 R_{da}}{R_{da}^2 + s^2 (x_{md} + X_{lda})^2}$$

$$c = \frac{s^2 x_{mq}^2 (x_{mq} + X_{lqa})}{R_{qa}^2 + s^2 (x_{mq} + X_{lqa})^2}$$

$$d = \frac{s\, x_{mq}^2 R_{qa}}{R_{qa}^2 + s^2 (x_{mq} + X_{lqa})^2}$$

With the alternating components of the field currents thus determined, the nominal voltages, which are proportional to the field currents, may be also determined by adding to the alternating component, the constant term supplied by the exciter. Since both the field current and the nominal voltage are expressed as fractions, they are numerically equal. Thus the peak value of the total nominal voltage, direct axis, at any time is, from (**3a**),

$$\begin{aligned} e_d &= I_d' + \Delta I_d \\ &= I_d' + \Delta i_d (c \cos s t - b \sin s t) \qquad (5) \end{aligned}$$

and, since there is assumed to be no constant excitation in the quadrature axis, the quadrature component is, from (**4a**),

$$\begin{aligned} e_q &= \Delta I_q \\ &= \Delta i_q (c \cos \alpha - d \sin \alpha) \cos s t - (c \sin \alpha + d \cos \alpha) \sin s t \qquad (6) \end{aligned}$$

The variable displacement angle δ will now be determined as a function of time. From Fig. 4 it is shown in Appendix A that two expressions for this angle can be written: one, as a function of Δi_q, t and α; the other, as a function of Δi_d and t. These will be identities involving terms of $\sin s t$ and $\cos s t$. Equating the respective coefficients, as in Appendix A, α is determined. Thus,

$$\cot \alpha = \frac{b\, d + (x_d - a)(x_q - c)}{d\,(x_d - a) - b\,(x_q - c)} \qquad (7)$$

Also, it is shown by equating the magnitudes (since the expressions represent the same angle) that the ratio of Δi_q to Δi_d is

$$\frac{\Delta i_q}{\Delta i_d} = \sqrt{\frac{(x_d - a)^2 + b^2}{(x_q - c)^2 + d^2}} \cot \delta' \qquad (8)$$

The variable displacement angle δ comprises the average component δ' plus an alternating component $\Delta \delta$.

Thus

$$\delta = \delta' + \Delta \delta \qquad (9)$$

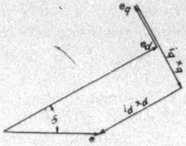

FIG. 4

The alternating component as a function of t and Δi_d is, as shown in Appendix A,

$$\Delta \delta = \frac{\Delta i_d \,[(x_d - a) \cos s t + b \sin s t]}{e \sin \delta'} \qquad (10)$$

It has been shown[1] that the torque of a synchronous machine connected to a power system of constant voltage is determined by the nominal voltages e_d and e_q, the terminal voltage e, the synchronous reactances x_d and x_q, and the displacement angle δ. It is

$$T = \frac{e\, e_d}{x_d} \sin \delta + \frac{e\, e_q}{x_q} \cos \delta + \frac{e^2 (x_d - x_q)}{2\, x_q\, x_d} \sin 2\,\delta \qquad (11)$$

This equation gives the torque not merely for steady state conditions, when $\Delta \delta = 0$, but for any condition within the limits of the premises, when the actual values of e_d, e_q and δ, existing at the moment under consideration, are substituted.

Substituting in Equation (**11**), e_d from (**5**), e_q from (**6**), δ from (**9**) and (**10**), and Δi_q from (**8**), the equation for the torque T at any time t is obtained, which contains a constant term T' and an alternating component ΔT. Thus

$$T = T' + \Delta T$$

where

$$T' = \frac{e\, e_d'}{x_d} \sin \delta' + \frac{e^2}{2}\, \frac{(x_d - x_q)}{x_d\, x_q} \sin 2\, \delta' \qquad (12)$$

It is convenient to express the alternating component of torque as a vector. Thus

$$\begin{aligned} \Delta \underset{\cdot}{T} = \Delta\, i_d \Bigg\{ & \frac{x_d - a}{e \sin \delta'}\, T_s' + \frac{a\, e \sin^2 \delta'}{x_d} \\ & + \frac{e \cos^2 \delta'}{x_q \sin \delta'} (c \cos \alpha - d \sin \alpha) \sqrt{\frac{(x_d - a)^2 + b^2}{(x_q - c)^2 + d^2}} \Bigg\} \\ + j\, \Delta\, i_d \Bigg\{ & - \frac{b}{e \sin \delta'}\, T_s' + \frac{b\, e \sin \delta'}{x_d} \\ & + \frac{e \cos^2 \delta'}{x_q \sin \delta'} (c \sin \alpha + d \cos \alpha) \sqrt{\frac{(x_d - a)^2 + b^2}{(x_q - c)^2 + d^2}} \Bigg\} \end{aligned} \qquad (13)$$

where T_s' is the synchronizing torque, given by Equation (**26**), corresponding to steady state operation.

The next step is to obtain the vector expression for the alternating component of velocity. Dividing (**13**) by this velocity will give the motional impedance. The velocity of oscillation is given by the rate of change of δ with respect to time. Thus, substituting (**10**) in (**9**) and differentiating, the velocity is

$$\Omega = \frac{d\, \delta}{d\, t} = \frac{d\, \Delta\, \delta}{d\, t} = \frac{s\, \Delta\, i_d}{e \sin \delta'} \left[- (x_d - a) \sin s\, t + b \cos s\, t\right] \qquad (14)$$

Taking the cos $s\,t$ term as reference vector, as before, (**14**) becomes

$$\underset{\cdot}{\Omega} = \frac{s\, \Delta\, i_d}{e \sin \delta'} \left[b + j\, (x_d - a)\right] \qquad (15)$$

Consider further the meaning of motional impedance. The well-known equation for torque consumed in any mechanical system involving inertial reaction, damping, and resilience is, for rotation,

$$T = I \frac{d\, \Omega}{d\, t} + T_d\, \Omega + T_s \int \Omega\, d\, t \qquad (16)$$

where

Ω = angular velocity
I = moment of inertia
T_s = resilience coefficient
T_d = damping coefficient

This is exactly analgous to the expression for voltage consumed in an electrical circuit containing inductance, resistance, and capacity. The familiar equation is

$$E = L \frac{d\, i}{d\, t} + R\, i + \frac{1}{C} \int i\, d\, t \qquad (17)$$

Now if the current in (**17**) is a sine wave, it can be expressed of course as a vector by the following familiar method.

Let

$$\frac{d}{d\, t} = j\, \omega$$

where ω = the angular velocity corresponding to the frequency of the current.

Then Equation (**17**) becomes

$$\underset{\cdot}{E} = \underset{\cdot}{i} \left(j\, \omega\, L + R + \frac{1}{j\, \omega\, C}\right)$$

or

$$\underset{\cdot}{i} = \frac{\underset{\cdot}{E}}{R + j\, \left(\omega\, L - \dfrac{1}{\omega\, C}\right)} \qquad (18)$$

Similarly, in Equation (**16**), if the total consumed torque is a sine wave, the velocity is

$$\underset{\cdot}{\Omega} = \frac{\underset{\cdot}{T}}{T_d + j\, \left(s\, I - \dfrac{T_s}{s}\right)} \qquad (19)$$

Since in the present problem the purpose is to study the character of the electromagnetic torques only as affected by the oscillatory motion, the inertial reaction does not enter the equations. In the present case it is as if, in Fig. 5, the total voltage E were a sine wave, thus giving a sine wave current, and as if only the component E_1 of the consumed voltage were to be investigated. The impedance which consumes the voltage E_1 would be

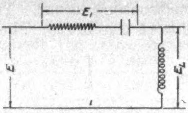

Fig. 5

$$Z_1 = \frac{\underset{\cdot}{E_1}}{\underset{\cdot}{i}} = R - j\, \frac{1}{\omega\, C} \qquad (20)$$

and, similarly, in the present problem the impedance to the oscillatory motion, *i. e.*, to the velocity of oscillation, offered by the electromagnetic reactions of the machines is, by (**19**),

$$Z_m = \frac{\Delta\, \underset{\cdot}{T}}{\underset{\cdot}{\Omega}} = \mathcal{R} + j\, X = T_d - \frac{j\, T_s}{s} \qquad (21)$$

Thus, the motional resistance, due to damping, is

$$\mathcal{R} = T_d \qquad (22)$$

and the motional reactance, due to the synchronizing torque, is

$$X = - \frac{T_s}{s} \qquad (23)$$

and

$$T_s = - s\, X$$

Dividing (**13**) by (**15**), and equating to (**21**),

$$T_d = \frac{b\,e^2 \sin^2 \delta' + \dfrac{e^2 \cos^2 \delta'}{x_q} \sqrt{\dfrac{(x_d - a)^2 + b^2}{(x_q - c)^2 + d^2}}\,[(c \cos \alpha - d \sin \alpha)\, b + (x_d - a)\,(c \sin \alpha + d \cos \alpha)]}{s\,[b^2 + (x_d - a)^2]} \quad \textbf{(24)}$$

and

$$T_s = T_s' + \frac{e^2 \sin^2 \delta'}{x_d} \left[\frac{a\,(x_d - a) - b^2}{(x_d - a)^2 + b^2} \right] + \frac{e^2 \cos^2 \delta'}{x_q} \frac{[(x_d - a)\,(c \cos \alpha - d \sin \alpha) - b\,(c \sin \alpha + d \cos \alpha)]}{\sqrt{[b^2 + (x_d - a)^2]\,[d^2 + (x_q - c)^2]}} \quad \textbf{(25)}$$

It is thus seen that the synchronizing torque, *i. e.*, the slope of the torque-angle characteristic, is equal to the steady state value T_s' plus an increment which is a function of the frequency of the oscillation and the average displacement angle δ'.

T_s' is obtained by differentiating (**11**) with respect to δ, and substituting δ' for δ. Omitting the second term, since there is no constant excitation in the quadrature axis,

$$T_s' = \frac{e\,e_d'}{x_d} \cos \delta' + e^2 \frac{(x_d - x_q)}{x_d\,x_q} \cos 2\,\delta' \quad \textbf{(26)}$$

Equations (**24**) and (**25**) are general. The type of machine usually found in installations where such oscillations exist, however, is the salient-pole type with an amortisseur winding. For such a winding, calculation shows that the resistance is so high at practical frequencies of oscillation that the winding has a negligible effect on the synchronizing torque. That is, c and d may be assumed to be zero in (**25**). For such cases,

$$T_s = T_s' + \frac{e^2 \sin^2 \delta'}{x_d} \left[\frac{a\,(x_d - a) - b^2}{(x_d - a)^2 + b^2} \right] \quad \textbf{(27)}$$

The main field winding and the amortisseur winding do produce damping, as given by (**24**). That is, c and d cannot be neglected in (**24**), although they are negligible in (**25**).

Consider a few special cases. When the resistance of the main field winding is very small compared to the leakage reactance, b becomes zero and, as shown in Appendix A,

$$a = x_d - x_d' \quad \textbf{(28)}$$

where x_d' = transient reactance, direct axis. Thus

$$T_s = T_s' + e^2 \frac{(x_d - x_d') \sin 2\,\delta'}{x_d\,x_d'} \quad \textbf{(29)}$$

Substituting (**26**),

$$T_s = \frac{e\,e_d'}{x_d} \cos \delta' + e^2 \frac{(x_d - x_q)}{x_d\,x_q} \cos 2\,\delta' + e^2 \frac{(x_d - x_d')}{x_d\,x_d'} \sin^2 \delta' \quad \textbf{(30)}$$

This condition—*i. e.*, zero field resistance, $b = 0$—gives the maximum synchronizing torque which can be obtained for the given average nominal voltage e_d' and terminal voltage e, at any oscillating frequency which is low enough to make c and $d = 0$. For more rapid oscillations c becomes significant, as shown later in Equation (**32**).

When the oscillations are very slow, so that the induced field currents are practically zero, a and b are zero, and

$$T_s = T_s' \quad \textbf{(31)}$$

That is, the oscillation occurs on the slope of the steady state angle-torque curve.

Another interesting case is the condition of very rapid oscillation in which the resistances of the direct axis winding and the quadrature axis winding are zero, that is,

$$R_{da} \text{ and } R_{qa} = 0$$

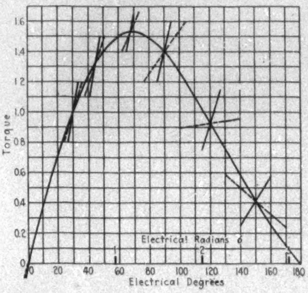

FIG. 6

Then, as shown in Appendix A,

$$a = \frac{x_{md}^2}{x_{md} + X_{lda}} = x_d = x_d'$$

$$b = 0$$

$$c = \frac{x_{mq}^2}{x_{mq} + X_q} = (x_q - x_q')$$

$$d = 0$$

Substituting these expressions in (**25**),

$$T_s = T_s' + e^2 \left[\frac{x_d - x_d'}{x_d\,x_d'} \sin^2 \delta' + \frac{x_q - x_q'}{x_q\,x_q'} \cos^2 \delta' \right] \quad \textbf{(32)}$$

Numerical Examples. a. The following constants are representative of a low-speed synchronous motor, say 300 kv-a. at 120 rev. per. min.:

$x_d = 1.0$, $x_q = 0.6$, $x_d' = 0.4$, $e = 1.0$, $e_d' = 1.4$, $e_q = 0$

The torque-angle characteristic for steady state operation, calculated from equation (**11**), is shown in Fig. 6. The slope of this characteristic at any angle δ is the synchronizing torque, as given by Equation (**26**). The synchronizing torque for oscillatory operation under the assumption that $b = c = d = 0$, is shown at different angles by the dotted line segments, as calculated for Equation (**30**).

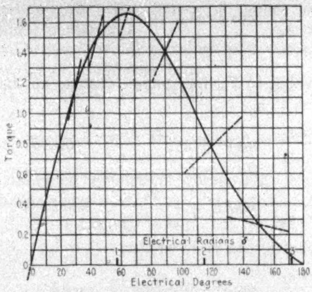

FIG. 7

If an amortisseur winding were added to the machine, the constants would be modified somewhat as follows: $x_d = 1.0$, $x_q = 0.6$, $x_d' = 0.25$, $x_q' = 0.3$, $e = 1.0$, $e_d = 1.4$.

Under the assumption that $b = d = 0$, the synchronizing torque, as calculated from (**32**), is shown by the full line segments in Fig. 6.

b. In Fig. 7 the characteristics are shown for $b = c = d = 0$, and for a higher speed machine with a somewhat lower transient reactance x_d' and a relatively smaller pole arc:

$x_d = 1.0$, $x_q = 0.5$, $x_d' = 0.3$, $e = 1.0$, $e_d' = 1.4$.

c. Fig. 8 shows the characteristics of a laminated cylindrical rotor machine: $x_d = 1.0$, $x\pi = 1.0$, $x_d' = 0.15$, $e = 1.0$, $e_d' = 1.45$.

These curves bring out some very interesting and important facts:

1. The synchronizing torque T_s under the oscillatory condition (indicated by the slope of the dotted lines) is exactly the same* as T_s' for steady state operation at $\delta = 0$ (*i. e.*, at no load, neglecting armature resistance); and departs only slightly from the steady state slope up to $\delta = 30$ electrical deg., which may be considered the practical operating range. Moreover, the dotted lines are the *maximum* slopes, neglecting the field resistance. The slope T_s for the transient condition, therefore, will be more nearly the same as the steady state slope T_s' than shown.

2. The practical approximation which has been used for some years by the authors is to divide full load torque by full load angle, which corresponds to the slope of a line through zero and the full load point on the curve. This, it will be noted, falls between the steady state slope and the dotted line, and is probably nearer the correct value in most practical applications than either of those limits, particularly for salient-pole machines.

3. It will be observed that the slope of the dotted line is still positive beyond the maximum power point of stable, steady state operation. This means that if a machine were operating beyond the angle corresponding to maximum, steady state power, say at 100 electrical deg., the machine would be stable under sudden changes, although the steady state characteristics at that point indicate instability.

4. Comparison of Figs. 6 and 7 with Fig. 8 shows that the difference between T_s' and the maximum slope T_s, indicated by the dotted lines, is much greater for cylindrical rotor machines than for those of salient-pole construction. An inspection of Equations (**29**) and (**30**) shows that the correction term for transient condition is the same in either case. The difference is in the middle term of (**30**), *i. e.*, the reaction torque term due to salient poles. When this is zero, as for cylindrical rotors, the correction term merely becomes a larger percentage of the total.

5. The addition of the effect of the quadrature axis winding, which comes into full play for very rapid oscillations, is shown in the full line segments, Fig. 6. It will be noted that the slope is steeper, and remains positive throughout the range between $\delta' = 0$ and $\delta' = \pi$.

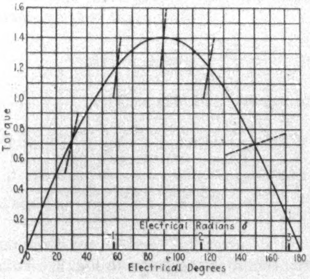

FIG. 8

B. SUDDEN ANGULAR DISPLACEMENT

A synchronous machine operating at a constant terminal voltage e, nominal voltage e_d', armature currents i_d and i_q, and an angular displacement δ', is suddenly displaced by an angle $\Delta\,\delta$ giving a total displacement δ. This will cause increments in the armature currents, so that the total current is

$$i_d = i_d' + \Delta\, i_d \qquad (33)$$

$$i_q = i_q' + \Delta\, i_q \qquad (34)$$

The increments $\Delta\, i_d$ and $\Delta\, i_q$ will induce corresponding increments of current in the short-circuited rotor windings in the two axes, and this also gives rise to increments in the nominal voltages. These are, from equations in Appendix A,

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*This is also evident from equation 29 for $\delta = 0$.

$$\Delta e_d = \Delta i_d (x_d - x_d') \tag{35}$$

$$\Delta e_q = \Delta i_q (x_q - x_q') \tag{36}$$

The total nominal voltages are thus

$$e_d = e_d' + \Delta i_d (x_d - x_d') \tag{37}$$

$$e_q = \Delta i_q (x_q - x_q') \tag{38}$$

There is assumed to be no constant component of nominal voltage in the quadrature axis.

The vector diagram for the conditions both before and after the sudden displacement, is shown in Fig. 9, from which the following relations are derived:

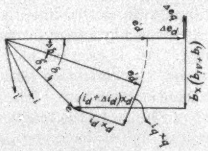

FIG. 9

$$i_d x_d = (i_d' + \Delta i_d)\, x_d = e_d - e \cos \delta \tag{39}$$

$$i_q x_q = (i_q' + \Delta i_q)\, x_q = e \sin \delta + e_q \tag{40}$$

Solving for Δi_d and Δi_q, and substituting in (37) and (38), respectively,

$$e_d = \frac{x_d}{x_d'}\, e_d' - \frac{x_d - x_d'}{x_d'} (e \cos \delta + i_d' x_d) \tag{41}$$

and

$$e_q = \frac{x_q - x_q'}{x_q'} (e \sin \delta - i_q' x_q) \tag{42}$$

Substituting (41) and (42) in (11),

$$T = \frac{e\, e_d'}{x_d'} \sin \delta + \frac{e^2 (x_d' - x_q')}{2\, x_d' x_q'} \sin 2\delta$$

$$- e \frac{x_d - x_d'}{x_d'}\, i_d' \sin \delta + \frac{x_q - x_q'}{x_q'}\, i_q' \cos \delta \tag{43}$$

The initial currents i_d' and i_q' are determined from the initial conditions as shown in Fig. 9. Thus

$$i_d' = \frac{e_d' - e \cos \delta'}{x_d} \tag{44}$$

$$i_q' = \frac{e \sin \delta'}{x_q} \tag{45}$$

Substituting these relations in (43),

$$T = \frac{e\, e_d'}{x_d} \sin \delta + e^2 \frac{x_d' - x_q'}{2\, x_d' x_q'} \sin 2\delta$$

$$+ e^2 \frac{x_d - x_d'}{x_d x_d'} \sin \delta \cos \delta' - e^2 \frac{x_q - x_q'}{x_q x_q'} \cos \delta \sin \delta' \tag{46}$$

Equation (46) gives the relation between torque and angle when the rotor is very suddenly shifted from the initial phase angle δ' to the new position δ.

The synchronizing torque T_s is obtained by differentiating (46) with respect to δ. Thus,

$$T_s = \frac{e\, e_d'}{x_d} \cos \delta + e^2 \frac{x_d' - x_q'}{x_d' x_q'} \cos 2\delta$$

$$+ e^2 \frac{x_d - x_d'}{x_d x_d'} \cos \delta \cos \delta' + e^2 \frac{x_q - x_q'}{x_q x_q'} \sin \delta \sin \delta' \tag{47}$$

The value of T_s, for sudden change, at the initial angle $\delta = \delta'$, is

$$T_{s1} = \frac{e\, e_d'}{x_d} \cos \delta' + e^2 \frac{x_d' - x_q'}{x_d' x_q'} \cos 2\delta'$$

$$+ e^2 \frac{x_d - x_d'}{x_d x_d'} \cos^2 \delta' + e^2 \frac{x_q - x_q'}{x_q x_q'} \sin^2 \delta' \tag{48}$$

Equation (48) can be re-written

$$T_{s1} = \frac{e\, e_d'}{x_d} \cos \delta' + e^2 \frac{x_d - x_q}{x_d x_q} \cos 2\delta'$$

$$+ e^2 \frac{x_d - x_d'}{x_d x_d'} \sin^2 \delta' + e^2 \frac{x_q - x_q'}{x_q x_q'} \cos^2 \delta' \tag{49}$$

The first two terms of (49) comprise the synchronizing torque T_s' at the angle δ' under steady state operation, as given by Equation (26). Hence,

$$T_{s1} = T_s' + e^2 \frac{x_d - x_d'}{x_d x_d'} \sin^2 \delta' + \frac{x_q - x_q'}{x_q x_q'} \cos^2 \delta' \tag{50}$$

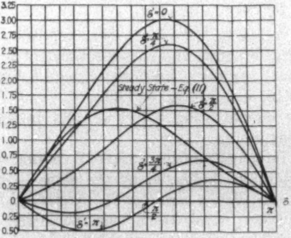

FIG. 10

It is thus seen that the synchronizing torque for very rapid displacements from the initial steady load angle δ' is equal to the steady state value T_s' plus an increment which is independent of the initial excitation and depends only on the terminal voltage, reactances, and the initial angle δ'. It is interesting to note that this equation checks with (32), which was derived, of course, under assumptions which are equivalent to those in Equation (50).

Numerical Examples. Equation (**46**) is applied to four different representative cases:

a. Laminated salient-pole rotor. (Low speed engine or compressor type.)

$x_d = 1.0$, $x_q = 0.6$, $x_d' = 0.4$, $x_q' = 0.6$, $e = 1.0$, $e_d' = 1.4$, $e_q' = 0$.

The torque T is plotted against displacement angle δ in Fig. 10 for various values of initial angle δ', and for the above voltages. Also, for comparison, the steady state torque-angle characteristic is shown, as calculated from Equation (**11**).

b. Amortisseur winding, salient-pole rotor. (Low speed engine or compressor type.)

$x_d = 1.0$, $x_q = 0.6$, $x_d' = 0.25$, $x_q' = 0.3$, $e = 1.0$, $e_d' = 1.4$, $e_q' = 0$.

The characteristics for this case are plotted in Fig. 11.

c. Laminated cylindrical rotor, Fig. 12. (High speed turbine generator.)

$x_d = 1.0$, $x_q = 1.0$, $x_d' = 0.15$, $x_q' = 1.0$, $e = 1.0$, $e_d' = 1.4$, $e_q' = 0$.

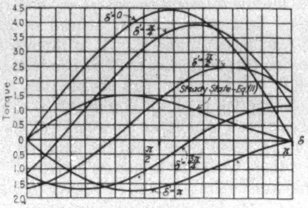

Fig. 11

d. Massive steel cylindrical rotor, Fig. 13. (High speed turbine generator.)

$x_d = 1.0$, $x_q = 1.0$, $x_d = 0.15$, $x_q' = 0.2$, $e = 1.0$, $e_d' = 1.4$, $e_q' = 0$.

These examples bring out some interesting points. In Fig. 10, example a, it is seen that if the machine is operating at a steady load corresponding to $\delta' = \dfrac{\pi}{4}$ (*i. e.*, 45 electrical deg.) and at approximately full load excitation $e_d' = 1.4$, a very sudden angular displacement of the rotor would cause a torque following the characteristic marked $\delta' = \pi/4$. Similar characteristics are also shown for other initial angles δ', but for the same nominal voltage e_d'. It is interesting to note that

1. The slope of these characteristics, that is, the synchronizing torque (which may be computed from Equation (**50**)), at the initial angle δ' corresponds to the dotted line segments in Fig. 6, since in the latter $b = c = d = 0$. That is, the resistance is zero and $x_q = x_q'$, as in the present case. In other words, the dotted lines in Fig. 6 are merely corresponding parts of the complete characteristics shown in Fig. 10.

2. The characteristics reverse between zero angle and δ'.

3. The maximum torque is attained when the initial angle is $\delta' = 0$, in which case it reaches 3.0 times normal torque at about 105 electrical deg. It will be noted also that when $\delta' = 0$, the slope of the transient curve is the same as that of the steady state curve, and continues to be so for 15 or 20 electrical deg.

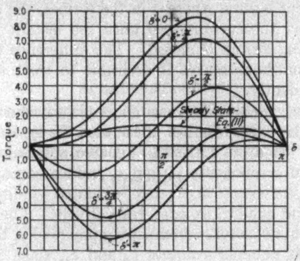

Fig. 12

4. All characteristics are zero at $\delta = 0$ and $\delta = \pi$.

The addition of an amortisseur winding significantly modifies the characteristics. Fig. 11, example b, shows the following points:

1. As in example a, the maximum slope occurs for $\delta' = 0$; unlike a, the slope of the transient characteristic at $\delta' = 0$ is much greater than the slope of the steady state curve at that point.

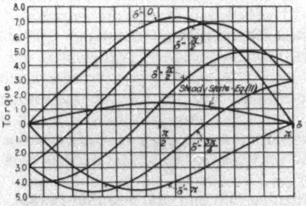

Fig. 13

2. The maximum torque is about 4.5 normal torque.

3. Unlike example a, the characteristics are not all zero at $\delta = 0$ and $\delta = \pi$, but show from 1.0 to 1.65 times normal at these angles, at which the steady state torque is zero.

Figs. 12 and 13 show characteristics for cylindrical rotor machines, examples c and d. The respective characteristics have the same general form as those for the salient-pole machines with the following exceptions:

1. The maximum torque is much greater, due to

the lower reactance. It is of the order of 8.0 times normal torque instead of 3.0 or 4.0 times normal.

2. There is a greater deviation from the steady state curve in the neighborhood of $\delta' = 0$.

C. SYNCHRONIZING OUT OF PHASE

Synchronizing out of phase gives rise to problems very similar to those occasioned by sudden angular displacement, as treated under B. The essential difference is that in the latter case the circuit is already closed, and carrying an initial current, when the displacement occurs; whereas in the former, the circuit is open and the current zero until the instant the synchronizing switch is closed. This difference in boundary conditions makes it desirable to treat the cases independently.

If the synchronizing switch is closed when the incoming machine is out of phase with the system by the angle δ, armature currents, of course, suddenly appear. The two components i_d and i_q induce corresponding current increments in the short-circuited windings in the direct and quadrature axes, respectively. The induced increments of rotor currents are, numerically, the increments in nominal voltages e_d and e_q, since both are expressed in per cent.

Thus, by Equations (**35**) and (**36**), the suddenly appearing currents i_d and i_q cause the following induced increments in the nominal voltages:

$$\Delta\, e_d = i_d\,(x_d - x_d') \qquad \textbf{(51)}$$

$$\Delta\, e_q = i_q\,(x_q - x_q') \qquad \textbf{(52)}$$

The vector diagram for conditions at the first instant is shown in Fig. 4. From the diagram and Equations (**51**) and (**52**),

$$e \cos \delta + i_d\, x_d = e_d' + i_d\,(x_d - x_d') \qquad \textbf{(53)}$$

and

$$e \sin \delta + i_q\,(x_q - x_q') = i_q\, x_q \qquad \textbf{(54)}$$

From these two relations,

$$i_d = \frac{e_d' - e \cos \delta}{x_d'} \qquad \textbf{(55)}$$

and

$$i_q = \frac{e \sin \delta}{x_q'} \qquad \textbf{(56)}$$

Substituting (**55**) and (**51**) and (**56**) in (**52**),

$$\Delta\, e_d = \frac{x_d - x_d'}{x_d'}\,(e_d' - e \cos \delta) \qquad \textbf{(57)}$$

and

$$\Delta\, e_q = \frac{x_q - x_q'}{x_q'}\, e \sin \delta \qquad \textbf{(58)}$$

The total nominal voltage e_d in the direct axis is the sum of the constant component e_d' supplied by the exciter, plus the induced component. Thus,

$$e_d = \frac{x_d}{x_d'}\, e_d' - \frac{x_d - x_d'}{x_d'}\, e \cos \delta \qquad \textbf{(59)}$$

The total nominal voltage e_q in the quadrature axis is $\Delta\, e_q$, since there is assumed to be no component furnished by the exciter. That is,

$$e_q = \frac{x_q - x_q'}{x_q}\, e \sin \delta \qquad \textbf{(60)}$$

Therefore, substituting (**59**) and (**60**) in Equation (**11**), the torque is

$$T = \frac{e\, e_d'}{x_d'} \sin \delta + \frac{e^2}{2}\, \frac{(x_d' - x_q')}{x_d'\, x_q'} \sin 2\, \delta \qquad \textbf{(61)}$$

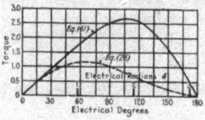

FIG. 14

It is very interesting that Equation (**61**) for torque under this transient condition is of exactly the same form as (**11**), excepting that in the present case the reactances are the *transient reactances*, x_d' and x_q', that is, the total leakage reactances, instead of the synchronous reactances x_d and x_q. It is interesting also that Equation (**61**) comprises the first two terms of (**43**), *i. e.*, when i_d' and $i_q' = 0$. It is not obvious, however, that the assumption $i_d' = i_q' = 0$ in (**43**) is equivalent to the open-circuit condition on which (**61**) is based, since with the closed circuit and the assumed nominal voltage e_d' there would be armature currents.

Since the same voltages e and e_d' appear in the Equation (**61**) as in (**11**), but with much lower reactances, the torque is correspondingly higher.

Numerical Examples. Four representative cases will be considered. In each case, a curve, calculated by Equation (**61**), is shown of the torque which would be obtained if the machine were synchronized out of phase by any angle δ. For comparison, the steady state torque-angle characteristic, Equation (**26**), is shown.

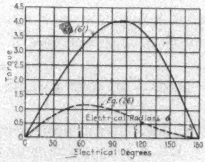

FIG. 15

a. Low speed engine or compressor type of, say, 300 kv-a., 120 rev. per min.

$x_d = 1.0$, $x_d' = 0.4$, $x_q = 0.6$, $x_q' = x_q$, $e = 1.0$, $e_d' = 1.0$.

Characteristics are plotted in Fig. 14.

b. Same as a, but with an amortisseur winding.

$x_d = 1.0$, $x_d' = 0.25$, $x_q = 0.6$, $x_q' = 0.3$, $e = 1.0$, $e_d' = 1.0$.

Characteristics are plotted in Fig. 15.

c. Medium speed water-wheel type of, say, 5000 kv-a., 300 rev. per. min.

$x_d = 1.0$, $x_d' = 0.27$, $x_q = 0.5$, $x_q' = x_q$, $e = 1.0$, $e_d' = 1.0$.

Characteristics in Fig. 16.

d. High speed steam turbine type generator of about 35,000 kv-a. with massive steel rotor.

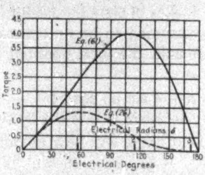

FIG. 16

$x_d = 1.0$, $x_d' = 0.15$, $x_q = 1.0$, $x_q' = 0.2$, $e = 1.0$, $e_d' = 1.0$.

Characteristics in Fig. 17.

It will be noted that these characteristics are of the same general nature as those shown in Figs. 10, 11, 12 and 13. It will be noted that a machine without amortisseur winding effect, *i. e.*, without a short-circuited winding in the quadrature axis, will suffer practically no more torque when it is synchronized at an angle less than, say, 20 deg., than would be exerted by the machine at that angle under steady operation. It increases very rapidly, however, above the steady state value for increasing angle, the maximum torque being obtained when the machine is synchronized at about 100 deg.

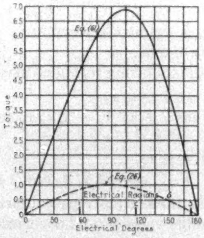

FIG. 17

Referring to Figs. 14 and 15, the addition of a short-circuited rotor winding in the quadrature axis very greatly increases not only the maximum torque but also the difference between the transient characteristic and the steady state curve at small angles.

It should be kept in mind that the torque given by Equation (**61**) is the average value existing in the first moment under the transient condition. There are, in addition, alternating components of normal and higher frequencies which are not taken into account here. While the latter are of large magnitude, nevertheless the frequency is so high that the torque does not have time, except in case of resonance, to produce much displacement, and therefore strain in the shaft. Hence the alternating component is rarely a serious factor. The unidirectional component given by Equation (**61**), however, has time to produce large strain in the shaft and coupling, where the fly-wheel effect on the other end of the shaft is comparable with that of the synchronous machine, as in the case of a turbine driven generator or a motor-generator set.

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Appendix A

When a three-phase system of currents of constant amplitude flows in the winding of a three-phase machine, there exists a uniformly rotating sinusoidal m. m. f. of constant amplitude, this m. m. f. rotating at synchronous speed. This is the speed at which the poles are rotating and by proper time phase of the currents, as at zero power factor, the rotating m. m. f. may be made to exist at every instant directly over the field poles, *i. e.*, the direct axis. If each of the three-phase currents pulsates in amplitude at a frequency, *f*, the m. m. f. over the field winding will likewise pulsate at the same frequency. The alternating component of this pulsating m. m. f. in acting on the field winding, which is short-circuited through the exciter armature, will induce currents in the field winding and we have essentially a transformer with the secondary short-circuited. The frequency impressed on this transformer is obviously the modulating frequency of the three-phase system of currents.

The equivalent diagram for this transformer is as shown in Fig. 3. In this diagram,

x_l = armature leakage reactance at normal frequency,

$X^3{}_{da}$ = field leakage reactance, in armature terms, for the winding in the direct axis, *i. e.*, the main field winding,

R_{da} = resistance of the main field winding, in armature terms,

x_{md} = mutual reactance in the direct axis,

Z_{d1} = equivalent impedance,

e_{d1} = voltage across Z_{d1},

s = modulating frequency as a fraction of normal frequency.

Then

$$Z_{d1} = \frac{j\, s\, x_{md}\, (R_{da} + j\, s\, X_{lda})}{R_{da} + j\, s\, (x_{md} + X_{lda})} \qquad \textbf{(1a)}$$

When a current $\Delta\, i_d$ flows through Z_{d1}, the voltage e_{d1} is

$$e_{d1} = \Delta\, i_d\, Z_{d1} \qquad \textbf{(2a)}$$

The current flowing in the field winding will be

$$\Delta \underset{\cdot}{I}_{da} = \frac{e_{d1}}{R_{da} + j\, s\, X_{lda}} \qquad \textbf{(3a)}$$

Substituting (**1a**) in (**2a**) and (**2a**) in (**3a**), and rationalizing,

$$\Delta I_{da} = \Delta \underset{\cdot}{i}_d \frac{s^2\, x_{md}\,(x_{md} + X_{lda}) + j\, s\, x_{md}\, R_{da}}{R_{da}^2 + s^2\,(x_{md} + X_{lda})^2} \qquad \textbf{(4a)}$$

The field current $\Delta \underset{\cdot}{I}_{da}$ is here expressed in armature terms; *i. e.*, it is the ampere-turns induced in the field winding as a fraction of normal *armature* ampere turns. The field current in field terms is numerically equal to the per cent nominal voltage which it produces. Since normal armature ampere-turns, existing in the field winding, produce a mutual flux or nominal voltage numerically equal to X_{md}, the current $\Delta \underset{\cdot}{I}_{da}$ will produce a nominal voltage,

$$x_{md}\, \Delta \underset{\cdot}{i}_{da}{}^{*}$$

Hence, the field current in field terms is

$$\Delta \underset{\cdot}{I}_d = \Delta \underset{\cdot}{i}_d \frac{s^2\, x_{md}^2\,(x_{md} + X_{lda}) + j\, s\, x_{md}^2\, R_{da}}{R_{da}^2 + s^2\,(x_{md} + X_{lda})^2} \qquad \textbf{(5a)}$$

If we let

$$\frac{s^2\, x_{md}^2\,(x_{md} + X_{lda})}{R_{da}^2 + s^2\,(x_{md} + X_{lda})^2} = a \qquad \textbf{(6a)}$$

and

$$\frac{s\, x_{md}^2\, R_{da}}{R_{da}^2 + s^2\,(x_{md} + X_{lda})^2} = b \qquad \textbf{(7a)}$$

$$\Delta I_d = \Delta \underset{\cdot}{i}_d\,(a + j\, b) \qquad \textbf{(8a)}$$

If Δi_d is taken as the reference vector, Equation (**8a**), as a function of time, is

$$\Delta I_d = \Delta i_d\,[a \cos s\, t - b \sin s\, t] \qquad \textbf{(9a)}$$

In the quadrature axis, similar relations may be obtained for such short-circuited rotor windings which exist in this axis. By similar reasoning, we get

$$\Delta I_q = \Delta \underset{\cdot}{i}_q \frac{s^2\, x_{mq}^2\,(x_{mq} + X_{lqa}) + j\, s\, x_{mq}^2\, R_{qa}}{R_{qa}^2 + s^2\,(x_{mq} + X_{lqa})^2} \qquad \textbf{(10a)}$$

If we let

$$\frac{s^2\, x_{mq}^2\,(x_{mq} + X_{lqa})}{R_{qa}^2 + s^2\,(x_{mq} + X_{lqa})^2} = c \qquad \textbf{(11a)}$$

and

$$\frac{s\, x_{mq}^2\, R_{qa}}{R_{qa}^2 + s^2\,(x_{mq} + X_{lqa})^2} = d \qquad \textbf{(12a)}$$

then

$$\Delta I_q = \Delta \underset{\cdot}{i}_q\,(c + j\, d) \qquad \textbf{(13a)}$$

The modulating current $\Delta \underset{\cdot}{i}_q$ is not, in general, in time phase with the modulating current, $\Delta \underset{\cdot}{i}_d$. Hence, if $\Delta \underset{\cdot}{i}_d$ is assumed to vary as a cosine function of time, *i. e.*, the reference vector, $\Delta \underset{\cdot}{i}_q$ must be taken as

$$\Delta \underset{\cdot}{i}_q = \Delta i_q \cos (s\, t + \alpha) \qquad \textbf{(14a)}$$

Then (**13a**) as a function of time becomes

$$\Delta I_q = \Delta i_q\,[(c \cos \alpha - d \sin \alpha) \cos s\, t - (c \sin \alpha + d \cos \alpha) \sin s\, t] \qquad \textbf{(15a)}$$

With the alternating components of the field currents thus determined, the nominal voltages which, in per cent, are numerically the same as the field currents, may be determined by adding to the alternating component the constant term supplied by the exciter. Thus the total nominal voltage in the direct axis is

$$e_d = I_d' + \Delta i_d\,(a \cos s\, t - b \sin s\, t) \qquad \textbf{(16a)}$$

Since there is assumed to be no constant excitation in the quadrature axis, the total nominal voltage in the quadrature axis is

$$e_q = \Delta i_q\,[(c \cos \alpha - d \sin \alpha) \cos s\, t - (c \sin \alpha + d \cos \alpha) \sin s\, t] \qquad \textbf{(17a)}$$

The displacement angle will now be determined as a function of time. It has been shown[1] that the vector diagram for a synchronous machine connected to a source of voltage e is as shown in Fig. 4. Armature resistance has been neglected since it may be shown to have a negligible effect on power angle characteristics for values of resistance which exist in commercial machines. In the diagram, i_d is the total component of current in the direct axis and is given by

$$i_d = i_d' + \Delta i_d \cos s\, t \qquad \textbf{(18a)}$$

where

i_d' = the average peak value of armature current in the direct axis.

Also, i_q is the total component of current in the quadrature axis and is given by

$$i_q = i_q' + \Delta i_q \cos (s\, t + \alpha) \qquad \textbf{(19a)}$$

where

i_q' = average peak value of the armature current in the quadrature axis.

From Fig. 4, the following relations may be obtained:

$$\begin{aligned} e_d &= e \cos \delta + i_d\, x_d \\ &= e \cos \delta + i_d'\, x_d + \Delta i_d\, x_d \cos s\, t \end{aligned} \qquad \textbf{(20a)}$$

and

$$\begin{aligned} e_q + e \sin \delta &= i_q\, x_q \\ &= i_q'\, x_q + \Delta i_q\, x_q \cos (s\, t + \alpha) \end{aligned} \qquad \textbf{(21a)}$$

The angle δ in these equations comprises two components, an average value δ' and a variation angle $\Delta \delta$. Hence

$$\delta = \delta' + \Delta \delta \qquad \textbf{(22a)}$$

and Equations (**20a**) and (**21a**) become

$$e_d = e \cos (\delta' + \Delta \delta) + i_d'\, x_d + \Delta i_d\, x_d \cos s\, t \qquad \textbf{(23a)}$$

and

$$e_q + e \sin (\delta' + \Delta \delta) = i_q'\, x_q + \Delta i_q\, x_q \cos (s\, t + \alpha) \qquad \textbf{(24a)}$$

From (**23a**)

$$e \cos (\delta' + \Delta \delta) = e_d - i_d'\, x_d - \Delta i_d\, x_d \cos s\, t \qquad \textbf{(25a)}$$

Expanding (**25a**)

$$e \cos \delta' \cos \Delta \delta - e \sin \delta' \sin \Delta \delta = e_d - i_d'\, x_d - \Delta i_d\, x_d \cos s\, t \qquad \textbf{(26a)}$$

---

*Here X_{md} is merely a numeric to change a quantity from one reference to another and the operator j is not required.

For very small values of $\Delta\,\delta$, Equation (**26a**) may be written

$$e\cos\delta' - e\,\Delta\,\delta\sin\delta' = e_d - i_d'\,x_d - \Delta\,i_d\,x_d\cos s\,t \qquad (\mathbf{27a})$$

or

$$\Delta\,\delta = \frac{e\cos\delta' - e_d + i_d'\,x_d + \Delta\,i_d\,x_d\cos s\,t}{e\sin\delta'} \qquad (\mathbf{28a})$$

Substituting for e_d from Equation (**16a**) and placing

$$I_d' = e_d' \qquad (\mathbf{29a})$$

we get

$$\Delta\,\delta = \frac{e\cos\delta' - e_d' + i_d'\,x_d + \Delta\,i_d[(x_d - a)\cos s\,t + b\sin s\,t]}{e\sin\delta'} \qquad (\mathbf{30a})$$

But $\Delta\,\delta$ is, by definition, alternating with respect to time. Also the bracket quantity on the right hand side is evidently alternating with respect to time. Since the other terms in the numerator are constant with respect to time, their sum must be zero, that is,

$$e\cos\delta' - e_d' + i_d'\,x_d = 0 \qquad (\mathbf{31a})$$

Then

$$\Delta\,\delta = \Delta\,i_d\,\frac{(x_d - a)\cos s\,t + b\sin s\,t}{e\sin\delta'} \qquad (\mathbf{32a})$$

From Equation (**21a**)

$$e\sin\delta = i_q'\,x_q - e_q + \Delta\,i_q\,x_q\cos(s\,t + \alpha) \qquad (\mathbf{33a})$$

Substituting (**29a**) in (**33a**), and expanding,

$$e\sin\delta'\cos\Delta\,\delta + e\cos\delta'\sin\Delta\,\delta = i_q'\,x_q - e_q + \Delta\,i_q\,x_q(\cos\alpha\cos s\,t - \sin\alpha\sin s\,t) \qquad (\mathbf{34a})$$

For very small values of $\Delta\,\delta$, Equation (**34a**) may be written

$$e\sin\delta' + e\,\Delta\,\delta\cos\delta' = i_q'\,x_q - e_q + \Delta\,i_q\,x_q(\cos\alpha\cos s\,t - \sin\alpha\sin s\,t) \qquad (\mathbf{35a})$$

Substituting for e_q from Equation (**17a**) and solving for,

$$\Delta\,\delta = \frac{i_q'\,x_q - e\sin\delta' + \Delta\,i_q\,\{[(x_q - c)\cos\alpha + d\sin\alpha]\cos s\,t - [(x_q - c)\sin\alpha - d\cos\alpha]\sin s\,t\}}{e\cos\delta'} \qquad (\mathbf{36a})$$

For reasons following Equation (**30a**),

$$i_q'\,x_q - e\sin\delta' = 0 \qquad (\mathbf{37a})$$

Then

$$\Delta\,\delta = \Delta\,i_q\,\frac{[(x_q - c)\cos\alpha + d\sin\alpha]\cos s\,t - [(x_q - c)\sin\alpha - d\cos\alpha]\sin s\,t}{e\cos\delta'} \qquad (\mathbf{38a})$$

Equations (**32a**) and (**38a**) are equations for the same angle. Hence the time phase angles must be the same and also the amplitudes.

From the time phase relations

$$\frac{b}{x_d - a} = \frac{d\cos\alpha - (x_q - c)\sin\alpha}{d\sin\alpha + (x_q - c)\cos\alpha} \qquad (\mathbf{39a})$$

and

$$\cot\alpha = \frac{b\,d + (x_d - a)(x_q - c)}{d\,(x_d - a) - b\,(x_q - c)} \qquad (\mathbf{40a})$$

Equating the amplitudes of (**32a**) and (**38a**),

$$\Delta\,i_d\,\frac{\sqrt{(x_d - a)^2 + b^2}}{e\sin\delta'} = \Delta\,i_q\,\frac{\sqrt{(x_q - c)^2 + d^2}}{e\cos\delta'} \qquad (\mathbf{41a})$$

and

$$\frac{\Delta\,i_q}{\Delta\,i_d} = \sqrt{\frac{(x_d - a)^2 + b^2}{(x_q - c)^2 + d^2}}\,\cot\delta' \qquad (\mathbf{42a})$$

Under certain conditions, the resistance of the field winding may be quite negligible compared with the reactance and it is interesting to examine the limiting case, *i. e.*, when R_{da} is zero. Equations (**6a**), (**7a**), (**11a**) and (**12a**) are then respectively.

$$a = \frac{x_{md}^2}{x_{md} + X_{lda}} \qquad (\mathbf{43a})$$

$$b = 0 \qquad (\mathbf{44a})$$

$$c = \frac{x_{mq}^2}{x_{mq} + X_{lqa}} \qquad (\mathbf{45a})$$

$$d = 0 \qquad (\mathbf{46a})$$

When the resistance of the field winding is zero, the total impedance of the equivalent circuit in Fig. 3 at normal frequency is the transient reactance, *i. e.*, $j\,x_d'$. Hence

$$j\,x_d' = j\,x_{ld} + j\,\frac{x_{md}\,X_{lda}}{x_{md} + X_{lda}} \qquad (\mathbf{47a})$$

The synchronous reactance is from Fig. 3,

$$j\,x_d = j\,x_{ld} + j\,x_{md} \qquad (\mathbf{48a})$$

From (**47a**) and (**48a**),

$$x_d - x_d' = \frac{x_{md}^2}{x_{md} + X_{lda}} \qquad (\mathbf{49a})$$

By similar reasoning for the quadrature axis we get,

$$x_q - x_q' = \frac{x_{mq}^2}{x_{mq} + X_{lqa}} \qquad (\mathbf{50a})$$

Hence, Equations (**43a**), (**44a**), (**45a**) and (**46a**) become

$$a = x_d - x_d' \qquad (\mathbf{51a})$$

$$b = 0$$

$$c = x_q - x_q' \qquad (\mathbf{53a})$$

$$d = 0 \qquad (\mathbf{54a})$$

NOMENCLATURE

All *armature currents* are expressed as fractions of the peak value (sine wave) of normal current.

All *field currents* are expressed as fractions of the field current corresponding to open circuit, normal voltage (sine wave) at normal frequency.

All *voltages* are expressed as fractions of the peak value (sine wave) of normal voltage.

Frequencies and *angular velocities* are expressed, respectively, as fractions of normal frequency and normal electrical angular velocity.

Angles are expressed as fractions of one radian.

Time is expressed as a fraction of the time corresponding to one electrical radian at normal frequency.

Reactance is expressed in per cent (as a fraction), that is, as a ratio of the voltage drop due to normal current to normal voltage.

e = terminal voltage,
e_d = nominal voltage, due to excitation in the direct axis,
e_q = nominal voltage, due to excitation in the quadrature axis,
I_d = field current, direct axis, in field terms,
I_d' = field current, direct axis, in field terms (average value),
I_q = field current, quadrature axis, in field terms,
I_{da} = field current, direct axis, in armature terms,
I_{qa} = field current, quadrature axis, in armature terms,
i_d = total direct component of armature current at the time t,
i_q = total quadrature component of armature current at the time t,
i_d' = steady, or average, peak value of the direct component of armature current,
i_q' = corresponding value of the quadrature component,
Δi_d = variation of i_d. See Fig. 1.
Δi_q = variation of i_q,
$\mathcal{R}$ = motional resistance, corresponding to the damping constant,
R_{da} = field resistance, direct axis, in armature terms,
R_{qa} = field resistance, quadrature axis, in armature terms,
s = frequency of modulation of armature current,
T = torque expressed as a fraction of that corresponding to the current and voltage (at unity power factor) on which the reactances are based,
T_d = damping constant, torque corresponding to unit electrical angular velocity,
T_s = resilience constant, torque corresponding to unit electrical angle,
T_s' = value of T_s for the synchronous machine at the average angle δ' under steady state operation,
X_{lda} = field leakage reactance, direct axis, in armature terms,
X_{lqa} = field leakage reactance, quadrature axis, in armature terms,
X = motional reactance,
x_d = synchronous reactance, direct axis,
x_q = synchronous reactance, quadrature axis,
x_l = armature leakage reactance, for either direct or quadrature axis,
x_{md} = mutual reactance, direct axis,
x_{mq} = mutual reactance, quadrature axis,
Z_m = motional impedance,
z_{d1} = equivalent impedance, direct axis. See Fig. 3.
z_{q1} = corresponding impedance, quadrature axis,
α = time phase displacement between the modulating current in the direct axis and the modulating current in the quadrature axis,
δ = displacement angle of the nominal voltage from the terminal voltage as a fraction of unit electrical angle (electrical radian). It is the actual space phase lag or lead of the rotor,
Ω = angular velocity of mechanical oscillation as a fraction of normal rotational velocity.

Bibliography

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3. Discussion of H. V. Putman's paper (2) by C. A. Nickle, Salt Lake City Convention, 1926.

4. Doherty and Franklin, A. S. M. E. *Trans.* 1920, Vol. 42, pp. 523 to 567.

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Discussion

J. F. H. Douglas: I wish to ask a question regarding the paragraph which reads, "It will be observed that the slope of the dotted line is still positive beyond the maximum power point of stable, steady-state operation. This means that if a machine were operating beyond the angle corresponding to maximum, steady-state power, say at 100 electrical degrees, the machine would be stable under sudden changes, although the steady-state characteristics at that point indicate instability." I am somewhat curious to know under what operating conditions, if any, this becomes of interest?

The curve in Fig. 7 involves not only sin δ but also sin $2\,\delta$; that is, it quite obviously contains a second harmonic. I believe this is fully recognized in the paper, and I simply wish to call attention to the fact that most textbooks do not recognize the existence of this second component, and that it is very important that it should be recognized inasmuch as it greatly increases torques in the stable operating range.

R. D. Evans: I was much interested in the question raised about the increased power limit obtainable by means of automatic control of excitation. I interpret the answer given by Mr. Nickle as giving essentially the same idea as what was incorporated in the term "artificial stability," a term coined by Mr. Shand in 1924. At that time it was thought impossible in actual operation to obtain a condition of increased power limits by that process.

Subsequently we made tests of a somewhat similar nature to that described by Mr. Nickle. These test results given in the closing discussion[1] on the Evans and Wagner paper at the 1926 Midwinter Convention, were the first experimental verification of the fact that increased power limits were actually obtainable. I might liken the condition of increased stability by control of the excitation to the process of maintaining equilibrium by the *artificial* process of juggling. Up to 90 deg., or actually somewhat less, it is possible to obtain stable operation *inherently*. Beyond that point, equilibrium may be maintained *artificially* but it is necessary to use automatic devices. This corresponds to the action of a juggler in making a corrective action after the system has started to pull out. This action on a power system is possible because of the time required for the system to pull out of step.

In the presentation Mr. Nickle described a mechanical system. We, too, have found such a system to be very good for the purpose of visualizing the actions taking place during the transients. Analytically, the actions are quite complicated, but they can be understood by a suitable mechanical model by adding inertia to the vector arms, and a spring connecting them. Mr. Griscom described a system of this general character in an article entitled "A Mechanical Analogy to the Problem of Transmission Stability", *Electric Journal*, May 1926. I notice that Mr. Nickle has described the addition of a dashpot to the arrangement which makes it possible to simulate the condition of the demagnetizing action in a machine, which changes the internal e. m. f. and brings about the change of machine reactance from leakage to synchronous reactance.

W. V. Lyon: Mr. Doherty and Mr. Nickle have presented in this paper an ingenious method for analyzing, what I believe to be, a very difficult problem. They have founded their analysis on what seem to be a reasonable set of premises. These premises have been so well chosen that in the subsequent mathematical work it is necessary to make but one simplifying assumption in order to arrive at a final result that is not unduly complicated. Whether or not this method of analysis produces accurate results can be determined only by laboratory experiment, and it is to be regretted that such data are not available at the present time. In fact we have at the Massachusetts Institute of Technology measured the torque-angle characteristic of a small synchronous motor when the load torque varies cyclicly. To be more exact we measured the power-angle characteristic, although there is but little difference between the two. Unfortunately I have had no opportunity to compare these results with Mr. Doherty's calculations.

In the third part of their paper where they consider the question of synchronizing out of phase, I should much prefer to see them follow the methods that have already been developed for computing the transient currents in a three-phase synchronous generator. The first shock on the machine, coming as it does within the time of one cycle, would probably occur before the rotor has swung more than a negligible amount. The methods to which I refer are based on the differential equations which apply to synchronous machines, whereas Messrs. Doherty's and Nickle's treatment has no such fundamental background. Since, however, the differential equations assume certain ideal conditions that do not exist, it is possible that their method will actually give better results. Here, again, laboratory experience only can decide.

I should like to suggest another method of attacking this problem. The premises upon which it is based are much the same as the authors have chosen. Briefly the assumption is that the vector diagrams which are used to explain the steady-state operation may also be used when the angular velocity of the rotor is not constant. The actual condition of operation can be resolved into two component conditions of operation as follows. First, consider that the armature is short-circuited and that normal excitation voltage is impressed on the field. The determination of the armature and field currents is a simple process even if the angular velocity of the rotor is slowly changing. Next, consider that the field winding is short-circuited and that normal polyphase voltage is impressed on the armature. Here we have an induction motor with an unsymmetrical rotor winding. The determination of the armature and field currents is again a fairly simple process which is well understood. It is only necessary to assume that the currents are determined by the actual angular velocity of the rotor and are not affected by its acceleration. Laboratory experiments alone can determine whether this assumption is reasonable. Under the actual condition of operation both of these components of current exist simultaneously and the resultant torque can be computed without much difficulty. We can then set up the differential equation which equates the electromagnetic torque developed equal to the sum of the torques acting on the shaft and that due to the acceleration of the rotor. Although I have had no opportunity to make this solution in detail I have gone far enough to see that there are no insurmountable difficulties in the path.

H. V. Putman (communicated after adjournment): It would seem that the damping torque calculated by the authors is not the actual damping torque of the motor. Actually, the damping torque is proportional to the rate of change of only that part of the displacement between the pole and the electrical field, while the damping torque calculated by the authors is proportional to the rate of change of the total displacement. Such a radical departure from the accepted ideas about this problem, is at least, worthy of further explanation.

There is another peculiar thing about this damping torque T_d calculated by the authors. It was obtained by substituting in a formula for the synchronizing torque derived under the assumption of steady-state conditions. They state that Equation (**11**) which is the synchronizing torque under steady conditions, gives the torque not merely for the steady state but for any conditions within the premises when the actual values of the nominal voltages and the displacement existing at the moment under consideration, are substituted. Substituting these values for the oscillatory condition in this formula, for the synchronizing torque, gives a vector expression of which one term is the synchronizing torque, and the other, so the authors claim, is the damping torque. It at least seems peculiar that one could obtain a damping torque by substituting in a formula for the synchronizing torque, derived under steady-state conditions and one would be inclined to question the premises which could lead to these conclusions.

If I understand the paper correctly, it seems to me that the fundamental assumption made by the authors, is unjustifiable. They assume that the whole phenomenon discussed in Part A can be handled as the result of two transformer actions, one taking place in line with the pole, and the other in line with the inter-polar space. The armature current has been resolved into two components, one in line with the *average* position of the field pole, the other in line with the *average* position of the inter-polar space. The modulation of these components of current causes the armature reactions produced by them to pulsate in magnitude. So the armature reaction produced by the direct component of current, for instance, pulsates in magnitude in line with the *average* position of the field pole. It is not in line with the field pole at every instant of time, as assumed by the authors, and hence, it would seem that the phenomenon can not be calculated as a simple transformer action if the damping torque is to be obtained correctly. If this assumption is not made, the problem might become more complicated but the damping torque, would, in all probability be found to depend on only that part of the displacement between the pole and the electrical field.

I think that the mathematical work from Equations (**14**) to (**21**) could be much simplified as follows:

ΔT is the pulsating motor torque resulting from an oscillation $\Delta \delta$ which was shown to be a harmonic function of $s\,t$; that is, $\Delta \delta$ is a function of the type

$$A \sin s\,t + B \cos s\,t \tag{1}$$

and the total motor torque is of the form

$$\Delta T = T_s \, \Delta \delta + T_d \, \Omega \tag{2}$$

but since $\Delta \delta$ is an harmonic function of $s\,t$, it is evident that:

$$\Delta \delta = - \frac{j}{s} \frac{d}{d\,t} \Delta \delta \text{ or } \Delta \delta = - \frac{j}{s} \Omega \tag{3}$$

Substituting (3) in (2) gives $\dfrac{\Delta \; T}{\Omega} = T_d - \dfrac{j}{s} T_s$

which is the author's Equation (**21**).

I found the explanation given in the paper for this part of the work, more confusing than clarifying, because of the confusion of the units involved. For instance, the well-known equation for torque consumed in any mechanical system, which is the authors' Equation (**16**), involves torque in foot-pounds, and time in seconds and angular velocity in mechanical radians per second. Also Equation (**17**) involves time in seconds and when one puts $\frac{d}{d\,t} = j\,\omega$, the differentiation is with respect to time in seconds.

I have been much interested in this percentage representation of the time unit, but it seems to me that the use of time expressed as a fraction of the time corresponding to one electrical radian at normal frequency, is actually somewhat cumbersome. For instance, if time is in seconds, one obtains the damping torque in units of torque per radian per second, and one can mark his answer exactly what it is. But how does one express damping torque in the time units used by the authors? They define it as "the damping constant, torque corresponding to unit electrical angular velocity." Unit electrical angular velocity is the angular velocity in electrical radians per second, divided by ω so that when one obtains the damping torque in the units used by the authors, he is rather puzzled as to how it is to be used until it has been transferred into electrical radians per second or some other tangible unit which can be defined.

R. H. Park: Mr. Putman states in his discussion that "Actually, the damping torque is proportional to the rate of change of only that part of the displacement between the pole and the electrical field, while the damping torque calculated by the authors is proportional to the rate of change of the total displacement. Such a radical departure from the accepted ideas about this problem, is at least, worthy of further explanation."

The explanation of this phenomenon is as follows: At any given frequency of motion of the rotor there will exist a harmonic electrical torque on the rotor of the same frequency and proportional to the amplitude of oscillation, the constant of proportionality depending in general on the frequency of oscillation. In general, there will be a difference in the time phase of the torque and the displacement. The total harmonic torque, however, may be broken up into two components, one in time phase and one in time quadrature with the displacement of the rotor. The component in time phase with the total displacement is referred to as the synchronizing component of torque. The component in time quadrature is referred to as the damping component of torque, because it is in time phase with the rate of change of displacement—*i. e.*, in time phase with the velocity. Therefore it is quite clear that at any given frequency of oscillation the electrical torque is capable of being expressed as

(a constant) × total displacement) + a constant)
× rate of change of total displacement).

It is also true, as shown below, that at any given frequency of oscillation the torque may be expressed as

(a constant) × (relative displacement of magnetic field and pole) + (a constant) × (rate of change of relative displacement of magnetic field and pole).

since, at any given frequency the relative displacement of the field and pole is in constant relation to the total displacement of the rotor provided that the oscillations are small as was assumed.

Mr. Putman also raises a question as to the reasonableness of the process by which the synchronizing and damping components of torque were calculated by substituting in the steady-state formula for synchronizing torque. The legitimacy of this method of calculation is explained most simply from the following considerations.

1. The electrical torque on the rotor depends only on the instantaneous distributions of flux and current in the machine.

2. Neglecting armature resistance, the distribution of flux and current in a machine, and therefore also the magnitude of torque, are known uniquely when the magnitude of the direct and quadrature nominal e. m. fs., terminal e. m. f., and the displacement angle between the rotor and the terminal e. m. f. are known. (It is to be noted that nominal voltage is to be interpreted as the percent armature flux linkages due to the direct component of field current, quadrature nominal voltage similarly and terminal voltage as the percent total armature linkages.)

3. Although the formula in question was originally derived in the study of the magnetic torque under steady conditions of operation, and was therefore expressed in terms of the nominal and terminal voltages and the displacement angle, nevertheless, since the torque, at any instant, actually depends only upon the instantaneous values of these quantities, it follows, as stated in the paper, that the formula may be extended in scope so as to cover variable conditions of operation.

Since the formula expressed the electrical torque completely, it must contain all component torques; thus it must contain both synchronizing and damping components. As shown in the paper, this is found to be the case. The correctness of the torque formula employed can, moreover, be demonstrated in a more explicit manner than given above. I propose to give such a demonstration in a paper to be presented before the A. I. E. E. in the near future.

C. F. Wagner (communicated after adjournment): During the discussion of this paper the question of the efficacy of voltage regulators and exciters in increasing the amount of power has arisen. This brings up the question as to whether the improvement so obtained could be attributed to the regulator or to the exciter. It is apparent that both must be sufficiently rapid; a long time lag in either regulator or exciter being approximately equivalent to conditions under hand regulation. It has been the experience of the Westinghouse Company with which I am associated that their standard vibrating voltage regulator, which is used with standard exciters, is sufficiently rapid even for quick-response exciters. This becomes apparent when it is known that the contacts of such a regulator close in a fraction of a cycle (at 60 cycles) under reduced potential. In light of these facts one must conclude that the improvement in power limits is due to improvements in exciters rather than improvements in regulators, the regulators as already developed being sufficiently satisfactory.

C. A. Nickle: Mr. Douglas has asked about the operation of synchronous machines above the steady-state power limit. In answering this question, a simple case with a cylindrical-rotor generator connected to an infinite bus will be considered. If the terminal voltage of the infinite bus is e, and e_1 is the nominal voltage of the generator, the power interchange between the generator and the bus is given by

$$P = \frac{e\,e_1}{x}\sin\delta$$

where x is the synchronous reactance of the machine. Evidently, when e_1, e, and x are constant, this expression has a maximum when $\delta = \pi/2$. If, however, e_1 is caused to vary in such a manner as to become a definite function of δ, the expression for power may have its maximum for values of greater than $\pi/2$ and the maximum power is increased. Operation beyond the steady-state power limit thus depends upon applying the proper excitation at the proper time.

By means of a new voltage regulator which we have developed, such requirements are fulfilled and machines have been caused to operate beyond the steady-state power limit by a considerable amount. To illustrate this, the following test may be cited. Two 435-kv-a. synchronous machines were connected to the same bus, one being driven as a generator and the other running as a motor. The rated voltage of these machines was 4000 volts and since, at this voltage, the possible power transfer would seriously overload the direct-connected, direct-current machines, all tests were run at a reduced voltage; *i. e.*, 2200 volts. The maximum power obtained in tests where the terminal voltage was held by hand-controlled rheostats and also by standard regulators, was 180 kw. The use of the new regulator increased this power to 480 kw. or almost triple the value which could be obtained by ordinary methods. The angular displacement between the rotors of the two machines when operating at these loads was considerably beyond 90 deg. as was verified by means of stroboscopic observations. The torque-angle characteristics for angular displacements beyond the steady-state limit thus have a physical significance as well as a theoretical one.

R. E. Doherty: Mr. Evans has referred to the term, "artificial stability," coined by Mr. Shand, as applying to operation beyond the "static" stability limit. Why coin a new term, since classical usage has long since specified such a state as "dynamic" stability, in contradistinction to "static" stability? It is the distinction between the stability of a boy riding a tricycle in one case and a bicycle in the other.

He refers also to the discussion which took place at the Philadelphia Convention in 1924[1], regarding power transmission, and states that "at that time it was thought impossible in actual operation to obtain a condition of increased power limits by that process," that is, by dynamic stability. There were a number of opinions expressed at that meeting regarding stability. I remember that I expressed this particular one: that, considering the then present stage of electrical engineering art, and the extent to which the studies under consideration projected beyond the limits of experience, we should "neither gamble that a voltage regulator will be able to insert a supporting prop under an otherwise falling system, nor depend for stability during load transients, upon possible, momentary, favorable conditions due to momentum and field transients. These may add up in the right direction, but engineers had better keep them up their sleeves . . . ." Mr. Nickle's investigation since that time has demonstrated that synchronous operation far beyond the steady-state limit is possible. Thus the mechanical momentum can be utilized in this connection to a much greater degree than was thought possible at that time.

I hope that the importance of Mr. Nickle's tests may not be overlooked. The greatest increase above the steady-state power limit which Mr. Evans and his associates state that they have obtained on test is about 20 per cent. I wish to call attention to the fact that in Mr. Nickle's test, the steady-state limit was 180 kw., and that, by the use of a new regulator which he has developed and which applies excitation not merely quickly, (*i. e.* not merely "high-speed excitation" but at the right time phase), it was possible to raise the power from 180 to 480 kw. And, in my opinion, he has written a new chapter in the story of long-distance power transmission.

Mr. Wagner refers to test results given in the closing discussion of the Evans and Wagner paper at the last Midwinter Convention, both as being "similar" to those mentioned by Mr. Nickle, and as being "the first experimental vertifications that increased power limits were actually obtainable."

How similar? They showed an increase of 20 per cent. Mr. Nickle's test showed an increase of 160 per cent beyond the power corresponding to the steady-state limit—*i. e.*, from 180 to 480 kw.

Over a year before, the paper by Doherty and Dewey, at the Pacific Coast Convention, September, 1925[2], had test results showing a 28 per cent increase in power above the steady state limit.

It is recognized generally that the problem of bringing about a quick change in the exciter voltage is important. The point which does not yet appear to be recognized in Mr. Wagner's discussion, is the very important part which the regulator plays. From his discussion, one is clearly led to the conclusion that his sole criterion regarding the efficacy of the regulator is whether its contacts close promptly on the occasion of a sudden voltage disturbance. Thoughtful consideration must nevertheless surely indicate that the subsequent behavior of the regulator is of equal importance. However, such questions as he has raised cannot be effectively settled by verbal discussion. Mr. Wagner's view would be immensely more convincing if, instead of submitting the time required for the regulator contacts to close, he had adduced some test results obtained by the use of the standard vibrating regulator which he mentions, such test results showing an increase in power over the steady-state limit comparable with those brought out in Mr. Nickle's and my discussion.

We are pleased to note that Professor Lyon considers the authors' premises to be reasonable; also that M. I. T. expects to make some experiments along these lines. Prof. Lyon mentions another possible method of attack which is interesting, and I hope that he may have an opportunity to carry this through.

Mr. Putman has raised some interesting points which the authors are glad to have the opportunity to clear up. Mr. Park has answered the question regarding the damping torque, and the particular angular velocity on which it depends; also he has shown why the torque formula referred to is applicable in general, and therefore in the present case. The authors acknowledge that there should have been further explanation regarding this point in the paper.

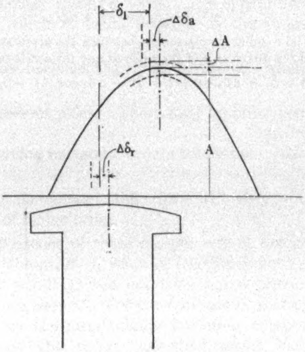

Fig. 1

With reference to his proposal to simplify the mathematics, both ways are now available, so the reader may choose to his liking.

Mr. Putman's statement regarding the authors' alleged assumption relating to the reference axes is interesting and requires detailed comment. The basic conception is a synchronous machine connected to an "infinite bus," and experiencing a periodic angular oscillation. Under such conditions the magnitude of the space fundamental component of armature m. m. f. will pulsate periodically. Moreover, on account of the variation of power factor during each oscillation of the rotor, the position of the m. m. f. wave with respect to synchronous space will vary periodically—at the same period as the pulsation in magnitude.

The question is, how shall these phenomena be expressed? One may choose the premises which the authors have actually chosen, or those which Mr. Putman understands that they have chosen, and the result will be the same; that is, one may assume the component of the fundamental m. m. f. wave over the pole (*i. e.* the direct component) and likewise the quadrature component, to vary harmonically; or, as Mr. Putman suggests, that the components in line with the *average* positions of the pole axis and the quadrature axis, vary harmonically. In the first case it is tacitly assumed by the authors that those variations of the m. m. f. *in line with the pole* other than the harmonic variations, are negligible, being second-order differences under the assumed extremely small oscillations. And it is these negligible differences to which Mr. Putman has apparently assigned an undue importance, as shown in the following:

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[1] A. I. E. E. Trans., 1924, p. 71.

[2] Trans., A. I. E. E., 1925, p. 972.

Referring to the accompanying figure, let δ_1 be the angular displacement between the average position of armature m. m. f. wave and the average position of the direct axis of the rotor, both of which positions are fixed references in synchronous space. Also, let the *total* armature m. m. f. wave vary harmonically in amplitude according to the expression

$$a = A + \Delta A \cos s t$$

and let the position of the m. m. f. wave vary harmonically about its average position according to

$$\Delta \delta_a = \Delta \delta_2 \cos (s t + \beta)$$

The harmonic oscillation of the direct axis about its average position is

$$\Delta \delta_r = \Delta \delta_3 \cos (s t + \beta_1)$$

The total angular displacement between the armature m. m. f. wave and the direct axis of the rotor at any instant is then

$$\delta = \delta_1 + \Delta \delta_2 \cos (s t + \beta) - \Delta \delta_3 \cos (s t + \beta_1)$$

Thus the component of m. m. f. which exists in the direct axis at all instants is

$$A_d = (A + \Delta A \cos s t) \cos [\Delta_1 + \Delta \delta_2 \cos (s t + \beta) - \Delta \delta_3 \cos (s t + \beta_1)]$$

Expanding, and taking advantage of the close approximation that for small angles

$$\cos x = 1 - \frac{x^2}{2}$$

$$\sin x = x$$

$$A_d = [A + \Delta A \cos s t] \left[\cos \delta_1 \left\{ 1 - \frac{< \Delta \delta_2 \cos (s t + \beta) - \Delta \delta_3 \cos (s t + \beta_1) >^2}{2} \right\} - \sin \delta_1 \{ \Delta \delta_2 \cos (s t + \beta) - \Delta \delta_3 \cos (s t + \beta_1) \} \right]$$

Neglecting second order terms,

$$A_d = A \cos \delta_1 + [\Delta A \cos \delta_1 \cos s t - A \sin \delta_1 \{ \Delta \delta_2 \cos (s t + \beta) - \Delta \delta_3 \cos (s t + \beta_1) \}]$$

likewise for A_q.

Hence the m. m. f. in the direct axis, that is over the pole, or in the quadrature axis, at all instants comprises a constant term plus a harmonically varying increment—which is the form taken by the authors.

The M. M. F. Wave of Polyphase Windings
With Special Reference to Sub-Synchronous Harmonics

BY QUENTIN GRAHAM[1]
Associate, A. I. E. E.

***Synopsis.**—The m. m. f. waves of fractional slot windings or other irregular windings are found to contain harmonic components having wavelengths greater than two pole pitches. These are designated as sub-synchronous harmonics since their harmonic order is lower than that of the synchronously rotating wave. They induce currents in the damper windings of synchronous machines which may produce noticeable loss. Some test data concerning losses are included. These harmonics have an effect upon reactance and, under certain conditions, they may cause vibration. An appendix covers the calculation of the m. m. f. of three-phase fractional slot machines.*

* * * * *

THE m. m. f. wave shapes of polyphase windings have been investigated by numerous writers.[2] The methods of analysis vary somewhat but the results are essentially the same. It is known, for example, that with the usual three-phase winding, the m. m. f. wave consists of a fundamental sinusoidal wave traveling uniformly at synchronous speed and certain odd multiples of the fundamental. Of these higher harmonic components, it is easily shown that the third or multiples thereof do not exist, and that the 5th, 11th, 17th, etc., travel against rotation while the 7th, 13th, 19th, etc., travel with rotation or in the same direction as the fundamental wave. The speed at which these components of the m. m. f. wave travel is inversely as their harmonic order. Thus the 5th harmonic travels at one-fifth synchronous speed, the 7th harmonic at one-seventh speed and so on. Each component moves through its own wavelength in the same interval of time.

In the present paper, the m. m. f. waves of certain particular types of windings are examined in some detail and it is shown that they possess additional harmonics which may have an important bearing on machine performance. The paper deals chiefly with fractional slot windings.

The term "fractional slot" is applied to machines in which the ratio of the number of slots to the number of poles is not an integral number. In machines of this type the m. m. f. wave varies from pole to pole. Thus when the m. m. f. for the complete armature is plotted and the wave analyzed, it is found to contain harmonic components having a wavelength greater than twice the pole pitch. That is, there are harmonics of lower order than the predominant component which travels at synchronous speed and which is normally spoken of as the fundamental wave. It becomes convenient, then, to use a new rotation in which the complete developed armature is taken as 2π radians and the fundamental component of the m. m. f. is a wave whose length is 2π. The complete m. m. f. wave contains components which are various multiples of this fundamental, one of which is the synchronously rotating wave. The synchronous component, which in usual notation is the fundamental wave, now becomes the $\frac{P}{2}$th harmonic, where P is the number of poles. There may be other component waves having harmonic orders below the $\frac{P}{2}$th or synchronous component and there will always be components of higher order.

The existence of these components of low harmonic order in the m. m. f. wave of fractional slot and other irregular windings has not been noted previously, as far as I am aware.[3] For convenience in distinguishing them from the usual higher harmonic components, I have used the term "sub-synchronous harmonics" since their harmonic order is below that of the synchronous component. This term is not entirely satisfactory since there may be important components whose order is above the synchronous component but which are not multiples thereof. The term "non-synchronous harmonic" may be used to designate any component other than the synchronous and of course is applicable to the 5th, 7th, etc., which occur in integral slot three-phase machines.

It is interesting to examine the sub-synchronous harmonics with reference to their speed of rotation and their direction. It is well known that the usual higher harmonics proceed around the armature at less than synchronous speed, each moving its own wavelength during a cycle of the current. It is not surprising, then, to find that the sub-synchronous harmonics travel at higher speeds, fulfilling the same condition of one wavelength of travel during one cycle of current. The direction is found to be with rotation in some cases

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[1] Designing Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[2] See, for example, "The Mathematical Treatment of the Magnetomotive Force of Armature Windings" by B. Hague in the JOURNAL of the Institution of Electrical Engineers, July, 1917.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 7-11, 1927.

[3] While the present paper was being prepared, the existence of these harmonics was mentioned in a footnote by Doherty and Nickle in their paper, *Synchronous Machines*, presented at the June, 1926, Convention of the A. I. E. E.

and against rotation in others, as will be shown later.

The method of determining the existence of any harmonic and of finding its magnitude will be shown in detail. The fluxes which correspond to the various harmonics will then be considered. The effect of these fluxes in both the stator and rotor will be discussed and it will be shown that they have an influence upon reactance and upon losses.

DETERMINATION OF M. M. F.

A balanced polyphase winding having an integral number of slots per pole and equal spacing of phase belt produces an m. m. f. wave in which the space distribution is repeated for every pair of poles. For this type of winding it is sufficient to plot the m. m. f. for two pole pitches and to base all analyses of armature reaction on an equivalent two-pole machine. For the type of windings under consideration, however, the m. m. f. distribution varies from pole to pole around the armature so it becomes necessary to consider the entire winding. The method of analysis used here consists in finding, first, the m. m. f. wave set up by a single coil. This is decomposed into its various harmonic components after which each component is added separately to the corresponding waves of the other coils.

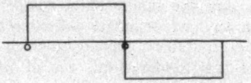

FIG. 1—M. M. F. OF FULL PITCH COIL

Consider for the moment a developed armature such as is shown in Fig. 1 with a coil having a pitch equal to half of the developed length. This may be, for example, a two-pole machine with a full pitch coil. The m. m. f. set up by this coil has a space distributed shown by the rectangular wave. It is well known that such a wave can be expressed as

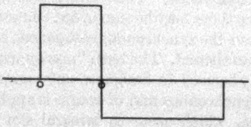

FIG. 2—M. M. F. OF SHORT THROW COIL

$$a = \frac{4}{\pi}\left(\sin\alpha + \frac{1}{3}\sin 3\alpha + \frac{1}{5}\sin 5\alpha \ldots \frac{1}{n}\sin n\alpha\right)$$

Suppose now that the throw of the coil is less than half of the developed armature. The m. m. f. form then takes the unsymmetrical rectangular shape shown in Fig. 2. The areas under both halves of this wave are equal and the ratio of the positive to the negative ordinates varies with the angle β which expresses the coil throw. The magnitude of any harmonic component of a wave of this type is shown, in Appendix *A*, to be

$$a_r = \frac{0.45}{n}\sqrt{1 - \cos n\beta} \qquad (5)$$

In this case both odd and even harmonics may be present. However, if the armature shown in Fig. 2 is used to represent a two-pole machine with short throw coils and the usual balanced arrangement of slots and phases, the even harmonics will not appear in the final m. m. f. wave due to a cancellation that takes place when the components of all coils are added. This is in agreement with the well-known fact that chording a winding changes the magnitude of the m. m. f. but does not introduce dissymetry.

Let another developed armature be represented by Fig. 3. In this case the complete machine has a large number of poles and the throw of the single coil shown is small compared to the complete developed armature.

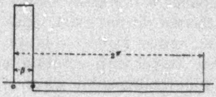

FIG. 3—M. M. F. OF SINGLE COIL OF A MULTIPOLE MACHINE

For purposes of analysis, however, the coil may be considered just as if it were a coil of extremely short throw on a two-pole armature. The angle of throw of the coil is β and the complete armature span is taken as 2π. The values of the different harmonics are found from (**5**). The component of lowest harmonic order will have a length equal to the complete armature; that is, it will be the fundamental wave and the other component waves will have harmonic orders which are multiples of its order.

The problem then resolves itself into locating all the other coils in their correct positions and adding together the waves of the same harmonic order. Appendix B shows the method of carrying out this calculation for the case of fractional slot, three-phase windings. Fig. 4 shows the fundamental wave and the $\frac{P}{2}$th or synchronous harmonic for a 10-pole machine. The magnitude of the fundamental is exaggerated in the sketch.

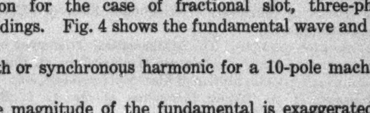

FIG. 4

If the same method of analysis were applied to a winding which is not of the irregular type, the harmonics of lower orders appearing in the m. m. f. of a single coil would cancel out when combined with those of the other coils. This is simply stating that when the armature winding for one pair of poles is the same as for every other pair of poles, the m. m. f. wave repeats itself for each pair of poles.

FRACTIONAL SLOT WINDINGS

Modern a-c. machines make extensive use of windings in which the number of slots per pole is not an integer. These fractional slot windings give perfectly balanced terminal voltages and usually produce voltage wave shapes which are noticeably free from harmonics. There are many design and manufacturing advantages in the use of these windings since the possible numbers of slots for a given machine are not limited to multiples of the number of poles.

The method of determining the possible numbers of slots for any combination of poles and phases and of distributing the coils so as to obtain balanced voltages, has been published[4] previously. For three-phase windings the rule is that the number of slots must contain the factor $3^{(r+1)}$ where r is the number of times the factor 3 appears in the number of poles. The distribution of coils into pole groups is illustrated by the chart in Fig. 5. The procedure to be followed in laying out such a chart can be explained more easily by the use of an example.

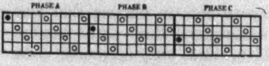

FIG. 5—WINDING CHART FOR MACHINE WITH 6¾ SLOTS PER POLE (*y*–4)

Chart covers one repeatable group for each phase. It is applicable to any machine having the same number of slots per pole and having a number of poles which is a mutiple of 4. The beginning of each phase is shown by the symbol ●

Let us take a 20-pole, three-phase machine having 135 slots. The number of slots per phase per pole expressed as a fraction is 9/4. Select a scale such that four divisions equal a slot pitch. Then nine divisions will represent a phase belt and 27 divisions a complete pole pitch. Locate the conductors a slot pitch apart on the chart, regardless of their relation to the phase belts, placing the succeeding poles under the first pole. Continue this until it is seen that the positions of the conductors along the horizontal scale are the same as those at the beginning of the chart. It will be found that four pole pitches must be covered before the winding begins to repeat. In any winding of this type the number of poles passed through before the winding repeats is the same as the denominator of the fraction expressing the slots per pole per phase. In the discussion that follows, y will be used to represent this number of poles.

The winding chart, Fig. 5, now shows the number of slots to be used in each phase group under each pole. The first pole will have 3 coils in phase A, 2 in phase B and 2 in phase C; the second pole will have 2 coils in phase A, 3 in phase B and 2 in phase C; and so on. Each group of 4 poles will be a repetition of the first 4 poles. Since there are 20 poles in the machine there will be 5 equal groups. These groups may be in series or parallel connection. The term "repeatable group" is used here to designate the coils of one phase in the 4, or in general, y adjacent poles.

In the determination of the m. m. f. waves as shown in Appendix B, the angular position of each coil in a repeatable group with reference to the initial coil must be found from the winding chart. In Fig. 5 the angular positions of the coils in phase A with respect to the first coil are as shown below. All angles are multiples of the slot pitch except that the angle π has been added to the actual physical position of coils in alternate poles to take into account the reversal of the direction of current in those coils. Letting S represent a slot pitch in angular measure the angles with respect to the nth harmonic wavelength are:

	Angles
Coil No. 1	o
Coil No. 2	$n\,s$
Coil No. 3	$2\,n\,s$
Coil No. 4	$7\,n\,s + \pi$
Coil No. 5	$8\,n\,s + \pi$
Coil No. 6	$14\,n\,s$
Coil No. 7	$15\,n\,s$
Coil No. 8	$21\,n\,s + \pi$
Coil No. 9	$22\,n\,s + \pi$

These are the angles to be used in adding the m. m. f. waves of the individual coils to obtain the quantity M used in Appendix B.

While the determination of the magnitude of any harmonic component in the final m. m. f. wave is rather tedious, the test for the existence of any harmonic is quite simple. It is shown in Appendix B that if

$$\frac{n + \dfrac{P}{2}}{\dfrac{P}{y}} = K_1 \qquad (21)$$

and if

$$\frac{y\,(P \pm 2\,n)}{6\,P} = K_2 \qquad (22)$$

where K_1 and K_2 are integers, including zero, the nth harmonic exists. Otherwise, although it is present in the coil m. m. f., it is cancelled when the summation is made.

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[4] See "Two- and Three-Phase Lap Windings in Unequal Groups," by E. M. Tingley, *Electric Review and Western Electrician*, Vol. 66, No. 4.

Table I shows the magnitude of the sub-synchronous harmonics in the m. m. f. waves of a number of typical fractional slot machines. All of these windings fulfill the conditions necessary to obtain balanced terminal voltages. The value of each harmonic is expressed in per cent of the synchronous component.

TABLE I

No. poles	No. slots per pole per phase	Per cent harmonics total	Per cent harmonics
10	4- 2/5	3.6	3.6_1
10	4- 4/5	5.2	5.2_1
22	1- 7/11	15.9	4.2_1 4.4_5 7.3_7
24	3- 3/8	6.6	3.4_3 3.2_6
26	2-10/13	14.4	2.1_1 7.7_5 2.2_7 2.4_{11}
44	2- 1/22	28.8	3.3_2 4.1_4 2.6_8 5.8_{10} 2.0_{14} 9.5_{16} 1.5_{20}
48	2- 1/4	11.0	11.0_{12}
72	1- 3/4	14.3	14.3_{18}

This table shows the value of the sub-synchronous harmonics in per cent of the synchronous m. m. f. The subscripts indicate the order of the harmonic. All machines listed are three-phase. Harmonics of orders higher than that of the synchronous component but not multiples of it are not shown, although they may exist and may be, in some cases, as important as those that are sub-synchronous.

EFFECT OF SUB-SYNCHRONOUS HARMONICS ON PERFORMANCE

When the existence of m. m. f. waves of low harmonic order has been established, there arises the question of their possible effects on performance. The equations in Appendix B show that these waves, like the usual higher harmonics, have speeds proportional to $\frac{1}{n}$. They set up corresponding flux waves, assuming constant permeance of the gap, which induce voltages of line frequency in the armature conductors. Thus they add to the reactance of the machine. Their speed relative to the rotor (of a synchronous machine) is such as to develop in the rotor windings voltages of frequency f_n where

$$f_n = f\left(\frac{2\,n}{P} \pm 1\right)$$

In a machine having a damper winding there will be secondary currents of this frequency which will react on the stator and reduce the value of the corresponding flux wave. The possible sources of loss, then, are $I^2\,r$ loss in the damper winding, if the machine has one, and iron loss in both the stator and the rotor.

Losses. It seems probable that the presence of sub-synchronous harmonics in the m. m. f. wave may cause appreciable rotor loss, particularly in machines having damper windings. It is important to note that the frequency of the induced rotor currents, for any sub-synchronous harmonic, is between zero and 200 per cent of the line frequency, as shown by the expression

$$f_n = f\left(\frac{2\,n}{P} \pm 1\right)$$

The damper winding loss of single-phase machines is caused by rotor currents of 200 per cent frequency and there are plenty of data to show that this loss may be an important factor in the efficiency. It is true that the magnitude of the non-synchronous component of m. m. f. in a single-phase machine is equal to the synchronous component and therefore is greater than in any of the polyphase machines under consideration. But with a total of, say, 20 per cent harmonic m. m. f., which may easily exist, causing rotor currents of various frequencies between zero and 200 per cent, it is reasonable to believe the losses may be appreciable. It is interesting to note in passing that the space harmonics of integral slot three-phase windings, those of 5th, 7th, 11th, 13th, etc., harmonic order compared to the synchronous component, cause rotor currents of six times normal frequency or multiples thereof and thus operate on relatively high impedance circuits and produce negligible loss.

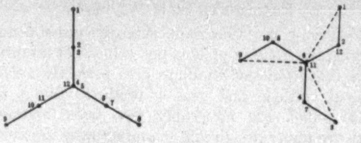

FIG. 6

A—STAR-CONNECTED WINDING

B—INTERCONNECTED STAR WINDING

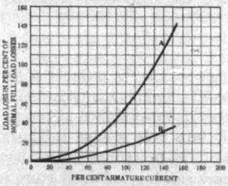

FIG. 7—MACHINE EQUIPPED WITH DAMPER WINDING

Curve A, loss with winding connected to produce sub-synchronous harmonics.

Curve B, loss with normal windings.

In order to obtain test data bearing on the question of losses, a normal three-phase machine with an integral number of slots was reconnected according to the diagram in Fig. 6. This type of connection, which has had some practical use, is usually called an interconnected star. It gives a simple means of changing a star-connected machine so as to have 86.6 per cent of the effective turns. In making this connection, however, the armature m. m. f. is unbalanced in such a way that sub-synchronous harmonics are set up. Table II gives the values of the various components. It will be seen that both the fundamental and the third harmonic have positive and negative waves. These arise because the space positions of the phases are no longer symmetrical, whereas the phase groups of fractional slot windings are always equally spaced. For our purpose, however, it is not necessary that a given harmonic have only one direction of travel.

Curve *A* in Fig. 7 gives the total load loss[5] of this machine with the interconnected star winding. Curve *B* gives the load loss with the winding arranged in star connection but with 120-deg. grouping so as to obtain the same synchronous m. m. f. per ampere as with the interconnected winding. The 120-deg. grouping gives an m. m. f. wave, free from sub-synchronous components, just as in the usual 60-deg. winding. The curves give a direct measure of the increased loss due to the presence of harmonics of lower order. At rated current the load loss increased to 3.6 times its normal value.[6] The sum of the four components given in Table II is 65.9 per cent which is more than double that of any of the machines listed in Table I. It is, of course, an exceptionally flagrant case of unbalanced m. m. f. but it indicates that machines with lower values of harmonics may be worthy of study.

TABLE II

No. Poles	No. slots per pole per phase	Per cent harmonics total	Per cent harmonics
8	4	65.9	$7.7 +_1$ $5.6 -_1$ $41.6 +_3$ $11.0 -_3$

Harmonics present with interconnected star winding. Values are given in per cent of the synchronous m. m. f. Positive and negative waves of the same harmonic order are present.

An attempt was made to simulate the conditions in the rotor due to these harmonics by applying various frequencies to the stator winding, with the normal star connection, and with the rotor locked. In this way currents of any desired frequency may be set up in the rotor bars and the loss in the rotor may be determined. It is realized that the distribution of current in the rotor is affected by the wavelength of the flux wave which induces the current, so that even though the frequency is the same, the loss may be slightly different. The test results, which were not particularly accurate, showed that about 70 per cent of the increased load loss could be accounted for as loss in the rotor.

A similar test was made on another machine giving the results shown in Fig. 8. In this case the load loss at rated current increased many times due to the fact that the normal load loss was almost negligible. While this was a much smaller machine than was used in the preceding test, the number of poles and number of slots were the same and the per cent of harmonic m. m. fs. given in Table II holds for this case also.

Load loss tests made on machines such as those listed in Table I have not given convincing data concerning the additional losses due to harmonics in the m. m. f. This has been due partly to the fact that most of the machines with high percentages of harmonics have been in the low speed class where the total load loss is small and difficult to measure with accuracy. A comparison of losses of similar machines, one with and one without harmonics, is not sufficient since a change in number of slots is always involved. With the present knowledge of load losses it is usually impossible to take into account these changes in magnetic proportions and thereby segregate particular losses. The data given in this paper are offered simply as evidence that there is a possibility of additional losses due to m. m. f. harmonics when irregular windings are used. Further study is needed to determine how important these losses are.

Reactance Voltage. The method of finding the reactance voltage due to any particular harmonic is shown in Appendix B for the case in which there is no opposing rotor current. The equations show that the voltages due to the various harmonics are not in phase with one another. The total reactance voltage due to m. m. f. harmonics may be found by adding together, in the proper phase relation, the various component voltages.

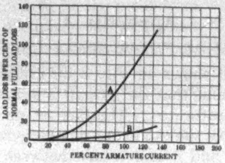

FIG. 8—MACHINE EQUIPPED WITH DAMPER WINDING

Curve A, loss with winding connected to produce sub-synchronous harmonics.
Curve B, loss with normal windings.

What may be said of the magnitude of the reactance voltage calculated in this manner? At first glance it would appear that fairly large values are to be expected considering the amplitude of the non-synchronous flux waves. However, when the actual interlinkages are worked out the result is found to be quite low. In the machines that have been examined up to the present time, all of them salient-pole synchronous machines, the total reactance voltage due to m. m. f. harmonics has been less than one per cent of rated voltage. It appears, therefore, that this part of the reactance is of no great importance. The whole subject of reactance, particularly that of fractional slot machines, is being investigated by other engineers and it is possible that, eventually, the factors treated here will assume more importance than appears likely at present.

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[5] The load loss is the difference between the measured short-circuit loss and the armature $I^2 R$ loss.

[6] In an unpublished report written by Mr. M. W. Smith during 1921, the existence of high load loss in machines with interconnected star windings was noted and was explained by means of a graphical plot of the m. m. f. wave.

Vibration. Severe vibration of the stator of a small machine having a fractional slot winding was largely responsible for the present investigation of m. m. f. forms. This particular machine had a fundamental wave of about four per cent of the synchronous component. This wave might be replaced by an equivalent two-pole rotor revolving at 3600 rev. per min. (for 60-cycle supply). Thus there were variations in magnetic attraction, tending to distort the stator at its weakest section, which occurred 120 times a second. Measurement by means of a torsiograph showed vibrations of exactly this frequency. Further tests showed that the natural frequency of vibration of the stator was almost identical with the forced vibration. A minor change in construction was sufficient to remove the natural frequency from the danger zone and stop the vibration. Such a coincidence of natural and forced frequency is probably rare but the experience indicates another of the possible effects of lower order harmonics.

ACKNOWLEDGMENTS

I am indebted to Mr. H. Brookins for assistance in working out the data in Table I and to Mr. H. V. Putman and Mr. C. M. Laffoon for much valuable criticism.

Appendix A

M. M. F. OF A SINGLE COIL

Let the coil shown in Fig. 3 be excited with unit current. Assume that the air-gap is uniform and that the iron is unsaturated. The flux set up by this coil must pass across the gap in one direction over the space between zero and β and return between β and 2π. We can then draw a rectangular wave, representing the density, in which the areas under the two half-waves are equal. With a uniform gap this curve may be used to represent m. m. f. also. Let the total m. m. f. of the coil be unity. Then if the positive ordinate of the wave is h, the negative ordinate will be $(h - 1)$. The wave may be expressed as

$$f(\alpha) = \Big[\, h \,\Big]_{\alpha = 0}^{\alpha = \beta} + \Big[\, h - 1 \,\Big]_{\alpha = \beta}^{\alpha = 2\pi} \qquad (1)$$

The value of the sine component of any harmonic of a periodic function is known to be

$$a_s = \frac{1}{\pi} \int_0^{2\pi} f(\alpha) \sin(n\alpha)\, d\alpha$$

or, in this case,

$$a_s = \frac{1}{\pi} \int_0^{\beta} h \sin(n\alpha)\, d\alpha + \frac{1}{\pi} \int_{\beta}^{2\pi} (h - 1) \sin(n\alpha)\, d\alpha \qquad (2)$$

Integrating,

$$a_s = \frac{1}{n\pi}(1 - \cos n\beta) \qquad (3)$$

Similarly, the cosine component is found to be

$$a_c = \frac{1}{n\pi} \sin n\beta \qquad (4)$$

The resultant value of the nth harmonic is

$$a_r = \sqrt{a_s^2 + a_c^2}$$

or,

$$a_r = \frac{1}{n\pi} \sqrt{(1 - \cos n\beta)^2 + (\sin n\beta)^2}$$

Solving,

$$a_r = \frac{0.45}{n} \sqrt{1 - \cos n\beta} \qquad (5)$$

The zero point of the resultant wave is displaced from the origin by the angle,

$$\theta_n = \tan^{-1} \frac{a_c}{a_s}$$

$$\theta_n = \tan^{-1} \frac{\sin n\beta}{1 - \cos n\beta} \qquad (6)$$

Appendix B

M. M. F. AND REACTANCE VOLTAGE OF FRACTIONAL-SLOT, THREE-PHASE WINDINGS

Let the coil shown in Fig. 3 be excited with a current equal to $\sqrt{2} \sin \omega t$. The maximum ordinate of the nth harmonic wave will be, from Equation (5),

$$\frac{0.45\sqrt{2}}{n} \sqrt{1 - \cos n\beta} \sin \omega t$$

At any point in space the ordinate of the wave will be

$$a_1 = \frac{0.636}{n} \sqrt{1 - \cos n\beta} \sin \omega t \sin n(\alpha - \theta_n) \qquad (7)$$

since the zero point of the wave is displaced from the origin by the angle θ_n.

For any other coil in the same phase the nth harmonic wave is expressed by

$$a_2 = \frac{0.636}{n} \sqrt{1 - \cos n\beta} \sin \omega t \sin n(\alpha - \theta_n - \phi_2) \qquad (8)$$

where ϕ_2 is the angle between that coil and the origin.

To find the combined nth harmonic wave due to the entire phase it is necessary to add the single coil waves in proper angular relationship. These individual coil waves are all equal in magnitude but are spaced in irregular fashion. To find the resultant wave it is convenient to consider the waves as vectors of equal length separated from the zero vector by various angles corresponding to the positions of the coils with respect to the reference coil. It is necessary to consider only the coils in one repeatable group instead of the whole phase, as will be shown later. The actual angles must be obtained from the winding chart. These vectors are then added and the resultant is determined both in magnitude and position. If M is taken as the ratio of the length of this resultant vector to the length of a single vector and $n\,\psi$ is the angle that the resultant makes with the zero vector, the equation for the m. m. f. of a repeatable group is

$$a_g = \frac{0.636\,M}{n}\sqrt{1-\cos n\beta}\,\sin\omega t\,\sin n(\alpha-\theta_n-\psi) \quad (9)$$

The quantities M and $n\,\psi$ must be determined in each case by performing the vector addition. This is the laborious part of working out the m. m. f. wave for any particular harmonic. Each phase contains $\frac{P}{y}$ repeatable groups. The angle between the beginnings of these groups is $\frac{y\,2\,\pi}{P}$. If y, which is equal to the number of poles per group, is an even number, the angle between the nth harmonic waves of adjacent groups is also $\frac{y\,2\,\pi}{P}$.

However, since y may be odd, the waves of adjacent repeatable groups may be reversed due to the fact that alternate pole groups are wound in opposite directions. This may be taken into account by writing the angular space between the nth harmonic waves of adjacent repeatable groups as

$$\frac{y\,2\,\pi}{P} + \frac{y\,\pi}{n}$$

or $\left(\dfrac{n+\dfrac{P}{2}}{\dfrac{P}{y}}\right)$ nth harmonic wavelengths.

Since both numerator and denominator of this fraction are known to be integers, we may write

$$\frac{n+\dfrac{P}{2}}{P/y} = K_1 + \frac{K_2}{P/y} \quad (10)$$

where K_1 is any integer and K_2 is any integer less than P/y including zero.

If $K_2 = 0$, the angle between the waves of the various repeatable groups is an integral number of wavelengths and the waves add arithmetically. If, on the other hand, K_2 is not zero, the $\frac{P}{y}$ waves are spaced by equal multiples of $\frac{y\,2\,\pi}{n\,P}$ and must add up to zero.

Thus when $\dfrac{n+\dfrac{P}{2}}{P/y}$ is an integer, the resultant wave for the whole phase becomes

$$a_{p1} = \frac{0.636\,M\,P}{n\,y}\sqrt{1-\cos n\,\beta}\,\sin\,\omega\,t\,\sin\,n(\alpha-\theta_n-\psi) \quad (11)$$

And, when $\dfrac{n+\dfrac{P}{2}}{P/y}$ is not an integer, the nth harmonic is not present in the m. m. f. wave.

The next step is to combine the wave set up by the first phase with the corresponding waves of the other phases. In a fractional-slot, three-phase winding the beginning of the second phase is at an angle $\frac{y\,2\,\pi}{3\,P}$ from the beginning of the first phase, as can be seen from the winding chart. The beginning of phase 2 is understood to be the point from which the subsequent pole groupings for that phase follow the same sequence as in the first phase when starting from the original reference coil.

The time phase of the current in phase 2 may be expressed by $\left(\omega\,t - \frac{y\,2\,\pi}{6}\right)$ since, as the winding chart shows, there are y pole phase groups between the starting points.

The equation for the nth harmonic wave of phase 2 is, then,

$$a_{p2} = \frac{0.636\,M\,P}{n\,y}\sqrt{1-\cos n\,\beta}\,\sin\left(\omega\,t-\frac{2\,\pi\,y}{6}\right)$$
$$\sin n\left(\alpha-\theta_n-\psi-\frac{2\,\pi\,y}{3\,P}\right) \quad (12)$$

and similarly, for phase 3,

$$a_{p3} = \frac{0.636\,M\,P}{n\,y}\sqrt{1-\cos n\,\beta}\,\sin\left(\omega\,t-\frac{4\,\pi\,y}{6}\right)$$
$$\sin n\left(\alpha-\theta_n-\psi-\frac{4\,\pi\,y}{3\,P}\right) \quad (13)$$

For convenience, let

$$C = \frac{0.636\,M\,P}{n\,y}\sqrt{1-\cos n\,\beta}$$

and

$$D = (\alpha-\theta_n-\psi)$$

Then, substituting C and D in Equations (**11**), (**12**) and (**13**) and expanding,

$$a_{p1} = \frac{C}{2} [\cos(\omega t - n D) - \cos(\omega t + n D)] \qquad \textbf{(14)}$$

$$a_{p2} = \frac{C}{2} \left[\cos\left(\omega t - \frac{2 \pi y}{6} - n D + \frac{2 \pi y n}{3 P}\right) - \cos\left(\omega t - \frac{2 \pi y}{6} + n D - \frac{2 \pi y n}{3 P}\right)\right] \qquad \textbf{(15)}$$

$$a_{p3} = \frac{C}{2} \left[\cos\left(\omega t - \frac{4 \pi y}{6} - n D + \frac{4 \pi y n}{3 P}\right) - \cos\left(\omega t - \frac{4 \pi y}{6} + n D - \frac{4 \pi y n}{3 P}\right)\right] \qquad \textbf{(16)}$$

The sum of the waves of the three phases will give the final resultant wave for the whole armature. Adding a_{p1}, a_{p2} and a_{p3},

$$\begin{aligned} a_n = \frac{C}{2} \Bigg[& \cos(\omega t - n D) \\ & + \cos\left(\omega t - \frac{2 \pi y}{6} - n D + \frac{2 \pi y n}{3 P}\right) \\ & + \cos\left(\omega t - \frac{4 \pi y}{6} - n D + \frac{4 \pi y n}{3 P}\right)\Bigg] \\ - \frac{C}{2} \Bigg[& \cos(\omega t + n D \\ & + \cos\left(\omega t - \frac{2 \pi y}{6} + n D - \frac{2 \pi y n}{3 P}\right) \\ & + \cos\left(\omega t - \frac{4 \pi y}{6} + n D - \frac{4 \pi y n}{3 P}\right)\Bigg] \end{aligned} \qquad \textbf{(17)}$$

It will be observed that if

$$- \frac{y}{6} + \frac{y n}{3 P} = K_1$$

where K_1 is any integer, including zero, Equation (**17**) becomes

$$\begin{aligned} a_n = {} & \frac{C}{2} [3 \cos(\omega t - n D)] \\ & - \frac{C}{2} \left[\cos(\omega t + n D) + \cos\left(\omega t + n D - \frac{2 \pi y}{3}\right) \right. \\ & \left. + \cos\left(\omega t + n D - \frac{4 \pi y}{3}\right)\right] \end{aligned} \qquad \textbf{(18)}$$

Since y is always an integer but is never a multiple of 3, the last term equals zero and the expression becomes

$$a_n = \frac{3 C}{2} \cos(\omega t - n D) \qquad \textbf{(19)}$$

Similarly, when

$$- \frac{y}{6} - \frac{y n}{3 P} = K_1$$

it follows that

$$a_n = \frac{3 C}{2} \cos(\omega t + n D) \qquad \textbf{(20)}$$

Suppose that neither of the foregoing assumptions is true. That is, $\left(- \frac{y}{6} \pm \frac{y n}{3 P}\right)$ is not equal to an integer. It has been shown that in order for the nth harmonic to exist in the m. m. f. wave of a single phase,

$$\frac{n + \frac{p}{2}}{P/y} \text{ must equal } K_1$$

That is,

$$n = \frac{K_1 P}{y} - \frac{P}{2}$$

Substituting this value of n in the expression

$$- \frac{y}{6} \pm \frac{y n}{3 P}$$

we obtain $\frac{K_1 - y}{3}$ when the plus sign is used and $\frac{K_1}{3}$ when the minus sign is used. When these values, neither of which is an integer, are substituted in Equation (**17**), both terms become zero.

The criteria for the existence of the nth harmonic in the final wave are, then,
first,

$$\left(\frac{n + \frac{P}{2}}{P/y}\right) \text{ must equal } K_1, \qquad \textbf{(21)}$$

and second,

$$\left(- \frac{y}{6} \pm \frac{y n}{3 P}\right) \text{ must equal } K_2,$$

which may be written

$$\frac{y (P \pm 2 n)}{6 P} \text{ must equal } K_2 \qquad \textbf{(22)}$$

where K_1 and K_2 are integers, including zero.

When the negative sign is used, the equation for the m. m. f. is

$$a_n = \frac{3\,C}{2} \cos(\omega\, t + n\, D) \tag{20}$$

When the positive sign is used,

$$a_n = \frac{3\,C}{2} \cos(\omega\, t - n\, D) \tag{19}$$

Substituting the values of C and D, the equations become

$$a_n = \frac{0.95\, M\, P}{n\, y} \sqrt{1 - \cos n\, \beta}\, \cos(\omega\, t + n\, \alpha - n\, \theta_n - n\, \psi) \tag{23}$$

and

$$a_n = \frac{0.95\, M\, P}{n\, y} \sqrt{1 - \cos n\, \beta}\, \cos(\omega\, t - n\, \alpha + n\, \theta_n + n\, \psi) \tag{24}$$

These expressions represent waves of constant magnitude which glide around the armature at speeds equal to one wavelength per cycle of current. Equation (**23**) is a wave traveling in one direction while (**24**) is a wave with opposite rotation. If n is made equal to $\frac{P}{2}$, the expression $\frac{y\,(P \pm 2\, n)}{6\, P}$ becomes an integer (zero in this case) when the minus sign is used. That is, Equation (**20**) represents the m. m. f. wave. Since a value of n equal to $\frac{P}{2}$ corresponds to the synchronous component of the m. m. f., it follows that for those cases in which the minus sign is used to satisfy the equation

$$\frac{y\,(P \pm 2\, n)}{6\, P} = K_2$$

the wave travels with rotation. When the positive sign is used the wave moves opposite to the direction of the rotor.

Reactive Voltages. Let a_n' equal the maximum ordinate of the nth harmonic m. m. f. wave. Then,

$$a_n' = \frac{0.95\, M\, P}{n\, y} \sqrt{1 - \cos n\, \beta}$$

The flux density at any point, assuming a constant permeance, p, is

$$B_n = a_n'\, p \cos[\omega\, t + n\, \alpha - n\, \theta_n - n\, \psi] \tag{25}$$

for a positively rotating wave.

The flux enclosed by the reference coil is

$$\Phi_n = a_n'\, p\, N \int_0^\beta \cos[\,\omega\, t + n\, \alpha - n\, \theta_n - n\, \psi]\, d\, \alpha \tag{26}$$

where N is a constant equal to the surface of the armature divided by $2\,\pi$.

Integrating,

$$\Phi_n = \frac{a_n'}{n}\, p\, N\, [\sin(\omega\, t + n\, \beta - n\, \theta_n - n\, \psi) - \sin(\omega\, t - n\, \theta_n - n\, \psi)\,] \tag{27}$$

The voltage induced in the coil is

$$e_n = -\, 10^{-8} \frac{d\, \Phi_n}{d\, t} \tag{28}$$

Differentiating,

$$e_n = \frac{a_n'\, p\, N\, \omega}{n\, 10^8} [\cos(\omega\, t + n\, \beta - n\, \theta_n - n\, \psi) - \cos(\omega\, t - n\, \theta_n - n\, \psi)\,] \tag{29}$$

which reduces to

$$e_n = \frac{2\, a_n'\, p\, N\, \omega}{n\, 10^8} \sin\frac{n\, \beta}{2} \sin\left(\frac{n\, \beta}{2} + \omega\, t - n\, \theta_n - n\, \psi - \pi\right) \tag{30}$$

The voltage per phase is found by adding vectorially the voltages of the individual coils. But the time angle between the voltages of any two coils is the same as the space angle between their m. m. f. waves. We can make use of the previous vector addition, therefore, and write the expression for the voltage per phase:

$$E_n = \frac{e_n\, M\, P}{y} \tag{31}$$

which is a voltage wave displaced from the voltage of the reference coil by the angle $n\, \psi$. Thus the complete expression for the phase voltage is

$$E_n = \frac{2\, a_n'\, p\, N\, \omega\, M\, P}{n\, y\, 10^8} \sin\frac{n\, \beta}{2} \sin\left(\frac{n\, \beta}{2} + \omega\, t - n\, \theta_n - 2\, n\, \psi - \pi\right) \tag{32}$$

Substituting the value of a_n',

$$E_n = \frac{2.68\, p\, N\, \omega\, M^2\, P^2}{n^2\, y^2\, 10^8} \sin^2\frac{n\, \beta}{2} \sin\left(\frac{n\, \beta}{2} + \omega\, t - n\, \theta_n - 2\, n\, \psi - \pi\right) \tag{33}$$

It will be convenient, usually, to express the reactance voltage in per cent of normal voltage. If $a_n'\, p$ is expressed as a percentage of normal flux we may write, from Equation (**32**), the expression for reactance voltage due to the *n*th harmonic as

$$\% E_n = \frac{(a_n'\, p)\, N_s\, M_n \sin \frac{n\,\beta}{2}}{N\, M_s \sin \frac{n_s\,\beta}{2}}$$

$$\sin\left(\frac{n\,\beta}{2} + \omega\, t - n\,\theta_n - 2\,n\,\psi - \pi\right) \qquad (\mathbf{34})$$

where N_s and M_s correspond to the synchronous harmonic.

Equation (**34**) gives the reactance voltage per phase due to the flux set up by any one harmonic in the armature m. m. f. It will be seen that the phase position of this voltage varies with the order of the harmonic. To find the total reactance voltage due to m. m. f. harmonics it is necessary to solve for each component voltage and then add all components in proper angular relation.

LIST OF SYMBOLS

α = distance along the armature in radian measure
a = ordinate of m. m. f. wave
a' = maximum ordinate of resultant m. m. f. wave
β = coil throw in radians
B = flux density
$C = \dfrac{0.636\, M\, P}{n\, y}\sqrt{1 - \cos n\,\beta}$
$D = (\alpha - \theta_n - \psi)$
e = reactance voltage of single coil
E = reactance voltage of one phase
f = frequency
K = an integer
M = ratio of resultant vector to single vector
n = order of harmonic
N = constant depending on machine dimensions
p = permeance
P = number of poles
S = slot pitch in angular measure
t = time in seconds
$\omega = 2\,\pi\, f$
y = denominator of fraction equal to slots per pole
θ = displacement of wave from the origin
ϕ = angular position of any coil
ψ = angle between resultant vector and zero vector

Discussion

P. L. Alger: This paper raises questions whose discussion might be carried to great lengths, since the ramifications of the effects of harmonics on various characteristics of the motor are very extensive. I will confine myself, however, to making two comments on this paper and then giving a few things from my own experience.

In the first place, Mr. Graham develops a formula for the magnitude of any particular harmonic, which contains the coefficient $\sqrt{1 - \cos n\,B}$. That, to my mind, is very hard to visualize, and it is much better to replace it by the exactly equivalent expression $\sqrt{2} \sin \frac{n\,B}{2}$. When this is done, the magnitude of each harmonic is seen to be proportional to the pitch factor for that particular harmonic. As the pitch factor is a very familiar thing to all designers, this expression tells the relative magnitudes of different harmonics almost by inspection.

In the second place, it appears obvious to me from physical considerations that with a perfectly balanced winding, every phase being like every other phase, the other phases can only introduce purely reactive voltages in the first phase, similar to the voltages induced by that phase in itself. That is, if you have a perfectly balanced arrangement, the currents in each phase being displaced in time by the same angle as they are in space position, there cannot be produced any two harmonic waves of the same order but opposite directions of rotation. So the *n*th pulsating harmonic made by phase 1 alone will either be converted into a revolving field of amplitude 3/2, due to the three phases, or else will be cancelled out altogether. For that reason, it is clear that Graham's equations showing a phase difference between the various harmonic voltages in a single phase must be wrong. There is no phase difference because all of them are purely reactive voltages.

Once we have admitted the existence of these harmonics and shown they are present and undesirable, the really interesting thing is how to avoid them. That Mr. Graham did not touch upon at all.

By rearranging the windings in various groupings, a great many effects can be produced. There are a number of arrangements possible for any particular winding, all of which give balanced phase voltages, but which have different characteristics. Systems of distribution can be worked out to give minimum vibration, minimum reactance or minimum load losses, or to meet other conditions, and some of those other arrangements have distribution factors so little different from the one that we ordinarily use, that they are more desirable than the ordinary arrangement.

An interesting case illustrating these possibilities is that of a 1000-h. p. induction motor, with 14 poles and 4 2/7 slots per pole per phase, which I had the opportunity of observing in 1923. When the motor was tested the first time, it developed a very severe load vibration, similar to that Mr. Graham observed on his motor. We determined the vibration to be of twice line frequency, and due to the 10-pole field produced by the irregular winding arrangement. By reconnecting the winding to the arrangement giving a minimum 10-pole field, the trouble was remedied without any mechanical change. In general, unbalanced magnetic pull occurs whenever two fields of nearly the same numbers of poles exist together, and so the way to avoid vibration is to arrange the winding to minimize that particular harmonic whose number of poles is nearest the fundamental.

The study of the best winding arrangement, taking into account all the factors, is very fascinating, but it is quite difficult to make a complete analysis of the problem. I have been working on this matter for some time, and I hope ultimately to be able to say for any given number of fractional slots per pole per phase what arrangement gives the best combined characteristics, taking into account noise, fundamental distribution factor, reactance and losses.

C. A. Nickle: I agree with Mr. Alger that in a rotor with uniform permeance, reactive voltages other than 90 deg. out of phase with the currents cannot be obtained. When such voltages exist, we must have reluctance torques in the machine, we must have consumption of power, and we cannot have reluctance torques in a uniform-permeance machine.

The author has made calculations of the leakage reactance introduced by sub-harmonics and he gets a value of approximately 1 per cent. Of course, if we add reactance voltages vectorially, we shall get a much smaller voltage than if we add them arithmetically. If all these reactance voltages were added in time phase, the total reactance voltage might be considerably greater, and I think might be worth investigating.

W. V. Lyon: Mr. Graham has applied the principles of harmonic analysis to an interesting and not uncommon problem, the most important aspects of which I believe are the extra losses and vibration which may occur when these irregular windings are employed. He has clearly shown how the problem may be attacked.

In the spring of 1926, apparently at about the time when Mr. Graham was preparing this paper, I analyzed the same problem with a class of graduate engineers at the General Electric Works at Lynn. Curiously enough, the illustrative problem that we worked through was the same that Mr. Graham has chosen, *viz.*, one in which there are 6¾ slots per pole. Also the winding arrangement was the same in both cases. The only difference was that ours was a stator having 54 slots and wound for 8 poles. We found no particular difficulty in obtaining a resultant *n*th harmonic reduction factor for this winding which was a combination of the ordinary pitch and breadth factors. We did not, however, determine the criteria for the existance of the *n*th harmonic nor for its direction of rotation.

One point that Mr. Graham has made is well worth emphasizing. When a uniform air-gap is acted upon by a magnetizing winding there will in general be produced component sinusoidal flux distributions which have two poles, four poles, six poles, eight poles, etc. By properly arranging the winding, certain of these distributions can be completely or partially eliminated. If, for example, a symmetrical three-phase winding having an integral number of slots per pole per phase and wound for 4 poles is used, there will be no components except the 4-polar, 20-polar, 28-polar, 44-polar, 52-polar, etc. Again in the 20-pole winding that Mr. Graham cites there will be 10-polar, 20-polar, 30-polar, 40-polar, 50-polar, 70-polar, etc., distributions.

I should like to add a little to what Mr. Graham has said about vibration. If the stator and rotor were perfectly rigid, the internal stresses between them due to the magnetic field could cause no vibration. Actually, however, both the stator and rotor have a certain amount of flexibility and thus these internal stresses may produce serious vibration. There are two distinct types of internal stress. One is due to the action of a single sinusoidal flux distribution and the other is due to the joint of two sinusoidal distributions of different wavelengths. If there is a single sinusoidal component of flux in the gap, the rotor will be acted upon by balanced radial forces which can cause no vibration in the rotor if its laminations are rigidly connected to the shaft. On the other hand the rotation of the flux distribution will produce a harmonic variation in the magnetic force between the stator and rotor in any fixed radial direction of such a frequency that it will go through one cycle while the magnetic field is moving one pole pitch. If the stator frame is flexible in a radial direction, this harmonically varying force may set up serious vibration. This is apparently the case that Mr. Graham cites.

The other cause of vibration may, I believe, be equally important. If there are in the gap two component distributions of different wavelengths the radial forces acting on the rotor may be unbalanced. Two polar distributions that differ by a single pair of poles will produce this effect. This unbalanced force on the rotor may be either constant or variable. If the distributions are moving properly with respect to each other the unbalanced force acting on the rotor will vary harmonically, and serious vibrations may result.

Quentin Graham: Mr. Alger has suggested the substitution of $\sqrt{2}\,\sin\frac{n\beta}{2}$ for the expression $\sqrt{1} - \cos n\beta$ which I have used. I agree that this change would aid in visualizing the magnitude of the harmonics.

Mr. Alger and Mr. Nickle have both pointed out the error contained in the last paragraph of Appendix B and also in the body of the paper where it is stated that the reactance voltages of the various harmonics are not in phase. I am giving below a number of corrections in the mathematics, changing the final equation on which this false conclusion was based. I am indebted to Mr. Nickle for having pointed out the particular points which were in error. The corrections are as follows:

Equation (**6**) should read

$$n\,\theta_n = -\tan^{-1}\frac{\sin n\beta}{1 - \cos n\beta}$$

which can be simplified to

$$n\,\theta_n - \frac{n\beta}{2} - \frac{\pi}{2}$$

Using this value for $n\,\theta_n$ Equation (**30**) may be simplified so that the quantity in the parenthesis becomes $\left(\omega t - n\psi - \frac{\pi}{2}\right)$

The statement following Equation (**31**) should give the displacement angle as $-n\psi$ instead of $n\psi$.

With this correction the quantity in the parenthesis in equations (**32**), (**33**) and (**34**) will be $\left(\omega t - \frac{\pi}{2}\right)$. It is clear then that the final paragraph, following Equation (**34**), is in error and that the reactance voltage components of the various harmonics must be in phase.

I have experimented to some extent with the rearrangement of windings, as Mr. Alger suggests, for the purpose of reducing some particular harmonic. This is usually accomplished at the expense of increasing some other harmonic but, as he says, the total result may be beneficial.

Mr. Lyon's discussion of the vibration problem is very interesting and his conclusions appear to agree with Mr. Alger's experience.

Transverse Reaction in Synchronous Machines

BY J. F. H. DOUGLAS[1]

Associate, A. I. E. E.

Synopsis*—The confusion existing at present in the theory of synchronous machines is shown to be due to insufficient experimental evidence of the behavior of this type of machine under transverse magnetizing (cross magnetizing) conditions. A method of testing which yields the needed data is described. The results of these tests on a particular machine are given and analyzed. It is proved, for the machine tested, that the effect of transverse reaction can be most accurately estimated by the use of a magnetomotive force diagram. Experimental constants useful in design are found. A theory of the operation of synchronous machines is proposed, and a diagram for finding the performance of a synchronous machine from experimental tests is given; which, it is hoped, will be more accurate than present methods.*

* * * * *

INTRODUCTION

THE subject of the performance of synchronous machines seems to be of perennial interest. Many theories have been proposed with their appropriate methods of computation. Nearly every text-book is a monument to the confusion of this subject, a confusion resulting from an insufficiency of facts. It is interesting to note how theory has become more sound as experimental facts have become more extensive.

When the open and the short-circuit tests were the usual compromise tests, we had the e. m. f. and the m. m. f. methods which were based upon them, with the Torda-Heyman method as a compromise. When the shape of the full-load zero power-factor saturation curve became known, the improved Kapp diagram and the Potier diagram were developed. A number of discrepancies between observed and computed results, together with theoretical studies of the variation of air-gap permeance, have led to various proposals for the perfection of the theory of this machine. In the judgement of the writer, the best method derived to date is that of Andre Blondel[2], and the best exposition of this method that of Professor V. Karapetoff[3]. As expounded, it is chiefly a design method. Although it is possible to apply it to experimental data, the directions are not clear enough to enable it to be easily applied for this purpose. Some recent refinements in this method have been made by Karapetoff, and Doherty and Nickle[4].

The elimination of guarantees of regulation at other than zero power factor, has led to the attitude that present methods are good enough for practical purposes. There has been a gradually increasing number of facts, however, that indicates that further improvements are necessary. Hunting frequencies and pull-out torques some thirty per cent in excess of predicted values, for example, indicate considerable error in customary theory. The Blondel theory does give better results in these cases. The phenomena of pull-in torque in motors and instability of generators (pole slipping) are entirely unpredictable except with some form of two-reaction theory such as that proposed by Blondel.

Experimental data for machines operating under demagnetizing conditions are easily obtainable; the methods for securing them and the general character of the results secured are well known, and a theory able to account for the results has been developed which is adequate also as a basis of computation. A very different situation holds for the effects in a synchronous machine under transverse (cross magnetizing) conditions. Enough facts are available, indeed, to show that magnetic conditions are then radically different. A plausible theory has been developed which is applicable to design with some measure of success. However, the writer is unaware of any systematic experimental data which will show what the effect of transverse armature reaction in a synchronous machine is in reality.

The conditions for such an experimental study may be stated very simply. In the first place, the load placed upon the machine should be wholly transverse in effect; that is, the armature currents must attain their maximum value when opposite a pole center. This condition must exist in order that the effects observed shall be due to one cause only, and not complicated by direct magnetization. This carries as a corollary, that the experimental method must have a flexible power factor control; that is, a quick and accurate adjustment of the phase angle of current is needed. In the second place, for an experimental study of transverse reaction, we must be able to measure the terminal voltage of the machine not only in magnitude, but in phase position with reference to the pole axis. In other words, the components of the terminal voltage with reference to the pole axis must be known. Lacking this information, unwarranted assumptions as to the phase angle of the reactance drop would have to be made.

AN EXPERIMENTAL METHOD FOR STUDYING TRANSVERSE REACTION

The basic idea on which the following experimental method rests is to measure not the voltages and currents of the machine tested but their components with

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[1] Asst. Prof. of Electrical Engineering, Marquette University, Milwaukee, Wis.

[2] *Trans.* Int. Elect. Cong., St. Louis, 1904, Vol. 1, pp. 620-635.

[3] Magnetic Circuit Articles 47-48.

[4] A. I. E. E. JOURNAL, July 1926 and Oct. 1926.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 7-11, 1927.

reference to the pole axis—that is, with reference to the no-load voltage of the machine. The second important idea in these experiments is the use of a wattmeter as a device for the measurement of components of voltage and of current.

The machine tested was a 15-kv-a., 6-pole, 60-cycle, synchronous machine rated at 190 volts with its two-phase connection, one of a set of two machines made by the Westinghouse company, for educational institutions. A description of the design details of this machine is in the A. I. E. E. JOURNAL[5]. The diagram of connections used is shown in Fig. 1. Machine A was the one tested for its performance, and it was loaded as a motor, by the d-c. generator belted to it. The load on the motor was regulated by the generator field rheostat. The d-c. generator returned its surplus power to the laboratory d-c. line, so that on occasion the set could be run from the d-c. machine as a motor. The current for the synchronous motor A was supplied from the mains of the Milwaukee Electric Co. with 60-cycle current. This current was supplied through a Scott connection and an induction regulator. The voltage supplied could be regulated for any value from 100 to 300 volts. By varying the impressed voltage, the motor could be made to take current at any phase angle desired.

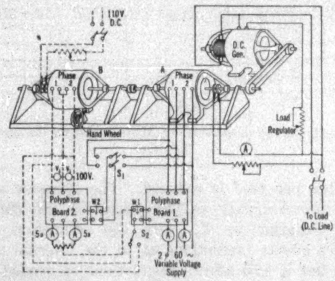

FIG. 1

The voltage of machine A was measured by wattmeter (2) excited with five amperes from machine B. Thus the reading of the wattmeter divided by five gave the component of voltage in phase with the excitation. The current in machine A was measured by wattmeter (1) excited with 100 volts from machine B. Thus the reading of this wattmeter divided by 100 gave the component of current in phase with the excitation. Since machine B was loaded on a Cutler-Hammer load rheostat, made of Advance wire wound on flat spools, it was assumed that the excitation of both wattmeters were in the same phase.

Machine B was connected as a two-phase machine, and a double throw switch $S2$ and a polyphase board $P2$ were provided. In this way the excitation of the wattmeters could be transferred to either phase of machine B. The components of current and voltage in machine A were measured, therefore, in two perpendicular phase positions. The double throw switch $S1$ and the polyphase board $P1$ enabled measurements to be taken on both phases of machine A. By adjusting the hand-wheel of machine B, the axes on which the components of current and voltage were read could be brought into any position. The hand-wheel was actually adjusted so that these axes were the same as the voltages of machine A at no load. Thus the wattmeters gave directly the direct and the transverse component of both current and voltage supplied to machine A.

A number of runs was made, each for a definite field current in machine A. In all the runs the induction regulator was adjusted so that the current supplied machine A was wholly transverse in character; that is, wattmeter (1) excited with phase (1) of A and phase (2) of B or *vice versa* read zero. In each run the load was varied from zero to full load. At each load the excitations were checked and all the wattmeter readings taken. The range of field currents used was from 3.25 to 23 amperes, giving saturations of from 100-300 no-load volts.

The data for the two phases were averaged to eliminate small unbalances. The polarities of all the wattmeter combinations were established definitely. The data were corrected for systematic error, including $I R$ drop. The data were corrected for other constant errors such as residual demagnetizing currents. The readings of transverse and direct voltage were plotted against transverse current and smooth curves drawn, eliminating small observational errors. The data are recorded in graphical form in Fig. 2, and represent the results after correction for the above mentioned errors. The errors were small, and the uncorrected data gave approximately the same set of curves.

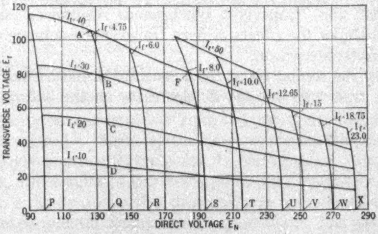

FIG. 2—TRANSVERSE AND DIRECT VOLTAGE WITH TRANSVERSE ARMATURE REACTION

DISCUSSION OF THE EXPERIMENTAL DATA

Fig. 2 shows the transverse voltage induced in the machine, under various amounts of transverse mganetizing current, plotted against the direct voltage induced. The lines sloping downward and to the right are loci for constant values of the transverse current (I_t) of 50, 40, 30, 20 and 10 amperes. The nearly vertical lines are for constant field currents. The direct voltage is designated by the symbol E_n meaning the voltage induced by the net magnetomotive force along the polar axis.

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[5] A. I. E. E. JOURNAL, May 1925, page 543.

The corresponding loci were calculated from the Blondel theory, using the following data. A demagnetizing reaction equivalent to 7.0 amperes of field current for 40 amperes of demagnetizing armature current, an armature reactance of 0.5 ohm, and a slope of the saturation curve of 31 volts per ampere of field current agree with experimental data in Fig. 5. It was assumed that the transverse reaction was 40 per cent of the direct, and that the armature reactance was a constant. The loci of E_t, for constant transverse magnetizing armature current of 40, 30, 20 and 10 amperes, were lines parallel to the E_n axis and passing through the points *A*, *B*, *C*, and *D* in Fig. 2. The loci of E_t for constant field current were straight lines perpendicular to the axis E_n and passing through the points *P*, *Q*, *R*, *S*, *T*, *U*, *V*, *W*, and *X*. Even with other assumptions as to the constants, the loci would be a system of lines parallel and perpendicular to the axis E_n. The discrepancy in Fig. 2 between the observed and computed loci is disappointing and does not give promise of any practical method of predicting the effects of transverse reaction from an e. m. f. diagram.

If we analyze Fig. 2 we see several important results. In the first place the transverse voltage for a constant field current is proportional to the transverse armature current. Their constant ratio may be denoted by the symbol X_{st} and may be appropriately called the "transverse synchronous reactance," since it includes the effects of both transverse reactance and transverse reaction.

In the second place, for a constant armature current the tranverse voltage E_t decreases as the saturation increases. The transverse synchronous reactance is not a constant for different saturations, therefore, but decreases from a value of 3.0 ohms with 3.25 amperes of field current to a value of 1.2 ohms with 23 amperes of field current. The explanation of this phenomenon is simple. The flux set up by transverse reaction is chiefly under the pole tips where the tooth and pole tip iron is affected by the amount of main flux as to saturation.

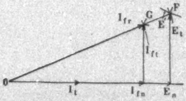

FIG. 3—DERIVATION OF M. M. F. FROM E. M. F. DIAGRAM

In the third place, the direct voltage E_n is decreased as the amount of transverse current is increased, even though the field current is constant. This points definitely to a demagnetizing action of the transverse reaction. The voltage E_n, therefore, is not a function of the field current alone. This is shown clearly at all loads and at all field currents, in spite of the fact that there was no demagnetizing current present in the armature. The explanation of this effect is also simple. The cross magnetizing or transverse effect operates to strengthen one pole tip of a pole and to weaken the other; but, owing to saturation, the weakening effect is larger than the strengthening effect. Consequently the total flux per pole is reduced. The decrease in E_n varies approximately as the square of the current.

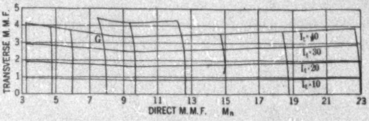

FIG. 4—M. M. F. DIAGRAM OF MOTOR UNDER TRANSVERSE LOAD

The variation in X_{st} with the saturation most seriously affects predictions as to the internal phase angles, but it also affects calculations of needed field current. The variation in E_n affects regulation and field current calculations alone.

It is a mistake to build up a theory of transverse reaction on the basis of an unsaturated magnetic circuit for the transverse flux, since such a theory leads to a constant value of the transverse synchronous reactance. The predicted values of transverse reactance may be correct for some one particular voltage, but at other voltages the performance will be off. For example, in Fig. 2, pull-out torques will be actually larger than computed values for voltages in excess of 130 volts.

The Blondel theory of transverse reaction is in reality a reactance method, since the vectors appearing in the Blondel diagram are *I X* drops. Since the reactances used replace the effects of reaction, the *I X* drops appearing in the diagram are properly termed synchronous reactance drops. It is but natural to turn to an m. m. f. diagram, to see if it gives superior, that is, more constant, results. To convert any point such as *F* in Fig. 2 into a corresponding point *G* in a m. m. f. diagram, we use the construction in Fig. 3 and the saturation curve in Fig. 5. We take the voltage E', the resultant of E_n and E_t, to the saturation curve and read there the corresponding resultant magnetomotive force expressed in equivalent field amperes, which we denote by I_{fr}. Since the flux inducing E' is set up by the resultant magnetomotive force, we draw I_{fr} in phase with E'. This magnetomotive force is made up of two components. One produced along the pole axis by the net action of field and direct reaction, we denote by I_{fn}, since it is expressed in equivalent field amperes. The other component of I_{fr} is the transverse m. m. f., which we denote by I_{ft} since it is expressed in equivalent field amperes. Each point in Fig. 2 was converted by this means into a point in the m. m. f. diagram shown in Fig. 4.

Fig. 4 consists of a system of approximately horizontal lines for constant armature current, intersected by a system of approximately vertical lines for constant field current. We may say, therefore, with substantial accuracy, that transverse reaction may be represented on a magnetomotive force diagram, by a vector proportional to the armature current, independent of the field current, and perpendicular to the armature current. Secondary demagnetizing effects do not appear on the m. m. f. diagram to any appreciable extent. This diagram proves that we may regard the magnetic flux set up in a machine as produced by a resulting magnetomotive force acting on a single magnetic circuit of substantially uniform saturation. There is no evidence in either Figs. 2 or 3 of any transverse flux component directly proportional to the armature current and independent of saturation. Stated in other words, the transverse leakage reactance X_t appears to be zero; that is, the leakage reactance appears to be less for conductors under the poles than for conductors between the poles. If a transverse leakage reactance existed, the lines in Fig. 4 would all slope upward and to the right. In any machine showing such characteristics, the two effects reactance and reaction could be separated, by trial.

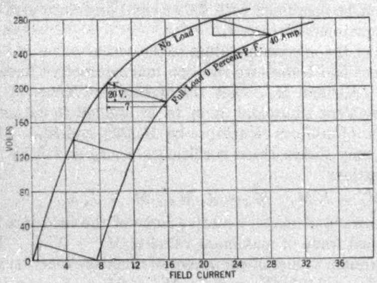

Fig. 5—Saturation Curves

Fig. 5 shows the saturation curves of the machine tested. From it the constants previously used were obtained. We define the experimental coefficients of armature reaction as the ratios of equivalent magnetomotive force expressed in field amperes to armature current. The direct coefficient $C_d = 0.175$, the transverse $C_t = 0.09$, for average conditions in the machines tested. It is worthy of note that the transverse coefficient is approximately one-half the direct coefficient. In solving for the performance of a synchronous machine, for which only the data like that in Fig. 5 were available, the writer would be inclined to use a value of $X_t = 0$ and a value of $C_t = (C_d/2)$. However, it is not permissible to generalize too much from the data on one machine.

Taking the dimensions of the machine and the winding data given by Mr. Quentin Graham[6], the writer has checked the various constants found. The resistance when increased by the A. I. E. E. allowance agrees with the measured value 0.25 ohm per phase. The armature reactance at zero power factor, of 0.5 ohm, checks 25 per cent higher than that computed using Hobart's values of slot and end-connection permeance (10 perms per in. and 2 perms per in.) and somewhat higher than the value computed from data on page 230 of the "Magnetic Circuit."[7] In the equations for direct and transverse reaction given in Karapetoff's "Magnetic Circuit," Art. 49 and 50, we shall for the present take M_d as $N_f I_{fd}$ and M_t as $N_f I_{ft}$, since the constants there derived are for equivalent field ampere-turns. Thus

$$N_f I_{fd} = K_d K_b K_w m n I_d; \quad N_f I_{ft} = K_t K_b K_w m n I_t \qquad \textbf{(1)}$$

We will take $N_f = 185$ turns per pole, $I_{fd} = 7.0$ from Fig. 5, $K_b = 0.907$ for two phases and six slots per pole per phase, $K_w = 0.925$ for 75 per cent winding pitch, $m = 2$ phases, $n = 22$ equivalent turns per pole per phase for winding undivided in group, $I_d = 40$. The equivalent transverse magnetomotive force corresponding to $I_t = 40$ was taken as 3.6 amperes of field current. Using the above data, the constants are

$$K_d = 0.875; \quad K_t = 0.451 \qquad \textbf{(2)}$$

The writer made a careful estimate of the wave of air-gap permeance, the waves of direct, transverse and field flux, analyzing them for their sine wave components, and arrived theoretically at the values

$$K_d = 0.755; \quad K_t = 0.405 \qquad \textbf{(3)}$$

It is plain, therefore, that the design constants need experimental correction.

A Theory of Synchronous Machines

It may be presumptuous from the test of one machine to announce a theory of the synchronous machine and give yet another diagram for its performance. However, the results of our tests point very definitely to such a theory and such a diagram, and they are offered for experimental confirmation or rejection. The writer does not have ready access to recent literature, and therefore does not know whether the theory proposed is new or not.

In Fig. 6 the magnetic effect of armature currents in position A is entirely different from that produced by currents in position B. With reference to the pole, the first is transverse; the second is direct. From considerations of symmetry, the armature currents should be resolved into components in these two positions. The magnetic effect of the transverse current in position *A* is obviously much smaller than that of an equal current acting in position *B* because it acts on a magnetic path of much higher reluctance.

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[6] Loc. Cit.

[7] Doherty and Nickle's work accounts for some of the discrepancy observed. See paper cited.

The magnetic effect of the two components of the armature current may be considered as two-fold; namely, that effect produced individually by each, —that is, as if the other magnetomotive forces were not there,—and that effect produced jointly through the cooperation of the field,—the direct and the transverse currents. Each of the components of armature current may be considered as producing a component of flux, mainly local in character, proportional to the current and independent of saturation, which we may call armature leakage fluxes. They cause a drop in voltage which we may call the leakage reactance drops, each leading its component of current 90 electrical degrees. The direct reactance drop caused by the direct component of current in position *B* is appreciable, because this drop is due to flux set up in the interpolar regions where there is little interference with the main flux. The transverse reactance drop is small and may be sometimes neglected for the reason that this flux is set up when the conductors are under the poles and therefore is merged with the main flux[8].

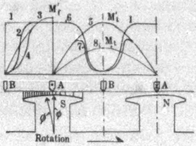

FIG. 6—THEORY OF ARMATURE REACTION

The effect of transverse reaction may also be stated as a distortion of flux. Thus, in Fig. 6 the flux is crowded from the center toward the left-hand pole tips. Although thus displaced in the teeth, air-gap, and pole shoe, the path of the main flux is generally the same under load as under no-load. It is the resultant flux ϕ', for example, in Fig. 6, which induces the voltage at the terminals plus the $I R$ and the two $I X$ drops. It is the resultant m. m. f. of the machine which produces this resultant flux, and consequently the no-load saturation curve should serve to correlate the induced voltage and the resultant magnetomotive force.

The resultant magnetomotive force may be taken as the vector sum of the three component magnetomotive forces, namely the field, the direct and the transverse magnetomotive forces. Since only sine waves combine to give a sine wave and only sine waves can be represented by vectors, it is obvious that the actual m. m. fs. must be reduced to equivalent sine waves acting upon a uniform active layer. The maximum value of the field, the direct and the transverse magnetomotive forces must therefore each be multiplied by some coefficient to reduce it to the equivalent sine wave m. m. f. applied to a uniform active layer. For the present it is sufficient to notice that the coefficient for the transversely reacting armature current is about one-half of that for the directly reacting armature current. Thus the armature reaction is not 90 degrees from the edge of the coil carrying maximum current, but each component of current must be computed separately. The direct reaction ($I_{fd} = C_d I_d$) can be combined algebraically with the field current I_f, to get the net m. m. f. along the polar axis I_{fn} in equivalent field amperes; the transverse reaction ($I_{ft} = C_t I_t$) must be combined with I_{fn} at right angles to give the magnetomotive force I_{fr}.

Let the maximum values of the actual waves of field, direct and transverse reaction magnetomotive force be M_f', M_d' and M_t'. Let the maximum values of the equivalent sinusoidal m. m. fs. as applied to an active layer of uniform reluctance be M_f, M_d, and M_t. Then we may define three coefficients J_f, J_d, and J_t by the equations

$$M_f = J_f M_f'; \; M_d = J_d M_d'; \; M_t = J_t M_t' \qquad (4)$$

The wave of magnetomotive force of the field is a flat topped wave of maximum value of $M_f' = N_f I_f$. The maximum values of the waves of armature reaction are[9]

$$M_d' = 0.9 K_b K_w m n I_d; \; M_t' = 0.9 K_b K_w m n I_t \qquad (5)$$

In Fig. 6, curve 1 is the rectangular wave of field magnetomotive force. Curve 2 is the wave of air-gap permeance for direct flux denoted by $P_d(X)$. Curve 2 is also the wave of equivalent field m. m. f. applied to a uniform active layer. Curve 3 is the fundamental sine wave of curve 2, and is the equivalent sinusoidal wave of field m. m. f. as applied to a uniform gap. It is higher than curve 2 because of a prominent fifth harmonic. The constant J_f is the ratio of the maximum value of curve 3 to that of curve 2 and is 1.13 for the machine tested.

If we assume the sine wave curve 3 to be the wave of direct m. m. f. M_f' applied to the machine with curve 2 of direct permeance, the equivalent wave of m. m. f. as applied to a uniform active layer will be curve 4 obtained as a product of curves 2 and 3. If curve 4 be analyzed for its fundamental sine wave, that would be the equivalent sine wave of direct reaction m. m. f. as applied to a uniform gap. This wave is not shown but is some 5 per cent lower than curve 3; that is, $J_d = 0.95$.

Let curve 5 be the wave of actual transverse m. m. f., and curve 6 be the wave of transverse permeance $P_t(X)$. The product of curves 5 and 6 will yield curve 7, the wave of equivalent m. m. f. applied to a uniform active layer. Curve 8 is the equivalent sine wave for curve 7 and is much lower than curve 5 because of a large third harmonic. The ratio of curve 8 to curve 5 is the coefficient $J_t = 0.51$.

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[8] Apparently all the fluxes except that due to (X_{ld}) are affected by saturation. This appears to be in contradiction to the assumptions of Doherty and Nickle, in Fig. 27 of paper cited.

[9] See Karapetoff's "Magnetic Circuit" Arts.

The coefficients J_f, J_d, and J_t are defined by the equations for a Fourier analysis, namely

$$J_f = (2/\pi) \int_0^\pi P_d(x) \sin\,(x)\, d\,x \qquad (6)$$

$$J_d = (2/\pi) \int_0^\pi P_d(x) \sin^2\,(x)\, d\,x \qquad (7)$$

$$J_t = (2/\pi) \int_0^\pi P_t(x) \cos^2\,(x)\, d\,x \qquad (8)$$

$P_t(x)$ is a trifle higher than $P_d(x)$ in the interpolar regions.

The design coefficients used by Professor Karapetoff are defined in terms of the coefficients above as

	Theory	Experiment[10]	
$K_d = 0.9\,J_d/J_f$			
$= 0.9 \times 0.95/1.13 =$	0.755	0.875	(9)
$K_t = 0.9\,J_t/J_f$			
$= 0.9 \times 0.51/1.13 =$	0.405	0.46	(10)

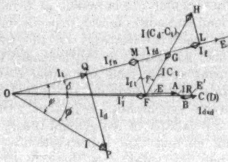

FIG. 7—PROPOSED VECTOR DIAGRAM OF A SYNCHRONOUS MACHINE

A better correspondence could be obtained with experiment if an empirical coefficient were substituted for the rational coefficient 0.9. For our test the value of the coefficient is indicated as approximately unity.

The experimental constants C_d and C_t are most useful even in design calculations; the formulas for these coefficients are

$$C_d = K\,J_b\,K_b\,K_w\,m\,n/J_f\,N_f;$$
$$C_t = K\,J_t\,K_b\,K_w\,m\,n/J_f\,N_f \qquad (11)$$

Here K is an empirical factor approximately unity, K_b and K_w are the breadth and winding pitch factors of the winding, m is the number of phases, n is the number of turns per pole per phase, N_f is the number of field turns per pole, and J_f, J_d and J_t are defined by the equations above. If it is not practical to estimate the waves of air-gap permeance in a particular case the values of the J coefficients found here might be used.

PERFORMANCE DIAGRAM

The problem of finding the performance of a synchronous machine may be formulated thus: Given the terminal voltage, output (input) current and power factor, the no-load saturation curve of the machine, and the constants of the machine, namely R, X_d, X_t, C_d and C_t; it is required to predict the location of the pole axis and find the needed excitation. For definiteness let us find the performance of the machine tested as a generator with normal voltage, normal current, and 86.6 per cent lagging power factor. Let us assume the constants $R = 0.25$, $X_d = 0.50$, $X_t = 0$, $C_d = 0.175$, $C_t = 0.090$.

In Fig. 7 we draw the terminal voltage E from the origin O to the point A, and the current $I = 40$ from O to P lagging E by 30 deg. We draw $A\,B$ equal to the $I\,R$ drop of 10 volts parallel to I. We assume the polar axis $O\,X$ and project P on $O\,X$ giving the direct reacting current $P\,Q$ of 28 amperes. We draw $B\,C$, the direct $I\,X$ drop of $28 \times 0.5 = 14$ volts parallel to $O\,X$. If there were a transverse $I\,X$ drop we would draw it perpendicular to the pole axis arriving at point D. In this case D and C are one point. The line $O\,D$ is the induced voltage E' scaling as 212 volts. We take E' to the saturation curve obtaining the resultant magnetomotive force $I_{fr} = 10.5$ equivalent field amperes, which we lay off along E' from the origin to point F.

We draw $F\,G$ perpendicular to I and equal to $C_t\,I = 0.09 \times 40 = 3.6$ equivalent field amperes, and $F\,H$ in the same direction equal to $C_d\,I = 0.175 \times 40 = 7.0$ equivalent field amperes. We draw the line $O\,G$, which is the pole axis. If serious error was made in assuming $O\,X$, we repeat the work to this point. We project the point H on $O\,G$ giving the point L. The line $O\,L$ is the field current $I_f = 14.2$ amperes. The angle between pole center and the voltage is 14 deg. and the pole center and current is 44 deg.

If we want the regulation we find from I_f that $E_o = 245$ volts and the regulation is 29 per cent. If we want the direct, transverse and net magnetomotive forces, we project F on the pole axis giving point M. The transverse reaction is $F\,M$, the direct reaction is $M\,L$, and the net magnetomotive force is $M\,O$. The formal proof of the above construction is implied in the theory given above; we need only note the following relations:

$$F\,M = F\,G \cos\psi = C_t\,I \cos\psi = C_t\,I_t = I_{ft} \qquad (12)$$

$$M\,L = F\,H \sin\psi = C_d\,I \sin\psi = C_d\,I_d = I_{fd} \qquad (13)$$

CONCLUSIONS

In addition to the conclusions enumerated above, it is hoped that it is now clear that the way is open for a systematic experimental study of the constants of synchronous machines. Studies are needed to determine:

1. Whether the performance of a synchronous machine can be predicted with a saturation curve and the five constants R, X_d, X_t, C_d, and C_t.

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[10] The theoretical coefficient may have to be modified in view of Doherty and Nickle's paper. Priority must be accorded them for certain features of our Fig. 7.

2. Whether there is any definite ratio between X_t and X_d, and if there is any definite ratio between C_t and C_d.
3. What the correlation is between the coefficients C_d and C_t and the design constants.
4. What values of slot and end connection permeance will best check the values of X_d and X_t.
5. What improvements can be made in the experimental method here proposed.

Thanks are due the following Marquette engineering students for assistance in carrying out the tests, making valuable suggestions as to procedure, and pointing out some of the conclusions: Messrs. R. M. Smith, C. McClurg, L. V. Sparks, M. Kempf, F. Stodola.

Discussion

H. V. Putman: I have been intensely interested in Professor Douglas's paper because in my own work I use Blondel's two-reaction theory exclusively for calculating the performance of salient-pole machines.

His method of measuring the phase angles between the terminal voltage, current and the pole axis by the use of two wattmeters is extremely interesting, but I do not see clearly how this same method could be applied to 3-phase machines (the machines he used were 2-phase machines), because in the 3-phase machine there would be no way to check the current by getting the zero reading on the wattmeter, to get it in phase with the pole axis. Maybe there is some way, and I should like to ask Professor Douglas if he has some similar way worked out for the 3-phase machine.

Professor Douglas has called attention to the fact that the transverse synchronous reactance decreases with increasing saturation. He also states that it is a mistake to build a two-reaction theory on the basis of an unsaturated magnetic circuit. We must not forget in this connection that not only the transverse synchronous reactance but also the direct synchronous reactance decreases with the saturation. Actually, I believe the direct synchronous reactance decreases more rapidly with increased saturation than does the transverse. Dr. Berg defines the direct synchronous reactance as the sum of the real armature reactance plus the reactance equivalent of the armature reaction, and he expresses it in this form:

$X + \frac{m}{c}$ where X is the reactance, m is the coefficient of armature reaction and c is the factor that depends on the saturation.

If one expresses m in percentage based on the no-load excitation of the machine, as is usually done, then c is unity for a condition of saturation that corresponds to no-load excitation. Under short circuit, where there is no saturation in the machine, c is less than unity, and at higher saturations it is much greater than unity, so that the synchronous reactance for high saturation is considerably less than one would measure from the short-circuit test. One should certainly not use the same value of synchronous reactance measured at full load under a condition of normal saturation. Nor would one use the same value for a condition of load at a leading power factor where there is a high degree of saturation present. Consequently, Professor Douglas should not calculate orthogonal lines for those shown in his Fig. 2.

It is always necessary to determine first, the point on the saturation curve corresponding to the condition under investigation. c is then known and consequently one can calculate the correct values of synchronous reactance to be used in Blondel's theory.

It is all right to have a theoretical structure based on the assumption of no saturation, provided it can be made to give the right answer, and I think this is true of Blondel's theory. It may not give the pull-out torque absolutely correct, but I feel sure that it will give it closer than 30 per cent when handled correctly. In his discussion he referred to the work of Mr. Doherty and now I wonder if he meant the sudden pull-out, which of course would be higher than calculated for the steady-state condition.

Professor Douglas says that in order to make an experimental study of the transverse reaction it is necessary that the load be placed so that the effect shall be wholly transverse; that is, the armature currents shall attain their maximum values when opposite a pole axis. I think this is at least interesting, but Blondel pointed out that this is not necessary. In his book on "Synchronous Motors and Converters" he gives the equation for the angular displacement of a synchronous motor as follows:

$$\text{Tan}\,\delta = \frac{I\,X s' - E_0 \sin\theta}{E_0 \cos\theta - I\,r}$$

δ is the angle between the current and the pole axis. The angle of displacement is this same angle plus the power-factor angle of the machine. I is the current and θ is the power-factor angle of the machine between the terminal voltage and the current. Xs' is the transverse synchronous reactance.

Notice that nowhere in this equation is the direct synchronous reactance involved at all, and Blondel called particular attention to this point: The angular displacement of a machine, under any condition of load and power factor, depends not at all on the direct synchronous reactance but only on the transverse synchronous reactance, so that it is only necessary to measure the angular displacement at any load and power factor whatever, and then substitute in this formula and calculate backwards to get the transverse synchronous reactance. Of course, this does not assume the validity of the general theory that Blondel bases his diagram on and that superposition is possible. If, however, one denies the validity of the Blondel diagram, he would say that this is also invalid.

However, the common use of Blondel's theory is in the calculation of characteristics which depend on the displacement. If, by intelligent comparison of calculations with test results, it is possible to determine the constants in such manner that they give the correct displacement characteristics, the theory fulfills its purpose.

There is one point in connection with Professor Douglas's experimental set-up which is not clear to me. I understood from his description that the terminal voltage of machine B is supposed to correspond in phase position to the pole axis of machine A. This would be true if the rotors and stators of the two machines were lined up; that is, if the rotors were in line on the shaft, and the stators were in line on the floor, and if there were no load on machine B. But machine B was loaded with 5 amperes. If B is also rated 15 kv-a. (which, incidentally, he did not state), this 5 amperes would produce a displacement of three or four electrical degrees. Was this corrected for by adjusting the hand-wheel, and if so, how could it be done without an oscillograph to show when the terminal voltage of machine B, when loaded on the rheostat, was in line with the no-load voltage of machine A?

Professor Douglas refers to the great confusion of statements on the subject of transverse reaction.

In 1918 we were discussing the subject of armature reactance and reaction at zero power factor. The problem then was simply to divide the reactance flux from the armature reaction flux. The total flux or interlinkages were known directly from the short-circuit test.

Now we are discussing the armature reactance and reaction at unity power factor referred to the pole axis. The problem is twofold. It is first necessary to find out the total interlinkages corresponding to the transverse synchronous reactance and then to divide this total into reactance and armature reaction. The first part of the problem can be handled experimentally by measuring the angular displacement. As I pointed out, that will give the value of the transverse synchronous reactance. It will give the total flux in the transverse field. So the accuracy with which the transverse synchronous reactance can be known will depend on how accurately the angular displacement can be measured. The second part of the problem, that is, the division of this flux, I think is not of great importance. It is usually only necessary to know the total transverse reaction for most problems, not the component parts of it. It will, however, be desirable to make the separation in order to understand more clearly the nature of the tranverse synchronous reactance and also to settle the question about the magnitude of the transverse reactance. This question is perhaps the one on which there is more difference of opinion than on any other relating to Blondel's theory.

Blondel himself considered the transverse reactance equal to the direct reactance. At least, he used only a single value of reactance in his diagram.

Dr. Steinmetz and Dr. Berg both used a transverse reactance much larger than the direct reactance. Dr. Berg states definitely in his "First Course in Electrical Engineering" that the direct reactance is only about 60 per cent of the transverse. That means that if an ordinary synchronous machine had 30 per cent reactance, for instance, the transverse reactance would be in the neighborhood of 50 per cent—not reaction but reactance.

Professor Karapetoff is also of the same opinion, if one may judge from his paper on "Variable Leakage Reactance." While he does not state definitely how much larger the transverse reactance is than the direct reactance, if one may scale his diagrams, he would agree substantially with Dr. Berg.

Professor Arnold, on the other hand, holds that the transverse reactance is considerably smaller than the direct. He says that when a phase belt is above a pole face, only that flux which links the phase belt without entering the pole iron is reactance flux. This means that there is practically no tooth-tip leakage in this position.

Personally, I never could see why, because a line of flux in trying to get around a phase belt found it convenient to enter a friendly pole face for a part of its journey, it had to have its name changed from "reactance" to "reaction." So I agree with Dr. Berg and not with Professor Arnold.

Messrs. Doherty and Nickle, after a thorough and elaborate investigation, conclude that the transverse and direct reactances are practically equal, differing by not more than 1 or 2 per cent.

And now Professor Douglas concludes that the transverse reactance is zero, or at least negligible.

C. A. Nickle: I am inclined to agree with Mr. Putman that the effect of saturation upon quadrature reactance is not quite as great as concluded in this paper.

I think that one question might be raised. The author points out that "the voltage E_n," that is, the voltage in the direct axis, "is not a function of the field current alone." This is true. This merely means that due to pole-tip saturation, instead of the quadrature and direct axes being in their geometric position, they have been slightly shifted, so that the quadrature current now has a mutual component with the field winding and reduces the flux in that circle. I think on that basis that it is reasonable to say that the curve in the main field winding, under these conditions, will have a mutual effect on the quadrature axis. If the quadrature axis has a mutual effect on the direct, it should work the other way. If such is the case, we would expect the values as measured to be much lower than the true values. If the transverse-current magnetomotive force produces a certain transverse flux and in addition to that flux we have another component which is 180 deg. out of phase, this flux being actually produced by the main field winding, the total flux that we measure in the quadrature axis is not the flux produced by the quadrature curve but is a composite flux. I think that quadrature reactance should at least be defined by the flux produced by the quadrature curve. Before we can say what transverse reactance is, we must separate the two components of flux so as to get that component of flux which is due to the main field winding.

It is found when the pole-tip saturates, we have the effect of having removed a small amount of iron from the tip of this pole. The two direct axes are no longer the geometric direct axes but have been shifted slightly to the left. Therefore, since the fluxes in the main field winding, especially at the right-hand end of this curve, are very high compared to the fluxes in the quadrature axis, it requires but a 1- or 2-deg. shift of the two axes so that the main field winding will give a component of flux in the quadrature path which is probably 50 per cent as large as the flux that exists there due to the current alone. I think when this is taken into account, we shall find that quadrature reactance is not affected so vitally by saturation as we would be led to believe from these tests. Large numbers of tests have been made in the past and the angles, pull-out torques, and so forth have checked very well.

R. E. Doherty: Prof. Douglas has offered something for us to think about though there are some things in the paper that are not altogether clear and the results are not what most of us who study these matters might have anticipated.

I would like to mention one point which Prof. Douglas has made, namely, the necessity of distinguishing between a rational method of analysis and one which is largely empirical. The theory of superposition applies only when no saturation is present. If saturation is present, it may be negligible to such an extent that one may apply a theory based on no saturation and get approximate results, or he may, as Mr. Putman has mentioned, shade the constants to take care of it and assume that it can be used; and then interpret the results accordingly.

So this is the point: If those pictures of saturation curves and methods referred to by Professor Douglas are intended to represent a philosophy underlying a method of making such calculations, it must be considered as a relic of the past, because I am pretty sure that nowadays not very many informed engineers so regard them. However, very many calculations on important machines are made according to a theory which, strictly, does not apply—let us say Potier's diagram applied to salient-pole synchronous machines. Why? Because it gives results that are as close, practically, as you can test them. It gives practically correct results in magnitudes of excitation, but not in phase angle. But it is an engineer's privilege to use such tools as he sees fit, so long as he gets results that he can depend upon. Thus, if you can compute the magnitude of excitation under load on a salient-pole machine, when some saturation exists, by using Potier's diagram, that is a perfectly justifiable procedure so long as you recognize its limitations and understand why it gives practically the same magnitude as Blondel's method, which of course is theoretically more sound.

I might say that the reasons why Potier's diagram will give practically the same magnitude of excitation as Blondel's treatment are explained fully in the literature.[2]

Mr. Putman mentioned the different opinions regarding reactance. It is not a question of difference of opinion regarding reactance, but difference of definition; that is, Dr. Berg and Dr. Steinmetz are right; and Blondel is right. Also Doherty and Nickle, Professor Karapetoff and Professor Douglas may be right. The only person who is wrong is one who, not clearly understanding the definition and physical significance of some particular value of reactance, uses it where it is not applicable. In the case of a synchronous motor, for instance, there is a number of different reactances which must be known separately in order to calculate the various operating characteristics. It is therefore meaningless merely to refer to the "reactance" of a synchronous machine.

Going a step further, there may be a legitimate difference in view regarding the segregation of armature leakage reactance into its components. According to the usual view, armature leakage reactance comprises three components: (a) *slot reactance*, about which there is little or no question; (b) *end-winding reactance*, that is, the reactance due to leakage flux around the ends of the coils, about the nature of which there is very little question; and (c) *zigzag* or *tooth-tip reactance*, about which there has been considerable question and discussion. Now if one can build up a consistent theory, using only (a) and (b) as comprising armature reactance, it would be a logical thing to do, provided everybody who used the theory understood that. On the other hand, since it is more in keeping with the established view of things to include all three of these terms in *armature reactance*, and include the remainder reactance effects of the armature current as the effective reactance of armature reaction, this point of view was taken in our paper on *Synchronous Machines—I*.[1] And we believe it is a comprehensive and logical treatment of the problem. It is a question of definition, as I stated at the outset, and if we wish to discuss the particular term which Professor Karapetoff treated, then let us discuss it as such, and not be confused by comparing its value with the whole, of which it is only a part.

Vladimir Karapetoff: To me, the principal value of Professor Douglas's paper lies in a novel experimental method which permits the obtainments of partial data in addition to the usual load data. It seems to me that further progress should lie not in juggling any more with vector diagrams and introducing more factors, but primarily in devising new experimental ways whereby we could get not only "bulk data," that is, the terminal voltage, current and power factor, but partial data as well. A synchronous machine is a comparatively complex aggregate of physical phenomena and if one provides a sufficient number of arbitrary factors, one can usually duplicate the performance of the machine with a sufficient degree of accuracy. However, the progress of the art requires checking those individual factors and not the final performance alone.

I judge from Professor Douglas's paper that he is familiar only with Blondel's early work. Since 1918 Blondel has done a considerable amount of work on synchronous machines. I refer in particular to the investigation, of which the purpose was checking the individual fluxes and voltage drops in the machine, rather than the final performance. Special, rather complicated experimental means were devised for that purpose.[3] Besides, Blondel presented several papers before the French Academy of Sciences, discussing several advanced phases of the theory of synchronous machines. In the third edition of Vol. II of my *Experimental Electrical Engineering* (1927) there is a fairly complete bibliography on the subject, and I hope that Prof. Douglas's paper will create a new stimulus for attacking the problem with modern experimental means and coordinating the results with a more rational theory.

J. F. H. Douglas: Mr. Putman mentioned the test of the 3-phase machine. We did not make such a test although we saw some simple modifications by which it could be done. We submit that the line voltage, perpendicular to the Y voltage, would give us the two reference axes necessary. For convenience, we chose the 2-phase connection which was available on the machine.

Mr. Putman mentioned that the direct synchronous reactance was also a variable. With this we agree. For this reason I advocated following Prof. Karapetoff's idea in this respect, treating it in a magnetomotive force diagram, where it will be a constant. My paper has no reference to sudden pull-out or suddenly applied torques. When I alluded to Mr. Doherty's paper, it was to a formula included which applied to steady-state conditions. This formula has a term which includes $\sin 2\,\delta$ and consequently makes use of a transverse coefficient different from the direct coefficient.

With reference to the formula advocated by Prof. Blondel quoted by Mr. Putman, I did not mean to imply that the transverse coefficient could not be computed unless the action were wholly transverse, but rather that under those conditions, having one system of causation present, the effect would be less subject to constant or systematic error. As a matter of fact, Mr. Nickle called our attention particularly to the fact that direct demagnetizing current might have an effect on transverse conditions. We foresaw that possibility and for that reason the tests were made as nearly as possible with the current fully transverse in its action.

And again, if we refer to Fig. 2 in the paper, we were looking for a demagnetizing effect of transverse reaction, and consequently if we had a large demagnetizing current present, we would not have been able to separate it with accuracy from that produced by the cross-magnetizing current.

With reference to the question raised as to how we lined up the two machines without knowledge of how the coupling between them was adjusted, I would say that, when we said that we brought the two machines so that the pole axes were in the same line, it was simply a short way of saying that we lined up the reference voltage with the no-load voltage of the machine tested. The machine B was loaded to the 5-ampere load, which was used, before the hand-wheel was turned, and so it was the terminal voltage under load that was adjusted to the same phase as the no-load voltage of machine A.

I will not respond further to Mr. Putman's and Mr. Doherty's remarks on reactance than to say that my attitude is entirely empirical. It is a question of how you define reactance. If you define reactance as including non-synchronous revolving fluxes, then undoubtedly the reactance will be larger under transverse than under demagnetizing condition. I think that for the purposes of calculation, the most useful division of the effect between the e. m. f. and the m. m. f. diagrams is that one giving factors which remain fairly constant. If we define reactance as due to those flux components unaffected by saturation, and if all flux components are affected by saturation in a given condition, as we found in the case of transverse reactance condition, we can say that the transverse reactance is zero. This same attitude is taken when we use the zero per cent power-factor characteristic to find the direct reactance and reaction components, although it is well known that the reactance thus determined is considerably larger than what can be computed with rational formulas.

Mr. Nickle's remarks deserve a better discussion than I can give. Pole-tip saturation should be important. I am not at all sure that his analysis is not correct. The case seems to me to be an exact analog of the corresponding case in d-c. machines. The treatment given in Prof. Karapetoff's "Magnetic Circuit" for d-c. machines could be applied almost bodily to the synchronous machines. In this theory there is a demagnetizing effect produced by transverse reaction but no cross-magnetizing effect produced by demagnetizing reaction. The loci of our e. m. f. vectors were parabolas proving the existance of a demagnetizing component of cross-reaction. However, on the m. m. f. diagram the loci were approximately straight lines. In other machines this result may not prove to be true. Since writing the paper, the writer and his students have used three other different methods, and have tested one other machine, the results being the same; the transverse effect is a constant only when placed wholly in the m. m. f. diagram. The question whether the transverse reactance is unaffected by saturation or whether it is constant in the m. m. f. diagram should be easily verified, since at, say, 50 per cent overvoltage the pull-out torques predicted by the two theories are quite different.

I am heartily in accord with what Prof. Karapetoff has said, and it means that there is still a lot of work to be done on the subject.

In particular the writer has assumed a reasonable active layer characteristic for the machine tested, and has by calculation been able to check the curves shown quite closely.

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[1] JOURNAL A. I. E. E., October, 1926, p. 974.

Starting Performance of Synchronous Motors

BY H. V. PUTMAN[1]
Associate, A. I. E. E.

***Synopsis.**—This paper deals with the theory underlying the starting performance of the salient-pole synchronous motor equipped with damper windings. The theory, while involving some approximations, is accurate enough for practical engineering calculations. Formulas are developed for the starting torque, pull-in torque and inrush. A method is also given for calculating the speed torque curve from standstill to synchronous speed.*

Due to the fact that the damper winding is not continuous around the periphery and due to the presence of the single-phase field winding, the rotor circuit is not a perfect polyphase secondary but is unbalanced to some extent. In order to take care of this unbalance, it is necessary, in addition to the usual system of positively rotating vectors, to employ a second system of negatively rotating vectors as is done in unbalanced three-phase problems.

The stator resistance has been disregarded in working out the general case of the unbalanced or partial polyphase secondary in order to obtain a torque formula which will be simple and at the same time accurate enough for practical calculations. Mr. Q. Graham is working on this problem and expects to present in an Institute paper, in the near future, the general solution including the stator resistance.

The use of the double squirrel-cage type of damper winding in salient-pole machines has been examined both theoretically and experimentally. Other methods of obtaining unusual starting performance are suggested and the results of some actual calculations presented.

INTRODUCTION

PROBABLY the earliest published work on the starting performance of synchronous motors was done by Carl J. Fechheimer in 1912[2]. This was a paper of great merit and a valuable contribution to the art. Not only did it give much valuable experimental data concerning the synchronous motor but it added much to our knowledge of the calculation of reactance. The discussion which followed the presentation of this paper showed clearly that the synchronous motor was well understood experimentally in 1912. Engineers knew *what* the motor would do under various conditions but the "why" was often in doubt. The theory developed in the present paper explains some of the "whys." Mr. Fechheimer limited his theoretical work to the conditions at standstill. In this paper the starting conditions are examined not only at standstill but through the whole starting period up to the pull-in point.

NOTATION

The following notation will be used throughout the paper. The subscript p used with a vector quantity denotes that the vector belongs to the positive system of vectors, while the subscript n denotes that the vector belongs to the negative system. For instance:

$\underset{\cdot}{E}_{ip}$ = Induced voltage, positively rotating,
$\underset{\cdot}{E}_{in}$ = Induced voltage, negatively rotating.

In a similar manner the subscripts p and n may appear with any of the vectors.

$\underset{\cdot}{E}_0$ = Impressed voltage,
$\underset{\cdot}{I}_1$ = Stator current,
$\underset{\cdot}{I}_{00}$ = Magnetizing current,
$\underset{\cdot}{I}_2$ = Rotor current,
$\underset{\cdot}{I}_b$ = Rotor bar current,
$\underset{\cdot}{I}_f$ = Rotor current in the field winding,
$\underset{\cdot}{I}_{2nn}$ = Negatively rotating rotor current due to E_{in}. (The subscript 2 indicates rotor current. The first subscript n indicates that the vector is a negatively rotating vector. The last subscript n indicates that the current is produced by the negatively rotating induced voltage E_{in}.)

Similarly,

$\underset{\cdot}{I}_{2np}$ = Negatively rotating rotor current due to E_{ip},
$\underset{\cdot}{I}_{2pp}$ = Positively rotating rotor current due to $\underset{\cdot}{E}_{ip}$,
$\underset{\cdot}{I}_{2pn}$ = Positively rotating rotor current due to $\underset{\cdot}{E}_{in}$,
$\dot{V}$ = $x_2 + j\, r_{2s}$,
$\dot{V}$ = Z_{bs}/Z_{fs},
a = Half the depth of double squirrel-cage damper bar (App. II),
b = Width of double squirrel-cage damper bar (App. II),
B = $1/x_1 + b_{00}$,
b_{00} = Magnetizing admittance,
f = Frequency in cycles per second,
j = $\sqrt{-1}$, indicates the imaginary term in vector expressions,
K = Ratio $\dfrac{\text{N. R. V.}}{\text{P. R. V.}} = \dfrac{\sin\theta}{\theta}$ (see Equation (**14**)),
K_c = Value of K corrected for effect of closed field circuit,

$$K^2 = \frac{4\,\pi\,p}{\sigma} \text{ (App. } II\text{)}$$

p = Operator $\dfrac{d}{d\,t}$ and denotes differentiation with respect to time (App. II),
r_1 = Stator resistance,
r_2 = Rotor resistance,
r_b = Rotor bar resistance,
r_{2s} = r_2/S,
r_f = Field circuit resistance,
r_{fs} = r_f/S,
r_0 = $r_1 + r_2$,
S = Slip,

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[1] Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.
[2] TRANS. A. I. E. E., 1912, Vol. 31, Part I, p. 529.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

i = Time in seconds,

T_p = Motor torque due to positively rotating flux and positively rotating rotor current,

v = Ratio Z_{bs}/Z_{fs},

x_1 = Stator reactance,

x_2 = Rotor reactance,

x_b = Rotor bar reactance,

x_f = Field winding reactance,

x_0 = $x_1 + x_2$,

z = $\sqrt{1 + 2\,B\,x_2 + B^2\,Z_{2s}^2}$

Z_2 = Rotor impedance = $\sqrt{r_2^2 + S^2\,x_2^2}$

Z_{2s} = $\sqrt{r_{2s}^2 + x_2^2}$

Z_{bs} = $\sqrt{r_{bs}^2 + x_b^2}$

Z_{fs} = $\sqrt{r_{fs}^2 + x_f^2}$

Z_0 = $\sqrt{r_0^2 + x_0^2}$,

Z_1 = Stator impedance = $\sqrt{r_1^2 + x_1^2}$,

θ = Bar span in electrical radians (Part I),

θ = $\tan^{-1}\dfrac{x_b}{r_{bs}} + \alpha$ = phase angle of rotor circuit with field closed (Part IV),

α = $\tan^{-1}\dfrac{v\cos\varphi}{1 + v\sin\varphi}$,

φ = $\tan^{-1}\dfrac{x_b}{r_{bs}} + \tan^{-1}\dfrac{r_{fs}}{x_f}$,

Φ = M. m. f. (Part I),

Φ = Self-inductive flux (App. II),

ω = Angular velocity in radians per sec.,

σ = Specific resistance of material used in double squirrel-cage damper bar (Abohms per cu. cm.).

Part I. Theory of the Partial Polyphase Rotor Circuit

The expression,

$$\cos\omega t + j\sin\omega t \tag{1}$$

represents a vector of unit length rotating forward in the positive direction with angular velocity ω and starting at the zero position at the instant of counting time.

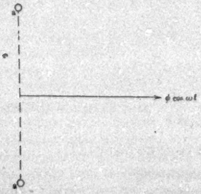

FIG. 1—M. M. F. PRODUCED BY A SINGLE COIL, PULSATING IN TIME, UNDIRECTIONAL IN SPACE

Similarly,

$$\cos\omega t - j\sin\omega t \tag{2}$$

represents a vector rotating backwards or in the negative direction with angular velocity ω.

The sum of these two rotating vectors is

$$2\cos\omega t \tag{3}$$

which is simply a pulsating quantity in one direction in space.

If, now, Equation (3) represents the m. m. f. due to a single-phase winding, it follows from the above that this m. m. f., which is unidirectional in space and pulsating in time, can be represented by, or split up into, two vectors constant in time but rotating in space, one rotating in the positive direction and the other in the negative direction.

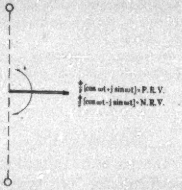
$\frac{\Phi}{2}[\cos\omega t + j\sin\omega t] = $ P.R.V.
$\frac{\Phi}{2}[\cos\omega t - j\sin\omega t] = $ N.R.V.

FIG. 2—M. M. F. PRODUCED BY A SINGLE COIL, RESOLVED INTO TWO ROTATING VECTORS

As an example of the above, let the field produced by a coil $a - a$, Fig. 1, be $\Phi\cos\omega t$.

In Fig. 2 the equivalent rotating vectors are shown.

$$\frac{\Phi}{2}[\cos\omega t + j\sin\omega t] = \text{positively rotating vector.}$$

$$\frac{\Phi}{2}[\cos\omega t - j\sin\omega t] = \text{negatively rotating vector.}$$

It should be noted that the length of each rotating vector is only half the maximum value of the stationary vector. In Fig. 1, when $t = 0$, $\Phi\cos\omega t$ is maximum and the current in the coil is, therefore, a maximum. In Fig. 2, then, time is counted from the instant the current in the coil is maximum, and at this instant it should be particularly noted that the positively rotating vector and negatively rotating vector are together in space.

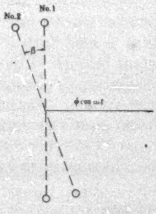

FIG. 3—M. M. F. PRODUCED BY TWO COILS SEPARATED BY ANGLE β, RESOLVED INTO TWO ROTATING VECTORS

THE PARTIAL POLYPHASE SECONDARY PRODUCED BY DAMPER BARS IN THE POLE FACE

Consider now the m. m. f. produced by two coils or pairs of bars shown in Fig. 3, as No. 1 and No. 2. Let time be counted from the instant the current in coil No. 1 is a maximum. Let the current in coil No. 2 lag behind that of No. 1 by an angle β equal to the space angle between the coils, as this would be the case with damper bars.

The m. m. f. due to coil No. 1 is

$$\left.\begin{aligned}\frac{1}{2}\,\Phi\,[\cos\omega t + j\sin\omega t] &= \text{positively rotating vector}\\ \frac{1}{2}\,\Phi\,[\cos\omega t - j\sin\omega t] &= \text{negatively rotating vector}\end{aligned}\right\}\quad(4)$$

To find the m. m. f. due to No. 2 coil, proceed as follows: Assume temporarily that the current in No. 2 is in time phase with that in No. 1. On this assumption the rotating vectors would be

$$\left.\begin{aligned}&\frac{1}{2}\,\Phi\,\{\cos(\omega t+\beta) + j\sin(\omega t+\beta)\}\\ &\qquad= \text{positively rotating vector}\\ &\frac{1}{2}\,\Phi\,\{\cos(\omega t-\beta) - j\sin(\omega t-\beta)\}\\ &\qquad= \text{negatively rotating vector}\end{aligned}\right\}\quad(5)$$

But the current in No. 2 lags behind that in No. 1 by an angle β. Hence it is necessary to substitute $(\omega t - \beta)$ for ωt in (**5**). This gives for the rotating vectors for coil No. 2 under the conditions specified above:

$$\left.\begin{aligned}&\frac{1}{2}\,\Phi\,\{\cos\omega t + j\sin\omega t\}\\ &\qquad= \text{positively rotating vector}\\ &\frac{1}{2}\,\Phi\,\{\cos(\omega t-2\beta) - j\sin(\omega t-2\beta)\}\\ &\qquad= \text{negatively rotating vector.}\end{aligned}\right\}\quad(6)$$

It should be noted that the positively rotating vectors for both coils No. 1 and No. 2 are in phase with each other. The negatively rotating vectors are, however, out of phase by an angle 2β.

To get the combined m. m. f. due to both coils No. 1 and No. 2 it is only necessary to add Equations (**4**) and (**6**) together.

This gives:

$$\left.\begin{aligned}\text{Positively rotating vector} &= \Phi\,(\cos\omega t + j\sin\omega t)\\ \text{Negatively rotating vector} &= \Phi\cos\beta\,\{\cos(\omega t-\beta)\\ &\qquad - j\sin(\omega t-\beta)\}\end{aligned}\right\}\quad(7)$$

It is seen that the negatively rotating vector has been shortened by $\cos\beta$ factor. This is due to the fact that the two coils constitute a partial or imperfect polyphase field. For a perfect polyphase field the negatively rotating vector would disappear entirely. The positively rotating vector and negatively rotating vector are not together in space when $t = 0$. This is because time was counted from the instant the current in coil No. 1 was maximum. If, now, time had been counted from the instant the current in coil No. 2 was maximum, the rotating vectors would have been:

$$\left.\begin{aligned}\text{Positively rotating vector} &= \Phi\,(\cos\omega t + j\sin\omega t)\\ \text{Negatively rotating vector} &= \Phi\cos\beta\,\{\cos(\omega t+\beta)\\ &\qquad - j\sin(\omega t+\beta)\}\end{aligned}\right\}\quad(8)$$

The phase angle of the negatively rotating vector thus depends upon the instant from which time is counted.

Consider, now, the m. m. f. produced by a number of coils uniformly distributed over an arc or angle θ, Fig. 4. Thus:

Let

$\Delta\beta$ = angle between coils

Φ = m. m. f. per radian periphery. The m. m. f. of each coil is then $\Phi\,\Delta\beta$ and there are

$$\frac{\theta}{\Delta\beta} = n \text{ coils.}$$

Count time from the instant the current in coil No. 1 is maximum. Since the positively rotating vectors are all in phase and each has a magnitude of $\dfrac{\Phi\,\Delta\beta}{2}$, and there are $\dfrac{\theta}{\Delta\beta}$ coils, the

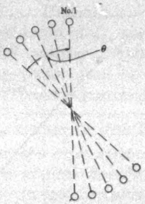

FIG. 4—M. M. F. PRODUCED BY A NUMBER OF COILS DISTRIBUTED OVER ANGLE θ TO THE LEFT OF COIL NO. 1

$$\text{positively rotating vector} = \frac{\Phi\,\theta}{2}\,(\cos\omega t + j\sin\omega t)\quad(9)$$

The negatively rotating vectors are all out of phase and the sum of them is:

Negatively rotating vector

$$\left.\begin{aligned}&= \frac{\Phi\,\Delta\beta}{2}\left\{\sum^{n\text{ terms}}\cos\omega t + \cos(\omega t - 2\,\Delta\beta)\right.\\ &+ \cos(\omega t - 4\,\Delta\beta) + \ldots - j\sum^{n\text{ terms}}\sin\omega t\\ &+ \sin(\omega t - 2\,\Delta\beta) + \sin(\omega t - 4\,\Delta\beta) + \ldots\end{aligned}\right\}\quad(10)$$

$$= \frac{\Phi \, \Delta \beta}{2} \left\{ \sum_{r=1}^{r=n} \cos \{ \omega t - 2 (r - 1) \Delta \beta \} - j \sum_{r=1}^{r=n} \sin \{ \omega t - 2 (r - 1) \Delta \beta \} \right\} \tag{11}$$

Now let $\Delta \beta \to 0$ and $n \to \infty$

The Σ's then become integrals as follows:

Negatively rotating vector

$$= \frac{\Phi}{2} \left\{ \int_{x=0}^{x=\theta} \cos (\omega t - 2 x) \, d x - j \int_{x=0}^{x=\theta} \sin (\omega t - 2 x) \, d x \right\} \tag{12}$$

$$= \frac{\Phi}{2} \sin \theta \, [\cos (\omega t - \theta) - j \sin (\omega t - \theta)] \tag{13}$$

which is the negatively rotating field.

The ratio of the magnitude of the negative field to that of the positive field is from (**9**) and (**13**),

$$\text{Ratio } \frac{\text{Negatively rotating vector}}{\text{Positively rotating vector}} = K = \frac{\sin \theta}{\theta} \tag{14}$$

K is really a measure of the single-phase action in the rotor. It shows how nearly the rotor circuit approaches the perfect polyphase condition. If, in Equation (**14**), θ is put equal to zero for the case of the single-phase rotor, $K = 1$, since $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$, so that the negatively rotating field is equal to the positively rotating field which is correct for the single-phase rotor. But if in (**14**) θ is made equal to π corresponding to a damper winding which is continuous around the periphery as in a squirrel-cage induction motor, then $K = 0$, and there is no negatively rotating field. Fig. 5 shows a curve which gives values of K corresponding to different values of the bar span θ.

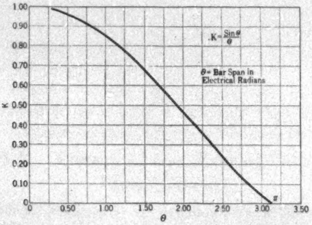

FIG. 5—CURVE SHOWING VALUES OF K AS FUNCTION OF THE DAMPER BAR SPAN. K IS A MEASURE OF THE SINGLE-PHASE ACTION PRODUCED BY THE DAMPER WINDING

It should be noted that the magnitude of the positive field is always equal to one-half the m. m. f. in the whole winding, assuming all currents in phase.

It can be shown in a similar manner that for a group of coils as shown in Fig. 6 covering an angle to the right of coil No. 1 (instead of to the left as in the previous case) that the vectors are:

Positively rotating vector

$$= \frac{\Phi \, \theta}{2} [\cos \omega t + j \sin \omega t] \tag{15}$$

Negatively rotating vector

$$= \frac{\Phi \sin \theta}{2} [\cos (\omega t + \theta) - j \sin (\omega t + \theta)] \tag{16}$$

Here again time was counted from the instant the current in coil No. 1, Fig. 6, was maximum.

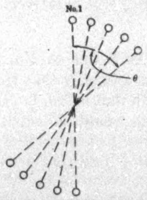

FIG. 6—M. M. F. PRODUCED BY A NUMBER OF COILS DISTRIBUTED OVER ANGLE θ TO THE RIGHT OF COIL NO. 1

Combining the results of Figs. 4 and 6 gives the m. m. f. for the arrangement of coils shown in Fig. 7, time being counted from the instant the current is maximum in coil No. 1, which is the middle coil of the group.

The angle or arc covered by the winding is in this case $\psi = 2 \theta$. To get the positively rotating vector it is only necessary to add (**9**) and (**15**).

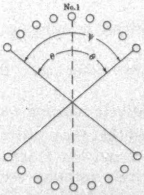

FIG. 7—M. M. F. OF A SYMMETRICAL ARRANGEMENT OF COILS

$$\text{Positively rotating vector} = \Phi \, \theta \, [\cos \omega t + j \sin \omega t] \tag{17}$$

To get the negatively rotating vector add Equations (**13**) and (**16**) which gives:

Negatively rotating vector

$$= \frac{\Phi}{2} \sin \theta \, \{ \cos (\omega t - \theta) + \cos (\omega t + \theta)$$

$$- j\,[\sin(\omega t - \theta) + \sin(\omega t + \theta)]\}$$

$$= \Phi \sin\theta \cos\theta\,[\cos\omega t - j \sin\omega t] \qquad (18)$$

Substituting $2\,\theta = \psi$ in (17) and (18) gives

$$\left.\begin{array}{l} \text{Positively rotating vector} \\ = \dfrac{\Phi\,\psi}{2}\,[\cos\omega t + j\sin\omega t] \\ \text{Negatively rotating vector} \\ = \dfrac{\Phi}{2}\,\sin\psi\,[\cos\omega t - j\sin\omega t] \end{array}\right\} \qquad (19)$$

Hence it is seen that if time is counted from the instant that the current is maximum in the middle of the belt of conductors, the positively rotating vector and negatively rotating vector are in phase with each other; that is, they are together in space. Or, more generally, it may be stated that regardless of the instant from which time is counted, the positively rotating vector and negatively rotating vector are together in space at the instant the current is maximum in the middle bar in the pole face.

Part II. Construction of the Vector Diagram of a Synchronous Motor With Partial Polyphase Rotor Circuit

It is well known, of course, that the ordinary vector diagram of a polyphase induction motor represents not only the time relation of the various quantities but the actual space relation in the machine as well. The same will be true of the vector diagram of a motor with a partial polyphase rotor circuit. It will be found convenient to represent by the vector for the rotor bar current the current in the bar in the middle of the pole.

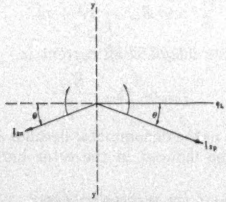

Fig. 8—Diagram Showing Simple Positive and Negative Rotating Systems

Relation Between Negatively Rotating System and Positively Rotating System

It has been shown above that if, due to a positively rotating sine wave of flux in the gap, there is induced a voltage in each of the rotor bars which causes a sine wave of current to flow in each of the bars, the resultant m. m. f. may be represented by two rotating vectors, one rotating positively and the other negatively. In Fig. 8, let e_i be the voltage induced in each of the bars.

Two current vectors are produced, the positively rotating vector which is I_{2p} and lags behind the induced voltage by angle θ, the phase angle of the damper bar circuit, and the negatively rotating vector which is I_{2n} and which lies along the reflection of I_{2p} about the y axis. Now, how does one know for sure the position of I_{2n}? It has been shown very clearly that I_{2n} must be so located that I_{2n} and I_{2p} will come together in space at the instant the current in the middle bar of the pole is maximum. Since the projection of any of the vectors on the y axis as they rotate gives their instantaneous values in time, this will occur when I_{2p} comes in line with the y axis. Since I_{2n} is rotating negatively, it will come in line with the y axis at the same time as does I_{2p}. Hence, it is seen that by making I_{2n} lie along the reflection of I_{2p} about the y axis, the condition that the positively rotating vector and negatively rotating vector shall be together in space at the instant the current is maximum, is fulfilled.

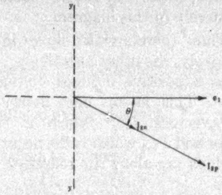

Fig. 9—Diagram Illustrating the Principle of Reflection

Principle of Reflection of the Negative System

It is evident that the conventional system of vector notation cannot be applied directly to the condition existing in Fig. 8. In fact, it cannot be applied to a vector diagram in which there are vectors rotating in opposite directions, so that, for the purpose of mathematical analysis it is necessary to construct the diagram in such a manner that the actual vectors in the diagram rotate in the positive direction. Such a diagram is shown in Fig. 9.

This diagram is obtained from that in Fig. 8, by applying the principle of reflection which may be stated as follows: If the negative system of vectors (in this case only the single vector I_{2n}, Fig. 8) be reflected about the y axis and made to rotate positively, the instantaneous values of the quantities represented by the vectors of the negative system remain unchanged. In other words, if in Fig. 9 I_{2n} rotates positively, it produces the same projections on the y axis (which projections are the instantaneous values) as does I_{2n} in Fig. 8 when rotating negatively. Now, the diagram in Fig. 9 has the advantage that it lends itself to analysis by regular vector equations. It has the disadvantage that it is a time diagram only and does not represent the space relations in the machine as does the diagram in Fig. 8. Either diagram can, of course, be obtained from the other by simply reflecting the negative system.

VECTOR DIAGRAM OF SYNCHRONOUS MOTOR WITH PARTIAL POLYPHASE ROTOR CIRCUIT IN FIG. 10

Fig. 10 gives the complete vector diagram of a motor. It is of the same type as Fig. 8, giving space as well as time relations. It is similar to the ordinary diagram of an induction motor except more complicated because of the negative system. All dotted vectors are actually rotating negatively in this diagram.

In the machine there are two fluxes in the gap, one rotating positively, the other negatively. The positive flux generates a positive induced voltage E_{ip} in the damper bars.[3] This, as has been described, produces two current vectors: I_{2pp}[4] which is the positively rotating vector and I_{2np} which is the negatively rotating vector and which lies along the reflection of I_{2pp} about the y axis.

In a similar manner the negatively rotating flux generates a negative induced voltage E_{in} in the damper bars which again sets up two current vectors. The larger of these in this case is the negatively rotating vector which is I_{2nn} and lags behind E_{in} by the phase angle of the damper bars. The other is I_{2pn} which is the positively rotating vector and lies along the reflection of I_{2nn} about the y axis.

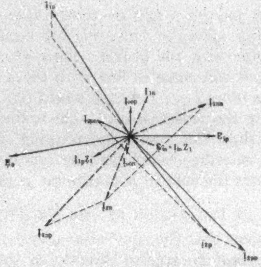

FIG. 10—MOTOR VECTOR DIAGRAM AT STANDSTILL WITH PARTIAL POLYPHASE ROTOR CIRCUIT

Dotted lines represent negatively rotating vectors.

The total positive m. m. f. in the rotor is represented by I_{2p} and is the sum of I_{2pp} and I_{2pn}. Similarly, the total negative m. m. f. in the rotor is I_{2n} and is the sum of I_{2nn} and I_{2np}.

The positive m. m. f. in the stator is the sum of $-I_{2p}$ and the magnetizing current I_{00p},[5] which produces the positive flux in the gap. Similarly, the negative m. m. f. in the stator is the sum of $-I_{2n}$ and the negative magnetizing current I_{00n} which produces the negative flux in the gap.

The impressed voltage on the motor is the sum of $-E_{ip}$ and the $I_{1p} Z_1$ drop. Since there is no negatively rotating impressed voltage (if the phases are balanced as assumed) the negative induced voltage E_{in} is equal to the $I_{1n} Z_1$ drop, as shown in the diagram.

Part III. Vector Equations—Derivation of the Torque Formula

In writing these equations the stator resistance will be disregarded. This leads to many simplifications which are impossible if it is included.

The positive secondary current is

$$I_{2p} = I_{2pp} + I_{2pn} \tag{20}$$

or

$$I_{2p} = \frac{E_{ip}}{Z_2} + \frac{K E_{in}}{Z_2} \tag{21}$$

where K is the ratio of the $\frac{\text{negatively rotating vector}}{\text{positively rotating vector}}$

given by Equation (**14**).

$$\text{Actually } K = \frac{\text{Magnitude of } I_{2pn}}{\text{Magnitude of } I_{2nn}} = \frac{\text{Magnitude of } I_{2np}}{\text{Magnitude of } I_{2pp}}$$

or

$$I_{2p} = \frac{E_{ip} + K E_{in}}{S Z_{2s}^2} (r_{2s} - j x_2) \tag{22}$$

where $r_{2s} = \frac{r_2}{S}$ and $Z_{2s}^2 = x_2^2 + r_{2s}^2$

The positive magnetizing current is

$$I_{00p} = j b_{00} \frac{E_{ip}}{S}$$

The S enters in the denominator because E_{ip} was taken as the voltage induced in the rotor bars, not in the stator.

$$I_{1p} = - I_{2p} + I_{00p}$$

or

$$I_{1p} = \frac{-E_{ip}}{S Z_{2s}^2} [r_{2s} - j (x_2 + b_{00} Z_{2s}^2)] - \frac{K E_{in}}{S Z_{2s}^2} (r_{2s} - j x_2) \tag{23}$$

---

[3] It is easier, in describing the diagram, to forget about the field winding. Its effect is discussed later. The diagram is perfectly general, applying to any machine with a partial polyphase rotor of any type.

[4] I_{2pp} is read as follows: Secondary current positively rotating produced by the positively rotating induced voltage. The 2 means secondary. The first p means positively rotating and the last p means due to the positive induced voltage.

[5] It is, of course, not entirely correct to represent the magnetizing current by a simple vector on account of the non-uniformity of the air-gap in a synchronous motor. To take care of this correctly would probably involve the introduction of harmonics of higher order than fundamental. The error is probably not large, although it is a fact that the magnetizing admittance in a synchronous machine is large, the exciting current usually being greater than the normal full load current of the machine. Possibly this point is worthy of further study and investigation.

$$I_{1p} Z_1 = j\, I_{1p}\, x_1 = \frac{-E_{ip}\, x_1}{S\, Z_{2s}^2}\left[x_2 + b_{00}\, Z_{2s}^2 + j\, r_{2s}\right] - \frac{K\, E_{in}\, x_1}{S\, Z_{2s}^2}\,(x_2 + j\, r_{2s}) \qquad (24)$$

but

$$E_0 = \frac{-E_{ip}}{S} + I_{1p}\, Z_1$$

or

$$E_0 = \frac{-E_{ip}\, x_1}{S\, Z_{2s}^2}\left[x_2 + B\, Z_{2s}^2 + j\, r_{2s}\right] - \frac{K\, E_{in}\, x_1}{S\, Z_{2s}^2}\,(x_2 + j\, r_{2s}) \qquad (25)$$

where

$$B = \left(\frac{1}{x_1} + b_{00}\right)$$

Similarly for the negative current,

$$I_{2n} = \frac{E_{in} + K\, E_{ip}}{S\, Z_{2s}^2}\,(r_{2s} - j\, x_2) \qquad (26)$$

The negative magnetizing current is

$$I_{00n} = j\, b_{00}\left(\frac{2\,S - 1}{S}\right) E_{in}\,\frac{1}{(2\,S - 1)} = j\, b_{00}\,\frac{E_{in}}{S} \qquad (27)$$

To understand this equation clearly it must be remembered that this magnetizing current is assumed to flow in the stator[6]; that the magnetizing admittance is inversely proportional to the frequency; and that b_{00} is given per unit volt in the stator while E_{in} is the voltage generated in the rotor bars. Since the negative flux actually moves backward on the rotor at a speed S, while the rotor moves forward at a speed $(1 - S)$, the negative flux really cuts the stator at a frequency of $(2\,S - 1)$, while it cuts the rotor bars at a frequency S. The exciting admittance in the stator for this negative flux is then

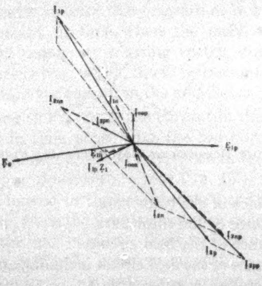

FIG. 11—MOTOR VECTOR DIAGRAM AT STANDSTILL, ALL VECTORS ROTATING POSITIVELY IN THE DIAGRAM

Vector equations are based on this diagram.

$$b_{00}\left(\frac{1}{2\,S - 1}\right)$$

since it is inversely proportional to the frequency and the voltage induced in the stator is $\left(\dfrac{2\,S - 1}{S}\right) E_{in}$.

The stator negative current is

$$I_{1n} = -\, I_{2n} + I_{00n}$$

or

$$I_{1n} = \frac{-E_{in}}{S\, Z_{2s}^2}\left[r_{2s} - j\,(x_2 + b_{00}\, Z_{2s}^2)\right] - \frac{K\, E_{ip}}{S\, Z_{2s}^2}\,(r_{2s} - j\, x_2) \qquad (28)$$

$$I_{1n}\, Z_1 = j\, I_{1n}\, x_1\,(2\,S - 1) = \frac{-E_{in}\, x_1\,(2\,S - 1)}{S\, Z_{2s}^2}\,(x_2 + b_{00}\, Z_{2s}^2 + j\, r_{2s}) - \frac{K\, E_{ip}\, x_1\,(2\,S - 1)}{S\, Z_{2s}^2}\,(x_2 + j\, r_{2s}) \qquad (29)$$

But

$$I_{1n}\, Z_1 = E_{1n}\,\frac{(2\,S - 1)}{S} \qquad (30)$$

Equating (29) and (30), reducing and solving for E_{in} gives:

$$E_{in} = -\, K\, E_{ip}\,\frac{V}{V + B\, Z_{2s}^2} \qquad (31)$$

where

$$V = x_2 + j\, r_{2s}$$

Substituting (31) in (25) and putting $x_2 + j\, r_{2s} = V$ gives:

$$E_0 = \frac{-E_{ip}\, x_1}{S\, Z_{2s}^2}\left[\frac{(V + B\, Z_{2s}^2)^2 - K^2\, V^2}{V + B\, Z_{2s}^2}\right] \qquad (32)$$

In Appendix I, it is shown that the numerical value of the bracket is:

$$\frac{Z_{2s}}{z}\,\sqrt{(z^2 - K^2)^2 - (2\,K\,B\, r_{2s})^2} \qquad (33)$$

where

$$z = \sqrt{1 + 2\,B\, x_2 + B^2\, Z_{2s}^2}$$

Substituting (33) for the bracket in (32) and making E_{ip} zero vector, (32) may be solved for the positive induced voltage.

---

[6] This assumption is not necessary. The point of view could be taken that the negative magnetizing current flows in the rotor and precisely the same result reached. Actually, the negative magnetizing current does flow in the rotor bars as can be seen from either diagram, Fig. 10 or Fig. 11. It will be noted that I_{2n} is larger than I_{1n}, which simply means that since the negative m. m. f. in the rotor is greater than that in the stator, it must supply the magnetization. Of course, just the opposite is true of the positive m. m. fs. I_{1p} is always greater than I_{2p} so that the positive magnetizing current is carried in the stator.

$$\frac{E_{ip}}{S} = \frac{E_0 Z_{2s} z}{x_1 \sqrt{(z^2 - K^2)^2 - (2 K B r_{2s})^2}} \qquad (34)$$

The positive torque which is due to the positive flux in the gap reacting on the positive current in the rotor bars is

$$T_p = \frac{E_{ip}}{S} \times \text{real part of } I_{2p} \qquad (35)$$

From (**31**)

$$E_{ip} + K E_{in} = E_{ip} \left[\frac{V (1 - K^2) + B Z_{2s}^2}{V \quad B+Z_{2s}^2} \right] \qquad (36)$$

Substituting (**36**) in (**22**) gives:

$$I_{2p} = \frac{E_{ip}}{S Z_{2s}^2} \left[\frac{V (1 - K^2) + B Z_{2s}^2}{V + B Z_{2s}^2} \right] (r_{2s} - j x_2) \qquad (37)$$

Now, the rationalizing factor in the denominator is $x_2 + B Z_{2s}^2 - j r_{2s}$, so that the problem of finding the real part of I_{2p} resolves itself into that of finding the real part of

$$(x_2 + B Z_{2s}^2 - j r_{2s}) [x_2 + B Z_{2s}^2 + j r_{2s} - K^2 (x_2 + j r_{2s})] (r_{2s} - j x_2) \qquad (38)$$

Multiplying this out and reducing the real part is found to be

$$Z_{2s}^2 r_{2s} [z^2 - K^2 (1 + 2 B x_2)] \qquad (39)$$

The real part of I_{2p} is, therefore,

$$I_{2p} \text{ (real part only)} = \frac{E_{ip} r_{2s}}{S Z_{2s}^2 z^2} [z^2 - K^2 (1 + 2 B x_2)]$$

Substituting in (**35**) gives the positive torque,

$$T_p = \left(\frac{E_{ip}}{S} \right)^2 \frac{r_{2s}}{Z_{2s}^2 z^2} [z^2 - K^2 (1 + 2 B x_2)] \qquad (41)$$

Substituting (**34**) in (**41**) and reducing gives,

$$T_p = \frac{E_0^2}{x_1^2} r_{2s} \frac{[(z^2 - K^2) - 2 K^2 B x_2]}{[(z^2 - K^2)^2 + (2 B K r_{2s})^2]} \qquad (42)$$

In this formula,

$$z^2 = 1 + 2 B x_2 + B^2 Z_{2s}^2$$

and

$$B = \frac{1}{x_1} + b_{00}$$

This is the torque of the motor produced by the positively rotating flux in the gap and by the positive current in the rotor bars. On the assumption of no stator resistance this is the total motor torque, since if there is no stator resistance there is no component of I_{2n} in phase with E_{in}. (See vector diagram Fig. 10). In other words, there is no current in time phase with the negatively rotating flux in the gap, so there can be no torque due to this flux.

If in (**42**) K is put equal to 1 for the case of a single-phase rotor circuit, the torque is

$$T = \frac{r_{2s}}{x_1^2 \{ B^2 r_{2s}^2 + (2 + B x_2)^2 \}} \qquad (43)$$

By differentiating this expression with respect to slip, it is easily seen that maximum torque occurs at

$$S = \frac{B r_2}{2 + B x_2} \qquad (44)$$

and the maximum torque is

$$T_{max.} = \frac{.5}{B x_1^2 (2 + B x_2)} \qquad (45)$$

Formulas (**44**) and (**45**) are useful in studying the conditions at pull-in as will be seen later.

By substituting the numerical values of the several constants of the machine in Equation (**42**) for different values of slip, a speed torque curve could be made for the case of open field circuit. This is of little practical value as machines are almost never started with the field open and even if they are started with the field open, it is usually closed before the motor reaches the pull-in point. This is necessary because a motor has very little pull-in torque with open field. Hence, it is necessary, before one can make a speed torque curve with closed field circuit, to study the effect of closing the field circuit on the other constants of the machine.

Part IV. Effect of Field Winding on the Characteristics of the Rotor Circuit

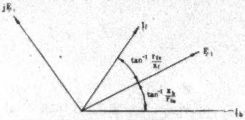

FIG. 12—TIME DIAGRAM OF ROTOR BAR AND FIELD CIRCUITS

Let:

x_b = reactance of the rotor bars,

r_b = resistance of the rotor bars,

x_f = reactance of the field circuit,

r_f = resistance of the field circuit including any external resistance in series with it,

I_b = rotor bar current,

I_f = field current.

All constants are, of course, referred to the stator.

Let E_i be the voltage induced in the rotor bars by a sine wave of flux. Then

$$I_b = \frac{E_i}{r_b + j S x_b} = \frac{E_i/S}{Z_{bs}^2} (r_{bs} - j x_b) \qquad (46)$$

With the direction of flux rotation assumed in Fig. 13, the induced voltage in the field is 90 deg. ahead of that in the bars, so that

$$I_f = \frac{j E_i}{r_f + j S x_f} = \frac{E_i/S}{Z_{fs}^2} (x_f + j r_{fs}) \qquad (47)$$

The time relations expressed in Equations (**46**) and (**47**) are clearly shown in Fig. 12. In other words, the current in the field circuit in time, is ahead of the current in the bar circuit by an angle

$$\left.\begin{aligned}\varphi &= \tan^{-1}\frac{x_b}{r_{bs}} + \tan^{-1}\frac{r_{fs}}{x_f}\\ &\text{or}\\ \varphi &= \tan^{-1}\frac{x_b\,x_f + r_{bs}\,r_{fs}}{r_{bs}\,x_f - x_b\,r_{fs}}\end{aligned}\right\}\qquad(\mathbf{48})$$

It will now be convenient to draw a space diagram, Fig. 14, representing the combined reaction of both the bars and the field on the stator. Both positively and negatively rotating fields will be required. It will also be convenient to draw this diagram as it would appear at the instant in time when the positively rotating vector and negatively rotating vector, due to the bar current, are in line, which, it will be remembered, occurs at the instant the current in the middle bar is maximum.

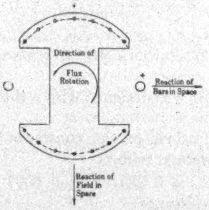
Direction of Flux Rotation
+
Reaction of Bars in Space
Reaction of Field in Space

FIG. 13—SPACE DIAGRAM OF ROTOR BAR AND FIELD CIRCUITS

Referring to Fig. 14, if the current in the field circuit had been in time phase with the bar current, its two vectors I_{fp} and I_{fn} would have been in line with the y axis pointing downward. (See Fig. 13.) They have, however, moved in their respective directions through an angle φ, since the field current is φ degrees ahead in time of the bar current. This is shown in Fig. 14. The total positively rotating m. m. f. in the rotor is represented by I_{2p} and is the vector sum of I_{bp} and I_{fp}. Similarly, the total negatively rotating m. m. f. is represented by I_{2n} and is the vector sum of I_{fn} and I_{bn}. Expressed mathematically, the first relation is

$$I_{2p} = I_{bp} + I_{fp}\,(\sin\varphi - j\cos\varphi) \qquad (\mathbf{49})$$

or numerically

$$I_{2p} = \sqrt{I_{bp}^2 + I_{fp}^2 + 2\,I_{bp}\,I_{fp}\sin\varphi} \qquad (\mathbf{50})$$

Now let

$Z_{2s} = \sqrt{x_2^2 + r_{2s}^2}$ = impedance of the rotor circuit-bars and field combined

and

$Z_{bs} = \sqrt{x_b^2 + r_{bs}^2}$ = impedance of the bar circuit alone.

Then obviously

$$\frac{Z_{2s}}{Z_{bs}} = \frac{I_{bp}}{I_{2p}}$$

or

$$Z_{2s} = Z_{bs}\frac{I_{bp}}{I_{2p}} \qquad (\mathbf{51})$$

But from (**50**)

$$\frac{I_{2p}}{I_{bp}} = \sqrt{1 + \left(\frac{I_{fp}}{I_{bp}}\right)^2 + 2\left(\frac{I_{fp}}{I_{bp}}\right)\sin\varphi} \qquad (\mathbf{52})$$

Now let

$$V = \frac{I_{fp}}{I_{bp}} = \frac{Z_{bs}}{Z_{fs}} \qquad (\mathbf{53})$$

where

$$Z_{fs} = \sqrt{x_f^2 + r_{fs}^2}$$

Substituting (**53**) in (**52**) and then (**52**) in (**51**) gives

$$Z_{2s} = \frac{Z_{bs}}{\sqrt{1 + V^2 + 2\,V\sin\varphi}} \qquad (\mathbf{54})$$

which is the impedance of the rotor circuit with the field closed.

It can be seen from Fig. 14 that the phase angle of the rotor circuit has been increased by angle α, where

$$\alpha = \tan^{-1}\frac{V\cos\varphi}{1 + V\sin\varphi} \qquad (\mathbf{55})$$

so that, if θ is the phase angle of the rotor circuit with field closed,

$$\theta = \tan^{-1}\frac{x_b}{r_{bs}} + \tan^{-1}\frac{V\cos\varphi}{1 + V\sin\varphi} \qquad (\mathbf{56})$$

and hence

$$\left.\begin{aligned}r_{2s} &= Z_{2s}\cos\theta\\ x_2 &= Z_{2s}\sin\theta\end{aligned}\right\}\qquad(\mathbf{57})$$

Equation (**57**) gives the values of the rotor resistance

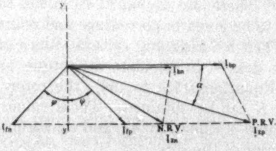

FIG. 14—SPACE DIAGRAM SHOWING CONTINUED REACTION OF FIELD AND FOUR CIRCUITS ON THE STATOR

and reactance with the field circuit closed. It can be seen from (**54**) that there is in general a decrease in the impedance of the rotor circuit when the field is closed. It is usually found that at starting, the rotor resistance is considerably decreased and the rotor reactance slightly increased. In some cases however, where the rotor bar impedance is very high—approaching that of the field—there is a very considerable increase in the reactance of the rotor circuit.

There is another effect of closing the field which is apparent from a study of Fig. 14. The value of K, which is a measure of the single-phase action in the rotor, is changed. Let the new value of K be K_c. It will be remembered that for the damper bars alone K is determined by the bar span and is equal to the ratio of $\frac{I_{bn}}{I_{bp}}$. Similarly, K_c is equal to the ratio of $\frac{I_{2n}}{I_{2p}}$ in Fig. 14.

From the geometrical relations in Fig. 14 it is easily determined that

$$K_c = \sqrt{\frac{K^2 + V^2 - 2\,K\,V \sin\varphi}{1 + V^2 + 2\,V \sin\varphi}} \qquad \textbf{(58)}$$

Now, the effect of closing the field circuit has been determined both upon the rotor resistance and reactance, and upon the single-phase action of the rotor. But it should be noted particularly that all of these equations contain the slip S, so that the effect of the field is different at every different value of slip. With the field open and only the damper bar circuit under consideration, it is possible to say the rotor resistance is this and the rotor reactance is that, but not so with the field closed. It is necessary to calculate the rotor resistance and reactance for every assigned value of slip from the combined effect of field circuit and bar circuit.

Part V. The Speed-Torque Curve—Pull-in Torque and In-rush.

The actual work involved in calculating a speed-torque curve is not so complicated or so great as might be assumed from the more or less elaborate theoretical investigation in the foregoing paragraphs. Especially is this true if the work is arranged in orderly fashion in a blank made for the purpose.

It is necessary to know all the constants of the machine, except the stator resistance which has been neglected. These are x_1, b_{00}, x_b, r_b, x_f, r_f.[7] They are, of course, to be given in percentage and referred to the stator. Then by assigning certain values of slip, for which it is desired to calculate the torque, and substituting in Equations (**53**) to (**58**), the rotor constants x_2, r_2, and K can be obtained. These values can then be substituted in Equation (**42**) and the torque obtained for each value of the slip.

The details of these calculations are made clear in Table I, where an actual example is worked out complete. The motor was rated 220 h. p., unity power factor, 2200 volts, three-phase, 60-cycles, 277 rev. per min. It had 1 $^{11}/_{13}$ slots per pole per phase in the stator and five ¼- by ¼-in. brass damper bars per pole in the rotor. It was designed with a short-circuit ratio of 1.00.

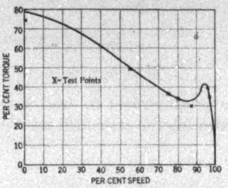

FIG. 15—COMPARISON OF TEST AND CALCULATED RESULTS ON 220-H. P., 277-REV. PER MIN., SYNCHRONOUS MOTOR

The actual values of the constants calculated for this machine were as follows:

$x_1 = 0.153$, $r_b = 0.384$, $r_f = 0.0097$ (not including any external resistance),

$b_{00} = 1.23$, $x_b = 0.184$, $x_f = 0.675$.

Fig. 15 shows the speed torque curve calculated in Table I. The test values are also indicated in Fig.15 by crosses. Both test and calculation were made with a starting resistance in the field circuit equal to twice that of the field resistance. The agreement between test and calculated values is unusually good, although the usual discrepancy does not exceed about 10 per cent.

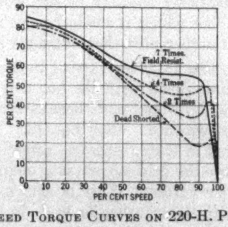

FIG. 16—SPEED TORQUE CURVES ON 220-H. P., 277-REV. PER MIN. MOTOR

Showing effect of different values of starting resistances in the field circuit, on the pull-in torque.

THE PULL-IN TORQUE

In Fig. 16 are shown several speed torques for the above motor calculated with different values of resistance in the field circuit. The one with two times the resistance of the field is the same as the curve in Fig. 15. It will be noted that all of the curves with values of starting resistance from 0 to 2 times show a distinct

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[7] It is not within the scope of this paper to present methods of calculating the several constants of the machine. The writer has found from experience that methods which give excellent results for one type of construction are not at all satisfactory for the other types. So much depends on the type of damper winding construction used, the method of making and connecting the damper end rings, and the type of damper slot, that no methods could be presented which would be universally applicable. These features vary so widely with the different manufacturers that it was thought better to present only the general theory and leave the calculation of the constants to the individual reader.

TABLE I

EXAMPLE CALCULATION OF SPEED-TORQUE CURVE

The last term in the denominator in equation (47) has been omitted in this calculation. This term will be found to be negligible.

	Slip = S	0.06	0.08	0.10	0.12	0.15	0.20	0.25	0.60	1.00*
	$r_{bs} = r_b/S$	6.40	4.80	3.84	3.20	2.56	1.92	1.54	0.64	0.384
	$Z_{bs} = \sqrt{r_{bs}^2 + x_b^2}$	6.40	4.80	3.84	3.20	2.56	1.92	1.54	0.69	0.425
	$r_{fs} = r_f/S$	0.485	0.364	0.291	0.242	0.194	0.145	0.116	0.048	0.0097
	$Z_{fs} = \sqrt{r_{fs}^2 + x_f^2}$	0.832	0.766	0.735	0.718	0.701	0.690	0.685	0.675	0.675
	$V = Z_{bs}/Z_{fs}$	7.70	6.27	5.23	4.46	3.65	2.78	2.25	0.965	1.63
(1)	x_b/r_{bs}	0.029	0.038	0.048	0.057	0.072	0.096	0.119	0.288	0.480
(2)	r_{fs}/x_f	0.719	0.540	0.431	0.359	0.287	0.215	0.172	0.072	0.014
(3)	$\tan^{-1}$ (1)	1.60	2.10	2.80	3.30	4.10	5.50	6.80	16.1	25.6
	$\tan^{-1}$ (2)	35.7	28.4	23.3	19.8	16.1	12.2	9.90	4.10	0.80
	$\varphi = \Sigma$:	37.3	30.5	26.1	23.1	20.2	17.7	16.7	20.2	26.4
(4)	$\cos \varphi$	0.795	0.861	0.898	0.920	0.938	0.952	0.958	0.938	0.895
(5)	$\sin \varphi$	0.606	0.507	0.440	0.392	0.345	0.304	0.287	0.345	0.444
(6)	$2\ V \sin \varphi$	9.33	6.35	4.60	3.50	2.52	1.69	1.29	0.688	0.56
	$1 + V^2$	60.0	40.2	28.4	21.0	14.3	8.72	6.06	1.93	1.39
(7)	Σ:	69.3	46.5	33.0	24.5	16.8	10.4	7.34	2.62	1.95
(8)	$\sqrt{(7)}$	8.33	6.82	5.75	4.95	4.10	3.23	2.71	1.62	1.40
	$Z_{2s} = Z_{bs}/(8)$	0.770	0.704	0.667	0.646	0.625	0.594	0.569	0.425	0.303
(9)	$1/V$ + (5)	0.736	0.666	0.631	0.616	0.619	0.664	0.732	1.38	2.03
	(4)/(9)	1.08	1.29	1.42	1.49	1.52	1.43	1.31	0.678	0.441
(10)	$\tan^{-1}$ (4)/(9)	47.2	52.2	54.9	56.1	56.7	55.0	52.7	34.2	23.8
	θ = (3) + (10)	48.8	54.3	57.7	59.4	60.8	60.5	59.5	50.3	49.4
	$\cos \theta$	0.658	0.583	0.534	0.509	0.488	0.492	0.507	0.639	0.651
	$\sin \theta$	0.752	0.812	0.845	0.860	0.873	0.870	0.861	0.769	0.759
	$r_{2s} = Z_{2s} \cos\theta$	0.507	0.410	0.356	0.329	0.305	0.292	0.288	0.271	0.198
	$x_2 = Z_{2s} \sin\theta$	0.578	0.571	0.564	0.556	0.545	0.516	0.490	0.326	0.230
	$V^2 + K_o^2$	59.3	39.5	27.7	20.3	13.6	8.04	5.38	1.25	0.72
	$K_o \times$ (6)	5.32	3.62	2.62	2.00	1.44	0.96	0.73	0.39	0.39
(11)	Diff.	54.0	35.9	25.1	18.3	12.2	7.08	4.65	0.86	0.40
	K_c^2 = (11)/(7)	0.78	0.77	0.76	0.75	0.72	0.68	0.63	0.33	0.205
	$B^2 \times Z_{2s}^2$	35.7	30.0	26.9	25.1	23.6	21.3	19.5	10.9	5.55
(12)	$2\ B x_2$	9.00	8.90	8.76	8.65	8.47	8.03	7.61	5.06	3.57
	$1 - K_c^2$	0.22	0.23	0.24	0.25	0.28	0.32	0.37	0.67	0.80
(13)	Σ:(3 terms above)	44.9	39.1	35.9	34.0	32.3	29.6	27.5	16.6	9.92
	(12) $\times K_c^2$	7.02	6.85	6.66	6.50	6.13	5.45	4.80	1.67	0.73
(14)	Diff.	37.9	33.3	28.2	27.5	26.2	24.2	22.7	14.9	9.19
(15)	$x_1^2 \times (13)^2$	47.2	35.8	30.0	27.0	24.4	20.6	17.7	6.50	2.30
	Tor. = r_{2s} (14)/(15)	40.7%	37.0%	33.5%	33.5%	32.7%	34.3%	37.0%	62.0%	79.0%

Constants—

$x_b = 0.184$ | $r_b = 0.384$

$r_f \times 3 = 0.02913$

$x_f = 0.675$ | $x_1 = 0.153$

$K_o = 0.57$ | $B = 7.77$

Machine

S. O. No. 013 E 47
H. P. 22
Amps. 48.3
Volts 2200
Phase 3
Cycles 60
R. P. M. 277

*Calculated with field dead short-circuited Test which was used in finding starting torque was with field dead short-circuited.

cusp in the curve. But by increasing the starting resistance up to seven times the cusp disappears entirely and the speed torque curve assumes quite a different character. So it is possible to distinguish at least two distinct types of speed torque curves, those with a cusp and those without a cusp, and this presents a real difficulty in attempting to define the pull-in torque.

In the case of a curve with a cusp it is quite obvious that if the motor has sufficient torque to accelerate the load up through the low point of the cusp, it will pull into step because beyond the cusp the motor torque increases rapidly to a point very close to synchronism, usually within 2 or 3 per cent. Hence, the motor torque at the low point of the cusp becomes the pull-in torque of the motor, and there is no difficulty in defining pull-in torque for this type of curve. However, if there is no cusp, as in the case of the curve with six times field resistance, the problem becomes one of determining at what slip the motor will pull in; knowing the slip, the torque is given by the speed-torque curve. But to calculate the slip at which it will pull in is a difficult problem and one which has been the subject of much theoretical investigation and discussion[8]. Experience shows, however, that an ordinary machine may be expected to pull in at a slip of 6 or 7 per cent, while there are rare cases on record where it has been necessary to bring the machine up to within 3 per cent before it would pull in. In such cases, however, there has usually been a large flywheel on the driven machine which cannot properly be considered as part of the motor, and it has been this flywheel which has been responsible for the low value of slip required for pull-in. So far as the definition of pull-in torque is concerned, the best one can do is to give the motor torque at some slip, say from 5 to 7 per cent, and state the slip at which it is given.

FORMULA FOR PULL-IN TORQUE

It is evident from what has been said that it is not possible to give an exact formula for the pull-in torque of a motor and the only correct way to obtain it is to calculate the speed torque curve. It is possible, however, to derive an approximate formula which will be found convenient for the designer. That portion of the speed torque in the neighborhood of synchronous speed may be considered as made up of two parts more or less independent, one due to the damper bars and one to the field winding. These two parts are shown in Fig. 17, part *a* due to the field winding and part *b* due to the damper bars. The dotted curve shows the combined effect of both.

Now the curve *a* in the neighborhood of synchronous speed is given approximately by formula (**43**), if $x_2 = x_f$ and $r_2 = r_{fs}$. The maximum point P can, therefore, be obtained from Equation (**45**) as follows:

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[8] See references listed under item 6, in Bibliography.

$$T_{max.} \text{ at point } P = \frac{0.5}{B\, x_1^2\, (2 + B\, x_f)} \qquad (\mathbf{59})$$

and from this formula one is able to estimate the pull-in torque from the proportion of the various constants and a knowledge of the starting resistance to be used in the field circuit. The approximate formula for pull-in torque may be written

$$T_{pull\text{-}in} = \frac{0.4 \text{ to } 0.8}{B\, x_1^2\, (2 + B\, x_f)} \qquad (\mathbf{60})$$

Experience shows that if the rotor bar resistance is about equal to the total reactance (rotor and stator) of the machine and a starting resistance equal to twice the field resistance is used a value as low as 0.4 may be expected in the numerator, while, if the rotor bar resistance is equal to about half the total reactance of the machine and a starting resistance equal to 4 or 5 times the resistance of the field (which will usually eliminate the cusp in the torque curve completely) is used, then a value of 0.8 can often be obtained.

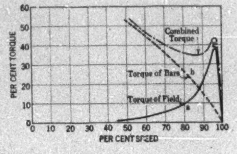

FIG. 17—CURVES SHOWING HOW THE TORQUE OF THE BARS AND THE TORQUE OF THE FIELD COMBINE TO GIVE THE PULL-IN TORQUE

THE IN-RUSH CURRENT

It is comparatively easy to derive from Equations (**20**) to (**34**) a formula for the in-rush—in fact, two formulas, one for the positive current I_{1p} and one for the negative current I_{1n}. From these two values the unbalanced three-phase currents in each of the three lines can be determined. This unbalance is usually small and both the formulas for I_{1p} and I_{1n} are rather involved and cumbersome. No attempt is made to guarantee anything but an average value of the in-rush in each of the three lines. The writer has found that this average in-rush can be calculated with sufficient accuracy from the induction motor formula,

$$I_s = \frac{\sqrt{1 + 2\, b_{00}\, x_2}}{\sqrt{r_0^2 + x_0^2}} \qquad (\mathbf{61})$$

where

$r_0 = r_1 + r_2$
r_1 = stator resistance
r_2 = rotor resistance (combined effect of bars and field winding)
$x_0 = x_1 + x_2$
x_1 = stator reactance
x_2 = rotor reactance (combined effect of bars field winding).

If the machine is started with the field open one should use r_b the resistance of the bars in place of r_2, and x_b the bar reactance in place of x_2.

When the calculation is made with field closed, formula (**61**) usually gives values slightly higher than test so that they are conservative, while if the field is open it is apt to give values too low.

For the 220-h. p. motor calculated above, the in-rush obtained from formula (**61**) is

$$I_s = \frac{\sqrt{1.579}}{\sqrt{0.228^2 + 0.388^2}} = 2.8^{9} = 120.5 \text{ amperes}$$

Test = 118 amperes

With field open

$$I_s = \frac{\sqrt{1.452}}{\sqrt{0.422^2 + 0.337^2}} = 2.07 = 89.2 \text{ amperes}$$

Test = 96 amperes

Part VI. Characteristics of Double Squirrel-Cage Windings in Salient-Pole Synchonous Motors

Any definite pole synchronous machine contains the necessary elements of the double squirrel-cage winding. The ordinary damper bars form a rotor circuit of high resistance and low reactance, while the field winding forms a circuit of low resistance and high reactance. At starting, the high damper bar circuit provides high starting torque and low in-rush because the resistance of the damper bars enters potentially into the impedance which limits the in-rush. This results in a high power factor at starting and consequently high torque for the kv-a. required. As the machine approaches full speed the field winding comes into play, and maintains a comparatively high value of torque up to within a few per cent of synchronism.

A machine designed with a low reactance, high resistance damper winding and provided with a large enough starting resistance in series with the field winding to remove the cusp from the speed torque curve, may be said to possess a double squirrel-cage rotor of a most effective type. It gives the usual double squirrel-cage characteristics as they are known in induction motor practise; low in-rush, high starting torque and comparatively high torque at low values of slip. These characteristics are well illustrated in Fig. 16 by the speed torque curve for the case of seven times resistance in the field circuit.

But when an actual double squirrel-cage winding is

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[9] This value is given in percentage based on the output current which is 43.1 amperes. The in-rush in amperes is 2.8 × 43.1 = 120.5 amperes.

put in the pole faces it is rather difficult to produce characteristics of this type. One difficulty the designer encounters is the limited depth of slot he may use in the head of a synchronous motor pole. The depth of the pole head cannot be increased appreciably without an excessive increase in the field leakage, and it is well known that in order to obtain any appreciable double squirrel-cage effect, that is, any appreciable change in the bar resistance with the frequency, a reasonable amount of slot depth is absolutely essential.

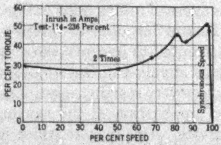

Fig. 18—Test Results on 225-H. P., 257-Rev. per Min. Synchronous Motor Equipped with Double Squirrel-Cage Damper Winding, Showing Typical Double Squirrel-Cage Characteristics.

It is also well known that the double squirrel-cage effect obtained depends on the resistance of the material used in the bar, the lower the resistance the greater being the effect. In other words, in order to obtain any appreciable decrease in the resistance of the bar as the frequency decreases (this is the object of the double squirrel-cage construction) it is necessary to employ a material with as low a specific resistance as possible, and hence copper. Especially is this true if the depth of the slot is limited. These facts are illustrated very clearly in Equation (**34**) of Appendix II, where a simple method is worked out for the calculation of the inverted *T*- or *L*-shaped bar. Equation (**34**) shows that the modulus of the vector upon which the double squirrel-cage effect or skin effect depends, involves the factor

$$a\sqrt{\frac{f}{\sigma}}$$

where a is the depth of the bar, (actually half of the depth).

f is the frequency

and

σ is the specific resistance of the material of the bar. Thus, it is seen that if the depth of the bar could be doubled the same skin effect would be obtained as with the original bar at four times the frequency. This shows clearly the handicap of limited depth. Also, it is evident that increasing the bar resistance has exactly the same effect as decreasing the frequency so that if the material of the bar were changed from copper to brass the same skin effect would be obtained as with the copper bar at a frequency of $\frac{60}{3.3} = 18.2$ cycles.[10] This simply means that with the limited depth available in the head of a synchronous motor pole a brass winding gives practically no skin effect.

There is also another point to be borne in mind. Any double squirrel-cage winding, because of the depth of the slot is bound to have a considerably higher reactance than a single cage winding of the same resistance.

The conclusion, therefore, which has been reached without making any figures at all, is that a synchronous motor equipped with a double squirrel-cage damper winding will inherently have a rotor bar circuit of low resistance and high reactance and hence poor power factor. The power factor will be further decreased by closing the field circuit. The in-rush will be limited almost entirely by the reactance of the machine, and may be expected to be low due to the excessive bar reactance. The starting torque will be comparatively low due to the low damper bar resistance and the poor power factor. The pull-in torque will be comparatively high and there will probably be very little cusp in the speed-torque curve even with a low value of starting resistance in the field circuit. It would be difficult, perhaps impossible, to design a double squirrel-cage winding in a synchronous motor so that the starting torque was equal to, or at least appreciably greater than, the pull-in torque.

Of course, there are some special applications such as pumps and fans, where low starting torques and high pull-in torques are desirable and for these the double squirrel-cage synchronous motor is especially suited. For most applications, however, it is better to have a motor with higher starting torque than pull-in torque and this, as has been pointed out, is a condition which would be very difficult to obtain with a double squirrel-cage winding.

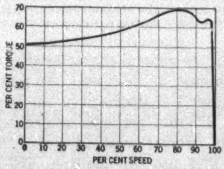

Fig. 19—Calculated Results of 220-H. P., 277-Rev. per Min. Synchronous Motor Equipped with Double Squirrel-Cage Copper Damper Bar of the Inverted T Type

Experimental confirmation of these conclusions is presented in Fig. 18 which gives a speed torque curve plotted from test results on a commercial synchronous motor equipped with a double squirrel-cage damper winding. This machine was rated 225 h. p., 257 rev. per min., 2200 volts, three-phase, 60 cycles, unity power factor. The characteristics of low in-rush, low starting torque and high pull-in torque are clearly shown.

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[10] Brass has a resistance of approximately 3.3 times that of copper at 100 deg. cent.

Theoretical confirmation of these conclusions is found in Appendix II where the resistance and reactance of a double squirrel-cage bar are calculated. This bar is so designed that it could be used in the 220-h. p., 277-rev. per min. motor calculated in Table I. The bar is $^9/_{16}$ in. deep overall, $^1/_{16}$ in. wide at the top, and $^3/_{16}$ in. wide at the bottom. It is made narrow in order to keep the resistance as high as possible and still use copper. Likewise, the bar extends clear to the top of the pole face to keep the reactance low. Both of these features combine to make the power factor of the bar as high as possible and yet it is only 0.46 as compared with 0.84, the power factor of the plain square brass bar. These values are shown in Table III, Appendix II.

A complete calculation of the performance of the above synchronous motor (220 h. p., 227 rev. per min.) when equipped with this double squirrel-cage bar has been made. The speed-torque curve and other data are given in Fig. 19. It will be noted that the performance is excellent, but the general characteristics of the double squirrel-cage motor, as discussed, are apparent.

TABLE II

TABLE SHOWING COMPARATIVE PERFORMANCE OF SINGLE SQUIRREL-CAGE AND DOUBLE SQUIRREL-CAGE MOTORS

Motor	Starting torque per cent	Pull-in torque per cent	In-rush per cent	Starting torque per kv-a.	Pull-in torque[11] per kv-a.	Avg. torque per kv-a.
225 h. p., 257 rev. per min., with double squirrel-cage—test results	28	52	236	0.118	0.220	0.169
220 h. p., 277 rev. per min., with plain brass dampers—test results	75	50[12]	246	0.304	0.203	0.253
220 h. p., 277 rev. per min., with double squirrel-cage T-shaped copper bars—calculated results	50	63	303	0.164	0.208	0.186

[11] It is unusual to state the pull-in torque per in-rush kv-a. but it is convenient to do so in comparing the merits of damper windings, in different machines having different values of stator reactance. The 225-h. p. motor had considerably higher stator reactance than did the 220-h. p. machine. This accounts for its lower values of torque and in-rush in percentage.

[12] Calculated value with starting resistance equal to seven times the field resistance.

Table II shows a comparison of the performance of the 220-h. p., 277-rev. per min. motor equipped with both the brass damper winding and with the double squirrel cage, and also the 225-h. p., 257-rev. per min. motor tested with the double squirrel-cage winding.

Part VII. Possibilities of External Damper Bar Circuit.

To obtain synchronous motor starting performance nearly equivalent to slip ring induction motor characteristics has probably been a secret ambition cherished in the heart of every engineer who has wrestled with the starting problem in salient-pole motors in the last two decades. The fact that this problem has not been solved in a commercial way may indicate either one of two things; either the problem is extremely difficult, or else there has not, up to the present time, been a sufficient market for motors of such excellent starting characteristics. However, it has been felt that if a motor with these characteristics could be developed, the field of application of the synchronous motor could be greatly extended, and it is probably this feeling which continues to revive interest in this problem.

Perhaps this same feeling has been responsible for the development and use of the synchronous induction motor in Europe. The high cost, low efficiency, and low pull-out torque of this type of machine has been largely responsible for its very meagre use in this country. Still the problem of obtaining the excellent starting performance of the slip ring motor and maintaining the excellent running performance of the salient-pole synchronous motor has remained unsolved.

Comparatively recently, the synchronous motor has found some new applications in steel mill service for driving certain types of rolling mills. For some of these applications the starting duty is severe and manufacturers have attempted to improve the sturdiness of the damper windings supplied in their motors with a view to meeting these severe starting conditions and extending the field of application of the synchronous motor in steel mill service. One feature which is limiting the application of these motors in steel mills is the low torque per kv-a. obtained in ordinary low speed synchronous motors, and consequently the high in-rush required. Another difficult feature is the absolute reliability required. In some cases manufacturers have been asked to guarantee continued operation on the damper winding for an indefinite period, in case of d-c. failure. This is a difficult requirement because the rolling peaks are usually much higher than the rating of the motor and this means that the machine must have a pull-out torque as an induction motor practically equal to its pull-out torque under synchronous operation. In other cases, it has been necessary to guarantee several starts in succession and the problem of absorbing the heat generated in the damper winding has become difficult. Perhaps here in this new application of synchronous motors an opportunity will be afforded for some entirely new developments in the starting arrangements of synchronous motors.

In this connection an investigation was made to determine the possibilities of an external damper bar circuit used in conjunction with a very high starting resistance in the field circuit—a resistance equal to or greater than the reactance of the field winding at full slip. Mr. Fechheimer[13] pointed out in 1912 the increased starting torque and the improved power factor at starting which results from the use of a high resistance of this type in the field circuit. These advantages would obviously be augmented by making the damper bar circuit with as high a power factor as possible also, that is, with high resistance and low reactance. The low reactance is also necessary for high pull-out torque as an induction motor, but the high resistance must be arranged external to the damper bar circuit so that it can be short-circuited for running as an induction motor. This is necessary because a very low value of damper bar resistance is necessary to maintain a low value of slip at full load and to prevent the machine from slowing down too much on the peak loads.

The simplest type of circuit which would provide an external resistance in series with the damper winding (this is probably a very old idea) is shown in Fig. 20.

An analysis of the possibilities of this circuit showed, as was expected, that the power factor of the damper winding could be increased by increasing the external resistance up to a certain point, but beyond that point the power factor began to decrease again. This was to be expected because in the limiting case of an infinite resistance it is evident that the power factor of the damper winding would be very poor with very low resistance bars. This simply means that the amount of power which could be drawn out of the rotor circuit into the external resistor is limited.

The analysis also showed that if the external circuit were split up into two or more circuits, as shown in Fig. 21, it would be possible to insert enough resistance to obtain a very good power factor in the damper winding before the limiting resistance is reached.

The mechanical construction of the end ring connections did not seem to present any serious difficulties. In attempting to apply this scheme to an actual design of a 2500-h. p., 62½-rev. per. min., 25-cycle synchronous motor it was found that heavy brass slip rings and metallic brushes, such as are used on rotary converters, were required. A difficulty was also encountered in connection with the resistor for the field circuit. It was found that a very high voltage would be developed across the resistor and consequently across the slip rings at start. This was sufficiently reduced by designing the field for 50-volt excitation instead of 125. The calculated performance of the machine showed that it would carry full load continuously as an induction motor with a power factor of approximately 50 per cent. Its pull-out torque was approximately 225 per cent as an induction motor and 250 per cent as a synchronous motor. The starting torque depended upon the external resistances used, but it was found possible to obtain 150 per cent starting torque with an in-rush of 300 per cent. The pull-in torque at five per cent slip was 175 per cent.

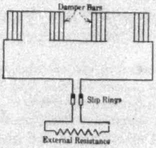

FIG. 20—SINGLE EXTERNAL DAMPER BAR CIRCUIT

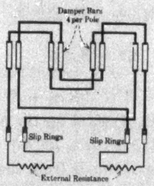

FIG. 21—DOUBLE EXTERNAL DAMPER BAR CIRCUIT

These figures seem to indicate great possibilities in the improvement of the starting performance of synchronous motors. No doubt developments to this end will take place rapidly if there becomes sufficient demand to warrant them.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the assistance of Mr. E. S. Perrine and Mr. H. R. Goss in making calculations and preparing data for the manuscript. Credit is also due to Mr. Q. Graham for many helpful suggestions in connection with the vector diagrams in Figs. 10 and 11, and also to Mr. M. L. Annappa who has contributed some valuable suggestions in regard to the theoretical work in Appendix II. Item 6 of the bibliography is due to Mr. Graham.

Appendix I

To show that the numerical value of the vector expression

$$\frac{(\underset{\cdot}{V} + B Z_{2s}^2)^2 - K^2 \underset{\cdot}{V}^2}{\underset{\cdot}{V} + B Z_{2s}^2} \qquad (1)$$

is

$$\frac{Z_2}{z} \sqrt{(z^2 - K^2)^2 - (2 K B r_{2s})^2} \qquad (2)$$

---

[13] See Bibliography, 2.

let

$$\left.\begin{aligned} Z_{2s} &= \text{modulus of } \dot{V} \\ M &= \text{modulus of } \dot{V} + B\,Z_{2s}^2 \\ \theta_1 &= 2\tan^{-1}\frac{r_{2s}}{x_2} = \tan^{-1}\frac{2\,r_{2s}\,x_2}{x_2^2 - r_{2s}^2} \\ \theta_2 &= 2\tan^{-1}\frac{r_{2s}}{x_2 + B\,Z_{2s}^2} \\ &= \tan^{-1}\frac{2\,r_{2s}\,(x_2 + B\,Z_{2s}^2)}{(x_2 + B\,Z_{2s}^2)^2 - r_{2s}^2} \end{aligned}\right\} \qquad (3)$$

The numerator of (1) may be written

$$M^2 \underline{/\theta_2} - K^2\,Z_{2s}^2 \underline{/\theta_1} \qquad (4)$$

But

$$M^2 = Z_{2s}^2\,z^2 \qquad (5)$$

where

$$z^2 = (1 + 2\,B\,x_2 + B^2\,Z_{2s}^2)$$

Substituting (5) in (4) gives for the numerator

$$Z_{2s}^2\,[z^2 \underline{/\theta_2} - K^2 \underline{/\theta_1}] \qquad (6)$$

The magnitude of this is from the cosine law

$$Z_{2s}^2\,\sqrt{z^4 + K^4 - 2\,z^2\,K^2\,(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)} \qquad (7)$$

But

$$\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = \frac{(x_2^2 - r_{2s}^2)\,[(x_2 + B\,Z_{2s}^2)^2 - r_{2s}^2] + 4\,r_{2s}^2\,x_2(x_2 + B\,Z_{2s}^2)}{Z_{2s}^4\,z^2} \qquad (8)$$

which reduces to

$$\frac{1 + 2\,B\,x_2 + B^2\,(x_2^2 - r_{2s}^2)}{z^2} \qquad (9)$$

Substituting (9) in (7) gives:

$$Z_{2s}^2\,\sqrt{(z^2 - K^2)^2 - (2\,K\,B\,r_{2s})^2} \qquad (10)$$

as the numerical value of the numerator.

The numerical value of the denominator of (1) is $Z_{2s}\,z$ so that the numerical value of the whole vector expression (1) is

$$\frac{Z_{2s}}{z}\,\sqrt{(z^2 - K^2)^2 - (2\,K\,B\,r_{2s})^2} \qquad (11)$$

Q. E. D.

Appendix II[14]

RESISTANCE AND REACTANCE OF AN INVERTED *T*- OR *L*-SHAPED DAMPER BAR

Consider a slot which is narrow at the top and wide at the bottom, shaped either like an inverted *T* or an *L* (Fig. 22).

Let

σ = specific resistance of the bar,

$f(x)$ = distribution of current in bottom part, x being measured from the bottom of the slot,

$F(x)$ = distribution of current in top part, x being measured from the division line 0 − 0.

The distribution of current in the wide part of the bar will be calculated first[15]. Consider the flux through an element $d\,x$ due to the m. m. f. in an element $d\,u$. This m. m. f. is:

$$b_1\,f(u)\,d\,u \qquad (1)$$

Therefore

$$d\,[d\,\varphi] = \frac{4\,\pi\,b_1\,f(u)\,d\,u}{b_1/d\,x} = 4\,\pi\,f(u)\,d\,u\,d\,x \qquad (2)$$

Now, the total flux through the element $d\,x$ is due to all such elements as $d\,u$ from the bottom of the slot up to the element $d\,x$. Thus:

$$d\,\varphi = 4\,\pi\int_0^x f(u)\,d\,u\,d\,x \qquad (3)$$

which is the total flux passing through any element $d\,x$ at distance x from the bottom of the slot.

The flux which produces the e. m. f. of self-induction at any point x in the conductor is all the flux which threads through the conductor from the top of the conductor down to the point x, plus the flux in the wedge and air-gap. Let Φ be the self-inductive flux in the wedge, air-gap and that which threads through the narrow part of the bar down to the division line 0 − 0. Then the self-inductive flux at point x is

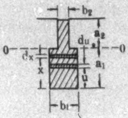

FIG. 22

$$\varphi = 4\,\pi\int_x^a\int_0^x f(u)\,d\,u\,d\,x + \Phi$$

and the e. m. f. of self-induction is

$$p\,\varphi = 4\,\pi\,p\int_x^a\int_0^x f(u)\,d\,u\,d\,x + p\,\Phi \qquad (4)$$[16]

---

[14] Anyone interested in the calculation of double squirrel cages for synchronous motors will find App. II of especial interest. This problem can be handled by differential equations but the method of integral equations here presented will be found simpler and more straightforward and gives directly both the resistance and reactance of the bar. The problem of "setting up" the integral equation is much easier than the corresponding problem of "setting up" the differential equation. Especially is this true for those of us who have been taught the fundamentals of inductance from the point of view of interlinkages. The same integral equation can be applied to the calculation of a bar comprised of two different materials either T-shaped or of uniform width.

[15] All this work is done in absolute electromagnetic units for simplicity.

[16] p = the operator $\dfrac{d}{d\,t}$ and denotes differentiation with respect to time.

Since the induced voltage is the same at all points in the cross-section of the bar, and is equal to the e. m. f. consumed by resistance and self-induction, it follows that the sum of the e. m. fs. consumed by resistance and self-induction is the same in all parts of the bar. Hence,

$$\sigma f(x) + 4\pi p \int_x^a \int_0^x f(u)\, du\, dx + p\Phi$$

$$= \sigma f(0) + 4\pi p \int_0^a \int_0^x f(u)\, du\, dx + p\Phi \qquad (5)$$

$p\Phi$ cancels out and the equation becomes

$$\sigma f(x) = \sigma f(0) + 4\pi p \left\{ \int_x^a \int_0^x f(u)\, du\, dx - \int_x^a \int_0^x f(u)\, du\, dx \right\} \qquad (6)$$

or

$$f(x) = f(0) + \frac{4\pi p}{\sigma} \int_0^x \int_0^x f(u)\, du^2 \qquad (7)$$

This is a simple integral equation and can be solved by the method of continued substitution. Substituting the right hand member of Equation (7) under the integral sign and putting $K^2 = \dfrac{4\pi p}{\sigma}$ Equation (7) becomes

$$f(x) = f(0) + K^2 \int_0^x \int_0^x [f(0) + K^2 \int_0^x \int_0^x f(u)\, du^2]\, du^2 \qquad (8)$$

$$= f(0) + K^2 f(0) \frac{x^2}{\underline{|2}} + K^4 \int_0^x \int_0^x \int_0^x \int_0^x f(u)\, du^4 \qquad (9)$$

Substituting again the right hand member of (7) under the integral sign in (9) and continuing the integration and repeating the process gives at once the infinite series:

$$f(x) = f(0) + K^2 f(0) \frac{x^2}{\underline{|2}} + K^4 f(0) \frac{x^4}{\underline{|4}} + \qquad (10)$$

or

$$f(x) = f(0) \cosh x K \qquad (11)$$

This is the current density in the wide portion of the bar, as function of the distance from the bottom of the bar.

Let

I_1 = current in the wide portion of the bar.

Then

$$I_1 = b_1 \int_0^{a_1} f(x)\, dx = \frac{b_1 f(0)}{K} \sinh a_1 K \qquad (12)$$

Now the flux which passes through an element dx in the top of the slot is due to the total current in the lower part I_1, and also to the m. m. f. in the top part from the division line 0 — 0 up to the element dx.

This m. m. f. is

$$b_2 \int_0^x F(u)\, du \qquad (13)$$

The flux through dx is, therefore:

$$d\varphi = \frac{4\pi \{I_1 + b_2 \int_0^x F(u)\, du\}\, dx}{b_2} \qquad (14)$$

The self-inductive flux at the point x is

$$\varphi = 4\pi \int_x^{a_2} \left\{ \frac{I_1}{b_2} + \int_0^x F(u)\, du \right\} dx \qquad (15)$$

and the e. m. f. of self-induction is

$$p\varphi = 4\pi p \int_x^{a_2} \left\{ \frac{I_1}{b_2} + \int_0^x F(u)\, du \right\} dx \qquad (16)$$

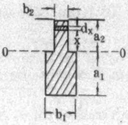

Fig. 23

Since the induced voltage is the same in all parts of the bar, it follows that

$$\sigma F(x) + 4\pi p \int_x^{a_2} \left\{ \frac{I_1}{b_2} + \int_0^x F(u)\, du \right\} dx$$

$$= \sigma F(0) + 4\pi p \int_0^{a_2} \left\{ \frac{I_1}{b_2} + \int_0^x F(u)\, du \right\} dx \qquad (17)$$

or

$$\sigma F(x) = \sigma F(0) + 4\pi p \int_0^x \left\{ \frac{I_1}{b} + \int_0^x F(u)\, du \right\} dx \qquad (18)$$

Therefore,

$$F(x) = F(0) + \frac{K^2 I_1 x}{b_2} + K^2 \int_0^x \int_0^x F(u)\, du^2 \qquad (19)$$

Using the same method of solution for this equation as was used for Equation (17), it easily reduces to

$$F(x) = F(0) \cosh x K + \frac{I_1}{b_2} K \sinh x K \qquad (20)$$

But

$$F(0) = f(a_1) = f(0) \cosh a_1 K \qquad \textbf{(21)}$$

Substituting (**21**) and (**12**) in Equation (**20**)

$$F(x) = f(0) \left[\cosh a_1 K \cosh x K + \frac{b_1}{b_2} \sinh a_1 K \sinh x K \right] \qquad \textbf{(22)}$$

This is the distribution of the current in the top of the slot as function of the distance x from the division line $0 - 0$. The total current flowing in the top is:

$$I_2 = b_2 \int_0^{a_2} F(x)\, d x$$

$$= b_2 \left[\frac{F(0)}{K} \sinh x K + \frac{I_2}{b_2} \cosh x K \right]_o^{a_2}$$

$$= \frac{F(0)\, b_2}{K} \sinh a_2 K + I_1 \cosh a_2 K - I_1 \qquad \textbf{(23)}$$

Substituting for $F(0)$ from (**21**) and for I_1 from (**12**) gives

$$I_2 = \frac{f(0)}{K} \{ b_2 \cosh a_1 K \sinh a_2 K + b_2 \sinh a_1 K \cosh a_2 K \} - I_1 \qquad \textbf{(24)}$$

The total current $I_0 = I_1 + I_2$ is, therefore,

$$I_0 = \frac{f(0)}{K} \{ b_2 \cosh a_1 K \sinh a_2 K + b_2 \sinh a_1 K \cosh a_2 K \} \qquad \textbf{(25)}$$

The voltage which produces this current is

$$E = \frac{4 \pi p I_0}{\rho} + \sigma F(a_2) \qquad \textbf{(26)}$$

where ρ is the reluctance of the leakage path above the conductor.

The effective impedance of the bar is, therefore,

$$Z = \frac{E}{I_0} = \frac{1}{I_0} \left[\frac{4 \pi p I_0}{\rho} + \sigma F(a_2) \right] \qquad \textbf{(27)}$$

$$Z = \frac{4 \pi p}{\rho} + \frac{\sigma K}{b_2} \left[\frac{\cosh a_1 K \cosh a_2 K + W \sinh a_1 K \sinh a_2 K}{\cosh a_1 K \sinh a_2 K + W \sinh a_1 K \cosh a_2 K} \right] \qquad \textbf{(28)}$$

where

$$W = \frac{b_1}{b_2}$$

This vector expression is the impedance of a T-shaped bar per unit length. The real part is the resistance, the j part, the reactance. Obviously the first term $\frac{4 \pi p}{\rho}$ is a pure reactance term and is the reactance of the bar due to the leakage flux in the slot above the bar and in the air-gap.

As a special case, let $a_1 = a_2$ which means that the narrow portion and wide portion are of the same depth. Neglect the first term. Then

$$Z = \frac{\sigma K}{b_2} \left[\frac{\cosh^2 a K + W \sinh^2 a K}{(1 + W) \sinh a K \cosh a K} \right] \qquad \textbf{(29)}$$

which reduces easily to

$$Z = \frac{\sigma a K}{a b_2} \left[\frac{Q + \cosh 2 a K}{\sinh 2 a K} \right] \qquad \textbf{(30)}$$

where

$$Q = \frac{1 - W}{1 + W} = \frac{b_2 - b_1}{b_2 + b_1} \qquad \textbf{(31)}$$

But

$$a K = a \sqrt{\frac{4 \pi p}{\sigma}} = a \sqrt{\frac{j 8 \pi^2 f}{\sigma}}$$

$$= 2 \pi a \sqrt{\frac{f}{\sigma}} (1 + j) \qquad \textbf{(32)}$$

Or

$$a K = A + j A \qquad \textbf{(33)}$$

where

$$A = 2 \pi a \sqrt{\frac{f}{\sigma}} \qquad \textbf{(34)}$$

Substituting (**33**) in (**30**) gives

$$Z = \frac{\sigma (A + j A)}{a b_2} \left[\frac{Q + \cosh 2 (A + j A)}{\sinh 2 (A + j A)} \right] \qquad \textbf{(35)}$$

This can be expanded to

$$Z = \frac{\sigma (A + j A)}{a b_2} \left[\frac{Q + \cosh 2 A \cos 2 A + j \sinh 2 A \sin 2 A}{\sinh 2 A \cos 2 A + j \cosh 2 A \sin 2 A} \right] \qquad \textbf{(36)}$$

This expression is simple enough to use for practical calculations.

As an example consider the calculation of the resistance and reactance of the copper bar shown in Fig. 23 which is about as large as one could use in a small size synchronous motor pole. The width of the bar has been made as small as is mechanically desirable, in order to keep the resistance as high as possible. The over-all depth of the bar is $\frac{9}{16}$ in., width at top $\frac{1}{16}$ in., at bottom $\frac{3}{16}$ in. The dimensions on Fig. 23 are given in cms. as the calculation must be made in absolute units.

σ for cu. at 100 deg. cent. = 2240 abohms per cu. cm. at 60 cycles.

$$A = 2\,\pi \times 0.715\,\overline{\sqrt{60 \div 2240}} = 0.736,\ 2\,A = 1.472$$

$$Q = \frac{0.159 - 0.477}{0.159 + 0.477} = -\ 0.50$$

$$\frac{\sigma(A + j\,A)}{a\,b_2} = \frac{2240\ (0.736 + j\,0.736)}{0.715 \times 0.159} = 20{,}500\underline{/45}\text{ deg.}$$

$$\begin{aligned}
\cosh 1.472 &= 2.294\\
\sinh 1.472 &= 2.064\\
\cos 1.472 &= 0.0986\\
\sin 1.472 &= 0.995\\
Q + \cosh 2\,A \cos 2\,A &= -\ 0.2735\\
j \sinh 2\,A \sin 2\,A &= j\,2.055\\
\text{Numerator} &= 2.065\underline{/97.6}\text{ deg.}\\
\sinh 2\,A \cos 2\,A &= 0.204\\
j \cosh 2\,A \sin 2\,A &= j\,2.284\\
\text{Denominator} &= 2.29\ \underline{/84.5}\text{ deg.}
\end{aligned}$$

Therefore,

$$Z = \frac{20{,}500 \times 2.065}{2.29}\ \underline{/45}\text{ deg.} + 97.6\text{ deg.} - 84.5\text{ deg.}$$

or

$Z = 18{,}500\ \underline{/58.1}$ deg.

and hence

$\therefore\ r$ eff. = 18,500 cos 58.1 deg. = 9750 abohms/cm. length

x eff. = 18,500 sin 58.1 deg. = 15,700 abohms/cm. length

On the basis of uniform current distribution the "nominal" reactance is

$$X = 17{,}750 \text{ abohms per cm.}$$

and the d-c. resistance is 4920. The ratio of $\dfrac{r\text{ eff.}}{r_{\text{d-c.}}}$ = 1.98 so that the T-shaped copper bar is an effective double squirrel-cage bar, even with a depth of only ⁹⁄₁₆ in.

It is interesting to compare these values of resistance and reactance with those of the ¼-in. square brass damper bar used in the 220-h. p., 277-rev. per min. synchronous motor calculated in Table I in the body of the paper, and to see what the characteristics of this motor would be if it were equipped with this double squirrel-cage damper bar.

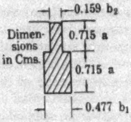

FIG. 24

Fig. 25 shows the dimensions of the ¼-in. by ¼-in. square slot actually used in the motor. Dimensions are given in inches. D-c. resistance at 100 deg. cent. = 18,400 abohms per cm. length.

Reactance = 8900 abohms per cm. length.

In addition to the slot reactance there is a tooth tip leakage reactance which in this case amounts to 3000 abohms per cm. This reactance corresponds to the first term in Equation (**28**). It must be added to the reactance of both types of bar. Table III gives a comparison of the resistance and reactance of the two bars at 60 cycles, taking the tooth tip leakage into account.

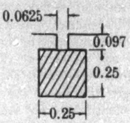

FIG. 25

TABLE III
TABLE SHOWING RESISTANCE, REACTANCE, AND POWER FACTOR OF PLAIN BAR AND DOUBLE SQUIRREL-CAGE BAR

Type of bar	60-Cycle resistance	60-Cycle reactance	P. F.
Plain square brass bar........	18,400	11,900	0.84
Double cage copper bar.......	9,750	18,700	0.46

All values given in abohms per cm.

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Discussion

R. H. Park: In the treatment of a complex problem such as the starting performance of a synchronous motor, it is necessary to employ simplifying assumptions to facilitate the calculation. At the same time, it is desirable to keep in mind just what assumptions have been employed.

On studying Mr. Putman's paper, I listed the assumptions that I found in it as follows: First, that the machine has a uniform air-gap. Second, that rotor bar reactance and resistance is equal for all rotor bars. Third, that only space fundamental of air-gap flux is considered. Actually, the rotor currents will produce a considerable amount of flux that is not space fundamental. This flux will be leakage reactance flux and will have a good deal to do in the determination of the distribution of rotor-bar currents. Fourth, the numerical value of all rotor-bar currents is assumed equal, even with the field closed. Fifth, the electrical phase angle in time of the rotor bar currents is assumed equal to their electrical space separation. This would be true in an induction motor, but actually, on account of the non-space fundamental air-gap flux, that is, leakage reactance flux, the phase angles will be different. Sixth, the effect of armature resistance is neglected. The effect of armature resistance will be important in determining the torque at half speed. In view of the approximations involved in these assumptions, it is, I think, particularly interesting that Mr. Putman is able to secure results which check tests.

P. L. Alger: Mr. Putman's ways of taking into account the width of pole arc of the machine, and the single-phase reaction of the field winding, are very interesting. And the close checks he gets with test results indicate that his method is at least approximately correct.

However, I feel that some of the bold approximations he has made seriously limit the generality of his conclusions. For example, he assumes the stator resistance to be zero, and thus entirely neglects the dip in torque at half-speed which occurs with any unbalanced rotor. Also, he asumes the current in every squirrel-cage bar to be the same, whereas, as a matter of fact, we know that the outside bars of a squirrel cage always carry more current during the starting period than the middle bars. Finally, he has combined the effects of the field and the squirrel cage by entirely neglecting the action of the squirrel cage in the field axis. That is, he has assumed the field winding to have such low impedance in the direct axis that the squirrel-cage current in this axis is negligible. These approximations are in addition to those he has mentioned in the paper.

Mr. Putman concludes from his study of double squirrel-cage synchronous motors that they are of no practical importance. While there is a measure of truth in this conclusion, there is much to be said on the opposite side of the argument. The difficulty of getting enough space in the pole tip to insert a satisfactory type of double squirrel-cage is the most fundamental part of the problem. The L-bar type of squirrel-cage Mr. Putman employed is not the best for this purpose, since the impedance cannot be made high enough to reduce appreciably the starting current with the field closed. By using an open-circuited, or idle, steel bar above the squirrel cage proper, a higher impedance can be obtained in the same space, and thus a material reduction of starting current can be secured. However, the reduction possible is not great enough to warrant the extended use of this construction.

The primary object of a double squirrel-cage is to reduce the starting current on full voltage sufficiently to avoid the use of a starting compensator. Therefore, all those machines whose starting currents are only about 20 per cent higher than permissible for full-voltage starting can be brought within the permissible class, and so can be made considerably cheaper by the use of the special construction. When the torques are compared on the basis of the same starting current, the two types then give comparable results.

Quentin Graham: I made an experimental investigation several years ago which showed a number of interesting facts concerning synchronous-motor starting performance. Chief among these was the enormous effect of the field winding on the speed—torque curve, a fact which is clearly shown in Mr. Putman's paper, and which, I believe, has not been appreciated fully by designing engineers. A typical set of curves illustrating this point is shown in Fig. 1 herewith. Curve *A* is the speed—torque characteristic of the motor with its field winding open-circuited so that the squirrel-cage winding furnishes all the torque. If the field winding is closed on itself the torque characteristics are as shown by Curve *B*. It will be seen that the torque at low speed is about the same but that it increases greatly for low values of slip. Curve *C* shows that with resistance in the field circuit the cusp is almost entirely removed although the torque is changed very little at the extremities of the curve. By making use of these characteristics, determined first experimentally, we have been able to use relatively high-resistance cage windings with consequent high starting torque and low starting current. The field winding takes care of the torque at low values of slip and the judicious selection of a field starting resistor prevents a dip in the curve at intermediate speeds. While these characteristics have been known and have been explainable in a general way by induction motor theory, Mr. Putman has published the first adequate mathematical treatment of the problem.

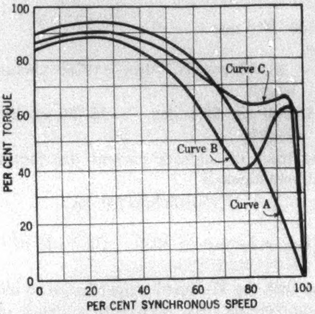

Fig. 1

Another result of our experimental work was that we obtained an entirely new conception of the pull-in problem. A number of investigators, both in this country and in Europe, have attacked this problem but in nearly every case they have been concerned with the oscillation which takes place when the field current is applied. They have attempted to find the maximum slip at which the motor could operate and still pull into step during the surge that takes place when the field is excited. The solution of this problem requires a knowledge of the inertia of the rotor and its connected load and depends also upon the point on the slip cycle at which excitation is applied. Our investigations, however, showed that we need not concern ourselves with this aspect of the pull-in problem except in rare cases. We found that if the motor could be brought to the upper branch of the speed—torque curve, that is, to a speed above the pull-out or unstable point on the curve, the application of excitation would always bring the rotor into synchronism. The problem thus became one of finding the induction motor characteristics rather than one dealing with the purely pull-in phenomenon.

Mr. Alger, in his discussion, has called attention to Mr. Putman's omission of the stator resistance and has pointed out that this may give an error in the torque at half-speed. I have developed the equation for torque including the stator resistance and have substituted values for a few cases. While there is a slight dip in the curve at half-speed, I have concluded that it is of negligible importance.

H. V. Putman: Before this theoretical investigation was started at all, the company with which I am associated (thanks to the painstaking efforts of Mr. Quentin Graham) has, for a number of years, accumulated a vast amount of experimental data on the starting performance of synchronous motors. These data were of great assistance in building a theoretical structure on which to base calculations of starting performance.

Both Mr. Alger and Mr. Park mentioned the fact that I neglected the stator resistance and Mr. Alger says that I did this without saying anything about it in the paper. I stated very clearly that this assumption was being made in order to simplify the theory. We were well aware of the effect of the stator resistance at half-speed and I stated that Mr. Graham had worked out the theory taking the stator resistance into account. He has calculated curves on this basis which show the dip at half-speed. However, a review of a great many speed-torque curves which we have made, disclosed the fact that none of them shows a distinct dip or cusp at half-speed. At half-speed there are usually a few test points which appear erratic. Sometimes there is a point above the curve, sometimes one below, but we have never been able to obtain a test curve which showed distinctly a dip at half-speed. The dip does exist, but the fact that it can't be obtained experimentally shows that it is so small as to be negligible. This is also borne out by the fact that since we have been building real damper windings, we have never had a case of trouble where a motor stuck at half-speed and refused to come up to full-speed. If there were any appreciable dip in the torque at half speed, it seems likely that, with the increased severity of synchronous-motor applications and hence more severe starting duty, we would have had some cases of sticking at half-speed.

Both Mr. Alger and Mr. Park say I assume the same current in each bar and Mr. Alger says that I neglect the effect of the squirrel-cage in the axis of the field winding. This shows that both Mr. Alger and Mr. Park do not clearly understand the theory of the symmetrical-coordinate method and they have not read my paper carefully. They have this theory mixed up with Blondel's two-reaction theory. One speaks of the direct and transverse axes in the two-reaction theory, but not when dealing with the symmetrical-coordinate method.

What I did assume, was this: There are two fluxes rotating in the gap, one in the positive direction and one in the negative. Both cut the damper bars. Due to the positive flux, there are damper-bar currents set up which are all equal, provided the damper bars all have the same resistance and reactance, and are apart in time phase by the space angle between the bars. These produce a positively rotating m. m. f. which I represent by I_{2pp} and a negatively rotating m. m. f. represented by I_{2np}. Similarly, the negative flux in the gap produces additional damper-bar currents all of which are equal and which in turn produce two more m. m. fs., a negative m. m. f. represented by I_{2nn} and a positive m. m. f. represented by I_{2pn}. Since I_{2pp} and I_{2pn} rotate in the same direction, they combine to make the resultant I_{2p} which is the positively rotating rotor m. m f. Similarly, I_{2np} and I_{2nn} combine to form the negatively rotating rotor m. m. f. From this point on I deal only with m. m. fs. But if one combines the bar currents due to the positive flux, with those due to the negative flux, the combined currents which result are not equal in each bar; neither are they apart in time by the space angle of the bars.

Mr. Alger's statement that I neglect the effect of the damper bars in the axis of the field winding, amounts to the same thing as saying that in an induction-motor diagram, if one represents the rotor-bar current by a single vector I_b, he is neglecting the effect of the bars which constitute a phase at right angles to I_b. But we all know that a polyphase m. m. f. or current can be represented by a single vector and the problem handled as though it were a single-phase, but because we handle it as single-phase it doesn't mean that we neglect the phase at right angles to it. If Mr. Alger will read my paper carefully he will see that I have made no such approximation.

Mr. Park says that my theory assumes that all bars have the same resistance and reactance. This is not necessarily true. My theory begins on the basis of a rotor having a definite rotor resistance, rotor reactance and single-phase action factor, K. It is true that it may be a little more difficult to get the rotor-bar resistance, reactance, and K, if the bars are all different, than it is when they are all alike. I showed in my paper how the value of K is obtained when they are all alike to give a general idea of the problem; however, I did not include in my paper any explanation of the calculation of the several motor constants as explained in one of the foot-notes in my paper. The handling of cases where damper bars of different resistances and reactances are used in the same field pole, is simply a problem in calculating the values of rotor resistance, rotor reactance and K. We often build machines with bars of at least two different materials and usually the bars at the tip of the pole are nearer the surface and have less reactance than those in the middle. In all such cases, it is only necessary to calculate an equivalent bar and then proceed as if all bars were like the equivalent bar.

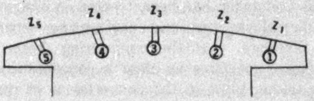

FIG. 2—SYMMETRICAL GROUP OF DAMPER BARS

An example calculation of this kind, is as follows: In the accompanying Fig. 2, let Z_1 equal the impedance of bars No. 1 and No. 5. Let β_1 equal phase angle of bars No. 1 and No. 5, that is, $\beta_1 = \tan^{-1} \frac{X_1}{r_1}$ where $Z_1 = \sqrt{x_1^2 + r_1^2}$. Similarly, let Z_2 equal the impedance of bars No. 2 and No. 4 and let β_2 be the corresponding phase angle. Let Z_3 be the impedance of the middle bar and β_3 its phase angle.

Calculate
$$\varphi_1 = \beta_3 - \beta_1$$
$$\varphi_2 = \beta_3 - \beta_2$$

Then calculate $X = \left(2 \frac{Z_3}{Z_1} \sin \varphi_1 + 2 \frac{Z_3}{Z_2} \sin \varphi_2 \right)$

$$Y = \left(1 + 2 \frac{Z_3}{Z_1} \cos \varphi_1 + 2 \frac{Z_3}{Z_2} \cos \varphi_2 \right)$$

Then calculate $B = \sqrt{X^2 + Y^2}$ and $\beta^1 = \tan^{-1} \frac{X}{Y}$

The equivalent bar has an impedance of $\frac{5}{B} Z_3$ and a phase angle of $\beta_3 - \beta^1$.

Small differences in the impedances of the several bars make only a very slight change in K. When it is thought necessary to calculate a new K, we have a fairly elaborate formula involving the several bar impedances, phase angles, and the space angle between the bars. Usually, this is not necessary.

I did not mean to say that the double-squirrel-cage synchronous motor has no practical value. What I did say, is that it is more desirable to limit the inrush by the use of a high-resistance damper rather than by the use of high reactance because this method gives a higher average torque per kv-a. In higher-speed motors where there is a greater depth of pole head, the double squirrel cage can be used to greater advantage and, as Mr. Alger says, is of use in bringing many ratings within the full-voltage starting class.

The Synchronous Converter

Theory and Calculations

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Synopsis.—*The first part of this paper is devoted to the purpose of presenting a clear conception of the internal voltages, currents, heating, and armature reactions as related to the physical structure of the simple converter and as related to the passage of time. These may be styled "space" and "time" relations. "Space" relations are indicated by means of diagrams representing as nearly as possible the physical structure of the converter. "Time" relations are shown by conventional curves and vectors.*

The accepted mathematical expressions for voltage ratios, heating effect of currents, and armature reaction imply that certain facts be disregarded. This is done in order to simplify their development.

The mathematical treatment developed in the appendixes to this paper includes most of these disregarded factors and makes it possible to evaluate them.

This treatment is based on the method of "harmonic analysis" by which any regularly repeating function may be represented.

THE general theory of the synchronous converter has been given less attention in the technical press than has that of the more widely used purely alternating current machines. Moreover, the synchronous converter has been treated in less detail in our technical colleges and engineering schools. It seems, therefore, that the engineering fraternity in general may not have as clear a conception of phenomena taking place in the converter as in the other types of electrical machines.

It is the purpose of this paper, first, to present a clear and simple explanation of the internal actions and reactions of the synchronous converter without resorting to laborious mathematics; and second, to set forth for those who will be interested the complete mathematical treatment by harmonic analysis.

The subject divides naturally into three parts:

Voltage Relations,
Heating Effect of Current,
Magnetomotive Force Relations.

VOLTAGE RELATIONS

The ratio of effective a-c. voltage for any number of phases may be determined by representing the voltage of each armature coil vectorially as in Fig. 1 where the voltage of the complete armature is represented by a regular polygon of vectors which is so related to the two field poles that the magnitude of the voltage of any coil or any phase is measured by the projection of the vector or vectors upon the axis $(y - y)$ which lies at right angles to the axis of the poles. This axis is usually called the "quadrature axis."

This combination vector diagram and pole drawing is in accord with the physical facts, in that maximum instantaneous voltage is indicated for a given vector (such as c) when it is directly under the center of a pole. Likewise the maximum voltage is generated in a coil when its sides are passing under the center of a pole.

It is evident that the maximum instantaneous voltage across any diameter $(A\ B)$ occurs when that diameter coincides with the "quadrature axis" which is also the brush or neutral axis. Hence the d-c. voltage at the commutator is numerically equal to the maximum instantaneous voltage of an a-c. diameter, and the ratio of effective a-c. diametral voltage to direct current is the ratio of the effective value to the maximum value of the a-c. wave. This ratio is $\frac{1}{\sqrt{2}}$ or 0.707 for the ideal condition of 100 per cent efficiency and a pure fundamental sinusoidal a-c. voltage.

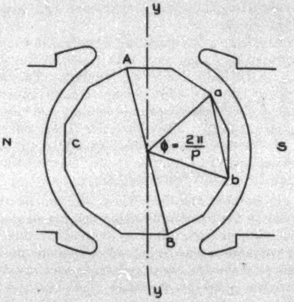

FIG. 1

If the converter has p phases per pair of poles, then each phase covers an angle $\phi = 2\ \pi / p$, and the cord $a\ b$ subtended by this angle is a measure, under ideal conditions, of the maximum a-c. voltage induced in the phase. It is easily seen from the geometry of Fig. 1 that the ratio of the d-c. voltage $A\ B$ to the maximum a-c. phase voltage $a\ b$ is

$$\frac{A\ B}{2\ (A\ B/2)\ \sin\phi/2} = \frac{1}{\sin\pi/p}$$

or in terms of the effective value of the a-c. voltage it is $\sqrt{2}/\sin\ (\pi/p)$; so that the normal voltage ratio of a

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

synchronous converter is primarily fixed by the number of phases p. It is well to point out, however, that this result has been arrived at by assuming a sinusoidally distributed field flux passing through zero at infinitely thin brushes; an infinitely distributed ring or full pitch drum winding; zero armature reaction; zero armature impedance; and a magnetic circuit of constant reluctance for all positions of the armature.

These ideal conditions are almost never realized in commercial machines because of practical limitations, some of which will be discussed.

The necessity of providing a neutral zone for commutation and the space requirements of the field coils both demand a shorter pole arc than corresponds to a sinusoidal distribution of the flux. The result is a slightly flattened a-c. voltage wave which has a higher ratio of effective to maximum; the latter always being equal to the d-c. voltage. It will be found that the majority of commutating pole converters have a no-load diametral ratio of approximately 0.725 and, due principally to resistance losses, a full load ratio of 0.74 to 0.75.

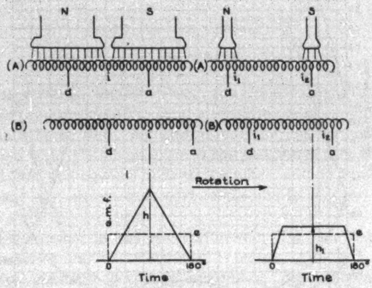

FIG. 2

The effect upon wave shape of varying the pole face arc can be made plain by considering two extreme cases shown in Fig. 2.

Here is represented poles having an arc of 180 deg. and others, a very short arc. It is assumed that the flux density is constant across each pole face and of such value that the same effective a-c. voltage is generated across a diameter of the armature winding.

It is evident that the diametral phase of winding $d\,i\,a$ in position A under the long arc poles has equal and opposite voltages generated in the two halves $d\,i$ and $i\,a$ so that the total voltage is zero. Now equal voltages are generated in all conductors and the number of conductors under each pole varies directly with time and position of the phase in space and therefore the total voltage of the phase $d\,i\,a$ will be a straight line function of time and space position as shown. The maximum e. m. f., h, is generated when the phase is central with the pole S, position B. The effective a-c. value of this voltage is e and the maximum or d-c. is h which appears continuously between two fixed points between the pole tips where brushes may bear on a commutator connected to the winding.

Now similarly under the short arc poles equal and opposite voltages are generated in $d\,i_1$ and $i_2\,a$ in position A and the resultant voltage is zero. When the phase has moved the distance $d\,i_1$ coils in it will be under the influence only of pole S and the total voltage between d, a will be constant until the tap d reaches the pole S. The net voltage of this diametral phase will be trapezoidal, as shown, with a maximum value h' and the same effective value e as in the previous case.

Thus the effect of increasing the pole arc, keeping the same effective value of a-c. voltage, is greatly to increase the d-c. voltage; and vice versa.

The regulating pole converter uses this principle for varying the d-c. voltage with constant effective a-c. voltage impressed. A variation of the net pole arc is secured by constructing each pole and pole face in two adjacent parts, one larger "main pole" and one smaller "split" or "regulating" pole. The exciting field of the latter may be varied, or even reversed, which results in a net effect equivalent to varying the total pole arc.

There is a number of factors which affect voltage ratio as shown in detail in the appendix. These are:

Number of phases,
Flux distribution or "field form,"
Flux shift relative to brush position,
Flux pulsation caused by armature reaction pulsating m. m. f.,
Flux pulsation caused by variation in reluctance due to position of slots,
Brush shift relative to field flux position,
Brush width determining the number of coils short-circuited,
Brush contact resistance,
Armature coil winding pitch,
Armature coil distribution,
Armature impedance.

HEATING EFFECT OF ARMATURE CURRENT

Before considering the explanation of armature heating, it is advisable to mention the fundamental definition of "motor" and "generator" action.

Generator action in any dynamo-electric machine is the act of current flowing through it in the same direction as, and by means of, the internal "induced" e. m. f.

Motor action in any dynamo-electric machine is the act of current flowing through it in the opposite direction to the internal "induced," and by means of an external impressed e. m. f.

In the converter it is convenient to regard the direct current as due to generator action, and the alternating current as due to motor action. The actual current in any conductor results from superposition of these two.

A bipolar converter, having for convenience a

Gramme ring armature winding, is represented in Fig. 3. The mechanical rotation of the armature is clockwise. Zero time is selected as the instant when the center of the phase *a b* coincides with the center line of the brush.

The voltage generated under each pole in the conductors passing under it is directed from the negative brush toward the positive brush through both of the armature circuits.

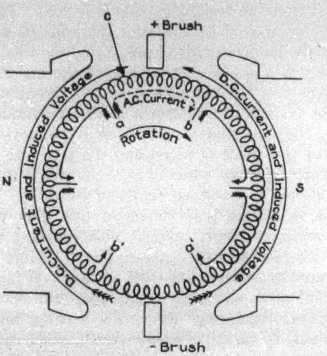
c
+ Brush
D.C.Current and Induced Voltage
D.C.Current and Induced Voltage
A.C.Current
a
b
Rotation
N
S
b'
a'
- Brush

Fig. 3

The specific case of a six-phase converter is assumed, where the phase *a b* occupies 60 electrical degrees.

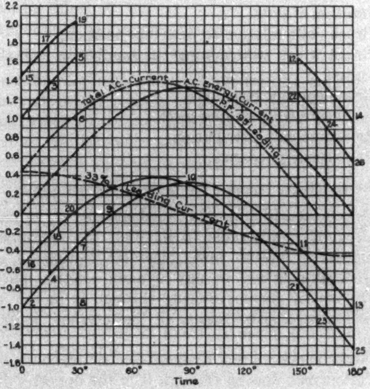

Fig. 4

With the phase in the position of zero voltage, corresponding to zero time in Fig. 4, there is no alternating current at unity power factor. But when the phase moves toward the pole *S* the voltage will be induced in the direction from the tap *b* toward *a*.

Now the a-c. motor, or input current, must flow from *a* to *b* in opposition to this induced voltage. At the same time the d-c. generator current is flowing from *a* to the positive brush in the same direction as that in which the alternating current is just starting to flow and the two currents are added. This condition holds true in any part of the trailing half of the phase which has not yet passed under the brush.

The magnitudes and relations of these currents in time are shown in Fig. 4. The direct current, taken as unity, is represented by the line 1, 6, 8, 13 for the tap coil *a* which is commutated at point 6 after the 30 deg. of time required for *a* to rotate to the brush.

The alternating current (dotted curve), since it flows in the same direction, adds to the direct current in coils not yet commutated; hence the current curve of the tap coil *a* is 1, 3, 5 (commutates), 6, 7, 9, 10, 11, 13.

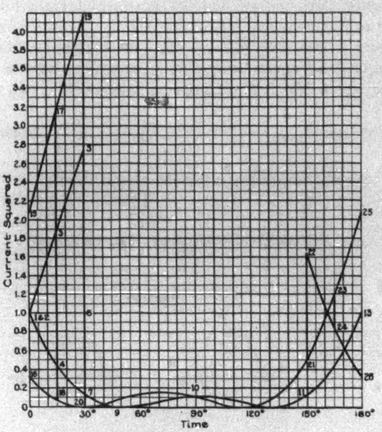

Fig. 5

The heating effect of this current is proportional to its square as shown in the corresponding Fig. 5 by the similarly numbered curve 1, 3, 5, 6, 7, 9, 10, 11, 13. The total or integrated heating is represented by the area under this curve.

In like manner the current in coil *c* half way between *a* and the phase center may be traced. This coil will commutate after 15 deg. of time. The current is 1, 3 (commutates), 4, 7, 10, 11, 13, Fig. 4, and the heating similarly numbered in Fig. 5.

The current and heating of the center coil which commutates at zero time is similarly 1, 2, 4, 7, 9, 10, 11, 13.

The current and heating of tap coil *b* may be traced through in the same manner. The direct current which has already been commutated flows in the opposite direction to the alternating current for 150 deg. of time required for the tap to rotate to the negative brush

The total current follows 2, 4, 7, 9, 10, 11, commutates, and follows 12, 14. One-half cycle or 180 deg. of time then finds tap *b* at *b′*, Fig. 3. It is evident that this current cycle is the exact equivalent of that shown for tap coil *a*; similarly, it is evident that any two coils equidistant from the center of the phase are subject to the same heating at unity power factor.

The effect of power factor upon the heat losses is more important in the converter than it is in any machine that carries alternating current alone.

Fig. 4 shows a certain amount of leading reactive current which, combined with the same energy current as before, produces the total alternating current shown. It is evident that this leading current has its highest values when the phase *a b* is in the position of zero time, and that this current adds to the direct and alternating energy currents in that part of the phase between the tap *a* and the d-c. brush.

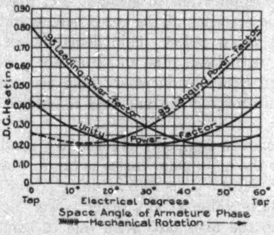

FIG. 6

The current in any coil throughout 180 deg. of time or rotation may be determined as before.

For instance, take the tap coil *a*. The current is represented by 15, 17, 19 (commutates), 20, 21, 23, 25. The heat loss at any instant of time is represented similarly in Fig. 5.

The current and heating of any coil between tap *a* and the center of the phase may be traced through the same curves except that commutation will take place sooner at a point between 15 and 19 determined by the distance of the coil from the brush.

The center coil of the phase which is commutated at zero time will carry current and heating indicated by curves 16, 18, 20, 21, 23, 25, Figs. 4 and 5.

The current and heating relations in the other, or leading half of the phase (center to tap *b*), follow a different course when the power factor is not unity. The tap coil *b*, for example, has already been commutated, so that the current starts at 16 and for 150 deg. of time follows the same curves 16, 18, 20, 21 (commutates) and then follows 22, 24, 26, Fig. 4.

The heating similarly follows 16, 18, 20, 21, 22, 24, 26 and the total heat is proportional to the area under it.

In like manner any other coil located between tap *b* and the center of the phase will carry current following the same variation except that it will be commutated later than the tap coil *b*. This is represented for one such coil by 16, 18, 20, 21, 23 (commutates), 24, 26 in both Figs. 4 and 5.

The averaged total heating in any coil in a phase, expressed as per cent of the loss with direct current alone in the armature, is shown in Fig. 6, for power factor = 1.0, 0.95 leading, and 0.95 lagging. It will be noted that the heat losses at unity power factor are symmetrically distributed while at any power factor other than unity the distribution is not symmetrical. Leading current causes the coils just "ahead" of the tap to overheat while lagging current reverses all these relations and overheats the coils just "behind" the tap. This fact furnishes a means of determining whether a converter armature winding has been damaged by leading or lagging power factor current.

Another important fact shown by these curves is that the heating of the coils near the tap is increased much more rapidly by low power factor than in any other type machine. In this instance of 0.95 power factor the tap heating increases from 0.43 to 0.80 or 85 per cent, whereas in any other device carrying alternating current alone, the increase is only 11 per cent. It may be stated in a general way that the tap coil heating of the converter is increased several times as fast by a change of power factor as in pure a-c. apparatus.

ARMATURE REACTION

Several of the best known textbooks treating of the armature reaction in the synchronous converter erroneously apply a "distribution factor" to the d-c. armature reaction, and neglect the factor $4/\pi$ which is the ratio of the sinusoidal component of a square wave to the amplitude of the wave itself. These errors lead to the false conclusion that the a-c. and d-c. armature reactions completely cancel each other in line with the interpoles.

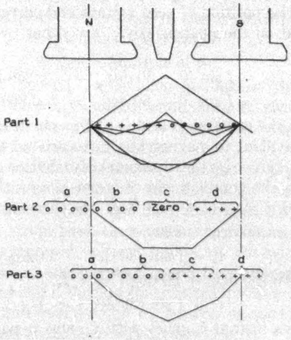

FIG. 7

As a matter of fact the armature reaction of a converter consists of a steady component and a series of even $M\ (p)$ time harmonics. Their effect on commutation and flux distortion requires that they be given due consideration in design.

A conception of the physical relations and magnitude of armature reaction in the converter may be formed by referring to Fig. 7. The direct and alternating currents and their m. m. fs. are considered separately, and then superimposed to form a resultant. A six-phase converter has been chosen, with full pitch, infinitely distributed windings. Thus each phase belt covers 60 electrical degrees and differs by the same amount in time phase from the adjacent belt.

In part I the direct currents are represented as flowing into the plane of the paper under pole N and out of this plane under pole S. According to the usual convention these currents are assumed to be paired so as to constitute turns about the neutral or quadrature axis. The m. m. f. of these direct currents is thus a triangular wave.

In a similar way the m. m. f. distribution due to the alternating current can be obtained, as shown in parts II and III, for two different positions of the armature.

Assuming for convenience unity power factor, and choosing c as reference phase, the currents may be represented as

$$i_a = I \sin (\omega\, t - 120 \text{ deg.}),\ i_c = I \sin (\omega\, t)$$

$$i_b = I \sin (\omega\, t - 60 \text{ deg.}),\ i_d = I \sin (\omega\, t + 60 \text{ deg.})$$

where $\omega\ t$ is the displacement of the midpoint of c from the quadrature axis. Thus for $\omega\, t = 0$ phase belt c is in the position of zero voltage and current, that is, centered on the neutral axis. The other two currents are

$$i_b = I \sin (0 - 60 \text{ deg.}) = -\ 0.866\, I$$

$$i_d = I \sin (0 + 60 \text{ deg.}) = +\ 0.866\, I$$

Thus the currents i_b and i_d at this particular instant are equivalent to magnetizing turns around the neutral axis, and give rise to trapezoidal distribution of m. m. f.

When the common tap of b and c is on the neutral axis 30 deg. later ($\omega\, t = 30$ deg.) the currents are

$$i_a = I \sin (30 \text{ deg.} - 120 \text{ deg.}) = -\ I,$$

$$i_b = I \sin (30 \text{ deg.} - 60 \text{ deg.}) = -\ I/2$$

$$i_d = I \sin (30 \text{ deg.} + 60 \text{ deg.}) = +\ I,$$

$$i_c = I \sin (30 \text{ deg.}) = +\ I/2$$

At this instant i_a pairs with i_d, and i_b pairs with i to form two bands of magnetizing turns about the neutral axis, and give rise to the polygon distribution of m. m. f. shown in part III.

These two m. m. fs., corresponding to different positions of the armature, may be superimposed on the d-c. diagram in part I and combined with it to produce the corded triple space harmonic of resultant reaction, which oscillates between these two extremes for every 30-deg. rotation of the armature, giving a 6th time harmonic.

These 6th harmonics of m. m. f. are responsible for the appearance of the "tap frequency pulsation" in the external circuit of the converter when the machine is loaded. They also increase the core loss and losses in the amortisseur winding.

There is a steady term of armature reaction under the interpole represented by the average of the two extremes shown in part I, Fig. 7. This must be compensated by extra turns on the interpole. Its small value compared to the d-c. reaction is the reason why interpoles were not adopted for converters until after their application to straight d-c. machines.

Note the presence of the cross magnetizing m. m. f. under the main poles. This demands an increased excitation with load, (due to saturation), which must be supplied either by increased field excitation, or by reactive alternating current and a change of power factor.

It is shown in Appendix III that the resultant reaction under the main poles varies as tan θ, where cos θ is the power factor. It is zero at unity power factor but has a definite value at any other power factor; being opposed to the field flux when due to leading power factor and vice versa when due to lagging power factor.

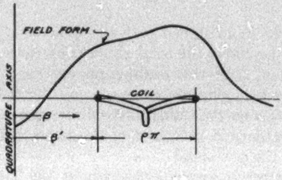

Fig. 8

The mathematical treatment of armature reaction by the method of harmonic analysis is given in Appendix III.

Appendix I

Voltage and Current Ratios

The A-c. Voltage. In Fig. 8 consider the armature coil of pitch ρ and n turns, which at any time t is located an angle β' from the quadrature axis. Let the flux density through which this coil is moving at synchronous speed $\omega\ t$ be specified by the Fourier series.

$$B = \sum_{1}^{\infty}{}_{\kappa}\ B_\kappa \sin \kappa\, (\beta + \gamma_\kappa) \tag{1}$$

The flux included by the coil is

$$\phi = \frac{2\, r\, l}{P} \int_{\beta'}^{\beta' + \rho\pi} B \,.\, d\,\beta \tag{2}$$

where

r = air-gap radius

l = effective length of armature stacking

P = number of poles

This flux induces in the coil the voltage

$$e = \frac{-n}{10^8}\frac{d\phi}{dt} = \frac{-4rln}{P\,10^8}\sum_{1}^{\infty}{}_{\kappa} B_\kappa \sin\frac{\kappa\rho\pi}{2}\,.$$

$$\cos\kappa\left(\beta' + \gamma_\kappa + \rho\frac{\pi}{2}\right).\frac{d\beta'}{dt} \qquad (3)$$

But if (β_r) is the position of the coil at $(t = o)$, then since it is moving at synchronous speed

$$\beta' = \beta_r + \omega t \text{ so that } \frac{\partial\beta'}{dt} = \omega = 2\pi f \qquad (4)$$

Substituting (4) in (3)

$$e = \frac{-4lf\tau n}{10^8}\sum_{1}^{\infty}{}_{\kappa} B_\kappa \sin\frac{\kappa\rho\pi}{2}\,.$$

$$\cos\kappa\left(\omega t + \beta_r + \gamma_\kappa + \rho\frac{\pi}{2}\right) \qquad (5)$$

where

$$\tau = 2\pi r/P = \text{pole pitch}$$

The total voltage of a phase belt of c such coils uniformly distributed around the armature by an angle σ between adjacent coils sides is

$$e_p = \frac{-4lf\tau n}{10^8}\sum_{1}^{\infty}{}_{\kappa}\sum_{0}^{C-1}{}_{r} B_\kappa \sin\frac{\kappa\rho\pi}{2}\,.$$

$$\cos\kappa\left(\omega t + \beta_o + \gamma_\kappa + \rho\frac{\pi}{2} + r\sigma\right) \qquad (6)$$

where β_o is the position of the first coil of the phase belt at $t = o$, and r refers to the rth coil after the first so that

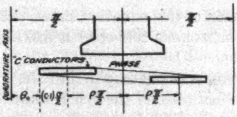

Fig. 9

$$\beta_r = \beta_o + r\sigma \qquad (7)$$

By the geometric method of Appendix IV the r summation may be evaluated, yielding

$$e_p = \frac{-4lf\tau nc}{10^8}\sum_{1}^{\infty}{}_{\kappa} B_\kappa \sin\frac{\kappa\rho\pi}{2}\left(\frac{\sin\frac{c\kappa\sigma}{2}}{c\,.\sin\frac{\kappa\sigma}{2}}\right)$$

$$\cos\kappa\left[\omega t + \beta_o + \gamma_\kappa + \rho\frac{\pi}{2} + (c-1)\frac{\sigma}{2}\right] \qquad (8)$$

If time is counted from the instant that the phase is centrally located with respect to the pole, then it is seen from Fig. 9 that

$$\beta_o = (1-\rho)\frac{\pi}{2} - (c-1)\frac{\sigma}{2} \qquad (9)$$

It is customary to write

$$C_{p\kappa} = \sin\frac{\kappa\rho\pi}{2} = \text{pitch coefficient of the } \kappa th \text{ harmonic} \qquad (10)$$

$$C_{d\kappa} = \frac{\sin\frac{c\kappa\sigma}{2}}{c\,.\sin\frac{\kappa\sigma}{2}} = \frac{\sin\frac{\kappa\pi}{p}}{c\,\sin\frac{\kappa\pi}{cp}} = \text{distribution coefficient of the } \kappa th \text{ harmonic} \qquad (11)$$

where p is the number of phases per pair of poles. If $\sigma \to o$ the distribution is said to be "infinite" and the coefficient C_{dk} approaches its limiting value

$$(C_{d\kappa})_{\sigma\to 0} = \frac{\sin\frac{\kappa\pi}{p}}{\frac{\kappa\pi}{p}} \qquad (12)$$

For lower values of k, Equations (11) and (12) are practically identical, so that in subsequent expressions, where the higher harmonics may be neglected, Equation (12) will be used, since it facilitates the calculations.

By (9), (10), and (11), Equation (8) may be written

$$e_p = \frac{-4lf\tau nc}{10^8}\sum_{1}^{\infty}{}_{\kappa} B_\kappa C_{p\kappa} C_{d\kappa} \cos\kappa\left(\omega t + \gamma_\kappa + \frac{\pi}{2}\right) \qquad (13)$$

and the fundamental component of e_p is

$$e_1 = \frac{4lf\tau nc}{10^8} B_1 C_{p1} C_{d1} \sin(\omega t + \gamma_1) \qquad (14)$$

As a matter of convenience in the argument of this discussion let us introduce the notation

$$M(p) = \text{multiple of } p \qquad (15)$$

It is of special importance to notice that the $\kappa = M(p)$ harmonics do not contribute to the terminal voltage of the phase, since by (11) the coefficient $C_{d\kappa}$ is zero for these harmonics.

If the fundamental is the only component present in the a-c. voltage applied at the slip rings, then such harmonic currents will flow as will consume by their impedance drops and their armature reaction, those harmonic generated voltages which are not wiped out by the winding coefficients C_{dk} and $C_{p\kappa}$. If resistance be neglected and if the flux contributed by the armature reactance and reaction be considered as part of the *resultant* field form, then we may say that

The resultant field form can contain only those harmonics for which

$$\left.\begin{array}{ll}\text{(a)} & C_{pk} = 0 \text{ or} \\ \text{(b)} & C_{dk} = 0 \text{ (includes all the } M(p) \text{ harmonics)}\end{array}\right\} \qquad (16)$$

and this statement holds, within its limitations, for a machine loaded or unloaded.

In commercial machines the pitch ρ is near unity, so that the *even* harmonics are wiped out thereby, and the neutralizing currents are therefore composed only of *odd* time harmonics, even though the field form contains very large even space harmonics.

In practise, the a-c. line voltage is nearly a pure sinusoid, but almost without exception there is a transformer bank between the line and the converter. The transformer may usually be considered as short-circuited at its primary terminals to all harmonics, since the impedance of the connected system is very low. The harmonic currents are then limited by the combined impedance of the transformer and converter, and the voltage drops in the transformer due thereto appear as a harmonic voltage across the slip rings. Thus due to the presence of the transformer the resultant field form may contain other harmonics than those allowed by (**16**), but they are of no great importance. They affect both the a-c. and d-c. voltages.

The D-c. Voltage. Fig. 10 shows the brush b shifted an angle ψ from the quadrature axis. Since the coils have a pitch ρ the mean position of the belt of conductors connecting adjacent brushes extends from

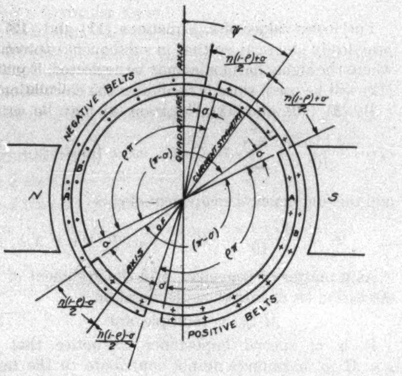

FIG. 10

$$\left(\psi-\rho\,\frac{\pi}{2}+\frac{\pi}{2}+\frac{\sigma}{2}\right)\text{ to }\left(\psi-\rho\,\frac{\pi}{2}+\frac{3\,\pi}{2}-\frac{\sigma}{2}\right) \tag{17}$$

The belt contains π/σ conductors. Its instantaneous position will vary periodically by half the angle σ between coil sides either side of the mean position specified above. This variation, which is partly responsible for the "tooth ripple" observed in the voltage wave, will be neglected in this discussion.

Equation (**8**) gives the voltage induced in a belt of c conductors at any instant t. By substituting

$$\left.\begin{aligned} \omega\,t &= 0 \\ c &= \pi/\sigma \\ \beta_o &= \psi-\rho\,\frac{\pi}{2}+\frac{\pi}{2}+\frac{\sigma}{2} \end{aligned}\right\} \tag{18}$$

we obtain the equation for the d-c. voltage.

$$E_{DC} = \frac{+\,4\,l\,f\,\tau\,n}{10^8}\sum_1^{\infty}{}_\kappa\, B_\kappa\, C_{p\kappa}\left(\frac{\sin\dfrac{\kappa\,\pi}{2}}{\sin\dfrac{\kappa\,\sigma}{2}}\right)\cos\kappa\,(\psi+\gamma_\kappa) \tag{19}$$

Here the $M\ (p)$ harmonics are not wiped out by distribution, but contribute to the d-c. voltage. But the even harmonics are completely wiped out by the distribution factor $\sin\dfrac{\kappa\,\pi}{2}$.

The Voltage Ratio. The ratio of the direct current to the effective value of the fundamental of the a-c. voltage, as obtained from Equations (**14**) and (**19**), is

$$\frac{E_{DC}}{E_1} = \frac{\sqrt{2}\cos(\psi+\gamma_1)}{\sin\dfrac{\pi}{p}} + \sqrt{2}\sum_2^{\infty}{}_\kappa\frac{B_\kappa\ C_{p\kappa}\ \sin\dfrac{\kappa\,\pi}{2}\ \sin\dfrac{\sigma}{2}}{B_1\ C_{p1}\ \sin\dfrac{\pi}{p}\ .\ \sin\dfrac{\kappa\,\sigma}{2}}\cos\kappa\,(\psi+\gamma_\kappa)$$

$$= \sqrt{2}\ \Delta/\sin(\pi/p) \tag{20}$$

Equation (**20**) is more specifically written, according to (**16**), by substituting m for κ and dropping the limits of summation, where m is any harmonic for which $\kappa = M\ (p)$.

An inspection of this equation shows that the voltage ratio of a synchronous converter is fixed by the following conditions:

a. The number of phases p in the ratio $\sqrt{2}/\sin(\pi/p)$, called the *normal* ratio of the synchronous converter.

b. The brush shift ψ, limited because of sparking for shifts large enough to affect the ratio more than a few per cent.

c. The flux shifts γ_1 and γ_m have the same effect, theoretically, as the brush shift ψ, but are more feasible practically since the brush may be left in the artificial neutral zone of the commutating poles, and sparking thereby avoided.

d. Superposition of those flux harmonics of odd order which are multiples of the number of phases per pair of poles. The voltages due thereto are cancelled out in the phase by the distribution of the winding, but appear in the d-c. voltage and thus change the ratio. Other harmonics in the flux may be used to change the ratio, but not so effectively, since they change both the a-c. and d-c. induced voltages, and consequently, they cause harmonic currents to flow which tend to neutralize the flux harmonics by armature reaction and reactance and increase the transformer heating, but may or may not increase the converter heating.

Note that the even multiples of p harmonics are ineffective since they are cancelled out entirely in the d-c. voltage by the factor sin $m\,\pi/2$. For this reason it is impossible to vary the voltage of a two-phase, four-phase, or two n-phase machine by superposition of multiple of p harmonics. However the voltage of a six-phase machine may be varied by superimposing the odd multiple of (3) harmonics, provided that the transformer connections are open-circuited to the flow of the third harmonic current and its multiples.

Such connections are equivalent to the use of a six-phase converter as a duplex three-phase machine.

Items *c* and *d* constitute the regulating pole method of voltage variation.

e. The armature winding factors are the angle σ between adjacent coil sides, the per cent pitch ρ and the pitch coefficients $C_{\rho 1}$ and $C_{\rho m}$. In commercial machines σ is usually less than 12 deg., and $1.00 > \rho > 0.95$.

In addition to the above there are several other influences on the voltage ratio which do not appear in the equation. They are:

f. The brush short-circuits the voltage across its bearing arc, but this decrease of net voltage is too small to be of any practical importance.

g. The armature impedance and brush resistance drops change the voltage ratio with the load.

h. The armature reaction contains a steady term and a series of even multiple of p harmonics. The steady term is of little practical importance, but the harmonics of armature reaction in conjunction with *g* cause the so-called "tap frequency pulsations" to appear in the terminal voltages.

i. The oscillation about its mean position of the belt of conductors between brushes, and the variation in reluctance of the magnetic circuit due to the passage of the slots under the poles, cause a "tooth frequency pulsation" to appear in the voltage. This tooth frequency pulsation is high enough to prove objectionable on account of telephone interference when the feeders run parallel to telephone lines. It may be eliminated by spiralling the armature slots.

In present-day practise it has been found more satisfactory for operating and economic reasons to vary the terminal voltage of the converter rather than change its ratio by flux distortion. This may be accomplished by the following means:

j. Tap changing transformers or voltage regulators.

k. Booster converters, in which a small alternator is connected in series with the converter. The voltage is controlled by manipulacion of the booster field excitation. In smaller machines the booster is usually a separate machine with a revolving field, but in the larger sets it is made integral with the converter proper and has a revolving armature. Direct-current boosters are not as satisfactory since they have the disadvantage of a commutator capable of carrying the full converter output.

l. Field controlled converters, in which a change in the excitation changes the power factor. The reactive component of current flowing through the external reactance of the transformers or of separate reactors, changes the voltage at the converter slip rings, and thus the d-c. voltage.

Current Ratio. The current ratio of the synchronous converter follows from the law of conservation of energy

$$\text{d-c. output} = \text{a-c. input} \times \text{efficiency} \tag{21}$$

$$\therefore E_{DC}\, I_o = p\, E_1\, I_1\, (1 - \zeta) \cos\theta \tag{22}$$

where

$\cos\theta$ = power factor

I_1 = eff. value of a-c. current

ζ = rotational losses as per cent of a-c. input.

$$\therefore I_1 = \frac{E_{DC}}{E_1}\,\frac{I_o}{p\,(1-\zeta)\cos\theta} = \frac{\sqrt{2}\,\Delta\, I_o}{p \sin\dfrac{\pi}{p}\,(1-\zeta)\cos\theta} \tag{23}$$

The effective value of the slip ring current is the vector sum of the currents in the two phases connected to that ring. Its absolute value, therefore, is

$$I_1{}' = I_1 \left| \epsilon^{j0} + \epsilon^{j\left(\pi - \frac{2\pi}{p}\right)} \right| = 2\, I_1 \sin\frac{\pi}{p}$$

$$= \frac{2\sqrt{2}\,\Delta\, I_o}{p\,(1-\zeta)\cos\theta} \tag{24}$$

Appendix II

ARMATURE HEATING

On the assumption of linear commutation and a brush overlap of *b* radians the direct current in the layer of conductors between adjacent brushes is a trapezoidal wave as shown in Fig. 11. The Fourier series of this wave reckoned from the quadrature axis is

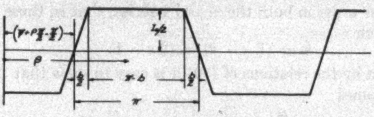

FIG. 11

$$i_{ac} = \frac{4\, I_o}{\pi\, b} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin (2\kappa - 1)\, b/2}{(2\kappa - 1)^2} \sin (2\kappa - 1) \left(\beta - \psi + \rho\frac{\pi}{2} - \frac{\pi}{2}\right) \tag{25}$$

The alternating current in a phase is, in general,

$$i_{ac} = \sum_{1}^{\infty}{}_{m} I_m \sin m\,(\omega\, t + \theta_m) \tag{26}$$

If the phase belt has *c* coils of pitch $\rho\,\pi$, uniformly distributed by the angle σ, then at any time t the position with respect to the quadrature axis of the $(r + 1)$ *th* coil of the belt is, by Fig. 9,

$$\beta = \left[\omega\, t + \frac{\pi}{2}\,(1 - \rho) - (c - 1)\frac{\sigma}{2} + r\,\sigma\right] \tag{27}$$

The substitution of (**27**) in (**25**) thus specifies the direct current in the $(r + 1)$ *th* coil at any instant. The resultant current in this coil is the difference of the alternating and direct current, and is

$$i_{r+1} = \sum_{1}^{\infty}{}_{m} I_m \sin m\,(\omega t + \theta_m)$$

$$- \frac{4 I_o}{\pi b} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin (2\kappa - 1)\, b/2}{(2\kappa - 1)^2} \sin (2\kappa - 1)$$

$$\left[\omega t - \psi - (c - 1)\frac{\sigma}{2} + r\sigma\right] \qquad (\mathbf{28})$$

The average loss per cycle for this $r + 1$th coil is

$$W_{r+1} = \frac{1}{\pi}\int_0^{\pi} R\, i_{r+1}^2\, d\,(\omega t) \qquad (\mathbf{29})$$

where

R = resistance of the coil.

The substitution of (**28**) in (**29**) yields integrals of the following types

$$\int_0^{\pi} \sin^2 a x\,.\,d x = \pi/2 \qquad \int_0^{\pi} \cos^2 a x\,.\,d x = \pi/2$$

$$\int_0^{\pi} \sin a x\,.\cos a x\,.\,d x = 0 \quad \int_0^{\pi} \sin a x\,.\cos b x\,.\,d x = 0$$

$$\int_0^{\pi} \sin a x\,.\sin b x\,.\,d x = 0 \quad \int_0^{\pi} \cos a x\,.\cos b x\,.\,d x = 0$$

$$a \neq b \qquad (\mathbf{30})$$

If, now, $2s - 1$, denotes those harmonics having the same order in both the m and κ series; that is, those for which

$$m = (2\kappa - 1) = (2s - 1) \qquad (\mathbf{31})$$

then by the relations of (**30**) it is easy to show that (**29**) becomes

$$W_{r+1} = \frac{R}{2}\sum_{1}^{\infty}{}_{m} I_m^2$$

$$+ \frac{R}{2}\left(\frac{4 I_o}{\pi b}\right)^2 \sum_{1}^{\infty}{}_{\kappa} \frac{\sin^2 (2\kappa - 1)\, b/2}{(2\kappa - 1)^4}$$

$$- R\left(\frac{4 I_o}{\pi b}\right) \sum{}_{s} I_{(2s-1)} \frac{\sin (2s - 1)\, b/2}{(2s - 1)^2}$$

$$\cos (2s - 1)\,(\lambda_{2s-1} + r\sigma) \qquad (\mathbf{32})$$

where

$$\lambda_{2s-1} = \left[- (c - 1)\frac{\sigma}{2} - \psi - \theta_{2s-1}\right] \qquad (\mathbf{33})$$

The last summation of (**32**) can not, in general, be evaluated since its limits and arguments are all indefinite. But the second summation on the right of (**32**) can be evaluated by the methods of analysis shown in Appendix V. It is

$$\frac{R}{2}\left(\frac{4 I_o}{\pi b}\right)^2 \sum_{1}^{\infty}{}_{\kappa} \frac{\sin^2 (2\kappa - 1)\, b/2}{(2\kappa - 1)^4}$$

$$= \frac{R I_o^2}{4}\left(1 - \frac{2b}{3\pi}\right) \qquad (\mathbf{34})$$

$$\text{for } \pi \geq b \geq 0$$

Since, in practise, b is of the order of ¼ radian it is seen that the effect of brush over lap is to reduce the losses in a conductor by the order of 3 per cent of the total armature current heating. It is therefore of no importance. Obviously, since we have shown the effect of brush over lap to be insignificant for any conductor it will also be negligible for the whole armature heating. Let us then, for simplicity, pass at once to the limit by allowing $b \rightarrow 0$ in our equations. Then (**32**) reduces to

$$W_{r+1} = \frac{R}{2}\sum_{1}^{\infty}{}_{m} I_m^2 + \frac{R I_o^2}{4}$$

$$- 2 R \frac{I_o}{\pi} \sum{}_{s} I_{(2s-1)} \frac{\cos (2s - 1)\,(\lambda_{2s-1} + r\sigma)}{(2s - 1)} \qquad (\mathbf{35})$$

The average heating of the whole armature is

$$W = \sum_{0}^{c-1}{}_{r} \frac{W_{r+1}}{c} = \frac{R}{2}\sum_{1}^{\infty}{}_{m} I_m^2 + \frac{R I_o^2}{4}$$

$$- 2\frac{R I_o}{\pi c}\sum{}_{s}\sum_{0}^{c-1}{}_{r} I_{(2s-1)} \frac{\cos (2s - 1)\,(\lambda_{2s-1} + r\sigma)}{(2s - 1)} \qquad (\mathbf{36})$$

Effecting the r summation as shown in Appendix IV, we obtain

$$W = \frac{R}{2}\sum_{1}^{\infty}{}_{m} I_m^2 + \frac{R I_o^2}{4}$$

$$- \frac{2}{\pi} R I_o \sum{}_{s} I_{(2s-1)} \frac{C_{d(2s-1)}}{(2s - 1)} \cos (2s - 1)\,[\psi + \theta_{2s-1}] \qquad (\mathbf{37})$$

where

$$C_{d(2s-1)} = \frac{\sin (2s - 1)\, c\sigma/2}{c\,.\sin (2s - 1)\,\sigma/2}$$

It is very interesting to note from Equation (**37**) that the total heating may be reduced by introducing harmonic currents of proper phase angle, θ_{2s-1}, such that

$$\frac{2 I_o I_{2s-1}}{\pi\,(2s - 1)} C_{d(2s-1)} \cos (2s - 1)\,[\psi + \theta_{2s-1}] > \frac{I^2_{2s-1}}{2} \qquad (\mathbf{38})$$

This inequality is a maximum for $[\psi + \theta_{2s-1}] = 0$ or π corresponding to $C_{d(2s-1)} = (+)$ or $(-)$

$$I_{2s-1} = \frac{2 I_o C_{d(2s-1)}}{\pi\,(2s - 1)} \qquad (\mathbf{39})$$

Since the sinusoidal component of the a-c. current bears a fixed ratio to the d-c. current, it is not possible to meet conditions (**39**) with the fundamental. Therefore, segregating it, we have as the equation of minimum armature heating,

$$W_{min} = \frac{R\,I_1^2}{2} + \frac{R\,I_o^2}{4} - \frac{2}{\pi} I_o\, I_1\, C_{d1} \cos\left[\psi + \theta_1\right]$$

$$- \frac{2}{\pi^2} R\,I_o^2 \sum_{2}^{\infty}{}_s \frac{C^2_{d(2s-1)}}{(2s-1)^2} \qquad \textbf{(40)}$$

One way to cause such currents to flow would be to introduce their order into the field form with proper phase and amplitude. The *M* (*p*) harmonics could not, of course, be introduced by this method, as was pointed out in Appendix I. As an example, suppose we have a six-phase converter with transformer connections open-circuited to the third harmonics and its multiples. Let us assume infinite distribution and consider the effect of introducing fifth and seventh harmonic currents in conformity with the "best compromise" conditions of Equation (**39**). Then

$$C_{d5} = 0.966 \times \frac{12}{5\,\pi},\; C_{d7} = 0.966 \times \frac{12}{7\,\pi} \qquad \textbf{(41)}$$

Therefore by (**40**)

$$W = R \left\{ \frac{I_1^2}{2} + \frac{I_o^2}{4}(1 - 0.022) - \frac{2}{\pi} I_1\, I_o\, C_{d1} \cos\left[\psi + \theta_1\right] \right\} \qquad \textbf{(42)}$$

where the saving is $0.022\,R\,I_o^2/4$, entirely negligible.

In the case of a particular conductor the conditions to be satisfied are

$$\left.\begin{aligned} \lambda_{2s-1} + r\,\sigma &= 0 \\ I_{2s-1} &= \frac{2\,I_o}{\pi\,(2s-1)} \end{aligned}\right\} \qquad \textbf{(43)}$$

If these relations hold for every odd harmonic from (**1**) to (∞) the heating in that particular conductor would be zero.

But by Equation (**22**) this would require that

$$I_1 = \frac{2\,I_o}{\pi} = \frac{2\,I_o}{p\,.\sin\frac{\pi}{p}\,(1-\zeta)\cos\theta} \qquad \textbf{(44)}$$

which is obviously possible only in the case $p \rightarrow \infty$, $\theta = 0$, and $\zeta = 0$; that is:

a. an infinite number of phases,
b. unity power factor,
c. zero losses.

In this theoretical discussion of heat reduction by superposition of harmonic currents we have assumed the possibility of their introduction. Practically, however, the difficulties encountered are enormous.

If the a-c. harmonics be neglected we have from (**22**), (**35**) and (**36**)

$$W_{r+1} = \frac{R\,I_o^2}{4}\left\{1 + \frac{8\,\Delta^2}{p^2 \sin^2\frac{\pi}{p}\,(1-\zeta)^2\cos^2\theta} - \frac{16\,\Delta\cos(\lambda_1 + r\,\sigma)}{\pi\,p\sin\frac{\pi}{p}\,(1-\zeta)\cos\theta}\right\} \qquad \textbf{(45)}$$

$$W = \frac{R\,I_o^2}{4}\left\{1 + \frac{8\,\Delta^2}{p^2\sin^2\frac{\pi}{p}\,(1-\zeta)^2\cos^2\theta} - \frac{16\,\Delta\,C_{d1}.\cos\left[\psi + \theta_1\right]}{\pi\,p\sin\frac{\pi}{p}\,(1-\zeta)\cos\theta}\right\} \qquad \textbf{(46)}$$

The usual text-book expressions are obtained herefrom by introducing the following simplifying assumptions.

1. Normal voltage ratio, $\Delta = 1$
2. 100 per cent pitch, $\rho = 1$
3. Infinite distribution, $\sigma = 0, c = \infty$
4. $\tau = r\,\sigma - (c - 1)\,\frac{\sigma}{2}$ = position of a coil from middle of the phase belt

$$W_{r+1} = \frac{R\,I_o^2}{4}\left\{1 + \frac{8}{p^2\sin^2\frac{\pi}{p}\,(1-\zeta)^2\cos^2\theta} - \frac{16\cos(\tau - \theta)}{\pi\,p\sin\frac{\pi}{p}\,(1-\zeta)\cos\theta}\right\} \qquad \textbf{(47)}$$

$$W = \frac{R\,I_o^2}{4}\left\{1 + \frac{8}{p^2\sin^2\frac{\pi}{p}\,(1-\zeta)^2\cos^2\theta} - \frac{16}{\pi^2\,(1-\zeta)}\right\} \qquad \textbf{(48)}$$

The sensitivity of the heating to the power factor (cos θ) requires that the converter be run at or near unity power factor.

It is obvious from Equation (**47**) that the heating is a maximum at one or the other of the outside conductors of the phase belt; that is, at the "tap" coils. At unity power factor it is the same for both tap coils, but at any other power factor it is increased at one tap and decreased at the other.

Appendix III

ARMATURE REACTION

Polyphase Armature Reaction. Each armature coil of *n* turns, pitch $\rho\,\pi$, and instantaneous current *i* gives rise to the rectangular wave of m. m. f. shown in Fig. 12 whose Fourier series is

$$F_1 = \frac{4}{\pi}\,n\,i\sum_{1}^{\infty}{}_{\kappa}\frac{\sin\kappa\,\pi/2}{\kappa}\sin\kappa\,\rho\frac{\pi}{2}\,.\sin\kappa\,\alpha \qquad \textbf{(49)}$$

where α is the space angle measured on the armature.

The total m. m. f. of a phase belt of c such coils uniformly distributed by the angle σ is

$$F_p = \frac{4}{\pi} n\, i \sum_{0}^{c-1}{}_{r} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin \kappa\, \pi/2}{\kappa} \sin \kappa\, \rho \frac{\pi}{2} \cdot \sin \kappa\, (\alpha - r\, \sigma) \qquad (50)$$

Evaluating the r summation by the method of Appendix IV,

$$F_p = \frac{4}{\pi} n\, i \sum_{1}^{\infty}{}_{\kappa} \frac{\sin \kappa\, \pi/2}{\kappa} \sin \kappa\, \rho \frac{\pi}{2} \cdot \frac{\sin \kappa\, c\, \sigma/2}{\sin \kappa\, \sigma/2} \sin \kappa \left[\alpha - (c-1) \frac{\sigma}{2} \right] \qquad (51)$$

The displacement angle $\kappa\,(c-1)\,\sigma/2$ may be eliminated by choosing the reference axis at an angle

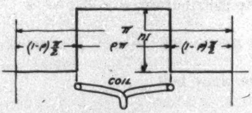

FIG. 12

$(\rho - 1)\,\pi/2$ from the midpoint of the phase belt. Then by (**10**), (**11**) and (**20**) we get

$$F_r = \frac{4}{\pi} n\, c \sum_{1}^{\infty}{}_{\kappa} \sum_{1}^{\infty}{}_{m} \frac{\sin \kappa\, \pi/2}{\kappa} C_{\rho\kappa} C_{d\kappa} I_m \sin m\, (\omega\, t + \theta_m) \sin \kappa\, \alpha$$

$$= \frac{4}{\pi} n\, c \sum_{1}^{\infty}{}_{\kappa} \sum_{1}^{\infty}{}_{m} \frac{\sin \kappa\, \pi/2}{\kappa} C_{\rho\kappa} C_{d\kappa} I_m \{\cos [\kappa\, \alpha - m\, (\omega\, t + \theta_m)] - \cos [\kappa\, \alpha + m\, (\omega\, t + \theta_m)]\} \qquad (52)$$

In this expression the angles for which

$$\left.\begin{aligned} \cos [\kappa\, \alpha - m\, (\omega\, t + \theta_m)] &= 1 \\ \cos [\kappa\, \alpha + m\, (\omega\, t + \theta_m)] &= 1 \end{aligned}\right\} \qquad (53)$$

are

$$\left.\begin{aligned} \alpha &= m\, (\omega\, t + \theta_m)/\kappa \\ \alpha &= -\, m\, (\omega\, t + \theta_m)/\kappa \end{aligned}\right\} \qquad (54)$$

and these angles move around the armature with the velocities

$$\left.\begin{aligned} d\, \alpha/d\, t &= m\, \omega/\kappa \\ d\, \alpha/d\, t &= -\, m\, \omega/\kappa \end{aligned}\right\} \qquad (55)$$

That is, the pulsating m. m. f. of amplitude

$$\left(\frac{4}{\pi} C_{\rho\kappa} C_{d\kappa} I_m/\kappa \right) c n$$

has been decomposed into two traveling waves of half this amplitude rotating in opposite directions at equal speeds.

If the machine has p phase belts per pair of poles, supplied by a balanced symmetrical p phase system of polyphase currents, then both the time and space angles between adjacent phase belts will differ by the angle $2\,\pi/p$. Thus to specify the m. m. f. of the qth phase belt, decrease both α and $\omega\, t$ in (**52**) by the angle $2\,\pi\, q/p$. The total m. m. f. for all p phases then is

$$F = \frac{2}{\pi} c\, n \sum_{0}^{p-1}{}_{q} \sum_{1}^{\infty}{}_{\kappa} \sum_{1}^{\infty}{}_{m} \frac{\sin \kappa\, \pi/2}{\kappa} C_{\rho\kappa} C_{d\kappa} I_m \left\{ \cos \left[\kappa \left(\alpha - \frac{2\,\pi\, q}{p} \right) - m \left(\omega\, t - \frac{2\,\pi\, q}{p} + \theta_m \right] - \cos \left[\kappa \left(\alpha - \frac{2\,\pi\, q}{p} \right) + m \left(\omega\, t - \frac{2\,\pi\, q}{p} + \theta_m \right] \right\}$$

$$= \frac{2}{\pi} c\, n \sum_{1}^{\infty}{}_{\kappa} \sum_{1}^{\infty}{}_{m} \frac{\sin \kappa\, \pi/2}{\kappa} C_{\rho\kappa} C_{d\kappa} I_m \left\{ \cos [\kappa\, \alpha - m\, (\omega\, t + \theta_m)] \sum_{0}^{p-1}{}_{q} \cos\, (m - \kappa) \frac{2\,\pi\, q}{p} - \cos [\kappa\, \alpha + m\, (\omega\, t + \theta_m)] \sum_{0}^{p-1}{}_{q} \cos\, (m + \kappa) \frac{2\,\pi\, q}{p} - \sin [\kappa\, \alpha - m\, (\omega\, t + \theta_m)] \sum_{0}^{p-1}{}_{q} \sin\, (m - \kappa) \frac{2\,\pi\, q}{p} - \sin [\kappa\, \alpha + m\, (\omega\, t + \theta_m)] \sum_{0}^{p-1}{}_{q} \sin\, (m + \kappa) \frac{2\,\pi\, q}{p} \right\} \qquad (56)$$

But by Equations (**101**) and (**102**)

$$\sum_{0}^{p-1}{}_{q} \cos\, (m \pm \kappa) \frac{2\,\pi\, q}{p} = \begin{cases} p \text{ if } (m \pm \kappa) = M(p) \text{ or } 0 \\ 0 \text{ if } (m \pm \kappa) \neq M(p) \text{ or } 0 \end{cases} \qquad (57)$$

$$\sum_{0}^{p-1}{}_{q} \sin\, (m \pm \kappa) \frac{2\,\pi\, q}{p} = 0 \qquad (58)$$

Hereby Equation (**56**) reduces to

$$F = \frac{2}{\pi} p\, c\, n \sum_{1}^{\infty}{}_{\kappa} \sum_{1}^{\infty}{}_{m} \frac{\sin \kappa\, \pi/2}{\kappa} C_{\rho\kappa} C_{d\kappa} I_m \{\cos [\kappa'\, \alpha - m\, (\omega\, t + \theta_m)] - \cos [\kappa''\, \alpha + m\, (\omega\, t + \theta_m)]\} \qquad (59)$$

where

$$\left.\begin{aligned} k' &= \text{any value of } k \text{ for which } \pm\, (m - k) = M(p) \text{ or } 0 \\ k'' &= \text{any value of } k \text{ for which } (m + k) = M\,(p) \end{aligned}\right\} \qquad (60)$$

It is to be understood that the summation is carried out with respect to these particular values κ' and κ'' of κ only.

For a single-phase machine κ' and κ'' include every integer value, and each may therefore be replaced by κ. It is then the equation of two traveling waves of equal amplitude rotating in opposite directions at equal speed. Their combination gives a standing wave of twice the amplitude. This means that in the single-phase machine there is no rotating component of armature reaction with respect to the armature.

Equation (**59**) gives the armature reaction with respect to the armature. It is necessary, however, to have it with respect to the field. Let time be counted from the instant that the phase is in its mid-position about the pole. Now the armature is running backwards at synchronous speed $w\,t$ relative to the poles (since in our convention positive rotation is the direction of rotation of the fundamental rotating field of armature reaction); so that the armature reaction stationary in space at any angle β from the quadrature or interpolar axis is given by the substitution

$$\alpha = \omega\,t + \beta \qquad (\mathbf{61})$$

Then (**59**) becomes

$$F = \frac{Z}{\pi} \sum_{1}^{\infty}{}_{m} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}\, I_m$$

$$\{\cos[(\kappa' - m)\,\omega\,t + \kappa'\,\beta - m\,\theta_m] - \cos[(\kappa'' + m)\,\omega\,t + \kappa''\,\beta + m\,\theta_m]\} \qquad (\mathbf{62})$$

where

$Z = 2\,p\,c\,n$ = total conductors per pair of poles. (**63**)

By the definitions of κ' and κ'' it is seen from (**62**) that the only sets of terms that contribute to the polyphase armature reaction are:

a. Those of constant magnitude, defined by $(\kappa' - m) = 0$

b. The $M\,(p)$ time harmonics, defined by $(\kappa' - m) = \pm M\,(p)$ and $(\kappa'' + m) = M\,(p)$. (**64**)

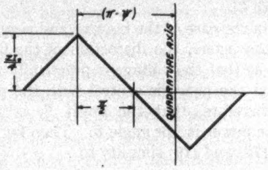

FIG. 13

Terms of the form $\kappa' - m = -M\,(p)$ are cancelled out by corresponding terms in $\kappa'' + m = M\,(p)$. All even values of κ are wiped out by the factor $\sin \kappa\,\pi/2$. Therefore the only terms that need be considered are those for which $\kappa > m$ and odd.

The components of m. m. f. in line with the interpoles and main poles are referred to as the quadrature and direct components respectively. They are given by $\beta = 0$ and $\beta = \pi/2$ respectively, and are, for the fundamental current,

$$F_Q = \frac{Z\,I_1}{\pi} \sum_{2}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}$$

$$\{\cos[(\kappa' - 1)\,\omega\,t - \theta_1] - \cos[(\kappa'' + 1)\,\omega\,t + \theta_1]\} + \frac{Z\,I_1}{\pi}\, C_{\rho 1}\, C_{d1} \cos\theta_1 \qquad (\mathbf{65})$$

$$F_D = \frac{Z\,I_1}{\pi} \sum_{2}^{\infty}{}_{\kappa} \frac{\sin^2 \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}$$

$$\{\sin[(\kappa'' + 1)\,\omega\,t + \theta_1] - \sin[(\kappa' - 1)\,\omega\,t - \theta_1]\} + \frac{Z\,I_1}{\pi}\, C_{\rho 1}\, C_{d1} \sin\theta_1 \qquad (\mathbf{66})$$

The average over a pole pitch is

$$F_{av} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} F\,d\,\beta = \frac{2}{\pi^2}\, Z\,I_1\, C_{\rho 1}\, C_{d1} \cos\theta_1$$

$$+ \frac{2}{\pi^2}\, Z\,I_1 \sum_{2}^{\infty}{}_{\kappa} \frac{\sin^2 \kappa\,\pi/2}{\kappa^2}\, C_{\rho\kappa}\, C_{d\kappa}$$

$$\{\cos[(\kappa' - 1)\,\omega\,t - \theta_1] - \cos[(\kappa'' + 1)\,\omega\,t + \theta_1]\} \qquad (\mathbf{67})$$

The D-c. Armature Reaction. It is convenient to consider the top layer of conductors connecting adjacent brushes as a "phase belt," having its returns in the bottom layer of conductors an angle $\rho\,\pi$ ahead. The d-c. armature reaction may then be calculated as for two such "phases," displaced by 180 deg. in space from each other, and carrying currents in opposite directions; thus, in a sense, also 180 deg. out of phase in time. Equation (**52**) gives the armature reaction of a phase belt carrying a current i. This equation can be applied to determine the d-c. armature reaction, if the tooth pulsations and brush overlap be neglected, and the following substitutions made:

I_o = d-c. brush current
i = $I_o/2$ for the first belt
i = $-\,I_o/2$ for the second belt 180 deg. from the first.
$c\,n$ = $Z/4$ conductors per belt

$$C_{d\kappa}' = \frac{\sin \kappa\,\pi/2}{(Z/4\,n)\sin(2\,\kappa\,n\,\pi/Z)} \cong \frac{\sin \kappa\,\pi/2}{\kappa\,\pi/2} \text{ for } \frac{Z}{4\,n} \text{ large} \qquad (\mathbf{68})$$

Then the armature reaction for the two belts is

$$F' = \frac{Z\,I_o}{2\,\pi} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}'$$

$$[\sin \kappa\,\alpha' - \sin \kappa\,(\alpha' - \pi)]$$

$$= \frac{Z\,I_o}{\pi} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}' \sin \kappa\,\alpha' \qquad (\mathbf{69})$$

where α' is reckoned from an angle $(\rho - 1)\,\pi/2$ behind the midpoint of the belt. But from Fig. 10 it is seen that the origin of α' is at an angle $\psi + \pi/2$ ahead of the quadrature axis, so that

$$\alpha' = \beta - \psi - \pi/2 \tag{70}$$

Equation (**69**) then becomes

$$F' = \frac{Z\,I_o}{\pi} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin^2 \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}' \cos \kappa\,(\beta - \psi + \pi) \tag{71}$$

For infinite distribution and unity pitch (**71**) reduces to

$$F' = -\frac{2\,Z\,I_o}{\pi^2} \sum_{1}^{\infty}{}_{\kappa} \frac{\cos\,(2\,\kappa - 1)\,(\beta - \psi)}{(2\,\kappa - 1)^2} \tag{72}$$

By Equation (**110**), Appendix V, the value of this triangular wave over a certain range of its angle is

$$F' = -\frac{Z\,I_o}{4} \left\{ 1 - \frac{2}{\pi}\,(\beta - \psi) \right\} \tag{73}$$

$$\text{for } 0 > (\beta - \psi) > -\pi$$

Hereby the quadrature and direct components of d-c. armature reaction are

$$F_Q' = F' \Big]_{\beta = 0} = -\frac{Z\,I_o}{4} \left(1 - \frac{2\,\psi}{\pi}\right) \tag{74}$$

$$F_D' = -F' \Big]_{\beta = -\pi/2} = \frac{Z\,I_o}{4}\,\frac{2\,\psi}{\pi} \tag{75}$$

The Resultant Armature Reaction. The resultant armature reaction is the sum of the a-c. and d-c. reactions as given by Equations (**62**) and (**71**). It is

$$F_R = \frac{Z}{\pi} \sum_{1}^{\infty}{}_{\kappa} \cdot \sum_{1}^{\infty}{}_{m} \frac{\sin \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}\, I_m$$
$$\{\cos\,[\,(\kappa' - m)\,\omega\,t + \kappa'\,\beta - m\,\theta_m\,]$$
$$-\cos\,[\,(\kappa'' + m)\,\omega\,t + \kappa''\,\beta + m\,\theta_m]\,\}$$
$$+\frac{Z}{\pi} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin^2 \kappa\,\pi/2}{\kappa}\, C_{\rho\kappa}\, C_{d\kappa}'\, I_o \cos \kappa\,(\beta - \psi + \pi) \tag{76}$$

Or for 100 per cent pitch, infinite distribution and the fundamental current only

$$F_R = \frac{Z}{\pi^2} \sum_{1}^{\infty}{}_{\kappa} \frac{\sin^2 \kappa\,\pi/2}{\kappa^2} \left\{ I_1\, p \sin \frac{\kappa\,\pi}{p} \right.$$
$$(\cos\,[\,(\kappa' - 1)\,\omega\,t + \kappa'\,\beta - \theta_1]$$
$$\left. -\cos\,[(\kappa'' + 1)\,\omega\,t + \kappa''\,\beta + \theta_1]\,) - 2\,I_o \cos \kappa\,(\beta - \psi) \right\} \tag{77}$$

The quadrature and direct components are given by substituting $\beta = 0$ and $\beta = \pi/2$ respectively; or can be obtained by adding the corresponding components of the a-c. and d-c. reactions from Equations (**65**), (**66**), (**74**), and (**75**). Thus, using (**23**)

$$F_{RQ} = \frac{Z\,I_o}{(1 - \zeta)} \left\{ \frac{2 \cos \theta_1}{\pi^2 \cos \theta}\,\Delta - \left(\frac{\pi - 2\,\psi}{4\,\pi}\right)(1 - \zeta) \right.$$
$$+\frac{2}{\pi^2}\,\frac{\Delta}{\cos \theta} \sum_{2}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/p\,.\,\sin^2 \kappa\,\pi/2}{\kappa^2 \sin \pi/p}$$
$$\left. (\cos\,[(\kappa' - 1)\,\omega\,t - \theta_1] - \cos\,[(\kappa'' + 1)\,\omega\,t + \theta_1]) \right\} \tag{78}$$

$$F_{RD} = \frac{Z\,I_o}{(1 - \zeta)} \left\{ \frac{2 \sin \theta_1}{\pi^2 \cos \theta}\,\Delta + \frac{\psi}{2\,\pi}\,(1 - \zeta) \right.$$
$$+\frac{2}{\pi^2}\,\frac{\Delta}{\cos \theta} \sum_{2}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/p\,.\,\sin \kappa\,\pi/2}{\kappa^2 \sin \pi/p}$$
$$\left. (\sin\,[(\kappa' - 1)\,\omega\,t - \theta_1] - \sin\,[(\kappa'' + 1)\,\omega\,t + \theta_1]) \right\} \tag{79}$$

Since k is always odd, and since $\kappa \pm 1 = M\,(p)$ by the definitions of k' and k'', it follows that

$$\frac{\sin \kappa\,\pi/p}{\sin \pi/p} = \frac{\sin\,[M\,(p) \pm 1]\,\pi/p}{\sin \pi/p} = \pm 1 \tag{80}$$

where the $-$ sign is to be associated with k'' and the $+$ sign with k'.

Except in the case of the booster and regulating pole types of converters, the distortion of the field flux is negligible, so that the voltage is practically zero at the instant that the phase is central with the pole. It is evident, therefore, that the angle θ_1 is nearly the same as the power factor angle θ. Then for the simple converter (**78**) and (**79**) simplify to

$$F_{RQ} = \frac{Z\,I_o}{(1 - \zeta)} \left\{ \frac{2}{\pi^2}\,\Delta - \left(\frac{\pi - 2\,\psi}{4\,\pi}\right)(1 - \zeta) \right.$$
$$+\frac{2\,\Delta}{\pi^2 \cos \theta} \sum_{2}^{\infty}{}_{\kappa} \frac{\sin^2 \kappa\,\pi/2}{\kappa^2}$$
$$\left. \times\,(\cos\,[(\kappa' - 1)\,\omega\,t - \theta] + \cos\,[(\kappa'' + 1)\,\omega\,t + \theta]) \right\} \tag{81}$$

$$F_{RD} = \frac{Z\,I_o}{(1 - \zeta)} \left\{ \frac{2}{\pi^2}\,\Delta \tan \theta + \frac{\psi}{2\,\pi}\,(1 - \zeta) \right.$$
$$+\frac{2\,\Delta}{\pi^2 \cos \theta} \sum_{2}^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/2}{\kappa^2}$$
$$\left. \times\,(\sin\,[(\kappa' - 1)\,\omega\,t - \theta] + \sin\,[(\kappa'' + 1)\,\omega\,t + \theta]) \right\} \tag{82}$$

Now $k \pm 1$ is always even, and since k' and k'' are only particular values of k it follows that the harmonics of armature reaction are always even and $M\ (p)$. Also, for every $k' - 1$ term there is a $k'' + 1$ term of the same order, because for every value of $M\ (p)$ values of k can be chosen such that $k' - 1 = M\ (p) = k'' + 1$.

Thus the armature reaction of a converter consists of a steady term and a series of even $M\ (p)$ harmonics. The pulsations in the m. m. f. due to these harmonics show up in the oscillograms as the so-called "tap ripple." This ripple is not present at zero load because the armature reaction is then zero. In the case of the quadrature reaction these pulsations occur under the interpole and seem to have some influence on commutation.

For unity power factor and zero brush shift the steady term of the direct component of armature reaction disappears; but contrary to general opinion the quadrature component does not vanish under these conditions.

Chief interest lies in the quadrature component since it acts on the neutral zone and must be compensated by extra turns on the interpoles. As an instance, take a six-phase converter for which $\zeta = 0$, $\theta = 0$, $\psi = 0$, and $\Delta = 1$. Then its quadrature component of armature reaction is

$$F_{6\phi} = Z\,I_o \,\{\, 0.037 + 0.013 \cos 6\ \omega\,t + 0.003 \cos 12\ \omega\,t + 0.001 \cos 18\ \omega\,t + \ldots \,\} \qquad \textbf{(83)}$$

Thus a 6th harmonic pulsation exists of about ⅓ the magnitude of the steady term. Due to this pulsation the armature reaction varies from about 10 per cent to 20 per cent of the d-c. armature reaction. Of course the ordinary series winding of the interpole does not neutralize these pulsations of m. m. f.

In design the average armature reaction over a pole arc is used as a criterion of the demagnetizing effect of the armature. If $2\ \lambda$ is the pole arc, then by (**77**) the average m. m. f. over this arc is

$$F_\lambda = \frac{1}{2\,\lambda} \int_{\pi/2-\lambda}^{\pi/2+\lambda} F_R\,d\,\beta$$

$$= \frac{Z}{\lambda\,\pi^2} \sum_1^{\infty}{}_{\kappa} \frac{\sin \kappa\,\pi/2}{\kappa^3}\, C_{\rho\kappa} \sin \kappa\,\lambda \left\{ I_1\, p \sin \frac{\kappa\,\pi}{p} \,.\, (\sin\,[(\kappa'' + 1)\ \omega\,t + \theta_1] - \sin\,[(\kappa' - 1)\ \omega\,t - \theta_1]) - 2\,I_o \sin \kappa\,\psi \right\} \qquad \textbf{(84)}$$

For zero brush shift, normal voltage ratio, full pitch, fundamental only, and Equation (**23**) this reduces to

$$F = \frac{2}{\pi^2}\ \frac{Z\,I_o}{(1-\zeta)}\ \frac{\sin \lambda}{\lambda} \tan \theta$$

$$= \frac{4}{\pi^3}\ \frac{Z\,I_o}{(1-\zeta)} \tan \theta\, \frac{\sin a\,\pi/2}{a} \qquad \textbf{(85)}$$

where

$$a = \left(\frac{\text{pole arc}}{\text{pole pitch}} \right) \qquad \textbf{(86)}$$

For $a = 2/3$ this gives,

$$F_{2/3} = 0.1675\, \frac{Z\,I_o}{(1-\zeta)} \tan \theta = 0.67\, \frac{Z\,I_o \tan \theta}{4\,(1-\zeta)} \qquad \textbf{(87)}$$

Appendix IV

THE DISTRIBUTION SUMMATIONS

The distribution of the armature coils leads to the two following definite series:

$$y_1 = \sum_0^{c-1}{}_r \sin n\,(x \pm r\,\sigma) \text{ and } y_2 = \sum_0^{c-1}{}_r \cos n\,(x \pm r\,\sigma) \qquad \textbf{(88)}$$

Expanding,

$$y_1 = \sum_0^{c-1}{}_r (\sin n\,x \,.\, \cos n\,r\,\sigma \pm \cos n\,x \,.\, \sin n\,r\,\sigma)$$

$$= \sin n\,x \sum_0^{c-1}{}_r \cos n\,r\,\sigma \pm \cos n\,x \sum_0^{c-1}{}_r \sin n\,r\,\sigma$$

$$= B \sin n\,x \pm A \cos n\,x$$

$$= \sqrt{A^2 + B^2} \sin \left(n\,x \pm \tan^{-1} \frac{A}{B} \right) = C \sin\,(n\,x \pm \beta) \qquad \textbf{(89)}$$

Likewise

$$y_2 = C \cos\,(n\,x \pm \beta) \qquad \textbf{(90)}$$

where

$$\left. \begin{aligned} A &= \sum_0^{c-1}{}_r \sin n\,r\,\sigma, \quad C = \sqrt{A^2 + B^2} \\ B &= \sum_0^{c-1}{}_r \cos n\,r\,\sigma, \quad \beta = \tan^{-1}\,(A/B) \end{aligned} \right\} \qquad \textbf{(91)}$$

The A and B summations may be evaluated geometrically from Fig. 14, where the vectors have unit length so that their projections on the x and y axes are the *cosine* and *sine* respectively of their direction angles $n\,r\,\sigma$. The figure has been drawn for four vectors, but the argument holds for any number of vectors, c. From the figure the following relations are evident.

$$R = 1/2 \sin \frac{n\,\sigma}{2} \qquad \textbf{(92)}$$

$$\alpha = (\pi - c\,n\,\sigma)/2 \qquad \textbf{(93)}$$

$$\gamma = (\pi - n\,\sigma)/2 \qquad \textbf{(94)}$$

$$\beta = \alpha - \gamma = (c - 1)\,\frac{n\,\sigma}{2} = \tan^{-1}\,(A/B) \qquad \textbf{(95)}$$

$$C = 2\,R \cos \alpha = \frac{\cos\,(\pi - c\,n\,\sigma)/2}{\sin n\,\sigma/2} = \frac{\sin c\,n\,\sigma/2}{\sin n\,\sigma/2} \qquad \textbf{(96)}$$

Therefore

$$y_1 = \sum_{0}^{c-1}{}_r \sin n\,(x \pm r\,\sigma) = \frac{\sin c\,n\,\sigma/2}{\sin n\,\sigma/2} \sin n \left[x \pm (c-1)\frac{\sigma}{2} \right] \tag{97}$$

$$y_2 = \sum_{0}^{c-1}{}_r \cos n\,(x \pm r\,\sigma) = \frac{\sin c\,n\,\sigma/2}{\sin n\,\sigma/2} \cos n \left[x \pm (c-1)\frac{\sigma}{2} \right] \tag{98}$$

Now if $\sigma = 2\,\pi/c$

$$\frac{\sin c\,n\,\sigma/2}{\sin n\,\sigma/2} = \frac{\sin n\,\pi}{\sin n\,\pi/c} = 0 \text{ if } n \neq M\,(c) \text{ or } 0 \tag{99}$$

But if $n = M\,(c)$ or 0 the fraction is an indeterminate

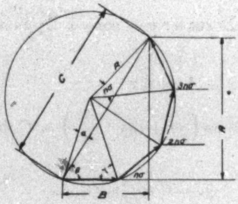

FIG. 14

which must be evaluated by differentiating numerator and denominator, thus

$$\left.\frac{\sin n\,\pi}{\sin n\,\pi/c}\right]_{n=M(c)} = \frac{D_n \sin n\,\pi}{D_n \sin n\,\pi/c} = \frac{c\,\pi \cos n\,\pi}{\pi \cos n\,\pi/c} = \pm\,c \tag{100}$$

Then in the limiting case of $x = 0$ and $\sigma = 2\,\pi/c$

$$\sum_{0}^{c-1}{}_r \sin n\,r\,\sigma = \frac{\sin n\,\pi}{\sin n\,\pi/c} \sin n\,(c-1)\frac{\pi}{c}$$

$$= \frac{\sin n\,\pi}{\sin n\,\pi/c}\left(\sin n\,\pi \,.\, \cos\frac{n\,\pi}{2} - \cos n\,\pi \,.\, \sin\frac{n\,\pi}{c}\right)$$

$$= 0 \tag{101}$$

$$\sum_{0}^{c-1}{}_r \cos n\,r\,\sigma = \frac{\sin n\,\pi}{\sin n\,\pi/c} \cos n\,(c-1)\frac{\pi}{c}$$

$$= \frac{\sin n\,\pi}{\sin n\,\pi/c}\left(\cos n\,\pi \,.\, \cos\frac{n\,\pi}{c} + \sin n\,\pi \,.\, \sin\frac{n\,\pi}{c}\right)$$

$$\left.\begin{aligned} &= 0 \text{ if } n \neq M\,(c) \text{ or } 0 \\ &= c \text{ if } n = M\,(c) \text{ or } 0 \end{aligned}\right\} \tag{102}$$

Appendix V

SUMMATION OF THE SERIES $\sum_{1}^{\infty}{}_n \frac{\sin^2 (2\,n-1)\,x/2}{(2\,n-1)^4}$

The Fourier Series of a periodic function is

$$f\,(x) = a_o + \sum_{1}^{\infty}{}_n \,(a_n \cos n\,x + b_n \sin n\,x) \tag{103}$$

where

$$\left.\begin{aligned} a_o &= \frac{1}{2\,\pi}\int_{-\pi}^{\pi} f\,(x)\,.\,d\,x \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\,(x)\,.\,\cos n\,x\,.\,d\,x \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\,(x)\,.\,\sin n\,x\,.\,d\,x \end{aligned}\right\} \tag{104}$$

For the case of the trapezoidal wave of unit amplitude shown in Fig. 15 we find that

$$f\,(x) = \frac{8}{\pi}\sum_{1}^{\infty}{}_n \frac{\sin (2\,n-1)\,b/2\,.\,\sin (2\,n-1)\,x}{b\,(2\,n-1)^2} \tag{105}$$

If $b = 0$ it becomes a square wave:

$$f_1\,(x) = \frac{4}{\pi}\sum_{1}^{\infty}{}_n \frac{\sin (2\,n-1)\,x}{(2\,n-1)} = 1 \text{ for } \pi > x > 0 \tag{106}$$

If $b = \pi$ it becomes a triangular wave:

$$f_2\,(x) = \frac{8}{\pi^2}\sum_{1}^{\infty}{}_n \frac{\sin (2\,n-1)\,\pi/2\,.\,\sin (2\,n-1)\,x}{(2\,n-1)^2} \tag{107}$$

The sum of the required series can be obtained by three successive integrations between the definite

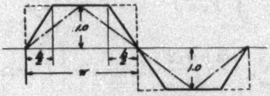

FIG. 15

limits of 0 and x of the square wave equation, and using the triangular wave equation to evaluate an intermediate series introduced by the first integration. Thus:

$$\int_0^x 1\,.\,d\,x = \int_0^x \frac{4}{\pi}\sum_{1}^{\infty}{}_n \frac{\sin (2\,n-1)\,x}{(2\,n-1)}\,.\,d\,x$$

$$\therefore -x = \frac{4}{\pi}\sum_{1}^{\infty}{}_n \frac{\cos (2\,n-1)\,x}{(2\,n-1)^2}$$

$$- \frac{4}{\pi} \sum_{1}{}_{n} \frac{1}{(2\,n-1)^2} \qquad (108)$$

But substituting $x = \pi/2$ into the triangular wave of unit amplitude, Equation (**105**), we get

$$f_2\left(\frac{\pi}{2}\right) = \frac{8}{\pi^2} \sum_{1}^{\infty}{}_{n} \frac{1}{(2\,n-1)^2} = 1 \qquad (109)$$

since $f_2\ (x) = 1$ at $x = \pi/2$. Therefore (**108**) becomes

$$\frac{\pi}{2} - x = \frac{4}{\pi} \sum_{1}^{\infty}{}_{n} \frac{\cos\,(2\,n-1)\,x}{(2\,n-1)^2} \qquad (110)$$

$$\int_0^x \int_0^x \left(\frac{\pi}{2} - x\right) d\,x$$

$$= \int_0^x \int_0^x \frac{4}{\pi} \sum_{1}^{\infty}{}_{n} \frac{\cos\,(2\,n-1)\,x}{(2\,n-1)^2}\, d\,x \qquad (111)$$

$$\frac{\pi\,x^2}{4} - \frac{x^3}{6} = \frac{4}{\pi} \sum_{1}^{\infty}{}_{n} \frac{1 - \cos\,(2\,n-1)\,x}{(2\,n-1)^4}$$

$$= \frac{8}{\pi} \sum_{1}^{\infty}{}_{n} \frac{\sin^2\,(2\,n-1)\,x/2}{(2\,n-1)^4} \qquad (112)$$

Therefore the required series is

$$\sum_{1}^{\infty}{}_{n} \frac{\sin^2\,(2\,n-1)\,x/2}{(2\,n-1)^4} = \frac{\pi^2\,x^2}{32} - \frac{\pi\,x^3}{48} \qquad (113)$$

LIST OF SYMBOLS

b = Brush width expressed in electrical radians
B_κ = Amplitude of flux density for the κth harmonic
c = Coils per phase belt
$C_{p\kappa}$ = Pitch factor of the κth harmonic
$C_{d\kappa}$ = Distribution factor of the κth harmonic
e = Instantaneous voltage induced in a coil
e_p = Instantaneous voltage induced in a phase
E_{dc} = D-c. induced voltage
E_1 = Effective value of the fundamental of a-c. induced voltage
F = Polyphase armature reaction
F' = D-c. armature reaction
F_R = Resultant armature reaction
Subscript D for direct component. Subscript Q for quadrature component.
i_{ac} = Instantaneous a-c. phase current
I_1 = Fundamental of a-c. current (eff. value)
I_m = Amplitude of the mth harmonic of a-c. current
I_0 = D-c. brush current
m = Time harmonic of the a-c. current
$M(\ )$ = Multiple of ( )
n = Number of turns per coil
p = Number of phases per pair of poles
r = Index for the $(r+1)$th conductor in a phase belt
R = Resistance of an armature coil
t = Time counted from instant when the phase is centrally located with respect to the poles
Z = Total number of conductors per pair of poles
α = Space angle measured on the armature
β = Space angle measured on the field
γ_κ = Flux shift of the κth harmonic of flux distribution
Δ = Distortion factor of the converter
ζ = Rotational losses as fraction of the a-c. input
θ_m = Phase angle of the mth time harmonic of a-c. current
θ = Power factor angle
κ = Space harmonic
κ' = Any value of κ for which $\pm(m-\kappa) = M\,(p)$ or 0
κ'' = Any value of κ for which $(m+\kappa) = M\,(p)$
ρ = Pitch expressed as a fraction
τ = Position of a coil from the middle of the phase belt
σ = Distribution angle of the coils
ψ = Brush shift from neutral position
ω = $2\,\pi f = 2\,\pi$ (frequency).

Discussion

E. B. Shand: From the standpoint of practical design calculations, I believe that the formulas in the complex forms given are not generally applicable, due to the fact that the accuracy of design calculations is limited by a number of factors not considered in the paper. An instance of this may be found in Equation (**20**) for voltage ratio. This expression may be simplified very greatly by introducing coefficients obtained directly from the converter field form without going through the process of actual harmonic analysis, which have a greater degree of accuracy than those of the calculated field form.

The expressions for the armature-loss constants vary from those ordinarily employed in that the effect of non-sinusoidal alternating currents is included. In working with converters the difficulty in determining actual armature losses is due to the fact that the phase relations between the alternating and direct currents within the armature are not expressed completely by the phase angle of the currents at the collector rings, due to the internal phase displacement within the converter itself. This difference in loss may be greater than 50 per cent in certain cases. Certain data and the manner of estimating the armature losses under these conditions have already been published.[1] I believe that Equations (**45**) and (**46**) should be modified to allow for these conditions.

In the discussion of armature reaction, it is stated that "the 6th harmonics of m. m. f. are responsible for the 'tap-frequency pulsation' in the external circuit of the converter when it is loaded." In my own experience with converters I have often found this pulsation to be just as great with no load on the converter when the reaction should be negligible, as with full load. This would preclude the generality of the above statement. In cases given some investigation, it has been found that the pulsation was due mainly to the fact that the converter with its 3rd and 5th, (etc.) harmonics in the generated voltage wave had been connected to a power system of low impedance and with a voltage closely approximating a sine wave. The harmonics had therefore to be absorbed to a large extent within the converter itself by pulsating fluxes produced by harmonic currents. These pulsating fluxes generated the voltage variations found in the d-c. voltage and which naturally must repeat themselves with every tap. These harmonic currents are particularly apparent when the converter is operating at very light loads.

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[1] "Operation of Synchronous Converters at Reduced Voltages", *Electrical Journal*, Dec. 1924.

One more comment might be applied to the paper in a general way. Its usefulness would probably be increased if the symbols used in the appendices were more completely defined.

Quentin Graham: I note that the authors have not discussed the average heating, or total loss, of all coils in the converter. They have shown very clearly how the heating of individual coils varies with coil position and power factor and might easily have gone a step further and included the average heating or loss. The latter is important if the efficiency is to be calculated under various conditions. I am submitting herewith a curve, Fig. 1, which shows the average heating constant for six-phase converters with the assumption of no rotational losses, and another set of curves, Fig. 2, shows the variation of the heating constant for six-phase converters under conditions of varying power factor and varying ratio of d-c. to a-c. power. The authors' equations contain a factor which depends upon this ratio although it is defined in terms of efficiency or rotational losses. If the ratio of d-c. to a-c. power is allowed to vary through a wide range as in Fig. 2 the results are applicable to cases in which mechanical power is either put into or taken out of the shaft. Such curves are useful in connection with booster converters, for example, in which the booster machine is coupled to the converter shaft.

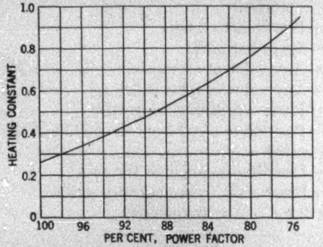

FIG. 1—CURVE SHOWING AVERAGE HEATING OR LOSS CONSTANT FOR 6-PHASE SYNCHRONOUS CONVERTERS WITH EQUAL INPUT AND OUTPUT

It is interesting to note that the minimum heating or loss does not occur when the input and output are equal, but that a small amount of power put in through the shaft is necessary to reach the minimum point on the curve. As the power factor becomes lower the minimum point recedes farther from the point of pure converter operations.

L. V. Bewley: The equations in this paper were derived to show explicitly the several factors which influence the characteristics of the converter, and were not intended to be directly applicable to routine design calculations.

The angles θ_m and θ refer to the displacement of the currents in the phase belts. The heating is a minimum when $\theta = 0$; although, as Mr. Shand points out, the external power factor of the machine will be lagging under these conditions due to its internal reactances.

The tap-frequency pulsation is caused by the harmonics of armature reaction and by a variation in the impedance drop of the alternating currents between brushes due to the transition of the phase belt across the brush. If the field form contains only the fundamental and M (p) flux harmonics and if there are no harmonics in the applied voltage, then the tap-frequency pulsation varies with the alternating current, and is zero at no-load. But if the field form contains other than the M (p) flux harmonics, or if there are harmonics in the applied voltage, then one component of the tap-frequency pulsation will be practically independent of the load. In many machines this latter condition predominates. The effects of the variation of the resistance in causing these pulsations has been very fully discussed by Mr. Neville in the November, 1917 issue of the *Electrician*, London, England. We have to thank Mr. Shand for bringing this question up in the discussion.

The average heating of the whole armature, which Mr. Graham asks about, is given in Equations (**36**), (**37**), (**38**), (**39**), (**40**), (**46**) and (**48**).

Equation (**23**), which gives the current ratio, contains a factor, ζ, which takes care of the rotational losses of the machine. The booster converter may be considered as included in our equations if ζ is taken as positive when the booster generates and as negative when the booster motors.

Also, in addition to what Mr. Graham brings up, if you want to carry things further and take up consideration of the variable-ratio or split-pole converter, the distortion of the flux causes motor or generator action depending upon whether the distortion factor Δ is less than, or greater than, unity. This is easily seen by averaging the armature reaction from the center of a main pole to the center of the next adjacent main pole. This average armature reaction, which is a measure of the torque, involves the expression $[\Delta/(1 - \zeta) - \cos \psi]$; it thus depends upon the distortion of the field form, the mechanical torque on the shaft, and the relative shift between flux and brush. Our equations are therefore rather general and easily extensible to other types of converters, although we specified that the scope of the paper was confined to consideration of the simple converter.

T. T. Hambleton: I agree entirely with Mr. Shand that the complication involved in the formula which we have developed would render it unsuitable for ordinary everyday use. The formula we already have in use is good enough.

There is another point of interest in addition to those mentioned by Mr. Graham relative to armature heating. This is a comparison of the increase in armature copper loss of the converter and other machines carrying only alternating current as the power factor is lowered.

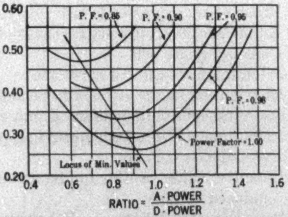

FIG. 2—CURVE SHOWING AVERAGE HEATING OR LOSS CONSTANT FOR 6-PHASE SYNCHRONOUS CONVERTERS FOR VARIOUS VALUES OF POWER FACTOR AND RATIOS OF A-C. to D-C. POWER

For values of the abscissa above 1.0 the converter performs some mechanical work (either externally or in overcoming rotational losses). For values below 1.0, some power is put in through the shaft.

A curve may be drawn using power factor as ordinates and per cent increase in copper loss as abscissas. A few points on these curves are as follows:

Power factor	Converter heating (tap coil), per cent	A-c. machine heating (all coils), per cent
1.00	100	100
0.97	165	106
0.95	190	111
0.90	248	123
0.80	375	156

This shows why the converter is essentially a unity-power-factor machine and illustrates the danger of departing from specified power factors.

Constant-Current Regulating Transformer Characteristics

Special Tests Show Radical Differences From Conventional Assumptions

BY H. C. LOUIS[1] and A. ALBAUGH[1]
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*It has generally been assumed that the total losses of a constant-current regulating transformer remain practically constant for all loads. Performance and test calculations have accordingly heretofore been made according to well-known conventional methods from the segregated losses determined from test or design data, assuming total losses constant. Extensive tests described in the paper show, however, that these losses are not necessarily even approximately constant, but for some transformers vary greatly with the load, the total losses at light loads greatly exceeding those at full load.*

The tests further show that the increased losses produce increased temperature rises at light loads, which may be sufficiently excessive to endanger the apparatus, although the same transformer may be able to carry full load with normal temperature rise.

The increased total losses and temperature rises at light loads are shown to be due largely to the increase of stray load losses with decrease of load, caused mainly by the leakage flux inducing excessive circulating currents in the laminations and structural parts of the transformer, such as the cage. Exploring coils and iron filings were used to investigate the amount and direction of the leakage flux.

Performance specifications should be based on the actual total losses for all loads and not on the conventional assumption of constant total losses, as has been done in the past.

It is pointed out that the characteristics described depend somewhat on the design, other features besides liberality in the use of materials being of importance. Consequently, measures should be taken by designers to correct these undesirable characteristics as much as possible. Users not only desire the most highly efficient apparatus consistent with cost, but must have apparatus of the highest reliability under all possible conditions, which cannot be expected of constant-current transformers endangered by excessive temperatures at light loads. The characteristics discussed are, therefore, not only of theoretical interest, but also of considerable practical interest to both designer and user.

THE general theoretical principles usually accepted and applied to performance calculations and tests of moving-coil constant-current transformers have been based on certain conventional assumptions which have heretofore been generally accepted without question. It has generally been assumed that the total losses of a constant-current transformer remain practically constant for all loads, that is, for all coil positions.

The reasons for this are based on simple fundamental considerations. Inasmuch as the applied voltage remains constant for all loads, and the resultant magnetic flux nearly so, it seems reasonable to assume that the core loss should also remain practically constant. Furthermore, inasmuch as the current in both primary and secondary remains practically constant, changes of load being changes in secondary voltage effected by variation of leakage reactance with position of the coil producing power factor changes only in current, the copper losses should also be practically constant. It was also assumed that the stray load losses did not vary greatly with the load.

The more exact reasoning leading to the assumption of constant losses is that the iron loss and copper loss both vary somewhat, but in such a way as to keep the total losses practically constant. Under the condition of full load, the magnetic leakage is a minimum, the entire magnetic circuit is excited to nearly its maximum density, and the core loss a maximum. At reduced loads, more of the flux passes across the open space between the legs, thus reducing the flux in parts of the core, consequently, decreasing the core loss. So the core loss would be greatest at heavy loads, and slightly lower at light loads. Considering the copper loss, the $I^2 R$ component remains constant, but the eddy current component is less at full load and slightly greater at light loads. On this basis the total losses would be practically constant for all loads.

Performance and test calculations were therefore formerly made according to well-known conventional methods from segregated losses determined from test or design data. The segregated losses were determined under conditions corresponding to full load, and the values thus determined used as the losses for other loads as well as for full load.

ACTUAL CHARACTERISTICS DIFFER GREATLY FROM THEORETICAL

Tests made by the writers, however, showed that the total losses were not necessarily even approximately constant, but for some transformers varied greatly with the coil position and consequently the load. In most cases the losses at light loads were found to greatly exceed those at full load. This condition is contrary to the usual performance of most electrical machinery. This increase of losses with decrease of load is plainly shown in Fig. 1.

This deviation of actual performance from that conventionally assumed is not only of theoretical interest but of great practical significance to both manufacturer and operator. It means that the losses at light

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[1] Both of the Consolidated Gas, Electric Light and Power Co., Baltimore, Md.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

loads are much greater than supposed, consequently the cost of supplying these losses appreciably increases the cost of current supplied to series lamps. It makes it desirable for users to operate constant-current transformers close to full load but this is not always practical under actual conditions.

Standard tests and manufacturers' guarantees have been based on the assumption of constant losses. This has given misleading fictitious values for the losses and efficiences at the lower loads. Referring to Fig. 1, at full load the losses are two kw. and the efficiency 96 per cent according to both methods. At one-quarter load, the actual loss is four kw., that is, twice that assumed by the conventional method, and the actual efficiency is 76 per cent as against 86 per cent based on the conventional method.

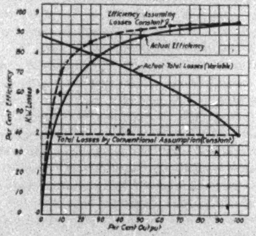

FIG. 1—ACTUAL AND CONVENTIONALLY ASSUMED VARIATION OF TOTAL LOSSES IN A CONSTANT-CURRENT TRANSFORMER WITH LOAD

TEMPERATURE CHARACTERISTICS UNUSUAL

Another consideration which is exceedingly important is the effect on the temperature rise of the transformer. The increased losses will produce increased temperatures at light loads, which may not only exceed the temperature guarantees, but may endanger the apparatus. Conventional temperature tests on constant-current transformers were formerly made under conditions corresponding to full load. Tests made by us, however, showed that temperature rises were higher at light loads than at full load due to the greater losses at light load. Transformers which have satisfactorily passed the standard temperature tests may develop excessive temperatures at light loads. This is shown in Fig. 2. This condition is contrary to our usual experience with electrical apparatus, which is that temperature rises are ordinarily greater at heavy loads than light loads.

Due to the arrangement of circuits, changing load conditions and necessary spare capacity allowances, it is not always practical to run all constant-current transformers at full load, but many of these must be run at light loads. Consequently, some of these may overheat and ultimately break down, even though they may be able to operate satisfactorily at full load without reaching excessive temperatures. The transformer of Fig. 2 shows a temperature rise in the iron of 55 deg. cent. at full load, which does not exceed the normal allowable limit. At one-half load the rise is 86.5 deg. cent. and at one-quarter load the rise is 93.5 deg. cent. which greatly exceeds the allowable limit.

FACTORS AFFECTING THESE CHARACTERISTICS

This variation of losses with load is more marked in some transformers than in others, depending somewhat on the design. As some of the older transformers were designed along very liberal lines, this variation was so small comparatively, as to justify the assumption of constant losses. The variation is most marked, however, in modern designs in which materials are worked to the limit, a condition typical of modern design in general, brought on by economic necessity and the stress of competition.

Liberality in the use of materials is not the only factor affecting the performance as described, but there are other factors which play an important part. We made a number of tests trying to analyze some of these, and to find some of the underlying causes and provide a theory to account for some of the particular characteristics described.

STRAY LOAD LOSSES

The results of our tests indicate that the stray load losses are mainly responsible for the effects described. The leakage fluxes vary greatly in amount with the load, causing the stray load losses to increase greatly with decrease of load. In some cases these may equal or exceed the total of the other losses. In most electrical apparatus the stray load losses, which are small at light loads and greatest at heavy loads, are usually considerably less than the other losses.

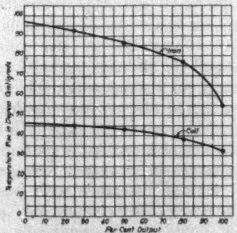

FIG. 2—EFFECT OF LOAD ON TEMPERATURE RISE OF CONSTANT-CURRENT TRANSFORMERS

The direction of the leakage flux has a very important bearing. As shown in detail later on, the leakage flux leaves the laminations not only by the edges, but a large proportion of it leaves from the sides perpendicular to the plane of the laminations, producing a heavy eddy current loss in these, with consequent heating. Furthermore, this is the part of the leakage flux which is most likely to encounter in its path metallic structural parts of the transformer, thus producing eddy currents in these parts. Fluxes leaving in the direction of the edges will produce comparatively small losses of this nature.

The total effect will therefore depend greatly upon the distribution of leakage flux resulting from the design. As the leakage flux increases with decrease of load, a transformer in which much of the leakage flux passes from the sides of the laminations will show greatly increased losses of this nature at light loads.

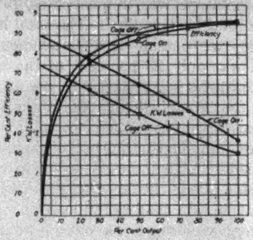

Fig. 3—Effect of Cage on Operation of Constant-Current Transformer

A marked example of losses in structural parts of a constant-current transformer was the stray load loss in the protective cage, which was greatest at light loads, tests showing the large value of $\frac{3}{4}$ kw. When the cage was removed, this cage loss was eliminated. Fig. 3 shows how the cage affected the total losses and the efficiency for various loads.

This marked variation of large stray load losses with load will therefore largely account for the great variation of total losses with load characteristic of some constant-current transformers. When the stray load losses are comparatively small the conventional assumption of constant losses is reasonably correct. However, when these are comparatively large the total losses will increase noticeably with decrease of load.

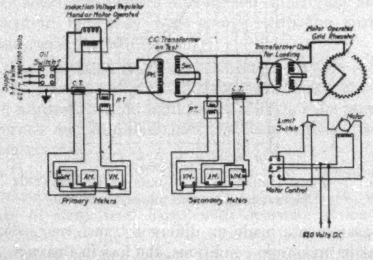

Fig. 4—Wiring Diagram of Constant-Current Transformer Test showing Meter Connections and Method of Loading

Efficiency and Loss Tests

Efficiencies and losses were determined by input—output method, tests being made on different makes and sizes of transformers at various loads and voltages. The scheme of connections used during the tests is shown in Fig. 4. This gave good control of voltage and load. Due to the use of transformers in the secondary circuit, the power factor in this circuit was slightly less than unity. The main effect of this was to materially reduce the maximum capacity of the transformer. However, as long as the secondary power factor was kept nearly constant throughout a particular test from no load to full load, the characteristics of the transformer were not altered.

The transformers tested were rated at 6.6 secondary amperes and at either 2300 or 2540 volts primary. Most of the tests were made on 60-cycle transformers at an actual frequency of 62.5 cycles, this being the frequency of our nominal 60-cycle system. A few tests were also made on 25-cycle transformers.

Readings were taken of voltage, current and kw. on both primary and secondary side, meters being read as closely as possible. Although absolute accuracy is not claimed for the instruments used and methods of measurement they were of sufficient practical accuracy for the purpose. This was shown by the general consistency of the loss measurements, and by the fact that the temperature runs gave results in line with the loss measurements. For example, as wattmeters are more subject to error at low power factors than at high power factors, the wattmeters on the primary side would tend to show higher losses at light loads than full load, due to the much lower power factors. However, not only was the actual possible error due to this cause far from being large enough to account for the great increase of losses shown by our results, but the greatly increased temperature rises actually measured proved that these represented actual additional losses rather than errors of measurement.

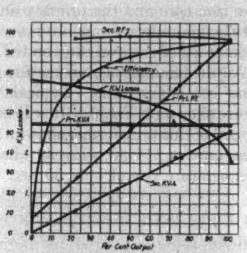

Fig. 5—Typical Performance Curves of Constant-Current Transformer

Efficiencies, power factors, and losses were calculated from the readings. Fig. 5 shows typical performance curves determined by actual test.

Temperature Tests

Temperature tests were also made on different makes and sizes of transformers under various conditions of load and voltage, to determine the effect of the load carried upon the temperature rise of various parts. These temperature runs were made with the same scheme of connections as for the efficiency tests shown in Fig. 4, that is, with test conditions the same as actual load conditions.

Temperatures were measured by means of thermometers and thermocouples placed in numerous locations on the iron core and the primary and secondary coils. The maximum iron temperatures were found to be on the center leg of the iron core, on the side parallel to the sides of the laminations, somewhere between the primary and secondary coils. Typical temperature rise curves are shown on Fig. 6. It will be noted that the floating coil of this transformer shows a temperature rise of 65 deg. cent. at ¼ load, whereas it had previously shown a temperature rise of only 55 deg. cent. at full load.

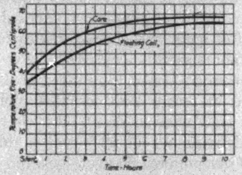

Fig. 6—Maximum Temperature Rise in Constant-Current Transformer Operating at One-Quarter Load

Exploring Coils used to determine the Amount and Characteristics of the Leakage Flux

Exploring coils consisting of several turns of small wire were wound around each of the three legs of the transformer. These coils were so arranged that they could be moved up and down the core. The voltage induced in these windings is a measure of the amount of flux at that particular point in the iron core. Moving the exploring core further up and further down the coil showed a change in voltage in the coil which represents an increase or decrease in magnetic flux in that core. The results obtained with the use of these exploring coils are shown in Fig. 7. It can be noted from the curves on Fig. 7 that the rate of change of flux throughout the entire distance between the primary and secondary coils is practically uniform. This means that the leakage flux leaving a narrow section near one coil, say on short circuit, is practically the same as the flux leaving a similar narrow section near the other coil. It was found from these tests that the flux per unit section between the primary and secondary coils was the same regardless of the position of the moving coil, provided, of course, that the primary voltage and secondary current and power factor were held constant.

To determine the amount of leakage flux leaving the core perpendicular to the core laminations, exploring coils were wound on the face of the sides on both the center leg and the two outside legs of the iron core. The voltage obtained on these cores is an indication of the amount of flux entering or leaving that portion of the circuit. When reduced to a unit area basis, it was found that the leakage flux leaving the center core perpendicular to the laminations was, in most cases, 45 per cent or more of the total leakage flux. The total leakage flux entering the two outside legs perpendicular to the laminations was only 25 per cent of the total transformer leakage flux.

The fact that so much flux leaves the center core perpendicular to the laminations accounts for the high temperatures found on that portion of the iron core and for at least some of the increased losses with decreased load, this being due to the fact that much more leakage flux occurs at no load than at full load.

Using a small exploring coil about one in. in diameter, the passage of the flux from the center leg to the outside leg was traced. With this coil it was found that a considerable portion of the leakage flux passes through the region normally occupied by the expanded metal protecting cage around the outside of the transformer. The amount of this leakage flux is sufficient to cause considerable induced currents in the screen. Careful measurements made on different transformers showed that in maximum conditions, the loss in the cage may amount to as much as 20 per cent of the total losses measured. This maximum loss occurs at short circuit when the leakage flux is greatest. As the load increases and the coils come closer together the loss in the screen is decreased slightly.

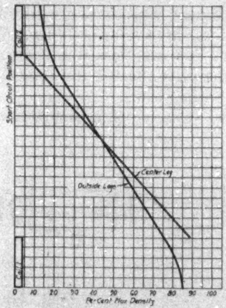

Fig. 7—Flux Densities in Cores of Constant-Current Transformer when Short-Circuited

Flux Distribution Shown by Iron Filings

The flux distribution was also studied by means of iron filings spread on paper placed around the core. Photographs of these were taken, Figs. 8 and 9 showing preroductions. These show the flux distribution in the air-gap between the center leg and outer legs of the core for two different transformers under load. The iron filings show how the flux leaves the center core and travels to the outside legs. Both of these show that not only does the flux pass from the edges of the center core to the outside legs, but a large amount of flux leaves the center core practically perpendicular to the laminations. On the other hand, on the outside legs, the greater proportion of this same part of the leakage flux passes around and enters at the extreme outside edges. That is, the leakage flux path to the outside legs is mainly to the inner and outer edges, the proportion of flux entering perpendicular to the laminations being considerably less than that in the center leg of the core.

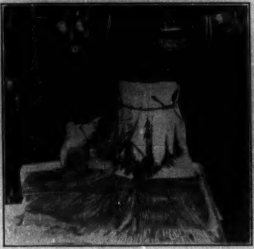

Fig. 8—Direction of Leakage Flux in Constant-Current Transformer under Load Illustrated by Means of Iron Filings

Conclusions

1. The results prove conclusively that the total losses of a constant-current moving-coil transformer are not constant, but may vary greatly with the load, increasing with decrease of load.

2. The increased losses at light loads may be sufficient to cause excessive and dangerous temperature rises in a transformer at light loads, although the same transformer may be able to carry full load with normal temperature rise.

3. This increase of total losses and increased temperature rises at light loads are due largely to the increase of stray load losses with decrease of load, caused mainly by the leakage flux passing from the core perpendicular to the sides of the laminations. These induce excessive circulating currents not only in the laminations themselves but in structural parts of the transformer, such as the cage.

4. Performance specifications, in order to be correct, should be based on the actual total losses for all loads. The conventional methods hitherto used based on constant total losses give highly misleading results. More attention should be given to the framing of specifications which should include such details as are necessary to show the true characteristics of the transformer, and to enable its fitness for particular operating conditions to be determined.

Some constant-current transformers are equipped with a light-load tap, which accomplishes reduction of secondary voltage by a change of ratio. This reduces the leakage flux and resulting stray load losses for a given load. While such a tap can be used to advantage under some conditions, tap changing is not always practical, such as under changing load conditions due to shifting of circuits. Specifications should not only cover the performance for the reduced capacity tap as well as for full winding, but should also clearly indicate on which each part is based. Some specifications do not indicate whether the performance at light loads is based on the full winding or on the reduced capacity tap.

5. Measures should be taken by designers to correct as far as possible the undesirable characteristics described. Aside from a more liberal use of materials, it should be possible to accomplish much by other design features A few possibilities are here suggested. The loss in the metallic cage can be eliminated by replacing this by a non-metallic guard. Losses in the laminations can be reduced by making these narrower. The use of a cruciform core, if practical, would also reduce the losses in the laminations by providing edges for the flux to leave in all directions thereby reducing to a minimum the flux leaving perpendicular to the sides of the laminations.

In this connection it is desired to point out that the extra cost of a properly designed constant-current transformer may be offset by the savings which will result from reduced losses. It may also be possible to get the desired characteristics without materially increasing the cost of the transformer.

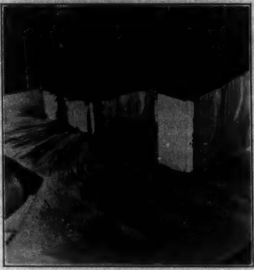

Fig. 9—Second Illustration of Leakage Flux by Means of Iron Filings Showing How This Leaves the Sides as well as the Inside and Outside Edges of the Laminations

Aside from the savings in losses, continuity of service, which is of special importance in street lighting service, requires reliability in operation of all equipment supplying this service. Such service is endangered by using transformers which may break down due to heating on light loads.

Discussion

E. D. Treanor: The authors' criticism of the conventional method of stating the characteristics of constant-current transformers is possibly justified in that this method was indefinite and allowed the possibility of misinterpretation by the purchaser and of neglect by the manufacturer. The assumption that the losses remain essentially constant at all loads was of course known to be not strictly accurate, but the method was of long standing and thought to be satisfactory for transformers which were usually operated near full load or on taps which approximate full-load conditions. It was a convenient assumption because it simplified and reduced the costs of tests and gave accurate results at the point of usual loading.

It seems to me that the most important point in the paper is the viewpoint that there is sufficient use of such transformers at quite low loads to justify more attention to light-load losses, as it is suggested that better characterisics may even justify increased cost if they cannot be obtained without it. On this basis the suggestions for improvement made by the authors and other possible methods should of course be studied to determine whether they are feasible economically on transformers which are now in somewhat limited use. It would seem quite difficult to prove that it is economical to operate constant-current transformer at half load or less even at the assumption that the losses remain constant.

Where the general data of the company with which I am affiliated have been made on the so-called theoretical basis, this has been very plainly stated, but in order to avoid any possibility of confusion such data will in the future be placed on the basis of input-output measurements and it will be shown where the data are based on normal windings and where on taps.

One other point is suggested as a possible danger from increasing light-load losses, that is, high temperature in coils or core structure. These transformers have not as yet been specifically covered by standardization rules of the Institute. Temperature limits, of course, should be considered with reference to the particular apparatus involved and the location of the heated portions with respect to insulation. Until such rules are laid down, the best guides would seem to be general information on other apparatus and experience on particular transformers. The maximum temperature reported at extremely low loads, while undesirably high, should not be injurious to the transformers which have been carefully designed to keep organic material from exposure to points of maximum temperature on metallic parts. The temperatures shown are not generally representative of modern designs.

There should be no difficulty in covering desired characteristics in losses and temperature by specifications so definite that no confusion can arise. However, when operating conditions compel the use of constant-current transformers at so much less than their normal load that the taps provided will not give reasonable characteristics, it would seem that the economical thing would be to provide transformers of a proper rating even if the larger transformer has the same losses at all loads.

J. B. Gibbs: It is well known that constant-current regulating transformers depend for their operation on the leakage flux between coils. This knowledge has enabled us to design and build regulators which operate satisfactorily and even to predict with good accuracy the reluctance of the average path which the magnetic leakage flux must follow. It has remained, however, for the authors of this paper to make a detailed study of the path of the leakage flux and of its effect on the regulator operation.

The ampere-turns in the winding of a constant-current regulator, or of any other transformer for that matter, cause a difference in magnetic potential between different parts of the iron circuit. This in turn causes a leakage flux to pass from one part to the other through the air space between the coils. The amount of this leakage flux depends upon the ampere-turns which produce it and on the length and area of the average path which it must traverse. If a constant-current regulator is to go to short circuit without an increase in secondary current, the designer must make the total leakage flux when the coils are at their position of maximum separation as great as the total flux induced in the regulator. The leakage leaves from the central core of the regulator in every direction, and quite a large part of it leaves in a direction perpendicular to the plane of the laminations. This part, of course, must pass directly through the outer laminations and it sets up considerable losses. The losses naturally depend upon the amount of iron affected, that is, upon the distance between the primary and secondary coil, and they are greater, as the authors have pointed out, under no-load condition. Under certain conditions, the temperature of the iron may be relatively high, especially under the no-load condition. The highest temperatures are usually confined to small parts of the iron, though, and it should be pointed out that the coils of this type of transformer are not wound on the iron; they are wound on a heavy insulating tube, and this is further separated from the iron by an air space. I never have heard of a case of damage to the coils on account of the temperature of the iron. In fact, all our tests seem to indicate that although the iron at certain points may become hot, the wire nearest to that iron is relatively cool. As to the coils themselves, the temperature test on such regulators is usually made with the secondary coil short-circuited and full voltage applied to the primary, which is the worst condition, as this paper shows. So that if the temperature test shows the coils at a safe temperature, you may be reasonably sure that the operating condition will show them at a still lower temperature. The authors' remarks about temperature as applied to commercial constant-current regulators seem to me rather unduly alarming.

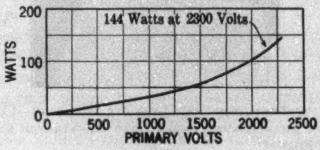

Fig. 1

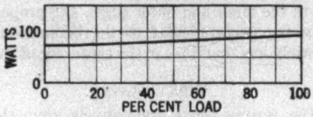

Fig. 2

The most desirable operating condition, as every one recognizes, is with full load on the regulator. This results in the highest efficiency and the highest power factor. The present paper shows also that it results in the lowest temperature. All these three factors operate in the same direction. Sometimes the condition of the circuits demands that the regulator be operated at less than full load. If this is done, efficiency and power factor are sacrificed to a certain extent and the temperature is increased above what it might be, but it is not increased to a point where a good commercial regulator will be endangered either in operation or in life.

W. B. Kouwenhoven: As Messrs. Louis and Albaugh show, the losses are mainly caused by the leakage flux. This leakage flux will naturally produce a high temperature in the iron, but it is a surprise to me that the temperature of the secondary winding should be as high as shown by the authors. I should like to know how the temperature of this winding was measured. Owing to the small current in the secondary, the size of the conductor is relatively small and I should expect that the eddy currents set up in this conductor by the leakage flux would be small.

A. F. Hamdi: During the summer of 1925 we were testing some constant-current transformers and it had been our habit to use the over-all method for getting efficiency. We did not use the accepted A. I. E. E. method, which means calculating the efficiency from the losses. The A. I. E. E. rules apply specifically to constant-potential transformers and were not suited for this purpose; and the paper and also the previous speakers have pointed out why those rules do not cover constant-current transformers. We hope that in the future revision of the standards, this thing will be taken care of.

In our tests the errors in efficiency were not quite as much as pointed out by the authors, namely, in Fig. 1 of the paper we find about a 10-per cent difference at 25-per cent load. In our tests we found discrepancies of about 1½ to 5 per cent in various transformers. The transformers we tested were oil-cooled, 10-kv-a. modern transformers.

I should like to ask the actual magnitude of the stray-load losses. In our case we found that the total losses varied by less than 20 per cent, whereas the stray-load losses themselves varied by over 100 per cent; in other words, they were about 100 watts at full load and went up to about 225 watts at light load.

I should like to discuss an empirical method which we have used for efficiency calculations, which apparently gives very good results. On a test of six transformers, we have been able to come within better than 1 per cent of the over-all method.

One thing, of course, to be kept in mind is that the entire core of the constant-current transformer is not excited to the same flux density.

In the new method we have assumed, arbitrarily, that one-half of the core is excited to a flux density corresponding to the primary voltage, while the other half is excited to a flux density corresponding to the secondary voltage. The total core loss at any load can therefore be obtained from the accompanying Fig. 1, which represents core losses at various voltages with open-circuited secondary. Fig. 2 represents the total core losses obtained from Fig. 1, following the above assumption.

Taking for instance the case of full load, the core loss is 95 watts, made up of 72 watts (half the core loss occurring at primary voltage—2300 volts) and 23 watts (half the core loss occurring at full load secondary voltage—1335 volts). Similarly at no load, the core loss is taken to be only 72 watts, as there is practically no voltage across the short-circuited secondary.

The next thing to calculate is the stray-load loss. This can be obtained from the short-circuited impedance test made with the two extreme positions of the coils: first with the coils locked close together and second with the coils away from each other as far as possible, corresponding to the full-load and the no-load positions respectively.

From the two values of losses so measured we obtain the stray-load losses by subtracting the d-c. $I^2 R$ losses involved and also the core losses which are obtained from Fig. 1. This last step is essential because contrary to the case of constant-potential transformers considerable voltage is necessary to perform a short-circuit test on constant-current transformers.

The two values of stray-load losses so obtained are plotted in Fig. 3. The straight line joining them gives the stray-load losses at various loads, if we assume that the coil displacement is a straight-line function of the load. This assumption is really fair, because the stray-load losses actually vary as the curve in dotted line superimposed on the straight line in Fig. 3.

The total losses of the transformer can therefore be obtained by adding to the d-c. $I^2 R$ losses (constant for all loads) the core losses as obtained from Fig. 2 and the stray-load losses as obtained from Fig. 3.

The efficiency is then calculated in the usual manner.

H. C. Louis and A. Albaugh: The question of loading constant-current transformers is a very serious one, especially when circuits are growing. In this case, it is necessary to provide certain spare capacity in the transformers. This capacity, of course, will vary with the assumed ultimate size of the circuit. So that to prepare for the final condition, transformers of apparently excessive size are often used.

Another reason is that when station-type transformers are used, it is highly advantageous to have them all of the same size, thereby providing the greatest standardization and flexibility. Some of the circuits for these transformers, therefore, will be more heavily loaded than others.

Regardless of the loading of these transformers, it is important that their characteristics be thoroughly understood by operators and designers and the main object of this paper is to present the most important of these characteristics.

Mr. Treanor mentioned that the temperatures shown in the curves are not generally representative of modern designs. The temperatures shown are fairly representative of the latest design station-type air-cooled constant-current transformers. However, oil-cooled transformers have lower temperature due to the presence of the cooling oil.

The advantage of taps on constant-current transformers is questionable. Some of our tests show that although operating on a lower capacity tap, the transformers do not have a higher efficiency than when operating at the same load on the full-capacity tap. At light loads on the low-capacity tap, losses were found to be actually higher than on the full-rating connection. The main advantage of the tap is the slightly higher power factor obtained.

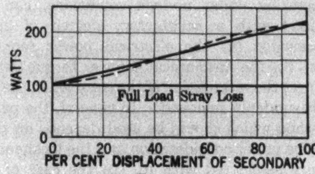

FIG. 3

Mr. Gibbs mentions that temperature tests are usually made with the secondary coils short-circuited and full primary voltage applied. Under these conditions, the temperature rise will be a maximum, but considerable care must be taken in these tests to locate the hottest parts, especially in oil-cooled transformers. The hot-spot temperatures of these transformers may be sufficiently high to cause sludging and for this reason, should be located.

Dr. Kouwenhoven's question about the temperature rise in the coils, is an interesting one. Undoubtedly, the great amount of leakage flux causes eddy-current losses in the copper conductors which will vary somewhat with the load and thus affect the temperatures. However, the copper temperature variation noted was mainly due to radiation and convection of heat from the iron core. The temperature of the coils on the transformers tested was measured by means of thermometers and thermocouples applied directly to the coils. Temperatures by rise of resistance measurements were taken but as these gave only the average temperatures and not the maximum temperatures, they were not used in the paper.

In reference to the losses, we have made some attempts to segregate them but the results were not very satisfactory. The stray-load losses at full load on 50-kw. air-cooled station-type transformers amount to approxmately 400 watts out of a total loss of about 1800 watts, while at short circuit, the stray-load losses amount to 2400 watts out of a total of 3500 watts. Of course, we are not certain of these values and do not put much dependence in them. They were worked up only in order that we might reach some conclusion for the apparent increase in losses of the transformer with decreased load.

Additional Losses of Synchronous Machines

BY C. M. LAFFOON[1] and J. F. CALVERT[1]
Associate, A. I. E. E. Non-member

Synopsis.—*In the case of high-speed turbine generators, the most reliable means of determining the losses under actual operating conditions is to measure the weight and temperature rise of the cooling medium and to estimate the small part of the losses which is dissipated from the frame to the surrounding medium. The temperature rise of the cooling medium can be obtained by means of temperature detectors located at the inlet and outlet sections of the generator. In order to obtain reliable values of the average temperature rise for the machine, it is necessary to have approximately uniform velocities at both inlet and outlet sections and to measure the temperature rise at a large number of incremental sections.*

The volume of cooling medium passing through the machine can be determined by (a) introducing a definite amount of heat energy into the cooling medium and measuring its temperature rise, or (b) measuring the mean velocity head at the outlet section of a properly designed stack.

Loss tests were made on five 3600-rev. per. min. turbine generators when operated as synchronous condensers. In the case of these machines, the additional losses including the increase in core loss at full kv-a. and zero per cent power-factor load varied from 3 to 22 per cent of the total losses. This corresponds to approximately 0.14 to 1.0 per cent of the generator input. The additional losses as measured under sustained short-circuit conditions were from 5 to 10 per cent less than the corresponding values for full kv-a. at zero per cent power factor.

It is suggested that data can be obtained for predetermining the magnitude of the total additional losses by measuring the loss in each structural part and determining graphically the magnetizing flux which produces it. The magnetizing flux distributions at different parts of the machine were plotted for different ampere-turn relations.

Additional studies are being made on methods of calculating the magnitude of the additional losses and the increase in core loss with load.

THE losses of a synchronous machine, when operating under no-load conditions, can be obtained with a satisfactory degree of accuracy by measuring: (a) the electrical power required to drive it; (b) the mechanical counter torque developed by it; or (c) the retardation of the rotor when the driving power is suddenly removed. In general, the particular test method which should be used depends on the type of machine under consideration and the local conditions which are associated with it. In the case of a high-speed steam turbine generator, it is preferable to operate it as a synchronous motor and measure the electrical input. The power factor of the load can be adjusted to unity, practically, so that the electrical input measurements can be made very accurately. This method eliminates the necessity of correcting for the coupling, and for motor losses when a separate driving motor is used. It has an outstanding limitation when applied to steam turbine generators, however, in that such machines are not usually adapted for self-starting as induction motors, but must be brought up to speed synchronously with an isolated generator set. In connection with large steam driven turbine generators, it is well to call attention to the fact that the direct-connected auxiliary generator furnishes a satisfactory means of driving the main generator and thus makes it relatively easy to measure the magnitude of its no-load and short-circuit losses. In the case of low-speed generators, the excitation loss is an appreciable percentage of the total losses; consequently, a direct-connected exciter, when available, can be used as a source of driving power for determining the no-load and short-circuit losses. With larger water-wheel driven generators which have large fly-wheel effect, the retardation method offers a satisfactory means of determining the no-load and short-circuit losses[2].

Various satisfactory methods have been derived for separating the total measured value of the no-load losses into the two principal components of (a) windage and friction, and (b) iron loss. Since the total value of the no-load losses can be accurately determined and segregated into the respective components, it is a relatively simple matter to estimate the magnitude of the no-load loss components quite accurately, provided the loss constants are based on test results from similarly proportioned machines. Obviously, this method of calculation is limited by the actual test data which are available. With the more analytical and scientific method of calculating the losses, it is necessary to make an exhaustive study of the effect of the different variables involved and then determine the necessary constants from tests on actual machines or models which reproduce the conditions that exist in the machine. A large number of such investigations has been and is being made by different individuals, organizations, and manufacturing companies, so that, so far as the no-load losses of synchronous machines are concerned, the field is being rather thoroughly investigated.

SYNCHRONOUS MACHINES UNDER LOAD CONDITIONS

When a synchronous machine delivers or receives current at constant terminal voltage and definite power-factor conditions, the flux and ampere-turn relations are appreciably different from the conditions which exist at no-load on account of the leakage reactance and demagnetizing action of the armature winding. Consequently, the armature current not only introduces other losses, but the iron and excitation losses are higher than at no-load. The friction and windage losses can be considered, for all practical

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[1] Both of the Westinghouse Elec. & Mfg. Co.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

[2] See Bibliography, 1.

purposes, constant at all loads, provided the air and speed conditions are not changed. The losses introduced by the armature current are the armature copper loss based on d-c. resistance and the additional losses which are produced in the materials of the main magnetic and electric circuits and in materials which are in close proximity to the armature winding. The excitation loss under different load conditions can be calculated or measured with a satisfactory degree of accuracy. The d-c. resistance of the armature winding can be satisfactorily measured under load conditions and thus the d-c. armature copper loss can be obtained. The question of increased copper loss due to the non-uniform distribution of the current in the armature conductors has been thoroughly investigated by different writers, and it can be assumed that this portion of the additional loss can be calculated with a satisfactory degree of accuracy.[3] The question of increased iron loss due to load will be considered in a later article. Several studies have been made by European writers in regard to the calculation and measurement of generator losses under load conditions.[4] But the data which were obtained apply particularly to machines of European design proportions. The design proportions and construction of European machines are widely different from present American designed machines and the additional losses are probably widely different in the two types of machines. The principal purposes of this article are (a) to discuss methods of measuring the losses of high speed synchronous machines under load conditions; (b) to give results of loss measurements on four, 3600-rev. per. min. turbine generators; and (c) to segregate the losses into the different components so as to ascertain the magnitude of the total additional losses.

METHODS OF MEASURING THE LOSSES OF STEAM TURBINE-DRIVEN GENERATORS UNDER LOAD CONDITIONS

The loading-back and calorimeter methods are the two outstanding means of measuring the losses of high-speed synchronous machines under different load conditions. In the loading-back method two duplicate machines, which are electrically and mechanically coupled together, are driven at synchronous speed. When one unit operates as a motor and the other as a generator at definite current, voltage and power-factor conditions, the only power required from an external source is that which is necessary to supply the losses of the two units. With this method of test, the total losses of the two machines can be measured with a satisfactory degree of accuracy, but there is question in regard to the division of the losses on account of the fact that the two machines do not have the same excitation and consequently do not have the same internal flux conditions. This method has the further limitation in that two machines are required and a complicated coupling or frame shifting mechanism is required if it is desired to operate at different power factors.

With the calorimeter method, the losses of the machine are determined by measuring the weight and temperature rise of the cooling medium. Obviously this method is only applicable to machines with forced ventilation, such as rotating machines and water- or oil-cooled transformers. When such an electric machine is operating at any given load condition, and constant temperatures have been reached, a major portion of the losses within the frame is carried away by the cooling medium, and the remainder is dissipated to the medium surrounding the frame of the machine. In the case of steam turbine driven generators, the magnitude of the loss to the cooling medium can be measured quite accurately and the small part of the loss which is dissipated from the frame to the surrounding medium can be estimated with a sufficient degree of accuracy to make this method the most reliable one for this class of machines. If the machine has a closed circuit ventilating system and the surface type of cooler for the cooling medium of the generator, the loss given up by the cooling medium can be obtained by measuring the weight and temperature rise of the water passing through the surface cooler. In addition to estimating the loss dissipated from the frame of the machine, it is necessary in this case to estimate the portion of the loss which is lost by the cooling medium in passing from the outlet of the machine to the inlet of the cooler, and also the loss dissipated from the frame of the cooler. With these three corrections to make, instead of one, the magnitude of the error in the estimated portion of the losses is probably three times as great as in the previous case. Moreover, the energy transferred from the cooling medium to the water cannot be measured with as great accuracy as that from the machine to the cooling medium on account of the fact that (a) the temperature rise of the cooling water is only about one-fourth of that of the cooling medium in passing through the machine and hence, for the same numerical error in the temperature rise, the percentage error would be four times as great; and (b) equally reliable measurements of the temperature rise of the cooling water cannot be made on account of the difficulty of getting a sufficient number of accurate-reading temperature detectors in intimate contact with the water at the inlet and outlet sections of the cooler. In general, the direct measurement for determining the weight and temperature rise of the cooling medium is the simplest and most reliable calorimeter method of determining the total losses of steam driven turbine generators under any load condition.

MEASUREMENT OF THE TEMPERATURE RISE OF THE COOLING MEDIUM

The temperature rise of the cooling medium can be obtained by measuring its temperature at the inlet and discharge sections of the machine or by measuring it directly by means of differentially connected temperature detectors. The measurement of the temperature rise of the cooling medium would be very easy to make if the temperature were constant at both inlet and discharge sections. In actual machines, neither the temperatures nor the velocities of the cooling medium are uniform at either the inlet or discharge sections of the generator. Consequently, in order to determine the average temperature rise of the machine, it is necessary (a) to divide the inlet and outlet sections into a large number of incremental sections and measure the temperature and the volume of the cooling medium which flows across each incremental section; or (b) to obtain uniform velocities at both inlet and outlet sections and then obtain the temperature at a sufficient number of incremental sections so that a simple algebraic average can be used. The latter is the preferable method to use on account of the fact that the velocity variable of the cooling medium is eliminated and the temperature rise can be obtained at a sufficient number of incremental sections by measuring the difference in e. m. fs. induced in thermocouple junctions or the difference in potential drops across resistance elements. Satisfactory results can be obtained by either type of detector provided the necessary precautions are exercised, and the choice of method will depend to a large extent on general preference and previous experience. When the thermocouple type of detector is used, the number of junctions in series can be chosen such that the indicating instrument operates at the most accurate part on its scale. Rubber insulation should be used on the wires between junctions in order to eliminate erratic results due to short circuits and grounds which are usually experienced when less reliable insulation is used. If the inlet air temperature periodically fluctuates over an appreciable range, the measured temperature rises will also be irregular because the pulsations are damped out or absorbed by the generator and do not appear to an appreciable extent in the outlet air temperature. The variation in inlet air temperature must be corrected for by taking a relatively large number of readings, or the temperature detectors must be compensated so that a close average temperature rise can be obtained.. In order to damp out variations in the temperature rise after the machine has reached constant temperatures, thermal storage capacity should be added to the detector located at the inlet air section. From a theoretical consideration, the heat storage capacity of the detectors which are located at the inlet air section should be equivalent to that of the machine but actually the mass of the heat storage materials for the inlet detectors cannot be made very large without obstructing the air flow. Marked improvement can be obtained, however, by making the heat storage capacity of the detectors located in the inlet air stream as large as practical, without obstructing or seriously disturbing the air flow. In order to obtain sufficient thermal storage capacity in the inlet detectors, and yet not interfere with the air flow, a satisfactory procedure is to measure the temperature rise between two arbitrarily chosen incremental sections at the inlet and discharge by means of a set of thermocouples connected in differential series. Correction for the variation in temperature at both the inlet and discharge section can be made by measuring the temperature variation of all other points of each section with respect to the arbitrarily chosen reference points, by means of another set of thermocouples connected in differential series but which have no appreciable thermal storage capacity.

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[3] See Bibliography, 2, 3, 4.

[4] See Bibliography, 5, 6, 7.

MEASUREMENT OF WEIGHT OR VOLUME OF THE COOLING MEDIUM

The calorimeter method can also be used to measure the volume of the cooling medium which passes through the machine. If a known amount of power is absorbed by the cooling medium and the corresponding temperature rise is measured, the volume of the cooling medium can be calculated from its specific heat constant, barometric pressure, and absolute temperature. If air is used as the cooling medium the volume can be calculated from the following formula[5].

$$V = \frac{0.177\,(273 + T_1)}{B}\,\frac{W}{T_2} \qquad (1)$$

where V = the mean volume in the meter in cu. ft. per min.,

T_1 = the mean temperature in deg. cent.,

B = room barometric pressure, and approximately the mean static pressure in the meter in inches of mercury,

W = watts input to the air,

T_2 = temperature rise of the air in deg. cent. due to W.

In order that satisfactory values of air volume be obtained with this method of measurement, the following conditions and requirements must be fulfilled:

a. Either uniform air velocities or uniform air temperatures must exist in the air stream at the sections where the temperature detectors are located,

b. The heat input to the air stream should be uniform over its entire cross-section, or else the air must be thoroughly mixed after heating so that uniform air temperature will exist at the outlet section,

c. The temperature of the heating elements must be sufficiently low, and the lead arrangements such that the percentage of heat radiated and conducted from these elements is negligible as compared to percentage of heat input which is convected away by the air stream,

d. The walls surrounding the air stream at the place where the heat is introduced should be insulated so that a negligible percentage of the heat input is dissipated to the surrounding atmosphere,

e. A sufficient length of time should elapse, after the heat is applied, before making the temperature rise measurements, in order for the walls, bus bars, and wiring connections to reach constant temperatures,

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[5] See Bibliography, 8.

f. The degree of accuracy in measuring the temperature rise of the air should be comparable with the accuracy of the temperature rise measurement of the generator under load conditions.

The volume of the cooling medium can also be obtained by measuring its velocity head at the discharge section. This necessitates the discharge of the cooling medium from the machine from a nozzle or stack which must be designed so that the velocities of the cooling medium at the outlet are practically uniform and the direction of flow is normal to the outlet section. When the cooling medium is discharged directly into a large room, the discharge velocity heads for the incremental section of the outlet section can be measured with an impact tube and an inclined pressure gage. From these velocity head measurements the respective velocities can be calculated in feet per minute for any specific cooling medium. The derivation of the formula for air as the cooling medium is as follows:

p = velocity head in inches of water for any incremental section,
a = area of any incremental section in sq. ft.,
v = velocity in ft. per min. for any incremental section,
q = volume in cu. ft. per min. for any incremental section,
θ = air temperature in deg. cent.,
$T = 273 + \theta$ = absolute air temperature,
K = constant,
α = density of air in lb. per cu. ft.,
β = barometric pressure.

Subscript *o* indicates values corresponding to standard air conditions which are assumed to be 25 deg. cent. or 298 deg. cent. absolute temperature, and a barometric pressure of 29.92 in. of mercury. Subscript 1 indicates values applying to an actual test condition, and subscript *i* indicates values applying to intake air conditions.

$$v_0 = \frac{1}{\sqrt{K \alpha_0}} \sqrt{p_0} = 4030 \sqrt{p_0}$$

for any incremental section,

$$v_1 = \frac{1}{\sqrt{K \alpha_1}} \sqrt{p_1}$$

$$\alpha_1 = \frac{\alpha_0 T_0 \beta_1}{T_1 \beta_0}$$

$$q_1 = a v_1$$

$$q_1 = 4030 \sqrt{\frac{T_1 \beta_0}{T_0 \beta_1}} \sqrt{p_1}$$

$$= 6 \times 10^3 \sqrt{\frac{273 + \theta_1}{\beta_1}} \sqrt{p_1}$$

Then the total volume Q_1 at the outlet, is

$$Q_1 = \sum_1^n q_1 = 6 \times 10^3 \sqrt{\frac{273 + \theta_1}{\beta_1}} \sum_1^n \sqrt{p_1} \qquad (2)$$

Since the variations in temperature of the outlet air are small, practically no error in the total volume is introduced by considering T_1 constant for all of the incremental sections. The total volume in terms of intake condition is

$$Q_i = Q_1 \frac{T_i}{T_1} = \frac{6 \times 10^3 (273 + \theta_i)}{\sqrt{\beta_1 (273 + \theta_1)}} \sum_1^n \sqrt{p_1} \qquad (3)$$

Since it is necessary to measure the temperature rise of the air passing through the machine, it is desirable to have the temperature at the outlet of the stack the same as at the outlet of the machine. The loss of heat energy from the walls of the stack can be made inappreciable by applying a sufficient amount of cork and felt insulation.

In comparing these two methods of measuring the volume of air passing through steam driven turbine generators, both require practically uniform velocities at particular sections. The calorimeter method requires special wiring, and considerable care in the thermocouple measurements. The velocity head readings at the outlet section of the stack can be easily obtained and consistently repeated. The number of points at which these pressures are read can be increased indefinitely and thus the accuracy of the results can be made as high as it is advisable to go. The number of thermocouple junctions cannot be increased beyond a particular value without restricting the air flow. The stack is simple to build, more sturdy, easier to maintain, and in general gives more consistent results than the calorimeter method of measuring air volumes. Both methods will give reliable results provided the elements are properly designed and the necessary precautions exercised in making the measurements.

LOSS MEASUREMENTS ON HIGH SPEED TURBINE GENERATORS

Temperature and loss measurements were made on five three-phase, 3600-rev. per. min., 80-per cent power-factor turbine generators with rating characteristics as indicated in Table I. The generators were operated as synchronous condensers, and the losses were determined by the calorimeter method for several different kv-a. loads. The power factor varied from 100 per cent at no-load to approximately zero per cent at 100-per cent kv-a. load.

The loss from the generator to the cooling air was calculated by the following formulas:[6]

$$K W_1 = \frac{Q_o \theta \times 10^{-3}}{1.765} \qquad (4)$$

$K W_1$ = Loss to the air in kw.

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[6] See Bibliography, 8.

TABLE I

Generator number	Rating		Remarks
	Kv-a.	Volts	
1	2500	600	Generator had same fan as generator No. 2. One conductor per slot type of stator winding.
2	3125	2400	Stator end plates of magnetic material.
3	3125	2400	Same as generator No. 2 except non-magnetic end plates.
4	6250	2400	Axial system of ventilation. Armature punchings of medium loss steel.
5	6250	2400	Multiple path radial system of ventilation. Larger fan than on generator No. 4. Armature punchings of low loss silicon steel.

Q_a = Air volume in cu. ft. per min. at standard temperature and barometric pressure conditions.

θ = Average temperature rise of the cooling air in deg. cent.

The loss dissipated from the frame by natural convection was calculated on the basis of a dissipation constant of 0.012 watts per sq. in. per deg. cent. temperature difference between the surface of the machine and the surrounding air.

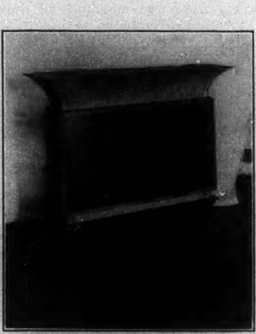


Fig. 1—Calorimeter

$$K\,W_2 = 0.012 - S\,(\theta_s - \theta_a) \qquad (5)$$

where

$K\,W_2$ = Loss in kw. dissipated from the surface of the frame,

S = Effective dissipating surface of the frame in sq. in.,

θ_s = Average temperature of the frame surface in deg. cent.,

θ_a = Average temperature of the surrounding air in deg. cent.

The total loss within the frame of the generator is the sum of the two losses as defined above.

The temperature rise of the cooling air for all of the machines was measured by means of thermocouples located at the inlet and outlet sections and connected in differential series. The volumes of cooling air for machines Nos. 1 to 4 inclusive were determined by the calorimeter method and Equation (**1**). Fig. 1 shows the type of calorimeter which was used to make the measurements. The volume of cooling air for generator No. 5 was determined from the velocity head measurements at the outlet of a discharge stack and the calculations were made by using Equation (**3**). Fig. 2 shows the stack as assembled on this generator for the test.

Fig. 2—Discharge Stack on Generator

Separation of the Losses

The total losses within the frame were separated into the following components:

a. Air friction and fan loss,
b. Iron loss at no-load,
c. Field copper loss,
d. Armature copper loss based on the d-c. resistance,
e. Additional loss.

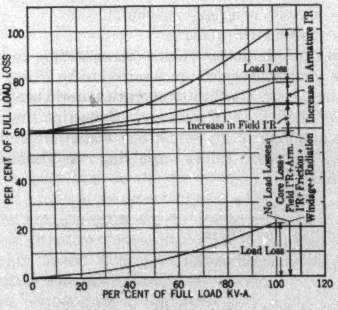


Fig. 3—Showing Ratio of Loss to Load

(Generator No. 1, 2500-kv-a., 600-volt, three-phase, 60-cycle, 3600-rev. per min.)

The total losses of the machine at no-load and full voltage conditions, as obtained by the calorimeter method, checked very closely with the results which were obtained from the electric input measurements when the machines were operated as synchronous motors at no-load and 100 per cent power factor. The iron loss was separated from the total no-load loss by measuring the kw. input to the machine when operated as a synchronous motor at 100 per cent power factor for several values of impressed voltage, and then extending a curve of either the voltage or a function of the voltage vs. total kw. loss to the zero voltage line.

The field copper loss was calculated from the measurements of the field current, and the voltage at the collector rings after constant temperature conditions were reached. The armature copper loss, based on the d-c. resistance of the winding, can be calculated with good approximation by estimating the average temperature of the winding from readings of imbedded temperature detectors on the ends and buried parts of the coils. A closer approximation can be obtained by measuring the resistance of the winding immediately at the end of each test run. By plotting the values of resistance against time and extending the curve back to the instant of time corresponding to the end of the test, the resistance of the winding at the final operating condition can be determined with a good degree of accuracy. Both methods were used in determining the armature copper losses of these generators. The additional loss was obtained by subtracting the sum of the above losses, items *a* to *d* inclusive, from the total value of the measured loss within the frame for each generator. The total losses within the frame and the segregated components are given in Figs. 3, 4, 5, 6 and 7, as a function of the per cent kv-a. load for all of the generators. Fig. 8 shows the variation of the additional losses as a function of the per cent kv-a. load at zero per cent power factor for all of the machines. The total and segregated values of the generator losses are given in Table II for both full kv-a. load at zero per cent power factor, and for sustained short-circuit conditions with full-load current flowing in the armature winding.

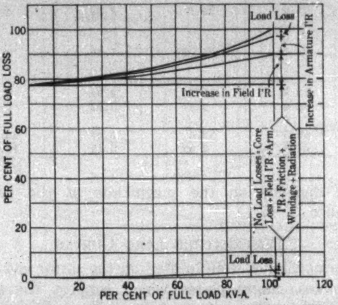


FIG. 4—SHOWING RATIO OF LOSS TO LOAD
(Generator No. 2, 3125-kv-a., 3300-volt, three-phase, 60-cycle, 3600-rev. per min.)

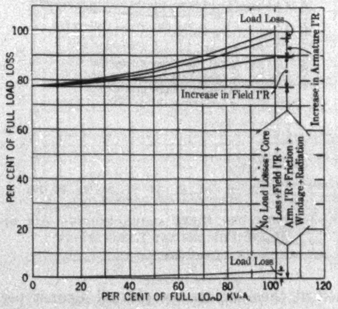


FIG. 5—SHOWING RATIO OF LOSS TO LOAD
(Generator No. 3, 3125-kv-a., 2300-volt, three-phase, 60-cycle, 3600-rev. per min.)

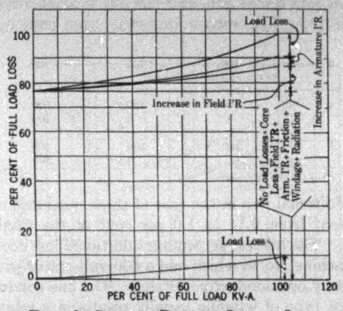


FIG. 6—SHOWING RATIO OF LOSS TO LOAD
(Generator No. 4, 6250-kv-a., 2400-volt, three-phase, 60-cycle, 3600-rev. per min.)

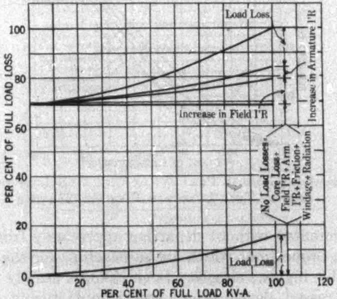


FIG. 7—SHOWING RATIO OF LOSS TO LOAD
(Generator No. 5, 6250-kv-a., 2400-volt, three-phase, 60-cycle, 3600-rev. per. min.)

It can be seen from the data in Table II that the additional loss varies from approximately 3 to 22 per cent of the total loss for these machines. Hence, if the total loss represents 4.5 per cent of generator input,

TABLE II

Generator number	Rating in per cent			Generator Losses Expressed in Per Cent				
	Kv-a.	Volts	Amp.	Total	Fan + friction + iron	Field copper	Armature copper	Additional
1	100	100	100	100	56.35	12.9	8.91	21.84
	..	0	100	59.5	28.0	3.06	8.48	20.0
2	100	100	100	100	74.2	15.6	7.39	2.80
	..							
3	100	100	100	100	74.0	15.56	7.55	2.9
	..							
4	100	100	100	100	73.8	12.7	3.92	9.5
	..	0	100	50.2	35.0	1.87	3.92	9.4
5	100	100	100	100	66.5	12.5	5.28	15.6
	..	0	100	68.4	46.8	2.48	5.15	14.0

the additional loss in the case of these machines would represent from 0.14 to 1.0 per cent of the total kw. input. The highest percentage additional loss occurred for machine No. 1, which had a 600-volt stator winding with only one conductor per slot. The one-conductor-per-slot type of winding usually results in a relatively large per cent additional loss on account of the higher current per slot and the greater concentration of current in the end connections of the winding. This loss handicap for low-voltage windings cannot ordinarily be eliminated without departing appreciably from the design proportions which are satisfactory for the more desirable higher voltages. Such changes usually result in increased development and manufacturing costs, and consequently, from the standpoint of generator costs alone, the use of special low-voltage windings should not be encouraged.

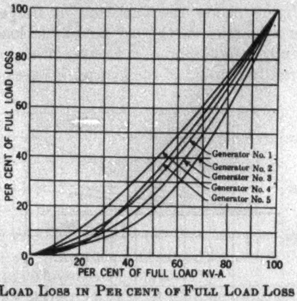


Fig. 8—Load Loss in Per cent of Full Load Loss vs. Per cent of Full Load for Five Generators

The additional loss of the machines at full kv-a. and approximately zero per cent power factor is of the same order of magnitude as the additional losses of the machine under sustained short-circuit conditions. The additional loss for full kv-a., full voltage conditions includes the increase in iron loss over the no-load values, whereas the additional loss at short-circuit conditions includes only the iron loss due to a magnetizing flux which depends on the magnitude of the leakage reactance.

Temperature Load Curves

Thermocouples were located on the stator end bells, end plates, finger plates which support the stator teeth, and on the surface of the stator teeth near the finger plates. The final temperature rises which were obtained at these points are plotted against per cent kv-a. load as shown by the curves in Figs. 9 to 13. The outstanding points in connection with these curves are as follows:

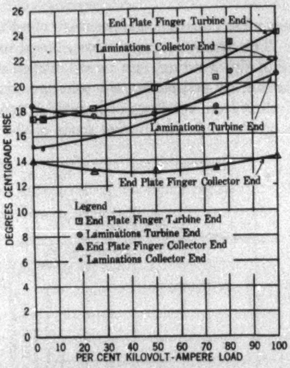


Fig. 9—Temperature Rises above Ingoing Air of End-Plate Fingers and Laminations

(Generator No. 1)

a. The final temperatures which were reached at full kv-a. load were very low. The temperature rise did not exceed 20 deg. cent. at any of the points on any machine. The temperature rise of finger plates was practically the same as for the adjacent stator tooth laminations. On the basis of these results, it can be concluded that the additional loss in these parts of these machines must not be very large.

b. In all cases, the temperature rise decreased slightly as the load increased from zero to approximately 25 per cent of the full load kv-a. rating. While this is of little or no practical importance, it is of interest from a theoretical standpoint because it shows that the resultant flux in these parts due to the stator and rotor m. m. fs. is smaller than at no-load due to the field acting alone.

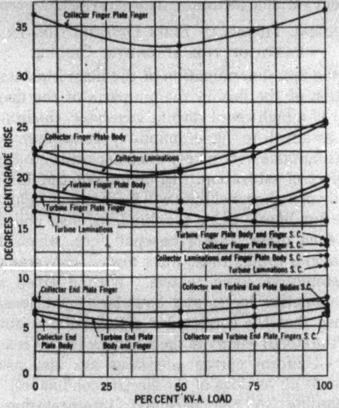


FIG. 10—TEMPERATURE RISE ABOVE INGOING AIR OF FINGER PLATES, END PLATES AND LAMINATIONS, MAGNETIC END PLATES
(Generator No. 3)

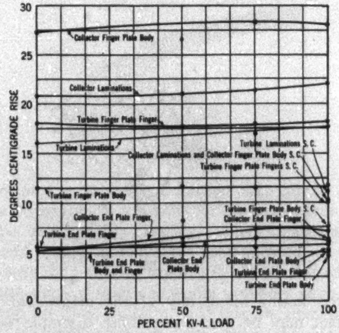


FIG. 11—TEMPERATURE RISES ABOVE INGOING AIR OF FINGER PLATES, END PLATES AND LAMINATIONS, NON-MAGNETIC END PLATES
(Generator No. 3)

MEASUREMENT OF THE LOSSES IN DIFFERENT PARTS OF THE MACHINE

In order to predetermine the magnitude of the additional loss of a synchronous machine, it is necessary to know the relative proportions of the loss that exist in the different parts of the machine and also how these loss components depend on the different variable factors. While the calorimeter method is quite satisfactory for measuring the total value of the additional losses of a high speed synchronous machine, it is not applicable for measuring the loss that occurs in the different parts of the machine, due to the fact that the loss in each part represents a too small percentage of the total value of the measured loss. A satisfactory method for determining the proportion of the additional loss which occurs in each part of the machine should measure the loss in that part alone and not measure it in combination with other losses of considerably greater magnitude. Such tests can be made on models which represent the different parts of the magnetic circuit and which are artificially subjected to the electric

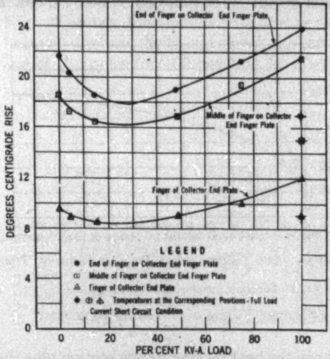


FIG. 12—TEMPERATURE RISES ABOVE INGOING AIR OF FINGER PLATES, END PLATES AND LAMINATIONS
(Generator No. 4)

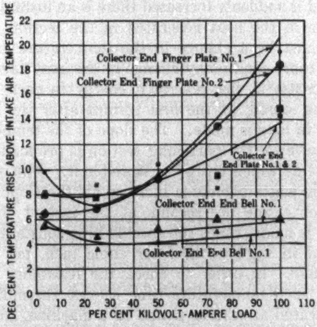


FIG. 13—FINAL TEMPERATURE RISES IN DEGREES CENTIGRADE ABOVE INTAKE AIR TEMPERATURE PLOTTED AGAINST PER CENT KV-A. LOAD

and magnetic conditions as exist in the machine. Such loss measurements can be made quite accurately, but in most cases it is difficult to reproduce the actual magnetic conditions which exist at different load and operating conditions. The losses which occur in the different parts of the machine due to any change in operating conditions can be obtained by measuring the energy absorbed by these parts when the machine is operating at constant temperatures and the change in operating condition is suddenly made. Under steady conditions, the heat flowing to any section plus the heat generated in that section must equal the heat flowing away from the section, since there is no change in stored energy under steady temperature conditions. When the load is suddenly increased there is an instantaneous increase in the heat generated in the section. Since there must be a change in the temperature gradient before there can be any change in the heat flow, all of the increase in the generated energy in each section must be stored, at the first instant after the sudden change in load is made. The slope of the temperature time curve for the particular section is proportional to the rate at which energy is stored in it. The increase in loss in watts per cubic inch equals the slope in degrees centigrade per second, at the instant the load change is made, times the specific heat constant of the material in watt-seconds, per cubic inch, per degree centigrade. In order to check the feasibility of obtaining such temperature time curves, thermocouples were located at various points on a machine, and temperature readings were obtained for sudden load changes by means of a reliable potentiometer and a very sensitive galvanometer. The curve in Fig. 14 shows the change in temperature of the stator tooth laminations with respect to time when the load on generator No. 5 was changed from no-load, no-voltage condition to no-load, full-voltage condition. The rate of change of temperature with respect to time at zero time is 0.01365 deg. cent. per sec., and with a specific heat constant of 55.8 watt-seconds per cu. in. per deg. cent., the loss in the stator teeth at the surface is 0.761 watt per cu. in. Since the density of the magnetizing flux in the stator teeth can be readily calculated, the loss constant can be obtained for the teeth with any kind of laminated iron as actually built in the machine. In a like manner, this method of measuring the loss and the analytical method of determining the magnetizing flux can be applied to all parts of the machine so that reliable loss constants can be obtained in terms of the different variable factors.

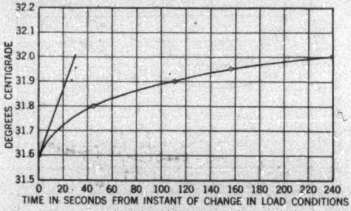


FIG. 14—TEMPERATURE RISE OF IRON VS. TIME

Couple No. 2—Location, turbine end lamination - - - $\dfrac{d\theta}{dt}$ in deg. cent. per sec., 0.01365 - - - *C*, 55.8 - - - - watts per cu. in., 0.761

GENERAL THEORY OF FLUX DISTRIBUTION IN AIR PARTS OF THE MAGNETIC CIRCUIT

In the first determination of the magnitude and distribution of the flux in the air parts of the magnetic circuit of a high speed turbine generator, the iron parts were assumed to have infinite permeability. Later approximations were made, when necessary, to consider the effect of saturation. In laying out the flux fields, a family of orthogonal lines will be shown at right angles to the flux lines. Since the flux lines close and do not cross each other, the orthogonal lines, which are at right angles to flux lines, must always converge at the magnetomotive force centers. With a given set of magnetomotive forces and boundary conditions, it is assumed that the correct flux distribution is obtained when the stored energy in the magnetic circuit is a maximum. Uniform current densities are also assumed to exist at all sections of the electric conductors.

According to the accepted interpretation, the magnetomotive force produces closed lines of force or flux around the magnetic centers. There is no force, however, acting along any of the orthogonal lines. This fact may be used to establish a simple relation between the orthogonal lines and flux lines which will greatly assist in the plotting of these curves. Referring to Fig. 15, the following notation will be used:

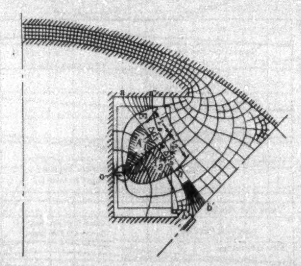


FIG. 15

A_1 and A_2 = the amperes in the respective shaded areas shown in Fig. 15.

$\Delta\,\phi$ = flux or number of lines of force going from $a\,a^1$ to $b\,b^1$ and is constant in value along this tube. This tube is to be considered of negligible width so that there is no difference in ampere turns acting on this circuit or path in the direction $a^1 - a$, $b^1 - b$, etc.

l_1 and l_2 = the lengths of the path in the parts shown.

Δd_1 and Δd_2 = the corresponding mean widths.

H_1 and H_2 = mean field intensity along l_1 and l_2.

The work done in moving a unit pole around the path o, m, p, o linking A_1 is all done along m, p or l_1. Similarly in moving a unit pole around o, p, n, o in linking A_2, the work is all done along p, n or l_2.

Then:

$$H_1 l_1 = \frac{4\pi}{10} A_1 \qquad (4)$$

$$H_2 l_2 = \frac{4\pi}{10} A_2 \qquad (5)$$

and since

$$\Delta \phi = H_1 \Delta d_1 = H_2 \Delta d_2 \qquad (6)$$

by substituting Equation (**6**) in (**4**) and (**5**),

$$\Delta \phi \frac{l_1}{\Delta d_1} = \frac{4\pi}{10} A_1 \qquad (7)$$

$$\Delta \phi \frac{l_2}{\Delta d_2} = \frac{4\pi}{10} A_2 \qquad (8)$$

Let

ΔP_1 and ΔP_2 = the respective permeances in the two parts of the circuit shown above.

Then

$$\Delta P_1 \,\alpha\, \frac{\Delta d_1}{l_1} \qquad (9)$$

$$\Delta P_2 \,\alpha\, \frac{\Delta d_2}{l_2} \qquad (10)$$

Then

$$\frac{\Delta P_2}{\Delta P_1} = \frac{A_1}{A_2} = \frac{\text{area}_1}{\text{area}_2} \qquad (11)$$

Since all the values in Equation (**11**) can be easily estimated, graphical solutions for the flux distributions can be made fairly readily, where mathematical solutions would become too complicated in most irregularly shaped fields existing in electrical machines. As mathematical solutions would probably have to be based on the same assumption as this one, namely, iron paths, of infinite permeability, even the increased accuracy in determining the field probably would be unwarranted in many cases on the basis of this assumption.

While the foregoing theory of flux distribution applies specifically to the magnetomotive force of the armature or field winding acting alone, it can be extended to cover cases in which there are magnetomotive forces in both elements of the machine. In applying it to an actual machine, the magnitude and distribution of the flux were determined for several different parts of the magnetic circuit, as shown in the following figures:

a. Fig. 16 shows the flux distribution in a portion of the air-gap with the field winding excited to give normal voltage, but with no current in the armature windings.

b. Fig. 17 shows the flux distribution in the air-gap when the machine is operating at 100 per cent kv-a., 100 per cent voltage, and zero per cent power factor.

c. Figs. 18 and 19 show the flux distribution in the air-gap when the generator is delivering 100 per cent kv-a. at 100 per cent voltage and power factor.

d. Fig. 20 shows the flux distribution in the air space between the end bells and the machine at the centerline of the poles for 100 per cent kv-a., 100 per cent voltage and zero per cent power factor. The instantaneous values of the currents in the three phases of the armature winding were 0, 86.6, and 086.6 per cent, respectively, of the maximum value at full load. Fig. 21 shows the flux distribution in the space as indicated by iron filings. The rotor was stationary, but its relative position with respect to the stator winding was the same as above. Both elements were supplied with d-c. values of the same magnitudes as the above instantaneous current values. The arrangement of the filings checks very closely with the flux distribution as obtained by the graphical analysis.

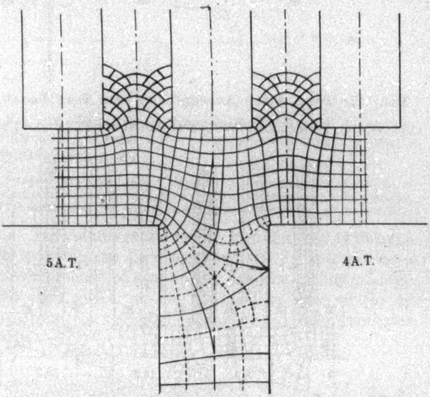
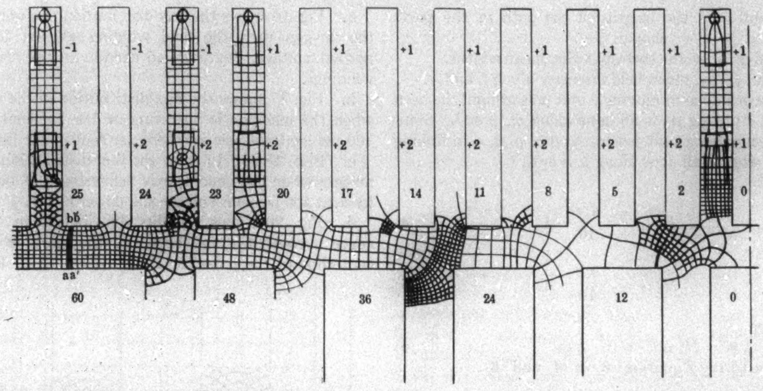


Fig. 16—Flux Distribution in the Air-Gap and Rotor Slot with Relative M. M. Fs. of 5 and 4 Ampere-Turns Across Gap Due to Rotor Winding and Zero M. M. F. Due to Stator Winding

e. Figs. 24 and 25 show the magnetomotive force diagram due to the end connections of the armature winding of a 3600-rev. per min. turbine generator, for specific instantaneous values of current. In obtaining these diagrams, it was assumed that the magnetomotive force acts at right angles to the end winding surface determined by the end connections. The results which have already been obtained from preliminary experi-


-1
-1
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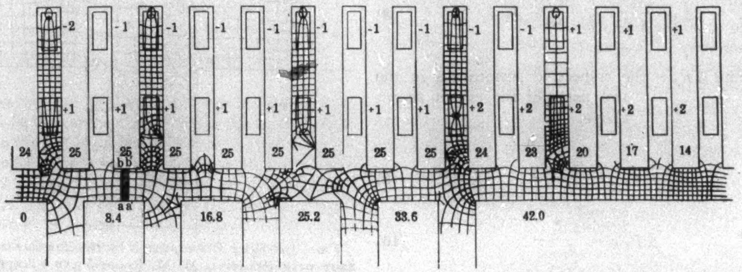
+2
+2
+2
+2
+2
+2
+1
25
24
23
20
17
14
11
8
5
2
0
bb'
aa'
60
48
36
24
12
0

FIG. 17—TOTAL FIELD AMPERE-TURNS PER POLE EQUAL 31,824 AND ARE TAKEN AS PROPORTIONAL TO 60 IN THESE CALCULATIONS. PER INCH PERPENDICULAR TO THE SHEET FLUX PER TUBE $a - a'$ TO $b - b'$ EQUALS $3.19 \times \frac{31824 \times 4}{60} = 6768$ LINES


-2
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+1
+1
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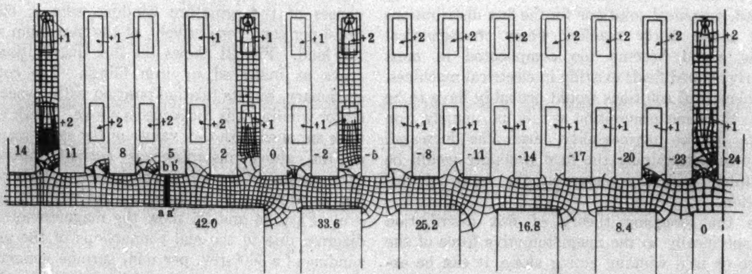
+1
+1
+1
+1
+1
+2
+2
+2
+2
+2
24
25
25
25
25
25
25
25
25
24
23
20
17
14
bb'
aa'
0
8.4
16.8
25.2
33.6
42.0

FIG. 18—TOTAL ARMATURE AMPERE-TURNS PER POLE = 13,260 AND ARE TAKEN AS PROPORTIONAL TO 25 IN THESE CALCULATIONS. Total field ampere-turns per pole = 22,277 and are taken as proportional to 42 in these calculations. Per inch perpendicular to the sheet the flux per tube $a - a'$ to $b - b' = 3.19 \times 22{,}277 \times \frac{4}{42} = 6768$ lines


+1
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+1
+1
+1
+1
+1
+1
+1
+1
+1
+1
14
11
8
5
2
0
-2
-5
-8
-11
-14
-17
-20
-23
-24
bb'
aa'
42.0
33.6
25.2
16.8
8.4
0

FIG. 19—TOTAL ARMATURE AMPERE-TURNS PER POLE = 13,200 AND ARE TAKEN AS PROPORTIONAL TO 25 IN THESE CALCULATIONS. Total field ampere-turns per pole = 22,277 and are taken as proportional to 42 in these calculations. Per inch perpendicular to the sheet the flux per tube $a - a'$ to $b - b' = 3.19 \times 13{,}260 \times \frac{5}{25} = 8460$ lines

mental and analytical investigations indicate that this method should be satisfactory for determining the necessary data and constants for calculating the losses of electric machines which cannot be calculated or measured directly.

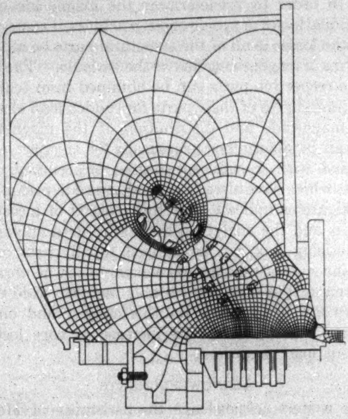

FIG. 20—TURBO GENERATOR END BELL LEAKAGE FLUX

Three-phase diamond coil armature winding section through radial plane of maximum flux densities for full load voltage and zero per cent power factor with armature current of 0.866 maximum value in two phases and zero in the third.

FIG. 21—ARMATURE 1200 AMPERES. FIELD 275 AMPERES. POWER FACTOR ZERO PER CENT

SUMMARY AND CONCLUSIONS

a. The magnitude of the losses of high speed synchronous machines can be determined with a satisfactory degree of accuracy for any load condition, by measuring the volume and temperature rise of the cooling medium which passes through the machine.

b. The temperature rise of the cooling medium can be determined by measuring the difference in e. m. fs. induced in thermocouple junctions, or the difference in potential drops across resistance elements located at the inlet and outlet duct sections of the generator. The velocities of the cooling medium must be practically uniform at both inlet and outlet sections and readings should be taken at a relatively large number of incremental sections to obtain an accurate value of the mean temperature rise.

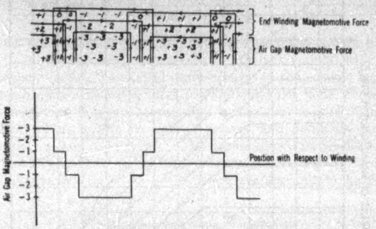
End Winding Magnetomotive Force
Air Gap Magnetomotive Force
Air Gap Magnetomotive Force
Position with Respect to Winding
-3
-2
-1
-1
-2
-3

FIG. 22—FULL PITCH CONCENTRIC WINDING. CURRENTS OF EQUAL MAGNITUDE IN BOTH WINDINGS AND WITH DIRECTIONS INDICATED BY ARROWS

Numbers and signs indicate the relative magnitude and directions of magnetomotive forces perpendicular to the surface through all the windings, on the assumption that the flux passes perpendicularly through that surface

c. The volume of the cooling medium passing through the machine can be determined: (a) by introducing a known amount of heat energy into the cooling medium and then measuring its temperature rise; or (b) by measuring the velocity heads at a relatively large number of incremental sections at the stack outlet. Both methods will give reliable results for high speed turbine generators provided the elements are properly designed and sufficient care is exercised in making the measurements.

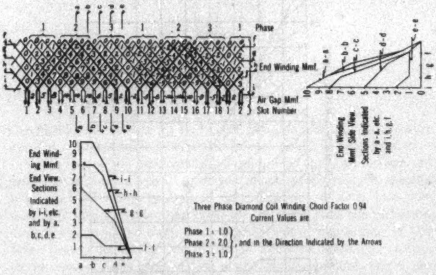
Phase
End Winding Mmf.
Air Gap Mmf
Slot Number
End Winding Mmf. End View. Sections Indicated by i-i, etc. and by a, b, c, d, e
End Winding Mmf Side View. Sections Indicated by a-a, etc. and i, h, g, f
Three Phase Diamond Coil Winding Chord Factor 0.94
Current Values are
Phase 1 = 1.0
Phase 2 = 2.0
Phase 3 = 1.0
, and in the Direction Indicated by the Arrows

FIG. 23—THE NUMBERS AND SIGNS INDICATE THE RELATIVE MAGNITUDE AND DIRECTION OF THE MAGNETOMOTIVE FORCES PERPENDICULAR TO THE SURFACE THROUGH ALL THE WINDINGS ON THE ASSUMPTION THAT THE FLUX PASSES PERPENDICULARLY THROUGH THAT SURFACE

d. In the case of the five 3600-rev. per. min. turbine generators which were tested at full kv-a. and zero per cent power factor, the additional loss including increase in core loss, varied from 3 to 22 per cent of the total losses. This corresponds to approximately 0.14 to 1.0 per cent of the generator input, on the assumption of 95.5 per cent generator efficiency. The additional loss at full kv-a. and approximately zero per cent power factor is 5 to 10 per cent greater than the additional loss measured under sustained short-circuit conditions.

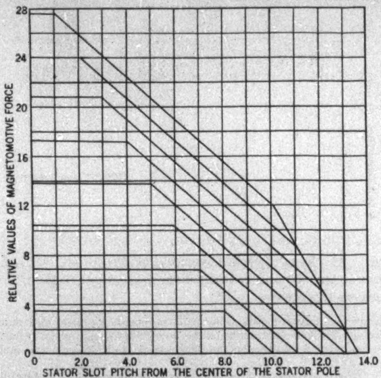

Fig. 24—End Winding Magnetomotive Forces, End View. Three-Phase, 54 Slots, Pitch 1-17. Equal Currents in Two Phases; Zero in the Third

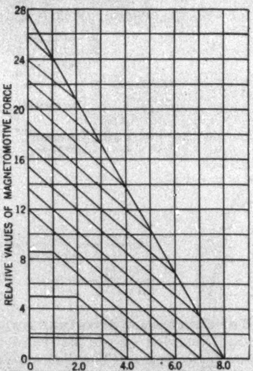

Fig. 25—End Winding Magnetomotive Forces, Side View. Three-Phase, 54 Slots, Pitch 1-17. Equal Currents in Two Phases; Zero in the Third

Since the increase in core loss is slightly larger for low per cent power-factor loads, it is probable that the additional losses of these machines, when carrying full kv-a., 80 per cent power-factor load, are approximately the same as for sustained short circuit with full load armature current. Since it is of considerable practical importance to know the relative magnitude of the additional losses of large power generating equipment, it is suggested that the manufacturers and users of large turbine generators cooperate in making loss measurements on several representative machines under definite operating conditions.

e. In order to predetermine the magnitude of the additional losses of synchronous machines, it is suggested that the losses in all of the structural parts be obtained in terms of magnetizing flux as the variable. The losses in the respective parts can be obtained from temperature time curves of these parts for sudden load changes. The magnitude and distribution of the magnetizing flux can be determined graphically for any part of the machine with a sufficient degree of accuracy. The results which have already been obtained in preliminary investigations indicate that this method of measuring the loss and the analytical method of determining the magnetizing flux can be applied to all parts of the machine so that reliable loss constants can be obtained in terms of the different variable factors. Additional studies are being made on this subject and on the problem of calculating the increase in core loss for different load conditions.

ACKNOWLEDGMENTS

The writers acknowledge the assistance of Messrs. A. M. Harrison and P. E. Watson and members of the 1926 Electrical Design School in laying out the flux fields. The tests on generators No. 2 and No. 3 were made by Mr. J. R. Taylor of the Power Engineering Department. Valuable assistance was received from Dr. J. Slepian of the Research Department in connection with the general theory of flux distribution.

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*The papers of Messrs. Stevenson and Alger appeared after this article was completed.

Discussion

I. H. Summers: Referring to Fig. 3, in which data are given for a 600-volt winding, one bar per slot, the load loss at 100 per cent kv-a. is slightly more than 20 per cent of the full-load loss. Referring to Fig. 4, in which data are given for a 2300-volt winding, 2-coil sides per slot, the load loss at 100 per cent kv-a. is approximately 3 per cent of the full-load loss.

I should like to ask the authors whether or not, in their opinion, this discrepancy can be materially attributed to either the eddy-current loss in the armature windings or the pole-face loss, or both?

W. F. Dawson: I wish to comment on the method of measuring losses by the calorimetric method. I published an article on this subject in the *General Electric Review*, February 1920. Messrs. Laffoon and Calvert have laid particular stress on the stack method. It is one means of measuring the volume of air discharge, but if one refers to the paper published by Barclay and Smith, (*Journal I. E. E.* London, Vol. 57, April 1919), he will find that they also experimented with the stack method and found that the distribution of air velocity across the section was very uneven, varying from 1050 ft. per minute to 1480 ft. per minute. It was necessary to insert several trays of expanded metal as baffles before even approximate distribution was secured. They selected the anemometer for measuring the velocity.

My experiments were commenced over eight years ago. We divided the discharge area into 100 or more rectangular sections and measured the velocity of each section with a manometer or hook gage. These results (see Fig. 3, *G. E. Review*, article) were very discouraging, as the readings varied from 0.05 in. of water to a maximum of 0.154 in., corresponding to velocities of 15 ft. per second and 26.3 ft. per second. We found also that all sorts of whirls and distortions occurred in the discharge pipes, even to an occasional indication of negative flow. These were corrected by placing a large wooden cross near the inlet end of the pipe, having an axial length of about twice the pipe diameter.

Another great difficulty was in averaging the temperatures of the inlet and outlet air. The temperatures of the inlet air were often influenced by the presence of adjacent steam pipes and turbine parts, and at times the variation of temperature across the inlet was greater than the difference between the average inlet and outlet temperatures. Special electrical resistance thermometers were used to average the temperatures of inlet and outlet air, the resistance wires being wound on wooden crossarms, so distributed and spaced as to give a true average of the air temperature. An improvement over the method of attempting to measure at the ordinary generator outlet was to place thereon a long straight pipe of suitable section, on the end of which was a specially shaped, calibrated orifice, similar to that described by Mr. Laffoon. This reduced the area of the outlet sufficiently to bring the discharge velocity up to about 4000 or 5000 ft. per min., corresponding to an air pressure of from 1 to 1.5 in. of water. Properly arranged, these orifices give, by a single reading, observed at the center, the actual accurate air velocity to within 1 per cent.

By using electric resistance thermometers giving true average temperatures, and by inserting electric heaters, and a third resistance thermometer beyond the electric heater, very satisfactory results were obtained, but the hook-gage readings were found unsatisfactory. When this method is used, it is not necessary to make allowance for the varying barometric pressure of the air. It is particularly adaptable to turbine alternators; usually there are two inlets and one discharge and the air stream is confined and guided in such a way that all the air that goes into the machine can be measured.

I show in the accompanying Fig. 1, a generator frame having two inlets and one discharge. The original suggestion for this arrangement is due to Mr. H. M. Hobart.

T_1 = Average temperature (deg. cent.) of inlet air

T_2 = Average temperature (deg. cent.) of outlet air

H = Watts energy supplied to electric heater

T_3 = Average temperature (deg. cent.) of air after leaving heater

$$\text{Watts Loss} = H\,\frac{T_2 - T_1}{T_3 - T_2}$$

$$\text{Cu. ft. air per min.}^* = \frac{\text{Watts loss}}{0.585\,(T_2 - T_1)} = \frac{H}{0.585\,(T_3 - T_2)}$$

Caution. From three to four hours are usually necessary for all parts to attain steady temperature. Only about 30 min. are necessary to produce steady temperatures in the electric heater. If readings are recorded three or four times per hour, it will be observed that T_3 increases rapidly when current is applied to the heater. This is due to radiant energy and demands certain precautions and corrections. The heater should be placed 4 to 5 ft. from T_2 and exactly midway between T_2 and T_3. In writing the expression $T_2 - T_1$, use the last reading before heater is energized. In writing $T_3 - T_2$, use the last reading with heater applied. Measurable convection losses should be allowed for.

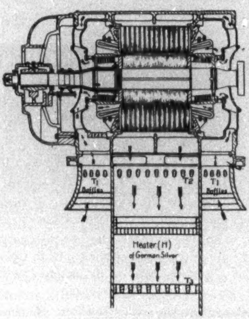

FIG. 1

If the loss of the machine be calculated by these equations, we are rid of all such disputes as correct barometric pressure, specific heat and effect of moisture. My experience with two-pole machines of note more than 6000-kw. rating is that the radiation or convection losses amount to probably 3 or 4 per cent of the total losses, which for machines of 1500 to 2000-kw. rating, never exceed 5 per cent of the rating of the machine, nor 4 per cent for machines of 5000-kw. rating.

Mr. Laffoon's paper has made a comparison between the losses, particularly the additional losses observed when a machine is being operated on short circuit and those observed when it is being operated over-excited at zero power factor. It seems, after all, that while that comparison is interesting, it doesn't tell us what we want to know. What the manufacturer wishes to know and what his customers wish to know is what is the ratio of the load losses measured on short circuit to those measured while the machine is operating at normal voltage and power factor in accordance with its rating? The few data that I have gathered indicate that at times at least the losses under normal operating conditions are less than they are on short circuit. I have not, however, confirmed that sufficiently to lay it down as a rule.

E. E. Johnson: Refer to Fig. 20 of the paper by Messrs.

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*Standard air at 29.92 in. Hg., 15.5 deg. cent. (60 deg. fahr.)

Laffoon and Calvert. The distribution of the end-bell leakage flux is three-dimensional, and I raise the question as to whether a field plot made on the basis of a two-dimensional distribution, as was done in Fig. 20, holds strictly. It may be that the results obtained from this field plot made on the basis of a two-dimensional distribution are sufficiently accurate for the particular case in question, although, in general, this is not necessarily true. It will be noted that all the areas in the plot of Fig. 20 are curvilinear squares; whereas, on the outer boundaries, they should perhaps not be so, considering that the actual flux distribution there is three-dimensional.

P. L. Alger: The two conclusions that I draw from the paper are: (1) That the method of measuring the total losses by means of the rise of air through the machine is not yet developed to the point where it can be called convenient; and (2) that the extra losses due to the armature leakage flux are not large in machines of the type described. This latter conclusion is not general, as it is evident that very markedly different losses will be obtained with different types of end construction. For example, if a chain-type armature winding had been used, the end losses would have been very much greater.

The most interesting thing about the paper to me is the temperature-rise method used by the authors to determine the separate losses in different parts of the machine. It seems to be quite feasible to determine the initial rate of temperature rise of each part separately, and, from this and the known heat capacity of that part, to determine the segregated losses. We have made some experiments along this line, and have found one of the principal difficulties to be the variation in the rate of heat generation in different parts of the same element of the machine. For instance, the losses in the clamping fingers are localized near the end, and the rate of rise of the tip of the finger is, therefore, much greater than that of the back. It requires considerable care to decide on what is the average rate of rise of each part from a relatively small number of temperature readings.

I should like to ask the authors whether they consider that the average rate of temperature rise of one tooth in the end packet of laminations, for example, can be determined with reasonable accuracy by using only two thermocouples.

E. H. Freiburghouse: The authors of the paper have favored the stack method for determining the stray-load losses of the generator. It has also been our practise at Schenectady to use that method in measuring the stray-load losses of large turbo generators.

I should like to ask the authors whether they have ever applied a baffle-type mixer in the discharge air to get uniformity of temperatures?

I am somewhat surprised to notice the comparison for generators 2 and 3, as to stray-load losses. Generator No. 2 (see Table I) had stator end plates of magnetic material, whereas in generator No. 3, which I assume is the same generator or one of the same rating at least, they used a non-magnetic ring for clamping the core. By Table II and by the curves in Figs. 4 and 5, I find that the load loss is even higher for the machine which had the non-magnetic end ring. This is somewhat of a surprise to me. We have made many experiments on model generators of the turbine type, in which we applied different materials for the end rings, even using wood to determine what the limit would be in the elimination of the loss. Based on our investigation, I believe that the load loss ought to be less in the case of the generator which has the non-magnetic end ring.

Mr. Alger referred to the influence of the rings on the rotor. We have found by the application of non-magnetic end rings on the rotor that the temperatures of the end structure of the stator were higher with the generator running on open circuit than they were with magnetic end rings, whereas on short circuit and normal load conditions, the losses were higher with the magnetic end ring. The temperatures in the end structure on short circuit were considerably reduced by the use of the non-magnetic rings on the rotor.

Fig. 20 indicates that losses should be obtained in the end plate. We see that the flux passes over into the end plates and is localized most intensely nearest the air-gap. I question whether the application of the magnetic end ring gave less loss than the non-magnetic ring.

W. H. Colburn: The purpose of my discussion is to emphasize the importance, from the standpoint of the user of the machine, of a foreknowledge of stray-load losses in general in all machines. For example, let us consider a particular instance of a synchronous converter used in a substation. Assuming certain values for cost of power and of generating and transmission plant tied up by the losses of the machine, we arrive at the conclusion that a gain of efficiency of 0.1 per cent in that equipment is worth about 45 cents per kw. This means that the user could afford to pay for a converter rated at 3000 kw. something like $1350 additional for the machine that would save that 0.1 per cent.

I will illustrate how apparently neglected items may run up in value. The analysis of a certain equipment which had a blower used with it indicated that the more efficient equipment could be justified on the basis of the capitalized efficiency. We then began to investigate the fan and found that the manufacturer had applied it without consideration of actual air and power requirements, so that the balance was thrown to the other equipment.

The same condition exists in connection with consideration of these stray-load losses. We know that it is a very difficult thing to predict them, as the Institute has recognized in its Standards of many years, where you find it repeatedly stated that no definite value could be placed on them. Recently the Standards Committee has asked that 1 per cent of the rating of the machine be added for these stray-load losses.

It is almost useless in the present state of the art to go into a discussion of the results of calculations of and tests for stray-load losses, because they are fraught with many difficulties, but some of these seem to indicate that 1 per cent may be quite wide of the mark. In some cases it seems that these stray-load losses are in the neighborhood of 0.5 per cent and in other cases they may run up to nearly 2 per cent. If this is the case, it is almost useless for the user to attempt to capitalize guaranteed efficiency.

I think these tests which the authors have made are very fine records to have. They are needed to check our design, but I think that it is even more important that we extend these tests to all conditions in and classes of machinery, and that we draw from our tests some indication of how we can attack these problems in advance of construction in order to predict accurately the values of these stray-load losses. Only then can the user of the machine determine whether he is actually getting what he is paying for.

F. D. Newbury: The fact that we have this paper indicates the desire on the part of manufacturers to know more about them. I am quite sure that that desire has long been present with other manufacturers.

The purpose of the present discussion, I think, is to establish, for the benefit of the Institute Standards Committee, a relation between the actual and measurable stray losses in a-c. machines, and particularly of the turbine type. We can only measure these losses under no-load conditions at short circuit or at full voltage and zero power factor. As Mr. Dawson pointed out, the thing of real interest is the ratio of the measurable losses under these conditions and the actual losses under full voltage and current and high power factor. That is a part of the problem still to be studied, and I am quite sure work that is going on will throw light on that interesting point.

The previous speaker referred to a 1-per cent value of the stray losses. The Standards Committee included that value in the 1925 edition for d-c. machines and not a-c. machines. The

Institute Standards for 14 years, at least, have included the full short-circuit losses as load losses for polyphase a-c. machines. The correctness of that practise has been pretty well established for salient-pole machines but it is still to be established for the cylindrical-rotor, turbine-type machines. I think opinion is tending toward acceptance of the short-circuit loss for cylindrical rotors also.

C. M. Laffoon and J. F. Calvert: In reply to the discussion by Mr. I. H. Summers: When a standard turbine-generator frame of a given rating is wound with a one-conductor-per-slot type of winding so as to obtain unusually low voltages, it is generally found that the additional losses are greater than for the two-conductor-per-slot type of winding. With the one-conductor-per-slot type of winding, the current per slot and the magnitude of the harmonics in the armature magnetomotive force are larger than for the more favorable two-conductor-per-slot winding. It is our opinion that these factors are responsible for the relatively larger eddy-current losses in the rotor pole face and other iron parts which are within the range of the pulsating fields. Both calculated values of the eddy-current losses and the measured temperature of the armature copper indicate that only a small part of the increase in additional loss occurs in the armature winding.

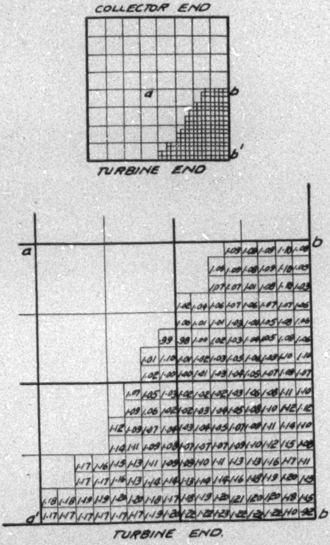

Fig. 2—Pressure Distribution Over Airstack Outlet

In reply to the discussion by Mr. W. H. Dawson: In the type of calorimeter test which he described in which a heater was used on the outlet of the machine, the heater is essentially a calorimeter-type volume meter, and as such probably introduces all the difficulties which are outlined in the paper. It was indicated in the paper that this method required (1) either uniform velocities for at least three sections in the air stream, or (2) a very complicated system for obtaining the mean temperatures of the intake and discharge sections of the heat sources (both the machine and the volume meter); while the use of the stack with the calorimeter test requires uniform air velocities at one less section in the air stream than does the heater or calorimeter-type volume meter. On the basis of results from a large number of tests, it is our conclusion that uniform velocities at several sections in the air stream are inherent requirements in calorimeter tests of this sort, and that the stack, rather than increasing the difficulty, simplifies it. Reference to Fig. 2 herewith will show that sufficiently uniform velocities were established over the outlet of the stack. The maximum variation in velocity pressure head from the average value is 12 per cent; hence the maximum variation in velocity from the mean value is approximately 6 per cent, (neglecting the one low point in the corner).

In the tests described in the paper, care was taken to avoid the convection of heat to the inlet air from transformers, steam pipes, leads, and other sources, so that practically uniform intake air temperatures were maintained. The possible errors due to the existing variations in temperatures and velocities in the intakes were checked on generator No. 5, and found to be a small fraction of one per cent.

The measurements of barometric pressures is a comparatively simple matter, and introduces no serious difficulty in the use of the stack volume meter. Since the variations in the specific heat of air with changes in temperature, pressure, and moisture content are known, corrections can be made if desired; but it did not appear that this difference was of sufficient importance to consider.

The fact that the calorimeter tests and electrical input readings were in agreement for a number of "no-load" 100 per cent power-factor tests at different voltages, is a satisfactory confirmation of both the volume-meter and air-rise measurements, because both these latter two readings could not be in error by just the right amount in each case, and because the electrical readings of input at 100 per cent power factor can be taken as quite reliable.

In reply to the discussion by Mr. E. E. Johnson: The determination of the end-bell flux field on the basis of a two-dimensional distribution as shown in Fig. 20 of the paper, does not hold strictly, but to consider it as a three-dimensional field, tremendously complicates the already difficult problem of field mapping. Fig. 20 checked the filing photograph very closely in the parts of the field where the flux density is greatest, but deviated somewhat where the density was small. Since the density decreased very rapidly toward the outer edge of the end winding, the errors introduced in either loss or reactance calculations on the basis of the two-dimensional field must be small with this type of winding and end bell.

In reply to the discussion by Mr. P. L. Alger: While the additional losses are not a large percentage of the total losses in many machines, yet in large machines they represent a direct loss to the customer and a limitation to the manufacturer of very considerable importance. Unfortunately, sufficient data were not obtained to tell how many temperature detectors would be necessary to determine the losses from time-temperature curves for the various parts of the machine. It appears, however, that flux mapping can be used to indicate the best location for these detectors, and the safest method would be to locate a rather large number on various radial lines throughout the machine. Then, if it could be shown during the first tests that some couples need not be read, they could be abandoned for later readings.

In reply to discussion by Mr. E. H. Freiburghouse: Before the air discharge stack was built, a model stack was tested under conditions which were intended to give distributions of both temperature and pressure with wider variations than those which would be found at the outlet from the machine. In the tests with models, two types of baffles, as well as a screen, were used in the air stream just preceding the entrance to the stack. The baffles, or air mixers, did not give sufficient improvement to warrant their application. The screen gave considerable improvement in extreme cases, but was not used, because of the undesirable loss of head, and consequent change in air volume from that existing under normal operating conditions. Had the tests on the actual machine shown the desirability of better distributions, undoubtedly, it would have been necessary to introduce a screen, and perhaps some form of baffle in the air stream at the entrance to the stack.

The tests on generators Nos. 2 and 3 were made to determine the effect of stator end-plate material on the magnitude of the additional losses. The same machine was used in both cases; magnetic plates were used on No. 2 and non-magnetic plates were used on generator No. 3. The total values of the additional losses of these two generators were very small and it appears evident from the end-plate temperatures that only a very small portion of the additional losses actually occurred in the end plates. Hence, it is impossible to draw any definite conclusions from the tests on these two machines in regard to the influence of magnetic and non-magnetic stator end plates on the additional losses of the machines.

Reduction of Armature Copper Losses

The Inverted Turn Transposition for the Reduction of Losses Due to Non-uniform Current Distribution in the Armature Conductors of Large A-C. Machines

BY IVAN H. SUMMERS*

Associate, A. I. E. E.

***Synopsis.**—A new method of reduction of armature copper losses is described, which consists in the inversion of the conductors of a multi-turn barrel type coil at one or more places in the end portions of the coil. Previous writers have described methods of transposing the conductors in the slot and of carrying insulated strands through successive positions in successive coils, but the new method now described presents distinct points of difference in theory and in construction from any of these earlier methods. The theory of the new form of transposition is briefly described, complete formulas are presented for the most useful cases, and illustrations of its use are given.*

* * * * *

It is the aim of this paper to present a method for the reduction of extra losses and heating in armature conductors of large a-c. machines due to non-uniform current distribution. The scope of the paper is limited to finitely laminated conductors with the laminations insulated throughout all the turns in a coil. It is further limited to coils wound either with no special twist or inversion at any point, or with one or more of the turns or half-turns inverted as illustrated in the various accompanying diagrams. With the aid of the formulas and tables the calculation of the extra copper losses becomes a simple matter for simple untransposed coils, and the gain to be derived from transposition can be determined by selecting the proper factor from the tables. It is found that in most cases it is a simple matter to select a transposed winding for a large machine so that the extra losses will be reduced to a negligible value and the transposition may be accomplished at very slight additional expense.

A considerable literature on the subject of reduction of eddy-current losses in the armature copper has accumulated, as is evident from an inspection of the bibliography. Nevertheless, the type of transposition here described, which consists in inverting the conductors of a multi-turn coil at one or more points in the end connections during the process of winding, is believed to be entirely new. European writers have chiefly described means of transposing the strands of a bar winding within the slot, thus enabling both ends of the bar to be solidly connected to adjacent bars. American writers have described the reduction of losses secured by the inversions occurring at both ends of the standard barrel type coil, as shown in Fig. 1, and have also described the further means of reducing the losses, which consists in carrying the strands through the several coils of a phase belt by special insulated connections. The earlier papers by Mr. W. V. Lyon and by Mr. H. W. Taylor (see Bibliography) have been of great assistance in furnishing foundations for the present paper.

The new method now described has some distinct advantages in construction over other methods, as it enables solidly connected machine wound coils of identical character to be made practically free from circulating current losses, whatever the number of turns per coil. By making the transpositions in the ends of the coil, space for them is obtained without sacrifice of slot room, and by properly locating them with reference to the already present inversions at the ends of the coils, almost complete avoidance of residual voltages can be secured. Diagrammatic illustrations of the various types of transpositions in the ends here considered are shown in Figs. 2 to 7 inclusive, and a reproduction of an actual coil similar to Fig. 6 is shown in Fig. 9.

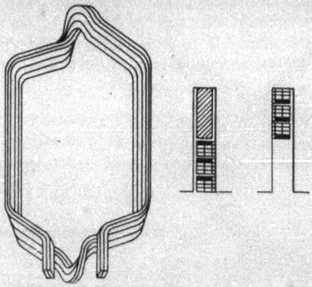

FIG. 1—STANDARD BARREL TYPE COIL

Showing successive positions in slot of a single strand—Type I, Table I

Consider a solid rectangular conductor placed in a rectangular slot in an iron body. It is clear that the reactance of a path at the bottom of the conductor is more than that of any other path in the conductor because the bottom path is enclosed by more flux. Furthermore, a path at the extreme top of the conductor

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*General Electric Co., River Works, West Lynn, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

is reactanceless with regard to the rest of the conductor, and in this respect has resistance only. Thus a larger current will flow at the top than at the bottom. Additional losses will ensue and the temperature will be higher than if uniform current distribution were maintained. This non-uniform current can be thought of as a uniform current with an eddy current superimposed on it so as to increase the total current at the top and decrease it at the bottom. When the term eddy current is used hereafter in this paper, it means this hypothetical superimposed current. The formulas for extra loss factor give the ratio between the loss caused by this eddy current and the loss caused by the uniform current.

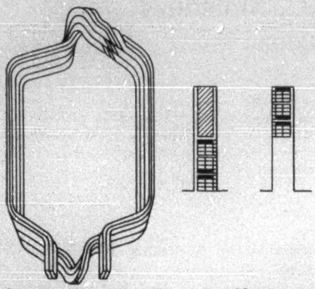

Fig. 2—Barrel Type Coil With Odd Number of Turns
All inverted at the end opposite connections—Type II, Table I

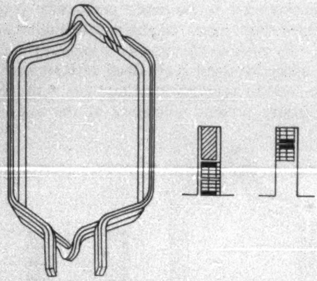

Fig. 3—Barrel Type Coil With Even Number of Turns
All inverted at the end of connections—Type II, Table I

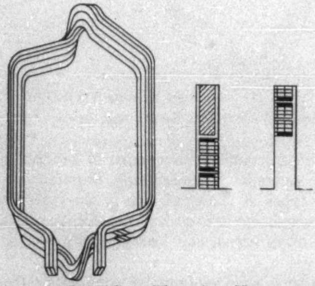

Fig. 4—Barrel Type Coil With Odd Number of Turns
All inverted at the connection end—Type III, Table I

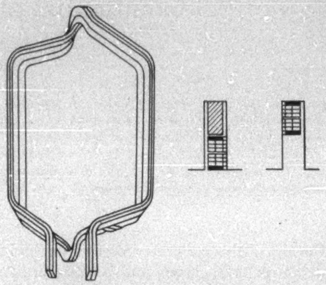

Fig. 5—Barrel Type Coil With Even Number of Turns
All inverted at the connection end—Type III, Table I

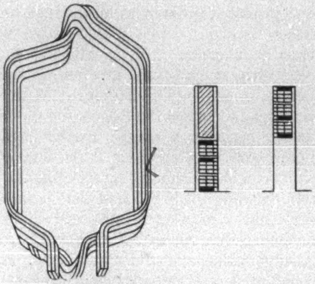

Fig. 6—Barrel Type Coil With Last Turn Only Inverted at Connection End—Type IV, Table I

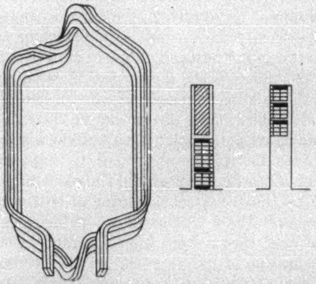

Fig. 7—Barrel Type Coil With First Half Turn Only Inverted at End Opposite Connections—Type V, Table I

If the conductor be laminated, conditions will be improved because the paths will be longer and the eddy currents reduced. Thus an elementary method of reducing the extra losses is simply to laminate the conductor. In addition, this makes the coil more flexible and easier to handle. For this reason most coils in large machines are laminated. Mere lamination, however, is not sufficient to keep the extra losses down to a negligible amount and for this reason various other schemes have been proposed.

The new method now proposed is to invert the laminations at various points in the coil by making a 180-deg. twist in the conductor. This can be arranged usually so that the voltage induced in the whole length of the strands by the leakage flux is more nearly uniform than it would be if there were no inversion. Some of the convenient ways of employing the inversion are illustrated in the accompanying diagrams. They are discussed later and formulas are given to calculate the extra losses which the coils will have.

In order to investigate mathematically the problem of inversion of the turns, certain simplifying assumptions are convenient as in all engineering problems. These are briefly indicated below:

1. Only the horizontal components of flux and magnetizing force are considered. Actually there are additional losses due to the vertical components but these have been found to be small in most practical cases. Horizontal is taken here to mean parallel to the bottom of the slot.
2. The vertical component of current is neglected. The current is always considered to flow straight into or out of the plane of the paper, except, of course, at the soldered joints at the ends of the coils.
3. The iron sides and the bottom of the slot are considered to have infinite permeability and no losses, as far as this problem is concerned.
4. The resistivity of the conductors is supposed to be constant and invariable over the depth of the slot.
5. The voltage induced by the leakage flux in the end connections is the same for each strand.

These assumptions are the same as the ones made by most writers on the subject. From the fundamental laws of electromagnetic theory and these assumptions, the differential equations of the problem can be derived. From these differential equations and their solutions, a general rule for finding the loss in any conductor or combination of conductors can be derived.

CIRCULATING CURRENT LOSS

For a neat statement of the problem and a derivation of rules for finding the loss in infinitely laminated coils (hereafter called circulating current loss) see the paper by W. V. Lyon in the May 1921 issue of the JOURNAL. The rule derived therein may be briefly restated as follows:

Form a quantity I_0 for each conductor which is right side up in the slot by taking

$$I_0 = I_b,$$

where I_b is the vectorial summation of all the currents in the slot below the conductor in question. For each conductor which is the other side up, take

$$I_0 = -(I_b + I_1),$$

where I_1 is the current in the conductor considered. It makes no difference which side up is called right side up. The I_0 used is the average for all the conductors which are in series, and which have strands continuously insulated throughout. Let I_2 be all the current in the bottom coil side of another coil which may be in the slot below the conductors in question and substitute the known value of I_2 in terms of I_1 and solve for $|I_0/I_1|$ and $\cos\alpha$, where $|I_0/I_1|$ is the numerical value of the ratio and α is the angle between I_0 and I_1. Substitute these values in the relation

$$L = \left|\frac{I_0}{I_1}\right|^2 + \left|\frac{I_0}{I_1}\right| \cos\alpha. \tag{1}$$

Then the heat loss in the coil will be determined from the relation

$$K = M + L\,N, \tag{2}$$

where M and N are certain complex hyperbolic quantities[1] and K is the ratio of the total heat loss in the coil to the loss that would exist if the uniform component of the current existed alone.

These quantities are discussed in Appendix B where it is shown that if the current has a distribution which is not widely different from uniform, the extra loss factor due to the eddy currents may be approximated by

$$k = (4 + 15\,L)\,D, \tag{3}$$

where D is a quantity depending on the dimensions of the slot and the coil. It is

$$D = 0.075\left[\frac{f}{60/\text{sec}}\right]^2\left[\frac{r\,b\,n^2\,(d/\text{in})^2}{m^2}\right]^2 \tag{4}$$

for a two-coil side per slot winding. Also

$$D = 1.19\left[\frac{f}{60/\text{sec}}\right]^2\left[\frac{r\,b\,n^2\,(d/\text{in})^2}{m^2}\right]^2 \tag{5}$$

for a one-coil side per slot winding, and

$$D = 1.19\left[\frac{f}{60/\text{sec}}\right]^2 [\,r\,(d/\text{in})^2\,]^2 \tag{6}$$

for strand loss calculation.

In these formulas f means frequency of the alternating current; r means the ratio of the width of the copper in the slot to the slot width times the ratio of the depth of conductor plus strand insulation to the net conductor depth; b means the ratio of the length of the slot to the length of a half-turn; d means the depth of a strand; n means the number of layers of strands in the depth of the slot; and m means the number of turns per coil[2]. The copper conductors have been assumed to be at 75 deg. cent. and the resistivity for this temperature has been used. The extra loss factors are inversely proportional to the square of the resistivity so that any other temperature may be evaluated if desired.

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[1] See Appendix B.

The quantity L which depends on the type of winding is calculated in Appendix A and is tabulated in Table I. The general method of calculating L for any type of winding is illustrated in Appendix A so that any reader who wishes to investigate types of windings outside the scope of this paper may do so.

Type	Description of winding.	See Fig.	Value of L
I	Multiturn Coil, two coil sides per slot with an involute at each end. (Standard coil with which others are compared.)	1	$L = \frac{m^2-1}{4}$
II	All turns inverted on the end opposite the connections.	2	$L = \frac{3}{4} - \sin^2\frac{1}{2}\theta$ (m being odd)
		3	$L = -\frac{1}{4}$ (m being even)
III	All turns inverted on connection end	4 and 5	$L = 0$
IV	Top turn only inverted on connection end.	6	$L = \frac{(m-1)(m-2)}{m}\left[\frac{m^2-5m+2}{4m} + 2\sin^2\frac{1}{2}\theta\right]$
V	First half turn only inverted on end opposite connections.	7	[illegible]
VI	Bar---bottom coil side only.		$L = \frac{m^2-1}{3}$
VII	Bar--- Top coil side only.		$L = \frac{2m^2-1}{3} - 2m^2\sin^2\frac{1}{2}\theta$
VIII	Bar--- Both coil sides.		$L = \frac{4m^2-1}{3} - m^2\sin^2\frac{1}{2}\theta$

Note: Top always refers to open end of slot

TABLE I—CIRCULATING LOSSES

It is shown in Appendix C that the extra loss factor in a laminated conductor of Type I may be calculated by the simple formula

$$k = 0.28\left[\frac{f}{60/\text{sec}}\right]^2\left[\frac{r\,b\,n^2\,(d/\text{in})^2}{m}\right]^2 \qquad (7)$$

for the first order of eddy currents with copper at 7 deg. cent[3].

The loss factors in any other type of transposed winding considered may be calculated by using the formulas for L given in Table I or by referring to the ratios given in Table II. Each formula in Table I and each ratio in Table II is given for a single coil, or group of coils, having a definite value of θ, which is the angular phase difference between the top and bottom coil side currents. Thus it is necessary to know from the pitch of the coils how they are distributed. Two factors must then be calculated and the weighted average taken to apply to the whole machine.

Extra loss factors obtained by the methods described are based on the assumption that the extra losses are not too large. Formula 7 may be used without sensible error provided the extra loss factor which it indicates does not exceed approximately 100 per cent. At this point the result is less than six per cent too large, but the error increases beyond this point. Even if the result is larger than 100 per cent it may still be used in combination with Table II provided the result for the type of winding under consideration is not too large. Thus it is clear that the approximate formulas are quite sufficient for any windings that would be allowed in practise.

It may be remarked in passing that the losses in bar windings may be calculated by formulas for coils of Type VI, VII and VIII given in Table I, and if the extra loss factor is large, the complete expression for M and N given in Appendix B should be used together with Equation (2) to obtain the total loss factor. Bar windings have been discussed fully elsewhere[4] and therefore are not treated in this paper.

m	Type II	II_a	II_b	II_c	II_d	III	IV	IV_a	IV_b	IV_c	IV_d	V	V_a	V_b	V_c	V_d
2	0.0164	0.0164	0.0164	0.0164	0.0164	0.262	0.262	0.262	0.262	0.262	0.262	0.0784	0.0784	0.0784	0.0784	0.0784
3	0.448	0.338	0.228	0.118	0.0074	0.118	0.0196	0.167	0.313	0.462	0.607	0.0565	0.0933	0.130	0.167	0.203
4	0.0041	0.0041	0.0041	0.0041	0.0041	0.0663	0.0197	0.207	0.393	0.579	0.765	0.195	0.226	0.257	0.289	0.319
5	0.162	0.122	0.0825	0.0426	0.0027	0.0426	0.0807	0.253	0.464	0.656	0.847	0.316	0.339	0.364	0.387	0.412
6	0.0018	0.0018	0.0018	0.0018	0.0018	0.0296	0.153	0.338	0.522	0.707	0.891	0.410	0.428	0.447	0.466	0.484
7	0.0827	0.0624	0.0421	0.0217	0.0013	0.0217	0.221	0.395	0.572	0.743	0.917	0.481	0.495	0.509	0.525	0.538
8	0.0010	0.0010	0.0010	0.0010	0.0010	0.0166	0.283	0.447	0.612	0.773	0.937	0.542	0.551	0.563	0.574	0.585
9	0.0502	0.0378	0.0254	0.0131	0.0008	0.0131	0.337	0.488	0.642	0.795	0.951	0.585	0.595	0.605	0.613	0.623
10	0.0007	0.0007	0.0007	0.0007	0.0007	0.0107	0.385	0.529	0.673	0.818	0.961	0.623	0.632	0.640	0.648	0.656

Note: Subscripts a, b, c and d refer to values of θ of 60°, 90°, 120° and 180° respectively.

TABLE II—RATIO OF EXTRA LOSS IN VARIOUS TYPES OF WINDINGS TO THAT IN TYPE I

STRAND LOSS

Up to this point the losses are supposed to be those which may be called circulating current losses and which would occur if the conductor were laminated by infinitely thin laminations. As intimated previously, there are additional losses which are due to the non-uniform distribution of current over the finite depth of the strands themselves. These losses may be approxi-

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[2] The formulas are dimensionally correct and the quantities may be measured in any consistent system of units. They are so arranged that if f is measured in units of 60 cycles per second, and d is measured in inches, the calculation will be simplified. Thus d/in means the ratio between d and one inch. For example, if d were seven cm., the ratio would be

$$\frac{7\text{ cm.}}{\text{one in.}} \equiv 7 \times \frac{\text{one cm.}}{\text{one in.}} \equiv 7 \times 0.394 \equiv 2.76$$

As another example suppose that f is 60 cycles per second: then

$$\frac{f}{60/\text{sec}} \equiv \frac{60/\text{sec}}{60/\text{sec}} \equiv 1$$

but if f is 3000 cycles per minute

$$\frac{3000/\text{min}}{60/\text{sec}} \equiv 50 \times \frac{\text{one sec.}}{\text{one min.}} \equiv 50 \times \frac{1}{60} \equiv \frac{5}{6}$$

This system of dimensional formulas has been discussed by V. Petrovsky (see Bibliography). He shows that it is unnecessary to have several formulas, one for each system of units.

[3] First order of eddy currents means the eddy currents induced by the uniform current. They are expressed by the first term of an infinite series as shown in Appendix B.

[4] See Bibliography.

mated by considering first that the current is uniform over the depth of the slot, and then calculating the eddy-current loss which would occur in the strands. This may be done by considering each strand to be a bar of a series-connected bar winding and applying formulas for coils of Type VI, VII, and VIII. It is shown in Appendix C that the strand loss may be approximated by

$$k_s = 6.0\, b\, \psi \left[\frac{f}{60/\text{sec}} \right]^2 [r\, n\, (d/\text{in})^2]^2 \qquad (8)$$

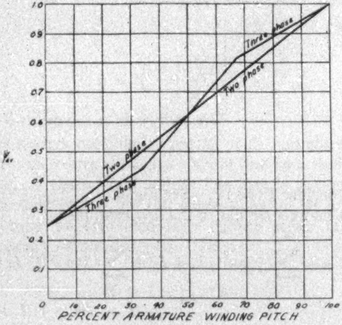

Fig. 8—The Factor ψ_{av} Used in Calculating Strand Loss Factor

This value is different for each coil that has a different θ, and in general there are two groups of coils in a machine each having a different value of θ. It becomes convenient to average ψ over a whole machine so that the strand loss may be calculated easily. This has been done and ψ_{av} is plotted against per cent pitch in Fig. 8.

The strand loss factor is small in a well-designed machine and may often be neglected when considering the effect of transposition. It is always present no matter what the transposition is and must be added to the circulating current loss factor to get the total extra loss factor for the first order of eddy currents. The strand loss may always be reduced by using finer laminations in the conductor.

To summarize the method of calculation of eddy-current losses it may be noted that an extremely simple slide rule calculation using Formula (7) will obtain the extra loss factor for a coil of Type I. The effect of transposition may be obtained from Table II by selecting a factor to apply to the extra loss factor for Type I. Finally the strand loss factor may be obtained by Formula (8) and added to the circulating current loss factor. The result is the total extra loss factor for the first order of eddy currents. This, multiplied by the normal copper loss of the winding, calculated for uniform current distribution, gives the extra loss due to eddy currents and is accurate, provided it be not so large that the first term of the infinite series discussed in Appendix B fails to properly describe it. If it should be as large as this, however, it would be too large to be considered permissible in a modern winding. A method of transposition would be selected to reduce it to a much smaller value.

An example will help to make these rules understandable. Suppose we are considering an hypothetical three-phase winding having coils of Type V, Fig. 7, with three turns, 17/24 pitch, and two coil sides per slot. Suppose the slot is 0.675 in. wide and each turn is composed of 30 strands, each 0.14 in. wide, 0.075 in. over the strand insulation, and 0.07 in. net depth, and arranged 3 wide and 10 deep. Suppose that the ratio of the length of the core to the length of a half-turn is 0.50, and the frequency is 60 cycles per second.

Then

$$\frac{f}{60/\text{sec}} = 1$$

$$\begin{aligned} r &= 0.667 \\ b &= 0.50 \\ n &= 60 \\ d/\text{in} &= 0.07 \\ m &= 3 \end{aligned}$$

and

$$k = 0.28\,[1]^2 \left[\frac{0.667 \times 0.50 \times (60)^2 \times (0.07)^2}{3} \right]^2 \equiv 1.07$$

A three-phase winding of 17/24 pitch has 87.5 per cent of the coils with an angle θ of 60 deg. and the rest with an angle θ of zero. Thus from Table II the proper factor is

$$0.875 \times 0.093 + 0.125 \times 0.056 \equiv 0.0884$$

Therefore, the extra loss factor for the whole winding is

$$0.0884 \times 1.07 \equiv 0.095$$

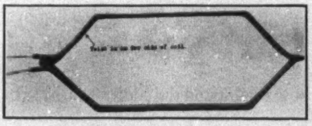

Fig. 9—Completed Five-Turn Coil of Type IV (Similar to Fig. 6)

or the extra loss due to circulating current is about 9.5 per cent of the ordinary copper loss, calculated for the square of the current and the ohmic resistance. Without the transposition, the formulas indicate an extra loss of about 107 per cent. The reduction from 107 per cent to 9.5 per cent by simply inverting the first half-turn when winding the coil is quite a remarkable gain when the simplicity of the transposition is considered.

The strand loss factor to be added to the above in either case is

$$k_s = 6.0 \times 0.50 \times 0.822\, [1]^2\, [0.667 \times 60 \times 0.0049]^2 \equiv 0.094$$

and the total eddy current loss is about 19 per cent of the normal copper loss. This is a fairly high value for strand loss factor but it can be reduced by using smaller strands.

In all practical cases where there is more than one turn per coil, the extra loss may be kept down to a small fraction of the normal loss by using the proper inverted turn transposition. Where there is only one turn per coil the Roebel or 360-deg. transposition may be used. The discussion of this type of bar is outside the scope of the present argument and it suffices to say that it reduces the circulating current loss in a conductor to practically zero.

In conclusion the author wishes to express his appreciation of the invaluable suggestions of Mr. P. L. Alger and his enthusiastic support. He also wishes to thank Mr. M. C. Holmes for assistance with the numerical work in connection with the tables and for helpful criticism.

Appendix A

In order to illustrate the method of deriving the formulas shown in Table I, two of the cases are derived below. The others may be obtained in a similar manner and will not be proved here. Some of them have been derived previously by other writers but they were all checked in the preparation of this paper.

Consider Type III with an even number of turns. See Fig. 5.

Applying the rule given in the body of the paper

$$I_0 = \sum_{1}^{\frac{m}{2}} {}^{s} (p-1)\, I_1,$$

where p = 2 s – 1 for all the odd half-turns in the lower[5] coil side,

$$I_0 = \sum_{1}^{\frac{m}{2}} {}^{s} [(p-1) I_1 + I_2],$$

where $p = 2\,s - 1$ for all the odd half-turns in the upper coil side,

$$I_0 = -\sum_{1}^{\frac{m}{2}} {}^{s} [(p-1)\, I_1 + I_1],$$

where $p = 2\,s$ for all the even half-turns in the lower coil side, and

$$I_0 = -\sum_{1}^{\frac{m}{2}} {}^{s} [(p-1)\, I_1 + I_1 + I_2],$$

where p = 2 s for all the rest.

Averaging these over the 2 m half-turns, the result is

$$2\,m\,I_0 = \sum_{1}^{\frac{m}{2}} {}^{s} [(2\,s-2)\, I_1] + \sum_{1}^{\frac{m}{2}} {}^{s} [(2\,s-2)\, I_1 + I_2]$$

$$- \sum_{1}^{\frac{m}{2}} {}^{s} [2\,s\,I_1] - \sum_{1}^{\frac{m}{2}} {}^{s} [2\,s\,I_1 + I_2].$$

This gives

$$I_0 = -\, I_1$$

Hence

$$\left| \frac{I_0}{I_1} \right| = 1,$$

$$\operatorname{Cos} \alpha = -\, 1$$

and finally

$$L = 0.$$

Now consider Type IV. See Fig. 6.

In a manner similar to the first example

$$2\,m\,I_0 = -\sum_{2}^{m} {}^{p} [(p-1)\, I_1 + I_1] + \sum_{1}^{m-1} {}^{p} [(p-1)\, I_1 + I_2]$$

$$- [I_2 + (m-1)\, I_1 + I_1].$$

This reduces to

$$I_0 = \frac{(2 - 3\,m)\, I_1 + (m-2)\, I_2}{2\,m}.$$

Since all the conductors are in series

$$I_2 = m\, I_1 \angle\, \theta.$$

This is substituted and the vectorial summation effected so that

$$\left| \frac{I_0}{I_1} \right| = \frac{\sqrt{(2-3\,m)^2 + (m^2-2\,m)^2 + 2\,(2-3\,m)\,(m^2-2\,m)\cos\theta}}{2\,m}$$

and

$$\operatorname{Cos} \alpha = \frac{(2-3\,m) + (m^2-2\,m)\cos\theta}{2\,m \left| \dfrac{I_0}{I_1} \right|}$$

and finally

$$L = \frac{(m-1)\,(m-2)}{m} \left[\frac{m^2 - 5\,m + 2}{4\,m} + 2 \sin^2 \frac{\theta}{2} \right].$$

Any combination of half-turns may be calculated in a manner similar to the above and the reader may verify the rest of the formulas given in Table I or he may make new formulas for other combinations.

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[5] That part of the slot nearest the closed end is referred to throughout this paper as the bottom and that part nearest the open end, as the top.

Appendix B

Non-uniform current distribution in armature conductors introduces changes in the effective reactance of the windings as well as in the resistance. The question of the reactance is not considered here. It is possible to set up equations containing complex numbers which express both the resistance and reactance factors, but since only the resistance is considered here M and N are taken[6] as

$$M = \text{real part of } q\,h \coth q\,h$$

$$N = \text{real part of } 2\,q\,h \tanh \frac{q\,h}{2}$$

where

$$q = \sqrt{\frac{8\,\pi^2\,j\,r\,f}{\rho}}$$

and h is the depth of the conductor.

It is always possible by means of the proper transposition to reduce the eddy current loss to a negligible fraction of the normal copper loss and it is therefore unnecessary to use the full expressions for M and N. They may be expanded into infinite series and all unnecessary terms dropped. Thus

$$M = 1 - \frac{4}{45}\,q^4\,h^4$$

$$N = -\frac{1}{12}\,q^4\,h^4$$

Let

$$q = (1 + j)\,g$$

so that

$$g = \sqrt{\frac{4\,\pi^2\,f\,r}{\rho}}$$

and

$$M = 1 + \frac{4}{45}\,g^4\,h^4$$

$$N = \frac{1}{3}\,g^4\,h^4\,.$$

M appears in every expression for the circulating current loss factor. The constant part of it represents the normal copper loss while the variable part represents the extra loss. Thus the extra loss factor is

$$k = \frac{4}{45}\,g^4\,h^4 + \frac{L}{3}\,g^4\,h^4$$

or

$$k = [4 + 15\,L]\,D$$

where

$$D = \frac{g^4\,h^4}{45}.$$

The quantity h is the depth of one turn, while d is taken as the depth of one strand, m is the number of turns per coil, and n is the number of layers of strands in the slot, so that for a two-coil side per slot winding

$$h = \frac{n\,d}{2\,m};$$

for a one-coil side per slot winding

$$h = \frac{n\,d}{m},$$

and for strand loss calculation

$$h = d\,.$$

In a laminated winding the current is forced to flow over the whole length of a half-turn by the voltages induced in the core portion only. The resistivity ρ is taken as $1/b$ times the actual resistivity for this reason where b is the ratio of the length of the core to the length of a half-turn. Copper at 75 deg. cent. is assumed. Substituting the value of ρ and g, the value of D for a two-coil side per slot winding becomes

$$D = 0.075 \left[\frac{f}{60/\text{sec}}\right]^2 \left[\frac{r\,b\,n^2\,(d/\text{in})^2}{m^2}\right]^2$$

and for a one-coil side per slot winding, it becomes

$$D = 1.19 \left[\frac{f}{60/\text{sec}}\right]^2 \left[\frac{r\,b\,n^2\,(d/\text{in})^2}{m^2}\right]^2$$

and for strand loss calculation

$$D = 1.19 \left[\frac{f}{60/\text{sec}}\right]^2 [r\,(d/\text{in})^2]^2.$$

Appendix C

CIRCULATING CURRENT LOSS

The formula for a coil of Type I is taken from Table I and substituted in Equation (3), resulting in

$$k = \left[4 + \frac{15}{4}\,(m^2 - 1)\right] D,$$

or

$$k = \left[\frac{15}{4}\,m^2 + \frac{1}{4}\right] D\,.$$

The constant term in this expression may be dropped without fear of sensible error and the result is

$$k = \frac{15}{4}\,m^2\,D\,.$$

Substituting the proper value of D

$$k = 0.28 \left[\frac{f}{60/\text{sec}}\right]^2 \left[\frac{r\,b\,n^2\,(d/\text{in})^2}{m}\right]^2.$$

Circulating current loss factors for any other type of winding may be calculated by selecting the proper factor from Table II and applying it to the result just obtained. That is, the effect of the transposition on a coil of Type I is to reduce the extra loss in the ratio given in Table II. This table has been made up with the use of Table I and Formula (3.)

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[6] See the paper by W. V. Lyon, Bibliography, 1.

STRAND LOSS

The strand loss in a one-coil side per slot winding is obtained by substituting

$$L = \frac{m^2 - 1}{3}$$

in Formula (3.) The result is

$$k_s = \left[4 + \frac{15}{3} (m^2 - 1) \right] D$$

and this is practically equal to

$$k_s = 5\, m^2 D \; .$$

When the proper value of D is substituted

$$k_s = 6.0\, b \left[\frac{f}{60/\mathrm{sec}} \right]^2 [r\, n\, (d/\mathrm{in})^2\,]^2 \; .$$

The factor b is introduced because the extra losses occur in the slot portion only.

In a two-coil side per slot winding

$$L = \frac{4\, m^2 - 1}{3} - m^2 \sin^2 \frac{\theta}{2}$$

and therefore

$$k_s = \left[4 + \frac{15}{3} (4\, m^2 - 1) - 15\, m^2 \sin^2 \frac{\theta}{2} \right] D \, .$$

Practically this is

$$k_s = 20\, m^2 D\, \psi$$

where

$$\psi = 1 - \frac{3}{4} \sin^2 \frac{\theta}{2} \, .$$

The correct value of D is now substituted and b is introduced with the result that

$$k_s = 6.0\, b\, \psi \left[\frac{f}{60/\mathrm{sec}} \right]^2 [r\, n\, (d/\mathrm{in})^2\,]^2$$

This is the same result that was obtained for the strand loss in a one-coil side per slot winding where θ is zero, and is therefore general. Various coils in a machine may have different values of θ and it becomes convenient to have an average value of ψ to apply to a whole machine. This has been done and ψ_{av} is given in Fig. 8. For a three-phase machine ψ_{av} varies linearly from 1.0 at 100 per cent pitch to 13/16 for 66⅔ per cent pitch and from there linearly to 7/16 at 33⅓ per cent pitch and from there linearly to 0.25 at zero pitch. For a two-phase machine ψ_{av} varies linearly from 1.0 at 100 per cent pitch to 5/8 at 50 per cent pitch and from there linearly to 0.25 at zero pitch.

TABLE OF SYMBOLS

I_0	A quantity used in the formation of equations for extra loss. The rule for obtaining I_0 is given in the body of the paper.
I_b	The vectorial summation of all the current in the slot below the conductor in question.
I_1	The current in the conductor in question.
$\left\lvert \frac{I_0}{I_1} \right\rvert$	The numerical value of the ratio indicated.
α	The angle between I_0 and I_1.
L	A quantity which depends on the type of coil and its transposition. It is defined by Equation (**1**).
K	The ratio of the total heat loss in the coil to that due to uniform current distribution.
k	The ratio of the extra heat loss in the coil due to eddy currents to that due to uniform current distribution. Also called extra loss factor due to circulating currents.
D	A quantity depending on the dimensions of the slot and coil, the frequency, the number of turns per coil, and the number of coil sides per slot. It is defined in Equations (**4**), (**5**) and (**6**).
f	The frequency of the alternating current.
r	The ratio of the width times the over-all height of the copper conductor in the slot to the slot width, times the net conductor height.
b	The ratio of the length of the slot to the length of a half-turn.
d	The depth of a strand.
n	The total number of layers of strands depthwise in the slot including both coil sides.
m	The number of turns per coil.
k_s	The ratio of the extra loss due to strand loss to that due to uniform distribution of current. Also called strand loss factor.
p	The order of the conductor under consideration. The one nearest the bottom of the slot in the coil side under consideration is $p = 1$. The effect of eddy currents is calculated by averaging I_0 over all conductors and is accomplished by finding I_0 for the general conductor p and summing up all such values.
s	Any integer. The summations in Appendix A are effected with respect to s between the limits indicated.
h	The depth of a conductor, or turn.
q	A quantity depending on frequency, resistivity of the conductor, width of conductor, width of slot, length of slot, and length of a half-turn. It is a complex quantity.
j	The square root of minus one.
g	The quantity q divided by $(1 + j)$. It is real.
ψ	A quantity used in strand loss calculation. It is $\psi = 1 - (3/4) \sin^2 (\theta/2)$.

ψ_{av} The quantity ψ, averaged over all the coils in a machine.

ρ The resistivity of the conductor.

θ Angular phase difference between the currents in the top and bottom coil sides in one slot.

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Discussion

G. H. Rockwood, jr.: In order to compute the losses in these transpositions which Mr. Summers has described, it is necessary to determine the percentage of slots in the winding having a given value of the angle θ between current in the top and bottom coil sides. Necessity for visualizing the winding may be done away with and the computation simplified by the following procedure:

In a three-phase winding of full pitch, the angle between the currents in the top and bottom coil sides of all slots is zero; for a two-thirds pitch winding, it is 60 deg.; for a one-third pitch winding, 120 deg., and for a zero pitch winding, 180 deg. Between these limiting values, the percentage of coils having a given value of the angle θ will vary linearly with pitch. We may, therefore, plot the data of Table II of the paper, as is done in the accompanying Fig. 1, and obtain a family of curves for each transposition. Such a plot will enable us to omit the rather laborious computation in the paper, and to read the reduction factor for circulating current directly as a function of the coil pitch.

P. L. Alger: Mr. Summers' idea of making an inversion in one or more turns at the ends of an armature winding seems to me to be a very simple and satisfactory method of avoiding eddy currents. In Schenectady, in building a considerable number of large machines, we have used it with very good success. The advantage of the method is that it enables a standard machine-wound coil to be made without the extra expense of special insulated clips, the connecting of adjacent coils, or any other complications. All other schemes that have been proposed before have involved some inconvenient departure from the simple standard manufacturing processes, and so their use has not seemed to be warranted unless the circulating-current loss without transposition was more than about 25 per cent. With the new method, we have found it desirable to transpose the winding whenever the circulating-current loss should exceed approximately 10 per cent without it.

It is true that the inverted-turn idea is very similar indeed to those which have been suggested by H. W. Taylor in England, and by others; but so far as I know, no one has proposed exactly the same scheme, nor has anyone worked out, completely, the eddy-current loss formulas for this case. I feel, therefore, that Mr. Summers has made a very distinct contribution to the art of electrical machine design.

J. R. Dunbar: I desire to point out that the method of transposition advocated by Mr. Summers was used by H. B. Dwight on experimental coils for machines that were being manufactured by the Canadian Westinghouse Company. At that time, it was decided that the reduction in eddy-current losses was insufficient to justify the added complication in the winding of the coils for the machines then being considered. I should like to ask Mr. Summers if it is proposed to use in machines now under construction any of the transpositions he describes?

F. D. Newbury: It may be of interest to point out a few of the more general considerations affecting transposition. There have been three methods advocated: First, there is transposition secured by means of certain connections between coils. That method has been applied for 10 or 12 years in large machines and is effective and economical in such cases. The second method is to transpose the conductors within the armature slot. That is a method which has been used in Europe very considerably but not in this country until recently. In Europe, designers have used partially closed slots; we have used open slots. They have used straight bar conductors joined by separate end connectors in these partially closed slots. This construction with transposition within the slot, is fairly simple. Designers in this country have taken up this construction in recent years in large machines because coils became so long that it was desirable to deal with half coils instead of complete coils, and it is only possible to deal successfully with half coils when it is possible to connect both ends of the conductor solidly in forming the complete winding. So in American practise, in our largest machines the transposition within the slot is coming into use.

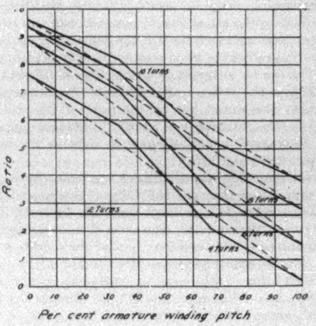

FIG. 1—RATIO OF EXTRA LOSS IN TYPE IV WINDING TO THAT IN TYPE I.

Full line—3-phase.
Dash line—2-phase.

It seems to me that this third method advocated and described by Mr. Summers is a very desirable construction for the moderate-sized machine where it is still desirable to use complete coils, machine-wound, and to secure transposition economically.

In looking over the bibliography at the end of this paper[2] there is another thought that occurs to me. It illustrates the greatly accelerated progress in the mathematical and theoretical bases of

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[1] *Heat Loss in Stranded Armature Conductors*, W. V. Lyon, TRANS. A. I. E. E., 1922, p. 199.

our design work. From this bibliography you will note that the earliest American paper was one by A. B. Field, presented in 1905. There was no other important contribution, at least, not in America, until Gilman's paper in 1920,—a period of fifteen years. In the six or seven years since that, we have had the important papers by Prof. Lyon, and now another one by Mr. Summers, which indicates the very healthy condition of progress in this branch of design.

C. M. Laffoon: I wholly subscribe to the views given in the paper; in fact, I used the method a few years ago.

W. V. Lyon: I agree with Mr. Summers that in the computation of the extra-loss factor for any winding that is properly designed, it is sufficiently accurate to use the first terms in the expansion of the hyperbolic functions. This is also the expressed opinion of H. W. Taylor. For anyone who might wish to compare the approximate with the exact values of these hyperbolic functions, I am appending a table of such values.

I should like to call attention to one minor correction. It is in regard to the use of the ratio r when calculating the strand loss. This ratio takes into account the effect of the insulation between the strands. The magnetic field that exists in the space between adjacent strands plays its part in affecting the distribution of current among the strands and thus is a factor in determining the circulating-current loss. On the other hand, this magnetic field does not influence the strand loss. The expression for the strand loss can be corrected by using either a different value for the ratio b or the ratio r. The effect of this correction will be to multiply the strand loss by the ratio of (the net depth of strand)^2 to (the gross depth of strand).[2] Making this correction in Mr. Summers' numerical example reduces the strand loss factor from 0.094 to 0.082.

I believe that there are still one or two points in the theory of these extra losses that are not fully appreciated. The extra circulating-current loss is zero when the current is equally divided among the strands. The extra strand loss is not a minimum for this condition, however, but for the condition when the current is more concentrated toward the top of the slot. As a simple illustration of this, consider two equal strands placed one above the other. The total loss in these strands is a minimum when the current in the lower strand is one-half of the total multiplied by the fraction $\left(1 - \frac{N}{M}\right)$ and the current in the upper strand is one-half of the total current multiplied by $\left(1 + \frac{N}{M}\right)$. As further evidence of this fact, consider the expressions for strand loss that were derived in my 1922 paper.[1] In case 5 of that paper, the circulating-current loss is much smaller than in case 6, but the strand loss is smaller in case 6 in about the ratio of 13 to 16 for full-pitch slots and in about the ratio of 10 to 13 for three-phase fractional-pitch slots. It is readily shown that in the windings which have a small circulating-loss factor, it is very nearly correct to compute the strand loss on the assumption that the current is equally divided among the strands, as Mr. Summers has done. The exact expression for the strand loss is readily deduced by following the method outlined in my 1922 paper. It is:

$$\left\{ \frac{1}{2\,n} \left(\sum_{\mathrm{D}} \left| I_b \right|^2 + \sum_{\mathrm{R}} \left| I + I_b \right|^2 \right) - \left| I_o \right|^2 \right\} T_r'$$

The notation is the same as in my paper. The value of n is the number of turns in the coil. The first term is the sum of the squares of the currents in the slot *below* the successive half-turns of the coil in which the strands are numbered in the direct order. The second term is the sum of the squares of the conductor current plus the current in the slot below the successive half-turns of the coil in which the strands are numbered in the reverse order. The last term is the square of the current I_o. Mr. Summers describes the method of computing I_o.

For a given depth of copper in the slot, the strand loss can be reduced to a reasonable value by increasing the number of strands. For a given number of strands it is nearly independent of the strand arrangement. The circulating-current loss, on the other hand, depends in a marked degree upon the strand arrangement. The problem then seems to be to reduce this latter loss to its smallest possible value.

With full-pitch slots, Mr. Summers' quantity, L, has its smallest value when $\frac{I_o}{I} = -\frac{1}{2}$, for which condition $L = -\frac{1}{4}$. This then gives the condition for minimum circulating loss. This value of $\frac{I_o}{I}$ can always be obtained, if the coil has an even number of turns, by inverting the end connections as shown in his Fig. 3. It can also be obtained if the coil has an odd number of turns provided the current in the lower coil side is in phase with that in the upper coil side. Figs. 2 and 3 herewith show how it may be accomplished when the coil has three turns and when it has five turns. Mr. Summers does not show this type of inversion. It will be found, however, in H. W. Taylor's paper. With these same inversions, the losses will be somewhat greater for fractional-pitch slots. There is another type of inversion, however, that will give more nearly the theoretical minimum loss for the case of an odd number of turns and fractional-pitch slots. In the case of three-phase, fractional-pitch slots, the minimum value of L is $-\frac{1}{18}$ if the coil has three turns. The arrangement is the same as shown in my Fig. 2, except that the lower conductor of the lower coil side is inverted. For three-phase, fractional pitch slots, arranged as shown in Fig. 2, L is zero. If the conductors have a value of $q\,h$ equal to 2.0, which is not excessive, the inversion of this bottom conductor reduces the circulating-current extra-loss factor from 0.309 to 0.245 for these fractional-pitch slots. Although fractional pitch has certain distinct advantages from the standpoint of flux distribution and wave-form of generated e. m. f., it prevents the attainment of the minimum copper-loss factor except in the single case of a winding having an even number of turns arranged as in Mr. Summers' Fig. 3.

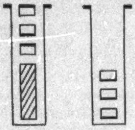

Fig. 2

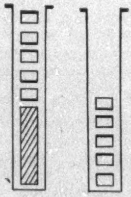

Fig. 3

There is one more case that is perhaps worth while mentioning. A certain large 25-cycle generator has a calculated extra-loss factor of 9.9 per cent. If it had been mechanically possible to invert the end connections, as in Mr. Summers' Fig. 3, the extra-loss factor would have been 0.6 per cent, a reduction of 8.5 per cent in the armature copper loss. For the same armature heating loss the rating of this generator might have been increased nearly 2000 kv-a. There might possibly have to be a slight modification of the field winding to compensate for the increased armature reaction.

TABLE OF APPROXIMATE AND EXACT HYPERBOLIC FUNCTIONS

$q\,h$	M Exact	$1 - \frac{(q\,h)^4}{45}$ Approximate	N Exact	$-\frac{(q\,h)^4}{12}$ Approximate
0.4	1.001	1.00058	0.002	0.00213
0.8	1.008	1.0091	0.034	0.0342
1.2	1.044	1.0461	0.169	0.173
1.6	1.137	1.146	0.512	0.546
2.0	1.309	1.356	1.147	1.333
2.4	1.566	1.738	2.074	2.766
2.8	1.874	2.368	3.17	5.12

H. W. Taylor (communicated after adjournment): When a coil consists of two or more turns, methods of reversing the order of laminations of the conductor in different parts of the coil have been previously known whereby the circulating-current loss may be reduced to a minimum, which minimum loss corresponds to that of a conductor at the bottom of the slot, consisting of half the laminations of the conductor actually used in the coil.

Sometimes it is found inconvenient, however, to arrange all the coil ends so as to change the order of the lamination in the top and bottom conductors as frequently as is required to provide a minimum loss. The author of this paper is to be congratulated upon having disclosed in Fig. 6 a method of winding multi-turn coils which provides a compromise between facility in winding the coil on the one hand and reduction of the circulating eddy-current losses on the other.

In order to consider the merits of the various forms of coil windings with laminated conductors, the writer has found it convenient to consider the circulating loss only and as if the conductor were infinitely limited. The term in the formula for extra loss in an infinitely laminated conductor which involves the position of the conductor in the slot is

$$\frac{4}{45} - \frac{(p^2 - p)}{3}$$

where p is the position of the conductor counting from the bottom of the slot.

In the following table, particulars have been tabulated for two- up to six-turn coils for the ordinary method of winding, for the most perfect method, and for Summers' method as described in the paper. In the table, parallel columns give equivalent values of p, the value of $(p^2 - p)$ and the value of the above expression (leaving out the denominator of 180 in all cases), as a figure of merit.

No. of turns in coil	Ordinary Method			Most Perfect Method			Compromise Method		
	p	(p^2-p)	Fig. of merit	p	(p^2-p)	Fig. of merit	p	(p^2-p)	Fig. of merit
2	1½	¾	61	½	−¼	1	0	0	4
3	2	2	136				1/3 or 2/3	−2/9	3
4	2½	15/4	241				¼ or ¾	−3/16	5
5	3	6	376				1 1/5	6/25	59
6	3½	35/4	691				1 1/3	10/9	83

It will be seen that the author's method is particularly applicable to two-, three-, and four-turn coils. It provides considerable improvement in five- and six-turn coils, but owing to the fact that in coils with this larger number of turns, the total depth of conductor would be relatively shallow, the loss in such coils would in practise, be small in any case.

I. H. Summers: Mr. Rockwood has presented a very interesting and valuable discussion. His curves should materially simplify the computation of extra-loss factors. They will be especially valuable in design work where it is highly desirable to have a quick and convenient method for checking the efficacy of proposed windings.

Mr. Dunbar has stated that H. B. Dwight experimented with transpositions similar to those I have described but gave up the idea because of the belief that the reduction in loss was insufficient to justify their use. Mr. Alger has answered this statement by pointing out that a considerable number of large machines has already been built using these transpositions with good success and that it is found economical in Schenectady to use the transposition whenever the extra circulating eddy-current loss would exceed about 10 per cent without it. It will be noted that some of the types of transpositions described are more effective than others. Naturally, in any given winding, that method will be selected which combines a sufficient reduction of eddy-current loss with economy in making the winding.

With regard to Mr. Newbury's points about the applicability of various types of transpositions to particular classes of windings, I may say that there is no gain to be expected from the inverted-turn transposition when it is applied to a one-turn coil. For machines using a one-turn coil, an effective form of transposition is already in extensive use, which completely eliminates the circulating eddy-current loss. The transposition in this case is accomplished within the slot, and a bar winding, consisting of half coils, is used.

H. W. Taylor's discussion is very interesting. His tabulation shows that type IV, Fig. 6, is particularly applicable for two-, three-, and four-turn coils, when the winding is full-pitch. I wish to remark that type V, Fig. 7, is quite applicable also for two- and three-turn coils; in fact it gives a lower loss than type IV if the coils are wound with fractional pitch. For coils of four or more turns, type IV is preferable in any case.

I am very much indebted to Prof. Lyon for calling attention to an error in the use of the ratio r. The correction for the strand insulation was not a part of the original manuscript but was sent in as an after-thought. Obviously, it should have been applied to the circulating-current loss but not to the strand loss. Perhaps the best way to take care of this is to define r simply as the ratio between the total width of copper in the slot and the slot width. Then, a new factor c should be introduced into the formula for circulating-current loss wherever it occurs, where c is the square of the ratio between (strand height including strand insulation) to (net strand height).

Prof. Lyon's remarks concerning the problem of strand loss are pertinent and it is interesting to note that his exact results lead him to the conclusion that my formulas are sufficiently accurate for practical windings. Perhaps it is in place to point out that the first duty of the engineer who sets out to provide a solution to a problem is to solve it in the most general and exact manner that he can devise. Prof. Lyon has done this in a neat and logical way. In so doing, he has provided a firm foundation for anyone who later wishes to apply his results. It is often possible in such cases to reduce the general solution to a more practical form which will be the simplest possible, consistent with the accuracy required. Thus, it may be simplified so that practical men will find great use for it. This is what I have attempted to show in my paper.

Graphical Determination of Magnetic Fields

Theoretical Considerations

BY A. R. STEVENSON, Jr.[1] and R. H. PARK[1]

Associate, A. I. E. E. Non-member

Synopsis.—*Three papers on the general subject, "Graphical Determination of Magnetic Fields," are being presented simultaneously to cover three phases of the subject: 1. Theoretical Considerations, 2. Comparison of Calculations and Tests, by E. E. Johnson and C. H. Green, 3. Practical Applications to Salient-Pole Synchronous Machine Design, by R. W. Wieseman. In the following paper, which is the first of this series, the authors have reviewed the history of the subject, have briefly stated the ordinary rules for plotting magnetic flux in air and in current-carrying copper, have developed additional rules for checking the accuracy of field plots, and have given theoretical methods for mathematically calculating the distribution of field in certain cases commonly encountered in practise.*

The authors have called attention to the great value of the mathematical work by the German engineer, Rogowski, and the graphical methods by the French engineer, Lehmann. These German and French articles contain the only extensive practical applications of the plotting of magnetic fields in current-carrying regions with which the present authors are familiar. Since it is much more difficult to plot fields in current-carrying regions, and since the majority of readers are less familiar with this phase of the subject, the greater part of the first paper, "Theoretical Considerations," is devoted to a study of such fields. As special examples, the mathematical solutions for the magnetic field between the poles of a salient-pole alternator and the magnetic field in a circular conductor in a circular slot are given in Parts I and III, respectively, of this first paper. Part II of the first paper contains a set of theorems which deal with various questions which have arisen while studying this subject—such as the proof that there is no refraction of the lines of force at a copper-air boundary, but a change of curvature when a line of force crosses such a boundary; and a general law for checking any field plot by the relation of the magnetic intensity and its rate of change to the current density and the radius of curvature of the magnetic field. The first two appendixes contain practical rules for the free-hand plotting of flux distribution in both air and copper; Appendix C contains an interesting discussion of the conception of vector potential, showing how easily it can be applied to the solution of practical problems.

The three papers in this group should be considered as one complete whole and the reader will find very interesting a comparison between the mathematical plots given in this paper on "Theoretical Considerations" and the experimentally determined plots in the second paper on "Comparison of Calculations and Tests," and the practical uses for these methods in the design of machinery given in "Practical Applications to Salient-Pole Synchronous Machine Design."

In the early days of the design of electrical machinery, magnetic distributions were largely a matter of guesswork, but in these days of more severe competition and closer refinement of design, an accurate knowledge of the magnetic distribution is a necessary fundamental of the greatest importance in determining the best designs.

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HISTORICAL INTRODUCTION

IN the design of electrical apparatus, the accurate calculation of the characteristics of a proposed design depends on the distribution of magnetic flux in the machine.

A few shapes lend themselves to rigorous mathematical analysis. One of the most famous examples of this is the calculation of the fringing coefficient due to the slots along the periphery of an induction motor air-gap, by F. W. Carter[2], who used the theory of functions of a complex variable to obtain a solution of Laplace's equation.

The mathematical complications encountered in analyzing the magnetic fields surrounding many of the irregularly shaped iron boundaries commonly found in electrical machinery, however, have led many authors to advocate and describe graphical and experimental methods. One of the best bibliographies on this subject is given by J. F. H. Douglas in an article[3] in which he reviews the many methods suggested by other authors, but especially emphasizes the use of high-resistance templets for experimentally plotting a field.

There are many articles[4] that describe the graphical method of plotting magnetic fields in air, but almost none of them give any suggestion of a method for plotting fields within current-carrying conductors. One of the best articles on the subject is "The Free-Hand Graphic Method of Determining Stream Lines and Equipotentials," by L. F. Richardson[5]. This author makes the statement that in a region occupied by current, the difference of successive chequer ratios in a direction perpendicular to the lines of force, divided by the mean chequer area, is equal to a constant times the current density in the region[6]. But he goes on to say:

"To draw chequers free-hand, so as to satisfy a difference relation of this sort between chequer ratios, is likely to be toilsome, and we will here consider only the case where $\Delta^2 v = 0$."

The remainder of Richardson's paper is given up to the sketching of various fields, in none of which does he take into account the effect of current-carrying conductors in the space occupied by the flux.

The most complete treatise on the free-hand plotting of fields of magnetic flux will be found in a series of French articles by Lehmann[7]. The 1909 article con-

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[1] Both of the Engineering General Department, General Electric Company, Schenectady, N. Y.

[2] See bibliography, 1.

[3] See bibliography, 5.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

[4] See bibliography, 8 and 10.

[5] See bibliography, 2.

[6] See Appendix B for a proof of this statement.

[7] See bibliography, 4, 7 and 9.

tains many valuable suggestions for plotting fields in air, and the 1923 articles give an elaborate set of rules for plotting magnetic fields in regions where the curl of the field is not zero—*i. e.*, in space occupied by current-carrying conductors.

It has always been said that there are no lines of magnetic equipotential in current-carrying space. This is true, but Lehmann pointed out that a system of lines orthogonal to the lines of force can be drawn. He called these lines gradients, and called attention to the fact that they all converge to one or more points in the copper which he called points of indifference.

Consider the field in and around a cylindrical conductor. The lines of force are circles. The gradients, or lines of no work, as the authors prefer to call them, are radial lines; and the point of indifference, or kernel, is the center of the wire. It is also seen that the lines of no work on leaving the copper become lines of equipotential.

This conception of lines of no work radiating from one or more kernels greatly facilitates plotting fields in copper. In Appendix B, a very brief set of rules for plotting the flux in the current-carrying regions is given by one of our associates, Mr. Lloyd P. Shildneck, using as an illustration the distribution of flux between the field poles of an alternator.

In June, 1918, Messrs. Doherty and Shirley published a paper[8] which made practical use of many sketches of magnetic fields in air. Since that time, the company with which the authors are associated has made considerable use of free-hand sketching in the predetermination of the air-gap fields of salient-pole machinery. Except in a few special cases, however, our attempts to sketch magnetic fields in regions occupied by current-carrying conductors were rather unsatisfactory.

In the fall of 1924, not yet having noticed Lehmann's 1923 article, we started to investigate this problem seriously. The task of sketching is greatly simplified when one knows in advance the approximate shape which the sketch should have. Not yet having gained the conception of a kernel from which lines of no work radiate, we had only vague and incorrect ideas as to the detail distribution of flux within the copper coils surrounding alternator field poles, which was the problem we set ourselves to solve.

After many attempts to make a reasonable looking sketch, we decided that it would be worth while to attempt a mathematical analysis of this field, in the hope of obtaining a picture which would at least suggest the general arrangement of the lines of force in the copper. The purpose of this article is to describe this mathematical method, the route by which it was derived, and the additional suggestions for free-hand sketching which resulted from this work.

Professor V. Karapetoff called our attention to a most valuable article by Rogowski[9] which we immediately adopted as a basis for our work. Unfortunately, this is out of print in German and had never been translated into English until recently, when a partial translation was given in an article by one of the present authors[10], which it will be necessary to read in order to understand Part I of this paper. Rogowski mathematically plotted the leakage field in and around transformer coils. The pictures shown in Rogowski's article called our attention to the kernels or points of indifference.

The two fundamental ideas which we obtained from Rogowski's work were:

1. The simultaneous solution of four relatively simple differential equations set up to satisfy the conditions in four adjacent regions into which the field was divided. This simplified the problem greatly because each region, taken by itself, could be analyzed easily; whereas, the alternative course, a consideration of all four regions at once, would have been a very difficult task.

2. This division into regions made it possible to represent the current distribution by a Fourier's series of only one of the space coordinates.

The contribution of the present article consists in the extension of Rogowski's method to more complicated magnetic shapes. We followed Rogowski by dividing the field of a salient-pole alternator into several regions which could be solved simultaneously. The current-carrying region was represented by an infinite series of reflected and re-reflected images, while the mathematical expression for the boundary region between the pole tips was obtained from a free-hand sketch. This combination of a strictly mathematical method with the theory of images and with free-hand sketching in the parts which can easily be sketched free-hand, into one consistent system of equations which gives mathematical expressions for the lines of force in the part of the field which cannot be easily sketched, is, we believe, a valuable engineering tool which can be applied to many other problems. Illustrations are given of its application to a problem with rectangular boundaries and also to a problem with circular boundaries. The method can probably be extended to an even wider range of problems by expressing the equations of Poisson and Laplace in more complicated systems of coordinates.

It has been our experience that, in certain cases, so much sketching and resketching is required before the kernel can be found by the free-hand method that this mathematical method could actually be worked out, by the use of comptometers, more quickly than the field could be sketched.

This theoretical work also led to additional rules for guiding the construction and checking the accuracy of free-hand sketches.

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[8] See bibliography, 6.

[9] See bibliography, 3.

[10] See bibliography, 11.

The body of the paper is arranged in three parts:

PART I. Calculation of the flux leakage between poles in an alternator with approximately parallel pole sides.

PART II. Point conditions in coplanar magnetic fields. (New rules for constructing and checking of free-hand sketches.)

PART III. Application of the method to polar coordinate systems.

Five appendixes have been included, the first two merely in the interest of completeness; but the last three contain valuable original material.

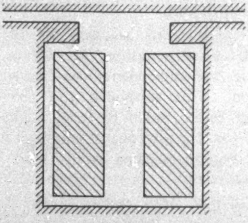

FIG. 1

APPENDIX A. Rules for plotting flux in air.

APPENDIX B. Method of plotting two-dimensional magnetic fields in space occupied by current-carrying conductors.

APPENDIX C. Vector potential in coplanar magnetic fields.

APPENDIX D. The relation of the scalar potential function to the vector potential function in coplanar magnetic fields.

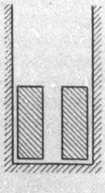

FIG. 2

APPENDIX E. Calculation of inductance from a knowledge of vector potential.

I. CALCULATION OF THE FLUX LEAKAGE BETWEEN POLES IN AN ALTERNATOR WITH APPROXIMATELY PARALLEL POLE SIDES

It is desired to calculate the flux distribution in the two-dimensional magnetic circuit of Fig. 1, which is supposed to represent a cross-section of an actual pair of field poles. The shaded area is intended to represent current-carrying copper. It is assumed that the current density is constant over the cross-section of the conductors and that the iron is of infinite permeability.

One step in the solution of the problem in which the actual magnetic circuit was replaced by an infinitely deep slot, as in Fig. 2, was suggested by one of our associates, Mr. C. J. Koch. Under this assumption, the complete magnetic circuit may be replaced by a double row of images extending to infinity, as shown in Fig. 2A. It was found that the solution used by Rogowski[11] for the flux in a transformer with pancake coils could be applied to this case, by using a Fourier's series for the infinite string of images.

The problem may be reduced, in fact, to a consideration of an infinite series of rectangular conductors parallel to an infinite plane of iron, as in Fig. 2B, and the solution is the same as Rogowski's for the case $\mu = \infty$ except that, since all the currents are of the same sign, there will be a constant term, α_0, in the Fourier's series for the current density, which requires an additional term, $2\,\pi\,\alpha_0\,x^2$, in the equation for R in current-carrying space, and it will also be found that the constant terms and coefficients of x will not drop out as they did in the transformer problem.

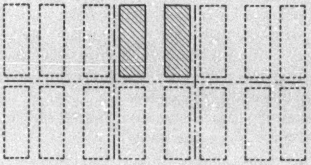

FIG. 2-A

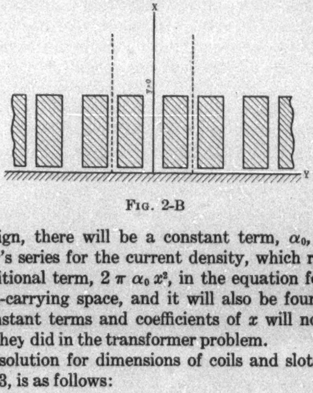

FIG. 2-B

The solution for dimensions of coils and slots given in Fig. 3, is as follows:

Region I:

$$R = \sum_{1}^{\infty} \frac{2\,\pi\,\alpha_n}{(k\,n)^2} [1 - \epsilon^{-kn(b-a)}] [\epsilon^{-kn(a-x)} + \epsilon^{-kn(a+x)}] \cos n\,\theta$$

Region II:

$$R = 2\,\pi\,\alpha_0\,(x - a)^2 + \sum_{1}^{\infty} \frac{2\,\pi\,\alpha_n}{(k\,n)^2} [2 - \epsilon^{-kn(b-x)}$$

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[11] For translation, see bibliography, 11.

$$- \epsilon^{-kn(x-a)}(1 - \epsilon^{-2kna} + \epsilon^{-kn(b+a)})] \cos n \theta$$

Region III:

$$R = 2 \pi \alpha_0 (b - a) [2 x - (b + a)]$$

$$+ \sum_{1}^{\infty} \frac{2 \pi \alpha_n}{(k n)^2} [1 - \epsilon^{-kn(b-a)} - \epsilon^{-2knb}$$

$$+ \epsilon^{-kn(b+a)}] \epsilon^{-kn(x-b)} \cos n \theta$$

wherein, for convenience, the y coordinate has been replaced by θ which varies from $- \pi$ to π across the total width of the slot. Also,

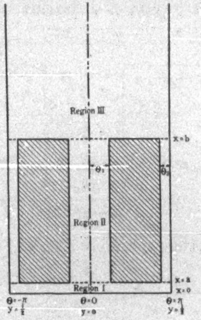

FIG. 3

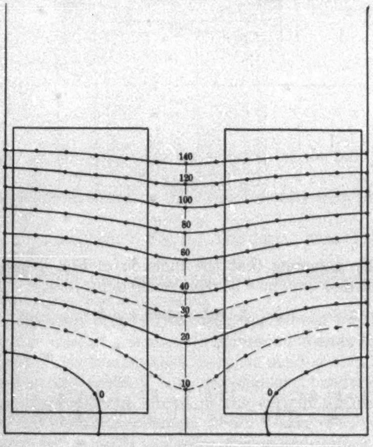

FIG. 3-A

$$k = \frac{2 \pi}{l} \text{ where } l \text{ is the width of the slot.}$$

$$\alpha_0 = \frac{\pi - (\theta_1 + \theta_2)}{\pi} i$$

$$\alpha_n = \frac{2 i}{n \pi} [\sin (n \theta_1) + (- 1)^n \sin (n \theta_2)]$$

where i is the current density in the copper and θ_1 and θ_2 are indicated in Fig. 3. The field distribution for this case is given in Fig. 3A.

In order to extend the solution to include the effect of the pole tips and the finite depth of slot, it was suggested that the field be calculated by replacing the actual magnetic circuit with a closed rectangular magnetic circuit of the same dimensions as though the pole faces came together, leaving no air-gap. The effect of the air-gap was to be simulated by a narrow strip of copper along the surface of the iron, assumed to carry a total current equal to the sum of the currents in the two field coil sides (see Fig. 4.).

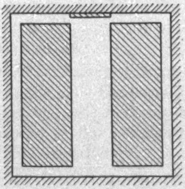

FIG. 4

A better method of solution is provided by first estimating one component of the field in the air-gap between the pole faces and using this estimated field component as a boundary condition.

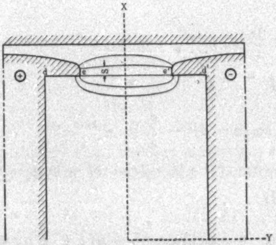

FIG. 5

The procedure is as follows:

The field at the air-gap is first calculated or plotted free-hand on the basis of a definite potential between the field poles.

The Y component of the field along the line $d\,d'$ (Fig. 5) is zero from d to e. It is approximately inversely proportional to the vertical distance s between two lines of force in the neighborhood of the line $d\,d'$ from e to e' and is again zero from e' to d'. The arrangement is symmetrical with respect to the x axis.

The variation of Y is calculated in this way from d to d', or from $\theta = -\pi$ to $\theta = \pi$, and is plotted to any convenient scale. The general character of the curve is shown in Fig. 6.

This curve of Y along the upper boundary of Region III can be expressed as a Fourier's series[12] of the form:

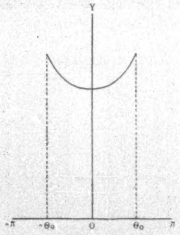

FIG. 6

$$Y = \beta_0 + \sum_{1}^{\infty} k\,n\,\beta_n \cos n\,\theta$$

Unfortunately, it seems necessary to use about a dozen harmonics in using this equation as a boundary condition.

The solution for the vector potential R at any point in the three regions is then given by three equations:

Region I:

$$R = \sum_{1}^{\infty} [a_n\,\epsilon^{-kn(a-x)} + a_n'\,\epsilon^{-knx}\,] \cos(n\,\theta)$$

Region II:

$$R = 2\,\pi\,\alpha_0\,(x-a)^2 + \sum_{1}^{\infty} [b_n\,\epsilon^{-kn(b-x)} + b_n'\,\epsilon^{-kn(x-a)} + 2\,K_n] \cos(n\,\theta)$$

Region III:

$$R = 2\,\pi\,\alpha_0\,(b-a)\,[2\,x - (b+a)\,] + \sum_{1}^{\infty} [c_n\,\epsilon^{-kn(c-x)} + c_n'\,\epsilon^{-kn(x-b)}\,] \cos(n\,\theta)$$

where the six constants of integration determined from the original Fourier's series for current distribution and the six boundary conditions, one of which is another Fourier's series, are as follows:

$$a_n = \frac{\beta_n\,\epsilon^{-kn(c-a)} + K_n\,[1 - \epsilon^{-kn(b-a)} + \epsilon^{-kn[2c(b+a)]} - \epsilon^{-kn2(c-a)}]}{1 - \epsilon^{-2knc}}$$

$$a_n' = a_n\,\epsilon^{-\kappa na}$$

$$b_n = \frac{\beta_n\,\epsilon^{-kn(c-b)} - K_n\,[1 + \epsilon^{-kn[2c-(a+b)]} - \epsilon^{-kn2(c-b)} - \epsilon^{-kn(2c+a-b)}]}{1 - \epsilon^{-2knc}} -$$

$$b_n' = \frac{\beta_n\,\epsilon^{-kn(c+a)} - K_n\,[1 - \epsilon^{-2kna} + \epsilon^{-kn(b+a)} - \epsilon^{-kn(2c+a-b)}\,]}{1 - \epsilon^{-2knc}}$$

$$c_n = \frac{\beta_n + K_n\,\epsilon^{-kn(c-b)}\,[1 - \epsilon^{-kn(b-a)} + \epsilon^{-kn(b+a)} - \epsilon^{-2knb}\,]}{1 - \epsilon^{-2knc}}$$

$$c_n' = \frac{\beta_n\,\epsilon^{-kn(c+b)} + K_n\,[1 - \epsilon^{-kn(b-a)} - \epsilon^{-2knb} + \epsilon^{-kn(b+a)}\,]}{1 - \epsilon^{-2knc}}$$

wherein the letters a, b, and c refer to Fig. 7, and

$$K_n = \frac{2\,\pi\,\alpha_n}{(k\,n)^2}$$

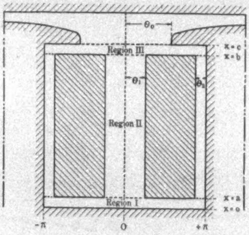

FIG. 7

By assuming that the function of Fig. 6 between $-\,\theta_0$ and $+\,\theta_0$ may be represented in the form,

$$Y = f + g\,\theta^2 + h\,\theta^4$$

the values of β_n become

$$\beta_n = \frac{\left[\left[\frac{1+h'\,\theta_0^4}{n} + \left(g' - \frac{12\,h'}{n^2}\right)\left(\frac{\theta_0^2}{n} - \frac{2}{n^3}\right)\right] \sin(n\,\theta_0) + \left[\frac{4\,\theta_0^3\,h'}{n^2} + \left(g'\frac{12\,h'}{n^2}\right)\frac{2\,\theta_0}{n^2}\right] \cos(n\,\theta_0) \right]}{\frac{\theta_0}{2} + \frac{g'\,\theta_0^3}{6} + \frac{h'\,\theta_0^5}{10}} \left[\frac{2\,\pi\,\alpha_0\,(b-a)}{(k\,n)}\right]$$

---

[12] The determination of the Fourier's series will be simplified if the curve (Fig. 6) between $-\,\theta_0$ and $+\,\theta_0$ is assumed of the form: $Y = f + g\,\theta^2 + h\,\theta^4$.

where:

$$g' = \frac{v}{f}$$

$$h' = \frac{h}{f}$$

Any inaccuracy in the determination of g' and h' will result in inaccuracy in the field plot only in the immediate vicinity of the pole tips.

In the above equation, θ_0 is the value of θ at the pole face edge, as indicated in Figs. 6 and 7, and k and α_n are the same as in the solution for the infinite slot.

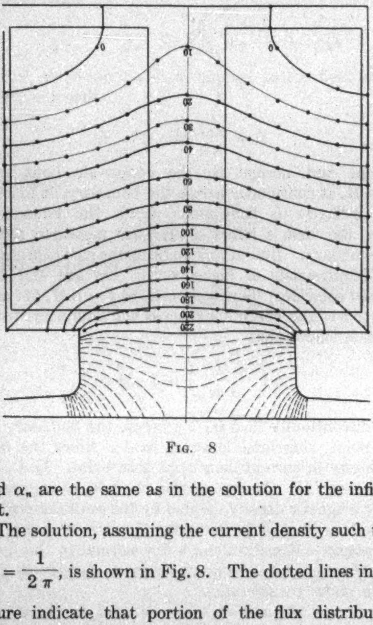

FIG. 8

The solution, assuming the current density such that $\alpha_0 = \dfrac{1}{2\pi}$, is shown in Fig. 8. The dotted lines in the figure indicate that portion of the flux distribution which was sketched in free-hand.

Although the problem has been worked out for the particular case of the flux distribution between two field poles, it is evident that the same method may be applied to any problem with rectangular boundaries. In case the magnetic circuit or the current distribution is not symmetrical about a central axis, it will be necessary to express R as a series of sines and cosines.

II. POINT CONDITIONS IN COPLANAR MAGNETIC FIELDS

The theory used in the preceding calculation can be made to answer various questions which arise in the free-hand sketching of magnetic fields, and also offers various criteria for judging the accuracy of the sketch.

The question is sometimes asked whether there is any refraction of the lines of force at a copper-air boundary. It will be proved that there is no such refraction, or discontinuity in slope, but that the radius of curvature of a line of force may change suddenly when the line crosses the boundary of a current-carrying region. This change in the curvature of the field at the boundary of current-carrying copper is readily detected on a correctly drawn field plot, especially where the lines emerge from the copper perpendicularly and the field intensity is low.

Theorem I. There is no refraction at a copper-air boundary. In studying the possibility of refraction at copper-air boundaries where the permeability is continuous, the classical problem of refraction at a boundary where the permeability is discontinuous will be considered, taking into account a flow of current on one side of the boundary.

Fig. 9 indicates the boundary of two regions, I and II, of permeabilities μ_1 and μ_2. The normal and tangential components of the magnetic field on either side, but immediately adjacent to the boundary, are H_{n1} and H_{n2}, and H_{t1} and H_{t2}, respectively.

In order that the total flux emanating from the space included by the dotted area shall vanish, it is necessary that the integral of the outward component of flux around the area must vanish. If the flux density is everywhere finite, in the limiting case in which the area is made vanishingly thin, the flux emanating from the ends of the area may be neglected. Therefore

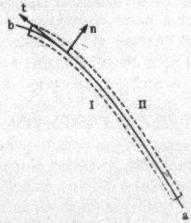

FIG. 9

$$\int_a^b \mu_1 H_{n1}\, dt = \int_a^b \mu_2 H_{n2}\, dt$$

and

$$\mu_1 H_{n1} = \mu_2 H_{n2} \tag{1}$$

From the application of the condition that the line integral of the field around the dotted area must equal 4π times the current included, and the assumption that the current density is finite, it follows in the limiting case, in which the area is made vanishingly thin, that the current included, and therefore the complete line integral around the boundary, vanishes, that is:

$$\int_a^b H_{t1}\, d\,t + \int_b^a H_{t2}\, d\,t = 0$$

Therefore,

$$H_{t1} = H_{t2} \tag{2}$$

From Equations (**1**) and (**2**), it is evident that the well-known rule that, at any point of a magnetic field, the normal component of flux density μH_n and the tangential component of field intensity H_t are continuous across any boundary, holds even when there is current flowing on one side of the boundary, provided that the current density is not infinite at the point under consideration.

Since the permeability of copper and air are approximately equal, it therefore follows that there can be no discontinuity in the normal and tangential components, or in the direction of the field at the boundary of the copper, even though it be carrying current, provided that the current density at the boundary is not infinite. Q. E. D,

Note: In the analysis of practical electrical problems involving coplanar fields, it is sometimes convenient to idealize to the extent of assuming the exciting currents to be concentrated in a ribbon of negligible thickness—that is, to assume that a finite amount of current flows into the plane of the field through a line drawn in that plane.

In this case, the current density is infinite for points on the line. It will be readily verified that, at a boundary along which a linear distribution of current density exists, Equation (**2**), Theorem I, becomes:

$$H_{t2} - H_{t1} = 4\,\pi\, i'$$

where i' is the linear current density at the point in question.

In particular, for a linear distribution of current density along an iron surface, we have:

$$H_{t1} = O$$

and thus:

$$H_{t2} = 4\,\pi\, i'$$

That is, the tangential field intensity along an iron surface faced with a distribution of linear current density depends only on the value of the linear density at the point in question. This relation, which is obvious as soon as appreciated, may be seen to necessitate the generally understood rule that equipotential lines shall enter such a surface at points separated so as to mark off sections including equal amounts of current. From the point of view that such equipotential lines when continued into the copper to form lines of no work shall enclose equal amounts of copper, this conclusion might appear to be open to question. A further rather obvious point that, however, is sometimes neglected, is that equipotential lines and lines of force must, in general, enter such linear distributions of current at a slant.

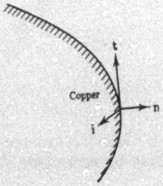

FIG. 10

Theorem II. There will be a change in the normal gradient of tangential field intensity at a boundary of current-carrying copper.

At a copper-air boundary the relation, curl $H = 4\,\pi\, i$, may be written in the form,

$$\frac{\partial H_t}{\partial n} - \frac{\partial H_n}{\partial t} = 4\,\pi\, i \tag{3}$$

where the vectors n, t, i form a right-handed system of unit vectors, as in Fig. 10, and t and n are respectively tangent and normal to the copper-air boundary. Since H_n is continuous across the boundary, it follows that $\partial H_n / \partial s$ is continuous across the boundary, where the term s refers to distance measured along the boundary. But, at any point on the curve, neglecting infinitesimals of higher order, the arc and the tangent coincide. Therefore, $\partial H_n / \partial s = \partial H_n / \partial t$ and $\partial H_n / \partial t$ is also continuous across the boundary, from which it follows that

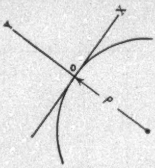

FIG. 11

$$\Delta \left(\frac{\partial H_t}{\partial n} \right) = 4\,\pi\, \Delta\,(i) \tag{4}$$

The discontinuity in $\partial H_t / \partial n$ across the boundary at any point, therefore, is equal to $4\,\pi$ times the discontinuity in current density at that point. Q. E. D.

Theorem III. At any point in the field, the quotient of the magnetic density divided by the radius of curvature of the lines of force, minus the rate of change of the magnetic density along a line normal to the lines of force, equals $4\,\pi$ times the current density in the region under consideration.

Equations (**3**) and (**4**) may be used to check the accuracy of a field plot sketch, but a somewhat more convenient relation, and one which involves the curvature of the field directly, is available.

Let the curve in Fig. 11 be a line of force within a field carrying current. It is desired to investigate its curvature at the point 0. Shift the axes of coordinates so that the origin coincides with the point in question, and rotate them until the x axis is tangent to the curve as shown.

Using these axes, the curve can be expressed by an equation,

$$y = f\,(x)$$

The relation between the curl of the field and the current density is

$$\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} = 4\pi i \tag{5}$$

And since X and Y are the components of the field intensity, the slope of the line of force can be expressed:

$$\frac{dy}{dx} = f'(x) = \frac{Y}{X} \tag{6}$$

or

$$Y = X\frac{dy}{dx}$$

Differentiate this[13] with respect to x;

$$\frac{\partial Y}{\partial x} = \frac{dY}{dx} = \frac{dX}{dx}\frac{dy}{dx} + X\frac{d^2y}{dx^2}$$

But, since the x axis is tangent to the field at the point in question,

$$X = H \text{ and } \frac{dy}{dx} = 0$$

Therefore,

$$\frac{\partial Y}{\partial x} = H\frac{d^2y}{dx^2}$$

Substituting in Equation (5),

$$H\frac{d^2y}{dx^2} - \frac{\partial H}{\partial y} = 4\pi i \tag{7}$$

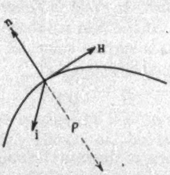

FIG. 12

but the radius of curvature of any curve $y = f(x)$ is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$$

and, since $dy/dx = 0$, at the point in question,

$$\frac{d^2y}{dx^2} = \frac{1}{\rho}$$

Making this substitution, Equation (7) becomes

$$\frac{H}{\rho} - \frac{\partial H}{\partial y} = 4\pi i$$

Or, since y is the coordinate normal to the curve, a more general form would be

$$\frac{H}{\rho} - \frac{\partial H}{\partial n} = 4\pi i \tag{8}$$

where the quantities H, n, i form a right-handed system of vectors, as in Fig. 12, and the radius of curvature ρ is to be measured to the position of the center of curvature on the vector n[14]. It should be noted that, in Equation (8), n is a unit vector normal to the field rather than a unit vector normal to the copper-air intersurface, as in Equations (3) and (4).

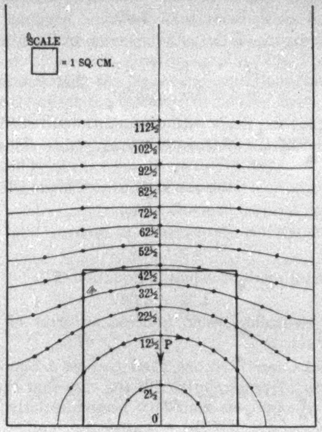

FIG. 13

Equation (8) provides a method of checking up with tolerable accuracy the shape of lines of force which have been sketched in free-hand.

Numerical Application of Theorem III. In order to test its probable accuracy, this method was used to check a field plot at a point inside a square conductor in the bottom of an infinitely deep slot (see Fig. 13). The lines of force had already been plotted from an exact formula for this case. Since the current is flowing downward perpendicular to the paper, the positive directions of H and of the normal are as indicated by the arrow heads.

Assume that the cross-section lines of the figure are spaced a centimeter apart. The tube (22½–12½) has a width of 1.11 cm. and contains 10 lines of force. Therefore, the flux density is 9.0 lines per sq. cm. The tube (12½–2½) has a width of 1.86 cm., and contains 10 lines. Therefore, its flux density is 5.38. The flux density at the point P is assumed to be the average of these two.

Therefore, $H = 7.19$ lines per sq. cm.

The distance between the center lines of tube

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[13] At the origin, the slope of the line $y = f(x)$ is zero and the partial derivative of Y with respect to x becomes equal to the total derivative with respect to x at that point.

[14] The value of ρ shown in Fig. 12 is negative because it is measured in the negative direction.

(22½-12½) and tube (12½-2½) is 1.5 cm. Since the difference in density is − 3.62, the rate of change of density with respect to distance measured along the normal is

$$\frac{\partial H}{\partial n} = \frac{-3.62}{1.5} = -2.41$$

The radius of curvature of the 12½ line at the point P determined with a pair of dividers is

$$\rho = +3.95$$

Substituting these in the left portion of Equation (**8**),

$$\frac{H}{\rho} - \frac{\partial H}{\partial n} = \frac{7.19}{3.95} + 2.41 = 4.23$$

The plot was made with an assumed current density in the coil of $1/\pi$ abamperes per sq. cm.[15]. Therefore,

$$i = \frac{1}{\pi}$$

and

$$4\,\pi\,i = 4$$

Substituting these values in Equation (**8**),

$$4.23 = 4.00$$

This inequality would indicate an error of about 5.75 per cent.

Special Case Where the Field Crosses a Copper-Air Boundary Perpendicularly. In the case that the field crosses a copper-air boundary perpendicularly, as in Fig. 14, the term $\partial H/\partial n$ is identical with the quantity $\partial H/\partial s$, where s refers to distance measured along the copper-air boundary, and $\partial H/\partial n$ is, therefore, continuous across the boundary. The discontinuity in H/ρ across the boundary, therefore, is equal to $4\,\pi$ times the discontinuity in current density across the boundary; that is,

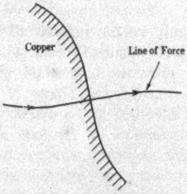

FIG. 14

$$\Delta\left(\frac{H}{\rho}\right) = 4\,\pi\,\Delta\,i$$

Or, since H is not discontinuous,

$$\Delta\left(\frac{1}{\rho}\right) = \frac{4\,\pi}{H}\,\Delta\,i$$

Hence, it follows that near the nucleus, where H is small, a discontinuity in current density at a given point may easily produce a large discontinuity in the curvature of the field at that point.

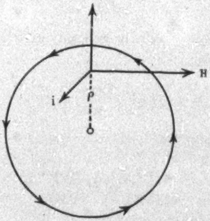

FIG. 15

Special Case Where the Field is Tangent to the Copper-Air Intersurface. If the field is tangent to the copper-air intersurface, this surface coincides with the x axis in Fig. 11. It should be noted that

$$\frac{H}{\rho} = \frac{\partial Y}{\partial x}$$

It is equal, therefore, to the rate of change of the normal component along the surface of the copper which is continuous across the boundary.

Therefore, in this case, any discontinuity in Equation (**8**) must be written

$$\Delta\,(\partial H/\partial n) = -\,4\,\pi\,\Delta\,i$$

Thus, in the case of an isolated circular wire carrying a longitudinal current, the field inside the wire is given by

$$H = -\,2\,\pi\,i\,r$$

and differentiating:

$$\frac{\partial H}{\partial n} = \frac{\partial H}{\partial r} = -\,2\,\pi\,i$$

If r_0 is the radius of the wire, the field outside the copper is given by

$$H = \frac{-\,2\,(\pi\,r_0^2)\,i}{r}$$

and differentiating,

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[15] This value of current density can be checked as follows: The area of the conductor is 36 sq. cm. Therefore, the total current is

$$I = \frac{36}{\pi}$$

$$m.\,m.\,f. = 4\,\pi\,I = 144$$

The width of the slot is

$$l = 12 \text{ cm.}$$

$$H = \frac{\text{m. m. f.}}{l} = \frac{144}{12} = 12 \text{ lines per sq. cm.}$$

Therefore, in the region where the lines of force are straight parallel lines, the width of the tubes containing 10 lines should be $10/12 = 0.835$ cm.

$$\frac{\partial H}{\partial n} = \frac{\partial H}{\partial r} = \frac{+2\,\pi\,r_0^2\,i}{r^2}$$

Therefore, at $r = r_0$,

$$\frac{\partial H}{\partial n} = -2\,\pi\,i \text{ inside the copper}$$

$$= +2\,\pi\,i \text{ outside the copper}$$

$$\Delta\frac{\partial H}{\partial n} = 4\,\pi\,i = -4\,\pi\,\Delta\,(i)$$

as would be expected since the field at the boundary is tangent to the copper.

In the case of an infinitely deep slot partly filled with copper (see Fig. 16), the curvature is zero, and we have

$$H = -4\,\pi\,i\,x$$

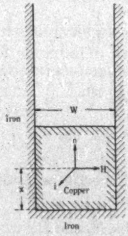


FIG. 16

and differentiating,

$$-\frac{\partial H}{\partial n} = -\frac{d\,H}{d\,x} = 4\,\pi\,i$$

while outside the copper the field is constant, so that $\partial\,H/\partial\,n$ vanishes. Hence, at this type of copper-air intersurface also,

$$\Delta\frac{\partial H}{\partial n} = -4\,\pi\,\Delta\,(i)$$

as would be expected.

III. APPLICATION OF THE METHOD TO POLAR COORDINATE SYSTEMS

One of the authors' associates, Mr. T. R. Rhea, has plotted the flux in and about a round bar[16] in a round slot, as shown in Fig. 17.

The tangential component of field intensity H_θ will be zero around the iron surface $\mu = \infty$ except at the air-gap. Here the value of H_θ will rise abruptly and follow some sort of curve to the other side of the air-gap. If H_θ, obtained from a free-hand sketch, is plotted against θ around the circumference (at $r = r_0$), a curve something of the shape shown in Fig. 18A would be obtained.

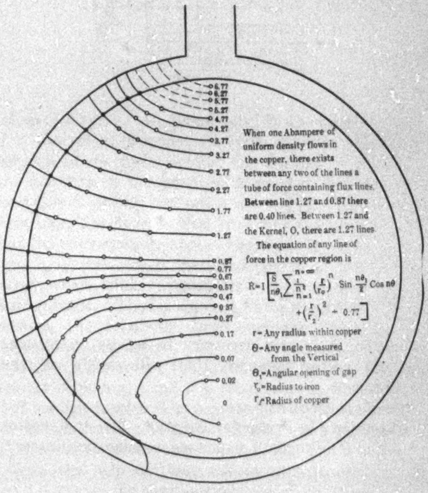
The sharp peaks on the curve are caused by the corners where the slot cuts into the circumference. To obtain a boundary condition that would exactly represent the boundary conditions of our problem, the Fourier's series for the above curve should be obtained, after this curve had been determined from an accurate and quantitative field sketch at the air-gap.

It is assumed, however, that these peaks may be neglected and the average value of H_θ across the air-gap used. This will greatly simplify the Fourier's series representing the boundary conditions, and will not greatly affect the calculations farther down in the regions which are to be investigated.

With this assumption, the boundary condition curve for H_θ will be of the form shown in Fig. 18B.

θ_1 is the angular opening of the slot.

The total m. m. f. across the slot is $4\,\pi\,I$, where I is the total current in abamperes. As the medium in the slot is air, the field intensity, assumed uniform across the gap, will be



When one Abampere of uniform density flows in the copper, there exists between any two of the lines a tube of force containing flux lines. Between line 1.27 and 0.87 there are 0.40 lines. Between 1.27 and the Kernel, O, there are 1.27 lines.

The equation of any line of force in the copper region is

$$R = I\left[\frac{\delta}{n\theta_1}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\left(\frac{r}{r_0}\right)^n \operatorname{Sin}\frac{n\theta_1}{2}\operatorname{Cos} n\theta + \left(\frac{r}{r_2}\right)^2 + 0.77\right]$$

r = Any radius within copper

θ = Any angle measured from the Vertical

θ_1 = Angular opening of gap

r_0 = Radius to iron

r_2 = Radius of copper

FIG. 17

$$H_{\theta_1} = \frac{4\,\pi\,I}{r_0\,\theta_1}$$

The Fourier's series expressing such a curve will be of the form,

$$H_\theta = C_0 + \sum_{n=1}^{n=\infty} C_n \cos n\,\theta \qquad (9)$$

---

[16] With alternating current flowing in the bar, the current distribution would not be uniform, but the problem of skin effect is outside the scope of this present paper and, for purposes of illustration, it is here assumed that the current density is uniform.

The constant C_0 is the average value of the curve, and has the value

$$C_0 = \frac{\dfrac{4\pi I}{r_0\,\theta_1} \times \theta_1}{2\pi} = \frac{2I}{r_0} \qquad \textbf{(10)}$$

The value of the general constant C_n may be found from the expression:

$$C_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} H_\theta \cos n\,\theta\, d\,\theta$$

$$= \frac{1}{\pi}\int_{-\theta_1/2}^{+\theta_1/2} \frac{4\pi I}{r_0\,\theta_1} \cos n\,\theta\, d\,\theta$$

$$= \frac{8I}{r_0\,n\,\theta_1}\sin n\,\frac{\theta_1}{2} \qquad \textbf{(11)}$$

Having obtained a Fourier's series for the boundary condition, it is now necessary to set up expressions for the vector potential that will satisfy all the conditions.

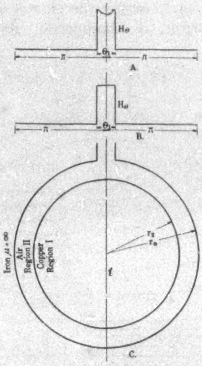


FIG. 18

The four conditions are:

Condition I: Poisson's equation for the vector potential R in Region I, expressed in polar coordinates[17], is

$$\frac{1}{r}\,\frac{\partial R}{\partial r} + \frac{\partial^2 R}{\partial r^2} + \frac{1}{r^2}\,\frac{\partial^2 R}{\partial \theta^2} = +4\pi i \qquad \textbf{(12)}$$

where i is the current density (assumed uniform).

Condition II: At the boundary between Regions I and II, both H_θ and H_r are continuous functions because there is no change in permeability.

Condition III: Laplace's equation for the vector potential in Region II, expressed in polar coordinates, is

$$\frac{1}{r}\,\frac{\partial R}{\partial r} + \frac{\partial^2 R}{\partial r^2} + \frac{1}{r^2}\,\frac{\partial^2 R}{\partial \theta^2} = 0 \qquad \textbf{(13)}$$

Condition IV: The outer boundary condition at $r = r_0$ has already been given in Equation (**9**).

A solution which will satisfy these four conditions is provided by the form:

$$R = R_1 + R_2$$

where R_1 is the vector potential of an isolated wire in space and R_2 is a function satisfying Equation (**13**) and so adjusted that Equation (**9**) is satisfied. That this solution satisfies the conditions imposed in both regions may easily be verified.

Choosing $R_1 = 0$, at $r = 0$, the function R_1 is, in Regions I and II, respectively,

$$R_1^{\mathrm{I}} = +I\left(\frac{r}{r_2}\right)^2 \qquad \textbf{(14)}$$

$$R_1^{\mathrm{II}} = +2I\left[\log_\epsilon\left(\frac{r}{r_2}\right) + \frac{1}{2}\right] \qquad \textbf{(15)}$$

A sufficient form for the function R_2 in both Regions I and II will be found to be:

$$R_2 = b_0 + \sum_{n=1}^{n=\infty}\left(\frac{r}{r_0}\right)^n (b_n \cos n\,\theta + a_n \sin n\,\theta) \qquad \textbf{(16)}$$

In Region I, the total vector potential is

$$R^{\mathrm{I}} = R_2 + R_1^{\mathrm{I}}$$

where the expression for R_2 and R_1^{I} would be taken from Equations (**14**) and (**16**).

In Region II, the total vector potential is

$$R^{\mathrm{II}} = R_2 + R_1^{\mathrm{II}}$$

From Equations (**15**) and (**16**),

$$R^{\mathrm{II}} = b_0 + \sum_{n=1}^{n=\infty}\left(\frac{r}{r_0}\right)^n (b_n \cos n\,\theta + a_n \sin n\,\theta) + 2I\left[\log_\epsilon\left(\frac{r}{r_2}\right) + \frac{1}{2}\right]$$

Maxwell has shown that

$$H_\theta = +\frac{\partial R}{\partial r} \qquad \textbf{(17)}$$

$$H_r = -\frac{1}{r}\,\frac{\partial R}{\partial \theta} \qquad \textbf{(18)}$$

Taking the partial derivative of (R^{II}) with respect to (r), we therefore obtain

$$H_\theta = +\frac{\partial R^{\mathrm{II}}}{\partial r}$$

$$= +\frac{1}{r_0}\left[\sum_{n=1}^{n=\infty} n\left(\frac{r}{r_0}\right)^{n-1} (b_n \cos n\,\theta + a_n \sin n\,\theta)\right] + \frac{2I}{r} \qquad \textbf{(19)}$$

This must satisfy the terminal conditions imposed by Equation (**9**), when $r = r_0$. Hence, Equation

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[17] See foot-note 25.

(**19**) must equal Equation (**9**) when $r = r_0$. Therefore,

$$+\frac{1}{r_0}\sum_{n=1}^{n=\infty}(b_n \cos n\,\theta + a_n \sin n\,\theta) + \frac{2\,I}{r_0} = C_0 + \sum_{n=1}^{n=\infty} C_n \cos n\,\theta$$

A comparison of these series, term by term, gives

$$a_n = 0$$

$$b_n = +\frac{r_0\,C_n}{n}$$

And from Equation (**10**), we already know that

$$C_0 = \frac{2\,I}{r_0}$$

All the constants except b_0 are now known, and the equations for vector potential in Regions I and II are

$$R^{\mathrm{I}} = b_0 + \sum_{n=1}^{n=\infty}\frac{8\,I}{n^2\,r_0\,\theta_1}\left(\frac{r}{r_0}\right)^n\left[\sin\frac{n\ \theta_1}{2}\right]\cos n\,\theta + I\left(\frac{r}{r_2}\right)^2 \qquad (\mathbf{20})$$

$$R^{\mathrm{II}} = b_0 + \sum_{n=1}^{n=\infty}\frac{8\,I}{n^2\,r_0\,\theta_1}\left(\frac{r}{r_0}\right)^n\left[\sin\frac{n\ \theta_1}{2}\right]\cos n\,\theta + 2\,I\left[\log_e\left(\frac{r}{r_2}\right)+\frac{1}{2}\right] \qquad (\mathbf{21})$$

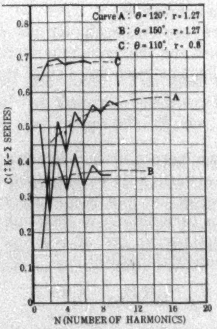

Fig. 19

where

r_0 = 1.43 cm.

I = 1 abampere = total current in conductor

θ_1 = 0:222 radians

r_2 = 1.27 cm.

It is not necessary to know b_0 in order to plot the field. Make the calculation on the assumption that $b_0 = 0$. This will make $R_1 = 0$ when $r = 0$. Later, b_0 will be given the proper value to cause R_1 to be zero at the kernel.

It is well known that curves of constant R are lines of force and that unit difference of R is one maxwell.[18] The field can be plotted, therefore, from the expressions for R^{I} and R^{II} in Equations (**20**) and (**21**).

The arithmetical evaluation of R is interesting, and a part of this work is therefore included. A number of terms of the series are calculated for particular values of θ and r. Unfortunately, for the larger values of r, the series does not converge rapidly. Plotting the sum of the series against the number of terms n was found to be a convenient method of determining the value to which the series converged. Several of these plots are given in Fig. 19. It is most important that the convergence value of the series be determined accurately, because it is subtracted from the value $(r/r_2)^2$, which, when r approaches r_2, is of the same order of magnitude. Thus it is a case where small differences between the relatively large numbers have to be determined accurately.

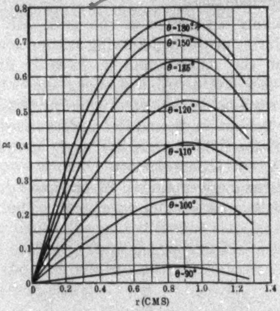

Fig. 20

After having found the value of R for a sufficient number of points, r and θ, a set of curves was drawn of R against r, with θ as a parameter. Also, a set of curves was drawn of R against θ, with r as a parameter. A sample of these curves is included, Fig. 20.

With the help of these curves and parameters of r and θ, curves for constant values of R could easily be plotted as lines of force in the magnetic field.

Neglect of the constant b_0 made the flux at the center of the figure zero and the flux at the kernel + 0.77. As it was desired to have the flux at the kernel zero, the sign of Equations (**20**) and (**21**) was changed and a value of + 0.77 assigned to the constant b_0, in order to make the entire flux plot positive.

The field plot in Fig. 17 is believed to be very accurate within the copper, but due to the slow convergence of the series it is not so accurate outside the copper in Region II.

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[18] See bibliography, 11, or Appendix C.

Appendix A

Brief Rules for Plotting Flux in Air

Although various authors have given rules for plotting flux in air, it seems well in the interest of completeness to briefly sum up the more important principles.

The method used by Messrs. Doherty and Shirley can best be described by reference to a particular problem, illustrated in Fig. 21, which represents a couple of alternator field poles.

In attacking a problem of this sort, Mr. Doherty and Mr. Shirley made the following assumptions:

1. That the magnetomotive force is due to an infinitely thin coil distributed along the pole[19]. The armature reaction is neglected while plotting the flux due to the field m. m. f.; and later, after another sketch is made of the field due to the m. m. f. of armature reaction, the two are superposed.

2. The magnetic potential of one pole due to the m. m. f. of its own winding was $+F$. The magnetic potential of the other pole due to its own m. m. f. was $-F$.

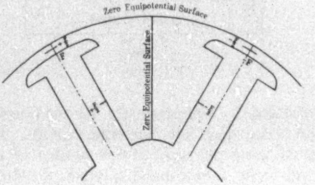

Fig. 21

3. Assuming infinite permeability, the armature and field pole faces are equipotential surfaces.

4. By symmetry, a zero equipotential surface is drawn midway between the two poles.

5. The armature surface is also assumed to be a zero equipotential surface.

6. At the surface of the left-hand pole, the potential is $+F$. As the air-gap is crossed, this drops to 0 at the surface of the armature. In crossing back across the gap from the armature surface to the right-hand pole, the magnetic potential drops to $-F$. Therefore, the total drop from one pole face to the other is $2F$.

7. In any place where the flux lines are known to be straight and parallel, as is approximately true in the air-gap, the potential drop is a linear function of distance. Thus, a line half-way across the gap represents one-half potential.

8. The equipotential planes $+F/2$ and $-F/2$ will curve from the center of the air-gap to the mid-points of their respective poles. The beginning and ends of the one-quarter potential planes can be similarly located. In sketching the traces of these planes through points known to be at the same potential, there must be no discontinuity in the gradual change in shape of adjacent potential lines—*i. e.*, when near the pole they follow its configuration, but those near the zero equipotential line must approach its rectangular shape.

9. After the equipotential lines have been sketched, the lines of force are drawn perpendicular to them. The whole surface is thus divided into chequers. If enough lines are drawn, these chequers become very small rectangles. The sketch is not correct until all the angles are right angles.

10. In space through which no current is flowing, all these rectangles must be similar—*i. e.*, the spacings of lines of force at two different places in the field must be proportional to the spacing of equipotential lines at the same respective points. It is recommended as a matter of convenience that the flux density be represented by such a number of lines as will make these rectangles curvilinear squares.

The above method of plotting magnetic fields is a cut-and-try method. The first few sketches will obviously be wrong, but there are sufficient conditions so that the final picture obtained after several readjustments will be approximately correct.

The actual flux density can be calculated at any point from the picture as follows:

a. The total m. m. f. per pole,

$$F = 4\,\pi\,n\,I \tag{22}$$

where n is the number of turns per pole and I is the field current in abamperes.

b. If there are m equipotential planes drawn between the pole and the zero equipotential surface, the potential gradient between any two of these surfaces is

$$\frac{d\,P}{d\,s} = \frac{4\,\pi\,n\,I}{m\,\delta} \tag{23}$$

where δ is the perpendicular distance measured in centimeters between adjacent equipotential surfaces at the point in question; *i. e.*, $m\,\delta$ = air-gap.

c. Since the permeability of air is unity, the density in lines of force per sq. cm. at every point is exactly equal to the potential gradient.

$$B = \frac{d\,P}{d\,s} = \frac{4\,\pi\,n\,I}{m\,\delta} \tag{24}$$

d. But, if the figure has been drawn so that the small chequers are square, the lines of force will have the same spacing, δ, as the equipotential surfaces. Therefore, the flux included in the tube between consecutive lines of force is

$$\Delta\,\phi = B\,\delta = \frac{4\,\pi\,n\,I}{m} \tag{25}$$

e. At the risk of criticism for repetition, attention is called to the fact that if δ is the spacing of lines of force, the area of a tube δ wide and 1 cm. thick is δ, and the density at any point is

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[19] This approximation is more accurate than neglecting the distribution of the coil entirely, as has been done by some others, and apparently does not very much affect the accuracy of flux distribution in the air-gap.

$$B = \frac{\Delta \phi}{\delta} = \frac{4 \pi n I}{m \delta} \qquad (26)$$

Equation (**26**) is identical, of course, with Equation (**24**), and merely represents another viewpoint.

In order to check the accuracy of the free-hand method, 23 different men determined the effective air-gap of a motor by making free-hand sketches of the flux. By Carter's equation, the correct value of the effective gap was 0.575. The results of the free-hand sketches were as follows:

Results Between	Number
0.570–0.605	8
0.605–0.640	6
0.640–0.675	4
Rejected for gross inaccuracy	5
Total	23

The above table indicates that the more accurate sketches gave a minimum reluctance. This agrees with Arnold's[20] statement: "The most nearly correct distribution of the flux will be shown by the sketch which makes the permeance of the flux tubes a maximum."

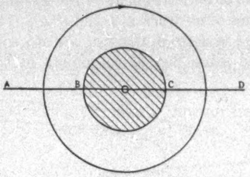

FIG. 22

It is probably obvious that the flux will arrange itself in such a manner as to follow the path of least resistance; but it is interesting to note that a comparison of the reluctances determined from several independent flux plots with the mathematically correct permeance confirms this theory so well.

Appendix B

METHOD OF PLOTTING TWO-DIMENSIONAL MAGNETIC FIELDS IN SPACE OCCUPIED BY CURRENT-CARRYING CONDUCTORS

Consider a section of an infinitely long cylindrical conductor, shown in Fig. 22. The work done in making a complete circuit about such a conductor with a unit pole is

$$W = 4 \pi I \qquad (28)$$

The difference of potential between two points is usually defined as the work done against the field in transporting a unit pole from one point to the other by any path whatever.

Let the potential at A be zero. Then the potential at D is

$$P_D = 2 \pi I \qquad (29)$$

But the potential continues to increase as the circle is completed. On returning to A after a complete circuit, the potential is no longer zero, but

$$P_A = 4 \pi I \qquad (30)$$

Every time the unit pole is taken around the conductor, the potential increases by this amount.

It might be asked, "How does the law of conservation of energy apply?" The answer is that in taking the unit pole around the conductor, a voltage has been induced. The product of this induced voltage multiplied by the current in the conductor represents electrical power which, integrated for the elapsed time, gives electrical energy. Thus the mechanical work done in carrying the unit pole about the conductor is converted into electrical energy.

Again referring to Fig. 22, it should be noticed that no work would be done in going from A to D by the path $A\,B\,O\,C\,D$, because this path at all points is perpendicular to the lines of force. Such lines, therefore, can be called lines of no work.

The following are the most important rules to be followed in plotting flux in regions containing current-carrying conductors:

1. The equipotential lines in the air space, when projected as lines of no work into the copper, must divide the copper into equal areas. It is thus seen that each particular part of the ampere conductors may be regarded as responsible for a particular part of the field.
2. The work done in carrying a unit pole along any line of force from one point to another is proportional to the current flowing in the area enclosed by the line of force and the lines of no work passing through the two points.
3. The spacing of lines of force must be inversely proportional to the copper enclosed and proportional to the length of the part of the tube enclosing the copper.[21]

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[20] See bibliography, 12.

[21] As stated in the Introduction, Mr. L. F. Richardson has given a rule for the plotting of fields in current-carrying copper to the effect that in a region occupied by current "the difference of successive chequer ratios in a direction perpendicular to the lines of force, divided by the mean chequer area is equal to a constant times the current density in the region." A proof of this relation is briefly outlined below.

Let n and t represent distance measured along lines of no work and along lines of force, respectively. Let small quantities of the order x be represented by the symbol $o\ (x)$. Then, referring to the adjoining figure (Fig. 23A), if n and t are small enough increments so that the field intensity and current density change

In Fig. 23:

$$H\,l = 4\,\pi\,C$$

If (k) is the number of lines in a tube, the width is

$$a = \frac{k}{H} = \frac{k\,l}{4\,\pi\,C}$$

The spacing between lines of force must be proportional, therefore, to the ratio l/C, where l is the length of the tube under consideration between the lines of no work and C is the total current included by this length of tube and the boundary lines of no work.

4. The line of half potential outside the copper, when extended as a line of no work into the copper, does not necessarily intersect every line of force at a point which would divide into two equal halves the work done in carrying a unit pole around the line. A similar statement can be made, of course, for the other potential lines.

BRIEF RULES FOR CONSTRUCTION OF FIELD FLUX PLOTS INSIDE THE COPPER

(BY L. P. SHILDNECK)

The following directions have proved very helpful in making flux plots of the fields of salient-pole machines.

Consider the case of a copper and iron distribution as given by a field pole (Fig. 24), with lines of no work $o\,a$, $o\,b$, $o\,c$, $o\,d$, $o\,e$, etc., drawn so as to divide the current into equal areas. Then the work done in transporting a unit pole from a to b is equal to the work done in transporting a unit pole around the path $a\,b\,o\,a$.

$$\begin{aligned} \text{Work } (a\,b) &= \text{work } (a\,b\,o\,a) \\ &= \text{work } (b\,c) = \text{work } (b\,c\,o\,b) \\ &= \text{work } (c\,d) = \text{work } (c\,d\,o\,c), \text{ etc.} \end{aligned}$$

$$\text{for work} = (\,a\,o)\ \text{work } (b\,o) = \text{work } (c\,o) = 0.$$

That is, the work done in going from one equipotential line such as a to another such as b, is equal to the m. m. f. between these two lines, or $4\,\pi$ times the current enclosed by the lines $a\,b$, $b\,o$, and $o\,a$. Consequently, work $a\,b$ equals work $a'\,b'$; but work $c\,d$ is greater than work $c'\,d'$, for more current is enclosed by $c\,d$. It is well to remember that the m. m. f. between any two points in the copper, such as c' and d', is proportional to the amount of current enclosed by the line of flux $c'\,d'$ and the lines of no work $c'\,o$ and $d'\,o$. Therefore, if a tube of flux is desired along $a'\,b'\,c'\,d'$, so that it may enclose the same flux at all points of the tube, the reluctance must vary in direct proportion to the m. m. f. And since the m. m. f. varies in direct proportion to the amount of current enclosed, the ratio l/a[22] for the curvilinear rectangles must vary in direct proportion to the amount of current enclosed. This relation makes it possible to extend the plot into the copper.

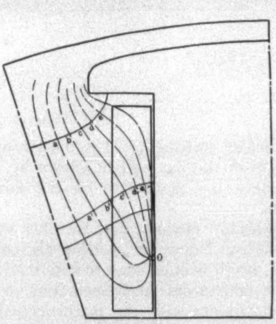

FIG. 24

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only slightly for points included in their variation, the relations exist:

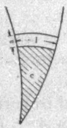

FIG. 23

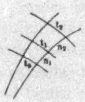

FIG. 23-A

$$H = o\,(0)$$

$$n,\,t = o\,(1)$$

$$H_1\,t_1 - H_0\,t_0 = 4\,\pi\,i\,n_1\,t_1 + o\,(3) \qquad \textbf{(30a)}$$

$$H_2\,t_2 - H_1\,t_1 = 4\,\pi\,i\,n_2\,t_2 + o\,(3) \qquad \textbf{(30b)}$$

where

H_n = field intensity along t_n,

i = current density at any point near n_1, n_2, t_1, t_2.

If n_1 and n_2 are chosen so that the flux between t_2 and t_1 is the same as the flux between t_1 and t_0, then

$$(H_1 + H_0)\,n_1 = (H_2 + H_1)\,n_2 + o\,(3) = 2\phi + o\,(3)$$

where

ϕ = the flux between t_1 and t_0, and between t_2 and t_1.
$= o\,(1)$.

Also,

$$n_2 - n_1 = o\,(2)$$

$$n_2\,t_2 = n_1\,t_1 + o\,(3) = n\,t + o\,(3)$$

Adding (**30a**) and (**30b**), there results:

$$(H_2 + H_1)\,t_2 - H_1\,(t_2 - t_1) - (H_1 + H_0)\,t_1 + H_0\,(t_1 - t_0) = 8\,\pi\,i\,n\,t + o\,(3) \qquad \textbf{(30c)}$$

But

$$H_0\,(t_1 - t_0) - H_1\,(t_2 - t_1) = (H_0 - H_1)\,(t_1 - t_0) - H_1\,(t_2 + t_0 - 2t_1) = o\,(3)$$

Therefore, **30c** becomes

$$(H_2 + H_1)\,t_2 - (H_1 + H_0)\,t_1 = 8\,\pi\,i\,n\,t + o\,(3)$$

Substituting (4) in the above, there results

$$\phi\left(\frac{t_2}{n_2} - \frac{t_1}{n_1}\right) = 4\,\pi\,i\,n\,t + o\,(3)$$

$$\frac{t_2}{n_2} - \frac{t_1}{n_1} = \frac{4\,\pi\,i\,n\,t}{\phi} + o\,(2)$$

Thus, omitting second order terms, we obtain a relation involving quantities of the first order on either side:

$$\frac{\dfrac{t_2}{n_2} - \dfrac{t_1}{n_1}}{n\,t} \quad \frac{4\,\pi}{\phi}\,i$$

which is the rule stated by Richardson.

[22] The reluctance is proportional to the length l of the path and inversely proportional to the area a of the path. Since unit thickness is chosen for the plot, the width of a tube represents its area.

The subsequent rules will aid materially in shortening the time necessary to obtain an accurate field plot. The reasons for following the directions in the order given will be obvious.

1. Draw bisectors of the angles at the points *a*, *b*, *c*, *d*, *e*, *f*, as shown in Fig. 25. These are the directions of the lines of flux at these points. Continue them wherever it is possible to do so with any degree of accuracy, as at *a*, *b*, *c*, *d*, entering the opposite side at right angles.

2. Divide the current into eight equal regions, as shown in Fig. 25. The reason for the peculiar division is that later the lines of no work will roughly coincide with the construction lines, thereby providing an easy method of sketching the lines of no work so that they will divide the current into equal parts. If it is found later that some of the construction lines are not placed to best advantage, of course others may be drawn.

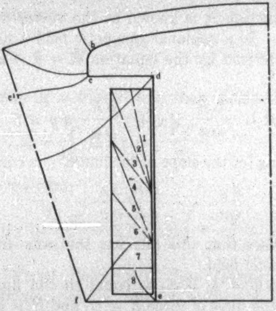

FIG. 25

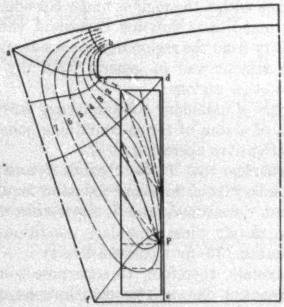

FIG. 26

3. Draw a trial set of seven lines of no work (see Fig. 26), crossing the lines of flux *a*, *b*, *c*, *d* at right angles, hugging the sharp projections closely, as at *b* and *c*, and keeping away from the inverted corners as at *a* and *d*. This trial set must divide the current region into eight equal portions. Arbitrarily choose some point *P* down in the corner closest to the iron, for the kernel. If no iron were present, it would lie in the center of the copper; if iron were touching the copper along one surface, the kernel would be at the copper-iron boundary; if iron were touching the copper along two surfaces, the kernel would be on both copper-iron boundaries, at the corner. Obviously, the kernel, in all practical cases, would lie somewhere between the center of the copper and the lower corner near the iron. Line No. 4 must divide the current into two equal regions. The advantage of the straight construction line between Regions 4 and 5, dividing the current into two equal portions, is evident. The other construction lines are also located so as to be most useful in enabling an accurate division of the current to be made by judging only small differences in area with the eye.

4. Starting from some line of flux $c\,c'$ (Fig. 27), draw lines of flux, making curvilinear squares along the line $a\,c'\,f$ in the region where there is no current. Then extend the lines of force into the current region orthogonal to the lines of no work.

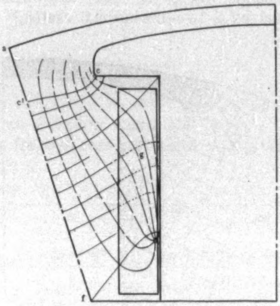

FIG. 27

5. The correctness of the plot may now be tested as follows:

a. Lines of flux must cross lines of no work at right angles.

b. Lines of flux must enter the iron at right angles (assuming infinite permeability).

c. All rectangles outside the current must be curvilinear squares (ratio l/a = unity).

d. Within the current region, the rectangles must have a ratio l/a less than unity and equal to the ratio between the current enclosed and the one-eighth portion of the total current. Thus at a point *g* (Fig. 27), if the flux line, and the two lines of no work intersected by it, enclose one-half of the eighth portion of current, then the length of the rectangle at this point should be one-half of the width.

6. If the ratios l/a are *not* proportional to the current enclosed, then the plot must be redrawn, either

changing the position of the kernel or else shifting the position of the lines of no work, and making the corresponding necessary changes in the lines of flux.

It is well to use tracing paper, for then each previous trial may be used for a guide.

Fig. 28 is the final plot. Any required accuracy may be obtained by continuing the process indefinitely. By this method, however, a large portion of the cut-and-try is eliminated because as many of the required conditions as possible are fulfilled in the first part of the construction, and after three or four trials a surprisingly accurate sketch can be obtained.

Appendix C

VECTOR POTENTIAL IN COPLANAR MAGNETIC FIELDS

Neither the problems nor the general methods in this appendix are new but it is believed that the application of the little used conception of vector potential to a group of familiar problems will be of interest.

In problems where the flux distribution can be represented by a two-dimensional sketch,[23] the vector potentials are all perpendicular to the plane of the paper and consequently can be added and subtracted exactly like scalar quantities. This makes the construction of field plots by lines of equi-vector potential very much easier than by the more usual method which employs the two components of the vector field intensity.

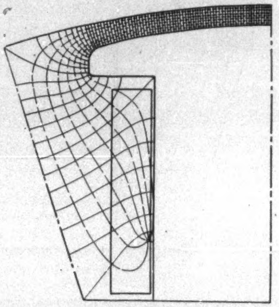

FIG. 28

Let X and Y be the x and y components of the field. Then, integration around any small area in the $x\,y$ plane will show that[24]

$$\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} = 4\,\pi\,i \tag{27}$$

where i is positive when directed upward out of the plane of the paper.

Let R be a function[25] such that

$$X = -\,\frac{\partial R}{\partial y} \tag{28}$$

$$Y = +\,\frac{\partial R}{\partial x} \tag{29}$$

Then

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} = +\,4\,\pi\,i \tag{30}$$

The function R is known as the vector potential of the field. In a coplanar magnetic field a line of force is characterized by the equation $R = \theta =$ a constant, because

$$d\,R = \frac{\partial R}{\partial x}\,d\,x + \frac{\partial R}{\partial y}\,d\,y = 0 \tag{31}$$

and solving for the slope of the line $R =$ a constant:

$$\frac{d\,y}{d\,x} = -\,\frac{\partial R/\partial x}{\partial R/\partial y} = \frac{Y}{X} \tag{32}$$

which shows that this line has the same direction as the magnetic field.

Also, $\Delta\,\phi = \phi_1 = \phi_2$ is equal to the flux included between the lines of force $R = \phi_1$ and $R = \phi_2$, for

$$\Delta\,\phi = \int_1^2 d\,R = \int_1^2 Y\,d\,x - X\,d\,y \tag{33}$$

may be seen to give the flux included between the points 1 and 2 whatever the path of integration employed.

In any problem in which the only sources of m. m. f., are currents within the regions under consideration, the boundary conditions imposed consist in variations of permeability from the region under consideration to the adjoining regions and in general from the adjoining regions to other regions.

This type of boundary condition may be recognized to consist of a sum of terms containing constants and partial derivatives operating on R.

It is then clear that if two solutions R_1 and R_2 satisfy these boundary conditions and the point Equation (**30**) for current densities i_1 and i_2 separately, their sum must also satisfy these boundary conditions and the point Equation (**30**) for a current density $i_1 + i_2$.

It is possible, therefore, to superpose solutions for vector potential due to two distributions of current density and the result may be extended to any number

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[23] Such fields are said to be coplanar in the plane of the paper.

[24] See bibliography, 11.

[25] In the above, it has been found convenient, in order to make the sign of R positive, to define it by the relations:

$$Y = \frac{\partial R}{\partial x}, \quad X = -\,\frac{\partial R}{\partial y}$$

instead of

$$Y = -\,\frac{\partial R}{\partial x}, \quad X = \frac{\partial R}{\partial y}$$

as in Rogowski's work.

of superpositions; and thus from a knowledge of the vector potential of an element we may by integration determine the vector potential of a complicated distribution of current density.

I. The Vector Potential of an Isolated Straight Wire of Circular Section. Let the radius of the wire be a and let the total current through the wire be I.

Then inside the wire the field intensity[26] is

$$H = \frac{2\,I\,r}{a^2} \tag{34}$$

and outside the wire:

$$H = \frac{2\,I}{r} \tag{35}$$

If we choose $R = 0$ at $r = 0$, then inside the wire:

$$R = \int_{r=0}^{r=r} H\,d\,r = \frac{I\,r^2}{a^2} \tag{36}$$

and outside the wire:

$$R = \int_{r=a}^{r=r} H\,d\,r + \frac{I\,a^2}{a^2} = 2\,I\left[\log_e\left(\frac{r}{a}\right) + \frac{1}{2}\right] \tag{37}$$

II. The Vector Potential of Two Isolated Straight Wires of Circular Section. Let r_1 and r_2 be the radii vector from wires 1 and 2, respectively, and let a_1 and a_2 be the respective radii of the conductors. Then inside conductor 1:

$$R = 2\,I_2\left[\log_e\left(\frac{r_2}{a_1}\right) + \frac{1}{2}\right] + I_1\,\frac{r_1^2}{a_1^2} \tag{38}$$

Inside conductor 2:

$$R = 2\,I_1\left[\log_e\left(\frac{r_1}{a_1}\right) + \frac{1}{2}\right] + I_2\,\frac{r_2^2}{a_2^2} \tag{39}$$

Outside both conductors:

$$R = 2\,I_1\left[\log_e\left(\frac{r_1}{a_1}\right) + \frac{1}{2}\right] + 2\,I_2\left[\log_e\left(\frac{r_2}{a_2}\right) + \frac{1}{2}\right] \tag{40}$$

a. In the *special case* where the currents are in opposite directions, $I_2 = -I_1 = I$, and $a_1 = a_2$, the vector potential outside the conductors is

$$R = 2\,I\,\log_e\frac{r_1}{r_2} \tag{41}$$

and the equation of a line of force, R = a constant, is

$$\frac{r_1^2}{r_2^2} = \epsilon^{R/I} \tag{42}$$

The well-known fact follows that the lines of force outside the conductors are circles.

In particular, if the centers of the wires are located at $x = -b$, and $x = b$, respectively, Equation (42) becomes

$$[x + b\coth(R/2\,I)]^2 + y^2 = \frac{b^2}{\sinh^2(R/2\,I)} \tag{43}$$

b. In the *special case* where the currents are in the same direction, $I_1 = I_2 = I$, and $a_1 = a_2 = a$,

$$R = 2\,I\left[\log_e\left(\frac{r_1\,r_2}{a^2}\right) + 1\right] \tag{44}$$

The equation of the line of force in this case is of the fourth degree.

III. The Vector Potential of a Straight Wire Near the Corner of two Infinitely Permeable Planes. The wire and images for this case are shown in Fig. 29. Outside the wire

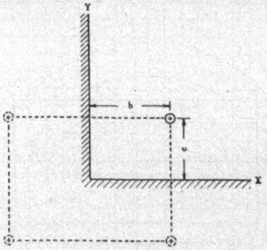

FIG. 29

$$\frac{R}{I} = \log_e\frac{(x-b)^2+(y-c)^2}{a^2} + \log_e\frac{(x-b)^2+(y+c)^2}{a^2} + \log_e\frac{(x+b)^2+(y-c)^2}{a^2} + \log_e\frac{(x+b)^2+(y+c)^2}{a^2} + 4 \tag{45}$$

Equations of the type of (45) may be solved by plotting since only the real roots are desired. In the case under consideration, R/I should be plotted as a function of x with y as a parameter. Points on a line of force are then determined by the intersections of the curves along lines of R/I = a constant.

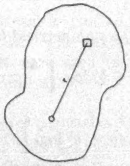

FIG. 30

IV. The Vector Potential of an Isolated Wire of Any Cross-Section in Air. Referring to Fig. 30, the vector potential of an element of the wire is, in general,

$$d\,R = i\,(\log_e r^2)\,d\,a + \text{a constant} \tag{46}$$

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[26] $H = \sqrt{X^2 + Y^2}$.

Vol. 46
1927

where r is the distance from the point at which R is determined to the element under consideration, i is the current density, and $d\,a$ the area of the element. The value of R for the whole wire is, then,

$$R = \int_S i\,(\log_e r^2)\,d\,a + \text{a constant} \qquad (\mathbf{47})$$

the integral being taken over the whole surface of the wire. If the current density over the cross-section is constant, we have

$$\frac{R}{i} = \int_S (\log_e r^2)\,d\,\mathrm{a} + \text{a constant}^{27} \qquad (\mathbf{48})$$

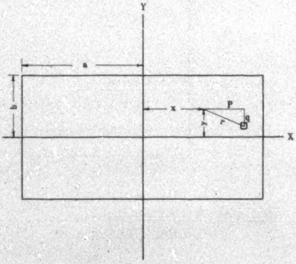

FIG. 31

a. For the *special case* of an isolated wire of rectangular section we have, referring to Fig. 31,

$$\frac{R}{i} = \int_{q=y-b}^{q=y+b} \int_{p=x-a}^{p=x+a} \log_e (p^2 + q^2)\,d\,p\,d\,q \qquad (\mathbf{49})$$

Integrating and adding the constant terms,

$$-4\,a\,b \log_e (a^2+b^2) - 6\,a\,b - 4\,a^2 \tan^{-1}\frac{b}{a} - 4\,b^2 \tan^{-1}\frac{a}{b}$$

In order to make $\frac{R}{i} = 0$ at the origin, Equation (**49**) can be reduced to the symmetrical form:

$$\frac{R}{i} = (x+a)\,(y+b) \log_e \left[\frac{(x+a)^2 + (y+b)^2}{a^2+b^2}\right]$$

$$- (x-a)\,(y+b) \log_e \left[\frac{(x-a)^2 + (y+b)^2}{a^2+b^2}\right]$$

$$- (x+a)\,(y-b) \log_e \left[\frac{(x+a)^2 + (y-b)^2}{a^2+b^2}\right]$$

$$+ (x-a)\,(y-b) \log_e \left[\frac{(x-a)^2 + (y-b)^2}{a^2+b^2}\right]$$

$$+ (x+a)^2 \left[\tan^{-1}\frac{y+b}{x+a} - \tan^{-1}\frac{y-b}{x+a}\right]$$

$$- (x-a)^2 \left[\tan^{-1}\frac{y+b}{x-a} - \tan^{-1}\frac{y-b}{x-a}\right]$$

$$+ (y+b)^2 \left[\tan^{-1}\frac{x+a}{y+b} - \tan^{-1}\frac{x-a}{y+b}\right]$$

$$- (y-b)^2 \left[\tan^{-1}\frac{x+a}{y-b} - \tan^{-1}\frac{x-a}{y-b}\right]$$

$$- 4\,a^2 \tan^{-1}\frac{b}{a} - 4\,b^2 \tan^{-1}\frac{a}{b} \qquad (\mathbf{50})^{28}$$

The field in and around a rectangular conductor of dimensions two units by four units, carrying a current of 80 amperes, *i. e.*, $a = 2$, $b = 1$, $i = 1$, has been plotted from Equation (**50**) by one of our associates, Mr. R. S. Arthur, and is shown in Figs. 32 and 33. Figs. 34 and 35, showing R as a function of x and y, were used in plotting Figs. 32 and 33.

b. *Special Case.* Infinitely thin ribbon.

For sufficiently small values of b, that is, if the rectangle degenerates into a ribbon, putting $2\,b\,i = i'$ = current per unit length of the ribbon and adding terms such that $\frac{R}{i'} = 0$ at $x = y = 0$, Equation (**50**) can be written

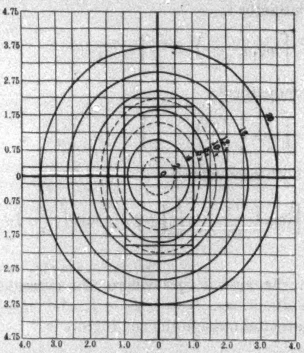

FIG. 32

$$\frac{R}{i'} = (a+x) \log_e \frac{(a+x)^2 + y^2}{a^2}$$

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[27] This result may be expressed in the form $\frac{R}{i} = \log D^2$ + a constant, where D equals the geometric mean distance of the point at which R is to be computed from the area of the section of the conductor.

[28] The variable terms in (50) and (51) may be checked against those given by Maxwell, "Electricity and Magnetism," paragraph 692, for the mean geometric distance of a point from a rectangle and a straight line, respectively.

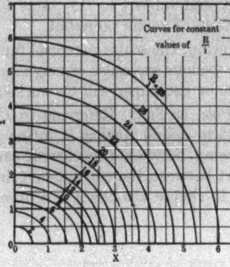

FIG. 33

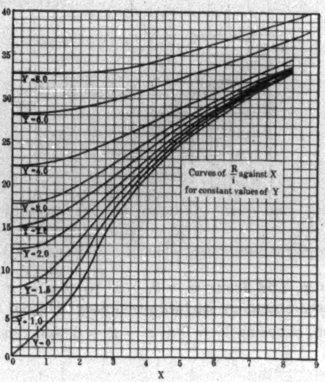

FIG. 34

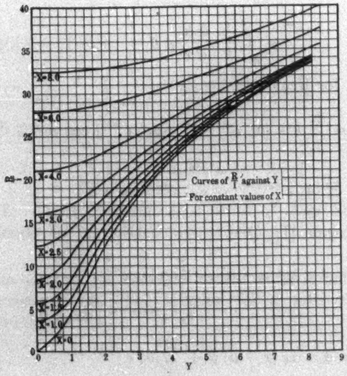

FIG. 35

$$+ (a - x) \log_e \frac{(a - x)^2 + y^2}{a^2}$$

$$+ 2\,y \left[\tan^{-1} \frac{a + x}{y} + \tan^{-1} \frac{a - x}{y} \right] \qquad \textbf{(51)}^{23}$$

Equation (**51**) corresponds precisely to the case of a vanishingly thin isolated conductor, or to a narrow strip of conductor on an infinite iron surface.

In the case of a ribbon conductor two units long, carrying a current of 20 amperes, the expression for vector potential is

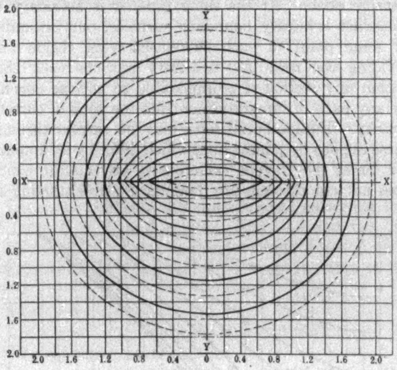

FIG. 36

$$R = (1 + x) \log_e [(1 + x)^2 + y^2] + (1 - x) \log_e [(1 - x)^2 + y^2]$$

$$+ 2\,y \left[\tan^{-1} \left(\frac{1 + x}{y} \right) + \tan^{-1} \left(\frac{1 - x}{y} \right) \right] \qquad \textbf{(52)}$$

The field for this case as plotted by one of the authors is shown in Fig. 36. The curves of R as a function of x and y which were used in plotting Fig. 36 are shown in Figs. 37 and 38.

Appendix D

THE RELATION OF THE SCALAR POTENTIAL FUNCTION TO THE VECTOR POTENTIAL FUNCTION IN COPLANAR MAGNETIC FIELDS

For points outside of regions in which current density exists, it is legitimate to construct a potential function V having the property:

$$H_n = \frac{\partial}{\partial n} V \qquad \textbf{(53)}$$

where H_n is the field intensity in a direction n and where n is distance measured in any direction from the point at which H_n exists.

Then, in particular,

$$dV = \frac{\partial V}{\partial x}\,dx + \frac{\partial V}{\partial y}\,dy$$

$$= H\ dx + H\ dy$$

$$V_2 - V_1 = \int_1^2 H_x\,dx + H_y\,dy \qquad \textbf{(54)}$$

where x and y form any convenient set of coordinate axes. The path of integration must be arranged so that it does not traverse regions in which current density exists. At the same time, it is necessary to construct arbitrary boundary surfaces, one beginning at some point of each current-carrying region and dividing the field into arbitrary sections such that within any section it is impossible to enclose current by any path of integration outside the zone of current density. Equipotential lines may be carried through these regions by joining lines which differ by an amount 4π times the current enclosed by any circuit which does not cut through any boundaries, and starting at one side of the boundary and ending at the other side at the same point.

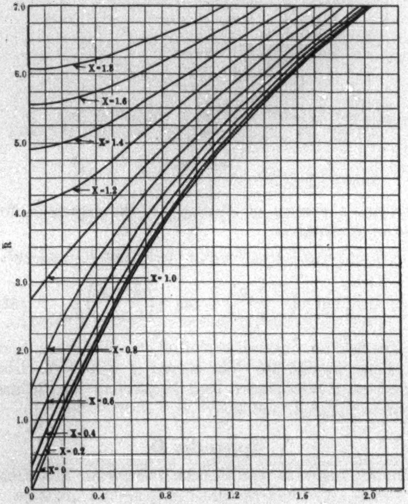

Fig. 37

If the vector potential of the field is known, we may use Equation (**54**) in the form:

$$V_2 - V_1 = \int_1^2 \frac{\partial R}{\partial x}\,dy - \frac{\partial R}{\partial y}\,dx \qquad \textbf{(55)}$$

Solutions of scalar potential due to particular distributions of current density will be superposable in the same way and subject to the same limitations as apply to solutions of vector potential.

In the following, the scalar potential functions which obtain in the various examples of magnetic fields given in other parts of the paper will be worked out briefly.

I. Field Pole, or Slot. Referring to Fig. 8, it will be convenient to divide Region II into a and b parts, the a part being between the current-carrying zones and the b part between the current-carrying zones and the iron, and to choose the line $x = a$ from $\theta = \pi - \theta_2$ to $\theta = \pi$, and from $\pi = \theta_2 - \pi$ to $\theta = -\pi$ as the arbitrary boundary surface previously referred to.

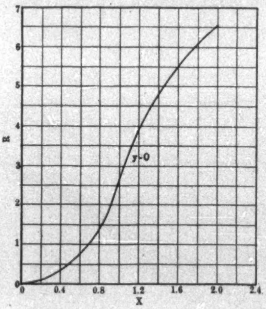

Fig. 38

Then, choosing $V = 0$ at $x = y = 0$, and remembering $y = \dfrac{\theta}{k}$,

$$V = \int_0^{\theta,x} k\,\frac{\partial R}{\partial \theta}\,dx - \frac{1}{k}\,\frac{\partial R}{\partial x}\,d\theta$$

$$= \int_0^{\theta} \frac{1}{k}\,\frac{\partial R}{\partial x}\,d\theta \qquad \textbf{(56)}$$

Thus, for Region I:

$$\frac{\partial R}{\partial x} = \sum_1^{\infty} k\,n\,[a_n\,\epsilon^{-kn(a-x)} - a_n'\,\epsilon^{-knx}]\cos n\,\theta$$

$$V^{I} = \sum_1^{\infty} [a_n\,\epsilon^{-kn(a-x)} - a_n'\,\epsilon^{-knx}]\sin n\,\theta \qquad \textbf{(57)}$$

For Region II:

$$\frac{\partial R}{\partial x} = 4\,\pi\,\alpha_0\,(x-a) + \sum_1^{\infty} n\,[b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\cos n\,\theta$$

$$V^{II^a} = 4\,\pi\,\alpha_0\,(x-a)\,y + \sum_1^{\infty} [b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\sin n\,\theta \qquad \textbf{(58)}$$

In order to obtain the scalar potential for Region IIb, it is convenient to subtract from the known potential at the iron surface, the potential difference from the point in question referred to that surface. Thus:

$$V_{IIb} = 4\,\pi\,\alpha_0\,(b-a)\,\frac{l}{2} - \int_0^x \frac{1}{k}\,\frac{\partial R}{\partial x}\,d\,\theta$$

$$= 4\,\pi\,\alpha_0 \left[(b-a)\,\frac{l}{2} - (x-a)\left(\frac{l}{2} - y\right)\right]$$

$$+ \sum_1^\infty [b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-c)}]\sin n\,\theta$$

$$= V_{IIa} + 2\,\pi\,\alpha_0\,l\,(b-x) \qquad (59)$$

For Region III:

$$\frac{\partial R}{\partial x} = 4\,\pi\,\alpha_0\,(b-a) + \sum_1^\infty [c_n\,\epsilon^{-kn(c-x)}$$

$$- c_n'\,\epsilon^{-kn(x-b)}]\cos n\,\theta$$

which gives:

$$V_{III} = 4\,\pi\,\alpha_0\,(b-a)\,y + \sum_1^\infty [c_n\,\epsilon^{-kn(c-x)}$$

$$- c_n'\,\epsilon^{-kn(x-b)}]\sin n\,\theta \qquad (60)$$

The solution for an infinitely deep slot may be obtained by inserting $c = \infty$.

II. Circular Slot. In polar coordinates, Equations (**54**) and (**55**) become

$$V_2 - V_1 = \int_1^2 H_r\,d\,r + H_\theta\,r\,d\,\theta \qquad (61)$$

$$= \int_1^2 -\frac{1}{r}\,\frac{\partial R}{\partial \theta}\,d\,r + \frac{\partial R}{\partial r}\,r\,d\,\theta \qquad (62)$$

Taking $V = 0$ at $\theta = 0$, there results:

$$V = \int_0^\theta r\,\frac{\partial R}{\partial r}\,d\,\theta \qquad (63)$$

For Region II:

$$\frac{\partial R}{\partial r} = \sum_1^\infty \frac{n}{r_0}\left(\frac{r}{r_0}\right)^{n-1}[b_n\cos n\,\theta] + \frac{2\,I}{r}$$

which gives:

$$V_{II} = \sum_1^\infty \left(\frac{r}{r_0}\right)^n b_n\sin n\,\theta + 2\,I\,\theta \qquad (64)$$

III. Scalar Potential of a Wire of Any Section. Evidently, for an isolated circular wire,

$$V = 2\,I\,\theta$$

The solution for several circular wires is obtained by superposition. The solution for a wire of any section is

$$V = \int_S 2\,i\,\theta\,d\,a \qquad (65)$$

the integral to be extended over the whole section in which i exists.

It may be found more convenient, however, to obtain V indirectly through (**47**) and (**55**).

Appendix E

CALCULATION OF INDUCTANCE FROM A KNOWLEDGE OF VECTOR POTENTIAL

In general, in order that it shall be permissible to employ the conception of inductance to a conductor of large section, it is necessary that the conductor consist of a bundle of smaller conductors, all of these conductors supposedly connected in series, or in such a way that the same current flows in each[29]. If there is a sufficient number of small conductors, it is permissible as an approximation to calculate the inductance of the system as the average inductance of a continuous distribution of small current filaments, the density of the filaments being in proportion to the density of small conductors. It will further be assumed that the current distribution may be regarded as continuous within the section of the large conductor.[30]

Let ϕ_0 be the total flux outside some particular line of force $R = R_0$ and between that line and the line with respect to which induced voltage is to be computed, where R is the vector potential function of the field.

Then

$$\phi_0 + R_0 - R$$

will be the flux outside any line R.

Let

n = the density of small conductors.

The average flux linkages per conductor are

$$\int_S \frac{(\phi_0 + R_0 - R)\,n\,d\,a}{N} = (\phi_0 + R_0) - \int_S \frac{R\,n\,d\,a}{N}$$

where

$$N = \int_S n\,d\,a$$

= the total number of small conductors in the section.

If I is the total current through the section, the component of average inductance due to flux up to the point that ϕ_0 is computed, is

$$L = \left(\frac{\phi_0 + R_0}{I}\right) - \int_S \frac{R\,n\,d\,a}{N\,I}$$

But

$$i = \frac{n}{N}\,I$$

$$I = \int_S i\,d\,a$$

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[29] From the restricted point of view that stored magnetic energy equals ½ × inductance × current squared, it is permissible to apply the conception of inductance to conductors which are not subdivided.

[30] If desired, this approximation may be corrected. A correction of this type is given by Maxwell, "Electricity and Magnetism," par. 693.

or

$$L = \left(\frac{\phi_0 + R_0}{I}\right) - \int_S \frac{R\, i\, d\, a}{I^2} \qquad \textbf{(66)}$$

Equation (**66**) may be put in the form,

$$L = 4\,\pi\,(P_0 - P) \qquad \textbf{(67)}$$

where P_0 and P are permeance factors;

$$P_0 = \frac{\phi_0 + R_0}{4\,\pi\, I} \qquad \textbf{(68)}$$

$$P = \frac{\int_S R\, i\, d\, a}{4\,\pi\,[\int_S i\, d\, a]^2} \qquad \textbf{(69)}$$

In (**68**) and (**69**), the quantity P_0 is the permeance factor that would obtain if the flux $\phi_0 + R_0$ linked all of the conductors, while the factor P takes into account the effect of partial linkages.

It is interesting that in the foregoing equation, the vector potential R need not be computed so that the minimum value of $R = 0$. Thus it is not necessary to calculate the value of R at the kernel. If the kernel is known to exist on some line such that one coordinate x or y, or r or θ, for example, is fixed, the calculation of the value of R at the kernel is comparatively easy. If, as in the field pole problem, however, the value of neither coordinate is known, then on account of the unsatisfactory convergence which usually obtains at the kernel, the calculation of the position and value of R at the kernel is a task of considerable difficulty. It is fortunate, therefore, that this calculation is not required.

Inductance of Two Parallel Cylindrical Conductors. As a simple example of formula (**66**), consider the case of two straight conductors of circular section carrying currents I and $-I$, respectively. From Formula (**38**), the vector potential at a point inside one wire is

$$R = I\,\frac{r_1^2}{a^2} - 2\,I\left[\log\left(\frac{r_2}{a}\right) + 1/2\right]$$

where

a = the radius of the section.

Let

$r_1 = r$

l = distance between centers of the conductors.

The component of inductance due to half the flux will be calculated. Thus ϕ_0 will be chosen equal to 0 for $r_1 = r_2$. But also, from (**41**), $R = R_0 = 0$ at $r_1 = r_2$. Thus, (**66**) gives

$$L = -\int_S \frac{R\, i\, d\, a}{I^2}$$

which may be put in the form:

$$L = -\frac{1}{\pi\, a^2}\int_0^a\int_0^{2\pi}\left[1 - \frac{r^2}{a^2}\right.$$

$$\left. + \log\left(\frac{l^2 + r^2 - 2\,l\,r\cos\theta}{a^2}\right)\right] r\, d\,\theta\, d\, r \qquad \textbf{(70)}$$

$$= \frac{2}{a^2}\int_0^a\left[1 - \frac{r^2}{a^2} + \log\frac{l^2}{a^2}\right] r\, d\, r$$

$$= 2\log\frac{l}{a} + \frac{1}{2} \qquad \textbf{(71)}$$

which is the usual expression for one-half the inductance of a circuit formed by two parallel Wires.

Application to Field Pole Problem. The use of formula (**69**) is illustrated below by applying it to the field pole problem. In this case, and considering all the copper as comprising a single coil side, there is

$$P = \frac{2\, i \int_{\theta_1}^{\pi - \theta_2}\int_{x = a}^{x = b} R^{\mathrm{II}}\, d\, x\, \mathrm{d}\, y}{4\,\pi\,[\alpha_0\,(b - a)\, l]^2} \qquad \textbf{(72)}$$

This integral evaluates to the expression:

$$P = \frac{b - a}{6\, l} - P'$$

where

$$P' = \sum_1^\infty \frac{\alpha_n}{16\,\pi^2\,\alpha_0^2\,(b - a)^2\, n}\left[(b_n + b_n')\left\{1 - \epsilon^{-kn(b-a)}\right\}\right.$$

$$\left. + 2\, K_n\, k\, n\,(b + a)\right] \qquad \textbf{(73)}$$

Thus, for the field pole shown in Fig. 8,

$$P' = 0.0045$$

For the case of an infinite slot, $c = \infty$, (**73**) becomes explicitly:

$$P' = \sum_1^\infty \frac{1}{8\,\pi^2}\left(\frac{\alpha_n}{n\,\alpha_0}\right)^2\left(\frac{l}{b - a}\right)\left[1\right.$$

$$\left. - \frac{(2 - \epsilon^{-2kna} + \epsilon^{-kn(b+a)})\,(1 - \epsilon^{-kn(b-a)})}{k\, n\,(b - a)}\right] \qquad \textbf{(74)}$$

Thus, for the "infinite" slot shown in Fig. 3A,

$$P' = 0.0037$$

In the case of an infinitely deep slot, it may be verified that the factor P' provides a correction which gives the increase of inductance due to concentration of current above the inductance which would be calculated on the assumption that the lines of force were everywhere perpendicular to the slot sides.

Circular Conductors in a Circular Slot. The expression for P' in the case of a circular conductor is very simple because for b_0 chosen $= 0$, the integral over the area of the copper of the term which involves the effect of the slot is zero and there remains only the term which would exist were the conductor isolated in space.

Thus, with $b_0 = 0$:

$$P' = \frac{i}{4\,\pi}\int_S \frac{R\, i\, d\, a}{(\int_S i\, d\, a)^2} = \frac{1}{4\,\pi^2\, r_2^2}\int_0^{2\pi}\int_0^{r_2}\frac{r^2}{r_2^2}\, r\, d\, r\, d\, a \qquad \textbf{(75)}$$

$$= \frac{1}{8\,\pi} \qquad (76)$$

Conductors in Air. The calculation of the inductance of conductors in air permits the development of special formulas involving the conception of geometric mean distance. Thus, for a system of positive currents with return currents within a finite distance, the vector potential calculated from (**47**) will be zero at any point which is an infinite distance from the system in question, if the constant term is taken equal to zero. Thus, if the voltage due to flux between the system of positive currents and infinity is to be calculated, and if the system of currents fulfills the requirements permitting the calculation of inductance, then:

$$L_1 = -\int_{S_1} \frac{R\, i\, d\, a}{I^2} \qquad (77)$$

where S_1 refers to the area of the sections of conductors carrying positive currents and

$$R = \int_{(S_1+S_2)} i \log r^2\, d\, a$$

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$$L_1 = \frac{-1}{I^2} \int_{S_1} [\int_{S_1+S_2} i \log r^2\, d\, a]\, i\, d\, a \quad (78)$$

The voltage due to flux from infinity to the return is

$$L_2 = \frac{-1}{I^2} \int_{S_2} [\int_{S_1+S_2} i \log r^2\, d\, a]\, i\, d\, a \qquad (79)$$

The total inductance of the circuit is

$$L = L_1 + L_2 = \frac{-1}{I^2} \int_{S_1+S_2} \int_{S_1+S_2} i \log r^2\, d\, a\, i\, d\, a \qquad (80)$$

which may be put in the symmetrical form[31]:

$$L = +2 \frac{\int_{S_1+S_2} \int_{S_1+S_2} i \log r\, d\, a\, i\, d\, a}{[\int_{S_1} i\, d\, a]\, [\int_{S_2} i\, d\, a]} \qquad (81)$$

For uniform current density, remembering that the current density is negative over the section of the return conductors, (**81**) becomes:

$$L = -2 \frac{-\int_{S_1} \{[\int_{S_1} - \int_{S_2}] \log r\, d\, a\}\, d\, a}{A_1 A_2} \qquad (82)$$

$$= -2 \frac{[\int_{S_2} \int_{S_1} \log r\, d\, a\;\; d\, a - 2 \int_{S_2} \int_{S_2} \log r\, d\, a\;\; d\, a + \int_{S_2} \int_{S_2} \log r\, d\, a\, d\, a]}{A_1\, A_2}$$

$$= 2 \log \frac{R_{12}{}^2}{R_{11}\, R_{22}} \qquad (83)$$

where

R_{11} = geometrical mean distance of section 1 from itself.

R_{22} = geometrical mean distance of section 2 from itself.

R_{12} = geometrical mean distance of section 1 from section 2.[32]

A_1 = area of section 1.

A_2 = area of section 2.

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[31] It is understood that, subject to the conditions imposed,

$$\int_{S_1} {}_1 i\, d\, a = -\int_{S_2} i\, d\, a$$

[32] The geometrical mean distance for sections of several types are given in Maxwell, loc. cit, p. 63.

Graphical Determination of Magnetic Fields
Comparison of Calculations and Tests

BY E. E. JOHNSON[1] and C. H. GREEN[2]
Associate, A. I. E. E. Non-Member

Synopsis.*—This paper, which deals with the experimental determination of magnetic fields, is presented as a verification of some of the results described by Messrs. A. R. Stevenson, Jr., and R. H. Park in a companion paper, "Graphical Determination of Magnetic Fields—Theoretical Considerations," and to introduce new and convenient methods of determining the flux distributions in air spaces and in regions occupied by current carrying media. The particular case dealt with is the determination of the leakage flux about the field poles of an alternator at no load, both in the copper and in the surrounding air spaces.*

TEST METHODS AND RESULTS

SINCE this investigation involves a large number of determinations of direction and intensity of magnetic flux in fields of intensity varying from, say, 50 to 40,000 lines per sq. in., it was essential to have a simple, reasonably accurate, and reliable device for the explorations. In deciding upon the methods to be employed in measuring magnetic fields, due consideration was given to those which have been used by various investigators.[3] Finally two devices were chosen for use. One, due to Professor Dellenbaugh,[4] and developed by C. H. Green, measures the intensity by means of a revolving search coil equipped with a collecting device or commutator, and operating at a high and constant speed. The direction of the field is determined by that position of the brush axis which gives either maximum or zero reading of the instrument. This device is shown in Fig. 1 and described in Appendix A. The other device used in this investigation measures the intensity of the magnetic field by the angular deflection of a small current carrying coil, the direction being determined by noting the position of the coil axis for zero deflection of the coil.[5] This device is shown in Fig. 2.

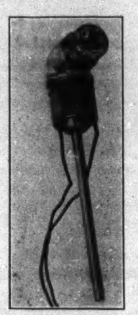

FIG. 1—FLUX MEASURING APPARATUS

This instrument consists of a small coil rotating at high speed which is inserted in the magnetic field to be measured. Connections are made from the coil to a commutator whose brushes are connected to an indicating meter. The revolving parts are enclosed in a non-magnetic casing. The entire instrument is about 6 in. long. The winding consists of two coils 2¼ in. long, of 175 turns each. The rotating parts are driven through a flexible shaft by a 3600-rev. per min. synchronous motor. See Appendix A for further details

In order to get an experimental check of the plot of field flux determined mathematically by Messrs. Stevenson and Park, a special pole structure having the required dimensions was constructed of laminated iron. This is shown in Fig. 3.

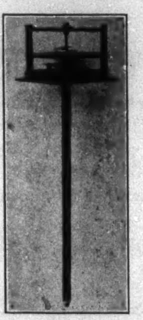

FIG. 2—COMPASS FLUXMETER

This device consists of a current-carrying coil which is inserted in the magnetic field to be plotted. The movement of the coil is opposed by a spring and flux direction is determined by the position of the coil axis for zero deflection. The coil and its shaft are enclosed in a non-magnetic case. The dimensions are as follows:

Length of hollow shaft, 6 in.; diameter of hollow shaft (outside), ¼ in.; diameter of hollow shaft (inside), 3/16 in., length of armature coil (inside the shaft), ½ in.; diameter of armature coil, 5/32 in.; number of turns in armature coil, 20

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1. General Electric Company, Schenectady, N. Y.
2. Raytheon Manufacturing Co., Cambridge, Mass.; formerly of the General Electric Co., Schenectady, N. Y.
3. Bibliography, 1, 2, 3, 4, 5.
4. Bibliography, 6.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

5. A similar device employing a bar magnet in place of the coil was first used. It indicated the direction exactly as does a compass, the field strength being determined by offsetting the needle from its position of rest in the field and counting the oscillations resulting. The instrument had the objection that the bar magnet became saturated at high field intensities, thus changing its strength.

The following description of the experimental methods employed and the results obtained is divided into two parts: first, the Flux Distribution in Air Spaces; and second, the Flux Distribution in Current Carrying Regions.

I. Flux Distribution in Air Spaces

The flux fields in the air spaces between the poles and between the poles and armature were explored by both types of measuring instruments and checked by the iron filing method. Fig. 4 is a reproduction of an actual photograph of the iron filing plot. The plot obtained by use of the instruments is shown and discussed in Part II.

Fig. 3—Experimental Set-up of Alternator Field Poles

The leakage flux at the end of the field poles of a generator is shown in Fig. 5A and B. The lines in Fig. 5B indicate the direction taken by iron filings and the arrows show the directions as indicated by the measuring instruments. Lines of constant magnetic intensity[6] of the end leakage flux are shown in Fig. 6.

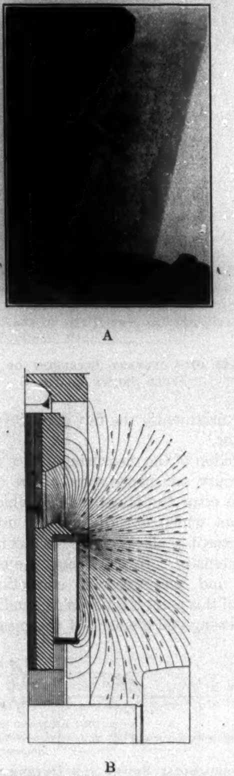
Fig. 4—Flux Field in the Interpolar Space of the Set-up Shown in Fig. 3, as Determined by the Iron Filing Method

II. Flux Distribution in Current Carrying Regions

Although a knowledge of the flux distribution within the current carrying regions of a field pole was made available by the mathematical method described in in the paper by Messrs. Stevenson and Park, no experimental determination had been made, as far as is known, until the present one.

It was thought that the practical difficulty of exploring the field within the copper might be overcome by the substitution of a liquid conductor such as mercury, in the place of the copper, thus, of course, reducing the winding to one turn. The idea worked out successfully, the method of doing it in the case of the field poles being perhaps best explained by reference to Fig. 7.

As indicated in Fig. 7, the field winding consists of one turn per pole. That part of the winding about pole *S*, which is labeled *A*, is a non-magnetic tank containing mercury. At the top of this tank is a copper plate perforated with ¼-in. holes, the plate being in electrical contact with the mercury. The holes in the copper plate are for the purpose of giving access to the inside of



A

B

Fig. 5—Magnetic Leakage Flux at One End of a Pole of a 6-Pole, 435-Kv-a., 1200 Rev. per Min., 4000-Volt Alternator

A. As shown by iron filing method

B. As determined by use of the instrument, shown in Fig. 1

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[6] These lines are lines along which the magnetic intensity is constant. They were useful in studying the effects of the rotating stray fields on the end structure of machines.

the conductor. The armature structure has been purposely omitted from this figure in order to afford a view of the set-up. With the current flowing through the one turn field coils, the conditions as existing in an alternator at no load are practically duplicated, except that due to the high losses in the mercury it is impossible to obtain conveniently the flux densities which exist in an alternator.[7]

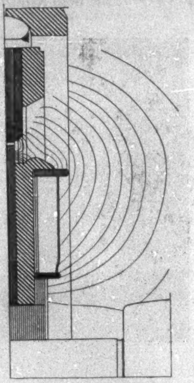

FIG. 6—LINES OF CONSTANT INTENSITY OF THE MAGNETIC FIELD SHOWN IN FIG. 5

The insertion of the measuring device displaces some of the mercury and hence some of the current. This involves an error the magnitude of which varies over the field but which, for the present investigation, is negligible except in places where the flux to be measured is of low intensity. In any event, the error is readily calculated and the necessary corrections made. A discussion of this point is given in Appendix A.

The exploring coil element of the measuring instrument must necessarily carry a current, the effect of which is to distort the field. This field distortion is practically negligible in most magnetic fields about electrical machinery. This point is discussed in Appendix B.

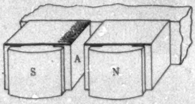

FIG. 7—EXPERIMENTAL SET-UP FOR DETERMINING THE FLUX DISTRIBUTION WITHIN THE FIELD WINDING OF AN ALTERNATOR

The plot obtained by the free hand method is shown in Fig. 8A, the plot obtained by the mathematical method outlined in the paper by Stevenson and Park is shown in Fig. 8B, and the experimental plot is shown in Fig. 8C. It may be of considerable interest to know that the free hand plot was done first, the mathematical plot second, and the experimental plot third, each independently of the other, the theory preceding the experiment.

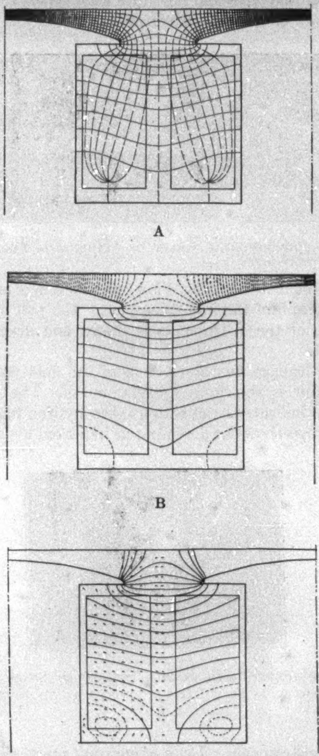

FIG. 8—FLUX DISTRIBUTION IN THE AIR-GAP AND INTERPOLAR SPACES OF THE SET-UP SHOWN IN FIGS. 3 AND 7

A. As determined by the free hand graphical method
B. As determined by mathematical method
C. As determined experimentally

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[7] The actual current employed in the single turn was 1000 amperes, thus producing an m. m. f. of 1000 ampere-turns per pole, which is considerably less than that which exists in an actual alternator.

Appendix A

DESCRIPTION OF THE FLUX-MEASURING DEVICE SHOWN IN FIG. 1

Complete investigation of both the useful and leakage flux distributions in the air spaces about an alternator requires the measurement of direction and densities in fields varying from, say, 50 to 40,000 lines per sq. in. The idea of using for this purpose a revolving search coil equipped with collecting device for commutator, is due to Mr. F. S. Dellenbaugh (Bibliography, 6).

The instrument developed by C. H. Green is designed to be simple and reasonably accurate, as well as portable and sturdy, for shop use. Fig. 1 shows the device, which is about 6 in. long overall, and therefore easily manipulated with one hand. The miniature armature runs within a protective stationary sleeve and is operated through a pair of small beveled gears driven in turn by detachable flexible drive shafts, from a 1/8th-h. p., 3600-r. p. m. synchronous motor. A dial graduated in degrees may be slipped over the knurled sleeve at the head end.

To obtain a maximum reading on an output meter, the brush position on the miniature commutator is varied by turning this friction sleeve between the fingers so that the direction of the flux being cut may be read off the dial.

The complete device is made of non-magnetic metal, with the exception of Bakelite where required for insulation. Brass is used for the casing, while aluminum monel, a stainless non-magnetic alloy of extremely high resistance, is used for the miniature armature spindle and commutator segments.

The spindle is four inches long, supported by a bearing at its outer end, and is 0.094 inch diameter. The winding consists of two coils each 2½ inches long, confined to the outer end of the spindle. Each coil is wound of 175 turns 0.002 inch enameled copper wire separated from each other and retained to the spindle with silk floss.

The completed armature is impregnated in lacquer to resist oil that might issue from bearings. Connections are made to the four-segment commutator, 3/8 inch diameter. The radio clearance of the armature within the ¼ inch outside diameter sleeve is 0.015 inch.

The miniature armature is calibrated in a magnetic field of predetermined intensity, so that a factor between milli voltage output and flux density may be applied to subsequent meter readings when the instrument is used in the shop.

Appendix B

INVESTIGATION OF THE ERROR INVOLVED DUE TO THE DISPLACEMENT OF CURRENT CARRYING MERCURY BY THE MEASURING INSTRUMENTS

It seems that in order to explore the magnetic field inside of a liquid conductor, such as mercury, it is necessary to displace some of the mercury. If the measuring instruments were infinitesimally small, there would be no appreciable displacement of the mercury and hence no error due to that source would be involved in the field plot so obtained. The instruments used in this investigation, however, have a diameter of ¼ inch and an error of some magnitude is to be expected.

The effect of the displacement of current carrying mercury is to remove a portion of the current. The removal of this current is equivalent to superposing upon the preexisting current in that portion a current of opposite direction and of the same density in that portion.[8] The magnetic field which results from the effect of the normal current distribution and of the superposed current is the same as that produced by the removal of the portion of current carrying mercury in the actual case.[9]

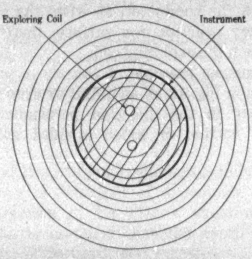

FIG. 9—FLUX DISTRIBUTION WHICH WOULD EXIST IF THE SPACE OCCUPIED BY THE MEASURING INSTRUMENT SHOWN WAS FILLED WITH A CURRENT FLOWING IN A DIRECTION PERPENDICULAR TO THE PLANE OF THE PAPER, AND NOT NEAR ANY OTHER ELECTRIC OR MAGNETIC CIRCUITS

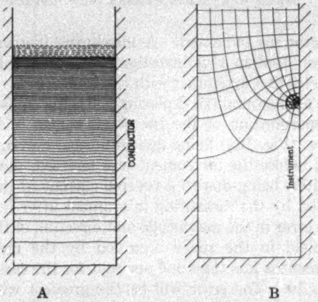

FIG. 10—FLUX DISTRIBUTION WHICH EXISTS IN AN INFINITELY DEEP IRON SLOT HAVING A LIQUID CONDUCTOR IN ITS BOTTOM

A. Without the measuring instrument

B. Due to a reverse current in the space occupied by the measuring instrument after insertion

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[8] The instrument is properly insulated with varnish to prevent a flow of current through it.

[9] Neglecting saturation, of course.

The problem, then, is to determine what effect the current in a cylindrical conductor has on an exploring coil placed in the magnetic field within that conductor for various configurations of the surrounding magnetic circuit.

If the lines of force in the conductor due to its own current are concentric with that conductor, there is no effect on the exploring coil, as may be seen by reference to Fig. 9. If, however, the lines of force are not concentric with the conductor, as when the conductor is brought very near an iron surface, there will be an an effect upon the exploring coil.

As an extreme example of the above, consider the case, shown in Fig. 10, of an infinitely deep slot extending infinitely in the direction perpendicular to the paper, the bottom of which is filled with a liquid conductor, as indicated. A measuring instrument of the sort used in this investigation is inserted in the conductor and in contact with the iron.

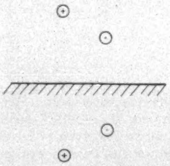

Fig. 11—Image of the Currents in the Measuring Instruments when Near a Plane Iron Surface

The field around the exploring coil is the equivalent of the superposition of the two fields shown in Fig. 10, the first (Fig. 10A) being due to normal current distribution before the instrument was inserted, the second (Fig. 10B) being due to a reverse current in the space occupied by the measuring instrument after insertion.

The error in the magnitude and direction of the magnetic field in the space occupied by the measuring instrument is less than one per cent for the case shown in Fig. 10. The error will be the greatest where the field to be measured is the weakest, which, in the case shown in Fig. 10, is at the bottom of the slot. The error there is about 30 per cent, but this error does not render the method useless because it can always be calculated so that the field plot obtained experimentally may be corrected.

Appendix C

ERROR DUE TO THE MAGNETIC REACTION OF THE MEASURING INSTRUMENTS UPON THE MAGNETIC FIELD TO BE MEASURED

The error due to this cause may be determined readily by making a plot of the magnetic field, which is caused by the current in the instruments and finding the magnitude and direction of the flux at the place occupied by the measuring instrument. If the instrument is near a particular plane iron surface, the magnetic field due to current in the instrument may be determined by the method of images; that is, the iron surface may be considered to be removed and another set of currents similar to those in the instrument placed at an equal distance from the iron surface and in the same direction, as shown in Fig. 11.

The error due to the cause encountered in this investigation is less than one per cent, except very near the kernel where the flux density of the field to be measured is zero.

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Graphical Determination of Magnetic Fields
Practical Applications to Salient-Pole Synchronous Machine Design

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***Synopsis.**—There are three methods of obtaining the flux distribution in a magnetic field.*

First: By test. Templets or models can be made of the field to be explored and the flux distribution can be obtained by test as described in a companion paper "Graphical Determination of Magnetic Fields—Comparison of Calculations and Tests" by Messrs. E. E. Johnson and C. H. Green.

Second: By mathematical analysis. This method is accurate and consistent results can be obtained. If the problem is very complicated, however, the mathematical solution is very laborious and sometimes impossible.

Third: By the graphical method. This method is quite accurate and it can be used for design calculations. It is comparatively easy and it is the quickest method of the three. The graphical method of plotting magnetic fields used in this paper is described in a companion paper, "Graphical Determination of Magnetic Fields—Theoretical Considerations" by Messrs. A. R. Stevenson, Jr. and R. H. Park.

This paper shows how the graphical flux plots can be used very successfully in design calculations. In fact, the performance characteristic of a salient-pole synchronous machine can not be predetermined accurately without the use of flux distribution coefficients.

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INTRODUCTION

THE distribution of magnetic flux is a very important factor in the design of electrical apparatus. In order to predetermine the characteristics of an electrical device, it is necessary in many cases to plot the flux distribution around the several parts. This is especially so with salient-pole synchronous machines.

Carter, Rogowski, Lehmann, and others have plotted magnetic fields graphically by drawing the potential and flux lines at right angles and by arranging the tubes of flux so that they form approximate squares with the potential lines. This is naturally a cut-and-try method, but usually the symmetry of the figure and the known conditions enable one to arrive at a correct or balanced plot with only a few trials.

As a rule, it is not difficult to determine if the plot is correct because, with a little practise, the eye can be trained to detect any irregularity in the squares formed by the flux and potential lines. The results obtained by some of the graphical flux plots were found to be in very close agreement with similar results obtained mathematically. Furthermore, the flux distribution data given in this paper have been used very successfully in the design calculations of synchronous machinery by a large manufacturing company for nearly ten years.

I. MAGNETIC FLUX DISTRIBUTION IN A SLOT

The first practical application of magnetic flux plotting to dynamo design was done by F. W. Carter in 1901[2]. Carter determined the value of the air-gap coefficient by introducing a fringing coefficient which assumes that all of the fringing flux is confined to a limited area instead of to the entire region over the slot. Carter found this fringing coefficient mainly by the use of the theory of functions of a complex variable to obtain a solution of La Place's equation. In this solution, Carter assumed that the depth of the slot and the width of the tooth were infinite[3].

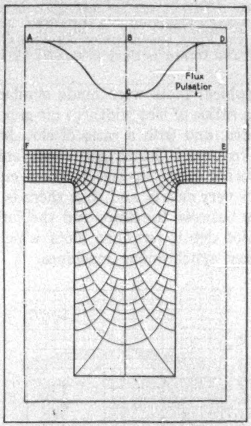

FIG. 1—FLUX DISTRIBUTION IN A SLOT

$$\frac{\text{Slot width}}{\text{Slot pitch}} = 0.5 \qquad \frac{\text{Slot width}}{\text{Air gap}} = 3.33$$

$$\text{Gap Co-efficient} = \frac{\text{Area } A\ B\ D\ E\ F}{\text{Area } A\ C\ D\ E\ F} = 1.26$$

$$\text{Flux pulsation} = \frac{BC}{AF} = 0.493$$

Calculation of Air-Gap Coefficients from Graphic Flux Plots. Fig. 1 shows the flux distribution in the

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[2] It is claimed that A. Potier in 1889 derived the permeance between a slotted and a plain surface in his study of the electrometer. It can be found in Vol. 2, page 563, of Potier's translation of Maxwell's treatise.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February, 7-11, 1927.

[3] A mathematical solution for the finite depth of tooth by Hadamard can be found in the *Annales de Chimie et de Physique*, 1909, Vol. 16, Second Series, page 403.

air-gap over one slot pitch. The line *A C D* shows how the flux density varies in the air-gap at the surface of a pole. It is evident that the area *A B D C* represents the amount of flux which is lost due to the slot. In other words, the effective air-gap is increased by the introduction of a slot and thus the air-gap coefficient (Fig. 1) is equal to the area *A B D E F* divided by the area *A C D E F*.

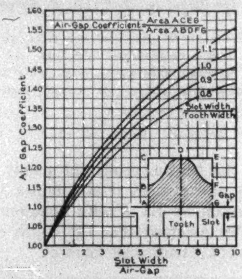

FIG. 2—AIR-GAP COEFFICIENT OBTAINED BY PLOTTING GRAPHICALLY THE FLUX DISTRIBUTION AROUND A TOOTH WITH FINITE WIDTH AND DEPTH AS USED IN PRESENT DAY MACHINES

Forty graphical plots were made similar to Fig. 1 with various ratios of slot width to air-gap, slot width to tooth width, and with a ratio of slot depth to slot width equal to four. The air-gap coefficients obtained by these plots are shown in Fig. 2. These curves check Carter's work very closely and thus there is practically no difference between the finite and the infinite tooth width and slot depth for open slots which are used in present-day synchronous machines.

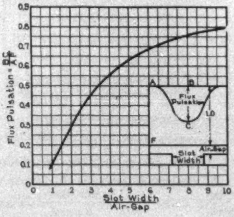

FIG. 3—MAGNITUDE OF FLUX PULSATION OBTAINED BY PLOTTING GRAPHICALLY THE FLUX DISTRIBUTION IN A SLOT WITH A FINITE WIDTH AND DEPTH AS USED IN PRESENT DAY MACHINES

Flux Pulsation Due to Armature Slots. The variation in the flux density or the flux pulsation $\left(\frac{B\,C}{A\,F}, \text{Fig. } 1\right)$ was obtained from the flux plots where the tooth width equaled the slot width and the results are shown in Fig. 3. This curve also checks Carter's work for the case of the infinite tooth width and slot depth. The flux pulsation caused by the armature slots produces a loss in the pole face. This loss is part of the open circuit core loss, and it can be calculated when the magnitude of the flux pulsation is known.

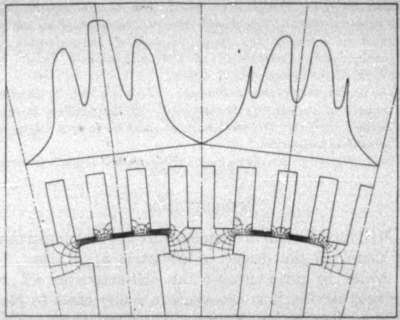

FIG. 4—FLUX DISTRIBUTION AT NO LOAD IN THE AIR-GAP OF A SALIENT POLE SYNCHRONOUS MACHINE FOR THE MAXIMUM AND MINIMUM PERMEANCE POSITIONS. 10 PER CENT FLUX PULSATION

If the number of teeth spanned by a pole varies when the pole moves through a tooth pitch, the pole flux will pulsate if the pole has no low impedance damper winding, etc. Fig. 4 shows a 10 per cent flux pulsation from the maximum to the minimum permeance positions in a machine which has a small number of wide stator slots and a small air-gap. This flux pulsation, if excessive, will increase the open circuit core loss and it may produce a magnetic noise. If the teeth are not spiralled, therefore, it is always desirable to have the number of teeth over the pole a constant, especially if a small number of teeth per pole is used.

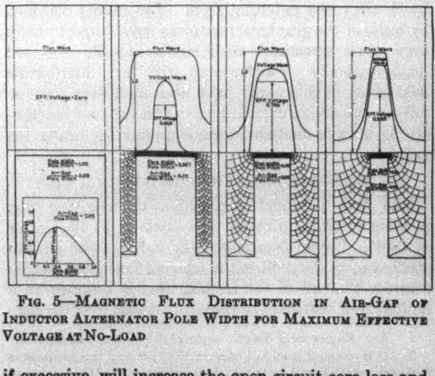
Flux Wave
EFF. Voltage = Zero
Pole Width / Pole Pitch = 1.00
Air-Gap / Pole Pitch = 0.05
Voltage Wave
EFF. Voltage
Pole Width / Pole Pitch = 0.667
Air-Gap / Pole Pitch = 0.05
1.0

FIG. 5—MAGNETIC FLUX DISTRIBUTION IN AIR-GAP OF INDUCTOR ALTERNATOR POLE WIDTH FOR MAXIMUM EFFECTIVE VOLTAGE AT NO-LOAD

Inductor Alternator Pole Width Which Gives the Maximum Effective Voltage. There seems to be an increasing demand for high frequency generators for supplying power to induction furnaces, and high-speed tools, for testing, and for experimental work. Usually if the frequency is above 2500 cycles per second, the inductor type alternator is used.

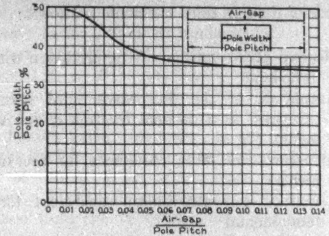

Fig. 6—Inductor Alternator Pole Width for Maximum Effective Voltage at No Load

The voltage induced in an armature coil of an inductor alternator depends upon the flux pulsation, as shown in Fig. 5. If the pole width is 100 per cent of the pole pitch, and if the air-gap is uniform, the flux wave will be a rectangle, and, obviously, the flux pulsation and the induced voltage will be zero. As the pole width is decreased, Fig. 5, the flux pulsation increases. The effective value of the induced voltage, however, increases until it reaches a maximum, and as the pole width is further decreased, the effective voltage decreases. Thus it is very desirable to shape the pole of an inductor alternator so that the voltage induced in the armature coil is a maximum. The insert in Fig. 5 shows how the effective voltage varies with the pole width when the air-gap is five per cent of the pole pitch. In this case the maximum effective voltage at no load occurs when the pole width is 38 per cent of the pole pitch. A number of these flux plots were made for various ratios of air-gap to pole pitch and the corresponding maximum effective voltages were obtained and plotted in Fig. 6. It can be seen that a pole width equal to half of the pole pitch should be used only when the air-gap is infinitely small. For a 10 per cent air-gap, the pole width should be 0.35 of the pole pitch to give the maximum effective voltage at no load. Under load the maximum effective voltage should occur when the pole pitch is a little less than given by the curve in Fig. 6.

II. MAGNETIC FLUX DISTRIBUTION IN THE AIR-GAP OF A SALIENT-POLE SYNCHRONOUS MACHINE AT NO LOAD WHEN EXCITED ONLY BY THE FIELD COILS

Fig. 7 shows the flux distribution around the pole at no load when a salient-pole synchronous machine is excited by its field coil. The full line flux wave was calculated from the flux plot at the surface of the armature and the dotted flux wave was obtained by test with an exploring conductor placed on the surface of the armature. The flux wave fundamental is 1.11 times the maximum value of the flux wave and the flux wave third harmonic is 0.085 times the maximum value of the fundamental.

Fundamental and Third Harmonic in the Air-Gap Flux Wave at No Load. Seventy-five hypothetical flux plots at no load were made of poles used in present-day machines whose pole faces were arcs of circles. The pole shapes used are included in the limits of the following three variables:

$$\frac{\text{Minimum Gap}}{\text{Pole Pitch}} \quad \text{from } 0.01 \text{ to } 0.05$$

$$\frac{\text{Pole Arc}}{\text{Pole Pitch}} \quad \text{from } 0.50 \text{ to } 0.75$$

$$\frac{\text{Maximum Gap}}{\text{Minimum Gap}} \quad \text{from } 1 \text{ to } 3$$

The flux waves were analyzed for their fundamentals and third harmonics, the values of which are shown in Figs. 8 and 9. In Fig. 8, the fundamental A_1 of the flux wave is expressed as a decimal fraction of the maximum value of the flux wave which is taken as unity. In Fig. 9, the third harmonic A_3 of the flux wave is expressed as a decimal fraction of the fundamental which is taken as unity. The polarity of the third harmonic is also given and it is considered minus when it is as shown in the insert of Fig. 9. The pole shape, which will have a flux wave with a zero third harmonic, can also be obtained from Fig. 9. For

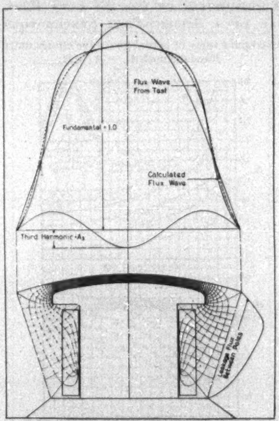

Fig. 7—Magnetic Flux Distribution Around a Pole at No Load When a Salient-Pole Synchronous Machine is Excited by its Field Winding

example, if the ratio of the minimum gap to the pole pitch is 0.02, and if the ratio of the maximum to the minimum gap is 1.5, then the ratio of the pole arc to the pole pitch should be 0.67 to obtain a flux wave at no load which has no third harmonic.

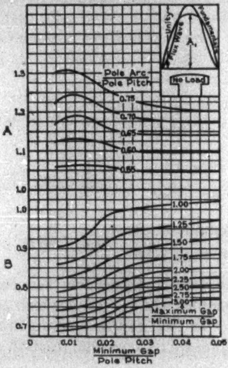

FIG. 8—FUNDAMENTAL OF THE NO-LOAD FLUX WAVE IN THE AIR-GAP OF A SALIENT-POLE SYNCHRONOUS MACHINE

Maximum value of actual flux wave equals unity
Fundamental $A_1 = A \times B$

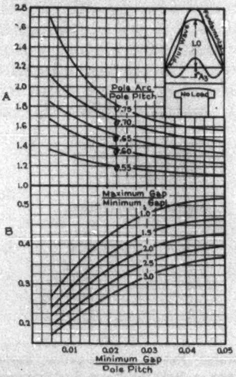

FIG. 9—THIRD HARMONIC OF THE NO-LOAD FLUX WAVE IN THE AIR-GAP OF SALIENT-POLE SYNCHRONOUS MACHINE

Maximum value of fundamental equals unity
Third harmonic $A_3 = A \times B - 0.6$

Calculation of the Open Delta Voltage and the Delta Circulating Current of a Synchronous Machine at No Load. A very interesting application of the flux wave third harmonic curves, Fig. 9, is the calculation of the voltage which appears at no load at the open corner of a delta-connected armature winding whose coils do not have a two-thirds pitch. It is well known that only the third harmonic or multiples of the third harmonic voltage can appear at the open delta. Since the multiples of the third harmonic flux wave are usually small, and since both the armature coil pitch and distribution further decrease the voltage produced by these flux multiple third harmonics, the multiple third harmonic voltages can be neglected. Thus, the calculation of the third harmonic voltage at the open corner of a delta-connected armature winding is very simple, if the amplitude of the flux third harmonic is known.

Let

E = Normal phase voltage of the armature winding at no load

A_3 = Amplitude of the flux wave third harmonic expressed as a decimal fraction of its fundamental Fig. 9

k_p = Armature coil pitch coefficient for the fundamental

k_d = Armature coil distribution coefficient for the fundamental

k_{3p} = Armature coil pitch coefficient for the third harmonic

k_{3d} = Armature coil distribution coefficient for the third harmonic

E_3 = Open delta voltage (third harmonic)

then

$$E_3 = 3 E A_3 \frac{k_{3p}\, k_{3d}}{k_p\, k_d}$$

This method of calculating the third harmonic delta voltage assumes that there is no saturation in the magnetic circuit and it ignores the effect of the stator and the rotor slots. These factors, however, are quite small in most machines. The three-phase machine whose pole shape is shown in Fig. 7 had 18 slots per pole, armature coil pitch 0.777, ratio of minimum gap to pole pitch 0.037, ratio of pole arc to pole pitch 0.674, ratio of maximum to minimum air-gap 1.22, and phase voltage of 3810 volts.

Thus

$$k_p = 0.939 \quad k_d = 0.955$$
$$k_{3p} = 0.49 \quad k_{3d} = 0.646$$

From

$$\text{Fig. 9, } A_3 = 1.44 \times 0.475 - 0.6 = 0.084$$

therefore

$$E_3 = 3 \times 0.084 \times 3810 \, \frac{0.49 \times 0.646}{0.955 \times 0.939} = 339 \text{ Volts}$$

$$E_3 \text{ by test} = 300 \text{ Volts}$$

The delta circulating current at no load is

$$I_3 = \frac{E_3}{\text{Third harmonic synchronous impedance}}$$

Calculation of Air-Gap Ampere-Turns, Using the Flux Distribution Coefficients K_ϕ and K_λ. In order to predetermine the no-load air-gap ampere-turns accurately for a pole whose pole arc radius is less than the radius of the armature face, two flux distribution coefficients must be obtained. It is well known that if the flux wave has a flat top, more flux (lines per pole) is required to give a certain effective voltage at the terminals of the armature winding than for a peaked flux wave. In order to obtain the flux per pole accurately, therefore, it is necessary to modify the flux equation which assumes a sinusoidal flux distribution and to introduce the flux distribution coefficient[4] K_ϕ which is the ratio of the area of the actual no load flux wave to the area of its fundamental.

Messrs. Doherty and Shirley introduced this flux coefficient K_ϕ in 1918[4], and they obtained the values of K_ϕ from flux distribution plots (see Fig. 36 of Messrs. Doherty and Shirley's paper) which assumed that the flux density varied inversely as the distance (in a straight line) from the pole face to the armature core. This, of course, is an approximate solution of the problem. Fig. 10 gives the values of K_ϕ which were obtained from the 75 hypothetical flux plots similar to Fig. 7 where the flux distribution was obtained by actually plotting the tubes of flux. These values of K_ϕ practically check the values of K_ϕ given in Messrs. Doherty and Shirley's paper.

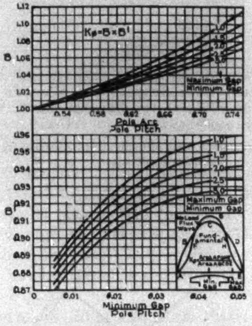

FIG. 10—FLUX DISTRIBUTION COEFFICIENT K_ϕ

The introduction of K_ϕ thus gives the actual flux per pole for any flux wave. The next step is to find the average air-gap density over the pole face in order to find the necessary ampere turns to force this flux across the air-gap. It is very convenient to know the flux which passes directly out from the pole and into the armature, as shown by the shaded area, in the insert of Fig. 11. This flux is equal to K_λ times the flux per pole where K_λ is the ratio of the area $G\ B\ C\ D\ F$ to the area $A\ B\ C\ D\ E$ in Fig. 11. The average air-gap density over the pole (Region $G\ F$, Fig. 11) is K_λ times the flux per pole divided by the area of pole face. The air-gap ampere turns can now be obtained accurately since the air-gap coefficient, Fig. 2, and the reluctance of the air-gap are known.

The two flux distribution coefficients K_ϕ and K_λ, calculated from the predetermined flux wave, made it possible to predetermine very closely the performance characteristics of a two-speed salient-pole synchronous motor[5]. This two-speed motor had irregular shaped poles arranged in pairs whose flux waves deviated appreciably from a sine wave at either speed.

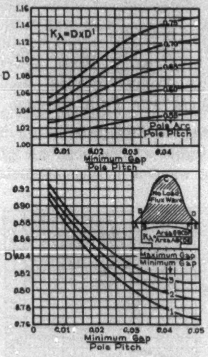

FIG. 11—POLE FACE FLUX COEFFICIENT K_λ

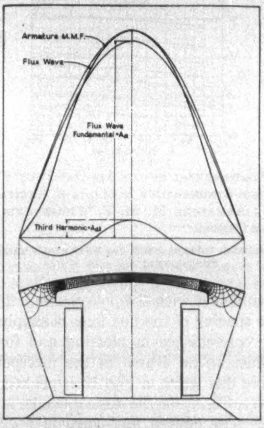

FIG. 12—MAGNETIC FLUX DISTRIBUTION IN THE AIR-GAP WHEN A SALIENT POLE SYNCHRONOUS MACHINE IS EXCITED ONLY BY A SINE WAVE ARMATURE M. M. F. WHOSE AXIS COINCIDES WITH THE POLE CENTER

Calculation of the Leakage Flux of a Salient-Pole Synchronous Machine. The flux distribution curve of the leakage flux between poles is shown in Fig. 7.

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[4] *Reactance of Synchronous Machines and Its Application*, Doherty and Shirley, A. I. E. E., TRANS. Vol. XXXVII, p. 1209.

[5] *A Two-Speed, Salient-Pole Synchronous Motor*, R. W. Wieseman, A. I. E. E., TRANS. Vol. XLIV, p. 436, 1924, Figs. 15, 16, 17, 18.

This curve is plotted on the interpolar center line and it neglects the small amount of leakage flux in the lower corner of the pole. The ratio of the area under the leakage flux distribution curve to the area under the main flux distribution curve in Fig. 7 plus 1 gives the leakage coefficient which is 1.16 for this pole. This coefficient should be increased slightly to allow for the small end leakage flux.

III. MAGNETIC FLUX DISTRIBUTION IN THE AIR-GAP WHEN A SALIENT-POLE SYNCHRONOUS MACHINE IS EXCITED ONLY BY A SINE WAVE ARMATURE MAGNETOMOTIVE FORCE WHOSE AXIS COINCIDES WITH THE POLE CENTER

Fig. 12 shows the flux distribution in the air-gap when a salient-pole synchronous machine is excited only by a sine wave armature magnetomotive force whose axis coincides with the pole center. The abrupt break in the spacing of the flux lines is simply a change in scale for convenience in plotting and for allowing more flux lines to be drawn in the interpolar space. The armature flux wave in Fig. 12 is peaked while the field flux wave, Fig. 7, for the same machine is decidedly flat topped. The peaked flux wave in Fig. 12 is the flux wave which balanced polyphase armature currents tend to produce at zero power factor. The flux wave fundamental, Fig. 12, is 0.94 times the maximum value of the flux wave and the flux wave third harmonic is 0.092 times the maximum value of the fundamental.

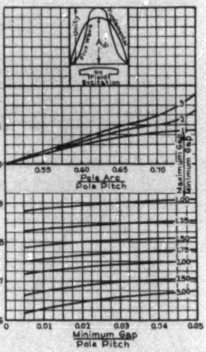

FIG. 13—FUNDAMENTAL OF THE AIR-GAP FLUX WAVE WHEN A SALIENT POLE SYNCHRONOUS MACHINE IS EXCITED BY ONLY A SINE WAVE ARMATURE M. M. F. WHOSE AXIS COINCIDES WITH THE POLE CENTER

Maximum value of actual flux wave equals unity
Fundamental $A_{d1} = A \times B$

Calculation of the Fundamental and the Third Harmonic in the Flux Wave Which Polyphase Armature Currents Tend to Produce at Sustained Short Circuit. Seventy-five hypothetical flux plots were made similar to Fig. 12 for the same range of pole shapes as previously described. These armature flux waves were analyzed for their fundamentals and third harmonics, the values of which are shown in Figs. 13 and 14. In Fig. 13, the fundamental A_{d1} is expressed as a decimal fraction of the maximum value of the flux wave which is taken as unity. In Fig. 14, the third harmonic A_{d3} is expressed as a decimal fraction of the fundamental which is taken as unity.

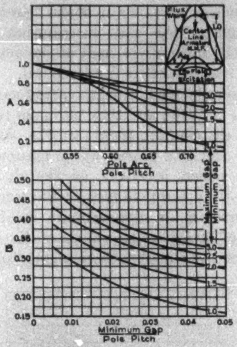

FIG. 14—THIRD HARMONIC OF THE AIR-GAP FLUX WAVE WHEN A SALIENT-POLE SYNCHRONOUS MACHINE IS EXCITED ONLY BY A SINE WAVE ARMATURE M. M. F. WHOSE AXIS COINCIDES WITH THE POLE CENTER

Maximum value of fundamental equals unity
Third harmonic $A_{d3} = - A \times B$

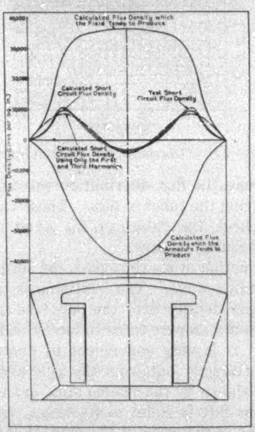

FIG. 15—VOLTAGE INDUCED IN EXPLORING CONDUCTOR PLACED IN THE AIR-GAP OF SALIENT-POLE MACHINE AT SUSTAINED POLYPHASE SHORT CIRCUIT.

Predetermination of the Flux Wave in the Air-Gap of a Salient-Pole Synchronous Machine at Sustained

Polyphase Short Circuit. When a polyphase salient-pole generator is short-circuited, the armature m. m. f. is in opposition to the field m. m. f. If the armature resistance is neglected, the sustained air-gap short-circuit flux, which travels in synchronism with the pole, can be obtained readily by subtracting the armature flux wave in Fig. 12 from the field flux wave, Fig. 7. This short-circuit wave is shown in Fig. 15. Fig. 16 shows the actual short-circuit flux wave obtained with an exploring conductor placed on the armature surface and this wave is also plotted in Fig. 15. The flux wave, Fig. 16, must include such flux waves which are not in synchronism with the pole and, therefore, this wave must be slightly different from the calculated flux wave in Fig. 15.

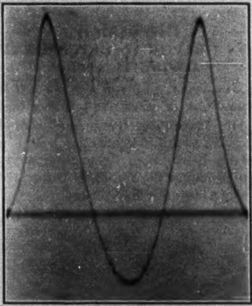

Fig. 16—Voltage Induced in Exploring Conductor Placed in the Air-Gap of a Salient-Pole Machine at Sustaining Polyphase Short Circuit

Calculation of the Stray Core Loss in the Armature Teeth of a Salient-Pole Synchronous Machine at Sustained Polyphase Short Circuit. The short-circuit flux wave can be predetermined approximately by combining the fundamentals and third harmonics of the field flux waves, Figs. 7, 8, and 9, with the armature flux waves, Figs. 12, 13, and 14. Fig. 15 shows the short-circuit flux wave obtained by this method. The stray core loss in the armature teeth of a salient-pole machine at sustained polyphase short circuit can now be predetermined. The problem can be simplified by neglecting the fundamental which is usually small and using only the resultant third harmonic flux.

The full-load flux wave of a synchronous condenser at zero power factor either over or under excited can also be predetermined by this method.

IV. MAGNETIC FLUX DISTRIBUTION IN THE AIR-GAP WHEN A SALIENT-POLE SYNCHRONOUS MACHINE IS EXCITED ONLY BY A SINE-WAVE ARMATURE MAGNETOMOTIVE FORCE WHOSE AXIS IS IN QUADRATURE WITH THE POLE CENTER

Fig. 17 shows the flux distribution in the air-gap when a salient-pole machine is excited only by a sine wave armature m. m. f. whose axis is in quadrature with the pole center. The effect of saturation and of the stator and rotor slots is neglected. The armature flux wave is made up principally of a fundamental and a large third harmonic. The flux wave fundamental is 0.54 times the maximum value of the armature m. m. f., and the flux wave third harmonic is 0.43 times the maximum value of the fundamental.

Seventy-five hypothetical flux plots similar to Fig. 17 were made of the air-gap flux which the armature currents tend to produce when the armature m. m. f. is in quadrature with the pole center. These flux waves were analyzed for their fundamentals and third harmonics, the values of which are shown in Figs. 18 and 19. In Fig. 18, the armature flux fundamental A_{q1} is expressed as a decimal fraction of the armature sine wave m. m. f., which is taken as unity. This method of evaluating the armature flux fundamental is done for convenience in design calculations. In Fig. 19, the armature flux third harmonic A_{q3} is expressed as a decimal fraction of the armature flux fundamental which is taken as unity.

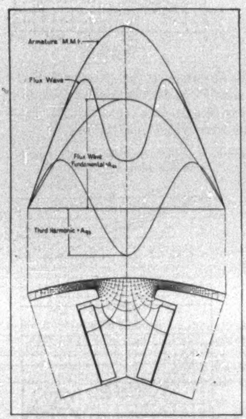

Fig. 17—Magnetic Flux Distribution in the Air-Gap When a Salient-Pole Synchronous Machine is Excited by Only a Sine-Wave Armature M. M. F. Whose Axis is in Quadrature with the Pole Center

Calculation of Displacement Angle of a Salient-Pole Synchronous Machine. It is well known that an a-c. generator rotor leads its voltage, and a synchronous motor rotor lags behind the line voltage. The power or displacement angle of lead or lag is caused by the armature flux distorting the field flux. The fundamental A_{q1} which can be obtained from Fig. 18 is thus a measure of this distortional effect[6]. The harmonics of the armature flux have no distortional effect on the field flux fundamental, and since the harmonics of the field flux are usually small, the displacement angle can be obtained by combining properly the armature flux fundamental A_{q1} with the field flux fundamental A_1.

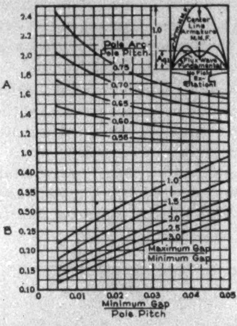

FIG. 18—FUNDAMENTAL OF THE AIR-GAP FLUX WAVE WHEN A SALIENT-POLE SYNCHRONOUS MACHINE IS EXCITED BY ONLY A SINE WAVE ARMATURE M. M. F. WHOSE AXIS IS IN QUADRATURE WITH THE POLE CENTER

Maximum value of armature m. m. f. equals unity
Fundamental $A_{q1} = A \times B$

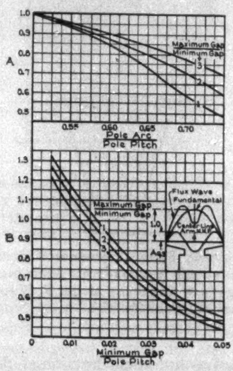

FIG. 19—THIRD HARMONIC OF THE AIR-GAP FLUX WAVE WHEN A SALIENT-POLE SYNCHRONOUS MACHINE IS EXCITED BY ONLY A SINE WAVE ARMATURE M. M. F. WHOSE AXIS IS IN QUADRATURE WITH THE POLE CENTER

Maximum value of armature flux fundamental equals unity
Third harmonic $A_{q3} = A \times B$

Furthermore, by combining the fundamentals and third harmonics A_1, A_3, A_{d1}, A_{d3}, A_{q1}, and A_{q3}, which can be readily obtained from Figs. 8, 9, 13, 14, 18, and 19, the air-gap flux wave of a salient-pole synchronous machine can be approximated for any load condition.

Additional practical applications of plotting magnetic flux distribution curves will be given in a future paper.

The author gratefully acknowledges the assistance of Mr. L. P. Shildneck in plotting some of the flux distribution curves.

Discussion

PAPERS ON GRAPHICAL DETERMINATION OF MAGNETIC FIELDS

(STEVENSON AND PARK, JOHNSON AND GREEN, WIESEMAN)
NEW YORK, N. Y., FEBRUARY 8, 1927

C. H. Linder: Any simple experimental method of magnetic field plotting is very useful in connection with a mathematical investigation of flux distribution. The accompanying Figs. 1 to 9 demonstrate the efficacy of the iron-filing method for determining the general form of a magnetic field.

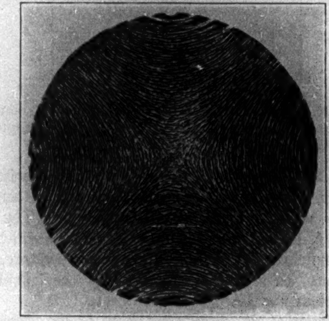

FIG. 1

This is a four-pole induction motor with the rotor removed and the stator windings excited with three-phase currents for a particular instant of balanced operation. The iron filings are aligned to indicate the 4 poles of the stator winding. The tuft of the filings around the circumference of the stator is due to the presence of the stator teeth.

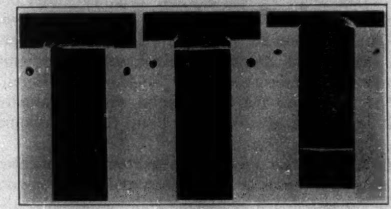

FIG. 2

The calculation of slot reactance is usually made on the assumption that a horizontal element of current in the slot produces no flux between the element and the bottom of the slot. The only flux produced by the horizontal element of current must cross the slot above the element, assuming no saturation. In these three cases the current is confined to a horizontal copper strip. Intense magnetic field exists above the element, whereas below no field is indicated by the filings.

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[6] *Synchronous Machines, Part I, An Extension of Blondel's Two-Reaction Theory*, by Messrs. Doherty and Nickle, A. I. E. E. October JOURNAL, 1926, p. 974.

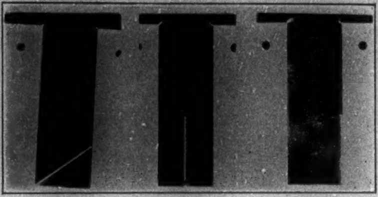

FIG. 3

The strip of current carrying copper along the side of the slot in the right hand figure represents, after a manner, the current in the field winding of a salient pole machine. The flux indicated by the iron filings is equivalent to the field pole leakage flux. Evidently, the flux density increases from zero at the bottom of the slot to a maximum at the top of the current element. The center and left hand exposures have no practical application, but merely indicate flux distributions with the current element in two positions.

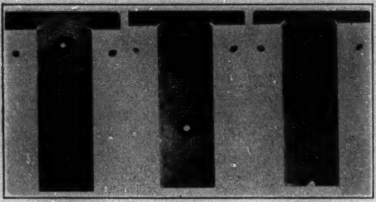

FIG. 4

By superposing the magnetic fields produced by a current carrying conductor located at a number of positions in the slot, the resultant magnetic field due to a current carrying armature coil can be obtained. Superposition of fluxes is only allowable where saturation does not exist.

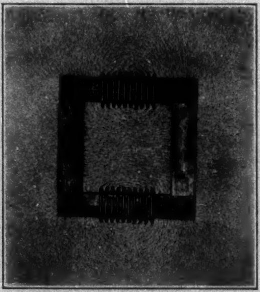

FIG. 5

A number of theoretical magnetic circuits have been studied, the magnetic field being plotted mathematically, graphically and experimentally. Such a circuit of rectangular form is shown here. The coils on the perpendicular legs are magnetized to send flux upward. Full potential, therefore, exists between the top and bottom horizontal arms. This accounts for the large amount of leakage flux in the perpendicular gap between the coils.

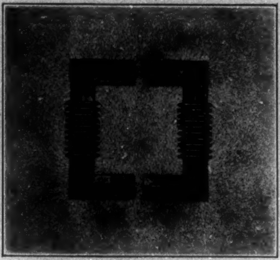

FIG. 6

This circuit is identical with No. 5 except for the air-gaps inserted in the horizontal arms.

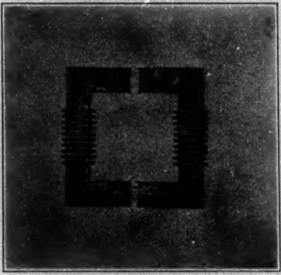

FIG. 7

This is similar to No. 6 except that the coils on the perpendicular arms are magnetized in the same direction; that is, with magnetomotive forces adding. The resultant flux around the circuit is the sum of that which would be produced by considering each coil alone, neglecting saturation. The flux density in the gap is very great, indicated by the tufting of the flux at the gaps.

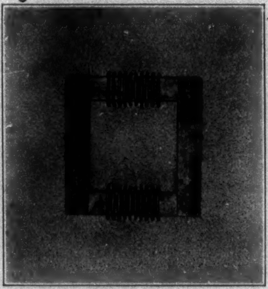

FIG. 8

The air-gaps are situated under the excited coils. Before this field was photographed extreme difficulty was experienced in an attempt to construct the field graphically. The difficulty was one of magnetomotive force distribution.

C. M. Laffoon: The graphical method of mapping electric and magnetic fields has been used by designing engineers for a large number of years to determine the physical dimensions and performance characteristics of electrical machines. In most instances, no particular effort was made to insure that the flux distribution satisfied all of the theoretical conditions, yet very accurate and reliable results were obtained. This was particularly true in calculating the magnitude and shape of the e. m. f. waves of rotating electrical machinery.

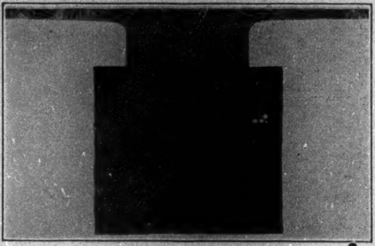

FIG. 9

The stator and a portion of two field poles of a synchronous machine with a fifth harmonic cosine of magnetomotive force impressed along the stator periphery. The flux distribution shown is the result. This particular circuit was employed in certain studies made in conjunction with the preparation of the paper "Synchronous Machines—Part I" by Messrs. Doherty and Nickle.

It has only been during the last two or three years that special consideration has been given to the problem of determining the distribution of the magnetic field in the space occupied by and adjacent to the electric conductors, with greater accuracy and refinement, by means of both mathematical and graphical methods of analysis. There can hardly be any question but that the mathematical method of analysis is the most rigorous and scientific, yet in most cases the graphical method is simpler and more convenient to use, and gives equally reliable results. It is for this reason that our own efforts have been confined to the use of the graphical method.

In Mr. Wieseman's paper, the graphical determination of magnetic fields has been applied to salient pole synchronous machines for the case in which the magnetization is produced by either the stator or rotor windings alone. The flux distribution for these cases are also given in the paper on *Additional Losses in Synchronous Machines*, by Mr. Calvert and myself. It is interesting to note that the results are essentially the same in the two papers. We have also applied the graphical method of determining magnetic fields to turbine-generators in connection with studies on additional losses and leakage reactances. In this connection, the distribution of the magnetic field in the following parts of the magnetic circuit of a turbine generator have been determined:

1. Air-gap space for the following load conditions:
   a. No load with rotor excited to give normal voltage,
   b. No load with the stator excited to give normal voltage,
   c. Full load at power factors of zero, 80, and 100 per cent.
2. End-bell space under the same load conditions as for No. 1.
3. Rotor and stator core at no load with the rotor excited to give normal voltage.

Part of these results are given in our paper on *Additional Losses in Synchronous Machines*. Some of the remaining cases will be referred to in a discussion by Mr. J. F. Calvert.

J. F. Calvert: In making flux plots, the greatest assistance that one can have is another drawing of a similar field. If a sufficient number of type cases can be solved then the solution of any particular problem becomes quite easy. It is probable that twenty or thirty of these reference solutions would cover practically all of the special types of problems which could be found in making the two dimensional figures for rotating apparatus. In electrical machines the arrangement of the magnetomotive

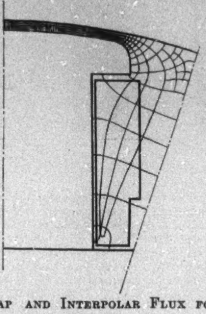

FIG. 10—AIR-GAP AND INTERPOLAR FLUX FOR A TEN-POLE MACHINE

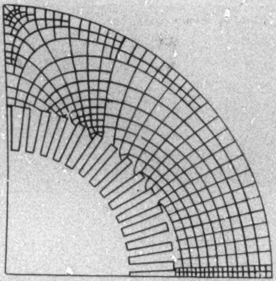

FIG. 11—FLUX DISTRIBUTION IN THE CORE OF A TWO-POLE TURBINE GENERATOR ON THE ASSUMPTION OF UNIFORM PERMEABILITY

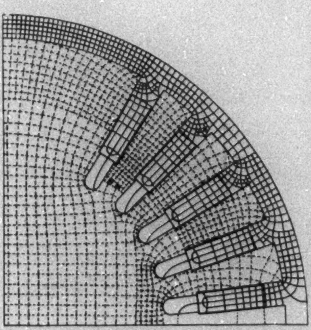

FIG. 12—FLUX DISTRIBUTION IN THE ROTOR OF A TWO-POLE TURBINE GENERATOR

forces and the iron surfaces are usually such as to render a mathematical solution either very difficult or in many cases wholly impossible at the present time. Therefore, it would seem that a good procedure would be to project the work by graphical solutions which should later be verified or corrected by mathematical solutions whenever possible.

Somewhat recently we have been doing work along similar lines to establish graphical solutions, but our work was done primarily in connection with loss studies. The theoretical basis of our work is described in the latter part of a paper on *Additional Losses of Synchronous Machines*. Examples are shown there of the application of the theoretical principles for plotting fields in the interpolar space on salient pole machines, and in the air-gap and end-bell zones of non-salient pole machines. In the latter solutions for non-salient pole machines, both the stator and the rotor windings are assumed to be carrying currents.

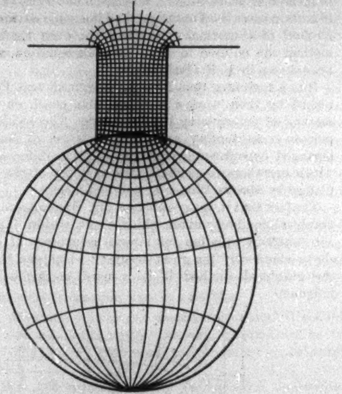
Fig. 13—Flux Distribution Across a Round Slot When the Conductor is in Contact With the Iron

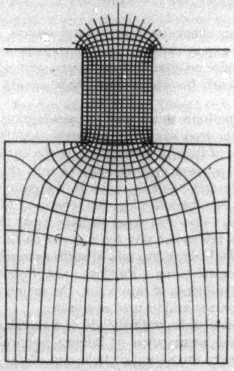
Fig. 14—Flux Distribution Across a Rectangular Slot When the Conductor is in Contact With the Iron

In Figs. 10 to 15 accompanying this discussion are shown graphical solutions to some other magnetic problems of interest. In the figures showing the flux distribution inside of the damper bars under steady conditions, the exact location of the kernel or center is rather difficult. The location of this point makes a considerable difference in the appearance of the picture, but usually makes very little difference in the total amount of flux and much less difference in the flux turn interlinkages. Mathematical solution, however, should locate these points beyond a doubt.

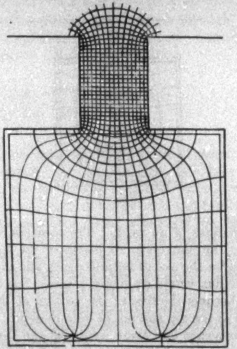
Fig. 15—Flux Distribution Across a Rectangular Slot When the Conductor is not in Contact With the Iron

J. S. Woodward (contributed after adjournment): In discussion of the paper by Messrs. Stevenson and Park, Mr. J. F. Calvert presented two figures showing the flux distribution in a slot containing a conductor carrying current. The correctness of these two flux plots presented was questioned, and Messrs. Stevenson and Park asked me to calculate the distributions by the mathematical theory, outlined in their paper. The result of such calculation is shown in Figs. 16 and 17 herewith.

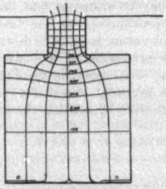
Fig. 16

Fig. 16 shows the flux distribution in a slot where the copper and iron are in contact with each other. Here, the calculations show there are two kernels, or foci, of the lines of no work, located in the lower corners of the slot.

Fig. 17 shows the flux distribution for the case of copper and iron insulated from each other, and here, there is but one kernel located on the vertical center line slightly above the bottom edge of the copper.

In both cases the flux lines at any appreciable distance from the kernel are very nearly flat, and the lines of no work are nearly vertical as they approach the bottom of the slot. Near the bottom, they turn sharply toward the kernel and are crowded together in the bottom of the slot. An enlarged view of the flux in the bottom of the slot is shown in the accompanying Figs. 18 and 19, which refer to Figs. 16 and 17, respectively. The sharp curvature of the lines of flux and also of the lines of no work is of particular interest as it illustrates the general statement made in Part II of the paper as to the action of these lines in the vicinity of the kernel.

The two cases under discussion are of interest, as they bring up the question of the location of the kernel. In free-hand plotting, the location of this point is important as it furnishes a basis for drawing in the lines of no work. The location of the kernel is determined by the boundary conditions of the copper.

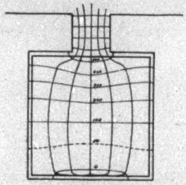

FIG. 17

In Fig. 16, were the slot of infinite depth and not partly closed at the top, the flux would go straight across the copper and the kernel would become the line at the bottom of the slot. If the slot were open at the top and not of infinite depth, the flux would be concave downward and there would be one kernel located on the vertical center line of the slot.

In Fig. 16, where the top of the slot is partially closed, the flux is concave upward, and this condition, carried to the bottom of the slot, causes the flux leaving the sides of the slot near the bottom to enter the iron before reaching the vertical center line and as a result there are two kernels, one in each lower corner of the slot.

In Fig. 17, if the slot were not partially closed at the top there would still be but one kernel, located on the vertical center line, but this kernel would be farther from the bottom edge of the copper than is the case in the figure. Partially closing the slot causes the flux near the top of the slot to have a curvature that is concave upward, thus tending to depress the kernel.

In Fig. 17, it may be observed that the curvature of the flux lines reverses approximately at the line marked 100. From there down, the curvature is concave downward; hence the effect of the insulation predominates and we have but one kernel, located as shown. It is possible to obtain two kernels in the general case of a partially closed slot, when the conductor is insulated from the iron, by making the insulation thinner or by making the depth of the slot smaller in comparision with the width.

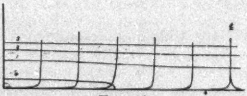

FIG. 18

In locating the kernel from a plot made by the mathematical formula, where the flux lines close to the kernel are very flat, two considerations may be followed. First, the general location of the kernel will be clearly indicated, as it is known to be in the region of lowest flux density, the density at the kernel being zero. Secondly, the flux lines surround the kernel, and the rate of change of flux, along any line passing through the kernel will change from positive to negative as the kernel is passed. Hence, by taking $\partial r/\partial x = 0$ for a value of (y) known to lie near the kernel, the (x) coordinate of the kernel will be obtained to a high degree of approximation. Then, taking $\partial R/\partial y = 0$ along this (x) coordinate will be determined to good accuracy. The accuracy can be made as great as desired by repeating this process.

Th. Lehmann (communicated after adjournment): By their ingenious extension of Professor Rogowski's method of calculation to interpolar fields, Messrs. Stevenson and Park have shown that in certain cases the Fourier method has some advantages over the method of conformal representation, even though the latter method can be used to solve Poisson's equation, as has already been shown by E. B. Christoffel[1].

But I maintain that Messrs. Stevenson and Park have furnished by their work a very welcome check on the graphical method of determining lines of force. The result of this comparison is satisfactory, and the same is true of the very cleverly arranged experimental check by Messrs. Johnson and Green, which corroborates perfectly the sketches of the lines of force plotted by Messrs. Wieseman and Shildneck.

The fact that it was possible for Mr. Wieseman to obtain by the graphical method values which check within 1 per cent with Mr. Carter's equation for air-gap reluctance of infinite teeth, shows eloquently the great accuracy which can be obtained by the graphical method if it is used methodically and with judgment.

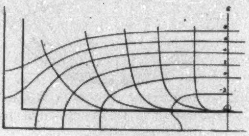

FIG. 19

Further, the charts and functional curves given in these papers for air-gap reluctance, tooth harmonics, reactances, etc., will be of service to practical engineers, and will save them a great deal of work, for which much credit should be given to the authors.

One might perhaps think that the assumption of infinite permeability in the iron would reduce somewhat the practical use of the sketches of lines of force given by the authors.

I am of the opinion that, in spite of this, these sketches are still interesting and valuable, even in the case where the poles are saturated, for the following two reasons. First, the saturation of the poles does not seem to have more than a slight influence on the interpolar flux for a given useful armature flux, even though the body of the pole absorbs as much as 30 per cent of the total ampere-turns. Further, even if one desires to take account of the influence of saturation, the sketches of the lines of force for $\mu = \infty$ can still be used as a basis.

I will later show how, with the aid of a sketch of lines of force obtained on the assumption of infinite permeability, one can deduce rigorously the actual distribution of the field when the circuit is saturated.

The sketches of lines of force given by the authors are, therefore, still of actual practical interest, even when there is considerable saturation in the iron, and it has seemed worth while to me to emphasize this fact.

J. F. H. Douglas: In one of the three papers a preference is expressed for graphical methods and I wish to say that with this new development of a means of treating the interior of copper, locating the flux and lines of zero work in that way, we have perhaps the most valuable contribution that has been made to the subject for many years.

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[1] E. B. Christoffel, Annali di Mathematica, 1867, ser. II, Vol. I, fasc. 1°, p. 89.

The test methods which I have used—namely, the use of high-resistance templets—are inadequate for such cases . Nevertheless, I cannot quite pass the matter without protesting that the difficulty attached to making tests with templets is somewhat exaggerated. It is a very speedy method, both for determining total reactance and also for determining flux densities to any point of the boundary.

I have used all three methods and templets, and I find the labor with templets somewhat less than in the case of using graphical distribution, and decidedly less than that using the functions of complex variable.

I do not think that the possibilities of the functions of complex variable, however, have been fully appreciated. Generally the writers feel it incumbent to evaluate the whole field of force. Where problems of boundary densities are concerned, which alone would be of interest in loss calculations, and where problems of the total reactance or permeance of the field is in question, then there is a very neat way of handling Schwartz's and Christophel's theorem, which I don't recall having seen in print. That is a method of graphical integration around the boundary. Schwartz's and Christophel's theorem is a very simple matter of formulating, provided the boundaries are straight line boundaries, but extremely difficult to integrate, except for the simplest cases. Nevertheless, on the boundary, the functions are wholly real and a graphical integration is easily possible.

I ask Mr. Wieseman one or two specific questions. I should like, either now or in the written closure, information as to whether Fig. 2 checks my 1915 work[2] and whether Fig. 3 checks[2] my 1924 work.[3]

I should like also to make one or two detailed comments. In Mr. Wieseman's paper is the statement: "If the number of teeth spanned by a pole varies, a certain amount of flux pulsation is necessary."

There is one shape of pole which will avoid all pulsations when used with armatures of any amount of slotting and every amount of slotting; namely that pole shape which gives a perfect sine wave on the equivalent armature circuit. Such a punching could be used with any amount of slotting opposite, without pulsation. The proof of this proposition was given in the appendix of the paper read by Douglas and Kane in Chicago two years this coming June.

A rough calculation of Mr. Wieseman's coefficient as $A\,q\,1$ and as $A\,D\,1$ for complete range of pole pitches, air-gaps, and pole-face curvatures used seems to lie within the range of 40 to 60 per cent. It seems that the problem of obtaining extremely high pull-out torque (I mean pull-out torques greatly in excess of present ratings) is not yet possible without a radical change of pole design.

It may be that there are some particular combinations of pole pitch, air-gap, and pole curvature which will give smaller ratios of the transverse coefficient, but it should now be clear that in order to develop extremely sturdy synchronous motors, it is needful that the ratios of $A\,q1$ to $A\,d1$ should be brought very much less.

Vladimir Karapetoff: (communicated after adjournment) The mathematical treatment in Appendix C may be given also in the language of vector analysis. While the authors are justified in using ordinary partial derivatives, so as to make the theory comprehensible to a larger circle of engineers, yet, with an ever increasing interest in vector analysis among younger engineers and physicists, the alternative abbreviated treatment, added below, may not, for the sake of completeness, be out of place. Several elementary works on vector analysis are now available, so that the exposition is given without proofs or definitions. The great advantage of this new "short-hand" language is that no axes of coordinates are used (crutches or scaffolding, as some vector analysis enthusiasts call them), and the quantities concerned are dealt with directly in their magnitude and direction in space, thus bringing out more clearly the physical relationships.

By the definition of vector potential

$$\mathbf{H} = -\nabla \times \mathbf{R} \tag{1}$$

According to the magnetic-circuitation law

$$4\,\pi\,\mathbf{i} = \nabla \times \mathbf{H} \tag{2}$$

Substituting **H** from (1) in (2), gives

$$-\,4\,\pi\,\mathbf{i} = \nabla \times (\nabla \times \mathbf{R}) \tag{3}$$

Since the vector potential is defined through its curl only, an additional condition may be imposed; namely, that **R** is a solenoidal vector ($\nabla\,.\,R = 0$).
Hence

$$\nabla \times (\nabla \times \mathbf{R}) = \nabla\,(\nabla\,.\,\mathbf{R}) - \mathbf{R}\,(\nabla\,.\,\nabla) = -\,\nabla^2\,\mathbf{R} \tag{4}$$

Consequently, Eq. (3) becomes

$$4\,\pi\,\mathbf{i} = \nabla^2\,\mathbf{R} \tag{5}$$

which is identical with Eq. (**30**) in the paper, ∇^2 being the Laplacian operator. If, in Eq. (2), H were expressed in rational units, the factor $4\,\pi$ would be entirely absent from the equations, thus still further simplifying the result.

In a two-dimensional field, according to Eq. (1), if the component of **H** in a certain direction is zero, **R** must be constant in the perpendicular direction. This follows directly from the definition of the curl as a line integral. Consequently, the equation of a line of force is **R** = const.

At a point in the two-dimensional field, consider the direction n in which R varies most rapidly. We then have from Eq. (1), for the absolute value of H:

$$H = \partial\,R / \partial\,n \tag{6}$$

so that

$$d\,R = H\,d\,n \tag{7}$$

This result indicates that an increment of R is equal to the flux between two lines of force to which the two values of R refer; Eq. (**33**) in the paper.

R. W. Wieseman: Perhaps the most interesting, as well as the most important, application of graphical flux plotting at the present time is the calculation of the quadrature synchronous reactance of a salient-pole machine. As a matter of fact, the value of the quadrature synchronous reactance can be predetermined only by a field plot. This quadrature synchronous reactance is one of the several coefficients which appear in the paper *Synchronous Machines*, Part II, 1926, by Messrs. Doherty and Nickle.

In a polyphase machine, the armature currents produce a sine wave of flux which travels in synchronism with the poles. When the armature m. m. f. axis coincides with the pole axis (for example, at zero power factor), the flux which the armature currents tend to produce is much more than the armature flux when the armature m. m. f. axis is in quadrature with the pole. Thus, the direct synchronous reactance is more than quadrature synchronous reactance.

Let the normal fundamental flux per pole = ϕ and let the normal armature currents produce a flux ϕ_q in the quadrature axis.

Let the fundamental of this armature flux = ϕ_{q1}

Let X_{lq} equal the armature leakage reactance (expressed as a decimal fraction) in the quadrature axis.

Then the quadrature synchronous reactance $X_q = \dfrac{\phi_{q1}}{\phi} + X_{lq}$.

The values of ϕ_{q1}, as well as many other coefficients, are given in this paper.

With reference to the Laffoon and Calvert discussion: I note that they also prefer the graphical method of obtaining the flux distribution and the flux distribution coefficients for design calculations.

Mr. Douglas stated that the templet method is very speedy, and that the labor involved with templets is somewhat less than in the case of using the graphical method. There are a few simple cases of magnetic-flux distribution where the templet method can be used to advantage, especially in illustrating flux fringing to students. When, however, the flux distribution is to be obtained in the many different magnetic parts of dynamo machinery, I prefer the graphical method, and I have found that the results obtained by this method are quite accurate.

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[2] *Potential Gradient and Flux Density*, by J. F. H. Douglas and E. W. Kane, TRANS. A. I. E. E., 1924, p. 982.

[3] *The Reluctance of Some Irregular Magnetic Fields*, by J. F. H. Douglas, TRANS. A. I. E. E., 1915, p. 1067.

Mr. Douglas requested information as to whether Figs. 2 and 3 checked his work. In Mr. Douglas' 1915 paper, the ratio of the slot width to the tooth width is very much larger than is used in Fig. 2 of my paper. Consequently, no check can be made. Fig. 6 in the article by Messrs. Douglas and Kane, in 1924, practically checks my Fig. 3.

Mr. Douglas stated that the flux pulsations can be avoided with armatures having any number of teeth, if the pole gives a perfect sine wave of flux on the equivalent armature circuit.

Salient-pole machines practically never have perfect sine flux waves and, therefore, this method of eliminating flux pulsations cannot be used.

A. R. Stevenson: Messrs. Laffoon and Calvert are right in saying that the graphical method is the most convenient one for use in the everyday designing of electrical machinery, but the mathematical method is of importance in the preparation of the 20 or 30 typical flux plots which Mr. Calvert mentioned as being of great assistance in sketching similar fields. Although these typical flux plots can be determined to any degree of accuracy by the graphical method, in some cases it takes a great deal of experimental sketching before the general outline of the distribution can be determined; whereas, in such cases, the mathematical method will sometimes give a more accurate answer with less work, in less time.

The best articles on graphical plotting are contained in the long series of articles by Lehmann, mentioned in the bibliography of our paper. He carried the graphical method of plotting much farther than we did, including in the problem the saturation in the iron. If anyone desires to study the graphical method further, he could not do better than to refer to Lehmann's work.

Mr. Calvert, in his discussion, has submitted six pictures of flux distributions sketched by the graphical method. The two showing the flux distribution in a square bar in a square slot did not look quite right to us. They are sufficiently accurate for all practical purposes; but we do not agree with the location of the kernels, and Mr. Woodward has submitting a discussion in which he shows the flux distribution in these same square bars in square slots, as determined by the mathematical method described in our paper.

Professor Douglas' discussion is of special interest because of the articles he has already published on this subject, especially with regard to the templet method of determining flux distribution and also because of his remarks about the use of the theory of functions of a complex variable in connection with these problems. The valuable work of Dr. Carter in this connection is well known, and is especially interesting because of the recent publication of another paper in which he applies the method to many new distributions which had not been attempted before.[4] The application of the Schwarz and Christoffel transformation is very difficult when it is necessary to integrate around more than four angles, and the suggestion by Professor Douglas of a graphical method of integration should be of great assistance in the application of the theory of functions of a complex variable to these problems.

The authors are very grateful to Dr. Lehmann for emphasizing that their sketches of magnetic lines of force are of actual practical value, in spite of the fact that saturation was neglected. The article by E. B. Christoffel, which he mentions, we think will be a valuable addition to the bibliography, although his discussion came in so late that we have not had a chance yet to look it up. The use of conformal representation for the solution of Poisson's equation has been done by St. Venat in the solution of the torsion of rods; see, for example, Love's "Theory of Elasticity." Herr M. Strutt, in the *Archiv fur Elektrotechnik*, April 7, 1927, has applied this method to the approximate solution of the case of a current-carrying rectangular iron conductor of constant (high) permeability.

Mr. E. E. Johnson: While the iron-filing method gives excellent indications of the form of the magnetic field, the field plots so determined must not be interpreted too strictly as regards field intensities. When making the iron-filing distributions, the filings, which are preferably of cast iron, are first distributed on some suitable plain surface, such as white paper. The m. m. fs. are then applied and the whole structure is gently tapped to allow the filings to take their proper conformations.

In the process of tapping, the iron filings in the near vicinity of highly magnetized iron surfaces have a tendency to skip along the paper and gather in clusters on those surfaces. Also, even in uniform magnetic fields of high intensity the filings cluster together in strings. This clustering leaves free open spaces, from which it might be inferred, on superficial examination, that the flux density is low at these points.

Mr. Green and I are very grateful for the comments of Mr. Laffoon and Mr. Calvert. The experimental method which we have used for determining the field distribution inside of current-carrying regions is not always convenient although there may be cases where it might with profit be employed. The method was used in the particular case of the alternator field poles as a check upon the work which Mr. Stevenson and Mr. Park did in their paper.

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[4] "The Magnetic Field of the Dynamo-Electric Machine." The *Institution of Electrical Engineers*, Vol. 64, No. 359, November, 1926.

Design of Reactances and Transformers
Which Carry Direct Current

BY C. R. HANNA[1]
Associate, A. I. E. E.

***Synopsis.**—It is usually necessary to place an air-gap in the core of a reactance or transformer which carries direct current in order to secure the greatest inductance. The work here reported points out a direct method of designing such reactances or transformers including the determination of the best value of the air-gap.*

* * * * *

INTRODUCTION

THE design of reactances or transformers in which considerable direct current flows is a problem of increasing importance. Interstage and output transformers for vacuum tube amplifiers, modulator chokes for radio-telephone transmitters, and reactances for rectifier filter circuits are examples. In all of these a high value of a-c. inductance is required, but the saturating effect of the direct current always causes the inductance to be lower than if it were not flowing. It is well known that in every such case an increase of inductance will result if an air-gap is introduced in the magnetic circuit. Where the steady m. m. f. is high, the best air-gap will be large; where it is low the best air-gap will be small, sometimes small enough so that the air spaces in the stacking of the core laminations are sufficient.

So far as the writer is aware, no direct method of predetermining this best air-gap has been presented. The usual method is to assemble a reactance or transformer and determine experimentally the best gap. The inductance for this best gap usually does not come to the required value, and a re-design is necessary. After several attempts, of course, the correct design can be determined, but a direct method of design is greatly to be desired.

The purpose of this paper is to set forth a straightforward method of solving this problem.

METHOD OF CALCULATION

Use is made of the permeability curves, both normal and incremental, for the core material used. As an example, the calculation will be carried through for four per cent silicon steel. Curves of Fig. 1 show the normal and incremental permeability for different values of B. Incremental permeability values are for very small minor loops, and were calculated from information given in a paper by Spooner[2]. Values for very small loops are used because in many cases the requirements are that the inductance shall be equal to, or greater than, a certain value for any applied alternating voltage, no matter how small, and it is well known that the incremental permeability (and therefore the inductance), is smallest when the flux variations are small. Examples are the modulator choke in a radio-telephone transmitter, interstage and output transformers in audio amplifiers, etc. In the case of reactors for filter circuits where the pulsations are always large, advantage can be taken of the larger incremental permeability corresponding to the greater flux variations. The calculations here given do not include this case, but may serve as a guide in the design of such reactances. The magnitude of the flux variations must be known or determinable for such calculations.

The following notation will be used:

B = Steady flux density in iron and air-gap, gausses.
N = Number of turns in winding.
I = Direct current, amperes.
A = Area of core section and air-gap, cm.^{2}
l = Length of iron path, cm.
a = Air-gap length, cm.
L = A-c. inductance, henries.

μ = Normal permeability = $\dfrac{B}{H}$

μ_Δ = Incremental permeability = $\dfrac{\Delta\, B}{\Delta\, H}$ where $\Delta\, B$ and $\Delta\, H$ are the increments from tip to tip of a minor hysteresis loop.

We have

$$B = \frac{0.4\,\pi N I}{\dfrac{l}{\mu} + a} \qquad (1)$$

and

$$L = \frac{0.4\,\pi N^2 A \times 10^{-8}}{\dfrac{l}{\mu_\Delta} + a} \qquad (2)$$

From (1)

$$N = \frac{B\left(\dfrac{l}{\mu} + a\right)}{0.4\,\pi I} \qquad (3)$$

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[1] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[2] T. Spooner, "Effect of a Superposed Alternating Field on Apparent Magnetic Permeability and Hysteresis Loss," *Physical Review*, 1925.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

Substituting in (2)

$$L = \frac{B^2 \left(\dfrac{l}{\mu} + a \right)^2 A \times 10^{-8}}{0.4\,\pi\, I^2 \left(\dfrac{l}{\mu_\Delta} + a \right)}$$

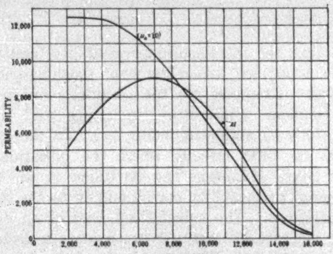

FIG. 1—NORMAL AND INCREMENTAL PERMEABILITY FOR 4 PER CENT SILICON STEEL

$$= \frac{B^2 \left(\dfrac{1}{\mu} + \dfrac{a}{l} \right)^2 l\, A \times 10^{-8}}{0.4\,\pi\, I^2 \left(\dfrac{1}{\mu_\Delta} + \dfrac{a}{l} \right)} \qquad (4)$$

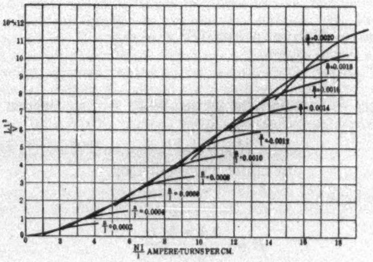

FIG. 2—4 PER CENT SILICON STEEL

Letting $l\,A = V$, the volume of iron in the core,

$$\frac{L\,I^2}{V} = \frac{B^2 \left(\dfrac{1}{\mu} + \dfrac{a}{l} \right)^2 \times 10^{-8}}{0.4\,\pi \left(\dfrac{1}{\mu_\Delta} + \dfrac{a}{l} \right)} \qquad (5)$$

Also from (1)

$$\frac{N\,I}{l} = \frac{B}{0.4\,\pi} \left(\frac{1}{\mu} + \frac{a}{l} \right) \qquad (6)$$

For any assigned value of $\frac{a}{l}$ (the per cent air-gap) equations (5) and (6) may be considered as parametric equations with B as the parameter, and a curve of $\frac{L\,I^2}{V}$ against $\frac{N\,I}{l}$ can be plotted. To do this, several values of B are assigned, and the values of μ and μ_Δ corresponding to B obtained from curves of Fig. 1. These values are substituted in equations (5) and (6) to determine corresponding values of $\frac{L\,I^2}{V}$ and $\frac{N\,I}{l}$. $\frac{N\,I}{l}$ represents the steady ampere turns for each centimeter of iron length and $\frac{L\,I^2}{V}$ is a quantity which if divided by the square of the current gives the inductance per cm.^{3} of core.

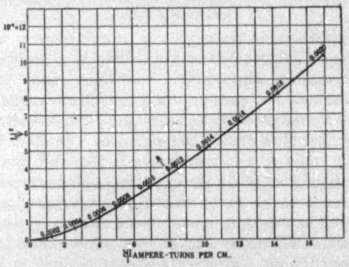

FIG. 3—4 PER CENT SILICON STEEL

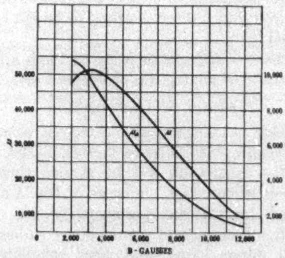

FIG. 4—NORMAL AND INCREMENTAL PERMEABILITY CURBES FOR HYPERNIK

The family of curves, each for a different value of $\frac{a}{l}$, is shown in Fig. 2. It is seen that if $\frac{N\,I}{l}$ is increased, by increasing N or I or by reducing l, $\frac{L\,I^2}{V}$

is greater for larger values of $\frac{a}{l}$. Evidently the envelope of the family of curves gives the relation between $\frac{L\,I^2}{V}$ and $\frac{N\ I}{l}$ if the best value of $\frac{a}{l}$ is chosen. Since each curve of the family corresponds to a certain value of $\frac{a}{l}$, the point of tangency with the envelope shows the value of $\frac{N\ I}{l}$ that requires this $\frac{a}{l}$. Hence along the envelope curve may be plotted a scale which shows the proper value of $\frac{a}{l}$. Fig. 3 shows the envelope curve with the $\frac{a}{l}$ scale along it.

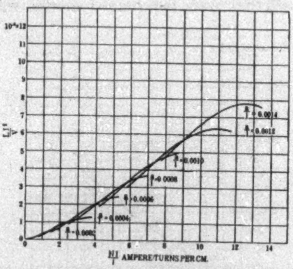

FIG. 5—HYPERNIK

Using this curve, it is quite simple to design reactances. Suppose a certain core size is chosen and a winding and air-gap are to be determined such that when a direct current I flows in the winding, the a-c. inductance will be L. The value of $\frac{L\,I^2}{V}$ is thus determined, and the corresponding value of $\frac{N\ I}{l}$ can be obtained from the curve. The core length l and the current I being known, N is determined. The value of $\frac{a}{l}$ can also be read from the curve, and thus the proper air-gap is determined.

To illustrate with a specific example, suppose

l = 14 cm.
A = 5.5 cm.^{2}
L = 12 henrys
I = 0.05 amperes

then

$$\frac{L\,I^2}{V} = \frac{12 \times 0.05^2}{14 \times 5.5} = 3.9 \times 10^{-4}$$

From curve

$$\frac{N\ I}{l} = 8.43$$

$$N = \frac{8.43 \times 14}{0.05} = 2360$$

Also

$$\frac{a}{l} = 0.0012 \qquad a = 0.0168 \text{ cm.}$$

It may be that to obtain the 2360 turns in the given winding space the resistance will be too high. Where this is the case, a larger sized punching or perhaps more punchings of the same size should be used, so as to increase the iron section. The calculation should be carried through again in the same way, a few trials usually being sufficient.

It frequently happens that the greatest inductance possible is desired for a given core size. If I has a definite value as before, it is readily seen from the curve that the greater the number of turns the greater the inductance for a given volume of iron, provided the air-gap is increased as shown by the curve. Hence a coil with the greatest possible number of turns as determined by the permissible resistance or capacity of the winding should be employed.

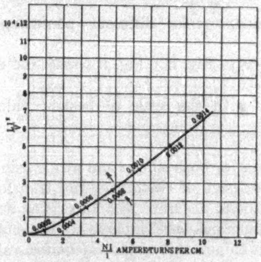

FIG. 6—HYPERNIK

For example, suppose with the core given in the first example, 4000 turns could be wound without exceeding the limiting value of resistance. If the current is as before

$$\frac{N\ I}{l} = \frac{4000 \times 0.05}{14} = 14.3$$

From the curve

$$\frac{L\,I^2}{V} = 8.2 \times 10^{-4}$$

from which

$$L = \frac{8.2 \times 10^{-4} \times 14 \times 5.5}{(0.05)^2} = 25.2 \text{ henries.}$$

The value of $\frac{a}{l}$ to obtain this is 0.0018. Therefore

$a = 0.0018 \times 14 = 0.025$ cm.

The same calculation has been carried through for 50 per cent nickle iron. Curves of Fig. 4 show the normal and incremental permeability, the latter having been calculated as before. The family of curves of $\frac{L I^2}{V}$ against $\frac{N I}{l}$ for different percentages of air-gap is shown in Fig. 5. Fig. 6 shows the envelope of this family with the scale of best $\frac{a}{l}$ values plotted along it.

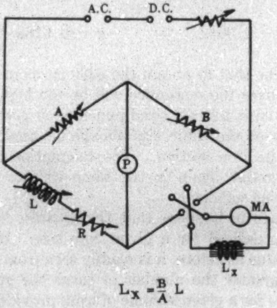

Fig. 7—A-c. Bridge

The curves for both silicon and nickle iron have been used in the design of reactances and transformers and have been found accurate to a surprising degree when annealed punchings were employed. Inductance values were within 15 per cent of the calculated values. The best air-gap, not being critical, was always close enough so that no change from the calculated value was necessary.

Alternate Method of Determining Curves

It is seen readily that the curves may be determined from experimental data instead of by computation. Each curve of the family is the relation of $\frac{L I^2}{V}$ against $\frac{N I}{l}$ for a given value of $\frac{a}{l}$. If a core of uniform section is secured and a winding placed on it, its a-c. inductance for different values of direct current may be measured by means of an a-c. bridge as shown in Fig. 7. Holding the air-gap fixed, the data for one curve of $\frac{L I^2}{V}$ against $\frac{N I}{l}$ can be determined. The air-gap may then be changed and another set of data obtained, and so on until the whole family of curves is determined. Then the envelope with its $\frac{a}{l}$ scale may be drawn.

Fig. 8 shows two curves of the family for 50 per cent nickle steel that were determined in this way. The envelope is also shown and is found to agree fairly closely with the calculated envelope of Fig. 6.

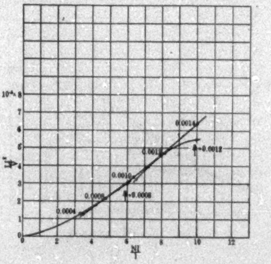

Fig. 8—Experimental Curve for Hypernik

In using the bridge, the direct current in the choke or transformer is varied by the resistance in series with the supply. A reversing switch is used to make sure that the induction in the core is that corresponding to the normal *B-H* curve. Small values of a-c. voltage are employed.

This method is best adapted to cases where the normal and incremental permeability values for the material are not available.

Discussion

D. C. Prince: The first question that came into my mind was, what is the real measure of these reactances? The reasoning that I went through is something like this: if we have a certain small variable current that is produced by a ripple voltage in the case of a rectifier or by whatever the alternating impulse is, and then we have a steady direct current that must be carried, the function of the reactor is to keep the ratio of the incremental current, the variable current to the direct current, as small as possible.

Or, looking at it the other way, we want to make the ratio of the direct current carried to the incremental current as large as possible. The incremental current will be inversely proportional to the reactance. So if we simply put a constant representing $2 \pi F L$, we find that we can consider that the measure of the performance of our piece of apparatus is proportional to the product of current and inductance.

Mr. Hanna has considered that the current should be squared, and I will be interested in hearing why he squared it, but while I am about it, I might as well carry the story on, upon the basis that we used, which was taking the first power of current instead of the square as our measure.

Referring to the accompanying Fig. 1, the product of inductance and current is taken vertically; the current is horizontal. Then for any particular air-gap we will get a curve of the sort, shown rising to a peak and then falling again. For the larger air-gap, the curve rises to a higher value at a higher current, and if we secure a number of such curves and then proceed to draw a curve that includes them, you will find that as the air-gap increases, the effectiveness of our inductance also increases, and it appears to approach a constant value somewhere in the neighborhood of corresponding to an air-gap of approximately 4 per cent of the total mean magnetic path.

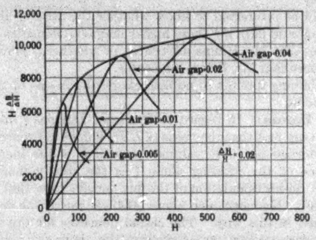

Fig. 1

That leads then to the question, "Why don't we always make the air-gap at least 4 per cent or more as long as the curve seems to be steadily climbing upward?" The answer to that is more or less suggested in Mr. Hanna's paper, although no direct attention was drawn to it. This small air-gap reactor giving its maximum inductance at a low value of current, when you design it is made almost wholly of iron, has hardly any copper in it. As you go in the direction of greater and greater air-gaps, you get larger and larger ratios of copper to iron, and if you are making a large piece of apparatus such as the reactor to smooth out the current from the rectifier, it becomes very important to strike a balance between the copper and iron. For very small pieces of apparatus of course that is less important, but for the larger things such as the chokes for radio transmitters and the filter reactor for rectifiers, the dominant thing is to strike a balance between copper and iron so as to get the most economical structure.

To that end we arrived at a plot slightly different from the one which Mr. Hanna has used. We plot inductance for the unit volume and air-gap, and then we have another plot giving the relation between the air gap and the ampere-turns as shown in Fig. 2 herewith. Taking such a plot as that, you simply assume a series of air-gaps for each one of which you can design a reactor immediately, and then having such a series the most economical one may be chosen, and the actual reactor built from it.

There is just one other point about the paper. Mr. Hanna has been dealing with very small increment of current, and as the alternating components increase, the effective reactance also increases, and we have thought it necessary, therefore, to have not one curve for the minimum variation in current but a group of curves so that we can design for what the demanded current pulsations are.

C. R. Hanna: I have converted one of the curves given in Mr. Prince's Fig. 2 to which he has called attention. The best air-gap is along the abscissas, one curve representing ampere-turns and the other $\frac{L}{N^2}$. These two curves could be properly converted into one if the scale of ampere-turns were plotted along the $\frac{L}{N^2}$ curve.

I took the lowest curve, interchanged air-gap with ampere-turns and multiplied ordinates by the square of the abscissas, converting his curves into the same form that I have given in my paper.

As has been brought out, he considered much greater values of steady magnetization or steady ampere-turns per centimeter than I did.

The lowest of the several curves was chosen because it represents the smallest percentage variation in current. That corresponds more closely to mine than any other because I considered very small minor hysteresis loops.

The derived curve when extrapolated came slightly lower than mine all the way. Of course, the difference in material might account for this, but I was wondering if it might not be in the way his maxima were arrived at.

D. C. Prince: As I understand it, Mr. Hanna has struck the one real difference in the fundamentals. The question is whether the curve in Fig. 1 goes through a series of peaks or is a curve that is tangent. Mr. Hanna has made his curve tangent to the family and not through the peaks, and his method is undoubtedly correct. That is, the two give slightly different values and in his terms you get slightly higher values if you take your curve tangent to the family than you do if you go through the peaks. However, he is dealing with small values of air-gap and the reactors that we actually build usually run more than 1 per cent air gap, and when you get more than 1 per cent air-gap, the tangent and the peak come so close together that the choice is immaterial.

C. R. Hanna: As Mr. Prince has pointed out, the envelope curve rather than the curve through the maxima should be used. Mr. Prince raises the question as to why $L\ I^2$ was used instead of $L\ I$ as in his work. I had no particular preference in the matter, but when the results are obtained by employing values of μ_Δ corresponding to very small changes in magneto-motive-force instead of those which correspond to a constant $\frac{\Delta I}{I}$, the results will inevitably be in terms of I^2 instead of I.

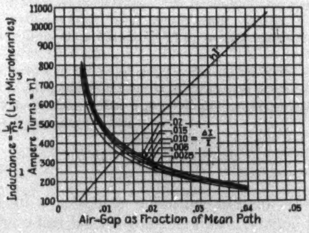

Fig. 2—Curve Showing Effect of Air-Gap on Inductance of Unit Reactor, (1-in. cube) With *n* Turns per Inch and Effect on Ampere-Turns for Least Pulsation Factor

J. F. H. Douglas: My interest in this topic is chiefly from the standpoint of Heising modulation choke coils. I wish to state that I think this paper is an important theoretical advance in the design of transformers and chokes carrying direct current. It gives a better gap for maximum inductance. However, I must point out that it is still a trial-and-error method for evaluating the necessary volume in the iron core.

The values of $L\ I^2/V$, which are roughly proportional to the energy storage per unit volume, indicate flux densities in the gap by rough calculations that I have made of only 24,000 lines per sq. in., or roughly 4000 gausses and represent an inefficient use of the iron material.

It is true in radio receiving apparatus that material cost is not such an important item, and the problem is economically handled in this particular case since the size of wire which is used though not stated here can hardly be much larger than No. 30 B & S gage, and the use of any smaller iron core would result in an impossible winding problem and an unworkable air-gap.

One of my senior students, Herbert Wareing, has designed and built and installed a Heising radio choke rated for an ultimate of 10,000 volts, 2 amperes and 60 henrys. The method that he used was one which, with slight modifications, I have taught for five years for low-frequency chokes and one which was derived from Professor Karapetoff.

I wish to say that I am opposed to empirical methods of design whenever rational foundation can be found. The rational foundation in Professor Karapetoff's method in the design of transformers and, I believe, chokes, is that of economical, specific, magnetic, and electric loadings and it should be gratifying to note its wide range of application.

There are other very important factors in the design of a Heising choke besides that of securing maximum inductance for a given core. Mr. Wareing reported to me that one important factor was leakage flux from the poles of the iron parts. Another was the distributed capacity between windings, and a third was the saturation which occurred when modulating low-pitched sounds of perhaps 20 or 30 cycles frequency.

The choke coil which he replaced resembled one of the old Edison bipolar dynamos, and it showed more inductance when the Keeper was entirely removed than when it was in place. The leakage flux alone saturated the legs at the corner point where they joined the yoke.

The choke that Mr. Wareing designed had a core weighing 220 lb. and copper coils of about 200 lb. The value of $L\,I^2/V$ was 74×10^{-4}, some six times greater than the maximum value recorded by Mr. Hanna's graph. The direct-current density was in the neighborhood of 60,000 lines per sq. in., roughly 10,000 gausses, the double air-gap 1¼ in., roughly 3 cm. the core 6 in. square, the length of the magnetic circuit, exclusive of air 46 in., and the ratio of air-gap to the mean magnetic path 2.7 per cent.

Obviously in Heising chokes of this rating we must use material efficiently, and a cut-and-try method for securing the iron core would not be entirely satisfactory. Since the installation of the choke coil, the music, by station W K A F where it is in use is reported to have much better quality of modulation than previously.

I would be interested to know whether the ratio of air gap to mean magnetic path for these higher magnetic densities checked the values that Mr. Hanna advocates, or whether it more closely checks those that Mr. Prince advocates.

Mr. Wareing intends to publish his results in one of the radio publications.

C. R. Hanna: In regard to Professor Douglas's discussion, I want to say first that the value of $L\,I^2/V$ does not represent the energy storage per unit volume because L is the incremental inductance of the winding while I is the steady magnetizing current. So his calculation of the flux density from that figure is not correct. He mentioned about 4000 gausses as the density he determined in this way. The actual densities for the range of ampere-turns per centimeter given in the paper are about 6000 or 7000 gausses. Of course the density is greater for larger values of $N\,I/l$. The scope of the paper was limited, however, to reactors and transformers having small steady m. m. fs.

Voltage Standardization of A-C. Systems

From the Viewpoint of the Electrical Manufacturer

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Synopsis.—*In outlining the history of voltage standardization, it is observed that there has been separate standardization of voltages of various types of apparatus, rather than standardization of a complete operating system. The results of a questionnaire answered by 22 operating companies are analyzed and the conclusion is drawn that the use of the old standard transformer voltages involved, in many cases, over-exciting the transformers or generators in order to maintain satisfactory voltage at the consumers' terminals. The reason is that the existing transformer voltage standards do not compensate for the line drop in transmission lines and feeders. The voltage standards of the International Electrotechnical Commission are set forth, indicating a partial agreement as to maximum system voltages with the proposed standards, although arrived at by a different method. The I. E. C. standardization, however, is not so complete as the proposed system.*

In describing the proposed standards, certain basic principles are laid down as the conditions which must be fulfilled.

The proposed system of voltage standards starts with already standardized utilization voltages at the low end of the scale and suggests tranformer voltage ratings and ratios which will allow proper voltage to be supplied to the consumers without over-exciting any of the transformers or generators in the system, and ties in the transformer and apparatus voltages with system voltages, based on the A. I. E. E. definition of rated circuit voltages. The proposed standards thus cover the whole field of voltages of a-c. apparatus of all kinds and harmonize them with system voltages in such a way that all reasonable operating requirements may be met.

The salient features of the proposed standardization are as follows:

The system voltage is the same as the highest rated voltage of transformers supplying the system; it thus corresponds to the A. I. E. E. rated circuit voltage and fixes test voltage on all apparatus used on the system.

Step-down transformer secondary voltages from 115 volts up to 69,000 volts will be multiples of 11.5, excepting transformers supplying 2400-volt systems, which will be rated 2400 volts. Thus, typical step-down transformers will deliver

460 volts
6,900 volts
23,000 volts
69,000 volts

For higher voltages, step-down transformers will have secondary voltages in multiples of 11, thus:

88,000 volts
110,000 volts
132,000 volts
154,000 volts

complying with well established practise.

In order to enable the step-down transformers to deliver these voltages, their primaries will be rated in multiples of 11, thus:

6,600,
22,000, etc., up to 66,000 and above that

multiples of 10½, thus:

105,000
126,000
210,000, etc.

Step-up transformers, excepting the 2400-volt class, will have their high-tension windings rated in multiples of 11½ up to 69,000, and multiples of 11 above that, whereas their low-tension winding will be rated five per cent lower than the system voltages or generator voltages.

Thus, step-up and step-down transformers will not be interchangeable, but each will have the proper ratio for its purpose. To make them interchangeable would require 25-per cent range, which would involve too great an expense if applied to all transformers. Transformers of 25-per cent range may be required in many cases where power flows in either direction but such transformers should be of special design and this extra cost should not involve the whole line of transformers. The tabulation of voltages gives also the present manufacturers, standards for apparatus voltages, such as oil circuit breakers, disconnecting switches, etc. These standards for 88,000 volts and above correspond to the system voltages of multiples of 11. Below 88,000 volts they are somewhat higher than the recognized system voltages given in the tabulation in order to meet existing conditions in these lower voltages. The tabulation also gives motor voltages in multiples of 11, while the generator voltages, in order to allow for line drop, are multiples of 12 up to 2400 volts, and multiples of 11½ from 6900 volts up.

The last section of the paper gives a discussion of the economic advantages of voltage standardization, indicates the magnitude of the investments involved, and gives a general idea of the savings which may be made by standardization.

* * * * *

INTRODUCTION

THE lack of a logical and coordinated voltage standardization of apparatus extending from the initial supply to the utilization device has resulted in an unnecessary expense to the industry. Standardization up to this time has been largely a matter of individual efforts on segregated types of apparatus. The result has been that while standards have been developed for the several types of apparatus, it is generally conceded that these do not harmonize in such a manner that standard apparatus can be coordinated readily into a complete operating system. For this reason there has been a considerable tendency for the users of apparatus to buy equipment of special voltage ratings to suit their own particular conditions. Since these conditions would naturally vary somewhat from operator to operator, the special apparatus ordered by them does not constitute a new but unofficial line of standards, but a truly special line of apparatus for each operating company.

Considerable attention has been given to this tendency to deviate from the established standards recognized by the A. I. E. E., N. E. L. A., and Electric Power Club, since although the use of special apparatus might be advantageous from the point of view of the individual operator, it nevertheless places a burden on

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the industry as a whole, including those purchasers who believe they are being supplied with special apparatus at standard price. Users of standard apparatus help to bear this as well as the users of special apparatus, since the development costs and extra supervision and manufacturing costs caused by the production of specials tend to raise the cost of standard apparatus. The special tools often required for the production of specials also raise the investment in manufacturing plant, and this extra cost is usually borne by the standard as well as by the special product. Furthermore, special apparatus entails longer deliveries both on the initial apparatus and on renewal and repair parts.

There is no need to elaborate further on the nature of the disadvantages coincident with the widespread use of specials, for these are generally well known. The importance of these disadvantages, however, is perhaps not so well known, and it is in the interests of a wider appreciation of the advantages to be gained through complete and acceptable standardization that various official bodies have been working the last few years. This is all the more important in view of the fact that the N. E. L. A. has recommended system interconnection. A more or less detailed review of the history of voltage standardization follows in a later section.

PURPOSE

This paper is to present the viewpoint of the manufacturers on voltage standardization, and its purpose in greater detail is:

1. To outline the history of voltage standardization, pointing out that it has been separate standardization of various types of apparatus rather than standardization of a complete operating system.

2. To show by the results of a questionnaire how representative operating practise lines up with the existing standards.

3. To discuss the need for standardization of the complete system and the conditions which such a standardization must meet.

4. To present what is believed to be a workable set of standards which meet these conditions and to demonstrate their application to a typical system.

5. To discuss the advantages of standardization from the manufacturer's viewpoint and to indicate the benefits accruing to the industry as a whole.

HISTORY

As indicated in the introductory matter, progress in the voltage standardization of the complete a-c. electrical operating system, including generation, transmission, distribution, and utilization, has been largely a matter of the standardization of the several divisions of apparatus rather than a concerted standardization of the whole operating system.

High voltage standardization dates back to 1899, when the Standardization Rules of the A. I. E. E. recommended that the rated circuit voltage should be measured at the receiving end and that the recommended values should be 1000, 2000, 3000, 6000, 10,000, 15,000, and 20,000 volts. In order to take care of line drop, generators were to be rated at 1150, 2300, and 3450 volts, thus allowing a total transmission regulation of 15 per cent.

The recommended voltages were extended in the 1902 Standardization Rules to include 30,000, 40,000, and 60,000 volts in order to keep up with progress in high voltage transmission, the voltage still being measured at the receiving end.

The 1911 Standardization Rules made a sweeping revision in that it was decided to measure the circuit voltage at the sending end. The recommended voltages were no longer based on multiples of 10, but on multiples of 11, and the list was extended to include 88,000 and 110,000 volts. It was also recommended that transformers should transform between the listed voltages and that their ratios should be exact multiples of five. Since that time high voltages have always been measured at the secondary of the sending end transformers and have remained on a basis of multiples of 11, although extensions and additions have been made to the recommended list.

In 1914 the manufacturers of a-c. motors reached an Electric Power Club agreement whereby motor voltages became standardized at 110, 220, 440, 550, and 2200 volts, these standards representating the voltages at which the performance guarantees would be made and checked. It was recognized that it would be necessary to allow for variations in the supply voltage, and the apparatus was designed for successful operation at 10 per cent above and below the rated voltage although the performance would be affected where the voltage makes these allowable variations from normal.

Standardization of lamp voltages came later, due to the fact that in the manufacture of carbon lamps it was impractical to predetermine the voltage rating of units. This had to be determined by test after the product was made, and the various operating companies were encouraged to standardize on different service voltages in order to provide a market for all the lamps which the rating test showed to fall into classes suitable for these service voltages.

The tungsten filament lamp, however, proved not to be subject to these limitations, for its voltage rating can be predetermined with considerable accuracy before manufacture. With the rapid displacement of the carbon lamp by the tungsten lamp, therefore, there was an increasing tendency to standardize lamp voltage, since it was possible and highly desirable to standardize the factory output of tungsten lamps.

The Lamp Committee of the N. E. L. A., along with the lamp manufacturers, became increasingly interested in this standardization and urged the member operating companies of the N. E. L. A. to converge where possible upon whichever of three proposed lamp standards, namely 110, 115, or 120 volts, they could adopt most economically. For the last twelve years the percentage demand for 115-volt lamps has steadily risen from 13.5 per cent to 44.6 per cent; and for 120-volt lamps, from 9 per cent to 31.3 per cent; while the percentage demand for 110-volt lamps varied between 20 per cent and 30 per cent during the period from 1913 to 1924 but is showing a gradual decline which reduced it to 16.5 per cent in 1925. The point of great significance, however, is that in those twelve years the percentage demand for odd voltage lamps has declined from 51.5 per cent to 7.6 per cent. Eighty per cent of this 7.6 per cent consists of 125-volt lamps which are probably used principally on d-c. systems.

Thus the trend in a-c. systems is quite definitely toward the use of 115-volt lamps, with 120-volt lamps a close second.

The tendency in the electric heating and household appliance field has been to express the voltage rating in terms of a voltage range rather than as a definite voltage. Actual examples of this method of rating are 100-114, 105-115, 111-120, and 115-120 volts taken from nameplates of well known appliances. This short list suffices to show that these voltage ratings cover ranges varying from 5 to 14 volts, and that little standardization has been attempted among the appliance manufacturers either as to the extent of the range or its location in the voltage scale.

The preceding outline gave a brief description of the situation which has developed with respect to utilization voltages during the past 10 or 15 years.

The operating companies are really interested in the standardization of service voltage rather than standardization of the utilization voltage, since their contract responsibility is to maintain a specified voltage at the service switch. The one depends on the other, of course, but it is natural that the power company should be interested in the supply and maintenance of its own definite commodity, which is voltage at the service switch rather than voltage at the lamp socket or motor terminals. In the interest of standardization of service voltage, the N. E. L. A. Subcommittee on the Standardization of Service Voltage undertook in 1920 to make an investigation of this question leading up to the recommendation of suitable standards, and preferably a single standard.

The factors which had to be taken into consideration in this investigation were:

1. Existing utilization voltage standards,
2. Existing distribution transformer voltage standards,
3. Regulation between transformer and service switch,
4. Regulation between service switch and lamp socket,
5. Economic advisability of modifying 1 or 2 or both.

What utilization voltage standardization had been accomplished by 1921 has been reviewed above, and may be summarized by:

Motors: Multiples of 110 volts, but will operate successfully on variations of ± 10 per cent in the supply.

Lamps: 110-, 115-, and 120-volt, the 115-volt predominating.

Appliances: No standardization.

The existing distribution transformer voltage standard was the triple rated 110, 115, 120 volt secondary. The great majority of these distribution transformers were of the 20:1 ratio, 2300-volt class, without taps in the primary, giving 240/230/220–120/115/110 on the secondary when suitably excited on the primary.

The average regulation between the transformer secondary and the service switch was found to be two per cent, and between the service switch and the utilization device one per cent for lamps and possibly double that for motors.

After considerable discussion between the Subcommittee, the Lamp Committee, and the member operating companies, it was tentatively recommended in the 1921 and 1922 Reports of the N. E. L. A. Electrical Apparatus Committee that the standard service voltage be such that the proper average voltage would be supplied to 115-volt lamps, with recognized departures for supplying the proper average voltage to 110-volt and 120-volt lamps, the latter being for only existing systems of that voltage. With any of these average service voltages it was intended to use 110-volt motors and electric household appliances having a voltage rating covering a range of from 110 to 120 volts, since these two classes of apparatus do not require the close correspondence between impressed voltage and normal rated voltage which lamps require for economical operation.

The "recognized departure" of service voltage corresponding to 110 volts at the lamp socket was admitted due to the number of companies that could not increase their delivered service voltage without exceeding the safe operating voltage of their generators; and the recognized departure of service voltage corresponding to 120 volts at the lamp socket was admitted due to the number of companies that felt they could not reduce their service voltage without losing considerable revenue on account of the reduced voltage running of lamps which their customers would be reluctant to replace before they were worn out. Furthermore, the lamp manufacturers were desirous of continuing the triple lamp standard which they had been at some pains to establish.

This scheme fitted in fairly well with existing distribution transformer voltages, except that on systems requiring 120 volts at the lamp socket the voltage at the transformer secondary must be between 123 and 124 volts to allow for the drop between transformer and lamp, and hence the excitation voltage impressed on the primary must be of the order of 2500 volts. This would work the iron rather heavily.

There was also some question raised as to the successful operation of 110-volt motors when the average service voltage was such that the average impressed voltage was 120 volts, since this average feature implied that at times the impressed voltage might be two or three volts above 120 and thus exceed the 10 per cent tolerance allowed in the Electric Power Club rule. Operating companies stated they had never experienced any trouble of this nature, however, and the report remained on the basis of 110-volt motors.

It was recognized that the standardization of service voltage would be possible and of value only if it could be coordinated with the rest of the system all the way back to the generator. Accordingly, the final disposition of the Subcommittee report was that it was recommended in 1923 to be brought to the joint attention of the A. I. E. E., Electric Power Club, and N. E. L. A., with the idea that these agencies might cooperate in the future toward the voltage standardization of the electrical system from generator to utilization device.

Such a sweeping standardization of the complete system was evidently needed, since it was stated in the N. E. L. A. Electrical Apparatus Committee Report for 1924 that a recent investigation showed that less than 50 per cent of the transformers 200 kv-a. and above, being bought by the operating companies, conformed to the transformer voltage standards adopted jointly by the N. E. L. A. and Electric Power Club as formulated in 1919 and amended in 1922. This investigation was prompted by several operating companies that communicated to the N. E. L. A. their feeling that systems should be so arranged and apparatus so rated that a higher voltage should be obtainable at transformer secondaries than was now possible. The results of the investigation indicated that the standards as adopted were not ideally suited to operating conditions, since the operating companies were finding it to their advantage to purchase transformers which varied from the recommended standards.

The causes which produced these conditions became the object of an investigation by the Transformer Subcommittee of the N. E. L. A. Electrical Apparatus Committee in 1925. By means of a questionnaire to representative operating companies they inquired into the actual operating voltage at each point in a complete system, in an effort to find out what apparatus ratings would be best suited to operating conditions, to what degree existing apparatus standards would suffice, and what changes would be necessary in order that operating companies would find it to their advantage to buy apparatus conforming to a new set of standards rather than to buy special apparatus. The questionnaire inquired specifically into the voltage ratings of all generators and transformers on the system, the tap voltage rating used on the transformers, the actual operating voltage at the generators and on the primary and secondaries of all transformer banks and at the service switch, for conditions of light and maximum load and on the longest feeder and the shortest feeder, and the range of feeder induction voltage regulators, if used.

Replies were received from 22 power companies, representing the two cases: (a) systems where the feeders were fed directly off the generator bus at generator voltage and the stepping down to service voltage accomplished through two or more transformations, and (b) systems where transmission at high voltage intervenes between the generator bus and the distribution system. While there were several discrepancies in the figures submitted, it was felt nevertheless that in the main they were reliable and representative of general practise.

The salient points which the questionnaire developed were as follows:

1. Eleven of the twenty-two companies had transformers on their system over-excited by more than five per cent above the connected tap rating. In five cases this over-induction was 10 per cent or more above the connected tap rating.

2. Three companies reported their generators to be operating at more than five per cent above their normal rating, the maximum being nine per cent above normal. Thirteen of the remaining companies operate their generators above normal but not more than five per cent above the normal voltage rating.

3. Whereas feeder induction voltage regulators were required to operate at greater buck than boost in five cases in order to maintain satisfactory service voltage, and to operate at equal buck and boost in seven cases, there were twelve cases where they were required to operate at greater boost than buck.

The evidence was plain that the existing voltage standards did not allow the maintaining of voltage at the point of utilization under load conditions unless the voltage at the various generation and transformation points was maintained above normal. The explanation of this was that whereas the existing transformer voltage standards provide for the regulation in the transformer itself by means of taps in the primary, insufficient provision is made for compensating for the line drop in the transmission line and feeders, for, according to the standards, the rated secondary voltage of each transformer is equal to the rated primary voltage of the transformer at the receiving end of the line which the former supplies. Since induction regulators are generally used at only one point in the network, it is often impossible to keep the voltage at all transformation points within the desired limits.

In order to remedy this situation, the Transformer Subcommittee suggested to the Electrical Apparatus Committee as a basis for criticism and discussion, a system of standards where the rated secondary voltage of each sending-end transformer would be five per cent higher than the rated primary voltage of the corresponding receiving-end transformer, in order to allow partially, at least, for the drop in the line. The proposed standards included voltage ratings for generators, synchronous condensers, induction motors, and switching, control, and protective apparatus, in addition to transformer ratings, since the idea of coordinating all of the apparatus on the system had been kept well in view from the beginning.

The Transformer Subcommittee had now reached the point at which the Subcommittee on Standardization of Service Voltage had arrived in 1923, namely, at the limit of its scope; and it was in order to refer the question to bodies in a position to come to authoritative agreements on the whole voltage question. Accordingly, action was commenced for making a very complete presentation of the entire problem to the industry as a whole, in order that concerted and general action might result after all interested parties had been thoroughly informed and afforded ample opportunity for presenting their views. The broadest vehicle for this presentation was considered to be a group of papers submitted before a convention of the American Institute of Electrical Engineers, these papers to be prepared by representatives of the operating companies, holding companies, consulting engineers, European engineers, and the manufacturers. From the discussion following these papers it was hoped that conclusive results would be obtained.

This paper is based on a memorandum which resulted from the cooperative action of the manufacturers. This memorandum will be quoted in full except for some introductory matter which has already been covered in the preceding paragraphs.

Before the presentation and discussion of the memorandum is taken up, however, some attention should be paid to the relation existing between American practise and the standards recommended at the 1926 meeting of the International Electrotechnical Commission, which took place in New York during April. These recommendations are summarized as follows:

I. Definition of "nominal high voltage": The nominal high voltage shall be the mean voltage at the consumers' terminals and shall be called nominal I. E. C. voltage of the network of that voltage range.

II. The maximum voltages at the generators and secondary terminals of transformers shall be considered as to be about 10 per cent higher than the mean voltages at the consumers' terminals. The values of the recommended nominal I. E. C. voltages and maximum voltages are included in the following table, the preferred voltages being in heavy type.

Nominal I.E.C. Voltages Mean Value at Consumers' Terminals	Maximum Voltages
1000	1100
3000	3300
6000	**6600**
10000	11000
15000	**16500**
20000	22000
30000	**33000**
45000	50000
60000	**66000**
80000	88000
100000	**110000**
150000	165000
200000	**220000**
300000	330000

It will be noted that these recommendations do not line up with present American practise, either as to the point at which the rated voltage of the circuit is measured, or as to the actual values of the voltage rating. The interested organizations and persons in America were at that time engaged in the consideration of a thorough revision of the whole subject of the voltage standardization of transmission and distribution circuits and of machinery and apparatus for those circuits. At the April meeting of the I. E. C. these studies had not arrived, however, at the stage where the American Committee had a definite program to offer. Neither did the American Committee feel justified in opposing progress on the part of other nations. It was believed that this situation was appreciated by the representatives of other nations and that the American Committee would on some later occasion be presenting before the I. E. C. the recommendations at which it would be arriving as the results of these studies.

PROPOSED SYSTEM OF VOLTAGE STANDARDS

The electrical manufacturers are interested in voltage standardization of apparatus to a perhaps even greater degree than the operators. While the latter are interested in the low prices, rapid shipments, and effective replacement and repair service made possible by standardization, nevertheless it is but natural they should be primarily interested in apparatus which will most perfectly meet their own particular requirements. Thus there is an inherent tendency for individual operating companies to set up standards of their own which might or might not agree with the practise of other operating companies.

The manufacturers, on the other hand, are desirous of a set of standards which will be universal for all customers, in order that their entire output might consist of standard lines of apparatus. The number of types could then be cut to a minimum, development costs reduced, and if proper standards were set up it would be possible to select standard equipment with the assurance that it could be incorporated into a system and form an operative whole.

With these interests in mind, the manufacturing companies which the authors represent have been at some pains to formulate a new set of voltage standards which are believed to be workable and possible of universal application. These will now be discussed in detail. The proposed standards were drawn up to meet the following five conditions:

(a) The new standards must provide apparatus capable of meeting most of the service requirements of a well designed and operated system.

(b) The voltages selected must closely resemble those now in use to permit a reasonable degree of interchangeability of new and old apparatus.

(c) The changes involved must not necessitate too great an expense in the development of new apparatus.

(d) Admittedly, universality of use is an essential end to be sought in all standardization. Usually its complete attainment involves an excessive expenditure, and so is not economically desirable. It should be sought in so far as it can be obtained without burdening the cost of standard apparatus to obtain characteristics which will be used rarely.

(e) The new standards must provide apparatus that will meet, in spirit as well as in letter, all requirements as set forth in the standards of the American Institute of Electrical Engineers. The apparatus should not only be capable of meeting the test requirements of the A. I. E. E. Standards, but should readily lend itself to operation within the limits as defined by them. The design of electrical systems contemplating the use of apparatus under conditions more severe than sanctioned by the A. I. E. E. Standards should be discouraged.

The first four of the foregoing principles are self-explanatory but the last will permit of further elaboration. We have to deal here primarily with the test voltage to which apparatus is subjected and its relation to the normal operating voltage of the apparatus. In general, the A. I. E. E. Standards specify a potential test of twice the rated voltage of the apparatus plus 1000 volts in the case of rotating apparatus and transformers (excepting current transformers) and 2¼ times rated voltage plus 2000 volts in the case of switching equipment, bus supports and current transformers. Except where specific mention is made to the contrary apparatus is designed to be operated at substantially its rated voltage, certainly not materially above it. The ratio of test to rated voltage has been selected to introduce a reasonable factor of safety, presumably as the result of theoretical consideration and operating experience. To operate normally above rated voltage is to decrease the recommended factor of safety, and to decrease the factor of safety is to introduce a hazard to life and property not sanctioned by the A. I. E. E. Standards. Or, to put it another way, those who operate apparatus above its rated voltage, operate it with a factor of safety less than the engineers responsible for the A. I. E. E. Standards deem advisable. Can any other conclusion be drawn? It can probably be said, without fear of contradiction, that the practise of encroaching on the universally accepted factors of safety is more prevalent in the electrical arts than in others. Who would contemplate normally operating a steam boiler above its rated pressure? Simply to ask the question is to answer it. The present system of voltage standards has in no small degree contributed to the present undesirable practise. In the case of an average system composed of generators, transformers and motors, rated in accordance with the present voltage standards, either the generators and transformers must be operated above their rated voltages or a sub-normal voltage will prevail at the motor when the system is loaded.

In addition to encroaching on the factor of safety relating to test voltage, the operation of generators and transformers for considerable periods of time at pressures exceeding their rated voltage may result in overheating and tends towards reduced life and lower reliability. Such operation may also distort the voltage wave and lead to interference with communication circuits. The operation of oil circuit breakers at pressures materially exceeding their rated voltage may involve marked reduction in their interrupting capacity.

As individual pieces of apparatus are interconnected electrically to form any operative system, it is incumbent upon the designers to determine the maximum voltage at which the system will be normally operated and to be certain that this will not materially exceed the accepted factors of safety for the individual pieces of apparatus of which it is composed. At present the most important rule—if not the only rule—in the A. I. E. E. Standards defining "Rated Circuit Voltage" appears in the Transformer Section (13-119). It has been suggested in the Standards Committee to issue a new general rule, resembling the present one, reading substantially as follows:

Rated Circuit Voltage: For the purpose of fixing a value to be used in designing and testing electrical apparatus, the rated voltage of a circuit (or system) is defined as the highest rated voltage of the apparatus supplying it. By "circuit voltage" is meant the voltage from line to line as distinguished from line to neutral. This voltage rating applies to all parts of the circuit. The actual operating voltage of the circuit may vary from the rated circuit but should not exceed it.

Such a rule as this and those pertaining to the individual pieces of apparatus would definitely set the rated voltages of apparatus, their test voltages and maximum operating voltage if they are to be operated as parts of a common system.

The present voltage standards, excepting apparatus rated 2300 volts and below, call for apparatus whose voltage ratings are expressed in even multiples of 11; for example, 6600-volt, 13,200-volt, 44,000-volt, etc. These voltages apply to motors, transformers (both their primary and secondary windings) and generators, with the exception that in some cases transformers are given a double rating. The second transformer rating simply permits their use at approximately five per cent over-voltage without any change in their voltage ratio. The transformer ratios referred to, of course, are no load or turn ratios. It is usual practise to equip step-down transformers, whose voltage rating exceeds 2300 volts, with four 2½ per cent primary taps below rated voltage. A simple system, employing the present voltage standards, is shown in Fig. 1.

In this elementary but representative system, the voltage ratings of generators, motors, and step-up and step-down transformers are shown as multiples of 11, excepting that the substation transformers supplying the 2300-volt distribution lines are rated 2300 volts, and the distribution transformers are given the usual triple rating of 2400-2300-2200 volts.

The voltage drops assumed are as follows:

132-kv. line	10	per cent approx.
66-kv. line	7½	per cent approx.
13.2-kv. line	5	per cent approx.
2300-volt line	5	per cent approx.
230-115 mains	3	per cent to lamps
and services	6	per cent to motors
Power transformers	5	per cent
Distribution transformers	2	per cent

This diagram shows that with the generator operated at rated voltage the voltage at the 115-volt lamps would be only 102 volts at full load on the system, and the voltage at motors 10 per cent low, even if each transformer is operated on its lowest high-tension tap and a regulator boosting 10 per cent is allowed for. The alternative at present is to over-excite the generating end of the system. It is this that is now so commonly done. If we assume motor and lamp voltages as fixed, it would seem obvious that the principal faults with the present system are that transformer ratios (ratios of primary to secondary) are too high and generator potentials too low.

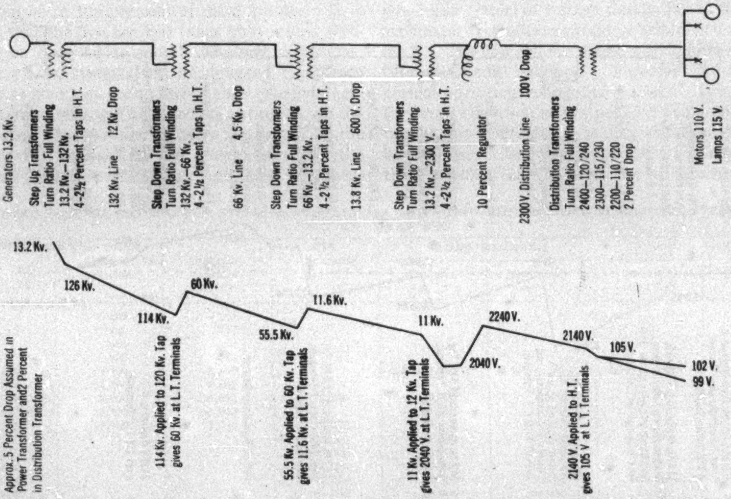
Generators 13.2 Kv.
Step Up Transformers
Turn Ratio Full Winding
13.2 Kv.—132 Kv.
4-2½ Percent Taps in H.T.
132 Kv. Line 12 Kv. Drop
Step Down Transformers
Turn Ratio Full Winding
132 Kv.—66 Kv.
4-2½ Percent Taps in H.T.
66 Kv. Line 4.5 Kv. Drop
Step Down Transformers
Turn Ratio Full Winding
66 Kv.—13.2 Kv.
4-2½ Percent Taps in H.T.
13.8 Kv. Line 600 V. Drop
Step Down Transformers
Turn Ratio Full Winding
13.2 Kv.—2300 V.
4-2½ Percent Taps in H.T.
10 Percent Regulator
2300 V. Distribution Line 100 V. Drop
Distribution Transformers
Turn Ratio Full Winding
2400—120/240
2300—115/230
2200—110/220
2 Percent Drop
Motors 110 V.
Lamps 115 V.
13.2 Kv.
126 Kv.
114 Kv.
60 Kv.
55.5 Kv.
11.6 Kv.
11 Kv.
2040 V.
2240 V.
2140 V.
105 V.
102 V.
99 V.
Approx. 5 Percent Drop Assumed in Power Transformer and2 Percent in Distribution Transformer
114 Kv. Applied to 120 Kv. Tap gives 60 Kv. at L.T. Terminals
55.5 Kv. Applied to 60 Kv. Tap gives 11.6 Kv. at L.T. Terminals
11 Kv. Applied to 12 Kv. Tap gives 2040 V. at L.T. Terminals
2140 V. Applied to H.T. gives 105 V at L.T. Terminals

FIG. 1—TRANSMISSION AND DISTRIBUTION CONNECTION AND VOLTAGE CHART—OLD STANDARD VOLTAGES—FULL LOAD

The new system of standard voltages here proposed contemplates a change in the principle of the present system. An example of the new system is shown in Fig. 2. Here, as in the present system, step-down transformers (in fact all used at the receiving ends of lines) will be equipped with the equivalent of four 2½ per cent taps in the primary windings below rated voltage. Step-up transformers, for location adjacent to generators, will be equipped with the equivalent of two 2½ per cent taps in the primary windings below rated voltage to provide additional range for compensating for the voltage regulation of the transformers and to avoid over-excitation of generators. A cursory study of the simple principles of voltage standardization as illustrated in Fig. 2 will show that the deficiencies of the present standards have been eliminated. Here, the windings from which power flows have voltage ratings approximately five per cent greater than those at which power is received. It is believed that ample provision has been made in the voltage ratings of apparatus, ratios of transformation, and transformer taps to meet most normal conditions of system design and operation. It will be noted that in Fig. 2, while the generators are operated at rated voltage, and rated voltage is supplied to lamps and motors, there are still remaining taps in reserve in the transformer banks, and only half the boost in the regulator is used. So much for the principle involved in the proposed system—now let us consider the actual voltage values it is proposed to assign to the several voltage classes. These are shown in the accompanying tabulation. They are admittedly a compromise with the present standards. The motor voltages now in use have been retained. In the lower voltage range generator voltages will not be changed; in the upper they will be increased approximately five per cent. Step-down transformers having primaries rated 2300 volts, in which class fall the millions of kv-a. capacity now in use for lighting service, will not be changed. It is proposed to change the ratios of all others so that they will readily conform to the new system. Even here it has been possible to retain ratios (when using tap connections) that will permit paralleling new with old units to a limited extent.

By the definitions previously quoted, the rated system voltage becomes the rated voltage of the generators or secondaries of the transformers supplying the system. It will be noted in the accompanying tabulation that above 4150 volts recommended system voltages are approximately multiples of 11.5 up to and including 69,000 volts; above they are multiples of 11.0. This is a compromise based on a study of existing systems and the cost involved in changing manufacturers' present standard designs. There are so many step-down transformers in the 66,000-volt class and below whose primaries are multiples of 11.0 that it was thought inadvisable to change their voltage ratings, and it was necessary therefore to increase the system voltage ratings and step-up transformer secondary voltages for these classes to multiples of 11.5. The manufacturers' present standard switching equipment (up to and including 73,000 volts) has ample margin in their voltage ratings to permit the proposed increase in system voltage ratings without necessitating changes in their design. Above 69,000 volts, the practise now in general use of standard system voltages that are multiples of 11.0 seems to be satisfactory. While there are theoretical reasons to indicate that a single standard based on voltages which are multiples of 11.5 should be established above 69,000 volts, general practise still indicates that this is not as necessary on the high voltages as on the low. The standardization of the higher values would probably necessitate the redesign of most of the present standard apparatus, particularly that of oil circuit breakers, the redesign of which, along with the special tools required for their manufacture, would be an expensive burden on the industry.

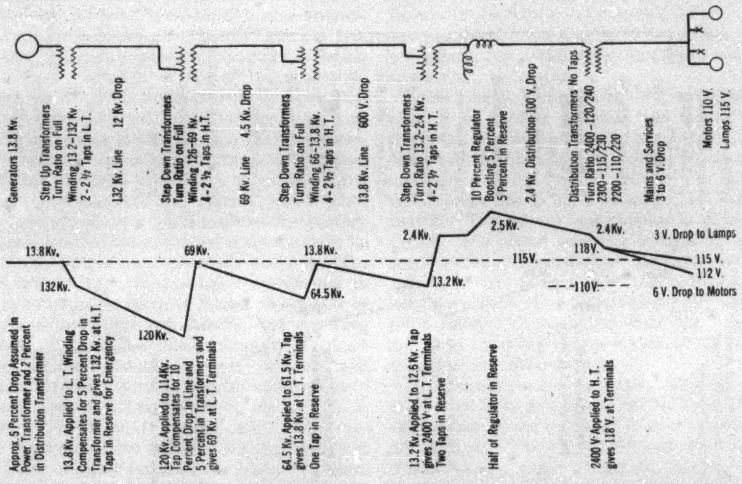
Generators 13.8 Kv.
Step Up Transformers Turn Ratio on Full Winding 13 2–132 Kv. 2-2½ Taps in L.T.
132 Kv. Line 12 Kv. Drop
Step Down Transformers Turn Ratio on Full Winding 126–69 Kv. 4-2½ Taps in H.T.
69 Kv. Line 4.5 Kv. Drop
Step Down Transformers Turn Ratio on Full Winding 66–13.8 Kv. 4-2½ Taps in H. T.
13.8 Kv. Line 600 V. Drop
Step Down Transformers Turn Ratio 13.2–2.4 Kv. 4-2½ Taps in H. T.
10 Percent Regulator Boosting 5 Percent 5 Percent in Reserve
2.4 Kv. Distribution 100 V. Drop
Distribution Transformers No Taps Turn Ratio 2400–120/240 2300–115/230 2200–110/220
Mains and Services 3 to 6 V. Drop
Motors 110 V.
Lamps 115 V.
13.8Kv.
132Kv.
120Kv.
69Kv.
13.8Kv.
64.5Kv.
13.2Kv.
2.4Kv.
2.5Kv.
2.4Kv.
118V.
115V.
110V.
3 V. Drop to Lamps
115 V.
112 V.
6 V. Drop to Motors
Approx. 5 Percent Drop Assumed in Power Transformer and 2 Percent in Distribution Transformer
13.8 Kv. Applied to L. T. Winding Compensates for 5 Percent Drop in Transformer and gives 132 Kv. at H.T. Taps in Reserve for Emergency
120 Kv. Applied to 114Kv. Tap Compensates for 10 Percent Drop in Line and 5 Percent in Transformers and gives 69 Kv. at L. T. Terminals
64.5 Kv. Applied to 61.5 Kv. Tap gives 13.8 Kv. at L. T. Terminals One Tap in Reserve
13.2 Kv. Applied to 12.6 Kv. Tap gives 2400 V. at L. T. Terminals Two Taps in Reserve
Half of Regulator in Reserve
2400 V. Applied to H. T. gives 118 V. at Terminals

Fig. 2—Transmission and Distribution Connections and Voltage Chart—New Plan—Full Load

The previously cited proposed definition of "Rated Circuit Voltage" provides that the operating voltage should not exceed the rated circuit voltage. This is an extremely difficult, if not impossible, condition to meet in normal operating practise. To fulfill the condition, the generating end of a system must be normally operated below the rated system voltage if provision is to be made to have a margin in the voltage range to overcome abnormal voltage drops between the sources of generation and utilization under emergency conditions, such as the loss of one or more of a parallel group of transmission lines. The A. I. E. E. Standards recognize a not dissimilar condition in the case of motors and transformers (see Standards 5-550, 7-700, 13-500). In the case of motors, the Standards specify that they should operate successfully at rated load at any voltage not more than 10 per cent above or below rated voltage; in that of transformers, that they should operate successfully at rated load when excited on full winding at five per cent above rated voltage. This might be interpreted as a recognition of the fact that under usual conditions it is not practicable always to operate apparatus at rated voltage. It is suggested that this recognition be extended to include other apparatus. To make the range 20 per cent, as in the case of motors, will be costly and probably unnecessary.

It is here suggested that a 10-per cent range be recognized in the case of generators, synchronous condensers, transformers, and apparatus (excluding motors) employed in generating, transmission and distribution systems with the exception of maximum rated apparatus (including under column titled "Apparatus"). It is recommended that these classes of apparatus, which are normally rated, be designed to operate successfully (but not necessarily within the guaranteed limits set for operation at rating) at rated load at any voltage not more than five per cent above or below rated voltage. Of course, apparatus will be operative below 95 per cent of rated voltage, but at reduced capacity. Such a change in the A. I. E. E. Standards will necessitate the manufacture of apparatus capable of operating at rated kv-a. output throughout the voltage range from 95 to 105 per cent of rated voltage. It is expected that the five per cent range above rated voltage will be reserved to meet emergency conditions. In other words, systems will be designed to operate normally at a voltage not to exceed their rated voltage. High potential tests will be based, as at present, on the rated voltage of apparatus, or, more accurately, on the "rated circuit voltage" of the system of which the apparatus forms a part. Maximum rated apparatus as given in the attached tabulation, *i. e.*, circuit breakers, disconnecting switches, etc., already contains the necessary tolerances in their maximum ratings and should not be used above such ratings.

It is expected that the rated voltages proposed here for step-up and step-down transformers, the essential feature of the plan, will make it possible to standardize apparatus so that it will meet most existing operating conditions without the necessity, which now prevails, of exceeding the established rating limits of the apparatus. Operators, under most conditions, will find it possible, when employing the proposed standard apparatus, to deliver rated voltage to apparatus at all points of their systems without exceeding the rated voltages of their systems.

The accompanying tabulation, with the foot-notes included with it, presents a brief summary of the proposed new system of voltage standards. A few further explanatory remarks might prove helpful in understanding it. The first column presents a list of the proposed "Rated Circuit Voltages." These set the maximum normal operating voltages of systems. It is proposed to permit operation at five per cent above these values but, as just mentioned, it is suggested that this margin be reserved for emergency operation. The second and third columns hardly need further explanation. In

PROPOSED VOLTAGE RATINGS FOR SYSTEMS, GENERATORS, SWITCHING, CONTROL AND PROTECTIVE APPARATUS, TRANSFORMERS, ETC.

Systems	Generators and syn. condensers (see "A")	Induction motors (see "B")	Apparatus (see "C" and "D")	Step-up transformers		Step-down transformers	
				Primary (see "E")	Secondary (see "E")	Primary (see "E")	Secondary (see "E")
	120	110					115
	240	220					230
	480	440					460
	600	550					575
2,400	2,400	2,200		2,300/3,980 Y	2,400/4,150 Y	2,300/3,980 Y	2,400/4,150 Y
4,150	4,150	3,800					
6,900	6,900	6,600	7,500	6,600	6,900	6,600	6,900
11,500 (F)	11,500	11,000		11,000	11,500	11,000	11,500
13,800	13,800	13,200	15,000	13,200	13,800	13,200	13,800
23,000			25,000	22,000	23,000	22,000	23,000
34,000			37,000	33,000	34,500	33,000	34,500
46,000			50,000	44,000	46,000	44,000	46,000
69,000			73,000	66,000	69,000	66,000	69,000
88,000			88,000	84,000	88,000	84,000	88,000
110,000			110,000	105,000	110,000	105,000	110,000
132,000			132,000	126,000	132,000	126,000	132,000
154,000			154,000	147,000	154,000	147,000	154,000
220,000			220,000		220,000	210,000	

GENERAL NOTE

Guarantees of efficiency, heating, overload, etc., and over-voltage tests of all apparatus should be based on the rated voltage of the apparatus with the exception of step-down transformers, the over-voltage tests on which should be based on rated secondary voltage; and a primary voltage five per cent greater than rated voltage.

SPECIFIC NOTES

"A"—Generators and synchronous condensers should be designed to deliver rated kv-a. output at rated power factor and frequency throughout a range of five per cent below to five per cent above rated voltage.

"B"—Induction motors should be designed to deliver rated h. p. throughout a range of 10 per cent below to 10 per cent above rated voltage at rated frequency.

"C"—Apparatus, as here used, includes oil circuit breakers, disconnecting switches, current transformers, insulators, bushings, and fuses. The voltage ratings of potential transformers should be the same as the recommended system voltage ratings; their secondaries should be rated approximately 115 volts to permit the employment of the now existing even ratios of transformation. Above 73,000 volts, apparatus should be designed to operate successfully at five per cent above rated voltage during emergencies. Up to and including 73,000 volts, apparatus is maximum rated. For maximum rated apparatus the operating voltage of the system on which it is used should not exceed the rated voltage of the apparatus even during emergency operation.

"D"—Lightning arrester voltage standardization recommendations have been omitted pending further study.

"E"—Transformers should be designed to operate during emergencies at five per cent above rated voltage, the over-voltage being obtained by over-excitation and not through the use of taps. They should be equipped with taps as follows:

(a) Step-up transformers should be equipped with the equivalent of two 2½ per cent full capacity taps in the primary windings to provide additional range for compensating for the voltage regulation of the transformers and to avoid over-excitation of generators.

(b) Step-down transformers should be equipped with the equivalent of four 2½ per cent full capacity taps in the primary windings to provide additional range for compensating for line voltage drop.

"F"—When possible, 11,500-volt systems should be discouraged in favor of 13,800-volt ones.

the fourth column the heading "Apparatus" is employed in a restricted sense, in the absence of a better term, to include oil circuit breakers, disconnecting switches, current transformers, insulators, bushings, and fuses. Years ago manufacturers standardized "apparatus" voltages up to and including 73,000 volts on the basis of maximum ratings, which is to say, without a five per cent over-voltage margin. The values were chosen after a study of the operating voltages of the then existing systems. The investigation revealed a conspicuous absence of voltage standardization in the range considered and maximum voltage values therefore were chosen somewhat above the more common voltages in use to include the numerous groups operating above the average voltages. The voltages of systems operating above 73,000 volts were fairly well standardized as multiples of 11.0, and so for these groups manufacturers standardized apparatus voltages as multiples of 11.0, but included a five per cent over-voltage operating margin for emergency operation. Higher values and greater over-voltage margins were considered but rejected because there seemed to be no logical reason or necessity to justify burdening users with the greater expense of higher voltage apparatus, particularly as all high-voltage apparatus is inherently expensive. Such is the explanation of why apparatus up to 73,000 volts is maximum rated and above nominal rated. The fifth and sixth columns, headed "step-up transformers," include the voltage ratings of transformers used at generating stations, where the potential drop between the generators and transformers is nil. The seventh and eighth columns refer to transformers used at the receiving ends of lines. Most of these will be step-down units. There are a few instances where step-up transformers must be installed at the ends of transmission lines. The voltages and taps of these should be selected from the seventh and eighth columns. Of course, the transformer voltages given in the tabulation are no load or turn-ratio voltages.

ECONOMIC ADVANTAGES OF VOLTAGE STANDARDIZATION

The proportion of standard and special apparatus now in use is indicated by Fig. 3, which shows the kv-a. of transformers produced by two manufacturers up to May 30, 1926, arranged according to high-voltage rating, for transformers 250 kv-a. and over, and 44,000 volts and above. The total kv-a. at standard voltages, namely, 44, 66, 88, 110, 132, 154 and 220 kv., is 51 per cent, and the kv-a. at special voltages 49 per cent.

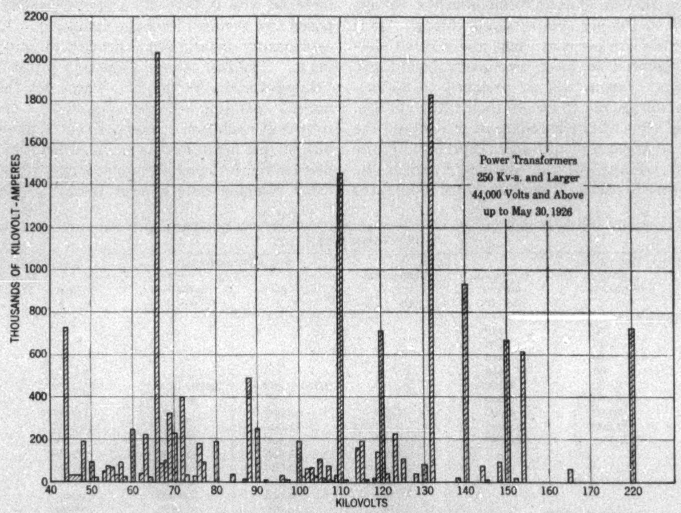

FIG. 3—CHART SHOWING TOTAL CAPACITY OF INSTALLED POWER TRANSFORMERS OF VARIOUS VOLTAGES

Since the chart does not take into account transformers which have special low voltage ratings, it appears that at least half the transformers are special as to voltages and the specials are of still greater percentage if other features are counted.

The situation is not quite so bad in most other lines of apparatus, but even industrial motors which might be expected to be as completely standardized as any line, due to the large demand and the number and the relatively small size of the units, are found to include

20 per cent special units. It is of interest to note in this connection that a study made by one manufacturer shows that these 20 per cent require 60 per cent of the total shop supervision, whereas the 80 per cent standard units require only 40 per cent of the supervision. Switchboards, on the other hand, are practically 100 per cent special when considered as a complete unit, although of course the equipment mounted on them is mostly standard. This is to be expected since hardly any two generating stations or substations are sufficiently alike to require the same switchboard.

Truck type switchboards are a notable exception to this, however, since the individual cells and trucks lend themselves very readily to standardization. Unfortunately, there is a tendency here for various operating companies to standardize on a design worked up by themselves, rather than to standardize on the manufacturers' designs, so that so far as the manufacturer is concerned, each order is special. An example of what this means in price is found in a recent case where a large order of trucks and cells built according to the purchaser's designs cost eight per cent more than if they had been built according to the manufacturer's standard designs for exactly the same duty.

It is of great interest to consider some figures which have been worked up on the development costs for some selected types of apparatus, on the assumption that each unit is completely special in every particular. These figures are presented because their magnitude is truly surprising and indicates in somewhat startling fashion the seriousness of this problem of the production of special apparatus in modern manufacturing establishments. By development cost is meant the cost of patterns, dies, special tools, supervision, engineering, and drafting. For various classes of electrical equipment such as large power apparatus, truck type switchboards, industrial apparatus, etc., these costs vary from 150 per cent to 900 per cent in excess of the first unit cost of a similar standardized line of apparatus.

In order to visualize the amounts which special apparatus has cost the electrical industry, data have been compiled from statistics in the United States Census and other sources that show the investments up to date in various classes of electrical apparatus used in the generation, transmission, distribution, and utilization of electricity.

From these it is estimated that there is roughly two billion dollars worth of electrical apparatus now in operation in the United States, of which somewhere between 20 per cent and 50 per cent is special, depending on the particular type, with the exception of lamps which are only about eight per cent special. If all the specials had been eliminated, the industry would have experienced, from two causes, a worth while reduction in this two billion dollars, the effects of which are roughly evaluable:

1. Standard apparatus would not have had to bear part of the development cost of the specials,

2. The increased sale of standards to take the place of the specials would have caused a further reduction in their price in accordance with the law of quantity production.

Estimates by various methods indicate that the effect of the use of special apparatus has been to increase the total investment in electrical apparatus by from 100 to 200 millions or from five to ten per cent of the total.

The proposed standardization of system voltages and apparatus voltages fitting in with the already standard utilization voltages should have a marked effect in the reduction of these extra investments in the future. This standardization should also make available considerable economies in the industry through facilitating interconnection and reducing the extra investments required for interconnection.

CONCLUSIONS

1. The present system of voltage standards is not suited to operating conditions since insufficient allowance is made for voltage drop in lines and transformers.

2. The extensive use of special apparatus for maintaining proper voltage at the load, supports this contention.

3. The use of special apparatus is undesirable since it raises costs, delays deliveries, reduces interchangeability of apparatus from point to point of a system or connected systems, impedes the policy of system interconnection recommended by the National Electrical Light Association, and reduces the efficiency of renewal and repair part service.

4. A new system of voltage standards is proposed whereby all apparatus on the system will be rated to conform to the following transformer ratings: Up to and including 69,000 volts, the primaries of transformers will be rated on the basis of multiples of 11 (except the 2300-volt class), and the secondaries on the basis of multiples of 11.5; above that voltage the primaries will be rated on the basis of multiples of 10.5, and the secondaries on the basis of 11. This will allow for approximately five per cent greater voltage drop in lines and transformers than is now possible without reducing the voltage at the load. All other apparatus on the system will be rated in accordance with these voltages.

5. It is expected that the adoption of these proposed standards would eliminate the use of apparatus which is special with respect to voltage, and would therefore help remedy a situation which has cost the public from $100,000,000 to $200,000,000.

6. It is believed that the plan set forth here will fulfill the five conditions set forth in earlier paragraphs and furnish an escape from the present embarrassing dilemma, and it is hoped that those interested in this question of a-c. voltage standards will accept and consider this proposal in the spirit in which it is offered,

namely, simply as a basis of discussion, as a seriously and comprehensively thought out plan of voltage standards for all apparatus employed in transmission systems. It will fully serve its purpose should it lead ultimately to a rational set of standards which can be employed with a reasonable degree of success in new systems and throughout an appreciable part of existing ones.

Discussion

For discussion of this paper see page 205.

Voltage Standardization
From a Consulting Engineer's Point of View

BY R. E. ARGERSINGER[1]
Member, A. I. E. E.

***Synopsis.**—The author points out certain reasons why specifications for purchased equipment have not followed the manufacturers' existing standards and recommends certain changes in standards for system and apparatus voltages. The importance of having transformers interchangeable as step-up and step-down units is pointed out and, by means of five per cent taps above and below rated voltages in both windings, it is proposed to obtain sufficient flexibility for such interchangeable use. The paper gives a comparison between transformers designed as suggested above and transformers designed according to the manufacturers' recommendations, and the suitability of the suggested tap range is pointed out.*

A recommendation is also made that the number of ratings of oil circuit breakers should be reduced.

VOLTAGE standardization is of value to the consumer principally in two ways; first, that equipment may be used under a variety of operating conditions, and second, that it may be obtained more quickly and at less cost.

If the first result can be secured, the electrical manufacturer's production problems should be so simplified that the second would follow readily. It should be remembered that in discussing the possibility of securing apparatus, such as transformers, at lower prices by standardization, comparisons should be based on the cost of equipment actually purchased at present rather than on the cost of the present standard transformers since it appears that less than half the transformers purchased are standard.

The present standards have not been closely followed because, first, the operating companies in general have given too little consideration to simplifying their requirements, and second, in designing for standardization the manufacturers have placed too much emphasis on cost reduction and have paid too little attention to flexibility of use.

The transmission line data published in the *Electrical World* for January 3, 1925, list eighteen different circuit voltages between 44,000 and 220,000 kv. Other variables have been brought into the situation until one manufacturer now lists 63 different oil circuit breaker ratings between 15,000 and 220,000 volts. Some of these are subject to two and some to three modifications for altitude so that something like one hundred fifty presumably standard breaker ratings are listed for the voltage range indicated, and these do not include the *H* breaker or other low-voltage indoor switches. On the other hand, in their laudable effort to reduce costs, the manufacturers have standardized a range of transformer taps that is not sufficiently flexible for general purposes and have limited too closely the allowable range of terminal voltages.

In the N. E. L. A. *Bulletin* for September, 1926, a proposed plan is offered as a remedy for existing conditions, but it appears to be open to several objections[2]. First, it is not simple; here are at least two, and in most cases three, standard equipment voltages for each system voltage. Second, it is not flexible. Standard transformers and breakers should be interchangeable through the greatest possible number of applications. Transformers should be rated in terms of standard system voltages and should be capable of use between any two systems whose voltages are included in the nameplate rating of the transformer, from which it follows that they should be interchangeable as step-up or step-down units. Third, it does not appear to offer sufficient simplification in manufacturing processes to yield enough benefit to the purchaser to cause him to specify standard equipment.

In an endeavor to meet these objections and at the same time to adhere as closely as possible to present practises, the following scheme is suggested:

First. Standard voltages should be as given in Table I.

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[1] Chief Electrical Engineer, Stone & Webster, Inc.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 7-11, 1927.

[2] See also *Voltage Standardization of A-C. Systems from the Viewpoint of the Electrical Manufacturer*, by C. F. Hanker and H. R. Summerhayes, A. I. E. E. Winter Convention, February 1927.

Second. All apparatus except motors should be rated at a standard system voltage.

All motors for use on systems of 600 volts and less should be rated at approximately 8½ per cent less than system voltage. All motors for use on systems above 600 and not exceeding 13,800 volts should be rated at approximately 4½ per cent less than system voltage.

TABLE I

VOLTAGE		
APPARATUS GENERATOR AND TRANSFORMER	MOTORS	SYSTEM
120	110	120
240	220	240
480	440	480
600	550	600
2300	2200	2300
3980	3800	3980
6900	6600	6900
11500	11000	11500
13800	13200	13800
22000		22000
33000		33000
44000		44000
66000		66000
110000		110000
132000		132000
176000		176000
220000		220000

TABLE II

	SELLING								
	EXISTING SCHEME 4-2½% HT TAPS			PROPOSED SCHEME			SCHEME 'A'	1-5% TAP ABOVE & BELOW EACH PRI & SEC VOLTAGE	
	RATED WINDING VOLTS	ACTUAL VOLTAGE	WINDING VOLT. ERROR	RATED WINDING VOLTS	ACTUAL VOLTAGE	WINDING VOLT. ERROR	RATED WINDING VOLTS	ACTUAL VOLTS	WINDING VOLT ERROR
GEN.	13200	13800	+5 %	13800	13800	0	13800	13800	0
1st S.U.T.	13200 132000	13800 131200	+5 % 0 %	13200 132000	13800 131200	+5 % -½%	13100 138600	13800 138700	+5 % 0 %
12KV DROP									
1st S.D.T.	119000 66000	119200 62800	0 % -5 %	113400 69000	119200 68800	+5 % 0 %	125400 69300	126700 66500	+1 % -4 %
45KV DROP									
2nd S.D.T.	59400 13200	58300 12300	-2 % -7 %	61000 13800	64300 13800	+5½% 0 %	62700 14400	62000 13600	-1 % -6 %
600V DROP									
3rd S.D.T.	11900 2300	11700 2150	-1½% -6½%	12600 2400	13200 2400	+5 % 0 %	13100 2415	13000 2275	-1 % -6 %
5% BOOST		2260			2520			2390	
100V DROP									
DISTR TRANS	2300 115	2160 106	-6 % -8 %	2300 115	2420 118	+5 % +2½%	2300 115	2290 113	0 % -2 %
DROP 3V TOLTS 6V TOLTS									
LIGHTS MOTORS	115 110	103 100	-10½% -9 %	115 110	115 112	0 % +2 %	115 110	110 107	-4½% -2½%
	BUYING								
GEN	13200	10900	-17½%	13800	8450	-39 %	13800	13630	-1 %
1st S.U.T.	13200 119000	10900 104000	-17½% -13 %	13200 132000	8450 89000	-36 % -32½%	14490 125400	13630 124000	-6 % -1 %
12KV DROP									
1st S.D.T.	132000 66000	116000 61100	-13 % -7½%	126000 69000	101000 58200	-19½% -15½%	138600 62700	136000 64800	-2 % +3½%
45KV DROP									
2nd S.D.T.	66000 13200	65600 13800	-½% +5 %	66000 13800	62700 13800	-5 % 0	69300 13100	69300 13800	0 +5 %
GEN.	13200	13800	+5 %	13800	13800	0	13800	13800	0
600V DROP									
3rd S.D.T.	12200 2300	13200 2365	+8 % +3 %	12870 2400	13200 2340	+3 % -2	13100 2415	13200 2315	+1 % -4 %
5% BOOST		2480			2465			2430	
100V DROP									
DISTR TRANS	2300 115	2380 117	+3½% +1½%	2300 115	2365 116	+3 % +1 %	2300 115	2330 114	+1½% -1 %
DROP 3V TOLTS 6V TOLTS									
LIGHTS MOTORS	115 110	114 111	-1 % +1 %	115 110	113 110	-2 % 0	115 110	111 108	-3½% -2 %

5% Drop in each power transformer -2% Drop in distribution transformer, Line drops as indicated

Third. Each winding of all large power transformers should have one full capacity 5 per cent voltage tap above and one full capacity 5 per cent voltage tap below rated voltage. Small power and distribution transformers should be rated in accordance with existing standards.

TABLE III

	OMITTING DISTRIBUTION TRANSFORMER	
	PERCENT SYSTEM VOLTAGE ERROR	PERCENT WINDING ERROR
EXISTING SCHEME	+5 -21	+8 -17½
PROPOSED SCHEME	+4½ -36	+5½ -39
SCHEME 'A'	+5 -6	+5 -6
SELLING ONLY		
EXISTING SCHEME	+4½ -11½	+4½ -7
PROPOSED SCHEME	+4½ -10	+5½ -½
SCHEME 'A'	+5 -6	+5 -6

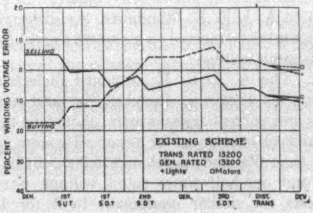

FIG. 1

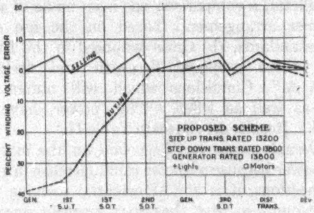

FIG. 2

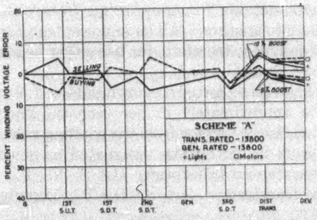

FIG. 3

Fourth. All generators, synchronous condensers, synchronous converters, arc rectifiers, small power and distribution transformers should operate at rated load without exceeding rated temperature rise through a range of from 95 to 105 per cent rated voltage.

All large power transformers should operate at rated load without exceeding rated temperature rise when excited at 7½ per cent above the rated voltage for the winding terminal to which the circuit is connected.

All motors should operate at rated load without exceeding rated temperature rise through a range of from 90 to 110 per cent rated voltage. All circuit breakers, disconnecting switches, fuses and instrument transformers should operate at rated current without exceeding rated temperature rise when used at 7½ per cent above rated voltage.

Fifth. Proposed definition of rated circuit voltage. Rated voltage of a circuit or system is the highest rated voltage of the apparatus supplying it and should be stated in terms of line-to-line voltage. In order to conform to A. I. E. E. Standards, the dielectric tests of all apparatus connected to a system should be based on rated circuit voltage.

Sixth. The A. I. E. E. standard dielectric tests for oil circuit breakers, disconnecting and horn gap switches and current transformers, should be modified so that all such equipment for use on all system voltages of more than 600 and less than 110,000 will be tested at two and one-half times rated voltage plus 2000 volts, tests for similar equipment built for 110,000 volts and above to remain as at present.

Following the general method outlined in the N. E. L. A. Bulletin, a comparison has been made of system transformer arrangements based on the use of (a) existing standards, (b) those proposed in the Bulletin and (c) those proposed herewith and designated "Scheme A." Conditions which will obtain when buying, as well as when selling, power, have been analyzed. Results are shown in Table II.

Considerable stress has been laid on the matter of buying power because with the rapid increase in system interconnections, it is almost essential to have transformers wound so that they may transfer energy in either direction. It is also desirable, in order to keep pace with the demands of a growing system, to have transformers suitable for transferring from one part of the system to another. All large power transformers should be designed, therefore, for use either as step-up or step-down units.

Curves have been plotted as shown in Figs. 1, 2, and 3 to indicate approximately in per cent the variation of the actual circuit voltage from the winding voltage at various points in the system designated as generator (GEN.) first step-up transformer (1st S. U. T.), first step-down transformer (1st S. D. T.), distribution transformer (DIST. TRANS.), device (DEV.), etc. Fig. 1 gives the variations in per cent between actual voltage and winding voltage when using transformers built to existing standards; Fig. 2, similar data, using transformers as proposed in the Bulletin; and Fig. 3, using transformers according to Scheme A

Table III shows the maximum variation in per cent between actual circuit voltage and rated circuit voltage measured at the transformers but omitting distribution transformers.

It is possible with transformers designed according to Scheme A that voltages approximately 13 per cent above rating (7½ per cent over-excitation) may be impressed on the winding. It appears that there are many transformers built to existing standards now in operation at equal over-voltages and the record of trouble does not seem to indicate the necessity of increasing the transformer dielectric test. In rating oil circuit breakers at a standard system voltage instead of at a considerable over-voltage, as is the present custom with breakers below 110,000 volts, encroachment should not be made on the existing insulation factor of safety and the proposed change in the circuit breaker dielectric test therefore has been offered.

While not strictly a matter of voltage standardization, there is an opportunity for a very considerable reduction in the number of standard circuit breaker ratings if careful standardization is undertaken. By following the proposed system voltages and by elimination of many current, as well as rupturing capacity ratings, the number of standard ratings and list of standard breakers could be so shortened that the increase in production of breakers having duplicate ratings should produce a considerable reduction in manufacturing costs.

CONCLUSIONS

The data seem to indicate that the so-called Scheme A is preferable for the following reasons:

First. The number of standard voltage ratings is reduced.

Second. The flexibility of apparatus is increased. The transformers are practically interchangeable as step-up or step-down units and should be suitable for use on any well-designed system of the same rated voltage.

Third. The rated voltages of the transformers are close to the corresponding rated system voltages.

Fourth. The actual operating voltages are close to the winding voltage.

Discussion

For discussion of this paper see page 205.

Standardization of Voltage Ratings
For Power Systems and Equipment

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***Synopsis.**—This paper undertakes to analyze the voltage practises and requirements of alternating current power systems with the aim of arriving at voltage standards that adequately correct and extend present standards.*

Utilization, or receiving terminals, is taken as the point of reference and for designation of system, circuit, and equipment voltages.

For this analysis and development of voltage standards a comprehensive chart, showing operating voltage limits of representative systems, is given in Plate 1. A tabulation summarizing the proposed standard voltage ratings is given in Table XI.

FUNDAMENTAL CONSIDERATIONS

In the preparation of this paper the authors endeavor to express the results of their experience in dealing with the voltage problems of a number of alternating current power systems well distributed throughout the United States. These systems, while not serving the largest metropolitan areas, do include a wide diversity in size and kind which makes them perhaps representative of usual requirements.

It is not the purpose of this paper to demonstrate the inadequacy of existing standards of equipment voltage ratings, a condition already recognized by manufacturers and operators. Nor is it the purpose to elaborate on the benefits of standardization of equipment voltages. The resultant economy through reduction in a multiplicity of types, through efficiency and convenience in manufacture, in the handling of spare parts, and in flexibility in use, is self-evident.

The purpose of this paper is to develop and propose a schedule of standard voltages believed to meet actual operating requirements and practises in alternating-current power systems. The authors endeavor to deal with the problem without undue adherence to A. I. E. E. or other existing standards wherever experience places the adequacy of any such standards in question. It is believed desirable that in any such standardization, the opportunity be taken to correct inadequacies of this kind.

It must be recognized that many systems, particularly among those that are larger and of pioneer origin, are non-standard, and probably for a long time will continue non-standard, as regards any revised general voltage standards that may be worked out. It seems impracticable to develop voltage standards with sufficient steps to include all of these systems. Some of these will doubtless continue with their individual standards, and move toward general standards only as the opportunity to benefit arises from time to time in the natural course of replacements or reconstruction. Voltage standards, when determined, will in practise, fall short of universal application. They will in effect be schedules of preferred voltages serving as a guide to the maximum practicable uniformity of practise.

The system or class voltages now recognized and established by usage in the United States are as given in Table I.

The inadequacies of present standards of equipment voltages seem to have come about from lack of sufficient appreciation of the relation between voltage levels and functions of a modern power system. For example, take the case of present standard transformers which were developed for distribution service purposes. Many users have purchased these distribution ratio transformers and applied them at supply substations and even for step-up purposes. Experience shows that they cannot be used successfully for all three of these functions. Many of the conditions obtaining on present systems, of equipment operating at voltages seriously in excess of rating, have doubtless come about in this way.

TABLE I
SYSTEM OR CLASS VOLTAGES

115
230
460
575
2300/4000 Y
4,600
6600/11,430 Y
11,000
7620/13,200 Y
13,200
22,000
33,000
44,000
66,000
88,000
110,000
132,000
154,000
220,000

It is reasonable and in accord with operating conditions to define the fundamental reference plane for all voltage standardization of a given class, or the "nominal system voltage," as the mean voltage at utilization terminals, that is, at receiving terminals. This is the plane where the central station company meets the consumer, the "counter" where the product of the industry is delivered. The consumer is interested only in the voltage at his utilization terminals, not at some

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point in the complex supply system which he does not understand. A power company which purchases its energy is interested in the voltage maintained at the point where it is received, not at some remote point in the system of the transmission company. The present habit of designating system voltage in terms of generation or supply voltage gives an undue appearance of non-uniformity of voltages among systems. This is because generation voltage is used extensively as a means of taking up the voltage "slack" of the system, so that efforts to maintain uniform utilization voltage may bring substantial differences between voltage values at the respective points of generation. The point of utilization is thus the point of practical voltage reference. It is the logical point to establish as nominal or designation voltage.

In the usual power channel between the point of utilization and the source, there are essentially three voltage steps or levels, that of the utilization and distribution zone, that of the high voltage distribution zone, and that of the remote source or transmission zone. In building up a practical schedule of standard voltage ratings for equipment to fit these three zones and to give essentially nominal voltage at receiving terminals, first consideration should be given to actual operating requirements and practises. However, standard equipment ratings now in use should be retained as far as may be reasonably practicable, although this will require that suitable correction be made in some of these standards which now limit effective and efficient power supply under existing and modern methods.

Obviously, standardization in voltages of utilization equipment such as lamps, appliances and motors, where the great bulk of equipment is used, is basic. Voltage standardization for distribution equipment is of next importance, voltage standardization for transmission and high voltage equipment is of least importance. The latter applies in that portion of the system where inequalities can best and most economically be adjusted because units affected are relatively few in number and of large size. It is logical then that a study intended to review, to revise, and to better coordinate existing voltage standards should progress from the utilization end of the system.

UTILIZATION REQUIREMENTS

The basic assumption in this paper is a standard lamp and appliance rating of 115 volts and a standard motor rating of 220 volts. No conclusions are drawn as to the suitability of 115 volts as a lamp standard, as compared with 110, 120 or any other voltage, but rather the 115-volt rating is taken as essentially averaging present usage and recommendations of the National Electric Light Association Lamp Committee. Being a fundamental of voltage standardization, this 115-volt rating should be confirmed or some other thoroughly considered value definitely decided upon. If by reason of further study the 115 volt lamp standard should be changed, corresponding adjustments must be made in those values here given which are built upon this basic value.

In practise small utilization demands are served extensively from combined three-phase motor and lighting secondaries arranged in the 3 wire delta or 4 wire star system. The delta circuits probably constitute the usual distribution arrangement; however, the more recent rapidly extending and advantageous use of 4 wire secondary networks establishes the star system as an important distribution circuit arrangement, particularly in areas of heavy load density. Lamps and appliances of 115 volts, 220-volt motors and distribution step-down service transformers, particularly of the 2300-volt class, are in extensive use on both of these systems. It is essential that standardization be such that lamps, appliances, motors, and service transformers can be used interchangeably on both delta and star systems.

Table II indicates voltage conditions obtaining when present utilization equipment designed for and rated at 115 volts (or 110/220 volts for motors) is used at each of the three voltages, 110, 115, and 120. The values in the table give an approximate picture of the necessary excitation range of equipment, without intending any expression as to the correctness of present nameplate ratings in respect to service results. The minimum departures from rated voltage obtain with the 115-230 volt delta and 115-199-volt star terminal voltages considered jointly.

TABLE II

OPERATING VOLTAGES OF UTILIZATION EQUIPMENT

Average approximate per cent departures from present nameplate rated voltages

Lamp socket or terminal voltage	Lamps rated 115 v	Motors rated 110/220 v	Service transformers rated 115/230 v
110-220 delta	− 5	0	− 3
110-190 star	− 5	− 15	− 3
115-230 delta	0	+ 5	+ 2
115-199 star	0	− 10	+ 2
120-240 delta	+ 5	+ 10	+ 7
120-208 star	+ 5	− 5	+ 7

THE DISTRIBUTION SYSTEM

By definition, as previously noted, nominal system voltage is the mean voltage of equipment at utilization terminals or receiving terminals. It follows that in the ideally designed and operated distribution system nominal voltage of 115 will always obtain at that lamp or appliance so located in the distribution system as to receive the mean voltage of all lamps and appliances, that is to say, the divergence from the standard of 115 volts at the lamp terminals will be the same percentage for that lamp in the distribution system nearest the supply source, as it is for that lamp in the distribution system most remote from the supply source.

A typical distribution circuit comprises house wiring, service taps, secondary mains, service transformers, primary mains, primary feeders, and feeder regulators. To these various parts of the circuit are assigned assumed percentage limits of voltage drop, which are believed to conform reasonably to good practise.

These assigned values of voltage drop for a distribution system are given in Table III.

In developing voltage standard schedules, it is necessary to lay down a background comprising the limits of the various factors that must be covered and coordinated in the spread of such standards. To be worth while, these standards must anticipate future development and growth so far as practicable; that is, standards established now must not only be adapted to present needs but also must be developed with thorough consideration of future needs. Plate 1 is a chart indicating these limits sufficiently, it is believed, to serve as a guide in building voltage standards, at least tentatively, although further survey and analysis of actual conditions is desirable if not essential to final standardization.

Plate 1 includes typical 2300 and 13,200 nominal voltage distribution with primary feeders serving primary load centers, remote from the supply source.

TABLE III

LIMITS FOR DISTRIBUTION SYSTEM VOLTAGE DROPS IN PER CENT

	Heavy load	Light load
House wiring	2.0	0.5
Service tap	1.0	0.25
Secondary main	2.0	0.5
Service transformer	2.5	1.0
Primary main	2.0	0.8
Primary feeder	8.0	2.5

Note: Heavy load is taken at 0.9 power factor. Light load is taken at 25 per cent of heavy load and at 0.75 power factor.

In the 2300-volt portion of the chart, there are assumed distribution or service transformers of the present 20:1 ratio and 115 volts delivered under all loads at the lamp receiving mean voltage. It will be noted that this system requires a variation in voltage at the supply station bus of from 2398 volts at light load to 2674 volts at heavy load, or a mean voltage of 2536.

In the 13,200-volt nominal system shown on the chart, if there are assumed service transformers of the present ratio, 120:1, and 115 volts delivered at the lamp receiving mean voltage as before, there will be required a variation in voltage at the supply station from 14,400 volts at light load to 16,010 volts at heavy load or a mean voltage of 15,200 volts. Obviously, these values are excessive and untenable. For reasons that will be explained more fully later, a voltage of 13,900, that is, approximately 5 per cent above nominal, is considered the upper limit of permissible mean voltage for a distribution system of this voltage class. It follows that a change from the present standard transformer ratio for this class is essential. Assuming a ratio of 110:1, that is, 13,200:120-volt distribution transformers, it will be noted that the system requires a variation in voltage at the supply substation bus of from 13,190 volts at light load to 14,705 volts at heavy load or a mean voltage of approximately 13,900 volts.

NOTE A: As regards voltage regulation calculations by referring to Plate 1 and Table III, if we call X the per cent voltage drop from the secondary of the step-down service transformer to the mean voltage lamp, then from this lamp the voltage will rise X per cent to the secondary of the service transformer, 2.5 per cent through the transformer and 1.2 per cent back to the primary feeder load center. The lamp having the highest voltage will have the voltage of this load center reduced by 2.5 per cent through the transformer and 1 per cent through the service tap. Similarly the lamp having the lowest voltage will have this load center voltage reduced by $2 + 2.5 + 2 + 1 + 2$ or 9.5 per cent.

Now calling C the percentage departure from normal voltage, for the lamps, we have:

$$+C \text{ (highest voltage lamp)} = (X + 2.5 + 1.2 - 2.5 - 1)$$
$$-C \text{ (lowest voltage lamp)} = (X + 2.5 + 1.2 - 2 - 2.5 - 2 - 1 - 2)$$
$$\begin{aligned} +C &= X + 0.2 \\ -C &= X - 5.8 \\ X &= 5.8 - C \\ C &= 5.8 - C + 0.2 \\ C &= 3.0 \\ X &= 2.8 \end{aligned}$$

That is, the lamp having mean voltage will be so located that the voltage drop from the secondary of the service transformer to the lamp will be 2.8 per cent.

It is an important consideration that, next to utilization equipment, step-down service transformers represent the largest single class of standardized equipment in use today on alternating current power systems. It is accordingly essential to practical standardization to utilize these present ratio step-down service transformers, so far as practicable.

As has been shown it is practicable to utilize present ratios in the 2300 volt class. These transformers, now so extensively in use on nominal 2300 volt distribution systems, can be used for delivering rated voltage to standard 115-volt lamps. While this requires increasing the excitation of the transformers above nameplate rated value, it does keep within limits which experience has shown permissible. Reference to Plate 1 will indicate the voltage limits encountered in practise. Adoption of a 120-volt lamp standard would necessitate carrying the excitation of these transformers still higher and probably excessively above the voltage limits for which they are suited. While no modification in present 2300-volt service transformer ratios is necessary and full benefit of interchangeability with existing equipment of this class can be retained, the nameplate rating should be revised to 120-volt secondary, *i. e.*, 2400:120 volts.

For service transformers in the 13,200-volt class, and in fact all distribution classes except the 2300- and 4600-volt classes, new ratios and ratings must be assigned if excessive bus voltages and excessive feeder regulator ranges are to be avoided. As has been shown the proper ratio in the 13,200-volt class is 110:1, 13,200:120, and similarly for other classes.

On the foregoing basis, step-down service transformer ratings as in Table IV are proposed.

From Plate 1 it also becomes evident that feeder regulators, oil circuit breakers, disconnecting switches, lightning arresters, and other miscellaneous equipment used on distribution systems, under limiting load conditions may be subjected to operating voltages approximately 10 per cent, and in some cases 15 per cent, in excess of nominal system voltage. The proposed ratings for general apparatus are somewhat adjusted where practicable, in order to retain some existing voltage ratings of this apparatus. It is understood that all equipment, transformers and generators included, is designed for successful operation, with an emergency tolerance of 5 per cent above voltage rating. The proposed ratings closely approximate and in most cases coincide with existing ratings for general apparatus. In practical application some elimination and consolidation to reduce the number of classes may be found feasible. It will be noted also that the proposed ratings extend so as to overlap and include, on the same basis, equipment used in the transmission zone of voltages. As will be shown later, these ratings are suitable for use in the transmission zone.

TABLE IV
PROPOSED STEP-DOWN SERVICE TRANSFORMER VOLTAGE RATINGS

Nominal system or class voltage	High voltage*	Low voltage*
115	..	120
230	..	240
460	480	480
575	600	600
2300/4000 Y	2400/4150 Y	2300/4000 Y
4,600	4,800	4,600
6600/11,430 Y	6600/11,430 Y	6600/11,430 Y
11,000	11,000	11,000
7620/13,200 Y	7620/13,200 Y	7620/13,200 Y
13,200	13,200	13,200
22,000	22,000	..
33,000	33,000	..
44,000	44,000	..
66,000	66,000	..
88,000	88,000	..
110,000	110,000	..
132,000	132,000	..

The high voltage winding has two 5 per cent full-capacity taps except that all taps are omitted in the classes 2300-volt and below.

*All values are at nominal except 4600 volt and below in the high side and 575 volt and below in the low side. These values are raised approximately 5 per cent to more nearly conform to voltages actually encountered in practise.

The proposed voltage ratings for general apparatus are given in Table V.

The receiving voltage at a centrally located supply substation should be logically nominal system voltage as by definition this receiving point becomes essentially a major utilization terminal.

A power company may supply several distribution substations and customer service substations from the same feeder. It is of course not feasible to deliver nominal system voltage to all simultaneously. Assuming a maximum limit of 10 per cent total voltage drop of feeders extending across a major load area and assuming nominal system voltage at the central or mean point of that load area, the feeder voltage at the beginning, or leading edge, of the receiving zone will be at approximately 5 per cent above nominal, the far limits at 5 per cent below nominal.

Under these conditions, it is obvious that step-down transformers at the leading edge of a major load area would operate on full winding, those at the load center would operate on a 5 per cent tap and at the far limits of the area on a 10 per cent tap.

As indicated on Plate 1, in order to maintain constant voltage at the mean lamps, feeder regulators may be called upon to operate between the limits given in Table VI. Present standardization of 10 per cent buck and 10 per cent boost for feeder regulators, is, therefore, reasonably adequate. Without the change in service transformer ratios in the 13,200-volt class, heretofore discussed, a feeder regulator operating range of at least 20 per cent all on the boost side, is required.

Table VI shows conditions imposed on feeder regulators.

It is assumed as the basis of the voltage chart, Plate 1, that constant voltage is held at the supply substation bus at the beginning of the distribution zone, by generator voltage variation. Essentially constant voltage, held by synchronous condensers or otherwise, at some such point imtermediate between utilization and generation, is necessary to avoid a prohibitive corrective range of feeder regulators. The profile on Plate 1 shows the relative values and spread of voltage progressively back through the system, for both heavy and light load conditions.

TABLE V
PROPOSED VOLTAGE RATINGS FOR FEEDER REGULATORS, OIL CIRCUIT BREAKERS, LIGHTNING ARRESTERS AND OTHER MISCELLANEOUS APPARATUS

Nominal system or class voltage	Apparatus voltage rating*
115	125
230	250
460	500
575	625
2300/4000 Y	2500/4330 Y
4,600	5,000
6600/11,430 Y	7500/13,000 Y
11,000	12,000
7620/13,200 Y	8500/15,000 Y
13,200	15,000
22,000	25,000
33,000	37,000
44,000	50,000
66,000	73,000
88,000	96,000
110,000	120,000
132,000	145,000
154,000	170,000
220,000	240,000

*These apparatus voltage rating values are approximately nominal plus 10 per cent.

It should be noted that these profiles are not plotted directly in terms of voltage drop, as customary. Instead, they are plotted in per cent variation from nominal voltage. The purpose is to bring out more clearly the improved relations maintained between voltage levels and nominal voltage and the corrective benefits throughout the system, gained by utilizing the transformer ratios proposed in this paper. If these voltage profiles were carried back from utilization terminals to generator terminals, using the present standard single ratio transformer throughout, the voltage departure from nominal would be much greater than shown. In fact, the departure from nominal would be increasingly upon the excess voltage side and at the generator terminals would reach 18 per cent above nominal at heavy load and 4.5 per cent above at light load.

TABLE VI

OPERATING VOLTAGE RANGE FOR FEEDER REGULATORS

Approximate per cent buck and boost

Location of regulator (Refer to Plate I)	2300-volt circuits	13,200 volt circuits	
	Present ratio distribution transformers 20:1	Present ratio distribution transformers 120:1	Proposed ratio distribution transformers 110:1
Distribution at end of load area	+ 6.5 (− 12.5)	+ 21.5 (+ 7.5)	+ 14.5 (− 1.5)
Distribution at center of load area	+ 6.5 (− 10.0)	+ 17.5 (+ 5.5)	+10.0 (− 3.0)
Distribution at beginning of load area	+ 6.5 (− 7.5)	+ 13.5 (+ 3.5)	+ 5.5 (− 5.0)
High voltage distribution at end of load area	+ 6.5 (− 6.0)	+ 14.0 (+ 0.0)	+ 6.0 (− 6.0)
High voltage distribution at center of load area	+ 6.5 (− 4.5)	+ 14.0 (+ 4.0)	+ 6.0 (− 4.5)
High voltage distribution at beginning of load area	+ 5.5 (− 2.0)	+ 13.5 (+ 6.5)	+ 5.5 (− 2.0)

THE HIGH-VOLTAGE SYSTEM

The preceding discussion, starting from utilization requirements and working back through the distribution system, determines the necessary bus voltages at the supply substation. If this supply is derived from transmission or high voltage distribution circuits, then the low voltage windings of supply substation step-down transformers must deliver this required voltage.

Plate 1 includes a representative high voltage distribution system (66,000 volts nominal), comprising high voltage service transformers, supply substation transformers, transmission circuits and step-up transformers. Plate 1 also extends further to include a representative transmission system, (132,000 volts nominal), serving this high voltage distribution system from a remote generating source. Generating equipment may be used on any or all of the several supply buses in the composite picture. Many combinations other than those shown on Plate 1 are possible. This chart is intended only to set forth representative limits adequate for voltage standardization purposes.

To the respective parts of representative transmission and high voltage distribution systems are assigned assumed limits of voltage drop as given in Table VII.

These values of Table VII determine the necessary voltage ratings of supply substation step-down transformers at ratios as given in Table VIII.

TABLE VII

LIMITS FOR HIGH VOLTAGE SYSTEM VOLTAGE DROPS IN PER CENT

	Heavy load	Light load
Step-down supply substation transformer	5.0	2.0
High-voltage step-down service transformer	5.0	2.0
High-voltage distribution feeder (to center of high-voltage distribution zone)	5.0	2.0
Transmission line (to beginning of high-voltage distribution zone)	5.0	2.0
Step-up transformer	5.0	2.0

Note: Heavy load taken at 0.9 power factor average, light load taken at 35 per cent of heavy load and at 0.85 power factor average. The values of per cent drop given above also take to account load diversities, power factors, load factors, and line charging characteristics.

Plate 1 indicates the necessary voltages on the generating station high-voltage buses which thus determines the high-voltage rating of step-up transformers. The low-voltage rating of these transformers must for standardization purposes be consistent with the voltage rating for generators as described in later paragraphs. The voltage ratings for step-up transformers given in Table IX will satisfy these conditions assuming excitation of the low-voltage winding to be normal when overcoming transformer regulation.

TABLE VIII

PROPOSED STEP-DOWN SUPPLY SUBSTATION TRANSFORMER VOLTAGE RATINGS

Nominal system or class voltage	High voltage*	Low voltage†
2300/4,000 *Y*	..	2500/4330 *Y*
4,600	4,600	5,000
6600/11,430 *Y*	6600/11,430 *Y*	6900/11,950 *Y*
11,000	11,000	11,500
7620/13,200 *Y*	7620/13,200 *Y*	8000/13,860 *Y*
13,200	13,200	13,800
22,000	22,000	23,000
33,000	33,000	34,500
44,000	44,000	46,000
66,000	66,000	69,000
88,000	88,000	92,000
110,000	110,000	115,000
132,000	132,000	138,000
154,000	154,000	162,000
220,000	220,000	..

All high voltage windings have two 5 per cent full-capacity and one 5 per cent reduced-capacity taps.

*Rated at nominal.

†4600 volts and below rated at nominal + 10 per cent.
6600 volts and above rated at nominal + 5 per cent.

At a generating station bus a step-up transformer is often used to serve a long high-voltage transmission to a remote load area. A high-voltage rating to provide for 10 per cent voltage drop in transmission seems a reasonable upper limit for standardization purposes.

Step-up transformers will operate normally on taps when located at the edge of the receiving or load area so that the transmission zone in effect is eliminated.

Table IX provides for the foregoing conditions.

While separate types are proposed to meet the foregoing three functional duties of transformers, it is recognized that by providing a broader tap range with increased iron to enable full voltage excitation on under-voltage taps, any two or all three of these types can be consolidated into a single standard. For example, to consolidate the service type and the supply substation type requires a high-voltage winding rated at nominal voltage with a 15 per cent full capacity tap range and 20 per cent total tap range and capable of normal excitation with full normal voltage impressed on the 95 per cent tap. Similarly, to consolidate all three types requires a high-voltage winding rated at 10 per cent above nominal voltage with a 25 per cent full capacity tap range and 30 per cent total tap range, and capable of step-down duty at normal excitation with full normal voltage impressed on the 85 per cent tap. The extent to which these three types should be consolidated, if at all, in final standards, is essentially an economic problem that must be decided by thoroughly weighing the benefits of interchangeability against the increased cost and other factors concerned.

TABLE IX
PROPOSED STEP-UP TRANSFORMER VOLTAGE RATINGS

Nominal system or class voltage	Low voltage*	High voltage†
115	115	..
230	230	250
460	460	500
575	575	625
2300/4000 Y	2300/4000 Y	2500/4330 Y
4,600	4,600	5,000
6600/11,430 Y	6600/11,430 Y	7200/12,500 Y
11,000	11,000	12,000
7620/13,200 Y	7620/13,200 Y	8400/14,500 Y
13,200	13,200	14,500
22,000	22,000	24,000
33,000	33,000	36,000
44,000	44,000	48,000
66,000	66,000	72,000
88,000	88,000	96,000
110,000	110,000	120,000
132,000	132,000	145,000
154,000	154,000	170,000
220,000	..	240,000

All high-voltage windings have two 5 per cent full-capacity taps.
*Rated at nominal.
†Rated at nominal + 10 per cent.

A study of the limits shown on Plate 1 indicates that voltage ratings of oil circuit breakers, disconnecting switches, lightning arresters, and other miscellaneous apparatus as previously shown in Table V meet the requirements and operating practises of the high voltage system.

While the proposed ratings raise the present test voltages for general apparatus in the higher voltage classes, redesign of this apparatus is not necessarily required. Much of this apparatus is now operating successfully at voltages in excess of nameplate rating. In determining test voltages for these various types of equipment, it is assumed of course that designers will make allowance for differences in fundamental characteristics. Furthermore, as rapidly as advance of knowledge will permit, it is essential that the effects of impulse and other transient voltage phenomena be taken to account.

GENERATION REQUIREMENTS

Generators, in many and perhaps the usual cases, deliver directly into supply buses and may operate at fixed voltage with feeder regulators or at variable voltage without regulators, within the limits shown on Plate 1. They must also coordinate with the low-voltage ratings of supply substation step-down transformers and step-up transformers, as previously proposed in Tables VIII and IX respectively. Furthermore, they must be suitable for use in industrial or isolated plant application.

Table X gives proposed generator voltage ratings to cover these requirements.

While synchronous condensers closely resemble generators in form, they are different in operating characteristics. They require special consideration in the matter of voltage rating and their comparatively limited use makes standardization at the present time probably premature.

SUMMARY TABULATIONS OF PROPOSED VOLTAGE RATINGS

The foregoing analyses are tabulated, for convenient reference, in Table XI, which assembles the proposed voltage ratings for equipment for alternating current power systems. In this tabulation are shown voltage steps and values considered sufficient to cover the range of system conditions reasonably subject to standardization.

TABLE X
PROPOSED GENERATOR VOLTAGE RATINGS

Nominal system or class voltage	Normal voltage rating*	Rated operating voltage range ± 10%
115	120	110–130
230	240	220–260
460	480	440–520
575	600	550–650
2300/4000 Y	2400/4150 Y	2200/3800 Y – 2600/4500 Y
4,600	4,800	4,400–5,200
6600/11,430 Y	6900/11,950 Y	6300/11,000 Y–7620/13,200 Y
11,000	11,500	10,500–12,500
7620/13,200 Y	8000/13,860 Y	7200/12,500 Y–8800/15,200 Y
13,200	13,800	12,500–15,200

*Rated at nominal + 5 per cent.

In reading the values in these tabulations, it should be borne in mind that ideal precision has been avoided as impracticable. It is believed that the frequent approximations employed are within reasonble limits of precision considering the many variables and inconsistencies being dealt with.

CONCLUSIONS

1. Present voltage standards for alternating current power systems and equipment are inadequate as they do not fit the needs and practises of representative power systems. This condition is generally recognized.

2. In developing general voltage standards, it must be recognized that there are some systems, particularly those of pioneer origin, which, because of large size and individual characteristics of voltage, will doubtless continue their individual standards and move toward general standards only as the opportunity to benefit arises from time to time in the natural course of replacements and reconstruction. General voltage standards will, in effect, be schedules of preferred voltages, serving as a guide to the maximum practicable uniformity of practise.

3. In developing voltage standards, a fundamental is to establish a suitable reference level or plane for voltage designation. The present practise of a reference level established at the point of generation is not suitable. As voltage "slack" of the system is most frequently taken up at the generating stations, this method of designation gives an undue appearance of non-uniformity of voltages as between systems. At utilization, or receiving terminals, is the point of maximum inherent voltage uniformity because this is the plane where the product is delivered and service is gaged. Uniformity of product at the point of delivery is the goal of system design and operation. Therefore, the fundamental reference plane for all voltages of a given class, or "nominal system voltage," is defined as the mean voltage at utilization terminals, that is, at receiving terminals.

4. Another fundamental is to segregate the power system along functional lines into zones of different voltage levels. It is especially essential in arriving at standards for transformer voltages and ratios that the correct relation between the zones of distribution, high voltage distribution and transmission be clearly understood.

5. A third fundamental is to adhere to present standards and practises in so far as reasonably adequate. However, where experience shows that any related standards including those of the A. I. E. E. are incorrect or insufficient, any needed corrections and extensions should be made.

6. A standard lamp and appliance rating of 115 volts and a standard motor rating of 220 volts are assumed. No conclusions are drawn as to the suitability of these voltages as compared with 110, 120 or any other lamp voltage. The 115 volt rating is taken as essentially averaging present usage and recommendations of the N. E. L. A. Lamp Committee. The definite establishment of a suitable value for lamp socket voltage is basic and should constitute an initial step in developing general voltage standards.

7. In practise, there are two outstanding systems of power and lighting secondaries, the three-wire delta and the four-wire star. It is important that standard utilization equipment be fully interchangeable as between these two systems of supply.

8. A chart is given, Plate 1, which brings out voltage relations between the essential features of power systems, together with operating voltage limits, as a means of determining the spread of operating voltages that must be provided for in general voltage standards. Basic assumptions for this chart include constant voltage held at the mean lamp terminal of each distribution feeder load area, by means of feeder regulators, and constant voltage held at some other plane intermediate between utilization terminals and generating station by generator voltage variation. The chart shows a profile of voltages progressively through the systems.

9. Tabulations in Table XI summarize the proposed standard voltage ratings for systems and various types of equipment. A foot note shows the relations, in percentages, between the proposed standard system and equipment voltages. However, in the tabulated values, frequent approximations are accepted in the interest of closer adherence to present standards. The more important features of the voltage standards proposed in Table XI include:

a. The present 20:1 ratio for 2300-volt distribution transformers is retained but normal rated voltages are raised from 2300:115 to 2400:120.

b. Both ratio and normal rated voltages are changed for the distribution transformers of the 13,200 volt class.

c. Three types of transformers are established for each voltage class, each having a ratio specific to its own functional duty. These types are service step-down, supply substation step-down, and step-up.

d. Transformer ratings are such, in relation to circuit voltages, that excitation on full winding is correct when the transformer is located at the beginning or leading edge of a load area. Taps in 5 per cent steps are provided for compensating regulation across the load area.

e. The present range of 10 per cent buck and 10 per cent boost for feeder regulators is retained.

f. Normal rated voltage for generators is raised approximately 5 per cent from present standards, and a normal operating range from plus 10 per cent to minus 10 per cent is required.

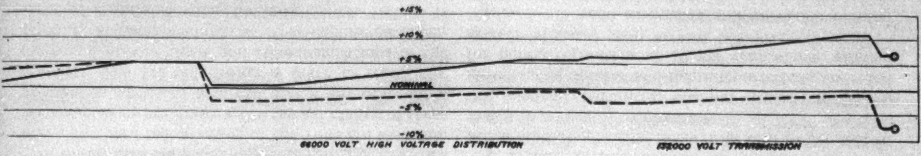
g. Present normal rated motor voltage is retained, and a normal operating range from plus 10 per cent to minus 10 per cent is required.

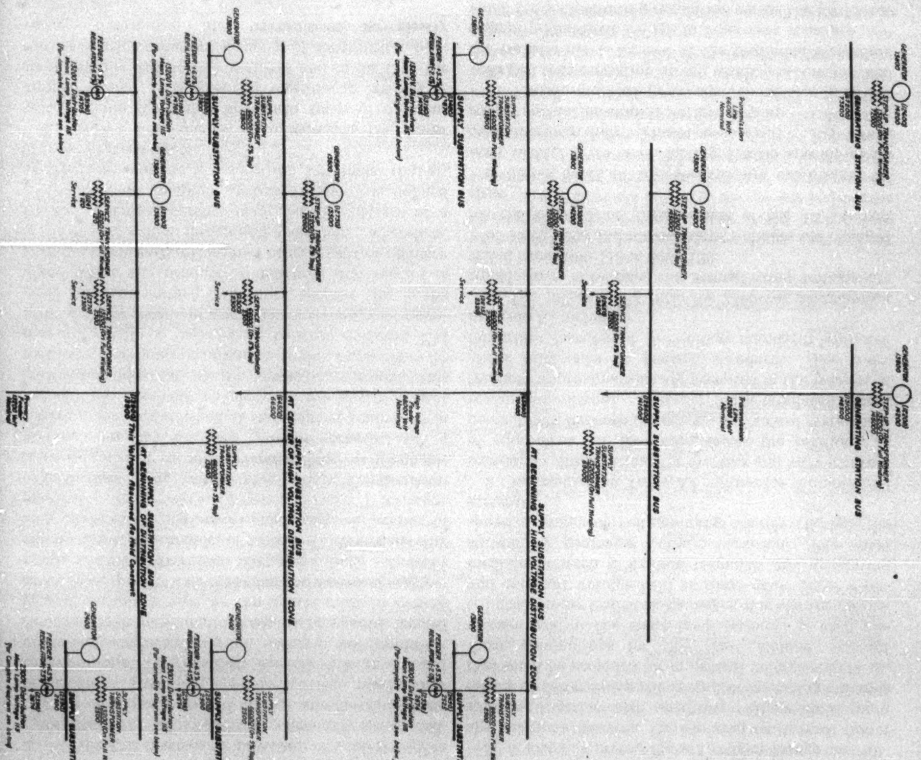
h. General apparatus, such as feeder regulators, switching equipment and lightning arresters, are rated for normal operation at 10 per cent above nominal system voltage.

i. A 5 per cent emergency tolerance above rated maximum operating voltage for all equipment is required.

10. The proposed standard voltage ratings for all equipment above 66,000 volts are higher than present

IN RELATION TO NOMINAL VOLTAGE




GENERATOR 13800
STEP-UP TRANSFORMER
GENERATING STATION BUS
Transmission Line 132000 Volt Nominal
Transmission Line 66000 Volt Nominal
SUPPLY SUBSTATION BUS
SUPPLY SUBSTATION BUS AT BEGINNING OF HIGH VOLTAGE DISTRIBUTION ZONE
SUPPLY SUBSTATION BUS AT CENTER OF HIGH VOLTAGE DISTRIBUTION ZONE
SUPPLY SUBSTATION BUS AT BEGINNING OF DISTRIBUTION ZONE
This Voltage Assumed As Held Constant
High Voltage Feeder 66000 Volt Nominal
Primary Feeder 13200 Volt Nominal
SUPPLY SUBSTATION TRANSFORMER
SERVICE TRANSFORMER
Service
FEEDER +5.5% REGULATOR
FEEDER +4.5% REGULATOR
2300V Distribution Main Lamp Voltage 115 (For complete diagram see below)
13200V Distribution Main Lamp Voltage 115 (For complete diagram see below)
HIGH VOLTAGE DISTRIBUTION
TRANSMISSION

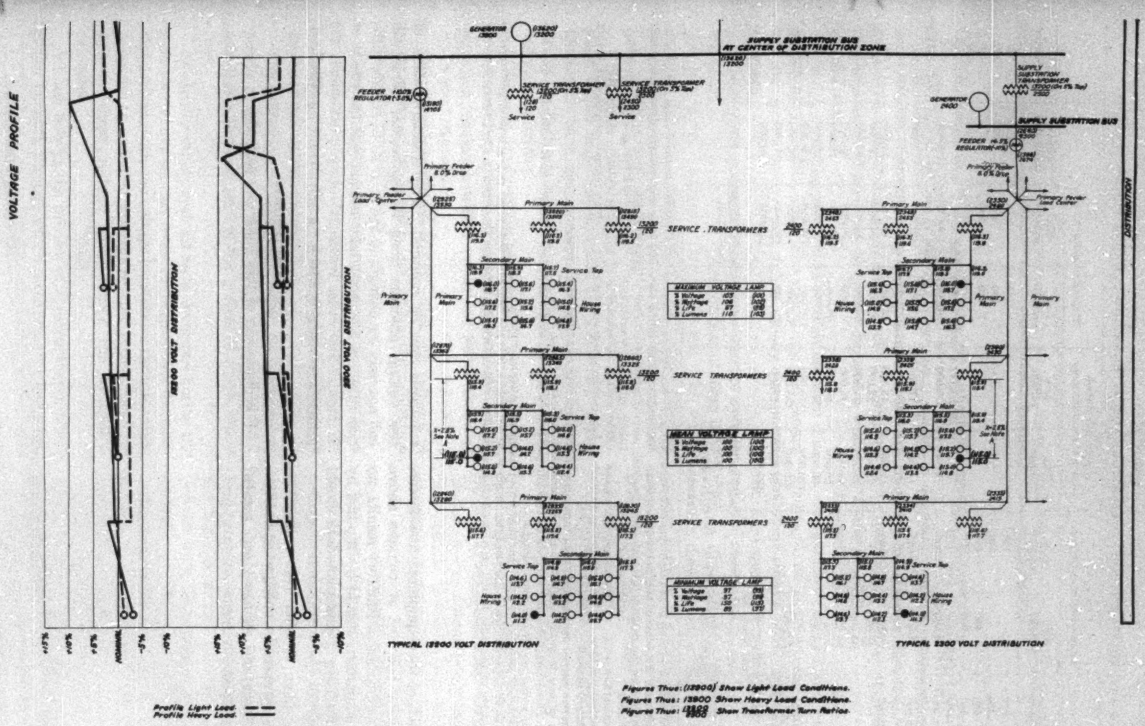

VOLTAGE PROFILE
13200 VOLT DISTRIBUTION
2300 VOLT DISTRIBUTION
+15%
+10%
+5%
NOMINAL
-5%
-10%
Profile Light Load
Profile Heavy Load
GENERATOR 13800
(13620) 13200
SUPPLY SUBSTATION BUS AT CENTER OF DISTRIBUTION ZONE
(13626) 13200
FEEDER +10.0% REGULATOR (-3.0%)
(13180) 14108
SERVICE TRANSFORMER 13200 (On 5% Tap) 120
(128) 120
Service
SERVICE TRANSFORMER 13200 (On 5% Tap) 2300
(2430) 2300
Service
SUPPLY SUBSTATION TRANSFORMER 13200 (On 5% Tap) 2300
GENERATOR 2400
SUPPLY SUBSTATION BUS
FEEDER +6.5% REGULATOR (-1.0%)
Primary Feeder 6.0% Drop
Primary Feeder Load Center
Primary Main
Secondary Main
Service Tap
House Wiring
SERVICE TRANSFORMERS
13200/120
2400/120
See Note
DISTRIBUTION
MAXIMUM VOLTAGE LAMP
% Voltage 105 (100)
% Wattage 106 (102)
% Life 87 (98)
% Lumens 110 (103)
MEAN VOLTAGE LAMP
% Voltage 100 (100)
% Wattage 100 (100)
% Life 100 (100)
% Lumens 100 (100)
MINIMUM VOLTAGE LAMP
% Voltage 97 (99)
% Wattage 97 (98)
% Life 130 (113)
% Lumens 89 (97)
TYPICAL 13200 VOLT DISTRIBUTION
TYPICAL 2300 VOLT DISTRIBUTION
Figures Thus: (13800) Show Light Load Conditions.
Figures Thus: 13800 Show Heavy Load Conditions.
Figures Thus: 13800/2300 Show Transformer Turn Ratios.

PLATE NO. 1

CHART SHOWING LIMITS OF VOLTAGE
FOR REPRESENTATIVE ALTERNATING CURRENT POWER SYSTEMS

TABLE XI

PROPOSED SYSTEM AND EQUIPMENT VOLTAGE RATINGS FOR ALTERNATING-CURRENT POWER SYSTEMS

Nominal system or class voltage*	Lamps and appliances	Motors†	Service step-down transformers		Supply sub-station step-down transformers		Step-up transformers		General apparatus	Generators
			High voltage	Low voltage	High voltage	Low voltage	Low voltage	High voltage		
115	115	110		120			115		125	120
230	230	220		240			230	250	250	240
460		440	480	480			460	500	500	480
575		550	600	600			575	625	625	600
2300/4000 Y		2200/3800 Y	2400/4150 Y	2300/4000 Y		2500/4330 Y	2300/4000 Y	2500/4330 Y	2500/4330 Y	2400/4150 Y
4,600		4,400	4,800	4,600	4,600	5,000	4,600	5,000	5,000	4,800
6600/11,430Y		6300/11,000Y	6600/11,430Y	6600/11,430Y	6600/11,430Y	6900/11,950Y	6600/11,430Y	7200/12,500Y	7500/13,000Y	6900/11,950Y
11,000		10,500	11,000	11,000	11,000	11,500	11,000	12,000	12,000	11,500
7620/13,200Y		7200/12,500Y	7620/13,200Y	7620/13,200Y	7620/13,200Y	8000/13,860Y	7620/13,200Y	8400/14,500Y	8500/15,000Y	8000/13,860Y
13,200		12,500	13,200	13,200	13,200	13,800	13,200	14,500	15,000	13,800
22,000			22,000		22,000	23,000	22,000	24,000	25,000	
33,000			33,000		33,000	34,500	33,000	36,000	37,000	
44,000			44,000		44,000	46,000	44,000	48,000	50,000	
66,000			66,000		66,000	69,000	66,000	72,000	73,000	
88,000			88,000		88,000	92,000	88,000	96,000	96,000	
110,000			110,000		110,000	115,000	110,000	120,000	120,000	
132,000			132,000		132,000	138,000	132,000	145,000	145,000	
154,000					154,000	162,000	154,000	170,000	170,000	
220,000					220,000			240,000	240,000	
			2400 v class and below–no taps others 2–5% full capacity taps		2–5% full capacity taps 1–5% reduced capacity tap			2–5% full capacity taps		

Notes: Except for service transformers up to and including 4800 volts, which are deviated to adhere to ratios now in use, the voltage ratings of all transformer primaries coincide with the value of nominal system voltage. Except for this deviation and some other approximations, to more closely coincide with existing standard ratings, the values in this table bear essentially uniform relations to values of nominal system voltage as follows:

Lamps and appliances—at nominal	Service transformers	Supply substation transformers	Step-up transformers
Motors —at nominal minus 5%	High voltage—at nominal	High voltage—at nominal	Low voltage —at nominal
General apparatus —at nominal plus 10%	Low voltage—at nominal	Low voltage —at nominal plus 5%	High voltage—at nominal plus 10%
Generators —at nominal plus 5%			

Generators and motors have a normal operating range of 10 per cent above and 10 per cent below rated voltage.

Feeder regulators have a normal operating range of 10 per cent boost and 10 per cent buck.

For transformers in general excitation is normal when the voltage impressed on the primary terminals under full rated load is sufficient to overcome regulation and maintain rated voltage on the secondary terminals.

All equipment has an emergency tolerance of 5 per cent above rated maximum operating voltage.

*As between the 6600/11,430 Y, 11,000 classes and the 7620/13,200 Y, 13,200 classes, it is recommended that so far as practicable, preference be given to the 7620/13,200 Y, 13,200 classes with the aim of eventual elimination of the 6600/11,430 Y, 11,000 classes. Possibly other eliminations may eventually be found advisable.

†Present nameplate ratings.

ratings. Because much of this equipment now in service is of necessity being operated at voltages above present ratings with reasonably satisfactory results, no extensive redesign of equipment should be imposed by these requirements. Revision of nameplate data should frequently be sufficient.

11. It is not undertaken to offer test voltage requirements for the proposed voltage standards. Doubtless, some increases and changes from present standards will be required. The standardization of test voltage practises must be deliberate. This will call not only for changes in the A. I. E. E, standardization rules but also must take into account the effects of is impulse and other transient voltage phenomena, the present knowledge of which, though advancing, is limited.

Discussion

For discussion of this paper see page 205.

Voltage Standards for Electrical Distribution

H. B. GEAR[1]
Fellow, A. I. E. E.

Synopsis.—*This paper discusses the necessity for standardizing voltages and advocates the utilization voltage as the most logical reference base. It suggests that the ratios adopted should be uniform at all voltages. It also proposes that there should be a recognized standard for transformers in which the direction of energy flow is subject to change.*

THE necessity of standardization of utilization voltages has been recognized and accepted since the days when electric lighting systems competitive with different lamp voltages and operating frequencies constituted such an obstacle to progress that standard voltages and frequency became a commercial necessity.

Systems operating at 55 volts for lighting were discarded for 110 volts, and a 220-volt rating was chosen for use where energy was taken chiefly for power purposes.

In later years, processes of lamp manufacture were so improved as to permit the concentration of lamp output into three voltages—110, 115, and 120 volts. A steady increase in the 115-volt output, and a decrease in the relative outputs of 110- and 120-volt lamps has been in progress since that time.

Utilization voltage standards are now so well recognized that a multitude of household and motor-driven appliances have been produced in quantities and at prices which would have been totally impossible without standardization.

Utilities in many states are required by regulatory bodies to adopt a utilization voltage standard and to maintain regulation within prescribed limits above or below such standard.

This to a considerable extent, fixes the voltages in other parts of the system and makes the utilization voltage the natural base of reference.

The proposal to establish ratings which are integral multiples of 115 is a recognition of the fact that the utilization voltage is the most logical base of reference in an electricity supply system.

The utilization voltage is one which must be kept as nearly constant as possible through all ranges of load, and this is the only part of the system of which this is true.

Electricity supply systems have developed during the past quarter of a century from simple groups of distribution feeders, with one voltage level above the utilization pressure, to extended systems serving large areas with two or three voltage levels above the utilization pressure.

We are now in the process of constructing a superpower network through which these areas are being tied together, and which, in some cases, adds another voltage level to those already in service.

Each additional transformation, with its accompanying line, has added to the drop in voltage and necessitated provision of taps or other means of compensating for the added drop.

The result has been that apparatus is being operated at voltages above that for which it was designed, special windings have been specified, generators are being over-excited at certain hours, and the manufacturers have felt it necessary to call a halt for the discussion of remedial measures.

As an illustration of what is taking place in some systems, they present a diagram of voltage drops in the various parts of a system having five transformations between the generator and the consumer, with a total of about 50 per cent voltage drop between generator and consumer.

The drops chosen for illustration are average values and do not represent cases which could be found in practise involving greater drops.

It is obvious that the maintenance of voltage regulation under such conditions is a difficult problem with the best of equipment, and when there is imposed the limitation that the apparatus must not be subjected to over-voltage, the limitation will quite surely in many cases be exceeded.

PROPOSED CHANGES IN PRACTISE

The manufacturers have presented a scheme for accomplishing this by increasing secondary voltage ratings to multiples of 115 instead of 110, as at present rated.[2]

This provides a no-load pressure about 5 per cent above that given by the present standard and a full-load pressure, which offsets the drop in the transformer and delivers at full load the same pressure which is derived from transformers of the present standard at no load.

Taps are to be provided in the primary coils, as at present, to care for situations where the drop in connecting lines requires more compensation.

It is proposed that generator regulation be held within a range, from plus 5 to minus 5 per cent of standard rating.

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[1] Commonwealth Edison Co.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

[2] *Voltage Standardization of A-C. Systems from the Viewpoint of the Electrical Manufacturer*, by F. C. Hanker and H. R. Summerhayes, A. I. E. E. Winter Convention, February, 1927.

SOME LIMITATIONS OF THE PROPOSAL

The example presented shows the application of the new plan under full-load conditions, where a single series of circuits is involved

The complete picture of an average system would reveal other circuits taken off at the generating station delivering energy at the generator voltage level. It would also include additional lines taken off at the 132/66/13.2-kv. levels, having various percentages of line drop, which must be taken into account in fixing the setting of transformer taps.

Inevitably there will be certain of these branches which will deliver a pressure above or below normal during certain parts of the day.

In the particular circuit shown, the pressure at quarter load would be from 10 to 12 per cent above normal at the utilization circuits, assuming that the generator pressure was reduced from 13.8 to 12.8, as proposed, during the light load period and the feeder regulator were run at full choke. In addition to this, the transformer and line equipment at the 69-kv. and 13.8-kv. levels are also operating at 10 to 12 per cent above the proposed maximum voltage ratings.

This, of course, can be avoided by the use of tap changing devices operable under load, and it is quite apparent that such auxiliary equipment must be included in the completed picture of the situation.

The necessity of having a considerable range of control between light-load and full-load conditions suggests the desirability of having as wide a range as is economically feasible in the generator voltage. It is proposed to get a range of 10 per cent by operating at pressures below normal (down to minus 5 per cent) during light load, and at pressures above normal (up to plus 5 per cent) during hours of heavy load.

This will prevent over-excitation of generators if transformer taps are connected to give the necessary boosting voltage to compensate for full-load drop. At light load periods, tap changers will be required where the total compensation in transformers is more than the combined range of generator and potential regulators.

It is proposed that the voltages above 69 kv. be left as multiples of 110.

The stage of development thus far attained may be such that it is a difficult matter to make a change now, but to those of us who have not until recent years been brought into pressures above 69 kv., it comes as something of a shock to learn that if we subject our 132-kv. equipment to any pressure above 132 kv., we are exceeding the manufacturer's rating and presumably taking a risk in operation which is not shared by the manufacturer.

The manufacturer's explanation that it is not necessary to use the multiples of 11.5 in fixing rated voltages at pressures above 69 kv. is lacking in any reasons sufficient to warrant a break of so fundamental a character in the proposed standards.

Whether the 132-kv. equipment in service has been designed for a maximum pressure of 132 kv. or not, much of the existing 132-kv. equipment is so related to the system of which it is a part that inevitably it must be subjected to pressures up to 138 kv. or higher, under the normal conditions of daily operation.

The proposed plan of adding 5 per cent to the secondary as a part of the fixed ratio of a transformer obviously can not be followed on lines where the direction of flow of energy is changed from time to time, as is often the case in tie-lines between power stations.

In such lines the transformers must meet the voltage requirements of both step-up and step-down transformers. Also, they are often so related to each other that, in order to transfer energy in the desired amount without displacing the general level of system pressure, they must be equipped with pressure taps adjustable under load.

This requires pressure taps giving a range of 15 to 20 per cent in either direction to deliver proper pressure at the bus of the receiving station.

OBSERVANCE OF STANDARDS

It is obvious that if a standard is to accomplish its purpose, it must be one which will be generally recognized as practicable of application and feasible in operation.

The failure of previously adopted standards to be generally observed seems to have resulted, in part, from a lack of adaptability of apparatus to working conditions, and perhaps from a lack of appreciation on the part of some engineers of the wide range of voltage drops which have been introduced in recent years into distributing systems.

The lack of adaptability has been met as nearly as is possible by the manufacturer's proposal to increase voltage ratings to a point where they will be similar to pressures which are normally encountered in practise.

The discussion of this subject in connection with the proposed changes has and will further serve to bring a greater number of engineers face to face with the situation in a way which will be beneficial.

The adoption of standards which automatically add voltage for full-load conditions will draw attention to the necessity of providing means of preventing overvoltage at light loads.

CONCLUSION

Voltage standards are a basic necessity for utilization equipment and are the basis of voltage throughout the system.

The manufacturer's proposal offers a decided improvement over present standards, but should treat all voltages on a uniform basis.

There should be a recognized standard for transformers used in interconnections where the direction of flow is subject to change.

When standards have been fixed which are applicable without radical change in existing equipment, they will, no doubt, be accepted and adopted by users of equipment.

Discussion

For discussion of this paper see page 205.

Voltage Standardization

Its Relation to the Interconnected Power Companies of the Southeast

BY H. J. SCHOLZ, W. W. EBERHARDT, and S. MURRAY JONES[1]
Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Voltage standardization has long been recognized as desirable, and to date no definite standards have been accepted and applied. An attempt is now being made to coordinate the recommendations of the different power companies and manufacturers with a view towards adopting definite voltage standards for all apparatus.*

A proposal has recently been submitted by the manufacturers which is not applicable in its entirety to conditions in the Southeast, and this paper contains a counter-proposal submitted as being more readily acceptable to the interconnected power systems of the Southeast.

I. INTRODUCTION AND JUSTIFICATION FOR VOLTAGE STANDARDIZATION

A proposed set of voltage standards appeared in 1922, but lack of cooperation on the part of both manufacturers and operators prevented any general acceptance of them. This was partially due to the fact that insufficient consideration was given to existing apparatus. In 1922 the power companies of the United States were valued at $4,500,000,000, while today they are valued at $9,000,000,000. This tremendous increase, appearing largely as new equipment, indicates the truth of the statement that the longer standardization remains a nonentity, the more complicated will be its final acceptance. Very few operating companies have attempted to eliminate what might be termed off-standard voltages in order to conform to the 1922 standards; in most cases, however, they cannot be justly criticized for this. Time was required for a realization of the savings to be had and as they have grown closer together, the benefits to be derived from standardization have presented themselves. Today most companies are willing to accept any such standards resulting in either financial gains to them or improvements to their service.

No time need be wasted in consideration of standards to be put into effect immediately. No operating company can afford to change its present equipment over night. Standardization in its final form will come about by virtue of a slow influx of semi-standard equipment which, when the absorption is complete, can be operated on standard voltages. This transition period will cover a number of years. Modifications or extensions to present properties made necessary in order to be in accord with any accepted standards must be consistent with proper financing and engineering practises, and any expense incident to such changes must not exceed the monetary values of standardization.

The efficiencies of electrical equipment are in most instances probably as high as will be attained, so that further large economies must be looked for in the development of radically new apparatus, in increasing the load factor of present equipment, and in standardization. The recent acceptance of lamp standards has gone far to bring about the present low cost and greater efficiency of lamps which the public now enjoys. In a like manner, actual savings in the cost of generating and transmission equipment will be evidenced by the standardization of voltages.

It is not to be expected that any expense incurred by carrying out the features of standardization will be returnable immediately in the form of huge savings, but economies will result from the enormous reductions in capital investment required over a long period of time, as compared to what would be necessary were the present policies to continue. The value of voltage standardization has also been emphasized by the benefits which may be obtained therefrom on interconnected systems. Interconnection will be practised more extensively in the future than at present, of course, because of the resulting economies. For example, in the territory covered by the interconnected Southeastern systems there are three distinct water-sheds covering very extensive areas over which the rainfall varies in different rivers in such a way that excess hydro capacity in one section can be used at times to assist systems in other sections in meeting their demands or in filling their storage reservoirs, thereby reducing the amount of reserve steam capacity required and also utilizing water which would otherwise go to waste. In addition to these economies, which are very great, the interconnected systems have been able to improve voltage regulation and service to their customers.

The economic and commercial justification for the standardization of all types of equipment has been proved many times and has ultimately resulted, in practically all cases, in simpler manufacturing methods and a lower cost of the product to the purchaser. There can be no doubt, therefore, as to the feasibility of standardization of voltages by the power companies. Any standards must be adaptable, however, to the large amount of equipment which has already been installed

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[1] All of the Alabama Power Company, Birmingham, Alabama.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

TABLE I

PRESENT RATED CIRCUIT VOLTAGES ON SOUTHEASTERN INTERCONNECTED SYSTEMS

Southern Power Co.	Carolina Pr. & Lt. Co.	Tennessee Elec. Pr. Co.	Alabama Power Co.	Georgia Ry. & Pr. Co.	Columbus Elec. & Pr. Co.	Central Ga. Pr. Co.
2,300	2400/4150 Y	2,300	2300/4000 Y	2300/4000 Y	2,300	2,300
	6,600					
6,600	7200/12,480 Y	6,600	6,600	6,600	5,500	6,600
			6900/11,950 Y			
11,000	11,000		12,000	11,000	11,000	11,000
13,200		13,200	13,200		12,000	
				19,000		19,000
	22,000	22,000	22,000	22,000		
		33,000		38,000		
44,000		44,000	44,000	44,000		
	60,000	66,000			66,000	66,000
88,000						
100,000	100,000	120,000	110,000	110,000	110,000	110,000

and, to be of any value, must be accepted by the majority of operating companies.

II. General Problems Confronting Southeast in Standardization

Table I gives the present rated circuit voltages of the interconnected companies in the Southeast and shows the conditions to be met in recommending any set of standards. It should be noted here that some of these systems are being operated at voltages slightly higher than those indicated in Table I. A map, Fig. 1, shows the transmission systems of these companies.

Before proceeding further, some of the general problems confronting the acceptance of a set of general voltage standards will be presented, and these problems should be borne in mind when referring to the standards which the writers have recommended as best suited for this territory. The generation in this section during a normal year is approximately 60 per cent from run-of-river hydro plants (some having storage capacity) and 40 per cent from steam reserve plants. The average annual load factor of the hydro electric stations is from 30 to 60 per cent, and of the steam electric stations from 10 to 40 per cent. Generating plants are located both adjacent to, and remote from, load centers, and may be operating during different seasons of the year as base load plants or as voltage boosting stations.

With such operations as outlined above, due mainly to location and types of plants, the annual load factor on the main high voltage circuits is low (approximately 30 per cent) as compared to systems the generation of which is entirely from base load steam plants, or from hydro plants where the river flow is not seasonal. For this reason a higher voltage drop on the transmission

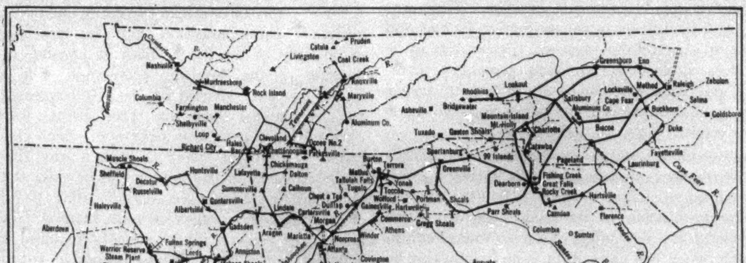
Pruden
Catula
Livingston
Coal Creek
Nashville
Murfreesboro
Rock Island
Knoxville
Maryville
Columbia
Farmington
Manchester
Shelbyville
Loop
Aluminum Co.
Cleveland
Ocoee No.2
Richard City
Hales Bar
Chattanooga
Parksville
Muscle Shoals
Sheffield
Decatur
Russelville
Huntsville
Lafayette
Chickamauga
Dalton
Calhoun
Summerville
Haleyville
Guntersville
Albertville
Gadsden
Aragon
Lindale
Cartersville
Morgan
Marietta
Aberdeen
Fulton Springs
Warrior Reserve Steam Plant
Leeds
Anniston
Birmingham
Bessemer
Magella
Jackson Shoals
Tuscaloosa
Sylacauga
Hogansville
Lock 12
Mitchell Dam
Lock 18
Cherokee Bluff
North Auburn
West Point
Marion
Upper Tallassee
Lower Tallassee
Meridian
Demopolis
Selma
Montgomery
Union Springs
Bartletts Ferry
Goat Rock
North Highlands
Columbus
Greenville
Troy
Eufaula
Credille
Laurel
Hattiesburg
Elba
Opp
Enterprise
Dothan
Blewitts
Bates
Flomaton
Mobile
Biloxi
Moss Pt.
Gulfport
Bay St. Louis
Pensacola
Gulf of Mexico
Panama City
Tallahassee
Valdosta
Chattahoochee
Tombigbee R.
Alabama R.
Flint
Burton
Terrora
Mathis
Tallulah Falls
Tugalo
Yonah
Toccoa
Chest a Tee
Dunlap
Wofford
Gainesville
Hartwell
Portman
Shoals
Commerce
Gregg Shoals
Athens
Winder
Norcross
Atlanta
Covington
Hampton
Bibb Jackson
Lloyd Shoals
Monticello
Griffin
Barnesville
Forsyth
La Grange
Thomaston
Macon
Ocmulgee R.
Rhodhiss
Lookout
Bridgewater
Asheville
Mountain Island
Mt. Holly
Tuxedo
Gaston Shoals
Charlotte
Spartanburg
99 Islands
Catawba
Greenville
Dearborn
Pageland
Fishing Creek
Great Falls
Rocky Creek
Parr Shoals
Camden
Hartsville
Florence
Columbia
Sumter
Greensboro
Eno
Salisbury
Aluminum Co.
Lockville
Cape Fear
Method
Raleigh
Zebulon
Selma
Goldsboro
Buckhorn
Blewett
Duke
Fayetteville
Laurinburg
Cape Fear R.
Pee Dee R.
Santee R.
Savannah R.
Augusta
Statesboro
Savannah
Charleston
Altamaha R.
Waycross
Jacksonville
ATLANTIC
OCEAN
LEGEND
Existing or under construction July, 1925
Proposed
Hydro-Electric Plants
Steam Plants
Sub & Switching Sts.
Storage Plants
Interchange
100 Kv. Lines
Lower Voltage Lines
Secondary Sub-Sta.
Scale of Miles
0
20
40
60

Fig. 1—Transmission Network of Power Systems of the Southwest

circuits may be considered economical and is at present accepted, as compared to that which is economical on lines between base load plants and load centers.

Lengths of transmission circuits and location of substations at many different positions with respect to the generating equipment, prohibits the acceptance of any definite point on the system as a reference for constant voltage except the consumers' bus bars. The receiving voltage at primary substations must be varied according to load conditions on the distribution lines, by regulating the generating units, or by using synchronous condensers to boost or buck the voltage as becomes necessary.

TABLE II

PROPOSED VOLTAGE RATINGS FOR SYSTEMS, GENERATORS, SWITCHING, CONTROL & PROTECTIVE APPARATUS, TRANSFORMERS, ETC.

	Generators & sync. condensers	Ind. motors	Apparatus	Step-up transformers		Step-down transformers	
				Primary	Secondary	Primary	Secondary
Systems	(see A)	(see B)	(see C)	(see D)		(see D)	
	120	110					115
	240	220					230
	480	440					460
	600	550					575
2,400	2,400	2,200		2300/3980 Y	2400/4150 Y	2300/3980 Y	2400/4150 Y
4,150	4,150	3,800					
6,900	6,900	6,600		6,600	6,900	6,600	6,900
7200/12,470 Y*			7,200	6,600	7200/12,470 Y*	6900/11,950 Y	7200/12,470 Y*
11,500	11,500	11,000		11,000	11,500	11,000	11,500
13,800	13,800	13,200	13,800	13,200	13,800	13,200	13,800
23,000			23,000	22,000	23,000	22,000	23,000
34,500			34,500	33,000	34,500	33,000	34,500
46,000			46,000	44,000	46,000	44,000	46,000
69,000			69,000	66,000	69,000	66,000	69,000
92,000			92,000	88,000	92,000	88,000	92,000
103,500			103,500	99,000	103,500	99,000	103,500
115,000			115,000	110,000	115,000	110,000	115,000
138,000			138,000	132,000	138,000	132,000	138,000
161,000			161,000	154,000	161,000	154,000	161,000
230,000			230,000		230,000	220,000	

GENERAL NOTE

Guarantees of efficiency, heating, overload, etc., should be based on the rated voltage of the individual apparatus as shown above. Voltage tests on all equipment should be based on the rated voltage of the circuit on which it is to operate. The rated voltages are in all cases the maximum permissible voltage for continuous operation.

SPECIFIC NOTES

A.—Generators and synchronous condensers should be designed to deliver rated kv-a. output at rated power factor and frequency throughout a range of 5 per cent below and five per cent above rated voltage.

B.—Induction motors should be designed to deliver rated h. p. throughout a range of 10 per cent below and above rated voltage at rated frequency.

C.—Apparatus as here used includes oil circuit breakers, disconnecting switches, current transformers, lightning arresters, insulators, bus bar supports, bushings and fuses; all to be maximum rated. The voltage ratings of potential transformers should be the same as other apparatus with their secondaries rated 115 volts to permit the employment of the now existing even ratios of transformation.

D.—Transformers should be designed and tested for operation on system rated voltages rather than transformer voltage ratings.

a. Step-up transformers should be equipped with the equivalent of three 2½ per cent full capacity taps in the primary winding, two 2½-per cent taps above and one 2½-per cent tap below rated primary voltages.

These taps are necessary to compensate for the inherent regulation of the transformers from no-load to full-load and also to provide sufficient range to make them adaptable to locations remote from, or adjacent to load centers.

b. Step-down transformers should be equipped with the equivalent of four 2½ per cent full capacity taps below rated voltage in the high-tension winding.

*For rural distribution lines.

A problem now actively confronting the companies in this territory is that of a rural distribution system, constructed as cheaply as possible, yet flexible enough to meet future demands in the most economical manner. This, in some cases, will call for a set of standards for equipment different from any others. The main problem will be that of suitable transformers having multiple ratios for adaptability on possibly as many as two or three different circuit voltages, and for locations close to the source of supply and at the end of the line.

The above comments will serve the purpose of an outline of the feelings and position of the companies in regard to standardization as a general proposition. In particular, the remainder of this paper covers a detailed analysis of the mechanics of adjustment to any proposed standards. The main problems confronting the companies concerned in this paper are those relative to generation and transmission since the bulk of their power is derived from hydroelectric and steam electric stations situated at a considerable distance from load centers. The problem of working out a set of standards for the lower voltage distribution lines can best be handled, it is believed, by the metropolitan companies the generation of which takes place near the load. Because of the diversity in present practises of equipment of this class, any set of standards satisfactory to such companies could probably be adjusted with equal facility to conditions here.

III. AUTHORS' PROPOSAL

In the September 1926 N. E. L. A. *Bulletin* there appeared a complete set of voltage standards recommended by the manufacturers.[2] These standards are not applicable in their entirety to the present operations in the Southeast. For the sake of simplicity in future discussion they will be referred to and used as a basis for the author's proposed standards which are shown in Table II.

Criticism of the manufacturers' proposed standards is not intended to be destructive, but rather a sincere attempt to show their applicability to the interconnected companies of the Southeast, and to acquaint those interested in this subject with the existing conditions. Quite naturally, comments are more directly bearing on the properties of the Southeastern Power and Light Company since it is with these properties that the writers are most familiar.

It appears to the writers that the manufacturers' definition of rated voltage is satisfactory, namely: "Rated Circuit Voltage: For the purpose of fixing a value to be used in designing and testing electrical apparatus, the rated voltage of a circuit (or system) is defined as the highest rated voltage of the apparatus supplying it. By 'circuit voltage' is meant the voltage from line to line as distinguished from line to neutral. This voltage rating applies to all parts of the circuit. The actual operating voltage of the circuit may vary from the rated circuit voltage but should not exceed it."

The manufacturers' proposed standards would permit a 5 per cent margin above rated voltage for equipment of one class and strictly forbid the employment of such a margin for other classes. The dividing point has been made between 69,000 volts and 88,000 volts to suit the present ratings, and as stated, to prevent any re-design of equipment above 88,000 volts. Restrictions discriminating between classes of voltage serve only to complicate definitions and destroy uniformity in tests and application.

The system rated voltages in the manufacturers' proposal are, with the exception of the first two (2400 and 4150), multiples of 11.5 up to 69,000 volts and beyond this are multiples of 11.0. If the history of the development of these voltage ratings is traced it will be remembered that practically all systems coming within the range of the tabulation started with maximum operating voltages which were multiples of 11.0. The range of operation to take care of peak load conditions was from maximum operating voltages in multiples of 11.0 at the generating end to voltages in multiples of 10.5 at the receiving end. As the load grew and the lines were extended it was necessary to raise the voltage at the generating end to values as high as 11.5 in order to maintain voltages in multiples of 10.5 at the receiving end. In order to accomplish this, it was necessary to over-excite generators and transformers, or, as has been done in recent years, to purchase transformers having taps to give voltages five per cent above the multiples of 11.0. The increase in potential at the source rather than the decrease at the load end of the line has been necessary and justifiable. Many thousands of transformers have been purchased with ratios, for example, of 110/44 kv. with four 2½ per cent taps below normal, in the primary winding. As loads on the distribution circuits were increased it was necessary to maintain higher than 44 kv. on the secondary by over-exciting the transformers. This was undesirable and recently transformers have been purchased having a ratio of 110/45 kv. with five 3 per cent taps below normal in the primary winding which resulted in better voltage at the distribution substations. The higher voltage transmission lines gradually became loaded so that the potential at some substations dropped as low as 100 kv. and with 110 kv. maintained at the generating plants, and primary substation transformers having four 2½ per cent taps below normal in the primary, and connected on the lowest tap, the secondary voltage could not be maintained at its rated value when carrying peak loads. The evolution was the same as in the case of distribution circuits, namely, raising the potential at the source by means of over-excitation, or by the purchase of step-up transformers having taps to deliver above rated voltages. Thus it will be seen that transmission circuits of the higher voltages have already reached the stage where it is desirable that rated voltages should be multiples of 11.5, as are the distribution voltages, and in fact must be such in order to utilize present equipment. It should be remembered, however, that maximum operating voltages in multiples of 11.5 will not occur at all generating stations, but only at those most remote from the load centers. The reason for this will be evident by referring to Fig. 1. In many instances several hydro plants are located on the same stream. Where these plants are distant from the load centers, separate lines from each of them to the load would be uneconomical and hence they are tapped to a group of circuits of one or more lines, extending from the most remote plant to the load.

During extreme emergencies, such as tornadoes, floods, etc., voltage ratings may be exceeded justifiably in order to maintain service; hence all equipment should have sufficient factors of safety to permit short time operation at five per cent above rated voltage. The operating companies should also realize that such practises, although necessary at times, are encroaching on the factors of safety of the apparatus and must be only for short time operation.

It will be noted that a voltage class of 103,500 has been included in the authors' proposed voltage standards. There are two large companies in this section with an installed transformer capacity of over one and a half million kv-a. falling in this class. To select the next voltage class above or below 103,500 volts would involve excessive costs in the rewinding or replacement of these transformers alone, which probably could not be justified.

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[2] See also *Voltage Standardization of A-C. Systems from the Viewpoint of the Electrical Manufacturer*, by F. C. Hanker and H. R. Summerhayes, A. I. E. E. Winter Convention, February, 1927.

IV. GENERATORS AND SYNCHRONOUS CONDENSERS

The rated voltages of generators and synchronous condensers, as listed in the manufacturers' proposal would, in general, be acceptable. At present, generators rated 6600 volts are of necessity being operated over peak load hours at 6900 volts. This probably shortens the life of the machine insulation, but since the circumstance prevails, future design at the suggested voltage will prove advantageous. The necessity for operating generators at five per cent above their rated voltage will only occur during emergency conditions. Synchronous condensers where operated on the tertiary of a three-winding transformer bank throughout the range of full leading and lagging kv-a. must of necessity be designed for rated kv-a. from five per cent below to five per cent above rated voltage.

V. INDUCTION MOTORS

The ratings of induction motors and customers' equipment should remain the same as stated in the manufacturers' proposal, due to the tremendous amount of apparatus at these voltages now in service, and the remainder of the system modified as necessary to maintain these voltages at the customers' terminals.

VI. APPARATUS

The term "apparatus" as used in this section is to be interpreted to include oil circuit breakers, disconnecting switches, current and potential transformers, insulators, bushings, bus-bar supports, lightning arresters and fuses. It would seem logical that the above listed apparatus should have voltage ratings exactly the same as the system voltage ratings. The ratings given in the manufacturers' proposal are merely those of present equipment which are adaptable to their proposed system rated voltages. In some instances these are similar to the system voltage ratings and in others they are not. The reason for variations in present ratings is apparently an attempt of the manufacturer to best fit their equipment to the multiplicity of voltages now in use. In order to accomplish the objective in standardizing voltages, the manufacturers would aid materially by rating their apparatus identically with system voltage ratings. This would tend to eliminate the present practise of gradually increasing circuit voltages.

Reference to the manufacturers' published ratings, particularly those for oil circuit breakers, horn gap and disconnecting switches, will show an appreciable variation in the ratings of apparatus for application on circuits of a given voltage. Closer attention should be given to the design and application of insulation in order to secure more uniformity in dielectric strength and flashover values of the various apparatus connected to circuits of given voltage ratings. A complete discussion of this matter and the factors of safety to be given different types of equipment involves entirely too much space to be more than mentioned in this paper, but unquestionably deserves serious consideration.

If the rated voltage is the maximum continuous operating voltage, lightning arresters should be rated the same as other apparatus. At stations where conditions of dynamic and static over-voltages require special design, the voltage rating should be that of the circuit and the nameplate should indicate that they are suitable for one location only. Present lines of apparatus, other than transformers, can no doubt be redesigned and re-rated where necessary to conform to any reasonable set of standards immediately upon their acceptance, without undue burden to either manufacturer or operating company.

VII. TRANSFORMERS GENERAL

The problems relating to step-up and step-down transformers are by far the most important of the entire subject of standardization. The system rated voltages which the authors have recommended have been made after careful investigation of their adaptability to present operations in this territory; likewise the proposed standards for transformers have been based on present operating experience and while they are not suitable for all operating conditions they are believed practical. Transformers to be entirely interchangeable between stations of the same voltage class would require five per cent more taps than indicated in Table II. It is recognized, however, that incorporation of sufficient taps in standard transformers to fulfill every operating requirement would penalize the standards and make them inconsistent with proper engineering practise. Due to this fact it is inevitable that there will always be a demand for certain classes of transformers to meet special operating conditions. The great majority of this special apparatus will be such as can be used on the standard voltages. Although a difference will be found in the high-tension and low-tension voltage ratings for step-up and step-down transformers, their test voltages should be based on the rated circuit voltage at which the equipment will operate and not on the actual primary or secondary voltage rating. The following comments apply especially to two-winding transformers, but three-winding transformers should come under the same classification of ratings, except that in most cases they will require special design due to short-circuit conditions or reactance requirements between windings when one winding is used for the operation of a synchronous condenser.

The voltage ratings of transmission and distribution transformers must be such that they will dovetail together to form a well balanced system from the generators to the consumer. Fig. 2 illustrates present operating practise and the application of the proposed voltage ratings of generators, step-up and step-down transformers.

Under present conditions with generators over-excited five per cent, it is permissible to take only a seven-per cent voltage drop in the transmission line and eight per cent in the primary distribution circuits. This limitation is due to the fact that: first, the generator voltage rating is the same as the rating of the primary windings of step-up transformers, and second, the voltage ratings of step-up and step-down transformers of transmission and primary distribution circuits are alike.

step-down transformers must be changed, necessitating the rebuilding of a large amount of present installed equipment.

VIII. STEP-UP TRANSFORMERS

Only generating station transformers will be considered in this section since it is believed that step-up transformers used in tying two systems together, or for like uses, will be entirely special and dependent upon their location and the purpose which they serve.

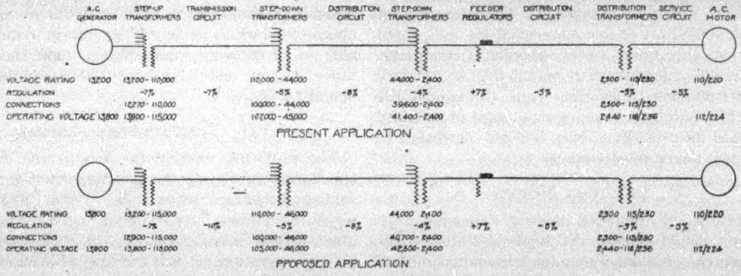
A.C GENERATOR
STEP-UP TRANSFORMERS
TRANSMISSION CIRCUIT
STEP-DOWN TRANSFORMERS
DISTRIBUTION CIRCUIT
STEP-DOWN TRANSFORMERS
FEEDER REGULATORS
DISTRIBUTION CIRCUIT
DISTRIBUTION TRANSFORMERS
SERVICE CIRCUIT
A. C. MOTOR
VOLTAGE RATING 13200 13200 - 110,000 110,000 - 44000 44000 - 2400 2300 - 115/230 110/220
REGULATION -7% -7% -5% -8% -4% +7% -5% -3% -5%
CONNECTIONS 12270 - 110,000 100,000 - 44,000 39600 - 2400 2300 - 115/230
OPERATING VOLTAGE 13800 13800 - 115,000 107,000 - 45,000 41.400 - 2400 2440 - 118/236 112/224
PRESENT APPLICATION
VOLTAGE RATING 13800 13200 - 115,000 110,000 - 46,000 44000 2400 2300 115/230 110/220
REGULATION -7% -10% -5% -8% -4% +7% -5% -3% -5%
CONNECTIONS 12900 - 115,000 100,000 - 46,000 40,700 - 2400 2300 - 115/230
OPERATING VOLTAGE 13800 13,800 - 115,000 105,000 - 46,000 42,300 - 2400 2440 - 118/236 112/224
PROPOSED APPLICATION

FIG. 2—PRESENT AND PROPOSED RATINGS OF GENERATORS, AND OF STEP-UP, AND STEP-DOWN TRANSFORMERS, WITH ILLUSTRATIONS OF REPRESENTATIVE APPLICATIONS UNDER FULL LOAD

Recently the Alabama Power Company has purchased step-down transformers rated 110/45 kv. to meet the limitation outlined under Table II and would not hesitate to go a step further to the use of 110/46-kv. transformers, if such a rating is adopted as a standard. In Fig. 2, illustrating the use of the authors' proposed standards, a 10 per cent voltage drop can be allowed in transmission lines and eight per cent in primary distribution circuits without over-exciting the generators. On certain systems in the Southeast it would be easier to make a transition from present ratings to the standards proposed by the writers, than to meet the standards proposed by the manufacturers because all that is required is an over-excitation of the generators and the operation of step-up transformers at five per cent above rated voltage. The disadvantage of reduced life of insulation of present transformer and generator windings, it is believed, is overbalanced by the operating advantages obtained. If the step-up transformers are operated at a maximum of 110,000 volts, the rating of

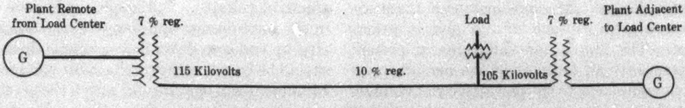
Plant Remote from Load Center
7 % reg.
G
115 Kilovolts
10 % reg.
Load
105 Kilovolts
7 % reg.
Plant Adjacent to Load Center
G

FIG. 3—DIAGRAM SHOWING STEP-UP TRANSFORMER

The generators are rated 13.8 kv. The transformers are rated 13. 2/115 kv. and their primaries (low-voltage side) have two 2.5-per cent taps above normal and one 2.5-per cent tap below normal. They have 7 per cent regulation. At full load the generator at the remote plant operates at 13.8 kv. The 12.9-kv. tap is used on the step-up transformer. With 7-per cent drop in the transformer and 10 per cent cut in the line, the voltage at the load is 105 kv. The generator near the load operates at 13.5 kv. and the 13.8-kv. tap is used in its transformer, giving 105 kv. at the load.

Step-up transformers at generating stations in the Southeast will require taps and ratios which will compensate for their inherent regulation and which will be adaptable also for stations located adjacent to, and remote from, the load centers.

A maximum transformer regulation of seven per cent will be assumed in further discussion. This figure should be sufficiently high to include the majority of step-up transformers purchased. One case where higher than seven-per cent regulation may occur is at seasonal generating plants where during one period of operation the plant is run at full capacity near unity power factor, and a second condition where the plant is carrying rated kv-a. capacity at low power factor, using the generators for voltage boosting.

Fig. 3 is included to indicate the ratios and taps required for step-up transformers on a basis of seven-per cent transformer regulation and 10 per cent maximum line drop. The voltage taps as shown in Fig. 3 will not only compensate for the transformer regulation, but are suitable for plants located at different points with respect to the load and for seasonal operation.

IX. Step-Down Transformers

Step-down transformers should have ratings and voltage taps for service in locations adjacent to, and remote from, the generating plants and to compensate for their inherent regulation. Step-down transformers will of necessity be shifted from one point on the system to another due to load growth, and should therefore be more flexible than step-up transformers which are rarely moved. The ratings which the writers have recommended will be found to vary from the manufacturers' recommendations only in so far as the changes which have been suggested in the system rated voltages. A maximum regulation of five per cent for step-down transformers has been assumed and this value will seldom be exceeded. The equivalent of four 2½ per cent taps as proposed, while not being sufficient to include all operating requirements, should include a large majority of the transformers purchased.

X. Transformers for Rural Distribution System

Although the problem of rural distribution is comparatively new, experience to date indicates that the most economical circuit voltage for rural lines is 7200/12,470 Y.

The manufacturers' publications state that transformers for this voltage class are given a test from high-voltage winding to low-voltage winding and core of 25,940 volts—(2 × 12,470) plus 1000 volts. On the basis of the proposed definition of "Rated Circuit Voltage," apparatus for rural distribution lines should be rated 7200/12,470 Y, and has been so listed in Table II. The advantage of this rating is that it is a multiple of 2400/4,150 Y, making it possible to use multiple windings on the secondaries of power transformers supplying distribution circuits which can be connected in parallel for 2400/4150 Y or in series for 7200/12,470 Y. This also allows a four-per cent voltage differential between supply transformers and customer's transformers. As the latter have a voltage ratio of 6900/11,950 Y to 115/230, the maintenance of 7200 volts at the supply end makes it possible to maintain rated secondary voltage on the customer's premises without the use of excessive taps on the primary windings of the step-down transformers.

XI. Conclusion

1. The writers have suggested modifications of the manufacturers' proposed standards to make them more adaptable to the interconnected systems of the Southeast. These changes consist of:

a. Addition of 103,500-volt class,

b. Uniform voltage ratings, multiples of 11.5 throughout,

c. Changes in apparatus ratings to make them identical with system voltage ratings,

d. Providing equivalent of one 2½ per cent full capacity voltage tap below normal, and equivalent of two 2½ per cent full capacity taps above normal in the primary winding of step-up transformers to take care of transformer regulation in excess of five per cent, and location of transformers adjacent to, or remote from, load centers,

e. Addition of 7200/12,470 Y voltage class for rural distribution systems.

2. The interconnected power companies in the Southeast are in accord with the proposition of standardization and it is believed that these proposed standards in general would be practicable and acceptable to the majority of operating companies in this section.

XII. Appendix

The appendix will be used for a detailed discussion of the reasons why some of the authors' proposed standards differ from those recommended by the manufacturers, and also to bring attention to instances where special equipment will be required.

A. An analysis of operating reports of companies in the Southeast would indicate many systems operating considerably above rated voltages, and with excessive voltage drop. For example, 110,000-volt systems during peak conditions will be found with voltages as high as 118,000 volts at the remote generating plants and as low as 98,000 volts at remote substations. There are reasons for this, some of which are probably justifiable and others which are not. Needless to say, a 20 per cent voltage drop on any system should not be tolerated except during emergencies, because voltages of 15 and 20 per cent above ratings of equipment not only shorten the life of insulation but encroach on the factors of safety.

In the standards proposed herein a 10 per cent voltage drop on transmission circuits has been assumed as a maximum. This value will be justifiably exceeded in at least one instance in the Southeast. The particular situation is one in which the load center is distant from the source of supply. During the peak hours of the day the load is carried from one set of plants 100 mi. distant, and the remainder of the time it is fed from a different source located some 60 mi. distant. Thus it will be evident that the yearly load factor of the transmission circuits is very low; hence a voltage drop of 15 per cent on these lines, when carrying peak loads, will be found economical. The maximum transmission losses at peak load in this illustration are but 9.8 per cent which, it will be admitted, are not beyond economic limits. The annual losses are approximately 3.0 per cent of the total energy transmitted.

To correct this condition to come within a 10 per cent voltage drop would necessitate one of two things, either the construction of an additional transmission line at a cost of not less than $750,000, or the installation of a $200,000 synchronous condenser station. With a new transmission circuit, the peak load line losses are reduced to 4.5 per cent and the yearly losses reduced to 1.5 per cent; capitalizing the difference in annual losses based on the power transmitted at 10 per cent would only justify the expenditure of $315,000, hence the line is uneconomical. A synchronous condenser likewise cannot be justified since the transmission losses remain practically the same and the condenser has only corrected voltage conditions.

With a 15 per cent line drop, the standard transformers cannot be used at the remote substations, and units having the equivalent of 15 per cent taps below normal in the primary winding, will be required at an estimated increase in price above standard transformers of from three to five per cent. These transformers are satisfactory, however, for operation at any point on the circuit, and proposed standards have not been penalized by the necessity of these special transformers to fit operating conditions.

B. It is very probable that some of the utilities having a large amount of what may be termed off-standard apparatus in service will be unable to justify a change to meet standards. This is especially true where to abide by the standards would mean lowering present circuit voltages. By reducing operating voltages, the power transmitted over present equipment would be reduced, of course, and the power limit of lines would be decreased. The problem of stability had not been given serious consideration until some few years ago, but as transmission distances have increased and lines become more heavily loaded these problems have been encountered and it is known that lowering the circuit voltage will decrease the static and transient power limits of a line.

A specific example where the reduction of circuit voltages from those recommended by the writers would bring considerable hardship is on the Alabama Power Company's system if it were necessary to reduce the operating voltage on transmission lines from a maximum of 115,000 volts down to a maximum of 110,000 volts. The generating plants with the exception of one steam plant are all remote from the load centers of Birmingham, Bessemer, and Anniston. The high-tension voltage at these load centers is, as would be expected, the lowest on the system, and at present varies between 105,000 and 108,000 volts. To reduce this voltage five per cent, as would be necessary were the maximum operating voltage 110,000 instead of 115,000, would require the installation of complete new transformers at these substations. Present transformers have taps as low as 100,025 volts which with 100,000 volts impressed would not give normal voltage on the secondary when operating at full load; thus the voltage must be maintained at approximately 105,000 volts. The total transformer capacity installed at these three substations is 140,000 kv-a. and even granting that these transformers could be rebuilt to deliver rated secondary voltage with 100,000 volts impressed on the primary, which we seriously doubt, without reducing their rating, the expense involved would be unjustified. These stations are at present heavily loaded and of different secondary voltages so that at least two complete new banks of transformers would be required were it possible to rebuild the present units.

Aside from the difficulties involved with transformers if the circuit voltage were to be reduced, such a procedure would involve the immediate construction of some 150 mi. of 110-kv. lines at a cost of not less than $1,500,000, which would hardly be justifiable.

This is only one specific case where the necessity of reducing operating voltages from those recommended would entail considerable hardship; there are several other such examples which could be cited. The only justification for such expenditures would be in the event of the acceptance of 110,000 volts as standard, requiring those companies operating at 115,000 volts to purchase all equipment of the next higher voltage class, but we do not believe that any standards are intended to penalize present operating conditions where there is such a prevalence of equipment now in service which could not be operated on the manufacturers' proposed standards.

If to be in complete accord with any standards involves increasing the circuit voltage, a careful analysis of conditions must be made. If spacings, insulation, corona limit and other electrical factors are satisfactory with increased voltage, the number of customer and other step-down substations must also be considered. The cost of relocation, retiring or rebuilding present transformers must be balanced against the increased carrying capacity of the line at the higher voltage, and decreased losses; also the cost of future apparatus with standard ratings balanced against the cost of off-standard transformers. The increases in costs of special transformers above the price for standard units will have to be given by the manufacturers. It is believed that on lines containing few substations it will be found practical to gradually work toward standard voltages. On other circuits containing numerous substations and customers, and where alterations in station layouts would be necessary to operate on a higher circuit voltage, such changes may not be feasible. These questions will require careful study for each different proposition and hence no definite conclusions can be made.

Discussion

For discussion of this paper see page 205.

Suggested Transformer Voltage Standards

Their Relationship to Pacific Coast Practise

Committee Report[1]

H. H. MINOR, Chairman

Synopsis.—*Explanation of authorship and basis of paper. General discussion of the proposed system voltage standards and the principles underlying them. Listing of transformer voltages and kv-a. connected now in use by P. C. E. A. Companies. Detailed discussion of changes in proposed standards to bring them into conformity with Pacific Coast practise. Summary and conclusion.*

THIS paper is presented by the Subcommittee on Transformer Standardization of the Electrical Apparatus and Overhead Systems Committees of the Pacific Coast Electrical Association as the result of a study of transformer ratios made last year and a further investigation carried on during the past few months. It is felt that it will be of interest as part of a general discussion of the subject, system voltage standardization, as it is a composite of the views of representatives of the several Member Companies of the P. C. E. A.

The work last year was carried on by means of questionnaires and by discussion at joint meetings of the two parent committees. The table of "Proposed Voltage Ratings For Systems" presented to the National Electric Light Association by a special committee appointed by the National Apparatus Committee to study data collected by that body was discussed, in so far as it applied to transformers, at some length. This year a survey has been made of the voltage ratings and kv-a. capacity of transformers in use by P. C. E. A. Companies. Further discussion of possible new standards and the probable application of those suggested to Pacific Coast conditions was invited. The following is an attempt to present an analysis of the data collected and to present an outline of the opinions of the P. C. E. A. members that have come to the committee.

The general view is that there is need of revision of the present transformer standards with respect to voltage rating. The five principles upon which such revision should be based, as set forth in the Memorandum of the Special N. E. L. A. Committee which was published in the September 1926 issue of the N. E. L. A. *Bulletin*, are excellent.[2] Of these, principle *B* which reads: "The voltage selected must closely resemble those now in use to permit a reasonable degree of interchange ability of old and new apparatus" is of particular importance. It will probably result that this is the vital point to be considered in any attempt at revision of the standards.

Principle *C*, which states that the changes involved must not necessitate too great an expense in the development of new apparatus, is also of extreme importance. This expense, however, should be the expense to the user of the apparatus and not simply the cost of development and manufacture. It should include the probable cost to the user because of the fact that principle *B* is not adhered to, sufficiently. Most of the present-day systems for the transmission and distribution of electrical energy are so greatly involved through interconnection on their own systems and with other systems that the voltages in use are fixed within rather narrow limits. That present standard transformers do not lend themselves to use on many of these systems without operating them at considerable variance with their rated voltages and those upon which tests and guarantees are based, is very true. This has naturally resulted in the purchase of a great amount of "special" or "off-standard" equipment.

To be successful, a new set of transformer standards must result in a decrease in the percentage of such specially designed transformers purchased. This result must not be obtained, however, by the inclusion in the standards of a great number of new designs.

Adherence to principle *B* will result in a great reduction in the special transformers but principle *C* will have to receive careful consideration so that the ultimate cost to the industry as a whole will be lessened. Thus the task before those to whom is delegated the fixing of standard voltages for systems is one of making such compromise as is necessary between an ideal or theoreti-

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 7-11, 1927.

[2] See also *Voltage Standardization of A-C. Systems from the Viewpoint of the Electrical Manufacturer*, by F. C. Hanker and H. R. Summerhayes, A. I. E. E. Winter Convention, February 1927.

cal standard and the voltage ratings of equipment now in use in order to arrive at a result that will be most satisfactory. That this is no easy task is evidenced by the study made by the N. E. L. A. Special Committee.

The proposed set of standard voltages and the explanatory notes with them are a wonderful improvement in many ways over the present standards. It may be the best possible compromise to which reference is made above. Certainly very careful thought has been given to its preparation, that the weaknesses of the present system of voltages shall be largely eliminated. The transformers and their taps listed will fit nicely into the somewhat ideal simple system used as an illustration in the Memorandum of the Special N. E. L. A. Committee. That such transformers would fit as well into existing systems may be open to question. In the West, where transmission distances are long, the full load drops encountered in the lines are considerably greater than the limits set in this example. The result is that in many cases step-down transformers are purchased with a much greater range of taps than the proposal allows in order to arrive at flexibility in the use of these transformers over a large system, or variations in their voltage ratings are specified to conform with existing drop conditions.

The accompanying tabulation gives an idea of the great variety of voltages and ratios in use by the P. C. E. A. Companies. Also is listed in part the kv-a. connected of transformers of the different classes shown. While this is not a complete listing, due to lack of information from a few of the Companies, it is felt that it presents a fair picture of the transformer situation on the Pacific Coast with respect to the possibility of arriving at a new set of standards that will fit into existing conditions.

A study of this tabulation shows that in the lower voltage classes, which include the great number of distribution transformers supplying utilization voltages, approximate conformity with the proposed standard exists. There are some changes, however, that seem advisable in view of present practise. In the higher voltage classes there is not such close conformity to the proposed ratings. The very great variety of voltages used is due in part to the above practise of specifying special transformers for use at different points on systems. For the most part, also, the larger systems have been built up by a gathering together of many small systems and interconnecting them by means of transmission lines. Each of these smaller companies had their own particular voltages and they were usually continued by the use of more or less "special transformers." This also has contributed to the great variety of transformer ratings.

The use of 2300-volt, delta-connected systems of distribution, supplying 110 to 115 volts to the consumer, has been the original set-up of most systems. As load

TABLE OF TRANSFORMER VOLTAGES IN USE AND KV-A. IN TRANSFORMERS CONNECTED TO LINES OF PART OF P. C. E. A. COMPANIES

Nearest voltage class	Primary voltages (Full winding)	Secondary voltages (Full winding)	Kv-a. connected
2,400	2000 to 2900 volts (2300 predominant)	110 to 850 volts (115-230 predominant)	1,055,200
4,150	3810 to 4800 volts (4000 predominant)	110 to 2300 volts (115-230 predominant)	23,721
6,900	6000 to 7900 volts (6900 predominant)	110 to 2300 volts (115-230 predominant)	469,753
11,500	10,000 to 12,500 volts (11,500 predominant)	110 to 2300 volts (115-230 predominant)	625,210
13,800	13,200 to 14,400 volts	110 to 850 volts	9,400
	15,000 to 18,000 volts	110 to 2400/4150 Y	421,000
23,000	20,000 to 26,400 volts	110 to 6900 volts	46,706
	20,000/34,600 Y and 22,000/39,000 Y.	2200 and 2300 volts	10,087
34,500	29,705 to 36,415 volts	110 to 4150 volts	257,266
		6900/11,950 Y	91,075
		7500/13,000 Y	44,500
	(Connected Y on	11,500 volts	13,500
	66-kv. systems)	13,200 volts	72,110
		17,000 volts	3,000
		22,000 volts	18,667
34,500	33,000 volts (Connected delta)	2400-11,500 volts	20,000
34,500	38,100 to 41,600 volts	110 to 4150 volts	139,092
		6900/11,950 Y	39,825
		7200/12,500 Y	200,000
	(Connected Y on	7500/1300 Y	8,350
	66-kv. systems)	11,500 volts	187,000
		16,500-11,500 volts	323,000
		22,000 volts	8,250
		2400-11,500 and 16,500-33,000 volts	580,000
69,000	50,000 to 76,200 volts	110 to 4150 volts	72,388
		6900/11,950 Y	138,333
		7200/12,500 Y	12,000
	(Connected Y on	7500/1300 Y	112,833
	110-kv. systems)	11,500 volts	181,817
		13,200 volts	164,167
		17,000 to 17,500 volts	9,900
		31,215/54,000 Y	26,667
		34,640/60,000 Y	135,500
		45,800/79,300 Y	185,000
69,000	50,000 to 66,000 volts	110 to 4150 volts	37,800
		6900/11,950 Y.	10,750
		11,500-16,500 volts	121,000
		20,000/34,600 Y	5,000
	(Delta connected)	22,000-11,000 volts	9,000
		36,000 volts	900
		2400-11,500 and 16,500-33,000	95,000
88,000	81,000 to 88,000 volts	2000 to 2400 volts	28,250
		11,500-2400 volts	167,500
	(Delta connected)	36,000-6600 volts	76,700
		55,00-2300 volts	37,500
110,000	110,000 volts (Delta connected)	9500/16,450 Y	37,500
132,000	115,470 to 127,000 volts	11,500 volts	206,739
	(Connected Y on	63,500 volts	166,667
	220-kv. systems)	66,000 volts	369,000
		72,000 volts	315,000
154,000	150,000 volts	66,000-16,500 volts	305,000
	154,000-115,000 volts	11,000	80,000

density increased it became necessary to provide greater capacity to existing lines. The logical set-up and the one taken by most of the Companies was to use 2300 volts connected Y or the so-called 4000-volt system. In many cases the lines were made three-phase, four-wire, the 2300-volt class transformers being connected Y-delta for three-phase service and from phase wire to neutral for single-phase service. As the territory covered grew and longer feeders became necessary, it was found best economy to discontinue stringing the neutral wire and make use of a special 4000- to 115/230-volt transformer for single-phase use on these lines, still employing three 2300-volt transformers connected Y-delta for three-phase service. This practise has been and is being extended rapidly. One manufacturer has catalogued a 4000-volt distribution transformer. It would seem advisable to include such a transformer in the proposed standards. Its rating, to conform with the Y voltage of the 2300-volt transformer, should be 3980 to 115/230, etc., for step-down transformers.

The next higher voltage class of transformers in use is the 6900-volt. As shown by the accompanying tabulation, there are a great many of this class in use on the Pacific Coast. There are, however, very few 6900-volt delta systems or lines. The almost universal practise is to use these transformers connected Y-delta on 11,500-volt systems, just as the 2300-volt class is used on 4000-volt systems. The present standards recognize this use by making the rating for distribution transformers of this class 6900/11950 Y to 115/230, etc

There are also in use many 11,500-volt class transformers on these same systems, either connected delta-delta for three-phase service or used separately for single-phase loads. There is, however, a discrepancy between the ratios of these transformers and the 6900-volt class in the present standards. The relationship between these two classes should be as 1.73 is to 1 instead of as 1.67 is to 1, which is present practise. The practical result has been that the 6900-volt class is connected on the 6585-volt tap and the 11,500-volt class on full winding to more nearly approach the proper secondary voltage when both classes are connected to the same line. This, in effect, cuts down the available taps on the 6900-volt transformer and the user is paying for more than he needs.

In the proposed standards the actual ratios of these two classes are changed. The step-down proposal is respectively for the 11,500-volt system and the 6900-volt system, 11,000 to 115/230, etc., and 6600 to 115/230, etc. The relationship between them is still as before: 1.67 to 1. No provision is made in the proposed standard, however, for Y-operation of the 6900-volt class transformer. To conform as nearly as possible with the proposed standards and also with the Pacific Coast practise, it will be advisable to make the proposed standard for these two classes substantially as below:

System	Step-up Primary	Transformer Secondary	Step-down Primary	Transformer Secondary
6,900	6,600	6,900	6,600/11,430 Y	6,900
11,950	11,430	11,950	11,430	11,950

This rating of the step-down, 6900-volt class transformer will require it to be tested for 11,950 volts under the recommendation of the Special N. E. L. A. Committee that high potential tests be based on "rated circuit voltage" of the system of which the apparatus forms a part. That this is correct for such transformers connected Y is undoubtedly true. Should there be sufficient use of 6900-volt transformers on 6900-volt circuits to warrant it, it may be advisable to include both the above suggestion and the 6900-volt step-down transformer for use on 6900-volt delta lines. This is not the case on the Pacific Coast as practically all of the 469,783 kv-a. listed in the accompanying table are used on 11,500-volt systems.

In the proposal of the N. E. L. A. Special Committee, the following recommendation is made in "Note F" accompanying the proposed Table of Voltages:

> "When possible, 11,500-volt systems should be discouraged in favor of 13,800-volt ones."

On the Pacific Coast this suggestion is not at all possible. The practise, mentioned above, of using both 6900- and 11,500-volt class transformers so universally on 11,500-volt systems cannot be changed without immense cost. To change to 13,800 volts would necessitate the replacement of more than 1,000,000 kv-a. in 6900- and 11,500-volt distribution transformers and almost as much capacity in station transformers serving them. It is the unanimous opinion of the P. C. E. A. membership that the 11,500-volt system should be retained with such changes as are necessary to bring it into conformity with the 6900-volt class step-down transformers, connected Y on such systems.

The failure to include a 6900-volt class of transformer for Y operation in the proposed table is really only a special case of a general condition. The very great use of Y-connected transformer banks for both step-up and step-down purposes seems to point to the advisability of including such transformers in a new set of standards. That this use of transformers is large on the Pacific Coast is shown by the accompanying table. Of transformers above the 13,800-volt class there were reported approximately 4,000,000 kv-a. connected Y and about 1,000,000 kv-a. connected delta. The proposal of the N. E. L. A. Committee only includes transformer voltages for use connected delta, with the single exception of the 2300/3980 Y for step-down and 2400/4150 Y for step-up high voltage ratings. That inclusion of transformers for Y-connection for the various system voltages chosen will approximately double the number of transformers in the standards, is evident, but it will not greatly increase the number of designs necessary and should not increase the cost of either class. If the proposed standards were to be adopted, those companies using Y-connected transformers would be forced to buy "special equipment" and the designs, patterns, etc., would still have to be made.

The "General Note" accompanying the Table of Proposed Voltage Ratings referring to guarantees of efficiency, heating, overload, etc., and over-voltage tests has caused some comment. It is felt that this note is correct with respect to over-voltage tests of primaries of step-down transformers being based on five per cent over rated primary voltage. This is necessary to care for the over-excitation of the transformer necessary to overcome transformer regulation drop. The same provision, however, would seem advisable for the primary or low voltage winding of step-up transformers. With these it is also necessary to over-excite approximately five per cent in order to supply rated secondary voltage at full load, due to transformer regulation. It would seem that this same result would be obtained, however, if the suggestion, mentioned before, that all apparatus have high potential tests based on "rated circuit voltage" of the system of which the apparatus forms a part, were adopted.

"Special Note E" states that transformers should be designed to operate during emergencies at five per cent above rated voltage, the over-voltage being obtained by over-excitation. Comment has been made that this statement is very indefinite. The question is asked as to just what constitutes such an emergency and how long such an emergency might be allowed to exist. In the Memorandum of the Special N. E. L. A. Committee the loss of one or more of a parallel group of transmission lines, causing abnormal drops between sources of generation and utilization, is cited as an emergency. One can well imagine such a condition lasting for many hours or even days when the lines are in mountainous and inaccessible regions. Under such "emergency" operation the transformer would be subject to the same conditions as would obtain for continuous operation in this over-excited condition. Another emergency that is often encountered on almost all systems is the sudden failure of apparatus or lines that will cause a large block of load to be disconnected from a long transmission system. The result is a very decided rise in voltage at points distant from the source of energy due to the decrease of line drop and transformer regulation.

Referring to the Memorandum of the Special N. E. L. A. Committee and particularly to Fig. 2A therein, let it be assumed that this represents part of a system at full load served from a 13,800-volt and 2400-volt distribution system. In order to approximate probable practical conditions let it also be assumed that there are four such 69-kv. radical feeders from the 132-kv. line, each carrying 25 per cent of its load. If, then, a fault should occur which would open up the 13,800 feeder switches and drop their load, the five per cent regulation drop of the 66,000- to 13,000-volt transformers would be eliminated as would the 4.5-kv. drop in the 69-kv. line and 25 per cent of the line drop and transformer regulation of the 132-kv. line and the 126,000- to 69,000-volt transformation respectively. This would result in approximately 18 per cent over-excitation of the 66,000- to 13,200-volt transformer. Thus this emergency, which would of course be of very short duration, would cause very much over the five per cent set up by "Note E." It is suggested that a more exact statement of the meaning of "emergencies" should be included in these notes.

As stated above, the long transmission distances in the West make it advisable to allow quite large line drops at full load, and this condition has made some of the companies specify quite a range of taps for their step-down transformers. The following is from one of the larger P. C. E. A. companies:

"While it may be possible in the eastern part of the country with concentrated loads in relatively restricted areas to keep transmission line drops within the limits indicated by the discussion of the proposed standards and consequently to maintain the transmission system voltage level with only four 2½ per cent taps in step-down transformers, we do not consider this economically practicable under the conditions prevalent on this coast. Our system, for instance, is spread over a veritable empire, with widely scattered power sources and load centers, and we find it necessary to provide taps for about 17½ per cent, particularly for transformers connected to the transmission systems. For instance, our present practise for transformers connected to our so-called 60-kv. system provides for two 5 per cent taps and one 7½ per cent tap, enabling these transformers to be used interchangeably on any part of the system. We therefore feel that more than 10 per cent in taps should be provided by these standards, either in 2½ per cent or larger steps as may be found necessary." Other companies operating over extensive territories find the same conditions confronting them and the practise of employing a greater than 10 per cent range in taps is quite common. Thus the "Special Note E (b)" might also be given more thought.

To summarize the foregoing briefly, Pacific Coast practise in the use of transformers points to the advisability of some change from the present standards. The proposed standards are a great improvement over the old and the principles upon which they are based are sound. There is need of the inclusion of a 3980-volt step-down distribution transformer and of a change in the 6900-volt and 11,500-volt class to bring them into conformity for operation on the same 11,500-volt lines. There is a distinct need of the standards including transformers rated for Y-connection as well as delta connections on standard systems. The "General Note" referring to over-voltage tests could be clarified as could the "Special Note E" referring to emergency operation. More study of the advisable range of taps on step-down transformers seems advisable. "Special Note F" suggesting the elimination of 11,500-volt systems is not at all in keeping with Pacific Coast practise.

In conclusion, it might be repeated that the task before those responsible for the formation of a set of voltage standards is a most difficult one. It must be a compromise between the ideal and the practical. Its result must be an economic gain to the industry as a whole.

Discussion

For discussion of this paper see page 205.

Voltage Standardization

BY A. HUBER-RUF[1]

Non-Member

Synopsis.—*A review of the points of view which in general influence standardization of voltages and a number of proposals of certain standards are given in this paper. The discussion pertains to classes of voltages, three-phase systems, d-c. systems, and nominal voltages. A plea is made for the use of the ratio of* $1/\sqrt{3}$ *for working voltages.*

I. GENERAL

THE standardization of voltages forms the most important basis for the economic manufacture of electrical machinery and apparatus, and for the installation of electrical plants.

In recognition of this fact, the electrotechnical bodies in various countries have drawn up standards, and the International Electrotechnical Commission has now undertaken the task of bringing into line the various standards set up in different countries so as to provide a better international basis for the manufacture of electrical plants, and to facilitate the exchange of energy between neighboring countries.

This report contains only a brief review of the points of view which, generally speaking, influenced the standardization of voltages, and gives a short summary of the results of this standardization.

A few remarks are made at the end concerning American practise so far as the writer of this report was able to obtain information on it on the occasion of the I. E. C. Conference in New York in April 1926 and from various other sources.

The following were the principal deciding factors in the selection of standard voltages:

1. Consideration of the voltages most used in the countries concerned,
2. Consideration of the voltages standardized in neighboring countries,
3. The fixing of as few voltages as possible, at suitable intervals.

The consideration of the three points mentioned presented some difficulties and the solutions proposed are consequently the results of compromise.

Exhaustive inquiries were made in various countries in order to settle the first question, concerning the extent to which the different voltages were employed. The importance of the various voltages, considered from the point of view of connected load in kilowatts, and the extent of the transmission plant was recorded graphically.

Fig. 1 gives an example of such a graphic record drawn up for Switzerland in 1919 for lighting and power systems.

Tables and graphical records were also made for the settlement of the second question, concerning standardized voltages in different countries.

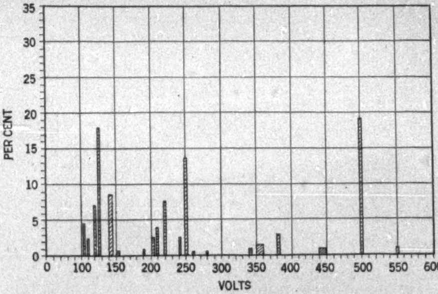

FIG. 1—FREQUENCY OF OCCURRENCE OF NOMINAL VOLTAGES IN LIGHTING AND POWER SYSTEMS IN SWITZERLAND (1919). THE VERTICAL LINES OR AREAS REPRESENT THE LOADS CONNECTED TO THE CORRESPONDING VOLTAGES IN PER CENT OF THE TOTAL LOAD CONNECTED TO THE SYSTEM IN QUESTION

Fig. 2 contains such a record for three-phase voltages in various countries.

The third point, "The fixing of as few voltages as possible, at suitable intervals," offered special difficulties.

It was generally recognized to be advantageous for three-phase working that the voltages should bear to one another the ratio $1:\sqrt{3}$ on account of the possibility of changing over the windings of machines and transformers from star to delta, and the increased possibility of utilization of material resulting therefrom.

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[1] Brown Boveri Company, Switzerland.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

Voltages have hitherto been standardized, however, mainly with the ratio 1:2, having regard, obviously, to the series-parallel connection of d-c. or single-phase a-c. machinery.

Series-parallel connection is also possible with three-phase windings but it presents considerable disadvantages as compared with delta-star connection. This point will be brought forward again later on as it is of great importance.

II. Classes of Voltages

In this report, distinction will be made between the following classes:

Class A, up to 99 volts,
Class B, 100 to 990 volts,
Class C, 1000 to 29,000 volts,
Class D, 30,000 to 100,000 volts,
Class E, over 100,000 volts.

Further particulars concerning the standardization of d-c. and three-phase voltages are given in the following, three-phase voltages being considered first as being more important as regards generation, distribution, and utilization of electrical energy.

III. Standard for Three-Phase Systems

Class A, up to 99 Volts. This class is of importance mainly for direct current and single-phase alternating current. There are already several standards existing in various countries. Hitherto the voltages falling within this class have not been dealt with by the I. E. C. They are of chief importance for the lighting of electric vehicles, etc., for telegraphy, and for electrotherapeutics.

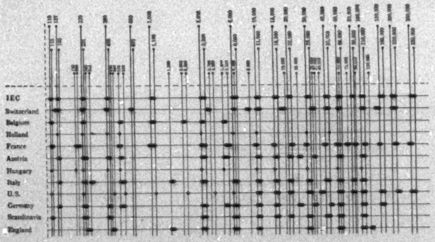

Fig. 2—Voltages for Three-Phase Systems. Comparison of the Standards of Various Countries

Class B, 100 Volts to 990 Volts. This is the most important class as it comprises house installations and the majority of apparatus utilizing electrical energy.

One hundred ten volts was used at first for lighting and power. When the load and the length of lines increased, the distribution voltage was raised successively and separate mains for higher voltages were frequently installed in towns for supplying electricity for power purpose. In many places this has led to complicated installations.

It has now been the practise for many years to supply energy for lighting, power, and heating, as far as possible from the same mains. Experience has shown that in this case the three-phase, four-wire system presents the most economic solution. For lighting and for small heating apparatus, about 220 volts between the phase leads and neutral is used while about 380 volts between star-connected phase leads is used for motors and larger apparatus.

Higher voltages are also suitable for power purposes, but lighting voltages much above 220 volts, which would result from the use of higher voltages for power in the same mains, are not to be recommended at present owing to the impossibility of manufacturing satisfactory lamps.

Two hundred twenty volts was also selected for three-phase because this voltage was widely used in d-c. and single-phase a-c. systems; lamps and small apparatus for domestic and industrial purposes can therefore be used for all systems, an important feature for the economic manufacture and use of apparatus of this nature.

In addition to 220–380 volts, it is advantageous, from the point of view of existing plants and special conditions, to fix two other standard voltages in Class B, one above and one below in the ratio $1:\sqrt{3}$, namely, about 127 and 660 volts. By changing over star-delta, motors and transformers can thus be used for two adjacent voltages. See Fig. 3.

The ratio 1:2 between adjacent voltages would be very desirable for direct current. Series-parallel connection for three-phase motors, however, necessitates subdivision of the windings and bringing *nine leads* out of the stator frame when the ratio 1:2 is used, whereas for changing over from star to delta, only the *six ends* of the three phases need be led out.

In European practise, motors are provided with terminal plates having six terminals corresponding to the beginnings and ends of the three phases. The change from delta to star can then be effected simply by changing over three connecting links on the outside terminal plate. There usually is not sufficient space on small and medium sized motors to accommodate nine terminals for changing over from series to parallel.

Changing over transformer windings from star to delta is also simpler than using series-parallel connections. Furthermore, three-phase motors and transformers are more reliable in operation when arranged for star-delta connection than when arranged for series-parallel connections, since the latter requires more internal connections and therefore offers more sources of trouble.

The International Electrotechnical Commission took these considerations into account by deciding on the standard voltages for Group II. The voltages standardized are 127–220–380 volts and also 110–190 volts. In addition to these 115–200 volts and 133–230–400 volts were designated as standard, but with the reservation that each country must decide upon either one or the other series. The most recommendable voltages, however, are 220–380 (Three-phase, four-wire system with 220 volts between phases and neutral for light and 380 volts between phases for power).

Class C, 1000 Volts to 29,000 Volts. From the comparison of the standards of various countries, (Fig. 2,) it is evident that the I. E. C. voltages, *viz.*, 3000, 6000, 10,000, 15,000, and 20,000 volts are standard in most countries, or at all events there are only relatively slight variations.

In the connection it should be noted that in each individual distribution system, generally only one or another of these voltages is used.

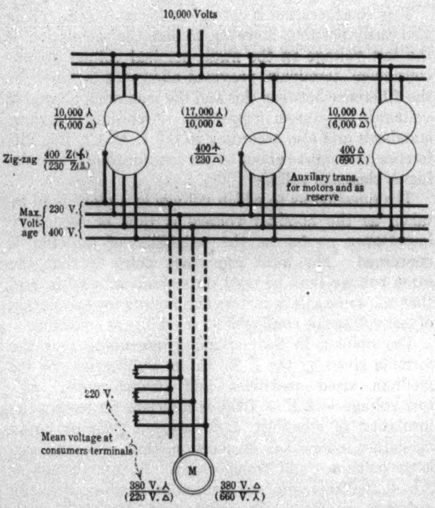

FIG. 3—THREE-PHASE DISTRIBUTION FOR LIGHT AND POWER MEAN VOLTAGE AT CONSUMERS' TERMINALS. (PRIMARY TERMINALS OF TRANSFORMERS)

To bring about a subsequent reduction in the comparatively large number of steps, the voltages in heavy type were specified as preferred standard voltages.

The following proposal, which had a similar object in view, was submitted to the I. E. C. by the Swiss Committee. In order to allow an increased range of application for this class of voltages by means of the star-delta change-over on machines and transformers, and at the same time to omit one step, it was proposed to alter the voltages 3000 and 6000 to some small extent and to bring together in one step the voltages 15,000–20,000. The resulting series was 3300, 5800, 10,000 and 17,300. Unfortunately this proposal (Appendix I) was not accepted at the conference in New York. The change would not have altered the size of machines, transformers, apparatus, and types of insulators, but the windings would have had to be altered slightly. The result of the alterations would have been a considerable simplification, however, and for this reason the question will be brought up again when it comes to standardizing windings and transformer taps.

Class D, 30,000 Volts to 100,000 Volts. The voltages of this class accepted as standard by the I. E. C. were the widely used values of 30,000, 45,000, 60,000, 80,000, and 100,000 volts, those in heavy type being preferred voltages.

The Swiss Committee also placed a proposal for this class (Appendix I) before the I. E. C., suggesting that the pressures of 30,000 and 60,000 volts should be changed to 33,000 and 58,000 volts so as to ensure the possibility of star-delta connection between these pressures and between 58,000 and 100,000 volts.

Considerations of the present state of affairs and doubts as to the utility of the proposal regarding these high voltages led to this proposal also being rejected by the I. E. C.

It was asserted in particular that it would not be economical to use, for example, transformers insulated for 100,000 volts in 60,000-volt installations. This was not intended for general practise, however. It should be made possible, however, for reserve transformers or even normal transformers, to be used in emergency cases in systems of the next lower voltage by changing over from star to delta.

In this connection an actual case should be mentioned which came to the knowledge of the writer of this report, after the Conference in New York. In the Queenston Power Station at Niagara, transformer units each comprising three single-phase transformers with a total capacity of about 50,000 kw. are changed over from 120,000 volts to about 60,000 volts with disconnecting switches, by means of the star-delta connection. The correct voltage is obtained by regulation on the generators. Although the voltages are approximately in the ratio 1:2, use was not made of the series-parallel connection of the windings, but of star-delta change-over instead; this in spite of the disadvantage that the voltage of the generators has to be regulated to a considerable extent.

Transformers of Class D usually have low-voltage windings belonging to Class C. If the change-over could be effected in both classes, the advantages, of course, would be correspondingly greater, as indicated in Fig. 3.

Class E, Over 100,000 Volts. The I. E. C. has decided on 150,000, 200,000 and 300,000 volts as standard voltages above 100,000 volts.

These voltages bear to each other and to 100,000 volts the ratio 1:2. It was assumed that it would be general practise in such plants to use a combination of three single-phase transformers instead of one three-core transformer, so that if necessary, series-parallel connection would be used with some advantage. Three-core transformers are also designed, however, for pressures over 100,000 volts. The example of the Queenston Plant shows that for groups of three single-phase transformers, also, the star-delta change-over connection is preferable to series-parallel connection, particularly so because it is frequently necessary to change over relatively quickly outside the transformer, and for this purpose single-phase transformers must each be provided with four high-voltage terminals for series-parallel connection.

It is often the practise in high-voltage and extra high-voltage plants to use lower transmission voltages when starting operation and during the first period of operation. This is done on the one hand to test the plant more carefully and on the other hand because the load demand at first usually is not equal to its subsequent maximum.

These considerations, too, indicate that the possibility of a simple change-over to a lower voltage is desirable with transformers, and that wherever possible the lower voltage should also be a standard voltage.

IV. STANDARD VOLTAGES FOR D-C. SYSTEMS

Class A, Up to 99 Volts. The fields of application are the same as mentioned in paragraph III A. The d-c. voltages also of class A have not yet been dealt with by the I. E. C.

Class B, 100 Volts to 1000 Volts. The more extensive d-c. systems are to be found in towns. New d-c. systems, however, are no longer installed on a larger scale for lighting and power as the economic advantages of three-phase, four-wire distribution are very considerable. A certain relationship, *i. e.*, common voltages for d-c. and three-phase systems, is very advantageous, however.

It might have been possible to obtain this relationship at 110 volts, but this was considered too low for lighting and domestic apparatus generally on account of the large cross-sections of conductor necessary. The fact that 110-volt lamps are more reliable than, for example, lamps for 220 volts, was not of such importance because the manufacture of lamps will be improved. The pressure derived from 110 volts between phase leads and neutral, *i. e.*, 190 volts for power mains, is, generally speaking, also too low.

In addition to 100 and 220 volts, 440 volts is also widely used in d-c. systems, and various combinations of 110 and 220 volts, namely 2 × 110, 4 × 110, and 2 × 220 volts, were accepted as standards by the I. E. C.

The I. E. C. has also designated as standard the pressures 115, 230, and 460 volts, but with the reservation that each country must decide upon either one or another series.

Higher voltages in this class and voltages in the next classes are used principally for electric traction which for the time being is not being taken into consideration.

V. NOMINAL VOLTAGES, VOLTAGES AT CONSUMERS' TERMINALS, MAXIMUM VOLTAGES AND TEST VOLTAGES

According to the standards in some countries the mean voltages at consumers' terminals are taken as nominal voltages for class B, while for classes C and D the maximum voltages at the generators and secondary terminals of transformers were fixed as nominal voltages. In favor of the second ruling, it is maintained that the calculation of the test voltage for a plant should be based on the nominal voltage and that the highest voltage normally occurring should be used as a basis for the calculation of the test voltage.

This consideration is correct as far as it goes. It is also easily possible, however, to base the calculation of the test voltage on the mean nominal voltage at con-consumers' terminals, provided allowance is made for the difference between this and the maximum allowable voltage in the system in question. According to various standards and also according to the I. E. C., this difference amounts to about 10 per cent for class C and the higher classes of voltage.

The question as to which voltage in a plant is to be taken as the nominal voltage is thus of secondary importance as far as the reliability of material is concerned. The most important point is that the same voltage shall be used throughout as a basis, and that the same safety factors and factors for calculation of test voltage be employed by the different countries.

The opinion in Switzerland is prevailing that the formula given by the I. E. C. in Publication 34 for medium sized machines and transformers, *viz.*, test voltage = $2\,E + 1000$, is sufficient for testing the insulation of windings, *i. e.*, when a solid or liquid insulation material is in question, thus including also large machines and transformers. A test voltage of $2\,E + 10{,}000$ is proposed for insulators where air is the dielectric medium. In other countries there is a question of increasing the factors to some extent. As this does not directly affect the standardization of voltages, further details will not be dealt with here.

The fixing of the *mean voltage at consumers' terminals* as nominal voltage was very strongly advocated in France on the grounds that this voltage usually figures in contracts for the supply of electricity and should therefore be considered as the nominal voltage. The I. E. C. accepted this proposal. This solution has the advantage that the nominal voltage of all classes is referred to the same point in the system, *i. e.*, to the mean voltages at the consumers' terminals (lamps, motors, and primary terminals of transformers).

This point is not made sufficiently clear in various regulations, including those of the Electric Power Club. The next section of the report will deal with this subject more fully. The decisions of the I. E. C. relative to the question and the table of accepted nominal voltages are given in Appendixes II and IIA.

Figs. 3 and 4 show diagrams of connections for three-phase installations. These give the maximum voltages at the generators and secondary terminals of transformers, and also the mean nominal voltages at the lamps, motors and primary terminals of transformers, for net-work of nominally 220–380, 10,000 and 100,000 volts.

Fig. 3 also indicates various possibilities of changing over on the primary and secondary sides of transformers and motors.

VI. REMARKS ON AMERICAN PRACTISE

The relations between Europe and America in the field of electrical engineering have always been relatively close. In the conferences of the I. E. C., and when drawing up standards for various countries in Europe, endeavors have been made to give due regard to American practise, also.

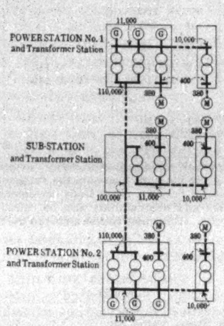

FIG. 4—DISTRIBUTION OF VOLTAGE. LIMITING VALUES

The following case is assumed: Two power stations operating in parallel. Generators for 11,000 volts at full load, one in each station connected to 11,000/400-volt transformer for direct distribution in the immediate vicinity; 380 volts at motor. One 11,000-volt outgoing feeder to a transformer station for supplying another district; as before, transformation ratio 11,000/400 volts and 380 volts at motor.

There are also 11,000/110,000-volt transformers in each power station supplying a substation situated about half way between them, containing 100,000/11,000-volt transformers. Energy is distributed from the substation by means of 11,000/400-volt transformers. There is also an 11,000-volt outgoing feeder for supplying a transformer station at 10,000 volts in another district. This station also transforms down from 10,000 to 400 volts for a pressure of 380 volts at motor terminals (220 volts for lightning).

The Conference in New York in April, 1926 made possible a still closer cooperation between electrical engineers in America and Europe. Earlier discussions of the I. E. C. led to the opinion that American practise regarding standard voltages shows many points of similarity to European practise. In particular, the view was held that the American standard voltages, 6600, 11,000, 22,000, 33,000, 66,000, 88,000, 110,000, and 220,000, represented the maximum voltages of the systems in question, in which 6000, 10,000, 20,000, 30,000, 60,000, 100,000, and 200,000 respectively were the mean voltages at the end of the lines, *i. e.*, at the primary terminals of the step-down transformers. It was thus supposed that the American standards agreed with the European standards, as these give as nominal voltages the same values, 6000, 10,000, 20,000, 30,000, 60,000, 80,000, 100,000, and 200,000 volts, at the consumers' terminals and at the primary terminals of transformers, these values being about 10 per cent lower than the maximal voltages. The difference of about 10 per cent between the mean nominal voltages and the maximum voltages on one and the same system was accepted as I. E. C. standard.

The view that the voltages, 6600 volts, etc., are an American standard maximum is confirmed apparently by some standards of the Electric Power Club. On page 185, for example, taps of five per cent (6270 volts) and 10 per cent (5940 volts) are quoted for 6600-volt transformers. This would obviously indicate that at the consumers' end of a line, *i. e.*, in this case at the primary terminals of the transformer, the pressure is about 5940 or 6000 volts according to the load and the length of the line. This would correspond exactly to the nominal voltage or mean voltage at consumers' terminals accepted by the I. E. C.

The low-tension voltage quoted by the Electric Power Club as standard for 6600-volt transformers is 220–440 volts. This appears not to agree with other standards defined by the Electric Power Club. On page 129, 220–440 volts are quoted, amongst others, as standards for motors. Lamps and other apparatus are also constructed for 220 volts. Such apparatus, however, is very seldom installed immediately near the transformer terminals, but some length of network is usually necessary, involving a voltage drop which must be taken into consideration. This drop for low voltage is assumed to be about five per cent on an average. The mean voltage at the secondary terminals of the transformer should therefore be raised by this amount. Thus, if motors are wound for 220–440 volts, the voltage at the secondary terminals of the transformers should be about 230–460 at full load. With this correction, the transformers would correspond to the I. E. C. standards. The latter standards do not yet contain data for increased voltages at the transformer secondary terminals for the low voltages but this question is to be settled at the next meeting.

The above remarks concerning the transformers for 6600 volts as defined by the Electric Power Club apply similarly to the transformers of other standard voltages, *i. e.*, 11,000 volts, 22,000 volts, etc.

VII. CONCLUDING REMARKS

The foregoing report emphasizes intentionally the importance of the *three-phase current* for the distribution of energy for lighting and power, as the standardization of voltage is influenced thereby.

The report also deals very fully with the question of star-delta change-over of machine and transformer windings, for the following reasons: In standardizing, great importance is rightly attached to voltages used hitherto. In addition to this, there is, however, the not less important question of adapting the standardized voltages to three-phase current, to be considered. If it is asserted that the change-over from star to delta is relatively seldom used at medium and high voltages and that it will not therefore be of great importance, it should be remarked that up until now it was not possible to make use of this change-over with many of the voltage used hitherto, as the resulting voltages are not standard. It is nevertheless frequently pointed out that the pressures 3, 6, 10 kv., and 30, 60, 100 kv. allow the change-over to a large extent. The approximation, however, especially at 3–6 and 30–60 kv., is not sufficiently close. Swiss electrical engineers are of the opinion that a closer agreement between the values would be of great advantage to manufacturers and electric supply companies.

These remarks should in no way lessen the importance of the fact that the I. E. C. has succeeded in fixing standard voltages to which all concerned are agreed. These voltages also form a basis for considerable simplification, as compared with conditions hitherto prevailing.

Appendix I

INTERNATIONAL ELECTROTECHNICAL COMMISSION

PROPOSALS OF THE SWISS COMMITTEE REGARDING STANDARD VOLTAGES FOR THREE-PHASE CURRENTS

A. Proposal. At the meeting held on the 21st April, 1925, at the Hague, it was decided to submit to the National Committees the following proposal of the Swiss delegate, M. A. Huber-Ruf, in regard to the standardization of high pressures. This proposal was inadvertently omitted from the Minutes R. M. 22.

The three-phase pressures proposed by the Advisory Committee on Standard Pressures, which bear to each other an approximate ratio of $1:\sqrt{3}$, should be modified and fixed as shown below, in order to make star-delta interconnection possible.

Two series should be formed, one starting from 10,000 volts and the other from 100,000 volts, as follows:

1st series.....	3300	5800	10000	17300
2nd series....	33000	58000	100000	

It is recommended that the pressures of 3000, 6000, and 15,000–20000 volts should be replaced by the pressures of the first series above, and pressures of 30,000 and 60,000 volts by those of the second series, in all new installations or when important additions or modifications are made to existing installations.

B. Explanation of the Reasons for the Proposal. The rational manufacture in series of three-phase machines and transformers requires above all the use of standard pressures interrelated by the ratio $1:\sqrt{3}$. This scale is the only one in which a standard pressure is obtained when the change from star coupling to delta coupling, or vice versa, is made.

The foregoing proposal satisfies this requirement for the *most commonly used* pressures and will also make it possible to establish in the future a standard series of high three-phase pressures as rational as that which has been fixed for low three-phase pressures on the basis of the ratio $1:\sqrt{3}$.

Appendix II

STANDARD VOLTAGES, R. M. 42

A. DECISIONS OF THE NEW YORK CONFERENCE OF I. E. C., 1926

1. *Voltages,* Class A.

TABLE I

	Voltage at Consumers' Terminals		
		Alternating Current	
Series	Direct Current	Single-phase	Three-phase
I	1 × 110	1 × 110	110
	2 × 110	2 × 110	127
	4 × 110	1 × 220	220
	1 × 220		
	2 × 220		
	1 × 440		
II	1 × 115	1 × 115	115
	2 × 115	2 × 115	133
	4 × 115	1 × 230	230
	1 × 230		
	2 × 230		
	1 × 460		

a. Each country must decide either on Series I or Series II.

b. The values given for three-phase are the voltages between line and neutral. The voltages between phase leads corresponding to the given values between line and neutral must also be considered as standard values, *e. g.*, 380 volts. (This note will be added to the table drawn up by the I. E. C.)

2. *Voltages,* Class B.

a. THREE-PHASE VOLTAGES
TABLE II

Nominal I. E. C. Voltages Mean Value at Consumers' Terminals	Maximum Voltages
1,000	1,100
3,000	3,300
6,000	**6,600**
10,000	11,000
15,000	**16,500**
20,000	22,000
30,000	**33,000**
45,000	50,000
60,000	**66,000**
80,000	88,000
100,000	**110,000**
150,000	165,000
200,000	**220,000**
300,000	330,000

b. Definition of "nominal high voltage:" The nominal high voltage shall be the mean voltage at the consumers' terminals and shall be called nominal I. E. C. voltage of the network of that voltage range.

c. The maximum voltages at the generators and secondary terminals of transformers shall be considered to be about 10 per cent higher than the mean voltages at the consumers' terminals. The values are included in the above table.

d. The maximum and minimum values of the voltages according to paragraph I, and the variations occurring under working conditions, should be considered at a later date.

e. Preferred nominal voltages. The voltages which are in heavy type are preferred high voltages.

Discussion

PAPERS ON VOLTAGE STANDARDS

(SUMMERHAYES AND HANKER, ARGERSINGER, SILVER AND HARDING, GEAR, SCHOLZ, EBERHARDT AND JONES, MINOR, HUBER-RUF)

NEW YORK, N. Y., FEBRUARY 9, 1927

B. G. Jamieson: This discussion was organized for the purpose of presenting this voltage question in such a broad light that it may arouse a unity of purpose and a spirit of compromise so that following this presentation there may be substantial progress towards either a settlement of the question or a definite statement that perhaps we are too far apart ever to settle it.

A few references may be made to previous work. This subject was recognized as preeminently important by the Electrical Apparatus Committee of the N. E. L. A. about 1921 and 1922, with particular reference to one of its several aspects, namely transformers. As a result the 1922 standard of the Electrical Apparatus Committee for the transformers was compiled and approved.

In 1924 the manufacturers began to complain that less than 50 per cent of the power transformers were being ordered in accordance with these standards, and an investigation by the Transformer Committee of the N. E. L. A. revealed that not only was a revision of the 1922 transformer standards necessary but that any such revision should take into account the whole system from the generator to the consuming device.

In 1926 the International Electro-Technical Commission held a meeting in New York, at which time standard system voltages was a major topic in the program, and it became evident that the United States was unable to present anything like a nationally endorsed schedule of system voltages. The recognition of this dilemma brought about a serious committee activity which has crystallized into this symposium.

As I said in the beginning, it is but an expression of a very earnest endeavor to bring about some unity of sentiment on this subject.

President Chesney in his address[1] on February 7 stressed the topic of standardization and the Institute's obligation in this work. In the appreciative responses to his appeal, a suggestion was offered, and that particular suggestion seemed to me to be particularly worth our consideration. The substance of it was that we accomplish our ends (meaning the greater efficacy in our work of attempts at so-called standardization) by a somewhat diminished stressing of that term. That is to say, instead of stressing the word "standard," let's see if we can't use some other word perhaps that will help in bringing about what we desire.

If, for example, we can convey to engineers that what we really are seeking is not in the nature of the absolute which the word "standard" always suggests nor of that fixed character, nor a futile academic ideal, but the ideal that engineers as economists all really appreciate, namely, that we ought to unify and thereby simplify our practise, perhaps this avowed concession to contemporaneous practise and, of course, fixed capital, will attract and soften some of the irreconcilable elements now preventing the desired unification. Operating practise will change with the development of the art, but we should, of course, maintain its orderly direction.

M. D. Cooper: Since the focal point of all endeavor along lines of voltage standardization is the ultimate service voltage, it may be interesting to review briefly the development of the present program of standardization in reference to lighting voltages and incandescent lamp demand.

The original Edison central stations practically all operated at 110 volts. The inherent spread in voltage encountered in the manufacture of carbon lamps induced a corresponding spread in voltage on the part of central station lighting companies so that in the early years of the century lamps were supplied in a large number of voltages between 100 and 130 volts. After the advent of the drawn-wire tungsten filament, the spread in voltage in lamp manufacture ceased. The outstanding voltages of greatest demand were 110, 115, and 120 volts, wherefore the lamp manufacturers started a movement to centralize the lamp demand on these three voltages.

The original 110 volts continued to be the favorite voltage until 1919, when the percentage of demand at this voltage was exceeded by that at 115 volts.

In 1923 the National Electric Light Association put forth the recommendations in reference to standardization of service voltage as quoted in the paper by Messrs. Hanker and Summerhayes.[2]

The percentage of total lamp demand at 115 volts has risen continuously, and for 1926 stands at 47 per cent. For the last seven years there has been a steady decrease in the demand for 110-volt lamps. This decrease has been accelerated in recent years by the increasing favor of the 3-phase, 4-wire network, which gives an undesirably low-power voltage when operated at 110 volts for lighting. In 1926 the demand for 110-volt lamps was only 12 per cent of the total.

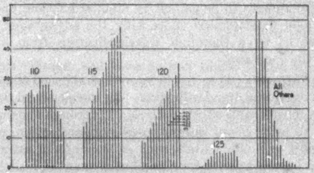

FIG. 1

There will always be a theoretical advantage in operation at 120 volts in preference to 115 volts. With growing load, however, it will be increasingly difficult to work line apparatus at sufficiently high voltages to maintain 120-volt service at peak load.

With the added impetus which will be given to operation of the new network system at 115/200 volts by the probable standardization of a line of 200-volt motors in the near future, we can anticipate an increase in percentage of the total demand at 115 volts at the expense of all other voltages.

It is often difficult to place any cash value upon any one particular step in a program of standardization. If we go back far enough in the history of the art, however, and draw a comparative picture between conditions then and now, we cannot fail to see the great advantage of standardization. For example, in the year 1900 standardization was just beginning to be felt in the matter of incandescent lamps. At that time there were 5 principal sizes of lamps ranging from 2 to 32 candle power and in most of these types there were available 4 types of filament construction, 3 different efficiencies of lamp, 3 different finishes of bulbs, 30 different voltages and 13 different bases. Multiplying all these factors together we find that there were more than 50,000 types of lamps in current demand. At the present time, with much greater range of size, *viz*,—10 to 1000 watts— with one standard efficiency, base, filament construction and bulb for every size of lamp—one type of bulb finish for the majority of the sizes—and only 3 voltages, the number of lamp types for general lighting has decreased from 50,000 to 54. The completion of the voltage standardization as recommended by the National Electric Light Association will still further reduce this number of types to only 18.

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[1] *Electrical Standardization*, address by C. C. Chesney, A. I. E. E. JOURNAL, March, 1927, page following p. 214.

[2] *Voltage Standardization of A-C. Systems* by F. C. Hanker and H. R. Summerhayes, A. I. E. E. JOURNAL, May, 1927, p. 438.

The writer had occasion to see what standardization means in ordinary business on a recent European trip. In Vienna there was a lamp manufacturer who maintained a factory stock of about 3,000,000 lamps. The biggest single item (lamps of the same description) in this stock was one of 3000 lamps. This condition resulted from a lack of standardization with its resultant complication in requirements of different voltages, style of lamp, bulb finish, bases, etc. In a factory stock in this country of approximately the same total size the largest single item was 40-watt, 115-volt lamps of which there were 198,000, or about 70 times as great a stock of the most popular item as could be maintained under the European conditions of non-standardization. The greater availability of the product under conditions of standardization makes, for much more convenient commercial service as well as greatly decreased cost of warehousing, less investment in stocks, less delay, duplication of shipments, etc.

No one needs to speak to the electrical industry in this country on the advantages to be gained by standardization, and I draw this parallel to emphasize more fully the advantages that can accrue to us by a thorough standardization from the lamp back to the generator.

C. E. Skinner: It seems to be well established that we need national industrial standardization. In the electrical field, there has been considerable effort, particularly through the work of the I. E. C. to bring about international standardization, especially with regard to fundamentals, such as the basis of rating. This effort has been continued for a number of years and further progress was made at the meeting of the I. E. C. in New York, April 1926, where this general question of voltage standards was considered.

Unfortunately, international accord seems difficult, due to the fact that in Europe they have been inclined to use the even thousands, while we in America have in general adopted voltages which are multiples of eleven. The lack of voltage standards is responsible for much of the confusion and non-standardization in international electrical trade. For example, a manufacturer of incandescent lamps doing a truly international business, is forced to manufacture something over 70,000 sizes, types and varieties. This makes mass production, as we understand it, impossible.

International standardization is of importance to everyone having anything to do with international trade and in trying to arrive at national standards we should continuously keep in mind the desirability of international accord wherever this may be at all possible.

A. H. Kehoe: I assume it to be generally accepted that the "mean lamp voltage" is the proper basis to use in any standardization of voltages. Some of the papers fail to indicate that standardization has not been rigorously adopted but is still on a tentative basis with three recognized lamp voltages, namely, 110, 115, and 120 volts. It appears that the immediate practical result to be sought is simplification or unification of practise, rather than hard and fast standardization which is likely to be too narrow to be suitable for future requirements. Last year's *Electrical World* statistics recorded over 50 per cent of the residential customer being supplied with 110 volts. Data presented today indicate 120-volt lamp use to be 35 per cent of this class of lamp. With these conditions existing it appears evident that simplification, particularly of the lower voltages, may be possible, but standardization of voltages or equipment unless broad enough to cover such conditions would be standardization in name only, similar to former cases mentioned in the papers.

I differ with statements in certain of the papers which indicate that adopted standards are endowed with properties whereby it becomes a moral duty to adopt them. It is recognized that contractual obligations, in the absence of other specifications, are controlled by existing standards, and the responsibility for the operation of apparatus outside of standards when not covered by specifications places a responsibility on the operator which would not otherwise be present. In practise, probably most of the departures are covered by specification, and I hold the real concern should be for the blind follower of standards regardless of their value for his conditions. For instance, consider the systems equipped with the present standard transformer—this group of papers indicates how inadequate they are for proper operating conditions. I disagree with the statement in the Hanker-Summerhayes paper that "The design of electrical systems contemplating the use of apparatus under conditions more severe than sanctioned by the A. I. E. E. standards should be discouraged." In certain instances, equipment is (and, in my opinion, should be) purchased with insulation in excess of the values which are standard at present, and if conditions warrant the use of apparatus with less than standard values, I see no moral reason why it should not be done. To discourage such action is merely to retard progress temporarily. When variations from standards become of economic importance, changes in standards should be considered. These cases do not have the hazards of a steam boiler, and are not incorporated in the safety requirements of the industry.

It appears possible to me for present conditions to be covered by modifying minor details of the proposals made in the different papers so that a large percentage in the industry can adopt a unified practise. To do this it will be necessary to recognize that we have three "lamp base" voltages, that important economic results are to be obtained by unification, and that the standards are very likely to require changes from time to time as developments in the art change the economics of the situation. If the first step is comprehensive, further simplification at a later date will doubtless be possible. What the future simplification is likely to be should be indicated only in a general way so that future progress will not be delayed if new conditions require changes from what is indicated at present.

W. F. Dawson: This standardization of voltages means much to all of us, but it appears that in starting out for standardization, existing practise has been ignored.

It is now proposed to make standard voltages 5 per cent higher than heretofore and to permit an operating leeway between plus 5 per cent or minus 5 per cent. Previously the plus or minus margin of 5 per cent was permissible but with the understanding that standard temperature guarantees did not apply; in other words, the departure from standard voltage was assumed to be an overload condition. Now apparently, it is proposed that in addition to raising the standard voltages 5 per cent, we are also to guarantee standard temperatures with 5 per cent margin, which is equivalent to raising the voltage standard 10 per cent. Many operators have come to consider dynamo-electric machinery as capable of overloads and special operating conditions much beyond the specifications. They found apparatus rather generously rated, but overlooked the fact that competition has been gradually driving manufacturers to adjust ratings more nearly in accordance with performance ability.

We are prepared to meet these new proposals, but venture the hope that one year, or five years from now, we shall not be confronted with another new "standardization" which will not be standardization, but actually "destandardization."

Many of the smaller turbine alternators are built in advance on stock orders and when special requirements are insisted upon, it means that these standardized and stock machines cannot be utilized. Special requirements mean that machines must be built to order, and involve special development which adds greatly to the cost. The time required for building the apparatus is also increased substantially.

J. H. Foote: Perhaps it is all right to standardize the voltages on the basis of utilization voltage. But we don't want to forget that the correct basis for apparatus design is not the utilization voltage or multiples thereof. In my opinion, the correct basis is the maximum voltage to which the apparatus will be subjected under the maximum load.

For instance, we may say that the correct utilization voltage is 115 volts in terms of a distribution system with 20-to-1 ratio transformers. We can say that the nominal for the distribution voltage is 2300 volts. That is fine, but don't buy 2300-volt generators and don't buy 2300-volt secondary transformers to feed a 2300-volt system, or you are in just the same trouble that you are in now.

That is what caused all this trouble—a generator which has a voltage rating and which was designed to supply that voltage at full load. That means that the generator will probably be a 2500-volt generator. The paper of Messrs. Silver and Harding bring that our very nicely.

Their paper seems to recognize the actual voltage situation more than the other papers in that they figure out what the voltage should be, but apparently have not the courage to say that voltage is impossible and, therefore, they will change the transformer ratio. To jump from the 2300-volt class to the 6600-volt class, a 6600-volt system is just three times a 2200-volt system. Therefore on the same basis on which we say a 2500-volt generator would be needed for a 2300-volt system, a 7500-volt generator would be needed for a 6600-volt system.

Just double that and you have instead of a 13,200-volt generator a 15,000-volt generator.

You say, "That is impossible." Well, it is done. Much 13,200-volt equipment is now operating under full-load conditions at potentials largely in excess of 14,000 volts, and the only reason they don't operate at 15,000 volts is that the machines would probably burn up if operated at that voltage at full load—and some of them do burn up once in a while. The temperatures run to excessive degrees, and the operators change the ratio of their distribution transformers from the 120-to-1 ratio, to a lower ratio by dropping down on the 5 per cent tap.

That means that in place of using the taps in our transformers for line regulation or for transformer regulation, we have to use that range because of the inability of the generator to produce adequate voltage.

Now that is the fundamental picture that comes to me when I think of this transformer standardization. I always have to think of the starting from the utilization voltage. Most of us are led astray in these profiles by starting at the generating end of the transmission system. We should start with the motor and the lamp and see what the voltage must be to supply their requirements adequately.

Now there are some more complications that make the present situation, I believe, besides this tenacious hold on the 11-multiple and the like. One is that most of us are using standard distribution transformers which are of straight ratio. Five or six years ago the Transformer Committee changed the ratio of the transformer stepping down to primary voltages from standard to a little off standard at that time. For instance, they changed from 13,200/2200 volts, which was the former standard, to 13,200/2300 volts, giving us 100 volts boost and helping out the situation somewhat.

Now we have all those transformers and we have the present generators and we have a situation. The solution that we must arrive at is something which will utilize as much of this equipment as possible; and I do not think that the solutions as proposed generally take adequate care of that situation.

There is another reason for our difficulty. We have without thought for many years brought transformers with either four 2½ per cent taps, or two 5 per cent taps, because it was standard and because the manufacturers found it convenient to limit the tap range to 10 per cent. Ten per cent has been shown in any number of transmission systems to be an inadequate range.

This is brought out in one or two of the papers, and most of us who operate systems involving ups and downs and transmission distances know that 10 per cent is not enough. The remainder of the required range is taken out by over-voltage apparatus.

We have found in the systems I am connected with, that a tap range of 16 to 17 per cent seems to be indicated as necessary in an interconnected system for transmission transformers. That does not mean distribution transformers. It is our opinion that distribution transformers should be without taps; that is, the generator voltage is adequate, and straight-ratio transformers and proper feed regulators are used, but there is no necessity for taps in a straight distribution transformer.

Another difficulty is that due to following present standards we have been using not only generator voltages but also transformer secondary voltages, which are too low to supply adequate voltage to the distribution systems.

Perhaps the last difficulty, which has not been really brought out in this meeting, is that in the years that have passed, people have adopted two different schemes of raising the voltage on their system. One is to use multiples of two. For instance, a man who had a 6600-volt system, some day when he needed it doubled his voltage, and then he had 13,200. The other man bought the 6600-volt transformer, and instead of doubling his voltage he *y*-connected it and got 11,400 or thereabouts.

Then a third man adopted 11,000 volts as his voltage. And so there are at least three voltages all in the same range and we wonder why they aren't the same.

Any new standard which we may adopt, which will displace those three lines of transformers or unify them must have a sufficient range of taps so that it will take care of the situation adequately and in following these proposed standards, limiting the taps to 10 per cent or so, that is impossible of solution.

In the solutions which are offered, I note a dread to change the name of the voltage. Instead of that we change the ratio of the transformer. I seriously question that we should do that. Transformers are rather flexible. We have thousands of them already installed and in supplying the new standard transformers to operate on the same system they must, of course, have approximately the same ratio. I think that, that has been brought out by Mr. Kehoe in his discussion.

There is another matter that seems to complicate the situation. That is, that all the new standards are proposed to be nominally rated. This was started four or five years ago and is a reversion to the old standard of rating at the receiving end and applying certain factors to take care of the drops and the like. It gets us into trouble.

For instance, take the 132,000-volt class of transformers. A voltage of 132,000 is largely a nominal voltage and means absolutely nothing. If a man buys a 132,000-volt transformer, we would not know what he was talking about unless he specified whether it would have one 5 per cent tap, or two 5 per cent taps in an extended winding supposedly to hold voltage under load.

That would mean that the extreme ratio of the transformer would give a no-load voltage of about 145,000 volts. If we rate that a maximum-rate transformer it would be a 145,000-volt transformer, and according to the present A. I. E. E. standards would be subject to a considerably higher test than a 132,000-volt transformer.

Therefore the proponent of the nominal rating says that it is to our advantage to rate apparatus nominally and supply over-voltage specifications in order that the A. I. E. E. test be not increased and, therefore, the expense of the equipment.

I think that that is a subterfuge. If the A. I. E. E. test at 132,000 volts is adequate for a transformer with a top-tap ratio giving 140,000 volts, then the thing to do is to change the standards. Instead of making them twice normal plus 1000, make them 1.9 normal plus 1000 or anything else. But why try to fool ourselves about this situation?

Another difficulty is that by nominally rating equipment, the manufacturer rates his insulated apparatus, such as the lightning arresters and oil circuit breakers and other apparatus, at that nominal rating. This means that any engineer who is desirous that the apparatus be adequate as regards test voltage must go into the next class if he is honest with himself and specifies a transformer at the maximum rating.

It seems to me that such apparatus should have the division point half-way between one class and the other class. This is particularly true of lightning arresters, because the performance of the arrester is absolutely dependent upon the actual voltage. Rating a lightning arrester at a maximum of 138,000 volts and a minimum of 126,000 or something like that, which is established practise, means that a system having 140,000 volts must either have special apparatus, or else must go into the next class which is 154,000.

If the division point between the 132,000 volts and the 154,000 volts were made at something around 147,000 volts just about half-way between, then the range of apparatus would fit the standard more adequately.

As I see it, the result, if we go ahead on any such basis as has been proposed, namely to change the distribution transformer ratios in order to keep the names correct, not a present standard and a present special, but a new standard, an old standard, and a present special. We are going to have three sets of transformers in a few years instead of two, and I should like to emphasize that in my opinion any new standard must be decidedly broader in scope than anything that has been offered yet, in order that it may not only include the present standard of transformers, but may also absorb as many of these present specials as possible.

This circuit-voltage rating is something I should like to mention. This A. I. E. E. definition, I believe, should be clarified, the definitions for the purpose of fixing a value to be used in designing and testing electrical apparatus. The rated voltage is defined as the highest rated voltage of the apparatus. That means that if you rate a machine at anything less than maximum voltage that is the rated voltage. That keeps the test down and cheapens the apparatus without changing the test definition, but the A. I. E. E. says, "This voltage rating applies to all parts of the circuit. The actual operating voltage may vary from the rated circuit voltage but should not exceed it."

We are proposing in these new standards to exceed the voltage. If that is all right, the A. I. E. E. rule should be changed. I think the A. I. E. E. rule is all right except that it should say, "It must not exceed it," instead of "should." I think we should bring the ratings of our systems up to what they really are.

To summarize the situation, as I see it: Adequate standards must be technically correct, or else engineers who really think they have worked out the solution adequately will continue as they are at present to buy so-called special apparatus. These new standards must include as much as possible of the present special standard apparatus. They must be based upon premises which will avoid the mistakes of the past, and I think the present standards are merely trying to make standard equipment which has been proved to be inadequate in the past.

They must recognize transformers as ratio machines only, with a certain maximum voltage rating. They must have an adequate tap range, which inter-connected systems find is at least 15 per cent, although full capacity is not necessarily required above 10 per cent.

Generators particularly must be rated at maximum voltage in order that injurious heating will not be experienced. Transformers should be rated at maximum voltage in order to clarify the situation and work in accordance with the present A. I. E. E. installation standards.

Finally, the general apparatus names should be such as to place the nominal rating of the apparatus midway between the maximum and minimum ratings of the new standard itself.

F. L. Hunt: It is not difficult to pick out details in the various papers which have been presented with which we disagree, but I believe, if possible, this discussion should be confined to the general features of the question before us.

I am willing to express my opinion as to one of the general points under discussion, and that is that we should base our standardization on the idea that power will flow in two directions on most of our important circuits. In general terms, I like the idea that Mr. Argersinger has proposed.

H. C. Sutton: In my opinion, generators should be designed to deliver full rated output throughout a range of 10 per cent above or below rated voltage. The manufacturers' proposal of 5 per cent voltage range does not give sufficient flexibility for operation.

There is one other point that I particularly want to stress, and that is, the difficulty due to the present rating of apparatus for voltages above 66 kv. For instance, step-down transformers have a voltage rating in multiples of 11.5 up to 69,000 volts. This same rate should apply for the higher voltage ratings. There is no logical reason for changing the multiple to 11 when the figure of 6900 volts is exceeded.

N. B. Ames: It seems to me that we have overlooked the main point in this proposition entirely. After all, is it a matter of the particular voltage to use or is it a matter of regulation? That seems to me to be the answer to the question. There are two very practical theories in conflict here, of course, (1) whether we shall have excessive reactance in our transformers and generators producing poor regulation and ample protection, or (2) whether we shall reduce this reactance and get better regulation and depend upon our protective devices to give us the protection.

Eugene Vinet: To my mind the essential thing to decide as a first step is what planes or levels of voltages we want to take as standards. That is to say, do we want to take, say, 33,000 volts as a standard or do we want to take 44,000 volts? Whether it is 33,600 or 31,500, etc., is only a matter of fluctuation or regulation as a result of the usual operation. What we have to decide are the planes of voltages that we want to use.

One thing which strikes me in these proposed voltages is that there are too many of them. The purpose of standardization is to reduce to a minimum. My feeling is that we should suggest only approximately 7 or 8 voltages. I speak from personal experience with the organization with which I am connected. Some two years ago we felt that there was a necessity for a standardization of voltages, as, owing to the growth of loads, a good many of the transformers became obsolete, and when we wanted to do some interchanging of transformers we were hindered because the voltages were too varied. We found on a survey of our conditions that we had 21 different voltages.

We have standardized on seven, which is a considerable reduction, and have tried to make everything as simple as possible. That was two years ago. Now every one is fairly well agreed that it has simplified matters considerably and has been of very great help in operation and interchangeability of material, not to mention the saving in money.

It seems to me that it might be well to make a definite issue of certain voltages, and debate them and see whether we can agree on them. For instance, some people favor 11,400, others 13,200. In our own case we have standardized on 11,400 volts star. The reason for that is because we have a great many rural lines which are growing all the time. We felt that the thing to be considered first was the distribution in preference to the generation, because we have got to give service and that should be the dictating factor.

In the case of rural distribution, very often the loads did not warrant voltages of 13,200. We can start at 6600 volts very often single-phase; then we make it three-phase, and then as the load grows we star it and get 11,400. That suits our purpose very nicely. We also standardized on 33,000 volts. It was a question whether we should go up directly to 66,000 volts, but for economical reasons we felt that it was advisable to make a step between 11 kv. and 66 kv. We have situations where we have relatively small towns to reach for service. In scattered communities we may have a town of 5000 people or thereabout to serve. We may have to build 20 or 25 mi. of line to reach that town with smaller places to be served from such a line. It would not be economical to build a 66,000-volt line. We can't build it at 11,000 volts because it won't give the service. Therefore 33 kv. is a very nice intermediate step. That was one reason for our 33 kv. Then we go up to 66 kv. and 132 kv.

The lower voltages are 115, 230, and 460. Then we have 2300 delta or 4000 star. Those are the only standard voltages which we have. We take care of voltage variations by means of taps.

I am mentioning these voltages because they may offer some ground for discussion. We have been very happy with these few voltages so far. I might mention in connection with the 6600 volts 11,400 star that it might be a good thing to go up to 6900 volts or 12,000 volts star. It occurs to me at this time that it might be a way of compromising with the advocates of 13,200 volts. There isn't any doubt that there will have to be compromises.

So far as we are concerned, we feel so much the importance and the benefit of standardization in voltages, that we certainly will do everything we can to cooperate in this movement and help get it over if possible.

E. C. Stone: It seems to me that there are certain fundamental principles brought out in this discussion that we can all well recognize. The first one is that standardization will start at the utilization voltage. We have heard there were three utilization voltages—110, 115, 120. I wonder if it means that we have to start with three different standards and build up.

The second point is that we must recognize the voltage regulation or drop in the transmission system. This varies widely on different systems, being small on some and large on others, but in any event, in the general problem of voltage standardization, the voltage drop in the various parts of the transmission distribution system must be recognized.

That means, I think, fundamentally that the old set-up of 10-to-1 ratios for transformers and multiples of 10-to-1 ratios is no longer acceptable, and we must break away from that system of ratios to something else.

The suggestion that I wish to offer for immediate consideration is that the essential mechanical features of power transformers be standardized. This is done to a considerable extent at the present time but might be carried further. By mechanical features I mean the core, bushings, end frames, tanks, terminal boards, terminal arrangements, number of taps, number and arrangement of outgoing leads. With these elements standardized, the actual number of turns in the winding, the ratios, and the exact location of taps in the winding might be left as the lowest-cost flexible link in the transmission system, to be worked out to meet to best advantage the local conditions which are peculiar to any particular system. It hardly seems reasonable to install on the system which has very low voltage drops, transformers which are designed with perhaps 15 per cent and 20 per cent full-capacity taps, to meet conditions on systems having long overhead transmission and correspondingly high-voltage drops.

B. G. Jamieson: Mr. Stone has enumerated some fundamentals that apply particularly to transformers, and there are others applying to transformers which will probably make the wisdom of such standardization apparent, as we find ourselves requiring exciting transformers and series transformers and tap changers and phase shifters and various other auxiliaries, that are now coming to be recognized as necessities with large transformers in the larger and more complex systems.

Of course there are standards to be considered in connection with generators. It is proposed by one of the authors that a simple way to get more flexibility without greater complication is to increase the range of generator voltage. That is something that might be discussed profitably. If, for example, we assume that generators will be built to deliver energy on the busses at 25,000 or 30,000 volts in the near future, perhaps we have more to think about in that connection than we now have when we are generating at pressures below 15,000.

It is quite a simple matter to specify any voltage and get any voltage in a generator so long as you stay below 15,000 volts, but before very long on account of general desirability of a lower current in these large machines, we shall probably see higher-voltage generators, and then it may not be so easy to get the 10 or 15 per cent range suggested by the simple device of varying the voltage.

There are other fundamentals in this problem. There is the general question of means of limiting the necessary voltage range, such as is accomplished in part by the use of synchronous condensers. The Pacific Coast companies are using 17½ or 20 per cent tap ranges partly because the use of the devices really makes it possible for them to keep within that range.

We need to know whether we must allow our systems to respond to this upward urge or trend of voltage that seems to be so evident. We need to know, in order to meet that, on what basis a gradation of steps in system voltages is to be founded. We need to know whether they are to be on a Y-delta basis or on a preferred-number basis or on the basis of approximation of our old standards or whatnot.

We need to know whether those steps can be met by apparatus having sufficient flexibility without being out of the pale of the standard class, or whether we must allow in our contemplated schemes for a very extended number of steps which will give us a minimum in flexibility requirements of individual apparatus forming part of the system.

We need to know whether or not the problem of high voltage resulting from open circuits is something that can be successfully taken care of.

In connection with the fundamentals of this matter, the Committee has made a survey and has been able to get almost universal assent to the basic acceptance of the principle of utilization voltage as a standard of reference in connection with any standard-voltage system. That point has been, we consider, gained and we hope it won't be upset, though of course if it need be, it will be I presume.

Another point referred to by Mr. Jones, namely the question of rated circuit voltage or, as he put it, the system voltage, was fixed by the Committee and approved by the Institute. Now it would be possible to undo that also, if necessary, but I mention that as another achievement of the Committee so that you will understand that at least two of the fundamentals necessary in consideration of a standard system voltage have been, we think, sufficiently settled so the discussion may proceed on that assumption.

C. E. Skinner: At the meeting of the International Electrotechnical Commission in New York last spring, some of our European friends brought to us the term "voltage zones" and it seems to me that this is a very apt term in connection with certain features of our standardization program. By "voltage zone" is meant a certain range of voltages between which all dielectric tests for apparatus are to be the same. This would be of distinct advantage in allowing manufacturers to stock such parts as outlet bushings and other features of design which have a definite limit due to the dielectric test. In many cases, it would probably be cheaper to take an outlet bushing, for example, from the next zone than to manufacture one for a specific purpose which happened to fall just beyond the particular zone. I would suggest that this question of voltage zones be kept in mind in connection with this standardization proposition, especially with regard to the application of dielectric tests.

R. K. McMaster: An important point is the breaking away from any attempt to have transformers of a single ratio suitable for use on both the 6600- and the 7200/12,470-volt systems. There should be a ratio of 2-to-1 between the voltage ratings of transformers for the 6600- and 13,200-volt systems and a ratio of 3-to-1 for the 2400/4150- and 7200/12,470-volt systems.

In the paper by Messrs. Silver and Harding the recommendation is made that 120 volts be the accepted secondary voltage for transformers of the service class. This is quite a step forward, and also will do much toward the consolidation of the 12,470- and 13,200-volt systems. There is only 6 per cent difference between these two voltages. It would be quite possible to have a tap at about that point, or even to use one of the standard taps for 13,200-volt transformers. In the same paper it is recommended that the voltage ratings of motors for use on 6600- and 13,200-volt volt systems should be reduced. This is a good idea and will go a long way toward avoiding step-by-step increases above these voltages.

One of the disadvantages of the 13,200-volt system is that twice this voltage, namely 26,400, is not standard. Might it not be that the real purpose of standardization would be served by adopting 16,500 volts as a standard voltage, this being half of 33,000 volts?

It is very important to maintain the ratio of 2-to-1 between 66,000 and 132,000 volts and between 69,000 and 138,000 volts, allowing the use of series-parallel transformer connections without complications due to a little lower ratio.

In the paper by Mr. Huber-Ruf voltage ratios based on the star-delta arrangement of motor and transformer connections are advocated. In some cases this is advantageous for motors. It is not, however, advantageous for high-voltage systems; not alone for the reason that there are already at least approximately standard voltages which do not allow for this, but also because of the grounding of the neutral which should be provided for at certain stations necessarily and at others as practicable to provide alternate points of grounding.

Regarding transformer taps in general the percentages of taps should be standardized, rather than the number of taps, so that transformers having a non-standard range of taps will parallel with standard transformers. Simple figures are desirable for tap voltages. An example of this would be the use of 64,500-, 63,000-, 61,500- and 60,000-volt taps for 66,000-volt transformers. There will be a readjustment of parallel operating conditions in any event in connection with standardization of percentages of reactance as well as standardization of voltages.

It is also desirable to give considerable attention from a standardization viewpoint to the phase angle between lines of all voltages, in the higher ranges to facilitate interconnections which are not thought of at the present time, and in the lower ranges to facilitate networks supplied from systems of more than one voltage.

For transformers of all ratios, with the exception of those involving 120 volts and small multiples thereof, it should be recognized that a zero phase angle is permissible, at least under certain conditions. The cases where a 30-deg. angle is desirable or permissible should be standardized so that there will not be more than a minimum possibility of a 30-deg. angle existing between systems of the same voltage in the same vicinity. It is also important to have the 30-deg. angle in the same direction wherever it occurs between systems of any two voltages operating under similar conditions.

Mr. Minor's paper mentions the use of transformers having a ratio suitable for connection directly between the phase wires of 4150-volt systems, permitting the omission of the fourth wire. Such transformers should be used wherever practicable, not only to eliminate the need for running the fourth wire, but also to facilitate the use of the combined light and power system with service voltages of 115 and 199.

P. H. Chase: I should like to ask what the manufacturers consider is the relative influence on transformer cost of standardization of reactance, also of such things as insulation of the neutral lead for full-line potentials as against treating it as a fully grounded neutral lead? On lower-voltage distribution transformers, such as those for subway installation, what is the influence of standardization on the number of phases? There are also certain dimensional and other manufacturing standardization points that, to my mind, must influence cost to a degree commensurate with the influence of voltage, as such.

H. L. Wallau: The point has been raised about reducing the number of voltage standards in use. I think that is something we can all consider. The figure mentioned by one of the previous speakers was a reduction from 21 to 7. I might say that in our own system we are gradually tending to 5, if we consider the 2300-4600 volt class as one. Five might be plenty for most of us. It may not be enough for all of us.

Messrs. Hanker and Summerhayes have enumerated five principles to which any proposed system of voltage standards should conform. These will not be challenged.

Undoubtedly the definition of "rated circuit voltage" will meet with general approval, and it is obvious that equipment should tested in conformity with the maximum line to line voltage to which it may be subjected.

The A. I. E. E. test, for transformers, Rule 6356, is twice line-to-line voltage plus 1000. An exception, Rule 6363 for "Transformers with Graded Insulation" is very vague. The manufacturers' present practise is to test at 2.73 times the voltage from line to ground. What test is proposed for this class of insulation? I quote in part "Such a rule . . . would definitely set the rated voltages of apparatus, their *test voltages* and maximum operating voltage."

If test voltages are to meet Rule 6356 even though induced in the windings must we not sacrifice graded insulation and its resultant economy? Should not the A. I. E. E. Standards Committee provide for a definite test voltage for transformers of this class?

Among low-voltage distribution systems of today are some involving transformers with windings for 2080/4160 and 2300/4600 volts delta. The proposed standards recognize only the 2300-volt class.

Cleveland, Detroit and, I believe, Chicago, have thousands of kilowatts of transformers connected of the above off-standard voltages. Too much is involved to discard these. By building transformers of this general class with coils for series or parallel connection, we establish the 4600-volt standard from the 2300-volt standard and by providing a 10 per cent tap, we can obtain 2070/4140 volts from these, which probably would be acceptable to operators.

For new systems the 11,500-volt standard may be abandoned in favor of the 13,800-volt. For many existing systems it must be maintained and transformers for both 6600 volts and 11,500 volts delta connection are required. The former group has been entirely eliminated.

To me, another grave defect in these proposed standards is the lack of reversibility of power transfer, due to the use of different turn ratios for step-up and step-down transformers. With interconnections growing apace, if full benefit is to be realized from them, power should be able to flow in the direction reverse from normal and the voltage delivered under this condition closely approximate the normal sending voltage.

Mr. Argersinger has most clearly indicated this disadvantage and suggested a remedy. It merits close study. His proposed voltage standards retain the values made familiar to us by long usage and his 5 per cent over-voltage tap automatically brings his system into practical agreement with the manufacturers'. However, he omits the 88- and 154-kv. ratings and adds a 176-kv. rating.

These changes would, I believe, be inadvisable because of considerable mileage in 88- to 90-kv. and of 140- to 154-kv. lines.

Also the 176-kv. standard would necessitate the design of a complete new line of equipment and there is a permissible argument that a project requiring at least 176 kv. should be developed at 220 kv. at the outset.

I am in general agreement with his views which, though differently expressed, result in standards not far different from those proposed, except for an additional 2½ per cent of over-voltage operation suggested and the omission of the emergency limitation. What constitutes an emergency and how long does it last? Mr. Argersinger's proposal is the more clear-cut.

Two taps per winding as suggested by him may not prove enough for certain long transmissions. It should be simple to provide others at a slight increase in cost, when required.

All power transformers should be equipped with externally operated ratio adjusters.

Referring to maximum voltage rated "apparatus," when standards are agreed upon, cannot this apparatus be derated to fit, that is, name plate data changed and equipment left as is?

F. C. Hanker: In considering the subject of voltage standardization an effort has been made to investigate the possibility of developing a practical system of voltages that will meet the needs of a large percentage of electricity supply systems.

There has been a tendency in the discussion to cite certain specific needs that appear of paramount importance for a particular district. Where there is sufficient justification for certain values they should unquestionably be adopted but we should carefully scrutinize these suggestions to be sure that they are not an expedient to care for a temporary condition. We should be sure that they conform to a logical plan.

The Pacific Coast has a condition where they are meeting distribution requirements with transformers arranged for 11,000-volt star connection and 6600-volt delta connection. The decision that we burden the entire industry with costs of a 6600-volt transformer that will be satisfactory for operation on 11,000 volts can only be determined by a survey to see whether the cost is justified. In the lower voltages the difference is not as great as it would be in the higher classes. It is very possible if the demand is sufficient that it would be justified. That same condition exists I feel throughout this standardization. We should study the conditions and where there is justification it should be recognized by being included in a standard line. Obviously it would be agreeable to everyone concerned if standards could be made flexible enough to meet all conditions. Unfortunately this cannot be done without increasing costs and only those that are suitable for general use would be included.

The objection to the star-delta proposal is greater for the higher voltages. For example if you take a transformer suitable for delta connection on 66,000 volts and star connection on 114,000 volts you must of necessity design the insulation for the higher voltage service. This means in the first place that the apparatus will cost from 35 to 40 per cent more for the star-delta combination than would be the case if it was designed for the 66,000-volt service. The design must be satisfactory for insulation to ground and insulation between turns when operated on the higher voltage. This adds to the expense of those transformers that are equipped only for the 66,000-volt service. In addition to higher cost you have a lower performance. In view of these disadvantages it does not seem desirable that the entire capacity of 66,000-volt apparatus should be burdened with the greater expense for the possible benefit to those systems that would use the transformers at the higher voltage.

It is generally recognized that the greatest return from standardization is in those classes where quantity production is a possibility. Every effort should be made to reach an agreement on the lower voltages applying to utilization equipment, distribution transformers, substation transformers, and possibly some of the lower transmission voltages that are generally used.

In the higher voltage classes it may not be possible to secure a general agreement on the requirements. The range that has been suggested by several of the groups varies from 5 per cent proposed by the manufacturers to a maximum of 25 per cent for those cases where reversal of power flow may be necessary. There are undoubtedly cases where a greater range than 5 per cent is necessary. We would suggest that a survey be made to establish the capacity of equipment that would come outside the proposed 5 per cent range. This study could be based on equipment already in service. It is probable that the curve would be somewhat similar to the "use-factor curve" showing the time generating apparatus is required to meet load conditions on a particular system. These curves show that the capacity required to care for the peak loads is in use only a relatively few hours during the year representing a high investment cost for these increments of load. If we establish the capacity of transformers operating at normal voltage and at voltages up to 25 per cent above normal, and determine a corresponding cost for transformers with different voltage ranges, we would then be able to establish the total cost to the industry that would result from the adoption of different zones. At the present time the range is based on opinion. Before a final decision is made it is recommended that a study similar to that proposed would be of value in establishing the most satisfactory range.

On the higher voltage transformers it is not the actual turn ratio that is so important as a standardization of the voltage classes. Such a standardization would minimize the number of designs necessary for the manufacturers and result in a reduction in development costs. With such a standardization the mechanical construction of the core, insulation structure, tanks and terminals could be standardized and advantage taken of this condition.

Ernest Pragst: I should like to comment on these papers and the discussion largely from the point of view of the manufacturer.

A number of years ago, the operators of public utilities and the manufacturers of electrical apparatus undertook to standardize the voltages of apparatus. Out of this work a set of standards finally emerged which were sponsored and issued by the National Electric Light Association.

The fact now seems to make itself apparent that when the standards were adopted, little or no consideration was given to the system as a whole. Each type of apparatus was standardized as to voltage with little or no consideration given to its operation in connection with other types of apparatus. Because of these oversights, we now find ourselves with an inoperative set of standards.

Now, the manufacturer has accepted the standards and has a number of standard lines of generators, transformers and motors. When he attempts to sell this standard apparatus, he discovers that his customer cannot operate it in a system without exceeding the limits for which it has been designed and guaranteed.

The operators of public utilities realized some time ago that the standards as adopted could not be used successfully, so they simply abandoned them. Each has sought his own solution in his own way, with the result that but little uniformity of practise now exists.

After listening to the many diverse opinions expressed I find myself in a quandary when I try to reconcile them. Some might have been led to believe that the manufacturers seeking a new set of standards will be next asking the discarding of present equipment. Nothing like this is contemplated. Moreover, nothing particularly radical is being asked.

In preparing the standards proposed by the manufacturers and presented by Messrs. Summerhayes and Hanker, I am sure every effort was made to depart as little as possible from the present standards. A comparison of the proposed standards with the present standards as issued by the National Electric Light Association will reveal the close similarity between the two. Generator voltages are such that the old can be paralleled with the new; transformer ratios are such that through the use of taps parallel operation of old with new transformers will usually be possible; motor voltages are to remain the same.

I am convinced that greater benefits will accrue to the operator than to the manufacturer through standardization and that the manufacturers are seeking only a workable set of standards that will meet an appreciable part of the requirements of the operating companies. With such diverse views as now exist (most of which are not without merit) an agreement will be reached only if we can realize the necessity of compromise and practise it to the utmost.

L. L. Elden: It is believed that an analysis of the value and quantities of equipment which have been found unsuited for operation under the voltage standards referred to, will be found to be only a relatively small percentage of the units in service and that such difficulties as have developed in this direction will be largely found in high-tension equipment.

The discussion which I will present is one in which Mr. Oliver of the New England Power Company and the writer have collaborated to some extent to present very briefly some views covering our experience in New England. One of us is operating a system utilizing moderate voltages in supplying a compact area, with interconnections to several adjacent systems. The other is operating an extensive transmission system reaching into five states and utilizing voltages of 66,000 and 110,000. The present transformer standards have been entirely satisfactory to the former, and with the addition of standard feeder regulators, high grade, efficient and reliable service has been maintained.

For the second system existing transformer standards have been found unsatisfactory, and apparatus has been purchased which does not conform to A. I. E. E. standards in order to meet operating requirements. Power-factor correction equipment has been found necessary at several points to insure a proper degree of regulation, with the result that substantially uniform voltage conditions are maintained throughout the system. In passing it may also be said that the voltage standards proposed by the manufacturers would still be unsatisfactory for this system.

In this discussion we have refrained from presenting any table of values, believing that the determination of final values applicable to the entire industry cannot be effected at this time. If broad principles applicable to the situation can be agreed upon, much will have been accomplished.

Any undertaking aiming to standardize voltages is bound to meet with many difficulties in view of the many interests affected. Many of the conditions to be met are not fully appreciated. On this phase of the question we may refer to the proposed basis of standardization outlined in the manufacturers' paper.

An analysis of the hypothetical system shown therein discloses the fact that as between full- and no-load conditions, an overall uncontrolled regulation of about 30 per cent would be developed, an amount which could at best only be divided between the generator and receiver if any load be served from, or near the generating station.

If the system is expanded and becomes an interconnected network with additional generating stations, difficulties immediately develop. For example, a second generating station connected to the 69,000-volt section of the system must experience voltage or reactive-power difficulties with changing load conditions on the main system. It is, of course, granted that the addition of regulating apparatus may obviate some of these difficulties.

The system described is hardly sufficiently comprehensive to illustrate the needs of practical application. Even if such a simple system exists, it may be short-sighted to design it so with no provision for expansion. If transmission systems or lines are interconnected, it is questionable whether a considerable difference of potential can be allowed to exist between any two parts. Probably it would always be found desirable to regulate the voltage very closely by power-factor control. Even if not, a step-down transformer may be installed close (electrically) to a step-up transformer, so close that the voltages at the two points are practically identical, and in this case also the proposed standards become inadequate. Some of these difficulties would be eliminated by the following suggested changes in the proposed transformer standards.

STEP-UP TRANSFORMERS

Adhere to proposed secondary voltage ratings and provide one 5 per cent tap above normal rating to maintain secondary rated voltages under load.

Provide two 5 per cent (each) reduced-voltage full-capacity taps.

Taps should preferably be located in the high-voltage windings (low-voltage taps in large transformers involve difficulties, particularly in regard to ratio-adjuster design on account of high current densities).

STEP-DOWN TRANSFORMERS

Increase proposed primary voltage ratings 5 per cent (which raises this voltage rating to agree with the rating of step-up transformers at zero regulation).

Provide three 5 per cent (each) reduced-voltage full-capacity taps.

Transformers should be designed to operate under emergency conditions at 5 per cent over-voltage (over-excited). Emergency conditions should be defined as existing a considerable length of time, perhaps several days.

Standardization to be effective must recognize that present-day developments indicate that interconnected systems covering large areas, with many sources of power supply are to be more and more important features of transmission and distribution practise in the near future and that voltage requirements in such systems must be studied from all points of view.

It is believed that standardization of voltages cannot be effective if based upon conditions assumed in any radial system. At least one of the papers has presented the question from the network point of view with favorable results.

Conservation of existing investments is no small portion of the main problem and any new standards must be devised to protect such investments.

Progress, however, has been made and agreement seems quite general upon certain items. Generators and transformers should be capable of operating at least 5 to 10 per cent above rated voltage.

Step-up and step-down transformers should be identical in operating characteristics and be equipped with similar taps ranging from 5 per cent above to 15 per cent below rated voltage. The definition of "rated voltage" is logical and acceptable. Certain other features require further study.

The point at which "system voltages" should be standardized must be agreed upon. European practise recognizes receiver voltage as most desirable. American opinion is divided upon this point, and before national progress can be made, agreement must be reached.

There is much to be said in favor of receiver voltage as it is at receiver locations that constant and normal conditions are expected to prevail. Elsewhere wide variation may exist. A difference of opinion exists between Mr. Oliver and myself on account of the difference in the practise of our respective interests.

Standards should be on same basis throughout the full range of systems and not change from receiver to sending values above 66,000 volts as proposed by the manufacturers.

In connection with test voltages applicable to apparatus specified under rated voltages, it is suggested that apparatus should be reclassified for test purposes. Apparatus including windings, for example transformers, should be tested substantially in accordance with present standards as such apparatus has proved most reliable. Oil switches, disconnecting switches, lightning arresters, high-tension bushings, etc., should be subjected to higher tests than at present. The many failures which occur in this apparatus justify this recommendation.

Further support to this theory is afforded from the data submitted in papers on surge investigations presented at this convention.

In general it is contended the switch ratings in the higher voltage classes (66,000 and over) are much too close to the operating voltages. The added cost of a higher voltage switch is sometimes considered prohibitive and as cost is too often a controlling element, the factor of safety secured is sometimes insufficient. Studies of dimensional data for switches in adjacent classes frequently show but small differences, which leads to the constructive suggestion that costs to the user might be lower all around if the manufacturers would eliminate certain classes of switches and utilize possibly 5 or 6 classes to cover the entire range of usage.

The following groupings are suggested:

For System Voltages of the Kv. Below	Use Switches of the Kv. Below
220 to 154	220
132 to 110	132
88 to 66	88
44 to 22	44
15 to 6.6	13.2

Substantial savings in manufacturing costs would result. Substantial savings would accrue to users through more general interchangeability, reduction in stock of parts, etc.

There never has appeared to be any justification for development of 33,000-volt switches as a separate class between 22,000 and 44,000 volts, or for certain other intermediate classes.

All switches should be designed with ample factor of safety in service at voltages 50 per cent above rated voltage. Puncture and flash-over tests should be based upon the 50 per cent excess voltages to enable apparatus to meet known impulse and surge values to be encountered under service conditions.

The net result of this proposal should be a better and safer product at no higher, if not actually less, cost than at present. This proposal is not out of line with present practise in other branches of the industry, namely, one size of tank for several transformer sizes, one motor frame for several different motor capacities, etc.

The suggested elimination of certain classes of switches leads to the further suggestion that certain system voltages might also be eliminated from the standards, for example, 4600, 11,000, 33,000, and 88,000. These might be considered exceptions and no new construction be undertaken for these voltages.

The responsibility for the present situation rests upon all interests; users and manufacturers, utilities, consulting engineers, designing engineers, and owners of projects, all of whom have contributed to present conditions. Much construction has been created in which through lack of foresight, knowledge or appreciation of future requirements, great sums have actually been wasted. Even today construction is going forward which is limited in its future usefulness and as designed represents substantial waste.

Failure to supply conductors of adequate capacity and suitable operating facilities leads to ultimate losses which are enormous when compared with the small additional investment required for an adequate arrangement. Isolated construction is frequently noted which involves factors preventing the use of any standard apparatus.

It may well be that certain interests delay or obstruct standardization in recommending that motors be designed for operation for 90-110 per cent of normal voltage and then operate at the lower value contrary to the intent of the present standard. Systems should not be designed for 20 per cent variation in voltage at receiver end as suggested. In short, responsibility for some of these matters must rest where it belongs.

Standardization of voltages has been and is really being impeded to a great extent by manufacturing interests who, for competitive purposes, create new classifications in design of apparatus which might well be eliminated through liberality in existing designs. Users' specifications may be responsible for certain of this undesirable effort, but a brief review of the many types of similar equipment which are offered from year to year is most convincing. Cooperative effort such as is being considered here should eliminate many of these conditions.

Assuming that standardization is really effected, will costs be reduced? We believe not, at least so they can be distinguished since it appears that in high-voltage construction, for example, most equipment is non-standard and is usually designed to meet requirements of individual systems, hence if such methods are to continue there is little opportunity for cost reduction. With years of progress in present switch design, rising prices are encountered.

On the other hand a review of many transactions involving purchases of apparatus under highly competitive conditions, manufacturing prices seem to have no anchor. Possibly we are really saving as much now as would be represented by increases in cost of higher rated apparatus which seems so necessary. It has been suggested that the cost of non-standardization in matters of voltages may total $150,000,000 to $200,000,000.

This is really not an excessive expenditure to be incurred in the development of a $10,000,000,000 industry, it being only 2 per cent of the total. What industry can show equal efficiency through an extended development period? We are really not so badly off as might appear from some of the comparisons which have been made.

Finally, if standardization is actually accomplished, the following queries suggest themselves:

(a) Will prescribed standards be followed by all? We believe the answer is "no," that special construction will still continue as controlled by the acts of individuals, and that there will always be special classes outside of the standards which we may create.

(b) Will not special voltages still be selected for certain projects as best harmonizing with local conditions, such as load, distance, preferred sizes and types of conductors, economic conditions to be met, etc.? We believe the answer is "yes."

It is common knowledge that many projects are designed and built as isolated units, to meet certain local conditions with no thought of future connection with other systems. Where is this condition more prevalent than in industrial plants where every standard is sometimes sacrificed to make a sale. In many more important undertakings operating conditions are seriously affected by economical limitations, imposed by investment restrictions supposedly to meet some theoretical or calculated load cycle to the entire disregard of future requirements.

P. H. Chase: It seems to me that Mr. Elden's remarks reflect the well-reasoned attitude of the central station man considering the problem from the broad point of view, looking forward to the day when we will have more interconnection, when the voltage regulation must be taken care of by voltage regulating means for power flow in both directions.

W. R. Bullard (by letter): Messrs. Silver and Harding have presented a comprehensive picture of a rational and practical method of assigning voltage ratings to different types of apparatus, so as to maintain the proper voltage levels at different points in the system. Under this method, the starting point for assigning voltage ratings is the lamp socket. This is as it should be, since the lamp-socket voltage is fixed by service requirements and the values of other voltages are very largely dependent upon the necessity for holding this voltage practically constant at its nominal value. Therefore, in connection with the general problem of voltage standardization, it is highly desirable that a single lamp-socket or utilization voltage standard be ultimately established, and a brief discussion of this phase of the problem may not be out of place.

A lamp-socket voltage standard of 115 volts was *assumed* by Messrs. Silver and Harding in building up the tentative assignment of voltage ratings. Of the two remaining voltages in general use—namely, 110 and 120 volts,—the popularity of the lower voltage is on the decline; 110 volts can therefore probably be eliminated from consideration as far as eventual standards are concerned. This leaves the choice of the ultimate standard between 115 and 120 volts, if the selection is to be made from existing standards.

Of these two voltages the latter has the advantage of providing a slightly higher copper efficiency in the low-voltage distribution circuits, while the former has the advantage of conforming, on the whole, more nearly to existing voltage standards of utilization and distribution devices and apparatus, as will be seen from the following:

Incandescent lamps are short-lived and are now furnished at the same cost and efficiency for both voltages. The selection of either 115 or 120 volts as a universal standard would therefore involve no difficulties as far as the manufacture of lamps is concerned.

In the case of motors and devices of the 220-volt class, existing designs are not entirely suitable for delta-connected distribution systems of either 115 or 120 volts at the lamp socket. They are, however, more suitable for the former than for the latter voltage in delta systems, since 240 volts, the delta voltage for the 120-volt standard, is nearly 10 per cent high for equipment rated at 220 volts.

Delta-connected distribution systems will doubtless continue to be used for many years. However, low-voltage systems of the 4-wire Y-connected type are rapidly coming into use in connection with underground distribution in business districts of both medium-sized and large cities, and therefore the relation of motor voltage standards to this type of system must be carefully considered. In Y-connected systems, the delta voltage corresponding to the 120-volt standard, or 208 volts, conforms more nearly to existing 220-volt apparatus designs than does the delta voltage corresponding to the 115-volt standard, or 199 volts. However, actual operating experience obtained in connection with several underground distribution systems of the 4-wire network type has demonstrated that 220-volt motors, both of ancient vintage and of present design, will with very few exceptions, function satisfactorily at 199 volts, in much the same manner as they will function satisfactorily at 240 volts in a 120/240-volt delta system.

In order to meet the demands of both delta and Y-connected systems and at the same time avoid developing two lines of motors and devices of the 220-volt class, the logical solution of this phase of the problem is to make slight changes in the future designs of this apparatus so as to make them conform to an intermediate voltage value between those of the two types of systems with the necessary tolerance above and below this value. This would be advantageous not only from the manufacturer's viewpoint but also from that of the customer, since the latter would be enabled to use his motors interchangeably in both types of systems. Assuming this procedure, there is little to choose between 115 or 120 volts as a universal standard as far as this class of apparatus is concerned. In one case the mean value between the delta voltages of the two types of systems would be 214.5 and in the other case, 224. Neither of these values conconforms exactly to the present 220-volt rating, but the relation is so close in each case that the necessary changes would be slight.

The situation is slightly different with respect to appliances of the 115-volt class. Many of the existing standard lines are designed to apply to a voltage range from 110 to 120 volts. This of course makes them entirely satisfactory for 115-volt distribution and in some measure out of line for a universal standard of 120 volts. Consequently, it seems fair to assume that the adoption of 120 volts as a single utilization standard would eventually bring about some general changes in the design of appliances whereas a 115-volt standard would almost exactly fit existing designs.

The most serious phase of the situation is encountered in the case of 2300-volt distribution transformers. The present voltage rating in this case,—namely 115/230/2300 volts,—is primarily suitable for use in systems having nominal 110 lamp-socket voltage. In 115-volt systems, it is quite generally necessary to over-excite these transformers by some 5 per cent or more in order to maintain 115 volts at the lamp socket. This is working out fairly well in practise, particularly since present transformer designs are probably liberal as to the allowable upper limit of operating voltage. However, a lamp-socket voltage of 120 would in many cases require an overexcitation of more than 10 per cent and even then it would be difficult in many systems to maintain normal voltage at the lamp socket with generators and station transformers of present voltage ratings.

It can of course be assumed that these difficulties would eventually be taken care of by some comprehensive method of voltage ratings in the future design of system apparatus. However, the number of standard distribution transformers of present ratings in service, and the present capital investment represented by other apparatus involved in the question of system voltage levels, are so great that existing equipment in this case must be given serious consideration.

On the other side of the picture is the fact that the 120-volt standard would provide a slightly higher copper efficiency in the low-voltage distribution circuits. The difference would indeed be slight in the case of a-c. systems, since it would represent only some 10 per cent of the secondary copper losses or, in usual types of a-c. distribution systems, a fraction of 1 per cent of the delivered energy. In d-c. systems the situation is somewhat more serious, and it is worthy of note in this connection that the d-c. systems provide a very large portion of the present market for 120-volt lamps and appliances. This suggests that a satisfactory solution to the problem might be the general adoption of 115 volts for a-c. systems and 120 volts for d-c. systems. This, however, has the serious disadvantage that in cities having both a-c. and d-c. distribution it would be necessary either to maintain two utilization voltage standards, or to depart from the standard voltage in one of the two systems. In practically all except the very largest cities, d-c. systems are being converted into or merged with a-c. systems, and the importance of establishing standards which will facilitate this process can easily be appreciated. Furthermore, although lamps and appliances are now furnished for both voltages, and would have little influence upon the question of which voltage should be selected, nevertheless a considerable simplification of manufacture and stocking of these articles would be brought about by the establishment of a single standard.

The ultimate solution of the problem must of course be based upon a very careful weighing of all the factors involved, and it will no doubt be a difficult matter to bring about, in any reasonable length of time the complete application of a single lamp-voltage standard. Nevertheless, in view of the large ultimate saving which would accrue to the industry, it can hardly be doubted that a well coordinated effort to fix upon such a single standard should be made in the near future.

The general problem of standardization of apparatus voltage ratings is largely dependent upon the establishment of such a single standard. Messrs. Silver and Harding have suggested a schedule of standard voltage ratings that is entirely practicable if the ultimate lamp-voltage standard is to be 115 volts. If, however, 120 volts should ultimately be selected it would seem from the foregoing that a change from the existing standard voltage ratings of distribution transformers would be necessary. The logical form for this change to take would be a change in ratio. For instance, the ratio of 18-to-1 might be adopted, this being an existing commercial rating in use in a number of systems. For a 120-volt standard this ratio would permit operating the distribution transformers at a more favorable excitation voltage than does the present standard 20-to-1 ratio in connection with 115-volt systems. Furthermore, with this one change in the schedule of voltage ratings suggested by Messrs. Silver and Harding, this schedule would apply as well to the 120-volt standard as it does in its present form to the 115-volt standard.

M. T. Crawford (by telegraph): I can endorse general plan of voltages proposed by Messrs. Silver and Harding as our distribution installations recent years conform thereto. There is considerable investment in 6600-volt/11,00-volt-Y systems in the northwest, which are very economical for rural distribution. I doubt the possibility of their eventual elimination. I suggest careful consideration in the discussion of Mr. Argersinger's scheme combining step-up and step-down types by adding taps.

H. Carl Wolf (communicated after adjournment): Voltage standardization has been discussed from practically every viewpoint except that of the public, and in the final analysis we all fall into this latter classification. Speaking broadly and collectively, the consumer wants standardization of voltages and wants it to begin at what is to him the most tangible point, his equipment. From data presented on the sales of lamps, it appears that lamp voltages will very soon be standardized at 115 or 120, either one of which provides a reliable starting point.

The consumer is interested in service and is willing to pay the price to get the very best service obtainable. He is also interested in flexibility, simplicity, sturdiness, and universality of equipment. Simplification of practise as to voltages is the most effective means of accomplishing these ends. A great deal of stress has been laid in discussions on this subject on the need for transformer taps in order to keep the voltage up during loads. Greater stress should, in my opinion, be laid on the regulation of voltage, thus necessitating more care in the design of lines and equipment in order to reduce voltage drop to the lowest point commensurate with the economics of the situation. With the development of equipment for changing taps under load, a larger number of taps might be justified, but until such time the other engineering features of the system should be stressed.

If we are to have simplified practise, the fewer the number of voltages agreed upon, the nearer will be attained the goal. It should not be forgotten that the electric industry is still in its infancy and present investment is still only a fraction of what will ultimately obtain. The present medley of voltages and practises should not be permitted to stand too much in the way of adoption of standards for the future. After all, this question of voltage standardization is nothing more than the preparation of a voltage budget within the limits of which good practise can move. Present equipment and present standards should not be rendered obsolete over night, but the industry should be given a goal toward which to work.

In selecting the voltages to be concentrated on and in considering the relative merits of the delta-Y or other systems of connections, the telephone situation should not be lost sight of. Joint construction is very desirable in a large number of cases and any standardization adopted should conform as far as possible to that operating practise which will reduce to a minimum inductive interference.

Combined Light and Power Systems
For A-C. Secondary Networks

BY H. RICHTER*

Associate, A. I. E. E.

***Synopsis.**—If the situation being created by the extensive use of combined light and power systems for secondary distribution is to be squarely met, widespread discussion is necessary.*

The increase of alternating-current low-voltage network systems employing automatic sectionalizing equipment has been very rapid. This move has been attended by a diversity of choice of combined light and power schemes for the secondary mains.

Carrying this condition to its logical conclusion may result in an extremely complicated situation for apparatus connected to these mains. There might thus be imposed on the industry as a whole a heavy expense tending to cancel a part of the savings attributed to the advent of the combined system.

Previous investigation of the probable effect of each of the principal light and power schemes on apparatus connected to the secondaries has been confined largely to general purpose motors. A study of the effects of six other types of equipment concerned showed, however, that general purpose motors are hardly more important than most of the other devices.

This analysis included not only the applicability of existing apparatus standards to each of the combined systems but also the probable developments that the future may bring. For a comprehensive comparison of the various schemes it was found necessary to consider the commercial as well as the engineering aspects.

If the operating companies decide to employ the combined light and power system universally for secondary networks, it is urged that they will soon apply the practise of standardization to the combinations of connection and voltage. In this regard due consideration should be given to the bearing of numerous trends in the industry.

The avoidance of such uneconomic situations as have attended failure to standardize in similar cases in the past makes all the more desirable a nation-wide discussion of this problem immediately. Such a discussion would also bring out the various points of view for the use of those groups which would choose the standard and promote its application.

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CLOSE contact with the discussion and use of the various combined light and power connection schemes that are rapidly being adopted as part of a-c. network systems brings the realization that:

a. The gross savings attributed to the combined scheme may be reduced by the expense incurred in the production of suitable utilization equipment,

b. This expense might range from a minimum of 75 million dollars to a maximum of 150 million dollars even under the favorable condition of a single combined scheme adopted universally, and

c. The latter sum may be exceeded if the three principal combinations are continued without the advantage of standardization on one system.

This gives good cause to wonder whether this may not be the proper time to investigate which method of supplying both power and light from the same secondary mains will give the greatest benefit with the least expense to the industry as a whole.

The rapidity with which this combined secondary system is spreading is indicated by the increase in the number of automatic network systems in use or under actual construction. Beginning about four years ago with a single network installation in New York City, the progress has been such that 11 cities will have in operation by the end of this year (1926) networks embodying combined secondary schemes of one design or another. Furthermore, early next year, this number will be increased by three and the possibilities of these networks are now being investigated in at least 16 other cities.

Where two networks are employed, one for light and the other for power, the distribution system may become very expensive and occupy too much space (ducts or pin positions). The combined secondary system, it is claimed, furnishes an economic solution for extending the a-c. network system into areas where lack of space would make very difficult the installation of two separate networks. And—just as in the d-c. three-wire system—the combined method permits the immediate servicing of any customer for either power or light, or both, from the same set of low-voltage mains.

In attempting to avail themselves of the advantages of this system, the operating companies have considered many different combinations of connections and voltages, each scheme admirably suited to the local conditions surrounding its development. Unfortunately, the three-phase schemes differ in the resulting voltages available for light and power, and the existence of the two-phase combined system further increases the number of possibilities.

All of the proposed arrangements that have received marked attention, except one, the two-phase, furnish some voltage that does not coincide with the voltages at present accepted as standard. While the exception gives voltages that do agree with existing standards, it has been claimed* that except for existing two-phase systems it has certain features which, from the standpoint of the industry as a whole, may outweigh all of its advantages.

There are indications that adoption of these various schemes on an extended scale may have considerable

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*Discussion of *Engineering and Economic Elements of Two-Phase, Five-Wire Distribution*, P. H. Chase, A. I. E. E. JOURNAL, XLIV, November 1925, p. 1249.

effect upon the operation of electrical devices now connected to secondary systems, and result in a more or less complete re-adaptation of many, if not all, of the devices for future applications. The magnitude of such a change in apparatus, for even one of the many types of combined schemes, can scarcely be realized without a careful study of all the factors involved. To gage the probable consequences of having several different schemes adopted for numerous large distribution systems is impossible without a careful study requiring the earnest cooperation of all the operating companies.

This paper is offered to show the result of an approximate analysis of what might be the effect on various types of apparatus were networks incorporating any one of the principal combined light and power systems adopted extensively. The analysis included consideration of such major lines as:

Lamps
Motors and Control Equipment
Appliances
Distribution Apparatus

This preliminary study revealed decided differences in probable expense to the industry when comparing the principal combined schemes. From a minimum of some $75,000,000 in one plan, this expense rose to as much as $150,000,000 in another. The considerable difference in probable expense of the various schemes, and the magnitude of even the minimum expense, urges the question whether there should not be initiated on the part of the operating companies a move to prevent the growth of an unnecessary number of new standards.

If the advantages of this step become universally apparent, this paper may be the starting point for a wide spread discussion of how to minimize the expense of adopting the combined secondary system for a-c. networks.

PRESENT STATUS OF APPLICATION OF COMBINED LIGHT AND POWER SECONDARY SYSTEMS

To better understand the nature of these combined systems and their extensive application, an outline of their present status may be of value.

Spread of Combined Secondary Systems. The steps that have preceded the spread of the combined light and power secondary system may be reviewed as follows:

1. In the usual radial distribution system, power loads greater than about five kw. each are mostly supplied by transformers separate from those carrying lighting loads, as shown in Fig. 1A. If the two loads are connected to the same transformer and its capacity is correct for both lighting and steady motor loads, this capacity, in conjunction with the design of the rest of the distribution system, is usually not great enough to keep the total voltage drop to the lamps within the limits required to prevent flickering of light.

2. Seeking to decrease the total number of transformers on their overhead radial systems, several companies use a single bank of three single-phase transformers for both power and light. This is done mainly in outlying districts when minimum installation cost is essential. Where better voltage regulation must be maintained at the lamps the lighting secondaries are separate from the power; also, transformer and secondary capacities are liberal. The transformer low-voltage windings are usually connected 230 volts delta and lighting loads at 115/230 volts are supplied from one of the transformers with mid-point grounded.

3. In one western New York city, a similar practise has been followed for the past few years in the downtown area, but the transformers are connected in star to give 120 volts at lamps and 208 volts three-phase at motors. These banks are usually installed in basement vaults of large buildings and each serves only the building to which it is assigned. Power service lines are separate from lighting lines. The customers are well satisfied with the voltage supplied, even though motors up to 40 hp., including elevator motors, may be located in a pent house on the roof.

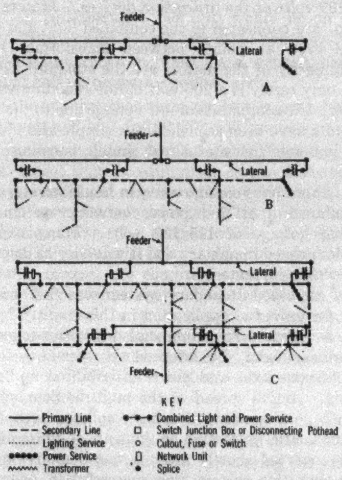

FIG. 1—TYPICAL DISTRIBUTION SYSTEMS

4. Some companies have experimented with the combined light and power method for secondaries, as well as for transformers and services, on radial systems. In most cases the practise has been abandoned upon the unexpected burning out of several motors, caused by heavy load occurring coincident with too low voltage.

5. For 14 years a city in California has operated a 120/208-volt three-phase four-wire system underground, using combined light and power secondaries as well as transformers. The latter are not banked and the secondaries are not bussed from block to block, but the low voltage mains are of ample size.

6. A city in Minnesota has for many years used on its overhead lines a four-wire combined secondary network system giving 230 volts three-phase delta for power, and 115/230 volts for lighting, taken from one of the three transformers in the bank. As the lighting load is carried by only one phase in each district, each network is of small size and confined to the transformers from but one feeder, as in Fig. 1B.

7. During the past 11 years a city in Tennessee has had a comparatively large three-phase underground network, supplied by numerous feeders and employing 115/199 volts at the utilization devices. Motors up to 50 hp. are connected to the combined light and power mains even at points between transformer banks. The majority of the motors are the standard 220-volt type; only rarely is a 200-volt motor encountered; and the very few complaints of low voltage due to the use of 199 volts have been remedied in a simple and inexpensive manner by means of small boosting autotransformers.

8. About one year ago a city in Louisiana started up a combined light and power network system with nominal voltage of 115/199 volts. Many expected troubles proved imaginary and it was easy to dispose of the few that did materialize.

The combined secondary system was first publicly urged for universal application in this country by Mr. W. C. L. Eglin,* who recommended that this scheme, in three-phase form, be adopted as standard for all miscellaneous light and power distribution and street lighting. Active spread of the multiple primary feed low-voltage a-c. network using combined secondaries and automatic protective equipment, as illustrated in Fig. 1C, started shortly after it was demonstrated in 1924† that this system was successful in operation, and more economical and reliable, than any previous a-c. system.‡

Past Discussion. The subject of combined light and power secondary systems has been discussed in detail in one A. I. E. E. paper specifically confined to combined secondary systems, in several papers concerned with networks, and in two Serial Reports, dated 1925 and 1926, by the Electrical Apparatus Committee of the National Electric Light Association.§

Some Combined Schemes. The combinations of connection and voltage for combined secondary systems that have been given the most study are indicated in Fig. 2,—*A, B, C, D,* and *F* applying to three-phase and *E* to two-phase. It should be noted that Fig. 2D is a combined light and power transformer system and not a combined secondary system. A complete discussion of the advantages and disadvantages of each of these combinations is contained in the 1925 and 1926 Serial Reports previously referred to. The schemes of Figs. 2E and 2F are receiving the most attention at the present time.

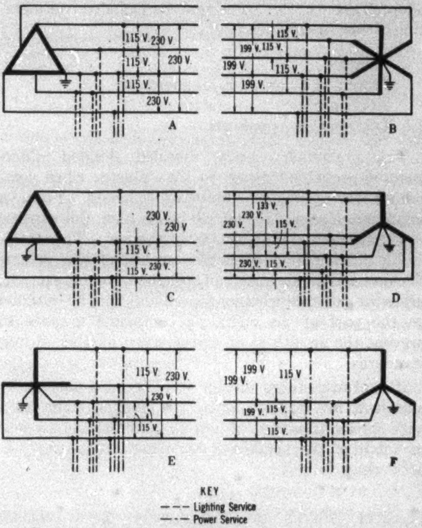

Fig. 1—Schemes for Combined Light and Power

Of the network systems now under construction, eleven will use Fig. 2F arrangement, two Fig. 2E, one Fig. 2D, and one Fig. 2C as a preliminary to Fig. 2F. For the majority of the fifteen networks planned and under consideration, Fig. 2F scheme is favored for the underground. For overhead network systems Fig. 2C is planned in one case. In the three-phase distribution systems to which Fig. 2F applies, 125/216 volts is being considered in only one place; for the remainder, about half prefer 115/199 volts at the utilization device and half 120/208 volts. While there are no present indications that the number of two-phase combined secondary systems will increase in the immediate future, unfortunately the groups favoring 115/199 volts and 120/208 volts for three-phase networks appear to have found no common ground for preventing complication of the apparatus situation in the future.

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*Paper before Association of Edison Illuminating Companies in 1922. See Bibliography.

†See A. H. Kehoe and W. R. Bullard in Bibliography.

‡Construction of such networks is now going on in 13 cities, which are, in chronological order of starting work: Manhattan, N. Y., New Orleans, La., Memphis, Tenn., Dallas, Tex., Philadelphia, Pa., Knoxville, Tenn., Atlanta, Ga., Bronx, N. Y., Miami, Fla., Ft. Wayne, Ind., Pittsburgh, Pa., Canton, O., and Brooklyn, N. Y. Construction has been started in one city in the east and a trial network is being installed in a western city. There are at least 10 networks of this type in successful operation in four of these cities. Adoption of such networks is now being planned for 3 other cities and is under consideration in at least 12 more. The number of cities thus totals 30. In two of them, overhead networks of a similar kind are also being considered.

§See Bibliography.

Probable Result of Present Tendencies. It often happens that a given set of conditions, usually viewed separately, take on a different aspect when correlated. This is largely true when the probable effects of present tendencies within each of the three groups—three-phase network, two-phase network, and radial—are considered collectively in the light of past experience.

A. *Three-phase Networks.* There are indications that a move will be made, as the number of systems using the two principal combined secondary arrangements increases, to change the design of apparatus used on these systems to suit each combination. Such developments are matters of common occurrence. Whenever a new idea is adopted on a relatively small scale it is fitted to conditions as found, in order that an immediate gain may be obtained from it. As the application becomes more extensive, the natural tendency is towards a change in the conditions to make it more simple or economical.

Thus, 115/199-volt star systems are likely to require changes eventually in the design of polyphase and other apparatus connected between phase-wires. It is estimated that if all underground distribution systems, and the systems feeding two-thirds of the total overhead miscellaneous distribution load in cities, should adopt networks using the combined method, this would affect up to about one-fifth of all phase-to-phase apparatus, the remainder being used outside of the network areas and on isolated plant systems.

The 120/208-volt star system will probably require changes in design for both apparatus connected between phase-wires and that connected from phase-wire to neutral. An effort to estimate what part of all the phase-to-neutral equipment would be thus affected might place it at about half.

B. *Two-phase Networks.* For the few two-phase systems, production and stocking of two-phase apparatus will no doubt continue, tending to keep up the prices of all polyphase equipment. In addition, the copper in a two-phase motor is about 6 per cent less effective than in a three-phase motor; in some cases this appears as a decrease of 0.6 per cent in power factor, 0.4 per cent in efficiency, and 11.4 per cent in starting torque, as compared with a three-phase motor. Furthermore, the companies operating two-phase systems may have somewhat higher annual charges than with the three-phase system*.

C. *Radial Secondary Systems.* It is probable that radial three-phase 110/220- and 115/230-volt distribution systems, some of which use single-phase feed, and other radial systems giving 110 volts or less at the utilization devices, will be confined to miscellaneous load systems in cities, towns, villages, and farms where load density is light (up to about 2500 kv-a. per square mile); to bulk loads fed by operating companies; and to isolated plant systems. These would thus ultimately take a little less than half of all phase-to-neutral apparatus and four-fifths of all phase-to-phase apparatus. As any change in standards would inconvenience the engineers of these systems and their customers, the present lines of equipment would probably be required in addition to all new lines.

In an effort to obtain a more tangible conception of the relative effect these four systems would have on the total quantity of apparatus, exclusive of lamps, connected to secondary systems, Table I was derived. This assumes, for phase-to-neutral apparatus, that:

a. Of all equipment, 6 per cent would be on two-phase systems and 94 per cent on three-phase systems;

b. Of the apparatus on three-phase systems, half on radial and half on network systems; and

c. Of the apparatus on three-phase networks, half on 115/199-volt and half on 120/208-volt systems.

The same assumptions were used with phase-to-phase devices, except that for equipment employed on three-phase systems 80 per cent were assumed on radial systems and 20 per cent on networks. This ratio takes into consideration, first, the preponderance of polyphase apparatus in industrial plants on the outskirts of cities, and, second, the condition that a considerable number of large plants might follow the example of one in Brooklyn which buys its power wholesale and is planning to install its own automatic network system employing the combined secondary scheme. Practically all such plants have hitherto been supplied by the radial method.

The right half of Table I gives an approximate idea of the relative values of apparatus that would be affected by these four systems. Assuming the sales of phase-to-neutral equipment for the entire industry to be 69 per cent of the total sales of all apparatus applied to secondary systems, the percentages in the left half of the table were multiplied by 0.69; similarly, for phase-to-phase equipment the percentages were multiplied by 0.31.

It thus appears, if existing tendencies continue, that about 23 per cent of all phase-to-neutral apparatus and 9 per cent of all phase-to-phase apparatus would be of non-standard voltage, while 6 per cent of the latter type of equipment would be two-phase. About 19 per cent of all sales for both types of apparatus might be for non-standard voltages; also, about 2 per cent of all sales might be for 220-volt two-phase equipment. In addition to showing what a miscellaneous demand may arise if present tendencies continue, Table I indicates that the relative quantity and value of non-standard apparatus which would be affected by the

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*P. H. Chase's paper (see Bibliography) shows the annual charges in one system to be from 2 to 3 per cent higher for two phase.

120/208-volt star system is much greater than that affected by the 115/199-volt system.

The requirement of supplying various standard lines of equipment for a plurality of distribution groups will have a tendency to increase apparatus development and distributing costs. It is only reasonable to expect, as these costs become greater due to uncontrollable economic factors, that an increase of prices in these lines is likely to occur; this presupposes that the quality is not to be lowered. That higher prices sometimes do not immediately follow changed conditions must not be allowed to obscure the fact that such causes contribute largely to price advances. It is thus clear how an unbridled growth in the number of connection schemes and voltages may eventually result in higher prices to all purchasers of apparatus connected to secondary systems.

Furthermore, an increase in the number of classes of apparatus and of types of distribution system may be expected to add complications in the choice of equipment having the proper rating, and confusion to customers by change of system in moving from city to city. With the higher prices, these factors would tend to reduce consumption of apparatus and electric energy.

TABLE I
PERCENTAGE OF APPARATUS AFFECTED BY VARIOUS DISTRIBUTION SCHEMES
ASSUMING EXISTING TENDENCIES

	Relating to number of units					Weighted according to total sales				
	Apparatus connected from					Apparatus connected from				
	Phase-wire to neutral		Phase-wire to phase-wire			Phase-wire to neutral		Phase-wire to phase-wire		
	Rating of apparatus		Rating of apparatus			Rating of apparatus		Rating of apparatus		
Scheme	115 volts single phase	120 volts single phase	199 volts three phase	220 volts three phase	220 volts two phase	115 volts single phase	120 volts single phase	199 volts three phase	220 volts three phase	220 volts two phase
1. Radial, three-phase	47.0			75.2		32.4			23.3	
2. Network, 115/199 volts	23.5		9.4			16.2		2.9		
3. Network, 120/208 volts		23.5		9.4			16.2		2.9	
4. Two-phase, rad. and net	6.0				6.0	4.2				1.9
Totals	76.5	23.5	9.4	84.6	6.0	52.8	16.2	2.9	26.2	1.9

TABLE II
ASSUMING THREE-PHASE NETWORKS ALL 120/208 VOLTS

1. Radial, three-phase	47.0			75.2		32.4			23.3	
2. Network, 120/208 volts		47.0		18.8			32.4		5.8	
3. Two-phase, rad. and net	6.0				6.0	4.2				1.9
Totals	53.0	47.0	0	94.0	6.0	36.6	32.4	0	29.1	1.9

TABLE III
ASSUMING ALL NETWORKS THREE-PHASE, 120/208 VOLTS

1. Radial, three-phase	47.0			75.2		32.4			23.3	
2. Network, 120/208 volts		53.0		24.8			36.6		7.7	
Totals	47.0	53.0	0	100.0	0	32.4	36.6	0	31.0	0

TABLE IV
ASSUMING ALL NETWORKS TWO-PHASE, 115/230 VOLTS

1. Radial, three-phase	47.0			75.2		32.4			23.3	
2. Network, two-phase	53.0				24.8	36.6				7.7
Totals	100.0	0	0	75.2	24.8	69.0	0	0	23.3	7.7

TABLE V
ASSUMING ALL NETWORKS THREE-PHASE, 115/199 VOLTS

1. Radial, three-phase	47.0			75.2		32.4			23.3	
2. Network, 115/199 volts	53.0		24.8			36.6		7.7		
Totals	100.0	0	24.8	75.2	0	69.0	0	7.7	23.3	0

Standardization as a Remedy. These troubles previously discussed could be avoided by the adoption, as a standard for combined light and power secondary systems, of that scheme which fits in best with existing standards. Such standardization would also result in this benefit—that, instead of spending time on problems introduced by various kinds of systems, all interested parties could confine their research to improving the one standard combined system.

PREMISES OF THE QUALITATIVE ANALYSIS

The result of an investigation into the effect of the various combined schemes upon the operation and manufacture of each of the types of apparatus connected to secondary systems will be presented as a Qualitative Analysis. Only the most important combinations have been chosen and each in turn has been assumed to be the standard. These are 115/199 volts three-phase star, 120/208 volts three-phase star, and 115/230 volts two-phase five-wire, since they are now in use; also 125/216 volts and 110/190 volts star because there appears to be some question as to the possibility of employing them.

In the absence of any definite data easily available for use in such an analysis, approximations must necessarily be made. Without some analysis of this sort as a guide, even though approximate, the complexity of the problem might result in unduly stressing some one aspect. It is not surprising that such undue emphasis might easily be given. For example, if motors alone are considered, there appears the astounding total of some 10,000 different ratings. These may be grouped as follows:

Capacities—at least 30 sizes in small and general purpose motors

Windings—three types in small motors and five in general purpose motors

Speeds—four standard speeds

Enclosures—three types

Power Supply—direct-current and alternating-current

Phases—three kinds

Frequencies—five

Voltages—seven.

To add yet another voltage rating would further complicate a situation that already seems burdensome to the industry. And yet a similar condition with regard to the other types of apparatus, such as control equipment, transformers, etc., stands out with equal prominence. This can be observed in the analysis that follows.

Due to the numerous unforeseen factors that may affect the situation in the future, it is possible that the conclusions to be drawn from this analysis are as close to the conditions of the future as would be those based on a laborious research into the exact effect of every factor on each type of apparatus. It is the result of an investigation lasting over a period of over two and one-half years.

It should be clearly understood that in submitting this analysis there was no inclination to favor any particular system of secondary connection. Its main function is to point out what factors will tend to cancel the tangible savings and other benefits that have been counted on by those companies which are adopting the combined system.

Qualitative Analysis for Apparatus Connected to Secondary System

EFFECT OF CHANGE TO COMBINED LIGHT AND POWER SYSTEM FOR THREE-PHASE STAR SECONDARY NETWORKS

This analysis applies to the seven classes of apparatus that would be directly affected by the application of a three-phase star connected combined system to network secondaries. These classes are:

1. Small motors,
2. General purpose motors,
3. Motor control equipment,
4. Safety switches,
5. Distribution transformers,
6. Miscellaneous apparatus, and
7. Electric heating devices.

The analysis was made on the assumption that all of the systems that may be networked would adopt this three-phase four-wire scheme, while all other systems would continue to be served by the three-phase radial method.

1. Effect on Small Motors. 110/220 volts is the present standard for this line, with allowable variation of 10 per cent plus or minus.

Polyphase Motors: 125/216 Volts. There would be no trouble with this voltage.

120/208 volts. Practically all standard 220-volt ratings could be used as they stand. However, it would be disadvantageous to put a double rating on the nameplate for, if performance guarantees at 208 volts must be given, these motors would be at a disadvantage as compared with lines designed for and rated at 208 volts. New designs could be reduced to a minimum, by having it generally agreed that 220-volt motors will be satisfactory for operation on 208-volt networks and that no change in the nameplate will be required.

Both study and experience have brought about a general assumption that five per cent voltage variation at utilization apparatus should be considered an extreme condition in low-voltage networks. If, as a result, five per cent plus or minus could be settled upon as the maximum tolerance for all types of apparatus connected to secondary networks, the extra expense due to adopting the combined system would be considerably reduced. This step would prevent penalizing a large number of customers for the relatively few cases where excessive voltage drop occurs in the interior wiring or distribution system.

115/199 or 110/190 Volts. A separate line of motors would have to be brought out and the expenditure involved would be fairly large; it would even be excessive for 190 volts if the greater production of the future is considered.

Single-Phase Motors, 2:1 Voltage Winding: For any voltage of the combined three-phase four-wire secondary system this class of motor must operate either across two phase wires or from phase-wire to neutral.

As this does not give a 2:1 ratio and the motor winding must be designed to operate satisfactorily on both the upper and the lower voltages, the tendency would be to design it for sufficient power on the series connection and with sufficient material to withstand the voltage to neutral on the parallel connection.

For instance, with the 120/208-volt scheme, instead of 208 ÷ 2 or 104 volts the parallel connection would have to withstand 120 volts. This means a range of 15 per cent in nominal voltage, and to this must be added 5 per cent to cover maximum regulation for the network. This would result in a motor considerably larger and from ten to fifteen per cent more expensive than the present standard. Double guarantees would be necessary and unusual care in application would be required to ensure that sufficient capacity be always available for the connection between phase-wires.

For these reasons, and also because the majority of single-phase small motors are used at the lower voltage (which would be phase-to-neutral voltage in the combined secondary system), it is hardly advisable to introduce 2:1 voltage winding motors for combined three-phase four-wire secondary networks. Any compromise using a double rating would also be unwise, as so many single-phase motors are sold to resale manufacturers who distribute their products throughout the country that it would be very embarrassing to these companies if they supplied motors with more than one rating.

Single-Phase Motors, Single Voltage Winding: Only a small percentage of these motors are used at the higher voltage. The present 220-volt ratings could be employed for 216 or 208 volts on network systems, but the risk at 199 or 190 volts would require the development of a separate line for these voltages.

On the few existing combined light and power systems where potentials higher than 115 volts are actually obtained at the motor terminals, 110-volt motors have caused almost no complaints. However, it is anticipated that if such systems become more numerous and are in the form of networks, whereby closer secondary voltage regulation is obtained, trouble may be experienced due to overheating, noise, increased starting current, and lower efficiency and power factor. This would be particularly true for 125 volts, which means that motors for this voltage would have to be redesigned almost immediately.

With small motors future results cannot be determined entirely from past experience of a more or less limited nature. For instance, in the case of 120 volts it is reasonably certain that difficulty would arise should customers begin to order 120-volt motors, making it necessary to give guarantees and eventually to mark the nameplates at 120 volts. To build a single line on the basis of such network voltages as 120 or 125 volts would result in unsatisfactory operation on the innumerable circuits that now deliver less than 110 volts to single-phase motors; for example, considerable trouble is already encountered on radial 110-volt systems due to low voltage at starting. Building and stocking a new line of 120- or 125-volt motors as a parallel to the present 110-volt line would require a fairly large expenditure. In addition, there would be considerable expense and confusion on the part of the utilization device manufacturers, for they would have to buy motors of both classes and carry large stocks at their factories and distributors' warehouses.

2. Effect on General Purpose Motors. 110/220 volts is the existing standard for these motors, with permissible variation of 10 per cent plus or minus.

The principal factors that limit the design of standard general purpose motors are:

a. Starting torque, especially in the smaller sizes;

b. Efficiency and power-factor (and principally the latter) which are of particular importance in low speed motors since heating ordinarily gives no difficulty if good performance is obtained; and

c. Heating, mainly in connection with high-speed motors as good performance is inherent in them.

The analysis, except for the sections on characteristic curves, is based on actual performance data for all sizes from 5 to 150 hp. in all lines.

125/216 Volts: There would be practically no difficulty with this voltage.

120/208 Volts: Starting Torque: Since starting torque varies approximately as the square of the voltage, the application of 208 volts to a 220-volt motor results in a reduction of about 11 per cent in starting torque. However, at least 50 per cent of the standard 220-volt ratings would still give satisfactory torque at 208 volts plus or minus 10 per cent.

For the remaining ratings it would be necessary to rewind the motors to provide the same guarantees. The expenditure in development and other costs would be large for the motors that would require rewinding. If the number of classes and sizes of motors that would have to be changed were relatively small it might be more expensive to keep a separate stock of motors for these ratings at 208 volts than to incorporate the rewound motors in one complete line good for 208 to 220 volts plus or minus 10 per cent. The latter course would, however, result in a reduction of the power factor of these rewound machines when operating on 220-volt circuits.

Were the principle of 5 per cent plus or minus voltage regulation in secondary networks established, these changes could be avoided, provided only a small number of 220-volt motors would require rewinding to operate satisfactorily on 208 volts. Where this procedure might result in too low a starting torque because of the reduced voltage at starting, the motors might be thrown directly on the line at 208 volts.

Starting these motors directly across the line at this voltage would probably not require any change within them. The starting current would be reduced at 208 volts, just as when using a reduced voltage starter on a

230-volt line. The starting torque, corresponding to the 208-volt point, would presumably be sufficient. That the motors could stand this method of operation mechanically seems assured because general purpose motors now in use, at least those up to 50 hp. in rating, can be and are connected directly across the line at the present standard voltages. Furthermore, with the frequency of starting usually encountered, there is also not likely to be any trouble so far as heating is concerned.

Should the number of motors that would need rewinding be fairly large, however, and this may come to be the average condition a few years from now, it might be desirable to establish a new line of motors rated at 208 volts. The performance in this case would naturally be improved as compared with the single line, but the maintaining of a separate stock for each of the two lines would prove extremely burdensome from both the investment and delivery standpoints.

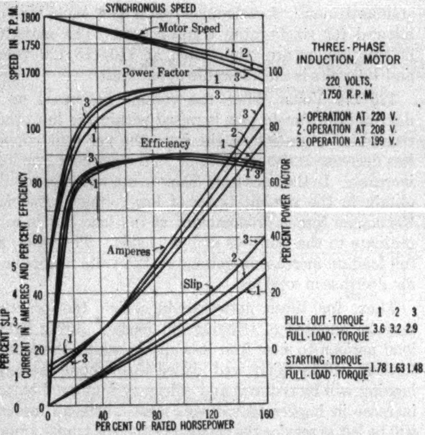

Fig. 3—Motor Performance Curves

Characteristic Curves: The following characteristics are taken from the curves in Fig. 3 of a typical standard 220-volt three-phase squirrel-cage induction motor showing performance at rated voltage and at 208 volts. For speeds lower than 1750 rev. per min. the performance is not quite as good and the differences are somewhat greater. The values for 50 to 60 per cent of full load are added as it is found that this is the average load on general purpose motors connected to typical underground distribution systems and these values are therefore the ones that will be of more interest during the immediate future. All values are approximate.

PERFORMANCE AT 208 VOLTS COMPARED WITH 220 VOLTS

	At 100 per cent of full load	At 50-60 per cent of full load
Speed	0.3 per cent lower	0.15 per cent lower
Power-factor	0.5 per cent higher	2.3 per cent higher
Efficiency	0.5 per cent lower	0.5 per cent higher
Current	5.2 per cent higher	3.0 per cent higher
Pullout Torque	11.0 per cent lower	

115/199 Volts: Starting Torque: The starting torque of a 220-volt motor is reduced 18 per cent when a voltage of only 199 volts is impressed. The number of 220-volt ratings that would give satisfactory torque at 199 volts plus or minus 10 per cent ranges from about 50 per cent of the total to a percentage considerably below this.

The factors that would determine whether a single complete line of motors or two separate lines would be preferable are similar to those treated under the heading 120/208 volts. The decision between one line or two lines in this case, however, might lean towards the latter on account of the difference in conditions underlying the comparison between this and the 120/208-volt system. Two of the factors against a single line are the large number of motors that would require rewinding, and the fact that with a single line to operate at 220 or 199 volts the power-factor when 220 volts is used would be possibly 2 per cent lower at full load and 4 to 5 per cent lower at half load. Due to the greater number of ratings involved the expenditure would be much larger than in the case of 120/208 volts.

Characteristic Curves: The differences listed below are taken from the curves of operation in Fig. 3 at 220 volts and 199 volts.

PERFORMANCE AT 199 VOLTS COMPARED WITH 220 VOLTS

	AT 100 per cent of full load	At 50-60 per cent of full load
Speed	0.6 per cent lower	0.3 per cent lower
Power-factor	1.0 per cent higher	4.0 per cent higher
Efficiency	1.0 per cent lower	1.0 per cent higher
Current	10.3 per cent higher	6.0 per cent higher
Pullout Torque	19.0 per cent lower	

Temperature Rise: There is no reason for applying any higher temperature rise motor on 199 or 208 volts than on 220 volts. The customers will probably prefer 40 degree rated motors for general purpose application if the combined systems adopted require the use of 199-, 208-, or 216-volt motors. Should a motor rated at 220 volts be operated at 208 volts or at 199 volts a higher temperature rise may be expected. As considerable variation in the results may be secured and induction motors in general must be considered, only very general statements can be made.

The accepted custom which permits a maximum voltage variation of 10 per cent from the nameplate rating is based on the assumption that when so operated at full load the motor may rise 10 deg. higher than at its normal voltage and rated load. It may thus be assumed that at 199 volts the standard 220-volt 40-deg. general purpose motors will usually operate at close to 50 deg. rise and that at 208 volts they will operate at about 45 deg. rise. These estimates will, of course, vary somewhat with the characteristics of the individual ratings.

110/190 Volts: A decrease to 190 volts from the present standard of 220 volts is very great. In addition, to provide for a further variation of 5 per cent (if this principle be accepted generally) widens the range to a point where difficulties might be encountered with the majority of existing motors. Should 10 per cent voltage variation be maintained as the accepted custom, the conditions would of course be even worse.

If the original guarantees were required, the adoption of 110/190 volts would necessitate two lines and the expense would be so great as to be uneconomic.

3. *Motor Control Equipment.* 220/440 volts is now standard for this apparatus, with allowable variation of 10 per cent plus or minus.

In the past the transformer secondaries of power banks have usually been connected in delta and two overload trip coils have sufficed for full protection of motors. A new National Electrical Code rule permits the use of two trip coils with a fuse in the third phase for motor starters on three-phase four-wire systems with grounded neutral such as are employed in the majority of combined network systems.

This rule may be used as an expedient to make unnecessary the equipping of existing control apparatus with additional trip coils when the secondary distribution system is changed to the combined light and power type. It is reasonable to expect, however, that considerable experience with or study of the combined secondary system for networks will bring about a demand for that form of apparatus which suits the application best.

It is thus justifiable to assume, regardless of the manufacturing costs involved, that extensive application of the combined system will finally result in a widespread demand for three-coil protection in starters for three-phase motors. Already there seems to be a move in this direction by numerous Underwriters' inspectors. This would necessitate rearranging all general purpose motor controller designs to provide the third trip coil, and that would entail a large item of expense.

125/216 Volts: The present standard coils could be used in most cases, but where the voltage is low special coils would be required.

120/208 Volts: 115/199 Volts: 110/190 Volts: Any of these voltages involves adding at least 33 per cent to the number of a-c. coils in stock and the maintaining of a double set of standards since the original standard of 220 volts will still hold for other than network systems. The expense of all necessary changes would be large, especially for 190 volts.

4. *Safety Switches.* 125/250 volts is standard at present for these switches and is the maximum rating.

To comply with the latest safety code ruling it will be necessary to change the construction of the standard triple-pole entrance switches by adding a neutral strap, for such services as will be three-phase four-wire. It is probable that this will apply to all standard sizes.

Where four-pole general purpose switches have been employed on delta-connected circuits, there might be a demand for three-pole switches with solid neutral connection. Motor starters having thermal cutouts may have to be redesigned to include an additional thermal cutout, for the same reasons as given under "Effect on Motor Control Equipment."

The expenditures thus incurred will be considerable.

5. *Distribution Transformers.* 115/230 volts is the standard now. A tolerance of 5 per cent plus is allowed for single-rated transformers. The majority of tranformers are triple-rated at 110/115/120 volts, and 120 volts is the allowable upper limit.

125/216 Volts: When the voltage impressed on a distribution transformer is raised within the limit that results in saturation of the iron, the per cent copper loss decrease is about the same as the per cent iron loss increase. In the sizes from 50 kv-a. up, which generally obtain in the present types of low-voltage networks, the copper loss is predominant at full load but is subordinate to the iron loss at light loads. Therefore, at full load an increase in voltage to 125 volts results in a net decrease in total loss.

Thus, for those networks in which the practise may be adopted of tripping out feeders during light-load periods to save iron loss and in which the transformers will be fairly well loaded throughout the day, heating will be reduced and efficiency improved by an increase in impressed voltage. But where all feeders will be left in service the entire day and the transformers will be lightly loaded for a considerable number of hours, the all-day efficiency will be decreased by voltage increase. In either case, regulation is improved.

The salient effect is on exciting current, which increases about 300 per cent when a 115-volt transformer is stressed to give 125 volts. As this is decidedly beyond the allowable limit for the majority of distribution systems, all ratings would have to be redesigned. The expense of this would be excessive.

120/208 Volts: Standard transformers will operate satisfactorily at this voltage. The exciting current of a 115-volt transformer, however, is still increased considerably when operated at 120 volts. The very commendable effort being put forth at the present time by the operating companies to reduce the exciting current on their lines may make such an increase undesirable. There is also the necessity of allowing for the voltage drop in secondary mains, services, and house wiring. Thus, the possibility that a line of 115/120/125-volt transformers would be required is apparent. The cost of such a change in all ratings would be very large. To this expense might be added that of continuing the present standard line of transformers, due to the probability that conditions on the majority of radial systems might make the new ratings unsatisfactory.

115/199 or 110/190 Volts: No changes would be necessary.

6. *Miscellaneous Apparatus.* This group includes carbon and oil circuit breakers, relays, meters, instruments, voltage regulators, rectifiers, rectigons, fan motors, condensers, and line material.

115/230 Volts, with tolerance of 5 per cent plus, is the existing standard for the majority of this equipment.

125/216 Volts: As 120 volts is now the allowable upper limit for most of the apparatus, much redesign would be necessary, resulting in excessive expenditure.

TABLE VI

EFFECT OF VARIOUS THREE-PHASE SCHEMES ON CLASSES OF APPARATUS

	Classes of apparatus				Weighted according to total sales of each class			
	Scheme				Scheme			
	110/190 volts	115/199 volts	120/208 volts	125/216 volts	110/190 volts	115/199 volts	120/208 volts	125/216 volts
Favorable	No. 4 No. 5	No. 4 No. 5 No. 6 No. 7	No. 2 No. 4 No. 6 No. 7	No. 2 No. 3 No. 4	1 27	1 27 39 8	29 1 39 8	29 4 1
Totals	2	4	4	3	28	75	77	34
Not favorable	No. 1 No. 2 No. 3 No. 6 No. 7	No. 1 No. 2 No. 3	No. 1 No. 3 No. 5	No. 1 No. 5 No. 6 No. 7	6 29 4 39 8	6 29 4	6 4 27	6 27 39 8
Totals	5	3	3	4	86	39	37	80
Net	−3	+1	+1	−1	−58	+36	+40	−46

Note: + Signifies net effect is favorable.
− Signifies net effect is not favorable.

120/208 Volts: With most of the apparatus there would be little serious difficulty in operation, but there would be considerable expense connected with changing the design of fan motors and rectifiers for 120 volts, and static condensers for 208 volts.

The application of condensers is comparatively new and growing rapidly, so that present estimates of the effect of a non-standard voltage on them should be multiplied by a large factor when considering the influence on future applications.

115/199 Volts: For this voltage the expenditure would be the least. The only change might be a new line of static condensers rated at 199 volts.

110/190 Volts: Some 220-volt apparatus could not operate satisfactorily on 190 volts and there would be considerable expense in redesign and extra stock.

7. *Electric Heating Devices.* 115/230 volts is the present standard, with allowable variation of 5 per cent plus or minus.

On a combined secondary system the lower voltage heating units are usually connected between the outside conductors and neutral, and balanced among the three phases as often as possible.

125/216 Volts: Since the upper limit would be exceeded at 125 volts, two separate lines of heating devices would very likely have to be maintained, both for industrial and household uses. The expense involved would be excessive and customers moving from one zone to another would undoubtedly experience trouble.

This voltage, if applied to the enormous number of heating appliances now connected on distribution systems, is so much in excess of their present voltage ratings that great trouble could be reasonably expected because of too high temperature or even burnout of elements.

120/208 Volts: There would be a small item of expense to allow for voltage variation up to 5 per cent above 120 volts in connection with a few types of equipment.

There is an advantage in having 230-volt elements for heavy duty service. In some cases the 230-volt elements in large electrically heated apparatus for industrial use, such as in furnaces and ovens, cannot be operated properly at 208 volts. This may or may not be a serious matter, according to the conditions encountered.

115/199 Volts: 115-volt elements are standard, but the condition described for devices operated at 208 volts applies more particularly with 199 volts.

110/190 Volts: There would be no difficulty with standard apparatus on 110 volts, but it is possible that a new line of 190-volt equipment would have to be developed, with considerable expense attached.

8. *Summary for Three-Phase Combined Systems.*

The study of the three-phase combined system for networks has included an analysis of the effect of the four voltages—all that have been seriously proposed as yet in this country.

The fact that 125 volts, the first voltage studied, has not yet been considered a recognized departure from the accepted standard lamp voltage, coupled with the excessive cost to consumers and operating companies for small motors, transformers, miscellaneous apparatus, and heating devices, would seem to weigh heavily against the general adoption of the 125/216-volt scheme.

The 110/190 volt system voltage of the analysis likewise appears to have serious disadvantages under existing conditions, due to similar excessive expense in connection with general purpose motors, small motors, and motor control equipment.

There remain for further consideration in the three-phase combined system group only the two voltages, 115/199 and 120/208. These will be studied later in the Quantitative Analysis.

Table VI summarizes the effects of the various three-phase voltage combinations on the different classes of apparatus discussed in the Qualitative Analysis. The numbers in the left portion of the table are those applied to the various classes in the Analysis. Basing a comparison between the various schemes only on the summary in the left portion might be incorrect, since the relative importance of each class of apparatus should be given due consideration. Hence, in the right portion of Table VI there has been substituted for each class a number proportional to the total annual sales of that product for the entire industry. The large adverse totals against 125/216 volts and 110/190 volts in this summary substantiate the proposition to eliminate those voltages; and the indicated parity between 115/199 volts and 120/208 volts points to the desirability of studying these two voltages further.

EFFECT OF CHANGE TO COMBINED LIGHT AND POWER SYSTEM FOR TWO-PHASE 115/230-VOLT SECONDARY NETWORKS

This portion of the Qualitative Analysis considers only 115/230 volts, as that is the only two-phase voltage so far suggested for combined secondary systems. Similar to the analysis of the three-phase schemes, it is assumed that this two-phase five-wire scheme would be adopted in all areas that could be networked and that in all other systems the present methods of distribution would be maintained.

1. Small Motors. The widespread retention of three-phase radial systems in addition to the general adoption of two-phase networks would, under conditions of the present, have an important effect on small motors as regards the industry as a whole. This would be due to the considerable expense in the production, distribution, and maintenance of an increased number of two-phase motors to parallel the line of three-phase motors.

2. General Purpose Motors. As with small motors, if all networks were two-phase the production and stocking of two-phase motors would be greatly increased, with a consequent very large increase in motor costs.

For equal amounts of material, that part which is active in a three-phase motor is practically 6 per cent more effective than in a two-phase motor. In performance, the actual results of this difference depend, of course, on the balance struck between the various motor characteristics when designing the machines. In any event, all parties would have to share this loss of 6 per cent in the effective use of materials entering into the construction of apparatus.

Regarding the feasibility of changing three-phase motors to two-phase, the potential should be reduced to 188 volts when a three-phase 220-volt motor is reconnected for two-phase, 220 volts. If operated at 230 volts the power factor would decrease from 4 to 6 per cent depending on the motor speed, and efficiency would increase somewhat, but the motor would generally operate within its temperature rating. Where extra insulation has been provided between phases in three-phase motors, similar insulation is required between phases after reconnection.

3. Motor Control Equipment. Due to the use of a grounded neutral in two-phase five-wire combined secondary systems, four overload trip coils may be required to give full protection, which means the addition of two coils and rearrangement of all controllers for motors intended for connection to networks. A fourth pole must also be added. The combined increased expense in development, manufacture, and stocking would be very large.

4. Safety Switches. A neutral strap and fourth pole must be added where entrance switches will be used on five-wire services; all standard sizes will probably be affected. Four-pole general purpose switches used on lighting feeders may require the addition of a solid neutral connection. Motor starters must have a fourth pole added and it is probable that where thermal cutouts are used two extra cutouts may have to be provided to make a total of four.

The entire extra expenditure would be very large.

5. Transformers. Since the majority of the feeders would eventually be three-phase, transformers would require Scott tap construction. Subway transformers of this type would have to be furnished. As the Scott connection for distribution transformers requires interlaced windings, the design is special.

There is some question as to whether a separate stock would have to be developed. If there were general agreement that these transformers would be used at full rating for other than network installations and at reduced rating in three-phase to two-phase network banks, one line would suffice. However, if the operating companies employing networks should desire to operate the transformers at full rated capacity and the other companies could see no value in the special design, a separate line for each of these groups would have to be carried in stock. The total extra expense would be very large.

6. *Miscellaneous Apparatus.* The main changes would be the addition of a fourth pole on certain oil switches and carbon circuit breakers, and the extra cost of two double-pole network protectors or one four-pole in the place of a triple-pole protector at each polyphase transformer bank. The expenditure involved would be very large.

7. *Electric Heating Devices.* The two-phase system uses devices of standard rating.

8. *Summary for Two-Phase Combined Systems.* The Qualitative Analysis for the two-phase 115/230-volt five-wire combined light and power system thus indicates the possibility of certain disadvantages with regard to small motors, general purpose motors, motor control equipment, safety switches, transformers, and miscellaneous apparatus.

To make it possible to gage how important these factors may be will require some form of quantitative analysis.

Quantitative Analysis

In any comprehensive study leading to a comparison between the most important of the combined secondary schemes thus far proposed, a qualitative analysis has its place. A quantitative analysis, however, may also be desirable, particularly to assist in forming a better conception of the increased cost that might be incurred if any one combination should be adopted for all network systems.

As a step in this direction, Tables II, III, IV, and V were developed. Employing the assumptions used for Table I, these tables offer an approximate measure of the percentage of all apparatus connected to secondary systems that would be affected by each of the three principal combined schemes and also by the radial system. The combined schemes chosen for consideration are the two-phase combination and the two principal three-phase arrangements selected for reasons previously explained in this paper.

The left portion of each table applies to number of apparatus units grouped as phase-to-neutral and phase-to-phase equipment. The right portions represent the percentages of the left parts weighted in proportion to total sales of the phase-to-neutral and phase-to-phase groups.

In Table II, all the three-phase network systems are assumed to be 120/208-volt, but allowance has been made for some two-phase systems still existing. Table III is similar to Table II with the exception that all two-phase systems are supposed to have been converted into 120/208 volt three-phase networks. Table IV gives the reverse condition, all systems operating as two-phase networks except the radial systems. In Table V the network system percentages of Table III are assumed to apply to 115/199 volt networks.

These tables indicate that apparatus to the value of about 37 per cent of the total sales would be different from the existing standard if all equipment in network areas were exactly suited to the 120/208-volt system; and that the 115/199-volt and two-phase schemes would each involve non-standard apparatus valued at only about 8 per cent of the total. As the preference for three-phase appears to predominate among operating companies and manufacturers, these tables show that the 115/199-volt scheme as a common choice for combined secondary systems would probably affect the least apparatus.

The main assumptions on which the Quantitative Analysis was based are listed in Tables VII, VIII, and IX. These tables bring out the changes that may be required in each of the seven apparatus classes of Table VI if any of the three principal schemes should be adopted as the standard for all networks. It was also assumed in this Analysis that the transition to network systems would be completed for the entire country within ten years.

Table X gives the results of the Quantitative Analysis. The relative index numbers represent the approximate expense to the entire electrical industry for small motors, general purpose motors, etc., and the totals summarize these expenses for each of the combined schemes considered.

The operating companies that have applied the combined light and power method of secondary distribution have been prompted mainly by the desire to effect a great saving to the entire electrical industry. Table X shows that if any type of combined secondary scheme is generally adopted for networks there will be a heavy expenditure tending to offset some of the gain. It also discloses that failure to standardize immediately will add further to this tendency.

Table X reveals the possibility that there may be a definite difference between the 120/208- and 115/199-volt three-phase systems, in favor of the latter; and that the adverse effect of the two-phase combined scheme may be relatively much greater than that of either of the three-phase combinations.

Relation of Trends in the Industry to This Problem

The Qualitative and Quantitative Analyses deal with the effect of the combined schemes themselves on apparatus as existing and as required in the future. If the task of choosing a standard scheme is undertaken, other influences should also be considered. For instance, what bearing may the numerous trends in the industry have on the contemplated standard? A discussion of a few of these tendencies will serve to illustrate the importance of giving them proper weight.

Trend of Low Voltages

The trend of system voltages throughout the country appears to have received the most attention among these factors. Both the voltages now existing at utilization devices and the trend of the various voltages should be of value in guiding the choice of a standard scheme. Only a thorough survey can give a measure of the extent to which each voltage is now in use.

TABLE VII
ASSUMPTIONS FOR QUANTITATIVE ANALYSIS
EXPENSE TO ENTIRE INDUSTRY
OVER TEN YEAR TRANSITION PERIOD

THREE-PHASE 115/199 VOLT SCHEME

Class No.	No changes required	Changes required
1. Small motors	Single-phase motors with 2:1 voltage winding would not be developed. Single-phase 110/220 volt motors satisfactory on 115 volts.	Rewind certain single-phase 220 volt ratings for 199 volts: a. Considerable development cost. b. Considerable shop cost increase. c. Some additional expense due to stock increase.
		Rewind certain polyphase 220 volt ratings for 199 volts: a. Considerable development cost. b. Some shop cost increase. c. Some additional expense due to stock increase.
2. General purpose motors	Less than 50% of present ratings satisfactory on 199 volts.	Rewind more than 50% of 220 volt ratings for 199 volts: a. Large development cost. b. Considerable shop cost increase. c. Some additional expense due to stock increase.
3. Control equipment		Add third coil for 3-coil overload protection; redesign all 3 coils for 199 volts: a. Considerable development cost. b. Considerable shop cost increase. c. Considerable additional expense due to stock increase.
4. Safety switches	No change in motor starters that do not use thermal cutouts.	Add neutral strap in all entrance switches; develop 3-pole general purpose switch with neutral strap; add third thermal cutout to certain motor starters: a. Considerable development cost. b. Some shop cost increase. c. Some additional expense due to stock increase.
5. Transformers	This system uses standard transformers.	
6. Miscellaneous apparatus	Relatively small changes required in apparatus other than static condensers.	New line of static condensers for 199 volts: a. Small development cost. b. Some shop cost increase. c. Some additional expense due to stock increase.
7. Electric heating devices	For 115 volts all apparatus is standard. Changes in industrial apparatus for 199 volts would involve relatively little expense.	

One method often used in gaging which voltages will tend to predominate in low-voltage distribution systems is the analysis of the lamp sales in the voltages of the 100-130-volt class. The curves in Figs. 4 and 5, applying to lamps sold during the past nine years, are based on data contained in reports of the National Electric Light Association Lamp Committees.

TABLE VIII
ASSUMPTIONS FOR QUANTITATIVE ANALYSIS
EXPENSE TO ENTIRE INDUSTRY
OVER TEN YEAR TRANSITION PERIOD

THREE-PHASE 120/208 VOLT SCHEME

Class No.	No changes required	Changes required
1. Small motors	Single-phase motors with 2:1 voltage winding would not be developed. Single-phase 110/220 volt motors probably satisfactory on 208 volts.	Rewind certain single-phase 110 volt ratings for 120 volts: a. Considerable development cost. b. Considerable shop cost increase. c. Large additional expense due to stock increase.
	Polyphase 110/220 volt motors probably satisfactory on 208 volts.	
2. General purpose motors	More than 50% of present ratings satisfactory on 208 volts.	Rewind less than 50% of 220 volt ratings for 208 volts: a. Some development cost. b. Some shop cost increase. Note: No stock increase.
3. Control equipment		Add third coil for 3-coil overload protection; re-design all 3 coils for 208 volts: a. Considerable development cost. b. Considerable shop cost increase. c. Considerable additional expense due to stock increase.
4. Safety switches	No change in motor starters that do not use thermal cutouts.	Add neutral strap in all entrance switches; develop 3-pole general purpose switch with neutral strap; add third thermal cutout to certain motor starters: a. Considerable development cost. b. Some shop cost increase. c. Some additional expense due to stock increase.
5. Transformers		Change standard rating to 115/120/125 volts: a. Some development cost. b. Large shop cost increase. Note: No stock increase.
6. Miscellaneous apparatus	Relatively small changes required in apparatus other than static condensers, fan motors, and rectifiers.	New line of static condensers for 208 volts; new lines of fan motors and rectifiers for 120 volts: a. Considerable development cost. b. Some shop cost increase. c. Some additional expense due to stock increase.
7. Electric heating devices	Changes in industrial apparatus for 208 volts would involve relatively little expense.	Modification of some 115 volt ratings for 120 volts: a. Small development cost. Note: No shop cost or stock increase.

Fig. 4* gives, for each year, the percentage of the sales in 115-volt lamps (the recognized lamp standard) as part of the sales in the entire 100-130-volt group, and also similar percentages for 110- and 120-volt lamps (the two recognized departures from the standard).

TABLE IX
ASSUMPTIONS FOR QUANTITATIVE ANALYSIS
EXPENSE TO ENTIRE INDUSTRY
OVER TEN YEAR TRANSITION PERIOD

TWO-PHASE 115/230 VOLT SCHEME

Class No.	No changes required	Changes required
1. Small motors	Single-phase 110/220 volt motors satisfactory on 115 volts or 230 volts.	
	No additional development required for polyphase motors.	Increased polyphase production cost and stock: a. Some shop cost increase. b. Some additional expense due to stock increase.
2. General purpose motors	No additional development required.	Increased production cost and stock: a. Some shop cost increase. b. Large additional expense due to stock increase.
3. Control equipment		Add 2 more overload protection coils and fourth pole: a. Very large development cost. b. Large shop cost increase. c. Considerable additional expense due to stock increase.
4. Safety switches		Add fourth pole and neutral strap in all entrance switches; add neutral strap to four pole general purpose switches; add fourth pole to motor starters, also two thermal cutouts to certain types: a. Considerable development cost. b. Considerable shop cost increase. c. Some additional expense due to stock increase.
5. Transformers		Add Scott tap to all ratings: a. Considerable development cost. b. Considerable shop cost increase. Note: No stock increase.
6. Miscellaneous apparatus	Practically no changes required in apparatus other than oil switches, carbon circuit breakers and automatic network units.	Modifications in oil switches, carbon circuit breakers, and automatic network units: a. Considerable development cost. b. Some increased shop cost. c. Considerable additional expense due to stock increase.
7. Electric heating devices	This system uses devices of standard rating.	

The approximate parallelism of the 115- and 120-volt curves appears to indicate that 120 volts is not displacing 115 volts as the preferred voltage. The slight convergence of these two curves between 1923 and 1925 is probably no more definite indication of a decided change in favor of 120 volts than was the divergence between the curves from 1921 to 1923 a sign that 115 volts would soon outstrip 120 volts.

The curves of Fig. 5 show another way of determining the probable tendency of system voltages. These curves give the total annual lamp sales for each of the three main voltages. In applying them it is assumed that the total number of lamps sold in a given voltage is a measure of the number of customers served at that voltage and hence of the extent to which the voltage is applied.

TABLE X
QUANTITATIVE ANALYSIS
EXPENSE TO ENTIRE INDUSTRY
OVER TEN YEAR TRANSITION PERIOD

Class of apparatus	Three-phase 115/199 volt	Three-phase 120/208 volt	Two-phase 115/230 volt
1. Small motors	19	30	8
2. Gen'l. purpose motors	18	2	26
3. Control equipment	32	32	50
4. Safety switches	5	5	17
5. Transformers	0	21	17
6. Miscellaneous apparatus	1	10	32
7. Electric heating devices	0	0.2*	0
Totals	$75,000,000	$100,000,000	$150,000,000

*Too small to influence total.

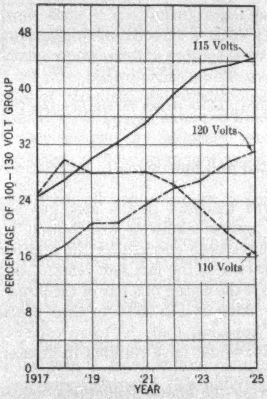

FIG. 4—PROPORTION OF LAMP SALES IN RECOGNIZED STANDARD GROUP TO TOTAL SALES IN 100-130 VOLT GROUP

Up to 1921 the total lamp sales for 120 volts appear to have kept pace with those for 115 volts, with the latter in the lead, but Fig. 5 shows that since 1921 there has been a more rapid upward climb of the 115-volt sales. If the tendencies indicated by these curves become the realities of the future, then it is likely that 115 volts will be the standard for many years to come.

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*Taken from Fig. 3, Page 3, of 1925-1926 Report of Lamp Committee, National Electric Light Association. Publication 256-47.

Furthermore, there appears to be little evidence at present of any trend that may change this situation. Noting the continued decrease in the number of 110 volts lamps sold since 1921, it is interesting to determine whether the companies that are dropping 110 volts may be adopting 120 volts instead of 115 volts. It is difficult to draw any such conclusion from Figs. 4 and 5, for the curves give no indication as to how much of the increase in 115-volt or 120-volt lamp sales is attributable to normal growth of connected load in 115- or 120-volt systems and how much to acquisitions from 110-volt systems.

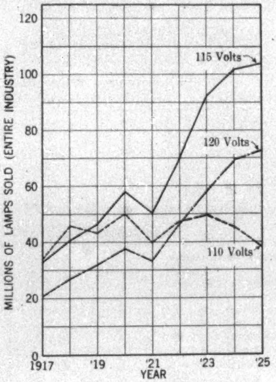

FIG. 5—TOTAL LAMP SALES IN RECOGNIZED STANDARD VOLT GROUP

While the tendency in the past has been towards a steady upward climb of distribution voltage, this seems to have been checked by the approach towards standardization in the last few years. As an illustration, there is the action of a few of the larger syndicates in standardizing on 115 volts for their properties, even though it appeared that the extra capacity available at 120 volts would have resulted in greater return on the investment.

Considering the above deductions, namely: (a) the increase in use of 120 volts will probably not overreach that of 115 volts, while the use of 110 volts is declining; and (b) seemingly, no trend or development important enough to change these conditions has been brought to light thus far; it appears that what had the semblance of an endless process of increase in voltage standards has been halted.

The subject of lamp standards brings to mind the method that was employed when the situation regarding lamp voltages was at least as difficult as is the combined secondary situation today—adoption of a single lamp standard with recognized departures. This precedent should be of value when considering the recommendation for a basis of standardization for combined light and power schemes to be made later in this paper.

GOOD PUBLIC RELATIONS

The maintenance of good public relations is acknowledged by operating companies as increasingly important. System changes are being made in the manner that disturbs the fewest number of customers. Universal adoption of a standard combined system may inconvenience some consumers at the start regardless of the kind of scheme chosen, but taking into account good public relations calls for minimizing the number thus involved.

The motor users that would be affected by any standard combined system constitute a relatively small part of the entire group of customers. The simple means that have been employed to remedy the motor troubles have been pointed out previously herein. Thus, from the standpoint of good public relations, consideration of the other utilization devices would seem to be of more importance.

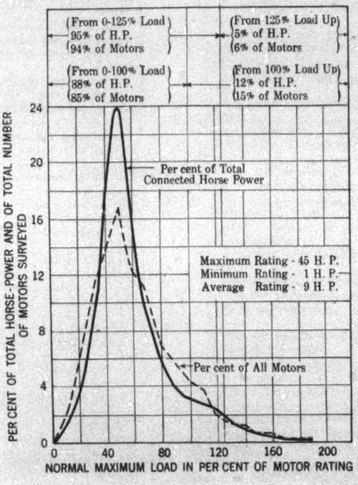

FIG. 6—LOAD SURVEY OF GENERAL PURPOSE MOTORS

STANDARDIZATION OF TRANSMISSION SYSTEM VOLTAGES

The present trend toward standardization of transmission system voltages will affect the ability of many companies to apply a given combined light and power secondary scheme chosen as the standard. This must be given due weight. There are numerous companies operating secondary systems at 110 and 115 volts that cannot raise the potential to 120 volts at peak load, the limit of voltage capacity of all apparatus on their systems having been reached. Usually 115 volts at the utilization device requires 120 volts at the transformer.

The choice of transmission voltage standards should take into consideration the possibility of adopting as the standard for combined secondary systems that scheme which conforms closest with *all* the requirements for a combined standard. These will be listed near the end of the paper*.

STANDARDIZATION ON SYSTEMS FOR DISTRIBUTION

There appears to be a tendency for operating companies to standardize on certain designs of systems for distribution when the concensus of opinion agrees on such designs as the best obtainable.

Thus, almost all direct-current distribution systems have the same pattern, some form of three-wire network at 120/240 volts. The majority of primary systems in cities have been changed from 2300 volts three-wire three-phase and 2300 volts three-wire or four-wire two-phase to 2300/4000 volts four-wire three-phase or something similar. Sixty cycles has practically become the standard frequency for alternating-current distribution systems. Most street lighting systems use series circuits and railway distribution systems are usually 600 volts, d-c.

Standardization on a single system for combined light and power secondaries in network systems would not therefore be a radical departure from existing tendencies.

OVER-MOTORING

Many operating companies favor making use of the present degree of over-motoring where possible, yet discourage its continuance into the future. These tendencies will improve the power-factor and make less costly both the plant required to render power service and the motor installations. To avoid any injustice to customers who have purposely allowed spare capacity in motors for future increase in loads, these operating companies stand ready to install the boosting auto-transformers whenever required.

Fig. 6 is the result of a load survey of all the general purpose motors served by a large underground distribution system. Load concentrations averaging 20,000 to 40,000 kv-a. per square mile prevailed. Every type of motor encountered was tested carefully and it may be said that the results are fairly typical for most of the underground areas where a-c. networks will be applied.

These curves indicate that for the majority of motors and motor hp. in such districts the maximum load is only 50 to 60 per cent of the rating, and that in but 10 to 15 per cent of the cases is 100 per cent or more load encountered. How the power-factor of underloaded motors is somewhat improved by operation at reduced voltage has been disclosed in the Qualitative Analysis.

TWO-PHASE DISTRIBUTION

The original alternating-current system was two-phase. The fact that the use of the newer three-phase system has increased so rapidly in radial distribution practise leads one to the conclusion that it has some important advantages over two-phase. At the threshold of selecting a standard combined scheme for network systems it may be desirable to investigate whether the choice of three-phase would likewise be on a sound economic basis.

In some quarters there appears to be the general impression that two-phase is becoming obsolete, but there is not much information available whereby this may be investigated. A recent survey of motors used in industrial plants* indicates a decrease in the number of two-phase motors by slightly more than 18 per cent in five years. As was brought out in the discussion of a recent Institute paper,† the assumption that approximately 6 per cent of all polyphase motors in use are two-phase may not be far from correct. Combining this rate of decrease with this present status gives only about 2 per cent as the percentage of two-phase motors 25 years from now.

It costs a little more to manufacture a two-phase motor than a three-phase motor of the same nameplate rating. At some time in the future, when two-phase motors are fewer in number than at present, this extra cost may become proportionately so large as to warrant an increase in the price of two-phase motors over three-phase; deliveries of two-phase machines would also be slower. At that time, the combination of these factors may have a very noteworthy influence on the demand for three phase in those areas now supplied with two-phase.

Consideration from all angles makes it appear unlikely that there will be additions to the two-phase group from companies now operating three-phase systems or contemplating new systems. Due mainly to the motor problem, several companies have expressed great opposition to any change to two-phase for networks. Although a two-phase five-wire secondary network with mains on both sides of the street involves only a small increase in total annual charges over a three-phase four-wire network,‡ this difference would be much increased in the many systems that would desire or be compelled to use only one side of the street for mains. In such cases the companies object to the fifth wire even if it can be pulled into the existing ducts.

SUPPLEMENTARY RATING FOR MOTORS

The supplementary load rating of 1.15 that has recently been generally accepted for application to standard 40-deg. general purpose motors is based on permitting operation at 1.15 times the rated full load provided the motor is operated at its rated voltage and frequency.

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*It is also pertinent to take into account the probable choice of an international standard for low voltages in distribution systems. See Report of New York Plenary Meeting, April, 1926, International Electrotechnical Commission, Publication 36, Page 167.

Electrical World, January 23, 1926, Page 206.

†A. I. E. E. JOURNAL, Vol. XLIV, November, 1925, Page 1252.

‡Paper by P. H. Chase. See Bibliography.

This rule has been interpreted as not applying to standard motors operated at either 208 or 199 volts. The same is true for 40-deg. 208- or 199-volt rated motors since these ratings have not been recognized as standard for general purpose motors. Hence, if the combined secondary system chosen as standard should employ a voltage less than 220 volts, it would be necessary to warn customers using motors in network areas to avoid applying the supplementary rating without a load check by the operating company.

It is likely, however, that the motor users in network districts receiving reduced voltage would also want the supplementary rating, as otherwise there would be a 15 per cent differential against their motors merely because they were not located in radial distribution districts. This would make the application of rated voltage to motors more necessary and so increase the tendency towards a change in motor design to fit the 115/199- and 120/208-volt schemes.

It has been previously noted that present lines of general purpose motors would probably require rewinding of almost 50 per cent of the ratings to give the original torque guarantees at 208 volts. If the supplementary rating were also applied, a large majority of the 220-volt ratings would have to be rewound to ensure satisfactory heating and characteristics at 208 volts. For operation at 199 volts, practicaly all of the ratings would need rewinding.

Standard two-phase motors need not be changed to permit application of the supplementary rating, but its use on three-phase motors at 199 volts might call for an entirely new line. A recalculation of the quantitative comparison between the costs of the 115/199-volt three-phase and the two-phase combinations was therefore made on that basis. The two-phase system still appeared to be considerably more expensive than the three-phase, the main reason for this being that general purpose motors constitute only one of six factors influencing the results, as can be seen from Table X.

Since the limiting feature in most applications of small motors, including fan motors, is not heating but torque, it is questionable whether the supplementary rating will even be applied to them.

FREQUENCY OF REDESIGN OF APPARATUS

The strides of the industry in the past seem to have brought about the impression that redesign of apparatus is not only frequent but periodic. It might be thought that these rapid changes will absorb the expense for such alterations in design as would be required if any one of the principal combined systems is chosen as standard.

Apparatus redesigns however, are very irregular in occurrence and are sometimes quite infrequent. In one case there was a lapse of 10 years for general purpose motors and in another a line of distribution transformers stood unchanged for 13 years. Where redesigns occur more often it is generally because individual parts must be modified in successive stages. Seldom does it become possible to combine many of these changes in one general redesign and thus minimize the expense of each individual change.

Even if demands on the part of the public or the operating companies for new classes of equipment should be coincident, it is only rarely that they would apply to the same parts. Hence there would still be the increases in shop cost and stocking, which are the main items of expense that would be incurred in an extensive change to combined light and power secondaries.

Based on this past experience there is little probability that redesigns of apparatus to suit a given combined secondary scheme could be included without increase of cost at the time of some other modification.

Weighing of the Principal Combined Systems and Recommendation of a Basis for Standardization

After determining the effect of each of the principal combined schemes on apparatus connected to the secondary system and the bearing of trends in the industry upon the establishing of a standard, the value of a given choice will depend on the proper weighing of these factors.

It may be well to list the main points to be considered in this regard, without attempting to arrange them in the order of importance.

1. The scheme chosen must involve the least possible expense to purchasers of apparatus affected; these are mainly the consumers.

The thought has often been expressed that it costs more to purchase and maintain the current-consuming equipment than it does to buy the power for operating it. If this is true, it may be even more important to adopt that policy which will result in the least increase in cost of this apparatus and its upkeep than it has been to make the greatest possible savings in the cost of generating and distributing the power.

2. The scheme should give minimum trouble to users and purchasers of apparatus, by conforming closely with present tendencies in the industry, and should not destroy the results obtained by years of effort, without due compensation.

3. It should be the cheapest to the operating companies when considered from all angles. This applies to total annual cost of the entire distribution system, and also to first cost if possible, as based on studies looking ten years into the future. Consideration of the effect of the scheme on future revenue may even require projecting these studies beyond the usual ten year period.

4. The scheme should be an improvement from the operating point of view, by giving the simplest outside plant and the least maintenance.

5. It should be such that non-standard systems can change to it with minimum expense and effort; for instance, in the case of changing two-phase systems to three-phase, or 110-volt systems to a higher voltage.

The completeness with which standardizing for combined secondary systems may be adopted would presumably be affected by the manner in which the plan is carried out. Having determined upon a single standard, its acceptance might thus be greatly helped if consideration were given to any other combination already employed by an appreciable number of companies.

It is therefore recommended that (a) *in determining upon a basis for standardization, that combined scheme be chosen as the standard which is preferable according to the most important of the requirements that have been listed above, and* (b) *if the next best combined system has already received wide application, it be adopted as a recognized departure.*

Concluding Suggestions

To have a complete study of these requirements made as promptly as the importance of the problem merits and to allocate to each its proper relative weight will require the concerted attention of the electrical operating companies. Nor can the manufacturing companies hope to avoid their proper share of any task that may arise as the result of a decision based upon that study. The full cooperation of both branches of the industry will be necessary to arrive at a solution involving the least burden to all concerned.

The following are a few suggestions for action whereby this purpose may be accomplished.

The active interest of the leading men in the industry should be enlisted. Their long experience with such problems and their intimate knowledge of the losses due to lack of prompt action in the past should be brought to bear. Their efforts would result in arranging in the order of importance not only the factors discussed in this paper but also those aspects that have not been brought out.

An intensive effort to improve the design of combined light and power systems for networks, in order to eliminate compromise as far as possible, would be of distinct value. The translator scheme is a praiseworthy step in this direction, but apparently it is not the final solution*.

Agreement among the operating companies on a standard scheme of connection and voltages, with an accepted departure if necessary, is recommended. Steps should be taken immediately to bring about such standardization before too much damage is done by the rapid increase of systems requiring non-standard apparatus.

Cooperative effort will be needed to encourage the active support of this agreement on the part of all operating companies. Experience with other problems shows that this is usually necessary, for at the beginning of any standardization there is often considerable inertia to overcome. A conservative campaign of education on the value of adopting one standard for combined secondary systems would decidedly aid this effort.

The manufacturing companies should make sure that none of their policies tends to promote the use of such schemes and voltages for combined light and power secondary systems as may differ from the standard that will be chosen.

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Appendix A

DETAILS OF NETWORK SYSTEM DESIGN APPLYING TO UNDERGROUND COMBINED LIGHT AND POWER SECONDARIES

Observations and analysis of the operation of combined secondary systems have shown that close attention to the details of system design is necessary to obtain the full benefits of the combined method of secondary distribution for networks. Without due regard to such details, complications detrimental to operating companies, or customers, or manufacturers, or all three, might result from some well-meant effort. For example, a certain part of the network system might be designed to better one condition at too great disadvantage to another part; or too expensive apparatus might be added to permit an economic advantage over d-c. network or a-c. radial systems, where continuity of service is not of paramount importance. Some types of network systems call for equipment that makes operation cumbersome or dangerous; for others, special apparatus might be considered necessary to meet a given local condition whereas a change of system plan would be more beneficial*.

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*One authority describes it as "complicated, costly, and inflexible," although it delivers standard voltages in a three-phase light and power system. For other opinions see A. I. E. E. JOURNAL, Vol. XLV, February, 1926, page 180.

*The desirable characteristics of network design, as distinguished from the necessary features, are discussed in "Evolution of Alternating-Current Secondary Networks," H. Richter, *Electric Journal*, July, 1925.

The following details of distribution system design, applying specifically to three-phase four-wire underground low-voltage networks, are suggested in the light of the present art. Some of these requirements are still controversial among those designing networks.

1. Where extra heavy motor load occurs in the same building with lighting load, the power service should be separate from the lighting service.

2. Service lines carrying power loads should be of ample size, and architects and electricians should be counselled to be particularly liberal in the design of power wiring in buildings.

3. Service lines should be as short as possible to cut down the voltage drop from the secondary mains to the utilization devices.

4. Where motors already installed require full rated voltage for normal operation, auto-transformers or booster starting compensators should be employed. This measure gives the least trouble to customers and the operating company.

There are several precedents for using such auxiliary apparatus. Among these are the small transformers or auto-transformers in street lighting systems to adapt the form of energy supplied to the form required by the lamps, in order to transmit by the most advantageous method. There are also the auto-transformers for serving 2300-volt three-phase motors from 2300/4000-volt star primary lines that have been changed over from 2300 volts delta.

The boosting apparatus employed should be well-built and have ample capacity. To reduce its cost it should be of indoor type and located inside the building where possible. Where manhole location is necessary, valuable space can be saved by mounting the auto-transformers on the ceiling or high up on a wall. When several power services coming from the same transformer bank all require full rated voltage for motors, one auto-transformer may be employed for all such services.

5. All three phase wires of a set of secondary mains should be put in the same duct. Where feasible, the neutral should also be installed in this duct.

6. The copper cross-section of secondary mains should be ample, that is 200,000 to 500,000 cir. mils.

7. Transformer banks should be of liberal capacity. As the saving in transformer capacity by diversity is usually large, some spare capacity to prevent high voltage drops on starting of the larger size motors can be afforded.

8. Care should be taken against spacing transformer banks too far apart.

9. High voltage for feeders (11,000, 13,200, and even 27,600 volt primary lines are being planned) will greatly assist in cutting down the voltage drop on motor starting. By this means one company will obtain a maximum voltage variation not exceeding 2 per cent when a 100 hp. motor starts up anywhere on the secondary network.

10. The copper cross-section of primary feeders should be of ample size, for instance, 1/0 to 300,000 cir. mils. This is inexpensive, due to savings by diversity and the ability to evenly load the feeders at all times.

11. There should always be connected to the network enough feeders to minimize the feeder voltage drop when the largest motor on the network is started. This usually fits in with the proper practise for securing continuity of service on the network on outage of one or more feeders.

12. The system design should be such that the voltage variation at lamps when a motor is started should be not greater than 2 per cent for the best class of service and 3 per cent for the next best.

Discussion

D. K. Blake: Mr. Richter's paper does not attempt to take sides. He tried very hard to avoid that. Some years ago, together with others, I made a similar study and obtained similar results, showing that the 115/199-volt system had the lowest cost, the 120/208 next, and then the two-phase system.

Of course this problem will not be settled in this meeting. It will have to be settled by the N. E. L. A., the Power Club, and the N. E. M. A., but I would like to state my positive convictions along these lines.

I believe, based upon my discussion with various operating engineers and knowing the manufacturers' conditions, that the 115/199-volt system will be the solution to our problem. I am just going to make three statements, without attempting to prove them. They are by no means all of the reasons.

The first is because of the lower cost. The second is because of the large number of consumers unaffected by retaining the 115-volt standard. When I stop and think of the data that were given in the *Electrical World* showing that we have somewhere around 14,000,000 domestic customers, about 2,000,000 commercial lighting customers, and about a half-million power customers, I think that in view of our attempts at good public relations, it is by far better to change things that affect the least number of consumers even though they may have more kilowatt demand and about an equivalent revenue.

The third thing is that I recognize the work that has already been done by the N. E. L. A. in attempting to make the 115-volt system a standard. Now of course if that is the answer, if the committee's work proves that 115/199-volt system is preferable, it means two lines of polyphase induction motors. I know positively that no manufacturer recommends the use of 220-volt motors on systems rated nominally 199 volts.

Now I want to say a word about the practise of using 220-volt motors on 208-volt systems. Suppose you decide on a 120/208-volt system, and use the old arguments of lower power factor and over-motoring, etc., to justify utilizing the 220-volt motor. You then have a system that 110-volt single-phase devices do not fit. I think those arguments are perfectly valid for existing conditions, and during the period of change-over, but I think it is wrong to continue to grow throughout the years without having a system supplying devices that do not fit the system voltage.

Therefore I can see no other answer than two lines of motors.

Now the manufacturers in the past have always, to the best of my knowledge, built additional lines of devices whenever there was a sufficient demand. I have no doubt in my own mind that they will do so in case it is the general opinion that two lines of motors would be desirable.

P. H. Chase: Mr. Richter's paper covers a very broad field and has so many angles both from the manufacturing point of view and also from the central-station public-relations and economic points of view, that it is difficult to make an adequate analysis.

I would like to take the liberty at this time of discussing a few of his assumptions and points of view. His paper, as I understand it, first presents a statement that the expense incurred in suitable utilization equipment may range from $75,000,000 to $150,000,000 even under the unlikely condition of a single combined scheme adopted universally.

Mr. Elden's remarks on voltage standardization could well be read into the record on this paper. It is open to serious question whether we ever can get to a single, universally adopted standard, because each operating company has its own local public policy, financial and construction conditions to meet. It may be true that one system compared with other systems theoretically may show an economic advantage a few per cent better. When that situation is reduced to terms of the existing system, as many of us have done, a change to another system may impair the quality of service, damage public relations, decrease flexibility, and involve expensive change-over costs for at best a small saving, which may not even be realized. This leads one to question whether a figure of $75,000,000 to $150,000,000 spread over a ten-year transition period is really important, compared with the expense that must be incurred to effect the change and with the hazards involved.

Now this figure of $75,000,000 to $150,000,000 on a ten-year transition period means $7,500,000 to $15,000,000 a year. That is compared with what other plan or plans? The paper does not give any figures showing what it will cost to continue with the present standards so far as the utilization of equipment is concerned. In other words, we have no base line with which to compare these figures. It seems to me that there should be further analysis to draw up a bill of the costs of continuing with a diverse standard basis. Certainly it will not be necessary to spend this $75,000,000 to $150,000,000 if the present standards are kept.

Possibly the central-station companies may have to spend money in continuing with the development of existing systems which may not be quite so efficient theoretically as one universal system might be. However, those operating systems will have avoided the very heavy costs, which this paper does not pretend to deal with, of changing over to some other system than the one they now have. The paper does not deal with the extra operating expenses that will exist during the transition period which, on a ten-year basis, are very appreciable. There is also extra investment during transition.

To come back again to this $75,000,000 to $150,000,000, are those gross figures, or are they net figures, after allowing for the expected manufacturing economies that would come from the adoption of a single standard? If these are gross figures, then they are misleading. If they are net, the public in one way or another must pay what seems like a high price for standardization unless it is assumed that in the overall problem there are marked savings over the present diversity of systems.

The paper is admittedly written from the manufacturers' point of view. The magnitude of the potential savings to the operating company of any proposed system over the existing system gets down to a very elaborate analysis. Let us take the X. Y. Z. Electric Light and Power Co. operating in a metropolitan area, covering suburban areas and some rural areas. As has been brought out, by far the largest number of customers are the residence customers, and all their needs can be supplied by single-phase, three-wire service. Power loads may be three-phase, three-wire; three-phase, four-wire; two-phase, four-wire, and so on.

Most of its distribution plant and most of the load is not in the dense portion of the territory. Also, possibly the dense portion of the territory requires underground construction; possibly the local conditions require only 10 or 15 per cent of the low-voltage load to be underground.

Such a company will not determine its system for all its territory from conditions pertaining to a small fraction of the load. Can it afford to start out on a policy that involves asking its customers to have a different type of motor in different portions of its territory? Is that company going to consider seriously, without a great deal of study, changing its miles and miles of aerial secondary distribution and its hundreds of thousands of services to residence customers to conform to a system that may be slightly advantageous from a theoretical point of view only for this central densely loaded area?

I think, as has been mentioned by Mr. Blake, this whole subject deserves and must have extended study by the N. E. L. A. and the Power Club before there can be an overall answer.

H. L. Wallau: Mr. Richter has pointed out to us the enormous expense to be charged against the electrical industry due to the redesigning of utilization equipment if any of the present a-c. low-tension network schemes is adopted. The tentative figures set up by him are appalling.

In Cleveland for over ten years we have been using the three-phase, 230-volt system with the four-wire lighting neutral (Fig. C-2 of his paper) with radial feed. This system was first installed in a new underground district. Later, a similar system was used to replace a three-phase, four-wire, 115/199-volt system, then at least 15 years old. This system has the advantage of using lamps, appliances, motors, control equipment, etc., as now standardized.

Can this system be modified slightly and become suitable for network use? I believe it can, but do not recall that the following scheme has ever been suggested. Referring to the accompanying diagram you will note three primary feeders which may issue from one or more substations and are supposed to have the customary over-current protection at the substation end. I have shown these connected to merely one transformer bank each. The transformer banks are shown connected delta-delta but might be Y-connected on the primary. A network protector is assumed cut into the secondary leads between the transformers and the network. Cut-outs, etc., have not been indicated. The cables forming the three-phase, 230-volt mains are continuous and the transformer banks consist of three units of equal capacity, or of one three-phase unit. The lighting neutrals carrying the out-of-balance current are *discontinuous*, there being one section per transformer bank. Between a pair of 230-volt phase wires there is connected a balance coil at each bank location, each successive balance coil being rotated so as to be across the next successive phase. The middle points of these balance coils are respectively connected to the lighting neutral of their particular section. We now have a system having 230 volts between any two-phase wires and 115 volts between any neutral and its corresponding phase wires. There is also a difference of potential of 115 volts between any two lighting neutrals, hence these neutrals should preferably remain discontinuous but could be made continuous by the insertion of a suitable iron-cored reactance between the junction points. So far we have not grounded this system. A ground is provided by means of a star-connected auto-transformer, wound with 133-volt coils, the common junction point of which is grounded. This provides a difference of potential of 133 volts between phases and ground and 66½ volts between neutrals and ground. If the secondary, three-phase mains are to be arranged for sectionalization at times, it would be desirable to have a grounding transformer for each section. If not so arranged, fewer might be used but at least two should be provided in case of failure of one. The total number would depend somewhat upon the extent of the interconnected network. The balance coils used need be only large enough to carry safely the maximum unbalanced load of any section. They would probably be standardized for any network as of the maximum size required, due to local conditions. In certain specific cases where a large amount of lighting load was connected to one section, as compared with adjacent sections, it would be advisable to select certain buildings and connect their lighting load across phases other than that normally supplying that section, additional balance coils being located either in the street or in the buildings selected.

The lighting neutral should be fused preferably to at least twice the value of the phase wires at both service entrance and in all house circuit panel boards, in order that internal faults might not cause the neutral to open, thereby unbalancing the voltage across the three-wire lighting system. Should a ground develop on a neutral within a building, the neutral fuse would, of course, blow. No ground should be installed on any of the house or service wires but the grounding transformers should preferably have their neutral leads securely bonded to the nearest water system in some adjacent building. I assume that the present network protectors may be used as now designed or with some slight modification; possibly a zigzag, instead of a star-connected grounding transformer, would be required to obtain certain necessary phase relations for relay operations.

Before this system could be adopted, certain changes in the code would probably have to be agreed upon, such as overfusing a lighting neutral in order that the neutral wire size need not be increased. There may be others.

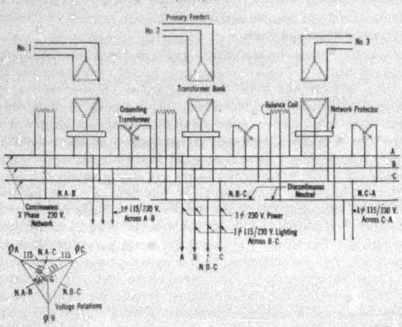

Fig. 1

I advance this scheme as being worthy of study at this time when standardization is still in the embryo. If feasible, I believe that the cost to the industry of additional equipment required, in the way of balance coils and grounding transformers, would be many millions lower than Mr. Richter's lowest estimate of the cost of the required changes in present equipment using the most favorable of the schemes at present contemplated. Perhaps Mr. Richter could contribute a comparative estimate later.

In the time at my disposal between the conception of this idea and the present, I have not thought of any vital objection to it. It may have some very vulnerable features which make it impractical. It should, therefore, be thoroughly checked for any such faults and if none can be pointed out, given serious consideration before being thrown into the discard.

A. H. Kehoe: I disagree with the conclusions which would be inferred from reading the paper, and claim that the 120/208-volt system is the preferable one to adopt. In discussions before the Institute on voltage standardization, it has been stated that 110-, 115-, and 120-volt systems are each being operated successfully throughout the country on a single-phase, three-wire basis. One unfamiliar with conditions might readily think in reading the paper that 120/240-volt supply existed only on some of the older d-c. systems. It should also be noted that items of cost are included for changes in single-phase equipment that is regularly sold by the manufacturers, who guarantee its successful operation for the same service which in the paper is indicated as causing these heavy redesign expenses. Such single-phase equipment, today, as in the past, is designed to operate successfully on 110-, 115-, and 120-volt services. It is difficult to understand how any charges for changes in these conditions will be incurred in time to affect the economics of the situation here under consideration.

The paper states in the qualitative treatment of the 120/208-volt system that there is a possibility of using either a 5-per cent or a 10-per cent maximum allowable voltage variation. The 5-per cent variation is ignored in the quantitative results. This omission is made regardless of the fact that it has been many times stated that the 120/208-volt system operating with a maximum terminal voltage variation of 5 per cent is the only one of the combined balanced light and power systems which can supply all of the standard utilization equipment within the limits covered by existing agreements at which the equipment is guaranteed to operate successfully. Needless to say, such a system is the only one which will avoid the expense which in the paper is estimated in the millions.

It is questionable whether third-leg protection on some motor circuits should be included in the costs, as the industry now uses this and will have to do so with any of the combined systems. Watthour meter costs, on the other hand, are not mentioned, presumably because the meters have been developed and are required in practise today. However, the watthour meter charge is an important one to consider in any three-phase case, and should not be overlooked when considering costs. A fact to note, in this connection, is that for the moment three-element meters cost more than three separate single-element meters.

I am not in accord with most of the statements made on public relations. It is paramount for good public relations that a customer never be inconvenienced from the viewpoint of our service to him, and that equipment which he wishes to buy, sell, or use can be readily utilized. Universal use covers more than equipment; it means service availability as well, on the fifth or fiftieth floor, if need be, without major reconstruction of the system while the consumer waits. The principal objection to the 115/199-volt system is due to public relations, in that it is impossible to guarantee delivery of a terminal voltage at which standard equipment is sold to operate successfully within its rating.

From time to time it has been stated that a special motor of 200 volts will be produced. Such a motor should not be produced unless it can be designed for universal application. However, if one is produced, it will be possible to operate it successfully on a well-regulated 120/208-volt system.

I am of the opinion that no single combined light and power system will be used for some time to come; but if one system should be generally adopted in the next few years, it will be the 120/208-volt, three-phase, four-wire with a maximum variation of 5 per cent in terminal voltage.

W. B. Kirke: I heartily agree with Mr. Kehoe on what he has to say in regard to the 120-volt versus the 115-volt system; I also am definitely on the 120-volt side.

If we look ahead to the future and consider possibly a 10- or 15-year period, we might make the assumption that in that time there will be a possible 10,000,000-kw. demand on network systems. If we ascribe a figure of $20 or $25 a kilowatt to the cost of secondary mains and subways, if it represents underground system, we shall have a capital investment of $250,000,000. A comparison of the kilowatt capacity of the 115-volt system with that of the 120-volt system has to be based primarily upon voltage regulations, and in round figures one is 10 per cent greater than the other. Ten per cent of $250,000,000 is $25,000,000 which you can add as a credit on the 120-volt system. That makes up the difference in cost between $75,000,000 and $100,000,000 as represented in the Table X, on the assumption that the other figures are correct. I seriously question the values as given.

L. L. Elden: In discussing this paper I must confess my inability to analyze some of the prospective savings which are computed for certain types of network systems. In some estimates which were presented recently it appeared that in a single system the cost of extensions during the next ten years was estimated to total something like $100,000,000 for distribution equipment if present methods of construction and supply were followed.

It was suggested that the adoption of the network system would mean a saving of approximately $7,000,000. I wonder if there is anybody here—engineer, commercial man, or otherwise—who will guarantee that over a period of ten years his estimate for such construction will fall within 7 per cent of the actual cost.

Such an estimate must take into consideration the future developments in the art, obsolescence of equipment, changing rates of interest, taxes, and many other contingencies. It is doubtful if anyone here would undertake such responsibility.

It has become increasingly evident that in the maintenance of favorable public relations, we must consider everything that we do primarily from the standpoint of the user and for that reason must carefully review any investment and its effect upon future operating and maintenance costs.

An important feature of the networks under discussion is the propriety of operating motors at subnormal voltages as appears necessary through the adoption of any four-wire, three-phase secondary networks where the lamp voltages range from 110 to 120 volts.

Manufacturers' data on motor operation show very conclusively that operation at the subnormal voltages referred to results in a reduction of efficiency if the motors are operated at anywhere near full load. This method of operation takes advantage of the 10-per cent tolerance factor provided by the manufacturers in their guarantees. From this point of view it may appear that our utilities are within their rights in providing for operating such motors at lower than normal voltages.

As this tolerance factor, however, is not intended to be used in such a manner since it is well known that there will be variations in almost any system voltage below normal voltage, it must be that in most cases the motor will be operated under conditions unfavorable to the customer.

It is unfortunate that this situation has arisen but while it is desirable to construct motors which are capable of operating within a 10-per cent range of voltage plus and minus from normal, it does not appear desirable to recommend the construction of systems on that basis as has been suggested in the consulting engineer's paper presented on this subject.

Considerable time has been spent upon a study of the motor situation in the hope that some development might be suggested to the manufacturers which would provide motors adapted to network operation and at the same time not require a change in present standards.

It appears that the bulk of the requirements will naturally fall in the 115/199-volt class, so that unless some special scheme can be developed, there appears to be a logical need for another motor possibly rated at 200 volts as Mr. Blake has suggested.

An extension of motor windings has been suggested with taps which may be utilized for either 200- or 220-volt operation as a possible solution. Motors are moved from one part of a system to another so that a motor arranged with a combination of taps as above suggested would be suitable for use in any location. If this method of construction is feasible, and I have been told by one designer that there is nothing to prevent such an arrangement being incorporated in motor construction except a slight additional cost, it may be the way out of the situation with which we are contending.

Mr. Blake has very definitely stated that manufacturers do not approve of the operation of 220-volt standard motors on 199-volt service. It appears, however, that manufacturers' representatives are guaranteeing such motors for operation on the lower voltages without hesitation. This appears to be a most undesirable procedure and in the end will be very destructive of any efforts which may be made to secure a definite standardization of motor ratings.

It is to be hoped that out of this situation something will be developed in the form of a motor product which will be universal in application and leave us free to develop networks at will without detriment to other interests.

D. K. Blake: It is by no means out of the question to have a motor that will operate successfully, a universal motor, on 199 or 220 volts. It may be a little difficult to do, but it seems that a great deal can be accomplished by a combination of parallel-*Y* and series-delta. I would merely suggest something like perhaps parallel-*Y* 195 volts, series-delta 225 volts. Not all motors can be built that way easily, but a large number can.

P. H. Chase: I would like to ask whether a motor of that type will run into considerable extra expense? It means heavier coil cost and it may affect the frame.

D. K. Blake: It is my understanding that it will not run into a very heavy expense except on some sizes and speeds. I can't say definitely because that has not been pursued far enough, but there is that hope.

W. B. Kirke: I would like to add one more point. In Brooklyn we have the 120-volt service standard. We attempt to keep that voltage within the limits of 116 to 124 volts.

Our complaints on utilization equipments connected from line to neutral due to 120-volt standard are practically nil. It may have some bearing upon the costs which Mr. Richter has given on rewinding of 120-volt parallel-connected or 220-volt series-connected motors. The network system that we intend to install will take care of loads up to 10-kw. demand from line to neutral. This will take care of the great majority of small motors. Above that capacity we expect to serve on a four-wire basis.

Our distribution transformers are operating above the 120-volt maximum rating. I should say 60 per cent of them are operating close to 124 volts or above, or rather 60 per cent are operating above 120 volts, and 40 per cent are operating below.

I would like to make one other reference to Fig. 4 showing the proportion of lamp sales. I believe the figures for 1926 on the 120-volt group are about 35 per cent, and on the 115-volt group approximately 47 per cent. This shows an increase of approximately 4 per cent during the past year at 120 volts and closer to a 3-per cent gain on 115 volts.

The parallelism is, therefore, converging, and there seems to be a very good economic justification for the increase in 120-volt service.

P. H. Chase: I would like to ask one specific question. These figures of $75,000,0000 to $150,000,000 stick in my mind very definitely. Mr. Richter explained that those figures were the net figures. In a ten-year period there would accumulate $75,000,000 to $150,000,000 deficit that somebody has got to pay. I don't imagine that the manufacturing companies will absorb that kind of a deficit out of profit and loss account. It will be passed on.

How many years after that ten-year transition period will it take to make up that deficit? In other words, I am asking in another way, what is the gross figure? After we have spent the $150,000,000, there must be some economies resulting at the end of the transition period that are going to pay back, we would hope very shortly, the money we spent.

I am much interested in that figure and how long it is going to take. If it is 25 years we must have a great deal of hope.

M. T. Crawford: (by telegraph) I suggest for discussion that consideration be given to the delta system, Fig. 2c, as it permits the supply of full normal voltage to all utilization equipment which will become more necessary with increased use of heating devices. Seattle has had multiple primary feed low-voltage networks in operation six years with success, using this delta system for combined light and power on recent work. We have no difficulty in balancing phase loads on primary feeders.

H. P. Seelye: (by letter) One cannot help but agree that the adoption of one system as a standard would be very desirable, if possible. It usually occurs, however, that standardization follows considerably behind utilization and is accomplished for the purpose of bringing order out of chaos but after the chaos is pretty well established. It would appear somewhat doubtful if the use of combined secondaries has yet reached such a point as to make a general agreement on a single standard possible, no matter how desirable it may be. A comparatively small percentage of the industry is using such secondaries as yet although consideration is being given to the subject quite universally. A satisfactory generally accepted standard cannot be impressed on such a situation but will come only after a wide experience with all the variations points to one type as most desirable.

The present trend seems to be quite generally toward the adoption of a Y-connected, 4-wire, three-phase scheme at either 120/208 volts or 115/199 volts. There seem to be enough arguments in favor of both these voltage combinations to make it quite certain that neither one will be universally accepted for some time. Companies will probably choose the voltage which corresponds best with their present standards and those which they will use elsewhere on the system (the combined secondaries will in most cases form only a part of the total secondary system). This might point to the adoption of one as a standard and the other as an accepted departure, as Mr. Richter suggests, but aside from the probable controversy as to which will be the standard and which the departure, can the desired result be gained by such a standardization? The two voltages are far enough apart so that most of the changes noted by Mr. Richter for both systems would be necessary. The manufacturer would be bound to furnish apparatus both for the standard and the accepted departure. A possible solution might be a compromise between the two, say 117.5/203.5 volts for apparatus which would be only about 2 per cent away from either utilization voltage.

Regardless of which voltage might be accepted as standard, there will no doubt be a demand by motor users for motors in the 200-volt range. This demand will probably not be entirely satisfied by 220-volt motors either with or without supplemental ratings or understandings as to reduced allowable voltage variation. It will be met by some manufacturer by 200-volt motors. It would seem the best practicable solution to accept the fact that there probably will be systems at both 115/199 and 120/208 volts (as well as at 115/230 volts) and to develop a line of apparatus, if possible, which will be suitable for both voltages with satisfactory rating and guarantees.

H. Richter: The question was raised by Mr. Chase as to whether the diverse conditions in the various distribution systems will allow the application of any standard for combined light and power secondaries. I wish to call attention to the fact that when standardization of frequencies was suggested many years ago, and of lamp voltages more recently, similar doubts were raised. Time has shown that in the general good of the industry the numerous local objections were relinquished. Basically, the conditions treated in the paper are the same as were those of the frequency and lamp-voltage problems.

There seems to be a doubt that the cost of continuing the present unstandardized conditions would exceed the total of $75,000,000 to $150,000,000. An approximate analysis made a short time ago gave a sum greatly in excess of $150,000,000. The assumptions included the usual process of developing equipment exactly fitted to each of the different systems.

The total expenditures given in Table II, incidentally, are net. In pointing out that the paper omits consideration of increase in investment and operating costs due to incidentals during the ten-year transition period, Mr. Chase lends support to those parts of the conclusions that suggest a comprehensive study by the leading men of the industry. His question as to how soon the huge totals against the combined system may be cancelled by economies introduced by this system can also be answered properly only by such a study.

I fail to see wherein there would be undue difficulty in standardizing on one combined scheme for secondary networks because of objections to operating in the congested area of a city a system differing from that in the remainder of the city or to changing the rest of the distribution system to conform with the network. What of the numerous d-c. underground systems now surrounded by extensive a-c. overhead systems? At least, with the Fig. 2F scheme, many of the motors can be used interchangeably on both radial and network systems. In one large city there are 120/240 volts direct current; 120/208 volts, three-phase, four-wire; 115/230 volts radial alternating current; and 115/230 volts, two-phase, five-wire. Of course, this is ideal but it shows what is done in practise for expediency.

Mr. Wallau's suggestion, like J. C. Parker's translator system, is another of those admirable attempts to derive a combined scheme for three-phase networks which I hope will eventually result in eliminating compromises with existing standards. It should be given due consideration, but already I see some of the vulnerable features that he seemed to feel impending as he closed his remarks. The necessity of leaving the neutral wires ungrounded does not meet the National Electrical Safety Code requirement to ground the neutrals at all services. The advantage of a thoroughly grounded solid neutral network over the entire system is also lost. Similar to the translator scheme, Mr. Wallau's suggestion involves that added complexity in the distribution system due to auxiliary apparatus which is directly opposed to the simplicity of the Fig. 2F scheme and may therefore be undesirable to the operating companies.

Mr. Kehoe objects to those assumptions in the paper that provide for changes in phase-to-neutral apparatus to suit the 120/208-volt system. Surely we cannot ignore the fact that on a nominal 120-volt system the apparatus must be guaranteed for at least 120 volts plus or minus 5 per cent. It is incorrect to think that the guarantees on all standard equipment are covered at these limits, which are 126 to 114 volts. This would ignore the electric heating devices rated at 115 volts plus or minus 5 per cent, or 121 volts maximum; miscellaneous apparatus, including fan motors, rectifiers, static condensers, etc., with the same rating; and distribution transformers rated at 110/115/120 volts, that is, 120 volts maximum.

It was even considered necessary to assume 10 per cent plus or minus for the limits on small motors, general-purpose motors and motor-control equipment. The reason was that this is the only standard that has been definitely agreed upon. It can be readily understood that such limits may not suit metropolitan network systems maintaining very close regulation at utilization devices. But the guarantees are formulated according to the requirements of the majority of the systems in the country. For years the apparatus connected to radial systems in large cities has similarly had the same limitations as equipment for outlying towns and villages.

Likewise, it is probable that the combined secondary scheme acceptable to the majority of companies operating networks may not be applicable to skyscraper services as well as to small stores and apartment houses, without some modification.

Mr. Kirke claims that the 5-per cent economic advantage of 120 volts over 115 volts will cancel the difference between the $100,000,000 expenditure for the 120/208-volt system and the $75,000,000 for 115/199 volts. In view of the decision of the industry to standardize on 115 volts for lamps and the indications that this decision is being put into effect, it would seem that the sum total of the disadvantages of going to 120 volts for phase-to-neutral apparatus must outweigh all the advantages.

In neglecting to be governed by this gain of 5 per cent, important consideration was apparently given to the condition illustrated by some figures published in the *Electrical World*. These state that the application of 110, 115, and 120 volts to residential customers is in the approximate ratio of 76 to 37 to 17.

The Europeans appear to be swayed by this economic factor and not only carry the process to its logical conclusion by using the 220/380-volt combined system but also point to the undue conservatism of our 115/230-volt separate light and power system.

Mr. Kirke thinks that the 1926 lamp sale totals will cause the 115- and 120-volt curves in Fig. 4 to converge further. It may be too early to make this prediction, as it is quite possible that there will be no further convergence. Furthermore, the 120-volt curve is not conclusive. No effort has been made to differentiate between the lamps on d-c. systems and those on a-c. I have obtained data from a reliable source that show that as of January 1, 1926, the percentage of domestic lighting customers using direct current to those served with alternating current was: for 110 volts—0.33 per cent; for 115 volts—0.25 per cent; and for 120 volts—25 per cent. Applying these corrections to the curves in Fig. 4 results in dropping the 120-volt curve considerably below the position shown. It is thus apparent that 115 volts is decidedly the standard at the present time.

If a standard combined system were chosen in conjunction with a recognized departure, Mr. Seelye believes the development of apparatus with guarantees for the departure would come about. I think this unlikely. At present, 120 volts is a recognized departure from 115 volts, yet the majority of electric heating devices, distribution transformers and those devices listed in the paper as miscellaneous are still rated at 115 volts plus or minus 5 per cent.

In the discussion of standardization of transmission voltages, mention was made that there had been omitted a review of the conditions that had brought about certain of the off-standard voltages. With these voltages so firmly entrenched, it certainly appears to be a difficult task to choose a series of transmission voltage standards satisfactory to the majority of systems. This situation is one more eloquent plea that the small differences here and there in the combined light and power secondary problem be reconciled as quickly as possible. There will thus be no past history to reveal the huge expenditures that might have been prevented by the adoption of a standard system at this time.

A New 132,000-Volt Cable Joint

BY DONALD M. SIMONS[1]

Member, A. I. E. E.

Synopsis.*—This paper describes what is believed to be a new form of high-voltage joint. The main novelties in the joint are that the metallic union of the conductors is insulated by wrapping on a single sheet of wide, impregnated paper by machine. The ends of the cable insulation are cut into a series of steps, or a taper, and knives on the machine cut the wide sheet of paper exactly to fit the steps or taper as the wide paper is being applied, until a smooth cylinder is built up to the original diameter of cable insulation. At this moment, the knives are removed, and at each end of the wide piece of paper, strips of tinfoil which have previously been cemented to the paper appear, and these strips gradually taper inward so that as the wide paper is applied, a flaring cone of metal is formed in the body of the insulation itself. This metallic cone acts as an electrostatic screen to control the longitudinal and radial stresses. It is formed automatically without any attention on the part of the splicer in the field, and it insures that all the insulation under stress is solid laminate paper insulation of the highest quality and breakdown strength, especially in the regions where the diameter is enlarged from that of the cable sheath to that of the joint sleeve. Test results are given.*

I. The Problem

THE problem to be faced (and our description will deal in terms of single-conductor joints only), can be explained in Fig. 1, in which we have diagramatically shown two single-conductor cable ends, the conductors of which have been mechanically joined by connector No. 1. For any single-conductor joint, the first problem is to insulate the region of No. 2, in order to build up the insulation over the connector to the diameter of the original cable. The classic methods of preparing the cable ends are to pencil (or bevel) the insulation as shown in the drawing, or to cut the surfaces into steps.

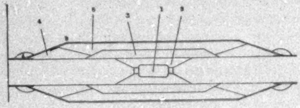

Fig. 1

When this is done, additional insulation must be applied, and this is shown as made up of insulation No. 3, which is ordinarily applied in tape form. In place of this, tubes could be used, or a combination of tape and tubes. The metal sleeve is then passed over the insulated union, wiped to the lead at each end, and the whole is filled with compound. This is sufficient for the lower voltages.

As voltages go up, however, a new point of weakness appears, due to the concentration of stress at the point where the diameter of the outer electrode changes, under the slope $A B$ in Fig. 1. For medium voltages, it becomes necessary to bell out the lead sheath, thus reducing the stresses at this point; and in fact joint casings have been designed so as to be practically a continuation of the bell of the lead, and act, themselves, as a flaring of the lead sheath. This, however, did not remove all the difficulties. As voltages became higher, it was found that the oil or compound in the region where the lead sleeve tapers under the slope, $A B$, would be overstressed, break down, and would lead to eventual breakdown of the cable at that point, or to a surface arc from the connector No. 1.

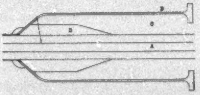

Fig. 2

This phenomenon is rather an interesting one, and is apparently due to the following: A liquid insulator has the characteristic that its breakdown strength decreases with thickness. Of course the breakdown voltage (in volts) will increase the thicker the layer of liquid, but its specific strength (in volts per mil) will decrease. A certain voltage is impressed between the conductor and the outer electrode, which, in the cable, is the cable sheath and in most of the joint is the cylindrical surface of the joint sleeve. In the region, $A B$, however, there is a taper, and proceeding from A to B, more and more of the voltage is impressed on the oil or compound in the joint, and less and less on the factory-made insulation of the cable. As we proceed from A to B, however, the thickness of the oil which is under stress becomes greater and greater. From A to B, the voltage impressed on the oil becomes greater and greater, and yet its breakdown *strength* becomes less and less. A point is reached therefore at which the oil becomes overstressed and breaks down, and this leads to failure of the cable. The author once had a striking illustration of this principle in developing some extra-high-voltage cable terminals. The occurrence is illustrated by Fig. 2, which represents the metallic end-bell at the bottom of a terminal. A is the conductor, B is the end-bell. The end-bell was filled in the space C with oil, and of course breakdown would have taken place in the region where the taper of the end-bell approached the cable surface had we not taken some precaution to relieve the condition. This region, therefore, was wrapped with saturated fibrous material in the mass marked D.

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[1] Standard Underground Cable Co., Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

The diameter of this wrapping could not be increased further as we desired to do because of a projecting flange at the right of *B*; but we hoped that we had enough there to relieve conditions. Voltage was applied, and breakdown took place as shown by the dotted line. In other words, the puncture took place through an inch of cable insulation, about 1½ in. of saturated fibrous material, and about 1 in. of oil, rather than in the cable itself where we had merely the 1 in. of insulation.

The method of preventing breakdown in the oil under the tapering sleeve consists in general in filling this region with solid material, such as paper tape, V. C. tape, or impregnated candle wicking, shown as No. 4 in Fig. 1. The reason that these materials are effective is twofold; in the first place, these saturated materials have a higher S. I. C. than the liquid oil, and thus the actual voltage to be withstood is less; and secondly, the insertion of these materials in effect splits the oil up into thin layers, and we thus get away from the reduction in breakdown strength of oil in thick layers.

The question is, how to apply this principle. In one well-known and successful joint for high voltages, this was accomplished by building up a tapering surface of candle wicking at each end of the joint, the sleeve being split in the middle in a plane perpendicular to the axis of the cable. These two half-sleeves could therefore be brought up against the candle wicking, and since this is more or less flexible, it would take the shape of the tapering surface, *A B*. There are two objections to this method, though it has been entirely successful for the purpose for which it was designed. The objections are the time required to apply this wicking, which is considerable, and the fact that paper tape and V. C. tape have a higher breakdown strength. If it is desired to fill this region with tape insulation of paper or V. C., this also takes a long time and the tape has the disadvantage of being very difficult to apply to a given curvature, and when it is applied there is very little flexibility, and it is thus practically impossible to make the curvature at the ends fit the surface *A B* which may have any desired curvature; the tapering surface of the wrapped material must therefore be metallized in some form, up to a certain distance, which again is a difficult process in the field.

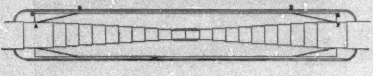

FIG. 3

It will be seen, therefore, that in a very high-voltage joint not only must the regions No. 2 and No. 3 of Fig. 1 be insulated, but steps must be taken to insulate the regions under the tapering section of the outer sleeve *A B*, namely the two regions No. 4. The taping by hand of the regions No. 2, No. 3, and the masses of insulation No. 4 at each end of the joint, and the application of metal to the outer surface of the latter sections, is something which requires time, the building up of large masses of insulation by hand application of thin tape being a slow process.

II. THE NEW JOINT

In the new joint which we will now describe, the insulations in No. 2, No. 3, and the two regions No. 4, as well as the metallization, is all done by applying *one sheet of insulation*, and this is applied by machine. The joint is shown in Fig. 3; the length of the joint is about 40 in., the distance from edge of sheath to edge of sheath being 38 in., and the inside diameter of the lead sleeve being 5 in., the length of the applied insulation *C E* is 35.5 in., and its thickness *E F* is 1 in.; the conductor diameter and cable insulation thickness are each about 1 in. An ordinary connector is used, and a few layers of ordinary hand-applied tape and saturated twine are applied over the beveled ends of the connector, filling up the lowest step until a smooth cylinder is obtained. From that point on up to the surface *C E*, a long roll of paper the full width of the joint is wrapped around, this being one continuous operation with one piece of paper shown diagramatically in Fig. 4. This long sheet of impregnated paper, which is 165 ft. long and 35½ in. wide, could be cut in advance to fit the steps, as shown in Fig. 4. Actually, we cut it by machine *as it is being applied*, thus making a very perfect fit. Referring to Fig. 4, lines *G H* and *I K* have been metallized (as will be described later) and also the region *H K L M*. It should be pointed out that Fig. 4 is quite diagramatic, as in the actual roll the ratio of length to width is about eleven times as great as in Fig. 4. When, therefore, this wide roll of paper is wrapped around the joint in its final position, it will be obvious that there will be formed in effect a solid tube of paper, containing metal cones. *A B* and *D F* in Fig. 3 are the metal cones imbedded in the tube, and the entire outer surface of the applied insulation *B D* will also be metal-covered due to the metallized region *H K L M*. After the wide roll has been applied, the usual metal sleeve is applied, which may be of any diameter desired as long as it clears the line *C E*, and then this sleeve can be filled with compound which has low dielectric loss. It may be a hard compound if desired, so that the joint itself will require no maintenance at all, in view of the fact that

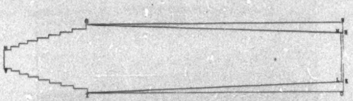

FIG. 4—LONG, UNROLLED, SINGLE SHEET OF IMPREGNATED PAPER, UGNOV, USED TO INSULATE THE JOINT OF FIG. 3 (TO VERY DIFFERENT SCALE) THE REGION TO THE LEFT HAS BEEN CUT SO AS TO FIT THE STEPS OF THE JOINT WHEN THE SHEET IS ROLLED AROUND THE JOINT. THE LINES *G H* AND *I K* ARE THIN STRIPS OF TINFOIL CEMENTED TO THE PAPER. THE REGION *H K L M* IS COVERED WITH TINFOIL.

the compound is entirely shielded from stress in the joint. There is no problem of a suitable jointing compound for this joint.

As to the details of applying the wide roll of paper to the steps, and the latter part containing the metal, this is done by the machine shown mounted in place in the manhole, Fig. 5. The picture was taken actually on the laboratory wall, but the stanchions are actual stanchions used and the spacing between joints is the same as that used in the case of a Philadelphia company's 75-kv. cable. The cable ends can be seen sweated together by the connector in the middle, with the insulation cut into a series of steps. At the top of the machine is the wide roll of metallized paper, and on the bottom rods of the machine can be seen the knives for cutting the paper to fit the steps; also the two V-blocks which bear against the steps and hold the cable central in the machine against the tension of the wide paper. Fig. 6 shows the process after the steps have been filled with paper, the full-width paper being wrapped on. The strips of tapering tinfoil can be seen at each end. The roll of wide paper is mounted on the drum and the drum revolved around the cable, feeding off the wide paper as it goes. The paper passes in and out of alternate rods and is put under tension, adjustable by the number of rods used, by the course of the paper on the rods, and by the location of the movable rods. Tension is also applied by pressure on the roll of paper.

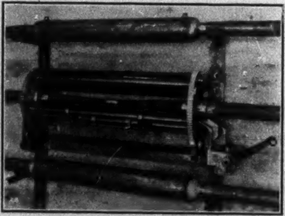

FIG. 5—MACHINE READY TO WRAP PAPER INSULATION IN STEPS

The two cable ends are stripped to have enough conductor exposed at each end to permit their being jointed together by a copper tube which is sweated to both of them. The lead sheath is then removed for the required distance from each end and the necessary number of steps are cut in the cable insulation by hand, though we have planned to have this also done by the same or another machine. The jointing machine is then clamped onto the cable, and in its latest form it is supported by brackets which are held by the same vertical racks that hold the cable hangers. After the steps have been cut, saturated twine and a few layers of paper tape are applied over the connector to build it up to a smooth cylindrical surface at about the level of the lowest step. In another form, no twine nor tape is used. Perforated tinfoil is then applied to the cable insulation for a few inches at each end adjacent to the lead sheath (*i. e.*, from the sheath to slightly beyond A and F respectively), and grounded to it so as to continue, electrically, the lead sheath out to the point where the tinfoil cone will eventually be formed. The foil is perforated so as to offer an easy path for the flow of fresh compound from the joint (and its reservoir, if used) to the cable insulation.

Then the wide roll of paper is put on the machine and the two small cutters are set on one of the tie-rods directly opposite the shoulders of the first step. The paper is drawn through the cutters by hand to cut a few inches of the right width, and this is fastened to the hand-applied tape around the connector. The machine is then started, being driven either by motor or by hand. As the paper is rolled onto the cable, the two knives cut it exactly to the width of the first step, the spare paper at each side being cut off and thrown away as it accumulates. When the space between the first steps is filled up, the cutters are moved out to the shoulders of the next step and the paper is cut to fit there, and so on, until the paper has been wrapped up to the diameter of the original cable.

At this point the tinfoil strips appear on the sheet of paper (points G and I of Fig. 4), and are wrapped over the tinfoil which extends out from the lead sheath at each end. The knives are removed from the machine and the machine revolves, wrapping on the wide roll of metallized paper continuously until it forms the completed tube, the metallized portions taking care of themselves. Compound is poured onto the wide roll at each revolution. The tension is maintained steadily, and thus the final result is a mass of solid insulation, free from air and of the highest quality.

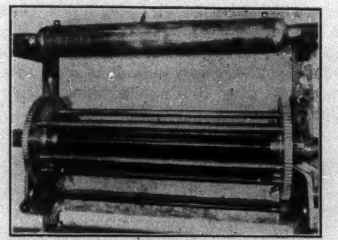

FIG. 6—SHOWING CABLE AFTER STEPS HAVE BEEN FILLED WITH PAPER AND FULL-WIDTH PAPER WRAPPED ON

The fundamental requirement of a joint is high breakdown strength, and the test results will be given later. We have, however, in this joint the fortunate combination of high breakdown strength with excellent and simple mechanical qualities. It will be seen that the whole operation is extremely simple, only one article being applied to insulate the entire joint, and the difference in time between this method and the method of hand or machine taping is very great. For instance, in an actual field installation in a rather small manhole and with a crude and rudimentary form of machine, the entire process of applying the metallized tube, including cutting the paper to fit the steps and wrapping it on the steps, was done in a minimum time of 1 hr. and 2 min., and a maximum time of about 1 hr. and 30 min. In the field, with the final design of machine, 75-kv. joints have been insulated in as short a time as 20 min. This means that the entire insulating of the joint with the exception of one or two layers of hand-applied tape over the connector was done in these respective periods of time.

One additional feature should be emphasized: The first joints made were somewhat different. Narrow tape used to be applied to the stepped region by hand and then the metallized tube was applied from that point on, making a much longer process because of the time required for the taping. It is extremely difficult to start a sheet of paper as wide as 36 in. without entrapping air, and there is always some curvature of the cable core, which greatly increases the trouble. In fact, the only reason good results were obtained was because even though air-gaps may have existed when the wide paper was first put on, the later tension applied squeezed out the air and compound and straightened the cable. The author thinks that it would be almost impracticable, however, to attempt the use by hand of any roll of paper wider than the 3-ft. When wide paper is applied to the steps by machine, however, the paper roll is essentially a straight cylinder after its application. It will be seen, therefore, that as it is applied, the cable core is straightened automatically and gradually, as it is held central against the tension of the paper by adjustable arms on the machine. As soon as the paper has filled up any particular set of steps, that region is straight, and it is believed that the method could be used for almost any length of joint which need be considered, because the cable core will be straightened, step by step, and when the final metallized tube is to be applied, a perfectly straight cylindrical surface will be available.

III. TEST RESULTS

In developing this joint, there have been made and tested over 100 of the metallized tube joints. The first 60 joints tested were in the form of short, straight pieces of cable, there being only about 1 ft. of lead between the wipe of the joint and the temporary test terminal, this straight sample being tested in oil. From that time on, due to the difficulties with temporary terminals for the higher voltages, we have jointed two 15-ft. sections of cable, bending the cable into a U, and applying complete out-door porcelain terminals to each end.

The joint is of such a type that its breakdown strength for voltages applied for a short time will be considerably above its long-time breakdown strength. For that reason, most of our tests have been at voltages which the joint could maintain for a period of hours. It hardly seems worth while to present a tabulation of the tests on all the hundred odd joints made, as this would involve details of their construction, a great deal of which would not be of permanent value, due to changes later made in the joint. The author will therefore merely give the tests on the final design of joints, after numerous small modifications had been made in view of his earlier work. Table I gives the test results on all of the joints of the final design of cable insulated with 30/32-in.

After completing this series of tests, tests on cable insulated with 24/32-in. were started and it was found that the breakdown strength of the joint was by no means as good, the failures invariably occurring in the joint rather than in the cable. After a series of experiments, it was found that with the higher stresses due to the thinner insulation, it was necessary to make the change in the field less abrupt as the diameter enlarged in the joint. The angle between the cone of tinfoil and the axis of the cable in the joint described above was 11 deg. For a cable with 24/32 in., excellent test results could be obtained if the slope was decreased to 7 deg., and Table II gives a tabulation of the tests made on cable with thin insulation and modified slope of foil. For purposes of convenience, since the tests took such long periods of time and tied up the whole laboratory, it was decided to accelerate the tests by keeping the voltage at 200,000 for 6 hrs. only, and then increasing

TABLE I
EXPERIMENTAL JOINTS
SINGLE-CONDUCTOR CABLE INSULATED WITH 30/32-IN. PAPER

Test voltage	Time		Location of failure	Remarks
	Hrs.	Min.		
200 kv.	28	0	Joint	
200 kv.	23	27	Cable	
200 kv.	31	50	Cable	
200 kv.	30	8	Joint	
200 kv.	15	38	Cable	
200 kv.	8	25	Cable	
200 kv.	17	40	Cable	
200 kv.	16	28	Cable	
200 kv.	30	30	Cable	
200 kv.	48	..	..	Test discontinued
200 kv.	40	17	Cable	

TABLE II
EXPERIMENTAL JOINTS
SINGLE-CONDUCTOR CABLE INSULATED WITH 24/32-IN. PAPER

Time of test					Location of failure
200 kv.	220 kv.	242 kv.	266 kv.	293 kv.	
6 hr.	1 hr.	1 hr.	0		Terminal
6 hr.	1 hr.	41 min.			Terminal
6 hr.	1 hr.	1 hr.	41 min.		Cable & joint
6 hr.	1 hr.	1 hr.	4 min.		Terminal
6 hr.	57 min.				Joint
6 hr.	1 hr.	1 hr.	1 hr.	6 min.	Cable

the voltage 10 per cent per hour. Unfortunately, it was not practicable to make a series of tests to tie together the two tables and make them directly comparable.

All voltages mentioned in this section are 60 cycles, a-c. (r. m. s.) the voltage being measured by a crest voltmeter, checked by a 50-cm. sphere-gap with the load on, and are between the conductors and lead sheath of a cable.

In addition to this experimental evidence, six of the joints have been in service since February 1926 at 75 kv. in Philadelphia, at the end of a cable line and where it connects to an overhead line about 40 mi. long, where the joints are thus exposed to all incoming voltage rises. One hundred forty of these joints have been in service also in Philadelphia since October 1926, and it is planned to use this joint on three of the experimental lines of 132-kv. cable shortly to be installed.

IV. METAL-BEARING PAPER

The principle embodied in the long sheet of paper which when applied forms a cone of metal, has a wider application than merely to joints. It may be applied wherever it is desired to reduce stresses by enlarging the surface of an electrode and at the same time to insulate this enlarged surface. It has an obvious application in connection with terminals, either temporary terminals or permanent terminals, for cables, or in connection with bushings in general. One form we have tried out is illustrated in Fig. 7, where the lines of metallization of the paper and a cross-section of the completed terminal are shown. By applying metal to a wide roll of paper as shown in Fig. 7A, and applying it to the cable, a complete torus can be formed which, itself, is insulated positively and definitely by solid insulation of high quality. This greatly reduces the stresses at the edge of the sheath, and at the same time insulates the points to which flashover is likely to take place. There is an endless variety of shapes which can be thus formed by different methods of applying the metal to the long sheet of paper.

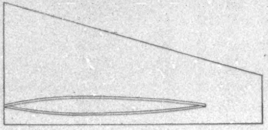

FIG. 7—SINGLE SHEET OF IMPREGNATED PAPER BEARING TWO STRIPS OF TINFOIL, CONVERGING AT THEIR ENDS, USED TO FORM THE TERMINAL SHOWN BELOW. (THE VERTICAL SCALE IS CORRECT; THE HORIZONTAL SHOULD BE INCREASED ABOUT ONE HUNDRED TIMES)

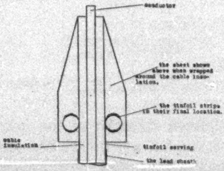
conductor
the sheet shown above when wrapped around the cable insulation.
the tinfoil strips in their final location.
cable insulation
tinfoil serving
the lead sheath

FIG. 7A—TEMPORARY TEST TERMINAL FOR A CABLE

In regard to actual details of how the metal is applied, this can be done in various ways. A thin strip of tinfoil can be applied on one or both sides of the sheet. The paper can be metallized by a spray process on one or both sides, on one side by a spray process and having this region perforated so that the metal would go through the sheet. It also could be done in a very perfect way by metallic inlays so that the surface would be entirely smooth and there would be no building up of thickness at this point. In our actual joints, we have used throughout the very simple method of cementing a 1-mil strip of tinfoil 3/8 in. wide to one side of the paper. We have never had any trouble with the tinfoil; it stays on through impregnation, it is not bothered by the jointing machine, and it has given no difficulty. In fact, we had so little difficulty with the wide paper rolls that when we were using the wide paper over the outer part, applying it over a hand-taped stepped region, we used the same roll for as many as four test joints, reimpregnating each time. Even with this handling the foil gave us no difficulty. Obviously the foil does not make a continuous cylinder but is really in the form of a flaring spiral, whose edges simulate a cone. The only trouble we have had at all with this simple construction has been in making terminals such as that shown in Fig. 7. In this figure the layers of foil are superimposed over each other so many times that there is a building up, and some spaces are formed between paper layers, tending to form wrinkles. For a construction such as in Fig. 7, special means should be taken.

V. CONCLUSIONS

The main purpose of this paper is to describe what is believed to be a new principle in the making of cable joints and to give the experimental results obtained with a joint of given dimensions. Whether the particular tests shown in Table I are sufficient for cable for 132,000 volts, three-phase is a point which may be debatable. If it should be considered that they are not sufficiently high, better results may be obtained by making a larger joint, and this is entirely practicable. Obviously, it will be seen that the joint could be applied also to three-conductor cable with some modifications, or without modifications to the three-conductor Type H cable.

This method of making joints is applicable to the jointing of paper-insulated cable impregnated with such a thin and fluid oil that the oil would escape if the lead is cut. It is merely necessary to surround the present machine with a tank full of clear oil and, without going into all the details, remove the lead sheath of the cable under the oil, thus preventing any loss of oil. The present joint can then be made under oil, and, in fact, there are some advantages of insulating under oil in any case, since the trapping of air is even more definitely impossible. The completed joint is then enclosed in a split oil-tight casing which can be bolted together, the machine and oil tank are removed, and finally an outer sleeve is wiped to the cable at each end around the inner sleeve and the space between them filled with oil.

The various features of the joint are covered by patents of the author, and the machine, by a joint patent held by him and F. D. Barbour, all being the property of the company with which they are both associated.

The cooperation and assistance of Mr. J. Cadwallader and Mr. W. C. Cadwallader, in the development of the joint, are gratefully acknowledged.

Discussion

D. W. Roper: Mr. Simons has introduced several new points in cable-joint design for which he should receive due credit.

We have used a few of these joints in Chicago. In making up the joints, the cable splicers and the engineers who were supervising the construction, had occasion to suggest one comparatively minor change which they thought would improve the joint. The change was not acted upon or received with any great degree of enthusiasm and we did not urge it. However, in giving some further thought to the subject and in looking over some of the patents that have been issued on joints to others, we discovered that the suggestion which we had made for a change in this particular joint was covered by a patent issued to another individual, and controlled by another manufacturer.

This situation appears to be getting somewhat worse and apparently it is going to become more complicated within the next few years. During the past year the industry has, for a consideration, acquired a fundamental patent covering one feature of central-station design which threatened to hamper the development of the industry, and after its acquisition threw the patent open to everybody without further charge.

If the cable joint patent situation develops into a somewhat similar state, it may be expedient for the industry to consider, in the same way, the taking over of the patents for a consideration, so that by a combination of the features, covered by the patents of various individuals, a much better joint can be made, than from the patents of any one individual or corporation.

T. F. Peterson: One point which the author emphasizes seems to me to be questionable. Failure of the composite structure consisting of oil, paper, etc., along the dotted line of Fig. 2 is attributed to the decreased strength of the so-called long oil path. This should be more fully demonstrated, or substantiated by proof before being adopted as to the true reason for the occurrence.

Consider the elementary case of two insulating materials in series,

lengths l_1, l_2 permittivities. K_1, K_2
gradients g_1, g_2 and applied voltage E

$E = g_1 l_1 + g_2 l_2$, and since D, density of electric displacement $= g K$ (1)

$$E = \frac{D\, l_1}{K_1} + \frac{D}{K_2} l_2 \qquad (2)$$

$$D = \frac{E}{\dfrac{l_1}{K_1} + \dfrac{l_2}{K_2}} \qquad (3)$$

$$\frac{D}{K_1} = g_1 \doteq \frac{E}{l_1 + l_2 \dfrac{K_1}{K_2}} \qquad (4)$$

When l_1, the thickness of oil path is small

$$g_1 = \frac{E}{l_2} \frac{K_2}{K_1} = \text{Average Gradient} \times \frac{K_2}{K_1}$$ which is maximum value possible.

From equation (4) it is evident that as l_1 increases g_1 decreases. It seems quite improbable that, even though oil with maximum stress is not broken down, a path can be found where a lower stress is actually in excess of breakdown value. Such a condition could exist only when rate of change of strength with increase in l_1 is greater than $\frac{d\, g_1}{d\, l_1}$. I doubt very much whether this is the case for series oil paths less than 1 in. (Breakdown gradient of oil remains fairly constant within this range.)

I should much prefer to consider failure as being due to overstressing a comparatively short path of oil ($k_1 = 2.5$) in series with a long path of paper ($k_2 =$ approx. 3.5). This is in accord with my experience with testing crotches, composite insulation, etc. The latter has lead to our avoiding short paths of oil in series with high-dielectric-constant materials rather than long ones.

Turning now to the joint proper, it is indeed surprising to note that the author considers the use of hard compound permissible. In Brooklyn we have a great mass of data on the migration of petrolatum compound from joints. This has seriously weakened penciled paths and necessitated refilling at 6-month intervals or continually by means of reservoirs. If this has been found necessary at 33 kv., how much more so is it necessary to insure keeping even a "sticky" oil in paper of a 132-kv. joint.

Mr. Simons complains of difficulties experienced in wrapping metal foil on conical surfaces. We have been doing this in the field for about 1½ years, without any apparent trouble.

I am somewhat skeptical concerning the use of the machine or single paper application to three-conductor joints in metal-sheathed cable. First, the general method does not lend itself very well to jointing sector cable. Then too, its use makes it impossible to maintain a lay or twist of conductors through joints. This is of considerable advantage in "phasing out" cable. It would seem that hand wrapping must needs continue in such cases.

In concluding I might say that I consider none of the fundamental principles used in the design of the joint as being new. We have used built-up conical structures of solid insulation wrapped with metal foil—thus getting a flared zero-potential surface—for several years; stepped insulation is standard in many joints; one-piece paper wrapping is an important feature of the Pirelli joint.

However, the method of construction which results in the practical introduction and use of all these principles, in one operation, is decidedly unique.

S. I. Oesterreicher: Aside from the new and comparatively simple method of making an almost factory-taped insulation in the field, it seems to me that this joint is no radical departure from other shielded cable joints.

The method of paper application may eliminate the much feared voids; however, the fact that the joint is also shielded, would indicate that there are other weak places besides voids in cable joints.

Naturally, one may ask, where?

Mr. Simons partly answers this by describing the breakdown of some extra high-voltage cable terminals which failed at a certain point on the flare of the end bell, where there was by far more insulation than in the cable proper. I have had similar disappointing experiences about the behavior of certain dielectrics. However, I do not believe that the insulation failed because I lowered the dielectric strength per unit thickness, but it failed because it had a greater stress along the line of breakdown than anywhere else.

In my opinion, the shape of the end bell, or that of a cable sleeve, or in more general terms the size and shape of two electrodes separated by a dielectric may considerably increase the dielectric layer stress at certain critical points.

In his classical work, Maxwell shows the field layer displacement upon unsymmetrical electrodes by his model condenser consisting of two electrodes, one a half plane, the other a full plane. While this model condenser might not be the exact duplicate of the conditions existing in a cable joint, the similarity is apparent.

Disregarding the ends and joints of a single-conductor cable, it may be represented by a concentric coaxial condenser of infinite length, or in a diagrammatic way by two symmetrical electrodes separated by a uniform dielectric. At the ends or at the joints the symmetry of electrode arrangement is disturbed, and the internal layer stress becomes distorted. With certain assumptions, Loebner, a research worker of the Duisburg Cable Works, obtained for cable ends identical curves to the ones of Maxwell's model condenser.

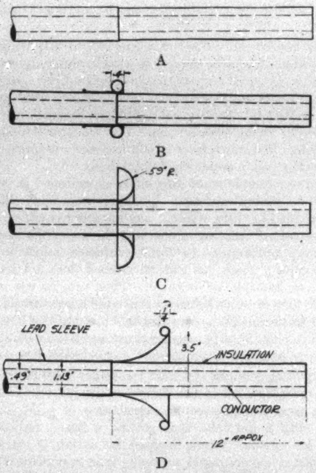

Fig. 1—Streamer Discharge Starts at A—30 Kv., B—40 Kv., C—60 Kv. and D—80 Kv., Plus or Minus 2.5 Kv.

From this data a nomogram was constructed by which—if the normal internal cable field intensity is known, the horizontal critical stresses may be readily found.

The practical importance of this investigation is readily understood by the following diagram from Loebner's work in which four sections of the same cable are shown. The outer sleeve of each section is formed to a different shape. While the straight sleeve shows streamer discharge at 30 kv., already, the funnel-shaped sleeve may be stressed to 80 kv. before streamer discharges appear.

Thus, I believe that a cable joint insulated in the most careful manner, may be unsatisfactory if the sleeve design is not fitted to the other characteristics of the joint.

While an electrostatic shield, if properly applied does distribute the unequal strains, it will not protect without reservations.

In view of this, it seems to me that the claim in the paper, that any diameter of a sleeve may be put over the insulated joint, providing it clears the paper insulation, should not be accepted in its strict sense.

R. J. Wiseman: For testing the joints, in Table I, Mr. Simons used 200 kv. In Table II, he started with 200 kv. and then raised the voltage in steps. Now we cannot evaluate just by looking at the tables, the relative advantages of this type of joint for the two different operating voltages. If we can only decide upon some single method of testing cable joints, and then evaluate to a specified voltage stress at the conductor rather than the average stress, I think we shall all get a better idea of how good joints can be. Today we do not do it, one company uses one method and another company a different method.

W. A. Del Mar: Dr. Wiseman has made a very good point, in my opinion, in making a plea for standardization of such tests.

One of the features that should be standardized is the internal pressure of the joint when it is being tested. Ordinarily, tests on joints have been made with the joint closed in the ordinary way. When voltage is applied, there is dielectric loss, which causes a temperature rise with consequent expansion, and internal pressure in the joint. The result is that the breakdown voltage in the joint is materially increased over what it would be with atmospheric pressure.

Under the term "dielectric loss" I include and refer particularly to the energy liberated just prior to failure by destructive ionization.

This situation can be averted either by attaching a bellows device to the joint, or by leaving an opening in the sleeve, either of which will introduce atmospheric pressure. If we do not do this, a high breakdown voltage may mean nothing more or less than a high dielectric loss or ionization in the joint, and this defect would be accounted a virtue on the basis of a high breakdown voltage.

C. N. Rakestraw: I want to discuss the fact that Mr. Simons apparently takes for granted that the type of joint should be a stepped or penciled joint, as far as the insulation is concerned.

As a good many of you probably know, in connection with our development work in Cleveland about four years ago we did a considerable amount of experimenting with just this type of joint, and in fact eventually did develop a joint of wrapped or taped insulation which apparently would stand up. Our experiments showed very conclusively that one certain number of steps was preferable to anything else, and a considerably less number than Mr. Simons has shown in his diagram. As a matter of fact, of course we later found a combination still better than that, and abandoned the idea of a stepped insulation entirely. We went to a conical connector obtained by undercutting the insulation, and the use of this conical connector increased the voltage of any joint on which it was used about 50,000 volts. So it seems to me that it is a step backward to use the insulation wrapped joint.

With a joint made by cutting the original cable insulation in steps, and then wrapping these with tape, I think it is universally found that a breakdown, when a breakdown can be accomplished, starts somewhere in the center of the joint, very often under the first step or the second step, and passes almost entirely within the original insulation, and then to whatever sleeve encloses the joint.

In Mr. Simon's joint, the end is protected at an angle of about 15 deg. Now, it doesn't seem to me that the difference made by wrapping the center of the joint with one single sheet of paper as compared with tape makes a great deal of difference; but it does make a great deal of difference if the insulation is carried unbroken to the center and the central connector is protected at an angle of about 15 deg. This, as I say, has shown an increase in breakdown voltage of about 50,000 volts on this type of cable.

E. D. Eby: This paper conveys the impression that there are insurmountable difficulties in the way of making a successful joint with tape, which is economically competitive with the joint insulated with a single sheet of paper. It would seem proper to correct this impression by referring to the fact, that practically all the joints which have been made on these high-voltage cables during the past year, have been made either with paper tape or specially processed varnished cambric tape. The 75-kv. joints made in Chicago were produced complete in about 4 hr. The 132-kv. joints now being made in Chicago, I am told, are being completed in about 6 hr. In our experience, both in factory tests and customer's tests, this type of joint has proved stronger than the cable in every case. From these facts, I think it is evident that the taped joint is entirely practical.

With reference to the practise of removing the cable insulation in steps, the merit of this is well illustrated in its application to some of the Cleveland-type joints in Philadelphia, which were made with the enlarged connector having the same diameter as the conductor insulation. Some of these joints failed when the cable line was tested, apparently because of incomplete filling of the under-cut in the insulation with the solder. These joints were successfully repaired by stepping the insulation and applying varnished-cambric tape, without having to renew the cable section.

I want to add a word to the subject of testing. It is highly desirable that our tests should be comparative and I would recommend, as a basis of tests for joints and terminals, the program of testing set forth in the A. E. I. C. rules for cables. We have been following these rules in our development work on joints so that our own tests could be compared. If this program were generally used, the different designs of joints could be readily compared.

Let me emphasize also that any high-voltage joint, as well as the adjoining cable, is greatly benefited if a thin mineral oil is used as a filler. The migration of the oil into the cable, if it is not already a so-called oil-filled cable, improves the cable by preventing the formation of voids in the insulation. By placing an oil reservoir in the form of an oil-filled joint in each end of the cable section, each length of cable is fed from two directions. Some operating companies have already found that the filling of joints with oil has sufficiently improved the strength of old cable so that the operating voltage could be materially raised and successful operation secured.

D. M. Simons: Mr. Peterson apparently does not agree with me in my explanation of the failure shown in Fig. 2. There are of course two effects, namely the decreased strength of oil in thicker layers, and also the difference in specific inductive capacity between the oil and the impregnated fibrous insulation. Mr. Peterson emphasizes the latter, while I emphasize the former. The actual truth is probably a combination of the two effects, but I cannot agree with Mr. Peterson that the breakdown gradient of oil remains constant up to paths 1 in. long, and would refer him to Peek's Dielectric Phenomena, Table LXIV for instance. A third effect is undoubtedly the tangential or longitudinal stress in the fibrous insulation and particularly along the dividing surface between the fibrous insulation and the oil, which stress is a function of the slope of the sleeve as emphasized by Mr. Oesterreicher in his discussion.

It is of course very necessary that the correct slope should be used in the tapering portion of a joint or terminal, and I brought out the importance of this in the text immediately following Table II. Theoretically, I agree with Mr. Oesterreicher that the slope should not be a straight line. Practically, however, with the small angles used, there seems no sufficient justification for using other than the straight-line construction for the slope. Mr. Oesterreicher questioned my statement that the diameter of the sleeve was of no consequence. Possibly he has forgotten that the entire joint is shielded inside by a metal coating of the cylindrical portion and by the tapering tinfoil at the ends. That is, the joint is completely shielded inside, and the outer sleeve is merely a mechanical covering, and its shape and dimensions have no electrical function.

I heartily agree with all the remarks about joint-testing standardization. It is usually possible to obtain a joint or to design a joint, which will stand any given test up to the demands of present cable practise, if the correct design principles are used. One of the most difficult questions for a given voltage however is to determine what the proper tests should be.

Mr. Del Mar's remarks on internal pressures are much to the point. I might add that some of our tests on joints were made with the joints as cold as minus 10 deg. cent. and up to about 20 deg. cent. without observing any effects on the strength.

Mr. Rakestraw tells us that in their design of joint, they are able to add 50,000 volts breakdown strength by abandoning stepped insulation and developing their enlarged connector with undercut insulation. He unfortunately does not mention whether the increase of 50,000 volts was in short-time strength or in long-time strength, and there is a vast difference in significance between the two. He also indicates that a certain number of steps seems better than either a greater or less number. It is difficult to generalize on such questions, and all that can usually be stated is that a specific number of steps or a particular type of connector is better with a certain design of joint. I formerly shared Mr. Rakestraw's belief that a certain number of steps was the best. While not desiring to generalize too much myself, I believe now that the greater the number of steps the stronger the joint, but that the percentage gain is so slight after a certain number of steps have been chosen as not to justify the increased labor of cutting more steps. In regard to the enlarged connector and undercut insulation, while this construction has increased the strength of the type of joint Mr. Rakestraw was discussing, it distinctly is not as effective as the stepped insulation with our type of joint, as determined by practical experience. I do not believe that it is safe to state that any one construction of connector is better than any other, but merely that certain types are best for certain designs of joints, and the only criterion is the strength of the completed joint, which in our case is indicated in the tables given.

I am glad that Mr. Eby has commented on their success with applying tapes by hand, as opposed to the use of wide paper. I may state to Mr. Eby that my paper was actually written before the development of the joint to which he refers, and that possibly my statements were a little too emphatic. In general, however, the application of wide paper is inherently quicker than that of narrow tape. For instance, if the applied insulation is to be 3 ft. long and tape 1 in. wide is to be used, the roll of tape must be wrapped about 36 times around the joint to form one layer of insulation, while one turn of wide paper will do the equivalent amount of work. I am sure that we all agree with Mr. Eby in his remarks about oil for filling joints, and the beneficial effect of such oil upon the insulation of the high-voltage cable which is being jointed.

Mr. Peterson questions the novelty of the joint, and possibly I was not clear enough in emphasizing exactly what new points were presented in the paper. Stepped joints and electrostatic shields at the ends of a joint are of course known, but I believe that a stepped joint with wide paper in the steps (particularly a stepped joint with wide paper cut to fit the steps at the moment of application), and shields formed by metal previously applied to sheets of insulation in a predetermined slope are new developments. In addition, I might include the machine-wrapping of wide paper, which process insulates from the connector to the outside and automatically forms the electrostatic shields.

As a matter of general interest, I might add that eight of these joints have gone into service at 132,000 volts, three-phase, within the 2½ months since the presentation of the paper, on February 24 in an experimental 132-kv. line, and have operated without incident to the present time.

Oil Breakdown at Large Spacings

BY DOUGLAS F. MINER[1]

Associate, A. I. E. E.

***Synopsis.**—Much work has been done on the breakdown of insulating oil at small spacings between electrodes. Information for electrode separations of several inches is not as complete. It has been found that sources of ionization external to the gap influence the gap breakdown, so that the design of electrode supports and parts is of great importance.*

Data on several sizes of spherically terminated rods or cylinders are presented. Short-time breakdown tests are shown to be quite erratic and a form of long-time test schedule was developed which gives more consistent results. The final test used is called a ten-minute-hold and yields values for a given condition representing the maximum voltage that can be held consistently. This is of special interest in design.

The empirical curves of oil breakdown are analyzed by mathematical methods. A general equation for breakdown voltage in terms of electrode diameter and separation is developed which agrees quite well with the experimental data.

Evidence is presented to show that water in globular form suspended in oil may increase the breakdown potential considerably with spherical electrodes if the separation is several times the diameter.

* * * * *

IN common with other dielectrics, whether gaseous, liquid, or solid, transformer and switch oils exhibit a non-linear relation between breakdown voltage and spacing of electrodes in the dielectric. The breakdown value of the standard test cup spacing gives no clue as to the breakdown at large spacings. In bulk, oil has a relatively low dielectric strength due to the rapid increase in ionization by collision. Ionization at the electrodes leads to local breakdown and consequent total rupture. Thus the shape of the electrodes and supports is of prime importance. It is essential that the breakdown between the electrodes chosen is not influenced by corona in the neighborhood, originating in some part of the support with a smaller radius of curvature than the electrode itself. In order to yield useful data on large oil spacings, work has been carried on under carefully controlled conditions.

A number of excellent studies of oil breakdown at small spacings (below two to four in.) have been made[2] and reliable data are available. At larger distances, however, the data are not as complete. The results to to be discussed help to fill out this region and will serve as guides to design where such spacings are necessary.

When the characteristics of materials are investigated, it is always a question whether the material tested should be in ideal condition—a perfect sample, or an average. This will depend on whether the object is to arrive at the maximum quality or at that which can be relied upon for a general run. In the case of insulating oil, extremely high dielectric strength can be obtained with carefully filtered and vacuum dried samples (50 kv. r. m. s. for standard 0.1-in. gap between one-in. diameter flat electrodes).

This quality is exceptional, however, and cannot be commercially maintained. Both the designer and the operator of apparatus have to rely on what more nearly approaches the allowable minimum instead of the maximum quality. For example, good oil of commercial grade may test 30 to 35 kv., r. m. s. for standard test, but in apparatus the oil may be allowed to deteriorate to the point where it tests 22 kv., r. m. s. before it is considered unsatisfactory. The weakest point in insulation or the poorest condition the oil may be in, under normal good practise, may determine the factor of safety of the insulation. These tests, therefore, were made at room temperature using transformer oil of good commercial quality testing 30 to 35 kv. for 0.1 in. in the standard test cup.

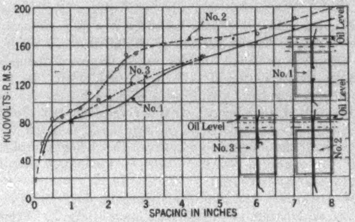

FIG. 1—OIL BREAKDOWN—EFFECT OF IONIZATION—$\frac{5}{8}$-IN. DIAMETER SPHERES. (EACH POINT IS AVERAGE OF THREE OR MORE TESTS)

I—Effect of Ionization on Oil Breakdown

Before considering the test data on large spacings, it will be necessary to present evidence to show why it is important to eliminate ionization other than that of the electrode itself. A set-up was made using $\frac{5}{8}$-in. diameter spherical electrodes, placed vertically in an insulating frame under oil. Comparison of breakdown was made with

a. $\frac{5}{8}$-in. sphere with $^1/_{10}$-in. diameter shanks,

b. $\frac{5}{8}$-in. spherical ended rods,

c. $\frac{5}{8}$-in. spherical ended rods with sharp wire attached.

Test Conditions. The oil was in very good condition, having been obtained fresh, and testing better than 35 kv. for 0.1-in. gap. The test tank was 3½ ft. in diameter and 5 ft. deep. Voltage supply was a 300-kv.,

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[1] Section Engineer, Experimental Section, Westinghouse Elec. and Mfg. Co., East Pittsburgh, Pa.

[2] Bibliography, 1, 5, 6, 7.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

100-kv-a., 60-cycle testing transformer with drum controller giving 500-volt steps on the high side. The tests were what is known as three-minute-hold tests in which the voltage is raised continuously to about 70 per cent of breakdown and then increased at the end of three-min. intervals by five per cent increments. The value used is the last voltage successfully held. The frame for holding the electrodes was made of bakelite paper micarta strips ½ in. thick and four in. wide with a length of 24 in. and width of 20 in. The arrangement is shown in Fig. 1. The sharp wire was a piece of No. 18 B & S copper wire wound around the upper electrode and bent down, the tip being two in. back of the end of the electrode and two in. out to the side.

The three curves of Fig. 1 show clearly the relative breakdown. At spacings up to 1½ in., the rods show slightly higher breakdown (15 per cent) but with separations from 2 in. to 5 in., a pronounced increase over the spheres is apparent. The rod curve can be brought down nearly to the sphere curve by the addition of the auxiliary source of ionization shown. This test, being typical of results obtained, shows the effect of the electrode support shape. For this reason the tests subsequently described were undertaken with electrodes of such a design that no metal part had a radius of curvature less than the end of the electrode. Spherical-ended cylinders or rods satisfy this requirement.

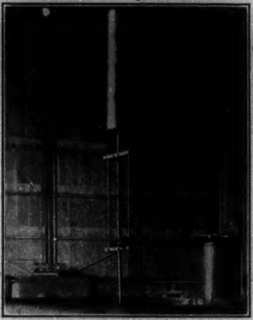

FIG. 2—ARRANGEMENT OF OIL TESTING ELECTRODES

II—Oil Breakdown at Large Spacings

In undertaking to obtain data under the limitations imposed, a considerable amount of study and work was put on the design and construction of testing apparatus. A description of this will be given to show difficulties encountered and methods used. The two chief difficulties were in bringing the high potential lead into the oil and in building a test frame that would not fail mechanically or electrically.

A small-sized wire (No. 18) was first used for a lead-in, entering the oil surface in the center of a 20-ft. diameter tank and connecting several feet below the surface to the electrodes under test. This form of lead showed great disturbance, especially at the oil surface, and flashover occurred over a radial distance of 10 ft. at 350 kv. This was due to the excess gradient adjacent to the oil surface which accompanies the distortion of flux due to difference in dielectric constants of air and oil. Excessive corona was evident above the oil surface. A 3⅛-in. diameter brass pipe was later used, being suspended from a string of suspension insulators. This brought about very little improvement, allowing tests up to 400 kv. only, before flashover occurred. This means that the average gradient over the 10-ft. distance is only about three kv. per in. whereas needle points 10 ft. apart flash at 1200 kv. or 10 kv. per in.

The final form of lead adopted is shown in Figs. 2 and 3. It consists of a micarta tube 8 ft. long (spliced) and 21 in. in diameter, covered with thin sheet metal smoothly applied and carefully soldered at the joints. The lower end was fitted with a toroidal piece of wood sprayed with copper. The torus (4-in. section) was cut away to fit flush with the outside of the cylinder. This device was suspended from insulators and electrically connected to a 3⅛-in. diameter brass tube running through the center. The testing rig was hung on this tube. Test voltages up to 700 kv. or more are successfully brought beneath the oil surface with this form of lead.

The first structure used to hold the rod electrodes was a crude affair built to determine the requirements of the problem. It consisted of a spruce board on which two porcelain pillars were mounted 31 in. apart and the rods clamped to these. This arrangement was tied to an 18-ft. wooden ladder and suspended horizontally in the 20-ft. tank. Ropes on the ladder aided in raising and lowering the outfit for adjustment.

The board soon failed by leakage and when the ground wire was arranged not to touch the board, the ladder and supporting rope finally showed leakage and burned badly. The porcelain pillars were also chipped and fused. From this it was shown that a highly insulating frame must be made with no metal parts to cause corona or parts of widely differing specific inductive capacity to cause stress concentration.

The second frame was built of micarta tubes and wooden end clamps arranged for a vertical gap. See Fig. 3.

Two micarta tubes 2-in. in diameter were used with split end pieces of maple clamped around the tubes with ¾-in. threaded wooden rods and micarta nuts. The upper electrode was a 3⅛-in. brass tube passing through the upper wooden pieces for an adjustable length. The frame was clamped to this tube so that the whole rig was hung from it. The lower electrode was a similar brass tube extending below the frame three ft. and ending in a chain which was dropped to the bottom of the tank for a ground connection. The inner ends of the brass tubes were closed with a hemispherical plug tapped to receive various-sized rods, so designed that no threads or parts were exposed with a radius of curvature smaller than the electrodes under test.

The conclusion drawn from these experiences is that for very high voltage tests under oil, wood, even the best seasoned hardwood commercially obtainable, is unsatisfactory as an insulator. It is suitable for a nonmetallic spacer or mechanical support but the insulation must be obtained from porcelain, micarta, fuller board, or similar high class dielectrics.

Two transformers were used in these tests, one 500 kv., 500 kv-a., 60 cycle, 5000 volts primary; and the other 1000 kv., 1000 kv-a., 60 cycle, 5000 volts primary. Both of these are single-terminal, one end grounded units.

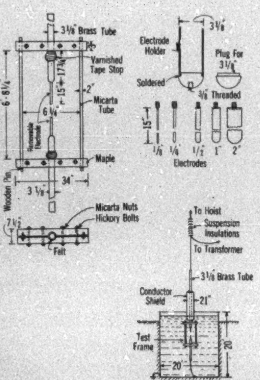

FIG. 3—TEST FRAME. ELECTRODES AND ARRANGEMENT OF APPARATUS

Voltage was measured by both crest voltmeter and ratio meter checked against sphere spark-gap values.

Test Methods and Results. Early tests demonstrated that with favorably controlled conditions (no neighboring ionization due to sharp corners, bolts, etc., and no adjacent field-distorting bodies) instantaneous breakdown values were obtained higher than those given in any existing data. This is of great value in determining the maximum strength of oil. Wide variations in test results were found, however. Later work involved the introduction of time tests and the tendency has been towards data on maximum holding voltages for longer times, one min., three min., five min., and finally 10 min. More consistent data, although much lower than instantaneous breakdown, are hereby obtained. From a design standpoint, these are the values usable in calculating oil insulation—the long-time holding voltages and not the occasional extra high voltages. The curves shown in this report are based on over 1850 observations or applications of potential, a large number being for 10 min.

The discouraging diversity of points shown in the instantaneous breakdown curves led to a study of test conditions, voltage measurement, etc., but reasonble control of these factors did not yield satisfactory results.

Curves of average values do not tell the whole story, especially with instantaneous breakdowns. We consequently have shown in some cases a number of representative points. The relation of average to maximum and minimum can be seen and the dispersion demonstrated.

Instantaneous Breakdown. It has been noted by several observers that oil breakdown for short-time application of voltage is quite variable, even when greatest care in controlling conditions is exercised. This deviation from an average value is far greater than with air[3]. This variation was shown by Hayden and Eddy to be a characteristic of oil apparently due to its chemical complexity. Other dielectrics, such as benzol, were not as erratic. Filtering, vacuum treatment, high temperature, etc., were found to be ineffective in changing this behavior.

Fig. 4, a typical curve, shows the individual breakdown points for one-in. diameter electrodes against spacing.

Voltage was raised continuously by induction regulator to breakdown in less than one min. It is at once apparent that the widest deviation occurs at small spacings (two in.). For example, with the one-in. electrodes, two-in. breakdown varies between 125 kv. and 320 kv., over 100 per cent.

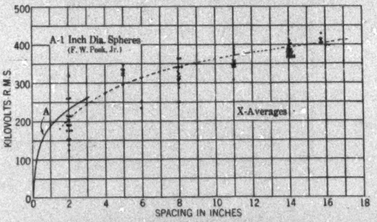

FIG. 4—INSTANTANEOUS BREAKDOWN VOLTAGES OF ONE-INCH DIAMETER RODS

The following table shows typical results obtained, being data for the two-in. diameter curve.

Spacing in Inches	Instantaneous Breakdown in Kv.
2	310, 285, 285, 285, 328, 260, 315, 190, 265, 315, 140, 180, 165, 175
4	320, 235, 310, 390, 275, 210
5	375, 340, 295, 285, 410, 365, 340, 295, 345, 325
8	360, 345, 390, 350, 345, 410, 450, 340, 470
11	425, 450, 390, 460, 340, 480, 475, 480, 480
12	380, 380, 380
14	410, 425, 370, 350, 450, 360, 460, 470, 450

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[3] Bibliography, 4.

This dispersion decreases in all cases with increased spacing, giving credance to the idea that with small gaps, there is an erratic lining-up of conducting particles or ionized oil. With longer distances, these variations are ironed out into an average state of conductivity. Thus the effect of poor oil will be more evident at small separation. The instantaneous data are presented to show that even an average of a large number of points cannot mean much when individual tests may depart 50 per cent in either direction from the average curve. After spending months trying to improve the consistency by a study of test conditions and voltage measurement, we were thoroughly convinced that the proper form of test of oil spaces is a time test in which the minimum consistent holding value of potential will be found.

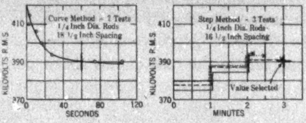

FIG. 5—BREAKDOWN VOLTAGES, ONE-MINUTE HOLD METHODS (CURVE AND STEP)

One-Minute and Three-Minute Tests. This test is defined as the maximum voltage that can be held for one min. without breakdown. Two methods were tried in determining these data, the step method and the curve method (see Fig. 5). After several series of tests made with these methods, the improvement in the matter of consistency hoped for, was not obtained. The step method was found to be preferable. The time was then extended to three min. Much better agreement between data taken at three different times some months apart is noted. The variation is still excessive for a relatively small number of tests. Fig. 6 shows a typical curve.

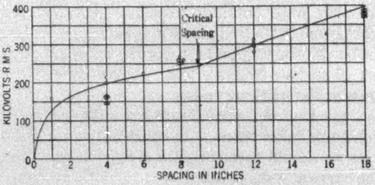

FIG. 6—BREAKDOWN VOLTAGES—THREE-MINUTE HOLD—ONE-HALF-INCH DIAMETER RODS

Ten-Minute-Hold Tests. Instead of increasing the number of tests, it was felt that a better method was to increase the time to 10 min., tending to eliminate the occasional freak values. This followed successful use of the method on transformer insulation tests. The method is very slow and tedious but seems to yield dependable results. A certain voltage is chosen below expected failure. This is applied for 10 min. and then taken off for five min. (to eliminate ionization), and is repeated five times. If no failure occurs, the voltage is raised a certain increment and the five 10-min. holds made. This proceeds until failure occurs during one or more of the five tests. Then the previous value is repeated five times and if checked (no failure), that value is selected as correct. Thus the least number of individual voltage applications necessary to determine one point is 15, requiring at least 225 minutes. Typical results of these tests are shown in Figs. 7 and 8, curves for points and one in. diameter electrodes. Fig. 9 gives a summary of 3-min. and 10-min. tests on logarithmic coordinates.

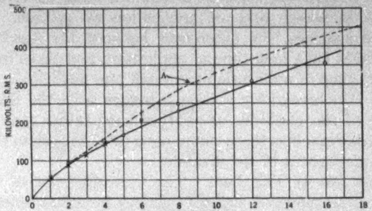

FIG. 7—BREAKDOWN VOLTAGES—TEN-MINUTE HOLD BETWEEN POINTS

A—Needles, F. W. Peek, Jr.
o —Solid curve drawn from data of present investigation (Kv.-53 $S^{0.7}$)
x —Data from W. H. Tobey, A. I. E. E. 1910

On some of the curves the data of various investigators have been shown for comparison. In Fig. 4, the curve for one-in. spheres given by F. W. Peek, Jr. appears slightly higher than the average curve of data obtained, but still well within the limits of individual points. Fig. 7 is an interesting comparison of data from three sources. The dash curve is taken from "Dielectric Phenomena," by F. W. Peek, Jr. and the solid curve is drawn from data of the present investigation (kv. = 53 $S^{0.7}$). The X's are from data of W. H. Tobey[4] and coincide nicely with the present curve. It seems probable that Peek's curve represents instantaneous breakdown and is consequently higher. The ten-minute-hold values are much lower.

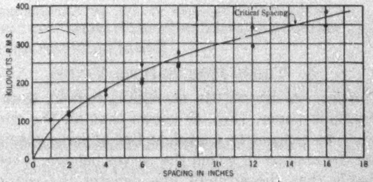

FIG. 8—BREAKDOWN VOLTAGES, TEN-MINUTE HOLD, ONE-INCH DIAMETER RODS

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[4] Bibliography, 8.

General Remarks on Test Results. From the results of the tests, the following may be inferred:

1. The instantaneous breakdown voltage for oils is not very definite, depending on too many factors that are sometimes not controllable.
2. The "time" tests show better consistency.
3. At short spacings, the breakdown voltage decreases with increased time of application of voltage. At large spacings, the differences between the sparking voltages for a long time of application and those for a short time of application are small and negligible.
4. The breakdown voltage increases with the diameter of the electodes at short spacing rather rapidly. At large spacings these values seem to be the same; that is, independent of the diameter.

Analysis of Test Data. The wide diversity of the test data rendered average readings for instantaneous tests unintelligible. Although results for the three-min. and 10-min. tests were more consistent, still there were some very pronounced deviations. In order to obtain conclusions that might prove of value in design, these data were analyzed so as to bring out some general relation that seemed to be representative of all the test data.

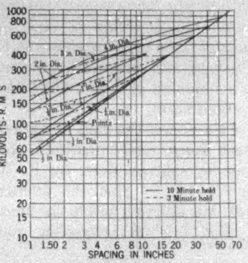

FIG. 9—BREAKDOWN VOLTAGES OF OIL IN ELECTRODES OF VARIOUS DIAMETERS ON THREE-MINUTE HOLD

A. THREE-MINUTE-HOLD VALUES

Plotting results of the three-minute-hold tests on logarithmic paper, (Fig. 9) shows that all the curves seem to be tangent to a straight line at large spacings. From the manner in which the curves behave, it seems probable that the straight line denotes the sparking voltage for electrodes with diameter equal to zero, *i. e.*, for needle points. That all other curves are tangent to and emerge into this straight line may be explained by the following speculation as to the nature of oil breakdown.

It is assumed that there is a definite breakdown potential gradient for oil. When this gradient is reached at the surface of the electrode, the oil surrounding the rod is ionized, the effect being to increase the effective size of the electrode. Whether a complete breakdown as shown by a spark over will follow this ionization or not depends on whether or not the increased size of the electrode causes the gradient to increase. Thus, if the spacing between the electrodes is large, the gradient decreases as ionization increases and in this case corona precedes a complete breakdown, so that if the applied voltage is not too high, only corona is observed. On the other hand, if the spacing is small, the gradient increases as ionization increases and a complete breakdown will follow as soon as the gradient at the surface of the electrode exceeds the breakdown value. It thus appears that for a given size of electrode there is a largest spacing—a critical spacing—at which no corona will occur before a complete breakdown.

Applying this to the case of two sharp points, it is seen that corona will always precede a complete breakdown. Thus, the sharp points are in effect two electrodes of varying diameter, their size being dependent upon the spacing and the applied voltage. For a given spacing the complete breakdown voltage is then the same as that for two electrodes having such a diameter that the given spacing is the critical one for such a size.

In the light of this speculation, we can readily explain why the curves for the electrodes with small diameters seem to intersect and finally emerge into the curve for sharp points. Thus the intersection of the ½-in. curve and the sharp point curve defines the critical spacing for ½-in. rod. At spacings greater than this, corona will occur first so that the breakdown voltage follows essentially the same curve given by the sharp points.

The points of tangency of the curves (on logarithmic scale) and the line denoting needle point might be thought of as those at which corona begins. These points as obtained from the curves do not agree with those that were obtained visually. The discrepancy might be due to the difficulties in making visual observations on the starting point of corona under oil or to the effect of surface irregularities. Above these points of tangency, the sparking voltage curves, on the basis of the above speculations, will follow a law different from that which holds below them. In constructing the sparking curves we should then expect a change in the direction of the curve at some point.

Mathematical Expression for Three-Minute-Hold Tests. As the plot on the logarithmic paper does not yield much information on the relations among the various curves, another plot was made on semi-logarithmic paper, since the general shape of the curves showed them to be of logarithmic form.

The straight line, *i. e.*, the probable curve for needle points on the logarithmic paper, becomes a curve on the semi-logarithmic paper. By trial it was found possible to represent the average test results approximately by a series of straight lines on the semi-logarithmic paper. The general form of the equations connecting the sparking voltage kv. and the spacing S is

$$\text{Kv.} = A \log S + B$$

or

$$\text{Kv.} = A \log \frac{S}{a}$$

where A and a are constants and Kv. = sparking voltage in kv., S = spacing in inches.

Moreover, when the various values of A and a were plotted on logarithmic paper against the respective values of the diameter of the rods, two parallel straight lines were obtained. Thus these two constants could be further expressed as functions of the diameter of the rods. These relations become

$$A = m\, d^n$$
$$a = c\, d^n$$

where m, c, and n are constants and d = diameter of rod in inches. Using these relations, it is then possible to express the sparking voltage in terms of the diameter and the spacing. Thus the general equation for the *three-minute-hold tests* is

$$\text{Kv.} = m\, d^n \log \frac{s}{c\, d^n}$$

With all the constants evaluated, this is

$$\text{Kv.} = 177 \sqrt{d} \log \frac{s}{0.16 \sqrt{d}}$$

It will be noted that the above equation does not become zero, for $S = o$. It will be noted, further, that the quantity whose logarithm is to be taken is large in all our cases. Thus if we add one to the number $S/0.16 \sqrt{d}$ and then take the logarithm, the values of the logarithm will not be increased greatly. In view of the fact that our data scatter more or less, such a modification will not change the results appreciably while the modified equations will give curves that do pass through the origin. Thus we have

$$\text{Kv.} = 177 \sqrt{d} \log \left(1 + \frac{S}{0.16 \sqrt{d}}\right)$$

for three-min. tests.

Curves calculated from this general equation have been plotted and found to be reasonably representative of the test data. For the smaller rods, these curves were calculated only up to the points where they intersect the probable needle-point curve. After that, since we assume the formation of corona, the curve is continued by the equation that represents the probable needle point, which, as already pointed out, is a straight line on logarithmic paper and hence is of the form Kv. = $D\, S^m$ in which D and m are constants. The numerical values substituted give the following as the equation:

$$\text{Kv.} = 53\, S^{0.7}$$

The following table gives the calculated values.

THREE-MINUTE-HOLD TESTS ON DIELECTRIC STRENGTH OF OIL

General Equation: Kv. = $177 \sqrt{d} \log \left(1 + \frac{S}{0.16 \sqrt{d}}\right)$

d = Electrode Diam., (in.)	Breakdown Kv.
$3\frac{1}{8}$	$312 \log \left(1 + \frac{S}{0.282}\right)$
2	$250 \log \left(1 + \frac{S}{0.226}\right)$
1	$177 \log \left(1 + \frac{S}{0.16}\right)$
$\frac{1}{2}$	$125 \log \left(1 + \frac{S}{0.113}\right)$
$\frac{1}{4}$	$88.5 \log \left(1 + \frac{S}{0.08}\right)$
$\frac{1}{8}$	$62.5 \log \left(1 + \frac{S}{0.057}\right)$
0	$53\, S^{7.0}$

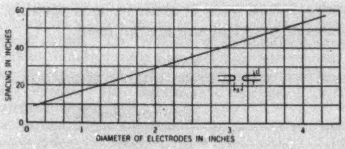

FIG. 10—CURVE OF CRITICAL SPACING TEN-MINUTE HOLD TESTS

Spacing of which curves of rods intersect and emerge into needle-gap curve (graphical solution from calculated curve)

B—TEN-MINUTE TESTS

Applying the same general equation, *viz.*,

$$\text{Kv.} = A \log \left(1 + \frac{S}{a}\right),$$

it has been found that the following equation agrees with test results quite well.

$$\text{Kv.} = \left[380 + 120 \left(1 - \frac{1}{d}\right)^4\right] \log \left(1 + \frac{d}{2} S\right)$$

in which Kv. = safe or holding voltage in kv.,

d = diameter of electrodes in inches,

s = spacing of electrodes in inches.

This is applied when ionization does not occur before breakdown. The same equation as before, kv. $= 53\, S^{0.7}$, applies when ionization precedes breakdown.

The intersection of these curves is called the critical spacing and might coincide with appearance of corona if accurate data were available. Fig. 10 was plotted showing the relation between calculated critical spacings and electrode diameter. The equation satisfied by diameters between ½ in. and 4 in. is $S = 5 + 12\, d$.

Breakdown curves for 3-in. and 4-in. diameter were calculated from the general equation. The equations for 10-minute-hold are as follows:

10-MINUTE-HOLD TESTS ON OIL

Electrode diam. (in.)	Breakdown Kv.
0	$53\,S^{7.0}$
¼	$500 \log_{10}\left(1 + \frac{S}{4}\right)$
1	$380 \log_{10}\left(1 + \frac{S}{2}\right)$
2	$388 \log_{10}(1 + S)$
3	$404 \log_{10}\left(1 + \frac{3\,S}{2}\right)$
4	$418 \log_{10}(1 + 2\,S)$

Conclusions. The principal result of all this work is the development of more suitable test methods and the fundamental recognition of the inconsistency of short-time tests. Values obtained by long-time tests are much more reliable from a design point of view. On the basis of this information obtained with ideal or standard electrodes, future work of great value is possible, using electrodes of practical and special form.

TABLE I
AVERAGE KV. FOR INSTANTANEOUS BREAKDOWN, TEST VALUES

Spacing (In.)	Electrode Diam. (In.)				
	1/8	¼	½	1	2
2	225	..	290	205	245
4	285	..	315	275	325
6	335	315	345	325	370
8	370	335	365	355	410
12	405	370	400	390	450
16	435	425	430	410	490

TABLE II
AVERAGE BREAKDOWN KV. FOR ONE-MINUTE-HOLD, TEST VALUES

Spacing (In.)	Electrode Diam. (In.)	
	1/8	1/4
2	185	190
4	230	235
6	265	265
8	305	275
12	355	330
16	370	375

III—Effect of Water in Large Quantities

Oil Breakdown. Low breakdown of oil samples is frequently ascribed to moisture or foreign materials. It has long been known that very slight percentages of moisture in oil, when in a dissolved state, lower the test value at small spacings. For example, one part in 10,000 will reduce the dielectric strength of dry oil to about 30 per cent of its original value.[5]

Some evidence exists, however, showing that under certain conditions water may increase the dielectric strength of a mass of oil. In Report No. 25 of Electrochemical Laboratories, Tokyo, Japan, Dr. T. Hirobe presents a curve (Fig. 11) which indicates a remarkable increase in breakdown between a point and a disk for a 300-mil gap in fiber-free oil. This increase from 21 kv. to 52 kv. takes place for an increase from zero to 0.028 per cent moisture content. Dr. Hirobe then goes on to show that with ½-in. spheres 150 mils apart there is a decrease from 90 to 61 kv. for the same moisture addition.

A number of tests were made by the author to find the effect, not of small traces of moisture, but of large percentages of actual water in visible globule form, such as may be present when leakage of rain into apparatus occurs.

Wet Oil Tests. A series of tests was made with ⅜-in. and ⅝-in. spherical electrodes using oil into which water had been stirred. The water was introduced in finely divided spray by definitely weighed amounts and agitated thoroughly before test. By this means the water was held in suspension in very fine globules less than one mm. in diameter.

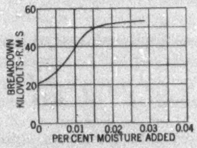

FIG. 11—EFFECT OF MOISTURE ON OIL BREAKDOWN
0.3-in. Gap—needle and disk (Dr. T. Hirobe, Report No. 25, Elect. Tech. Lab., Tokyo)

Preliminary Tests. Two ⅜-in. spheres were placed vertically in a large glass jar holding 25 lb. of oil. The gap was one inch. Various percentages of water were added and the test values were as follows:

Per cent Water	Instantaneous Breakdown	
0	56, 56, 54, 54	avg. 55 kv.
¼ (20 cu. cm.)	60, 60, 60	avg. 60 kv.
½	56, 56, 56, 56, 56, 58	avg. 56.3 kv.
¾	60, 62, 61, 62, 63, 62	avg. 61.6 kv.
1	61, 64, 59, 62, 64	avg. 62 kv.

More extensive tests were then run in a metal tank holding about 1000 lb. of oil. Percentages of water of ⅛, ¼, and ⅜ were introduced and instantaneous breakdown tests made. It was found that ⅜ per cent was about the maximum amount of water that could be held in suspension long enough for test. Instantaneous (rapid rise) values were obtained. As usual, a great divergence of points resulted, the average of which has no great significance unless hundreds of tests are made. With only a few shots (10), these averages are inconsistent, the ⅝-in. sphere breakdown being less than ⅜-in. spheres for some cases.

Average curves for the various water contents, (Fig. 12), were plotted and seemed to indicate that addition of water may increase the breakdown of oil gaps through gradient equalization. Small amounts (⅛ per cent or less) may possibly decrease the breakdown at small separation, probably because the water is either in partial solution or so finely divided that it has the elements of a low resistance path. Increase in water content causes agglomeration and the larger globules distribute the stress like a string of condensers. This effect increases with increase in voltage.

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[5] Bibliography, 1.

Another effect noted at high gradients is the throwing out of water from the strongest field. This gives a clarifying action that will ultimately improve the oil considerably if breakdown does not occur before this action is complete. These statements do not contradict the usual information about standard tests on oil with a test cup having 0.1-in. separation of electrodes for here the water bridges the gap and causes failure through conduction. In the curves for ⅜-in. spheres this appears to be the case below 1½-in. with ⅛ per cent water.

The curves show a greater increase in breakdown for the smaller electrodes (⅜-in. diameter). These results differ from Dr. Hirobe's in that he could obtain no increase with spherical electrodes. It may be concluded then that with spacings several times the electrode diameter, water in globular form evenly distributed may give a higher breakdown value than commercially dry oil, behaving entirely different from the case of dissolved water in small gaps.

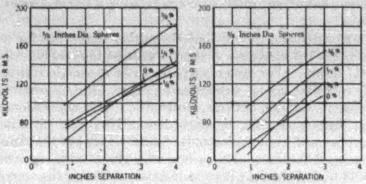

Fig. 12—Oil Breakdown Voltages—Instantaneous Averages with Various Amounts of Water Added

The curves of oil breakdown under various conditions as described in this paper show the range of values to be expected but are not intended as accurate data. Exactly duplicable results on oil are impossible. It is the purpose of this account rather to emphasize two points: First, that short-time tests are hopelessly erratic and that the time test of several minutes' duration is preferable in establishing values of use in design; and second, that the experimental data on different sizes of electrodes are related and are subject to mathematical analysis, one form of which is presented herewith.

The author gratefully acknowledges the assistance given by Mr. A. P. T. Sah in analysis of test results.

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Discussion

O. R. Schurig: The problem attacked by Mr. Miner, *i. e.*, that of oil breakdown at large spacings of from 1 in. to 15 in. and more, and when subjected to sustained voltage stresses, is of real importance to engineers because (1) the gaps in high-voltage apparatus are commonly of the order of 1 in. or more and (2) because the voltage stresses for a good part of the time are of a sustained nature. The designers of oil-immersed transformers and circuit breakers must therefore take into account the behavior of oils between widely spaced gaps and under sustained voltage stresses. Mr. Miner's data represent an advance of knowledge in this field.

At the same time, however, designers must also provide for sufficient dielectric strength at small spacings and under suddenly applied voltages. In oil circuit breakers, for instance, the sudden formation of ionized gases during circuit interruption has virtually the effect of shortening the clearances, for a few instants, between live and grounded paths of the oil vessel. Moreover, the temporary stresses acting on the oil at such times will often be higher than those occurring during normal operation under sustained voltages. Hence, in the design of such apparatus and in the selection of oils, engineers will need to take into account the dielectric strength of small gaps and for sudden voltages as well.

The second item that I wish to discuss is the point stressed by Mr. Miner that the results of short-time tests[1] are hopelessly erratic. I believe that he does not intend to have that statement apply to tests properly conducted with small gaps, such as the customary 0.1-in. gap between 1-in. disks. If reasonable care is taken in cleaning and drying the electrodes before the test, results of good reliability are obtained.

The following table gives the short-time breakdown voltages with a 0.1-in. gap of three samples of oil of various degrees of purity.

Sample No.	Breakdown voltage kv.	Ave.	Probable error of ave. in kv.
1	51, 59, 59, 58, 62, 62, 48, 48, 47, 54	55	1.7
2	10, 40, 48, 5, 23, 5, 22, 12, 12, 21	20	3.7
3	30, 28, 31, 24, 29, 30, 22, 22, 35, 29	28	1.1

The breakdown values of samples No. 1 and 2 are taken from the test values from which the curve of the accompanying Fig. 1 (which will be discussed later) has been plotted, sample

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[1] Meaning tests in which the voltage is raised fairly rapidly from zero to the point of breakdown, at a rate of, say, 3000 volts per sec.

No. 1 is the oil sample before being mixed with water and sample No. 2 is the same oil after being mixed with water and standing 1.8 days. Sample No. 3 is a transformer oil containing fibrous particles which were purposely mixed in the oil. By the "probable error," as given in the last column of the table, is meant an error of such a value that the probability of having a greater error is just equal to that of having a lesser error. This probable error is calculated from

$$P.\ E. = \frac{0.85\ a.\ d.^{*}}{\sqrt{N - 1}}$$

where $a.\ d.$ is the average deviation of the breakdown values from the mean and N is the number of breakdown values, in this case 10.

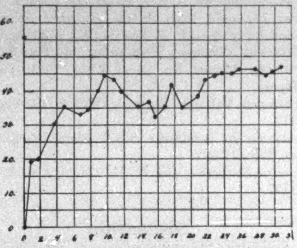

FIG. 1—THE RATE OF SETTLING OF WATER IN A TRANSFORMER OIL AS INDICATED BY THE BREAKDOWN VOLTAGE OF A 0.1-IN. GAP BETWEEN 1-IN. FLAT DISK ELECTRODES.

3 per cent of distilled water mixed in 1.5 liters of new oil by shaking for 4 min.

Number of breakdowns per point 10.

Rate of increase of voltage 3000 volts per sec.

Oil at room temperature, 20 deg. cent.

Breakdown voltage of oil before mixing with water; 55 kv.

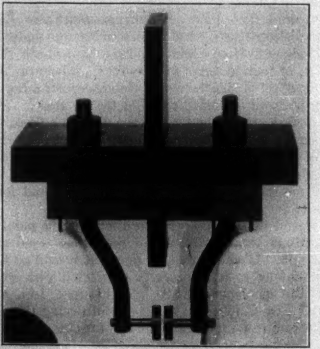

FIG. 2—ELECTRODES AND MOUNTING; 1-IN. DIAMETER BRASS DISKS, WITH 0.1-IN. GAP.

The electrodes were made for placing on a glass jar 3 in. by 5 in. by 7 in. high.

The third point that I wish to discuss refers to the effect of water on the dielectric strength of oils. Mr. Miner has presented data on the dielectric strength of oil containing water for wide gaps. I wish to outline the results obtained on the dielectric strength of oils mixed with finely divided particles of water.

The tests in question, made with some of the customary high-grade mineral insulating oils, tested with a 0.1-in. gap between 1-in. plane disk electrodes, showed that a thoroughly shaken up mixture of the oil with, say, 3 per cent of water gave initially less than 5 kv. dielectric strength, *i. e.*, when tested immediately after mixing, as is well known. After standing, however, the mixture began to separate with a consequent rising of the dielectric strength until after several days, the water had settled to the bottom, the oil had cleared up and had *regained a relatively high dielectric strength although there was a layer of water under the oil and in contact with it.* This result was consistently repeated in the absence of impurities other than water.

A typical curve is given in Fig. 1, showing the dielectric strength recovery of an oil-and-water mixture allowed to stand after throughly shaking up.

Fig. 2 is an illustration of the electrodes and their support.

The tests were made at Schenectady, as part of a research program aimed at determining the effects of various factors, such as water, carbon, heat, time, exposure to air, fibers, electric field, metals, etc., on the dielectric strength of circuit breaker and transformer oils.

V. M. Montsinger: Generally the breakdown voltage of oil is so erratic that few, if any, investigators have felt like trying to express it by a mathematical formula. The answer to this appears to be, as pointed out by Mr. Miner, that too small spacings have been used, spacings below the critical breakdown value.

I was interested to see that the author finds that for certain conditions, the dielectric strength varies as the spacing raised to the 0.7 power. For design purposes we have used in transformer work a similar law of strength varying as the $^2/_3$ power of the spacing or distance between electrodes. This expression seems to be one of nature's dielectric laws. Not only oil but solid dielectrics with similar electrodes follow somewhat the same law. However, if the electrodes are large flat planes with edges having large radii and the field is approximately uniform, the strength is practically a linear function of the spacing for both oil and solids. In other words, the strength varies almost directly with the spacing. This fact should be kept clearly in mind in generalizing on the laws of dielectrics.

The question of why the difference in the laws suggests itself naturally. The explanation for this is probably about as follows: When the electrodes are of such a shape as to produce a distorted field, this distortion becomes more and more pronounced as the spacing increases. Thus, for example, if the electrodes are small rods or spheres, at small spacings the dielectric field causing breakdown is more nearly uniform than when the spacing is increased. Finally, when the distance becomes great enough, the distortion becomes so great that the electrodes act like needle points, even though they may be many times larger. Mr. Miner's explanation of why they act like needle points above the critical spacing, by the ionization theory, seems a logical one.

The main points which I wish to emphasize are (1), that for uniform dielectric fields the strength of oil is approximately a linear function of the spacing and (2), for distorted fields the dielectric strength varies approximately as the $^2/_3$ power of the spacing or 0.7, as Mr. Miner states in his formulas. I prefer to use $^2/_3$ because it is a value very easy to remember. Roughly speaking, this means that if the spacing in oil is trebled, the strength is doubled, or to be more correct is increased 2.08 times. As regards uniform fields, in practise this condition is seldom, if ever, found. Consequently the $^2/_3$ power law is more nearly applicable than the first power or linear function law.

R. J. Wiseman: In regard to Mr. Miner's paper, where he refers to the effect of water or moisture in his oil, I wonder if he has taken into account the possibility that under high stress, with wide spacings, there is a certain attraction of the moisture toward the surface of the conductor, causing an increase in the diameter of that conductor which will decrease the stress temporarily until breakdown occurs. I think that is partially the explanation of why we are getting a higher breakdown voltage with a wide spacing and a large amount of moisture present.

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*Precision of Measurements and Gaphic Methods by H. M. Goodwin, McGraw-Hill Book Co.

I was very glad to see that Mr. Miner has tackled this problem from the point of view of ionization. We all know rather definitely ionization occurs in gases. Some of us have a pretty definite idea of ionization in solids. This is a sort of connecting link between the two, and I hope further work along this line will be carried on.

Mr. Roper suggested that we find a new way of testing cables for their quality. We are all interested in cables and want to know how we can determine what is a good cable and what is not. We make a good many tests today in order to pick out good cable. Another test means additional work on the part of the cable manufacturer; it is costly to both the purchaser and manufacturer. Apparently the tests today are not giving us what we want. When the cable leaves the factory, as far as we know, it is 100 per cent perfect. When it gets into service, it apparently breaks down. Is it a question entirely of tests in the factory, or is it also a question of operation in the field?

I hope we shall be able to find a new test that will help us to determine high-quality cable, but if we are going to have additional tests, let us drop some of the tests we now make which are unsatisfactory, rather than simply add one or more new tests to the number we now have.

F. W. Peek: Those of us who have had experience with oil as used in apparatus know that it is not only the most reliable and consistent insulation but also one of the best. However, oil is very erratic under test, and I wish to point out a few of the reasons.

As used in apparatus, the oil spaces are always divided by barriers of solid insulation. These barriers change oil from an unreliable to the most reliable insulation by preventing lining up of conducting particles. Some of the principal causes of erratic changes in the breakdown voltage of oil are occluded gases and foreign particles such as moisture or fibrous materials. Foreign particles may line up in the electrostatic fields. In the case of large electrodes such as plane surfaces the lines of force are straight lines. In such a field particles readily form chains from electrode to electrode along these lines. A short gap with large electrodes has very high dielectric strength for very good oil but very low strength for poor oil. The percentage change is thus very great for such a field. Impurities are easily detected by such a gap. As the spacing is increased the chance of lining up becomes less.

The other extreme in electrodes is the needle gap. The lines of force are curved, and are very concentrated at the points or at the electrodes. Any particles in this field will tend to go toward the electrodes to make them larger and store a greater amount of energy. Those that strike the electrodes will be charged to the same potential and immediately repelled or shot away. There is no tendency to line up. In fact this action purifies the oil between the terminals. So, with a needle type of gap there will be very little difference in dielectric strength, whether the oil is very good or very bad. This is especially so if the spacing is large.

The same effect occurs with spheres or other forms of electrodes at large spacings. I had an experimental apparatus which showed this very nicely. It consisted of a test tube filled with a light-colored oil in which an emulsion was made with colored water. A wire was extended through a cork and down the center of the tube. About two-thirds of the lower part of the test tube was then placed in a jar of clear water. The wire acted as one electrode, the transparent water as the other.

As soon as voltage was applied the oil immediately cleared up. An examination showed the particles of colored water around the inner surface of the test tube as well as at the bottom of the tube. The water particles had been attracted to the inner electrode and then repelled to the opposite side where, because of the less dense field, they gradually settled down to the bottom.

F. M. Clark: The hope has been expressed by one of the previous speakers that continued research will eventually result in a thorough understanding of the dielectric phenomena occurring in oils which are subjected to high or low voltage stress.

The average theorist, in considering the insulation problem, becomes very largely discouraged as soon as he starts to look up the previous literature history. Solids themselves are bad enough and show a very erratic nature; oils we know are blamed much more. However, it is probably true that a large part of the blame, instead of being placed upon the insulation, should be placed upon the various investigators who have not carefully considered the physical and chemical characteristics of the materials tested. Oils for example must be considered as chemical mixtures, the constituents of which are variable to a large degree. To understand thoroughly the behavior of a mixture we must know something of the behavior of each of its constituents. This is a difficult problem involving much fundamental research. If, however, we work on "commercial grade" oil such as Mr. Miner has carefully stated he used in his experiments, we have increased the difficulties of the problem by increasing the number of constituents of the mixture on which we are working. And worse still, the added constituents are variable in amount from day to day since the chief "foreign" material in commercial-grade oil consists of water and fibrous material over which we have ordinarily but little control. It may be claimed that if the dielectric strength is maintained, the water content may be considered negligible or fixed. Yet we have some evidence that wet oil may have high dielectric strength if properly tested.

In the same way, much of the difficulty besetting the solution of insulating phenomena, is to some extent increased by broad conclusions drawn from limited data. Thus Mr. Miner states in his paper that solids and liquid insulators show a non-linear dielectric strength-thickness relation. If, by this, Mr. Miner means oiled solids, tested with small electrodes or oils tested with small electrodes, he is largely correct. Yet it has been shown in a previous publication (The Dielectric Strength-Thickness Relation in Fibrous Insulation—*G. E. Review*, Vol. 28, page 286 for 1925) by V. M. Montsinger and myself that this is very largely a function of the testing method for oiled solids at least. Thus with large electrodes, oiled paper shows a linear strength-thickness relation. Moreover, untreated papers also show a dielectric strength very largely independent of thickness. In certain unpublished researches which we have carried out at Pittsfield, it appears that oils behave likewise and show a linear strength-thickness relation when large electrodes are used for the test. We hope to present this research to the Institute at a later date.

Mr. Miner advocates a long-time test in preference to the standard, rapidly applied test on oils. A more reliable test result is claimed. With carefully prepared oils, I believe that considerable of the erratic nature of oil breakdown might be eliminated. Unless care is taken, long-time tests merely indicate the time and voltage relation necessary to "suck" sufficient impurities into the field to bridge the electrodes. The test becomes one for impurities (such as fibres) rather than a test on the oil itself. This is illustrated clearly by the researches detailed in the Seimen's *Zeitschrift* for January, 1925, page 29. The short-time test comes nearer being a test on the oil itself. The fact that benzol gives more reliable test results is merely an indication of the effect of purity. With benzol, a degree of freedom from foreign substances can be obtained which cannot be approached in oil unless extreme care is taken in preparing the sample.

Another contributing cause to the confusion at present existing in the study of dielectric phenomena is the lack of proper definition of the experimental conditions. The dielectric strength-thickness relation for oil can very largely be upset by varying the position of the testing electrodes. With 10-in. plane electrodes, the test results are largely dependent on whether the electrode face is vertical or horizontal.

In the consideration of insulation data from the standpoint of fundamental theory, we must be very careful in accepting results based on ordinary commercial-grade material. In the same way, the designing engineer must be very careful in accepting data based on material which appears to approximate his operating conditions. Thus the dielectric strength of oil can be made low by a variety of methods some of which may or may not be applicable to his special machine. It is very difficult and dangerous to attempt to apply laboratory oil data to the explanation of fundamental theory, or even to the design of electrical machinery, if the tests involved concern foreign material dissolved or suspended in the oil, rather than tests on the oil itself.

Jacob Katzman (communicated after adjournment): Mr. Miner circumvents the difficulty he encountered in the erratic action of oil for short-time tests by lengthening the testing time, and apparently the effect of the time factor on the breakdown voltage is not taken into account in the final conclusions. His results for 10-min. hold tests are obviously more consistent than his instantaneous results, consistent enough to be mathematically expressed. It seems to me that this subterfuge may seriously mislead us if we are not on guard. Mr. Miner correctly recognized, (1), that the greatest variations obtained occur at small spacings, and (2), as shown under General Remarks on Test Results, that at short spacings, the breakdown voltage decreases with increased time of application of voltage. Is not the second phenomenon the reason for the first? Is it not likely that that condition which would really give a comparatively high breakdown for instantaneous voltage application, is brought to within the average breakdown by this extension of time?

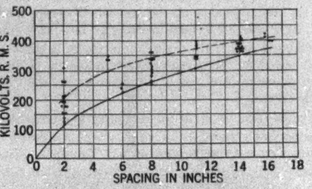

Fig. 3—Combined Curves of Breakdown Voltages for One-Inch Diameter Rods

Solid curve is from Fig. 8 of original paper and represents breakdown voltages for ten-minute hold.

Dash curve and circles are from Fig. 4 of original paper and is for instantaneous breakdown voltages.

The fallacy in ignoring the time effect is more readily appreciated when considering the behavior of a solid dielectric, such as oil-impregnated paper, under similar conditions. A number of apparently uniform dielectrics when given an instantaneous breakdown test, show very erratic results, perhaps more so than oil. Some will break down at voltages two or more times higher than others. However, every dielectric has a definite life at some particular voltage, and samples of one kind of dielectric show no more variable life than is shown by the instantaneous breakdown voltages. But, as is well known, the life of the dielectric varies inversely as the seventh power of voltage, and, hence at the high testing voltages, the life is rather small. If, therefore, instead of several seconds of voltage application, ten minutes is used, or, better still, five times ten minutes, then, of course, materials having variations of strength of 10 or even 20 to 1 will break down within that length of time, the weakest samples will break first, and the others will follow in the order of their strength. In other words even though all their variations in strength are still existent, the time was made long enough so that the life of almost all the samples will be spent at the voltage applied. We have, therefore, only apparently succeeded in obtaining uniform results by disregarding the important element of time.

That this is actually the case also with the oil can be seen from Mr. Miner's own findings. He finds that short-spacing breakdown will occur at smaller voltages if applied for longer time. At these smaller spacings, time is, therefore, an important factor. Now, by plotting Mr. Miner's curve of Fig. 8 on the same graph sheet with his curve of Fig. 4 retaining at the same time all the points showing the large variations at instantaneous breakdown we get the curve shown in the accompanying Fig. 3. From this combined figure it becomes immediately obvious that for the 10-min. hold, he used in every case a voltage lower or equal to the lowest voltage obtained at instantaneous breakdown, and by extending the time to five times ten minutes, succeeded in breaking down the oil that might otherwise show the highest instantaneous breakdown voltage.

The whole process seems to me to be akin to throwing away a micrometer because it shows up inaccuracies, and using instead a carpenter's rule to avoid detection of these inaccuracies. For it is evident that by making a time test the properties of the oil are not changed, and its molecular arrangement or arrangement of particles of impurities is not affected or controlled thereby. The varying instantaneous voltage breakdowns are merely replaced by varying lengths of time of breakdown at time-hold tests.

From the same combined curves too, it is hard to see how Mr. Miner could deduce that at large spacings length of time of voltage application does not affect voltage breakdown. Between instantaneous and 10-min. hold a difference of 10 per cent is seen at 16-in. spacing. Of course this is not as large as the 100 per cent at 2-in. spacing. It should be noted, too, that to get these changes, there was a change of length of time from a few seconds to many minutes or probably a ratio varying from 100-to-1 to 1000-to-1.

It is interesting to note also that at the larger spacings, where time is not as important an element, his results show practically the same variation for long as for short-time tests.

D. F. Miner: I was very much interested in Mr. Schurig's remarks about the erratic results obtained on large gaps compared to small gaps. It seems that we must define what we mean by "large" or "small." We found a great deal of variation in tests obtained at 2-in. spacing, which we in our tests thought was small. But if you call a gap of 0.1 in. or 0.2 in. small, then I certainly agree with him. If we had to put up with the divergence of results on small gaps in our standard oil-test cup that we find on real large gaps of several inches, we certainly would not complete very many standard oil tests per day.

There seems to be a good deal in the literature and in the data presented by Mr. Schurig as to the effect of large quantities of water on oil. In other words, if it doesn't improve the quality of oil, at least it doesn't harm it under certain conditions, but that would not be a good reason for recommending to operating people: "Dump water into the oil in your apparatus." The harm of the water in the oil is not in the decrease of the oil strength itself, but the effect on the insulation; it gets on the insulating surfaces and leads to creepage breakdowns.

I was very much gratified to have the confirmation of test results by Mr. Montsinger on breakdown formula for point gaps. I agree with him that the two-thirds power of separation is a much better figure to remember than 0.7. It appears that large-curvature electrodes, at large spacings, behave almost like needle gaps, as far as the actual voltage breakdown is concerned, but of course with sharp points or small electrodes, corona occurs at an earlier voltage. So that the reason for increasing the diameter of high-potential parts is to suppress ionization and corona rather than to increase the total breakdown voltage.

In connection with the argument as to whether tests should be made on commercial grades of dielectrics, such as oil, or whether we should purify our oil to the greatest possible extent and obtain data on that, there is a divergence of opinion. In this particular investigation we were interested in information that a design engineer could use. Now, it does not help him much to know that a certain gap once stood 300 kv. but the next time it might go down to 150. The 300-kv. value is not going to be of use. He has to design for minimum breakdowns, with a factor of safety then added to that. So that theoretical considerations in the true dielectric strength of oil did not enter into this problem. It was an engineering problem which was part of a series of tests on other electrodes besides spherical, that is, actual shapes occurring in apparatus, to develop design information.

Maxwell's Theory of the Layer Dielectric

BY FRANCIS D. MURNAGHAN[1]
Non-member
(with an Introduction by J. B. Whitehead)

Synopsis.*—Maxwell stated in his "Electricity and Magnetism" that for a non-homogeneous dielectric built up of n plane layers of varying thicknesses and varying ratios of resistivity to specific capacity, the charging current will be given by a linear differential equation of order n-1 involving the nth derivative of the applied e. m. f. This equation and its detailed solution are given here and the two cases of greatest practical interest—namely the case where the applied e. m. f. is constant and the case where it is a pure harmonic—are treated fully. By a Fourier analysis the general case can be reduced to these two.*

INTRODUCTION

The phenomenon of dielectric absorption has recently attracted renewed interest by reason of its importance in problems of the insulation of high voltage electrical circuits. It is recognized that absorption causes energy losses in the body of the insulation; these losses increase the temperature, leading to deterioration and final failure of the insulation.

No completely satisfactory theory of the nature of dielectric absorption has been proposed. In recent years a number have been offered invoking newly discovered physical phenomena in the fields of gaseous ionization, dissociation, and conductivity in liquids, and the electromechanics of atomic structure. These theories, for the most part, however, are mere speculations and none of them is subject to exact experimental test. The earliest theory proposed to account for dielectric absorption is that of Clerk Maxwell. It shares with all other theories the disadvantage that it has never been satisfactorily supported by experiment. It has nevertheless maintained its position as the most satisfactory theory yet offered, largely because it invokes only well recognized properties of dielectrics, namely, specific inductive capacity and conductivity, and makes no appeal to new properties nor any molecular nor sub-atomic phenomena.

All students of dielectric theory are familiar with Maxwell's treatment. He assumes a dielectric built up of a number of plane strata of different materials, stating that a medium formed of a conglomeration of small pieces of different materials would behave in the same way. He does not support this latter statement by further analysis, however. The most familiar manifestation of absorption is a sustained but continuously decreasing current when a continuous electromotive force is applied to a condenser having a solid dielectric. The same type of phenomenon occurs when the condenser is discharged. In this case it is the outflow of the familiar residual charge of the condenser. In the case of Maxwell's layer condenser, the different values of the specific capacity and resistivity in successive layers accounts for the relatively long time necessary for complete charge or discharge.

Many experimental studies have aimed to determine the law governing the gradual decay of the charging current of a condenser. Some of these have indicated a simple negative power of the time, others an exponential relation, and still others more complicated relationships. Maxwell did not extend his analysis to the derivation of the form of the function controlling the decay of the charging current of a layer dielectric nor, apparently, has this extension ever been made, under his theory, for the completely general case. It may be readily shown, however, that for the case of two layers the charging current decreases in accordance with a simple exponential function of the time with negative exponent. It has been generally assumed by supporters of Maxwell's theory that in the general case of any number of layers, or of a dielectric consisting of a mixture of any number of different materials, the complex form of the charging current curve is due to the superposition of a number of separate exponential terms with negative exponents. This assumption is supported by Maxwell's statement that for n layers of materials having different values of the ratio of resistivity to specific capacity there will result for the charging current a differential equation of order $n-1$, involving the nth order derivative of the applied electromotive force. Max-

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[1] Johns Hopkins University.

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well, however, does not derive the expression supporting this statement, and this omission has always constituted a difficulty for those seeking experimental means for testing the validity of his theory. Dr. Murnaghan has now supplied this deficiency and has derived in an elegant and satisfactory manner the expression for the charging current for the most general case of a layer dielectric. Equation (**14**) shows that this current is made up of a constant term, giving the final conduction current, plus $n - 1$ negative exponential terms of which the coefficients and constant parts of the exponents are given in terms of the constants of the various layers. This confirms Maxwell's statement and provides a ready means whereby one may use the convenient charging current as a basis for analysis and experimental check.

The extension of Maxwell's expressions to the alternating case has always involved great difficulty. Dr. Murnaghan has now made this extension in its most general form. His equations may be readily transformed to the simplest cases and they should offer frequent and convenient opportunity for application to experimental problems.

The National Research Council, through its division of Engineering and Industrial Research, has organized a Committee on Electrical Insulation. This committee during the past two years has been making a review of the widely scattered literature on dielectric behavior, in an attempt to point out the direction in which experiment may be prosecuted with the best chance of profitable result. As a result of conversations with the writer in connection with this review, Dr. Murnaghan has been good enough to undertake the analysis which he now presents and to consent to its being offered under the auspices of the committee mentioned above. The committee also has in progress reviews of our present knowledge of dielectric behavior from the standpoints of resistivity and dielectric strength, and in connection with all these reviews it is proposing problems for experimental research. Of particular interest in connection with Dr. Murnaghan's paper is a series of experimental investigations now under way at Johns Hopkins University on the origin and nature of dielectric absorption, one of the first phases of which will be an attempt to test the validity of Maxwell's theory. The investigation was suggested by the work of the Committee on Electrical Insulation, N. R. C., and is receiving substantial financial support from the Engineering Foundation.

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CONSIDERING unit cross-section of the plane strata of Maxwell's layer dielectric, let X_1, X_2, etc., be the electric intensities in the several strata; f_1, f_2, etc., the displacements; k_1, k_2, etc., the reciprocals of the specific inductive capacities; r_1, r_2, etc., the specific resistances. Then at any instant the conduction current in the first layer is X_1/r_1, while the displacement current in the same layer is $\dfrac{1}{4\,\pi\,k_1}\,\dfrac{d\,X_1}{d\,t}$, so that we have $u = (X_1/r_1) + (1/4\,\pi\,k_1)\,d\,X_1/d\,t$ where u is the current density in the outer circuit and so in each layer. Denoting the thicknesses of the various layers by a_1, a_2, etc., the e. m. f. across the first layer is $a_1\,X_1$, that across the second layer is $a_2\,X_2$, and so on for each layer. If there are altogether n layers, the total e. m. f. across the dielectric is connected with the various intensities by the relation

$$E = a_1\,X_1 + a_2\,X_2 + \ .\ .\ .\ + a_n\,X_n \qquad (1)$$

It is more convenient to deal with the displacements f than with the electric intensities X; on introducing the abbreviations

$$4\,\pi\,k_1/r_1 = b_1;\ 4\,\pi\,k_2/r_2 = b_2;\ .\ .\ .\ 4\,\pi\,k_n/r_n = b_n$$
$$4\,\pi\,k_1\,a_1 = \alpha_1;\ 4\,\pi\,k_2\,a_2 = \alpha_2;\ .\ .\ .\ 4\,\pi\,k_n\,a_n = \alpha_n \qquad (2)$$

the equations expressing the fact that the current density in each layer is the same, take the form

$$(D + b_1)\,f_1 = (D + b_2)\,f_2 = \ .\ .\ .\ = (D + b_n)\,f_n = u \qquad (3)$$

where, for convenience, we have used the symbol D for the sign of differentiation $d/d\,t$. Also equation (1) takes the form

$$\alpha_1 f_1 + \alpha_2 f_2 + \ .\ .\ .\ + \alpha_n f_n = E \qquad (4)$$

Solving for one of the displacements, say f_1, from (4) we have

$$\alpha_1 f_1 = E - \alpha_2 f_2 - \ .\ .\ .\ - \alpha_n f_n$$

and on substituting this in the equation $(D + b_1)\,f_1 = (D + b_2)\,f_2$ we obtain

$$[(\alpha_1 + \alpha_2)\,D + (\alpha_1\,b_2 + b_1\,\alpha_2)\,]\,f_2 + \alpha_3\,(D + b_1)\,f_3 + \ .\ .\ .\ + \alpha_n\,(D + b_1)\,f_n = (D + b_1)\,E$$

In addition to this equation connecting the $n - 1$ displacements $(f_2, f_3,\ .\ .\ .\ f_n)$ we have the $n - 2$ equations

$$\begin{aligned}(D + b_2)\,f_2 - (D + b_3)\,f_3 &= 0\\ (D + b_2)\,f_2 - (D + b_4)\,f_4 &= 0\\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ &\\ (D + b_2)\,f_2 - (D + b_n)\,f_n &= 0\end{aligned}$$

From the $n - 1$ equations now in our possession all but one of the displacements, say f_2, may be eliminated by the usual algebraic process. The determinant of the coefficients is quite simple; for convenience of writing let us take $n = 4$ although it will be quite obvious that the reasoning is general. The determinant of the coefficients being

$$\begin{vmatrix}(\alpha_1 + \alpha_2)D + (\alpha_1\,b_2 + \alpha_2\,b_1) & \alpha_3\,(D + b_1) & \alpha_4\,(D + b_1)\\ (D + b_2) & -\,(D + b_3) & 0\\ (D + b_2) & 0 & -\,(D + b_4)\end{vmatrix}$$

we expand it in terms of its first row. The result is

$$\alpha_1(D+b_2)\,(D+b_3)\,(D+b_4)+\alpha_2(D+b_1)\,(D+b_3)\,(D+b_4)+\alpha_3(D+b_1)\,(D+b_2)\,(D+b_4)+\alpha_4(D+b_1)(D+b_2)(D+b_3)$$

In the general case of n layers we may write the determinant of the coefficients of our equations conveniently in the form

$$(D + b_1)(D + b_2) \ldots (D + b_n)$$

$$\left[\frac{\alpha_1}{D + b_1} + \frac{\alpha_2}{D + b_2} + \ldots + \frac{\alpha_n}{D + b_n}\right] \quad (5)$$

This is a polynomial in the sign of differentiation D of degree $n - 1$, the coefficient of D^{n-1} being $(\alpha_1 + \alpha_2 + \ldots + \alpha_n)$. This sum of the α's occurs so frequently that it will be well to have a single symbol for it; denote it simply by α and write the determinant of the coefficients in the form $\alpha\,\varphi(D)$, so that $\varphi(D)$ is a polynomial of degree $n - 1$ in D, the coefficient of the highest power in D being unity. The equation for f_2 is, then,

$$\alpha\,\varphi(D)\,f_2 = (D + b_1)(D + b_3) \ldots (D + b_n)\,E$$

where the operator $(D + b_2)$ is missing from the product on the right. The equations for the other displacements are similar; they may be conveniently written in the symbolical form

$$\left[\frac{\alpha_1}{D + b_1} + \frac{\alpha_2}{D + b_2} + \ldots + \frac{\alpha_n}{D + b_n}\right] f_r = \frac{E}{D + b_r} \quad (6)$$

which means that the various denominators $(D + b)$ are first removed by multiplying through by the common denominator $(D + b_1)(D + b_2) \ldots (D + b_n)$. It follows at once from (6) and (3) that the current density u is given by the equation

$$\left[\frac{\alpha_1}{D + b_1} + \ldots + \frac{\alpha_n}{D + b_n}\right] u = E \quad (7)$$

or, in non-symbolic form,

$$\alpha\,\varphi(D)\,u = (D + b_1)(D + b_2) \ldots (D + b_n)\,E \quad (8)$$

Equation (8) is the equation to which Maxwell referred but which he rather unfortunately neglected to give explicitly. There is every probability that he arrived at it by the following reasoning; we may write the equations (3) in the symbolic form

$$f_r = u/(D + b_r)$$

and on inserting these values in (4) we arrive immediately at (7).

The equations (6) and (8) are linear differential equations with constant coefficients and are of the order $n - 1$ provided all the ratios of capacity to resistivity for the various layers are different; it is apparent from the symmetrical form of the left-hand side of (6) that the order in which the layers are supposed arranged is of no importance and if two of the b's, b_1 and b_2 let us say, are equal, we may combine the first two terms $\alpha_1/(D + b_1)$ and $\alpha_2/(D + b_2)$ of the left-hand side of (6) into the single term $(\alpha_1 + \alpha_2)/(D + b_2)$ and imagine that instead of the two layers 1 and 2 we have a single equivalent layer whose α is the sum of the two α's of the layers which it replaces. We shall suppose from this on, therefore, that all the b's are different. We shall also find it more convenient to consider the equations (6) rather than the equation (8) for the current density to which Maxwell called attention, the reason being that a determinate solution of a differential equation is not possible unless some information, such as the initial value of the unknown and its various derivatives, is at hand. Now the initial values of the displacements were shown by Maxwell to be all equal, the dielectric being supposed initially uncharged, his reasoning being as follows. From (3) we have, on integrating with respect to the time from 0 to Δt, results such as

$$\Delta f_1 + \int_0^{\Delta t} b_1 f_1\,dt = \Delta f_2 + \int_0^{\Delta t} b_2 f_2\,dt$$

each side being equal to the charge per unit area $\int_0^{\Delta t} u\,dt$ which flows through the dielectric in time Δt. If we let the time interval Δt tend to zero and assume that the displacements f remain finite during the sudden imposition of an e. m. f., we have the limiting equation $\Delta f_1 = \Delta f_2$, and if f_1 and f_2 were zero before the e. m. f. was applied this says that their initial values, *i. e.*, the values immediately after the imposition of the e. m. f., will be equal. If $E(0)$ denotes the initial value of the applied e. m. f., we find from (4) that the common initial value of the displacements is

$$f_0 = E(0)/\alpha \quad (9)$$

where

$$\alpha = \alpha_1 + \alpha_2 + \ldots + \alpha_n \quad (10)$$

We now proceed to solve the equations (6) in the particular case where a constant e. m. f. of amount unity is applied at time $t = 0$ to the uncharged dielectric, the initial values of the displacements f being accordingly $1/\alpha$. A possible solution of (6) in this case where $E = 1$ may be found by giving f_r the constant value $1/b_r\,(\sum_r \alpha_r/b_r)$ but this is not the solution of the problem before us since it does not give all the f's the same initial value. The difference between this value of f_r and the solution we need, however, will satisfy the homogeneous equation

$$\left[\frac{\alpha_1}{D + b_1} + \ldots + \frac{\alpha_n}{D + b_n}\right] f_r = 0,$$

$$\text{i. e.,} \quad \alpha\,\varphi(D)\,f_r = 0 \quad (11)$$

which, being a linear homogeneous differential equation with constant coefficients of order $n - 1$, has for its general solution a combination of $n - 1$ exponential functions of the time with arbitrary constant multipliers. To determine the exponents of the exponential functions we observe that since $\varphi(D)$ is a sum of n terms each of which has all the $(D + b_r)$ but one as a factor the value of $\varphi(-b_r)$ is $\alpha_r (b_1 - b_r)(b_2 - b_r) \ldots (b_n - b_r)/\alpha$. Now let us suppose the quantities $(b_1, b_2, \ldots b_n)$ arranged in ascending order of magnitude; then $\varphi(-b_1)$ *will* be positive since all its factors are positive but $\varphi(-b_2)$ will be negative since one of its factors, $(b_1 - b_2)$, is negative. $\varphi(-b_3)$ will be positive since two of its factors are negative, and so on. This tells us that the polynomial φ of degree $n - 1$ has all its zeros real and negative and that they lie in the intervals between the negative values of the b's. Let us denote these zeros by $(-\beta_1, -\beta_2, \ldots -\beta_{n-1})$ where the β's are supposed arranged in ascending order of magnitude. We have, then, the series of inequalities,

$$b_1 < \beta_1 < b_2 < \beta_2 < \ldots < b_{n-1} < \beta_{n-1} < b_n \qquad \textbf{(12)}$$

and the most general solution of (**11**) is a linear combination with constant coefficients of the exponential functions $\epsilon^{-\beta_1 t}, \epsilon^{-\beta_2 t} \ldots \epsilon^{-\beta_{n-1} t}$. Remembering the relation $(D + b_r) f_r = (D + b_s) f_s$ connecting any two of the f's and noting that $D(\epsilon^{-\beta_r t}) = -\beta_r \epsilon^{-\beta_r t}$, we write the general solution of (**11**) in the form

$$\frac{A_1}{(b_r - \beta_1)} \epsilon^{-\beta_1 t} + \ldots + \frac{A_{n-1}}{(b_r - \beta_{n-1})} \epsilon^{-\beta_{n-1} t}.$$

In other words, the most general solution of the equations (6) (with $E = 1$), which obeys the relations (**3**) is

$$f_r = \frac{1}{b_r \sum_r (\alpha_r/b_r)} + \frac{A_1}{(b_r - \beta_1)} \epsilon^{-\beta_1 t}$$

$$+ \ldots + \frac{A_{n-1}}{(b_r - \beta_{n-1})} \epsilon^{-\beta_{n-1} t} \qquad \textbf{(13)}$$

where the A's are arbitrary constants. All that is necessary to complete the solution is to so determine the values of these constants that the f's may all assume initially the common value $1/\alpha$. Before proceeding to do this we may remark that it follows at once from (**3**) and (**13**) that the current density u is given by the expression

$$u = 1/(\sum_r (\alpha_r/b_r) + A_1 \epsilon^{-\beta_1 t} + \ldots + A_{n-1} \epsilon^{-\beta_{n-1} t} \qquad \textbf{(14)}$$

where the A's have the numerical values we are about to determine. The first term in this expression is the value to which u tends as t increases indefinitely and gives what is known as the permanent or conduction current.

The equations which determine the numerical values of the A's are found by putting $f_r = 1/\alpha$ and $t = 0$ in (**13**). They are

$$A_1/(b_r - \beta_1) + A_2/(b_r - \beta_2) + \ldots + A_{n-1}/(b_r - \beta_{n-1})$$
$$= 1/\alpha - 1/\alpha_r (\sum_r \alpha_r/b_r) \qquad \textbf{(15)}$$

where r is to be assigned, in turn, the values $(1, 2, 3, \ldots n)$ so that there are n equations in all. As there are only $n - 1$ A's to be found, these equations cannot be independent of one another; in fact if we multiply the equation written above by α_r and add all the results obtained by giving r the values 1 to n, both sides of the total vanish identically, showing that a set of A's satisfying $n - 1$ of the equations will satisfy the equation remaining. We may confine our attention, therefore, to the first $n - 1$ of the equations (**15**). A direct solution of these $n - 1$ equations would not be very elegant, however, on account of the lack of symmetry involved in the omission of one of the equations and it is better to proceed as follows:

Consider the quotient of the two polynomials of degree n

$$(x + b_1)(x + b_2) \ldots (x + b_n)/x\,\phi(x)$$

where $\varphi(x)$ is the function defined by the statement that $\alpha\,\varphi(D)$ is the expression (**5**). This quotient can be written in the form $1 + \dfrac{g(x)}{x\,\varphi(x)}$ where $g(x) = (x + b_1) \ldots (x + b_n) - x\,\varphi(x)$ is a polynomial of degree less than n. From its very definition the quotient $g(x)/x\,\varphi(x)$ has the value -1 when x is assigned any one of the n values $(-b_1, -b_2, \ldots -b_n)$ and, furthermore, since the zeros of φ are the $n - 1$ numbers $-\beta_r$, we have $g(-\beta_r) = (b_1 - \beta_r)(b_2 - \beta_r) \ldots (b_n - \beta_r)$. Now the usual method of analysis of the quotient of two polynomials into a series of simple fractions[2] tells us that

$$\frac{g(x)}{x\,\varphi(x)} = \frac{g(0)}{x\,\varphi(0)} + \sum_r \frac{g(-\beta_r)}{-\beta_r\,\varphi'(-\beta_r)(x + \beta_r)}$$

where $\varphi'(x)$ denotes the derivative of $\varphi(x)$ with respect to x. Giving x the n values $-b_s$, in turn, we have the n equations

$$-\sum_r \frac{g(-\beta_r)}{\beta_r\,\varphi'(-\beta_r)(b_s - \beta_r)} = 1 - g(0)/b_s\,\varphi(0);$$

$$s = 1, 2, \ldots n. \qquad \textbf{(16)}$$

Now $g(0)$ has the value $b_1 b_2 \ldots b_n$ and $\varphi(0)$ has the value $b_1 b_2 \ldots b_n [\sum_r (\alpha_r/b_r)]/\alpha$, so that $g(0)/\varphi(0)$ has the value $\alpha/\sum_r (\alpha_r/b_r)$. On substituting this in (**16**) and comparing the result with (**15**) we see that the desired values of the A's are furnished by the formulas

$$A_r = -\frac{1}{\alpha}\left[\frac{g(-\beta_r)}{\beta_r\,\varphi'(-\beta_r)}\right]$$

$$= -(b_1 - \beta_r)(b_2 - \beta_r) \ldots (b_n - \beta_r)/\alpha\beta_r\,\varphi'(-\beta_r) \qquad \textbf{(17)}$$

Since

$$\varphi(x) = \frac{1}{\alpha}(x + b_1)(x + b_2) \ldots (x + b_n)$$

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[2] See note at end of paper.

$$\left[\frac{\alpha_1}{x + b_1} + \ldots + \frac{\alpha_n}{x + b_n}\right]$$

and since the factor in square brackets vanishes when $x = -\beta_r$, we have

$$\varphi'(-\beta_r) = -\frac{1}{\alpha}(b_1 - \beta_r)\ldots(b_n - \beta_r)$$

$$\left[\frac{\alpha_1}{(b_1 - \beta_r)^2} + \ldots + \frac{\alpha_n}{(b_n - \beta_r)^2}\right]$$

so that

$$A_r = \frac{1}{\alpha\,\beta_r\left[\dfrac{\alpha_1}{(b_1 - \beta_r)^2} + \ldots + \dfrac{\alpha_n}{(b_n - \beta_r)^2}\right]} \tag{17 bis}$$

This form is not particularly suited for purposes of calculation but it has the advantage of showing that all the constants A_r are positive. The expression for the initial value of the current, *i. e.*,

$$\frac{1}{\sum\limits_r \dfrac{\alpha_r}{b_r}} + A_1 + A_2 + \ldots + A_{n-1}$$

follows directly from the equations (**3**). Denoting initial values of the subscript zero, we have $(D f_1)_0 = u_0 - b_1 f_0; \ldots (D f_n)_0 = u_0 - b_n f_0$ and on substituting these in the equation $\sum \alpha_r (D f_r)_0 = (D E)_0$ obtained by differentiating (**4**) and then setting $t = 0$, we find

$$\alpha\, u_0 - (\sum_r \alpha_r\, b_r) f_0 = (D E)_0$$

so that

$$u_0 = \frac{(D E)_0}{\alpha} + \frac{(\sum_r \alpha_r b_r)\, E(0)}{\alpha^2} \text{ by (9)}$$

In the case of a constant unit e. m. f. we have

$$u_0 = \frac{\sum_r \alpha_r . b_r}{\alpha^2}$$

Upon substitution of the values given in (**17**) in the expressions (**13**) and (**14**), the displacements and current density at any instant are determined and the problem may be regarded as completely solved. We proceed to give an example of the method of procedure in the simple case of two layers.

When $n = 2$, $\varphi(D)$ is the simple linear expression $D + (\alpha_1 b_2 + \alpha_2 b_1)/\alpha$. Hence there is only one exponential term and the value of β_1 is

$$\beta_1 = (\alpha_1 b_2 + \alpha_2 b_1)/(\alpha_1 + \alpha_2) \tag{18}$$

It is rather noteworthy that when the thickness of one of the layers gets very small so that α_1, say, tends to zero, the value of β_1 tends to the definite limit b_1. For numerical calculations it is well to express β_1 *directly* in terms of the capacities and conductivities of the layers. In Wagner's notation[3] we have $b_1 = 4\pi k_1 \lambda^1 = \gamma \lambda_1/\epsilon_1$; $\alpha_1 = \gamma a_1/\epsilon_1$ where $\gamma = 36\pi . 10^{11}$ is a constant depending on the choice of units. Hence β_1 has the form $\gamma(a_1 \lambda_2 + a_2 \lambda_1)/(a_1 \epsilon_2 + a_2 \epsilon_1)$; the reciprocal of β_1 measures the time taken for the transient part $A_1 \epsilon^{-\beta_1 t}$ of the current to reduce to the fraction $1/\epsilon$ of its original value, and is called the time constant of the dielectric. For two equally thick layers the time constant has the expression

$$T = (\epsilon_1 + \epsilon_2)/\gamma(\lambda_1 + \lambda_2)$$

while in general if the second layer is m times as thick as the first one, $T = (m\,\epsilon_1 + \epsilon_2)/\gamma(m\,\lambda_1 + \lambda_2)$. For paper ϵ_1 may be taken around 2 while λ_1 is around 10^{-11} and for glass the values are around 8 and 10^{-15} respectively. If $m = 1$, so that the layers are equally thick, $T = 0.09$ sec., while if $m = 2$, so that the layer of glass is twice as thick as the layer of paper, $T = 0.05$ sec.; if $m = 1/2$ so that the layer of paper is twice as thick as the layer of glass, $T = 0.16$. The larger the ratio of the thickness of the paper to that of the glass the larger is the time constant, the theoretical limit when $m = 0$ being $\gamma\,\epsilon_2/\lambda_2$ which is about 700 sec.

The constant A_1 is given by (**17**); since $\varphi' = 1$ it is $(\beta_1 - b_1)(b_2 - \beta_1)/\alpha\,\beta_1$ which reduces to

$$A_1 = \alpha_1 \alpha_2 (b_2 - b_1)^2/(\alpha_1 + \alpha_2)^2 (\alpha_1 b_2 + \alpha_2 b_1)$$

$$= a_1 a_2 (\lambda_2 \epsilon_1 - \lambda_1 \epsilon_2)^2/(a_1 \epsilon_2 + a_2 \epsilon_1)^2 (a_1 \lambda_2 + a_2 \lambda_1)$$

$$= (\lambda_2 \epsilon_1 - \lambda_1 \epsilon_2)^2/(a_1 \epsilon_2 + a_2 \epsilon_1)^2 \left(\frac{\lambda_1}{a_1} + \frac{\lambda_2}{a_2}\right) \tag{19}$$

It will be observed that if the thickness of one of the layers, say the first, tends to zero, A_1 also tends to zero on account of the term λ_1/a_1 in the denominator; if, however, λ_1 is very small in comparison with λ_2, it is possible that a_1 will be small in comparison with a_2 without the term λ_1/a_1 becoming significant.

In Wagner's notation, the constant term in the expression (**14**) for the current has the form $1/(\sum_r a_r/\lambda_r)$ which is, for the two-layer dielectric, $\lambda_1 \lambda_2/(a_1 \lambda_2 + a_2 \lambda_1)$. The initial value of the current is $\dfrac{\alpha_1 b_1 + \alpha_2 b_2}{(\alpha_1 + \alpha_2)^2}$ or $\dfrac{(a_1 \lambda_1 \epsilon_2^2 + a_2 \lambda_2 \epsilon_1^2)}{(a_1 \epsilon_2 + a_2 \epsilon_1)^2}$; for the case of two layers of equal thickness d this is $\dfrac{\lambda_1 \epsilon_2^2 + \lambda_2 \epsilon_1^2}{d(\epsilon_1 + \epsilon_2)^2}$.

SECTION 2. THE ALTERNATING CASE

Here we suppose an e. m. f. of the form $E = e^{i\omega}$ applied at time $t = 0$. Since $(D + b_n) e^{i\omega t} = (i\omega + b_n) e^{i\omega t}$ and so on, our equations (**6**) for the various displacements take the form

$$\alpha\,\phi(D) f_r = \frac{(i\omega + b_1)\ldots(i\omega\, b_n)}{(i\omega + b_r)} e^{i\omega t}$$

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[3] See article in *Die Isolierstoffe der Electrotechnik* edited by Schering, H., pp. 1-59. Berlin, 1924.

A particular solution of these equations, which is consistent with the equations (**3**), is

$$f_r = \frac{B\, e^{i\omega t}}{i\,\omega + b_r} \tag{20}$$

where the constant B is determined by the equation

$$\alpha\, \phi\,(i\,\omega)\, B = (i\,\omega + b_1)\,.\,.\,.\,(i\,\omega + b_n) \tag{21}$$

i. e.,

$$B = 1/\left[\frac{\alpha_1}{i\,\omega + b_1} + \frac{\alpha_2}{i\,\omega + b_2} + \,.\,.\,.\, + \frac{\alpha_n}{i\,\omega + b_n}\right] \tag{21bis}$$

The general solution of the equations (**6**) is of the type

$$f_r = \frac{B\, e^{i\omega t}}{i\,\omega + b_r} + \frac{B_1\, e^{-\beta_1 t}}{b_r - \beta_1} + \,.\,.\,.\, + \frac{B_{n-1}\, e^{-\beta_{n-1} t}}{b_r - \beta_{n-1}} \tag{22}$$

where the factors β in the exponents are the same as before and the $(B_1, B_2, \,.\,.\,.\, B_{n-1})$ are arbitrary constants. The values to be assigned to these must be such that the initial values of the displacements f are all $E\,(0)/\alpha$, (see (**9**)); or, since here $E\,(0) = 1$, $\frac{1}{\alpha}$. The $n-1$ constants $(B_1, \,.\,.\,.\, B_{n-1})$ must therefore satisfy the n linear equations

$$\frac{B_1}{b_r - \beta_1} + \frac{B_2}{b_r - \beta_2} + \,.\,.\,.\, + \frac{B_{n-1}}{b_r - \beta_{n-1}} = \frac{1}{\alpha} - \frac{B}{b_r + i\,\omega};\; r = 1, 2, \,.\,.\,.\, n \tag{23}$$

That these equations are not independent follows at once on multiplication by $(\alpha_1, \,.\,.\,.\, \alpha_n)$ and addition when both sides vanish. To find the values of the B's, consider the analysis of the quotient $(x + b_1)\,.\,.\,.\,(x + b_n)/(x - i\,\omega)\,\varphi\,(x)$ into its simple fractions. Writing it in the form $1 + \frac{g\,(x)}{(x - i\,\omega)\,\varphi\,(x)}$ where $g\,(x)$ is of lower degree than n, we find

$$\frac{(x + b_1)\,.\,.\,.\,(x + b_n)}{(x - i\,\omega)\,\varphi\,(x)} = 1 + \frac{(b_1 + i\,\omega)\,.\,.\,.\,(b_n + i\,\omega)}{\varphi\,(i\,\omega)\,(x - i\,\omega)} - \sum_r \frac{(b_1 - \beta_r)\,.\,.\,.\,(b_n - \beta_r)}{(\beta_r + i\,\omega)\,\varphi'\,(-\beta_r)\,.\,(x + \beta_r)}$$

Setting $x = (-b_1, -b_2, \,.\,.\,.\, -b_n)$ in turn in this identity, we obtain the n relations

$$\sum_r - \frac{(b_1 - \beta_r)\,.\,.\,.\,(b_n - \beta_r)}{(\beta_r + i\,\omega)\,\varphi'\,(-\beta_r)\,(b_s - \beta_r)} = 1 - \frac{\alpha\, B}{b_s + i\,\omega};\; s = 1\,.\,.\,.\,n;$$

and on comparing these with (**23**) we see that the $n - 1$ quantities $(B_1, \,.\,.\,.\, B_{n-1})$ are given by the formulas

$$B_r = -\frac{1}{\alpha}\,.\,\frac{(b_1 - \beta_r)\,.\,.\,.\,(b_n - \beta_r)}{(\beta_r + i\,\omega)\,\varphi'\,(-\beta_r)} \tag{24}$$

From this it follows that

$$B_r = A_r \frac{\beta_r}{\beta_r + i\,\omega} \tag{25}$$

where the A's are the constants of the unit e. m. f. case whose values have been given in (**17**).

If we introduce the time constants T_r which are the reciprocals of the β_r, our equation (**25**) takes the form

$$B_r = A_r/(1 + i\,\omega\, T_r) \tag{25bis}$$

The two-layer dielectric in an alternating field.

Here the equation for the constant B is

$$B = \frac{(i\,\omega + b_1)\,(i\,\omega + b_2)}{(\alpha_1\, b_2 + \alpha_2\, b_1) + (\alpha_1 + \alpha_2)\, i\,\omega} = \frac{(i\,\omega + b_1)\,(i\,\omega + b_2)}{(\alpha_1\, b_2 + \alpha_2\, b_1)\,(1 + i\,\omega\, T)} \tag{26}$$

This is but a particular case of the general formula (see (**21**))

$$B = \frac{(i\,\omega + b_1)\,.\,.\,.\,(i\,\omega + b_n)}{\alpha\, \phi\,(i\,\omega)} = \frac{(i\,\omega + b_1)\,.\,.\,.\,(i\,\omega + b_n)}{\alpha\,(i\,\omega + \beta_1)\,.\,.\,.\,(i\,\omega + \beta_{n-1})} = \frac{(i\,\omega + b_1)\,.\,.\,.\,(i\,\omega + b_n)}{b_1\,.\,.\,.\,b_n\left(\sum_r \frac{\alpha_r}{b_r}\right)(1 + i\,\omega\, T_1)(1 + i\,\omega\, T_2)\,.\,.\,.\,(1 + i\,\omega\, T_{n-1})} \tag{27}$$

Since

$$\alpha\, \beta_1\,.\,.\,.\,\beta_{n-1} = \alpha\, \varphi\,(0) = b_1\,.\,.\,.\,b_n\left(\sum_r \frac{\alpha_r}{b_r}\right)$$

In Wagner's notation (**26**) takes the form

$$\frac{\left(\lambda_1 + i\frac{\omega\, \epsilon_1}{\gamma}\right)\left(\lambda_2 + i\frac{\omega\, \epsilon_2}{\gamma}\right)}{(a_1\, \lambda_2 + a_2\, \lambda_1)\,(1 + i\,\omega\, T)}$$

or

$$\frac{\left(\lambda_1 + i\frac{\omega\, \epsilon_1}{\gamma}\right)\left(\lambda_2 + i\frac{\omega\, \epsilon_2}{\gamma}\right)}{a_1\, \lambda_2 + a_2\, \lambda_1 + i\,\omega\, \frac{a_1\, \epsilon_2 + a_2\, \epsilon_1}{\gamma}}.$$

Similarly, the general expression (**27**) for n layers becomes

$$B = \frac{\left(\lambda_1 + i\,\omega\,\frac{\epsilon_1}{\gamma}\right)\ldots\left(\lambda_n + i\,\omega\,\frac{\epsilon_n}{\gamma}\right)}{\lambda_1\ldots\lambda_n\left(\sum_r \frac{a_r}{\lambda_r}\right)(1+i\,\omega\,T_1)\ldots(1+i\,\omega\,T_{n-1})} \qquad (27^{bis})$$

In the two-layer cases the constant B_1 in the complete expression

$$u = B\,e^{i\omega t} + B_1\,e^{-\beta_1 t} = B\,e^{i\omega t} + B_1\,e^{-t/T}$$

has the value

$$B_1 = \frac{A_1}{1+i\,\omega\,T} = \frac{(\lambda_2\,\epsilon_1 - \lambda_1\,\epsilon_2)^2}{(a_1\,\epsilon_2 + a_2\,\epsilon_1)^2\left(\frac{\lambda_1}{a_1} + \frac{\lambda_2}{a_2}\right)(1+i\,\omega\,T)}$$

from (19) (28)

When $\omega = 0$ the expression (27^{bis}) reduces to the conduction current $\frac{1}{\sum_r \frac{a_r}{\lambda_r}}$ under a constant unit e. m. f.

If the dielectric were uniform, the electric intensity would be $\frac{1}{\sum_r a_r}$ and so we write $\lambda = \frac{\sum_r a_r}{\sum_r \frac{a_r}{\lambda_r}}$ and call it the effective conductivity. For very high frequency we see from (21) that B tends to $\frac{i\,\omega}{\alpha}$ *i. e.*, $\frac{i\,\omega}{\gamma \sum_r \frac{a_r}{\epsilon_r}}$.

Writing $\epsilon = \frac{\sum_r a_r}{\sum_r \frac{a_r}{\epsilon_r}}$ for the capacity constant of the dielectric for very high frequencies, we may consider the difference

$$(\Sigma\,a_r)\,B - \left(\lambda + i\,\omega\,\frac{\epsilon}{\gamma}\right) = \bar{\lambda} + i\,\omega\,\frac{\bar{\epsilon}}{\gamma},$$ say, as

due to the lack of homogeneity of the dielectric. Writing B in the form

$$B = \frac{(i\,\omega + b_1)\ldots(i\,\omega + b_n)}{\alpha\,\phi\,(i\,\omega)}$$

the difference in question is the product of $\sum_r a_r$ and

$$\frac{(i\,\omega + b_1)\ldots(i\,\omega + b_n)}{\alpha\,\phi\,(i\,\omega)} - \frac{b_1\ldots b_n}{\alpha\,\phi\,(0)} - \frac{i\,\omega}{\alpha}$$

On reducing to a common denominator the numerator lacks the term in ω_n and also the constant term. Thus in the case $n = 2$ where $\phi\,(i\,\omega) = i\,\omega + \beta_1 = i\,\omega + \frac{1}{T}$

the numerator is $i\,\omega\left[\frac{b_1 + b_2}{T} - \frac{1}{T^2} - b_1\,b_2\right]$ or

$\frac{i\,\omega\,\alpha_1\,\alpha_2\,(b_2 - b_1)^2}{(\alpha_1 + \alpha_2)^2}$ since $\frac{1}{T} = \frac{\alpha_1\,b_2 + \alpha_2\,b_1}{\alpha_1 + \alpha_2}$.

Hence $\bar{\lambda} + i\,\omega\,\frac{\bar{\epsilon}}{\gamma}$

$$= \frac{(a_1 + a_2)\,i\,\omega\,\alpha_1\,\alpha_2\,(b_2 - b_1)^2}{(\alpha_1 + \alpha_2)^2\,(\alpha_1\,b_2 + \alpha_2\,b_1)\left(i \quad + \frac{1}{T}\right)}$$

$$= \frac{(a_1 + a_2)\,i\,\omega\,\alpha_1\,\alpha_2\,(b_2 - b_1)^2}{(\alpha_1 + \alpha_2)\,(\alpha_1\,b_2 + \alpha_2\,b_1)^2\,(1 + i\,\omega\,T)}$$

In Wagner's notation this is

$$\frac{i\,\omega\,(a_1 + a_2)\,(\lambda_2\,\epsilon_1 - \lambda_1\,\epsilon_2)^2}{\gamma\,(1 + i\,\omega\,T)\,(a_1\,\lambda_2 + a_2\,\lambda_1)^2\left(\frac{\epsilon_1}{a_1} + \frac{\epsilon_2}{a_2}\right)}$$

Writing this in the form $\frac{i\,\omega\,\epsilon\,.\,k}{\gamma\,(1 + i\,\omega\,T)}$ the "absorption constant" k has the value

$$k = \frac{a_1\,a_2\,(\lambda_2\,\epsilon_1 - \lambda_1\,\epsilon_2)^2}{\epsilon_1\,\epsilon_2\,(a_1\,\lambda_2 + a_2\,\lambda_1)^2} \qquad (29)$$

Writing $a_2 = m\,a_1$ this becomes $\frac{m\,(\lambda_2\,\epsilon_1 - \lambda_1\,\epsilon_2)^2}{\epsilon_1\,\epsilon_2\,(\lambda_2 + m\,\lambda_1)^2}$; regarded as a function of m it has a maximum value when $m = \frac{\lambda_2}{\lambda_1}$, the corresponding value of k being $\frac{(\lambda_2\,\epsilon_1 - \lambda_1\,\epsilon_2)^2}{4\,\epsilon_1\,\epsilon_2\,\lambda_1\,\lambda_2}$. If the dielectric consists of paper with impurities in the form of moisture, we may set $\lambda_1 = 10^{-11}$, $\epsilon_1 = 2$ (paper) and $\lambda_2 = 10^{-4}$, $\epsilon_2 = 80$ and k becomes 0.025 m.

We may indicate, in conclusion, some of the calculations for the three-layer case although some of the formulas become quite complicated. Here there are two exponential terms and $-\beta_1$, $-\beta_2$ are the zeros of the quadratic polynomial

$$\begin{aligned}\alpha\,\phi\,(x) = (\alpha_1 + \alpha_2 + \alpha_3)\,x^2 + \{\,&\alpha_1\,(b_2 + b_3) \\ &+ \alpha_2\,(b_3 + b_1) + \alpha_3\,(b_1 + b_2)\,\}\,x \\ &+ \alpha_1\,b_2\,b_3 + \alpha_2\,b_3\,b_1 + \alpha_3\,b_1\,b_2\end{aligned}$$

We have $\phi\,(i\,\omega) = (i\,\omega + \beta_1)\,(i\,\omega + \beta_2) = (1 + i\,\omega\,T_1)\,(1 + i\,\omega\,T_2)/T_1\,T_2$; the numerator of the expression

$\bar{\lambda} + i\,\omega \dfrac{\bar{\epsilon}}{\gamma}$ has two terms, one involving ω and the other ω^2. On using the relations

$$\frac{1}{T_1\,T_2} = \frac{\alpha_1\,b_2\,b_3 + \alpha_2\,b_3\,b_1 + \alpha_3\,b_1\,b_2}{\alpha_1 + \alpha_2 + \alpha_3};$$

$$\frac{1}{T_1} + \frac{1}{T_2} = \frac{\alpha_1(b_2+b_3)+\alpha_2(b_3+b_1)+\alpha_3(b_1+b_2)}{\alpha_1 + \alpha_2 + \alpha_3}$$

we find

$$\bar{\lambda}+i\,\omega\frac{\bar{\epsilon}}{\lambda} = \frac{\dfrac{i\,\omega\,(a_1 + a_2 + a_3)\,[L + M\,i\,\omega]}{(\alpha_1+\alpha_2+\alpha_3)(\alpha_1\,b_2\,b_3+\alpha_2\,b_3\,b_1+\alpha_3\,b_1\,b_2)^2}}{(1 + i\,\omega\,T_1)\,(1 + i\,\omega\,T_2)}$$

where

$$L = \alpha_1\,\alpha_2\,b_3^2\,(b_1 - b_2)^2 + \alpha_2\,\alpha_3\,b_1^2\,(b_2 - b_3)^2 + \alpha_3\,\alpha_1\,b_2^2\,(b_3 - b_1)^2$$

$$M = \alpha_1\,\alpha_2\,b_3\,(b_1 - b_2)^2 + \alpha_2\,\alpha_3\,b_1\,(b_2 - b_3)^2 + \alpha_3\,\alpha_1\,b_2\,(b_3 - b_1)^2.$$

We hope in a second paper to give a rigorous proof of the Boltzmann-Hopkinson Principle of Superposition, from which all the formulas given above for the alternating case can be deduced, and to apply it to the case $E = t$ where an e. m. f. with constant time gradient is applied.

Note on the analysis of a quotient of two polynomials into simple fractions.

The quotient $G(x)/F(x)$ where $G(x)$ is a polynomial of degree less than n and $F(x) = (x - x_1)\ldots(x - x_n)$ may be written in the form

$$\frac{G(x)}{F(x)} = \frac{A_1}{x - x_1} + \ldots + \frac{A_n}{x - x_n}$$

where $(A_1, \ldots A_n)$ are constants to be determined. To find them we multiply up by $x - x_1$, say, and then let $x = x_1$ giving

$$\frac{G(x_1)}{(x_1 - x_2)\ldots(x_1 - x_n)} = A_1.$$ Now $F(x) = (x - x_1)\,H(x)$, say, where $H(x) = (x - x_2)\ldots(x - x_n)$ and so the ordinary rule for the derivative of a product gives

$$F'(x) = H'(x)\,.\,(x - x_1) + H(x)$$

Whence

$$F'(x_1) = H(x_1) = (x_1 - x_2)\ldots(x_1 - x_n)$$

Hence

$$A_1 = \frac{G(x_1)}{F'(x_1)}$$ and, in general, $$A_r = \frac{G(x_r)}{F(x_r)}.$$

We have supposed here, as is the case in the text, that no two of the numbers $(x_1, x_2, \ldots x_n)$ are equal.

Discussion

Vladimir Karapetoff: The phenomenon of absorption in a composite dielectric, each layer of which separately shows no absorption, has been graphically explained by Grünewald[1]. Referring to the accompanying sketch, consider the simplest case of two slabs $A\,B\,C\,D$ and $C\,D\,E\,F$, of equal thickness and of equal insulation resistance to direct current. Let the material I be of much lower permittivity than the material $I\,I$. Let $A\,B$, $C\,D$, $E\,F$, represent very thin layers of metal foil, to insure the positions of equipotential surfaces. Assume the metal sheet $A\,B$ grounded at G and a d-c. potential of value $N\,F$ suddenly applied to the surface $E\,F$. The instantaneous distribution of potential may then be represented by the broken line $H\,K\,F$. The permittivity of the layer I being lower, a major portion, $S\,K$, of the total voltage is applied across it, and a minor portion, $K\,C$, across $I\,I$. The slopes of the two lines are inversely as the permittivities, or directly as the elastivities of the two materials.

At the first instant, there can be no electric charge on the surface $C\,D$, and the displacements of electricity through both dielectrics must be equal to each other. The line $P\,R$ indicates this condition, the ordinate $H\,P$ representing the displacement of electricity or the charge on the plates. It will be seen that the positive and negative charges on the plate $C\,D$ cancel each other, leaving equal and positive charges on $A\,B$ and $E\,F$.

Now, let the same voltage remain applied indefinitely, causing a conduction current through the dielectrics. Since, by assumption, the resistivities of the two materials are equal, the final distribution of potential will be represented by the straight line $H\,L\,F$. The voltage gradient in the material I has been reduced in the ratio of $L\,S$ to $K\,S$, and consequently the displacement of electricity will be reduced in the same ratio, say from $H\,P$ to $H\,P^{I}$. The potential gradient in $I\,I$ has been increased from $C\,K$ to $C\,L$, and the displacement increased in the same ratio, say from $N\,R$ to $N\,R^{I}$. Thus, the middle plate $C\,D$ receives a positive charge $S\,Q^{I}$ and a negative charge $S\,T^{I}$, or a net negative charge $Q^{I}\,T^{I}$. An electroscope connected to $C\,D$ would indicate this charge, corresponding to the difference $K\,L$ in the voltages at the first instant and under steady-state conditions.

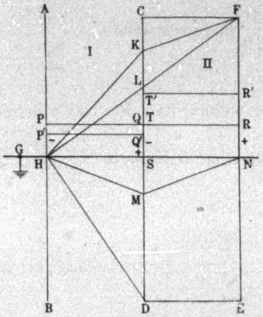

Fig. 1

Let the potential $N\,F$ be suddenly removed and the plate $E\,F$ immediately grounded. The new distribution of potential will then be represented by the line $H\,M\,N$, the potential of the middle plate being negative because it carries a negative charge. Numerically, $S\,M = L\,K$. The charge on $C\,D$ will be equal to $-\,Q^{I}\,T^{I}$, and that on each of the grounded plates $+\,0.5\,Q^{I}\,T^{I}$. Should the plate $E\,F$ be left grounded indefinitely, the charge $Q^{I}\,T^{I}$ would ultimately leak to the ground in both directions.

Let the plate $E\,F$, however, be grounded for an instant only, and then an equal and opposite voltage, $N\,E$, applied to it.

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[1] *Arch. für Elek.*, 1923, Vol. 12, p. 98.

The new instantaneous distribution of potential will be $H\ D\ E$, where $M\ D = S\ K$. In other words, the new distribution of potential may be considered as a result of superposition of the distribution $H\ K\ F$ in the neutral state and of the distribution $H\ M\ N$ due to the accumulated charge on $C\ D$.

It will thus be seen that the new distribution involves a greater stress in the material I than without absorption. Moreover, the absorbed change can reach $C\ D$ only through conduction which involves heat loss. The materials themselves have no property of absorption, but because the ratio of their conductivities is different from that of the permittivities, a residual charge, alternately positive and negative, accumulates on the dividing surface as the applied voltage changes its sign. This graphical interpretation may be carried further, to include more layers or layers of different thicknesses and different resistivities. Some years ago, the writer designed a kinematic model to show dielectric absorption according to Grünewald's representation. Certain linkages were connected through a spring and a dash-pot. At the first instant, as the point N on the linkage was suddenly raised to F, the linkage assumed the shape $H\ K\ F$. Then the spring gradually moved point K to L, overcoming the resistance of the dash-pot. Moving the point F suddenly to E gave the configuration $H\ D\ E$. Some parts of the model were made but the work was never completed due to a pressure of other investigations.

While Maxwell's theory of absorption has considerable methodological value, its practical usefulness is limited by the fundamental assumption that the layers which constitute a dielectric possess no absorption when tested separately. This may be true of such pure materials as white paraffin and xylol, but dielectrics used in practise show considerable absorption even in their constituent parts, such as paper, oil, mica leaves, etc. It is, therefore, not clear how to apply Maxwell's theory to a given case of say molded insulation, built-up mica, paper and oil, etc.

For this reason, it seems necessary for the present to assume the existence of the phenomenon of absorption in a given piece of insulation as a fact, and simply try to express mathematically the observed behavior, having made plausible assumptions about the shape of the functions expressing variations of current and flux density with the time. This method is used in the writer's paper on the subject presented at the 1926 Winter Convention.[2]

D. W. Roper: I should like to comment upon the necessity for such fundamental research. The industry at the present time is forging ahead of the science in cable dielectrics, particularly in high-voltage cables, and in this country that means paper-insulated cables. We are not yet able to test a cable and predetermine with reasonable accuracy its operating characteristics. A cable which has passed all of the requirements of the specifications agreed upon by the manufacturers and committees of the various associations will fail in service, and sometimes within a few months after being placed in service. Two instances of that sort have occurred within the past year. They were not confined to any one operating company, nor to any one manufacturer. In each case, after one or two failures had occurred, the manufacturers looked over their factory records and found evidence which caused them to be suspicious of certain other sections of cable that had been sent out from their factories, so they arranged with their customer to remove and replace certain other sections that had not yet failed, but which the manufacturers thought would probably fail, and the expense of the change was borne by the manufacturers. Now there is perfectly definite evidence that the manufacturer, from his factory records, can tell something about the cable that was not shown by any test that we know how to make, either in the factory or after it is placed in service.

We can make life tests on cable, and if we do, we get a fairly definite line between the voltage and the time. If we plot the logarithm of the kilovolts against the logarithm of time, we obtain a straight line, and for the types of cables that we are getting nowadays the kilovolts vary inversely as the seventh root of the time. Now, if you test that cable for a part of its life, there is no way of which we know at the present time to determine what fraction of that life has been spent, except by looking at the clock and the voltmeter. There is no electrical test that can be made to determine deterioration.

The lower part of that curve is of great interest because if you simply extend that curve, you get a finite time for zero voltage; but we know that if you get the voltage low enough, the time becomes infinitely long.

We have that problem before us continually. As a company increases in size and builds new generating stations, it scatters them about the territory which it supplies, so as to reduce the transmission distances, among other things, and that means that every time a new station is built, it is necessary to take out of service some cable which is no longer of value in its original location and remove it to some new location. As the voltages increase and we operate a little closer to the limit, there is every prospect that we shall at some future time be taking out of service cable which has lost so much of its life that it would hardly pay to reinstall it in a new location, but there is no way of telling that except to go ahead and install it and then replace it if it fails.

There is one other point: In connection with the cable failures in Chicago and the determination of their cause, we found it expedient to train some of our younger engineers in methods of systematic and careful inspection and analysis of cable in the vicinity of the failure and at points on the same section remote from the failure, so as to tell whether the cause of the trouble was one which existed throughout the entire length or which was local. A local case of trouble might for example, be due to irregular impregnation,—spotty impregnation, as the manufacturers call it. As these young engineers have competent supervision and instruction, you might say that their ability is proportional to their experience, and along certain lines the circumstances have been such that they had ample experience.

A few years ago we had some trouble with the joints on 33-kv. 3-conductor cables, and we had to remake the joints so as to use a different joint-filling compound. The one used was unsuitable for the purpose. In remaking these joints, we carefully examined the ends of the factory insulation in the joint, in some cases cutting back a few inches or a few feet, if necessary, and in a few cases replacing entire sections, because examination disclosed the evidence of ionization, and when those evidences of ionization were sufficiently definite and numerous throughout the insulation at one point, we removed that cable either by cutting it back in the manhole or by replacing the entire section. That gave us a very good opportunity for getting an average idea of the insulation of the entire line and from the evidence so gathered, the engineers making the inspection predicted the failure of the line very shortly from ionization. They reported that the life had been well spent at the operating voltage. The prediction was verified within six months.

The point to which I wish to call attention is that we can get this evidence and this prediction from a *visual examination*, but we cannot make any tests whatever, that we know now how to make, which will warrant the same prediction. We are now making, I believe, all the tests that we know how to make, on the data that we have available, and apparently what we need is more fundamental research that will enable us to make more tests and determine some of these features that we can now determine by visual examination or other methods which are not electrical tests.

Joseph Slepian: I should like to raise the question as to how much actual information about dielectrics such mathematical investigations can give. Essentially, Dr. Murnaghan shows,

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2. (A. I. E. E. TRANS., 1926, Vol. 45, p. 124.)

following Maxwell, that if a dielectric is composed of n layers of dielectric with different dielectric constants and resistivities the relation between voltage and current is one which satisfies an nth order differential equation and hence there must be a certain final relation between current and voltage. But it does not follow that if for any particular dielectric the voltage and current do satisfy such a relationship that the dielectric must be composed of n layers.

They are infinitely many hypothetical structures of the dielectric which will give the same relationship between voltage and current. For example, if we insist on explaining the behavior of the dielectric by inhomogeneities, we will have equally great success by imagining the inhomogeneities to be in the form of little spheres. Wagner, I believe, has carried out a mathematical investigation of such a dielectric. A year ago Professor Karapetoff also considered the mathematical theory of a dielectric with what he called particles, each particle having dielectric properties expressed by means of a first order of differential equations.[3] By suitable hypotheses as to the distribution of these particles, any relationship between voltage and current satisfying the principle of superposition may be derived.

The fact that an actual dielectric has voltage-current relationships which correspond to the results obtained by such analyses throws no light upon the structure of the dielectric itself at all, since all these hypotheses lead to the same results. By merely measuring voltage and current no information can be obtained which permits one to decide which hypothesis, if any, is the actual truth.

Hence I want particularly to point out the need of other kinds of information than relations between current and voltage. Before we can attach importance to any inhomogeneity theory, we must somehow show by other means the existence of the inhomogeneities which the theory postulates. In this connection I am glad to mention the work of Professor Joffé of Leningrad, whom I was fortunate enough to hear not long ago in Pittsburgh. Professor Joffé described some experiments upon crystals of rock salt in which, a priori, you wouldn't expect any inhomogeneity at all. In spite of this apparent homogeneity Professor Joffé did observe absorption, and after voltage had been applied to the crystal long enough for the absorption current to die down to nearly its zero value, he investigated the distribution of the potential through the crystal. By a very skillful technique which consisted of shaving away with insulated knives exceedingly thin layers of the dielectric he found that the distribution of potential through this dielectric consisted of a very small gradient through the body of the material with almost all of the potential concentrated in thin layers immediately next to the electrodes. Thus he showed the actual existence of a layer, next to the one electrode which had such different characteristics from the rest of the dielectric as to cause the voltage to be highly concentrated on that layer. The most significant thing about Joffé's experiment is that the layer in question had its peculiar properties only because of its relation to the electrode. If after the layer was cut off voltage was applied to the remaining crystal a concentration of potential upon the layer next to the electrode would again be obtained. The inhomogeneity which is active here is not in the material to begin with. It is an inhomogeneity which is produced by the electric current itself. For an understanding of the phenomena in dielectrics, it is not sufficient to treat the dielectric as if it had a simple ohmic conductivity. The nature of the carriers of current must be considered and account taken of the space charges which these carriers produce as a result of the flow of current.

In the absence of evidence other than the voltage-current relationships, the layers of Maxwell, spheres of Wagner, particles of Karapetoff, must be considered as mathematical fictions or conveniences, or simply individual preferences as to the manner of describing the phenomenon that is going on. Personally, I prefer to say that the homogeneous dielectric has its own complex properties rather than to assume a hypothetical complex heterogeneous structure, built up of parts having hypothetically simple dielectric properties.

Donald Bratt: In regard to the physical conditions of the problem, it is well to remember that Maxwell's idea was nothing more than to show that there is no such thing as "absorption" of charge in a dielectric. He did show, that the phenomena known as "absorption" are the result of lack of homogeneity and confined himself to the simple case of two plane condensers in series, subjected to a sudden application of a d-c. voltage. He further indicates, that the nature of these phenomena remains the same even if the dielectrics assume a geometrical shape other than plane. The proof of this statement could, perhaps, be given in strict mathematical language, there is nothing however to make one believe that the phenomena in physics should change their *nature* by changing the geometrical configuration of the boundaries. In all problems where phenomena obey for instance the well-known Laplace differential equation a purely mathematical transformation exists, which will refer any boundary to the cubical element on which we base our physical laws, as well as our calculus. Maxwell did not, apparently, consider it necessary to emphasize this.

It is not safe to base an extended analysis on Maxwell's work, for the further reason that Maxwell omitted the magnetic permeability from this problem to get a simpler analysis of the initial displacements due to a suddenly impressed d-c. voltage. He found these displacements all equal initially. Had Maxwell wanted to go a little further, it would have been necessary for him to emphasize that there cannot be any such thing as a displacement until the wave starting from one end of the dielectric has had time to reach the other end.

The idea of a wave does not, of course, occur unless magnetic permeability is introduced.

The term "dielectric absorption" is doubtless derived from a supposed analogy with heat. It was this idea Maxwell tried to refute by showing how different were the laws governing dielectric phenomena from those governing the flow of heat. On page 458 (Vol. I) in his work Maxwell says (referring to his above-mentioned analysis): "The object of the investigation is merely to point out the true mathematical character of the so-called electric absorption and to show how fundamentally it differs from the phenomena of heat which seem at first sight analogous."

To give a general solution of a stratified condenser, consider first the conventional picture of a dielectric: a non-inductive resistance in parallel with an ideal condenser. Should a d-c. voltage be suddenly impressed on such a condenser, the initial rush of current would be infinite. The same would happen if several such condensers were connected in series. To avoid this difficulty it is necessary to make certain restricting assumptions on the form of the impressed voltage, as will be shown below.

Take, now, Dr. Murnaghan's equation (7)

$$\left[\frac{\alpha_1}{D+b_1} + \cdot\cdot\cdot\cdot\cdot \frac{\alpha_n}{D+b_n}\right] u = E$$

which I prefer to solve for the current, writing, symbolically

$$i = \frac{E\ (t)}{\dfrac{1}{g_1 + p\,c_1} + \dfrac{1}{g_2 + p\,c_2} + \cdot\ \cdot\ \dfrac{1}{g_n + p\,c_n}}$$

where i is the total charging current at any time t $E\ (t)$ is the external voltage impressed, and may for the moment be assumed to be perfectly unrestricted.

g_k and C_k are the conductance and capacity of the kth condenser.

p is the differential operator $\dfrac{d}{d\,t}$ (Heavisides notation)

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[3] *Theory of Absorption in Solid Dielectrics*, by V. Karapetoff, A. I. E. E. Trans., 1926, p. 124.

Re-arranging, we obtain an expression

$$i = \frac{A\ (p)}{B\ (p)} E\ (t)$$

where A and B are algebraic polynomials in p; A being 1 degree *higher* than B.

To reduce still further, perform the division $\frac{A\ (p)}{B\ (p)}$; we get

$$\frac{A\ (p)}{B\ (p)} = M\ p + \frac{G\ (p)}{H\ (p)}$$

where M is a constant directly obtained by the division, G and H are polynomials in p of the *same* degree (*i. e.* $(n-1)$ if there are n condensers in series.)

According to the Heaviside Expansion Theorem, we now have

$$\frac{G\ (p)}{H\ (p)} = K_0 + K_1 \frac{p}{p - p_1} + \ldots\ K_{n-1} \frac{p}{p - p_{n-1}}\text{; where}$$

$$K_0 = \frac{G\ (o)}{H\ (o)}$$

$p_1, p_2 \ldots p_{n-1}$ are the roots of $H\ (p) = 0$ which are all negative and real, no two roots being alike, by assumption.

$$K_r = \frac{1}{p_r} \cdot \frac{G\ (p_r)}{\left[\frac{\partial}{\partial p} H\ (p)\right]_r}$$

so that

$$i = \left[M\ p + K_0 + K_1 \frac{p}{p - p_1} + \ldots\ K_{n-1} \frac{p}{p - p_{n-1}} \right] E\ (t)$$

The operation $\frac{p}{p - p_r}$ performed on $E\ (t)$ gives a solution

$$\frac{p}{p - p_r} E\ (t) = E\ (t) + e^{p_r t}\, p_r \int_o^t e^{-p_r u} E\ (u)\ d\ u$$

Further

$K_0\ E\ (t)$ means nothing but $K_0\ E\ (t)$ since K_0 is a constant

$M\ p\ E\ (t)$ means $M \cdot \frac{d\ E\ (t)}{d\ t}$

so that the solution can be written

$$i = M\ d \frac{E\ (t)}{d\ t} + E\ (t) \left[K_0 + \sum_1^{n-1} K_r \right]$$

$$+ \sum_1^{n-1} e^{p_r t}\, p_r \int_o^t e^{-p_r u} E\ (u)\ d\ u$$

The displacement f, of the rth condenser would be,

$$f_r = \frac{i}{A\left[\frac{1}{r_r} + p \cdot \frac{1}{4\ \pi\ k_r}\right]}$$

which solves into

$$f_r = \frac{4\ \pi\ k_r}{A} e^{-\frac{4\pi k_r}{r_r} t} \int^t e^{-\frac{4\pi k_r}{r_r} t}\ i\ d\ t$$

and can be calculated when i is known.

It should be noticed, that these solutions for i and f, are perfectly general, inasmuch as the voltage $E\ (t)$ has not been subject to any restrictions.

Our immediate concern, as physicists, would now be to check the theory by experiment. It would then be a question of the best form to adopt for the test voltage $E\ (t)$, and particularly the form of $E\ (t)$ near $t = 0$.

There are three principal forms that may occur:

(1) D-c. voltage is entirely inadmissable, as it would introduce wave motion that has not been considered above.

(2) Alternative (sine-wave) voltage is better, but gives a finite value of current for $t = 0$ in the formula above, and is therefore not recommended.

(3) Voltage, possessing zero derivative at $t = 0$ should be used, for instance $E\ (t) = E_0\ (1 - \cos\ \omega\ t)$.

In other words: an a-c. voltage super-imposed on a d-c. voltage so as to produce a gradual smooth rise of voltage at the first moments would probably be the best to use when testing the theory.

To sum up: The weak spots in any theory of the dielectric omitting the magnetic permeability will be found around discontinuous points in the external voltage-curve or its first derivative. In particular it seems inadmissible from a physical standpoint to base a general solution on the solution for a suddenly impressed d-c. voltage.

Vladimir Karapetoff: I am glad that Mr. Bratt brought up the question of discontinuity in the current at the first instant. This discontinuity is not confined to Dr. Murnaghan's paper; any problem on a combination of resistances and capacitances leads to a similar inconsistency if you omit the magnetic flux. The system then has no inertia, and theoretically the current rises instantly from zero to a finite value.

W. B. Kouwenhoven: We are carrying on an investigation of Maxwell's theory at Johns Hopkins University under the auspices of the Engineering Foundation. We are endeavoring to obtain two or more perfect dielectrics having no absorption but different conductivities and dielectric constants. We will then make a mixture of these two dielectrics and if Maxwell's theory is correct the resulting mixture will show absorption. This work is still in its early stages and we are not ready to report any results as yet, but simply to say that we are making progress.

A. F. Puchstein: (communicated after adjournment) Dr. Murnaghan uses the concepts of the physicist rather than those of the engineer. Would it not have been easier for the engineering reader if the set-up had been based on the idea of capacitances instead of electric intensities and displacements?

On this basis, the same value of current passes through all of the dielectrics, then for a two-layer arrangement

$$i_1 = i_2 \qquad (1)$$

Since each layer may be regarded as a condenser shunted by a resistance we have the relations,

$$i_1 = c_1 \frac{d\ e_1}{d\ t} + \frac{e_1}{r_1} \qquad (2)$$

$$i_2 = c_2 \frac{d\ e_2}{d\ t} + \frac{e_2}{r_2} \qquad (3)$$

in which e_1 and e_2 are the potential drops across the different layers; r_1 and r_2 are the insulation resistances; and c_1 and c_2 are the capacitances of the condensers formed by each layer.

In addition, the total potential drop e is the sum of the several drops, and this sum is at all times equal to the internal voltage E of the supply source minus its internal $i\ r$ drop. It is assumed that the system is initially uncharged. From this we have

$$e = e_1 + e_2 = E - i\ r \qquad (4)$$

At this point, the theory here given diverges from that of Dr. Murnaghan in that he assumes the displacements at the first moment to remain finite, with no further restriction, except that initially $\Delta f_1 = \Delta f_2$, while here a limit is set by the $i\ r$ drop in the supply source. The letter f stands for displacement.

There is an obscurity of statement where we read, "* * * the polynomial φ degree $n - 1$ has all its zeros real and negative and that they lie in the intervals between the negative values of *b*'s. Let us denote these zeros by $(-\beta_1, -\beta_2, \ldots -\beta_{n-1})$***."

This may be correct, but it carries no meaning to one not initiated into its mysteries.

To solve the above equations, equate (**2**) and (**3**), combine (**2**) and (**4**), use D in place of $\frac{d}{d\,t}$, and solve for e_1 when

$$e_1 = \frac{E}{r}\,\frac{c_2\,D + \dfrac{1}{r_2}}{\dfrac{1}{r}\left(c_1\,D + \dfrac{1}{r_1}\right) + \left(c_2\,D + \dfrac{1}{r_2}\right)\left(c_1\,D + \dfrac{r_1 + r}{r\,r_1}\right)} \tag{5}$$

Equation (**5**) may be solved by the rules of operational calculus or by the rules for solving simultaneous equations. The latter method is used here. We may solve for both or either e_1 and e_2, and obtain the other by interchanging the subscripts. This last is the simplest and will be used here.

If we put

$$a = r\,c_1\,c_2, \tag{6}$$

$$b = c_1\left(1 + \frac{r}{r_2}\right) + c_2\left(1 + \frac{r}{r_1}\right) \tag{7}$$

$$c = \frac{r + r_1 + r_2}{r_1\,r_2} \tag{8}$$

then equation (**5**) becomes, since E is independent of time,

$$e_1 = \frac{E/r_2}{a\,d^2 + b\,D + c} \tag{9}$$

The roots of the denominator in (**9**) are

$$m_1 \text{ and } m_2 = -\frac{b}{a} \pm \sqrt{\left(\frac{b}{2\,a}\right)^2 - \frac{c}{a}} \tag{10}$$

and the solution,

$$e_1 = A\,\epsilon^{m_1 t} + B\,\epsilon^{m_2 t} + \frac{E}{r_2\,c} \tag{11}$$

where A and B are the constants of integration. To determine these, we notice that e_1 is zero when t is zero, then from (**11**)

$$0 = A + B + \frac{E}{r_2\,c} \tag{12}$$

Also, when t is zero, from (**2**) and (**4**),

$$i = i_1 = \frac{E}{r} = c_1\,\frac{d\,c_1}{d\,t}. \tag{13}$$

Substituting e_1 from (**11**) in (**13**),

$$\frac{E}{c_1\,r} = A\,m_1 + B\,m_2. \tag{14}$$

Solving (**12**) and (**14**),

$$A = \frac{E\,(r_2\,c - m_2\,r\,c_1)}{r_1\,r_2\,c\,c_1\,(m_2 - m_1)} \tag{15}$$

$$B = -\frac{E}{r_2\,c}\left\{\frac{r_2\,c - m_2\,r\,c_1}{r_1\,c_1\,(m_2 - m_1)} + 1\right\} \tag{16}$$

To obtain e_2, interchange all subscripts 1, 2, except those of m_1 and m_2.

This theory is easily extended to the case of n layers and to alternating currents, though the solutions are more complicated.

F. D. Murnaghan: I have been very interested in the various comments on my paper but shall confine myself here to some remarks on Mr. Puchstein's communication since it is almost entirely mathematical in character. His idea of allowing for the internal resistance in the supply source is quite interesting and may be at once cared for by the general method of my paper. The E in equation (**7**) is now to be replaced by $E - r\,u$ so that our fundamental equation for the current is

$$\left[\frac{\alpha_1}{D + b_1} + \frac{\alpha_2}{D + b_2} + \ldots + \frac{\alpha_n}{D + b_n} + r\right] u = E$$

It is now of the nth order instead of the $(n-1)$th as before. In the case of two layers the determining equation for the exponents is

$$r\,D^2 + D\left\{r\,(b_1 + b_2) + \alpha_1 + \alpha_2\right\} + r\,b_1\,b_2 + \alpha_1\,b_2 + \alpha_2\,b_1 = 0$$

Our α's are the reciprocals of Mr. Puchstein's *c*'s while our *b*'s are the reciprocals of the product of his *c*'s and *r*'s, *i. e.*, $b_1 = \frac{1}{c_1\;r_1}$ etc. This equation checks, on substitution of these values, with his results (**6**), (**7**), (**8**). The same method takes care of the general case of n layers.

Space Charge and Current in Alternating Corona

BY C. H. WILLIS[1]

Associate, A. I. E. E.

***Synopsis.**—1. The physical nature of ionization in a corona discharge in air is studied; by means of the corona spectrum, the saturation current in air around the corona voltage, and the influence of the material of the wire on the saturation current. The results indicate that nitrogen only is ionized in a corona discharge in air; that the ionization of the nitrogen results in the separation of an electron from the nitrogen molecule; and that the electron quickly attaches to a molecule or group of molecules, probably water or oxygen, to form an ion.*

2. The free charge in the neighborhood of the wire called the space charge, is found to be alternating in character and to have a definite boundary. The space charge formed on any half-wave returns to the wire on the next succeeding half-wave.

3. The mobility of the ions is calculated from the boundary of the space charge. A limiting value of about 10 cm./sec. per volt/cm. is indicated for the positive ions. The negative ions show no sign of a limiting value and the mobility varies from about 1.6 to 10 cm./sec. per volt/cm. as the maximum impressed voltage rises from the corona voltage to twice the corona voltage.

4. Ionization is found to occur at lower voltages on the positive half-waves than on the negative half-waves and the ionization on the positive half-waves becomes much more copious with the beginning of ionization on the negative half-waves.

5. A formula for the corona current based on theoretical considerations, is developed by the aid of certain empirical assumptions. This formula gives excellent agreement with the observed currents measured in large cylinders. A calculation of the corona current for a 100-mile three-phase transmission line gives a satisfactory agreement with the values of current as measured by W. W. Lewis.

* * * * *

INTRODUCTION

IN 1913 J. S. Towsend published an explanation of corona on the basis of ionization by collision,[20] which has been widely accepted. In developing this theory Towsend assumed ionization by collision for both electrons and ions. However, recent work on the ionizing ability of charged molecules [31, 32, 49] has led to considerable doubt as to the possibility of any action by the positive ions in corona. Also recent studies of corona[46] have developed a number of anomalies in the behavior of gases under corona conditions. Among these anomalies may be mentioned the facts that: *a*, oxygen forms corona at a gradient below that required by nitrogen, but in the air the oxygen takes no part in the corona discharge,[42, 46] *b*, no simple relation seems to exist between the ionizing potential and the corona gradient. For instance, helium with an ionizing potential of 24.5 volts forms corona at a gradient much below that required by nitrogen with an ionizing potential of 17.0 volts.[44, 46] (The ionizing potential is the voltage through which an electron must drop unimpeded to gain sufficient energy to produce ionization.) A number of such instances can be pointed out and the results are no more rational if ionization is assumed to take place in two stages, first excitation and then ionizing the excited molecules.

It is the purpose of this investigation† to obtain further experimental evidence about the physical nature of the breakdown of the gases in corona and to obtain information about the space charges formed by the corona which may lead to analytical expressions for the corona current, corona loss, and extra capacity due to corona.

DESCRIPTION OF APPARATUS

The electrode arrangement chosen was a wire and large plane, as this gives a representation of half of a two wire transmission line, and permits an easy adjustment of the distance between electrodes. See Fig. 1. *P P* represents a plane 270 cm. by 420 cm. made of copper screen wire, number 16 mesh. *E E* represents a copper electrode 30 cm. by 91.5 cm. and about 0.2 cm. back of the plane. *W W* represents the corona wire 355 cm. long. The sizes of wire used and the distances used between the wire and plane, which are represented by *c*, are given in the corresponding data.

The potential source was a 5-kv-a. 60-cycle, 125-volt motor-driven generator, connected to a 10-kv-a., 120/240-100,000-volt, 25-cycle transformer. The diagram of connections is shown in Fig. 2. The transformer was used only in the 240-100,000 volt connection.

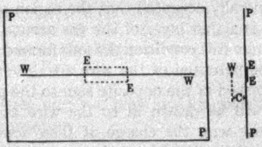

FIG. 1

The electrode *E E* is connected to a battery through a galvanometer *G* and a protective resistance *R*. By making *E E* positive with respect to the grounded plane *P P*, a portion of any negative ionic charge arriving at the plane will be drawn through the meshes of the wire to the electrode, and cause a deflection of the galvanometer *G*. By reversing the potential of the electrode *E E*, a portion of the positive ions arriving at the screen will be drawn to the electrode *E E* and cause a deflection of the galvanometer *G* in the opposite direction. This is essentially the same method for detecting the corona charges as that described by Whitehead in 1912.[13]

The potential on the wire was regulated by resistance

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[1] Princeton University, Princeton, N. J.

†This work was carried out at Johns Hopkins University.

20. For references see Bibliography.

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in the generator field. This method may cause variations in the wave form but these were not serious over the range of voltage used as shown by oscillograms. In order to maintain constant voltage the field and driving motor of the generator were supplied from a storage battery.

The high-tension voltage was in most cases determined by measuring the primary voltage. The ratio of transformation was determined by a potential transformer, and as the load was small and almost constant this ratio was assumed to hold for all measurements.

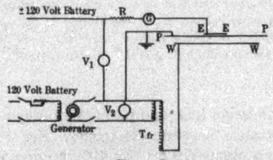

FIG. 2

All voltmeters used were of the dynamometer type and were calibrated on direct-current by a potentiometer and standard cell. The galvanometer was a Leeds and Northrup type *N* instrument with a period of 20 seconds and a sensitivity of 370 megohms with the scale as used.

THEORY OF SPACE CHARGE

(Wire and Concentric Cylinder)

It is generally accepted that the region of ionization in corona is a thin layer of the gas around the corona wire. Under this condition the ions formed of the same sign as the potential of the wire are driven out while the ions formed of the opposite sign to the potential on the wire will be drawn in to the wire at once. On reaching the wire the charge of these ions flows immediately through the conductor of the potential source to the opposite electrode. It is the flow of this charge that constitutes the corona current.

During the time that the charge of opposite sign to the potential of the wire is passing from the region of ionization to the wire and then to the opposite electrode the charge of the same sign as the potential of the wire moves very little because the charges move much more slowly through air than through a conductor. The charge of the same sign as the potential of the wire is then left as a space charge immediately surrounding the wire when the charge of the opposite sign reaches the opposite electrode. The energy represented by the corona current flowing through the high tension circuit is now stored as an electric displacement between the space charge and the charge on the opposite electrode. See Fig. 3A.

So far the corona wire and the opposite electrode have behaved just as a condenser but there is this important difference; the space charge is free to move under the action of the electric field, and, as the space charge does move out, the energy stored in it is dissipated, reappearing in other forms.

If the source of potential is continuous this space charge will reach the opposite electrode and all of the energy will be dissipated. However, if the source of potential is alternating the space charge may or may not have time to reach the opposite electrode, before the potential reverses, depending on the frequency and the distance between the electrodes.

Under these conditions the behavior of the space charge may be accurately represented by a circuit composed of a resistance in series with a capacity. Fig. 3A represents the condition just after a layer of space charge has been formed, and Fig. 3B represents the condition after this layer has moved out a distance from the wire. It must not be assumed that the entire space charge is formed instantly and in one layer for its formation starts when the voltage passes the corona voltage and continues certainly past the crest of the voltage wave. Figs. 3A and 3B refer to any particular layer of the space charge.

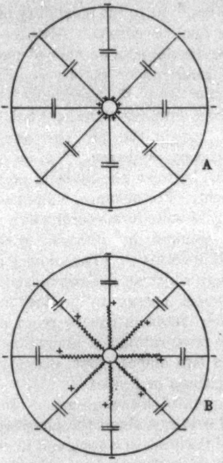

FIG. 3

An ion formed at any point on a voltage wave will have the time of the remainder of that wave for its outward journey, and the entire succeeding wave in which to return. All ions will therefore have more time in which to return to the wire than they have for their outward path, since ionization does not begin until the instantaneous voltage has risen to the corona voltage. If then we neglect diffusion and the slowing down of ions due to the formation of molecular aggregates, all of the space charge which does not reach the opposite electrode before the voltage reverses should return to the wire at which it was formed during the voltage half-wave succeeding that on which it was formed.

The returning space charge of course meets the outgoing space charge and any recombination between these two is equivalent to a return of the space charge to the wire. Recombination therefore increases the probability of the return of all of the space charge to the wire.

The corona loss is due to the movement of the space charge through the electric field surrounding the wire. We may concentrate our attention on any average ion, of the space charge. Beginning just after its formation at the surface of the wire, and assuming that the ion has the average velocity of thermal agitation this ion will be accelerated by a force due to the electric field. As the ion moves through a free path its velocity increases due to the acceleration. At the first collision between the ion under consideration and a neutral molecule, the ion imparts to the neutral molecule, a portion of the energy gained from the electric field, even though the collision be perfectly elastic. However, the ion will start its second free path with a velocity greater than the average velocity of thermal agitation and therefore will have a higher velocity at the end of its second free path than at the end of its first free path. It will therefore lose more energy in the second collision. In this manner the velocity of the ion rises above the mean velocity of thermal agitation, to a value, called the terminal velocity,[40] such that the ion loses as much energy at a collision as it gains during a free path. (The ion requires a number of collisions to reach this terminal velocity, and the terminal velocity of course varies with the field in which the ion travels.)

If the terminal velocity of the ion attains a value sufficient to ionize a neutral molecule, ionization occurs and the ion loses practically all of its energy, and must then start the process of building up its terminal velocity again. From this it is evident that as the space charge moves through the electric field, the energy stored in the space charge is converted into heat in the elastic collisions between ions and molecules, or in case of an inelastic collision may produce a disturbance of the atomic structure resulting in ionization, or sometimes only the radiation of light. This conversion of electric energy into heat, light, and free charge constitutes the corona loss.

In the case the space charge does not reach the opposite electrode, some of the energy stored in the space charge is not dissipated because the ions do not travel through the entire potential difference. The energy stored in the space charge and not dissipated by the ions as they travel out is returned to the system and results in the extra capacity effect observed in corona.

Spectrum of Corona

The spectrum of the corona light was photographed by means of a sensitive spectroscope through the courtesy of Dr. R. W. Wood. The spectrum showed the band spectrum of nitrogen but no trace of the oxygen spectrum even at a voltage twice the corona voltage. These spectrograms were unfortunately destroyed by a fire in the Physics Laboratory. However, the results were in accord with the spectra of the arc in nitrogen as studied by K. T. Compton[39] and the point discharge in air as studied by U. Yoshida and H. Hirata.[42] They have therefore not been repeated.

It has been found by K. T. Compton[39] that the the dissociation of nitrogen occurs at a much higher voltage than that required for ionization and that the dissociation is accompanied by the appearance of the line spectrum. The corona spectrum, a band spectrum, therefore indicates that the ionization in corona is the breaking off of electrons from the neutral molecule and not a dissociation of nitrogen molecules. The results of the corona ionization are electrons and charged nitrogen molecules. The absence of the oxygen spectrum indicates that the nitrogen alone is ionized, or that the oxygen does not play any appreciable part in the corona ionization.

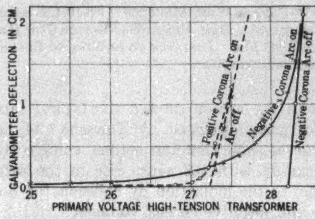

Fig. 4—Magnesium Wire Mercury Arc 15 Cm. above Wire, Temperature 26 Deg. Cent., Barometer 76 Cm., Wire Diameter 1.14 Mm. Distance from Wire to Plane 4.0 Cm. Transformer Ratio 401.

Saturation Current in Air Near the Corona Voltage

In order to obtain further information on the physical nature of the ionization in corona, the saturation current of the air was measured as the voltage approaches the corona value. Whitehead,[11] and Lee and Kurrelmeyer[46] found no effect on the corona voltage due to ionization of the air by external means. A quartz mercury arc was therefore used to increase the conductivity of the air and give an appreciable saturation current.

It was found that a quartz arc placed about 10 cm. from the wire and in a position central to the electrode *E E* would cause sufficient ionization to give an appreciable deflection of the galvanometer due to the charge drawn through the plane. With the electrode *E E* positive and the wire close to the plane, the alternating high-tension voltage was raised by small steps past the corona voltage, and the galvanometer deflections observed at each step. These galvanometer deflections give a measure of the space charge formed on the voltage half-waves when the wire is negative. These observations were repeated with the mercury arc illuminated, care being taken to have the arc illuminate the side of the wire nearest the plane. A typical set of observations on a magnesium wire are shown in Fig. 4. In a similar manner observations were made on the space charge produced during the half waves when the wire was positive. These results are also shown in Fig. 4.

A measurement of the lower portions of the current curve with the arc on showed the typical saturation curve for gases discussed by Towsend (34, page 2). These curves show that the corona voltage coincides approximately with the sharp rise in the saturation curve due to the ionization by collision for both the positive and negative corona voltages.

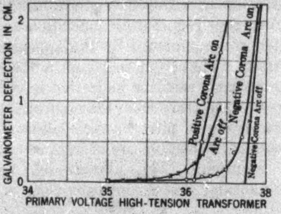

FIG. 5—COPPER WIRE MERCURY ARC 10 CM. FROM WIRE. TEMPERATURE 15.1 DEG. CENT., BAROMETER 77.0 CM., WIRE DIAMETER 1.583 MM. DISTANCE FROM WIRE TO PLANE 6.0 CM. TRANSFORMER RATIO 401.

However, the curve with a negative wire starts to rise at much lower voltages than the curve with a positive wire. This has been interpreted as indicating that with the wire negative the photoelectrons emitted by the wire cause ionization but with the wire positive the action is due to the charges coming in from the ionized gas. These incoming charges come from a lower gradient and have attached to molecules, forming ions. They, therefore, do not cause ionization at such low gradients nor so copiously as the electrons. This conclusion is substantiated by the following observations.

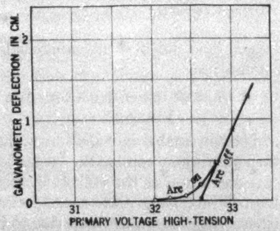

FIG. 6—NEGATIVE CORONA ON COPPER WIRE. WIRE SCREENED FROM ARC. ARC 10 CM. FROM WIRE. TEMPERATURE 16.9 DEG. CENT., BAROMETER 76.1 CM., DIAMETER OF WIRE MM. 1.583 DISTANCE FROM WIRE TO PLANE 4 CM. TRANSFORMER RATIO 401

In Fig. 5 we have curves taken on a copper wire with the arc directly in front of the wire so that it did not shine on the side of the wire nearest the plane. The crossing of the curves for positive wire with the arc on and arc off is probably due to the fact that the arc caused the brass wires in the plane between the wire and electrode (see Fig. 1), to emit negative electrons and therefore reduced the number of positive ions drawn through the plane to the electrode. In the other cases shown the arc did not shine directly at the plane behind the wire and this effect was therefore greatly reduced so that it became inappreciable.

The two curves with the arc on, shown in Fig. 5, are quite similar and indicate that the ionizing agent was the same type of carrier in both cases. Since there is no positive carrier of the dimensions of an electron we must either assume that ionization in both cases was due to ions of molecular dimensions, or that we have electrons liberated from the metallic surface in the case of the negative wire. Recent experiments on the ionizing ability of molecular ions have not shown any evidence of ionization by charged molecules.[32, 49] However, Horton and Davies[31] have shown that the action attributed to the positive ions may be due, in some cases, to the emission of electrons by the metal electrodes under the bombardment of the positive ions. If this be the case in corona the saturation curve should rise much more rapidly for a magnesium wire than for a copper wire when the wire is not illuminated by the arc. To test this conclusion curves 6 and 7 were taken for negative corona on copper and magnesium wires respectively.

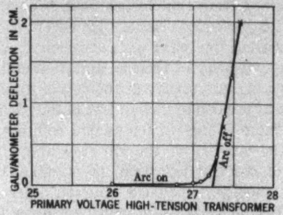

FIG. 7—NEGATIVE CORONA ON MAGNESIUM WIRE. WIRE SCREENED FROM ARC. ARC 10 CM. FROM WIRE. TEMPERATURE 26 DEG. CENT., BAROMETER 76.0 CM., DIAMETER OF WIRE 1.14 MM. DISTANCE FROM WIRE TO PLANE 4.1 CM. TRANSFORMER RATIO 401

In addition to having the arc behind the wires so it did not illuminate the side of the wire next to the plane, the wires were screened from the rays of the arc by a glass tube one cm. in diameter and with a two-mm. wall. These results are shown in Figs. 6 and 7.

The ordinates for the curve on magnesium wire are not materially different from those on copper. While this evidence is negative and may not be considered incontrovertible, it seems highly probable that the corona gradient is a field intensity where the positive molecular carriers become active ionizing agents as Towsend has assumed. The ionizing action of molecular carriers is quite unknown and probably entirely different in nature from the action of electrons. This is a possible cause of the anomalies in the behavior of the different gases.

BREAK IN THE DISCHARGE CURVE FOR POSITIVE CORONA

As previously shown, when corona forms at the crest of the voltage wave, the ions produced have a quarter period for their outward journey and the next succeeding half period in which to return. Under these conditions all of the space charge produced in any half wave returns during the next half wave and there is a definite boundary beyond which none of the space charge is found. (See Figs. 14 and 15.) Typical curves of the galvanometer deflections (proportional to the charge reaching the plane) against voltage are shown in Figs. 8 and 9. Fig. 8 gives the type of curves found when the wire is close to the plane. Fig. 9 shows the curves found when the wire is distant from the plane.

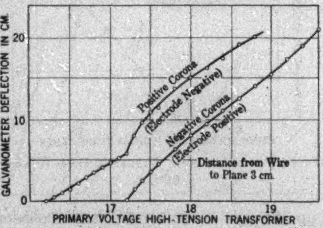

FIG. 8—CORONA DISCHARGE CURVES. TEMPERATURE 21.6 DEG. CENT., BAROMETER 75.4 CM., FREQUENCY 59.5 CYCLES. WIRE DIAMETER 0.440 MM. TRANSFORMER RATIO 401

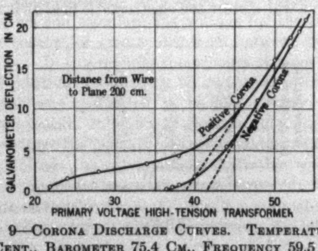

FIG. 9—CORONA DISCHARGE CURVES. TEMPERATURE 24.4 DEG. CENT., BAROMETER 75.4 CM., FREQUENCY 59.5 CYCLES, WIRE DIAMETER 0.440 MM. TRANSFORMER RATIO 401

The break in the curves for the positive corona shown in Fig. 8 is due to the start of negative corona on the opposite half waves. In order to establish this point definitely a battery was connected in series with the high voltage winding of the transformer. In this way the a-c. voltage may be given either a positive or a negative bias.

The start of the positive corona appears at a constant value of the positive gradient, while the break of the positive corona appears at an approximately constant value of the negative gradient coinciding with the start of the negative corona, as shown in Table I.

An inspection of the curves shown in Fig. 10 (for positive corona with varying distances between wire and plane) shows that the portion of the positive discharge curve previous to the break is found only at distances from the wire to plane less than 8 cm. for a one-mm. wire. With greater distances than this the first appearance of the positive corona is not detected but the point determined corresponds to the break in the positive discharge curve at which point the positive discharge becomes much more copious and also much more mobile.

TABLE I

	Positive gradient at start of positive corona	Negative gradient at break of positive corona
Curve A.	79.0 Kv/cm.	81.7 Kv/cm.
B.	79.0	82.3
C.	78.9	81.6

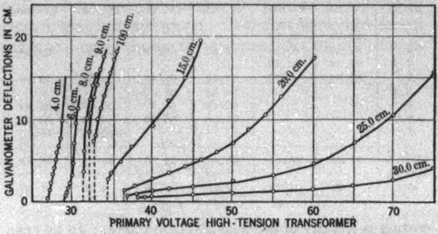

FIG. 10—GALVANOMETER DEFLECTIONS VS. PRIMARY VOLTAGE FOR CORONA ON POSITIVE HALF-WAVES. TEMPERATURE 18.9 DEG. CENT., BAROMETER 76.9., DIAMETER WIRE 1.00 MM. RATIO OF TRANSFORMER 401

This break in the discharge curve for positive corona is of considerable interest in view of the fact that other observers have found only one voltage for alternating corona, but different voltages for the positive and negative continuous corona. Whitehead and Brown[30] using the free charge in the air to detect the corona found the alternating corona to coincide with the negative continuous corona or the higher value. This is unquestionably due to the fact that their device for detecting free charges was not sufficiently sensitive to detect the first appearance of positive corona. Those observers using visual and aural methods to detect corona undoubtedly determined the voltage for negative corona, because the volume of the discharge becomes much more copious at this voltage. We were unable to detect any sound or light with the first appearance of positive corona. It seems probable that Whitehead and Isshiki[33] determined the voltage at which positive corona first formed but this is not certain. The size of their apparatus put them very near the boundary of the positive space charge at atmospheric pressure. So far we have made no observations under conditions which should give negative corona at the lower voltage.

CORONA GRADIENTS

The positive and negative corona gradients as determined by a wire and parallel plane on alternating current are given in Table II.

TABLE II

CORONA GRADIENTS FOR WIRE AND PARALLEL PLANE CORRECTED TO 20 DEG. CENT. AND 76 CM. PRESSURE FREQUENCY = 60 CYCLES

Distance from wire to plane in Cm.	Wire diameter 0.0440 cm.		Wire diameter 0.1000 cm.		Wire diameter 0.1584 cm.	
	positive corona kv./cm.	Negative corona kv./cm.	positive corona kv./cm.	Negative corona kv./cm.	positive corona kv./cm.	Negative corona kv./cm.
3.	81.37	86.71				
4.	81.49	85.88	63.53	69.89	56.87	59.64
4.5	81.12	85.15				
5.	81.06	84.70	63.45	68.45	56.39	59.89
5.5			63.38	68.15		
6.			63.48	68.17	56.47	58.84
6.5			63.58	68.10	56.44	58.74
7.					56.51	58.75
Mean...	81.26	85.61	63.49	68.35	56.54	58.97
Max. % dev...	0.28	1.29	0.17	2.26	0.58	1.13

The extreme constancy of the gradients for positive corona is worthy of especial attention here. In no case does the maximum deviation from the mean represent as great a deviation in voltage as one-tenth of the smallest scale division on the meter as used. There are two factors entering into this. The first is that the galvanometer deflections for positive corona before the break lie quite accurately on a straight line. Therefore several readings above the corona voltage can be taken with the voltmeter needle coinciding with a line on the scale and the corona voltage determined by extrapolating these values back to the axis. This gives an exceedingly accurate determination of the point at which the positive corona starts. The second factor influencing the relative constancy of the positive corona is the fact that the negative corona gradient seems to change with varying distances from the plane. The gradient for the wire nearest the plane is in every case about 2 per cent higher than the value a few centimeters further out. This may be due either to the distortion of the field due to returning positive charge or it may be due to the fact that the negative gradient is higher with a more rapidly divergent field.

In support of this latter explanation it may be mentioned: *a*, that with large wires (less divergent fields) in concentric cylinders the negative gradient drops below the positive gradient;[33] *b*, with the wire screened with a glass tube it was found that the gradient for negative corona was reduced until it coincided approximately with the gradient for positive corona, while the positive gradient was not appreciably affected. These facts seem to confirm K. T. Compton's[40] explanation of the increased strength of the air around small wires. Compton shows that the terminal velocity of an electron builds up through a number of elastic collisions thus requiring a voltage distance relation to produce ionization. His calculations seem to be of the proper order of magnitude but a quantitative check is hardly to be expected until more is known about the ionizing action of ions and electrons.

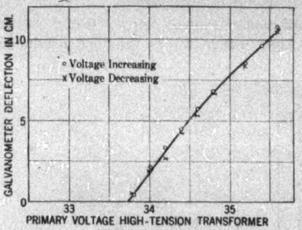

FIG. 11—POSITIVE CORONA. DISTANCE FROM WIRE TO PLANE 4.0 CM.

DISTORTION OF THE ELECTROSTATIC FIELD DUE TO RETURNING SPACE CHARGE

It is to be expected that the space charge of a previous half wave will distort the electrostatic field as it returns to the wire. Evidence of this is found in the fact that the negative corona ceases at a lower value of the effective voltage than that at which it starts, when the wire is far enough from the plane to permit the return of of the positive space charge. However, when the wire is so close that the positive space charge reaches the plane and does not return, the voltage for the start and stop of corona coincides.

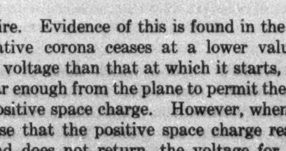

FIG. 12—NEGATIVE CORONA. DISTANCE FROM WIRE TO PLANE 16.0 CM.

This effect is shown in Figs. 11 and 12. Fig. 11 with the wire four cm. from the plane, shows that the curve of positive discharge for increasing voltage coincides with the curve for decreasing voltage. The same was found true for the negative corona at this distance. Fig. 12 with the wire 16 cm. from the plane shows that the curve for negative corona with decreasing voltages extends below the start of the curve for increasing voltages. The first appearance of positive corona cannot be detected at this distance, and the break in the positive corona curve due to the negative corona, which can be detected behaves just as the curve for negative corona, shown in Fig. 12.

BOUNDARY OF THE SPACE CHARGE

The curves shown in Figs. 10 and 13 give galvanometer deflections against voltage for a number of distances between the wire and plane. The start of positive corona for distances below seven cm. is given by the intersection of the discharge curve with the axis. Calculating the gradient for the start of positive corona, we find this constant, and using this value of the gradient we can calculate the voltage for the start of positive corona for greater distances between the wire and plane. In this manner the primary voltage for the start of positive corona with 15 cm. between the wire and plane was calculated to be 33.6 primary volts. At a high-tension voltage 400 effective volts above the corona voltage the primary voltage would be 34.6 volts. At this voltage the galvanometer deflection was three cm. (See Fig. 10.) In a similar manner we can find the galvanometer deflection at 400 effective volts above the corona voltage for all distances between the wire and plane. We thus obtain the relation between galvanometer deflections (or space charge reaching the plane) and the distance between wire and plane for a given rise in voltage above the corona voltage. Curves of this type for positive corona are shown in Fig. 14. However, they refer to a different size of wire from that used for the curves in Figs. 10 and 13.

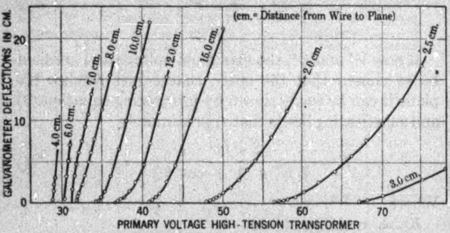

FIG. 13—GALVANOMETER DEFLECTION VS. PRIMARY VOLTAGE FOR CORONA ON NEGATIVE HALF-WAVES. TEMPERATURE 18.9 DEG. CENT., BAROMETER 76.9 CM., DIAMETER WIRE 1.00 MM. RATIO OF TRANSFORMER 401

The three small curves shown in the lower left hand corner of Fig. 14 give the charge reaching the plane for voltage increments insufficient to cause negative corona. These curves intersect sharply with the axis and this intersection is the boundary of the space charge at that voltage. Thus with a voltage 100 volts above the positive corona voltage no ions can be detected at a distance greater than 5.2 cm. from a 0.1-cm. wire. If the voltage rises 200 effective volts above the positive corona voltage the boundary of the positive space charge is 5.6 cm. from the wire. A few readings were taken at 40 and 80 cycles showing that the boundary of the space charge depends on the frequency. However, the machine used was not suited to these speeds. The results shown all refer to 60 cycles.

If the voltage rises 400 volts above the positive corona voltage, negative corona also forms. The start of negative corona influences the quantity of the positive discharge greatly and it also greatly increases the mobility of the charge as shown by the fact that the sharp drop in the curve comes at a much greater distance from the wire. Above the negative corona voltage the discharge curves for positive charge do not cut the axis sharply but approach asymtotically. This is believed to be due to diffusion. With the great increase in the density of the charge the effect of diffusion would be more important. Also as the voltage is carried higher and higher above the corona value. the time during which the first charges can travel out approaches nearer and nearer to the time they have to return, that is a half wave. Therefore we would expect more and more charge to be carried out by diffusion as the voltage rises above the corona value. However, the discharge curves descend quite rapidly in every case and this marks the practical boundary of the space charge.

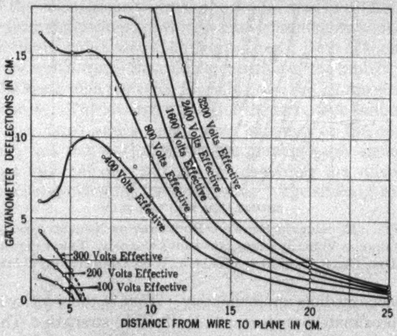

FIG. 14—CURVES SHOWING BOUNDARY OF POSITIVE SPACE CHARGE AS VOLTAGE INCREASES ABOVE CORONA. TEMPERATURE 24 DEG. CENT., BAROMETER 75.5 CM., DIAMETER WIRE 0.440 MM.

The maximum shown in the curves for 400 and 800 volts above the corona value is probably due to the fact that with a very small distance between the wire and plane most of the negative charge reaches the plane and therefore the effect on the positive charge is less important with small distances. This maximum should become less prominent as the voltage is raised because in that case the last charges formed will have a shorter time to travel out and therefore the amount of charge returning to the wire will be greater as the voltage is raised.

The curves shown in Fig. 15 are constructed in a similar manner for the negative charge, except that in the case of the negative charge the start of negative corona can be detected at any distance from the wire to plane by the break in the discharge curve for the positive corona. At great distances from the wire to plane, this is the first appearance of positive charge. These curves for negative charge all show a sharp intersection with the axis and a definite boundary of the negative space charge. Since the positive charge is more mobile than the negative charge (except for very high voltages) and since the positive corona starts at lower voltages for the wires used, the negative ions which fail to return to the wire are probably destroyed by recombination. This prevents the curves for negative corona from becoming asymtotic to the axis.

MOBILITY OF IONS IN CORONA DISCHARGE

Ions moving through a gas under the action of an electric field obey the laws for bodies moving in a viscous medium approximately. That is, the velocity is approximately proportional to the field strength. The mobility K is defined as the velocity of an ion in a unit field. K is usually given in cm./sec. per volt/cm. The velocity S at any field strength E will therefore be,

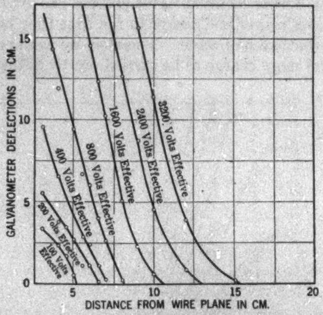

FIG. 15—CURVES SHOWING BOUNDARY OF NEGATIVE SPACE CHARGE AS VOLTAGE INCREASES ABOVE CORONA. TEMPERATURE 24 DEG. CENT., BAROMETER 75.5 CM., WIRE DIAMETER 0.440 MM.

$$S = K E$$

The distance $d\,r$ traveled in an element of time $d\,t$ is

$$d\,r = K\,E\,d\,t$$

If now we consider the ions which travel along the shortest path from the wire to the plane, the field along this path, neglecting the distortion due to the ions themselves, is approximately

$$E = 2\,c\,V/r\,(2\,c - r)\log 2\,c/a \qquad \text{Since } \frac{c}{a} >> 1$$

where c is the distance from the axis of the wire to the plane, a is the radius of the wire, r is the variable distance measured from the axis of the wire, and V is the potential difference between the wire and plane. (See Russell, Alternating Currents, Vol. I, p. 99.) Therefore

$$d\,r/d\,t = K\,c\,V/r\,(2\,c - r)\log 2\,c/a \qquad \textbf{(3)}$$

If now we assume V to be an alternating voltage

$$V = V''\sin w\,t$$

and let V' be the voltage necessary to produce corona such that

$$V' = V''\sin\theta'$$

The time which the first charges formed will have for their outward path will be

$$t = \frac{\pi - \theta'}{w}$$

If now V'' and E'', the maximum voltage and gradient be so chosen that the space charge just reaches the plane it can be easily shown by integrating equation **(3)** and substituting limits that approximately

$$K = \frac{2\,\pi\,f\,c^2}{3\,E''\,a\,(1 + \cos\theta')} \qquad \textbf{(4)}$$

Where

K = mobility of ions

c = distance from wire to plane

a = radius of wire

f = frequency

E'' = electrostatic gradient at the surface of the wire for the voltage V''

V'' = maximum voltage necessary to just drive the space charge to the plane

θ' = angle whose sine is V'/V''

V' = maximum voltage necessary to start corona.

The quantities which must be determined experimentally are a, f, c, V', and V''. a, c, and f can be measured directly. V' can be calculated, for any distance between wire and plane, from the maximum gradient for corona when the wire is near the plane. (In the case of negative corona, V' is the observed voltage for the break in the discharge curve for positive corona which can be measured for any distance between wire and plane.) V'' is the quantity most difficult to determine experimentally. This is due to the fact mentioned above that the boundary of the space charge is not very distinct on account of diffusion. However, it will be noticed that the upper part of the discharge curves becomes fairly straight for all distances between the wire and plane. This is shown well in Fig. 9. It is assumed that the straight portion of those curves represents the passage of charge to the plane on the same wave on which it is formed and without the aid of diffusion. Therefore the straight portion has been extrapolated back to the axis, and the voltage at which it intersects the axis taken as the voltage V'' at which the charge begins to reach the plane on the same wave on which it is formed.

The mobilities of the positive ions calculated in the manner outlined above are given in Table III. The corresponding mobilities for the negative ions are given in Table IV.

The curves in Fig. 16 show the variation of the mobility of positive ions plotted against the difference between corona voltage and the voltage at which the mobility was calculated. The corresponding curves for negative ions are shown in Fig. 17.

The initial values of the mobility check very well with

TABLE III
MOBILITIES OF POSITIVE IONS K'
FREQUENCY = 60 CYCLES

Distance from wire to plane in Cm.	Wire dia. 0.0440 cm.		Wire dia. 0.1000 cm.		Wire dia. 0.1584 cm	
	$V'' - V'$ primary volts	K' cm./sec. / volts/cm.	$V'' - V'$ primary volts	K' cm./sec. / volts/cm.	$V'' - V'$ primary volts	K' cm./sec. / volts/cm.
5.5	0.3	1.8				
6.	0.5	2.1				
6.5	0.4	2.5				
7.	0.6	2.7	0.3	1.7		
7.5			0.7	1.8	1.2	1.3
8.					1.3	1.4
8.5					1.9	1.5
10.	0.5	5.7				
12.	1.9	6.7			1.7	3.0
14.					1.9	4.2
15.	6.5	7.4	1.4	6.6		
16.					4.0	4.8
20.	16.9	8.8	10.7	7.3	12.0	5.6
25.	30.8	9.8	23.2	9.2	24.5	6.6
30.			37.5	9.5	41.0	7.5

TABLE IV
MOBILITIES OF NEGATIVE IONS K''
FREQUENCY = 60 CYCLES

Distance from wire to plane in Cm.	Wire dia. 0.0440 cm.		Wire dia. 0.1000 cm.		Wire dia. 0.1584 cm.	
	$V'' - V'$ primary volts	K'' cm./sec. / volts/cm.	$V'' - V'$ primary volts	K'' cm./sec. / volts/cm.	$V'' - V'$ primary volts	K'' cm./sec. / volts/cm.
5.	0.0	1.7				
5.5	0.3	1.7				
6.	0.5	2.0				
6 5	1.3	2.0				
7.	2.2	2.1				
8.	3.3	2.5	0.3	2.2	0.2	1.6
8.5					0.8	1.5
9.	4.1	3.0	1.0	2.4	0.6	1.7
10.	5.0	3.5	2.6	2.6	1.0	2.0
11.			4.2	2.8	3.0	2.3
12	7.2	4.5	5.7	3.1	4.0	2.6
13.			6.8	3.5		
14.					7.5	3.0
15.	10.9	5.9	8.0	4.4		
16.					8.5	3.8
20.	19.8	8.0	15.0	6.3	14.5	5.1
25.	31.5	9.7	24.4	8.1	25.0	6.5
30.			35.3	9.7	36.5	7.8

the values given by the physicists, varying from 1.3 to 1.8 cm./sec. per volt/cm. for the positive ions and from 1.6 to 2.2 cm./sec. per volt/cm. for the negative ions. The mobility of the positive ions rises quite rapidly with the appearance of negative corona and then becomes fairly constant at a value around 10 cm./sec. per volt/cm. Loeb[48] calculates from Langevin's equation that the mobility of a charged nitrogen molecule at atmospheric pressure would be 9.85 cm./sec. per volt/cm. The value here found is a surprisingly good check of this predicted value. The error is much below the probable error in our values of the mobility.

No explanation can be offered of the lower values found for the larger wire. However, it must be borne in mind that there are several approximations underlying these calculations. The most serious of these are: *a*, the fact that a sine wave voltage was assumed for the calculations whereas the oscillograms showed a ratio of maximum to effective value of 1.52; *b*, the fact that an electrostatic distribution of gradient has been assumed for the calculations whereas the actual gradient differs far from that, when the voltage rises considerably above the corona value, as will be shown later. Both of these errors would cause the calculated value of the mobility to be lower than the value actually possessed by the ions however. The lowered values for the large wire cannot be considered as due to a greater humidity on the day when this run was taken for the humidity was somewhat less that day than for the day when the values were determined for the small wires. The effect of the humidity could not be studied because there was no means of controlling the humidity and the normal variations in the room were not great.

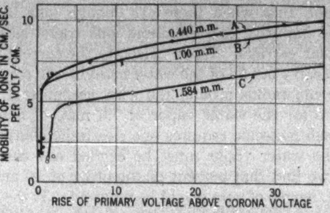

FIG. 16—MOBILITY OF POSITIVE IONS

A—Barometer 75.5 cm., Temperature 23.3-24.5 deg. cent., Humidity 8.6 mm. Hg.

B—Barometer 76.9 cm., Temperature 17.8-20 deg. cent.

C—Barometer 76.2 cm., Temperature 17.6-23 deg. cent., Humidity 7.4 mm. Hg.

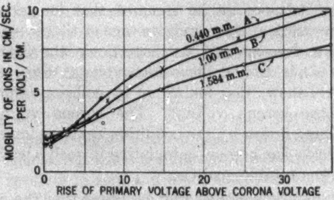

FIG. 17—MOBILITY OF NEGATIVE IONS

A—Barometer 75.5 cm., Temperature 23.3-24.5 deg. cent., Humidity 8.6 mm. Hg.

B—Barometer 76.9 cm., Temperature 17.8-20 deg. cent.

C—Barometer 76.2 cm., Temperature 17.6-23 deg. cent., Humidity 7.4 mm. Hg.

A few measurements were made with 40 and 80 cycles to determine the aging of the ions but any variation due to this cause was masked by the variation due to the voltage increase above corona.

The sharp rise in the mobility curve for the positive ions just above the voltage for negative corona suggests that the ions, as they become much more copious are washing out some impurity from the air such as moisture, and that when there are enough ions to wash out the impurity the remaining ions travel as simple molecular carriers. If we calculate the current necessary to carry out all of the water molecules in a cylinder of one cm. radius around the wire, assuming a vapor pressure for the water vapor of 7.6 mm. of mercury; that each molecule requires one electronic charge; and that the water vapor must be carried out each half cycle, we find that current of the order of 10 amperes per cm. length of wire would be required. If the water vapor is carried out it must be a process extending over a number of cycles, but even this does not seem probable.

The curves for the mobility of negative ions do not show a sharp break as found for the positive ions. However, the curves continue to rise more steeply than the curves for positive ions and there seems less evidence of a final maximum value. This is to be expected as the mobility of an electron as predicted by Loeb[47] for atmospheric pressure is of the order of 1500 cm./sec. per volt/cm. The electrons evidently do not travel a very great portion of their paths before attaching to form ions, even in the high field around a corona wire. This is to be expected from the work done on electrons in moist air at low pressures.

At a voltage practically twice the corona voltage the mobility of the positive and negative ions is the same and around 10 cm./sec. per volt/cm. as calculated from the boundary of the space charge.

THEORY OF CORONA CURRENT

In explaining corona on the basis of ionization by collision it must not be assumed that ionization by collision starts at the corona voltage or that it starts so sharply as does corona. An inspection of the saturation curves of air around the corona voltage, Figs. 4 to 7 inclusive, shows that ionization by collision is present below the corona voltage. The corona voltage is determined rather by the condition that the ionization by collision has become cumulative in such a way as to lead to an unstable condition.

This may be illustrated by assuming that the potential gradient around the wire draws to the wire ions due to the ambient ionization of the atmosphere. As these ions pass through the layer in which corona will form some of the ions will cause ionization by collision if the potential gradient is high enough. Also some of these newly formed ions due to the ionization by collision, in their turn will produce further ionization by collision, and any group of ions will in this way tend to reproduce itself indefinitely.

However, if the new ions formed by any group are less numerous than the original group, the descendants of this group will soon disappear and the current will be approximately the saturation current of the air. On the other hand if the new ions formed by any group are more numerous than the original group the descendants of this group will increase indefinitely and the saturation current of the air will be similarly magnified.

This serves to show that a small increase in the gradient just at the corona voltage would lead to an infinite current or a short-circuit condition; provided the gradient remained at the increased value. (The exposition of the current given above is not rigid because of the assumptions for the average value. However, the rigid treatment given by Towsend[9] shows the same characteristics for the current but in more complicated mathematical language.) In the case of corona, the gradient does not remain constant at the wire when ionization begins, for the space charge driven out from the wire increases the gradient at a distance from the wire, where the gradient is low. Thus the space charge absorbs a part of the impressed voltage when ionization is taking place, as shown below.

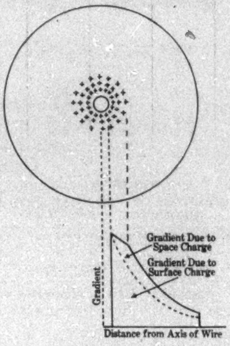
Gradient Due to Space Charge
Gradient Due to Surface Charge
Gradient
Distance from Axis of Wire

FIG. 18

Consider a wire inside a concentric cylinder as shown in Fig. 18. When ionization begins the charge of the same sign as the wire is driven out into space. This layer of charge raises the gradient at all points outside the layer and thus absorbs a part of the impressed voltage and reduces the gradient at the wire, where ionization is taking place. Thus there is established a condition of equilibrium between the space charge around the wire and the rate of formation of new charge, for any voltage. The space charge serves to shield the wire so that the gradient at the wire never rises above the critical value at which corona starts. In the alternating case the conditions are further complicated by the return of the space charge formed on any previous half-wave.

In order to obtain some idea of the nature of the corona current with an alternating electromotive force we will consider a wire of radius a inside a large cylinder of radius b with an impressed voltage.

$$v = V'' \sin w t$$

When any transient conditions have passed, and the instantaneous voltage is above the corona voltage there are three charges that must be considered. First, there is the surface charge on the wire which has a limiting value q and is constant for all values of voltage above the corona voltage (because the gradient at the surface of the wire is limited to the corona gradient by the formation of new space charge); second, there is the space charge Q' formed on this same voltage half-wave and moving out from the wire; third, there is the space charge Q'' which was formed on the previous voltage half-wave and is now returning to the wire.

For simplicity we will assume that the charge Q', instead of being distributed through space is concentrated in a cylindrical layer of radius L', such that its effect on the gradient in the ionizing layer around the wire is equivalent to the actual distribution, and that similarly Q'' is concentrated on an equivalent layer of radius L''.

We may now divide the impressed voltage into three parts. First, a component v''' which is supported by the surface charge q' on the wire; second, a component v' which is supported by the space charge Q' which has just formed and is now traveling out from the wire; third, a component v'' due to the space charge Q'' which is now returning to the wire. This last component is negative in sign.

We have then

$$v''' = 2\, q \log b/a \tag{8}$$

$$v' = 2\, Q' \log b/L' \tag{9}$$

$$v'' = 2\, Q'' \log b/L'' \tag{10}$$

and also that

$$v = v''' + v' - v'' = V'' \sin w t \tag{11}$$

Substituting the values of v''', v', and v'' in equation (11) and differentiating with respect to time we have

$$\frac{d v}{d t} = - \frac{2\, Q'}{L'} \times \frac{d L'}{d t} + \frac{d Q'}{d t}\, 2 \operatorname{Log} .\, b/a$$

$$+ \frac{2\, Q''}{L''} \frac{d L''}{d t} - \frac{d Q''}{d t}\, 2 \operatorname{Log} .\, b/a = w V'' \cos w t$$

(The derivatives of the charge with respect to time are multiplied by log b/a because the charge is formed and absorbed at the surface of the wire and not on the equivalent cylinder.)

The current i is

$$i = \frac{d Q'}{d t} - \frac{d Q''}{d t} \tag{12}$$

Transposing and substituting we obtain

$$i = \frac{1}{2 \operatorname{Log} b/a} \left[\frac{2\, Q'}{L'} \frac{d L'}{d t} - \frac{2\, Q''}{L''} \frac{d L''}{d t} + w V'' \cos w t \right] \tag{13}$$

A solution of this equation when the proper values are substituted for the Q's and L's seems at present impossible. However, some information about the corona current can be obtained from a discussion of the different terms.

The first term in the expression for the corona current, namely:

$$\frac{Q'}{L' \log b/a} \times \frac{d L'}{d t}$$

represents the rate at which new space charge must be formed to compensate for the outward movement of the space charge already formed. This term does not change sign during a given voltage half-wave and is of the same sign as the voltage half-wave.

The second term in the expression for the corona current,

$$\frac{Q''}{L'' \log b/a} \times \frac{d L''}{d t}$$

represents the rate at which space charge must be formed to compensate for the return motion of the space charge formed on the previous voltage half-wave. The motion of this charge is opposite to that of the outgoing charge so that the current required by this term is in the same direction as that required by the previous term and these two terms add their effects as shown by the negative sign before this term.

The third term in the expression for the current

$$\frac{w V'' \cos w t}{2 \operatorname{Log} b/a}$$

represents the current due to variation in the impressed voltage, and is the current due to electrostatic capacity. This term is of the same sign as the voltage wave on the rising part of the wave but reverses in sign at the crest of the voltage wave.

We see therefore that on the rising portion of the voltage half-wave all terms in the current relation add their effects, but when the voltage is decreasing the last term reverses in sign and tends to cancel the first two terms. The current will then decrease rapidly past the crest of the voltage wave and will probably reach zero slightly past the crest of the voltage wave. The maximum value of the corona current should come ahead of the crest of the voltage. If the frequency be high, the current will reach zero just past the crest of the voltage wave; if the frequency be low the outward motion of the space charge may produce a greater effect than the fall in the voltage and the current may continue down to the corona voltage, on the decreasing side of the voltage wave. Any recombination between the outgoing and returning space charges tends to reduce the terms representing the motion of these charges and causes the current to reverse nearer the crest of the voltage wave.

These facts suggest the possibility of expressing the total space charge and therefore the current in terms of the charge necessary to support the maximum voltage. This is attempted in the next section. Holm[42] has also devised a method for doing this. However, we think the approximation given below represents more closely the theoretical conditions than Holm's method.

Some precautions necessary in an experimental study of alternating corona are also evident from the discussion of equation (**13**). When the outer cylinder is too small some of the space charge will reach it. In this case the returning space charge is reduced in value and the current is therefore reduced in magnitude. If the cylinder be further reduced in size the outgoing space charge may begin to reach the cylinder before the crest of the voltage wave. In this case the current is increased again and we are approaching the conditions found in d-c. corona. It seems therefore, that there should be some size of cylinder which would give a minimum current for a given rise of voltage above the corona voltage. For a somewhat larger cylinder there should be a maximum current and then any further increase in the size of the cylinder would lead to a reduction in the current. No attempt has been made to check this experimentally.

In taking oscillograms of the corona current care should be taken to allow sufficient room for the space charge, if its full effect is desired. The oscillograms of Bennett[19] were taken in a cylinder so small that the effect of the space charge is almost if not entirely lost. They do not, therefore, represent the normal alternating condition. It will be seen later that when the space charge returns the average current on the opposite half-waves must be equal. This is far from true in most of Bennett's oscillograms.

The corona current curves of Whitehead and Inouye[38] were also taken in a cylinder too small to allow the full effect of the space charge. There is evidence in some of these curves that a part of the space charge was returning but it is also certain that a part of it did not return. It is impossible to know how nearly these curves represent the normal alternating condition.

The curves taken on transmission lines,[15,16] are the only ones allowing sufficient room for the full effect of the space charge and in these the corona current is a very small part of the total current, so that its true shape is difficult to determine. It would be very interesting to see the changes in the wave form of the current as the effect of the space charge is reduced by the size of the cylinder. Also the ratio of the average to the effective value of the corona current is of practical value.

APPROXIMATE FORMULA FOR CORONA CURRENTS

As seen above, an exact solution of equation (**13**) for the corona current seems impossible. However, an approximate solution may be obtained by assuming that all of the space charge is formed previous to the crest of the voltage wave and that it is just sufficient to support the rise in voltage above the corona voltage at the crest of the voltage wave. In order to make the calculations possible we must also assume that at the crest of the voltage wave this space charge is concentrated on a layer of equivalent radius L' as was previously done. (The method used for approximating L' will be given later.)

As shown above, the surface charge per unit length of the corona wire never exceeds the value q which it attains just at the corona voltage V'. Any increase in the voltage causes ionization and the formation of a space charge around the wire which absorbs the excess of the voltage above the corona value.

If we measure the current into a wire from zero voltage up to a voltage above the corona voltage we observe a current relation as shown in Fig. 19. There is a sharp break in the current curve at the corona voltage V'. At any voltage V above the corona voltage the current may be resolved into two parts; I' a constant part due to the surface charge on the wire and I due to the space charge around the wire. This part due to the space charge we will call the corona current. Let Q represent the maximum value of the space charge per unit length of wire in statcoulombs, l the length of the wire in cm., and f the frequency in cycles per second. We have then that the corona current (average value) is

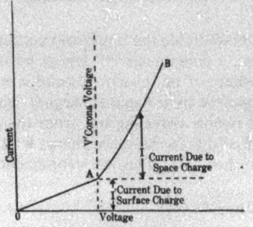

FIG. 19

$$I_{av} = 4\,Q\,f\,l \text{ (Statamperes)} \tag{14}$$

Since the space charge Q must support the excess of the voltage above the corona voltage, (by our assumption above) we have, if we assume Q concentrated on a cylinder of radius L',

$$Q = \frac{V'' - V'}{2 \log b/L'} \tag{15}$$

where V'' and V' represent the maximum values of the impressed and the corona voltages respectively in statvolts, b is the radius of the outside cylinder in cm., and L' is the radius of the equivalent space charge layer in cm. See Fig. 20.

We may determine L' as follows. Let the impressed voltage be,

$$v = V'' \sin w\, t,$$

and let θ' be the value of $w\, t$ when the instantaneous voltage has risen to the corona value such that,

$$V' = V'' \sin \theta'$$

We have then

$$\cos \theta' = \sqrt{\frac{V''^2 - V'^2}{V''^2}} \qquad \textbf{(16)}$$

We will now assume that the entire space charge is formed at an equivalent instant t''' such that

$$\cos w\, t''' = \frac{\cos \theta'}{4} = \frac{1}{4\, V''} \sqrt{V''^2 - V'^2} \qquad \textbf{(17)}$$

See Fig. 21. (The introduction of the factor 4 must be considered as a purely arbitrary assumption. However, it is justified by the fact that the first charge formed will tend to neutralize the returning space charge. Therefore the center of the formation of the space charge will be displaced toward the crest of the voltage wave. The introduction of the factor 4 is nearly equivalent to assuming that the formation of the space charge is due to the rise in the impressed voltage with the assumption that the first half of the space charge formed neutralizes the returning space charge and that therefore the center of formation of the useful space charge is at the point where three-quarters of the entire charge has been formed. Oscillograms would be of little use in determining the center of formation of the space charge because there is no way of distinguishing between the current due to the return of the previous space charge and the formation of new space charge.)

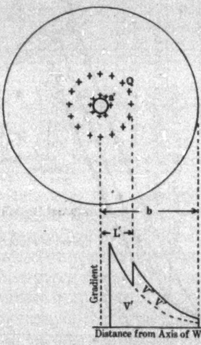

FIG. 20

Calculating now by equation (**4**) the distance L' that the charge formed at the equivalent instant t''' will travel by the time the voltage reaches the crest value, we have

$$L' = \sqrt{\frac{K}{2\, w \log b/a}} \sqrt[4]{V''^2 - V'^2} \qquad \textbf{(18)}$$

where K is the mobility of the ions, and the other symbols have the values assigned to them above. K is practically an arbitrary constant and the value of 4 cm./sec. per volts/cm. was adopted because it gave the best agreement with the observed values. However, this value for the average mobility of the positive and negative ions agrees well with what might be expected from the measurement of the maximum mobility in the first part.

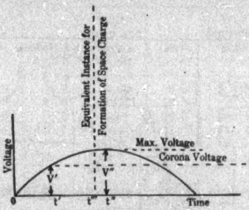

FIG. 21

It must also be noted that the value of K is dependent on the arbitrary factor 4 introduced above. We might combine these two arbitrary constants but we would then lose sight of the theoretical significance of the two parts of the arbitrary constant. From theoretical considerations the first constant was thought to be approximately 4 and having introduced this, a value of K was chosen to give the best agreement with the observations.

Our formula for the corona current in practical units is now,

$$I_{av} = \frac{f\, l (V'' - V')}{4.5 \log b/L'} \times 10^{-11} \qquad \textbf{(19)}$$

and L' has the value

$$L' = \sqrt{\frac{1}{\pi\, f \log b/a}} \sqrt[4]{V''^2 - V'^2} \qquad \textbf{(20)}$$

I_{av} = average value of the corona current in amperes
f = frequency in cycles per second
l = length of the corona wire in cm.
V'' = maximum value of the impressed voltage in volts
V' = maximum value of the corona voltage in volts
b = radius of outer cylinder in cm.
a = radius of corona wire in cm.

In order to apply this to a two-wire transmission line we have the following changes:

V'' = maximum value of impressed voltage to mid-point between wires (ground)
b = distance between wires, in cm.

This formula is not applicable unless all of the space charge formed on one half-wave returns on the next.

When a wire and concentric cylinder are used the cylinder should be so large that the space charge does not reach the cylinder. The size of cylinder required of course depends on the frequency, size of wire, and the rise of the voltage above the corona voltage. The radius required in any case can be determined by calculating the distance traveled on any half-wave, by the ions formed just as the instantaneous voltage reaches the corona voltage. This distance may be calculated by assuming the ions to have a mobility of 10 cm./sec. per volt/cm. For 60 cycles on a No. 10 wire with a voltage twice the corona voltage the radius of the cylinders should be about 30 cm. This was checked experimentally.

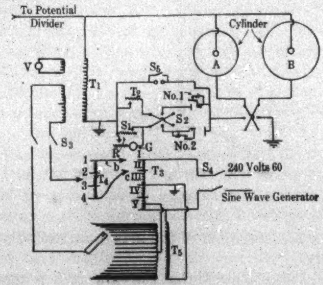

Fig. 22

In case parallel wires are used the space charge should not go more than half way between the wires for it will then meet the space charge from the opposite wire. The necessary distance between the wires may be calculated by using the gradient along the line joining the wires and assuming a mobility for the ions of 10 cm./sec. per volt/cm. as above. For No. 10 wires with a 60-cycle impressed voltage twice the corona voltage, the distance between the wires should be about 60 cm. This value was also checked experimentally.

Measurement of Corona Currents

Corona currents were measured in two large cylinders by rectifying the charging current to a central section of the cylinder through three-electrode vacuum tubes with grid connected to plate. The current was measured by a calibrated D'Arsonval galvanometer which could be connected to measure either the positive or negative waves. A diagram of connections is shown in Fig. 22.

Power was supplied by a 50-kv-a. 60-cycle motor-driven generator. The voltage was maintained by a Tirrill regulator. However, the speed was subject to fluctuations of about 1 per cent. By means of the tap-changing transformer and auto-transformers shown, the voltage could be varied by steps of one volt from zero to 240 volts. This arrangement was used to avoid wave distortion at varying voltages. The voltage was measured from a calibrated tertiary coil on the 10-kv-a. 120/240 to 100,000-volt high-tension transformer. The transformer was used only on the 240-volt connection.

The galvanometer *G* was shunted by the combination of resistances *R* and *S* for varying its range. It was calibrated by connecting a known e. m. f. and a known *R* in series with the galvanometer and its shunt. The opposite side of the circuit was balanced by a resistance *T* equal to the resulting resistance of the galvanometer and its shunt. By means of the switch *S* 2 the galvanometer could be thrown in series with tube 1 or tube 2. A reversing switch in the galvanometer circuit was provided (not shown in figure) to keep the galvanometer deflection in the same direction for both tubes.

Switch 1 permitted the rectifying and measuring device to be thrown in series with either cylinder *A* or cylinder *B*. Cylinder *A* was 61 cm. inside diameter and composed of a central section 62 cm. long and a guard ring on each end of approximately the same length. The central section in which the current was measured was screened from stray fields by a gauze wire around the outside. The separation between the central and end sections was about five mm.

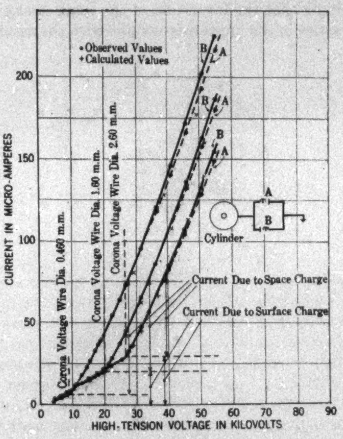

Fig. 23—Corona Current Curves. Average Value of Half-Waves. Diameter Cylinder 61 Cm. Active Length of Wire 62 Cm. Frequency 64 Cycles.

Cylinder *B* was 155 cm. in diameter. It was composed of a central section 154 cm. long and guard rings on each end of approximately the same length. As in cylinder *A*, the central section in which the current was measured was carefully screened. The separation between the central and end sections was about five mm. as in cylinder *A*.

Measurement of the insulation resistance and capacity to ground of the central sections of cylinders *A* and *B*, and of the plate voltage required for the rectifying tubes, showed that the error caused by the admittance to ground of the central sections of the cylinders was less than 0.5 per cent.

The charging current of the central section of each cylinder was measured for three sizes of wire, up to a voltage at least twice the corona voltage. These values are shown in Figs. 23 and 24. The currents plotted are rectified half waves, and therefore represent half of the total current expressed in average values.

The readings of the positive and negative half waves were equal except when the space charge began to reach the cylinder. The symmetry of the current waves is no indication that the positive and negative space charges are equal because any half wave measures the outgoing charge of one sign and the returning charge of opposite sign. This is not true when the cylinder is small. With small distances between the wire and cylinder the current corresponding to the formation of negative space charge exceeds the current corresponding to the formation of positive space charge. This may be explained as due to a decrease in the return of the negative space charge or on increase in the formation of the negative space charge. The voltage corresponds roughly to the voltage at which the negative charge should reach the cylinder and it seems most probable that this difference is due to a decrease in the return of the negative space charge. This indicates that the negative space charge reaches the cylinder first, which is in accord with our previous determinations of the mobility of the negative space charge for this voltage. For the large cylinder it was not possible to raise the voltage to such a value that the positive and negative current half waves became unequal.

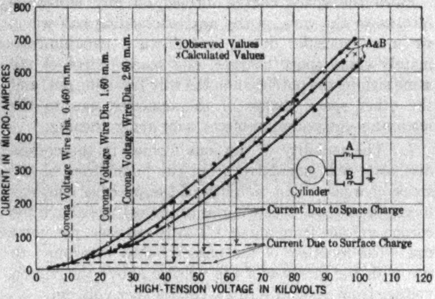

FIG. 24—CORONA CURRENT CURVES. AVERAGE VALUE OF HALF-WAVES. DIAMETER OF CYLINDER 155 CM. EFFECTIVE LENGTH OF WIRE 154 CM. FREQUENCY 64 CYCLES

CALCULATION OF CORONA CURRENTS

In the calculation of the corona currents the corona voltage is determined experimentally from the break in the current curves. A comparison of the observed values of the corona voltage with the values calculated from the formula of Whitehead and Brown, Peek, and Whitehead and Isshiki are given in Table V.

The ratio of the maximum voltage to the effective voltage at 240 volts for the generator used had been previously determined by oscillograms to be 1.46. This value is used in the calculations of the corona current. The calculated values are shown by crosses on the curves of the observed corona current Figs. 23 and 24.

TABLE V

Cylinder dia. Cm.	Wire dia. cm.	Observed kv. eff.	Corona voltage calculated by formula of			Eq. 2 kv. eff.
			White-head and Brown kv. eff.	Peek kv. eff.	White-head and Isshiki kv. eff.	
61	0.046	8.7	10.27	9.93	9.92	9.35
61	0.160	20.0	20.6	19.8	20.2	18.7
61	0.260	26.5	27.6	26.0	26.8	24.7
155	0.046	10.5	11.95	11.55	11.53	10.88
155	0.160	23.0	24.5	23.2	23.7	22.0
155	0.260	30.0	33.5	31.6	32.6	30.1

The agreement is entirely satisfactory in all cases. Any differences between observed and calculated currents are of the order of the probable experimental error. The agreement is even more impressive when we notice that the voltage range is quite wide. In Fig. 23 the voltage was raised to nine times the corona voltage for the small wire. This indicates that the corona current formula is of the correct mathematical form.

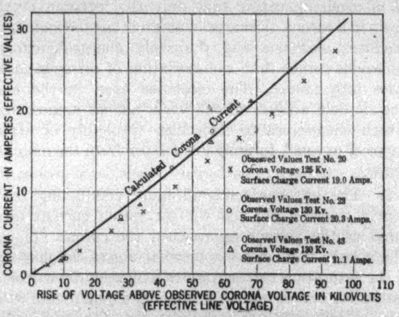

FIG. 25—CORONA CURRENT CALCULATION. TESTS OF W. W. LEWIS, A. I. E. E. TRANS., VOL. 40, P. 1059. THREE PHASE LINE WIRES IN VERTICAL PLANE. LENGTH 101.5 MI. SPACING 12 FT. CONDUCTORS NO. 0, SEVEN STRAND. DIAMETER 0.19 IN. FREQUENCY 30 CYCLES

The corona currents are also calculated for the three-phase transmission line studied by W. W. Lewis.[27] This line consisted of three 110,000-cir. mil, seven-strand conductors spaced 12 ft. in a vertical plane. The length was 101.5 miles, and the radius of the conductors was 0.19 in. The corona voltage is here again determined by the break in the charging current curves given by Lewis. In calculating the corona current, voltages to neutral are employed. The measured currents are in effective amperes. In order to convert the corona current from average to effective values the factor 1.11 is used—(the ratio for a sine wave.) This is open to criticism but no better method presented itself. A comparison of the observed and calculated values for this transmission line is given in Fig. 25.

The agreement is surprising when we consider the possible causes for discrepancy. Among these causes may be mentioned; 1, the fact that the voltage rose about 10 per cent along the line so that different parts of the line were at different stages of corona formation; 2, the space charge around the wire with corona on it distorted the electric field and therefore the neutral of the system (this may cause a shift of the voltage away from the corona wire); 3, the ratio of the effective to the average value of the current is probably not 1.11 as assumed; 4, the frequency of these tests was 30 cycles while the formula for corona current was developed for 60 cycles.

SUMMARY

1. The spectrum of corona shows the band spectrum of nitrogen indicating that only the nitrogen breaks down in a corona discharge in air and that the ionization produces electrons and positively charged nitrogen molecules rather than a dissociation of nitrogen molecules into atoms. The electrons even in the high electric field around corona wires almost immediately attach themselves to molecules (probably of water vapor or oxygen), forming ions as seen from the mobility below.

2. The air was rendered more conducting by a quartz mercury arc and the saturation current measured as the voltage approached the corona voltage. The corona voltage lies somewhat above the first rise of the saturation current due to ionization by collision. Saturation curves taken on copper and magnesium wires show no influence due to the material of the conductor. This indicates that the positive ions are active ionizing agents as Towsend has assumed.

3. Corona forms on the positive half-waves of an alternating impressed voltage about 2 per cent below the voltage required to form corona on the negative half-waves. (This has been tested only at atmospheric pressure for wires between No. 25 and No. 10 B. & S. gage.) The first appearance of positive corona does not give appreciable sound nor light, and ions due to this corona do not penetrate the air to a distance greater than about 8 cm. (for a No. 10 wire at atmospheric pressure).

4. The amount of ionization on the positive half-wave becomes very much more copious with the appearance of corona on the negative half waves.

5. The observed voltage for positive corona was found to be much less subject to errors due to surface irregularities and the divergence of the field, than the observed voltage for negative corona. This is to be expected from similar work on the continuous corona.

6. The ions formed around a corona wire, during any half-wave, which are of the same sign as the potential of the wire, are driven out from the wire forming a space charge. This space charge moves out from the wire until the voltage reverses. It then returns to the wire. Thus the space charge is able to penetrate the air around the wire only to a definite distance, or may be said to have a boundary. This boundary of the space charge varies with the corona voltage, the rise in voltage above the corona voltage, and the frequency. Diffusion reduces the sharpness of the boundary of the space charge. (See Figs. 14 and 15.)

Since the space charge formed on any half-wave returns to the wire on the next succeeding half-wave, we must consider during a half-wave two approximately equal space charges, one space charge of the same sign as the potential on the wire traveling out, and the other space charge of the opposite sign to the instantaneous potential of the wire and returning.

7. The mobility of the ions formed in the corona discharge was calculated from the boundaries of the space charges. The mobility of the positive ions was found to increase with the rise of the voltage above the corona voltage from about 1.3 to 10 cm./sec. per volt/cm. The mobility of the negative ions was found to rise with the increase of the voltage above the corona voltage from about 1.6 to 10. cm. sec. per volt/cm.

The curves of mobility of positive ions plotted against rise in voltage above the corona voltage indicate a limiting value of the mobility of about 10 cm./sec. per volt/cm. This is in good agreement with the value 9.85 calculated by Loeb[48] from Langevin's equation. The curves for the mobility of negative ions do not indicate that a limiting maximum value is probable.

8. An analysis of the conditions of corona formation indicate that the gradient near the surface of the wire can never rise above the value it has at the corona voltage. This requires that the surface charge on the wire be constant for all voltages above the corona voltage and that the space charge support any rise in the voltage above the corona voltage. The differential equation of the corona current based on these conditions does not admit of a simple solution.

9. An approximate formula for the average current due to the space charge has been derived by the aid of certain empirical assumptions. This formula gives a good agreement for the corona currents measured in two large cylinders. The corona currents were calculated by this formula for a 101.5-mile three-phase transmission line and the agreement is reasonable, with the measurements made on this line by W. W. Lewis.[37]

The author wishes to express his appreciation of the facilities provided by the Johns Hopkins University, and the assistance given by members of the faculty. He is particularly indebted to Dean J. B. Whitehead for his valuable suggestions and criticisms, and to Dr. W. B. Kouwenhoven for his advice on problems of measurement. The author also wishes to express his appreciation of the assistance given by Mr. S. K. Waldorf, graduate student in Engineering, in the measurement of corona current.

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Discussion

Joseph Slepian: One thing that Mr. Willis brings out and which is worth emphasizing is that the beginning of corona is not the point where ionization by collision begins. This is important because very frequently in the past it has been assumed that the beginning of ionization by collision is also the beginning of breakdown or of corona under various conditions. Mr. Willis has shown by his experiments here that the corona comes at a higher gradient than that at which appreciable ionization by collision occurs and that corona sets in when a sufficiently great cumulative reaction takes place between the ionization produced by collisions of the negative ions and that produced by collisions of the positive ions. Hence the gradient at which the corona begins is not a constant of the gas but also depends upon the geometrical configuration of the electrodes; that is, it depends also upon the radius of the wire and the distance to the other electrode.

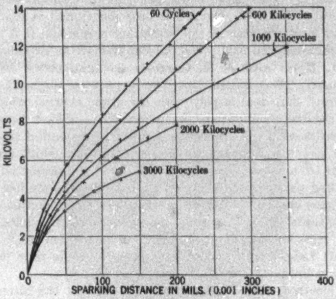

Fig. 1—Two-c. m. Sphere-Gap Sparking Voltage at Various Frequencies

Alexander Nyman: I should very much like to see this work extended to include operation at high frequency.

Probably many people are familiar with the fact that at high frequency the corona begins to appear at much lower voltage than at 60 cycles; in fact, perhaps two or three times lower at frequencies as high as 1,000,000 cycles. At frequencies as high as 5,000,000 or 10,000,000 cycles, corona begins even at lower voltages than this; it is possible to observe a corona on sharp points at 1000 or 1500 volts. This fact became apparent in the first place during the measurement of voltage on static condensers and on insulators for condensers for use on power transmitting stations. A sphere-gap calibrated at high frequency showed values as illustrated in the accompanying figure, from which it is evident that the sparking voltage at 1,000,000 or 2,000,000 cycles is considerably lower than at 60 cycles. At 10,000,000 cycles these voltages are lower yet.

Now, it is evident that the discharge between sphere-gaps is not of the nature of corona, but the reduction between the sphere-gaps brought to the attention a similar reduction of discharge voltage between sharp points and it was then even more conspicuous.

With the spacing of ½ in. and a frequency of 10,000,000 cycles, a sparking distance of 4000 or 5000 volts is quite usual between edges which are not strictly sharp. Of course, with sharper edges this voltage is considerably lower. The question will arise "How do we know that this is a correct voltage measurement?" The process to ascertain this fact is rather lengthy, but consists in general of the following:

A static condenser, which remains constant at different frequencies, and a measurement of current and frequency gives a measurement of voltage.

The measurement of current may be again in doubt, but can be assured by a proper design of an ammeter, suitable for this specific purpose.

W. A. Del Mar: On the third page of Mr. Willis' paper there is this statement: "The results of the corona ionization are electrons and charged nitrogen molecules. The absence of the oxygen spectrum indicates that the nitrogen alone is ionized, or that the oxygen does not play any appreciable part in the corona ionization." On the sixteenth page there is something which appears to be contradictory. Under "Summary," paragraph one it says: "The electrons, even in the high electric field around corona wires, almost immediately attach themselves to molecules, (probably of water vapor or oxygen), forming ions as seen from the mobility below." Oxygen is here mentioned as one of the types of ions, whereas in the preceding paragraph it is said to have practically no part. I should appreciate an explanation.

H. J. Ryan and J. S. Carroll: (communicated after adjournment) Three papers have been presented to the Institute, heretofore, that deal largely with the space charges which surround conductors in alternating corona[a,b,c]. These papers are not mentioned in the bibliography given at the end of the paper and no reference is made to their conclusions, which are quite different from those here presented. The facts presented in the preceeding papers were obtained by quantitative studies of the *whole* space charge that surrounds a conductor in corona due to high voltages at low frequencies,—generally 60 cycles. The studies reported in the papers presented at Seattle,[b] 1925, and Salt Lake City,[c] 1926, included the cyclic time element and the radial positions of the space charges.

The electrodynamic behavior of the ions in the alternating corona cycle is far too complex and too little understood, as yet, to enable anyone to formulate a theory of the space charge such as attempted in this paper. The author is apparently convinced that the integrity of his theory is ample to warrant him in setting forth the complete *quantitative* behavior of the space charge from *qualitative* observations of diffuse remnants of alternating space charges with the determinations of the radial positions incomplete and the cyclic time elements wholly omitted.

We submit that one cannot inform himself reliably about these space charges by that sort of procedure and that he certainly cannot help others. Surely knowledge of all facts that it is possible to obtain must come first; the facts must be studied thoroughly to develop an understanding of their mutual relation and then only can a theory be formulated and put forth to assist others to acquire, with economic effort, such understanding.

Dependable knowledge as to these space charges is of undoubted importance in high-voltage engineering. We regret that Mr. Willis has made it so difficult for us to join in a more hearty welcome to him with his results, as a worker in this field in which we hope we can all continue to work unremittingly. We earnestly trust that we will soon be able to resolve our differences with understanding, so that we can present the facts about these space charges, without confusion, to the membership of the Institute.

C. H. Willis: I appreciate Dr. Slepian's emphasis of the fact that the corona gradient is not the beginning of ionization by collision. In ionization by collision we are measuring average effects, but the probability of ionization at any particular collision between an ion and a molecule depends on the actual free path which the ion has just traversed and also on the type of collision. The saturation current in the gas increases as the voltage rises and the explanation of this increase is ionization by collision. We cannot, however, pick any particular voltage as the point where ionization by collision begins.

In corona, the current rises abruptly at a certain voltage. The explanation is that at this voltage the ionization has become cumulative to such an extent that it would lead to a short circuit, but for the action of the space charge.

Mr. Nyman speaks of the lowering of the voltage at high frequency. His remarks were quite interesting to me. The only work I know of in this regard is the work by Gorton and Whitehead, in which they found the corona gradient to be affected to about two or three per cent at frequencies up to 3600 cycles per second. I should be very much interested in seeing further work if it has been published.

In regard to Mr. Del Mar's question, I believe my statement was that the nitrogen played the main role in ionization by collision. We have in the air a mixture of N_2 molecules and O_2 molecules. There are, of course, two ways in which we may produce ionization. We can dissociate the molecules into N plus and N minus. This would give a line spectrum. Therefore, this is not the form of ionization which occurs in corona, nor does that occur except at very much higher gradients than the corona gradient. But the band spectrum indicates that the N_2 molecule breaks up into a positively charged N_2 molecule and an electron. We must remember that the electron has a mass of $^1/_{18}$ hundredth of the hydrogen molecule and therefore, is very much smaller than a nitrogen molecule. The probable mobility of the nitrogen molecule is around 10 cm./sec. per volt/cm. I had expected in this work to find that the negative space charge would have a very high mobility and would penetrate the air to a very much greater distance than the positive space charge. To my surprise, I found that the two charges penetrated to practically the same distance, and the conclusion which follows is that the electron attaches itself to a molecule very quickly forming an ion.

The nitrogen molecule is ionized and the electron then attaches itself either to a water or oxygen molecule to produce an ionic carrier rather than an electronic carrier. The reason that I conclude that the electron attaches to water vapor or oxygen is the fact that water vapor and oxygen have a very much higher electron affinity than nitrogen. We would expect that the nitrogen would not attract the electrons, from the work done by physicists.

The author wishes to apologize for the omission of the paper by Messrs. Ryan and Henline, from his bibliography. The bibliography however is by no means complete, as only those papers are given which seemed to have a direct bearing on our work. The other papers mentioned by Messrs. Ryan and Carroll were published after this work was planned and the bibliography prepared.

We feel that the criticism of Messrs. Ryan and Carroll that our work was qualitative is hardly justified. The method of determining the total space charge by measuring the rectified current is probably the most accurate method, and is certainly not subject to the very serious criticisms which may be made against the other methods which have been employed, (*i. e.*, that these methods either limit the space charge or distort the field or both.)

We also feel that any serious criticism of our generalizations in regard to the space charge, which are used in developing the expression for the average value of the corona current, should include a demonstration of the inaccuracy of the expression for the corona current.

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Tests on High- and Low-Voltage Oil Circuit Breakers

BY PHILIP SPORN[1] and HARRY P. ST. CLAIR[1]

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Synopsis.—*Data from a large number of tests on several types of oil circuit breakers are given in this paper. The tests were made on breakers with the following ratings: (a) 150 kv., 1,500,000 kv-a.; (b) 35 kv., 250,000 kv-a.; (c) 7000 volts, 7500 kv-a.; (d) 132 kv., 1,250,000 kv-a., and (e) 132 kv., 750,000 kv-a. These tests were made on power systems having sufficient connected capacity to make the tests conclusive. Complete data are tabulated and oscillograms are shown. Some valuable conclusions resulted from the tests.*

INTRODUCTION

THE American Gas and Electric Company has carried out a number of tests on high- and low-voltage oil circuit breakers, these tests falling into three principal groups which are as follows:

1. The first group of tests was brought about by the purchase from the Brown Boveri Company of a number of 150-kv. and 35-kv. breakers, the acceptance of which was made conditional upon the results of rupturing capacity tests. The 150-kv. breaker, described and illustrated more fully in later paragraphs, is of the round tank multiple break type equipped with oil filled 150-kv. bushings. A total of 10 breaks per pole is employed using simple ball type butt contacts.

Although it was not possible to obtain sufficient short-circuit current at any point on the interconnected 132-kv. system of the American Gas and Electric Company to test the breaker at its full rated interrupting capacity of 1,500,000 kv-a., it was felt, nevertheless, that a series of tests at the maximum capacity available, approximately 750,000 kv-a., would serve to indicate whether the breaker would be acceptable for the intended service. The Sunnyside Substation of the Ohio Power Company at Canton, Ohio, was selected as the logical place to carry out the tests, since not only was it possible to obtain a maximum concentration of short-circuit capacity at that point, but also because the particular point being a substation, any short circuit placed on the system there would be divided between the various generating stations on the interconnected system through the intervening lines between Sunnyside Substation and the generating stations.

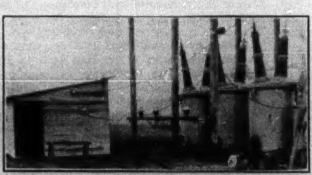

FIG. 1—BROWN BOVERI 150-KV. BREAKER IN POSITION FOR TEST WITH SHELTER FOR CURRENT TRANSFORMERS AND TRIPPING RELAYS

The 35-kv. breaker was of the plain break type, two breaks per pole, with all three poles in one rectangular tank. Several of these switches had been purchased subject to the results of tests to be made at 22 kv. (the only voltage of that class available at Sunnyside Substation) and carried to the full breaker rating of 250,000 kv-a.

2. The second group of tests, conducted at Schenectady, using the regular testing equipment of the General Electric Company, was made on one unit of the Reyrolle compound filled switchgear, type C-1-ORD, rated at 7000 volts, 400 amperes, and having a guaranteed rupturing capacity of 75,000 kv-a. As in the case of the Brown Boveri breakers a number of these units had been purchased subject to satisfactory performance under short-circuit tests. These tests, more fully described later, were made first at 2300 volts beginning with less than rated duty and carried to a point considerably beyond the rating, and later at 6600 volts at more than full rating.

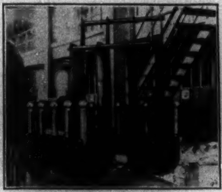

FIG. 2—UPPER RIGID CONTACTS (INVERTED) AND ONE SET OF BARRIERS

3. The third group of tests was made on two General Electric Company breakers, the breakers selected being two 132-kv. breakers, one of them an FHKO-39-B and the other an FHKO-136-B. The first, that is, the FHKO-39-B, had a rated rupturing capacity of 1,250,000 kv-a. This breaker, as is well known, is of the round tank, explosion chamber type. The other, the FHKO-136-B, is rated at 750,000 kv-a. and is of the oval tank explosion chamber type. It was felt that a short-circuit test on this breaker at the Sunny-

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[1] Both of the American Gas and Electric Co., 30 Church St., New York City.

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side Substation, where a capacity practically equal to the breaker rating was available, would serve as an excellent check on the design principles embodied in other high-voltage breakers of the same type.

TESTS ON THE BROWN BOVERI TYPE[2] A F 24/1 A 150,000-VOLT OIL CIRCUIT BREAKERS

The general appearance of the 150-kv. Brown Boveri breaker is shown in Fig. 1. This illustration also shows the method of mounting the breaker and of bringing the leads on the short-circuit side to a small shelter in which were placed the 2500-ampere current transformers and the relays used in tripping the breaker. The foundation for the breaker consisted of heavy timbers placed directly on the ground as shown.

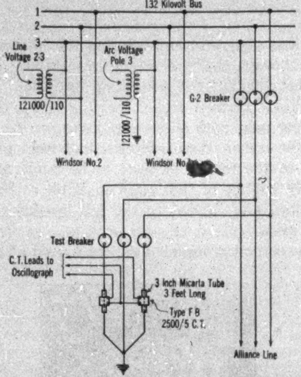
132 Kilovolt Bus
1
2
3
Line Voltage 2-3
121000/110
Arc Voltage Pole 3
121000/110
G-2 Breaker
Windsor No.2
Windsor No.
Test Breaker
C.T. Leads to Oscillograph
3 Inch Micarta Tube 3 Feet Long
Type F B 2500/5 C.T.
Alliance Line

FIG. 3—DIAGRAM OF TEST CONNECTIONS FOR THE BROWN BOVERI 150-KV. BREAKER TESTS AT SUNNYSIDE SUBSTATION

The breaker tanks have a diameter of 65½ in. and a height of 8 ft. 10 in. from the truck wheel to the center line of the horizontal operating shaft. The ball type contacts are shown in Fig. 2, which is a view of the upper stationary contacts in an inverted position. The upper contacts are rigidly attached to the lower end of the bushing. Fig. 2 also shows an assembly of flash barriers placed between the contacts.

Diagram Fig. 3 shows the test connections and relative location of equipment. The test breaker was connected solidly to the Alliance 132-kv. circuit at the first tower outside of the substation yard, the G-2 switch on this line serving as a back-up breaker. The current transformers used for obtaining current records were Westinghouse type FB 2500 amperes, 2300 volts indoor busbar type with the insulation reinforced by adding a micarta tube three in. in diameter, three ft. long, with a ⅜-in. wall, and wrapping the bare 4/0 conductor with varnished cambric to a thickness of approximately ½-in. before placing the tube and current transformer over it. It was felt that this insulation would withstand any voltage that could be built up on the grounded side of the switch even though the ground connection was not of particularly low resistance. The breaker tanks were grounded to separate ground pipes driven near the tanks and not tied in with the short-circuit ground connection.

Professor H. E. Dyche who, assisted by Professor E. R. Rath, both of the University of Pittsburgh, was engaged to take oscillographic records and to furnish

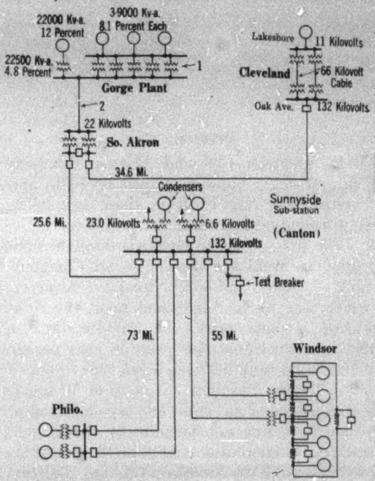
22000 Kv-a. 12 Percent
3-9000 Kv-a. 8.1 Percent Each
22500 Kv-a. 4.8 Percent
1
Gorge Plant
2
22 Kilovolts
So. Akron
Lakeshore
11 Kilovolts
Cleveland
66 Kilovolt Cable
Oak Ave.
132 Kilovolts
34.6 Mi.
Condensers
Sunnyside Sub-station
(Canton)
25.6 Mi.
23.0 Kilovolts
6.6 Kilovolts
132 Kilovolts
Test Breaker
73 Mi.
55 Mi.
Windsor
Philo.

FIG. 4—SYSTEM DIAGRAM SHOWING CAPACITY AVAILABLE FOR BROWN BOVERI AND GENERAL ELECTRIC OIL CIRCUIT BREAKER TESTS

Explanation of Diagram:

Lakeshore	7 generators, total 226,000 kv-a. 2.16 per cent on 30,000 kv-a. Two 30,000-kv-a. banks, 7.2 per cent each. Two 66 kv. cable circuits each 8 mi., 1/0 cu. 6.1 in. flat spacing.	
Oak Avenue	Two 30,000-kv-a. transformer banks 4.8 per cent each.	
Gorge Plant (1)	Several transformers total 6.6 per cent on 30,000 kv-a.	
(2)	Several 22-kv. circuits total reactance 13.7 per cent on 30,000 kv-a.	
South Akron	Two banks 11.4 per cent each on 30,000 kv-a.	
Sunnyside	Two 7500-kv-a. condensers 11.5 per cent each. Two transformer banks 11 per cent each on 30,000 kv-a. (132-kv. to 6.6-kv. windings).	
Philo	Two 42,100-kv-a. generators, 13.7 per cent each. Two 45,000-kv-a. banks 8.9 per cent each.	
Windsor	Five 30,000-kv-a. generators 15 per cent each. Two 50,000-kv-a. banks 8.4 per cent each. Reactors each 30,000 kv-a., 5 per cent. Note: On all tests using full capacity available, all reactors were short-circuited.	
Transmission Lines	Cleveland-Akron	200,000-cm. copper, 16.6 ft. effective spacing.
	Akron-Canton	200,000-cm. copper, 13.6 ft. effective spacing
	Windsor-Canton	200,000-cm. copper, 15.9 ft. effective spacing.
	Philo-Canton	336,400-cm. ACSR, 16.6 ft. effective spacing.

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[2] See also *Electrical World* of May 9, 1925.

the necessary equipment, had added to an original Siemens and Halske oscillograph a number of automatic features making it possible to start the film at an exact predetermined time delay after the actual closing of the test breaker control circuit, and to automatically stop the film after a predetermined travel. This feature made it possible to record as many as four shots on one film without reloading the instrument even though the shots were made in rapid succession. A film taken in this manner is shown in Fig. 6.

Complete results of all tests including data on the system set-up are summarized in Table I. The system data given will be sufficient, when combined with the system diagram Fig. 4, to show the set-up for each test.

Beginning with a standard duty cycle of 2-O C O shots with a two-minute interval and with a system set-up calculated to give approximately 225,000 kv-a., the duty on this breaker was increased by steps until on Tests No. 16 to No. 26 the full system capacity available was applied.

After Test No. 5 the oil was drained from one tank in

TABLE I
RESULTS OF TESTS ON 150-KV. BROWN BOVERI OIL CIRCUIT BREAKER

Duty cycle and system set-up	Test number	Test voltage	Recovery volts		Current			Duration of short, ½ cycles		Short-circuit kv-a.		Remarks
			Peak value line 2-3	Per cent initial	Closing Peak	Closing R.M.S.	Initial in arc R.M.S.	*Total	Arcing	Closed	Opened	
2-OCO 2-min. interval 4 gen. at Windsor 2 lines, 2 banks reactors in........	1	140,000	177,000	89.2	..	..	..	..	..	..	..	Opened O. K. trace of smoke.
	2	140,000	192,000	97.0	1990	1190	940	43	20	288,000	288,000	Ditto
1-OCO to test oscillograph. Same set-up as Test 1......	3	150,000	No record	..	Could not be read		1020	44	15	..	283,000	Ditto
2-OCO 2-min. interval. Same set-up except no reactors at Windsor and 2 condensers at Canton..............	4	132,000	Ditto	..	3740	2160	1260	38	14	492,000	288,000	Ditto
	5	130,000	104,400	57.8	Could not be read		1320	37	13	..	297,000	Ditto
8-OCO in rapid succession. Interval approx. 10 sec. System same as Test 4...........	6	135,000	No oscillogram taken			..	..	..	..	..	..	Ditto
	7	135,000	Ditto		..	..	..	..	..	..	..	Ditto
	8	135,000	No record	..	4160	2400	1420	36	12	560,000	332,000	Ditto
	9	135,000	Ditto	..	Could not be read		1435	38	14	..	335,000	Ditto
	10	135,000	Ditto	..	3630	2160	1435	40	15	520,000	335,000	Ditto
	11	135,000	Ditto	..	4270	2490	1320	37	14	582,000	309,000	Ditto
	12	135,000	Ditto	..	3740	2170	1435	36	13	507,000	335,000	Ditto
	13	135,000	Ditto	..	3310	2010	1455	†36	†13	470,000	340,000	Ditto
2-OCO 2-min. system as in Fig. 4 except Cleveland off and 22,000-kv-a. gen. only at Akron. 4 gen. at Windsor...	14	134,000	134,000	70.7	Could not be read		2340	40	14	..	534,000	Opened O. K. Some smoke, slight oil throw. Noticeable jumping of all three tanks.
	15	134,000	111,000	58.5	6920	4000	2180	38	13	930,000	505,000	
2-OCO 2-min. interval. System set-up as in Fig. 4 except 22,000-kv-a. gen. at Akron omitted..........	16	135,000	196,000	103.0	..	No record				..	..	Ditto
	17	135,000	166,500	87.3	..	No record				..	..	Opened O. K. Much smoke on tank No. 1 and violent jumping. Other tanks same as above.
2-OCO 2-min. system same as Test 16 except only 4 gen. at Windsor.......	18		Oscillogram no good			..	..	..	..	..	..	Opened O. K. Some smoke. Considerable jumping.
	19		Ditto		..	..	..	..	..	..	..	Ditto
7-OCO 1-min. interval. System same as Test 18........	20	132,000	262,000	140.0	8000	4680	2830	34	14	1,070,000	645,000	Ditto
	21	132,000	164,000	87.0	7930	4580	2800	37	16	1,040,000	640,000	Ditto
	22	132,000	No record	..	†7750	†4480	2830	No record		1,020,000	645,000	Ditto
	23	132,000	180,000	96.5	6660	4020	2830	40	17	926,000	645,000	Ditto
	24	132,000	No record	..	†7000	†4150	2860	No record		946,000	652,000	Ditto
	25	132,000	144,000	77.0	Could not be read		2920	39	16	..	666,000	Ditto
	26	132,000	217,000	116.0	8450	4900	2920	34	13	1,120,000	666,000	Ditto

*Estimated from current and voltage record.
†Estimated, record cut off.

order to permit examining the contacts. Finding the contacts only slightly burned and the oil only slightly discolored, it was decided to go ahead with the tests without any change of oil in the other two tanks and without dressing the contacts.

The series of eight shots comprising Tests No. 6 to No. 13 inclusive, presents an unusual and interesting test in that the eight shots were given as rapidly as the breaker could be closed, the interval averaging approximately 10 sec. The short-circuit kv-a. opened on this series averaged approximately 330,000 or approximately one-fourth of the rated capacity of the breaker. The oscillograms for this series of tests are not reproduced here but are of the same general form as those shown in Fig. 6. The first two shots were intentionally not recorded on the oscillograph, but the last six were all recorded on one film. The voltage element on the oscillograph was burned out at the beginning of this series of tests, but the current record was obtained in each case.

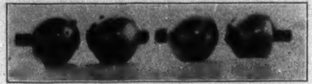


FIG. 5—BALL TYPE CONTACTS OF BROWN BOVERI 150-KV. BREAKER AFTER TEST NO. 15

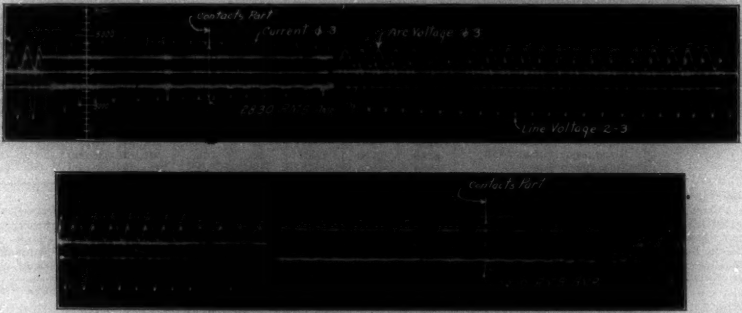


FIG. 6—OSCILLOGRAMS FOR TESTS 20 AND 21

Oil samples taken after this series tested 27 kv. as compared with a test of 28 kv. on the oil originally supplied to the breakers, and showed only slight discoloration.

It might be of interest to note that considerable trouble was experienced in recording the two currents on the oscillograph apparently due to stray currents in the secondary leads. The difficulty was not overcome until an entirely separate ground consisting of a single ground pipe had been made and the test current transformers completely isolated from the station ground.

Tests No. 14 and No. 15 represent a standard 2-OCO duty cycle to which was applied the full system capacity as shown on Fig. 4 with the exception that Cleveland was not connected and at Akron there was connected one 22,000-kv-a. generator only. This set-up was calculated to give a short circuit of approximately 525,000 kv-a. Fig. 5 shows the burning of the contacts which were removed for inspection after Test No. 15. Oil samples taken at this time showed slightly more discoloration than those taken after Test No. 5, but the oil was in very good condition and tested an average of 24 kv.

In an attempt to obtain a record of the internal pressure on this breaker during the tests, engine type pressure indicators were connected to the tanks through a fitting in the center of the manhole cover at the bottom of the tank. This method of obtaining a pressure record, however, was very inadequate as the pressure in the tanks was insufficient to cause any appreciable deflection. As near as could be determined, the indicators registered a pressure of approximately 10 lb. per sq. in. on all tests.

During the interval between Test No. 15, the final test on March 8, and Test No. 16, the contacts were removed and dressed and fresh oil supplied to the breaker.

For Tests No. 16 to No. 26 inclusive, the full system capacity shown in Fig. 4 was applied with the exception that the 22,000-kv-a. generator at Akron was omitted and that after Test No. 17 only four generators were available at Windsor instead of the original five. This set-up was calculated to give approximately 745,000 kv-a. with five generators at Windsor and approximately 725,000 kv-a. with four generators at Windsor.

While no current records were obtained on Tests No. 16 and No. 17, so that the exact value of ruptured kv-a. cannot be given, with the five generators at Windsor (only four available for Tests No. 18 to No. 26) and with a somewhat higher voltage, it is estimated that the duty on these two shots was approximately 700,000 kv-a.

On Test No. 17 which was the second shot of a standard 2-OCO duty cycle, No. 1 pole gave off considerable smoke and threw a small quantity of oil from the vent indicating more distress than on any of the other shots during the entire series. Nine shots at full system capacity were applied after this test without any repetition of this apparent distress so that the reason for it on this shot is not at all clear. An examination of the barrier between the individual breaks in tank No. 1 after the completion of the tests showed that one of them had been somewhat charred on both sides opposite the breaks. This barrier was of a material different from that furnished with the original breaker, as it had been obtained locally and placed in the breaker to replace one of the originals which had become broken in assembly. Subsequent examination failed to reveal any puncture, however, so that the distress shown is still unexplained unless it was simply due to the generation of more than the usual amount of gas and smoke caused by the burning of this barrier.

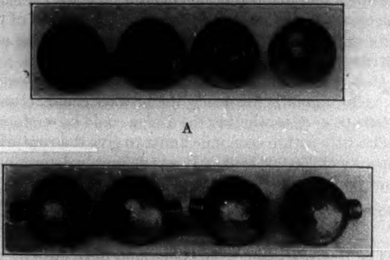


FIG. 7—(A) CONTACTS OF BROWN BOVERI 150-KV. BREAKER AFTER COMPLETION OF TESTS
(B) SIDE VIEW OF SAME CONTACTS

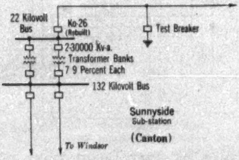


FIG. 8—CONNECTIONS FOR TEST ON 35-KV. BROWN BOVERI BREAKER

After Tests No. 18 and No. 19, which comprised another standard duty cycle of 2-OCO shots, a final series of 7-OCO shots at one-minute intervals was applied. The oscillographic records of Tests 20 and 21 are shown in Fig. 6. On all of the shots at the higher values of kv-a. a considerable movement of the tanks was observable due, it is believed, at least partly to the resilient character of the timber foundations. This last series was intended to consist of eight shots instead of seven, but, on account of the instability of the timber foundations causing considerable movement each time the breaker opened, it was agreed that between each of the shots the test breaker would be given a blind shot; that is, it would be closed and opened with the back-up breaker open so as to make sure that the tanks had not been thrown out of alignment to such an extent as to prevent tripping. This proved to be a wise precaution as the breaker failed to trip on the blind shot made after Test No. 26 and no further tests were made.

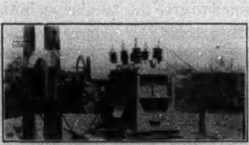


FIG. 9—BROWN BOVERI 35-KV. TYPE *A F* 12/36 OIL CIRCUIT BREAKER AS SET UP FOR 22-KV. TEST

The contacts taken from this breaker after Test No. 26 are shown in Fig. 7. Although considerably burned, there was nothing, in the authors' opinion, in the condition of these contacts which would prevent the breaker from continuing in service even without any dressing of the contacts. The oil was considerably discolored, but tested an average of 25 kv. as compared with 30 kv. for the fresh oil supplied to the breaker before Test No. 16.

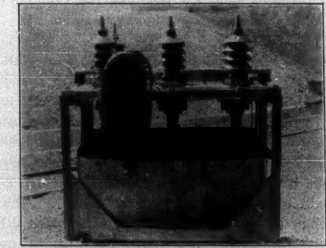


FIG. 10—FRONT VIEW OF 35-KV. BREAKER AFTER TEST. NOTE CRACK IN COVER AND RIVETS SHEARED OFF AROUND TOP OF TANK

TESTS ON THE 35-KV. BROWN BOVERI TYPE A F 12/36 OIL CIRCUIT BREAKER

The general appearance of this breaker and the location and arrangement for test are shown in Fig. 9. As shown on Fig. 8, the breaker was connected temporarily to one of the 22-kv. circuits radiating from the main 22-kv. bus at Sunnyside Substation.

The 132-kv. bus was supplied only from four generators at Windsor with two 132-kv. lines and two transformer banks at Windsor in service.

The contacts on this breaker are of the plain break type with spring mounted arcing contacts and are shown in Figs. 10 and 11. The most distinctive feature of this breaker of course is the fact that all three poles are placed in one rectangular tank and separated only by internal barriers.

It was planned to give this breaker an initial standard duty cycle at about 175,000 to 200,000 kv-a. and then increase in one or two steps to the full rating of 250,000 kv-a. The short-circuit kv-a. for the first test as calculated from the system set-up was approximately 180,000 kv-a. at 0.2 seconds and approximately 200,000 kv-a. at 0.1 sec. Tests on the over-all time required to trip the breaker from the closing of the trip circuit until the breaker contacts separated indicated that the duration of the short circuit would be between 0.1 and 0.2 sec. Arrangements were made for taking oscillographic records of current, line voltage, and arc voltage the same as on previous tests on 150-kv. breakers.

Fig. 11—Interior of 35-Kv. Breaker after Test. Showing Contacts and Broken Barriers

The planned series of tests was carried out, however, only to the extent of the first shot, as this resulted in a failure accompanied by the oil catching fire. Unfortunately, due to a mishap on the oscillograph, no record was obtained of the current and voltage values on this shot. As stated above, however, calculations indicate that the short circuit was probably in the neighborhood of 190,000 kv-a. Fig. 10 shows how the breaker tank was dropped on this shot, leaving the contacts exposed to the air. In this view is also shown the break in the cover casting as well as the two halves of the rivets which hold the tank to the angle iron ring at the top and which were sheared off, thus allowing the tank to drop. The arcing at the exposed contacts of course immediately turned into a short circuit, igniting the oil and causing the back-up breaker to open. Although not visible in the pictures the lower half of one of the bushings was stripped of porcelain by the arc. Fig. 11 is a close-up view of the contacts and the interior of the tank showing the manner in which the tank lining and barriers were displaced.

It is regrettable that no determination was possible as to the amount of capacity the breaker could actually interrupt or as to whether the design could be reinforced sufficiently within economic limits to give it a rupturing capacity of 250,000 kv-a. It was definitely determined, however, that the breaker was not up to its rating.

TESTS ON REYROLLE TYPE C 1-ORD-7000-VOLT OIL CIRCUIT BREAKER

The Reyrolle armor clad compound filled switchgear, the circuit breaker portion of which was tested with the results given below, is shown very clearly in Fig. 16. The upper enclosed compartment contains the bus while the lower compartment just beneath the bus contains the built-in current transformers designed to take care of the individual feeder controlled by the switch. Fig. 16 shows the breaker racked completely out in which position hinged covers automatically close the busbar openings.

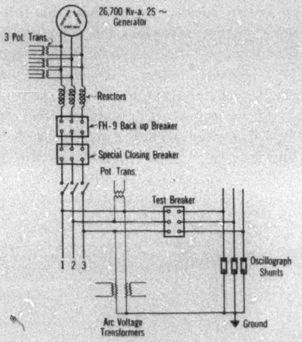


Fig. 12—Diagram of Test Connections for Reyrolle Switchgear

This breaker was purchased subject to the breaker being able to meet the guaranteed rupturing capacity of 75,000 kv-a. at 2300 volts. To determine this, arrangements were made to test one of these units at the factory of the General Electric Company at Schenectady using the regular testing equipment consisting of the 26700-kv-a., 25-cycle test generator and auxiliary equipment.

Fig. 12 shows the arrangement of equipment and connections for this test including the current and potential transformers used for obtaining voltage and current records. The tests were carried out under the supervision of American Gas and Electric Company representatives, the actual testing personnel of course being the regular staff of men employed by the General Electric Company for carrying out their own tests.

The complete results of the tests on the Reyrolle breaker are summarized in Table II. In connection with this table it is well, perhaps, to point out the method employed in obtaining from the oscillographic records the total r. m. s. value of interrupted kv-a. when the current wave is not symmetrical, particularly so in view of the fact that the method employed by the authors, and the one they believe is standard in this country, is apparently different from the method employed in England. The method employed to obtain the values shown in Table II consists of resolving the displaced wave into its a-c. and d-c. components and taking the r. m. s. of these two components as the total r. m. s. value of current interrupted. The d-c. component is equal to the distance from the normal zero line to the actual center line of the displaced a-c. wave at any particular point. For example, in the case of a displaced current wave having a major ordinate of 20 mm. and a minor ordinate of 10 mm., the d-c. component is the difference between the major ordinate and half the sum of the two ordinates, or five mm. To carry this example further, the r. m. s. total current for the wave in question would be $\sqrt{\left(\frac{15}{\sqrt{2}}\right)^2 + 5^2}$ which is equal to 11.7 mm. As to the method of calculating the total kv-a. interrupted, since the three poles were all in one tank, the average of the three measured current values was used instead of the highest value which of course is used in the case of the individual tank breaker.

TABLE II

RESULTS OF TESTS ON REYROLLE TYPE C1-ORD-7000-VOLT, 400-AMPERE OIL CIRCUIT BREAKER

Duty cycle	Test no.	Test voltage	Recovery volts		Initial current in arc			Kv-a. interrupted	Remarks
			Peak value arc pole 1 and 3	Per cent initial	A-c. comp.	D-c. comp.	Total R.M.S.		
2-OCO 2-minute interval...			2340	124	9,100	..	9,100		Opened O. K. Some oil spilled. Hissing sound of escaping oil and gas.
	1	2300	..	..	9,330	3,200	9,870		
			2280	121	9,550	1,500	9,670	38,000	
			2470	131	9,280	..	9,280		Opened O. K. Oil spilled on tests 1 and 2, 1 pint. Hissing sound. Oil test 26.3 kv. (before test, 29.6 kv.)
	2	..	..	..	9,550	2,000	9,500		
			2170	115	9,400	3,000	9,860	38,000	
2-OCO 6-minute interval...			*2290	122	14,350	6,170	15,650		Opened O. K. Some oil spilled. Hissing sound as before.
	3	..	..	..	12,600	1,440	12,700		
			2570	136	12,900	8,430	15,400	58,000	
			1880	100	11,950	..	11,950		Opened O. K. Oil spilled on tests 3 and 4, 1½ pints. Hissing sound, wooden barriers broken and displaced. Oil test 24.9 kv.
	4	..	..	..	11,200	..	11,200		
			1880	100	11,300	..	11,300	45,700	
2-OCO 2-minute interval...			*2380	127	23,900	6,600	24,800		Opened O. K. Some oil lost. Hissing sound not much greater than on previous shots.
	5	..	..	..	20,100	11,100	23,000		
			2770	147	22,300	16,400	27,700	100,000	
			1830	97	16,700	4,400	17,200		Opened O. K. Oil spilled on tests 5 and 6, approximately 1 quart. Barriers broken and displaced again. †Oil test 26.2 kv.
	6	..	..	..	14,800	5,050	15,600		
			1880	100	15,400	2,180	15,600	64,300	
Started to make 2-OCO....			5520	102	9,420	8,030	12,380		Circuit opened for 7 cycles (0.3 sec.) but tank burst causing short circuit and oil fire and general wrecking of switch. Duty 90 per cent above rating.
	7	6600	..	..	7,350	930	7,400		
			5850	108	9,600	7,420	12,200	122,000	

*Estimated—Curves off film.

†New oil supplied before test No. 5., tested 29.3 kv.

It was planned to give this breaker 2300-volt tests consisting of standard duty cycles at 40,000 kv-a., 60,000 kv-a., and 75,000 kv-a., followed by 6600-volt tests at 75,000 kv-a. and 100,000 kv-a. On the first duty cycle in which the breaker opened 39,200 kv-a. no distress was evident beyond the hissing sound of escaping oil and gas. Approximate figures as to the loss of oil and other data are recorded under Table II.

The second duty cycle on which 58,000 kv-a. and 45,700 kv-a. respectively were obtained on the two shots, was handled by the breaker apparently as easily as the first duty cycle. Upon lowering the tank, however, it was found that the wooden barriers separating the three poles were broken from their fastenings and displaced, the screws fastening these barriers to the tank lining at the side and at the bottom having been orn out of the wood. Also considerable burning was isclosed on this test, both on the arcing contacts and omewhat on the main contacts.

It will be noted that the kv-a. interrupted on test No. 4 was considerably less than that on test No. 3. This is due to the difference in the total duration of hort circuit which is evidenced by the oscillograms hown in Fig. 13. It will also be noted that there was a ix-minute interval between tests No. 3 and No. 4. The reason for this interval was the accidental jarring out of the manual trip lever of the breaker on test No. 3, preventing the closing of the second shot until the lever had been reset. Furthermore, this jarring out of the trip lever is responsible for the short duration of only two cycles in the short circuit on the first shot, thus accounting for the large difference in value, the decrement on the first shot being much smaller than on the second.

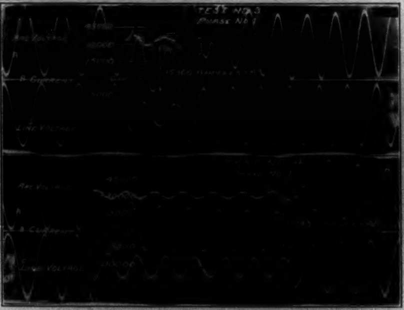


FIG. 13—OSCILLOGRAMS TAKEN ON TESTS NOS. 3 AND 4

FIG. 14—OSCILLOGRAMS TAKEN ON TESTS NOS. 5 AND 6

On the final duty cycle at 2300 volts the breaker successfully cleared a short circuit of 100,000 kv-a. on the first shot and 64,300 kv-a. on the second shot. Before these two tests were made the wooden barriers, found broken after the last test, were repaired and replaced and new oil was supplied to the breaker. The oscillograms for this duty cycle (Fig. 14) show that on test No. 5 the total duration of current was only about 1⅛ cycles as against 3½ cycles for test No. 6. The reason for the short duration and consequent high value of current on test No. 5 was the fact that the d-c. time delay relay used as part of the regular station testing equipment for tripping the test breaker was not reset after the last test, resulting in energizing the trip circuit of the test breaker as soon as the auxiliary contacts closed. This relay was properly reset before test No. 6 so that the normal duration of short circuit and the smaller value of current were obtained.

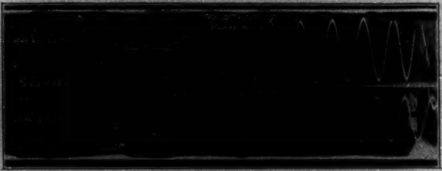


FIG. 15—OSCILLOGRAMS TAKEN ON TEST NO. 7

The wooden barriers found broken after test No. 4 and repaired prior to test No. 5 were again found completely broken from their fastenings and displaced after test No. 6. As before, the arcing contacts, both upper and lower, were quite badly burned and considerable burning was also evident on both the upper main contacts and on the lower contact bar.

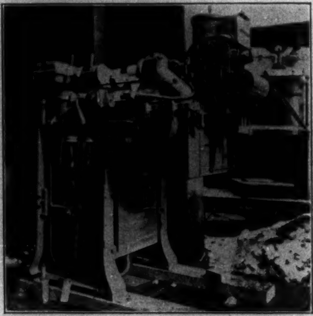


FIG. 16—REYROLLE TYPE C1-ORD—7000-VOLT COMPOUND FILLED SWITCHGEAR SET-UP FOR TEST. SWITCH RACKED OUT

Before proceeding with the test at 6600 volts it was decided, due to the badly burned condition of the arcing contacts after the preceding six shots at 2300 volts, that new arcing contacts, both upper and lower, should be made up and installed, that the main contacts should be carefully dressed, and that new wooden baffles should be made up and fastened in the tank. New oil of course was supplied.

Based on a duration of short circuit of seven or eight cycles for which it was intended that the time delay relay should be set, the generator circuits were arranged to give a calculated value of 75,000 kv-a. for test No. 7. For some unknown reason the short circuit lasted only 2½ cycles instead of seven or eight, and the kv-a. which the breaker attempted to open was 122,000. The oscillogram shown in Fig. 15 bears evidence that the short circuit actually was cleared for a period of seven cycles after which complete failure and blow-up of the breaker took place. Apparently the breaker cleared the short circuit but the gas pressure was so great that the tank was completely ruptured, one side being thrown against the exposed 6600-volt temporary terminals underneath the current transformer chamber causing a short circuit which immediately set fire to the oil. The appearance of the breaker after the fire had been put out is shown in Fig. 17. The force of the explosion which ruptured the tank was so great that the tank was split not only along the welded seams but on two edges where the steel had been bent but not welded.

Fig. 17—Reyrolle Switchgear after Test No. 7

As will be noted from the table, the rupturing capacity of this switch was exceeded by more than 60 per cent on test No. 7 at which value the result obtained might well have been expected. On test No. 5, on the other hand, the breaker successfully opened 100,000 kv-a. at 2300 volts, which, aside from the breaking of the barriers and rather extensive burning of the arcing contacts (the contacts tested were not designed for 2300 volt service), ought certainly to be considered a creditable performance.

Although the breaker more than met its guaranteed rupturing capacity, certain design features which were standard with the manufacturers on breakers of heavy duty were incorporated in the breakers actually installed, the principal ones being those of increasing the thickness of the tank from ¼ in. to $^5/_{16}$ in., increasing the depth of the tank two in., and the substitution of a much heavier butt type-arcing contact and the employment of steel barriers over which is placed a wooden lining instead of wooden barriers fastened to the tank lining. The authors have no doubt but that the breaker as finally obtained has a rupturing capacity considerably in excess of the originally guaranteed capacity.

TESTS ON GENERAL ELECTRIC 132-KV. OIL CIRCUIT BREAKERS

Due to the large number of 132-kv. breakers that are employed on the system of the American Gas and Electric Company, the question of rupturing capacity of different types of breakers, particularly of 132-kv. rating, is naturally a very vital one to us and it takes a test to make the final check on rupturing capacity. This was one reason for the third series of tests. Another reason was the desire already mentioned to carry out tests close to the rupturing capacity of the breaker and thus determine if possible the correctness of some of the fundamental design principles and therefore whether designs based on these principles for rupturing capacities beyond any possible test values could be relied upon with a certain degree of safety. Still another reason was to determine experimentally whether a cycle more strenuous than the standard duty cycle on the breaker was too dangerous or whether it could be employed safely.

Tests on the FHKO-39-B 132-kv. Breaker. Arrangements for the test included, among other things, first, tapping the Alliance Line at the first tower the same as for the Brown Boveri 132-kv. test, and running this tap through the FHKO-39-B and FHKO-136-B breakers which were connected in series to the short-circuit point; second, building a temporary house with dark room for the oscillograph equipment, including in this house a platform insulated for 132,000 volts to carry one of the General Electric oscillographs used for measuring currents by means of shunts placed

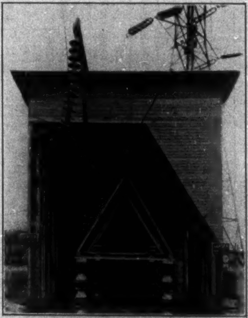


Fig. 18—House Built for Sheltering the General Electric Oscillograph Shunts and Prof. Dyche's Current Transformer

TABLE III
RESULTS OF TESTS ON GENERAL ELECTRIC TYPE FHKO—39B, 135-KV. BREAKER

Duty cycle and system set-up	Test number	Test voltage	Recovery volts Across pole 2				Current						Duration of short ½ cycles		Arc length inches	Short-circuit kv-a.			Remarks
			G. E. Co.		Dyche		Closing				Initial in arc r. m. s.					Closed G. E. Co.	Opened		
							G. E. Co.		Dyche										
			Peak volts	Per cent initial	Peak volts	Per cent initial	Peak	R.M.S.	Peak	R.M.S.	G. E. Co.	Dyche	Total	Arcing			G. E. Co.	Dyche	
Trial shot 1-OCO 2 gen. at Windsor..............	1	132,000	*82,500	76.5	90,000	83.5	Films could not be read						39.0	15.0	..	..	..	..	Opened O. K.
8-OCO 30- to 40-sec. interval except between No. 5 and No. 6. 2 minutes to change film. 2 gen.-2 lines 2 banks at Windsor 1 gen., 2 banks, 2 lines at Philo..................	2	132,000	96,500	90.0	105,500	93.2	1870 2455 2050	1365 1565 1380	2390	1530	1080 1150 1120	1160	39.0	16.0	11	357,000	263,000	265,000	do
	3	132,000	No G. E. film	..	95,200	88.3	..	..	2500	1490	..	1100	37.8	16.0	..	‡341,000	..	252,000	do
	4	132,000	No G. E. film	..	86,700	80.4	..	..	2710	1580	..	1175	39.0	16.0	..	‡361,000	..	269,000	Opened O. K. Trace of smoke.
	5	132,000	No G. E. film	..	88,500	82	..	..	2290	1440	..	1130	39.4	16.0	..	‡329,000	..	258,000	do
	6	132,000	96,500	90.0	90,000	83.5	1975 †	1400 ..	2910	1580	1080 1140 1070	1120	37.0	16.0	11	320,000	260,000	256,000	do
	7	132,000	No G. E. film	..	95,200	88.2	..	..	2080	1370	..	1130	39.0	16.0	..	‡312,000	..	258,000	do
	8	132,000	No G. E. film	..	88,500	82.0	..	..	1830	1290	..	1100	47.0	19.0	..	‡295,000	..	252,000	do
	9	132,000	98,000	91.5	No record	..	†	.. .. ..	2870	1660	1080 1175 1135	1100	43.0	19.0	12.9	‡380,000	268,000	252,000	do
2-OCO 2-min. interval full system §..............	10	132,000	83,000	77.5	78,000	72.5	4760 6350 6560	3250 3670 3860	8680	5030	2720 2930 3010	3410	35.5	13.0	11.6	885,000	670,000	780,000	Opened O. K. No smoke.
	11	132,000	*81,000	75.0	156,000	145.	4675 6840 6475	3300 2990 3785	8680	5050	Film blank	3410	34.0	13.0	..	910,000	Film blank	780,000	Opened O. K. Some smoke.
2-OCO 1-min. interval full system §..........	12	132,000	78,000	73.0	No Dyche film	..	6360 5740 6900	3900 3620 4080	..	..	2730 2840 2790	..	38.0	14.0	8.8	930,000	640,000	..	Opened O. K.
	13	132,000	No films	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	do

TABLE III—*Continued*

RESULTS OF TESTS ON GENERAL ELECTRIC TYPE FHKO—39B, 135-KV. BREAKER

Duty cycle and system set-up	Test number	Test voltage	Recovery volts Across pole 2				Current						Duration of short ½ cycles		Arc length inches	Short-circuit kv-a.			Remarks
			G. E. Co.		Dyche		Closing				Initial in arc r. m. s.								
							G. E. Co.		Dyche							Closed	Opened		
			Peak volts	Per cent initial	Peak volts	Per cent initial	Peak	R.M.S.	Peak	R.M.S.	G. E. Co.	Dyche	Total	Arcing		G. E. Co.	G. E. Co.	Dyche	
7-OCO 1-min. interval except between No. 17 and No. 18–2 min. to change film. Full system§	14	132,000	66,700	62.5	74,700	69.3	7550 7540 5800	4375 4740 3730	8920	4900	2220 2360 2290	2880	52.0	15.0	8.8	1,080,000	538,000	640,000	Opened O. K.
	15	132,000	No films	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	do
	16	132,000	No films	..	86,700	80.5	..	..	8820	5150	No film	3510	37.0	15.0	..	‡1,178,00	..	803,000	do
	17	132,000	78,000	73.0	No Dyche film	..	7025 6240 7860	4180 2820 4220	..	..	2780 2960 2840	..	36.0	14.5	10.5	965,000	675,000	..	Opened O. K. Puff of smoke Tank No. 3.
	18	132,000	81,500	76.0	104,000	96.5	5840 6960 5530	3470 4100 3620	8350	4970	2780 2960 2840	3450	36.0	12.0	8.8	940,000	675,000	789,000	Opened O. K. Some smoke.
	19	132,000	100,000	93.5	118,000	109.5	8350 6170 7450	4825 3960 4340	7190	4440	2680 2840 2740	3380	36.0	13.0	7.6	1,100,000	650,000	773,000	do
	20	132,000	*81,000	75.5	90,000	83.5	5700 7170 7320	3740 4150 4250	7890	4570	2780 2980 2870	3450	34.0	12.0	10.5	970,000	680,000	789,000	do
2-CO 2-min. inteval full system†	21	132,000	111,200	104.0	No Dyche film	..	5040 5450 5250	3420 3820 3510	..	..	2915 2980 2960	..	30.0	11.0	9.3	875,000	680,000	..	Opened O. K.
	22	132,000	91,200	85.0	104,000	96.5	7155 7320 5550	4455 4360 3670	9050	5320	2900 3020 2960	3610	31.0	13.0	9.9	1,020,000	690,000	826,000	do
4-OCO 2-min. interval full system§ short circuit ungrounded	23	132,000	*63,700	59.5	No Dyche film	..	5700 8340 7460	3580 4800 4340	..	..	2625 2840 2740	..	40.5	14.0	9.9	1,100,000	650,000	..	do
	24	132,000	93,500	87.5	No Dyche film	..	5550 5740 7180	4375 2820 4170	..	..	2760 2900 2790	..	34.0	13.0	9.9	1,000,000	665,000	..	do
	25	132,000	100,000	93.5	Do	..	6240 7600 6220	3740 4450 3870	..	..	2785 2960 2870	..	37.0	13.5	9.9	1,020,000	675,000	..	Opened O. K. Some smoke.
	26	132,000	*86,700	80.6	Do	..	7155 7170 5380	4190 4220 3760	..	..	2810 3010 3010	..	40.0	14.0	11	965,000	690,000	..	do

*Obtained from line voltage (arc voltage record N. G.)

†Curves off film.

‡Closed short-circuit kv-a. from Dyche Films.

§"Full System" means the set-up shown in Fig. 4 minus the 22,000-kv-a. generator at Akron.

directly in the short circuit leads; and third, the building of a special insulated triangular framework support for mounting the shunts, together with the shelter, all of which is shown in Fig. 18. In addition to the three oscillographs supplied by the General Electric Company, a fourth machine, Professor Dyche's, was provided in order to obtain parallel records of voltages and current, the latter by means of the same current transformer used in the previous Brown Boveri test. The system set-up for these tests was that shown in Fig. 4 with the omission of the 22,000-kv-a. generator at Akron.

Since it was planned on these tests to take a number of shots with the short-circuit point ungrounded, it was necessary to carry the leads from the shunts on 132,000-volt insulators to the oscillograph house and, as stated above, mount the oscillograph on an insulated platform. It was also necessary to carry a 132,000-volt insulated connection from one terminal of one of the permanently installed Windsor line potential transformers located in the 132-kv. yard, back to the short-circuit side of the 136-B breaker.

In an attempt to duplicate as nearly as possible on the FHKO-39-B the tests carried out on the Brown Boveri 150-kv. breaker, the program for testing consisted of a preliminary trial shot followed by a series of eight shots in rapid succession at approximately one-fourth the rating of the breaker, a standard duty cycle at full system capacity, a special duty cycle consisting of 2-CO shots with two-minute interval to obtain a shorter duration of short-circuit current, and finally several special duty cycles including a series of 7-OCO shots, also with full system capacity. Most of these tests were to be made with the short-circuit point grounded, permitting the taking of records both on the shunts and on the current transformer with its limited insulation. After these it was planned to take a number of shots with the short circuit ungrounded and with the current transformer disconnected from the circuit.

Table III gives a complete summary of the results of all of the tests on the 39-B breaker including the data from both the General Electric Company oscillographic equipment and Professor Dyche's equipment. Using the data given in Table III on the system set-up for each test and by referring to Fig. 4, the exact system connections for any test may be obtained, omitting of course the 22,000-kv-a. generator at Akron, shown on Fig. 4.

FIG. 19—OSCILLOGRAMS OF TEST NO. 10 ON FHKO-39B BREAKER (PROF. DYCHE)

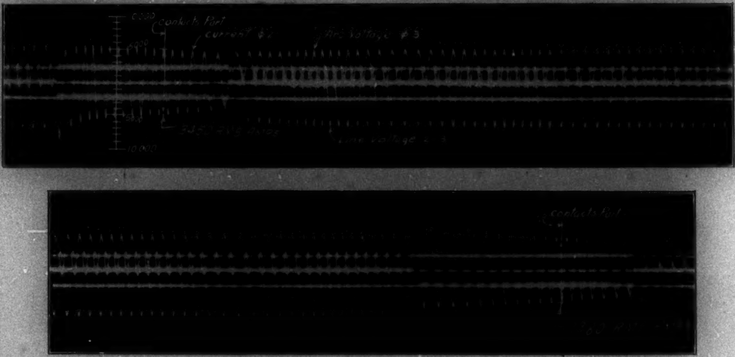


FIG. 20—OSCILLOGRAMS OF TEST NOS. 18 AND 19 (TAKEN ON ONE FILM BY PROF. DYCHE)

During all of the tests on this breaker no inspections of the contacts were made at any time. Typical oscillograms of these tests are shown, both those obtained by the General Electric Company and those from Professor Dyche's oscillograph. Of the latter, Fig. 19 shows the first shot of the standard 2-OCO duty cycle comprising tests 10 and 11 which were

TABLE IIIA
RESULTS OF COMPARATIVE TESTS MADE ON GENERAL ELECTRIC CO. LINE SHUNTS AND PROF. DYCHE'S CURRENT TRANSFORMER. TAKEN AS IN FIG. 24

Line shunt no.	G. E. Co. test no.	Current as measured by Prof. Dyche's oscillograph	Meter reading on ammeter in C. T. sec. circuit	Line current	Current as measured by G. E. Oscil.	Ratio of G. E. reading to meter in per cent	Ratio of Prof. Dyche's reading to meter in per cent	Ratio of Prof. Dyche's readings to G. E. readings in per cent
1.	23 HX	2040	4.77	1910			107.0	
1.	24 HX	2040	4.76	1905			107.0	
1.	25 HX	2040	4.76	1905	1860	97.7	107.0	109.5
1.	26 HX	2040	4.74	1895	1800	95.0	107.5	113.0
1.	27 HX	1112	2.73	1090	1070	98.3	102.0	104.0
1.	28 HX	1112	2.74	1097			101.0	
1.	29 HX	1112	2.74	1097			101.0	
1.	30 HX	2230	5.42	2170	2175	100.0	103.0	102.5
1.	31 HX	2230	5.39	2160	2040	94.5	103.0	109.0
3.	32 HX	1880	4.82	1925	2000	104.0	97.8	94.4
3.	33 HX	1970	4.81	1920	1910	99.5	103.0	103.0
3.	34 HX	1075	2.67	1070	1145	107.0	100.0	94.4
2.	35 HX		2.61	1042				
2.	36 HX	1043	2.60	1040	1093	105.0	100.0	95.3
2.	37 HX		4.69	1875				
2.	38 HX	1910	4.70	1880	1790	95.3	101.5	106.5
2.	39 HX	1810	4.68	1870	1885	101.0	96.8	96.3
2.	40 HX	1020	2.59	1035	1070	103.5	98.5	95.3
				Total Average		−100.1	102.25	102.1

the first tests using the full capacity of the system.

After the completion of this duty cycle oil samples were drawn from the middle of each tank and from the bottom of tank No. 1. Dielectric tests on these samples gave for tank No. 1, 24.5 kv. average; for tank No. 2, 21.3 kv.; for tank No. 3, 27.6 kv; and for the oil drawn from the botton of tank No. 1, 11 kv. No change was made in the oil.

The special duty cycle represented by tests 12 and 13 at one-minute intervals was not made intentionally as such but was supposed to be the beginning of the series of seven shots at one-minute intervals. Due to trouble with the oscillographs it was necessary to interrupt this series after the second shot.

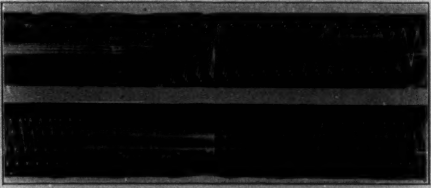


Fig. 21—Oscillograms of Test No. 26 (G. E.)

The special duty cycle of seven shots at one-minute intervals was then successfully carried out as shown under test Nos. 14 to 20 inclusive in the table. Due to the number of shots and the short interval between shots, it was not possible to record each shot on a separate film, although records were obtained on all but one of these shots from either one or the other of the oscillographic equipments. Fig. 20, which shows Professor Dyche's record of tests 18 and 19, is a reproduction of only a portion of the original film on which five shots were recorded. All of the shots were cleared by the test breaker with no evidence of distress, with no throwing of oil, and with very little smoke visible except on test No. 17 when No. 3 tank gave off quite a puff of smoke. This tank, however, as well as the other two tanks, did not give off more than a small amount of smoke on any other tests and the inspection following the completion of the tests did not reveal any unusual condition in No. 3 tank or anything different from that in the other two tanks.

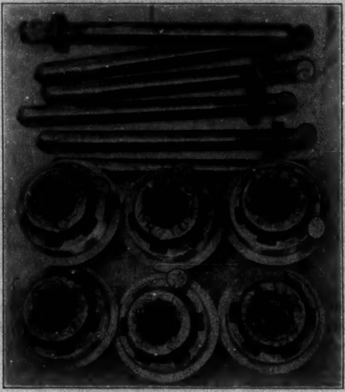


Fig. 22—Full Set of Contacts from FHKO-39B Breaker after Completion of Test Nos. 1 to 26

In order to obtain a higher value of current by decreasing the total duration of the short circuit, a duty cycle consisting of 2-CO shots was made as covered by test Nos. 21 and 22. While this type of duty cycle did decrease the total duration of short circuit to approximately 30 one-half cycles, the increase in ruptured kv-a. as shown in Table III was not very large.

All of the shots up to and including test No. 22 were made with the short circuit grounded. The remaining four shots, tests 23 to 26 inclusive, were then made with the short circuit ungrounded, so that it was necessary to disconnect the current transformer supplying Professor Dyche's oscillograph and confine further oscillographic records to the General Electric equipment, the current recording oscillograph as stated above being mounted on a platform insulated for 132,000 volts. Fig. 21 shows oscillograms obtained by the General Electric equipment on test No. 26 of this series.

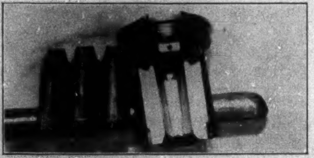


FIG. A

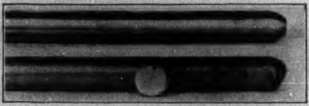


FIG. B

FIG. 23—(A) ROD AND CONTACT SEGMENTS FROM ONE POLE OF 39 B BREAKER AFTER TEST NO. 26

(B) COMPARISON OF NEW ROD WITH ROD FROM 39B BREAKER AFTER TEST NO. 26

At the conclusion of the tests the oil was immediately drained from the tanks and the contacts from phase 3, which had given off the puff of smoke on test No. 17, were removed from the tank. It was found that the burning was confined almost entirely to the arcing ring below the current-carrying segments and that the burning of the rod was confined to the arcing tip and was such that no beads or pits were left which might cause the rod to stick in the contacts. It was quite evident that the contacts were in sufficiently good condition so that without any dressing at all the breaker could be kept in service and carry its rated current even though 26 short circuits had been interrupted, 17 of them at full system capacity.

Fig. 22 shows all six contacts and contact rods taken from the FHKO-39 breaker after the completion of the tests while one of the contacts and contact rods with part of the segments removed to give a better view of the burning are shown in Fig. 23A. Fig. 23B, a comparison between a new contact rod and one of the rods taken from this breaker after the test, also shows the extent and character of the burning which was fairly smooth and without beads. The average test on the oil taken from this breaker after the completion of the tests was 19.25 kv.

An examination of Table III will reveal a comparatively large discrepancy between the values of current as recorded by the General Electric oscillographs and by Professor Dyche. The maximum discrepancy occurs on test No. 22 in which Professor Dyche obtains a value of 3610 amperes for the initial r. m. s. current in the arc as against a value of 3020 for the same phase on the General Electric oscillograms. Differences of somewhat lesser magnitude will be found in most of the other tests, particularly on the full system capacity shots.

In order to discover whether one or the other of the equipments was in error or whether the observed discrepancy was merely the possible error inherent in the oscillographic method of recording currents, a calibration check was made using the set-up in Fig. 24. A heavy current transformer for stepping down from 220 volts to three volts was obtained, together with a 2000-ampere instrument type current transformer and an ammeter calibrated with the current transformer, both of General Electric make. With this set-up, a current was passed through the circuit, first approximately 1000 amperes and then 2000 amperes and simultaneous readings were obtained on both oscillographic equipments as well as the calibrated ammeter. The results of these tests made under steady state conditions which are shown complete in Table IIIA give a maximum variation between Professor Dyche's record and that of the General Electric oscillographs of 13 per cent, which is nearly as large as the discrepancy found on the actual short-circuit test.

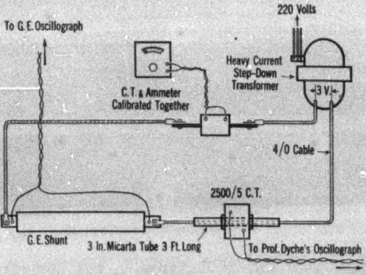

220 Volts
To G. E. Oscillograph
Heavy Current Step-Down Transformer
3 V.
C. T. & Ammeter Calibrated Together
4/0 Cable
2500/5 C. T.
G. E. Shunt
3 In. Micarta Tube 3 Ft. Long
To Prof. Dyche's Oscillograph

FIG. 24—SET-UP FOR CHECKING G. E. SHUNTS AND PROF. DYCHE'S CURRENT TRANSFORMER

Since check tests (with special current transformer and its own meter) showed the two oscillograph equipments to read within a very few per cent, it is difficult to explain what caused the discrepancy of some 15 per cent on test No. 22 and a smaller percentage on certain other tests. A great deal of study was given to circuit set-ups in the endeavor to locate the cause of this discrepancy but nothing definite was found. It is probably fair to assume that some condition peculiar to the particular set-up was the real cause, although unfortunately nothing definite was determined.

Plotting of the readings of the two oscillographs against the readings of the standard current transformer showed that the discrepancy was not consistent, one equipment reading high over part of the current range, then showing low results over another part; while the other equipment read low, then high over the same ranges. In view of the fact, however, that a certain amount of difference in calculated results is to be expected from such causes as differences in inaccuracy of reading waves of different heights and lengths, different methods of calibrating instruments, etc., a reasonably close agreement between results was shown. With these various considerations, therefore, it is probably not far from correct to assume that the true current values were represented by the mean of the two values indicated by the General Electric Company and Professor Dyche's equipments.

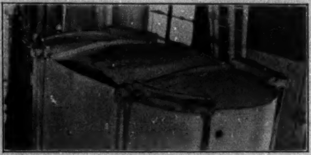


FIG. 25—MIDDLE TANK OF 136 B AFTER TEST NO. 31 SHOWING ACTUAL OPENING WHERE WELDED SEAM FAILED

Since the records in the case of the Brown Boveri 150-kv. circuit breaker tests, however, were taken with the same set-up as that used by Professor Dyche on the present tests, it is believed that the values obtained by Professor Dyche may be used as a fair comparison with the results obtained on the Brown Boveri breaker.

Tests on the FHKO-136B 132-kv. Breaker. After the completion of the tests on the FHKO-39B breaker, tests were made on the 136B which, as previously mentioned, was connected in series with the 39B, the latter then being used as a back-up breaker.

On account of the difficulties encountered in testing the 136 breaker, these tests were spread out over a considerable period of time and divided into three series, the first taking place on December 6, 1925, the second on January 10, 1926, and the third on May 23, 1926.

It was planned to subject this breaker to a series of eight OCO shots in rapid succession at approximately one-fourth of the breaker rating followed by one or more standard duty cycles at the full system capacity, and then to carry out one or more special duty cycles, such as four shots at full system capacity with one-minute intervals.

The first attempt to carry out the above tests on this breaker was made on December 6, 1925. Complete results of the tests made on this date, as well as of the tests made on the following January 10, 1926, are given in Table IV. As will be noted in this table, after subjecting the breaker to a trial shot and three of the proposed series of eight shots at approximately one-fourth of the breaker rating, the breaker failed by splitting open along the welded seam at the bottom of the middle tank, permitting the oil in that tank to escape. The character of this failure is clearly shown in Fig. 25 in which the tank is suspended and the view is from below. The tests were discontinued of course for that day and the tanks opened to permit examination of the contacts.

It was found that one of the insulating cylinders normally surrounding the explosion chambers had broken from its fastenings and was lodged on the cross-head. The other insulating cylinder in this pole had not fallen down but was partially broken from its fastening. It was apparent also that the explosion chamber insulation had failed, permitting the arc to cut through the throat bushing to the lower edge of the steel explosion chamber. Fig. 26A shows the broken throat with insulating collars in place. Marks on one of the fibre rings show evidence of burning by the arc. Fig. 26B shows the same throat bushing with one collar removed, revealing the place where the arc punctured through.

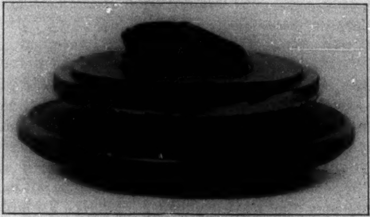


FIG. A

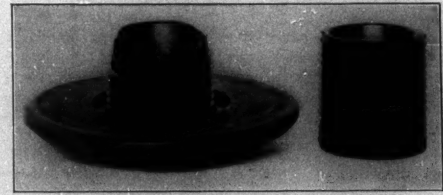


FIG. B FIG. C

FIG. 26—(A) THROAT BUSHING AND INSULATING COLLARS TAKEN FROM 136B BREAKER AFTER FAILURE ON TEST NO. 31

(B) SAME THROAT BUSHING WITH ONE COLLAR REMOVED SHOWING WHERE PUNCTURE OCCURRED

(C) THROAT BUSHING FROM OPPOSITE SIDE OF SAME POLE SHOWING BURNING

TABLE IV
RESULTS OF TESTS ON GENERAL ELECTRIC TYPE FHKO—136 B—135-KV. BREAKER, DECEMBER 6, 1925 AND JANUARY 10, 1926

Duty cycle and system set-up	Test number	Test voltage	Recovery volts Across pole 2				Current						Duration of short ½ cycles		Short circuit kv-a.			Remarks
			G. E. Co.		Dyche		Closing				Initial in arc R. M. S.							
							G. E. Co.		Dyche						Closed	Opened		
			Peak volts	Per cent initial	Peak volts	Per cent initial	Peak	R.M.S.	Peak	R.M.S.	G. E. Co.	Dyche	Total	Arcing	G. E. Co.	G. E. Co.	Dyche	
Trial shot 1-OCO 2 gen. at Windsor..........	27	143,000	..	..	No record	..	..	..	Could not be read	..	715	770	36.0	14.0	..	177,000	191,000	Opened O. K., part of G. E. oscillograph equipment insured by high voltage from shunts.
Beginning of 8-OCO 30-sec. interval. 3 gen. at Windsor. Reactors in circuit.	28	140,000	100,000	93.0	Ditto	..	..	..	2500	1440	950	930	41.0	14.0	†350,000	230,000	226,000	Opened O. K., slight jar.
	29	..	..	..			Breaker did not close					..	..	..	..	..	..	
	30	140,000	115,000	107.0	No record	..	..	..	1930	1110	704	700	55.0	18.0	†270,000	171,000	170,000	Opened O. K., heavier jar.
	31	140,000	90,000	84.0	Ditto	..	..	..	2030	1470	950	film cut off short	No record		†357,000	230,000	..	Heavy smoke on middle tank which split open around bottom weld letting oil escape. Test discontinued.
Trial shot 1-OCO 3 gen. at Windsor..............	*32	137,000	No record	..	Ditto	..	.. 1700 1940	.. 985 1120	1790	1100	.. 702 685	780	41.0	13.0	265,000	167,000	185,000	Opened O. K.
Beginning of 8-OCO 30-sec. interval 3 gen. at Windsor..............	33	134,000	Ditto	..	Ditto	..	1780 1640 1840	1070 973 1060	1570	990	700 668 705	740	No record		256,000	164,000	172,000	Opened O. K.
	34	132,000	No G. E. films	..	Ditto	..	..	..	1490	920	..	710	Ditto		†211,000	..	163,000	Opened O. K.
	35	131,000	Ditto	..	Ditto	..	..	..	2040	1180	..	720	Ditto		†268,000	..	193,000	Sharp report. Fire from vents on pole 3, much smoke, High side bushing broken on this pole. Tank bulged, oil level dropped 3 in. Test discontinued.

*Tests 32 to 35 inclusive made on January 10, 1926.
†Closed short-circuit kv-a. from Dyche Films.

Another throat bushing also bearing evidence of puncture is shown in Fig. 26C. The steel explosion chamber itself showed marks of the arcing on the inside edge of the bottom opening, and retained the imbedded half of the six fibre screws which were broken off and which, together with the explosion chamber throat bushing, were used to support the insulating cylinder around the explosion chamber. The contacts taken from this breaker, two of which are shown in Fig. 27, indicated by the small amount of pitting that the actual short-circuit duty was very light.

On the trial shot an accident occurred to the General Electric oscillographic equipment which may be of interest to describe. On all of the tests made on the 39B the oscillograph mounted on the insulated platform was kept entirely separate from the other equipment, using a separate 125-volt battery for motor control, etc. At the beginning of these tests through some delay the separate 125-volt battery was not available so that on the first shot the necessary current for operating the oscillograph motor and arc was taken from the battery connected to the other oscillographs. This was done partly through an oversight as it was fully realized that considerable voltage might be built up on the short-circuit side of the breaker due to the resistance drop of any momentary unbalanced current which might flow throught the ground connection. This was definitely proved on the first shot taken under these conditions. As soon as the circuit was closed, considerable fireworks ensued with resultant serious damage to the General Electric equipment, all of the elements being burnt out in the insulated oscillograph and some of the elements in the other two oscillographs, a total of six out of nine being completely burned out. This burning was not only confined to the vibrators proper but in some cases the field coils too were burned out. Naturally, no records were obtained on the General Electric oscillographic equipment in this series.

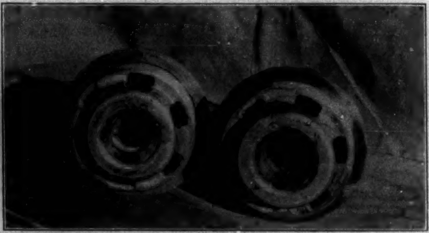


FIG. 27—CONTACTS FROM 136B BREAKER AFTER TEST NO. 31

In going over such evidence as was available as to the cause of the breaker failure, two things were apparently certain; first, it seemed established beyond a doubt that material used in the throat bushing was inadequate from the standpoint of dielectric strength and perhaps also from a mechanical standpoint; and second, the tank, and particularly the weld, seemed to be weak. The first, that is, the faulty bushing, through its electrical breakdown, probably caused the mechanical breakdown which in turn ruined the explosion chamber assembly and at the same time allowed open arcing with the result that there was created a pressure sufficiently high to open a weld which was none too strong in the first place.

The tanks were returned to the factory and rewelded, a stronger weld being employed in the new set-up. New types of throat bushings slightly different in design but principally different in the employment of new material, supposedly stronger from a mechanical and electrical standpoint, were also supplied. The breaker, embodying these changes, but no others, was submitted to a second series of tests on January 10, 1926.

The set-up for this second series of tests on the 136B breaker was the same as on the first series with the exception that the 39B back-up breaker instead of being connected directly to the Alliance line was connected by means of a short temporary 132-kv. line to the end of the 132-kv. station bus. The results of this series of tests, which along with the first series are summarized in Table IV, proved to be almost an exact repetition of the results obtained on the previous series, with the exception that the failure on the third shot of the proposed series of eight was evidenced by the breaking of a 132-kv. bushing on the high side of the breaker, permitting oil to leak out, and by a sharp report with fire issuing from the vents on pole 3.

The second series of tests being thus ended, the oil was immediately drained from the tanks and the interior examined. As in the case of the failure on December 6, it was found that the insulating cylinders on both the explosion chambers in No. 3 tank (the one on which the bushing was broken) were broken from their fastenings and both were down on the cross head. It was noted also that this tank showed considerably more bulging than the other two. Upon removing the explosion chambers from all of the tanks it was found that throat bushings were again broken in both No. 2 and No. 3 tanks. In this case, however, the breaks were such that it did not seem possible that they were caused by electrical puncture. These broken bushings are shown in Fig. 28. The manner in which the high-voltage bushing was broken is illustrated in Figs. 29 and 30, the break at the lower end of the bushing not being discovered until the bushing was taken apart.

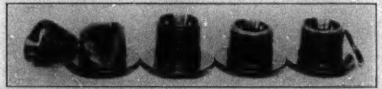


FIG. 28—THROAT BUSHINGS FROM 136B BREAKER AFTER TEST NO. 35 (SECOND FAILURE)

The final explanation adopted as the reason for the failures of December 6 and January 10, and the one which served as the basis for the changes that were made prior to the tests of May 23, was as follows:

The fairly long insulated cylinder placed over each explosion chamber was supported only and entirely at the bottom, partly by means of a number of fibre screws tapped into the bottom of the steel explosion chamber and partly by the explosion chamber throat itself. This assembly is shown in Fig. 31. When the breaker opened on short circuit the generation of gas caused some internal pressure, throwing oil against the flat sides of the tank and springing these sides out to a certain extent. On the rebound the oil was made to exert force in the opposite direction causing considerable thrust against the insulating cylinders over the explosion chambers. Since these long insulating tubes were supported only at the bottom, the cantilever strength was insufficient to withstand this shock and the supports had to give way, thus breaking the explosion chamber throat and allowing the cylinder to drop.

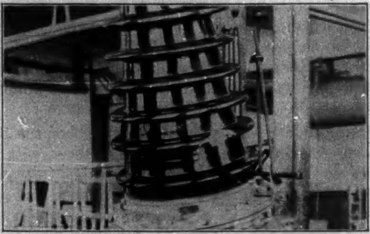


FIG. 29—UPPER HALF OF 132-KV. BUSHING FROM POLE NO. 3 OF 136B BREAKER AFTER TEST NO. 35

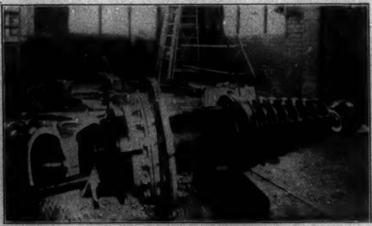
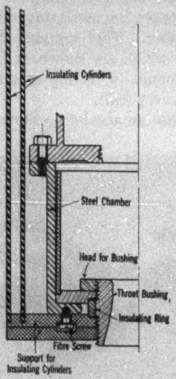
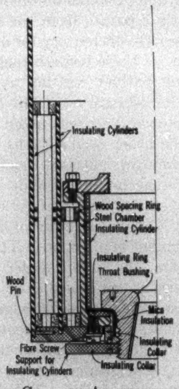


FIG. 30—BUSHING FROM POLE NO. 3 OF 136B BREAKER AFTER TEST NO. 35. NOTE BREAK AT LOWER END

While there was no conclusive evidence to show whether the electrical puncture encountered on the throat bushings during the test of December 6 was caused by mechanical failure or was the cause of the mechanical failure, at the same time there is a slight preponderance of evidence, especially after an analysis of the tests of May 23, that would show that mechanical failure preceded electrical failure. Following out this theory a new assembly for the explosion chamber parts, including the insulating shield, was worked out and is shown in Fig. 32. It will be noted that this assembly differs in a number of respects from the former, but the principal new features are, first, that the insulating cylinder is supported at the bottom again by means of fibre screws but tapped directly into the steel pot and kept quite separate from the explosion throat. Further, although not shown in the sketch, the number of these


Insulating Cylinders
Steel Chamber
Head for Bushing
Throat Bushing
Insulating Ring
Fibre Screw
Support for Insulating Cylinders
FIG. 31—EXPLOSION CHAMBER ASSEMBLY USED IN 136B BREAKER ON TESTS 27 TO 35 INCLUSIVE (DECEMBER 6 AND JANUARY 10) EXCEPT THAT THROATS USED ON TESTS 32 TO 35 WERE OF THE TYPE SHOWN ON FIG. 48


Insulating Cylinders
Wood Spacing Ring
Steel Chamber
Insulating Cylinder
Insulating Ring
Throat Bushing
Wood Pin
Mica Insulation
Insulating Collar
Fibre Screw
Support for Insulating Cylinders
Insulating Collar
FIG. 32—EXPLOSION CHAMBER ASSEMBLY AS REDESIGNED FOR TESTS 36 TO 65 (MAY 23, 1926)

supporting screws was increased from six used originally, to 12 in the new assembly. Second, an entirely new part was installed consisting of two wooden rings, one at the top of the steel pot and the other at the bottom, exactly filling the space between the inner insulating cylinder and the steel at these two points, thus taking all cantilever strain off from the lower end of the cylinder. These two upper and lower wooden rings are joined together by several connecting wooden uprights making a rigid structure.

With these changes carried out the third series of tests on the 136B breaker was arranged for May 23, 1926.

The testing arrangements on this series were the same as on the last, with the exception that the system capacity available was somewhat less, the set-up being in accordance with Fig. 4 with the following changes: first, no generating capacity was available from Akron, and second, at Cleveland the capacity was reduced from seven to five generators with only one transformer bank stepping up from 11 kv., one 66-kv. cable circuit, and one transformer bank stepping up form 66 kv. to 132 kv. at South Akron. With this reduced capacity the calculated value of short circuit available at 0.16 sec. after the beginning of the short circuit was reduced from 775,000 kv-a. to 685,000.

The complete results of this final series of tests on the 136B breaker, made on May 23, 1926, are summarized in Table V. In view of the discrepancies which were found between readings on the General Electric oscillograph and on Professor Dyche's during the 39B tests, arrangements were made in these final tests to have one element of the General Electric oscillograph supplied from the current transformer which was supplying Professor Dyche's oscillograph.

The program for testing was laid out in the same manner as that which had been attempted for the two preceding series. After making the preliminary trial shot to determine whether all oscillographic equipment was functioning properly, the series of 8-OCO shots at approximately one-fourth the breaker rating in rapid succession was begun. A little trouble was experienced at first when the breaker failed to close on the second shot of this series and when, after making slight adjustments on the mechanism, it failed to close again on the third shot of a second attempt to make the series. After this the mechanism was not satisfactorily adjusted, and the 8-OCO shots at approximately 30-sec. intervals were carried out. No distress was apparent on any of the shots, only a slight trace of smoke being visible and no oil being thrown.

The next test was a standard duty cycle of 2-OCO shots with a two-min. interval at the full system capacity available. Following this a duty cycle consisting of 2-CO shots with a two-minute interval was given to the breaker with the idea of obtaining a somewhat higher current due to decreased time between the beginning of the short circuit and the first half cycle of arcing. As will be noted from the value of interrupted kv-a. on both of these duty cycles, this procedure did result in increasing the current from approximately 2300 to approximately 2500 amperes. On both of these duty cycles, which include tests 48 to 51, the short circuit was cleared without signs of distress on the part of the breaker.

As a special duty cycle with a larger number of shots at closer intervals, the breaker was subjected to 4-CO shots at one-min. intervals with the maximum system capacity available. These four shots which averaged 600,000 kv-a. interrupted were handled without any distress by the breaker.

After these tests a number of additional shots were taken with the short circuit ungrounded. Professor Dyche's equipment, of course, was disconnected from the current transformer, which was not insulated for high voltage, and General Electric records only were obtained, taking advantage of the oscillograph mounted on the insulated platform. An attempt was made also to further increase the short-circuit current by means of CO shots on which the test breaker was tripped through an auxiliary switch on the KO.39 back-up breaker, thus decreasing the total duration of the short circuit. The first series made in this manner consisted of 4-CO shots at one-min. intervals, records being obtained only on the first and last shots. It was found, however, that the adjustment of the auxiliary switch trip on the back-up breaker did not speed up the tripping of the test breaker as much as had been anticipated.

The next series of tests consisted of 4-OCO shots at one-min. intervals at full system capacity, the first and last of these being recorded in Table V.

In a final attempt to approach nearer to the rating of the FHKO-136B test breaker a further adjustment was made on the auxiliary switch of the 39B back-up breaker so as to speed up considerably the tripping of the test breaker. With this adjustment and with the system still ungrounded a final duty cycle consisting of 2-CO shots at two-min. intervals was carried out. The attempt to speed up the tripping was quite successful in this case as it will be noted from the table that the total duration of short circuit was reduced to approximately 31 half-cycles and the interrupted kv-a. was increased to 625,000, a larger value than on any of the previous shots. These shots were handled by the breaker with no distress, only a small amount of smoke being given off and no oil being thrown. Oscillograms taken on this duty cycle, tests 64 and 65, are shown in Fig. 33.

After the completion of the tests, samples of oil were drawn from the middle of the tanks and tested for dielectric strength, averaging 15 kv. as against approximately 30 kv. obtained for the original oil. Upon draining the oil and inspecting the interior of the tanks it was found that all of the insulating cylinders around the explosion chambers were in place and that no

TABLE V
RESULTS OF TESTS ON GENERAL ELECTRIC TYPE FHKO—136 B—135-KV. BREAKER, MAY 23, 1926

Duty cycle and system set-up	Test number	Test voltage	Recovery volts across pole 2				Current						Duration of short ½ cycles		Arc length inches	Short circuit kv-a.			Remarks
			G. E. Co.		Dyche		Closing				Initial in arc R. M. S.						Opened		
							G. E. Co.		Dyche										
			Peak volts	Per cent initial	Peak volts	Per cent initial	Peak	R.M.S.	Peak	R.M.S.	G. E. Co.	Dyche	Total	Arcing		Closed G. E. Co.	G. E. Co.	Dyche	
Trial shot 1-OCO 3 gen. at Windsor................	36	140,000	114,000	110.0	97,800	86.0	1570 1440 1700 *1370	915 1020 980 970	1620	1060	693 720 693 730	740	57.5 56.0 55.0 56.0	22.0 22.0 22.0 22.0	13.5	247,000	175,000	178,000	Opened O. K. No smoke, slight jar
Attempt to get 8-OCO. Second shot did not close.....	37	140,000	126,000	110.0	109,000	96.0	1440 1680 1440 *1430	955 1175 980 1005	1780	1110	785 805 785 765	780	52.0 51.0 55.0 51.0	20.5 19.0 21.5 19.0	10.0	285,000	195,000	189,000	Ditto
Second attempt to get a 8-OCO—failed to close after 2 shots..................	38	140,000	126,000	110.0	No record	..	1830 1440 1700 *1370	1085 1020 1110 970	1430	965	760 805 785 730	760	53.0 54.0 52.0 54.0	17.5 19.0 18.0 19.0	11.7	269,000	195,000	184,000	Ditto
	39	140,000	No G. E. films	..	No record	..	..	..	1840	1070	..	750	55.0	19.0	..	260,000	..	182,000	Ditto
8-OCO 39-sec. interval except 2-min. interval between 43 and 44 to change film. 3 gen. at Windsor..	40	140,000	114,000	100.0	†	..	1700 2040 1570 *1940	1020 1200 1070 1130	2100	1240	785 805 785 730	785	53.0 53.0 52.0 53.0	21.5 22.0 22.0 22.0	11.5	291,000	195,000	191,000	Ditto
	41	140,000	No G. E. films	..	135,000	121.0	..	..	1740	1100	..	765	54.5	22.0	..	*267,000	..	186,000	Opened O. K. Some smoke.
	42	140,000	No G. E. films	..	127,000	114.0	..	..	1860	1100	..	750	56.0	22.0	..	*268,000	..	182,000	Ditto
	43	140,000	134,000	118.0	135,000	121.0	1830 2040 1960 *1940	1070 1190 1165 1130	1970	1150	760 805 785 730	760	54.5 55.0 57.0 55.0	20.5 21.0 22.0 22.0	11.0	288,000	195,000	184,000	Ditto
	44	140,000	107,000	94.0	103,000	92.7	1700 2040 1960 *1960	1020 1190 1230 1120	1870	1090	745 805 785 730	760	54.0 54.0 55.5 54.0	21.5 22.0 22.0 21.0	11.7	298,000	195,000	184,000	Ditto
	45	140,000	No G. E. films	..	119,000	107.0	..	..	2040	1180	..	760	53.0	22.0	..	286,000	..	184,000	Ditto
	46	140,000	Ditto	..	..	..	..	..	1600	1030	..	760	57.0	22.0	..	250,000	..	184,000	Dito
	47	140,000	100,000	88.0	No record	..	1700 1560 1570 *1485	1045 1020 1110 950	1770	1090	785 805 785 730	755	61.5 60.0 54.0 60.0	23.5 22.0 15.5 22.0	13.0	269,000	195,000	183,000	Ditto
2-OCO 2-min. interval. Full system..................	48	138,000	120,000	106.0	103,500	103.0	5240 6000 6020 *5700	3120 3480 3480 3300	6880	4000	2220 2380 2320 2200	2280	46.0 46.0 48.5 46.0	14.0 14.0 15.5 14.0	6.8	830,000	569,000	545,000	Cleared easily. Moderate jar, no smoke.
	49	138,000	103,000	91.0	78,700	78.3	5630 6000 4450 *5030	3300 3520 3060 2980	6540	3820	2140 2330 2320 2200	2310	52.5 50.0 47.5 50.0	18.5 16.0 13.5 16.0	8.5	841,000	556,000	552,000	Ditto

TABLE V—*Continued*

RESULTS OF TESTS ON GENERAL ELECTRIC TYPE FHKO—136 B—135-KV. BREAKER, MAY 23, 1926

Duty cycle and system set-up	Test number	Test voltage	Recovery volts across pole 2				Current						Duration of short ½ cycles		Arc length inches	Short circuit kv-a.			Remarks
			G. E. Co.		Dyche		Closing				Initial in arc R. M. S.					Closed G. E. Co.	Opened		
							G. E. Co.		Dyche										
			Peak volts	Per cent initial	Peak volts	Per cent initial	Peak	R.M.S.	Peak	R.M.S.	G. E. Co.	Dyche	Total	Arcing			G. E. Co.	Dyche	
2-OCO 2-min. interval. Full system..................	50	173,000	95,000	85.0	118,200	105.0	6550 4800 6550 *4570	3800 3190 2770 3040	4840	3140	2440 2590 2590 2380	2450	39.0 37.0 35.5 37.0	14.0 14.0 12.5 14.0	8.0	901,000	615,000	582,000	Cleared easily. Moderate jar.
	51	137,000	82,000	73.0	98,700	86.3	4700 5520 5500 *5250	3090 3290 3200 3100	5620	3270	2440 2540 2540 2330	2490	41.0 38.0 36.0 38.0	15.0 14.0 12.0 14.0	8.0	780,000	602,000	592,000	Ditto
4-CO 1-min. interval. Full system..................	52	137,000	85,500	76.5	97,200	87.4	5750 5280 6550 *5030	3390 3190 3780 3060	5520	3250	2320 2450 2410 2330	2440	40.5 41.0 41.0 41.0	14.5 16.0 16.0 16.0	9.5	895,000	582,000	580,000	Cleared easily. Moderate jar, No smoke.
	53	137,000	92,500	82.5	99,000	86.6	4970 5520 4850 *5150	3120 3290 3090 3080	5900	3490	2450 2540 2510 2420	2600	40.5 38.0 39.0 38.0	17.0 16.0 17.0 16.0	9.2	780,000	602,000	617,000	Ditto
	54	137,000	95,000	85.0	No record	..	5750 5400 6280 *4580	3420 3220 3620 7840	5570	3310	2410 2500 2540 2330	2450	39.0 40.0 36.0 40.0	13.0 16.0 12.5 16.0	9.5	858,000	602,000	582,000	Cleared easily, some smoke after ¼ min.
	55	137,000	105,000	94.0	No record	..	6550 4800 5500 *3890	3800 3240 3160 2680	5000	3140	2410 2450 2440 2370	2490	41.0 39.0 38.0 39.0	14.0 14.0 13.5 16.0	10.0	901,000	582,000	592,000	Cleared easily, slightly more smoke.
4-CO 1-min. interval. Full system. Ungrounded short..................	56	137,000	112,000	100.0	..	..	7600 4440 6930	4400 3100 4000	..	..	2380 2500 2480	..	39.0 39.0 37.0	15.0 15.0 13.0	9.5	1,041,000	592,000	..	Opened O. K. No smoke.
	†59	..	90,000	80.5	..	..	4450 5280 5900	2900 3100 3450	..	..	2130 2160 2170	..	57.0 56.0 56.0	18.0 18.0 17.0	11.0	818,000	515,000	..	Opened O. K. Some smoke (tests 57 - 58 opened O. K.)
4-OCO 1-min. interval. Full system. Ungrounded short circuit............	60	..	99,000	88.3	..	..	4580 5520 5500	2960 3360 3180	..	..	2410 2540 2500	..	42.0 42.0 36.5	19.0 19.0 15.5	8.0	797,000	602,000	..	On this test and also on 61 and 62 opened O. K. Some smoke.
	†63	..	..	..	..	..	4450 5640 5230	2900 3340 3090	..	..	1900 1955 1935	..	60.0 62.0 62.0	‡17.0	..	792,000	464,000	..	Opened O. K. More smoke.
2-CO 2-min. interval. Full system ungrounded short circuit..................	64	..	..	..	..	..	5500 4200 5500	3160 2440 3230	..	..	2590 2640 2640	..	31.0 31.0 30.0	‡17.0	..	765,000	625,000	..	Opened O. K. Slight smoke.
	65	..	..	..	..	..	4450 5640 5230	3010 3260 3040	..	..	2590 2640 2590	..	33.0 31.5 31.5	‡17.0	..	772,000	625,000	..	Opened O. K. Some smoke.

*Values obtained from CT in series with shunt in line No. 2.

†No films for tests 57, 58, 61, and 62.

‡No arc voltage record, values assumed

damage had been done to any of the throat bushings. The burning of the contacts which is shown in Fig. 34 was found to be confined almost entirely to the arcing tips of the contact rods and to the arcing bell of the explosion chamber, the contact segments themselves being in a very clean condition.

In view of the fact that the breaker handled a total of 30 shots without any inspection or even a change of oil, and that on 18 of these shots the duty was not far below the actual rated interrupting capacity, and further in view of the excellent condition that the breaker and contacts were in at the end of the tests, it does not appear rash to state definitely that the breaker was fully adequate to meet its guaranteed rupturing capacity and that the troubles in the explosion chamber assembly encountered in the tests of December 6 and January 10 were completely eliminated in the new assembly developed.

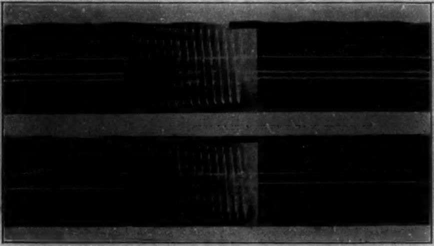


FIG. 33—OSCILLOGRAMS OF TESTS 64 AND 65 (G. E.)

VALUE OF THE TESTS

While the carrying out of tests on oil circuit breakers is as a general rule highly interesting, it is at the same time a very expensive affair and further very often results in a considerable upset of the system. Even if no actual physical damage results there is always the damage worked indirectly as a result of the effect of the short circuits on the system voltage and perhaps upon apparatus susceptible to voltage changes or dips. The authors believe therefore that before a test of this sort is undertaken the question should be raised as to the benefits that may be expected from the test and that these benefits should be weighed to make certain that they are sufficient to overbalance the possible harmful effects. Further, the tests having been carried through, it is very vital that the question should be raised again as to what value has actually been obtained.

Reviewing the results of the tests on the Brown Boveri breakers in the light of these data, it can be stated that in the case of the 150-kv. breaker tests the following benefits were obtained:

1. While the question was not definitely determined in the affirmative as to whether a multi-break breaker could be designed and built to handle successfully rupturing capacity in the order of 1,000,000 kv-a., it is believed at the same time that the performance of the breaker when rupturing a short circuit of the order of 75 per cent of that value was such that there appeared no doubt that the limit of the rupturing capacity of the breaker had not been reached.

2. Definite information was obtained as to the ability of the breaker to go through a cycle much more severe than the standard duty cycle. It was shown that for the system in question, if operating conditions called for it, the breaker tested could be plugged in on a short circuit four or five times in rapid succession with perfect safety.

3. In all, 26 short circuits were placed on the 132-kv. system of which 13 were at approximately full system capacity. So far as is known no appreciable damage of any sort resulted to the system. There were minor exceptions. One was that of the breaking of jewels on meters connected in secondaries of current transformers that fed heavily into the short circuit and which, through an oversight in some of the first tests, had not been removed from the circuit. A strain choke coil on the circuit supplying the full short-circuit capacity collapsed, and half of the primary of one of the current transformers on a 132-kv. circuit supplying the short circuit was short-circuited by arcing between turns. But with these exceptions no further damage of any kind was experienced.

When the tests were originally contemplated and the test procedure was being discussed, doubt was expressed by some of the operating people as to the advisability of purposely placing severe short circuits on a healthy system. The view that finally prevailed, however, was that a system such as ours was at all times in danger of having a short circuit placed on it with the full system capacity and if it was not in a position to stand up under such a short circuit, the sooner that condition was found and remedied the better off the system would be. It was satisfying to find that the system was able to go through all these short circuits without any appreciable damage.

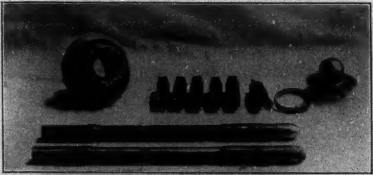


FIG. 34—ARCING BELL COMPARISON OF NEW AND USED CONTACT ROD, AND EXPLODED VIEW OF CONTACT SEGMENTS TAKEN FROM ONE POLE OF 136B BREAKER AFTER TESTS 36 TO 65

4. Until the carrying out of the tests in question no check was available as to the system short-circuit capacity. Many calculations and much design work

and specification work had been done, however, on the basis of calculated values and it was felt that sooner or later some of these calculations ought to be subjected to test to determine whether the actual values were in agreement within reasonable limits. The tests demonstrated that the system calculations were correct at least to within 10 per cent.

In the case of the 37-kv. breaker of the Brown Boveri type, the principal benefit obtained was, of course, a complete demonstration that the designer of the breaker had completely missed his mark in giving the breaker a guarantee of 250,000 kv-a. Further, it indicated that perhaps under no conditions would it be possible to build economically a single tank type breaker for that voltage and the rupturing capacity in question, namely, 250,000 kv-a. It would be interesting if this point, at some future time, could be definitely established or disproved.

As regards the Reyrolle tests, the benefits that were obtained as a result of the tests can be summarized as follows:

1. It gave confidence in the engineers' original decision to install this equipment although so far as was known no equipment of that type had at that time been placed in service in this country. The equipment, as is well known, however, is widely used on the continent and particularly in England.

2. As regards the design itself, it showed that the system of baffling, such as was originally proposed, which consisted of wooden linings with wooden barriers fastened to the linings, was inherently weak and was not advisable for breakers that were expected to rupture even such a moderate amount of power as 75,000 kv-a.

Coming now to tests of the General Electric Company breakers, it is believed that the following can be listed as distinct benefits resulting from the tests:

1. A further check was obtained on the system short-circuit capacity and on the calculations that had been employed in the past.

2. In all, 64 short circuits were placed on the 132-kv. system of which 35 were at approximately full system capacity. So far as is known, no appreciable damage of any sort resulted to the system. Full advantage, of course, was taken of the experience gained during the Brown Boveri tests, so that the troubles encountered at that time were not met with during these tests. It was highly satisfying to find that the system went through all these 64 tests without any damage.

3. The tests served as a very thorough check on the explosion chamber type of breaker and particularly on the assembly that was standard before the tests were undertaken. It showed the weaknesses of the original assembly and demonstrated the complete efficacy of the remedial measures that were finally applied to overcome these difficulties. This was of great value not only from the standpoint of future breakers but also in connection with some of the breakers that were already in service and that were employing the original explosion chamber assembly. An example of this occurred during the summer following the spring in which the last tests were made when perhaps six or seven 132-kv. breakers of the KO-39-B type failed in operation, the failure consisting of the inability of the breaker to go back in circuit after opening a number of times under short-circuit conditions. An examination of the explosion chamber assembly, which was the cause of the failure, showed that the failure encountered was exactly similar to that which had occurred during the tests on the 136-B breaker. As a direct result, therefore, of the experience that was obtained during the tests the new type of explosion chamber assembly was substituted for the old type and no trouble of any kind has been experienced since.

4. A benefit that must not be overlooked is the fact that a breaker of high rupturing capacity was subjected to a test very close to its rated values and on a cycle which might, on the basis of the present tentatively adopted standard for the derating of breakers for other than a standard duty cycle, be interpreted to have subjected the breaker to a duty considerably in excess of its rating. The test clearly showed that at least for the particular type of breaker in question, after certain changes had been made, the guaranteed limit could be handled by the breakers successfully. If now it is assumed that the design was carried through on a certain rational basis and on the basis of definite empirical and other fundamental data, then it may be safely considered that the tests of the breaker in so far as such data entered into its design, served as a check on it and on other breakers designed on the same principle.

The authors desire to acknowledge the great assistance and cooperation they have received in the carrying out of these tests, and without which the tests would have been impossible, from the Cleveland Electric Illuminating Company, The Northern Ohio Power and Light Company, The Ohio Public Service Company, and the West Penn Power Company whose systems were either tied in with the test circuit or through whose cooperation in carrying a certain portion of the load it was possible to make available the capacity gathered together for the tests.

The authors also wish to acknowledge the great help received from the American Brown Boveri Corporation and from the Reyrolle Company in furnishing the switches and assisting in the tests and to the General Electric Company for their cooperation in the making of the Reyrolle tests and for furnishing the switches, the test equipment and operators, and for other assistance rendered in connection with making the tests on their breakers. Finally, acknowledgment is due to the operating department of The Ohio Power Company and Professors H. E. Dyche and E. R. Rath for their great help in making and recording the tests.

Discussion

J. D. Hilliard: The management of the American Gas and Electric Company, and other companies allied in making the tests, showed real courage in permitting repeated and severe short circuits to be thrown on the system—short circuits of greater magnitude than any to which high-voltage system had hitherto been intentionally subjected,—and they deserve the thanks of the electrical profession. The operating and construction departments also deserve credit for their efforts in carrying out the testing program in an efficient and expeditious manner.

In consenting to the publication of the full details without reserve of the Canton tests made by Messrs. Sporn and St. Clair, the General Electric Company has taken the stand that it believes that the engineers of the power companies of the country should be fully informed regarding circuit-breaker operation. The General Electric Company's engineers realize the value of field tests as supplementing the tests made in its testing laboratory; they realize that the only test which absolutely determines the interrupting capacity of an oil circuit breaker is the test made repeatedly at the full rated capacity, both in current and voltage, and they realize also that the only thing proved by such a test is the interrupting capacity of the breaker upon that particular system and under the particular conditions existing at the time the test was made; that if the test had been made upon another system at the same voltage and current interrupted, results might have been entirely different. The latter remark is not "theorizing," but is based upon years of experience in circuit-breaker testing and is a fact.

In their paper, the authors have drawn certain conclusions which seemed logical to them; they have assumed that because to them, a breaker performed satisfactorily, at part rated current or part rated voltage, it would perform satisfactorily at its full rating. Such a conclusion is illogical and is, in a large number of cases, contrary to the facts. A breaker may interrupt more than its rated kv-a. at a voltage below its maximum rating, and be blown to pieces at a small part of its kv-a. rating at its maximum voltage rating. Many tests have proved the truth of this latter statement and have repeatedly proved it at the first shot of the higher voltage. The explanation is simple. The excessive current at the lower voltage produces a strong electromagnetic blowout effect which instantly ruptures the circuit, while at the higher voltage and much smaller current, the blowout effect is weak, the arc hangs, producing a continuous generation of gas which almost instantly blows off the tank. It is my belief that in reporting tests upon oil circuit breakers, all authors should stick to observed facts in the tests, that speculation is almost sure to mislead and may do a great deal of harm by giving confidence in apparatus which is in fact not the reliable piece it is assumed to be. The immediately proceeding remarks are general and not intended to apply to any particular breaker or make of breaker. It is a statement based on conclusions drawn from observing many tests.

It is not thought necessary to make extended explanations on the happenings in the case of the K-136-B breaker, except to say that the causes of the trouble are known, the remedy has been applied to our full satisfaction and the tests have proved that the explosion-chamber breaker is all we ever claimed it to be.

The tests upon General Electric Company's breakers confirmed the empirical formula upon which the interrupting-capacity rating is based, they confirmed our observations as to the burning of the arcing contacts in our own testing laboratory and it was this latter experience which influenced us to consent to the making of the full number of shots, (26 and 30 respectively), without an examination of contacts until the end of the tests. The tests confirmed also our laboratory tests for oil throw, as not a drop of oil was thrown from any tank during any shot on the K-39-B breaker and the final test on the K-136-B breaker. In short, except for the very slight burning of the arcing contacts, there was nothing to indicate that the breakers had undergone a test.

Evidently through an oversight, a misstatement of fact appears in the test of the Reyrolle breaker. I refer to the statement "For some unknown reason the short circuit lasted only 2½ cycles, etc.". As a matter of fact the reason was known at the time of testing and is previously explained as being due to the latch on the breaker not holding. The same thing had happened at previous shots. I believe it is due to the General Electric Company, whose generator and testing organization was used to make the tests that this statement of fact be made.

Since the data are given for the Brown Boveri and General Electric K-39-B and K-136-B breakers, it is possible to make in a way a comparison between them and since Prof. Dycke recorded the Brown Boveri tests, we shall take his records on the K-39-B and K-136-B breakers.

	Brown Boveri	General Electric	
		K-39-B	K-136-B
Rating of breaker......	1,500,000 kv-a. at 150,000-volt	1,250,000 kv-a. at 132,000-volt	750,000 kv-a. at 132,000-volt
Break in series.........	10	2-explosion chamber	2-explosion chamber
Total number of shots made...............	26	26	30
Maximum load interrupted..............	70,000 kv-a. (est)	826,000 kv-a.	617,000 kv-a.
Percentage of rated interrupting capacity interrupted..........	46.6	65.8	81
Ratio max. line voltage before shot, to rated voltage (per cent)....	89.5	100	104.5
Half cycles of arc at 2800 to 2900 amperes	13 min. to 17 max.		
Half cycles of arc at 3380 to 3610 amperes		11 min. to 15 max.	
Half cycles of arc at 2280 to 2600 amperes			12.5 min. to 17 max.
Oil throw.............	some	none	none
Contacts inspected and dressed during tests..	yes	no	no
New oil used during tests	yes	no	no
Signs of burning of any part of breaker other than arcing contacts.	yes	no	no

While no statement is made as to the speed of operation or arc lengths in the Brown Boveri breaker, it is believed from the arc duration that each of the ten arcs was not substantially shorter than each of the two arcs in the General Electric K-39-B breaker and in any event each of the two arcs of the General Electric K-39-B breaker had a shorter arc duration than each of the ten arcs of the Brown Boveri breaker. No conclusions are drawn from the above facts. I shall state, however, that the above observations agree with test results obtained in our testing laboratory on a breaker of similar design. I believe it should be stated that the observations and conclusions in reference to the General Electric Company breakers as stated in the paper are those of the authors, and do not necessarily express the opinions of the General Electric engineers.

Electrical engineers should understand that in testing oil circuit breakers they are dealing with very erratic phenomena and because of this fact should be slow in drawing conclusions from any series of tests or generally applying such conclusions.

The information gained by means of its present testing generator has been so valuable to the General Electric Company that it is building, and will have in operation in early summer, the argest testing generator in existence; a generator that will have a sustained short-circuit capacity for circuit-breaker interrupting tests of over 500,000 kv-a., three-phase, with provision for the addition of units of equal or larger capacity as they may be required. The new laboratory will be equipped to test breakers of all voltages and currents and to observe and record phenomena taking place during interruption.

The interruption of a given number of volt-amperes by the breaker—for example 100,000 kv-a.—does not necessarily impose the same stresses as result from the interruption of the same kv-a. at other times at the same point on the system, at other points on the system or on different systems. In other words, volt-amperes are not equally "hot" at all times and places, due to a number of causes. It seems probable that this difference in the difficulty of interruption depends largely upon the magnitude of the voltage "kick" at the end of each half wave of arc, and upon the speed at which each half wave of this transient voltage is built up. The need for making many tests on a device and making them under the most severe operating conditions is clearly indicated.

The Canton tests were very valuable in determining interrupting constants on that particular system and upon the existing conditions, but the "duty" was light as measured by other tests under other conditions. Certainly there are few, if any, other places in the world where so much power is available at 132,000 volts. In concluding my discussion, however, I wish to emphasize again the importance of conservation in drawing conclusions from one set of tests on a given breaker. Until experience is gained under conditions giving a vicious recovery voltage "kick," one is very likely to label a breaker safe which, as a matter of fact, is far from being so.

To give a concrete illustration of what may be expected, I shall cite one particular test in which the length of arc drawn in the same breaker operating with the same kind of oil, at the same speed, on the same system—*but a different part thereof*,—at the same voltage and current interrupted, consistently gave an arc nearly three times as long in one case as obtained in the other. In one case, the breaker was safe; in the other, it was severely stressed and if the break distance had not been large, it would have been blown up.

J. B. MacNeill: Looking over the data on the several makes of breakers given for the 132,000-volt tests, the thing that strikes one is that the duration of arcing time is comparable for all designs. For instance, with the 150-kv. Brown Boveri breaker, the average test voltage (so-called, in the paper),—that is, the system voltage prior to the short circuit,—for all the tests is 134,700 volts; the average duration of arcing in the breaker is 14½ half-cycles or 7¼ cycles on a 60-cycle circuit. In other words, the breaker handled approximately 18,600 volts per cycle of arcing.

Now, turning to the final set of tests on the General Electric 136-B breaker, (135-kv.), the average test voltage of 139,000 volts was somewhat higher, the average time of arcing was 16½ half-cycles on 60 cycles, or, the breaker handled 16,800 volts per cycle of arcing.

On the KO 39-B, rated at 132,000 volts, the average test voltage was 132,000 volts and the average time of arcing 13.3 half-cycles. This breaker handled voltage at the average rate of 20,000 volts per cycle of arcing.

So we have those three values for comparison; 18,600 volts per cycle, 16,800 volts per cycle, and 20,000 volts per cycle. They are all of the same order of magnitude. What are the relative dissipations of energy in the two types of breaker? Personally, until I saw the paper, I had expected to see considerably less duration of arcing on the 10-break breaker. While the data on volts handled per cycle are not conclusive regarding the operation of the breaker, in fact, the breaker with 10 breaks made a very successful test, but, looking forward to higher powers, the question naturally arises whether the dissipation of energy on 10 breaks isn't considerably greater than on 2 breaks. The indication from these tests is that 2 breaks will handle voltage about as fast as 10 breaks. Possibly these comparisons don't represent the ultimate development to which the two types may lead, but that is the conclusion that I draw from the data submitted here.

The length of the arcs in the breakers may vary and the dissipation of energy may not be five times in one case what it is in the other. The length of arc, I believe, was given for the General Electric tests but not for the Brown Boveri tests.

Another thing to which I invite attention is this: These tests are quite interesting from the point of view of the type of short circuit placed on the system. Some of the short circuits were grounded and some were made with the short circuit ungrounded. Most data that have been accumulated on previous tests on other voltages has indicated that ungrounded short circuits hang on longer. You get instantaneous conditions in the breaker when one pole has come to a zero of current whereby that pole may be subjected to as high as 87 per cent of line voltage. If the short circuit and the source of power are grounded, that particular pole that is open first cannot be subjected regularly to more than 58 per cent of line voltage. Consequently, we have been led to expect longer durations of arcing with short circuits ungrounded than with short circuits grounded. This is the first series of tests of any magnitude that I have seen in which the indications seem to be that an ungrounded short circuit is no more severe than a grounded short circuit.

Referring to Table V, we see that certain of the tests, (56 to 65), were made with the system ungrounded, and that they were made with different kv-a's ranging from 460,000 to 625,000 arc kv-a. The average duration of arcing on these tests is 16.7 cycles. Referring, now, to similar tests made on grounded short circuits, tests (52 to 55), the duration of arc is 15.4 on the same voltages, 134,000 kv-a.

The conclusion I should draw from this would be that at this voltage there is not much difference between a grounded and an ungrounded short circuit. This is of particular interest to the operating people at this time because of the discussions that have arisen as to the application of breakers on grounded-neutral systems. Several of the large systems have installed great quantities of apparatus for 220-kv. service, using 187-kv. apparatus. So far they have got away with it but there has been a great deal of discussion as to whether they were justified in buying under-rated apparatus. This is the first actual test information brought forward that indicates practically no difference between the two types of short circuit. If this data can be substantiated by further data, it would seem that the operator's practise of using undervoltage apparatus would be pretty well justified and that special apparatus was hardly necessary.

G. A. Burnham: It is possible to draw an entirely erroneous conclusion from witnessing the moving picture of an oil circuit breaker test.

The film of the 6600-volt test on one of our competitor's breakers showed a breaker tested to destruction. In making a judgment on this breaker's ability, one should bear in mind that this picture showed the oil circuit breaker undergoing a test at a value far above its rating.

With reference to Mr. Hilliard's comments on testing, we feel very much as he does. Apparently their vast experience in testing has led them to the conclusion that the variables in circuit-breaker operation are so great that a test on a particular circuit breaker at one place perhaps does not lead to a general conclusion as to how well that breaker may operate under other conditions. From this, we are led to believe that when comparison tests are to be made, (particularly on oil circuit breakers in which so much difficulty is encountered in arriving at analytical results), the test should be made in the same place; as near at the same time as possible; with the same system setup; and, if possible, the tests be run under what might be called a "master supervisor." It is obvious that exactly the same instruments should be used to record the results.

In making comparison between the Brown Boveri breaker and the General Electric Co.'s breaker in the 132,000-volt class, it is perhaps, as Mr. Hilliard has said, impossible to draw a conclusion without giving the most careful analytical study. The Brown Boveri breaker I believe handled all energy that could be imposed upon it at the time by the American Gas & Electric Co.'s system. It may have had slightly less duty, according to records, than the General Electric Co.'s breaker; nevertheless, we do not know how much more the Brown Boveri breaker would have handled had it been given the opportunity to display its ability. It apparently operated without sign of distress, and no doubt would have handled considerably more energy. Both Mr. MacNeill and Mr. Hilliard have referred to the fact that the duration of arcing was about the same in both breakers. Both gentlemen have therefore concluded that the arc length per arc in the Brown Boveri breaker was as great as the arc length per arc in the General Electric Co.'s breaker. From these statements, it would be a logical deduction to assume that the Brown Boveri breaker would have five times the total length of arc and for that reason would probably generate greater destructive forces. Such a conclusion would be incorrect as it rests upon the false premise that the speed of the moving elements was the same. The fact is that the moving element of the Brown Boveri breaker is slower and calculation shows that the total sum or actual arc length for the Brown Boveri 10 breaks is almost exactly equal to the total sum or actual arc length in the General Electric Co.'s 2 breaks. Furthermore, were Mr. Hilliard's assumptions correct, it is evident that the 10-break breaker would have to be built enormously stronger than the 2-break breaker. Analysis of facts will show that this older design of Brown Boveri breaker, successful as it was, was not of so heavy construction as the General Electric Co.'s breaker.

We believe this special test confirms the very satisfactory actual service results which have been had for some years in the United States with these multiple-break oil circuit breakers. It is our opinion that the multiple-break principle gives a more efficient handling of the arc resulting in less gas evolution, and less evolution results in lower pressures or destructive effects.

Philip Sporn: Mr. Hilliard stated in his discussion that a breaker performing satisfactorily at a low value of current may not perform satisfactorily at the maximum rating. There could be, of course, no argument on this point. What we should like to point out is that the engineer who tests his breaker and finds that it is satisfactory even at half rating is on safer ground than the one who makes no test at all.

Mr. Hilliard further stated that it is dangerous to speculate regarding the ability of a breaker to perform on the higher voltage from results obtained on a lower voltage. With this, again, we are in agreement. On the other hand, if a breaker is going to be used at other than its rated voltage, it should be tested at that voltage as that is the only way of finding out whether it can actually perform satisfactorily under those conditions.

Another statement made by Mr. Hilliard was that the test showed that the explosion chamber is all that it was claimed to be. Here, again, we agree. The explosion-chamber assembly is now all that it was claimed to be.

In connection with the table in which were shown comparisons between Brown Boveri and General Electric Company breakers, statement was made that the Brown Boveri breaker had the oil in it changed whereas the General Electric Company had no change. We should like to point out that this change in the case of the Brown Boveri breaker was made on one pole only, no changes whatever having been made on the other two poles. There was also a statement made to the effect that some burning occurred on the Brown Boveri breaker. Here, again, we should like to point out in fairness to the Brown Boveri Company that the burning in question was a slight amount of charring on a barrier that was made up in the field, out of micarta bought in a local shop. No trouble of any kind was experienced on any of the material that originally was supplied with the breaker.

Mr. MacNeill has raised the question as to whether, in view of the time of arcing being practically the same in the case of the General Electric as in the Brown Boveri breaker, this would not indicate that the relative rate of dissipation of energy was five times as great in the General Electric breaker as in the Brown Boveri breaker. We do not see that the tests have shown this. The fact of the matter is that the Brown Boveri breaker was physically lighter than the General Electric breaker. The above carefully considered may not lead to Mr. MacNeill's conclusions.

Another point brought out by Mr. MacNeill was the fact that previous to our tests it had been generally believed that ungrounded short circuits lasted considerably longer than grounded short circuits, whereas apparently in our tests this was not the case. It seems to us that there were not enough data obtained on this point to warrant any conclusions, nor do we see that this throws any conclusive light on the application of what we term 187-kv. breakers on 220,000-volt systems. For one thing, in a good many cases the breakers that were actually applied had clearances and length of stroke equivalent to 220,000-volt service, but bushings for 187,000-volt service. Such a piece of apparatus is, of course, underrated, but it certainly is not a straight 187-kv. piece of apparatus and the data which we have presented do not, we believe, throw very much light on the advisability or inadvisability of such practise.

Mr. Burnham has brought up a fact in regard to the Reyrolle breaker and motion pictures of the tests which were shown. It will be recalled that considerable fire and smoke ensued when the breaker exploded. We believe it was definitely pointed out in Table II that in the test where the breaker exploded, contacts of the breaker actually opened a short circuit of 122,000-kv-a. or a short circuit 63 per cent in excess of its guaranteed rating. The resulting pressure was enough to wreck the breaker tank. It would appear that little else could be expected under such conditions.

One more point in connection with the Canton tests we should like to bring out further and that is that, in all, we placed something like 90 short circuits on the system without apparently the slightest damage to it. While it may not be desirable in order to test the system to seek short circuits definitely, at the same time it is well to know that the system is so designed and assembled that it can withstand these trials when it may be called upon to withstand them without danger of everything breaking loose. So long as short circuits may occur on a power system it is essential that the system be so built that it can stand up under them and can come out of the short circuit unharmed except for the particular minor portion that can be affected by it. It seems to us that such a knowledge is bound to help the morale of an operating organization; that the less these things are feared the more likely it is that they will be handled properly when they do occur.

Klydonograph Surge Investigations

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***Synopsis.**—Since its recent development, the klydonograph has been used to investigate surge conditions on a number of transmission systems. Quantitative data, which show the characteristics of the actual surges present on transmission systems, are presented. In addition, these data are discussed in relation to various existing theories and practises regarding the production and the elimination of transient high voltages. The correctness of some of these ideas and the fallacy of others are indicated. The paper is subdivided, according to the nature of individual investigations, as follows:*

I. Open-wire Systems.
II. Cable Systems.
III. Lightning Arresters.

INTRODUCTION

THE development of the klydonograph[2] in 1924, by Mr. J. F. Peters, gave promise of settling the long-standing question of the prevalence of surges on transmission systems. This instrument and the results of the first few applications were described at the Annual Convention of the Institute in 1925.[3] Data from an extensive investigation on the 220-kv. lines of the Southern California Edison Company were presented by Mr. R. J. C. Wood[4] at the Pacific Coast Convention in the same year. Since the spring of 1925, 26 three-terminal klydonographs have been in almost continuous operation on many systems of widely varying characteristics. Work has been concentrated on open-wire systems during the lightning seasons and on cable systems at other times. This paper is presented to place the information obtained, before the engineering public.

In most cases, the klydonographs were connected and calibrated to measure the crest values of surges between conductors and ground. Therefore, the surge voltages have been referred to the "normal operating crest voltage to neutral." Where the term "times normal" is used, it refers to the magnitude of the surge, on this basis. In addition to measuring magnitude, the klydonograph indicates the polarity of a transient voltage; that is, whether it is positive, negative, or oscillatory, where these terms have their usual meaning. When an oscillatory voltage is recorded, a superposition of positive and negative figures is obtained on the klydonograph record. The number of cycles, if few, or the approximate duration of oscillatory surges may be estimated from the appearance of the figures.

The klydonograph figure resulting from a negative voltage, is less than half the size of that produced by a positive voltage of the same magnitude. It has been the practise to adjust the potentiometers, through which the klydonograph is connected to a system, so the operating voltage gives a potential of approximately three kilovolts crest at the instrument terminals. This produces a normal voltage line on the klydonograph film. Thus negative surges, up to about 2.5 times normal, are obscured beyond detection by this normal voltage line. However, since surges of this voltage class are not of great importance, it is considered preferable to continue this method of connecting the klydonograph, even at the sacrifice of these low-value negative surges. This arrangement permits the measurement of voltages of from seven to 10 times normal. Beyond this, the magnitude is estimated from the degree of spreading of the figure, or from the violence of the instrument flashover.

In addition to the investigations for the purpose of measuring transient voltages, arrangements were made to record the performance of lightning arresters on certain systems. The arrester-discharge current and the surge voltages to ground at the arrester terminals were the quantities measured.

The data have been divided into three parts, according to the nature of individual investigations:

I. Open-wire systems,
II. Cable systems,
III. Lightning arresters.

Each section is complete in itself. Of course, the surge condition found on open-wire lines are closely related to the subject of lightning arresters.

I. OPEN-WIRE SYSTEMS

Surges on open-wire transmission systems are of interest to the operating engineer, chiefly on account of the flashovers and consequent interruptions which follow. Although high voltages have a deteriorative effect on apparatus insulation, and although the life of this insulation probably is shortened by such surges, the effect on continuity of service usually is regarded as the more important point. The comparative freedom of high-voltage apparatus from failures justifies this opinion.

Consequently, in summarizing the data, the number of surges, rather than the number of indications, has been considered. For example, if high voltages appeared on more than one conductor, or at more than one point of a line simultaneously, they have been regarded as one surge. Obviously, such a surge could cause only one interruption, regardless of the voltages

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[1] Westinghouse Electric & Mfg. Company.

[2] The Klydonograph, J. F. Peters. *Electrical World*, April, 19, 1924.

[3] *The Klydonograph and Its Application to Surge Investigations*, J. H. Cox and J. W. Legg. TRANS. A. I. E. E., Vol. XLIV, 1925.

[4] *220 K. V. Transmission Transients and Flashovers*, R. J. C. Wood. TRANS. A. I. E. E., Vol. XLIV, 1925.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

existing on individual phases or at various points. The maximum value and its polarity have been used in classifying the magnitude and the polarity of each surge, in the tables.

Surges on transmission lines are usually due to one of three causes, lightning, switching, and short circuits due to mechanical reasons. However, it will be noticed that the causes of many surges are listed in the tables as unknown. No doubt, these were also due to lightning storms and switching operations, but incomplete operating records and discrepancies in recorded times of occurrence prevented the detection of many of these causes. However, the great majority were low in magnitude and, therefore, not of great importance.

All surges identified as due to lightning are listed in Table I, those to switching in Table II, and those to other causes, including unknown, in Table III. The tables show the relations between frequency of occurrence, cause, and magnitude, for each system investigated. In addition, the principal line characteristics, which are likely to have affected surge conditions, are included in Table I. A given system is designated by the same number in all Tables. Aggregate data for all systems are classified according to surge causes in Table IV.

In order to facilitate comparison of the results from various systems, all data have been brought to a common basis. This basis is ten years operation of one three-terminal klydonograph at one point of each system. The relatively long period of ten years was chosen to avoid fractions for high-magnitude surges, which were of infrequent occurrence. Thus, the actual number of surges of each classification, recorded on any given system, has been multiplied by the necessary constant. Since the lightning season usually is regarded as six months, the constant used where lightning surges were involved is one-half the value applied to all other surges. Although many investigations lasted only a few months, and although some were conducted at such times that no lightning data were obtained, the tests were sufficiently extensive to warrant some important conclusions.

Lightning. As shown by Table I, the majority of lightning surges were positive. However, all the highest surges were negative, but they were infrequent. The only positive surges over 10 times normal occurred on systems 4 and 21. These only slightly exceeded 10 times normal, while negative surges sometimes were considerably beyond this value. In three cases, direct strokes were known definitely to have caused the large negative records. In view of the fact that an induced surge is of opposite polarity, and that a surge, due to a direct stroke, is of similar polarity to the charged cloud, it is concluded that those clouds, which cause surges, are of negative polarity[5].

The occurrence of a flashover following a surge depends upon the wave-front, as well as upon the magnitude, of the transient voltage. The maximum possible potential, induced by lightning, is the product of the field gradient destroyed by the stroke, and the height of the line. It has been estimated that gradients as high as 100 kv. per foot (330 kv. per meter) are reached near the earth's surface; 60 kv. per foot (200 kv. per meter) has been measured. It is universally assumed that cloud-field gradients are constant for the height of a transmission line. Thus, extremely high induced potentials are possible. However, the extreme voltages are induced only near the discharge path. From this point to the boundary of the field, the gradient decreases rapidly.[6] The rate of rise of potential, on that part of the line directly in the cloud field, is equivalent to the rate of collapse of this field. Where negative clouds are concerned, it is believed that the collapse takes place in a time of the order of three microseconds. Upon the release of the bound charge, it divides and travels along the line in both directions. On points of the line outside the cloud field, the rate of rise of potential is determined by the front of the traveling wave. With the rates of discharge mentioned above, the front of this traveling wave is determined chiefly by the space configuration of the charge. Klydonograph data indicated that lightning surges have wave-fronts varying from a few to 200 microseconds. The higher-voltage surges had the steeper wave fronts. This conforms with the above discussion. Also, it is well known that the potential, which can be applied to any insulation, increases with the rate of application of the voltage. However, the time lag of the flashover of insulators decreases as the potential, in excess of the 60-cycle flashover voltage, is increased. Thus the potential, that can be applied at any given rate of application, is limited, and similarly the maximum voltages, reached by lightning surges of even the steepest wave fronts, are limited by insulator flashovers. These flashovers permit the charge to pass to ground. The limitation of voltage is indicated by the fact that, on systems of widely different voltages, the maximum surges were approximately the same number of times the operating voltages. However, the data are not conclusive on this point, owing to the limitations of range of the klydonograph. More definite information was obtained on a 55-volt signal circuit. Since this circuit was only 2.5 miles (4.0 km.) long, any surge induced upon it would have been detected. This line flashed over repeatedly during lightning storms, but the potentials never exceeded 10 kv.

In this connection, it is believed that the flashover voltage of 220 kv. transmission-line insulation, at the steepnesses of wave-front of lightning surges, is comparable to the maximum potential ordinarily induced by lightning. Thus, lines of this voltage should be practically immune to lightning, with the exception of direct strokes.

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[5] *Transmission Line Voltage Surges*, J. H. Cox, TRANS. A. I. E. E., 1927, pp. 330.

[6] *Lightning and Other Transients on Transmission Lines*, F. W. Peek, Jr., TRANS. A. I. E. E., Vol. XLIII, 1924.

TABLE I—LIGHTNING SURGES ON OPEN-WIRE SYSTEMS

	System								Surges per station in 10 years, according to voltage magnitude											
									Not causing flashovers						Causing flashovers					
No.	Voltage, kv.	Length in miles	Neutral Grounding	Ground wires	Minimum Insulation	Number Klyd. Stations	Months tests	Type *	Below 2.5 times normal	2.5–3.4	3.5–4.9	5.0–6.9	7.0–9.9	Over 10.0	Below 2.5	2.5–3.4	3.5–4.9	5.0–6.9	7.0–9.9.	Over 10.0
1	220	270	Solid at 7 points	One	12 10 in. disks	4	3	P												
								N												
								O												
2	140	245	Free	None	10 10 in. disks	4	8	P	64	4					6	6	2	2		
								N				2				2		2	2	2
								O		2					19	34	11			
3	140	174	Free	None	10 10 in. disks	1	4	P	225	30	15	15			15	15				
								N		30	15									15
								O	15						45	15		15		
4	120	148	Solid at 2 ends	None	10 10 in. disks	4	8.0	P	64	6	8	6			30	8	6	4	8	8
								N		2	2								2	6
								O	2	4							4	2		
5	120	148	Solid at 2 ends	One	10 10 in. disks	4	8.5	P	88	4	2				25	2	2			
								N	5										2	4
								O												
6	110	87	Solid gen. end	Two	10 10 in. disks	2	3.5	P	336	8					304		8			
								N							8		8		8	
								O	8	8					82					
7	110	72	Solid at 2 ends	None	6 10 in. disks	1	3.5	P	126						108		36			
								N									18			
								O												
8	100	Network	Solid	One	6 10 in. disks	1	3.5	P	34											
								N												
								O												
9	66	66	Solid at 2 ends	None	6 10 in. disks	1	1.5	P	43	43	43				43	86				
								N										43	43	
								O												
10	66	16	Solid gen. end	One	7 10 in. disks	2	2.5	P	389	27		67	27		13		27		13	
								N											27	
								O					27					13	27	
11	66	53	Solid gen. end	None	Pin	2	6.0	P	59	4					23	12	4	8	4	
								N								4	12			
								O	4						12	20	4	8		
12	66	Net.	60 Ohms at 3 points	One	6 10 in. disks	2	3.5	P	306	13	13				27					
								N												
								O	80						13	13		13		
13	66	27	Solid gen. end	One	5 J. D. disks	2	2.5	P							12				12	12
								N										12	24	
								O												
14	66	14	Free	None	8 10 in. disks	2	Special tests	P												
								N												
								O												
15	66	53	Free	One	Pin 70 kv.	1	2.5	P	24						144					
								N									24			
								O							24	24	24			
16	66	200	Free	One	Pin	1	1.5	P	84	42					42	42	42			
								N											42	
								O							42	84				
17	66	Net	1500 Ohms	None	7 10 in. disks	1	2.0	P	189	63	32				189	32	64			
								N							63	32	32			
								O	32						32	32		32		
18	60	140	1000 Ohms ends	Two	7 J. D. disks	2	3.0	P	80											
								N											10	
								O	10						10					
19	44	Network	Solid	One	Pin	1	5.0	P							36	24				
								N										12		
								O												

*P = Positive: N = Negative. O = Oscillatory.

TABLE I—*Continued*

LIGHTNING SURGES ON OPEN-WIRE SYSTEMS

	System								Surges per station in 10 years, according to voltage magnitude											
									Not causing flashovers						Causing flashovers					
No.	Voltage, kv.	Length in miles	Neutral Grounding	Ground wires	Minimum Insulation	Number Klyd. Stations	Months tests	Type *	Below 2.5 times normal	2.5–3.4	3.5–4.9	5.0–6.9	7.0–9.9	Over 10.0	Below 2.5	2.5–3.4	3.5–4.9	5.0–6.9	7.0–9.9.	Over 10.0
20	33	12	Free	None	Pin 45 kv.	2	7.5	P	24											
								N												
								O							40	8				
21	33	Net.	Solid at 1 Point	None	Pin 45 kv.	1	2.0	P	120	120	30	60			120	30				30
								N												30
								O	150						270	90	30	30		
22	25	20	19 ohms at 8 points	Two	Pin 37 kv.	2	3.0	P	44	11	22	22			11					
								N										11		11
								O			22					22	11			
23	24	Net	Solid	None	Pin	2	10	P							21					
								N												
								O												
24	23	Net.	Solid	None	Pin	1	2	P	410	32	64				252	32			32	
								N												
								O	32		32									
25	22	Net.	7 Ohms	None	Pin	2	3	P												
								N												
								O												
26	13.2	2.2	7 Ohms	None	Pin	2	3	P	57						181		11		23	
								N												
								O							23					
27	6.6	25	Free	One	Pin	1	3.0	P	460	140									20	
								N									20			20
								O	20						200	60			20	

*P = Positive. N = Negative. O = Oscillatory.

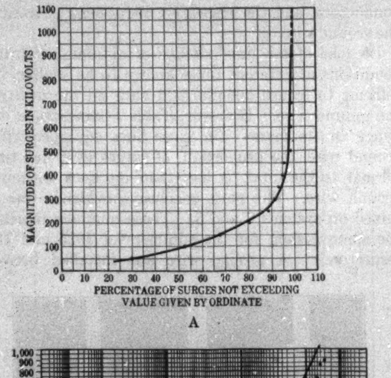
The number of important surges which appear at a given point of a line during one lightning storm, was found to be low. More than two surges in excess of seven times normal voltage seldom were recorded at one klydonograph station. Also, many apparently severe storms were experienced without recording these higher voltages. Of course, their presence on a line depends, to a certain extent, on its length; the greater the exposure, the greater is the probability of lightning voltages at some point. The distance, from the point of origin, over which a surge maintains a magnitude of several times normal, is not definitely known. However, this appears to be of the order of a very few miles. It is hoped that accurate information on this point will be secured in the near future. In the meantime, the fact that only about two surges per storm occur, has considerable significance in connection with the application and operation of protective equipment.

Table I shows a rather striking freedom from surges on the lower voltage lines, which operating statistics hardly verify. Higher, and more frequent surges, since lower induced-voltages must be considered, would be expected. For example, a lightning surge of 100 kv. would be 1.8 times normal on a 66-kv. line, but 5.4 times normal on a 22-kv. line. It is true that low-voltage lines are better protected from the field of influence of the cloud, because they are invariably nearer the ground than high-voltage lines. Consequently, lower induced voltages are to be expected, but this factor is not great enough to account for the results mentioned above. A number of possible explanations has been considered but, to warrant definite conclusions, further investigation is necessary.

Certain localities are nearly immune from lightning. Sometimes these are found even adjacent to regions where lightning is severe. On a system with lines extending in opposite directions from the same station, lightning conditions were found to be severe on one of these lines, and mild on the other. The freedom from trouble of the latter had been attributed, to certain features which had been incorporated in its construction. This illustrates the error of drawing conclusions regarding the efficiency of protective devices from an individual application.

In Table I, a distinction is made between those surges which caused flashovers and those which did not. As would be expected, all the higher surges caused flashovers. A wide variation exists in the amount of insulation used on lines of the same voltage. However, a measure of the flashover value of the average insulation, for the wave-fronts produced by lightning, is indicated. Surges, which were over seven times normal and did not cause flashovers, were recorded on only one system. This was No. 10, a 66-kv. system with a minimum insulation of seven suspension units.

The majority of the low-voltage surges are listed as "not causing flashovers." However, several surges of low magnitude are listed as "causing flashovers." Doubtless, the latter were of much higher magnitude at some point of the line remote from the klydonograph. It was found that high-voltage surges did not travel far. In traversing a few miles of line, they were always damped to a small fraction of their initial value. In several instances, a decrement from over 1000 kv. to about 150 kv. was observed over a distance of 35 miles (56 km.). In one case, a surge five times normal was recorded five miles (eight km.) from a direct stroke, which caused a flashover to ground in the middle of a long span. Thus, it appears that the higher-voltage surges are damped below the corona voltage in about 10 miles (16 km.). When this has occurred, the surge may travel a considerable distance without much attenuation. Frequently a surge of the order of 1.5 or 2.0 times normal, traveled 50 miles (80 km.), and occasionally, the entire length of a 250 mile (400 km.) line.



A

B

Figs. 1a, b—Relative Percentage of Lightning Surges of Various Magnitudes

The table shows that lightning voltages are unidirectional. When a flashover occurred, the klydonograph often indicated an oscillatory surge. Most of the oscillatory records are found in the column "causing flashovers." However, there are a few in the other column. These were probably due to surges producing lightning arrester discharges or flashovers on connecting lines, which were overlooked in collecting the information.

Fig. 2—Klydonogram of a Direct Stroke of Lightning

The lightning data are plotted in Figs. 1a and 1b. The voltages produced by lightning are a function of the heights of lines, and not of the operating voltages, except as limited by the flashover of insulators. Therefore, in view of the variation in the operating voltages of the lines tested, the curves have been drawn with ordinates in kilovolts rather than "times normal" although the latter is a better criterion of the severity of a particular surge. The plotted results include all the data from the 27 tests. Fig. 1a is plotted in Cartesian coordinates and shows the relative percentages of lightning surges of different magnitudes. Fig. 1b represents the same quantities plotted on probability paper. The exceedingly small percentage of the total of the higher-valued surges, is illustrated in Fig. 1a. The reasonable conformity of the plotted results to a straight line in Fig. 1b indicates that sufficient data to represent true conditions were obtained, and that the magnitudes of lightning surges follow the law of probability. The plotted results deviate slightly from the straight line at the higher values. This is explained by the fact that, on the lower-voltage lines, insulator flashovers limited the magnitudes of many surges, which otherwise would have reached higher values.

Various authors have discussed the value of the ground wire for reducing potentials induced on transmission lines by lightning. The theory of ground wire performance is relatively simple. It rests on the assumption that the charge on the ground wire can pass to ground as rapidly as the cloud discharges. The conclusions regarding the rate of lightning discharges indicate that this assumption is justified. Calculations based on this theory show that, with typical spacings, the ground wire reduces by 25 to 45 per cent, the potentials induced on the conductors. Using small-scale models, F. W. Peek, Jr. obtained test results which indicate reductions as high as 50 per cent.[6] The percentage protection against flashovers is much greater than the percentage reduction of induced voltages. This is for the reason that, on high-voltage lines, many of the induced surges which cause flashovers are not greatly in excess of the flashover voltages of the lines. It is readily conceivable that complete freedom from induced-voltage flashovers might result from a 50 per cent reduction of these voltages.[7]

FIG. 3—KLYDONOGRAM OF SWITCHING SURGE

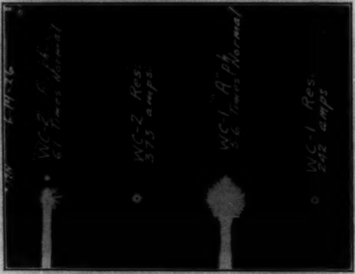


FIG. 4—KLYDONOGRAM OF A LIGHTNING ARRESTER DISCHARGE

Induced lightning voltages, simultaneously recorded on the three phases of vertically-spaced lines, were highest on the top phase and lowest on the bottom phase, particularly where no ground wire was installed. Operating records also indicated that most flashovers occurred on the top conductor. According to the theory of the ground wire, with the usual spacing arrangement, the greatest protection is afforded the top conductor, which needs it most. Hence, the relative voltages appearing on the conductors, where a ground wire is installed, are a compromise between the respective induced voltages, as affected by the heights of the conductors, and the respective reductions effected by the ground wire.

Because of the impossibility of obtaining lightning conditions at different times known to be similar, it is difficult to secure definite field data on the utility of the ground wire. However, Table I indicates a tendency in its favor. On those lines equipped with a ground wire, the proportion, of surges over five times normal to the total, is less than on lines without a ground wire. The most conclusive evidence was obtained on systems 4 and 5. These numbers represent the same system for the summers of 1925 and 1926 respectively. A ground wire was installed between these dates. It is noticed that, after the ground wire was installed, there were no induced surges above five times normal. The number of direct strokes remained practically the same. However, it is not expected that a ground wire will render the line immune to direct strokes. It may be argued that these results are due to the vagaries of lightning, rather than to the efficiency of the ground wire. But, the fact that there were nearly as many direct strokes the second year, indicates that the lightning season was nearly as severe. Furthermore, the operators were of the opinion that the improvement in their operating record was more pronounced than can be accounted for by the difference in the lightning encountered.

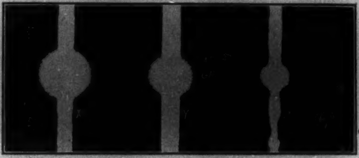


FIG. 5—KLYDONOGRAM OF AN ARCING GROUND SURGE

It is hoped that these quantitative data and the accompanying discussion will contribute to the solution of lightning problems. As every engineer knows, these problems are a very important consideration in the operation of transmission lines.

Switching. As shown in Table II, switching surges are not a serious problem where the present accepted factors of safety for insulation are used. The maximum switching surge recorded was six times normal. Surges of this kind over 4.5 times normal were found on only five of the 27 systems investigated. On 15 of these

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[6] *Lightning and Other Transients on Transmission Lines,* F. W. Peek, Jr., TRANS. A. I. E. E., Vol. XLIII, 1924, p.1205.

[7] *Discussion at Niagara Falls Convention, May 1926.* J. H. Cox, TRANS. A. I. E. E., Vol. XLV, 1926, p. 788.

TABLE II. SWITCHING SURGES ON OPEN-WIRE SYSTEMS

System		Type of surge*	Number of surges per station in 10 years, according to voltage magnitude															
			Opening switches at klyd. stations				Closing switches at klyd. stations				Opening switches at other stations				Closing switches at other stations			
No.	Voltage kv.		Below 2.0times normal	2.0–2.9	3.0–4.4	4.5–6.0	Below 2.0	2.0–2.9	3.0–4.4	4.5–6.0	Below 2.0	2.0–2.9	3.0–4.4	4.5–6.0	Below 2.0	2.0–2.9	3.0–4.4	4.5–6.0
1	220	U	1180	50			800	70			570	60	10		350	10		
		O	210	20			170	50			200	10			130			
2	140	U	38				45		4	8	79	15			79	11	8	
		O									4	4			11	8		
3	140	U	90			30		30	120	30	30				60			
		O				30		30		60								
4	120	U	196	32	24	4	236	28	8		120	20			104	20	12	
		O	4	8	8			4	12									
5	120	U	260	42	18	7	242	35	18	7	35	4			21	7		
		O			4	4	4	4	4									
6	110	U	1080	16	32		345											
		O	82				131								16			
7	110	U	36								72				36			
		O																
8	100	U	206		34		69	34			137		34		172	34		
		O									34							
9	66	U	172	86				86								86		
		O																
10	66	U	376	27			700				912				1230	54		
		O																
11	66	U	258	70	39	8	258	140	24									
		O		39	23	16												
12	66	U	27		53			27										
		O	346	106			373	133	27									
13	66	U	24	120	96		48	48										
		O	24					72	72									
14	66	U	120	180	270		480	90	30		180		60		270			
		O		90	60		30	30				30						
15	66	U	530	192			96											
		O																
16	66	U	83															
		O																
17	66	U	63				126								63			
		O																
18	60	U	80	20			80				20				40			
		O	20					40										
19	44	U	72												24			
		O													24			
20	33	U	176	32			192				16							
		O																
21	33	U	60	60			60											
		O	60															
22	25	U													44			
		O									22				22			
23	24	U	18				66								6			
		O																
24	23	U	690	63	126		380	63	63		126				126			
		O																
25	22	U																
		O																
26	13.2	U	158	23			90								68			
		O																
27	6.6	U																
		O																

*U = Unidirectional. O = Oscillatory.

systems, no switching surge over three times normal appeared. The majority of all switching surges were below this value.

Not all switch operations produce surges. On system 1, the tests covered a period during which 3600 high-tension switching operations were performed. Approximately three-quarters of these caused no surges that were detected at the stations, although there were klydonographs at each of the high-tension switching points. Of those surges recorded, 93 per cent were less than two times normal. Thus, only 1.75 per cent of the switching operations caused surges over two times normal. The maximum surge recorded on this line was 3.2 times normal. It is estimated from the data for the 27 systems, that, on the average, less than one-half of all switching operations caused surges above normal voltage.

Fig. 6—One Potentiometer for a 120-Kv. Klydonograph Installation

Although the results, obtained on systems of the same and different voltages, varied a good deal, the data have been reduced to an average for all systems, in Table IV. Again, to avoid fractions, the surges have been brought to the basis of 10 years at one point of each of 27 systems. As shown in this table, 80.8 per cent of all switching surges recorded, were less than two times normal, 93.0 per cent were less than three times normal, and 99.2 per cent were less than 4.5 times normal.

The higher-voltage surges were recorded on lines of 66 kv. to 140 kv. No surge as high as 4.5 times normal appeared on systems below 66 kv. With one exception there was none as high as three times normal. The absence of the higher surges on the lower-voltage lines is attributed to the fact that, in these cases, the klydonographs were usually connected to busses with many connected lines. As mentioned above, the highest surge recorded on the 220-kv. system was 3.2 times normal.

Little difference between the switching surges on systems with free and grounded neutrals, was detected. A grounded conductor on a free neutral system during switching operations accentuates the surges produced, by the factor 1.73. This is indicated by the surges listed for systems 2 and 3. The actual surges over 4.5 times normal, recorded on system 2, were caused by closing operations, when one conductor was grounded. They were all 5.2 times normal. On system 3, the actual surges over 4.5 times normal, listed under "closing switches at klydonograph stations," were caused by closing a 100-mile (160-km.) line and synchronizing with the power plants. All of these were 4.6 times normal.

Load switching does not cause surges as high as idle-line switching. This was found to be the case on all systems. In Table II, the switching surges have been grouped according to opening or closing operations at the klydonograph stations, or at other points. In general, de-energizing operations at the klydonograph station caused the higher surges. The high surges listed for closing operations on systems 2 and 3 are an exception to this. These were discussed above. Moreover, the large weighting factors necessary, particularly for system 3, give them more prominence than is warranted. With these exceptions, the surges caused by de-energizing idle lines are appreciably the highest of all switching surges. Of all the surges due to switching operations at the klydonograph station, 57.5 per cent were caused by opening operations and 42.5 per cent by closing operations.

It was found that switching surges occasionally traveled considerable distances. This is because they were low in magnitude, and thus not affected by corona. In other words, they were governed only by the resistance term of the attenuation constant.[5] Surges due to closing operations were recorded at distant stations oftener than those due to opening operations. Of all the surges recorded at distant stations, 53 per cent were caused by closing and 47 per cent by opening operations.

Fig. 7—Potentiometer for a Three-Phase Installation on a 12-Kv. Cable System

It is interesting to note that changing taps on a transformer under load produced no surges. In general, charging electrolytic lightning arresters did not cause surges. However, on one system where the arresters were not equipped with charging resistors, oscillatory surges as high as 2.4 times normal sometimes resulted. On other systems an occasional surge about 1.3 times normal was produced.

Switching surges were quite abrupt when generated, the fronts being of the order of one microsecond. They were damped rapidly and the fronts became more

TABLE III

OPEN-WIRE SYSTEM SURGES OF WHICH THE CAUSES WERE INTERRUPTIONS, MECHANICAL SHORTCIRCUITS, AND UNKNOWN

System			Number of Surges per station in 10 years, according to voltage magnitude											
			Overload Interruptions				Short circuits				Unknown			
No.	Voltage kv.	Type of surge*	Below 2.0 times normal	2.0–2.9	3.0–4.4	4.4–6.0	Below 2.0	2.0–2.9	3.0–4.4	4.4–6.0	Below 2.0	2.0–2.9	3.0–4.5	4.5–6.0
1	220	U									600	20		
		O					20				270			
2	140	U	11	8			11				146	48		
		O		8			15	45	38		15	8		
3	140	U	30								240	120		30
		O					30				30		30	
4	120	U	4				4				1000	144	4	
		O									20	32		4
5	120	U	14								1117	455	21	7
		O									32	4		
6	110	U	131				33				345	16		
		O					33		16		16	33		
7	110	U												
		O												
8	100	U	103				103				310	34		
		O	34				34						69	
9	66	U									430	86	86	
		O												
10	66	U					27				134			
		O												
11	66	U	31								24	16	8	
		O									8	31		
12	66	U	27				53				106		27	
		O	27					27	27	27	532	53		
13	66	U									72	48		
		O										48		24
14	66	U												
		O												
15	66	U									1440	48		
		O					96				48			
16	66	U		83							83			
		O						83	83			83		
17	66	U	252										63	
		O												
18	60	U	100	20							100	20		
		O	260	20	20						20			
19	44	U	72				24	24						
		O												
20	33	U	96				32				112			
		O					48	16						
21	33	U									180	60	60	
		O		180	60		180	60				120		
22	25	U	44								396	66		
		O												
23	24	U									54			
		O												
24	23	U	63								252			
		O												
25	22	U												
		O												
26	13.2	U	135	68							180	23		
		O					23				23			
27	6.6	U	40					40			920	120		
		O					200	160	80			40	40	

*U = Unidirectional. O = Oscillatory.

sloping. The greater the distances from the switch the longer were the fronts. The wave fronts of the switching surges recorded were from one microsecond to a few hundred microseconds.

Switching surges are of short duration. The table shows that 86.6 per cent were unidirectional and that 13.4 per cent were oscillatory. The only system contrary to the average was No. 12. Here, the majority of the surges were oscillatory. These were caused by interrupting the charging current of appreciable sections of line with disconnect switches. When reflections occurred and produced an oscillatory record, the surge lasted only one or two cycles. Further, the initial voltage was always appreciably higher than the succeeding half cycles. This indicates rapid damping, and consequently, the time of application of the high voltage was of short duration.

In view of (a) the moderate magnitude, (b) the low frequency of occurrence, and (c) the short time duration of the higher surges due to switching operations, they are not of serious importance either from the standpoint of continuity of service or of the effect on the life of apparatus insulation.

Flashovers and Interruptions. Surges, due to flashovers other than lightning, to overload interruptions, and to unknown causes, are listed in Table III.

Flashovers in general are objectionable from the standpoint of service interruptions. In some cases, they result in damage to the line in places where the repairs require considerable time and inconvenience. Also, the question of the effect of flashovers at one point, on the rest of the system, has often been raised.

Apparatus and line failures have been blamed on the suspected presence of high-frequency high-voltage oscillations. Klydonograph records show that, on grounded neutral systems, no serious high-voltages are produced by flashovers and no sustained high-frequency disturbances are produced by flashovers or by other causes. Sometimes small surges occur, due either to the fault or to the resulting switch operation—there is no way to distinguish between these causes. However, these surges are small and unimportant.

In the table, surges are classified as unidirectional or oscillatory. The latter classification really includes two types of surges. One is a highly damped oscillation of one or two cycles, the successive half cycles being much reduced in magnitude. These surges are really similar to unidirectional ones, in their effect. They sometimes occur in connection with switching, lightning, or flashovers on grounded neutral systems. Their oscillatory nature often is due to reflections in the circuits.

Oscillatory surges of the other type are sustained high-frequency oscillations. These occurred only on ungrounded neutral systems after flashovers, which result in arcing grounds. The frequency of these ranged from 2000 to 30,000 cycles per second; their maximum voltages were sometimes 4.5 times normal. The oscillations resulting from arcing grounds extended to all parts of the system.

It is known that the breakdown voltage of air is reduced, if the frequency of the applied voltage is high enough to maintain the ionization of the air from one cycle to the next. Thus, the ionization is cumulative, and the breakdown is progressive. Such a breakdown

TABLE IV
SUMMARY OF SURGES IN 10 YEARS AT ONE STATION ON EACH OF 27 OPEN-WIRE SYSTEMS

Cause	Type*	Below 2.0 times normal	2.0–2.9	3.0–4.4	4.5–6.0		
Opening a switch at a station where a klydonograph was located...	U	6000	960	700	49		
	O	750	260	95	20		
Closing a switch at a station where a klydonograph was located...	U	4300	650	280	45		
	O	710	360	120	60		
Opening switches at other stations....	U	2300	100	100			
	O	230	44				
Closing switches at other stations....	U	2700	220	20			
	O	200	8				
Shortcircuits and flashovers other than those due to lightning......	U	290	64				
	O	680	390	240	27		
Interruptions....	U	1200	180				
	O	320	210	80			
Unknown....	U	8200	1300	270	37		
	O	1000	450	140	28		
		Below 2.5	2.5–3.4	3.5–4.9	5.0–6.9	7.0–9.9	Over 10.0
Lightning which caused a flashover....	P	1600	290	200	26	112	50
	N	71	38	110	68	160	88
	O	810	400	84	110	47	
Lightning which did not cause a flashover....	P	3200	550	230	170	27	
	N	5	32	17	2		
	O	350	14	54		27	

*U = Unidirectional. O = Oscillatory. P = Positive. N = Negative.

will occur at a point below the 60-cycle breakdown voltage.

In recent years, certain devices, which were designed to prevent insulator flashovers from such disturbances, have been placed on the market. But sustained high-frequency oscillations occur only on isolated neutral systems, and there, only after a flashover. Obviously, it is futile, even on free neutral lines, to install devices designed to prevent flashovers due to this type of disturbance. Since, on long lines, the service is already impaired, protection against the second flashover is of minor importance. It is encouraging to know that no sustained high-frequency high-voltage disturbances need be feared on grounded-neutral systems.

As shown in Table III, no high-magnitude surges resulting from interruptions other than those due to flashovers, were recorded. The surges found were low in value. In most cases they were less than twice normal. The highest recorded were less than 4.5 times normal and these occurred on only two systems. These results are consistent with the data on switching operations. Surges due to switching load currents always were low.

Those surges for which no cause was found are also listed in Table III. As mentioned before, they were probably caused by switching or lightning, but incomplete operating records prevented the detection of the causes. They were relatively low in magnitude and warrant no further consideration.

In five locations, klydonographs were installed on both sides of transformers. In two of these tests, no surges which were known to have passed through the transformers were detected. In the others, simultaneous surges sometimes were recorded on both sides, but they were not noticeably more severe on one side than on the other. The transmission of a surge through a transformer depends on the electrostatic relations of the parts. Thus, it should not be possible for a large surge to pass through from the low to the high side and still be important. This was found to be the case. Where the transmission was in the other direction, the surge was approximately the same "times normal" on both sides. This indicates that the reduction in the surge was of the same order as the ratio of the transformer. In several cases, lightning surges were recorded simultaneously on both sides of a transformer. However, each of these might have originated on its own side, but due to the same stroke. The actual data relating to the transmission of surges through transformers were rather meagre and, therefore, not conclusive.

The tests on open-wire systems show that, except for lightning and arcing grounds no high-voltage disturbances of particular importance to the operating engineer are present.

II. CABLE SYSTEMS

The alleged presence of high voltage transients on transmission systems has been blamed for many cable failures in the past, both as a direct cause and a strong contributory factor. This naturally resulted in considerable controversy. It was a case of theoretical possibility versus meager and often conflicting negative evidence obtained in actual operation. Lack of a plausible alternative theory to explain failures and lack of suitable recording devices to investigate the surges made this situation possible and permitted it to survive for many years. That cable authorities have more or less discarded their former stand on this matter is indicated by the present general interest in the subject of dielectric breakdown and related problems in connection with cables. A number of theories which attempt to explain failures as due to the peculiar properties of dielectrics or the conditions under which they work in cables, have been advanced. Also, during the past two years, surges on cable systems have been investigated with the klydonograph. The information obtained in fourteen of these investigations is presented here.

In order to bring the data from various systems to a comparative basis, the records have been weighted to represent one year's operation of one three-terminal klydonograph on each system. The total number of voltage indications has been counted; for instance, if two or more abnormal voltages were recorded simultaneously on different phases or at different points of a system, each has been considered individually. While these methods of analysis have some undesirable features, it is believed they provide the most satisfactory comparison from the point of view of the effect of surges on insulation, which is the prime consideration.

Summarized data are shown in Tables V and VI. The former classifies the surges according to their causes; the latter demonstrates the variation between particular systems. In order to visualize these results as applied to the average system, they may be regarded as representative of the total number of surges to be expected at a given point of the average system in a period of fourteen years. Of course, this assumption has limitations—as will be seen later.

The klydonographs were usually connected to a system at generating stations or substations, and measured the surge voltages between the conductors and ground. All magnitudes were calculated with "times normal crest voltage to ground" as a unit. The highest voltage recorded was 4.6 times normal. Only 10 surges or 0.4 per cent of the total were over four times normal. Of these, six were on one cable-and-open-wire system, where certain contributory operating conditions prevailed. Only one of these high surges was on a pure-cable system.[8] Nearly 99 per cent of the total were under three times normal

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[8] It is realized that a "pure-cable" system possibly does not exist literally. However, the term is used here to classify those systems where cables strongly predominated, or where open-wire transmission lines were sufficiently remote to have no effect on surge conditions at the points investigated.

TABLE V

SUMMARY OF SURGES IN ONE YEAR AT ONE POINT ON EACH OF 14 CABLE SYSTEMS

Cause	Number of surges									
	Times normal voltage				Unidirectional		Oscillatory		Total	Per cent total
	1.1 to 1.9	2.0 to 2.9	3.0 to 3.9	4.0 to 4.9	1.1 to 2.5	2.6 & over	1.1 to 2.5	2.6 & over		
Closing a switch at a station where a klydonograph was located	1050	92		1	1120	23			1143	42.4
Opening a switch at a station where a klydonograph was located	340	25		1	360	6			366	13.6
Switching at other points of the system	117	13	5		120	15			135	5.0
Cables failures, automatic interruptions	300	49	13	5	150		185	32	367	13.6
Unknown	660	12	5	3	645	23	12		680	25.4
Total	2467	191	23	10	2395	67	197	32	2691	
Per cent total	91.6	7.1	0.9	0.4	89.0	2.5	7.3	1.2		100

TABLE VI

SHOWING THE VARIATION IN SURGE CONDITIONS ON INDIVIDUAL CABLE SYSTEMS. FIGURES ARE FOR ONE YEAR'S OPERATION OF A THREE-TERMINAL KLYDONOGRAPH ON EACH SYSTEM

System Cable and open wire	Uni-directional				Oscillatory			
	No. of surges				No. of surges			
	1.1 to 2.5 times normal	2.6 & over	Max. times normal	Per-cent total	1.1 to 2.5	2.6 & over	Max. times normal	Per-cent total
24 Kv	13		2.4	68	4	2	3.0	32
26 Kv	30		1.7	100				0
26 Kv	250	30	4.3	91	27		2.4	9
26 Kv	140	7	3.3	73	27	26	4.6	27
33 Kv	12	4	3.1	89	2		1.8	11
66 Kv	215	13	4.7	96	10		2.4	4
Cable only								
11 Kv	140	2	2.6	91	15		1.8	9
13 Kv	320		1.8	98	5		1.3	2
13 Kv	320		1.5	97	11		1.4	3
13 Kv	600	9	2.6	100				0
13 Kv	105	2	4.5	69	48		2.3	31
26 Kv	38		2.6	44	46	2	2.6	56
33 Kv	87		1.4	100				0
45 Kv	125		1.6	97	2	2	2.6	3
	2395	67		92	197	32		8

voltage. Also 92 per cent of the total were unidirectional and, therefore, of brief duration. Most of the abnormal voltages were caused by normal switching operations, and it is believed that many of those tabulated as unknown were also due to switching. Klydonograph clock errors and incomplete operating records undoubtedly prevented linking up a surge with its cause in many cases.

However, the surges produced when failures occurred were considerably more severe than those due to switching. Practically all the oscillatory surges, and particularly the higher-voltage ones, were caused by insulation breakdowns. That these surges were the effects, and not the causes, of insulation failures is strongly indicated by the fact that they did not occur unaccompanied by a short circuit. Surges of this nature invariably appeared on the two phases of the system other than the faulty one, with about the same magnitude on each. The theory of production of surges of this nature involves the characteristics of the arc. This theory calls for a maximum of 2.5 times normal voltage for surges due to arcing grounds on grounded neutral systems[9]. The results for all except one system check very closely with this value. A maximum of 2.6 times normal was recorded on these systems. Allowing for errors of measurement, the agreement is very satisfactory.

The relatively greater severity of surges due to short circuits is because of the following considerations. These surges affect a large part of a system. In many cases, appreciable voltages were recorded many miles from the location of the failure. As a rule switching surges failed to travel far. In addition, surges accompanying short-circuits usually were oscillatory, while switching surges were unidirectional. Thus it is seen that, for a given magnitude, surges resulting from short circuits are more severe on insulation. This is due to their longer time of application and to their greater energy which results in more of the system insulation being stressed. But the elimination of insulation failures would automatically eliminate surges of this class.

In addition to the surges included in the tables, there were many recorded when energizing and de-energizing short bus-sections or leads to the klydonograph equipment, or, in other words, where very small charging currents were involved. These reached a maximum of 4.5 times normal voltage, those for energizing operations averaging somewhat lower magnitudes. However these voltages appeared only on the sections being switched, involved very little energy, and usually were not the result of normal switching operations. For these reasons they are not included in the data. Certain physical conditions, which give the proper proportions between inductance and capacity, with high leakage resistance, are apparently necessary for the production of the higher voltages. Consequently, the great majority were of the order of two times normal or less. Surges, due to actual cable switching or operations involving appreciable currents, failed to reach three times normal voltage. In general, the production and magnitude of switching surges appear to be haphazard affairs and, for a given operation, no relation between the magnitudes on the various phases can be detected.

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[9] *Voltages Induced by Arcing Grounds*, J. F. Peters and J. Slepian; TRANS. A. I. E. E., Vol. XLII, 1923, p. 478.

The large number of small surges shown for certain systems is somewhat due to the influence upon the tabulated data, of special switching tests or other-than-normal cable switching operations. The latter were necessary, in some cases, for changing the plates or films in the klydonographs.

It will be noted in Table VI that the surges on cable-and-open-wire systems were somewhat higher than those on pure-cable systems. This is partly due to the presence of lightning voltages on open-wire systems, although surges of this nature were eliminated from the data where known to be such. Incidentally, none of these known lightning surges were of a magnitude to make them important. Again, transmission line switching usually results in higher surge voltages than similar operations involving cables. One surge 4.5 times normal was recorded on a pure-cable system. However this was due to energizing a switch group, which is a low energy operation as discussed above, but, at the same time, routine switching. Otherwise, the highest voltage observed on a pure-cable system was 2.6 times normal. On one particular system the surges identified with short circuits were considerably higher than the average. For instance, this system accounted for 26 of the 32 oscillatory surges of 2.6 times normal voltage and over. Its only apparent difference from the other systems, all of which were grounded solidly or through a low resistance, is the presence of a 75 ohm resistor in the neutral ground. No doubt, additional data will reveal more definitely the conditions or phenomena underlying these exceptional results. The inclusion of the data from this system has exaggerated to a certain extent the seriousness of surges on the average system.

An interesting phenomenon has been observed in connection with cable failures on four or five occasions. During three to six hours before the breakdown a succession of surges of the order of 1.7 times normal voltage has been recorded on two phases of a system, with the final breakdown occurring on the third phase. In other words the fault made itself evident some time before the short circuit developed. The klydonograph records indicated that this was not a continuous process but occurred more or less irregularly at intervals of a few minutes. Apparently, impulsive discharges through temporary punctures of the insulation took place. These created sufficient unbalance to raise the other phases to approximately the delta voltage above ground for an instant, without developing into a short circuit severe enough to cause an interruption. This action has been noticed only on 26-kv. and 33-kv. systems, and in each case the system neutral was grounded.

Apparently, transient high-voltages are a very minor contributory factor to cable-insulation failures. Considering the test voltages which cables are required to withstand for an appreciable time, it also appears that surges cannot be held responsible for the failures encountered in practise. This conclusion is reached by virtue of the low magnitude, the short time duration and the low frequency of occurrence of surges. It might be argued that it is wrong to assume that very high voltages occur nowhere on a particular system because none appeared at certain points studied for a limited period of time. However, the data were obtained on several systems and under a wide variety of operating conditions. The rarity of surges of even four times normal voltage naturally leads one to doubt very strongly the existence of higher transient voltages on cable systems. Of course it is conceivable that surges, of the order of magnitude found in these investigations, might cause a failure, if the insulation at any point were to reach such a condition that a slight increase in potential would break it down. But it is doubted that these surges could create this abnormal condition and its elimination would be the primary problem.

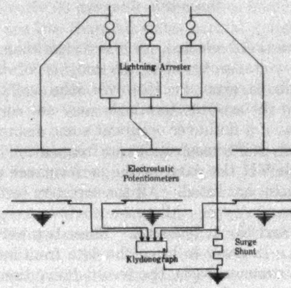

Fig. 8—Diagram of Connections for Lightning Arrester Tests

III. Lightning Arresters

The klydonograph has proven valuable for recording the performance of lightning arresters under operating conditions.[10] The magnitude and nature of the arrester discharge current as well as the same quantitites for the impressed voltage surge can be determined. The current measurement is obtained by recording the voltage drop across a non-inductive resistor, which is inserted in the arrester ground lead. A 10 to 15 ohm resistor is suitable. Since the minimum recording voltage of the klydonograph is about 2000 volts, the detection of currents above 150 to 200 amperes is possible with this arrangement. On three-unit arresters the bases must be insulated from their supporting foundations to force current through the resistance shunt. The voltages measured at the terminals of the arrester are the maximum values reached by the surge at that point. These exist for the very brief interval required for the arrester to come into operation, after which they are reduced by the discharge. However, the extreme speed of the klydonograph permits the measurement of these short-duration peak values. A diagram of connections appears in Fig. 8. Fig. 9 shows a typical installation.

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[10] 10. Arrester Tests with the Klydonograph, L. R. Golladay. *Electrical World*, Sept. 4, 1926.

Unfortunately, in most of the tests it was not feasible to install equipment to measure the surge voltages on all three phases. Where this was the case, the highest conductor, or the highest and the lowest conductors, were selected, for the reason that induced lightning voltages vary as the heights of the conductors above ground. However, the voltages impressed on the arrester terminals may not correspond to this law, if a flashover occurs at some distance from the arrester, or if transpositions are frequent.

FIG. 9—TYPICAL INSTALLATION FOR MEASURING ARRESTER DISCHARGE CURRENT

In Table VII the data on the performance of lightning arresters are listed. All the arresters tested were of modern valve type. All were autovalve except No. 2, system 1, and No. 1, system 6. These two were different types. In order to bring the data from individual tests to a common basis, the records have been multiplied by a factor to make them represent the operation of each arrester for one lightning season of six months. Classification has been made according to the maximum recorded voltage of each surge. The divisions are: (a) above 3.5 times normal; (b) 3.5 to 2.6 times normal; (c) below 2.5 times normal; (d) discharges recorded on arresters on which no potentiometers for measuring voltage were installed.

The basis of most apparatus insulation tests is twice the operating phase-to-phase voltage. Consequently, lightning arresters are adjusted to relieve surges above this magnitude. However, all surge calculations have been based on the normal crest voltage to neutral. Therefore, a surge 2 × 1.73 = 3.5 times normal in the tables, corresponds to twice the phase-to-phase crest voltage. Discharges for surges below this value are unnecessary, and, in some respects, objectionable. This is particularly true for the lower-voltage surges in view of their greater frequency of occurrence.

As shown in Table VII, with few exceptions, the operation of the arresters was satisfactory for surges above 3.5 times normal. Some of these, for which no discharges were recorded, were caused by switching. As switching surges often are not of the same polarity on all three phases, the current discharge path may have been from phase to phase through the neutral interconnection of the arrester units. For such a discharge no indication results on the klydonograph. Again, some surges, over 3.5 times normal, only slightly exceeded this value. In these cases the magnitudes possibly were exaggerated by the limitations of accuracy of the klydonograph. During the tests, only three lightning surges, which undoubtedly were high enough to require

TABLE VII

SUMMARY OF LIGHTNING ARRESTER INVESTIGATIONS, ON A BASIS OF ONE LIGHTNING SEASON'S OPERATION OF ONE ARRESTER

System number	Arrester number	Voltage	Weeks of test	Number of potentiometers	Surges 2.5 times normal or less				Surges 2.6 to 3.5				Surges 3.5 or over				No voltage measurements		
						Discharges				Discharges				Discharges			Discharges		
					Number	250 amps. or less	250 to 500 amps.	500 amps. or over	Number	250 amps. or less	250 to 500 amps.	500 amps. or over	Number	250 amps. or less	250 to 500 amps.	500 amps. or over	250 amps. or less	250 to 500 amps.	500 amps. or over
1	1	140*	22	2	67				1										
	2	120	24	3	210	5	17		28		10		6		2	3			
2	1	66	26	1	58	1	3		13		2	2	6		1	1			
	2	66	26	1	54	1	2		7				7	2	3	2			
3	1	66	11	1	126	2	7		4		2		4		4				
	2	66	11	0													9	20	13
4	1	66	4	1	26				7		7								
	2	66	4	0													0	0	0
5	1	33	23	1	26		2												
	2	33	23	1	32		1		1		1								
6	1	25	5	2	73	5	16	83	5		5		5			5			
	2	25	9	2	26								6			3			

*This is a 140-kv. arrester connected to a 120-kv. system.

a discharge and for which none above the minimum recording point resulted, were recorded.

A considerable number of discharges occurred when the accompanying maximum surge voltages were below 3.5 times normal. This was particularly true on arrester No. 1, system 6 and, to a lesser extent, on arrester No. 2, system 1. In tests where voltage measurements were made on only one or two phases, higher voltages on the other phase or phases likely were responsible for many of the discharges of this type.

The function of lightning arresters, as ordinarily installed, is the protection of station apparatus. They do not necessarily prevent line flashovers. If a high-voltage lightning surge originates at some distance from an arrester, the line insulators may flash over before the surge reaches the arrester. This point is discussed in Part I. To protect against insulator flashovers it would be necessary to distribute arresters at short intervals along a line.

The results of these tests show that the operation of lightning arresters in the field confirms predictions based on laboratory tests. However, the data indicate that the occasions, upon which high-voltage lightning arresters are called upon to operate, are relatively infrequent.

CONCLUSIONS

1. Surge voltages due to lightning are unidirectional. The clouds which produce surges are of negative polarity, resulting in positive induced-voltages and negative direct-stroke voltages.

2. The maximum values, reached by lightning surges on transmission lines, are limited by the flashover of the insulators. It is believed that the flashover voltage of 220-kv. transmission line insulation, at the steepnesses of wave front of lightning surges, is comparable to the maximum potentials ordinarily induced by lightning.

3. The flashover voltage of the average insulation of lines up to 140 kv. is about seven times normal for lightning impulses.

4. Seldom more, and often less than two surges, comparable in magnitude to the insulator flashover voltage, appear at a given point of a line during a storm.

5. The frequency of occurrence of the higher surges does not seem to be greater for low-voltage than for high-voltage lines.

6. High-voltage surges are damped below the corona voltage in traversing a few miles of line. At low magnitudes they may travel long distances.

7. The quantitative measurements with the klydonograph agree with the theories regarding induced voltages and the protection against these afforded by the ground wire.

8. Switching surges occasionally reach six times normal voltage, but 99.2 per cent of all produced are less than 4.5 times normal. Less than 50 per cent of all switching operations create a disturbance.

9. Switching idle lines produces higher surges than similar operations involving load currents. Opening operations result in higher voltages than closing operations.

10. Switching surges usually are unidirectional. When oscillatory, they are highly damped and therefore of short duration.

11. Switching surges are not of serious importance either from the viewpoint of continuity of service or of their effect on apparatus insulation, where the accepted factors of safety for insulation are used.

12. Flashovers and shortcircuits produce no serious voltage surges where the system neutral is grounded. The nearest approach to sustained high-frequency high-voltage oscillations is the arcing ground on isolated-neutral systems, where this form of disturbance reaches a maximum of about 4.5 times normal voltage.

13. Except for lightning surges and arcing grounds no high-voltage disturbances, of particular importance to the operating engineer, appear on transmission lines.

14. Nearly 99 per cent of the surges found on cable systems are less than 3.0 times normal voltage. The maximum voltages are of the order of 4.5 times normal; 92 per cent of the total are unidirectional.

15. Where open-wire lines are combined with cables, the surges are somewhat higher than on pure-cable systems.

16. The surges resulting from short circuits are *relatively* the most injurious to cables, owing to their longer time of application to insulation, and to their greater energy which results in more of the system insulation being stressed. However, except under certain rare conditions, their magnitude never exceeds 2.5 times normal, and their frequency of occurrence is low.

17. Apparently, transient high voltages are a minor contributory factor to cable-insulation failures.

18. In the investigations of the performance of lightning arresters in actual service, it was found that arresters in general give satisfactory operation, that is, they relieve all surge voltages above the standard test voltages for equipment insulation. Discharge currents up to 2500 amperes occur in practise. From these tests it is concluded that the field performances of arresters confirms predictions based on laboratory tests.

19. Lightning arresters do not protect a line against flashovers at distant points.

ACKNOWLEDGMENT

The authors wish to express their appreciation of the excellent co-operation accorded by the various operating companies, upon whose systems these tests were performed. Without this co-operative attitude, such extensive investigations would have been impossible.

Discussion

For discussion of this paper see page 348.

Transmission Line Voltage Surges

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Associate, A. I. E. E.

Synopsis.—Records of the transients actually occurring on transmission lines of widely varying characteristics have been obtained recently with the klydonograph.

1. These records substantiate many of the theories of transients on lines.

2. They indicate the incorrectness or incompleteness of some of these theories.

3. They suggested modifications or extensions of these latter theories.

This paper is a coordination of those theories which agree with test data obtained up to the present time. Since the results of the klydonograph surge investigations are presented in a companion paper, only the data required to make this paper complete in itself are included.

INTRODUCTION

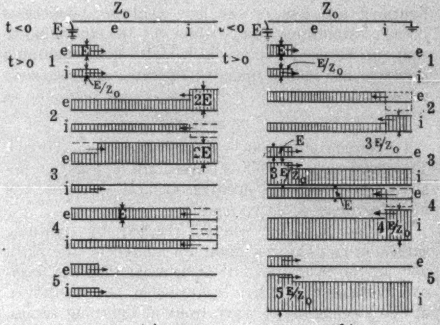
THERE has been a great deal of discussion regarding the high-voltage transients which appear on transmission systems. Switching operations and lightning are the most important causes of these transients. Due to their extremely short duration, there was, until recently, no instrument available for obtaining definite information regarding their characteristics. Their nature had been deduced from operating experiences such as flashovers and apparatus failures. Although electric circuit theory became quite well known, different assumptions in the premises led to different conclusions—hence the divergence of opinion.

Since the recent advent of the klydonograph and the cathode-ray oscillograph, field data, secured under a wide variety of conditions, have been obtained, with the former in America and with the latter in Europe. These data do much toward clarifying the situation. In this paper an attempt is made to coordinate the author's experiences and certain theories of surges, both switching and lightning, which are substantiated by field results.

I. SWITCHING

Ideal Case. The well known theory of traveling waves will be briefly reviewed. For simplicity the ideal case will be considered first. This is the case where a single line is switched from a limitless source, and the potential is applied or removed instantly. At the instant of such an operation, a sheer-front voltage wave, equal to the applied voltage, proceeds out along the line. It is accompanied by a current wave, equal in magnitude to the voltage divided by the surge impedance. These waves, when they reach a terminal, reflect with full magnitude with the same or opposite sign, according to whether the line is open or short-circuited, and according to whether the voltage or current wave is considered. The reflected wave will then add to, or subtract from, the initial wave. The maximum voltage possible from this effect on a homogeneous line is twice the applied voltage. This is shown in Fig. 1. A transformer at the end of a line, due to its high open-circuit impedance, presents the same condition to a surge upon its first impression, as an open line.

In the case where the end of the line is neither open nor closed, but joins a line of different surge impedance, there is a reflected wave and a transmitted wave. Where E is the voltage of the initial wave, Z_1 the surge impedance of the first line, and Z_2 that of the second line, the reflected wave is equal to $\frac{Z_2 - Z_1}{Z_1 + Z_2} E$, and the transmitted wave is equal to $\frac{2 Z_2}{Z_1 + Z_2} E$. This effect is shown in Fig. 2. The usual open air line has a surge impedance of the order of 550 ohms, and a cable 50 ohms. Taking the case of a composite line, it is seen that, if a switch is closed on the cable section, there will be a reflection where the wave enters the open wire part, and the transmitted wave will have a magnitude of nearly 2 E. Then if the open wire line is open ended, or ends with a transformer, the wave will reflect giving a total potential of nearly 4 E. Obviously, if a line divides into two or more branches, the branches



(a) (b)

FIG. 1—TRAVELING WAVES ON TRANSMISSION LINE

(a) End open (b) End short-circuited

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*Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the A. I. E. E. Winter Convention, New York, N. Y., February 7-11, 1927.

present to the line the same effect as a line of lower surge impedance.

Practical Case. In the practical case of alternating-current switching, the point of the applied wave at which contact is made, is a more or less haphazard affair. There is a tendency to arc over at the crest of the wave, especially at the higher voltages now used. However, with the usual closing speed of contacts, and the breakdown strength of oil, this tendency is not the controlling factor. Thus the traveling wave, initiated by a switch closure, will not always have a magnitude as great as the normal crest voltage. Of course, the reflections are governed by the original wave.

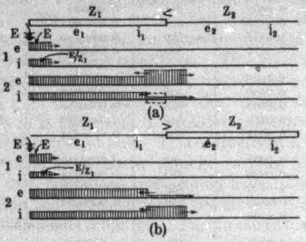

FIG. 2—REFLECTIONS AT THE JUNCTION POINT IN A COMPOSITE LINE

(a) Passing from cable to open wire
(b) Passing from open wire to cable

It must be remembered that the above discussion applies to the ideal case. Although the application of voltage by a switch closure in practise is a rather abrupt phenomena, it is not instantaneous. The voltage on the part being energized rises from zero to the full value in a time of the order of 0.1 microsecond to a few microseconds; that is, the front of the wave sent out extends over from 100 ft. to a mile of line at the start. There are, however, two factors entering into the practical case, each of which tends to lengthen the wave fronts of surges and to lessen the voltages obtained.

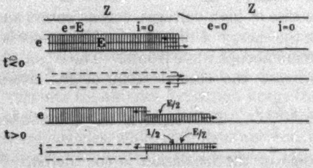

FIG. 3—A LINE ENERGIZED FROM A SIMILAR LINE

Limitation of Source. In practise, when a line is switched, it is not done in connection with an infinite power source. The nearest approach to this condition is closing a single line on a high-capacity bus; that is, a bus which has many other connected lines. When a line is closed to a similar line, the wave sent out is equal to $\frac{1}{2}E$, as shown in Fig. 3. Here the maximum voltage with reflection will be E. When a line is closed on a bus having more than one other line energized, the wave sent out is greater than $\frac{1}{2}E$, and approaches E as a limit. Another common case is where a line is energized from a transformer. Whenever there is a voltage wave, there must be an accompanying current wave equal to E divided by the surge impedance Z_0. It is impossible, however, for current to build up instantly in a transformer. The rate of rise will be limited by the leakage inductance of the transformer, and the wave front will be sloping. If the front is more than twice as long as the length of the line, there can not be a complete reflection, since, after the foot of the wave has traveled to the distant end and back, it will disturb the conditions at the terminal point considered.

Fig. 4 shows the approximate circuit of a line being switched from the high-tension side of a transformer. The potential of the line is determined by the relation

$$e = i\,Z_0 \qquad (1)$$

The relationship between the currents and voltages in the circuit is

$$R\,i + L\,\frac{d\,i}{d\,t} + e = E$$

or upon substituting,

$$\frac{d\,i}{d\,t} + \frac{R + Z_0}{L}\,i = \frac{E}{L} \qquad (2)$$

where E is the voltage on the low-tension side of the transformer referred to the high-tension, R and L are the equivalent resistance and leakage inductance of the transformer, and Z_0 is the surge impedance of the line with respect to ground. With E constant, the solution of (2) gives the equation of i:

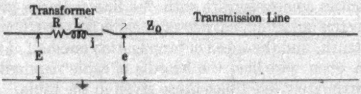

FIG. 4—APPROXIMATE DIAGRAM OF TRANSMISSION LINE AND TRANSFORMER

$$i = \frac{E}{R + Z_0}\left[1 - \epsilon^{-\frac{R + Z_0}{L}t}\right] \qquad (3)$$

From equation (1) it is evident that the voltage wave has the same shape as the current wave and its front extends over the same length of line. The current, from equation (3), will rise to almost its maximum value in a time equal to twice the time constant $T = \dfrac{L}{R + Z_0}$. Thus, to accomplish a complete reflection, the line must have a minimum length of

TABLE I

LENGTHS OF OPEN-WIRE LINES NECESSARY FOR COMPLETE REFLECTION OF A SURGE GENERATED BY SWITCHING A LINE FROM A TRANSFORMER

Line kv.	Transf. capacity kv-a.	Amps.	Per cent R	Per cent X	R ohms	L henrys	Z_0 ohms	$T = \frac{L}{R + Z_0}$	$v\,T$ miles
220	60,000	157	0.7	9.0	5.7	0.194	500	0.000383	71.0
220	75,000	197	0.7	9.0	4.5	0.155	500	0.000307	57.0
220	100,000	262	0.7	9.0	3.4	0.116	500	0.000230	43.0
220	150,000	393	0.7	9.0	2.3	0.0777	500	0.000154	29.0
220	215,000	565	0.7	9.0	1.6	0.0540	500	0.000108	20.0
132	60,000	262	0.7	9.0	2.0	0.0692	525	0.000131	24.0
132	75,000	328	0.7	9.0	1.4	0.0480	525	0.0000912	17.0
132	100,000	437	0.7	9.0	1.2	0.0416	525	0.0000790	15.0
132	150,000	656	0.7	9.0	0.8	0.0276	525	0.0000525	10.0
110	45,000	236	0.7	9.0	1.8	0.0637	525	0.000121	21.0
110	60,000	315	0.7	9.0	1.4	0.0480	525	0.0000915	17.0
110	75,000	394	0.7	9.0	0.9	0.0308	525	0.0000585	11.0
110	100,000	525	0.7	9.0	0.8	0.0289	525	0.0000550	10.0
110	120,000	630	0.7	9.0	0.7	0.0241	525	0.0000458	8.5
66	30,000	262	0.9	8.0	1.0	0.0307	550	0.0000558	10.0
66	45,000	393	0.7	9.0	0.7	0.0230	550	0.0000418	7.8
66	60,000	525	0.7	9.0	0.5	0.0172	550	0.0000313	5.8
66	75,000	656	0.7	9.0	0.4	0.0138	550	0.0000251	4.7
66	100,000	875	0.7	9.0	0.3	0.0103	550	0.0000189	3.5
33	10,000	175	1.0	7.0	1.1	0.0201	575	0.0000349	6.5
33	20,000	350	1.0	7.0	0.5	0.0101	575	0.0000175	3.3
33	30,000	526	0.9	8.0	0.3	0.00769	575	0.0000133	2.5
33	45,000	790	0.7	9.0	0.2	0.00583	575	0.0000101	1.9
33	60,000	1050	0.7	9.0	0.1	0.00424	575	0.00000732	1.4
22	5,000	131	1.2	5.0	1.2	0.0127	575	0.0000221	4.1
22	10,000	262	1.0	5.0	0.5	0.00636	575	0.0000110	2.0
22	20,000	524	0.8	6.0	0.2	0.00397	575	0.00000690	1.3
22	30,000	787	0.7	7.0	0.1	0.00291	575	0.00000506	0.9
22	45,000	1180	0.7	7.0	0.08	0.00212	575	0.00000369	0.7

$v\,T$, where v is the rate of wave propagation, 186,000 miles (300,000 km.) per second. Table I shows the lengths of open-wire lines, of various voltages, necessary for complete reflections, when energized from transformers of capacities commensurate with the line voltages given. Since the surge impedance of a cable is approximately one-tenth, and the speed of propagation one-half of that of an open wire line, the lengths of cable required are approximately five times those given in the table. For a double reflection, giving nearly $4\,E$ as discussed above, the cable must be connected to an open wire line twice as long. With the lengths of wave fronts generated on cables, it is evident that the lengths of cable and open-wire line necessary in a composite line for this double reflection are not common conditions. Further, with such lengths, the next factor discussed influences the situation.

Attenuation. The second factor tending to limit the surge potential is the attenuation of the wave. The attenuation factor of a line is:

$$\epsilon^{-\left(\frac{r}{2Z_0} + \frac{g\,Z_0}{2}\right)x}$$

where r is the series resistance and g the parallel conductance per unit length of line, Z_0 is the surge impedance, and x is the distance traveled in the same linear units used for r and g.

This reduces the wave as it travels down the line. In any case where there are abrupt wave fronts there are high-frequency components. The resistance of the conductor is higher to these high frequencies due to skin effect. Thus, the first term of the attenuation constant, $\frac{r\,x}{2\,Z_0}$ for the higher frequencies is many times its value using the d-c. resistance. This will tend to rapidly damp out the high-frequency components and slope the wave.

In the case of high voltages, corona enters and helps dissipate the energy of the surge[7]. This effect is higher for the more abrupt wave fronts. The effect of corona is to increase the second term of the attenuation constant $\frac{g\,Z_0\,x}{2}$. This term is negligible when only the leakage of the insulation is considered. For the higher voltages this second term is far greater than the first. Thus, when a surge above the corona voltage is generated, its energy is dissipated rapidly as the wave travels along the line. When the voltage has been reduced below the corona point, the wave proceeds, being governed by the first term of the attenuation constant, $\frac{r\,x}{2\,Z_0}$. This attenuates the wave more slowly.

It has been found in tests that extremely high voltage surges fall to a fraction of their original value in a distance of only a few miles. In several instances, a surge was recorded at one station with a magnitude of 1000 kv. and at a station 35 miles away with a magnitude of 150 kv. It is believed that most of this attenuation took place in the first few miles. Lower valued surges which traveled 250 miles have been recorded.

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[7] For references see Bibliography.

Characteristics of an Arc. The considerations thus far discussed do not take into account any characteristic of the arc, which always takes place at the contacts of a switch. This may affect the results considerably.

Fig. 5a shows the static characteristic of an arc in air; that is, the variation between volts and amperes under slowly varying conditions. This gives the effect of a negative resistance and an approximate equation is $V = V_0 - I\ r_0$, where V is the voltage across the arc and r_0 is the value of negative resistance, or the slope of the straight line. This characteristic, however, is not maintained under rapidly varying conditions. Under such conditions there would have to be a rapid ionization and de-ionization of the air and the speed of these phenomena is limited. Fig. 5b shows the dynamic characteristic of the alternating current arc in air. This characteristic is a function of both current and frequency. With small currents and low frequencies the characteristic approaches that of Fig. 5a. With heavy currents the intense ionization keeps the atmosphere of the arc conducting between alternations even at lower frequencies, while with light currents at higher frequencies there is not time for the de-ionization necessary to give the static characteristic. It is this characteristic, giving a low value of negative resistance at high frequencies, which minimizes the possibility of sustained high frequency oscillations on transmission lines. It is acknowledged that, if the negative resistance of the arc, in a sustained short circuit, were greater than the positive resistance of the connected circuit, oscillations with indefinitely increasing amplitude would be possible[2, 8]. Tests have proved that these do not exist. Where the arc is in rapid air currents or a magnetic field the tendency is toward the static characteristic. An arc in oil also has a negative resistance characteristic, so the following discussion applies to oil-circuit-breaker, as well as air-break, switching.

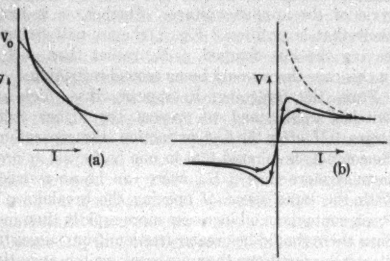


FIG. 5—CHARACTERISTICS OF AN ARC IN AIR
(a) Static (b) Dynamic

Effect of the Arc in Switching. Where power currents are switched, the arc persists over any high-frequency oscillation and conditions are controlled by the 60-cycle wave. This is also true in the usual case of charging currents since appreciable currents usually are involved. In the case of closing either, the arc does not ordinarily extinguish after once striking. Where power current is interrupted the arc will extinguish at the zero point of the current wave which is also at a low point of voltage. Also, when charging currents are interrupted, the arc will first extinguish at the zero point of the current wave, but this will be at the maximum of the voltage wave. In this case, surges three times normal are possible. For short lengths of line or station busses, where exceedingly small currents are involved, the arc maintains more of its static characteristic and higher potentials are possible.

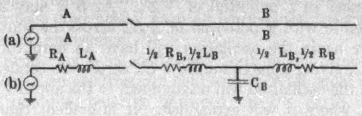


FIG. 6—TRANSMISSION LINE SUPPLIED FROM ALTERNATING CURRENT SOURCE

Switching Charging Currents—Larger Values. The above considerations may be better explained in connection with the following diagrams; Fig. 6a represents one phase of an open end line. Fig. 6b represents the approximate equivalent circuit. Fig. 7a shows the wave of voltage at the supply side of the switch. Fig. 7b shows the voltage on the line, or condenser. The arc between the switch contacts will first extinguish at 1, the zero point of the current wave and the crest of the voltage wave. This will leave the line charged to a maximum of E_m in one direction. The switch contacts are assumed to be separating continuously. From 1 to 3 the voltage across the switch contacts varies with the impressed voltage to a maximum of 2 E_m at 3. There will be no restriking if at this time the contacts are too far apart to break down at 2 E_m. If not, reignition will occur, and there will be a resulting oscillation about the point of the applied

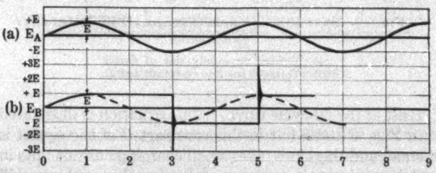


FIG. 7—SWITCHING CONTROLLED BY 60 CYCLE CONDITIONS
(a) Voltage on supply side of switch
(b) Voltage on line side of switch

voltage E_A, or $-E_m$, and with an amplitude of $2E_m$, the amount of the transition to the new condition. It is obvious that the potential will thus reach a maximum of $-3E_m$ across the condenser. Since there is appreciable current the negative resistance of the arc is low and it will not extinguish during the high-frequency oscillation. This oscillation will then damp down to the applied voltage. Again, it will extinguish at that point, as shown between 3 and 5, leaving the condenser charged to a potential of $-E_m$. At 5 it may again re-ignite with a potential of $2E_m$ across the contacts. Here again the oscillation will have an amplitude of $2E_m$, giving a maximum of $3E_m$ across the condenser, and the oscillation will damp down to the applied voltage, where it will extinguish. It is evident that, no matter how often the arc reignites, the potential can never be higher than $3E_m$.

Switching Charging Currents—Small Values. When small currents are involved, it is possible under certain circumstances for the arc to extinguish at the zero point of the high-frequency current wave, and ignite at the crest of the 60-cycle wave. This situation is illustrated in Figs. 6 and 8 but in this case part B of the circuit is small and therefore takes small charging current. As in the previous case the arc between the contacts will extinguish at the crest of the 60-cycle wave at 1, Fig. 8b. The condenser will then be left charged to a potential of E_m. It is assumed that after one half-cycle, at 3, the switch separation is such that $2E_m$ will just reignite the arc. There will then be an oscillation with an amplitude of $2E_m$ about the applied voltage E_A, or $-E_m$. The potential across the condenser will thus reach a maximum of $-3E_m$. When this oscillation has damped to a value slightly less than $2E_m$ the arc may extinguish at the zero point of the high-frequency current wave and leave the condenser charged to slightly less than $-3E_m$. Again the voltage across the contacts will vary between points 3 and 5 to a maximum of $4E_m$ at 5 when it is assumed that the switch contacts have separated to a position where $4E_m$ will just reignite the arc. This time the oscillation will have a magnitude of something less than $4E_m$ about E_m which will bring the voltage across the condenser to a maximum of slightly less than $5E_m$. This process may be carried on with an increase of nearly $2E_m$ with each restriking of the arc.

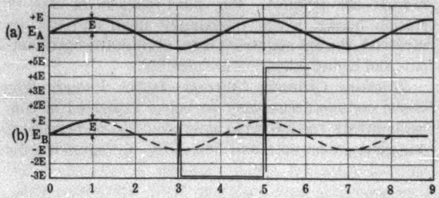


FIG. 8—SWITCHING CONTROLLED BY HIGH-FREQUENCY CONDITIONS

(a) Voltage on supply side of switch
(b) Voltage on line side of switch

While it is conceivable for the process discussed in the last paragraph to continue beyond two restrikings, it is extremely unlikely; and even two should be rare. There are many factors which tend to prevent the high frequency condition from controlling the situation. The theory assumes that the charging current to the section being disconnected is broken at the zero point of the high-frequency current wave after the first interruption. In the usual case of line switching the inductance and capacitance are both linear functions of the length of circuit. Thus, when the capacity is small, giving small currents, the inductance is also small and the frequencies are high. Usually these frequencies are high enough to maintain the arc over the high-frequency oscillation even though the currents are extremely small. The worst condition as discussed assumes that the rate of opening of the contacts is such that the breakdown is increased nearly $2E_m$ each half cycle of the applied voltage. Further, it is just as likely that in position 3 Fig. 8, the arc will extinguish leaving the bus charged $+E_m$ rather than $-3E_m$. In this case there would be no second restriking.

From the foregoing, it appears that high-speed switches would tend to prevent the higher voltage surges. If, after the first extinction, the separation of the contacts is so rapid that in one half-cycle its breakdown is more than $2E_m$, there can be no restriking. With the same speed of opening the breakdown between contacts in oil increases more rapidly than in air. Thus there should be greater freedom from surges from oil switch operations than air-break switch operations.

Closing operations should not cause potentials as high as opening operations. This is for the reason that the switch contacts are drawing continuously closer and therefore there should be at most only one restriking. This would limit the potentials obtainable to $2E_m$ on homogeneous lines. However, since switch contacts, especially disconnect switch contacts, are not perfect arcing contacts, it is possible for the second striking to be at a higher voltage than the first. This makes possible surges slightly higher than $2E_m$ from closing. It is fortunate that the higher surges come upon de-energizing since in this case the element of highest potential is being disconnected from the system and a flashover is not as serious. Further, there is usually the least apparatus connected to the part being disconnected.

In all the above it is assumed that the neutral of the system is held at ground potential. In the case of an isolated neutral system the potentials discussed above are aggravated by the amount of the shift of the neutral; that is by a maximum of 1.73.

Experimental Data. During the past three years, field tests using the klydonograph as a surge recorder have been conducted on a large variety of open-wire and cable systems. These have given definite information as to the surges actually present on lines. The complete results of these tests are being given in a companion paper by Messrs. McAuley and Huggins and the writer. Certain of these data, however, will be discussed here in connection with the above theories.

Switching in general caused no serious surges. The majority of switch operations produced no overpotentials and the majority of the surges which were produced were less than two times normal. Occasionally higher voltage surges occurred, but these were rare. This would be expected due to the small chance of the simultaneous presence, during a switching operation, of all the conditions and events necessary for their production as discussed above. Surges due to switching load currents or energizing lines were of the lower values. The higher surges always resulted from low energy operations, such as switching short lengths of line or busses. Many of these higher ones were caused by de-energizing busses or open oil switches with disconnect switches. As examples: In a test on a 220-kv. grounded-neutral system, with instruments at four points, about 3600 high-tension, oil-switch operations were performed. Approximately one-fourth of these caused surges. The largest surge recorded was 3.2 times normal due to de-energizing a 40-mi. section of the line. The next highest were two 2.7 times normal due to opening a bus. On a 140-kv. isolated-neutral system in ten months tests at four points there were five surges between 3.0 and 4.6 times normal. On a 120-kv. grounded-neutral system during ten months normal operation with recorders at four points there was one surge six times normal, one 4.0, one 3.7, and five between 3.7 and 3.0 times normal. All of the above were caused by idle-line switching, and the higher ones by de-energizing operations.

On cable systems, due to the higher charging currents per unit length, the higher attenuation constant, the longer initial wave fronts, and the denser networks usually encountered, milder surge conditions would be expected. This is what was actually found: The maximum surges recorded due to cable switching were of the order of 2.5 times normal. In one test, after a year's operation, a surge 4.5 times normal was recorded. However, this was due to energizing a switch group and not strictly a cable operation. On another test, high surges resulting from opening a bus section prompted a series of tests. The highest values were as follows:

Opening bus with disconnect switches—4.5 times normal.

Closing bus with disconnect switches—2.9 times normal.

Opening bus and transformer with oil switches—4.3 times normal.

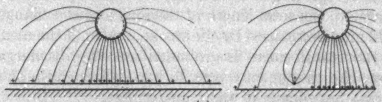
Closing bus and transformer with oil switches—2.9 times normal.

Both of these conditions are low-current operations. Although the current would be larger with the transformer connected, the natural period of the circuit would also be longer, thus permitting the action discussed under "Charging Currents—Small Values."

II. LIGHTNING

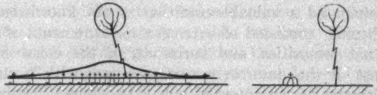
The principal characteristics of lightning of interest in connection with its effect on transmission lines are: the gradient of the field produced by the cloud charge; the nature of the discharge, that is, its rate, and whether it is unidirectional or oscillatory; and the polarity of the cloud. Ryan, De Blois, Norinder, Creighton, Peek, Simpson, and others have all contributed towards our present knowledge.

From any isolated charged body in the atmosphere there is an electrostatic field ending in an equal charge of opposite polarity gathered on the earth. As a charged cloud approaches a transmission line, a charge, opposite in sign to that on the cloud, gathers on the line. Since the cloud approaches slowly, the charge passes up from the ground through the neutral or leaks over the insulators. This charge in the presence of the field keeps the line at zero potential with respect to the earth. Fig. 9a illustrates this condition. Upon the stroke of lightning the field disappears and, if the collapse be rapid, leaves the charge bound on the line. If the collapse is instantaneous this raises the line potential to a voltage corresponding to the field before the stroke, but opposite in sign. The magnitude of this voltage is the product of the change in field gradient and the height of the line. Upon the disappearance of the field, the charge divides in the center and propagates a traveling wave in each direction as shown in Fig. 9c. Each of these waves has a magnitude equal to one half of that of the initial voltage directly under the cloud. It is readily seen that the rate of rise of voltage, and the maximum magnitude, on that part of the line occupied by the charge before the stroke, is



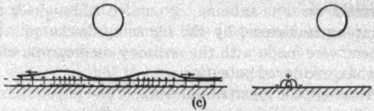
(a)

Cloud, and Bound Charge on a Line, Before a Stroke



(b)

Bound Charge and Voltage on a Line After a Stroke



(c)

Traveling Waves After a Stroke

Fig. 9

determined by the rate of discharge of the cloud, as well as the value of the field destroyed. The reason for this is that while the cloud is discharging, part of the charge is traveling out and increases the length of the surge, but reduces its magnitude. On the remainder of the line to which the traveling wave goes, the rate of rise of voltage is determined by the wave front, that is, the space configuration of the charge liberated, as well as by the rate of discharge of the cloud. It is obvious that for this phenomenon to take place the cloud must discharge rapidly, that is, in a few microseconds. Otherwise the charge will have time to spread out over the line gradually, without causing an appreciable rise in voltage. The amount of this rise is determined by the ratio of the length of the line occupied by the bound charge to the total length of the line, and the change of field gradient caused by the stroke. Also, if the neutral of the transformers is grounded and the discharge is slow, the charge may pass to earth through the short-circuit impedance of the transformers.

Norinder. In 1924, Dr. H. Norinder published the results of his tests in Sweden.[11] These tests, which contributed a valuable addition to our knowledge of lightning, consisted of extensive measurements of the actual intensities, and variations of the cloud field. Most of the data were measurements with a static voltmeter of the potential induced on highly insulated wires by the cloud field. Other measurements were made with a cathode-ray oscillograph of the potential induced on an antenna, grounded through a high damping resistance, by the lightning discharge. Still others were made with the ordinary oscillograph of the discharge-induced potential.

Although the inherent variation of the physical conditions under which lightning occurs is reflected in the data, the salient points in Norinder's results were as follows: the gradient of the cloud field at the height of the average transmission line was often of the order of 30 to 45 kv. per ft. (100 to 150 kv. per m.) and occasionally 60 kv. per ft. (200 kv. per m.). Dr. Norinder estimated the gradient near the lightning path to have been 90 to 120 kv. per ft. (300 to 400 kv. per m.); the cloud was just as often positive as negative. One of his charts showed the potential reversing during the passage of the cloud. The range of influence of the usual stroke was over 1.3 mi. (2.3 km.) and under 6.2 mi. (10 km.); the discharge was non-oscillatory; and the discharge was relatively slow, the induced potential on the antenna rising to a maximum in the order of 0.01 to 0.02 sec. From the results Dr. Norinder drew various conclusions some of which pertained to the operation of transmission lines.

Since both the data and the conclusions were somewhat at variance with current opinion on the matter, both were criticized rather severely.[12] The most disturbing feature of these data was the long wave fronts indicated. If lightning strokes were as slow as 0.01 sec., it would be impossible for surges as actually experienced to be induced on transmission lines. In a time of this order the bound charge could travel the length of the line many times and distribute itself over the line or leak off through the terminal apparatus. It is evident that the strokes which produce the surges experienced must take only a few microseconds to discharge the cloud. While some of the conclusions seem rather loosely drawn, there does not seem to be anything wrong with the methods used in obtaining the data. The ordinary oscillograph is not fast enough to record wave fronts of only a few microseconds. However, the fact that some of the oscillograms showed a wave front considerably longer than the minimum which the ordinary oscillograph would have recorded satisfactorily, indicated that these long wave fronts actually occurred. While the above facts seem to be inconsistent, recent information shows that they are not. This will be discussed later.

Simpson. In March, 1926, Dr. G. C. Simpson presented to the Royal Society an excellent paper on lightning. It is not feasible to repeat the complete theory here and the reader is referred to the original paper.[13] Briefly, the author stated that when a point on a positive cloud reaches a state of electrical intensity at which ionization begins, the point proceeds towards earth, forming a conducting channel of ionized air. The point of this channel attracts negative electrons from the surrounding atmosphere. These electrons being very mobile pass up the channel and spread out into the cloud. This leaves the channel positive and it thus burrows its way to the earth. The channel may branch but each fork remains pointed and the stress high. The rate at which the channel can grow is determined by the rate at which the cloud can absorb the negative electrons. As the electrons pass into the cloud they soon become negative ions, after which their speed is reduced; thus they tend to block up the mouth of the channel and prevent further flow. This accumulation of ions disperses in the cloud, the field is again established, and the flow continues if the ions in the channel have not had time to recombine entirely. Thus the flow will be slow or even intermittent. The intermittent flow is most marked in a discharge from one cloud to another.

With a negative cloud the process is different. If the formation of a channel toward earth at the point of highest stress is assumed, the point of the channel attracts positive ions from the surrounding atmosphere. These ions, being relatively immobile, are unable to pass up the channel rapidly. They thus neutralize the point which reduces the stress and the point spreads out. In this way the charge in the cloud is held until the stress at the earth's surface is high enough to start a channel. This channel operates as discussed above, for the positive cloud, with the following exceptions: The negative electrons are able to spread out readily in the ground, and the channel instead of passing into a region of lower stress continues always into a region of greater stress. The average stress is much higher. There is not the tendency to branch, as the field concentrates towards a smaller area, until the cloud is reached where it branches out and discharges a large volume of the cloud in a single flash. Thus the negative cloud discharges rapidly and violently.

Dr. Simpson substantiated the above theory with laboratory studies of discharges in conjunction with a large number of photographs of actual lightning strokes. Out of 442 photographs studied, 242 showed branches downward indicating positive cloud discharges, three were negative strokes with branches upward, and 173 were unbranched. Since the negative branches are within the cloud, in general they would be obscured. It was further pointed out that in all cases branches were less intense than trunks and that many of the 173 apparently unbranched strokes likely had branches downward which were obscured by the rain. Dr. Simpson divided the unbranched discharges into equal parts which made in all 328 positive strokes and 89 negative strokes, or nearly 4 to 1. He expressed the opinion that even this is too small a ratio.

Quoting Dr. Simpson, "If this reasoning is correct lightning flashes between earth and a negatively charged cloud will be much more intense than flashes to a positively charged cloud, although the two clouds may be charged to the same intensity. In fact one would expect on this reasoning that discharges from positively charged clouds would be frequent but weak, while discharges from negatively charged clouds would be infrequent but very strong."

Experimental Data. In transmission line surge investigations with the klydonograph it has been found that there are surprisingly few surges induced in a given locality during a thunder storm. Instead of surges of varying magnitude to correspond to the numerous lightning flashes usually observed during a storm, there result only from one to perhaps 10 surges; usually not more than two. Further, it has been found that practically all of these lightning surges are positive. The few negative surges recorded are always high and abrupt. Of these negative surges some are definitely known to have been caused by direct strokes. Bearing in mind that an induced surge is of opposite polarity to the cloud and that a direct stroke is of the same polarity, the above results indicate, at first sight, that all clouds are negative. The number of surges recorded, however, is not consistent with the numerous flashes visible during a storm.

These data, when analyzed in the light of Dr. Simpson's theories and Dr. Norinder's results, seem to substantiate both. Most of the lightning strokes are from a positive cloud. Perhaps as many clouds are negative, but the positive clouds discharge more frequently. These positive strokes are relatively slow, having a wave front of the order of 0.01 seconds, as shown by Norinder. They are slow enough to permit the bound charge to distribute itself over the line or to escape through the transformer neutral, as discussed above. Even on an isolated neutral line, where the charge does not escape, its distribution over the usual power line prevents a greater rise than a few thousand volts, which is negligible. It may be serious on communication lines, which are isolated from ground and insulated for low voltages. Tests on communication lines have shown more negative surges than tests on power lines. The effect of positive strokes may also be important on isolated neutral lines which are so short that the entire line is in the field destroyed by a single stroke. According to Norinder, this length would be of the order of two miles. Further, the effect of a positive stroke is less severe, due to the fact that less of the cloud is discharged and therefore less of the field is destroyed by a positive stroke. The negative clouds discharge violently but less frequently. They discharge a greater volume of cloud and hence destroy a larger field. The time of discharge is of the order of three microseconds, which is too brief to permit a bound charge on the line to dissipate, and hence high voltages are induced. Likely, Dr. Norinder's apparatus was not sufficiently rapid to detect these negative strokes in their true shape. Klydonograph records also indicate that direct strokes have abrupt wave fronts. These records show that induced strokes have wave fronts from a few microseconds to over 100 microseconds. As mentioned above, the measured rate of rise of voltage due to induced strokes depends upon whether the measurement is made at the position of the initial bound charge or at some other point of the line. The records agree with both Simpson and Norinder in that they indicate that the stroke is unidirectional. An oscillatory surge on the line may result, however, if the stroke causes a flashover.

It was found that the maximum surge potentials appearing on lines of various operating voltages are approximately a constant times the normal voltage. That is, the maximum surges are from 10 to 15 times the normal crest voltage above ground. This indicates that the lightning stroke is not faster than the insulator flashover at these high over-voltages. When the insulator flashes over the charge is released and the voltage can go no higher. It should be remembered that the higher the over-voltage, the less the time lag of breakdown of any dielectric.

III. CONCLUSIONS

The following conclusions may be stated:

Switching. 1. If there were no attenuation, a switch closure on a cable connected to an open-ended, open-wire line would reflect with nearly four times the applied voltage.

2. In practise, increase in the attenuation constant, due to steep wave fronts and to corona at high voltages, prevents high-potential steep-front waves from traveling long distances.

3. The limitations of the source from which a line is switched limit the steepness of wave front possible.

4. The characteristics of the arc at the switch contacts influence the surges caused by switch operations.

It has been shown that over-potentials caused by low-energy charging current operations are more severe than those caused by power or heavy charging current operations. Thus de-energizing short lines or busses causes the highest surges.

5. No switching operations cause surges high enough, or with a duration long enough, to affect the operation of properly insulated lines. Only rarely is a surge above 2.0 times normal produced.

Lightning. 1. Positive lightning strokes are frequent but weak. They are slow, of the order of 0.01 seconds, and hence do not induce surges on transmission lines.

2. Positive strokes, even though slow, may produce surges of importance on isolated low-voltage lines, such as communication lines.

3. Negative lightning strokes are less frequent but more violent. They discharge in about three microseconds and hence produce high voltage surges on transmission lines.

4. The field gradient is often as high as 60 kv. per ft. and may reach 100 kv. per ft. Thus a surge of over 2000 kv. might be induced upon a line of ordinary height with sufficiently high insulation. Eighteen hundred kv. has been recorded by the klydonograph.

5. The time lag of an insulator flashover is less than the time of discharge of a negative stroke and thus the impulse flashover voltage of the insulators limits the possible potential.

6. The stroke of lightning itself is unidirectional. If an oscillatory surge due to lightning is recorded, it is a line oscillation resulting from a flashover.

ACKNOWLEDGMENTS

The author is pleased to acknowledge his indebtedness to Mr. J. F. Peters and Dr. J. Slepian for their numerous suggestions and to Mr. P. H. McAuley for his suggestions and help in the preparation of this paper.

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Discussion

For discussion of this paper see page 348.

The Measurement of Surge Voltages
on Transmission Lines Due to Lightning

BY EVERETT S. LEE* and C. M. FOUST*
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*This paper, after referring to the work of previous investigators in the application of the photographic Lichtenberg figures to the measurement of surge voltages, describes results of additional work in this field by the authors.*

Laboratory calibrations of photographic Lichtenberg figures, using the cathode ray oscillograph and the lightning generator, are shown. Data are presented relative to the accuracy obtainable with these figures as a means of measuring surge voltages.

An extension of instrument design is described in which two recording elements are used to give greater certainty of result.

Means for connecting a surge voltage recorder instrument to a transmission line by an insulator-string potentiometer are described, and calibration of the instrument with potentiometer is given up to 1400 kv.

Specimen field records of surge voltages up to 2000 kv. are shown.

* * * * *

INTRODUCTION

THE results of continued recent study and use of the photographic Lichtenberg figures as a means of measuring voltages of short time duration in the order of microseconds, particularly surge voltages on transmission lines due to lightning, are creating a confidence in these figures which is gratifying both to the engineer who is called upon to make such measurements and to the engineer who uses the results in design and application. It was in 1777 that Dr. G. C. Lichtenberg[1] first described the figures in sulphur dust caused by the presence of a charged electrode. In 1888, Trouvelet[2] and Brown[3] showed that the same figures would be produced on a photographic plate. Several investigators[4, 5, 6] have since devoted much time to studying the nature of these figures, although at the present time their exact mechanism is still an uncertainty.

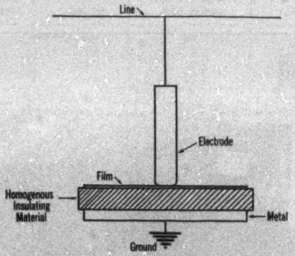

FIG. 1—ARRANGEMENT OF ELEMENTS FOR PRODUCING PHOTOGRAPHIC LICHTENBERG FIGURES

Directly connected recorder

But it remained for Mr. J. F. Peters[7] in 1924 to suggest the application of these figures to the measurement of surge voltages and to develop a suitable instrument based on this application which he called the klydonograph. In June, 1925, Messrs. Cox and Legg[8] presented before the A. I. E. E. the results of field tests with this instrument and described extended developments in the instrument design. In September, 1926, Mr. K. B. McEachron[9] presented before the A. I. E. E. the results of a most detailed study of the calibration of the photographic Lichtenberg figures using the Dufour cathode ray oscillograph as the means for determining with certainty the wave shape of the impressed voltage. Thus the discovery made 150 years ago has recently been applied to advantage.

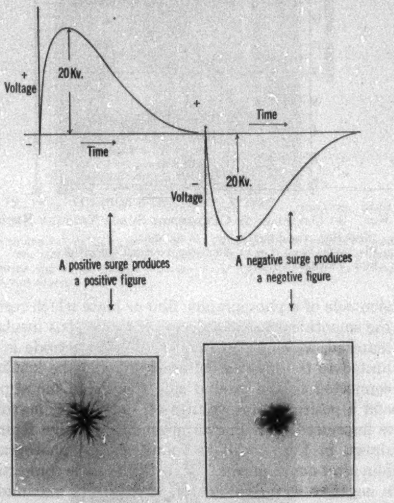
+
Voltage
20 Kv.
Time
-
+
Time
-
Voltage
20 Kv.
A positive surge produces a positive figure
A negative surge produces a negative figure

FIG. 2—SHOWING APPEARANCE OF POSITIVE AND NEGATIVE PHOTOGRAPHIC LICHTENBERG FIGURES PRODUCED BY POSITIVE AND NEGATIVE SURGE VOLTAGES OF SAME MAGNITUDE AND WAVE SHAPE

Directly connected recorder

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*Both of General Engineering Laboratory, General Electric Co., Schenectady, New York.

1. For references, see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 7-11, 1927.

To the work previously described, this paper contributes additional correlative data, describes an extension of instrument design, and shows that the art has advanced to a stage where surge voltages on transmission lines in the order of 2,000,000 volts may be recorded with a reasonable degree of accuracy.

GENERAL

As now used, the klydonograph or surge voltage recorder consists of an electrode bearing upon the emulsion side of a photographic film or plate which rests on the smooth surface of a piece of homogeneous insulating material, as shown in Fig. 1. If the electrode is connected to the line side of a circuit, and the insulation connected to the ground side through a metal plate, and a positive surge voltage of, say, 20 kv. maximum is impressed from line to ground, a positive figure as shown in Fig. 2 will be found on the photographic film after development. If, with the same connections, a negative surge voltage of, say, 20 kv. maximum is impressed from line to ground, a negative figure, as shown in Fig. 2, will be found on the photographic film after development

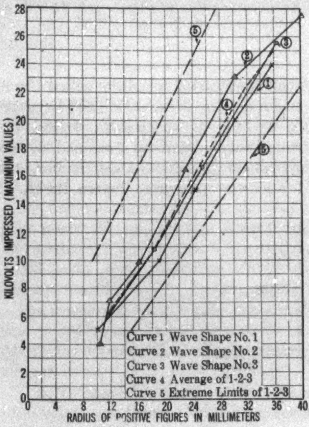

FIG. 3—CALIBRATION CURVES FOR SURGE VOLTAGE RECORDER

Insulation, varnished paper 1/8 in. thick
Electrode, brass, 1/8 in. diameter, rounded spherically
Film, Eastman No. 152

It has been found that figures will be produced even though the time duration of the impressed voltage is only a fraction of a microsecond, also that the size (radius) of the figure is a function of the magnitude of the maximum value of the impressed voltage, while the shape and configuration of the figure is a function of the wave shape of the impressed voltage. The problem of the instrument engineer therefore becomes one of deciphering the figures into terms of voltage and wave shape.

MAGNITUDE OF VOLTAGE

The calibration of Lichtenberg figures for a given instrument to determine magnitude of voltage is obtained by impressing voltages of different values and observing the size of the resulting figure. This can be done for as wide a range of wave shapes as are available.

Table I and Fig. 3 of this paper give results of the authors' calibrations obtained on a film-type instrument with varnished paper insulation and a brass electrode ⅛ in. in diameter rounded spherically. The wave shapes varied from one-half cycle of a sine wave at 60 cycles (wave shape No. 1) to surge voltages rising to their maximum value in two microseconds (wave shape No. 2) and in four microseconds (wave shape No. 3). The surge voltages were impressed from sections of a 500-kv. rectifying type lightning generator (Fig. 4), the circuit for wave shape No. 2 being as shown in Fig. 5 and for wave shape No. 3, as shown in Fig. 6. The wave shapes were determined by Dufour cathode ray oscillograph Figs. 7 and 8. Wave shape No. 2 is shown in Fig. 9. Wave shape No. 3 is shown in Fig. 10.

When calibrating the surge voltage recorder or when photographing the wave shapes with the cathode ray oscillograph, these instruments were connected between ground and the various voltage taps as shown in Figs. 5 and 6. The magnitude of the voltage was measured by sphere spark gap similarly connected.

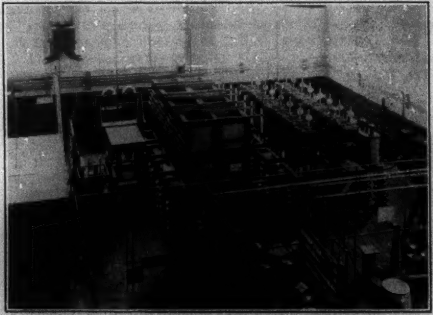

FIG. 4—500-KV. RECTIFYING TYPE LIGHTNING GENERATOR USED AS SOURCE FOR SURGE VOLTAGES IN STUDYING PHOTOGRAPHIC LICHTENBERG FIGURES

The cathode ray oscillograph used with this generator is located in the sheet-iron house at the left of the photograph

Fig. 11 shows a set of positive and negative figures for 5, 10, 15, and 20 kv. taken with wave shape No. 2. It is from such figures as these that the calibration curves are obtained. The radius of a positive figure is measured from the figure center to the most distant streamer tip.

TABLE I
CALIBRATION OF POSITIVE PHOTOGRAPHIC LICHTENBERG FIGURES

60-Cycle Wave Shape—Wave Shape No. 1

Kv. impressed (max. values)	Number of figures	Positive figures average radius mm.	Average deviation from mean. Per cent plus and minus	Maximum deviation from mean. Per cent plus and minus
5	35	10.3	12	45
10	34	19.0	5	16
15	36	24.3	5	15
20	36	30.2	8	37
24	31	35.6	5	12 disregarding slips 125 regarding slips
Two-Microsecond Wave Front—Wave Shape No. 2				
4	29	10.6	9	25
7.2	39	11.9	22	60
10	36	16.3	10	35
16.6	36	22.8	10	26
23.2	36	30.4	5.5	19
27.6	37	40.3	14	26
Four-Microsecond Wave Front—Wave Shape No. 3				
5.95	36	11.8	12	41
10.75	35	18.3	6.5	20
16.7	36	25.3	5.3	15
25.6	36	36.4	6.2	24

Average Deviation For All of Above Wave Shapes

Kv. impressed (max. values)	Number of figures	Average deviation from the mean. Per cent plus and minus	Maximum deviation from the mean. Per cent plus and minus
5	100	13.7	32
10	105	8.5	31
15	108	7.2	28
25	104	10.2	33

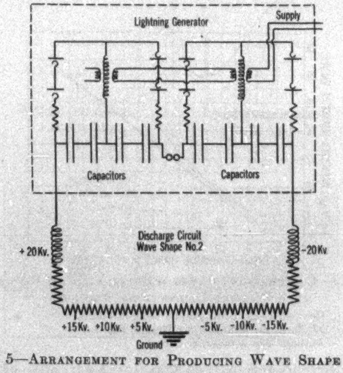
Lightning Generator
Supply
Capacitors
Capacitors
Discharge Circuit
Wave Shape No.2
+20Kv.
-20Kv.
+15Kv. +10Kv. +5Kv.
-5Kv. -10Kv. -15Kv.
Ground

FIG. 5—ARRANGEMENT FOR PRODUCING WAVE SHAPE NO. 2

The capacitors of the lightning generator discharge through the inductance and resistance shown in the external discharge circuit, in which the balanced arrangement of circuit constants with respect to the grounded point eliminates local oscillations

Referring to Table I, it is seen that the average deviation from the mean for 100 figures on the three wave shapes investigated at the different voltages is within ± 15 per cent, while the maximum deviation from the mean is in the order of ± 30 per cent.

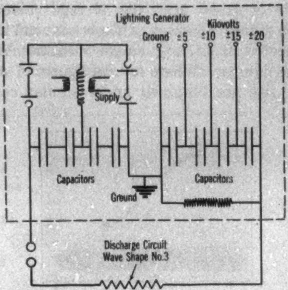
Lightning Generator
Kilovolts
Ground ±5 ±10 ±15 ±20
Supply
Capacitors
Capacitors
Ground
Discharge Circuit
Wave Shape No.3

FIG. 6—ARRANGEMENT FOR PRODUCING WAVE SHAPE NO. 3

The capacitors of the lightning generator on the left, discharge into the capacitors on the right through the resistor shown in the external discharge circuit

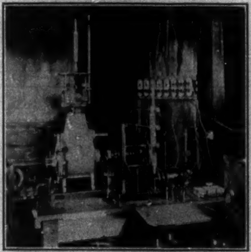

FIG. 7—THE DUFOUR CATHODE RAY OSCILLOGRAPH, SIDE VIEW

The timing switch shown at the right of the oscillograph is for low-speed work. The timing switch used to discharge the lightning generator when calibrating the surge voltage recorder is of high-speed type of special design, not shown

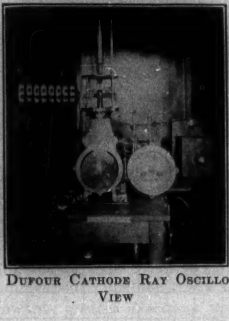

FIG. 8—THE DUFOUR CATHODE RAY OSCILLOGRAPH, FRONT VIEW

These results are shown graphically in Fig. 3. For any voltage all figures obtained on 100 measurements were within the extreme limits as shown. These limits are determined by one figure out of one hundred, and are quite outside of the values which may be reasonably expected from the average values shown. It appears that an accuracy of 25 per cent can be reasonably expected from any single measurement made with these figures. Where several figures of somewhat the same size are obtained under similar conditions, the agreement of these among themselves permits a more exact interpretation.

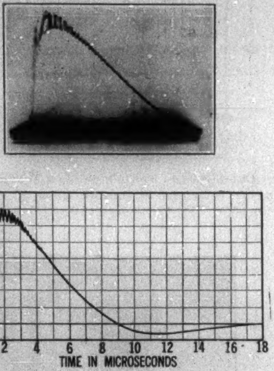

Fig. 9

(Above)—Cathode Ray Oscillogram of Wave Shape No. 2

(Below)—Cathode Ray Oscillogram of Wave Shape No. 2 Transcribed to Rectangular Coordinates

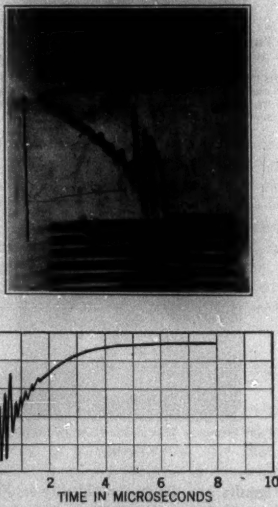

Fig. 10

(Above)—Cathode Ray Oscillograph of Wave Shape No. 3

(Below)—Cathode Ray Oscillogram of Wave Shape No. 3 Transcribed to Rectangular Coordinates

Messrs. Cox and Legg, in Fig. 39 of their paper[8], show a calibration curve for an experimental model of a film-type klydonograph. The wave shapes impressed varied from 25- and 60-cycle a-c. sine wave, to a surge voltage which attained its maximum value in five microseconds, the latter surges being obtained from a given network and their shape determined by calculation. The magnitude of the voltage was determined by sphere spark gap.

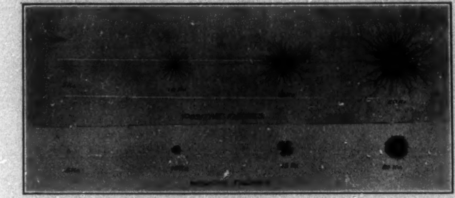

Fig. 11—Positive and Negative Lichtenberg Figures for Different Voltages. (Wave Shape No. 2)

McEachron, in Fig. 7 of this paper[9], shows calibration curves for Lichtenberg figures obtained with Eastman's super-speed portrait films, placed on a glass plate, and using a cylindrical brass electrode one cm. in diameter with square edges. The shape of the impressed voltage was determined by Dufour cathode ray oscillograph and the magnitude by sphere spark-gap. The range of wave shape was from a long wave wherein 22 min. were required to reach a maximum value of 25 kv., to a short wave where the time to reach maximum value was 0.1 microsecond.

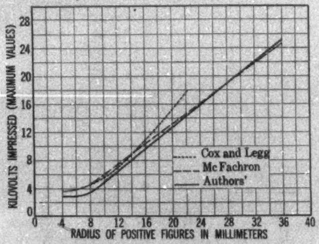

Fig. 12—Calibration Curves of Surge Voltage Recorders for Positive Lichtenberg Figures

Cox and Legg, Bibliography Item 8, Fig. 39
McEachron, Bibliography Item 9, Fig. 7
Authors' Fig. 3 (average curve)

The results of the calibrations reported by Messrs. Cox and Legg, and McEachron, are shown combined with the authors' in Fig. 12. These results show remarkable agreement for the work of different observers in different laboratories with different instruments and circuits, and give added weight and certainty to the calibration of the Lichtenberg figures as regards magnitude of voltage.

WAVE SHAPE OF IMPRESSED VOLTAGE

In studying surge voltages, the wave shape as well as the magnitude is of importance, for on this depends the duration of the voltage. At the present time, the determination of the wave shape from the Lichtenberg figure characteristics is not as definite or as certain as the determination of the magnitude from the figure size and herein there is room for added study. At the present time the figures recorded with unknown wave shapes can be compared with figures recorded with known wave shapes as determined by cathode ray oscillograph. This allows prediction of the time duration to within a general order, but not with the exactness required. The work by McEachron in this regard, as shown in Fig. 5 of his paper[9], wherein he designates the figures as Type I, Type II, and Type III, is to be commended. Further study along these lines tending towards greater exactness in the interpretation of figure characteristics is desirable.

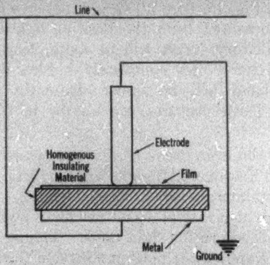
Line
Electrode
Homogenous Insulating Material
Film
Metal
Ground

FIG. 13—ARRANGEMENT OF ELEMENTS FOR PRODUCING PHOTOGRAPHIC LICHTENBERG FIGURES
(Oppositely connected recorder)

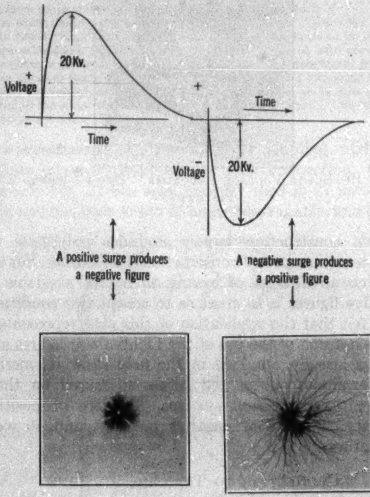
+
Voltage
-
20 Kv.
Time
+
Time
-
Voltage
20 Kv.
A positive surge produces a negative figure
A negative surge produces a positive figure

FIG. 14—SHOWING APPEARANCE OF NEGATIVE AND POSITIVE PHOTOGRAPHIC LICHTENBERG FIGURES PRODUCED BY POSITIVE AND NEGATIVE SURGE VOLTAGES OF SAME MAGNITUDE AND WAVE SHAPE
(Oppositely connected recorder)

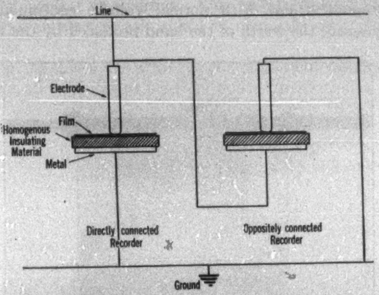
Line
Electrode
Film
Homogenous Insulating Material
Metal
Directly connected Recorder
Oppositely connected Recorder
Ground

FIG. 15—ARRANGEMENT OF ELEMENTS FOR PRODUCING BOTH POSITIVE AND NEGATIVE FIGURES FOR ANY SURGE VOLTAGE

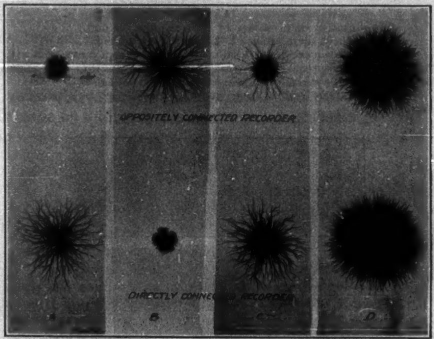
OPPOSITELY CONNECTED RECORDER
DIRECTLY CONNECTED RECORDER
A
B
C
D

FIG. 16—PHOTOGRAPHIC LICHTENBERG FIGURES OBTAINED WITH TWO-RECORDER TYPE INSTRUMENT

A—Positive surge voltage 20 kv. maximum
B—Negative surge voltage 20 kv. maximum
C—Oscillatory surge voltage 20 kv. highly damped
D—Oscillatory surge voltage 20 kv. slightly damped

DIRECTLY AND OPPOSITELY CONNECTED RECORDERS

From Fig. 11 it is clearly evident that the negative figures are quite inferior to the positive figures for purposes of voltage measurement since for a given voltage they are less than half the size of the positive figure; also McEachron has shown[9] that the negative figures present a greater deviation for differing wave shapes. There is also another serious objection which is that when the instrument with a moving film is connected to a transmission line with normal voltage continuously impressed, the width of the band produced by the line voltage (see Figs. 17 and 18) is enough to hide negative surges up to values as high as 2.3 times normal line voltage and to give uncertainty to values somewhat above this. This would result in erroneous conclusions as regards the number of negative surges recorded.

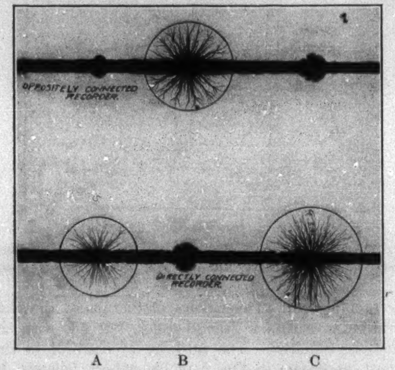

Fig. 17—Figures Obtained with Two-Recorder Type Instrument Showing Line Voltage Band

A—Positive surge voltage 13 kv. maximum (20 mm.)
B—Negative surge voltage 14 kv. maximum (22 mm.)
C—Positive surge voltage 17 kv. maximum (25 mm.)

The circles shown are drawn with the figure center as a center, and with the circumference touching the most distant streamer tip. The radius of this circle is the measure of the magnitude of the voltage

The line voltage band is from 60-cycle source of 2.84-kv. maximum value

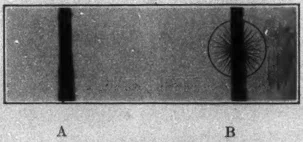

Fig. 18—Photographic Lichtenberg Figures Obtained with Two-Recorder Type Instrument Showing Hidden Negative Figure

A—Directly connected recorder B—Oppositely connected recorder

Negative surge voltage 9.0 kv maximum. The negative figure is hidden under the line voltage band. Its presence is indicated by the full-size positive figure on the oppositely connected recorder

The line voltage band is from 60-cycle source of 3.0-kv. maximum value

To overcome these objections, Mr. Foust conceived the idea of connecting two recorders in parallel with the connections of one opposite from that of the other, thus insuring a large positive figure for every surge. Referring again to Figs. 1 and 2, the results of directly connecting the recorder to the line are shown. If, however, the recorder is connected oppositely, that is, the electrode to ground and the metal plate to the line, as in Fig. 13, the positive surge will record a negative figure and the negative surge will record a positive figure as in Fig. 14.

If an instrument is made up with two recorders, one connected directly and one connected oppositely, as shown in Fig. 15, then all surges, positive or negative, can be measured from the positive figure. In addition, oscillatory surges will be more clearly recorded, and negative surges completely hidden by the line-voltage band will be shown distinctly as positive figures. These features are shown in Figs. 16, 17, and 18.

Fig. 19 shows an instrument of the two-recorder type. It uses an Eastman film eight feet long and eight inches wide as standard with "Cirkut Outfits." It is driven by a clock at a rate of ½ in. per hour, so as to give a continuous record for eight days. Timing is obtained by photographing the hour numbers on the film.

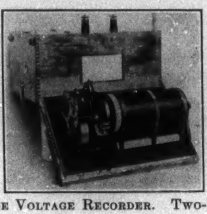

Fig. 19—Surge Voltage Recorder. Two-Recorder Type Instrument

This construction largely excludes polyphase instruments, because of constructional difficulties, but the obvious advantages of having all figures available as positive figures is so great as to accept this condition. It is felt that the application of this idea represents a real extension of the use of the Lichtenberg figures and results already obtained in the field show its merits. For example, out of 103 surges measured on three different transmission systems, 31 were of positive polarity, 26 were of negative polarity, and 46 were oscillatory.

Connection to Transmission Line

The voltage range of the instrument shown in Fig. 19 is from 2.8 to 25 kv. maximum. Above 25 kv. maximum, so-called "slips" occur in the figures as shown in Fig. 20, for which condition calibration curves do not apply. The arcover of this instrument on a two-microsecond wave, wave shape No. 2, Fig. 9, is 35 kv. maximum. Thus, some provision must be made for connecting the instrument to transmission lines up to values where the normal maximum voltage to ground is 180 kv. maximum for a 220-kv., 3-phase line, and where the maximum values of surges may be ten times this value.

Messrs. Cox and Legg[8] describe an electrostatic potentiometer and antenna coupling. The authors have investigated and used insulator coupling. Of the various schemes proposed for such connection, that shown in Fig. 21 has been recently used in 27 installations and has been found to be simple, reliable, and easy to calibrate. The instrument is connected in parallel across several insulators of an insulator string with added protection over the line insulation as desired. The instrument is placed in a sheet metal housing, Fig. 22, equipped with suitable entrance bushing and automatic grounding device when the door is opened. The door is equipped with a padlock. This housing protects the instrument from the weather and insures safety against tampering. The metal housing also acts as an electrostatic shield to eliminate stray field effects.

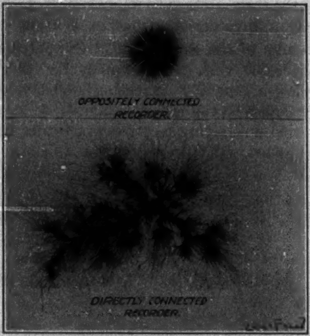

FIG. 20—PHOTOGRAPHIC LICHTENBERG FIGURES ABOVE THE INSTRUMENT RANGE

Positive surge voltage 33 kv. maximum wave shape No. 2.

The black lines clearly evident in the positive figure (below) are commonly called "slips" and their presence indicates the figure to be of uncertain calibration. However, such figures can be stated with certainty to be above a given voltage value depending upon the instrument design.

The negative figure (above), though symmetrical and appearing to be suitable for voltage measurement, nevertheless is not so in this range because of the great variation in figure size with wave shape. (See bibliography item 9, Figs. 6 and 7)

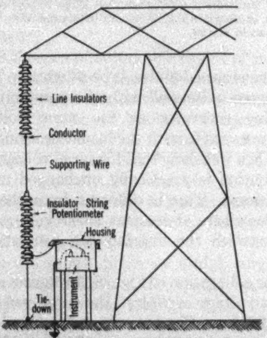

FIG. 21—ARRANGEMENT FOR CONNECTING SURGE VOLTAGE RECORDER INSTRUMENT TO TRANSMISSION LINE

Fig. 23 shows the arrangement of housing, insulator string, and connecting leads for a field installation.

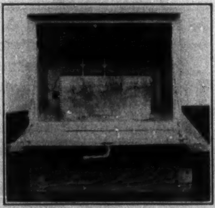

FIG. 22—SHEET METAL HOUSING FOR SURGE VOLTAGE RECORDER

It is important to have the instrument connecting leads short, preferably not longer than five feet. From Fig. 24 it is seen that the figure size decreases considerably with a longer lead, and if the instrument is used with leads of different length than that with which it is calibrated, the resulting error is large.

ADJUSTMENT OF INSULATOR STRING POTENTIOMETER

The adjustment of the insulator string potentiometer is made by adding a sufficient number of insulator units in series with the normal line insulators to give adequate protection, and to provide enough insulator units across which the surge voltage recorder instrument may be paralleled to obtain a satisfactory line voltage band. This procedure may be carried on in the laboratory by impressing normal voltage at normal frequency across the entire insulator string with the surge voltage recorder in position.

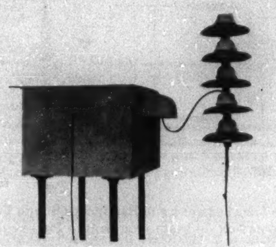

FIG. 23—ILLUSTRATION OF FIELD SET-UP OF INSULATOR STRING POTENTIOMETER AND SURGE VOLTAGE RECORDER IN SHEET METAL HOUSING

Table II gives the number of insulators which have been used successfully in the insulator string for different line voltages.

TABLE II

Line voltage between conductors three-phase kv.	No. of insulators line insulation	No. of insulators instrument string potentiometer	No. of insulators in parallel with instrument (included in col. 3)
66	4	9	2
110	8	13	2
140	10	15	2
220	14	20	4

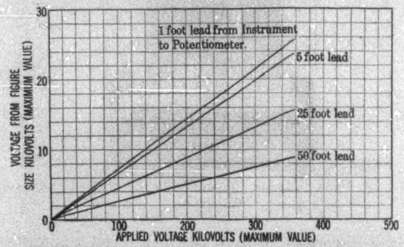

Fig. 24—Calibration of Surge Voltage Recorder and Potentiometer to Show Effect of Length of Instrument Lead

Calibration of Insulator String Potentiometer

The multiplying factor of the potentiometer can be calculated for normal voltage and frequency from the data obtained in adjusting the potentiometer string.

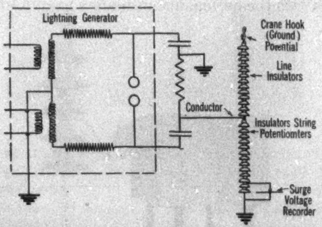

Fig. 25—Arrangement for Producing Surge Voltages for Calibration of Insulator String Potentiometer

For example, a 110-kv. line has a maximum value of voltage to ground of 90 kv. If the line-voltage band is three kv., then the potentiometer multiplying factor is 30.

The question then arises: Does this ratio hold for surge voltages? To answer this question, surge voltages were impressed across an insulator string potentiometer whose 60-cycle multiplying factor was 60. This was a string of twenty insulators, four of which were in parallel with the surge voltage recorder. The source of the surge voltages was a lightning generator of the non-rectifying type discharging into an external circuit as shown in Fig. 25. This circuit had to be used rather than the circuits as shown in Figs 5 and 6 in order to attain the requisite voltage. The magnitude of the voltage was determined by sphere spark gap. The time of rise of the surge voltage to its maximum value calculates to be in the order of a fraction of a microsecond.

The results of the calibration of the potentiometer up to 1,400,000 volts are shown in Fig. 26. These results show a generally decreasing multiplying factor from the higher to the lower voltages. At the higher voltages the multiplying factor is practically the same as that obtained with 60-cycle voltage.

Results of tests with circuit arrangement with the rectifying type of lightning generator (Fig. 5) to give a wave similar to that shown in Fig. 9 are also shown in Fig. 26. Tests were made with the insulator string potentiometer both dry and wet with spray. These were at the highest voltage which could be obtained with this generator for this type of work. The results seem to agree quite well with the non-rectifying type of lightning generator at the same voltage. The results of tests made with the insulator string when dry and also when wet show that for surge voltages the voltage distribution is practically unchanged under these two conditions. This is different from the condition at 60 cycles where at least at lower voltages the differences between the distributions wet and dry are greater.

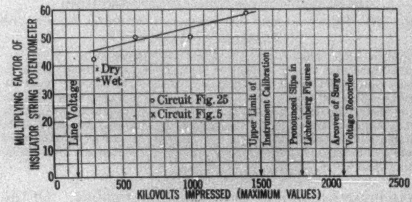

Fig. 26—Surge Voltage Calibration of Insulator String Potentiometer

Potentiometer for 220-kv., three-phase line
Number of insulators in string, 20
Number of insulators paralleled with the surge voltage recorder, 4
Instrument, as shown in Fig. 19 housed in housing as shown in Fig. 22.
Set-up as shown in Fig. 23

From the calibration of Fig. 26, for figures showing an instrument voltage of 25 kv., the surge voltage on the line is 1500 kv. Figures with slips would indicate surge voltages from 1500 kv. to 2100 kv. Film arcover at 35 kv. on the instrument would indicate surge voltages on the line of 2100 kv. or over.

The results of these calibrations indicate that surge voltages up to ten times normal maximum value, line to ground, on a 220-kv. line can be measured with considerable certainty as regards magnitude.

SPECIMEN FIELD RECORDS

On Fig. 27 are shown some Lichtenberg figures obtained on a transmission line installation. The surge record is from 3 p. m. until 10 a. m. of the next day. During this time there were lightning storms in the vicinity of the line. It is clearly seen that these figures have the same characteristics as those produced with laboratory equipment. (The circles are drawn for voltage measurement, see Fig. 17.) The figures on the left and right are interpreted to be from oscillatory surges of highly damped nature, such as shown in Fig 16C which is known to be from an oscillatory surge voltage. The oscillatory nature of these surges is derived from the presence of both positive and negative Lichtenberg figures on both recorders. The middle figure indicates a unidirectional surge voltage of negative polarity such as shown in Fig. 16B.

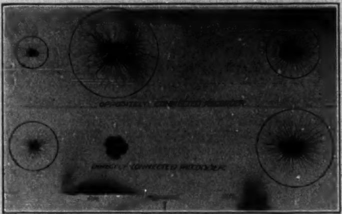

FIG. 27—LICHTENBERG FIGURES OBTAINED ON A TRANSMISSION LINE INSTALLATION DURING A LIGHTNING STORM

It is noted that there is no line-voltage band upon the film. This sometimes occurs and it is thought that this is due to the variation in voltage distribution across the insulator string potentiometer at normal line voltage and frequency.

Fig. 28 shows a photographic record of surge voltages obtained on a 220-kv., three-phase power transmission line, using a surge voltage recorder of the two-recorder type (Fig. 19) with an insulator string potentiometer as described above. The normal maximum value of the voltage to ground is 180 kv. and the multiplying factor of the potentiometer was 60. The record shown is from 11 a. m. on one day to 2 p. m. the day following. The line-voltage bands show when the line voltage was on and off during this period.

The record shows distinctly a high surge voltage at 4:20 p. m. on Friday and the weather reports indicate severe lightning in the vicinity of the installation at this time. The loss of the line-voltage band some thirty minutes before this surge shows that the line was de-energized at 3:50 p. m. A close examination of the original film reveals a surge at 4:03 p. m. but this is not distinguishable from the print. The figure obtained at 4:20 p. m. on the oppositely connected recorder is a positive "slip" (see Fig. 20) and therefore represents a voltage of negative polarity on the instrument of between 25 kv. and 35 kv. Using a potentiometer multiplying factor of 60, this figure represents a surge voltage on the line of from 1500 to 2100 kv. The corresponding figure on the directly connected recorder is predominently negative. Since some positive figure characteristics are discernible on the directly connected recorder, however, the surge must have been oscillatory and of a highly damped nature (see Fig. 16) with a first half-cycle of negative polarity and the second of positive polarity and very much lower voltage.

At 10:30 p. m. on the same day another surge was recorded. A lightning storm was in progress at this time and the line excitation had been removed about fifteen minutes before this surge. Positive figures were obtained on both recorders. The figure on the oppositely connected recorder indicates an initial half-cycle of negative polarity of 780 kv. The figure on the directly connected recorder indicates the second half-cycle to be of positive polarity of 270 kv.

The weather records for Saturday morning show another lightning storm in progress. The surge record reveals two surges, one at 8:11 a. m. and one at 8:18 a. m., the line having been de-energized at 8:11 a. m. From the print, these two surges are not so clearly distinguished, though from the original film the record is clear. The figure obtained at 8:11 a. m. on the directly connected recorder is of positive characteristics and on the oppositely connected recorder of negative characteristics. The line surge was therefore unidirectional and positive in polarity. The figure on the directly connected recorder is a positive "slip" and therefore indicates a line voltage of between 1500 and 2100 kv.

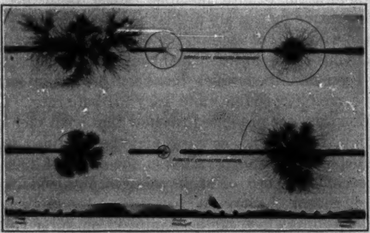

FIG. 28—SPECIMEN RECORD OF SURGE VOLTAGES ON A TRANSMISSION LINE DURING LIGHTNING STORMS

The figure obtained at 8:18 a. m. is positive on the oppositely connected recorder. The instrument voltage corresponding to this figure is 21.5 kv. and this gives a line voltage of 1290 kv. of negative polarity.

This specimen record shows how the figures may overlap on the slowly moving film when the surges occur in quick succession. Even under these conditions, however, it is generally possible to analyze the figures with considerable accuracy when the original film is used and when the figures from the two recorders are available.

Practically all figures obtained on transmission lines have been of the type II class[9], and may be placed, therefore, within the wide range of wave fronts which vary roughly from that of a slow 60-cycle wave to a surge which comes to its maximum value in a fraction of a microsecond's time.

In connection with the surge voltage values obtained from the figures shown on the specimen record (Fig. 28), it is interesting to note that they compare favorably with the laboratory results of insulator flashover tests. The value 1800 kv. for the lightning sparkover of a 14-unit insulator string given by Mr. Peek[10] seems to be close to the upper limit of voltages actually measured on the line by means of the recorders.

SUMMARY

It has been shown that surge voltage recorders using the positive photographic Lichtenberg figures have given essentially the same calibration data under a variety of conditions; also that the accuracy of such an instrument is in the order of 25 per cent, with a somewhat better value possible for those measurements wherein several similar observations may be obtained.

An extension of instrument design has been described wherein two recorders are used together, which allows the use of the positive figure as a voltage measure of all surge voltages, thus insuring greater certainty of result. A more comprehensive analysis of the figure characteristics is also possible, since both positive and negative figures are available.

A means of connecting the surge voltage recorder to a transmission line of higher than instrument voltage has been described which has been proved in service to be simple, reliable, and easy to calibrate. Calibration data are presented to show that with such connection, reasonable accuracy may be obtained in recording voltages up to values in the order of 2000 kv. A specimen record of such voltages obtained in the field is shown.

The records which can be obtained from surge voltage recorder instruments connected as desired along a transmission line will allow the facts regarding surge voltages on transmission lines to be determined with reasonable exactness.

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Discussion

KLYDONOGRAPH SURGE INVESTIGATIONS

(COX, MCAULEY, AND HUGGINS)

TRANSMISSION-LINE VOLTAGE SURGES

(COX)

THE MEASUREMENT OF SURGE VOLTAGES ON TRANSMISSION LINES DUE TO LIGHTNING

(LEE AND FOUST)

NEW YORK, N. Y., FEBRUARY 10, 1927

K. B. McEachron: In discussing the paper by Messrs. Cox, McAuley, and Huggins, and also the paper by Mr. Cox, I wish to raise a question concerning the conclusions. It does not seem to me that the data given support the definite conclusions drawn. For instance, the first conclusion in the Cox, McAuley, and Huggins paper states that the voltages due to lightning are unidirectional, and that the cloud which produces surges is of negative polarity resulting in positive induced voltages and negative direct-stroke voltages. The evidence which we have at the present time supports this conclusion but sufficient evidence to make such a positive statement appears to be lacking.

For instance, consider the direct stroke on a transmission line. Negative records have been obtained due to direct strokes but it does not necessarily follow that whenever a steep-wave negative surge appears on a transmission circuit, it was the result of a direct stroke, nor does it necessarily mean that all direct strokes must be negative. Simpson's theory, which has been mentioned in the paper by Mr. Cox, appears to bear out the conclusions given by the author, but it is necessary to remember that it is a theory, and in the case of lightning phenomena a very large amount of experimental data is necessary to prove that such a theory is correct.

I would like to draw attention particularly to a statement made on the second page of the paper by Messrs. Cox, McAuley, and Huggins. "It has been estimated that gradients as high as 100 kv. per ft. (330 kv. per meter) are reached near the earth's surface." On the same page the statement is made "It is believed that the flashover voltage of 220-kv. transmission-line insulation at the steepness of wave front of lightning surges is comparable to the maximum potential ordinarily induced by lightning." This same statement is also made in the conclusions.

Such a statement seems rather premature. Conditions have been experienced during the summer of 1926 on a 220,000-volt transmission circuit with fourteen disks of line insulation which were sufficient to flash over the line insulation several times. This indicates that 220,000-volt systems are not immune from lightning troubles provided those systems are where the lightning occurs.

Although the line flashovers may have been due to direct strokes, yet the evidence is not clear and both positive and negative surges have been found whose potentials were between 1500 and 2100 kv.

Concerning the matter of attenuation which is given in the paper by Mr. Cox there is a factor in connection with the theory of bound charges which I believe has not been fully recognized.

Assume for instance a long transmission line with a charged cloud one mile in extent over the middle of the transmission line. When this cloud discharges, a voltage appears on the transmission circuit which is equal to the potential gradient multiplied by the height of the line assuming, of course, that the entire cloud discharges in zero time. Underneath the edge of the cloud, the voltage distribution along the conductor will conform to the distribution of potential gradient before the cloud discharges. As a matter of fact, however, the cloud requires time to discharge and therefore the charge will begin to move out along the line having moved a distance equal to the time required for the cloud to discharge. If this time is less than 2½ microsec. the voltage on the transmission line underneath the middle of the cloud will reach an actual potential equal to the potential gradient, times the height of the conductor. A klydonograph located at this point would indicate a voltage of E volts, while another klydonograph located on the transmission line a few miles away would not indicate a maximum voltage of more than $\frac{E}{2}$ volts. This reduction in voltage[1] is due to the fact that all of the energy in the bound charge is electrostatic, while the energy of the traveling wave is half electrostatic and half electromagnetic.

The attenuation to be considered is that which takes place with reference to the traveling wave, rather than between the initial wave and the traveling wave. Measurements which have been made indicating that the voltage drops to a fractional part of its original value when passing over a distance of 15 or 20 mi., are probably due largely to this reduction in voltage which occurs when the bound charge becomes a traveling wave.

In connection with the paper by Messrs. Lee and Foust, considerable work has been done with the calibration of these figures with respect to their appearance, which the authors referred to in their paper. In Fig. 11, the first two positive figures are of the appearance which we have called type 3 figures, and the last two positive figures are of the type 2 class. From the information available it would be possible to predict roughly that these figures were made with a wave of the order of 2 or 4 microsec. Fig. 1 herewith gives the calibration curve[2] which shows roughly how such an estimation of the wave front can be obtained. Locating on this curve a line representing type 2 figures for 15 or 20 kv., and type 3 figures for 5 and 10 kv., the curved line with the arrows shown on the figure is obtained. This line indicates a wave front of between two and three microsec. to reach crest value.

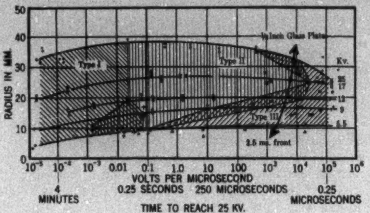

FIG. 1

The statement is made that practically all figures obtained on transmission lines have been of the type 2 class and therefore come within the wide range of wave fronts which vary roughly from that of a slow 60-cycle wave to a surge which comes to its maximum in a fraction of a microsec. time. Referring again to the calibration curve given, the most abrupt wave front giving a type 2 figure was of the order of 1 microsec. to reach its crest value and this only for one particular voltage, namely 17 kv. as measured on the instrument. Thus the evidence indicates that for the waves measured, at least surges of more abrupt front than 1 microsec. did not occur and most of them were considerably slower than this.

A. H. Schirmer: Mr. Cox's paper, as well as the paper by Messrs. Cox, McAuley, and Huggins, presents a number of interesting conclusions, which in general are substantiated by the results obtained from a number of tests made on telephone lines during the past summer. These tests were made on a 5-mi. open-wire line carrying eight wires. Approximately twenty recorders were in use during the major portion of the lightning season.

In no case did we obtain record of more than two surges during a single storm, and usually only one surge per storm was recorded. It should be noted that in our case the klydonographs were connected directly to the line, there being no line voltages to contend with, so that when only one surge was recorded, it means that during that storm no voltage in excess of 2000 volts was obtained on the line. The results also show that the surges produced by lightning are either unidirectional or highly damped oscillatory surges. Our data also agree with the conclusion given in the papers that lightning arresters do not protect the line against flashovers, even at relatively short distances from the arrester. For example, in one case six recorders were placed on a line wire within 1000 ft. of the protector. The first recorder, located at the arrester, showed no evidence of potential. The second recorder, located only 66 ft. from the protector, indicated a voltage of approximately 8000. The third recorder, located 224 ft. from the protector, showed voltage above the range of the instrument, and the fifth recorder, located 700 ft. from the protector, flashed over between the electrode and the ground plate.

Our data, however, do not support the conclusion that the clouds which produced surges are always of negative polarity, resulting in positive induced voltages or negative direct-stroke voltages. None of the telephone wires under test was directly hit by lightning during the period of observation. Nevertheless, extremely high surges of both positive and negative polarity were recorded. Both the positive and negative surges were of relatively steep wave fronts, which also does not support the conclusion that the surges from positive lightning strokes are slow, of the order of 0.01 sec. A summary of our results shows that five storms produced positive surges and three storms produced negative surges, while three other storms showed positive and negative surges. These last surges we have attributed to discharges between clouds, rather than between cloud and earth. Even in these cases, voltages induced were well above the range of the instrument.

As pointed out before, many high-voltage surges were obtained on the telephone line. However, these surges were of extremely short duration, and in no case, even where the recorders flashed over only a short distance from the protectors, did these surges cause any damage to telephone cables, or permanently ground telephone arresters, as usually occurs from lower voltages imposed on telephone lines either by induction from, or momentary accidental contact with, electric light and power circuits. While in the latter cases the voltage may be applied for only 0.1 sec., still, this time is so much longer than the period of application of the lightning potentials that the effects are more severe even though the maximum voltages reached are substantially smaller than the lightning voltages.

Fig. 8, in the paper by Messrs. Cox, McAuley, and Huggins, gives a diagram of connections for lightning-arrester tests. This diagram shows the klydonograph connected to the same ground as the lightning arresters. With this connection, the klydonograph merely records the drop in potential across the lightning arrester, and not the potential of the line to ground. During heavy discharges the potential of the protector ground connection may be many volts above true earth. In order to obtain a record of the effectiveness of protection at a given location, it is necessary to connect the ground plate of the klydonograph to an independent ground.

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[1] Time Voltage and Current Characteristics of Lightning Arresters, K. B. McEachron, *G. E. Review*, Vol. 29, p. 678, 1926.

[2] This curve is taken from "Measurement of Transients by Lichtenberg Figures" A. I. E. E. TRANS., 1926, V. 45, p. 712.

R. W. Atkinson: Messrs. Cox, McAuley, and Huggins in their paper state that transient voltages are a rare and unimportant cause of cable failure. These may of course be comparatively rare without being comparatively unimportant. Even a few failures per hundred miles per year for a given cause are evidently a matter of great concern to operating companies.

The writer shares what he believes is a general agreement in the industry with the opinion expressed by these authors, that transient voltages are not a major cause of cable failure. However, both of these papers throw a great deal of light on the fact, which has been known that a certain number of cable failures have resulted from transient voltages, particularly from unusual transient voltages. For instance, it is shown in these papers, that certain types of transients are of relatively low frequency so that they travel with substantially sustained peaks throughout the system. It is also pointed out that some of these are of the same character and thus materially more harmful than a single impulse. Probably the most direct application of this is in connection with the testing after installation with alternating current which fortunately is now becoming a thing of the past.

Thus, while a transient voltage of a given number of times the operating voltage may be only slightly or not at all destructive, a transient of the same number of times a test voltage may be very destructive.

There is an effect of even comparatively small over-voltage transients which is usually overlooked, but which, as has been pointed out previously, may well be of importance. The shape of the curve of power factor versus voltage of a cable is different depending upon the direction of the cycle, that is depending upon whether the voltage is being raised or lowered. To whatever extent this change of power factor with voltage is a measure of ionization of air spaces within the cable, this simply means that the amount of ionization at a given voltage depends on the previous history, and is greater if the cable has been subjected previously to a higher voltage. The action of a transient voltage has been likened to a trigger, which releases the energy from the normal operating voltage in such a way that its effect may be as severe as would be a steadily maintained operating voltage of considerably higher magnitude.

Published records show not only that there are individual instances where cable failures have been directly traceable to transient voltages but information is published in at least one instance concerning a system where a change with respect to the amount of transient voltages existing, produced an important effect on the number of cable failures experienced.

I wish to emphasize that transient voltages are important in connection with cable operation, and that it will be very valuable to continue to obtain information as to the transient conditions in various systems so that these may be evaluated in terms of what might be termed an equivalent steady operating voltage. Where a system is exposed to such transient voltages, it is obviously just as important to make the apparatus on it capable of meeting any transients which cannot be prevented, as it is important to have the apparatus and cable meet the steady operating condition.

Yet one more word as to a specific instance where the data shown by these authors explain that cable may be subjected to damaging transient voltages. If a cable insulated for relatively low voltage is connected to an overhead line insulated for considerably higher voltage, it is obvious that lightning potentials sustained by the cable may in this case be especially destructive.

F. W. Peek: The klydonograph is of special interest to me because it affords further means of checking conclusions regarding lightning which I arrived at several years ago. I am going to tell you a little about my work to show how well it is in agreement in the most important respects with the results of Messrs. Cox, McAuley, Huggins, and Lee and Foust.

Several years ago I undertook to determine, if possible, the order of the voltage and other characteristics of lightning, the gradients produced, etc.; the magnitude and character of lightning on transmission lines; the strength of apparatus and line insulators when subjected to lightning voltages; and the value of the ground wire and other protective apparatus. With such information it is possible to make lines that are immune from lightning. Whether or not immunity is secured is, to a great extent, an economic problem. The results of my investigation have already been given to the Institute.[3,4] As far as I know it was the first time that estimates of the actual values of lightning voltages on transmission lines were ever made. The rules for pre-determining lightning voltages and the lightning strength of insulators, etc. are quite simple. I will give a brief summary of my conclusions.

The voltage of a lightning flash is of the order of 100,000,000, the current 80,000 amperes, and the energy 4 kw-hr. The maximum gradient is 100 kv./ft. (330 kv./m.). The discharges are usually non-oscillatory and some times take place in a few microseconds.

Lightning disturbances on transmission lines are generally steep waves of a few microsec. duration though the lower voltage disturbances may be of larger duration. The induced voltage increases directly with the height of the conductors. It is found by multiplying the apparent gradient by the height of the line, but is limited by the lightning flash-over voltage of the insulator. Thus $V = g \alpha h = Gh$. When h is the height in feet, g is the gradient in volts, and α is a factor depending upon how rapidly the cloud discharges. G is the apparent gradient. The maximum possible gradient is 100,000 volts/ft. (330,000 volts/meter). This value can usually apply only in case of a direct stroke. In practise, because of the time required to discharge the cloud, G is usually less than 50,000. A voltage wave is reduced in a few miles by corona losses. After the wave is well under way the voltage is also reduced to one-half because part of the energy becomes magnetic. The lightning flash-over voltage of insulators and the strength of insulation is always higher than the 60-cycle values. An insulator flash-over curve made by artificial lightning has been checked to 1,800,000 volts by natural lightning with good agreement.

The ground wire, by cutting lightning voltages in half, has the effect of increasing the line insulation and at the same time reducing the stress on apparatus. It must be properly installed.

If a line is over-insulated, apparatus failures may result if protective gaps or arresters are not used.

Reasons that more trouble is not experienced are that most high voltages originate at some distance from the station, and a large number of induced voltages are from slowly discharging clouds.

By comparing the lightning strength of apparatus and insulators with possible lightning voltages the probability of failures and outages can be estimated. Whether a line is made immune or not is a question of economics.

It can be seen that my conclusions agree, in most respects, very well with the conclusions arrived at in these papers.

I was particularly interested in the value of 1800 kv. given by Lee and Foust for the lightning flashover voltage of a 14-unit insulator string as measured by klydonographs on an actual line. Estimates were made of the maximum possible and usual highest lightning voltages for such a line, and are given in Table I in my paper[6]. The insulators arc-over values are given as 1800. For conductors 40 ft. high the maximum possible voltage is given

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[3] F. W. Peek, Jr., *Lightning and Other Transients on Transmission Lines*, TRANSACTIONS, A. I. E. E., 1924, Vol. 43, p. 1205. Lightning, *Journal Franklin Institute*, Feb. 1925.

[4] F. W. Peek, Jr., *Lightning (A Study of Lightning Rods and Cages with Special Reference to the Protection of Oil Tanks)*, A. I. E. E. TRANS., 1926, p. 1131.

Either of these values is high enough to cause insulation flashover. The maximum voltage would be limited to 1800 kv. by the insulator. This is the value actually measured by Lee and Foust. The values in Table I were made long before such a line had been built. Table I also shows the effectiveness of a ground wire on such a line.

H. B. Vincent: I want to ask a few questions of Messrs. Cox, McAuley, and Huggins on their paper. In the data submitted were the lines in all cases energized, or were there any as 4000 kv. and the usual highest 2000 kv. without ground wires cases where the records were taken of the lightning voltage while the line was not energized?

There is no mention made of any so-called control devices in connection with the insulators, which might or might not affect the flashovers? Referring to their Table I, in case No. 2, which is a 140-kv. line, they recorded 2 flashovers with a voltage over 10 times normal, and on another line of the same kv. they had 15. Now, did the first line only have 2 and not 15 because there were fewer lightning storms, or were the insulators installed with some control device which presumably decreased the possibility of flashovers? If such records were kept, I am wondering whether it would throw any further light on the question of flashovers. I am not questioning the value of the klydonograph, which is beautifully shown and recorded. But, inasmuch as the cause is tied in with the effect (the effect being the flashover of the insulators), I simply question whether anything has been lost sight of in not mentioning the matter of control devices.

The operating man is also interested in the amount of damage that is done, as much as he is in whether the voltage is ten times or twenty times normal. I am wondering whether the data could not be amplified to show the effect a little more clearly, in addition to the cause.

R. G. Hooke: The Public Service Electric and Gas Company of New Jersey, with which I happen to be associated, have been using the klydonograph for nearly three years, and I want to amplify a little the data that have been given; particularly that in the paper by Messrs. Cox, McAuley, and Huggins. In their Table V, they give the relation between causes of surges and magnitudes.

I compared this roughly with our own records, because it is given as the average that might be expected on any one system, and I wanted to see how nearly our system lined up with that average. I found that the percentages due to different causes were rather interesting in that we had only about half as many surges resulting from switching at the klydonograph station, whereas we had twice as many as they give caused by switching at other points in the system. From that table also it appeared that we had an unusually high percentage of surges of the larger magnitudes. This is brought out particularly, perhaps, in their Table VI.

I would call attention to their remarks that, "On one particular system the surges identified with short circuits were considerably higher than the average. For instance, this system accounted for 26 of the 32 oscillatory surges of 2.6 times normal voltage and over. Its only apparent difference from the other systems, all of which were grounded solidly or through a low resistance, is the presence of a 75-ohm resistor in the neutral ground." They also say that "The highest voltage recorded was 4.6 times normal. Only ten surges or 0.4 per cent of the total were over four times normal. Of these, six were on one cable-and-open-wire system, where certain contributory operating conditions prevailed."

The system referred to, happens to be that operated by Public Service Electric and Gas Company in what we call our Southern Division. We have in Trenton approximately 10 mi. of 26-kv. cable, connected through about 32 mi. of open wire, to a cable network in Camden. In the latter city there are about 20 more mi. of 26-kv. cable, including ties with the Philadelphia Electric Company. I am going to give the details of two or three of the higher surges in this area because I think they are of particular interest.

We have recorded about 500 surges there in two years; 4.6 per cent of these have been between three and four times normal and 1.4 per cent have been above four times normal. The highest was 4.7 times normal. That highest surge was caused by a cable failure in Trenton. A report was received at our office that a cable was smoking. We sent a man out to investigate, but before he could get there the cable failed. Simultaneously, we lost a cable in Camden not far from the klydonograph location. The disturbance, therefore, originated in Trenton and traveled some 40 mi. to cause trouble at the other end of the system. This, however, is the only case in our records of a failure which may have resulted from a high surge. On the contrary, with one exception, all of the higher over-potentials appear to have been the result of cable breakdowns rather than their cause. The basis for this statement is the fact that insulation failures frequently are not accompanied by surges, but the surges, when they do occur, are always coincident with faults. This means that if we eliminate the cable faults, we shall practically eliminate the high voltages, as is brought out in the paper.

The one exception that I mentioned is interesting. It was a surge of 4.6 times normal and was the result of an accidental relay operation. A line was tripped out which happened to be carrying a fairly heavy load, together with the synchronizing power between our system and Philadelphia. We have no satisfactory explanation for the resulting surge. Similar switching operations are quite frequently performed. We often separate from Philadelphia intentionally and we never have had any such occurrence on any other occasion.

We have also had a klydonograph located in another transmission system containing something like 135 mi. of open wire and 51 mi. of cable. This system was operated for a good many months with a solidly grounded neutral, but it has now been in service for about six or eight months with a 75-ohm resistance in the neutral. At the time we changed the operating condition, we made some fairly extensive ground tests with different neutral resistances; we tried 75 ohms, 150 ohms, and 300 ohms, throwing grounds directly on the system. The object of these tests was not to see what surges we would get, but to try out the functioning of our relays. The surges which occurred, however, are very interesting.

While operating with a solidly grounded neutral, three transients of above four times normal were recorded due to the deenergizing of low-capacity equipment in the substation where the klydonograph was located. Surges of this nature are mentioned by the authors and since they are of very short duration, they are not considered to be important. They are, therefore, excluded from the following table which gives the results obtained with the different neutral resistances. Southern Division data are included for comparison.

	Central Div.				Southern Div.
Neutral resistance, ohms....	0	75	150	300	75
Number of surges recorded..	240	115	9	15	498
Per cent from 1 to 1.9 times normal voltage..........	84.6	75.6	55.5	46.7	84.5
Per cent from 2 to 2.9 times normal..................	15.4	21.8	44.5	46.7	9.5
Per cent from 3 to 3.9 times normal..................	0	2.6	0	6.7	4.6
Per cent from 4 to 4.9 times normal..................	0	0	0	0	1.4

These figures indicate very clearly that the higher the resistance in the neutral, the greater will be the percentage of surges of the higher magnitudes. Under no conditions, however, have transients in excess of four times normal occurred in the Central Division.

Comparing this with the data obtained in the Southern Division, as indicated in the table, it appears that nearly twice as high a percentage of surges between three and four times normal have been recorded in Camden on the 75-ohm resistor, and that seven, or 1.4 per cent of the total, have been above four times normal, whereas there was none of this magnitude in the Central Division. Obviously, there is some difference between these two transmission systems and the high surges on one cannot be explained on the basis of the neutral resistor. It happens that at numerous points in the southern part of New Jersey, the soil is dry and sandy and we find it difficult to obtain low-resistance grounds at our stations and substations. Since the high surges which have occurred seem, in general, to be due to faults in certain areas, while faults in other parts of the system cause much less voltage disturbance, we are of the opinion that the ground resistance at the point of fault is of particular importance. In fact, it is of more importance than the resistance which we find it necessary to use in the neutral in order to keep on friendly terms with the Telephone company whose lines parallel certain of our transmission circuits.

To summarize, then, with different neutral resistances, we have found that the higher the resistance the more surges might be expected of magnitudes of between two and three times normal, but there is very little likelihood of any transients of above four times normal occurring even with 300 ohms in the ground connection unless there are other causes either in the constants of the system or in the ground resistance at the point of fault. In connection with this general discussion, it is of interest that in one case an arcing ground in a cable joint caused surges of less than 2 times normal for a period of about 30 hr., the figures being obtained intermittently until complete failure finally developed.

In studying our particular problem, namely the high surges which occur in our Southern Division, we are now planning to install three or four more klydonographs in the area, attempting to find out whence the surges come and whither they travel. We have had only one instrument in that whole system thus far. We have, however, carried on some cooperative work with the Philadelphia Electric Company. They installed a klydonograph on their side of the Delaware River about 4½ mi. from our klydonograph station. We found that for a surge to be recorded simultaneously at both places it must be over two times normal on one side or the other. Attenuation amounted to about 45 per cent in the 4½-mi. cable tie. We think that is very interesting because with the klydonograph at one point if we get such an attenuation we don't know very much about what the surges may be elsewhere. However, in spite of the surges which we have—and I think they have been as serious as any company has experienced—we tend to disagree with those who charge cable breakdowns to high-voltage transients. We feel that the cable which we are now able to buy, in view of the voltages which it stands on test, should be almost entirely unaffected by surges up to four or even five times normal. I don't see any way to draw definite conclusions on this point, but it would be very interesting if some manufacturing company could conduct tests to determine the effect of transient voltages on cable insulation.

J. H. Cox: In the development of the klydonograph, we found the probable error of a record to be about ± 15 per cent, with a possible error of about 30 per cent. It is encouraging to note that the Messrs. Lee and Foust check this value.

There are two principal points in which the authors' procedure has differed from ours. They have used two oppositely connected electrodes for each connection, and they have used a different form of potentiometer for connection to lines. The use of two oppositely connected electrodes has certain advantages as pointed out by the authors and it is to be commended under certain conditions. Its principal advantage is that it gives a positive measure of the maximum voltage when the potential is a damped oscillation and the initial impulse is negative. There is also some advantage in having a positive figure for a negative surge, but it must not be inferred that the negative figures are entirely useless. The hiding of the small negative figures by the normal voltage band in the single-element instrument, can be prevented, if desired, by setting the potentiometers to eliminate the normal voltage band. In any case, negative records above 2½ times normal will be detected and it is questionable if surges on transmission lines below this value are very important. The advantages of the double connected arrangement are gained at the sacrifice of certain others. The instrument has a higher electrostatic capacity and hence imposes a greater burden on the necessary electrostatic potentiometer, which at best has a low capacity itself. It makes a more expensive arrangement both in installation and in operation as it eliminates the possibility of the multi-electrode instrument. With the inherent inaccuracy of spring clocks, having the records of three terminals on one film makes the time record much more complete. A double-electrode arrangement was considered when the first film-type klydonograph was designed, but it was discarded at that time in favor of our present arrangement. However, the method to be preferred for a particular test is determined by individual opinion and the nature of the test.

While the authors have gone to greater refinement in the instrument they have chosen the opposite trend in the potentiometers used. The use of a string of suspension insulators as a potentiometer was suggested for approximate work by Cox and Legg in their paper on page 869 in the 1925 TRANSACTIONS, A. I. E. E. When the klydonograph was first developed this form of potentiometer was investigated by Mr. Peters and discarded as unsatisfactory. The scheme works well as long as the surroundings remain constant, and as long as the insulators are dry, or wet and clean, but when the insulators become slightly dirty and get wet, as is inevitable in service, the leakage conductance disturbs the ratio. This can be readily seen by a comparison of the electrostatic capacities involved. The capacity of the singly connected film-type klydonograph is about 8 micro-microfarads per terminal. I have been told that the capacity of the doubly connected klydonograph is 23 micro-microfarads. Its greater capacity is due to the fact that it has two terminals and a ground sheet connected to one of them. The capacity of a single 10-in. insulator disk is about 25 micro-microfarads, which value is divided by the number of insulators in series. It is easy to see that a variable leakage path in parallel with a capacity which is smaller than the connected instrument will seriously disturb the ratio. As mentioned by the authors, this disturbance should be less for impulsive applications than for the normal frequency.

The authors mention the effect of a change in the lengths of the leads used. This is inevitable where such small capacities are involved. In an actual test it is often difficult to keep the lengths of the leads down to 5 ft. Altogether, the most desirable form of potentiometer is that having the highest capacity. At best this is none too high. The capacity of a 6 ft. ring of 2-in. iron pipe mounted 12 in. above a ground plate is from 100 to 200 micro-microfarads.

We have recently made use of condenser bushings as potentiometers. The klydonograph is connected from one of the condenser steps to ground. This scheme gives excellent results. The arrangement is not subject to as many variable leakage paths as the others. Furthermore these paths are not as important since the capacity across which the klydonograph is connected is from 250 to 800 micro-microfarads. No calibration in the field is necessary as the bushing can be designed for a definite ratio and this ratio remains fixed.

H. L. Wallau: (communicated after adjournment) The following data as to the magnitude of voltage surges experienced when switching high-voltage (66-kv.) cable circuits may be of interest.

The circuit was as follows:

No. 1 Breaker—Connecting 11-kv. generating plant bus to 30,000-kv-a. transformer bank stepping up to 66 kv.

No. 2 Breaker—Connecting high-tension side of transformer bank to 66-kv. cable circuit.

No. 3 Breaker—Connecting 66-kv. cable to low-tension side of 66-kv./132-kv. transformer bank. Length of cable circuit 8¼ mi. 3 single-conductor 500,000 cir. mil. cables 30/32-in. paper insulation 9/64 lead, spaced vertically 6½-in. centers.

No. 4 Breaker—Connecting high-tension side of bank to 132-kv. 4/0 three-phase circuit 47 mi. long.

Tests were made at no load.

The magnitudes of the over-voltage surges were obtained from oscillograph records.

CLOSING BREAKERS

C, indicates breaker closed before test
O, indicates breaker open before test
X indicates breaker closing under test

Breaker number				Max. over-voltages on 66-kv. cable, per cent
1	2	3	4	
X	O	....	....	0.0
X	C	C	C	15.6
X	C	O	....	65.0
X	C	C	O	73.5
C	X	C	C	25.3
C	X	C	O	60.0
C	X	O	....	62.3
C	X	C	O	123.5
C	C	X	O	0.0*
C	C	X	C	25.9*
C	C	X	C	42.5
C	C	C	X	10.0

*66-kv. neutral ground open at far end.

OPENING BREAKERS

C, indicates breaker closed before test
O, indicates breaker open before test
X indicates breaker opening under test

Breaker number				Maximum over-over-voltages on 66-kv. cable, per cent
1	2	3	4	
X	O	....	....	0.0
X	C	C	C	0.0
X	C	C	O	6.1
X	C	O	O	71.4
C	X	C	C	50.7
C	X	C	O	63.0
C	X	C	O	74.0*
C	X	O	O	105.0
C	C	X	O	0*
C	C	X	C	9.5
C	C	X	C	17.5*
C	C	C	X	37.0

*66-kv. neutral ground open at far end.

The maximum over-voltages obtained were for both closing and opening operations when the first 66-kv. breaker (No. 2) was actuated. For the closing operation the highest surge obtained when the 132-kv. bank had been previously closed on the cable circuit through the No. 3 breaker, thus simultaneously energizing both the cable circuit and the transformer bank. For the opening operation the next highest surge resulted when deenergizing the cable circuit only.

A. L. Atherton: (communicated after adjournment) We who deal with lightning arresters have what is perhaps the most indefinite problem in the electrical industry today. Although it is recognized that the results secured thus far do not cover all conditions and are not extensive enough to justify the drawing of final conclusions, there are several points of vital importance to the lightning-arrester question which warrant notice.

(a) First of these is the amazingly small number of times a lightning arrester is called upon to operate in a season. In the old days, when electrolytic lightning arresters were generally used, the manufacturers' recommendations for setting of the series gaps were rather indefinite, it being left to the user to adjust the gaps so that the arrester operated frequently but not too frequently. It was general practise to set the gaps so that the arresters operated quite often. The customer had invested some money and wished to have some visible evidence that he had bought something. It was not infrequently the case that these arresters would operate many times in a single storm and possibly several hundred times in a year. Observe now the date of Table IV, keeping in mind that voltages less than double normal line-to-line voltage, which corresponds to 3½ times normal voltage to ground, are not in the dangerous class and do not require an arrester operation. It is to be presumed that the only times when the arrester would be required to operate, would be for those voltage values greater than 3.5 times normal for which a flashover did not occur along the line. The total number of these is less than two per year per location. If we include those cases when the line insulators flash over, the total number of voltages in excess of 3.5 times normal is less than 6 per year per location. Even including all cases of flashover, to consider the condition of line insulation so greatly increased beyond present practise as to eliminate flashover, the total is less than 18 per year per location. This is a very startling reduction below the previous conception of how many times an arrester should operate in a season.

These figures are averages for all voltages and the weight of data lies at the higher voltages. The variation with voltage will be touched on later.

(b) Along this same line is another point almost equally startling. For years we have considered that it is desirable to provide as short a path as possible between lines to take care of switching transients. It appears clear from the information available here that switching transients are practically negligible. Wherever the length of line involved is appreciable, the voltage is low, and in general the voltages in excess of the value 3½ times normal voltage to ground for which an arrester operation might appear to be required, occur on very short lines and therefore are practically negligible because of short duration. The function of a lightning arrester is definitely the protection against lightning voltages.

(c) All of this arouses renewed interest in the time-honored question as to whether arresters are required on the higher voltage systems, and if not, where the dividing line can be placed. From the data given, it is clearly evident that voltages dangerous to the insulation of any apparatus at present made or contemplated for commercial use may readily be induced in the line conductors, providing the circuit is located in a territory where lightning conditions are at all severe. It appears that the magnitude of voltage which reaches a station is determined in general by the flashover value of line insulation. Insulators flash over even on lines for the very highest voltages. One way to look at this question then is that the need for lightning arresters is dependent on the ratio between insulation strength of the apparatus and flashover of the line insulators both presumably under the same conditions of transient voltage. Considered on this basis, the line insulators probably offer the same order of protection to terminal apparatus throughout the voltage range and, if this line of thought is correct and complete, lightning arresters are as necessary at the higher voltages as at the lower.

To get a further idea as to the relative need for protection in the various voltage classes, and to try to take into account the number of overvoltages per year which was neglected in the line of thought we just followed, we may total the data of Table I by voltage classes as follows:

System		Surges per station per year		
		Without flashover 3.5 times normal and above	With and without flashover, 3.5 times normal and over	Without flashover, 3.5 times normal and above plus all with flashover
No.	Voltage			
26 and 27	6.6–13.2	0	6	29
20 to 25	22–23	5	8	23
9 to 19	44–66	2	8	18
1 to 8	100–220	1	3	13

In this summary, fractional values are given as the next higher whole number.

There may be indications of a slight trend toward higher figures for the lower voltages, but there is no indication that the figures are of a different order of magnitude. On this basis, the need for arresters seems to be about the same for all voltages, neglecting any differences there may be in factors of safety of apparatus insulation and relative importance of serial continuity.

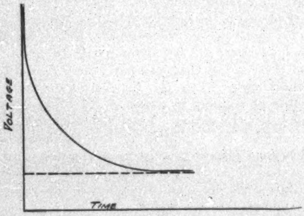

FIG. 2

Assuming, for the moment, that the need is equal, we must not take this to mean that the justification for the use of present-day arresters is equal for all voltages. Justification for use still depends on economics and the cost per kv. of present-day arresters increases very rapidly with voltage in the higher ranges.

To get the correct idea from these thoughts, we must keep in mind that experience with and without lightning arresters at the middle voltage classes, 33 or 25 kv. and down, has clearly demonstrated that good service cannot be rendered without lightning arresters if the circuit is in a location where lightning is prevalent.

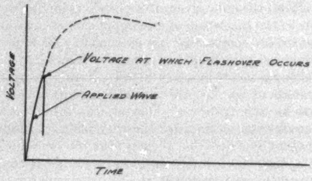

FIG. 3

We must also keep in mind that we cannot draw definite and final conclusions at this time. The data are not nearly complete enough to eliminate the possibility of large errors. It is interesting however to note the trend of opinion as the information accumulates.

(d) One further point in this connection bearing particularly on the question of line insulator flashover and the measurement of transient voltages which I should like to stress is the rather general present practise of referring to "Impulse Flashover Value" for insulators or insulation. It seems to me that this is a very dangerous practise. When we say impulse flashover voltage, we give the definite impression that there is one such voltage value for any particular piece of insulation. This is not the case. The impulse flashover voltage may be anything between the 60-cycle flashover voltage value and some value very close to infinite dependent on the duration of the application. If we apply a million volts to a path, and flashover occurs after one microsecond, it is evident that we can prevent flashover not only by reducing the voltage value but also by reducing the time. A curve something like that shown in the accompanying Fig. 2, doubtless exists. It is difficult to see how we can justify speaking of such a curve by one voltage value and it is very easy to see how such a loose practise may get us into misconception and future trouble.

For example, if we determine the "impulse flashover voltage" for a string of insulators by applying a rapidly increasing voltage and measuring the voltage at the flashover by means of a sphere gap or klydonograph, we measure the voltage value as indicated in Fig. 3, herewith. We get the same kind of value as a maximum by klydonograph tests in service at the time of flashover. When the klydonograph says 10 times normal voltage for a flashover, the induced voltage may have been on the way to 100 times normal when the flashover stopped it. If we refer to this kind of value as the "impulse flashover voltage," we give the impression that lower voltage values will not cause flashover. Suppose, however, that a transient voltage appears which never reaches this crest value but persists for some microseconds. It is entirely possible that such a transient will cause flashover, and if transient voltages on transmission circuits may be of appreciable duration, such as 10 to 20 microseconds, as is likely, this voltage value may be very much below "the impulse flashover voltage" above referred to. It is not unlikely that some of the flashovers with relatively low recorded voltages may be explained in this way.

There is a growing interest in the proper proportioning of line insulation, apparatus insulation, and protective equipment to make the design of systems consistent as a whole. If we base such system design on a single voltage value, as the tendency now is, we are liable to be in error. Even if we could assume that a test, as in Fig. 3, gives one point of the curve of Fig. 2, we would still have no assurance that this curve would have the same shape for various kinds of insulation, and when we are dealing with such widely different things as air flashover around insulators and dielectric puncture under oil in transformers, the difference is not likely to be negligible.

The manufacturers have been asked to give impulse flashover voltages for various kinds of apparatus and line insulation, particularly when applications are to be made on the very high-voltage systems in lightning territories. Such statements are meaningless and can only lead to misunderstandings and disagreements. Let us learn from past engineering experience that inaccuracies of speech are dangerous and adopt precise terms in this matter as we have had to in others. If we must specify a value for impulse flashover voltages, let us specify also the duration of this voltage or better yet, a curve of time against voltage.

V. E. Goodwin: (communicated after adjournment) I fully agree with the comments of Mr. Atherton and wish to emphasize the importance of a proper understanding of the term "impulse flashover" or "impulse failure" of apparatus. It is, for instance, common practise to speak of insulators as having a definite flashover or of the impulse strength of an insulated structure such as a transformer. The flashover of an insulator may be taken as illustrative, although this discussion applies equally well to the puncture of insulation or to other apparatus subjected to the action of transient phenomena.

Whether or not a given insulator will flash over depends on both the voltage and the time of application. The lower the voltage applied, the longer the time of application to cause failure. This means that the body of the wave is important since the flashover may occur after the wave front has passed. On the other hand, if the voltage rises to a high enough value the insulator will flash over on the wave front, thus preventing further rise in voltage.

When stating that a piece of apparatus has a certain impulse flashover, it is necessary to specify the wave used in determining this value and state whether the arcover occurs on the front, or if after the front has passed how long a time elapsed before the arcover took place.

The terms "time lag" and "impulse ratio" are open to the same criticism as the term "impulse strength." Time lag as ordinarily understood is an indefinite term since the wave is not specified and the lapsed time may involve a simultaneous voltage increase, as in the front of the wave or it may be a constant or decrease in voltage as in the body or tail of the wave.

One method of measuring time lags is to apply various overvoltages above that which this specimen will withstand continuously, and determine the times required in each case to cause breakdown. For this method to be satisfactory, the potential must rise to its maximum value in a time short compared to that required to cause breakdown. Until the cathode-ray oscillograph became available, this method could not be used for it was not possible to determine the time relations satisfactorily.

The other and more common method for measuring time lag is to determine the time required for the breakdown to occur on a steep wave front after the potential has reached a value which it could withstand if continuously applied. With this method, the exact wave front must be known and several points taken. Comparison of curves of these two methods for determining time lag will show quite different results.

The term "impulse ratio" does not have a definite value unless the impulse applied to the test specimen is prescribed and it is known that the breakdown occurred on the wave front and not after the voltage has reached its maximum value.

The terms "impulse strength, time lag, and impulse ratio" have come to have a variety of meanings and are terms which the Institute might well define to prevent confusion.

H. H. Plumb: (communicated after adjournment) Mr. Cox has brought out some interesting data in his first paragraph under *Experimental Data*, applying to lightning discharges. His deductions appear to be somewhat in conflict with other conclusions in the paper, and I wish to submit a different deduction which will harmonize the conclusions.

That there are surprisingly few discharges indicated by the klydonograph during a thunder storm, and the figures showing positive, supports the deduction that only negatively charged clouds discharge with sufficient severity and swiftness to make a record on the klydonograph. The few negative Lichtenberg figures recorded may have been direct strokes in every case. The evidence thus analyzed shows that in a thunder storm, many discharges take place, from both positively and negatively charged clouds; the discharges from positive clouds are relatively slow and fail to register on the klydonograph; the negatively charged clouds are discharged swiftly and with severity, and these usually register, with a positive Lichtenberg figure if by induced or bound charge, but negative if by a direct hit on the line. This view is in complete accord with Dr. Simpson's theory and Dr. Norinder's results.

J. H. Cox and P. H. McAuley: Mr. McEachron has objected to certain of our conclusions as somewhat premature with present available data. It should be obvious that the inherent variations in the physical conditions under which lightning occurs are reflected in any data obtained, and that the reliability of the conclusions is relative to the amount of data from which they are drawn. Moreover, deductions from data of this nature are invariably influenced, to a certain extent, by individual experience. Several of the discussions illustrate this point. To make a paper complete, however, results must be summarized and conclusions indicated. As Mr. Peek has pointed out, the magnitude of an induced surge is influenced by the rate of discharge of the cloud and is less than the product $G\ h$. Thus, although gradients as high as 100 kv. per ft. (330 kv. per meter) are reached near the earth's surface, it is entirely possible that surges high enough to flash over 220-kv. insulation seldom are *induced*. Furthermore, these higher gradients are present only in the vicinity of the lightning stroke. The division of an instantaneously released charge into two traveling waves, each with a surge voltage of one-half that of the initial voltage, is one of the most elementary laws of wave propagation and was not neglected by the authors. The reduction referred to was from more than 1000 kv. to about 150 kv. This indicates a rapid attenuation in spite of a maximum initial reduction factor of one-half.

Mr. Schirmer has reported some results which appear to conflict with Simpson's theory of lightning. These, however, are not so inconsistent when it is remembered that the voltages measured on telephone lines are low compared to those on power lines. Mr. Schirmer has some interesting data which show that the drop across the protector ground on telephone circuits is relatively high. The drop across power-line lightning-arrester grounds has not been investigated, but tests on this are now being started.

Mr. Vincent has asked some questions regarding test conditions which may have affected the results. In some cases the lines were not energized during lightning storms. This did not appear to influence the results very greatly. The only influences of having the line energized are that terminal conditions are different, and that the net surge voltage to ground becomes the sum of the instantaneous applied voltage and the surge potential. Lines 2 and 3 in Table I were of similar construction, neither being equipped with control devices. The difference in the number of surges over ten times normal was due to difference in the lightning encountered. The only claims made for flashover control devices are that they increase the flashover voltage of the line, and therefore higher surge voltages should be recorded on lines so equipped. In our experience with the klydonograph, however, we were unable to detect this difference.

Mr. Plumb has made certain comments regarding the first paragraph under *Experimental Data* in the paper *Transmission Line Voltage Surges*. This paragraph merely includes a number of statements regarding the data recorded and what these indicate at first sight. The conclusions stated by Mr. Plumb are discussed in the remainder of Part II of the paper. For instance, it is explained that positively charged clouds discharge too slowly to cause surges on transmission lines and that the surges experienced, both induced and direct, are caused by negatively charged clouds.

E. S. Lee and C. M. Foust: Several phases of the work on Lichtenberg figure measurements of surge voltages have been touched upon during the discussion, particularly by Mr. Cox, and we wish to add a few remarks relating to the points raised.

Regarding the use of the positive Lichtenberg figures to record all surges whether of positive or negative polarity, it is felt that this practise is advantageous for three reasons:

1. Because negative surge voltages up to approximately 2.5 times normal give negative Lichtenberg figures upon a directly connected recorder which are entirely obscured by the normal line-voltage band, and the interpretation of negative figures somewhat above this value may be uncertain.

2. Because negative Lichtenberg figure sizes are dependent upon the rate of voltage rise to a much greater degree than the positive figures.

3. Because the availability of both figures for all high-voltage

surges permits a more accurate determination of the nature of the surge producing the figures.

We do not consider the limitation of the two-recorder type of instrument to a single-phase instrument to be a disadvantage, since the certainty of the result obtained is greater. The use of the three-electrode type of instrument as a three-phase instrument requires that the conditions be arranged so that there is no interference between electrodes.

We have investigated both insulator and electrostatic-potentiometer methods of connecting the instrument to the transmission line and have used the insulator-potentiometer method for several reasons, among which are the following:

1. Availability of normal line insulator units, ease of installation, and small space occupied.
2. Requisite safety is assured by the use of an insulator-potentiometer string having a greater number of units than normal line insulation.
3. All measurements are made across a portion of the normal line insulation.
4. Laboratory tests have not demonstrated that the electrostatic-potentiometer method may be relied upon for any greater degree of accuracy than the insulator-string method.

While there is considerable evidence to support the contention that for 60-cycle voltages the distribution over a string of insulators may vary with conditions, such as indicated by the decrease in flashover under wet conditions, there also appears to be considerable evidence which indicates that this change in distribution does not take place where the applied voltages are of very steep wave front. This seems reasonable because of the much larger charging current present with very steep waves and the consequent unimportance of the low surface-leakage current. Data have been published which indicate that for steep waves the flashovers wet and dry do not differ greatly. Our laboratory test on insulator strings of various lengths seems to bear this out. Using a wave front of about two microseconds the distribution of voltage across the individual units of a string of six insulators was found to be independent of the magnitude of the voltage and similar to the 60-cycle distribution. These investigations check our ratio results on various insulator strings at 60 cycles and impulse voltage, and indicate that our ratio of line voltage to instrument voltage will be constant and identical both wet and dry on impulse voltages, and dry on 60 cycles.

The 60-cycle ratio with the string wet will vary somewhat due to the increase in surface-leakage currents. The use of the electrostatic potentiometer, however, does not seem to rectify this condition because the rings are still necessarily supported by porcelain insulators, again supplying surface-leakage paths which tend to disturb the normal frequency distribution. As regards the high capacitance of the electrostatic potentiometer, this may be a disadvantage since the higher the capacitance, the greater the distortion of the surge voltage from normal.

Mention has been made of the lead effect shown in Fig. 24 of the paper in connection with the insulator-string potentiometer. The results shown on this graph however, were, not obtained upon an insulator string but on an electrostatic potentiometer. While this potentiometer did not have the capacitance of some of those which have been used in practise, the results obtained do demonstrate a characteristic which will be obtained in any type of capacitance voltage-dividing arrangement used. In such devices, connecting leads should always be as short as possible. Therefore, Mr. Cox's statement "Altogether the most desirable form of potentiometer is that having the highest capacity" must necessarily be modified, since recognition must be given to the fact that high-capacitance potentiometers distort the line transient.

A New Electronic Rectifier

BY L. O. GRONDAHL[1] and P. H. GEIGER[1]
Associate, A. I. E. E. Non-Member

***Synopsis.**—A new rectifier utilizing a partially oxidized disk of copper as a rectifier unit is described. The disks may be arranged into groups suitable for all fields of rectifier applications.*

The rectification appears to take place at the junction between the copper and the oxide without observable physical or chemical changes, and is similar in character to rectification by the hot-cathode type of rectifiers.

The physical characteristics of assembled rectifiers and a method of designing the same for special purposes are outlined and some of the design problems discussed.

It is pointed out that the new rectification phenomenon is of a radically different nature from those observed in structurally somewhat similar contact rectifiers. The usual theories of contact rectification, which are based on electrolysis or thermoelectricity, are not applicable to the present case. The new phenomenon is discussed in the light of more recent theories based on electron affinities of copper and copper-oxide, which are in better accord with the observations.

Some applications are given for which the rectifier seems to be especially suited.

THE present paper is a discussion of the results of a development which has been based on a phenomenon discovered by one of the writers and reported at the meeting of the American Physical Society held at Washington, April 23-24, 1926.[2]

In the course of an investigation of copper oxide formed on a piece of copper, during which current was passed through the oxide in a direction at right angles to the surface of separation, it was observed that the resistance of the combination was less when the current flowed from the oxide to the copper than when it flowed in the reverse direction. In the first unit, the ratio of the resistances in the two directions was about 3 to 1. The phenomenon was so different in nature from anything that had been observed in other known types of rectifiers that an intensive study and experimental investigation was undertaken during which it became more and more evident that the new device has characteristics which make it very probable that it will find general application as a rectifier.

DESCRIPTION

A rectifier element consists of a disk of copper on which has been formed a layer of copper oxide, as shown in Fig. 1. A good electrical connection is made with the exposed surface of the oxide layer by means of a terminal member of soft metal, such as lead or metal foil. The copper disk and the terminal member of soft metal are conveniently made in the form of washers and assembled on a bolt to provide a good connection between the outer surface of the oxide and the soft metal washer.

Fig. 1 shows such an assembly of a half wave rectifier.

In the new rectifier, the rectification appears to be restricted to a microscopically thin layer at the junction between the copper and the oxide, and takes place under entire absence of electrolytic action or other observable physical or chemical changes.

Any number of individual elements may be assembled in series and in parallel into rectifier groups for any desired value of current and voltage. The two standard methods of connecting rectifiers for full wave rectification are shown in Fig. 2. Fig. 3 shows an assembly of four copper oxide rectifier elements into a group for full wave rectification, the connections being the same as in *b* of Fig. 2. Such an assembly may be used without a central tap in the transformer.

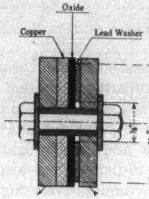

FIG. 1—ASSEMBLY OF SINGLE HALF WAVE RECTIFIER

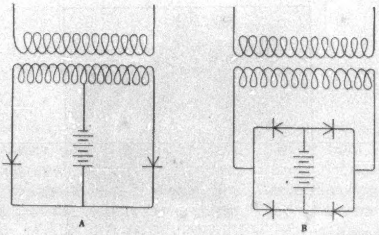

FIG. 2—ARRANGEMENTS OF RECTIFIERS FOR FULL WAVE RECTIFICATION

With good ventilation, such a unit will supply a unidirectional e. m. f. of 6 volts and a current which depends on the area used. The current density that may be used depends on the effectiveness of the ventilation that is provided. In order to dispose of the power lost in the rectifier, it may be provided with ventilating fins as illustrated in Fig. 4. With current densities greater than two amperes per square inch, a forced air

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[1] Research Department, Union Switch and Signal Company, Swissvale, Pa.

[2] L. O. Grondahl, *Phys. Rev.* 27, p. 813, June 1926.

Presented at the A. I. E. E. Winter Convention, New York, N. Y., February 7-11, 1927,

draft or immersion in oil is necessary. A rectifier provided with ventilating fins and immersed in oil has been operated continuously at 3.5 amperes per sq. in. The necessity of making special provision to dissipate the heat developed is due to the fact that for a given capacity, the volume and therefore the radiating surface of the rectifier itself are comparatively small.

CHARACTERISTICS OF SINGLE RECTIFIERS

Although at first thought the fact that contact is made with the exposed surface of the oxide layer would suggest a certain similarity to contact rectifiers such as those of the "cat-whisker" type used in radio, a careful investigation seems to point definitely to the junction between the copper and copper oxide as the seat of rectification. The ordinary "cat-whisker" type contact rectifier has a comparatively high resistance and is entirely unsuitable for the supply of any considerable amount of power. Those who have worked with such rectifiers will realize that the apparatus, even discounting its power limits, is unsatisfactory for most uses on account of the instability of the contact and the erratic behavior of the unit as a conductor. The present rectifier is consistent in its behavior, does not depend on a point contact, and the resistance is so low that the rectifier is capable of carrying large currents. The whole area at the junction between the copper and the oxide participates in the rectification and there is nothing that would suggest the idea of a sensitive spot such as is characteristic of the contact rectifier.

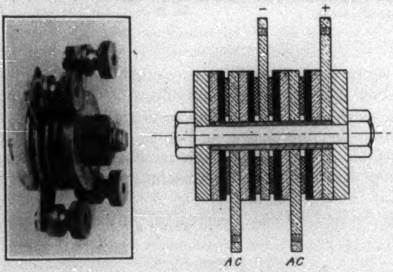

FIG. 3—ASSEMBLY OF FULL WAVE RECTIFIER

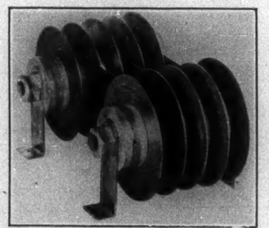

FIG. 4—ASSEMBLY SHOWING SERIES PARALLEL CONNECTIONS USED FOR HIGH CAPACITIES

The following curves illustrate the points that are emphasized in the preceding paragraph. Fig. 5 shows the relation between current and electromotive force in the two directions through the copper oxide. In this figure, the part of the curve that represents the current in the high-resistance direction has been drawn to a scale 1000 times as great as the remainder of the curve. The scale for currents above the horizontal axis is in amperes; the scale below the horizontal axis is in milliamperes.

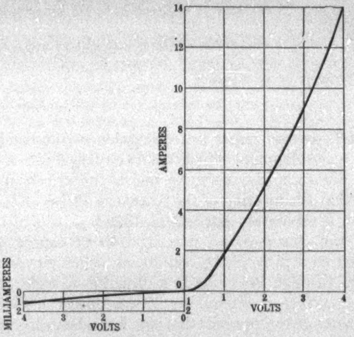

FIG. 5—CURRENT VOLTAGE CHARACTERISTICS OF COPPER-COPPER OXIDE ELEMENT

Fig. 6 gives the relation between resistance and electromotive force. The negative values of electromotive force correspond to the high-resistance direction. The resistances approach a common value as the voltage approaches zero. As the voltage is increased from zero, the high resistance increases and the low resistance decreases, at first very rapidly, and then at a decreasing rate as the voltage increases. The low resistance con-

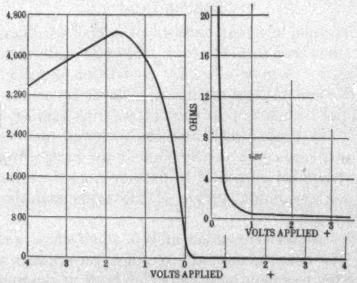

FIG. 6—RESISTANCE VOLTAGE CHARACTERISTICS OF COPPER-COPPER OXIDE ELEMENT

tinues to go down practically along an exponential curve while the high resistance increases to a maximum beyond which it decreases slowly with further increase in voltage. The low resistance is shown in the curve in the upper right hand corner with the scale magnified 200 times. The ratio between the two resistances increases up to values well beyond those at which the rectifier may be allowed to operate.

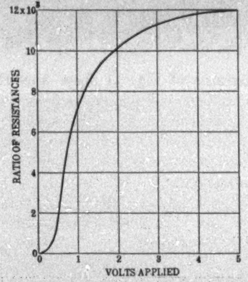

Fig. 7—Relation between Ratio of Resistance and Voltage Applied to Rectifier

The rectification ratio, which is obtained by dividing the low resistance at any particular value of e. m. f. into the high resistance at the same e. m. f., is shown in the curve of Fig. 7. This ratio bears a definite though not a simple relation to efficiency. The relation between the two is complicated by the fact that in practise the voltage across the elements of the rectifier varies from zero to a maximum which is different in the two directions. The efficiency of a full wave rectifier built with four washers is shown in Fig. 8. This is an average unit. The efficiency here shown is the ratio of d-c. watts output to a-c. watts input. True power efficiencies of over 80 per cent have been observed. Since the rectification ratio at very low voltages approaches unity, it follows that the efficiency of the unit as a rectifier at very low voltages approaches zero. At the voltages that are common in the usual applications of a rectifier, the ratio is so high that variations are not often important. A few principles that have to be kept in mind in the design of rectifiers are given below.

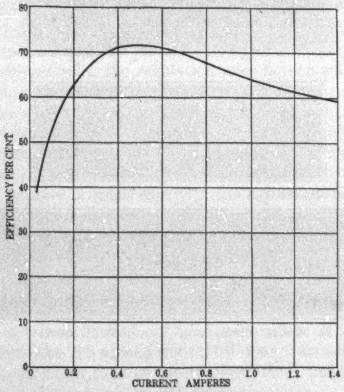

Fig. 8—Efficiency of Full Wave Rectifier Consisting of Four Disks

Design

For most applications, the losses due to reverse current should be taken into account although they are small. This may be seen from the following considerations.

In a full wave rectifier connected as shown in *b* of Fig. 2, the voltage across each element during the part of each cycle when the voltage is applied in the high-resistance direction is considerably greater than the voltage applied in the low-resistance direction. Usually the voltage applied in the high-resistance direction is such that the rectifier is working well beyond the maximum of the high-resistance-voltage curve (Fig. 6). For these reasons the ratio of the current in the low-resistance direction to the current in the high-resistance direction is considerably less under actual working conditions, than the ratio as shown in Fig. 7 of the high to the low resistance measured at the same voltage.

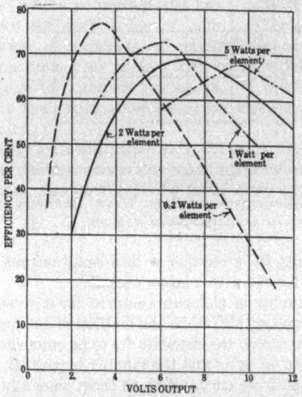

Fig. 9—Efficiency with Various Values of Constant Power

Fig. 9 shows the true power efficiency of a rectifier made up of four 1½-in. elements as shown in Fig. 3. The power output was kept constant for each curve, the designating number of each curve being one-fourth the total power output. Points to the left of the maximum efficiency represent an excess of losses in the low resistance direction; points to the right, an excess of losses in the high-resistance direction. If a given rectifier is used to supply a practically constant output voltage, as is required for battery charging service, the losses in the high-resistance direction remain nearly constant, while the losses in the low-resistance direction decrease with a decrease in charging rate. For this reason, it is sometimes advisable to use a larger number of elements for a rectifier with a small output than is required for one with a larger output.

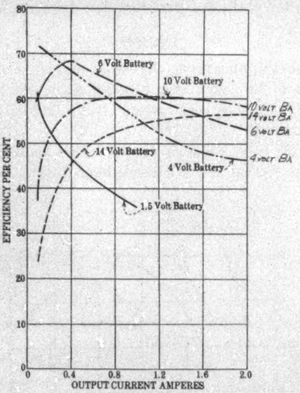

Fig. 10—Efficiency of Full Wave Rectifier used as Battery Charger

The number of elements required for a given power output depends also upon the method of cooling. The manner in which the elements are to be connected, *i. e.*, the number in series and the number in parallel, may be determined from curves such as those shown in Fig. 9. To obtain maximum efficiency in a complete unit, the number of elements in series between any two terminals of the rectifier is found by dividing the desired voltage output by the voltage giving the maximum efficiency on the corresponding curve. Enough elements are to be connected in parallel to give the desired current output, keeping the output per washer at the value previously decided upon. If the method of connecting that is shown in *a*, Fig. 2 is used, the number of washers in series should be doubled, thereby operating each element on the same portion of the characteristic curve as in the four-cell type rectifier.

For applications requiring a current of a few tenths of an ampere or less, such as supplying the plate current of vacuum tubes, it is sometimes necessary in order to obtain the maximum efficiency to use an element of less than 1½-in. diameter, or else to use a smaller output per washer than would be used in other applications. This can be seen from the curves, for the maximum efficiency (1½-in. washers) occurs at low current values only for the curves of small values of output per element.

Characteristics of Rectifier Groups

Fig. 10 gives the efficiency of a rectifier group used as a battery charger. Since in the charging of a battery we are interested in the average value of the direct current, efficiency is taken as the ratio of d-c. volt-amperes to the a-c. watts, and is less than the power efficiency.

The oscillograms of Fig. 11 show how the relation between the battery voltage and the a-c. voltage impressed on the rectifier affects the wave form of the charging current. These currents result from a combination of the steady battery voltage and the fluctuating voltage supplied by the rectifier. The portion of the cycle during which charging current flows into the battery increases with the increase of the applied voltage.

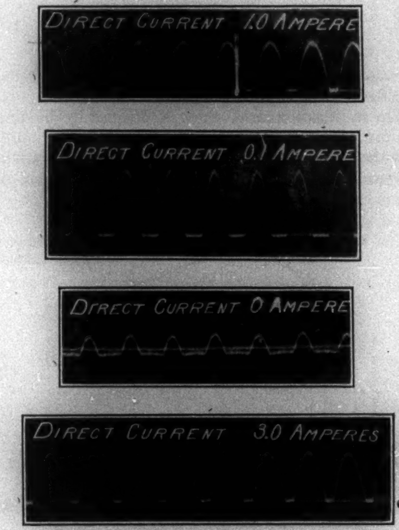

Fig. 11—Oscillograms of Charging Current in Six-Volt Battery with Different Values of A-C. Voltage Applied to the Rectifier

The form factor of the rectified wave has been found to vary in different units between 1.13 and 1.25. The form factor of a pure sine wave is 1.11. The oscillograms of Fig. 12 represent the wave forms of the rectified current from a full wave and a half wave rectifier in a non-inductive load. On account of the fact that the resistance varies with the voltage applied, the low values of current are a little lower than they are in a sine wave. The distortion is barely noticeable.

The rectifier may be used at any ordinary frequency without any effect on its operation. It has been tried with measuring instruments and found to give good rectification up to a frequency of over 3,000,000 cycles per second[3]. Above 100,000 cycles per second, there is a gradual decrease in rectification ratio which may be due to capacity.

The effect of temperature on efficiency may be compensated in various ways and the following is an illustration of what may be done by the proper choice of the size of the unit and by the use of a reactive ballast. The problem under consideration was to supply a certain constant amount of rectified power at all temperatures from 0 deg. cent. to + 80 deg. cent. with a constant voltage supply. The results are shown in the curve of Fig. 13.

FIG. 12—OSCILLOGRAMS OF RECTIFIER CURRENTS IN NON-INDUCTIVE CIRCUIT

The voltage regulation of a rectifier depends primarily on the effective resistance of the unit. It may be pointed out that just as in a battery of storage cells or of primary cells, the regulation may be controlled by varying the number of cells that are put in parallel, so in the case of this rectifier it is possible to control the regulation. Within reasonable limits, practically any excellence of regulation can be obtained by building into the rectifier the necessary amount of copper. In a test of a rectifier of small capacity, the regulation between no load and full load was changed from 16.5 per cent to 8.5 per cent by doubling the amount of copper in the rectifier.

THEORY

While the investigation of the new phenomenon has not yet been carried on to a point where it is fully understood, it is safe to say that it cannot be explained by application to it of the theories which are usually advanced in connection with contact rectifiers.

One of these theories is based on thermoelectricity. Since the oxide in our rectifier is very often not over 0.0015 in. in thickness, it is difficult to imagine any considerable temperature difference between the two surfaces. In addition, experiment shows that the asymmetric resistance is concentrated at or very near the surface of the junction between the oxide and the copper and that the heating of this junction produces an e. m. f. which is in the wrong direction for the rectification that actually takes place. The thermoelectric explanation is, therefore, not tenable.

Another explanation that has been adopted by some physicists is based on electrolysis. This explanation is probably applicable to some contact rectifiers. Where it is applicable, the rectifiers have characteristics that are easily recognized. They require some time after the e. m. f. is applied to reach their steady state. The current is very irregular and shows frequent and very sudden variations. After operation for a comparatively short time, products of electrolysis appear and the rectifier deteriorates. The rectifier under consideration has none of these characteristics. After the application of the e. m. f. it is immediately operative in its steady state. The current is smooth as would be expected with a conductor that has a definite value of resistance for each value of e. m. f. There are no indications of products of electrolysis even after operation for a year or two with current densities of 0.5 ampere to 1 ampere per sq. in. It seems safe to say that the explanation based on electrolysis must be rejected.

Schottky's[4] theory, involving the work required to carry an electron across the boundary between the two substances, also fails to give a satisfactory explanation as it is probable that the electron affinity of copper is greater than that of copper oxide.

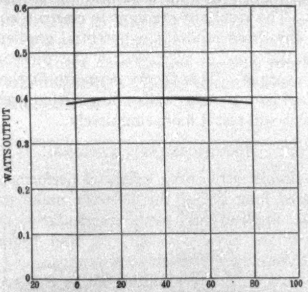

FIG. 13—OUTPUT POWER AT TEMPERATURES BETWEEN 0 DEG. CENT. AND 80 DEG. CENT.

Jolley in his book on Alternating Current Rectification mentions a theory which assumes a double layer, each half of which is made up of one constituent of the crystal. This condition may exist in the present instance at the surface where the two substances are joined to each other. It may be that beyond the last layer that contains oxygen atoms, there is a layer of copper atoms all, or most, of which are in chemical combination with the oxygen. It is not easy to get a mental picture of the action of such a double layer. It may have value when taken together with the work function considerations.

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[3] Data obtained through the courtesy of the Research Laboratory, Westinghouse Electric and Manufacturing Company.

[4] Schottky, *Zeits. fur Physik*, 14, p. 63, March 1923.

One of the present writers[5] has proposed a theory based on the fact that the copper and the oxide are in very intimate relationship. The transfer of an electron from copper to oxide, or vice versa, may then take place without passing through the whole potential drop represented by the electron affinity of either substance, but only through a potential drop corresponding to their difference. Under this condition it is conceivable that even at room temperature and without any application of e. m. f. a great number of electrons are able to escape from the copper and into the copper oxide. The copper then serves the same purpose as the hot wire filament in a vacuum tube, and maintains an atmosphere of electrons in the oxide in excess of the normal amount. On account of the short distance between the electrodes, the comparatively large area, and also probably assisted by the dielectric constant of the oxide, the resistance to the flow of electron current in the direction from the copper to the oxide is small.

When the e. m. f. is applied in the opposite direction, there is a tendency to drive the electrons back into the copper. This is opposed by the ready diffusion of electrons from the copper into the oxide so that the electrons become concentrated near the surface of the copper. The resultant gradient in electron concentration in the oxide produces a potential gradient which opposes the flow of electrons in the direction from oxide to copper. This theory seems to fit the voltage resistance curves very well. Experiments are under way which will test it more completely.

Practical Applications

The simple structure, excellent performance, and promise of long life of this rectifier make its use in practical applications seem particularly desirable. Engineers are already recognizing and beginning to exploit these characteristics.

In discussing the possibilities of the new rectifier, a prominent engineer suggested the idea of an entire automatic substation in the form of a tank mounted on a pole with the transformer, rectifier, and suitable switches housed in the tank. The complete absence of the requirement of servicing the rectifier, the length of its useful life, which gives promise of being very great, possibly even equal to that of the transformer, make this idea seem very likely to become practical.

The rectifier gives a smooth rectified current which looks very much as though it had been commutated by means of a perfect commutator. It is very constant in its characteristics, requires no electrolyte, and does not involve any moving parts or contacts to be made or broken; in fact, it consists only of parts that are solidly bolted together. It requires no attention or servicing, and can be built into units to meet any reasonable requirement of current and voltage. Each element represents a relatively small increment of current and voltage and they may be assembled into groups just as storage cells are assembled into batteries. The following are meant to be suggestive illustrations, rather than a comprehensive list, of its immediate uses.

1. *Instruments.* A practical use to which the rectifier has been put is in connection with d-c. instruments used on a-c. circuits. Here it is found exceedingly convenient. With a wavemeter, for instance where it has been common practise to use a thermocouple meter, the rectifier is advantageous. A thermocouple takes some time to reach a condition of equilibrium so that after every setting of the wavemeter, the operator has to wait for the instrument to reach a steady state. The rectifier responds instantly and in wavemeter work, therefore, it is possible to proceed very much more rapidly with a rectifier meter than with a thermocouple meter. It is also very sturdy, so that a temporary overload causes no injury to the rectifier. In general, the rectifier with a d-c. instrument is very convenient for reading small alternating currents. As is seen from the curves, the rectifier is not sensitive at low values of power so that there is a limit below which the rectifier is very inefficient. This limit is very low and the instrument can be used satisfactorily down to a few micro-amperes. Here it is important to match the impedances of the rectifier with the instrument and of the rectifier-instrument combination with the source. The impedance of the rectifier changes with the power so that an instrument that is well matched for one range is not necessarily satisfactorily matched for another range. The desired results may be obtained by matching impedances for the lowest range and the scales can be adjusted to take care of the higher ranges.

The direct-current output of the rectifier at low values of power is approximately proportional to the square of the a-c. input.

For reasonable precision in measuring instruments, it is necessary either to provide temperature compensation for the resistance variations in the rectifier or to choose the constants of the instrument so that the resistance variations of the rectifier are unimportant. This has been done very satisfactorily in special applications. Both the high and low resistance of the rectifier have a high temperature coefficient. The temperature change in the efficiency of a rectifier is due in part to the changes in resistance and in part to the changes in pressure due to the unequal expansion of the bolt and the other parts from which the rectifier is made. Pressure changes may be used to compensate partly for the resistance changes. Compensation may also be accomplished by introducing in the circuit resistances which have an effect that is opposite to that of the rectifier itself.

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[5] L. O. Grondahl, *Science*, September 24, 1926, Vol. 64, No. 1656, pp. 306-308.

For use with very sensitive instruments, the rectifier should be protected against illumination. Illumination not only changes the resistance, but produces a small e. m. f. in the rectifier.

2. General Battery Charging. The application of chargers are as numerous as the applications of storage batteries. The automobile starting battery may be used as an illustration since it is very often found necessary to give it an extra charge. This is usually a very inconvenient thing to do on account of the necessity of getting into the battery case and making the necessary connections. A small transformer and rectifier could be installed either in the automobile or on the wall of the garage with a plug on the instrument board or in some other convenient place so that the connection necessary for charging would be very simple. In such an application, the rectifier is especially practical on account of the fact that it is very sturdy and requires no attention. The charging of telephone and other storage batteries may be arranged as desired in accordance with the principles already laid down.

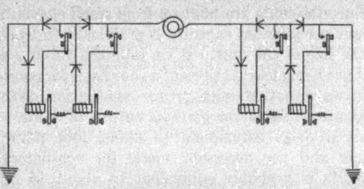

Fig. 14—Circuit Arrangement using Rectifier for Duplex Telegraphy

3. Control Apparatus. The control of electric circuits is usually accomplished by means of electromagnets. Electromagnets are more easily operated by means of direct current than they are by means of alternating current and the provision of a rectifier with each magnet makes this possible. Thus, for instance, d-c. switch magnets and circuit breaker magnets can be used on a-c. lines.

4. Telegraphy. A system of duplex telegraphy has been proposed which is given in outline in the diagram of Fig. 14. This makes it possible by the use of alternating current and a rectifier to polarize the line so that any telegraph line can be duplexed by simply adding a sending instrument and a receiving instrument and four small rectifiers at each station of the line. If the rectifiers are of the half-wave type, the operation of one sending key will transmit the upper half wave, which will be received by the sounder which is associated with another rectifier which also transmits the upper half wave. When the other sending key is used, the lower half wave is transmitted and this operates the sounder at the other end which is associated with a rectifier that transmits the lower half-wave. Such a system seems practical when one can use rectifiers that are reliable and the capacities of which are easily adapted to the purpose in question.

5. Detectors. The rectifier in its usual form is not suitable for a radio detector, but can be used in a similar way in circuits which involve larger amounts of power. For instance, if it is desired to get a current pulse through a transformer by making and breaking the current in the primary, a rectifier can be used to make the pulse uni-directional. In such cases, the rectifier serves the same purpose as a detector on a very much larger scale.

6. By-pass for Field Switches. With motor or generator field switches, provision has to be made to guard against the injurious effects that may result from the sudden release of the energy of the magnetic field. A rectifier connected between the terminals of the switch in such a direction that it opposes the flow of the direct current serves as a low resistance for the inductive surge that accompanies the opening of the switch and is very effective. A very high resistance rectifier may be used since the voltage of the field discharge is great and the energy loss in the rectifier may be made negligible. It is already in use in a number of similar applications for the protection of relay contacts.

7. Edison Direct Current Systems. In Edison direct-current systems a rectifier of this type has peculiar advantages due to the fact that it is static and can be assembled into units of any desired capacity. A large unit might be made up of a number of smaller standard units constructed so that the capacity of the rectifier may be altered as required by the load. Such a rectifier is entirely noiseless and the only moving parts are the fans or pumps necessary to carry away the heat.

8. Radio. An interesting field for rectifier manufacturers has been the radio field. Here again, the rectifiers are used as battery chargers.

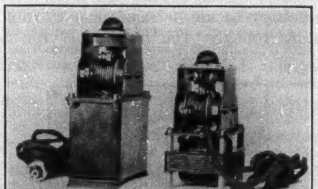

Fig. 15—"A" Battery Trickle Charging Unit

"A" batteries are very often maintained by what is known as the trickle charge method and tube rectifiers for this purpose have recently appeared on the market. With some "B" battery chargers it is necessary to disconnect the "B" battery and connect the various groups in parallel. The present rectifier can be built in the proper voltage and current capacities to charge either "B" or "A" batteries and to charge them at a normal rate or at a trickle charging rate as desired. To charge an "A" battery at a trickle charging rate, a small transformer and a rectifier consisting of 4 to 16 copper disks may be used. Such a unit is shown in the photographs of Figs. 15 and 16. The unit may be assembled in a case together with the "A" battery itself. Rectifiers have been designed to meet the demand for 2 ampere chargers and 5 ampere chargers. A 2 ampere charger is shown in Fig. 17.

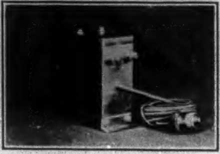

Fig. 16—"A" Battery Trickle Charging Unit

A "B" battery charging unit for any e. m. f. up to a 115 volt "B" battery may be had by connecting the rectifier with the necessary ballast reactance to the 110 volt house lighting circuit. For a 135 volt battery, it is necessary to use a transformer to step up the alternating voltage. The transformer can be built with the necessary reactance so that the size of the unit does not need to be any greater in the second case than in the first. With such a unit, it is only necessary to reverse the switch to disconnect the battery from the receiving set and connect it to the rectifier.

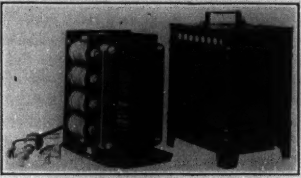

Fig. 17—Two-Ampere, Six-Volt Battery Charging Unit

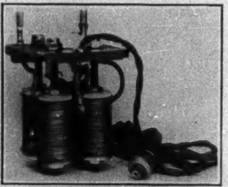

Fig. 18—"B" Battery Charger

The more interesting application of rectifiers in radio is probably in battery eliminators. For this purpose, the rectifiers have to be built with the necessary voltage and current capacity to supply not only the power to operate the tubes, but the power that is lost in the filter. Units have been built which give satisfactory service as substitutes for both "A" batteries and "B" batteries. Fig. 18 shows the interior of a "B" battery charger, which is the same as the rectifier that is used in a "B" battery substitute.

General Discussion of Power Units

Since the units are built up from small elements, the weight per kilowatt capacity is practically independent of the size of the unit. If we take the 1.5-in. washers that are being used at present as our basis for consideration, we find that with proper ventilation, 200 such disks are sufficient to give an output of 1 kw. Two hundred disks correspond to about four pounds of copper and the necessary metal for ventilators and supports is probably equivalent to about 16 pounds more so that the total weight per kw. capacity is about 20 pounds. This weight may be assembled in a space of approximately 400 cu. in. The capacity per cubic foot is, therefore, approximately equal to four kw.

Discussion

G. W. Janson: Schemes such as that shown in Fig. 14 have been tried at various times by communication engineers, in their efforts to increase the output of telegraph circuits. The possibilities of these arrangements have, however, been generally limited by factors other than the effectiveness of available rectifying means. However, there are probably many other valuable uses for a rectifying element having the excellent characteristics described in this paper.

Several questions have occurred to me which would be of interest to those seeking to adapt the element to purposes other than converting an a-c. supply to a d-c. supply.

What is the maximum voltage per element which may be applied before the rectifier passes a-c.? Are the characteristics of the rectifier permanently changed after that maximum voltage has been applied? Has the rectifier been used for application to loads of varying resistance and are the voltages under such conditions affected differently from those shown in the curves? Are there any polarization effects?

Another question that might be of importance is whether any time lag exists between the application of a potential to the flowing of direct current. I wondered, for example, whether the resistance ratio curves and the data on resistance at various voltages were determined by d-c. methods or by a-c. methods. If by a-c. methods, would those ratios be different for the steady d-c. state?

A. G. Oehler: When a rectifier is made of a number of elements connected in series, does the voltage or the power rectified in each unit divide up equally over all of the elements? What factors of design or what inherent characteristics limit the voltage which can be rectified by this rectifier?

George Crisson: The system of signaling described by Messrs. Grondahl and Geiger is of interest because it accomplishes duplex operation by very simple means which do not involve the problem of line balance.

Certain effects occur in this system, however, which are not encountered in the telegraph systems now in use. These effects, which are due to the fact that the velocity of propagation of a wave over a telegraph wire is not infinite, put definite limits on the distance over which signaling can be carried on without the use of repeaters.

No reason is seen why the system should not operate satisfactorily when the length of the line is a small fraction of the wave-length. The operation of any key will cause changes in the strength of the current in the corresponding instrument at the other end of the line so that signals can readily be sent but will not affect the instruments in the second channel.

As the distance from the generator to the end of the circuit increases, the behavior of the system becomes more complex. Operating a key has less effect on the corresponding instrument at the far end, and begins to produce changes in the current of the distant instrument in the second channel. This effect is due entirely to the fact that a finite time is required for a wave to travel from the generator to the key and back. It is quite distinct from the weakening of signals due to losses in the line.

The system would become inoperable when the distance from the alternator to the key has increased to one-eighth of the wave-length, or the total length of the line has reached one-quarter of the wave-length, assuming that the alternator is located at the middle.

Assuming that a frequency of at least 200 cycles would be required to avoid operating difficulties caused by the periodic nature of the current, the greatest length of line workable by this system without repeaters would be about 200 mi. for non-load open-wire lines, and a much shorter distance, in the neighborhood of 50 mi., for non-loaded circuits in cable. Practically, the lengths would have to be still less to allow for a safe operating margin and for various factors not considered in this simple treatment.

This system could not be applied to composited lines without modifying the compositing apparatus, which separates the telephone and telegraph currents at each end of the line in such a way as to encroach seriously upon the range of frequencies required for the telephone.

Of course, for classes of service permitting the use of a frequency lower than 200 cycles, the workable length of the line would be increased and the difficulty of applying the system to composited lines would be somewhat reduced.

Joseph Slepian: Four to five years ago in meditating about the usual thermionic rectifier—that is, a rectifier consisting of two electrodes in a high vacuum, one electrode heated—it occurred to me that probably other types of rectifiers were based upon essentially the same phenomenon, that is, the presence of three suitable materials, two metallic conducting electrodes, and an intervening material. In the case of the thermionic rectifier the part of the intervening material is played by the vacuum.

In this combination of three materials, the electrodes are good conductors of electricity. The intervening vacuum by itself is an insulator but if electrons are somehow supplied to it, it becomes a good conductor. In other words, electrons can move freely in this vacuum if they are provided.

Now it seemed to me that it might be possible that other insulating materials than a high vacuum would have this same property, so that it might be possible to take two metals and put an insulating material in between them, this insulating material having insulating properties ordinarily not so much because it obstructed the flow of electrons but because it, itself, lacked free electrons. Now, if one of the electrodes was able to supply free electrons to the insulating material, and the other not, a rectifier would be obtained.

Of course if such a rectifier was to be practical, this insulating material would have to carry electrons rather freely, but the electrons would have to be supplied to it by the adjoining electrodes. It seemed, of course, that this intervening insulating material would have to be very thin, because you could not expect the electrons supplied to this material to move through it so freely as they will do in a vacuum. I thereupon set out to investigate the properties of thin insulating films between unlike electrodes, and for a little over two years I examined all kinds of combinations to get a very thin layer of insulating material.

I considered films formed chemically, and I got results that were frequently promising, and which led me to continue my work, but I never got anything that looked practical, or appeared to be worth following in greater detail.

Then I heard that a man from one of our neighboring companies, the Switch & Signal Co., had devised a rectifier consisting of copper and copper oxide for which great claims were made. I rather scoffed at it when I first heard about it, as I had already experimented with copper oxide. I had placed sheets of copper oxide against various metals and observed some rectification, but always the rectification was small and rather erratic.

A little time later some of these oxidized copper washers were supplied to me at the laboratory and I proceeded to test them, not expecting very much, and was quite amazed at the results. The rectification was steady to an unbelievable degree. After the experience of my two years work it seemed absolutely revolutionary. I had never seen or heard of anything like that in a rectifier of this type, and I looked into the rectifier more closely and was interested to find it had just the elements I was looking for. The ideal that I had been working for seemed to be realized in this rectifier that Mr. Grondahl had provided.

The rectifier consists of two bodies of metallically conducting material; namely, the copper and the oxide, and an intervening insulating layer. I have proved the existence of this layer between the oxide and the copper by making capacity measurements. The electrostatic capacity has such a magnitude as to indicate that the rectification takes place in a layer less than a 0.0001 cm. thick between the oxide and the copper.

It is most astonishing that a rectifying layer is obtained by such simple means as forming the oxide on the copper. It is also very astonishing that this rectifying layer is obtained only between the copper oxide and the metal on which it was formed. A piece of copper oxide by itself clamped against a piece of copper will give some rectification, but of an altogether different order and quality from that obtained where the oxide is formed on the copper itself.

B. O. Adkerson: Does the use of foils of other metal than lead, *e. g.*, aluminum and gold, change the resistance of the rectifier in the two directions? If the resistance is affected what is the amount of such change?

Fig. 6 in the paper shows the voltage-resistance characteristic curve of the rectifier between plus and minus 4 volts, and in reply to a previous discussion it was stated that about 30 volts was the maximum that one element would sustain. Does the resistance decrease continuously to zero as the negative potential is increased to this maximum? If it does not, what is the resistance at the point of discontinuity,. *i. e.*, just before puncture occurs? When the rectifier is once rendered inoperative by exceeding the maximum voltage, will it automatically reform upon reducing the voltage, similar to an electrolytic rectifier, or is the element useless unless the copper washer is again oxidized?

Dr. Eccles' rectification theory, mentioned in Jolley's Alternating Current Rectification, indicated that rectification may occur at hot points of contact between or in the rectifying materials. Does the difference of potential across the rectifier, as determined by a rapid oscillograph, differ in any way from that across a pure resistance of equal ohmic value, when current is made and broken through it, *i. e.*, is there any evidence of a potential difference other than the $R\ I$ drop caused by the passage of the current?

Does the instantaneous response of the rectifier, mentioned in connection with wave-meter work, mean that the time required for the rectifier to reach a steady state is negligible, as considered from a practicable point of view, or that there is no perceptible change of current with time as determined by the oscillograph? If there is a change of current what is its approximate magnitude, is the current increasing or decreasing with time, and does the change occur in both directions through the rectifier?

L. O. Grondahl: The first question is with reference to maximum allowable voltage. The curve that gives the relation of the resistance in the high-resistance direction to the voltage shows that the resistance decreases with increasing voltage above two volts. We think of three volts in the high-resistance direction as being a normal counter-voltage for the rectifier to withstand. We are using them, however, in large units up to six volts with additional radiating surface. Within the limits mentioned it is only a question of carrying away the heat. When higher voltages are used the losses in the high-resistance direction make it difficult to carry away the heat generated.

The resistance curves in the paper were taken with direct current. There is no polarization. There is a very slight change in current in the high-resistance direction, but it is immaterial and a large part of it is explainable as due to rise in temperature. It is not in the right direction to be caused by polarization. We have tried rather carefully to find a back current such as exists in a cell that polarizes. We have not been able to find anything of that kind.

That answers the question also in regard to the various loads. The effect of varying the load is illustrated in the efficiency curve. In that case the various loads were obtained by simply changing the load resistance.

There seems to be no tendency for one unit to pick up the load and carry it to the exclusion of the other units. It divides itself exactly in terms of the resistances of the elements. As far as our experiments show up to the present time the rectifier behaves as if it were a pure resistance phenomenon.

In some rectifiers, one rectifier will pick up the load for a little while, and then another rectifier in the series will take its place so that the load darts back and forth between the elements. That does not happen here at all. The load is evenly and steadily distributed.

We have a unit in the laboratory that we have been using for eight or ten months now, supplying one ampere at 1500 volts for oscillators. It has been a reliable supply without any maintenance.

On the question of forming first of all, there is no such phenomenon as forming in connection with the rectifier as far as we have been able to determine. When I say that it operates instantaneously, I mean that the time required for the current to start and for the final condition to be set up, as far as rectification is concerned, is as nearly instantaneous as you can get it with the inductance that you have in the circuit. It depends upon the inductance in the circuit rather than upon the characteristics of the rectifier. The rectifier is a resistance pure and simple as far as its observable qualities are concerned, certainly in operation.

The change in current in the high resistance direction that I mentioned is of very small value and probably has no practical importance at all. If you have 1 milliampere high-resistance current (for instance, back leak, when the alternating current is off and the battery is discharging through the rectifier) it may grow in a little while up to 1.5 milliampere. That is something we have not yet explained. However, there is no effect similar to forming.

You can put the rectifier on direct current in either direction for any length of time, take it off and put it on in the opposite direction, and it behaves just as though nothing had happened to it. You can start it on an a-c. circuit for rectifying after it has been rectifying for a long time, and it behaves exactly the same as when you start it after it has been resting for several weeks.

The question of hot spots: if you go up to very high voltages, say 30 or 40 volts per disk so that you puncture the surface, then you find that it acts like every other dielectric—it punctures in a spot—but we have not found any other evidence of the rectification taking place in spots, or the current being carried in spots.

The lead is used as a contact on the outer surface, because it is more impressionable than other metals that are easily obtainable and makes a lower resistance contact, a slightly lower resistance contact than we get, for instance, with aluminum foil. It is not quite as low as we get with gold foil. Gold foil is inconvenient to handle and sheet lead is practical. So we use the lead. All resistance changes are continuous.

J. F. Dreyer, Jr.: I should like to ask what degree of uniformity exists in these units? Do they vary much in resistance?

L. O. Grondahl: The degree of uniformity depends entirely upon how well we are able to make the manufacturing process uniform. For our train-control work, for instance, we hold them within 10 per cent on the output. That is, those are our specification limits.

H. J. Rosenberger: Four questions came to my mind: (1) What oxide of copper is used? (2) How do you put it there? (3) How do you keep it there? (4) Does it make any difference how thick that layer is?

L. O. Grondahl: The oxide is the cuprous oxide, the red oxide of copper. It is formed in a furnace and it is formed at such temperatures and under such conditions that it stays.

The thickness is of no consequence at all except that the resistance increases with the thickness since the current has to flow through a thicker layer of oxide. Whether you have two mills of oxide or you have ten mills of oxide, you have identically the same kind of rectifier except for that increase in resistance in the low-resistance direction. The rectification takes place at the junction between the copper and the oxide or so close that we have not been able to distinguish it from the junction.

We have worked as low as 0.001 or 0.0015 in. of oxide, but for practical purposes we use 0.002 or 0.003 in.

D. E. Trucksess: I should like to know if there is any relation between the pressure on the contact and the output. I understand that after the rectifier has been operating for a while the output drops off. Can that output be increased by increasing the pressure?

L. O. Grondahl: The pressure applied seems to have an effect in that it reduces the contact resistance on the outer surface of the oxide. It is necessary to have a contact that is uniform over the whole surface as nearly as possible in order to get the low resistance.

As time goes on, there may be a slight reduction in pressure due to mechanical changes, which may be compensated for by tightening the bolt. As far as we know the pressure is important only in reducing contact resistance.

Measurement of Telegraph Transmission

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***Synopsis.**—Various factors contribute to impair the quality of telegraph signals. For instance, there may be interfering currents either induced in the circuit or brought in by conduction, the proportioning of the circuit elements may be imperfect or batteries and relays may be out of adjustment. The result in any case is to distort the telegraph signals so that the received signals are not a true copy of those transmitted. The paper describes methods for measuring this distortion and for analyzing the results so as to indicate the nature and extent of the impairment and its probable cause.*

THE object of this paper is to describe methods of making measurements of the over-all transmission efficiency of the common types of telegraph circuit and to explain how the results may be analyzed. The most direct and practical method of measuring the efficiency of such circuits consists in determining the lengthening or shortening of the dots, dashes, and intervening spaces comprising the signals by comparing the signals delivered by the receiving relay with the sent signals. This paper will be limited to a discussion of this effect which will be called *distortion*, and a description of methods which have been devised for measuring it.

The electrical characteristics of the circuit and the wave shapes of the operating currents are not dealt with in this paper except to the extent that their effects are included in the results of the measurements of over-all transmission efficiency.

In recent years considerable work has been done by engineers of the Bell System in connection with devising and applying convenient and accurate methods and means for making quantitative tests of telegraph signal distortion. This includes comparatively elaborate and refined apparatus intended primarily for laboratory use and simpler devices suitable for field work. The technical staff has made extensive use of the new devices, and measuring sets for use by the field forces are now under commercial trial.

Various other methods have been used in the past with a view to testing telegraph transmission and the results obtained have been of considerable value. Since little has been published on this subject, a brief discussion of several older methods will also be given.

GENERAL DISCUSSION OF TELEGRAPH DISTORTION

The following discussion applies directly to telegraph circuits employing only two different current values[2] thus including substantially all important long-distance wire circuits with the exception of some submarine cables which are operated with three-current values. It may also be applied to many radio telegraph circuits. Considerable modification would be required in order to adapt the ideas outlined herein to the case of circuits employing three or more different current values.

The operation of telegraph circuits by hand or machine involves impressing signals comprising parts of different length or duration at the sending end of the circuit and the reproduction of these signals at the receiving end. The interpretation of these signals depends upon the correctness of the length or duration of the individual signal parts, commonly referred to in the case of a two-element system as "marks" and "spaces" (corresponding respectively to the "closed" and "open" positions of the transmitter). Loudness or strength of the local response is practically never a factor in telegraph transmission since in nearly all cases use is made of local circuit arrangements at the terminals which provide sufficient strength and also avoid any change in strength due to variation in the line circuit. Excessive "lag," that is, time required for transmission over a circuit, is objectionable and, in the case of some printer circuits, variation in lag degrades transmission. Consideration of lag is usually not of importance, however.

An ideal or perfect telegraph circuit reproduces signals at the receiving end exactly as they were impressed at the sending end as regards length of the component marks and spaces, and any change in these lengths during transmission may be considered as lowering the quality. Therefore, the departure from perfection of the received signals, *i. e.* the lengthening or shortening of marks and spaces which occurs during transmission, is a measure of the degradation in transmission quality.

Definitions. It has been found desirable to subdivide distortion of telegraph signals into certain components. The main reason for doing this is that the components are largely due to different and distinct causes and require different treatment for their proper control in both design and transmission maintenance work. Fortunately, it is convenient to separate these components in connection with distortion measurements.

In explaining these components let us suppose a given signal such as the letter C in the American Morse code to be sent at regular intervals over a telegraph circuit and suppose that the signal is formed in such a way that each repetition is substantially perfect at the transmitting end. If the distortion of each of the unit marks or dots of a large number of successive signals

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[1] Dept. of Dev. and Research, American Telephone and Telegraph Company, New York, N. Y.

[2] See *Certain Factors Affecting Telegraph Speed*, H. Nyquist, TRANS, A. I. E. E., Vol. XLIII, 1924, pp. 412-22.

Presented at the A. I. E. E. Winter Convention, New York, N. Y., February 7-11, 1927.

is measured at the receiving end and tabulated, it is, in general, found that the distortion differs not only from dot to dot in a given repetition of the signal, but also that it differs from signal to signal for a given dot. Let us obtain the average of a large number of distortions for a given dot and consider each individual distortion as being made up of two components, one the average and the other the individual departure from the average. The average distortion of a given part of a large number of successive signals will be called the *systematic* distortion. The individual departure of one distortion from the average will be called the *fortuitous* distortion.

It is found of great value to subdivide the systematic component of the distortion still further. To understand this subdivision, assume that we are dealing with a telegraph system in which markings and spacings are sent by means of currents which are equal in magnitude but opposite in sign. It is, of course, possible with such a system to transmit the marking by means of negative current and the spacings by means of positive current, or vice versa. The change from one method of transmission to the other is accomplished by interchanging the positive and negative batteries at the transmitting end and at the same time interchanging the connections to the marking and spacing contacts of the receiving relay.

Now let us assume that the systematic distortion is brought about by the fact that the positive battery at the transmitting end is stronger than the negative battery. Further, let us assume that the circuit is such that this will result in lengthening of the marks when positive current is used for transmitting marks. Then, when negative current is used for transmitting marks, shortening the marks by substantially the same amount will result. When the systematic distortion is of such a nature that interchanging the functions of the two current values employed changes the sign of the systematic distortion but not its magnitude, the distortion will be referred to as *bias*, inasmuch as it indicates a lack of symmetry in the circuit.

Now assume a similar telegraph system in which the battery voltages are equal but which gives rise to distortion due to the fact that the current at the receiving end of the circuit is slow in building up. If the current does not have time to reach its final value on the short impulses, the first dot following a long space may be shortened. In this case it is obvious that interchanging the functions of the positive and negative current does not alter either the sign or the magnitude of the resulting distortion, the first dot of the C signal being shortened whether it is formed by means of positive current or negative current. If the systematic distortion is such that it changes neither sign nor magnitude on interchanging the functions of the two current values employed, it will be called *characteristic* distortion.

In general, it will be found, in measuring the systematic distortion, that neither of the two simple conditions considered above exists by itself. When the functions of the two currents are interchanged, it is nearly always found that the magnitude of the systematic distortion is changed but the sign may or may not be. This phenomenon may be described in a simple manner by saying that both bias and characteristic distortion are present and that the bias is reversed but that the characteristic distortion is not. In other words, it is convenient to say that the total systematic distortion, when the circuit is normal, is given by the expression

$$C + B$$

where C is the characteristic distortion and B is the bias and that with the reverse condition the total systematic distortion is given by the expression

$$C - B$$

The separation of the two components is then easily effected by simply adding and subtracting these measured values of the systematic distortion and dividing by two.

We are now in a position to give a definition of the components of the systematic distortion for the general case. Let us call the systematic distortion measured with the circuit normal S_1 and with the circuit altered so as to interchange the functions of the two current values employed S_2. Then the *characteristic* component is defined as $(S_1 + S_2)/2$ and the *bias* is defined as $(S_1 - S_2)/2$.

It should be noted that, in practically all cases, individual factors which cause distortion do not produce pure bias, characteristic distortion, or fortuitous distortion but rather a combination of these. One reason for this is that the effect of a particular factor depends on the extent to which the wave shape has been affected by other factors. Furthermore, distortion produced by a given factor in a particular repeater section depends on the impressed signal combination and this combination is, of course, changed by any distortion experienced previously; this is of importance mainly in connection with circuits made up of a number of repeater sections. As a result, the amount of one component of distortion as determined by the method outlined above depends to a secondary extent on the amount and sign of the other components. Distortion-corrrecting devices such as the Gulstad vibrating circuit[3] also tend to prevent linear addition of increments of distortion. It will be apparent from the foregoing that in order to obtain accurate data on total distortion, measurement should be made over the entire circuit with all components present.

In practical field work it is most generally desired to obtain a measure of the maximum total distortion which may be expected to occur at fairly short intervals. This maximum distortion is reached or exceeded when comparatively large characteristic and fortuitous components combine with bias in such a manner as to cause a comparatively large total distortion. Since bias is the most readily corrected or neutralized distortion, it is nearly always determined separately.

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[3] Described in *Metallic Polar Duplex Telegraph System for Cables*, Bell, Shanck and Branson, A. I. E. E. TRANS., Vol. XLIV, 1925, p. 316.

Many different signal combinations have been used in transmission measuring. The Morse letter C has been used for the most part since this was found to be a fairly severe combination from the standpoint of characteristic distortion. Miscellaneous signals such as occur in actual operation are, in general, preferable to anything else for the measurement of total distortion and are, therefore, used in some methods. For the measurement of bias, the use of "reversals" (a stream of dots and unit spaces) is convenient and gives good approximate data.

Distribution of Different Values of Distortion. It is of interest to note that the distribution of distortions of different magnitude is generally in fair agreement with the normal distribution curve of the theory of probability. This has been shown by results obtained in tests on a number of representative telegraph circuits in the Bell System. A distribution curve showing the quality of telegraph transmission of a particular circuit may therefore be constructed by measuring the total distortion of a number of parts of signals, obtaining the average distortion A, and the probable deviation d from the average. The latter is obtained from the following formula:

$$d = 0.6745 \sqrt{\frac{\Sigma\, r^2}{N-1}}$$

where N is the number of measurements, r is the difference between the distortion obtained in a particular observation and the average distortion A. The distribution curve may then be plotted from the formula given in Fig. 1.

Fig. 1 shows a typical curve plotted from the results of tests of a particular circuit at a manual operating speed (dotting rate about 12 per second). The basic idea of this distribution curve is that within a particular range of distortion (that is, on the horizontal axis) the probability of the distortion of any particular signal impulse falling in this range is indicated by the area under the curve in that range. Half of the area under the curve is included between the vertical lines drawn at distance d on each side of the vertical line passing through the peak of the curve, it being equally probable that a given impulse will have a distortion within and without this range.

If the distribution of distortion is in accordance with the normal distribution law, the two parameters A and d completely determine the grade of transmission. A circuit with good transmission has small average distortion and a high narrow curve whereas a circuit with poor transmission has large average distortion or a low flat curve or both.

The definition of perfect telegraph signals and the method of specifying the distortion or departure of signals from perfection could be formulated in other ways than that followed in the preceding discussion. However, it is believed that the criteria which have been set forth are very good for manual operation and are fairly good in connection with present printing systems and circuits[4]. As regards the latter, for complete information it would be desirable to determine not only the distortion as defined above but the relative displacement of marks and spaces because synchronism is involved in the operation of these systems.

Effect of Distortion on Operation. A large quantity of data has been collected in connection with tests of different telegraph systems and tests which have been made solely for the purpose of determining the effect of different amounts and kinds of distortion upon manual and printer operation. The effect of distortion will be discussed only very briefly here, however.

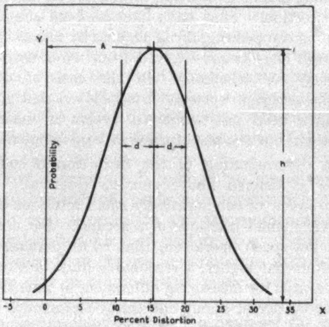

FIG. 1—PROBABILITY CURVE DERIVED FROM TELEGRAPH DISTORTION MEASUREMENTS

Scale of ordinates is so chosen that area under curve is unity. To find probable percentage of all marks or spaces whose distortions fall within a range of one per cent, read off corresponding ordinate from curve and multiply by 100. A = average distortion. d = probable deviation from the average distortion. $y = \frac{h}{\sqrt{\pi}} e^{-h^2(x-A)^2}$

It is thought that the limit which distortion may reach without seriously impairing service is about 35 per cent., as a round average figure, for either manual or printer circuits as used in the Bell System. In order for telegraph operation to be satisfactory, it must, in general, afford communication with very nearly 100 per cent accuracy. In the case of printers, there is no effect whatever until distortion becomes large enough to cause errors. In manual operation, however, the strain on the receiving operator due to distortion, as well as the effect upon accuracy of reception must be taken into consideration.

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[4] See *Printing Telegraph Systems Applied to Message Traffic Handling*, A. H. Reiber, TRANS. A. I. E. E., Vol. XLI, 1922, pp. 39-51, and *Printing Telegraph Systems*, J. H. Bell, TRANS. A. I. E. E., Vol. XXXIX, 1920, pp. 167-230.

METHODS AND MEANS OF MEASUREMENT

Several methods of observing telegraph transmission quality, which have been employed in the maintenance of commercial telegraph circuits and in the design and development work of the Bell System, will be described below. Generally speaking, methods 1 to 4, inclusive, are older methods in which standard telegraph apparatus is used in connection with making certain observations and accordingly are discussed only briefly. Methods 5, 6, and 7 are newer methods employing telegraph transmission measuring apparatus specially designed for the purpose and will be treated more fully.

The more elaborate and refined methods have been used to advantage in connection with development work on d-c. and carrier telegraph systems including balancing artificial lines and arrangements for minimizing distortion. They have likewise been employed in designing compositing arrangements by means of which telegraph circuits are derived from wires used simultaneously for telephony. In the case of the d-c. metallic telegraph system,[5] it is believed that it would not have been practicable to evolve a satisfactory design without the use of these or equivalent methods.

The determination of the distortion of telegraph signals as defined above consists essentially in the measurement of comparatively small intervals of time, *i. e.*, very small fractions of a second. For comparatively low-speed operation, that is, at manual speeds and somewhat higher, a reasonable limit of sensitivity of apparatus for measuring differences in time intervals is of the order of about one thirty-thousandth second for refined laboratory tests. For general field work, a sensitivity of the measuring apparatus of about one thousandth second is usually sufficient. These sensitivities are found to be well within the unpredictable variations in circuit performance from time to time.

Measurements of distortion by means of the methods described herein have given results which furnish a good criterion of the quality of telegraph circuits, data thus obtained being, in general, reasonably consistent with observations by highly trained telegraphers in the case of manual circuits, and with printer performance in the case of printer circuits. It will be appreciated that these data, being practically independent of judgment on the part of the observer, are considerably more accurate and dependable than those obtained by older methods in the case of manual circuits. As regards printer circuits, some inconsistency is to be expected, for reasons which were brought out in the discussion of distortion. However, experience to date indicates that relations between the results of measurement of distortion and printer performance are fairly consistent and the new methods should be of considerable value for use in connection with printer circuits.

1. Listening Tests. On manually-operated circuits, the quality of transmission may be observed by means of listening tests. In making such tests, a good sender should send signals at the distant end of the circuit while a competent operator at the receiving end listens to a sounder. The receiving operator then forms judgment as to whether the signals are biased or unsteady and whether transmission is satisfactory or not. Unsteadiness, of course, indicates the presence of considerable characteristic or fortuitous distortion or both. It is very convenient to make these observations since no special apparatus is required. In many cases it can be done without interfering with the normal operation of the circuit.

Experience has indicated, however, that there is considerable uncertainty in connection with such observations since they depend upon personal judgment. Skilled operators with considerable experience in passing judgment on telegraph signals often disagree and a particular operator does not form consistent opinions from time to time. It is, of course, impossible to detect small distortions or differences, as is required in connection with development work. In lining up circuits for service by means of listening tests, improper adjustments may be made in order to overcome faults of the sender or to suit the tastes of individual receiving operators.

2. Meter Observations. Ammeters and voltmeters which are not sufficiently fast to follow all the individual signal impulses faithfully are used to a considerable extent for detecting bias and unsteadiness of circuits when a steady stream of dots and short spaces, or reversals, is sent over a circuit. A measure of the bias is obtained by taking the difference between the meter reading for biased signals and the reading for unbiased signals. The presence of fortuitous distortion is indicated if the meter needle vibrates unsteadily or if sudden deflections occur occasionally. This method is particularly useful in maintaining multi-section circuits, especially those operated at high speeds. It is usually necessary to use the meters in local circuits rather than directly in the line and to take certain precautions to avoid misleading results.

3. Tape-Recorder and Similar Methods. A tape recorder, such as that used in the Wheatstone system of telegraphy, giving a graphic record of telegraph signals has been of considerable use in telegraph transmission investigations made by Bell System engineers but it has been almost entirely superseded by improved testing apparatus. Tape records properly taken and analyzed furnish complete information on the signal distortion (as defined above) and displacement of marks and spaces. The process is complicated and very laborious, however, the results not being available for a considerable time, and the accuracy with available apparatus is not great. However, a quick but rough idea of the quality of transmission may be obtained by merely inspecting the tape records.

In using the recorder, it is preferable to impress a perfect, regularly recurring test signal at the distant end and take two tapes, one with normal signals and the other with "inverted" signals, that is, with marking and spacing interchanged. A number of signals may then be measured and the components of distortion may then be computed.

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[5] See Note 3.

Tape recorders have been used in connection with making transmission observations on working telegraph circuits also. Two recorders are employed, one at the sending end of the circuit under test and the other at the receiving end. The effect of transmission over the circuit can be ascertained by comparing received marks and spaces with the corresponding sent marks and spaces.

It has been found practicable with some refinement of standard apparatus to obtain an accuracy of about ± 3 per cent at a speed of 15 dots per second with recorders, corresponding to ±0.001 second.

A recording voltmeter has been employed to advantage in determining the stability of transmission over telegraph circuits. In doing this, a continuous record is made of received signals while reversals are sent over the circuit. The record will show not only interruptions of material extent but gradual changes in bias.

Other instruments, such as Morse recorders, oscillographs, and "undulators," have also been employed for making records to show the quality of signals.

4. Tests With Printers. The start-stop and multiplex printing telegraph apparatus,[6] which is in general use in this country, is adaptable to simple manipulation which gives a very good indication of the quality of transmission for printer operation. A test which is commonly made is a determination of margin of orientation. Another test which is used in some cases is called a determination of bias margin.

In order to explain these tests, certain basic principles involved in the operation of these systems will be briefly outlined using a typical system for illustration. In the typical system, each printer operation requires five units of line time, five impulses being sent, one after the other, from segments on the sending "distributor" and received in the same sequence on corresponding segments of a distributor at the receiving end. The brushes of the sending and receiving distributors, of course, rotate in approximate synchronism. The receiving segments are shortened so that only the middle portion of each incoming signal unit is used for operating the selecting arrangements and, therefore, distortion must exceed a certain amount before there is any effect whatever upon the accuracy of the received message. A phase or "orientation" adjustment is provided so that each segment may be traversed by the rotating brush so as to receive a signal from the line at the proper time. This permits the obtaining of orientation margin by rotating the ring of receiving segments a short distance, first one way and then the other, until errors appear in the printed copy.

In order to obtain a measure of the quality of transmission of the line circuit, the orientation margin is first determined locally and then a similar determination is made with signals transmitted from a distant station. As the printer may be assumed to be operating on substantially perfect signals on the local test, the difference between the margins found in the two tests is a measure of the distortion of the line circuit.

In making a test of bias margin, the printing apparatus is first tested locally; then with the most favorable orientation setting, the signals are biased in one direction and then in the other until errors are noted in the printed record. Similar tests are then made with the distant station sending over the line circuit. Bias may be impressed at the receiving end, in which case the difference between the range which can be impressed without causing failure and the local bias range is a measure of the excellence of the signals as normally received over the circuit. When bias is impressed at the sending end of the circuit, the corresponding difference is a measure of the amount of distortion with which signals could be repeated into the line without causing failure; this test is, therefore, of most value in checking up parts of long circuits.

For printer circuits, this method has the advantage that measurements are made with apparatus identical with that used in operation and the results are readily interpretable in terms of performance. Good data on total distortion for use in connection with printer operation may be obtained with miscellaneous signals. By noting how failure occurs for various signal combinations in the neighborhood of the limits of the range, some information may also be obtained as to the components of the distortion.

5. Bridge Methods of Measuring Systematic Distortion. In this subdivision there will be described arrangements which have been employed advantageously for a number of years in making accurate measurements in connection with development work. These devices are in the form of bridge arrangements in which galvanometers give direct indication of the amount of systematic distortion. Simple computations from readings made before and after reversing certain connections give data on bias and characteristic distortion. These devices do not measure the total distortion and are therefore not well suited to use in connection with transmission maintenance.

As these arrangements are more accurate than any others which have been available, they have been very useful in showing the effect of small changes in circuit elements. With these devices, it is possible to make design tests on single section circuits, whereas, with less refined methods, it would be necessary to use a number of sections to obtain values of the distortion which could be measured.

One of these arrangements requires synchronism between the sending and receiving ends and is, accordingly, generally used in looped tests; that is, where both sending and receiving apparatus is located at one point. Another arrangement, which is considerably simpler, does not require synchronism and can be used for either looped or straightaway tests.

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[6] See Note 4.

A. *Synchronous bridge arrangement.* The synchronous arrangement for measuring systematic distortion employs a differential method in which signals received over the line are compared with substantially perfect signals in a circuit of the Wheatstone bridge type.

The basic principles of this device may be understood from Fig. 2. As indicated, relay 1 is operated by the incoming or distorted signal and relay 2 is operated by a perfect signal which is sent locally. If the armatures of these two relays move exactly together, there will be no deflection of the galvanometer inasmuch as the four outer resistances all have the same value and the four inner resistances are also equal to each other. It will also be seen that if both relays repeat perfect signals, but with a phase difference, the average current through the galvanometer will be zero. (There is no net effect of armature travel time if it is the same for both relays.) If a particular mark repeated by relay 1 is distorted systematically, there will, however, be a preponderance of current in one direction or the other through the galvanometer every time this mark occurs. Now, if a switching device be provided to close the galvanometer circuit only shortly before the beginning of this mark and open it shortly after its end, a slow-moving galvanometer will indicate directly the sign and magnitude of the distortion, provided the bridge is properly proportioned. Measurement of normal and inverted signals and simple computation will, therefore, afford data on bias and characteristic distortion of this part of the signal and similar procedure will give corresponding data for the other parts.

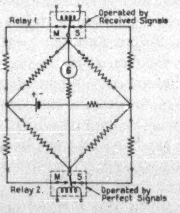

Fig. 2—Synchronous Distortion-Bridge Arrangement

This device has been used for very accurate measurements of the effect of duplex unbalance and some other kinds of interference by sending the interfering current from a distributor running at a speed slightly different from twice that employed with the bridge and noting the maximum deflection of the galvanometer as it swings slowly back and forth. The bridge is readily adaptable also to the measurement of relative displacement of signal parts or lag, as well as distortion, which may be desired, for instance, in connection with printer operation.

Several bridges of this type have been built which are capable of indicating differences in distortion of about one thirty-thousandth second. A multiplex printing telegraph distributor has been adapted to the sending of signals and performing the selecting operations.

For a detailed description of these arrangements, it is suggested that reference be made to U. S. Patents Nos. 1,435,328 and 1,548,059.

B. *"E" signal bridge.* This arrangement has the advantage of comparative simplicity and portability and also of being usable for straightaway measurements of systematic distortion. It consists of a simple Wheatstone bridge circuit so arranged that the average current through the meter is proportional to the distortion when an "E" signal having a certain ratio of marking to spacing is used. The accuracy is substantially the same as that of the synchronous bridge arrangement.

In applying this method, an "E" signal with, for example, a space four times as long as the mark is sent repeatedly at the distant station and the bridge arrangement shown in Fig. 3 connected to the receiving end so that the bridge relay repeats the incoming signals. When the tongue of this relay is on the marking contact *M*, the galvanometer current is in one direction and of strength 4; when the tongue is on the spacing contact, it is in the opposite direction and of strength 1; the average is, therefore, zero. A resistance is connected from the tongue of this relay to the right corner of the bridge so as to avoid any effect of time required for the tongue to travel from one contact to the other. The average current through the meter will be such as to cause it to indicate directly the magnitude and direction of distortion of the unit part of the "E" signal when proper voltage is used.

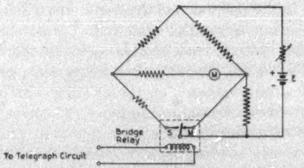

Fig. 3—E-Signal Distortion-Measuring Bridge

It is of interest to note that results obtained with the "E" signal have been nearly as satisfactory as those obtained with more complex signals such as the Morse C.

6. Speed-of-Failure Measurement. The speed at which total failure (100 per cent. distortion) occurs is of interest in connection with telegraph transmission work. The speed of failure must, in general, be materially higher than the operating speed to provide a margin for variations and in order that the circuit may handle signals which are considerably distorted before being impressed upon the circuit.

The "speed-of-failure meter," the essential features of which are shown in Fig. 4, is a convenient arrangement for obtaining a measurement of the breakdown speed of circuits with a regularly-recurring signal such as the Morse letter C. It may also be used for determining the speed of a regularly recurring signal. As will be seen from the figure, the condenser *C* is charged by the battery *E* while the relay armature is on marking contact and is discharged through the galvanometer *M* when the armature is on spacing contact. The time constants of the circuit with the armature on either contact must be small in comparison with the duration of greatly distorted dots and spaces sent at the speed of failure. Then, if a recurring signal is operating the relay at a certain comparatively low speed, the number of impulses passing through the meter is directly proportional to the speed and therefore a slow-moving meter may be made to indicate speed directly.

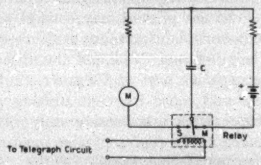

FIG. 4—SPEED-OF-FAILURE METER

In measuring the speed of failure, the speed is gradually increased from a low value while the indication of the meter is noted carefully. When the speed reaches the breakdown point, the deflection of the meter, which had been increasing gradually, is suddenly reduced. Although the deflection will then increase further for a time as the speed is increased (until another part of the signal fails), there is no difficulty in noting the breakdown speed with fair accuracy.

7. *New Method for Convenient Measurement of Total Distortion.* This method, which has recently been devised, allows the quick, convenient and accurate measurement of the total distortion of regularly-recurring signals, including both fortuitous and systematic effects. To accomplish this the problem was approached in a different way from that followed previously, the plan being to provide arrangements which would give a response only when the total distortion of signal parts under observation exceeded a particular amount determined by the observer. As has been brought out, since the distribution of distortions of different value is generally in reasonable agreement with the normal curve of probability theory, a few measurements by this method serve to give fairly complete information regarding transmission performance of a circuit.

Two types of transmission measuring sets have been designed for use in the new method, one of these being a simple arrangement intended primarily for field work and the other a more elaborate device suitable for laboratory tests. These sets are arranged so that a condenser is charged during each mark (or space) and a vacuum tube arrangement provided to give an indication whenever the condenser charge exceeds a predetermined amount. In this way, the amount of distortion which is exceeded with a certain frequency, or the frequency with which distortion exceeds any particular value, may be determined. In addition, the sets are arranged for the measurement of bias separately. Synchronism between sending and receiving apparatus is not required so that straightaway as well as looped tests may be made. Since the principles of operation of the two arrangements are nearly alike, the operation of the simpler type of set only will be taken up in detail.

A. Measurement with field-type set. The principles of this set for measuring total distortion of marks and spaces will be described in connection with Figs. 5 and 6, respectively. Fig. 5 shows a charging and discharging arrangement which produces a voltage across the condenser *K*, the value of which depends upon the duration of the mark.

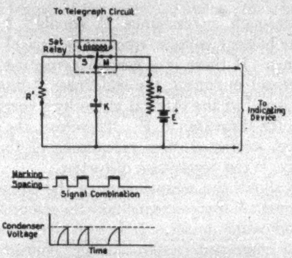

FIG. 5—CIRCUIT FOR MEASUREMENT OF MARKS

Suppose that the relay is operated by an undistorted normal "C" signal, such as is illustrated in the figure. During each mark the condenser *K* will be charged, as indicated at the bottom of the figure, to a value which depends on the voltage of the battery *E* and the value of resistance *R*. It will be practically discharged through the low resistance *R'* during each space. By suitably proportioning *K* and *R*, the voltage across the condenser *K* can be made to assume, during the time of a mark, any value less than the voltage of the battery.

Now, if one of the marks is distorted, the condenser voltage will be greater or less than that for an undistorted mark, depending upon whether the mark is lengthened or shortened. If, for instance, all of the marks are lengthened, the voltage at the conclusion of each mark will be greater than that for undistorted marks.

The voltage which the condenser attains during a distorted mark may be adjusted to the value for undistorted marks by changing the value of resistance *R*, the amount of change being a measure of the amount of distortion.

As is well known, in a circuit of this type, the voltage of a condenser of capacity *K*, after being charged for time *t* through a resistance *R*, equals $\left(1 - e^{-\frac{t}{KR}}\right)$ times the battery voltage, *e* being the base of Naperian logarithms. If there is a certain percentage increase (or decrease) in time *t*, the condenser voltage may be brought back to the original value by a corresponding increase (or decrease) in *R*. Therefore, the change in resistance, expressed as a percentage of the original resistance, is the percentage distortion.

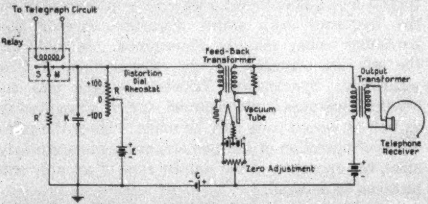

FIG. 6—FIELD-TYPE SET FOR MEASURING TOTAL DISTORTION ARRANGED FOR MEASUREMENT OF SPACES

Fig. 6 shows the arrangement of the circuit for the measurement of spaces and includes the essential features of the indicating device. It is seen that condenser *K* has been connected to the marking contact and the resistance *R'* to the relay armature, so that the condenser will be discharged during the marking interval and allowed to charge during the spacing interval. The duration of the space thus determines the amount of charge during the spacing interval. The resistance *R* may be used to control the rate of charge of the condenser and the percentage distortion of spaces determined in the same way as described above in the case of marks.

By reference to the part of Fig. 6 which shows the indicating device, it is seen that the input or grid circuit of the tube is connected across condenser *K* with a grid-bias battery *C* in series. The grid-bias voltage is of such value that the plate current is zero until the voltage of condenser *K* reaches that for an undistorted dot at which time the plate current suddenly increases. This is due to the feed-back effect which is obtained by coupling the input and output circuits of the tube together by means of a transformer. Therefore, the circuit commences to oscillate at an audible frequency whenever the potential of the grid reaches a certain value. This oscillation is allowed to persist for only a short time with the result that a click is heard in a telephone receiver connected in the output circuit whenever the voltage of the condenser exceeds that for undistorted marks.

The arrangements described above are suitable for the measurement of total distortion of marks and spaces of almost any recurring signal combination. In case the signal contains only unit signal parts, as for instance, when measuring the marks of a "C" signal, the distortion indicating dial is set at a position where occasional clicks are present, say several per minute, this value of distortion being taken as the representative maximum lengthening. Now, as the dial is moved towards the negative part of the scale, the clicks will become more and more frequent until a position is reached where clicks are missing only occasionally, this being taken as the representative maximum shortening. When measuring spaces of the "C" signal, undesirable noises are present in the receiver due to the presence of the two long spaces. In some cases this causes difficulty in measuring at high speeds but is not particularly bothersome at manual speeds. If desired these long parts may be measured by adding a lumped resistance in series with the rheostat so that the voltage across the condenser for undistorted parts of the new length is restored to the original reference value.

This set is also arranged to measure directly percentage bias when reversals are used as the test signal, the circuit arrangement for this purpose being shown by Fig. 7. A bridge circuit, containing a meter, is connected to the receiving relay. Equal and opposite currents flow through the meter when the relay armature is on the spacing contact *S* and marking contact *M*, respectively, provided that resistance *R* is one-half of resistance *R'* and resistances *r* are small. If the relay is adjusted to repeat unbiased reversals, the average current through the meter will be zero and the meter needle will vibrate through a small amplitude about the zero mark. With biased reversals, the average current through the meter is no longer zero, but it can be brought back to zero by adjusting resistance *R*. The meter will always indicate zero when resistance *R* has been adjusted to correspond to the amount of bias, the percentage

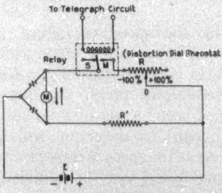

FIG. 7—CIRCUIT FOR MEASUREMENT OF BIAS

change in R being equal to the percentage bias in terms of an undistorted dot length.

It will be recalled that this relation also holds for resistance R when measuring total distortion so that only one calibrated rheostat is necessary. This rheostat is designed to cover the range from plus 100 to minus 100 per cent., with resistances corresponding to 5 per cent distortion for each step. This is sufficiently close for most field work. Intermediate values may be roughly estimated, however.

Before making a series of measurements, the set is calibrated with substantially perfect reversals. In doing this, the relay is adjusted to repeat the signals unbiased and the vacuum-tube circuit is adjusted so that clicks are just produced in the receiver for about half of the dots. The latter is called the zero adjustment and is accomplished by a slight variation in the grid-biasing potential.

In making a measurement over a circuit the bias is first measured, then the marks and spaces of both normal and inverted Morse "C" signals are usually observed to determine the representative maximum to taldistortion. If desired, a rough idea of the value of the characteristic component may be obtained by measuring the total distortion on both "C" signals and reversals and taking the difference, since in the latter case there is practically no characteristic distortion present.

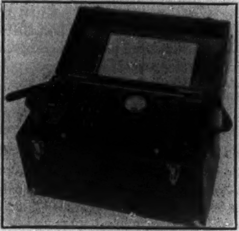

FIG. 8—FIELD-TYPE SET FOR MEASURING TOTAL DISTORTION

Fig. 8 is from a photograph of one of the field type measuring sets. As may be seen both a neutral relay and a polar relay are provided so that either may be used as the receiving relay. The distortion-indicating dial, meter, zero-adjusting dial, switching keys, and jacks are mounted on a panel which may be locked. Underneath the panel are mounted the necessary batteries, resistances, condensers, transformers, and vacuum tube. A Western Electric 215-A tube which requires only small currents is employed. Accordingly, it is possible to employ a dry cell to energize the filament, and small B batteries. The set is entirely self-contained with the exception of a source of biasing current for the polar relay when receiving with it in an open-and-close local circuit.

B. Laboratory type of measuring set. The essential features of the laboratory type of measuring set are illustrated by Fig. 9. The condenser C_1 corresponds to condenser K of the field-type set and the variable resistance R_1 in its charging circuit corresponds to the distortion dial. Although R_1 might be used for the distortion dial in this case, it has been found more desirable to use a potentiometer P for this purpose. This potentiometer is connected into the charging circuit of condenser C_2 and permits of varying the applied voltage, and consequently, the value of charge taken by this condenser. For a given battery voltage the amount of the charge stored on condenser C_1 depends upon the length of the mark (or space), but that stored on C_2 depends, for practical purposes, only upon the setting of potentiometer P. At the end of the mark, the two charges are first combined so that they tend to neutralize each other and the residue discharged through the meter M and resistance R_3. These operations are performed in the proper sequence by the relays D_1 to D_3, inclusive. The potentiometer calibration holds true for any particular speed of signaling, provided the resistance R_1 is adjusted to the proper value for that speed.

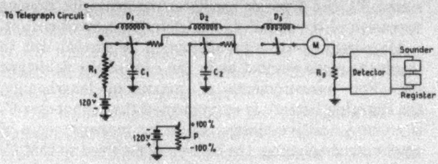

FIG. 9—LABORATORY-TYPE TELEGRAPH TRANSMISSION MEASURING SET

Schematic Circuit Diagram.

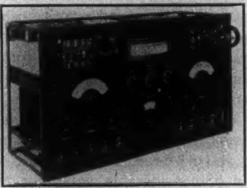

FIG. 10—LABORATORY-TYPE SET FOR MEASURING TOTAL DISTORTION

To measure average distortion of a recurring test signal the potentiometer is adjusted until the meter M indicates zero, under which condition the charges of the two condensers are on the average equal and neutralize each other. By taking readings with normal and inverted signals, the bias and characteristic distortion may be calculated as explained previously. If an "E" signal is used, good results will be obtained in this manner. However, with other signals the results being average values may be misleading, as positive distortion of one impulse tends to offset negative distortion of others. If reversals are used for the signal, it may be assumed that there is no characteristic distortion and the measurement will give the bias.

To measure total distortion, the detector shown connected across R_3 is used instead of the meter, this detector being responsive only to positive discharges through R_3. Whenever such a charge is impressed across R_3, the detector oscillates momentarily causing operation of the sounder, or register associated with it.

Measurement of total distortion is carried out in much the same manner as in the case of the field-type set. For measurements of maximum lengthening, the charging battery is so connected that condenser C_1 is given a positive charge, while for measurements of maximum shortening, the battery is reversed so that C_1 is given a negative charge. By means of the register, data may conveniently be obtained for plotting distribution curves of distortion.

The complete set as shown in Fig. 10 contains numerous switching arrangements to adapt the set to various conditions of measurement. It also includes a speed-of-failure meter. Incidentally the set is provided with an arrangement which can be used to suppress dashes and long spaces so that measurement may be made of miscellaneous printer signals.

The distortion dial is graduated in 2 per cent steps and intermediate values may be estimated. In the measurement of bias and characteristic distortion, the set may be readily adapted to the detection of much smaller differences.

CONCLUSION

A number of methods have been described for measuring the transmission quality of telegraph circuits. Which method to use in a particular case depends on the circumstances. For the purposes of transmission maintenance, the need has existed for a convenient method of fair accuracy for measuring the total distortion. In this connection it is believed that the method and means described under 7-A above offers considerable promise. Considerable use has been made of this type of apparatus by the development and engineering staffs of the Bell System and experience indicates that it will probably be of considerable value for the following purposes, particularly in connection with manual systems: (1) Obtaining quantitative data with a view of determining whether or not telegraph circuits are satisfactory for service, (2) Making routine checks of telegraph transmission, (3) Lining up circuits for service, and (4) Locating and diagnosing troubles. In the case of printer circuits it should be of most use in connection with item 4.

Discussion

J. H. Bell: The old method of evaluating the quality of telegraph signals was by running a tape at a particular speed and measuring each dot and each space and comparing with the length of a perfect dot. That required a good deal of time to carry out. The experimenter in the laboratory, after making each change in his circuit, had to measure his tape before he could proceed any further.

With these newer tools we can change a coil or a relay and at the same time listen to the effect upon the circuit, so that the tools make for much quicker development work than hitherto.

The telegraph circuits in this country have a great many repeaters, and, as you will find in the paper, the maximum allowable distortion between terminals is 35 per cent. That does not mean that 35 per cent is an allowable distortion in one section. The distortion is cumulative from section to section, so that the permissible distortion in one section must be kept down to say, 4 or 5 per cent.

The old method did not permit of measuring 5 per cent distortion with any degree of accuracy, as pointed out in the paper; about plus or minus 3 per cent was the best that could be done. That was due to the variations in the speed of the tape running through the Wheatstone receiver.

In the older days when the Wheatstone was the system for carrying heavy traffic, it was perfectly satisfactory if one got all the dots, and was able to distinguish between the dots and the dashes; the human factor could then come in and make up the discrepancy and translate the tape without any difficulties.

However, today with the growth of machine telegraphs, it is necessary that the amount of distortion be kept down to a definite limit, and these new tools will certainly be a great aid in carrying out our experiments. Even in hand-operated systems, I question whether any operator can detect a few per cent distortion.

H. W. Drake: A rather significant statement is made on the first page of this paper, which leads me to ask a question. The statement I refer to is that "in the case of some printer circuits, variation in lag degrades transmission. Consideration of lag is usually not of importance, however."

The question I wanted to ask is whether these admirable means which have been devised in the Bell System for measuring transmission include the possible measurement of this variation in lag.

The reason for asking that question is the fact that I am aware of a good deal of work that is being done by the commercial telegraph companies, and this matter of determining variation of lag has been one of the important factors in that work. I think that I would not be stretching the case to say that this paper is probably in the nature of a progress report and that the authors would be the last to have it taken as the final statement of what can and must be done in the measurement of telegraph transmission.

H. Nyquist: We have successfully used both the synchronous distortion bridge arrangement shown in Fig. 2 and the Wheatstone recorder in the measurement of lag and its variation. The field type of measuring set is not readily adaptable to this kind of measurement.

Telegraph Traffic Engineering

BY H. MASON[1] and C. J. WALBRAN[1]
Member, A. I. E. E. Non-Member

THE subject, telegraph traffic engineering, covers a field so wide that to compress it within the limits of a single paper necessitates an arbitrary limitation of the points to be discussed. For this reason there is here presented an outline of only the following four points:

1. Wire layout.
2. Traffic routing.
3. Office layout.
4. Operator assignment.

WIRE LAYOUT

As the economic conduct of any business activity is dependent upon a system of adequate, accurate, and up-to-date records, so the economic use of a large telegraph wire plant requires a somewhat elaborate scheme for recording the kind and amount of use made of the various facilities as well as for directing changes to be made in such use. The Western Union Telegraph Company has over a million miles (1,600,000 km.) of wire used for the handling of commercial telegrams and for other telegraph company activities. This figure does not include the mileage of wire owned but used by the railroad companies under contracts nor does it include the conductor mileage in underground and aerial cables used in city-wide distribution. This mileage of wire can be divided as to usage, as follows:

Wire used as	Miles of wire	Number of msgs. carried per month
Automatic trunks	266,000	20,100,000
Morse trunks	165,000	4,450,000
Way wires	296,000	3,270,000
Spare and protection	174,000	
Dispatcher and test	30,000	
Other uses	92,000	
	1,023,000	

"Other uses" cover wires in the service of disseminating quotations of stocks and commodities either by Morse or by ticker; wires leased to firms and especially to Press Associations for their use in Morse or automatic service, the so-called "private wires;" and wires given over to some small scattering uses, battery, clock, telephone tie lines, and the like.

In reading the foregoing table one must guard against taking the figures in the right hand column as representing the messages filed with the company by the public. The figures are quoted to indicate the relative density of load on the circuits of different sorts and a given message is counted again each time it is relayed from one circuit to another. That company's report to the Interstate Commerce Commission showed for 1925 a total of 146 million messages filed. It will be noted from the table how vastly more economical is the use of wire in automatic service. There a density of 76 monthly messages per mile of wire is achieved while in Morse trunk service 27 is the prevailing figure and on way wires only 11 can be counted on. Yet over 29 per cent of the whole wire plant is given over to way service, chiefly as the result of adherence to the policy of providing universal telegraph service.

The traffic engineer should be responsible for the amount and general location of wire used and for the economic assignment of circuits to these wires. The material, method of erecting, and the detailed location of pole lines and wires are properly the consideration of other departments but these, broadly, must be designed to provide what is required in the way of operating circuits. To this end there must be established a section of the traffic department whose duty it is to maintain a record of wires and of assignment of circuits to those wires, and long experience has shown that it is entirely impractical to maintain such a record unless the same section is given authority to designate the assignment of all wires and to insist that field forces recognize and conform to its orders. This section receives from divisional headquarters in the field suggestions as to improving operating arrangements and is itself continually busy with investigations looking toward improved layouts.

Monthly reports of the volume of traffic handled on each circuit terminating in all important offices are received. These reports show the loads, preferably by half-hourly periods, throughout a typical day and segregate the business into full rate, day letter, and night letter traffic. With these data and a knowledge of the practicable operating speeds of various circuits the circuit layout engineer can determine how best to utilize the wires at his command and can justify to that authority which appropriates the money, his requests for additional new wire construction. He daily issues orders in a well defined form, copies of which are sent to every field office which may be interested in the changes. The orders designate changes in circuit assignment together with the date upon which they are to be made effective. These changes are thereupon indicated on the chart of wires which he maintains as his primary record. This chart, with a line for every wire, shows the route followed, all towns being indicated and notations made as to all switchboards at which each wire is cut in either for operation or for test. The material and gage of the wire are shown and at each junction office the cross connections

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[1] Both of the Western Union Telegraph Company.

Presented at the A. I. E. E. Winter Convention, New York, N. Y., February 7-11, 1927.

are indicated. These latter are customarily in lead pencil in order that alterations may readily be made as orders for circuit layout changes are issued. The chart for the million miles of wire before mentioned is 65 feet (19.8m.) wide and 24 feet (7.3m.) high and, of course, has to be maintained in sections.

In determining upon advisable layouts many facts must be taken into consideration and these vary with the type of circuit involved.

Taking up first the simplest of the circuits, the way wires, we can define as such all circuits that have intermediate offices or "drops" between the terminal stations. Almost invariably they are operated "single Morse," that is by the opening and closing of a circuit in which unidirectional current may flow, though in rare cases where the circuit is very long duplex operation with its greater reliability due to use of current of one polarity for "marking" and of the other polarity for "spacing" is used between the relay office and some repeater point at which a "half-set" connects it to a single Morse wire on which are located all the operating drops.

The volume of business which may be handled on a given way wire is to a considerable extent dependent upon the operating ability of the personnel at the various offices and particularly upon the experience and the judgment of the operator customarily working this wire at the relay office. Then, too, in an effort to give universal service provision must be made for handling business from offices so small that it obviously cannot be in itself profitable. For these reasons the provision of way wires is largely a matter of determining the most economical way to give service under particular conditions rather than a determination of the economy of giving such service. Broadly speaking, it is found that with a number of small offices, say ten or twenty on a way wire, a load of about 150 messages per day is all that is consistent with a satisfactory speed of service. Naturally where distances are great, as particularly in the western part of the country, and the way wires of necessity are very long, a somewhat greater density is enforced before relief is given. An ordinary way wire operated "single Morse" with current supplied generally by motor generators at each end of the line, can comfortably be operated up to about 150 miles (240 km.). Many circuits exceed that length but it is the tendency to so locate relay offices that wires may be shortened to about this figure. The longest way wire in the Western Union plant today is from Denver, Colorado, to Farmington, New Mexico, 497 miles (800 km.), with an intermediate single line repeater at Alamosa. Arrangements are now under way for the installation of equipment at Alamosa to permit relaying at that office this and adjacent way wires, thence handling the business to Denver on an automatic circuit.

If iron wire is used, the possible limit of mileage in a circuit depends upon the amount of deterioration and as the older wires are customarily put into way service it is not unusual to find cases where such circuits run up to 18 ohms per mile (11.2 ohms per km.), though new No. 8 gage iron would measure about 12.5 ohms. Where the abnormal resistance of an iron wire is found to be largely in the joints, conditions may be improved by soldering, brazing or welding as suggested in the paper presented by Mr. Stanley Rhoads (TRANS, A. I. E. E., 1921, Vol. XL, p. 301). Naturally, the use of No. 9 gage copper, averaging about five ohms per mile (3.1 ohms per km.), improves conditions greatly but insulation plays its part and even with the best of wires it is found that satisfactory service cannot in general be maintained on a way circuit of over 300 miles (480 km.) in length without repeaters.

As the load on a way wire grows, relief may be provided by constructing a new wire from the relay office to a point about midway of the load thus giving room for approximately 100 per cent growth. Quite often, however, the major portion of the load of a congested way wire will be from an individual office in which case the new construction may be placed between that town and the relay office thus establishing a trunk, naturally much underloaded at first, but which, it may be hoped, will gradually grow in usefulness until the day comes when the load justifies duplexing it. Occasionally, as a territory develops, it becomes necessary to establish new relay offices in order to avoid an uneconomical arrangement of many long way wires. The form of an engineering study of the features involved in determining upon the propriety of such a step might alone easily suffice as material for a paper the length of this.

The next general class of wires comprises the Morse operated trunks. With the present development of the printing telegraph, the establishment of new Morse operated trunks is indeed rare except in the manner just above indicated where a way office becomes sufficiently important to have an exclusive wire. Since several printer channels but only one Morse circuit may be put on a wire and, too, since it is easier to secure and quicker to train automatic than Morse personnel, it is not surprising that the duplex Morse trunks are diminishing with the growth of the automatics. The present field for duplex Morse trunks is largely confined to those offices which are maintained in connection with grain, cotton, and other commodity exchanges. In these situations where the messages are short and where the operators are especially selected very substantial loads may be secured. But better loads could be carried by automatics, and we may confidently look forward to the time when, with an improved technique and with customers' prejudices swept away, all important trunks, including those to exchange offices, will be operated automatic.

Broadly speaking when a duplex Morse trunk load grows to over 300 messages each way per day, the limit for a satisfactory speed of service has been reached and it is time to consider measures for relief. Generally it will be found more economical to install automatic equipment, with its attendant maintenance charges but greater capacity, than to construct new wire. Particularly is this so if the offices involved already have such maintenance in connection with other circuits.

The single Morse trunk, the outgrowth of the way wire as previously described, occupies a wider field. Even here the evidence of trial installations suggests that a single channel printing mechanism of simple structure and substantial operation is, or shortly will be, available with economic advantages over Morse operation so that we may reasonably look forward to the time when manual Morse operation will have disappeared from all towns large enough to warrant a trunk circuit for an outlet and which enjoy 24 hour electric light service to provide the necessary operating power.

A load of 300 messages per day, counting both directions together, is about all a single Morse trunk may be expected to move with a satisfactory speed of service. Relief in general would be secured by duplexing the wire, requiring a more elaborate power plant to supply the two polarities needed by this method.

In taking up the third and most important class of circuits, the "automatic" trunks, it may be well to point out that the word universally used in the industry is somewhat of a misnomer. Really what is meant by "automatic" circuit is a circuit operating printing telegraphs. As a perforator operator does the sending and a receiving operator arranges the blanks in the machine or gums down the tape as the case may be, checks the number of words and calls for the necessary corrections, the only human operation that has in fact been eliminated is the actual typing up on the receiving end. The productiveness of the labor under the two systems is, however, marked. An experienced automatic operator works at between 55 and 60 words per minute while the average output per Morse operator is between 12 and 15 words.

The tendency toward automatic operation on trunk circuits is to be noted, the largest company handling about 80 per cent of its trunk line traffic by this means. In present usage the relation between practical line speeds and customary operator speeds sets a limit of three channels per circuit, these being duplexed so that six messages can be handled simultaneously, three eastbound and three westbound. However, four have been operated in many instances and the indications are that further improvements in the art will result in increasing to at least four and possibly to six channels in the majority of cases. It appears that the average automatic operator can be safely expected to perforate at the rate of 60 words per minute, a word averaging 5 characters and a space. The five unit code used in automatic telegraphy uses 2½ cycles per character so a channel speed of 60 words per minute corresponds exactly to a speed of 15 cycles per second. Therefore a three channel circuit has a line speed of 45 cycles. The present practise indicates that this speed may be maintained on wires up to about 1000 miles and involving probably three repeaters. Longer circuits are operated two channel at line speeds of from 25 to 30 cycles, and such circuits can be and are operated over the greatest distances covered. Specifically, the longest automatic land line circuit regularly operated is one of the New York-San Francisco wires, which is 3779 miles (6100 km.) along the route followed with 13 repeaters located at Washington, Parkersburg, W. Va., Cincinnati, St. Louis, Little Rock, Texarkana, Tex., Dallas, Sweetwater, Tex., El Paso, Tucson, Ariz., Yuma, Ariz., Los Angeles, and Fresno, Cal.

In all of these longer automatic circuits at least one and perhaps two of the repeaters are of the rotary regenerative type.

It is possible, by suitable installation of apparatus, to drop off one channel of an automatic at any repeater point, or to intercept one or more channels and work them both ways. Thus the Boston-Cincinnati circuit is equipped for three channel operation one channel working between those cities and two channels cut at Pittsburgh giving Pittsburgh two channels to Boston and two to Cincinnati. Extensive use is made of this ability in situations where the traffic is somewhat lighter than would justify the use of an exclusive wire. For instance, the business of Dayton for New York or of Newark for Chicago would not economically justify a direct wire though ample to support one-third of a wire. So a three channel circuit is set up between Dayton and New York, two channels being cut at Columbus, whose loads happen to fit the situation, giving one Dayton-New York channel, two Dayton-Columbus channels and two Columbus-New York channels. The Newark situation is taken care of by cutting through a channel of one of the Chicago-New York circuits to one on the New York-Newark wire, giving two outlets each way from New York and one through circuit Newark to Chicago.

Perhaps enough has been said under this heading of "wire layout" to give some conception of the involved factors to be taken into consideration in administering the wire plant. With many existing wires on many routes, with varying circuit limitations as to lengths and loads, with alternative possible methods of operation, with different existing and potential relay offices to care for a given territory and with the possibility of combining channel loads on wires in different combinations, it is evident that it is a real problem of economic engineering to properly assign circuits. With a wire plant in commercial service carrying annual charges of perhaps thirty million dollars an improvement of but one per cent in average efficiency would, in an ever growing business, provide a sum of $300,000 per year, enough to pay for a considerable engineering staff to achieve that one per cent saving.

TRAFFIC ROUTING

When, as elsewhere described, direct trunks have been established between all of the cities whose interchange of business either originating or relayed is sufficient to warrant such connection, it is evident that, in general, a number of possible routes will present themselves over which might be handled the business between any two offices which are not directly connected. The economies which can be achieved by judiciously selecting these routes are found greatly to outweigh the cost of the office force at headquarters necessary to determine the facts and to suitably instruct the field. Several considerations affect the choice of these authorized routes; the most important, likewise the most obvious, is that the fewest possible number of relays must be involved. This for the double reason that each relay increases the labor costs and at the same time degrades the speed of service.

Another, and quite important feature, is the relative percentage of full load carried by the various trunk groups involved. For example, a message filed at *A* and destined to *Z* may be handled with one relay through either *M* or *N*. Now, the trunks from *A* to *M* and from *A* to *N* may be equally loaded and both may have ample room for the proposed *A–Z* message. The operating force at *A* therefore, unless instructed, might choose either *M* or *N* as the outlet. If, however, the trunks from *M* to *Z* are already running to capacity while those from *N* to *Z* have a wide margin for increase then, obviously, the *A–Z* message should be routed through *N*. It has been found that due consideration of this factor may postpone for a considerable period, sometimes several years, the construction of additional wire to relieve congested trunk groups as, for instance, between *M* and *Z* in the quoted instance.

A less controlling ground for selection may be the method of operating, because the cost per message is cheaper on the automatic circuits than on the Morse circuits. Lastly, relaying through one office may be cheaper than relaying through another office because the cost per message handling in the first is lower than in the second. This difference may be due to the size of the office, to a more fortunate load factor due to more even fall of traffic throughout the hours of the day, or possibly even to closer operator assignment reflecting a superior grade of management. These differences, however, will be quite small and as a rule the other reasons for selecting a route will have determined the matter before this last mentioned point comes into consideration. It is probably a fact that in a perfectly ideal plant the relay point for business between *A* and *Z* should be located geographically half way between those offices. Practically, this thought is given little or no weight in determining routing instructions. It is, however, made a universal practise to route from *Z* to *A* through the same relay office as was used from *A* to *Z*. A theory was advanced and, in fact, some years ago a slight effort was made toward putting it into effect, that always provided the number of relays was not increased, a message should be routed on its first sending to the office geographically nearest its destination. The idea was that there was more probability of ample facilities and less probability of interruption on all of the available routes between points geographically near together than between points more widely separated. This plan, of course, insures that relayed traffic between *A* and *Z* would generally take a different route eastbound from that used westbound. Actually, this scheme is unsound in theory and impractical of application. Under it, it will be found impossible in all or even most cases to keep the eastbound and the westbound loads on a given trunk route equal which is obviously desirable from a wire economy standpoint, else the more heavily loaded side will require the construction of new wire for relief before this relief is needed in the opposite direction. Then, too, so much of the telegraph business is between regular and active customers that the peculiar form and phraseology of their messages becomes more or less familiar to the operating forces who are therefore in a position more accurately and readily to handle them. Evidently, then, to have the eastbound and westbound business between such firms relayed in the same office presents some slight advantage.

Following the considerations set down above the engineer charged with the responsibility of authorizing routes from each point to every other point in the country can readily reach his conclusions on the basis of the records of direct trunks and of their loads which have already been described. Then there remains the matter of issuing instructions to the field.

The earliest route guides consisted merely of a list of the telegraph offices, a separate list for each state, with a statement after each entry of the office which acted as relay for it. The inadequacy of such an arrangement is evident, for the card would show that Gorman, Md., was relayed by Pittsburgh while the office at Billings, Mont., would be wholly uninformed thereby as to whether to send the message to Helena, Denver, or Minneapolis, his only three outlets. Then, in the larger offices, the local forces made up route guides of their own, taking as a basis the "tariff book," which is the telegraph directory, and lists all the telegraph offices by states, entering the name of the relay point to which they should send messages for each of the listed destinations. This was faulty because decision as to route lay with the local people whose knowledge of the conditions was not complete and also it offered a very unsatisfactory arrangement for ease of consultation.

The most modern method, which has some five years' experience to justify it, consists of issuing from the home office in New York routing instructions for every office of sufficient size to warrant three or more trunk outlets. These instructions are in the form of lists printed on narrow slips much the size and shape of a newspaper column which can be slid into suitably designed metal leaves mounted on the wall or on racks in the operating rooms. Several duplicate copies may be displayed if necessary. They are prepared specifically for the office in which they are used and they tell the local forces what they are to do with each message rather than what happens to the message at points beyond their reach. That is to say, if the route clerk at Billings has a message addressed to Gorman, Md., on one of these instruction sheets headed "Maryland" he finds the entry "Gorman" and after it the direction, "Minneapolis." The Billings operator then sends the message to Minneapolis and has no further interest in how it is routed.

In the Western Union system there are 87 offices at which such route guides are displayed. The tariff book lists 65,000 names of places in the United States. Not all names are listed in any one guide, for about three-quarters are eliminated by the expedient of determining for each state which outlet covers the greatest number of places, then listing only the other places with the direction that those not listed be routed to that outlet. For instance, most Maryland points are relayed at Philadelphia, Baltimore, or Washington, all of which cities are best reached from Billings through Denver but fifteen towns, including Gorman, are relayed by Pittsburgh which has a direct trunk to Minneapolis. Therefore Billings' route guide for Maryland consists of a statement that all points go to Denver except the list of names that go to Minneapolis. So, though Maryland has 1400 listed offices, the Billings guide carries but fifteen of them.

To prepare these instruction lists a somewhat elaborate organization is employed. Linotype slugs are made for each entry on a linotype machine in the executive offices at headquarters and these are held in galleys especially shaped, the galleys being filed in metal racks also designed for the purpose. Changes are continually being made and when any change is made either in trunk routes or by reason of new offices being added to the list, such changes are made up on slugs which are put in the galley in lieu of or in addition to those already there and as many copies are struck off as may be necessary. These are made on a proof press which is the most satisfactory way of taking a few copies from type slugs carried in galleys rather than in chases. These instruction slips are then mailed to the respective offices. A record is kept of them and it is required that the receiving office remove the slips which they are to replace and mail the old slips back to New York. This not only serves as a receipt for the new slips but is almost certain evidence that the local forces have not erroneously removed a perfectly good slip and put the new slip which really belongs elsewhere, into its place.

A considerable office force is needed to effect this procedure. The benefits realized are those of more efficient use of wire plant, reduced number of relays, and improved speed of service due to elimination of misroutes. While it is difficult if not impossible to prove the monetary saving, it seems beyond question an advisable expenditure to those who have had an opportunity to compare the routing of business before the advent of this method with that of the present time. Since all the linotype slugs for all of the instructions for all of the offices are kept permanently on file in order that reprints may readily be made when changes in individual items occur, it is obvious that a considerable amount of type metal is involved. For the 87 offices mentioned, a total of about 1.3 million names are carried involving some 45 tons (41,000 kg.) of type metal.

Some provision must, of course, be made for routing over unusual channels when confronted by unusual conditions. A telegraph company has always to contend with inoperative circuits due to wire troubles, sometimes more, sometimes less, but always in some part of the country to such a degree as to exceed the possibility of restoration by spare wires paralleling those interrupted. Then, too, extraordinary files of traffic occur which when scheduled for a trunk group of limited carrying capacity may well overload it. To take care of these two classes of emergencies, those of wire failure and those of extraordinary file, a department is maintained centering in New York with branches in several of the important telegraph centers, known as the Dispatching Bureau.

The Dispatching Bureau is the only authority recognized as supervening over the authorized routing instructions. The Dispatching Bureaus are equipped with wires picking up all of the more important centers of the country and used solely for dispatcher circuits. A dispatcher is hourly advised of the traffic conditions of each of the important offices in his territory. He is advised both as to the number of messages on hand and the filing time of the oldest. He is also informed of all circuit failures which it has not been possible to make good with spare paralleling facilities. With this information at hand he is in a position to and he does instruct the offices as to temporary routings to reduce congestion which may have arisen. Having knowledge of the conditions on all groups he is able to avoid the earlier unfortunate practise of having the office *A* send on the *N* trunk business destined to *Z* and authorized to go to *M*, because his *M* trunk was overloaded or in trouble when at that particular time the *N–Z* trunk, unknown to him, was already badly congested or out of service. The dispatcher is also authorized to divert a wire from one assignment to another assignment even though it leaves the first assignment short. This is a function which cannot safely be left to the local forces because of their natural ambition to keep their own offices clear even at the expense of taking down a long through wire to make good a short local wire. The installation of the Dispatching Bureaus has proven of great benefit in expediting the movement of traffic under abnormal conditions and of increasing the useful distribution of wire when storm conditions prevail.

OFFICE LAYOUT

If we are to separate the engineering of a telegraph company into various parts among which are traffic engineering and fundamental plan engineering, we find that the matter of determining operating room layouts falls within both of these classifications. Practically, it works out that in the larger offices being newly established and planned for continuing service through a relatively long period of years, that part of the enengineering force which has to do with the estimates of future growth and fundamental plans undertakes a major part of the office planning. Even in this case, however, the details of the initial equipment are arranged by, or at least are approved by, those specifically responsible for the traffic engineering.

Prior to the general use of printing telegraphs we had the "quad room," so-called, which really more often was merely one end of the operating room, in which were located the duplex repeating sets, the duplex terminal sets, the quadruplex sets, such repeaters as there might be, together with the main switchboard and the loop board. Now, with the high proportion of printing telegraph circuits, the so-called "automatics," this older arrangement has been departed from in a measure. In a large modern office we have a terminal room, usually on a floor separate from the operating room, in which are located the main and loop switchboards, the repeaters, both relay and rotary, and the Morse terminal equipment, whether single, duplex, or quadruplex. The terminal apparatus for the automatic circuits including the main line relay and the duplexing equipment in general present practise, are carried to the operating room floor and respectively located on the distributor tables appurtenant to each automatic circuit. These distributor tables are set at the head of the line of alternate sending and receiving positions making up the two or three channels worked on that circuit and while other arrangements might be possible, the desirability of this layout is so great that the floor plan of the operating room is customarily forced to conform to this arrangement. The Morse operating positions, either single or duplex, and carrying only local circuits from the quad room, present an easier problem and may be placed around the room as conditions permit although, of course, for the best handling of physical messages within the office, proper grouping is desirable.

In the normal operating room we would expect to find the following sections:

a. Automatic trunks
b. Morse trunks
c. Way Wires
d. City lines
e. Tube center
f. Routing center

The equipment for recording and delivering to customers over the public telephone and for operating the telephone circuits to branch offices which are so served, is usually in a "phone room" separate from but near to the operating room and connected therewith by some type of message conveyor, either belts, tubes, Lamson carriers, or gravity chutes.

As previously suggested the automatics will all be arranged in rows. At the head of the row is the distributor table with the artificial line for duplexing, the main line relay, the phonic wheel motor and the tuning fork that drives it, coupled to the rotating brushes and the segmented face plate which constitutes the distributor, and sundry miscellaneous apparatus. Then in line come the tables at which sit the four or six operators needed to man the two or three channels, all facing in the same direction. These are invariably arranged in alternating perforating and printing positions, in such order that the *A* channel perforator is to the right of the *A* channel printer. Two such rows are placed back to back with a "pickup" belt between. These belts, open on top and continuously moving, take the received messages as they are removed from the printer (or from the gumming desk if the position is equipped with the newer tape printer rather than the older page printer) and carry them to the ends of the rows where other mechanical conveyers take them to the routing center. An aisle is left between parallel rows as near to 5 feet 6 inches in width as the floor plan of the building permits.

The Morse positions are also aligned in a similar manner though their arrangement is much more flexible as the only limitation as to adjacency is that for duplex operation the sending position be immediately to the left of the receiving position. Quite generally a pair of duplex positions are so equipped that they can be used to table two single Morse wires. These rows, like the automatics, are arranged back to back and have a pickup belt between them.

In handling commercial business on way wires it is not the custom for way offices even on the same way wire to handle messages directly one to the other, partly because railroad operators generally have other duties and it is easier for one to unload his traffic on the main office which is always there rather than to try to raise the man at the other small office; partly too, because a main office relay copy is desired for check control to protect the company's revenues.

An experienced main office operator covering a way wire will become familiar with the train schedules, meal hours, and other facts regarding the offices and the operators with whom he works and he can turn this knowledge to good advantage with considerable conservation of line time when he is trying to dispose of business to these points. Evidently it is not conducive to economy to spend many minutes calling on a wire to raise an office which is closed for lunch or where the operator is out arranging baggage to put aboard an approaching train.

Improved operating efficiency at the main office on way wires is achieved through the use of "concentrator units" a device which is nothing more nor less than a small switchboard in which the line terminates and on which a lamp lights if a prearranged code is sent on the wire by any of the outer offices. These outer offices may communicate with each other or with, in many cases, the head railroad office, without calling in the telegraph company's main office operator. The concentrator units are so situated that four operators, two on each side of the table, may at any time reach any of the eight lines terminated in the concentrator by plugging into the jack associated with the corresponding signal lamp, jacks and lamps being multipled on the two sides of the unit. Arrangements are also provided, either in the loop board or immediately adjacent to the concentrator unit, whereby any given line can be taken out and tabled individually during the busy hours of the day if the load during those hours warrants the entire attention of one operator.

It is still the general practise to run "number sheets" on all circuits which is to say that between the main office and each office on the line a series of numbers is run and after the transmitting of each message one of these numbers is crossed off at both the main office and the distant office. These series of numbers appurtenant to any one wire are in general kept upon a single number sheet. The message itself bears the same number. At the end of the day the numbers are compared between offices and the whole is intended to guard against lost messages. Were it not for these number sheets obvious economies could be secured by terminating all of the way wires in one switchboard, multipled if necessary, and handling the traffic very much as calls are handled on a telephone switchboard, but the impracticability of moving the number sheets around between the various operators involved, according to present practise, limits the concentration to units as before described. Upon these units a simple rack is used to hold the number sheets belonging to the wires terminating therein and the operator who for the moment is working with an office on a specific wire reaches for and puts before him the number sheet of that wire and makes the necessary notation thereon.

.Of course not all way wires are in concentrators and some are put in concentrators during slack hours but given individual positions during busy hours. The arrangement of these Morse positions in the room does not differ from that of the Morse trunks.

The "city lines" are the short single Morse wires to branch offices throughout the city. They are worked on positions quite the same as the way wires.

The growth of these various methods of reaching branch offices is interesting. Originally, of course, Morse was used entirely but beginning some fifteen years ago, the telephone was introduced. Its advantage lay partly in the fact that it is faster than Morse even when all proper names and questionable words are spelled out, "S for Sugar, M for Mary, I for Ida, T for Thomas, H for Henry, Smith," etc., and the message then repeated back. Even more advantage resulted from the greatly widened field from which branch office personnel could be selected, an important consideration in view of the fact that quite generally besides transmitting messages, they have to meet the public, keep accounts, and generally represent the company. For all the substitutions of phone for Morse on city lines and automatics on the trunks it is nevertheless true that the number of available operators for manning the remaining Morse wires is decreasing chiefly because of the failure of the younger generation to learn the code and enter into this profession.

In their turn the telephone city office lines are now giving way to the short line printer because of the somewhat greater accuracy and the considerably greater output which may be achieved by this means.

Pneumatic tubes have been used for moving messages between main and branch offices for many years. Roughly, a speed of 1000 feet per minute may be expected and not more than a five minute run may be allowed without unwarrantably degrading the speed of service. From this it follows that in general, branch offices cannot be served by tube if more than a mile from the main office. The installation costs of tubes are quite high but their operating costs are extremely low as compared with any other method. For that reason great savings can be made by means of tubes in an office with a sufficient number of messages per day depending upon the distance and upon other conditions. After tubes have once been installed, then growths of several hundred per cent can be taken care of without any appreciable increase in cost. In general it is found economical to use tubes to offices where the loads are in excess of from two hundred to six hundred messages both ways per day.

It is beyond the scope of the present paper to describe the methods used in moving messages from one part of the operating room to another as is necessary in connection with the relay and delivering functions. Suffice it to say that for this intra-office routing it is found, in general, best to pick up business of all sorts from the incoming wires and carry it automatically to a single "route center" located as near the center of the operating room as conditions will allow. At the route center a slow moving belt carries the business before the requisite number of route clerks who dispatch it to the "drops" nearest the proper outgoing wires. Largely this is done by belt conveyers but sometimes the route clerks lodge the business in pigeon holes whence it is taken by "routing aides" by hand to the outgoing wire or perhaps it is dispatched by Lamson carrier or by tube. In any case "routing aides" move the messages from the "drops" to the outgoing operator positions.

The pneumatic tubes are grouped at a "tube center" generally immediately adjacent to the route center. Where the number of tubes warrants, instead of having the clerk who inserts the carriers into the tubes load the messages into the carriers, the carriers are all loaded at one location and distributed to the tube heads by a conveyer belt. The carriers being variously colored, the tube attendants readily insert them into the proper tubes. On very busy tubes an automatic device is employed which insures that successive carriers are not despatched in less than the permissible time interval and yet conserves tube capacity by assuring that when business is on hand it is sent just as promptly as is consistent with proper carrier speed in the tubes. According to the fall of traffic one or several messages may be enclosed in a single carrier. The tubes are 2¼ inches in diameter, the carriers of course less, and good practise indicates that not more than a dozen messages should be inserted in a single carrier.

The matter of delivering telegrams direct to the patron and receiving them direct from him over a public telephone system presents many angles. The success of a widespread application of this method depends of course upon the attitude the public takes toward it. This attitude in turn may be affected both by the physical conditions and by the development work carried on by the telegraph company's publicity forces. For instance, in a manufacturing town where the telegraphing is largely conducted by the offices of the mills and the mills themselves are naturally scattered at some distance from the center of the town, the telephone pickup and delivery is looked upon with considerable favor. In other districts where branch offices are located within a few steps of all of the biggest users, considerable resistance is felt toward an effort to make telephone deliveries. There is no question that telephone delivery direct to the customer is more expeditious than branch office and messenger boy handling and since speed is practically the sole incentive for using the telegraph, the telephone seems to be the logical method of effecting delivery and the public prejudice against it can, in large measure, be attributed simply to habit, and can be overcome to some extent by proper publicity work. As extremes of these conditions might be pointed out, on the one hand, the city of New Haven, Connecticut, where 55 per cent of all telegraph deliveries are made over the public telephone and on the other hand Manhattan Island where no effort is made to deliver by telephone any telegrams to addresses south of 59th Street.

Operator Assignment

Operator assignment studies are predicated to such a considerable extent upon certain fundamental facts regarding telegraph traffic that an outline of these features seems to be essential to an understanding of this problem.

After a message is filed at a branch office a certain amount of time is required to complete the primary record concerning this message before it can be sent to an operating position for transmission. Similarly, when a message is received at the main office on any circuit and this message is destined for retransmission by the main office to another office a certain amount of time is required to get this message through the distributing center and deliver it at the proper sending position. This time interval is known as the "office drag" and is a variable depending upon the size and type of the office.

When a message arrives at a given circuit for transmission to the distant terminal, other messages may be on hand awaiting transmission. All the traffic for this circuit is then sorted by classes in filing time order, the earliest full rate message being transmitted first.

The message last received will therefore be subject to some delay before it is transmitted and a decision as to how long it may be so delayed before it is transmitted is in order. If we assume as permissible an average delay of not more than five minutes we may find that with the given load one operator can handle all the traffic within the time limit. On the other hand, if the average delay is fixed at one minute it may be necessary to use no less than four sendings to handle the same volume of traffic.

The latter figure will obviously result in a tremendous increase in the cost of handling this traffic and in determining the time limits within which all traffic must be transmitted after it is received in an office, the company's policy must be based on the relation between the cost of the desired speed of service and its value to the patron, as reflected in the volume of traffic which may be expected as a result of this speed of service.

Having decided upon the average and the maximum delay to be maintained we may now proceed to develop the load which we may reasonably expect an operator to handle. At this point we find that if an operator working on a single channel between two offices is given such an amount of traffic as will cause her to work more than 43 minutes out of an hour, the desired speed of service cannot be maintained and it becomes necessary to cover a second sending position.

As the load increases the second operator absorbs more and more of the excess and we find that we can still maintain the speed of service up to the point where each of the two operators is busy for 52 minutes out of the hour. Assuming a channel capacity of 60 messages per hour we can expect to handle 43 messages per hour on a single channel to a given office but when two channels are in use to this office up to 52 messages per hour can be transmitted on each channel without degrading the speed of service.

This increase in available work time per operator continues as the number of operators in the group increases up to the point where 10 channels are in use, when approximately 100 per cent of the operators' time is usefully employed and each channel is working at the full capacity of the apparatus. As automatic trunk groups are largely of two or three channels one cannot normally expect to get 100 per cent capacity from the equipment even during the busy hour. It is only in such large groups as the Chicago-New York circuits where fourteen duplex channels are provided that the capacity of the equipment and the time of the operators is fully utilized. In times of stress, of course, by arbitrarily degrading the speed of service through increasing the time limits, the available operator work time is increased to such an extent as to clear up abnormal accumulations of traffic without increasing the operator staff.

Further limits are placed on the amount of traffic which can be handled over a given channel or channels, by the safe operating speed at which the circuit can be worked. By safe operating speed is meant a speed which can be maintained day by day without introducing too many errors or interruptions.

Detailed analysis of the circuit load reports which record the volume of traffic filed for transmission on each circuit or circuit group during each half hour of the day, shows that not only are there wide fluctuations in the amount of traffic to be handled in any half hour period, but there are variations due to the day of the week as well as seasonal changes. Estimates of the traffic to be handled tomorrow or next week, and the operator requirements based on this traffic must of necessity recognize these variations.

Analysis of the actual messages flowing regularly over a given circuit shows that the number of characters in the average message is practically a fixed quantity for each class of traffic handled on the circuit, but this average length is not the same for all circuits.

Having decided upon the operating speed of the circuit in words per minute and knowing the average length of the message to be handled on it the theoretical capacity of the channel in messages per hour can easily be determined. From this theoretical capacity we must deduct allowances for

a. Lost channel time due to temporary wire and equipment failures.

b. Operating or apparatus errors which necessitate the correction of messages which have already been transmitted.

c. Errors which the operator is conscious of having made and corrects before the complete message is transmitted.

When these deductions have been made it is possible to state that under the given conditions a certain number of channels must be manned to handle the estimated volume of traffic.

It will be noted that up to this point we have apparently assumed that the operators can work at any circuit speed. The fact is that while Morse and phone operators are in general limited only by their ability and not by the circuit characteristics, the automatic operator is limited by the speed at which the channel operates. Several thousand stop watch observations on perforator operators indicate that the present maximum operating speed of 65 words per minute on automatic circuits is not only well within the range of the average operator but that this rate can be sustained without undue fatigue.

In actual operating room practise, it has been found desirable to locate all the trunk circuits in one group, all city circuits in a second group with a third comprising the way circuits. Interchange of operators from one position to another within the given group is facilitated by a further division of the three major groups into automatic, Morse, and telephone operating sections, and the operating staff is in general assigned by methods.

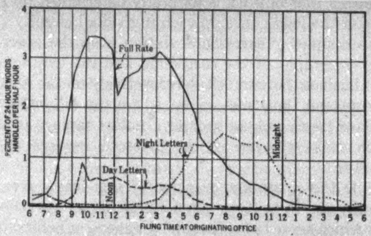

Fig. 1—Load Curve—Entire Telegraph Plant

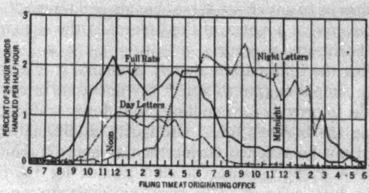

Fig. 2—Load Curve—Atlantic Coast to Pacific Coast

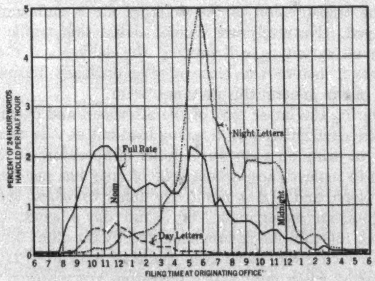

Fig. 3—Load Curve—Pacific Coast to Atlantic Coast

An operator's tour of duty is nominally eight hours, but as each operator receives a fifteen minute rest period after she has been at work approximately two hours, and a second fifteen minute period about midway of the last four hour working period, both of which are on company time, their actual working period, is only 7.5 hours. Advantage is taken of the relief periods to minimize the effect of fatigue, particularly on the automatic circuits by alternating the kind of work. If an operator begins her tour of duty at a sending position she will be assigned to a receiving position after the first relief period. Upon her return from the lunch relief she will again be assigned to a sending position.

Reference to the load curve for the entire telegraph plant shows that while the traffic is heaviest during ordinary business hours, *i. e.*, 9:00 a. m. to 5:00 p. m., there is a considerable volume remaining to be handled up to 10:00 and 11:00 p. m. and the beginning of any operator's tour of duty is naturally determined by the shape of the load curve on each individual circuit.

As examples of extreme deviation from the average are given graphs of the eastbound and the westbound file of business between cities respectively on the Atlantic coast and the Pacific coast where the difference in standard times and the greater absolute savings to the patron make the use of the deferred services more extensive.

While the shape and position of this load curve is generally the same for a given circuit minor variations occur which serve to set these peaks ahead or behind the normal time position. Our obligation to give service requires us to anticipate these changes and the positions are covered in advance of the time when the change in load is expected to occur.

With the information already available it is a relatively simple matter to assign the operators in such a way as to provide for handling the full-rate traffic filed during the business day. Ordinarily, additional staff would not be assigned during the day hours to handle deferred traffic because the staff needed for full-rate traffic has sufficient idle time available to permit of handling the deferred traffic during these idle periods if the transmission of the deferred file is sufficiently delayed. Naturally we cannot delay this traffic to such an extent as will prevent the delivery of these messages before the close of the ordinary business day but in general this does not occur.

During the latter part of the afternoon and evening we are confronted with a falling load but as it is necessary to bring on a certain number of operators at this time to replace those whose tour of duty has just been completed and as it is manifestly unfair to release them without an opportunity for a full day's work, the falling full-rate file is supplemented by the night letter file in such a way as to keep this staff busy until after midnight when only a few operators are required to handle the balance of the file.

A complete assignment of the staff required for the entire 24 hours may now be made for each of the circuits or circuit groups. The total number of operators required to handle the traffic on all circuits including relief operators may then be determined and to this total is added a sufficient number to provide for operators who are either late, or fail to report, those on vacations, etc.

Roughly, and subject to considerable variation, in specific instances, the total number of operators needed in an office to conduct operations throughout the 24 hours, to cover vacation and sickness reliefs, absentees and those tardy, can be taken as 2.8 times the number of operating positions covered during the busiest half hour of the day.

Discussion

J. H. Bell: There is one question I would like to ask Mr. Mason. On page three this statement occurs: "an experienced automatic operator works at between 55 and 60 words per minute while the average output per Morse operator is between 12 and 15 words per minute."

I question whether those two are comparable. My recollection is that it is much easier to operate a Morse system by hand at about 20 words per minute, and human nature being as it is, I rather think that the operator would prefer to operate about 20 words per minute rather than 12 or 15. I would like to know whether 55 to 60 words per minute is the output of the automatic operator?

H. Mason: The point is well taken. The way it is stated in the paper the figures are not literally comparable. I agree that a Morse operator might work at 20 words as well or perhaps better than he would at 12, but our actual output is of the order of from 12 to 14 words per minute. In the case of the automatic operators 55 words per minute represents an average performance but not the output. The human nature feature comes in here. The operator is perforating against a machine. That machine is set to go at a given speed, generally about 55 words per minute. She must keep up with that machine or the auto control stops the transmission in a way evidencing her failure.

I am not prepared to say that the performance throughout the country is universally as high as 55, but individual operators can perforate at double that speed and we have no mechanisms which would take that rate of perforating. A recent figure shows the average output per Morse operator hour to be 28 equated messages, while that of an automatic operator is 62 equated messages. These figures reduce to approximately 15 words per minute and 32 words per minute respectively.

J. H. Bell: Reference was made to the delivery of telegrams. Nothing was said about the messenger service. I would like to mention that in one of the British journals I read that they have started a new method of delivering telegrams. Instead of having the boys go out with each message that comes, they have divided the town into 25 or 30 separate routes. A boy leaves every five minutes from the telegraph office and he takes all the messages along one particular route. That makes for a slight increase in delay in delivery of some messages, but it brings the average delay much more uniform.

H. Mason: In the company which I represent the line of organization is not such that the messenger delivery problem comes under the control of the engineers for which reason I am not so well fitted as would be some other, to discuss this question. I do not think, however, that anywhere in our service is such a method as is suggested by Mr. Bell used.

We do have routes and send boys over them with messages just for those routes in-so-far as the delivery of night letters in the early morning is concerned, and also full-rate messages for which we have instructions, as we have in some cases, not to make delivery until the customer's office is open.

However, other than that, so far as I know, we do not at the present time indulge in the suggested practise but our full-rate meassages are taken out practically as they fall. That does not mean that the messenger boy always goes out with one, quite often he has two or three, but he is not definitely held until several come for that zone.

The matter of deliveries of the day letters may be held up. For instance, if it is several blocks from the office, the delivery clerk may very well hold up a day letter waiting for some preferred business to come along, and send it out with the same boy, but not as a policy.

Developments in the Manufacture of Copper Wire

BY JOHN R. SHEA[1] and SAMUEL McMULLAN[1]
Non-Member Non-Member

Synopsis.*—This paper covers interesting developments in the manufacture of copper wire and contains a description of a copper rod and wire mill designed to meet the new requirements. It also includes a brief survey of the copper rolling and wire drawing art at the time the investigation was started; a summary of tests made in varying the practise in rod rolling and wire drawing, and an outline of the work done by the Bell System engineers in developing and designing new types of wire drawing machinery. The economies in floor space and plant investment due to the use of more compact and higher-speed machinery are outlined. Some of the outstanding features in plant arrangement which contribute to more efficient operation are discussed in the concluding pages.*

RAPID developments in the various branches of the communication business are constantly leading to important investigations in line with more efficiently and economically meeting the increasing demands of the service. In this connection, one of the more recent and very interesting investigations indicated the possibility of effecting substantial improvement in the process of manufacturing copper wire. Accordingly, a comprehensive study of all the factors concerned was undertaken which resulted in the construction of a rod and wire mill at Chicago, Illinois, embodying many unique and improved operations, a schematic layout of which is illustrated in Fig. 1.

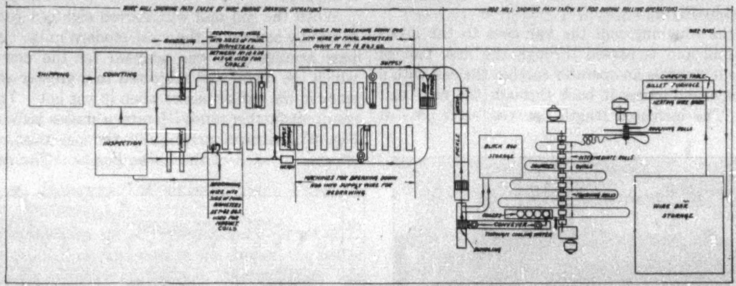
WIRE MILL SHOWING PATH TAKEN BY WIRE DURING DRAWING OPERATIONS
ROD MILL SHOWING PATH TAKEN BY ROD DURING ROLLING OPERATIONS
BUNDLING
REDRAWING WIRE INTO SIZES OF FINAL DIAMETERS BETWEEN No 10 & 26 B&S GA USED FOR CABLE
MACHINES FOR BREAKING DOWN ROD INTO WIRE OF FINAL DIAMETERS DOWN TO No 18 B&S GA.
SUPPLY
SHIPPING
COUNTING
INSPECTION
WEIGH
MACHINES FOR BREAKING DOWN ROD INTO SUPPLY WIRE FOR REDRAWING
REDRAWING WIRE INTO SIZES OF FINAL DIAMETERS USED FOR MAGNET COILS
WIRE BARS
CHARGING TABLE
BILLET FURNACE
HEATING WIRE BARS
ROUGHING ROLLS
PICKLE
BLACK ROD STORAGE
INTERMEDIATE ROLLS
FINISHING ROLLS
COILERS
CONVEYER
THROUGH COOLING WATER
BUNDLING
WIRE BAR STORAGE

FIG. 1—SCHEMATIC LAYOUT OF WESTERN ELECTRIC CO.'S COPPER ROD AND WIRE MILL AT CHICAGO

At the outset the sources of copper and its transportation were studied and it was found more economical to ship wire bars to Chicago for conversion into wire than to locate a wire mill near some of the large refineries and ship wire to the factory. It was also considered that this plan would reduce the investment in wire during the process of manufacturing cable and telephone apparatus.

ROD ROLLING MILL

The rod rolling mill equipment consists of a billet heating furnace, a roughing mill, an intermediate mill, a finishing mill, coilers, conveyers, and pickling tubs. The mills are water-cooled and equipped with a downdraft exhaust which carries the fumes produced during the rolling operation to an air washer where the copper dust is removed before the air is discharged.

The 225-pound wire bars as received in cars from the refineries are unloaded onto skids in the train shed and transported by an electric truck to the charging end of the billet heating furnace. Here they are transferred in groups of six by a hoist to the charging table, where a compressed-air pusher moves them along through the furnace which holds 120 bars. The bars are brought up to the required temperature for rolling as they move through the furnace, which is heated by fuel oil. When the bars reach the opposite end of the furnace they are withdrawn at about 1600 deg. fahr. with a pair of tongs through the discharge door and pushed into the roughing mill one at a time. These tongs operate on a trolley suspended from a beam, which is in line with the first groove of the mill.

The roughing mill consists of three motor-driven rolls, one above the other. The bar, after passing through the first groove between the top and middle roll, drops upon feed rolls set in the floor and is returned through the second groove, between the middle and bottom roll; then raised into position and passed through the third groove, which is in the same rolls as the first pass.

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[1] Both of the Western Electric Company.

Presented at the A. I. E. E. Winter Convention, New York, N. Y., February 7-11, 1927.

Five passes are made in this manner until its cross-section is reduced sufficiently for it to enter the intermediate mill. As the bar enters the roughing mill it is 54 inches long and about 4 inches square. When it leaves this mill it has been rolled into an oval cross-section and is about 124 feet in length. Formerly the last pass on this mill was handled manually, and recently a mechanical repeater has been added as illustrated by Fig. 2.

FIG. 2—VIEW OF ROUGHING MILL SHOWING REPEATER ON LAST PASS

From the roughing mill the bar goes to the intermediate mill and is passed through the first pair of rolls. As it emerges an operator catches the end with a pair of tongs and passes it back through the next pair of rolls. The increased length between each pass at the intermediate and finishing mills is allowed to run out in a loop on a sloping iron covered floor on each side of the rolls. This catching and returning is repeated at each set of rolls until the original copper bar finally emerges a round, quarter-inch rod about 1200 feet long. This last pass goes through a guide pipe into a coiler, Fig. 3. The reductions in cross section are illustrated by Fig. 4. An appreciable amount of copper oxide scale is carried off with the cooling water, and deposited in a reservoir from which it is later salvaged.

FIG. 3—VIEW OF INTERMEDIATE AND FINISHING MILLS AND COILERS

The coils are automatically unloaded from the coilers on to a conveyer, which carries them through cooling water in a tank underneath the floor. Eighty-two seconds after entering the roughing mill the bar is a coil of ¼ in. rod ready to proceed on its way to the pickling tanks. The mill has a capacity of 70,000,000 pounds annually on a 48-hour per week basis.

While the diagram and illustration of the intermediate and finishing mills indicate for simplicity that the rod follows only a single path, in actual operation sufficient material is kept in the mill practically to maintain at least two rods in the finishing mill. This is illustrated graphically by that part of Fig. 5 which covers the finishing mill. Referring to line A–A^1, 11 reductions are being made in this mill at the same time, two for each of the first four pairs of rolls and three on the final rolls. At this period in the cycle of operation 800 hp. is required.

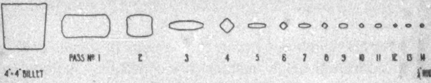

FIG. 4—ROD MILL REDUCTIONS—4-IN. x 4-IN. BILLET TO ¼-IN. ROD

When the rod mill was started eighteen passes were in use by several of the most modern mills. A sixteen pass arrangement was adopted for the new mill, in which the metal was subjected to a greater amount of work in the earlier passes when it was hot. Later, as a result of further study, fourteen passes were adopted. Fig. 6 illustrates graphically the per cent reduction effected at each of the above passes. The reductions plotted as the abscissa are in terms of reduced area in cross-section at each pass and the passes reading from left to right are plotted as ordinates.

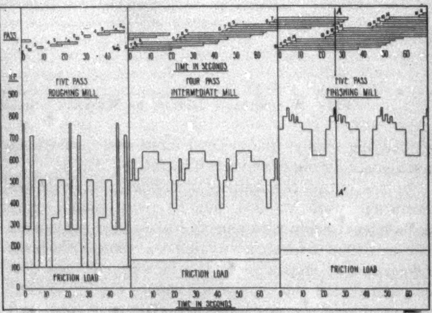

FIG. 5

It is obvious that careful planning must be done in changing the number of passes in a mill in order not to exceed the safe working capacity of the mill rolls and stands. Such calculations have been made using rolling mill formulas[2]. Based on the more sturdy mill installed at the Chicago plant the first four passes of the eighteen pass arrangement would operate at about 82, 100, 105, and 90 per cent of the safe working load of the mill. These same passes calculated on the basis of the sixteen and fourteen pass arrangement operate at 86, 87, 90, 85 and 96, 96, 90, 90 respectively. This indicates that a further reduction may be made in the number of passes in the mill provided other features are improved; *i. e.*, roll adjustment.

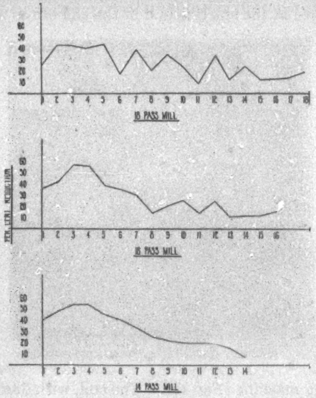

FIG. 6

RELATION BETWEEN WORKING AND PHYSICAL PROPERTIES

It has often been stated that the more passes (*i. e.*, the more gradual working) given the copper, the better the physical qualities of the rod. Actual tests (see Table I) made on representative lots of ¼-in. rod fail to confirm this impression.

TABLE 1

Lot	Number of passes	Elongation per cent	Tensile strength lb. per sq. in.
1	18	35.8	33,752
2	18	40.0	31,445
3	16	37.1	32,468
4	16	41.0	32,160
5	14	42.0	32,391
	Average of 5 lots	39.5	32,243

The averages indicate that a fourteen-pass rod is superior in elongation, and better than the total average in tensile strength.

CLEANING OF ROD

When the coils emerge from the tank through which the rod coiler apron conveyer passes, they are cool enough to handle and after being tied with wire, several are lifted together by a monorail crane, and placed for thirty minutes in a pickling tank containing from 5 to 10 per cent free sulphuric acid, in order to remove the black oxide caused by oxidation of the hot copper in the air during rolling. The solution is maintained at approximately 120 deg. fahr., and the copper content varies from one to three grams per 100 cu. cm. Experiments have shown a difference of less than 10 per cent in pickling time between the minimum and maximum acid used, the greater solubility being obtained from the weak solution. Actual results obtained were checked with Sidell's Table of Solubilities (see Fig. 7). While a variation from the minimum to maximum acid concentration does not materially affect the pickling time, a variation in temperature has a decided effect as may be seen from Fig. 8.

ELECTROLYTIC PLANT

Fig. 9 shows a plant in which the copper is reclaimed from the pickling bath at about the same rate as it is absorbed. This is accomplished by electrolytic deposition according to principles worked out and practised in the large refineries which produce electrolytic copper.

The electrolytic system operates best with a minimum content of about 1 per cent copper and 5 per cent acid and a maximum of 3 per cent copper and 10 per cent acid. The copper and acid contents are kept as low as practicable to minimize "carrying out losses"[3] during the pickling operation. About 775 pounds of acid, and 430 pounds of copper are recovered per day

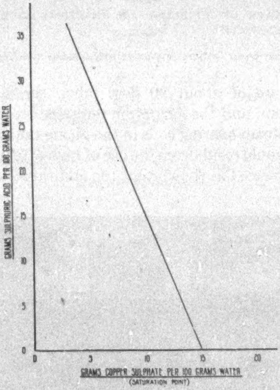

FIG. 7—SOLUBILITY CURVE OF COPPER SULPHATE IN A SULPHURIC ACID SOLUTION (TEMP. 25 DEG. CENT)

Reference Table of Solubilities by Sidell, *Chem. & Met. Eng.*, Vol. 21, p. 181-2, 1919.

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[2] "Pass Limitation in Rolling Mill Practice," *Machinery*, July, 1918.

"The Theory and Practise of Rolling Steel," Wilhelm Tafel.

[3] Pickling solution carried out when coils are removed from tank.

from the electrolyte. The anodes are operated at a current density of five amperes per square foot with a rate of deposition of about 0.00261 pound of copper per ampere-hour.

The heat generated in the plating tanks under normal operating conditions maintains a minimum temperature of about 90 deg. fahr., throughout the acid system, and the maximum temperature is obtained through steam heating coils in the pickle tanks. Faster pickling would result from the use of higher temperatures but experience has shown that the additional steam and gas released above 120 deg. fahr., results in unsatisfactory operating conditions.

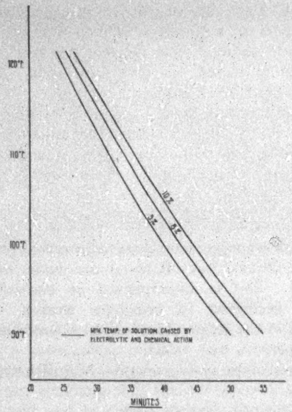

FIG. 8—RATE OF PICKLING AT PRACTICAL ACID CONCENTRATION

Max. temp. without excessive evaporation and fumes

FIG. 9—ELECTROLYTIC RECOVERY OF COPPER FROM ROD MILL PICKLING SOLUTION

The coils of rod after pickling are thoroughly washed with lake water[4] at a pressure of about 70 pounds per square inch to remove loose copper dust and acid, and then immersed in an alkaline fat solution to neutralize any trace of acid and to provide a protective coating against oxidation until converted into wire.

WIRE MILL

The coils, after being pickled and washed, are carried by monorail cranes to the wire-drawing machines where they are converted into wire of the desired size. The dies used in the heavy wire drawing machines are pulled into place at the starting end of the coil of rod on a die stringing machine (Fig. 10). The coil, with dies strung into position, is then placed in a heavy wire-drawing machine.

FIG. 10—HEAVY WIRE DIE STRINGER

The heavy gages of wire, such as line wire, are drawn with one set-up on a heavy wire-drawing machine; medium sizes, used in lead covered cable, are made by taking the wire as it comes from the heavy machine and redrawing it on the intermediate machine; and finer sizes, commonly known as magnet wire sizes, are produced by redrawing intermediate sizes.

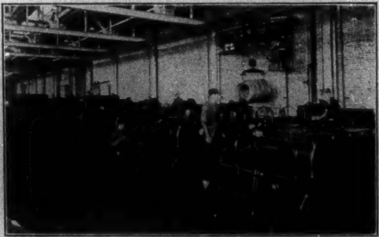

FIG. 11—BATTERY OF NO. 1 WIRE-DRAWING MACHINES

The present capacity of the wire mill is approximately 42,000,000 pounds annually, and the sizes range from 0.165-in. line wire to 42 B. & S. (0.00247-in.) gage magnet wire. Provisions have been made in the construction of the building and its foundations so that the mill may be expanded in capacity when needed.

The No. 1 or heavy wire-drawing machine shown by

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[4] Lake water is relatively free from mineral salts which would corrode the rod and affect the wire drawing compound.

Figs. 11 and 12 draws line wire, heavy toll cable sizes, and supply wire for the loop cable wire machines. This ten-die machine with its auxiliary equipment and operating area occupies a floor space of 270 sq. ft. and runs at 1500 to 2000 ft. per min. as compared with 470 sq. ft. for a commercial nine-die machine running about 1000 ft. per min.

A battery of these machines costs much less than an installation of commercial machines of the same capacity, and in addition, effects a considerable economy in floor space.

Fig. 12—Close-Up View of No. 1 Machine

The commercial types of ten-die intermediate machines for drawing cable wire require about 130 sq. ft. of floor space as compared to 90 sq. ft. for a twelve-die multiple head machine. The former is a single-unit machine and the latter a four-unit machine operating at twice the speed and capable of producing about five times the output of the commercial equipment. This new multiple unit machine costs more than regular equipment, but considering the four units, the cost is materially less per unit, and very much less on an output basis.

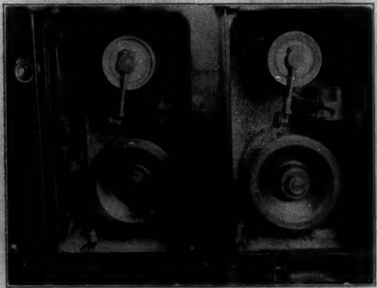

Fig. 13—Close-Up View of Units of No. 3 Wire-Drawing Machine

The magnet wire-drawing machine is a high-speed twelve-die multiple head machine of eight wire drawing units, occupying 90 sq. ft. of floor space including the operating area. A close-up view of two heads of this machine is shown in Fig. 13. Fifty-one sq. ft. of floor space are required for a single unit (one head) commercial machine of the same die capacity. The saving in investment in this case is even greater than for the heavy and intermediate types of machines. The use of these compact machines and overhead monorail equipment for transporting material instead of using trucks with large aisles has permitted the installation of the wire drawing mill in less than one-fourth of the building area which would have been required if commercial equipment had been purchased.

General Plant Features

The present connected load of the motors in the rod and wire mill is about 6000 horse power for which it was necessary to enlarge the factory power plant. A 700-ft. tunnel connects the power plant with the rod and wire mill in which are laid pipes for carrying hot and cold water, steam, gas, and air and lead covered power cables.

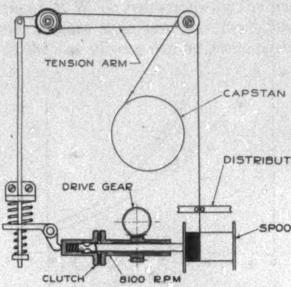

Fig. 14—Automatic Tension Mechanism of No. 3 Wire-Drawing Machine

The basement under the rod mill houses the electrolytic equipment, control boards for the roughing and intermediate mills, pumps for cooling water, and exhaust fans connected with an air washer for removing the fumes from the rod mill. A tunnel which passes beneath the intermediate and finishing mills connects with a room which houses the drives for the four rod coilers, the coiler control boards, the finishing mill control board, and the main power panel. In the wire mill basement are six large tanks which hold the compound used to lubricate and cool the wire-drawing dies. This compound is supplied under pressure to the wire-drawing machines on the floor above and returns by gravity.

All the wire-drawing machines are controlled by push buttons mounted on the machines, which connect with compensators in the basement. The 100-hp. motors driving the large wire-drawing machines are mounted in a tunnel and are connected to the machines above by chain drive.

This arrangement permits accessibility for maintenance of the electrical equipment with a minimum of interference to production, prevents the wire-drawing operators from having access to the electrical equipment and reduces accident hazard to a minimum.

DEVELOPMENTS IN WIRE-DRAWING EQUIPMENT AND METHODS

The rod and wire mill just described was designed following a comprehensive survey of wire-drawing processes and equipment used in this country and abroad. In connection with these studies, extensive laboratory investigations were undertaken relative to the characteristics of different types of commercial machines especially from the standpoint of operating efficiency, investment, and floor space requirements. As a result of these investigations, it developed that marked improvements could be effected if wire could be produced commercially at higher machine speeds and with more compact machine equipment.

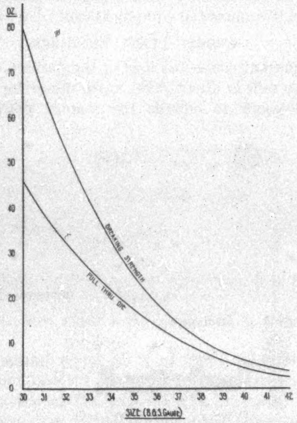

FIG. 15

While the design of the drawing mechanism in the new machine was very important, it was also essential that the finished wire be taken up on spools instead of coils. After considerable experimental work, a sensitive take-up device was developed to permit spooling at a constant drawing speed.

This spooling mechanism is illustrated by Fig. 14 in which the spool spindle is driven by a slipping clutch member controlled through a tension arm, on which an idler pulley is located over which the wire passes on its way from the drawing capstan to the take-up spool. The take-up mechanism rotates the core of an empty spool at a speed synchronous with the speed of the wire as it leaves the drawing capstan. As the spool fills and the speed tends to increase, the wire on the tension arm tightens and compresses the tension arm against a spring adjusted for the proper gage of wire. This in turn reduces the pressure of the clutch driving the take-up spindle permitting the spool of wire to readjust its speed.

This device is extremely sensitive as illustrated in the drawing of No. 42 B. & S. wire at 2000 ft. per min., in which case the control arm must be adjusted to operate between 90 and 150 grams, since the pull required is 87 grams and the breaking strength of the wire is 170 grams. This device is so flexible that it can be adjusted to a drawing tension of from 9 pounds for No. 25 wire to 3 ounces for No. 42 wire. Fig. 15 illustrates its operating range on wire sizes No. 30 to No. 42, showing the gradual narrowing of the limits as the sizes decrease. A larger machine used for drawing loop cable wire from No. 18 to No. 30 B. & S. gages contains a similar mechanism.

The use of this sensitive device and a clutch which would slip without overheating as the spool filled, together with improvements in the wire drawing compound and the shape and quality of the diamond dies later described, permitted the drawing of wire at speeds ranging from 2000 to 3000 ft. per min.

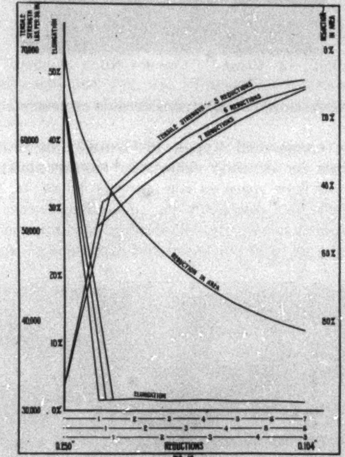

FIG. 16

WIRE DRAWING COMPOUND

At low speeds it was discovered that the compound for lubricating wire-drawing dies required little attention but as the speeds were increased the necessity for close analytical control was evident. The compound consists of an emulsion of soap, tallow, and water, the percentage of the soap and tallow being varied depending upon the size of wire and type of machine on which it is used.

It is important that the degree of emulsification[5] be carried far enough to break the tallow into particles about one micron in diameter, so that the material will stay in suspension in the water. If the tallow content is increased beyond a certain point it holds in suspension in the solution a large amount of the copper dust which flakes off in a very fine state during the wire-drawing operation and this clogs the dies and causes breakage during the wire drawing. Ordinarily this copper dust settles out of the solution while in the large cooling tanks and a considerable amount is salvaged in this manner.

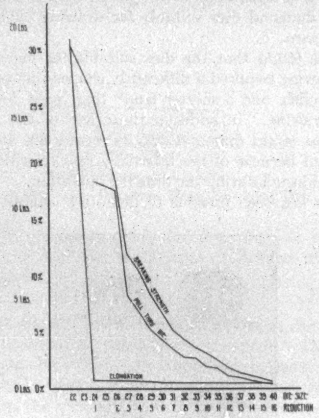

FIG. 17

EFFECT OF DRAWING ON COPPER

Tests were made to determine if the drawing of the smaller cable and all magnet wire sizes[6] in Brown & Sharpe (A. W. G) steps was obtaining the maximum reduction possible per die. These tests showed it was feasible to make much heavier than A. W. G. reductions at the first draft when annealed wire or soft copper rod was being drawn. It also showed that the elongation[7] of the rod or annealed wire was rapidly reduced to the drawing minimum after the first pass, and remained at that point throughout the process.

Fig. 16 illustrates the effect of a five-die reduction on elongation and tensile strength. It may be seen that the elongation drops very rapidly at the first die when a reduction in area of about 42½ per cent is made, and the tensile strength increases rapidly because of the cold working of the metal.

This same figure shows the tensile strengths obtained when five-, six-, and seven-die reductions are used to produce line wire of 0.104 diameter from the same supply. Here the elongation loss is about the same in each case, but the tensile strength is greater with the heavier reductions. The five-die arrangement is satisfactory according to the results shown on the curve, but the heavy reduction at the first die often results in rough or slivered wire. The six-die arrangement, therefore, gives the greatest factor of safety. The seven-die arrangement is less satisfactory since the elongation and tensile strength in the finished wire are so close to the requirements.

The use of A. W. G. reductions for the finer sizes of cable and magnet wire provides flexibility, since a change in the size of wire can be accomplished simply by increasing or reducing the number of dies used. Tests were conducted to determine the gain by using heavier reductions and annealing the wire before redrawing, and Fig. 17 shows the increased reduction possible at the first die when the metal is plastic. In this test, an annealed No. 22 gage wire of 31 per cent elongation was reduced to No. 24, two gages, in one draw. The soft copper permitted a double reduction at the first die, but the elongation dropped during the operation to less than 1 per cent; the second reduction on this test was from No. 24 to No. 26 gage and the pull

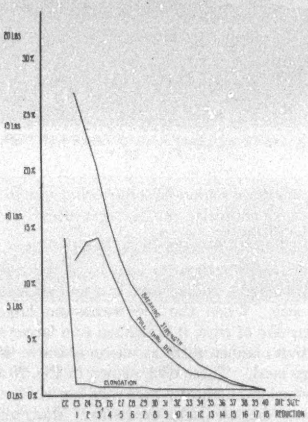

FIG. 18

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[5] "The Theory of Emulsions and Emulsifications," W. Clayton.

[6] A. W. G. ("American Wire" or "Brown and Sharpe" gage) reductions are not used in converting the rod to line wire; these are generally specified in B. W. G. and N. B. S. gages.

[7] See Figures, 16, 17, 18, and 19 showing the elongation of the rod or wire dropping to about 1½ per cent at the first die reduction and remaining practically constant.

required for this pass practically coincides with the breaking strength of the wire. Wire drawing under such conditions is impractical because the annealing operation is much more expensive than drawing hard wire from No. 22 to No. 24 in two passes.

Fig. 18 illustrates the results obtained when drawing annealed wire with A. W. G. reductions. The large margin of safety between the pull required and the breaking strength of the material again disappears after two reductions. Fig. 19[8] illustrates practical drawing conditions adopted for drawing wire to finished sizes without annealing during the process.

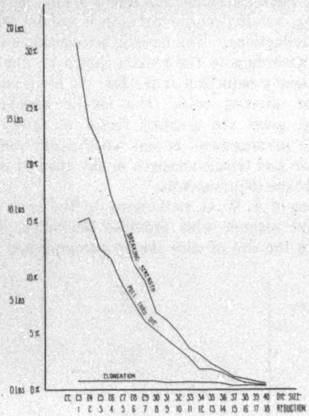

Fig. 19

Chilled Iron Dies

The dies used for drawing heavy wire are cast with a tapered hole from chilled cast iron and reamed to the desired size. When the die wears too large for a particular size of wire, it is reamed to a larger size and used in that manner until the dies goes above the maximum size used. These dies, shown in Fig. 20 are used for drawing line and heavy gage wire for which the cost of diamond dies would be excessive. Many alloy steel dies have been tested as substitutes for chilled iron dies for copper wire drawing, but so far have failed to replace them, due to excessive cost. For the wire sizes smaller than No. 16 down to as fine as No. 42 B. & S. diamond dies as described below are used.

Diamond Die Study

It was necessary to make an extensive study of the manufacture of diamond dies because dies through which wire could be satisfactory drawn at low speeds failed to draw to gage and without excessive breakage of the wire as the speeds were increased. At this time practically all commercial diamond drilling was done in Europe, Belgium being the hub of the diamond cutting industry and the art was new to this country. The diamonds generally used for wire drawing dies are obtained from South Africa[9], Australia, and Brazil, and made into diamond dies in Europe.

In view of the difficulty of obtaining dies for drawing wire at high speeds and the large investment in dies required for the proposed wire mill, it was decided to undertake a laboratory investigation of the manufacture of diamond dies suitable for drawing cable and magnet wire.

It was found that the dies suitable for high-speed wire drawing required a differently shaped approach, a better polish, and a shorter land[10] than used for low-speed drawing. In addition the origin of the stone, the shape of the diamond and its setting are all very important because of the internal strain to which the die is subjected during the drawing operation.

It has not been possible to definitely establish any quantitative relationship as to the effect of high-speed drawing on the wear of dies except that about the same number of million feet of wire may be expected from a properly lubricated die irrespective of the drawing speed. Under such conditions, the high-speed die naturally runs a shorter time, but length of life is not

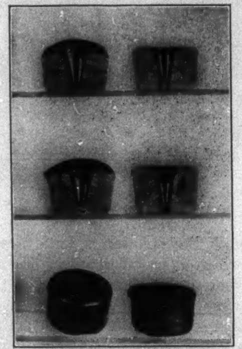

Fig. 20—Dies Used for Drawing Line and Heavy-Gage Wire

the important factor; tonnage of a satisfactory quality with a minimum plant and labor investment is the prime consideration.

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[8] Slight irregularities in the curves are due to variations from the mean in the diameters of the dies used during the tess.

[9] The South African and Australian diamonds are the more suitable for wire drawing. There are two types of the former, the smooth brown premier which is not suitable for dies because of its tendency to crack and split, the other commonly known as the Jager, a product of the Jagerfontein mines. These stones, very irregular in contour and light gray to black in color, are most suitable for dies. The Australian diamonds are gray to brown to almost black in color and can be distinguished from the Jager. Many of the Brazilian diamonds are a dark gray similar to graphite in color and not being translucent are difficult to inspect for seams, cracks, or inclusions.

[10] See Figure 21.

Fig. 22 shows a diamond before drilling, a stone drilled and lapped, ready for mounting, and a die in the final mounting ready for use. Fig. 21 gives an outline of the shape of the working surfaces of a wire-drawing die.

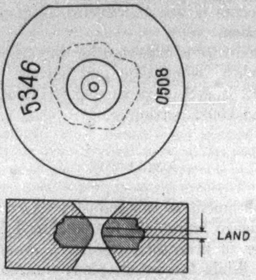

Fig. 21—Diamond Wire Drawing Die

ANNEALING

Hard copper wire is obtained by using the wire as it comes from the wire drawing machine. This same wire may be softened by annealing, or medium-hard wire can be produced by annealing hard wire at such a point in the drawing operations that the final draws will give the desired degree of hardness[11].

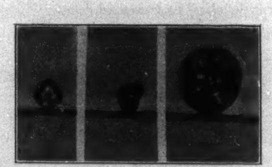

Fig. 22—Showing Diamond Before Drilling, Drilled and Lapped Ready for Mounting, and Die Ready for Use in Final Mounting

In a recent commercial type of annealing furnace, Fig. 23, wire may be bright annealed, but it requires a drying operation in order to remove the water through which it passes in leaving the furnace. The retorts of these furnaces are water-sealed and filled with steam to exclude the outside atmosphere, which would discolor hot copper. To obtain bright wire, it is passed under water into the retort to exclude the air and is generally taken out and cooled under water or in an atmosphere of steam or gas, which excludes oxygen until the wire is relatively cool.

A special steam-seal annealing furnace for small spools of wire was developed on an experimental basis

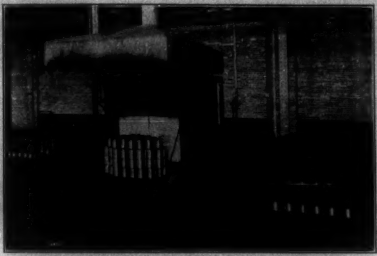

Fig. 23—Water-Seal Annealing Furnace

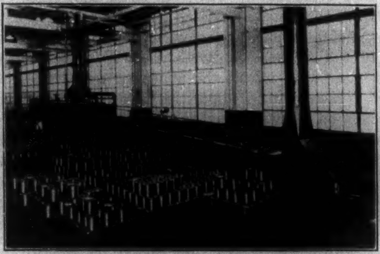

Fig. 24—Steam-Seal Annealing Furnace

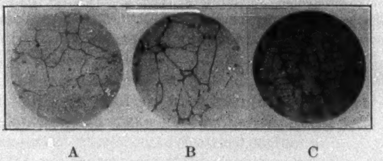

Fig. 25—Photomicrographs of Wire Bar (Magnification 33)

A. High Set—Oxygen, 0.035 per cent
B. Level Set—Oxygen, 0.05 per cent
C. Low Set—Oxygen 0.12 per cent

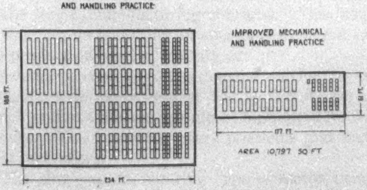
CONVENTIONAL MECHANICAL AND HANDLING PRACTICE
186 FT.
234 FT.
AREA 43,524 SQ FT
IMPROVED MECHANICAL AND HANDLING PRACTICE
61 FT.
177 FT.
AREA 10,797 SQ FT

Fig. 26—Diagram of Wire-Drawing Plant

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[11] "Experiments in the Working and Annealing of Copper," F. Johnson, *Journal Institute of Metals*, Volume XXVI, No. 2, 1921.

from which the wire was obtained bright annealed and free from moisture. In this furnace the spools were submerged in water to displace the air, raised into the charging end which was under water, thence to the muffle to be heated and then along a cooling tube to the discharge opening. Air was excluded from the retort and cooling chamber at the discharge end by means of a steam jet.

The success of the small furnace led to the construction of a larger machine (Fig. 24) for annealing cable wire on spools. The spools are placed in perforated metal baskets which are charged into the furnace at a specified time interval, pushing each other through the retort and along the cooling tube to the discharge end.

INSPECTION OF RAW MATERIAL AND FINISHED PRODUCT

Wire bar made from electrolytic refined copper is used as a material in the manufacture of wire. This material is practically free from silver and other elements which ordinarily exist in the ore, and which have a detrimental effect on the electrical or physical properties of the finished product. A small percentage of silver[12] seriously affects the annealing qualities of the wire. Traces of other impurities have a very detrimental effect on the wire drawing properties. During the refining process, the molten bath is oxidized in order to carry off the foreign material in the form of slag, and it is very important that the oxygen content be later reduced to a very small point if bars of proper set are desired. Fig. 25 shows three photomicrographs of wire bar containing varying amounts of cuprous oxide[13]. Ordinarily the surface condition on top of the bar is a good index of the oxygen content. If the bar is level set or slightly convex on top, it is usually a satisfactory material. If it is low set or concave it usually contains a large amount of copper oxide, which caused the metal to shrink in solidifying[14]. When excessive shrinkage occurs it has an adverse effect during the rolling operation.

The finished wire is inspected for dimensional limits, tensile strength, elongation, and surface condition. Limits for 42 B. & S. gage wire 0.002475 are 0.00245 minimum and 0.0025 maximum.

CONCLUSION

The establishment of this industry as a part of the plant at Chicago represents the combined effort of a large number of inventors, engineers, designers, and mechanics. While the actual plant was built within a comparatively short period, the advances which have been made in the art represent several years' effort. Briefly, the development of compact and high-speed wire-drawing machines has required a much smaller investment in buildings and equipment as compared with a plant of the same capacity using commercial equipment. A comparison of the relative floor area, based upon the conventional and the improved types of wire drawing equipment, is illustrated by Fig. 26. The supervisory force in charge of the operation of this new mill must be given a considerable share of the credit for its successful operation.

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[12] "Effects of Silver on the Recrystallization Temperature of Copper," Caesar and Gerner, A. S. M. E., Volume 38, 1916.

[13] "Microscopic Structure of Copper," H. P. Pulsifer, Mining and Metallurgy, January, 1926.

[14] "Copper Refining" Lawrence Addicks.
"Metallurgy of Copper" H. O. Hofman.

A-C. Elevator Motor Drive

BY E. B. THURSTON[1]

Fellow, A. I. E. E.

Synopsis.—*There are very few data available on the problems of using alternating current when applied directly to a motor on an elevator. It is apparent, also, that eventually there will be no d-c. power transmitted for elevator service.*

Because of these facts and the size of this industry it seems that such data should be available. A paper covering the entire field in detail would be excessively long and it is therefore the aim of this paper to cover the subject in a general way, giving such outstanding facts as are felt to be of most interest at this time.

It is hoped also to correct a false impression that is sometimes found to exist—that an a-c. elevator is not practical for car speeds above 350 ft. per min. Without question, this understanding was correct six or seven years ago but it is desired to call attention to the fact that for the past five years many a-c. elevators have been installed with car speeds in excess of 500 ft. per min. and today some are operating as high as 700 ft. per min., and nothing has appeared to indicate that there is a limit of car speed other than for any other type of control.

A brief outline of the necessary requirements of the elevator machine is given because as yet the development of a-c. elevators has depended upon the success of this unit.

The desirable characteristics of the motor are given somewhat in detail, the important ones being positive speed control, elimination of exposed and sliding contacts, speed ratios of at least 6:1, a rotor of low kinetic energy, quiet under operation, allowing torque characteristic changes, smooth control of speed changes, liberal temperature range, high power factor, a maximum torque capacity, and maximum practical starting torque per ampere.

The desirable characteristics of the controller which permit high speed elevator operation with economical and reliable service and a minimum number of shut-downs, may be outlined as follows: Full magnetically operated but with a minimum number and types of magnets, types of magnets that guarantee against magnetic hum or chatter requiring no oil immersion and giving a constant pull. The controller parts in general should be as interchangeable as practical, with oilless bearings and a minimum of auxiliary parts and contacts. As a whole, the controller and its wiring must be simple and easily understood.

The principles of control allowing the high-speed elevator operation are rapid but smooth acceleration and retardation, a forced slowing down of the elevator by the motor irrespective of the operator and allowing the simultaneous or overlapping braking action of the slowing down and stopping means.

The brake magnet must be one guaranteeing against magnetic hum or chatter, giving a constant pull, and must be positive and rapid but not violent in action.

The curves which were taken by power companies serve to show the high power factor and a minimum of line disturbance.

INTRODUCTION

STATISTICS show that the elevators of the larger cities actually carry more passengers than the horizontal transportation systems. Furthermore, there is probably no industry so vital to the public and of corresponding size with its equipment and principles so little understood by the engineering profession. Naturally they are justified in asking for more reliable data.

Referring to published papers on elevators, we find there are a few on elevator service, d-c. elevators, etc., but without question there is need for more information on these and similar subjects. Upon searching for papers with information about a-c. elevator equipment, its history and problems, it is found there are so few available that it would be very difficult to get a fair understanding of this phase of the industry.

Far from a general understanding of the present art of a-c. elevators, we hear such expressions as these: "They are all right for low-speed elevator service;" "They cannot be operated smoothly;" or, "A-c. magnets and motors are too noisy to use for passenger service." These and similar expressions serve to show that it is easy to forget that new developments are possible.

Today, it has been proved illogical in connection with any development to say "It cannot be done," although this statement has been made in connection with high-speed a-c. elevators.

When we stop to realize the tremendous change from direct to alternating current in general applications, and other developments that have been made in recent years, it is natural to expect that the elevator industry has progressed accordingly. It is felt, however, that this industry did not keep pace with developments in general previous to 1920, with the result that since that time the developments have been tremendous, especially with alternating current, in order to bring this industry to its present state.

The fact that passenger a-c. elevators are operating at 700 ft. per min. and have been proved practical and reliable should make information regarding their development interesting to electrical engineers.

It is not the purpose of this paper to cover the subject in detail but rather to give a general discussion of the more important problems with the hope that other papers with more details will follow.

HISTORY

About 20 years ago there was developing a demand for direct-connected a-c. elevators, especially from many of the larger manufacturing concerns. This was due to their plants operating on alternating current and to the production basis of manufacturing.

A large percentage of this demand was for the higher capacity with car speeds of 100 to 200 ft. per min., which at that time was considered high speed for freight service. This presented many problems that had not been met before.

The single-speed, slip-ring type of motor, being the only one considered practical for elevator service, was

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[1] Haughton Elevator and Machine Co., Toledo, Ohio.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

used and proved quite satisfactory for the demands of service at that time. Due to continual starting, stopping, and reversing, the major problems were those of preventing the rotor working loose and the burning and rapid wearing of the slip-rings.

The brake problem was considered the most serious and difficult. Many devices were developed with none having all the desired features.

The mechanically operated brakes gave the best operating service but were not by themselves considered safe. The magnet-operated brakes were very violent in action and noisy. A brake that proved quite popular and reliable was a dual type, using the mechanically-operated principle for the service and a magnetic-operated emergency brake for safety.

At this time a-c. elevator controllers were mostly mechanically operated, magnetically-operated controllers not having been developed to a reliable stage.

About 15 years ago the high resistance squirrel-cage motor began to find favor and was used for service requiring the lower horse powers. This type of motor at first met with considerable opposition, probably more because of its name than any other one thing, although there were many motors of this type manufactured with characteristics such that objections to their use were well warranted.

At this time these squirrel-cage motors were started directly across the power lines, requiring only a very simple controller, with the result that an elevator of this type was found to be the most reliable obtainable.

Its operation limited its application, however, because of the high instantaneous starting current drawn from the line, and its too sudden starting of the car. It is now apparent that to have added a resistance starter or its equivalent would have overcome these objections, but we must not forget that at that time one of the chief objections to the slip-ring type of motor was that it required a starting device, and there was no real satisfactory and reliable magnetic starter available for elevator service. Thus the high-resistance, squirrel-cage motor across the line elevator gained its reputation for reliability.

About 10 years ago the fact was apparent that the a-c. elevator for all car speeds and service was inevitable. While passenger elevators were at this time available for car speeds up to 300 ft. per min., they could not be considered as satisfactory for reliability, smoothness and noise. It was necessary, therefore, to plan for a large amount of research and development work.

The equipment offered at this time generally used a 3:1 motor speed ratio and a motor composed of a single stator and rotor with slots sufficiently large in each to permit two insulated and independent windings. This required not less than five slip-rings. Although it is difficult to accelerate a slip-ring motor smoothly and rapidly with varying loads, it is even more difficult to retard from a higher to a lower synchronous speed, smoothly. It was also found impossible by any known method to reduce the motor noise sufficiently to make it satisfactory for all elevator service.

Experience indicates that it is impossible to accelerate an elevator at high speed with a slip-ring motor rapidly and smoothly with varying loads except by what might be called a dual acceleration.

Let us suppose that we have an elevator fully loaded; to accelerate it in the up direction will require approximately the maximum torque available and a controller resistance may be used that produces maximum torque at standstill. This resistance may be cut out by any known method, either series relay, speed control, or definite time, and this operation will be satisfactory.

When starting the fully loaded elevator in the down direction, however, the load is tending to drive the motor in that direction and its maximum torque added to the load tendency to start gives too violent an action, so it is necessary to use additional resistance for a smooth start under all loads. With this additional resistance it is impossible to utilize the full available torque of the motor when used in connection with the series relay or speed control method of acceleration.

If we now use a definite time control for acceleration, it must be set slow in order to lift full load and not short circuit the resistance too rapidly, causing the motor to stall. This results in too slow an acceleration when lifting load, if the resistance is such as to give smooth operation when lowering the load. The only way found to insure the desirable results was by the use of a rapid definite time control acceleration for all surplus resistance down to that which produces maximum torque at standstill and from this point by series relay or speed control.

With this type, it was found that with fairly rapid and smooth acceleration it was possible to insure lifting a 25 per cent greater load with a given motor.

This, however, increased the cost considerably, made the controller more difficult to keep in adjustment, and still used slip-rings.

During the next five years there were what might be termed radical developments, the results of which allowed the installation early in 1922 of the first a-c. elevators running above 500 ft. per min.

With these developments and further experience the range of speed has been raised to 700 ft. per min.

DEFINITION OF A-C. ELEVATOR

In order that there may be no misunderstanding as to the type of equipment considered when the term, a-c. elevator, is used, it should be defined as an elevator with alternating current applied only directly to the elevator motor, controller and brake with no conversion.

MACHINE

In the development of a-c. elevators for the higher car speed service, there were four vital units to be considered, each a complete problem by itself—the machine, the motor, the controller, and the brake.

It was immediately apparent from known principles of a-c. motor construction that a suitable geared elevator machine designed for the high car speed service must be produced before much headway could be gained with the other units.

It was not, nor is it now, practical to produce a gearless type, a-c. motor that is satisfactory for elevator service. The geared type elevator machine for the higher car speed service had been more or less neglected because a gearless type d-c. motor had been developed for this class of service.

It soon became evident that the geared elevator machines, designed primarily for the slower car speed service, would have to be re-designed for the higher car speeds and would require refinements for passenger service that had not been available theretofore.

Some of these important refinements were: a more sturdy machine throughout, more accurate machine work especially in connection with the gear, antifriction bearings for the thrust of the worm shaft, antifriction bearings for the drum shaft, and gears which would be perfectly adjustable under load and running conditions of the elevator.

The reason for the more sturdy machine and accurate workmanship is to reduce vibration that may produce noise objectionable for this class of service. The antifriction bearings are necessary to increase the efficiency and the adjustable gear to give the proper running position. As further explanation for the adjustable gear, it will be appreciated that it is impossible to babbitt a gear in place and insure having the correct gear tooth contact under operating conditions with the load of car and counterweights on the machine.

Inasmuch as the slower and moderate speed passenger elevators do not always warrant as high a degree of refinement as the higher speed passenger elevators, it is desired to confine the remainder of this paper to a-c. elevators with car speeds greater than 350 ft. per min.

THE MOTOR

The a-c. motor being used today for the higher speed passenger service represents the result of the accumulated experience of the last 15 years. It has been a very gradual and conservative development.

It is at once evident that the motor must have more than one speed and these speeds must be positive and practically constant whether the motor is running as a motor or is driven as a generator. A direct-connected elevator motor is always required to operate under both conditions, and any motor which varies much in speed with load variations must be eliminated.

Accordingly, the induction type of a-c. motor is the only one that has been found suitable for elevator service and giving the positive multi-speed control.

As was stated in the history, the slip-ring type of induction multi-speed motor was preferred until about 1919 at which time the multispeed high resistance squirrel-cage motor was indicating advantages.

Past experiences gave a basis for arriving at what was essential for a motor in order to produce what would be considered a satisfactory a-c. elevator for all service and speeds. Tests indicated that so far as is known the single stator with two independent windings and a high resistance squirrel-cage rotor does fulfill more of the requirements than any other type available.

The more important of these requirements are outlined and explained as follows:

First, it must be practical to manufacture motors of at least 6:1 speed ratio and still have a motor of practical size to install on the elevator machine. As far as is known, 3:1 ratio was the maximum for slip-ring type motors. Squirrel-cage motors of from 25 to 150 h. p. with 6:1 ratio have been in regular elevator service for over five years.

Second, the motor must be as small as possible, especially the rotor. This is necessary in order to keep the kinetic energy at a minimum and allow rapid acceleration and retardation with a minimum of power consumption and starting current. It is apparent that the single stator squirrel-cage motor would be best fitted for this requirement.

Third, it must be possible to design a motor sufficiently quiet in operation to allow its use in hospitals, hotels, apartments, office buildings, and private residences. Because of the insulated polar windings in the stator and the rotor, the slip-ring type of motor requires slot combinations such that it is impossible to reduce noise sufficiently, while the squirrel-cage motor allows practically any slot combinations.

Fourth, the motor must allow torque characteristic changes at the installation. This is necessary because it is practically impossible to determine the exact requirements in advance, and if it were possible, it would be impractical to design a motor for each application. These changes can be accomplished in either the slip-ring or squirrel-cage motors after the general motor characteristic requirements are known.

Fifth, the motor must allow a smooth and positive control of speeds, whether accelerating or retarding, and still not require an excessive number of switches or a complicated and expensive type of controller. This is necessary in order to obtain simplicity and reliability. As outlined in the History, the squirrel-cage motor is best suited for this requirement.

Sixth, all sliding and exposed contacts should be eliminated. This is very essential because of the operating conditions and requirements. It is a generally recognized fact that the squirrel-cage type of motor is the most reliable.

Seventh, the motor must have sufficient radiation to permit the handling of maximum elevator service without abnormal temperature rise. This is one of the most important problems of design in connection with high-resistance squirrel-cage motors for elevator service. It is possible to change the heating of a motor on elevator service a large per cent by changing its characteristics. It should also be recognized that standard commercial motor parts cannot be adapted to all elevator service. One of the most important facts to consider when designing a motor for elevator service is that its average running speed is usually less than one-half its full rated speed. This results in less than one-fourth the volume of air that a constant running motor would have. And last its radiating surface must be carefully considered.

Eighth, it is very important to have a relatively high running and starting power factor. The reason for this is to reduce to a minimum the power line disturbance when starting or lifting maximum loads.

Ninth, a motor should be capable of producing the maximum starting torque that is practical to obtain with given mechanical dimensions. This is necessary in keeping the kinetic energy at a minimum and increasing the acceleration and retardation efficiency.

Tenth, the motor should produce the maximum practical torque per ampere in starting. What is meant by this is that it should produce the maximum starting torque obtainable without increasing the frame size. This is also necessary to obtain the most efficient acceleration and retardation. In this connection attention is called to the fact that it is more important to have efficient acceleration and retardation than efficient full speed operation in order to obtain the most economical elevator service. This fact is also very forcibly shown in connection with d-c. elevators.

Exhaustive tests under actual operating conditions were necessary to develop a motor that would give these requirements. The facts found by these tests resulted in the development of a new motor of entirely new characteristics and one that was very different from any commercial motor available.

CONTROLLER

Next to the development of this motor the important problem was the development of a controller that would give certain required characteristics and could be adapted to control this motor as required.

The important requirements for a satisfactory and reliable a-c., high-speed elevator controller are outlined as follows:

First, it should be full magnetically operated. This is self-evident because it would not be practical to operate at high speeds with a mechanical control.

Second, it should have a minimum number of magnets. This is necessary for simplicity and reliability.

Third, the magnets should be interchangeable, as far as practical. This is essential for reliability and low maintenance.

Fourth, it is very necessary to insure continuously against magnetic hum or chatter. This is because of magnetic noise being so objectionable in connection with elevators. In view of this it seemed essential to use a polyphase magnet of a non-sealing type although this was a radial departure from previous practise.

Fifth, it was very desirable to use a type of magnet that did not require oil immersion, because oil around a controller has always been found objectionable.

Sixth, the magnets should be such as to give a constant pull throughout their range of action. This is necessary in order to eliminate auxiliary retarding devices and yet insure against too violent an action in closing the switches.

Seventh, to insure low maintenance and reliability in operation, the units, contacts, and parts should be interchangeable to the extreme.

Eighth, all bearings should be oilless to prevent the collection of dust and to insure long life with a minimum amount of attention.

Ninth, experience indicates that a series or speed control of acceleration does not lend itself to elevator requirements; therefore a minimum number of dash pots or retarding devices should be used and, if dash pots, they should be large, sturdy, and oilless to insure against variation in their retarding action under all conditions of service, care, and atmospheric changes.

Tenth, there should be a minimum of auxiliary contacts. This is required to insure against frequent or long shut-downs, because experience has always proved that small or auxiliary contact troubles more often do not indicate themselves and therefore are very difficult to locate, the result being long interval shut-downs while locating the fault. Attention is called to the fact that it is not the cost of a replacement contact of an elevator as much as the time out of service, that is most important.

In the interest of reliability, then, there may be mentioned: sturdy construction, minimum number of magnets, minimum of auxiliary contacts, a minimum of power contacts for service rendered, and contacts that give long life.

After considerable research it was found that a polyphase rotating-magnetic-field rotary type of magnet would give the above requirements. It was found that using this type of magnet, however, would make the controller somewhat more expensive to manufacture than with other known types; nevertheless experience has proved that this increased first cost was more than justified by the results obtained.

Further, it is interesting to note that this type of magnet made possible other desirable features. It has inherent phase reversal protection, inherent phase failure protection, allows the locking of the switch contacts in the off position, allows the forcing open of contacts by power should they mechanically stick closed, or vice versa, and gives the car control switch control over two separate and independent circuits.

Incidentally, this type of magnet allows a very simple system of wiring which is not what was originally anticipated.

Figs. 1 and 2 are front and rear views of a controller using this type of magnet, and show the simplicity of wiring and the interchangeability of parts.

PRINCIPLE OF CONTROL

After the development of the controller, the next and probably the most vital considerations to decide were the principles of control.

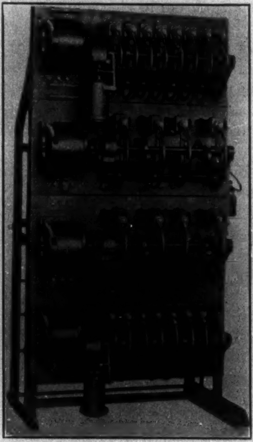

FIG. 1—FRONT VIEW OF A-C. ELEVATOR CONTROLLER

It was evident that in entering a new field of application (because it is believed there were no a-c. elevators in operation with car speeds above 400 ft. per min., prior to 1920), there were many problems to overcome. The principal and most important one of these was safety, which led to conclusions that proved very important.

When it is desired to stop quickly, the neutral or reverse position of the car control switch should produce this result, or in other words when traveling at full speed and immediately moving the car control switch to the neutral or reverse position, all of the normal stopping means must act together. This principle was old with d-c. elevators but had never been available with a-c. elevators. This result or its equivalent is essential for smooth, rapid, positive, and safe operation of a-c. elevators at the higher car speeds.

It will also be appreciated that this equipment has no run-away characterisitics, because there is no generator action from an induction motor when disconnected from the power line. Should the power fail, all devices will immediately return to the stop position regardless of the load, speed, or direction.

It was also found desirable for the control to be such as to tend to increase the power factor of the motor, thereby aiding in reducing any line disturbance under operating service to a minimum. It is probably of interest to know that many of these elevators are operating very satisfactorily in buildings having a common transformer for lights and power, and experiencing no trouble from light flicker.

THE BRAKE

It is rather generally understood that an elevator magnet brake is applied by a compression spring or by gravity and released and held released by a magnet while the elevator is operating.

The major problem is the development of a suitable a-c. magnet that will give reliable operation and still be such that it will be suitable for all classes of work. The important necessary requirements are outlined as follows:

First, it must insure against a magnetic hum or chatter under all conditions of service, such as low voltage, want of adjustment, collection of dirt, etc.

Second, it must be one that does not require oil immersion for cooling or reducing noise. Further objections to the use of oil immersion are its effect in varying brake operation with atmospheric changes and the liability of getting oil on the brake drum.

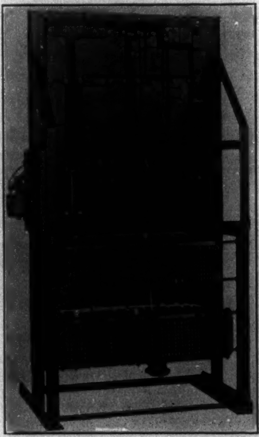

FIG. 2—REAR VIEW OF A-C. ELEVATOR CONTROLLER

Third, it must insure against the violent action so common to a-c. magnets.

These requirements prompted the following: that it should be polyphase, non-sealing, giving a constant pull, and be reciprocating in action.

After considerable research a magnet was developed that fulfilled all of the above features.

A sectional view of this magnet is shown in Fig. 3. Its principle of operation is the taking advantage of the end pull on a polyphase induction motor when the rotor is axially shifted out of line with the stator. This characteristic everyone is familiar with, and it is also generally known that the synchronous rotating magnetic field of such a motor is always constant.

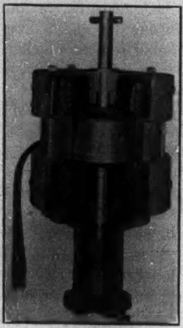

FIG. 3—NON-SEALING A-C. BRAKE MAGNET

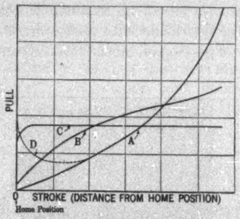

FIG. 4—CHARACTERISTIC CURVES OF A-C. MAGNETS

Curves *A*, *B*, and *C* are for the rotating-field type of magnet. Curve *A* is for a cylindrical coil. Curves *B* and *C* are for tapered cores, Curve *C* being the most desirable design. Curve *D* is a characteristic of the ordinary a-c. magnet.

Now if only the rotor iron is used with no closed winding on the rotor, there will be little rotative action but the end pull action remains the same, and its value will be absolutely steady and entirely free from a cyclic vibration.

With a laminated core having a surface parallel to the axis, the end pull curve characteristic is as shown by curve *A* in Fig. 4. The abscissa of this curve is shown as inches of stroke or movement from the home position, shown at zero, with the extreme right hand end showing the position at which the core is about to leave the stator. The ordinates are shown as the end pull in pounds necessary to maintain any particular position or stroke.

It was found that the core surface can be tapered slightly and produce curve *B* and by a different tapering to produce a curve similar to *C* which was the desired characteristic curve for this particular application.

It should be remembered that other types of a-c. reciprocating magnets have a characteristic pull curve similar to curve *A* except that the lower or home position will follow curve *D*. With this it is seen that the magnet could not be required to operate and hold more than the lowest point of curve *D* and with the maximum as shown by curve *A* it inherently produces the violent action of a-c. magnets so familiar to electrical engineers.

TEST RECORDS

The following test records were taken at random from files and it is hoped they will prove of interest.

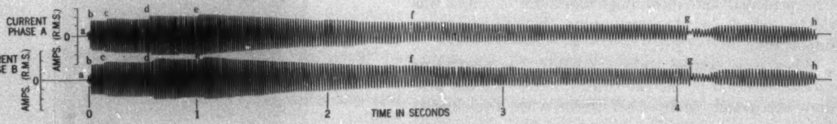

FIG. 5—OSCILLOGRAM SHOWING CURRENT IN A-C. ELEVATOR MOTOR, STARTING, RUNNING, AND STOPPING

Taken on an elevator rated 1500 lb. at 425 ft. per min, with a two-phase, 220-volt motor having 3:1 speed ratio. The elevator was run from the third to the fifth floor with 1000 lb. in the car. At *a* the motor was started on its low-speed connection. At *b* it was transferred to the high-speed connection. Positions *c*, *d*, and *e* show respectively the closing of the first, second, and third accelerating switches. At *f* the elevator is in full speed. Position *g* shows the transfer to low-speed connection and *h* shows the disconnection for stopping.

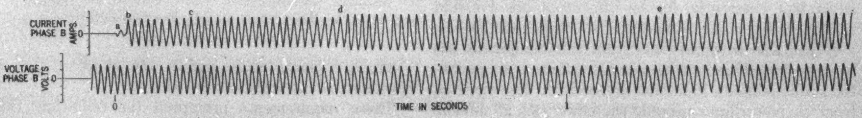

FIG. 6—OSCILLOGRAMS SHOWING ELEVATOR OPERATION WITH OVERLOAD

The conditions are the same as those in Fig. 5 except that the car carried 2000 lb., an overload of 33 per cent.

Figs. 5, 6, and 7 were traced from tests made by one of the largest power companies and are of an elevator rated for 1500-lb. capacity at 425 ft. per min. The motor is a 3:1 speed ratio, 220 volts, two-phase. Fig. 5 is an oscillogram showing the amperes in each phase with time in seconds as abscissa. In this particular operation the elevator was run from the third to the fifth floor with 1000 lb. in the car. Starting from the left at zero time, position *a*, the motor was connected for its low speed. Position *b* shows the current on transferring to the high-speed connection. Position *c* shows the closing of the first accelerating switch. Positions *d* and *e* show the closing of the second and third accelerating switches. Position *f* shows that the elevator has attained approximately full running speed and that this has taken place in 2½ sec. Position *g* shows the transfer to low speed for slowing down or stopping and position *h* shows the point where motor was disconnected for stopping, thus completing a two-floor travel in less than five sec.

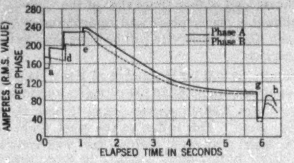

FIG. 7—CURRENT CURVES OF ELEVATOR LIFTING OVERLOAD

The load is 2000 lb. The points *c*, *d*, *e*, *g*, and *h* correspond to similar points in Figs. 5 and 6.

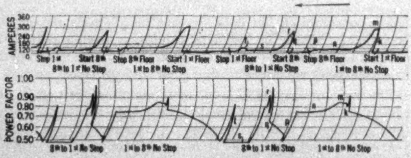

FIG. 8—PERFORMANCE CURVES OF ELEVATOR IN FULL RUNS FROM FIRST TO EIGHTH FLOORS

Elevator is rated 2500 lb. at 510 ft. per min. with a three-phase, 220-volt motor with 4:1 speed ratio. The motor carried 2031 lb. Position *k* is starting upward; *m*, accelerating upward; *n*, running full-speed upward; *p* slowing and stopping after upward run. Point *q* is starting downward; *r*, accelerating downward; *s*, running full-speed downward; *t*, slowing and stopping after downward run.

Fig. 6 shows oscillograms of the voltage and current in one phase with *a*, *b*, *c*, *d*, and *e*, representing the same positions as shown in Fig. 5, but with a more rapid time scale and lifting a load of 2000 lb. in the car. It should be noted this is an overload of 33 per cent.

Fig. 7 is a curve plotted by the power company from an oscillogram of the amperes in each phase while the elevator is operating from the third to the fifth floor with a load of 2000 lb.

Figs. 8, 9, and 10 were taken by another large power company and are curve-drawing instrument records; they are to be read from right to left. This elevator has a rated capacity of 2500 lb. at 510 ft. per min., the motor being a 220-volt, three-phase and having a 4:1 speed ratio. The same letter on each curve represents the same operation in all three figures. Position *k* is the point of starting in the up direction, *m* is accelerating in the up direction, *n* the full speed running in the up direction, *p* the slowing down and stopping in the up direction, *q* the point of starting in the down direction, *r* is accelerating in the down direction, *s* the full speed running in the down direction, and *t* the slowing down and stopping in the down direction.

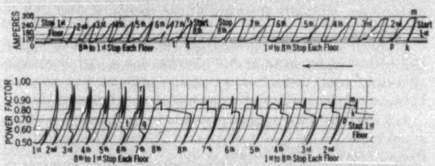

FIG. 9—ELEVATOR PERFORMANCE STOPPING AT ALL FLOORS

Elevator, load, and notations are the same as in Fig. 8

It should also be noted all these tests were with a load of 2031 lb.

Fig. 8 shows full runs between first and eighth floors, the upper curve being for amperes and the lower one for power factor.

Fig. 9 shows full runs between first and eighth floors but stopping at all intermediate floors.

It is understood that this installation in actual service is showing an average power factor in excess of 80 per cent.

Fig. 10 is of particular interest because it shows the effect on the current when the elevator is traveling at full speed in each direction and an instantaneous full reverse movement of the car control switch is made.

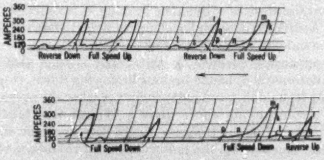

FIG. 10—EFFECT OF REVERSING ELEVATOR WHEN RUNNING AT FULL SPEED

Elevator, load, and notations are the same as in Fig. 8

Without giving detail thought to the matter, there has been a general impression that an a-c. elevator motor does not return power to the line as a generator when the load is tending to drive it above its synchronous speed. In order that this impression may be corrected, it is found from experience that on elevators with car speeds from 400 to 700 ft. per min., the motor will supply the controller and brake losses and in addition return from 25 per cent to 40 per cent of its rated hp. back to the power line as a generator.

Discussion

P. A. Lindemann: Unfortunately Mr. Thurston's paper does not give speed-time curves of the high-speed geared a-c. elevator. I should like to know the details of the stopping period of a fully loaded elevator having a car speed of 700 ft. per min. and the brake applied without appreciable slow down by the motor.

I rather expect an unevenly divided period of deceleration with too abrupt an ending due to the absence of dynamic braking, and too great a pressure on the shoes at low rotor speeds. I wonder if this assumption is correct?

It would also be interesting to know as to the thrust performance on a-c. elevators operating at car speeds of over 500 ft. per min.

To me it seems that the electrical efficiency would be higher, and cost of maintenance lower, were the compensator principle of control used, instead of the series resistance type which is liable to unbalance the phases due to loose and broken grids.

J. Lebovici: We would appreciate some description of the construction of the motor mentioned in this paper and would like to know in what respect it differs from a high-resistance Rotor squirrel-cage motor. We would appreciate having some speed-torque characteristics of the motor; also the slow-down characteristic.

We also would like to know if the motor is of two-speed, two-winding type.

We notice the statement that an elevator motor must allow torque characteristic changes at the installation. I would like to know how these changes could be made at the installation outside of varying, of course, the resistance in series with the motor stator.

We also notice that the number of switches or the number of magnets should be kept to a minimum. While we agree with this statement, we believe that the number of steps of acceleration should be as high as possible.

We would like to call attention to the development of a controller using an induction regulator for the purpose of applying a varying voltage in an infinite number of steps to the elevator motor. Such a control approaches the variable-voltage control of the d-c. motor.

E. B. Thurston: The question by Mr. Lindemann in regard to the stopping characteristics of a 700 ft. per min. a-c. elevator, if understood correctly, is a very important one. While he mentions the absence of dynamic braking, it is believed the information desired is the stopping characteristics, when the controller is moved quickly to the off position.

The apparatus covered by this paper is such that there is always automatic dynamic braking in slowing down, stopping, or reversing from high speed independent of what the operator does.

The accompanying curve gives a typical test taken by an oscillograph. Curve *A* shows where the operator is slowing down the elevator by the motor only and running a short distance *C* on the slowest speed point and finally stopping at point *D* by the action of the brake.

Curve *B* is one showing the action when the operator quickly moves the car control switch to the off position.

It is, of course, to be appreciated that by adjustment any rate of retardation desired can be obtained.

It is felt this feature of combined braking action is part of the development that made the a-c. elevator possible for higher speed service.

As to his next question in regard to the thrust performance on a-c. elevators, experience has indicated that there is no difference in in the use of a-c. from that with d-c. power. If the question is in regard to the geared-type machine for the higher car speeds, we would cite as an illustration, a machine designed for either duty of 10,000-lb. capacity at 150 ft. per min. or for 2500-lb. capacity at 600 ft. per min. It is only necessary to change the gear ratio, to change from one to the other rating, and it surely is self-evident that the thrust loads, the tooth load and machine stresses are less on the 600 ft. per min. than on the 150 ft. per min. service.

Experience has proven that the wearing qualities of the gear in the higher speed service is much better.

The thrust load may be calculated by different methods and it will be found that for full load lifting the 10,000-lb. service will have a thrust load of four times that of the 2500-lb. service, and for all allowable acceleration and retardation rates, the thrust loads of the 10,000-lb. service will be more than twice that of the 2500-lb. service.

Incidentally, it will be of interest to know that the 10,000-lb. service requires from 20 to 25 per cent greater motor horse power than that of the 2500-lb. service, even though the actual work done is the same. This is due to the much higher efficiency of the higher speed gear ratio, and is a fact well known in the gear art.

The question of compensator principle of control or resistance was raised and experience has indicated as yet that the resistance has worked out the best. It does not increase materially the power consumption but does increase considerably the starting power factor, which is a very vital point in connection with power companies, especially where the lights and power are supplied from the same transformers.

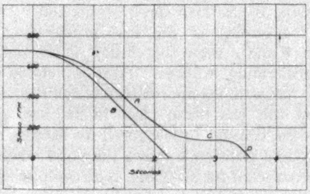

FIG. 1

The first question by J. Lebovici in regard to the motor characteristics is a very logical one, but it would be too long to go into the discussion of that at this time. It is hoped that in the next year or two a complete paper on motor development can be given.

His next question is in regard to the winding of the motor. The one to which this paper refers has a single stator, with two windings in the primary and with the high-resistance squirrel-cage rotor. However, as far as safe operation is concerned it is immaterial whether it is a single winding reconnected, or a double-type motor, providing it can be made to produce the other desirable characteristics mentioned in the paper.

His next question is in regard to a minimum number of magnets. This is one of the desirable features of the rotating type of magnet allowing the reduction to a minimum. Referring to Fig. 1 of the paper, it will be noted there are four magnets, and that each magnet can operate four or eight accelerating switches and results in a very reliable controller.

He next refers to a controller that uses an induction regulator for giving a varying voltage. This is a comparatively new development, and it is hoped in the near future we may have a paper by those who are building this type of controller.

Since this brings up the question of smoothness or the elimination of the sensation of accelerating or retarding steps, it is desired to state that it has not been found difficult entirely to eliminate these sensations with the equipment covered in this paper.

Stroboscopic Method of Testing Watthour Meters

BY H. P. SPARKES[1]

Member, A. I. E. E.

Synopsis.—*This paper deals with an optical method applied to watthour meter testing. The method as presented overcomes, to a great extent, personal error, and lessens the time required through the use of measured light impulses. It gives instantaneous comparison between watt-seconds on two measuring devices.*

The objects of this method are:

First: To reduce the time of laboratory tests, acceptance tests, and re-calibration.

Second: To reduce personal error, and to increase the accuracy of the test.

Third: To provide a device that gives precision instrument accuracy.

Fourth: To make time devices in precision tests unnecessary.

INTRODUCTION

THE present-day ideal of calibrating and checking watthour meters requires maximum accuracy with minimum loss of time. This paper deals with a device that reduces the time required per meter and increases the accuracy of the test by eliminating the human error factor and giving large indications of small increments of speed. In reality, this device is a wattmeter with a light vernier scale for measuring watt-seconds and giving instantaneous indications of meter speed.

This device measures watts with a high degree of precision, then transfers the measurement into a corresponding number of light impulses per second. The meter disk is calibrated in watt-seconds by means of marks placed on the circumferential edge of the disk.

In operation, a load is placed on the meter and the light impulses are then synchronized with the lines on the disk. When thus synchronized the markings appear to be stationary. The error of the watthour meter is then read on a balance indicator. By this method the accuracy of the meter is checked.

For calibrating the watthour meter the frequency of the light impulses is kept proportional to the meter load. Then if the meter is running at an incorrect speed, the markings on the disk will appear to move. For high speed they will progress and for low speed they will retrogress. To calibrate the meter it is adjusted until the markings appear stationary.

This apparent standing still and moving of the disk markings is the stroboscopic effect, which is more or less familiar to most engineers.

PRINCIPLES OF THE DEVICE

To illustrate the principles, it may be well first to refer to Fig. 1 which shows a portable outfit with hand adjustment only for controlling the frequency of the light impulses. This outfit consists of two principal parts. The first is the light-impulsing machine consisting of a driving motor on whose shaft are mounted a commutator which makes and breaks the light circuit and a magneto whose voltage varies as its speed. A hand rheostat is used to adjust the motor speed. The second part is the balance indicator. This is similar to a polyphase wattmeter except that one element is replaced by a standard d'Arsonval d-c. voltmeter element. In operation the two elements mechanically oppose or buck each other. To the a-c. element is connected the same load that passes through the watthour meter under test. The d-c. element is connected in series with the magneto and its torque is proportional to the speed of the motor or the frequency of the light impulses.

When a meter is to be checked it is connected to a load as shown in Fig. 1. The speed of the motor is then adjusted by the hand adjuster until the markings on the meter disk appear stationary when viewed by the impulsed light. The error in meter speed is then read from the balance indicator.

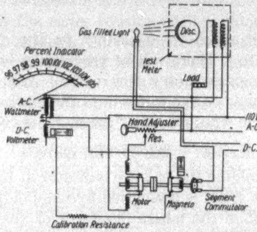

FIG. 1—CONNECTION DIAGRAM FOR PORTABLE STROBOSCOPIC WATTHOUR METER CHECKER

May be used with calibrator by means of transfer switches not shown.

The functioning of the parts may be explained as follows: At the given load the meter revolves at a certain speed. This load also causes a certain torque on the a-c. element of the balance indicator. The speed of the motor is adjusted to synchronize with the disk markings. This causes the magneto to generate a certain voltage and this acts on the d-c. element of the balance indicator. If these two elements exactly balance, then the meter is running at the correct speed for the given load. If the meter is running too fast, the frequency of the light impulses must be increased to bring them in synchronism with the meter disk. On

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[1] Westinghouse Electric & Mfg. Co.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

increasing the light frequency the speed of the magneto is increased which increases its voltage and this increases the torque on the d-c. element of the balance indicator causing it to read high.

Conversely if the watthour meter is running too slow, the balance indicator will read below normal.

The foregoing illustrates the method of checking the accuracy of a meter. For calibrating, the method is varied as follows: The speed of the motor is adjusted so that the balance indicator reads 100 per cent speed or no error. Then if the watthour meter is running at an incorrect speed the markings on the disk will appear to rotate. There will be progression for a fast meter and retrogression for a slow meter. Adjustments may then be made on the meter until the marks appear stationary which will mean that the speed is correct.

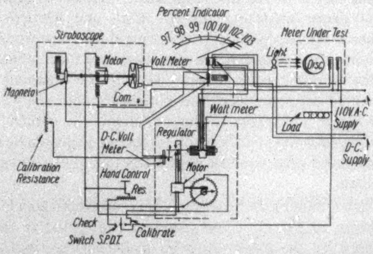

FIG. 2—CONNECTION DIAGRAM OF STROBOSCOPIC WATTHOUR METER CALIBRATOR AND CHECKER

LABORATORY FORM OF THE DEVICE

The job of holding the balance indicator at the point of 100 per cent speed is performed by hand in the portable device but in the laboratory form of the device this is performed by an automatic regulator. Such a regulator is necessary where a high degree of accuracy is desired as it eliminates the necessity of maintaining a balance by hand. This regulator is shown in Fig. 2 which is a design of the laboratory device and connections. The regulator consists of three main parts. The first part is a wattmeter of the Kelvin-balance type which takes the same load as does the tested watthour meter. The second part is a standard d-c. voltmeter of the d'Arsonval type. This d-c. meter is actuated by the magneto already mentioned and its torque is proportional to the speed of the stroboscope motor. These two elements are mechanically connected in opposition and by means of contacts and a reversing control motor (the third part), they control the series resistor of the stroboscope motor. By this arrangement the speed of the stroboscope motor and the frequency of the light impulses are maintained proportional to the watthour meter load.

The foregoing is a general description of this device and in the following paragraphs further descriptions will be given of the four main parts, (1) the watthour meter to be tested, (2) the regulator balance, (3) the light impulsing machine, and (4) the balance indicator.

THE WATTHOUR METER TO BE TESTED

The watthour meter to be tested must have its disk marked with a number of equally spaced lines, usually 300. Fig. 3 is an illustration of a commercial five-ampere, 115-volt watthour meter with marks on the edge of the disk. These marks are carefully graduated so that there are 300 equidistant marks filled with black. However, the black may be omitted, as the lines are visible and are much sharper without the black.

Disks for new meters and old meters may be marked with precision at a very low cost. The major expense will be in changing the disks. Standard disks may be used; but the graduations must be perfect and of the proper number to match the range of speed of the calibrator.

THE LIGHT-IMPULSING MACHINE

The light-impulsing machine is composed of a high-speed series motor (a-c.), a commutator and brush ring, and a speed indicating magneto, all of which are mounted on a heavy bed plate. The shafts are coupled with a high-speed pin coupling of large diameter, the flexible portion being mounted on the motor shaft.

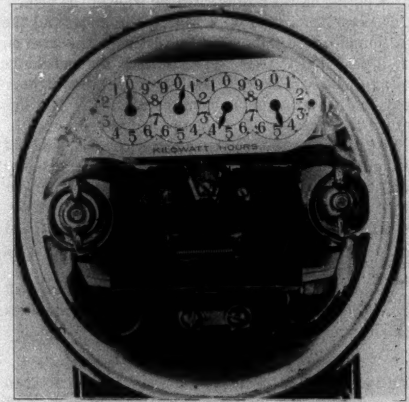

FIG. 3—WATTHOUR METER WITH STROBOSCOPIC LINES ON DISK

The motor is of the high-speed, high-torque type so that it will follow the regulator quickly. This feature would cause the regulator to hunt if it were not properly damped. A magneto with a straight line speed—voltage curve is used and direct connected to the motor by means of a special coupling. This magneto must have annular ball bearings and a very well made commutator.

The light-impulse commutator mounted on the non-flexible end of the coupling has eight sections equally spaced consisting of four metal bars and four insulated sections. The metal bars are held in place by shrink rings which short-circuit them, forming a simple flashing commutator. The brush support is a ring section with one permanent and one adjustable brush. The adjustable brush may be adjusted to the proper "cut-off" or "light-on" position to produce a clear vision of the lines. At slow speed this adjustment is proportional to the ratio of width of space to the width of line on the disk. At high speed such adjustment is not required. It may, therefore, be set for the low-speed point and fixed, namely at 5 per cent of full load as this is about the lowest point at which the stroboscopic effect has been found satisfactory.

A special lamp is connected with a battery or rectifier unit in series with the commutator, so that for each revolution of the magneto, the light is on and off four times. This means that the lamp will have to flicker at a maximum frequency of 300 × 25 = 7500 cycles per minute. This is too fast for standard lamps, and a high-speed lamp such as used in aero signaling was developed in miniature for this work. It is necessary to have a cooling medium and hydrogen gas is used. It was found that 15,000 light periods per minute could be recognized by means of the stroboscope when using this lamp. The possibility of air leaking into the lamp makes it necessary to enclose the lamp completely and to vent the housing to provide for explosion. The explosions are very powerful and danger exists unless properly guarded lamps are used. The current consumed by the lamp is above normal, because the gas absorbs a large portion of the heat at a high rate.

A flood-light may be used for gang or group testing, either for one bench or for several. In fact, the overhead lighting in the laboratory may be changed to operate with this system. Where the room has exceptionally good daylight, it may be best to use a small hand lamp with a focus beam; or the operator may use dark glasses. Dark glasses seem to protect the eyes, preventing eye fatigue.

BALANCE INDICATOR

The balance indicator acts as a check on the regulator when calibrating, and as an error indicator when checking. In the laboratory it should be mounted directly above the tester's position at the test bench. It is similar to an indicating polyphase wattmeter except that one element is a d-c. voltmeter of the d'Arsonval type and is connected to buck the watt element mechanically. Both elements have a uniform scale and, as a result, the pointer indicates the difference which may be calibrated in watts or in per cent at one load. The author is devising an instrument which will read per cent for all loads. The connections for the balance indicator are shown in Figs. 1 and 2.

THE AUTOMATIC REGULATOR

In making tests, no means is provided for keeping the watthour meter load absolutely constant. Therefore, in calibrating, the speed of the stroboscope motor must be varied as the watt load changes and for great accuracy this must be done by an automatic regulator. This regulator keeps the speed proportional to the load. As already mentioned the a-c. measuring element of the regulator is connected to measure the same load as that which the watthour meter measures, and the d-c. voltmeter element is connected to the magneto. These two elements are mechanically connected in opposition. The measuring elements, by means of contacts and the reversing motor, control the series resistor of the commutator motor. The contacts are of the standard three-point type such as are used in graphic meters.

As the wattmeter element is of the Kelvin-balance type, it has a straight-line scale of watt values. This means that there must be a buck-balance with a similar straight-line scale. The standard d-c. voltmeter of the d'Arsonval type may therefore be used for a buck balance. With these two elements mechanically connected in opposition, they will find a point of balance over their entire range. This means that the speed must be proportional to the watts; otherwise the balance will close its contacts, causing the control motor to adjust the series resistor until a balancing speed is obtained through the voltmeter element. This part of the scheme is the heart of the device. Taps for voltage, a range for various current capacities, and a changeover switch on the voltmeter element for various makes of meters should be provided.

When a precision wattmeter is used, the torsion head should be left intact, so that it may be checked with a potentiometer, the calibration of the device being thus established from this point. The torsion spring will have practically no effect, as the balance operates at zero torque, and with practically no movement. On the other hand the torsion head may be set to balance part of the meter torque and thus eliminate the necessity for some of the switches and taps.

The voltmeter element should be of the finest workmanship; also it should be connected to the wattmeter through suitable mechanical linkage and the entire balance must be properly damped.

This part of the device should be built so that it may be located near a standardizing bench, thereby facilitating a check test on the outfit.

OPERATION

The general diagram of connections, as shown in Fig. 2 gives an idea of the electrical connections used for the laboratory set. A source of alternating current is required for loading the watthour meter, the Kelvin balance, and the wattmeter element of the per cent indicator. A small source of direct-current is required for the light. This may be a battery or rectified alternating current with a wave filter in the circuit.

Several load switches are required to cover the testing range. The major switches cover full load, light load and 50 per cent power factor.

To make a check or "as found" test throw the regulator switch to the check position which cuts out the regulator and cuts in the hand control. Then adjust the set until the lines stop moving and read the per cent indicator.

In calibrating, the power is supplied to the load deflecting the watthour meter, Kelvin balance, and per cent meter. This upsets the balance, starts the watthour meter, and gives an indication on the per cent indicator. The Kelvin balance then closes its contacts which control the motor on the rheostat. This decreases the resistance in the commutator-motor circuit until this motor reaches a speed at which the voltage generated by the magneto produces a torque on the voltmeter element that bucks the Kelvin balance and equals the torque developed by the watts in the meter circuit. The contacts then open and regulate the speed of the commutator motor by increasing or decreasing the series resistor.

The speed of the commutator is the same as that of the motor, so that the light is impulsed at a proportional speed. This speed depends upon the number of segments on the commutator. When the frequency of the light impulses is the same as that of the movement of the lines on the watthour meter disk, or the disk line movement is synchronized with the light, the lines will appear to stand stationary, which is the well-known stroboscopic effect. If they are not synchronized, there will be progression for high speed on the meter or retrogression for low speed on the meter, indicating that the meter is out of step or calibration. In some of the standard meters, moving the adjustment screw in the direction of the line motion will correct the error.

While this action has been taking place, the balance meter has been indicating the difference in calibration, as it functions the same as does the regulator. The two elements are bucking and as a result, when speed is proportional to watts, this instrument should read zero or 100 per cent. When the regulator is in use for calibrating, the balance indicator is simply a check on the regulator; but when the hand control is used for checking, this indication reads watts error plus or minus, slow or fast.

At this point it may be well to bring out the fact that the disk markings may appear stationary at harmonic values of speed, but the image is very poor at such values. At the calibration values the image is very sharp and clear. Also, noting the action of the per cent indicator and the range of calibration of the watthour meter will prevent mistakes.

After the above mentioned operations have been taken care of, note the per cent indicator to see if the regulator is functioning properly. Then observe the line movement and adjust the tested meter until the lines stop. Then test for the other load and power factor by simply throwing the proper switch, as the regulator will take care of the change.

The meter under test may be connected after the set is started if quick-connection test blocks are used, as the regulator will take care of the system when the test meter is out of the circuit by stopping the commutator motor, exactly as it would for a no-load condition. This means that no time is lost while changing meters. Because of a possible short circuit at the lead ends, this procedure is not advisable if leads are used to connect the test meter.

ACCURACY OF TEST

As personal errors of switching are eliminated in this laboratory set the accuracy will be materially improved. A second point is that a precision wattmeter may be used, if desired, giving maximum accuracy while calibrating. Observation of the motion of the lines is made through a cylinder type of lens which apparently speeds up the motion in case of slow speed. The author has tried several types, and this cylinder lens gives the best results, although a telescope with cross hairs will give minute measurements.

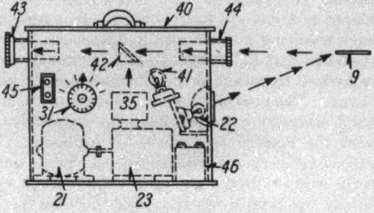

FIG. 4—SCHEMATIC DRAWING OF PORTABLE STROBOSCOPIC WATTHOUR METER CHECKER

It has been found by test that progression of one line one division in 30 sec. is quite perceptible with the unaided eye. Therefore the error which may be discerned equals a movement of 1/30 division in one second.

Full load on the meter used is 500 watts and the disk constant is ⅓. Therefore at 500 watts the disk will revolve at the following speed:

$$n = \frac{500}{3600} \div \frac{1}{3} = 0.4167 \text{ rev. per sec.}$$

As there were 300 equal divisions on the disk the number of divisions per second is $0.4167 \times 300 = 125$ divisions per sec. But the error which can be read equals 1/30 division per second. Expressed in per cent this is $1/30 \div 125 \times 100 = 0.026$ per cent. Translated into watthours at full load this equals:

$$500 \times 0.026 \div 100 = 0.13 \text{ watt-hr.}$$

Discussion

W. B. Kouwenhoven: The method described by Mr. Sparkes is very ingenious. The arrangement that he has provided, for changing the speed of his flashing lamp so as to bring it into synchronism with the marks on the meter disk, makes it possible to use the device to check over a wide range of speed.

The method should give accurate results, as the torque of the wattmeter measuring the a-c. load, is balanced by the torque of a d-c. voltmeter element supplied with current from a magneto driven from the flasher shaft. Care must be taken to see that the armature reaction of the magneto is negligible, otherwise errors may be introduced.

I regret to note that the author has found it necessary to put 300 marks on the meter disk. This requires a high-speed lamp of special construction, and special apparatus in the laboratory always means trouble. If, however, he had used fewer marks and a standard lamp, which would operate at a low speed, he could not have obtained such a high degree of accuracy for his device.

I think that his statement, that his method is excellent for gang meter testing is open to question.

As I see it, the advantages of Mr. Sparkes' method, depend upon the fact that it is easier to detect a departure from zero than it is to read a meter accurately. Mr. Sparkes' method also does away with the necessity of calculation and meter reading, as the scale provided in his device reads per cent error directly. It should give good results in the hands of the unskilled meter tester.

B. J. Brown: I am very much interested in some points made by Dr. Kouwenhoven, particularly to anything that would give trouble in the future in the apparatus, especially such that develops after service.

In regard to the number of marks on the disk, can they be reduced? And the lamp, could it be changed?

And also its use in gang testing? There are other devices on the market. I would like to hear somebody's opinion regarding them.

H. J. Blakeslee: Mr. Sparkes has made a distinct advance that doubtless will eventually be of benefit, to meter manufacturers and electric light and power companies alike.

For many years efforts have been made to reduce the time required for testing watthour meters. Mr. Brown has just spoken of the group methods of testing. There are a number of such methods in use, and also a number of methods for making at least a part of the test automatic and so eliminating the personal error.

Group methods reduce the time required not only for determining the accuracy of the meters, but also for making the necessary records and for removing the covers, hanging the meters in position, connecting them in circuit, and reversing those operations. It has been found that the actual time required for testing a meter is small in comparison with the total time required for the entire procedure.

However, every advance of this kind in reducing the time required for the test is an incentive to reduce the time for the balance of the operation, and for that reason I do not wish at all to disparage Mr. Sparkes' contribution. I think that efforts will follow to catch up on the other end of the process.

I do not understand just what Mr. Sparkes means by his statement that he is able to make tests to an accuracy of 0.0026 per cent. As I understand it, in the apparatus which Mr. Sparkes has described, the accuracy is dependent upon the precision of the per cent indicator and the accuracy to which it can be read. Possibly I am mistaken about that.

One of the most desirable features of Mr. Sparkes' method is the fact that the meter is tested at speed so that any possible errors of start and stop do not enter into the result.

One other point which Mr. Sparkes has not brought out, but which should be borne in mind, is that his method is a distinct contribution to human welfare. Anyone who has intimate knowledge of the tediousness involved in counting meter revolutions day after day, will appreciate how relief from that monotony may add to the desirability of the meter tester's work.

G. A. Sawin: Referring to Mr. Sparkes' paper, the item that will most interest the operating man is his statement that this testing device will reduce personal errors and save time. Just how is this to be accomplished?

To illustrate, supposing we paint a picture of the meter man of the present and a possible meter man of the future. Today the meter man goes into the premises with a rotating standard and his load-box, connects up, and then counts the revolutions of the disk through his test meter. In order to avoid stopping and starting errors, and possible personal errors, he has to take a reading of three-quarters of a minute to a minute long and usually checks himself three times. I think that is the rule of most operating companies. He does the same thing at full load and at light load. Then if the meter is incorrect, he adjusts it a little bit and counts revolutions all over again, and he continues this same process until he gets the meter correct. Now the meter man of the future comes in. He has a box in his hand, which is probably about the same size as his present rotating standard, maybe smaller, and his load-box. He connects up, puts the testing device in front of the meter, adjusts the hand rheostat until the lines of the disk stand still, and reads his indicator. He has obtained his meter accuracy. He will probably check himself once, to make sure that he is right. He does the same thing at full load and at light load. You can see how much quicker that is, than the present method.

Now take your meter room where you have the laboratory device: in this case the flickering of the light is held constant by the standard meter. The tester knows that if the lines are going one way the meter is fast; the opposite way it is slow. He puts a screw-driver into the full-load adjustment, turns that adjustment gradually; watches the lines slow down and finally stand still. The meter is calibrated. Of course, he will check himself to make sure he is correct, but there are no counting revolutions of the disk, no long waits, no stopping and starting errors.

As Mr. Sparkes says, the device is still in the experimental stage, but I think it marks a step forward. If we could only get away from that old bugbear of counting revolutions, it would be a blessing. I think Mr. Sparkes has earned a great deal of credit for showing us a possible way toward that end.

A. E. Knowlton: I should like to ask Mr. Sparkes to give an answer to this question: Does the logarithmic manipulation of the torque of the per cent indicator assure getting the same degree of accuracy of the percentage indication on a 50 per cent power-factor test as on a 100 per cent power-factor test?

The other question I would like to ask is: Does the limitation of the use of harmonic values of speed prevent accurate testing of the meter at 200 per cent or even 300 per cent of load, for which the claim is made, that a large number of meters now available are quite reliably accurate?

P. van Santen Kolff: Mr. Sparkes evidently needs in the tachometer a magneto that for a certain range of speed will give a perfectly straight curve. The particular magneto used by Mr. Sparkes has that characteristic. A much higher speed could be got, and still give a straight line curve, but, naturally slow speed tends to give more permanent and better commutation than high speed.

Mr. Sparkes might have added that the speed of his motor is by no means limited. If I understood his paper, all that Mr. Sparkes desires to do is to create a certain number of impulses, which may run to a maximum of maybe 8000 per min. for the flashing of his light; and by mounting more contact points or cams on his commutator, he naturally can reduce the maximum running speed of his motor and subsequently the speed of his magneto to any speed he likes. If a magneto at 600 rev. per min.

would not serve his purpose, he could reduce that speed to 300 rev. per. min.

Mr. Sparkes wanted a lamp which would flicker very rapidly. Filling the lamp with hydrogen gas is ideal for accomplishing this. Nevertheless, if the bulb becomes cracked or for some reason oxygen and hydrogen become mixed in just the right proportion, a very dangerous explosion may result.

In Mr. Sparkes' work, danger from explosions must be absolutely eliminated. So his lamps, made up for him by our lamp laboratories, have very small bulbs, in fact, the bulb is about the size of an olive, and in these bulbs is less than atmospheric pressure of hydrogen gas. The danger of explosion is still there, but in Mr. Sparkes' apparatus he has the lamp enclosed in a fixture with a piece of plate-glass about ten times as thick as is needed, so the danger is so small as to be negligible. Furthermore, if his later experiments show it is necessary, we may fill these lamps with helium gas, which is almost as good for the lamp, and with it there is no danger from explosion.

H. P. Sparkes: Dr. Kouwenhoven's first question concerns the characteristics of the magneto. In developing this machine as far as it is developed, care was used in choosing the proper speed range. We selected from the curve of the magneto that portion which we may term "straight." The machine has been operated from 4 per cent load on the watthour meter to approximately 200 per cent. In the range of the magneto we are using that section of the curve which is straight for such operating conditions. The straight-line range runs from approximately zero to 1200 rev. per. min.

The next point brought out under Dr. Kouwenhoven's discussion was the question as to whether the number of lines could be reduced or not. All electrical men are familiar with the fact that after passing approximately 30 cycles, the vision of light interruption ceases; in other words, the eye is not sensitive enough to record over 30 cycles. Keep in mind that when operating this machine at light load, we must have a large number of lines passing the focal center of the eye within a given period, otherwise we cannot get the stroboscopic effect, we see the actual motion. It is therefore necessary to keep the stroboscopic lines on the disk in the vicinity of 300 or better. Experimental tests were made at 200, and at present we are working at 300. Some tests were made at 360, but care will have to be used in selecting the number of lines to match the watthour constant of the meters now being used. I make that remark because in calibrating with this device it will be necessary to supply a switch to take care of the various constant values as applied to various makes of meters.

I believe the point about the mechanism running at high speed is covered in the past remark concerning the lines.

For a reply to the question on special lamps see Mr. Beggs' remarks.

Another point which I wish to emphasize is the fact that the machine entirely eliminates personal error other than bad vision.

As to the point of gang testing, the term is applied in several different ways. You may make a gang test where the meters all stop and start at the same time. You start them all, time them, stop them, go over and adjust those that are out of calibration. With the stroboscopic machine you turn on the machine with the gang test running and in place of checking, you go along and calibrate while the load is on the meters. It eliminates the cut-and-try method.

As to Mr. Blakeslee's remarks, I would like to bring out one point in his statement. The meters, when placed on acceptance test, may be checked and passed, if they are within limits of required accuracy, without breaking the seals or removing the covers. The lines are placed on the circumferential edge and have a greater visibility than the present calibrating mark on the disk. This means that there will be considerable saving in time in the actual set-up for the test. That is, it will not be necessary to remove the metal covering, because the light beam can be projected through the glass opening on to the disk and a check made without touching the original manufacturers' seals.

The accuracy figures given in the paper, 0.0026, are based on calculations of the ultimate accuracy, and I believe in the paper I remark that the accuracy is based entirely upon the regulator, and as the regulator is composed of a precision wattmeter, you may see quickly that the accuracy of the entire machine or the overall accuracy depends entirely upon the linkage in the machine, 0.0026 being the ultimate accuracy according to visibility. Those figures are taken on what you may see through a 16-power cylindrical lens; in other words, that is the maximum.

Mr. Knowlton asks the question as to 50 per cent power factor. At 50 per cent power factor, we have the same change in the stroboscopic machine as you have in the watthour meter, so there will be no effect to introduce an error at this point, the stroboscope will follow the load with the meter and will act as though the meter had simply had the 50 per cent power-factor load placed on it, and the stroboscope will follow right through with it. I have made a number of tests at this point with the machine and no errors were found.

The last point brought out by Mr. Knowlton is an important one, concerning the harmonic adjustment. In developing this machine, I found that the eye is subject to a number of tricks. For instance, we are running the machine at twice the speed of the meter. The result is that the number of lines will apparently be double the actual number because you are taking a picture with your eye, of the line in two positions in place of one. Reverse the condition, with the meter running at 200 per cent and the stroboscope at 100, and you will be taking the opposite condition, every other line. In fact, you can check harmonic values from zero up to about 200 per cent. But realize what happens when you strike those harmonic values you are possibly working at one-half of 0.0026, which makes the point unstable so that it is impossible to hold it; you have reduced your vision by the multiplier of your harmonic, and as a result your vision is exceedingly poor. At some harmonic values you have to reduce the light in the room or the external source of light to practically total darkness to observe some of the harmonic values. I have used colored glasses to determine some of these harmonic values.

Now, a point that hasn't been discussed is the question of using a lens multiplier at light load. Remember at light load you are getting an impulse-light. The light condition actually goes off and on to the human eye, meaning that the retina of the eye and the iris will get a flicker of light. Now the eye will automatically adjust itself to an intermediate position. You cannot follow the stroboscopic action at exceedingly light load by simply glancing at it. You have to give the eye several seconds, possibly, to see the extreme light-load conditions. By using a cylindrical lens in front of that moving object, it apparently speeds up the action to the eye and you get away from the eye delay.

In the criticism of the paper, the point was brought out: Does the stroboscopic machine have any effect upon the eye, such as injury to sight or fatigue to the eye? I would like to bring out one point here, and that is this: The optician today, with his methods of correcting eye trouble, often places an image on the edge of a wheel and asks you to watch that. The effect is that your eye rotates, exercising the muscles. In the case of the stroboscope operating at slow speed, you exercise the muscles in the eye. I have been operating this machine over a period of a year and I have had no eye trouble from it.

Theory of Action of the Induction Watthour Meter and Analysis of its Temperature Errors

BY D. T. CANFIELD[1]
Associate, A. I. E. E.

Synopsis.*—The question of changes in the registration of watthour meters due to variations in temperature is receiving considerable attention at the present time from manufacturers and public utilities alike. This paper discusses the development of a temperature-compensated watthour meter. The effect of certain changes in the fundamental constants of the meter circuits and the materials of certain vital parts are shown to point out the necessity of two independent compensating devices.*

The compensating devices found most effective consist, first, of a permanent magnet flux diverter mounted on bimetal strips in such a way that it shunts more or less of the permanent magnet flux around the disk, on a decrease or an increase in temperature, respectively, and second, a moving lag plate controlled by bimetal strips arranged in such a way as to cause the plate to move up or down with an increase or a decrease of temperature, respectively.

In Appendix I is given the construction of a theoretical vector diagram of an induction type watthour meter showing the relative phase positions of the various fluxes, voltages, and currents that are present.

In Appendix II is given a discussion of the sources of temperature errors in watthour-meters as derived from an analytical study of this diagram.

An attempt is also made in this paper to summarize these sources, the reason for their existence, and their effect upon the registration of the meter, in convenient tabular form.

* * * * *

INTRODUCTION

THE interchange of large blocks of power between utilities, the need for accurately determining water rates of turbines, and the increasing number of consumers who use large amounts of energy, are making various refinements in watthour meter practise desirable. In loads of this character, any real increase in precision is more than desirable.

As a natural consequence, it follows that the task of developing a temperature compensated watthour meter is typical of the problems to which American meter manufacturers are devoting their attention. Moreover, the study of temperature errors in watthour meters is in line with the suggestions made by the subcommittee on Instruments and Measurements of the American Institute of Electrical Engineers.

The writer was engaged to investigate the effectiveness of temperature compensation by thermostatic control of the lag adjustment and drag element of a watthour meter.[2]

At the time the solution of this problem was undertaken, very little of a definite nature was available in the literature of the art concerning the real sources of the change in registration of watthour meters, due to changes in ambient temperature. Subsequently, however, there appeared a very excellent discussion and classification of these sources by Messrs. I. F. Kinnard and H. T. Faus.[3]

Since a complete understanding of the action of the compensating devices later to be described is contingent upon an equally complete understanding of the sources of the errors they compensate for, it will not be out of place to repeat a similar classification and discussion in this paper.

Table I gives a summary of such a classification and discussion. This table was derived from a theoretical analysis of the vector diagram of Appendix I, the discussion given in Appendix II, and a study of the registration curves of meters at different temperatures.

DETERMINATION OF THE RESULTANT EFFECT OF GROUP I ERRORS

Since in Group I there are some effects producing an increase in speed and others producing a decrease in speed, it is necessary to determine the resultant effect of Group I errors as a whole. To do this, a set of typical performance curves was obtained, such as those shown in Fig. 1. These curves are the per cent registration curves of a watthour meter under different temperatures and lagging power factors. They were obtained

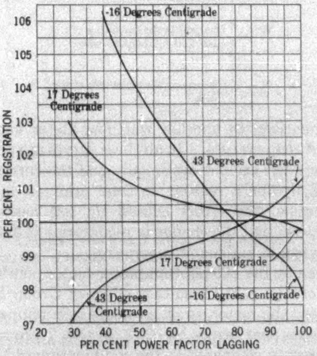

FIG. 1—TYPICAL PERFORMANCE CURVES OF AN UNCOMPENSATED METER, AT LAGGING POWER FACTORS

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[1] Assistant Professor of Electrical Engineering, Purdue University, Lafayette, Indiana, and Consulting Engineer for Duncan Electric Mfg. Co.

[2] Developed by Mr. Jesse Harris, Chief Development Engineer of the Duncan Electric Manufacturing Company.

[3] TRANS. A. I. E. E., 1925, p. 275.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 7-11, 1927.

TABLE I

A SUMMATION OF THE SOURCES OF ERRORS IN WATTHOUR METERS DUE TO VARIATIONS IN TEMPERATURE AND THEIR EFFECT UPON REGISTRATION

Combination I—Assumes an increase in ambient temperature with lagging power factor

Group	Error	Description of error	Effect on meter speed	Effect of decreasing power factor	Reasons for the changes in speed
	1	Changes in the magnetic properties of the permanent magnet.	Increases	Independent	Decrease in permeability, reluctance increases, dragging flux decreases, meter speeds up.
	2	Changes in the magnetic properties of the magnetic circuits of the potential and current elements.	Decreases	Independent	Decrease in permeability, reluctance increases, driving fluxes decrease, meter slows down.
I	3	Changes in length of the air-gap of the permanent magnet.	Decreases	Independent	Magnet gap widens, causing the flux passing between the gap to decrease. Meters using this flux for dragging will speed up, while meters using a shunted portion of this flux will slow down.
	4	A shift in the phase position of the exciting current.	Increases	Independent	The exciting current shifts down due to decreased iron losses, causing a shift in I_p R_1 This in turn increases E^1 which must be accompanied by an increase in Φ. (See Fig. 12.)
	5	An increase in the magnitude of the exciting current.	Decreases	Independent	Increase in reluctance of iron cuts down Φ, exciting current then increases but not enough to increase Φ to its original value. This means a decrease in E^1 and therefore in the speed of the meter.
I	6	Changes in the choking effect of the lag and light-load plates.	Increases	Independent	The choking effect of these plates upon the main flux decreases. More of the main flux acts as a driving flux. Meter speeds up.
	7	Changes in the resistance of the potential windings.	Decreases	Independent	Shift of I_p R_1 due to shift in exciting current causes E^1 to decrease and consequently Φ. Speed decreases.
II	8	Changes in the resistance of the potential windings.	Decreases	Increasingly	Reacts to decrease the 90 deg. relation bietween the driving fluxes. Speed decreases.
II	9	Changes in the phase position and magnitude of the exciting current.	Decreases	Increasingly	Both effects shift the voltage V^1 (See Fig. 12) so as to decrease the 90-deg. relation between the driving fluxes. Speed decreases.
	10	Changes in the resistance of the lag and light-load plates.	Decreases	Increasingly	Increase in resistance of these plates decreases the 90-deg. relation between the driving fluxes. Speed decreases.
	11	Changes in the resistance of the disk.	Decreases	Increasingly	Reacts to decrease the 90-deg. relation between the driving fluxes. Speed decreases.

by comparison to another watthour meter kept at a constant temperature. In all these curves the potential coils were left excited continually and the current coils warmed up for a reasonable time prior to checking. As a result, the errors caused by self heating are negligible so that the change in registration is due solely to a change in ambient temperature.

As seen by the 100 per cent power-factor points of the curves of Fig. 1, the resultant effect of Group I errors is to speed the meter up on an increase in temperature. This means that errors No. 2, 3, 5, and 7 are small in the aggregate compared to the combination of No. 1, 4, and 6. Fig. 1 also shows that as the lagging power factor decreases, the meter slows down at high temperatures and speeds up at low temperatures. This substantiates the reasoning with respect to Group II errors.

Since, as just shown, the resultant effect of Group I and Group II errors is opposite, it is obvious that at some lagging power factor the meter will be independent of changes in temperature. In the meter of Fig. 1, this lagging power factor is approximately 80 per cent as the curves meet here in a common point. This point will vary up and down the power-factor scale, depending on the relative magnitude of the resultant effects of Group I and II errors.

If, however, leading power factors are used in place of lagging power factors, the resultant effect of Group II errors will be reversed and, therefore, Group I and Group II errors will not oppose each other. As a result, there is no leading power factor at which the meter is independent of temperature.

This is illustrated by the curves of Fig. 2 where the broken lines are the corresponding leading power-factor curves to the solid or lagging power-factor curves.

It should be noticed that the leading power-factor curves, although similar in form, are opposite in direction to the lagging power-factor curves and therefore do not meet in a common point.

The effect of leading power factors, as far as single-phase meters are concerned, is of little importance, since single-phase meters are rarely subject to leading power factors, but with two-element polyphase meters this is not necessarily true. In the first place, polyphase meters are more apt to be used where leading power factors exist, and in the second place, a polyphase meter at lagging load power factors above 86.67 per cent has one element operating on a leading power factor. The two elements would tend, therefore, to compensate each other as far as temperature errors are concerned.

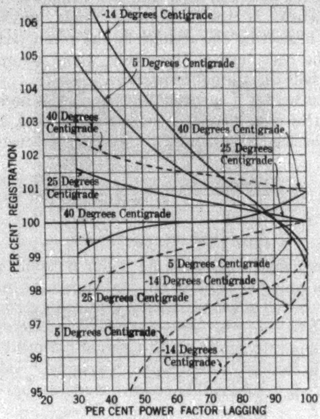

Fig. 2—Effect Upon the Typical Performance Curves of Leading Power Factors

This probably accounts for the fact that polyphase meters in general have less over-all temperature error at low power factors than do single-phase meters of the same make.[4]

Effect of Changes in the Constants and Materials of the Meter

Any change in the constants of the meter circuits, in the materials of which they are made, or in the design of the meter will change the relative magnitude of the several errors and thereby change the typical performance curves. The effect of some of these changes will now be illustrated.

Fig. 3 shows the effect of a brass phasing plate in the meter of Fig. 1. Since brass has a lower temperature coefficient than copper, its resistance will change less with a given change in temperature than the copper plate, and, therefore, the error due to this change will be less pronounced. Comparing Fig. 3 with Fig. 1, it is seen that the spread of the curves in Fig. 3 is less and the neutralizing or crossing point has been lowered. This means that the resultant effect of Group II errors has been reduced so that they do not overcome the resultant effect of Group I errors as quickly as in Fig. 1. Furthermore, the spread of the curves at the lower power factors has been greatly reduced. This indicates that error No. 10 is the predominant one of the Group II errors. This figure also illustrates that it is possible by proper calibration to get virtually perfect performance at some one temperature, 20 deg. cent. in this case.

The substitution of a lag plate which has nearly zero temperature coefficient has been suggested, but, unfortunately, all the metals which have low temperature coefficients also have high specific resistance. Even with brass, it is necessary to use a plate nearly four times as thick as the copper plate it replaces, in order to obtain the proper lagging. The design of the average present-day meter has a relatively small space available for the lagging device with the result that the limit to which this idea can be carried without materially changing the dimensions of the meter, is quickly reached.

In order to determine the effect of error No. 8, two otherwise similar meters were made up, one with its potential coil resistance increase above normal and the other with its potential coil resistance decreased below normal.

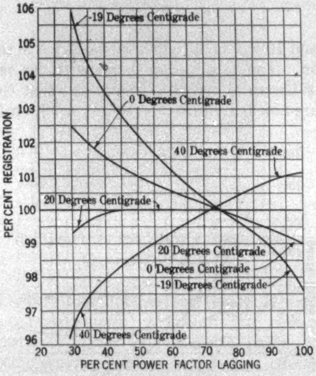

Fig. 3—Effect Upon the Typical Performance Curves of Substituting a Brass Phasing Plate for the Customary Copper Plate

The first of these meters was obtained by winding the potential coil with the same number of turns of No. 33 wire instead of the customary No. 30. Neglecting a small change in the mean length of turn, the resistance of these coils would be approximately twice the normal resistance.

Since increased potential losses mean a decrease in the angle between the applied voltage and the flux set up thereby (see Fig. 12), it follows that greater lagging must be applied to this meter. It was found that to obtain the proper lagging, a larger or thicker lag plate was required than that normally used.

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[4] See the author's paper, "Watthour Meter Accuracy as Affected by Temperature Changes," *Experiment Station Bulletin* No. 22, Purdue University.

Turning now to Table I, it is evident that both errors No. 8 and 10 will be affected and moreover in the same direction. The result is an increase in the resultant effect of Group II errors. This means that Group II errors are now better able to overpower Group I errors, thereby raising the crossing or neutral point on the power factor scale.

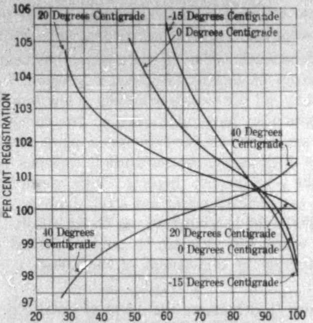

Fig. 4—Effect Upon the Typical Performance Curves of Increasing the Resistance of the Potential Winding Above Normal

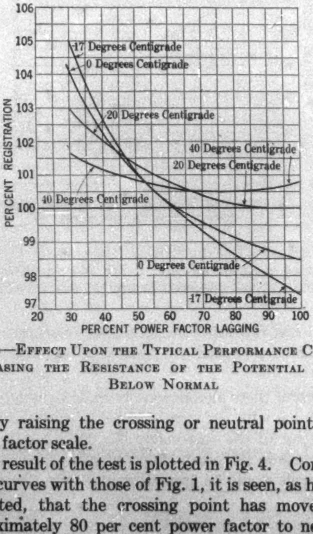

Fig. 5—Effect Upon the Typical Performance Curves of Decreasing the Resistance of the Potential Winding Below Normal

The result of the test is plotted in Fig. 4. Comparing these curves with those of Fig. 1, it is seen, as had been predicted, that the crossing point has moved from approximately 80 per cent power factor to nearly 90 per cent. This is a desirable change as far as high power-factor operation is concerned, but it should be noticed that at the lower power factors, the spread of the curves has been greatly increased.

Although this meter is compensated to some extent at high power factors, it is accomplished at the expense of accuracy at low power factors, and, furthermore, any compensation which depends upon an increase in the losses is fundamentallly wrong.

The second meter referred to above was obtained by winding the potential coils with two strands of No. 30 wire instead of the customary single strand. As before, neglecting the change in the mean length of turn, the resistance of these coils would be approximately half the normal resistance.

Also as before, a change in the lag plate is necessary, but in this case in the opposite direction. A No. 16 brass phasing plate was substituted for the customary copper plate. The result of these changes is just the opposite to that obtained with an increase of resistance.

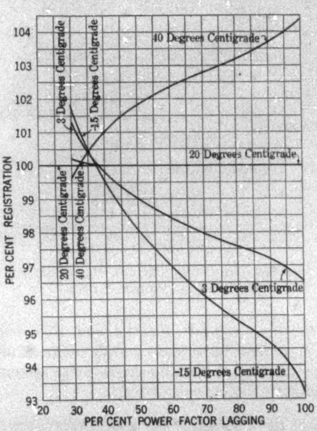

Fig. 6—Effect Upon the Typical Performance Curves of Lagging Both the Potential and Current Fluxes

The effect upon the performance curves is shown in Fig. 5. Comparing these curves to those of Fig. 1, it is evident that the crossing occurs at lower power factors and with less variation at very low power factors.

This is obviously a step in the right direction as far as Group II errors are concerned, but it is equally obvious that it is not a complete solution as it would be impossible to have zero resistance in the potential circuit, so doing away with the lag plate entirely.

In Fig. 5, the curves show the meter running fast at all temperatures on low power factors. This is due to over lagging of the meter when it was adjusted and calibrated. Had it been properly lagged at the customary 50 per cent power factor, the curves as a whole would have been more nearly coincident with the 100 per cent registration line, but the spread of the curves at either end would not have been changed appreciably.

As previously mentioned, changes in design will also affect the typical performance curves. As an example, consider the simultaneous lagging of both the potential and current fluxes. This is done by over-lagging the potential flux and then lagging the current flux sufficiently to produce the required 90 deg. relation between them at unity power factor.

As the temperature increases in a meter so modified, it will tend to lessen the lagging of both the potential and current fluxes. By proper proportioning of these two lag plates, it might be possible to shift these two fluxes by approximately the same amount for a given change in temperature so that the angle between them would remain at 90 deg.

Fig. 7—The Compensating Device Used for Group I Errors—Cover Removed

Fig. 6 shows the result of an attempt to apply this principle. Comparing these curves with those of Fig. 1, a slight improvement is found in the slope of the curves.

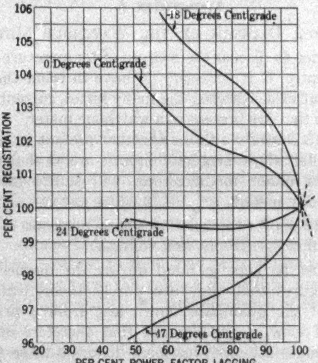

Fig. 8—Effect Upon the Typical Performance Curves of the Compensating Device For Group I Errors

This means that Group II errors have been partially compensated for. The effect, however, is not very pronounced.

Type of Compensating Devices Found to be Most Effective

The compensating device found to be most effective for Group I errors in the meters investigated consisted of a flux diverter for the permanent magnet. This diverter was mounted across the gap of the magnet on a bimetal strip, so that it moved out or in as the temperature went up or down. See Fig. 7.

The law which this diverter obeys depends on the relative magnitude of the magnetic pull upon it and the spring and thermal action of the bimetal strip. These, in turn, depend on the position, size, and shape of the iron diverter and bimetal strip.

The effect of this device on the meter of Fig. 1 is shown in Fig. 8. This figure shows that by this means it is possible to over-compensate for Group I errors, as the crossing point has moved beyond the 100 per cent power-factor point. By moving the diverter away from the gap, the crossing point may be made to occur at 100 per cent power factor exactly. It should be noticed that the removal of Group I errors leaves Group II errors unrestrained, as shown by the excessive spread of the curves at low power factors.

From what has just been said, it is evident that two independent compensating devices are needed, one for Group I errors and another for Group II errors, to completely compensate a meter for changes in temperature.

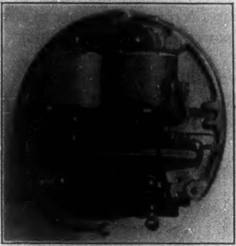

Fig. 9—The Compensating Device Used for Group II Errors

The compensating method found most effective for Group II errors consisted in mounting the phasing plate on bimetal strips in such a way as to cause it to move up and down with a rise and fall of temperature. See Fig. 9.

The effect of this device alone on the meter of Fig. 1 is shown in Fig. 10.

The effect of both of these devices combined in the same meter resulted in the performance shown in Fig. 11, which, when compared to Fig. 1, shows a marked improvement.

The solid curves are the average curves of ten separate tests made on this meter at frequent intervals over a period of four weeks. Between tests the meter received average handling and transportation and was not found to be appreciably affected by this treatment.

This would indicate that although movable compensating devices are used, they are rigid enough to withstand ordinary handling.

The dash lines represent the maximum deviation from 100 per cent registration of any one of six compensated meters, each tested three times between -18 deg. cent. and 40 deg. cent. and 100 per cent power factor and 30 per cent power factor.

In other words, the solid curves of Fig. 11 illustrate the best performance obtained thus far in any one meter as a whole by careful adjustment. The dash lines on the other hand represent the poorest performance of any of six different meters taken severally.

This would indicate that compensated meters of this type might be produced and calibrated by the customary methods of production and testing and have none of its temperature power factor registration curves outside the range bounded by the dash lines of Fig. 11, and will on the average be better.

CONCLUSIONS

To completely compensate a meter for changes in temperature two independent compensating devices are needed, one to compensate the meter at unity power factor or Group I errors and another to compensate the meter at any power factor other than unity, or Group II errors.

The compensating device found most effective for Group I errors is a permanent magnetic flux diverter mounted on a bimetal strip across the gap of the magnet in such a way that it moves in or out with decrease or increase in temperature.

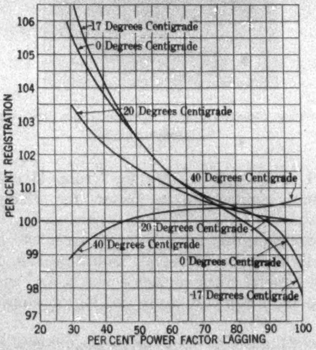

FIG. 10—EFFECT UPON THE TYPICAL PERFORMANCE CURVES OF THE COMPENSATING DEVICE FOR GROUP II ERRORS

The compensating method found most effective for Group II errors consisted in mounting the phasing plate on bimetal strips in such a way that it moves up or down with increase and decrease in temperature.

A meter equipped with these devices and carefully adjusted will not vary over one-half of one per cent between 100 per cent and 50 per cent power factor over a temperature range of 40 deg. cent. to -18 deg. cent. From 50 per cent power factor to 30 per cent power factor, a change of about one per cent in registration takes place. Six meters equipped with these devices did not vary more than ± ¼ per cent at unity power factor and did not exceed ± 1 per cent at thirty per cent power factor over the above mentioned range of temperature.

It should be noticed that even this change is relatively small when compared to the variation in an uncompensated meter.

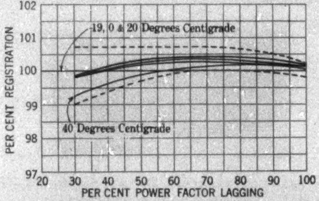

FIG. 11—EFFECT UPON THE TYPICAL PERFORMANCE CURVES OF BOTH THE DEVICES PROPERLY ADJUSTED

Ordinary handling and transportation of meters was not found to have any effect upon the compensating devices.

Appendix I

VECTOR DIAGRAM OF THE WATTHOUR METER

In order to discuss the sources of temperature errors intelligently, it is necessary to have in mind the relations that exist between the many fluxes, currents, and voltages that are present in a watthour meter.

Fig. 13 shows the fluxes that are present in a watthour meter whose magnetic circuits are as shown, and with but minor modifications, will suffice for most present-day meters.

Fig. 12 shows the relative phase relations of these various fluxes, currents, and voltages.

It must be kept in mind during the discussion of these figures that they are purely theoretical, that many of the quantities shown are, from a practical point of view, negligible, and that therefore only an approximate attempt was made to draw these figures to scale.

Considering, first, the potential circuit with the lag plate removed, V is the applied voltage which causes the no-load current I_0 to flow. This current has both a core loss and magnetizing component not shown in the figure.

The current I_0 produces the flux Φ and what leakage flux exists, Φ_L. The flux Φ sets up a back e. m. f. at right angles to it, which requires the primary component E to balance it. By adding E to the voltage drop of the potential winding, which consists of $I_0 R_1$ in phase with I_0 and $I_0 X_1$, at right angles to I_0, the diagram closes on V.

The flux Φ divides at a (see Fig. 13), into two fluxes Φ_1 and Φ_2, of which Φ_1 is much the larger, due to the low reluctance of its path as compared to that of Φ_2. However, vectorially $\Phi_1 + \Phi_2 = \Phi$ as shown in Fig. 12.

If, now, the lag plate is placed in circuit as shown, Φ_2 will induce in it a voltage E_2. The voltage E_2 causes a current I_2 to flow. Since the lag plate is only one short-circuited turn, the voltage E_2 is used up in the $I_2 R_2$ and $I_2 X_2$ voltage drops of the lag plate.

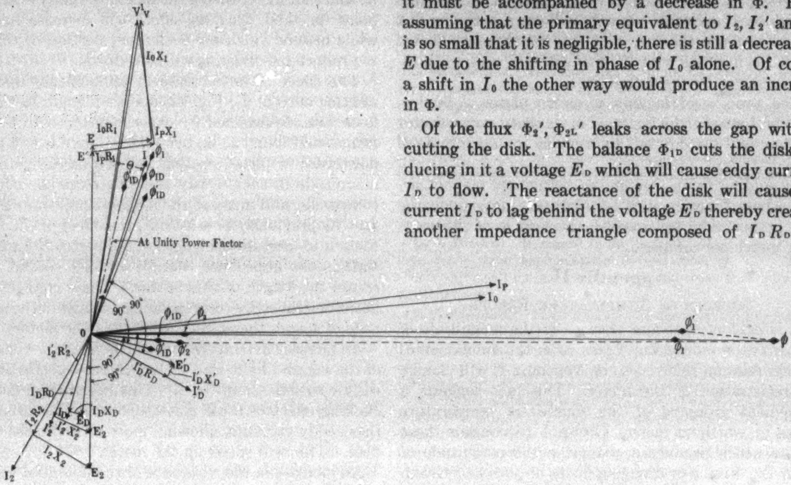

FIG. 12—A THEORETICAL VECTOR DIAGRAM OF AN INDUCTION TYPE WATTHOUR METER

In like manner, the flux Φ_2 will induce in the light-load plate a voltage E_2'. This voltage will cause a current I_2' to flow. Since the light-load plate is also only one short-circuited turn, the voltage E_2' will be used up in the $I_2' R_2'$ and the $I_2' X_2'$ voltage drops of the light-load plate.

The currents I_2 and I_2' will both produce a m. m. f. which when combined with the m. m. f. producing the flux Φ_2 will produce some other flux Φ_2'. Some of this change is transmitted to Φ_1, which now becomes Φ_1'. Assuming for the moment that the sum of these new fluxes, Φ_1 and Φ_2', is again vectorially equal to Φ, the relation will be shown by the heavy lines in Fig. 11. As will be pointed out later, this assumption is not quite true, but is a very close approximation[5].

The currents I_2, I_2', and I_D must have a primary equivalent to balance them. This primary load current will change the position and magnitude of I_0 to I_p. This change will also change $I_0 R_1$ to $I_p R_1$, and $I_0 X_1$ to $I_p X_1$, both in magnitude and position. Assuming for the sake of simplicity that E is fixed in position and V fixed in magnitude, these changes will have the effect of moving V to V^1 and decreasing E to E^1. This decrease in E is the fallacy in the above assumption for it must be accompanied by a decrease in Φ. Even assuming that the primary equivalent to I_2, I_2' and I_D is so small that it is negligible, there is still a decrease in E due to the shifting in phase of I_0 alone. Of course a shift in I_0 the other way would produce an increase in Φ.

Of the flux Φ_2', Φ_{2L}' leaks across the gap without cutting the disk. The balance Φ_{1D} cuts the disk, inducing in it a voltage E_D which will cause eddy currents I_D to flow. The reactance of the disk will cause the current I_D to lag behind the voltage E_D thereby creating another impedance triangle composed of $I_D R_D$ and $I_D X_D$ closing on E_D. The eddy currents I_D produce a flux Φ_{1D} which reacts with the flux Φ_{2D} to produce a flux Φ_{1D}'. In other words, the disk reacts upon the potential circuit in much the same way as the lag plate and

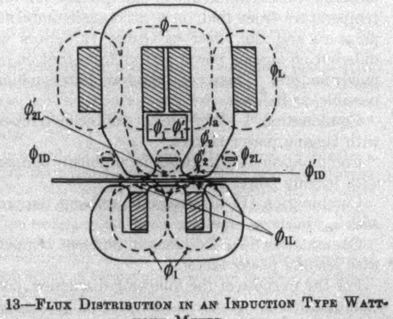

FIG. 13—FLUX DISTRIBUTION IN AN INDUCTION TYPE WATTHOUR METER

light-load plate did. In effect it is a third secondary.

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[5] "Theory and Operation of Split-Phase Magnet," *Electrical World*, Vol. 68, No. 21, Nov. 18, 1916.

The line current I in the series coil which in general will lag behind the applied voltage V^1 by some angle θ will produce a flux Φ_I which will not be exactly in phase with I but will lag behind slightly. Part of this flux will leak across the gap. The balance Φ_{ID} will cut the disk, inducing in it an e. m. f. E_D' which sets up eddy currents I_D'. These eddy currents act in a similar way to the eddy currents I_D to produce the impedance triangle consisting of $I_D' R_D$, $I_D' X_D$, and E_D'. The flux set up by these eddy currents will react with the flux Φ_{ID} to produce the flux Φ_{ID}'. At unity power factor when I is in phase with V^1, Φ_{ID}' will take up some position as shown by the dotted line.

The process of lagging a meter places a 90-deg. relation between the flux Φ_{ID}' in its unity power factor position and the flux Φ_{1D}' by varying the magnitude of I_2.

By adding to the meter the permanent magnet circuit whose function it is to produce a drag upon the disk proportional to its speed, the circuits of the watthour meter are complete.

Appendix II

SOURCES OF TEMPERATURE ERRORS

It is evident that any changes in the magnitude of the driving or dragging fluxes or in the fundamental 90-deg. relation referred to in Appendix I, will change the registration of the meter. This fact suggests a convenient grouping of the sources of temperature errors in watthour meters, Group I to contain those factors which produce a change in the magnitude of either the driving or dragging fluxes or both and Group II to contain those factors which produce a change in the phase position of the driving fluxes.

Furthermore, the effect of Group I and Group II factors will be different on an increase in ambient temperature from that on a decrease in ambient temperature and the effect of Group II factors will be different on lagging power factors from that on leading power factors. There are, therefore, four combinations possible, as follows:

Combination I—Increase in ambient temperature with lagging power factors,

Combination II—Increase in ambient temperature with leading power factors,

Combination III—Decrease in ambient temperature with lagging power factors,

Combination IV—Decrease in ambient temperature with leading power factors.

For the purpose of the following discussion, consider Combination I. Also consider each effect as existing independently of every other effect, although, of course, they will exist simultaneously in the meter.

It is well understood that an increase of temperature will produce a decrease in the permeability of magnet steel, with a consequent increase in reluctance and decrease in flux. In the case of the permanent magnet this will speed up the meter as it is the dragging flux that is reduced, but in the case of the potential and series elements it will decrease the speed of the meter, as here it is the driving flux that is decreased.

Since most permanent magnets are bent, expansion caused by an increase in temperature will tend to widen temporarily the gap between the poles. The effect of this upon the speed of the meter will depend upon the design of the drag magnet circuit. In general, it can be said that meters using the flux in the gap between the poles for their dragging effect will increase in speed while meters which use a shunted portion of this flux to produce the dragging will slow down.

The effect of an increase in temperature upon the exciting current I_0, Fig. 12, due to changes in the iron losses was determined by experimenting with a transformer and found to be two-fold. There is first a shift downward in phase position and second an increase in magnitude in such a way as to decrease the core loss component and increase the magnetizing component of I_0. As previously pointed out, the effect of the first of these is to increase the length of the vector E^1 through a shift in the impedance triangle and the second to decrease the length of this vector. These changes must be accompanied by corresponding changes in the magnitude of Φ and, therefore, the speed of the meter.

An increase in temperature will increase the resistance of the lag and light-load plates, thereby decreasing the eddy currents set up in it. This in turn will decrease choking effect of these plates upon the flux producing these eddy currents, allowing more of it to pass to the disk. This will speed up the meter.

An increase in the resistance of the potential winding will cause the exciting current to shift more nearly in phase with the applied voltage. This shift as before decreases E^1 through a shift in the voltage triangle of the potential coil. This must be accompanied by a decrease in Φ and, therefore, the speed of the meter.

In Group II, there is first an increase in the resistance of the potential winding. This will cause the speed of the meter to decrease since it causes a decrease in the 90-deg. relation between the driving fluxes.

The effect of the change both in phase position and in magnitude of the exciting current will cause the meter to decrease in speed since both tend to decrease the 90-deg. relation between the driving fluxes.

The increase in resistance of the lag and light-load plates causes a decrease in the lagging effect and a consequent decrease in the speed of the meter.

An increase in the resistance of the disk will cause a decrease in the eddy currents I_D and I_D' (Fig. 12) set up in the disk and a consequent decrease in its speed. This change, however, is balanced by a corresponding decrease in the dragging effect of permanent magnet which speeds up the disk by exactly the same amount.

Hence there is no Group I error due to the change in resistance of the disk.

An increase in the resistance of the disk will also cause

I_D and I_D' to shift towards E_D and E_D', respectively. This will shift Φ_{1D}' and Φ_{ID}' in the same sense, thus tending to maintain the 90-deg. relation between them.

These two fluxes do not shift by the same amount, however. Since I_D' is the only current producing a m. m. f. affecting the phase position of Φ_{ID}', its effect is more pronounced on Φ_{ID}' than is the m. m. f. of I_D upon Φ_{1D}'. The result is a decrease of the 90-deg. relation between these two fluxes and a consequent decrease in the speed of the meter.

This effect can be demonstrated by using disks of different resistances in the same meter. In the particular experiment tried, a maximum shift in the position of the disk in the gap accounted for only 50 per cent of the change in registration due to a change in the resistance of the disk. The balance must be due to a greater shift in Φ_{ID}' than in Φ_{1D}', thereby decreasing the 90-deg. relation between them and consequently the speed of the meter. This is a Group II error.

Table I gives a summary of the above discussion using Combination I. The first seven errors comprising Group I are independent of the power factor, operate at all power factors, and are constant at any given temperature. The remaining errors comprising Group II are a minimum at unity power factor, but increase with a decrease in the power factor.

The effect of Combination II on Table I will be to reverse the effect of Group II errors, of Combination III to reverse the effect of both Group I and Group II errors, and of Combination IV to reverse the effect of Group I errors.

Since the analysis of the sources of error in this investigation is only incidental to the development of methods of compensation, not enough work was done to claim that the above discussion and the summary in Table I cover all of the possible sources of error due to temperature. For example, the effect of temperature on the light-load plate has not been considered, except as it functions as a part of the artificial lagging of the meter.

Discussion

W. H. Pratt: I think it should be emphasized that the present paper, except so far as it describes a particular mechanism for effecting compensations, is a restatement of material most effectively presented by Kinnard and Faus at the midwinter convention two years ago. Previous to that time, meters having much smaller Group II errors than those used for a background of this paper were used in predominant numbers, and since then the whole output of one of the largest producers of meters has been substantially without Group I errors. So that, presented at this time, the background of this paper must not be looked upon as a picture of general present-day practise.

The treatment of potential-circuit resistance, while perhaps not in error, tends, I think, to give a wrong impression and seems to be somewhat of an apology for high resistance in the potential circuit, whereas it was shown beyond peradventure in the Kinnard-Faus paper that resistance in the potential circuit is the outstanding source of Group II errors. Of course it is not possible to produce meters with zero resistance in the potential circuit, but this is not necessary. It is quite possible to make meters with a resistance so low that the use of manganin lag plates is convenient and the residual Group II errors are of almost vanishing value.

The results shown in Fig. 11 of the paper are excellent, but it appears that they are the results of painstaking individual adjustment. At present meters are in use which, however, even surpass this Fig. 11 performance, not as the result of adjustment but by reason of proper regard taken in the design of the various sources of error. They are inherently without errors exceeding 0.2 or 0.3 per cent, within the limits of application ordinarily prevalent. To be sure, they are portable test meters, though similar meters for switchboard use arranged for connection to 2-stage current transformers are also in service. Incidentally, manganin lag plates are used in these meters. It does not appear that the author advocates for general use the forms of compensation he describes. It seems that it would be necessary to use them with great caution, for there must be an ever present menace of maladjustment and disarrangement. Be that as it may, were I to venture a prediction in regard to future meter practise, I should say that in all probability in the not distant future, meters which have temperature characteristics not greatly inferior to the best performance here shown will be in general and extensive use.

As pointed out in the Kinnard-Faus paper, sources of error should be eliminated, and only when further elimination is impossible should recourse be had to compensation.

B. J. Brown: In regard to the paper by Mr. Canfield, how long have these compensating devices been in use? Time shows any small mechanical troubles, and often time only will develop them.

I. F. Kinnard: In my opinion, the most outstanding feature of the paper is the fact that the conclusions reached by Mr. Canfield agree substantially with the findings made by Mr. Faus and myself several years ago, in so far as classification of temperature errors into two distinct groups is concerned.

It is rather unfortunate that Mr. Canfield's analysis is entirely qualitative, since the listing of so many errors as he shows in Table I is likely to be misleading and does not give any sort of an idea as to just how important each error is from the standpoint of the meter's operation. In looking down this list, we see under Error 1, "Changes in the magnetic properties of the permanent magnet." As a matter of fact, this is by far the greatest source of Group I errors with which we have to contend. The other six errors listed under this group are of very little practical importance. We have determined this from exhaustive tests on a large number of meters, by means of which was made a quantitative study of the magnitude of the various errors involved. This error due to the changes in the magnetic properties of the permanent magnet with temperature, is something that is quite fundamental and cannot readily be eliminated. It therefore seems feasible to use some sort of a compensating device in order to keep the braking flux constant over wide variations in temperature.

Now in regard to Class II errors or those errors which cause a shifting in the phase relationship of fluxes with variations in temperature; Mr. Canfield has given an exhaustive list of possible sources of error of this nature. I have found, however, that there is only one really important source of error of this nature with which we have to contend,—one which not only causes a shifting in phase relationship of fluxes with varying temperatures but which is reflected in various other factors of the meter's performance. I am speaking of the resistance of the potential windings. In an ideal meter, we should have all reactance and no resistance in the potential windings; that is, we want the flux to be in quadrature with the applied voltage. If it is not, we have to use some sort of a device to "lag" the useful portion of this flux through a small angle, with which all meter men are familiar. The only practical reason, therefore, for our having to put up with temperature errors of this nature is the existence of resistance in the potential windings.

I say this without hesitation, because meters are in actual service in which the potential coil resistance has been reduced to the point where the lagging can be accomplished by means of a manganin lag plate and the resulting errors are no longer of any commercial or practical importance whatsoever.

It is true that the compensation for both Class I and Class II errors, which the author has described in this paper, will work providing everything is in very nice adjustment, and he should be complimented on this solution. As someone has mentioned in a previous discussion, however, the real way to get rid of errors of any kind is to eliminate them at their source rather than use a compensation whereby it is endeavored to force two wrongs to make a right.

A. R. Rutter: The paper presented by Prof. Canfield is of great value, I think, not only in adding additional data to our literature on the watthour meter by discussing the temperature compensation, but by discussing the vector diagrams of the induction type watthour meter in dealing with the various vectors of eddy currents, of fluxes and of the current, and of voltage. The induction type watthour meter is usually regarded as a very mysterious piece of apparatus, so that a paper which presents a good analysis of the performance of the meter is a real addition to the literature on the subject.

Prof. Canfield refers to Fig. 6 of the paper and states that the results obtained in using a method of compensating for the Class II errors by lagging both the current and voltage fluxes gives a slight improvement for these errors. I think it is possible to interpret from his curves in Fig. 6 that if the compensation for Class I errors, (which are shown in his Fig. 8) were added to the compensation for Class II errors (as shown in Fig. 6), it should produce a very well compensated meter. I am familiar with results of this method, which produced meters which were carefully compensated for Class II errors by lagging both the current and voltage fluxes. This method has the advantage that compensation is obtained by changes in material and not by shifting mechanical parts.

C. T. Wallis: I have been working with alloys which change their magnetic properties with temperature, and observations over a period of about seven years have shown these to be very constant indeed.

With regard to the use of bi-metallic compensators which depend upon the expansion and contraction of two dissimilar metals these are rather erratic in their action. Considered strictly from a mass-production standpoint the air-gap between the magnet and the shunt which is carried on the bi-metallic strip must be adjusted very carefully at some definite temperature. On the other hand, the use of a permanently fixed magnetic shunt which changes its properties with temperature has none of the above disadvantages.

D. T. Canfield: In reply to Mr. Brown's question, I should like to say that the meters described in my paper have not been in commercial use at all. They have been used in my laboratory, however, for a matter of 2.5 or 3 years.

As Mr. Kinnard points out, I have no method other than a guess in my process of analysis of determining the relative magnitude of the several errors mentioned in Table I. I have no doubt whatever as to the overall error of the meter as a whole. That has been very definitely determined by me on all makes of meters a number of times, so that I do know, and did know at the time that this work was started, the amount of overall error that actually existed.

I claim that this resultant effect, in the last analysis, is the important feature. It is instructive and interesting only more in an academic way than in a practical way to analyze this overall error into its several component parts, and that is why I made no attempt to do it quantitatively.

As Mr. Pratt points out, the ideal way to eliminate error is to do away with the source of error, but as he also points out, it is not possible to eliminate the errors completely in this way. No claim is made in my paper that the method of compensation used by me is the only one. In fact, my Fig. 5 shows that the method proposed by Mr. Pratt is effective in reducing Group II errors.

Mr. Rutter's remarks concerning Fig. 6 and the method of compensation therein illustrated are well founded. The particular meter used in this experiment was of an obsolete variety which had the desired compensation as far as Group II errors were concerned but was decidedly off on Group I errors. As far as this figure is concerned the compensation of Group I errors would not materially affect the slope of the curves.

This does not detract, however, from the merits and advantages of this method of compensation. It is necessary only to obtain the proper proportion of the counteracting lagging devices to give theoretically perfect performance.

As Mr. Wallis points out, it is necessary to carefully form and age the bi-metal strips used on the compensating devices. When this is done, I have found no indication of subsequent change.

Current Analysis in Circuits Containing a Resistance Modulator

BY L. S. GRANDY[1]
Associate, A. I. E. E.

Synopsis.*—The function of a resistance modulator is to produce in an electric circuit a current which is a copy of an exciting impulse such as speech or light waves. The principal example is the carbon granule telephone transmitter.*

It is desirable that the electric current be an exact copy of the exciting impulse. There is inherent in such a device however, a distorting effect, for the current copy is produced by reason of Ohm's law and thus is an inverse function of the modulated resistance and not a true copy of it. The amount of distortion arising from this effect depends upon the electrical constants of the modulator and its associated circuit.

The study quantitatively analyzes this distorting effect by two methods in a circuit containing a modulator, a battery, and a resistance for single frequency modulation, and by one of the methods for double frequency modulation. An analysis is also developed for a special test circuit.

The study shows that the relation between modulators and circuits is a design problem.

INTRODUCTION

In the sense used in this study a modulator is a device used to transform electrical energy supplied to it into a time copy of a desired mechanical impulse or so to modulate an electrical current that the result will be an electrical copy of an exciting wave. The exciting impulse may be any function of time. If the current to be modulated is supplied by a constant difference of potential it may be modulated by introducing into the circuit either a resistance or an electromotive force varying in accordance with the exciting impulse. The result will be a modulated current by reason of Ohm's law. The present subject is limited to modulation by the resistance method.

Two applications of the resistance modulator are the carbon-granule telephone transmitter and the selenium light-sensitive cell. Either of these pieces of apparatus forms a unit in a system which fails to transmit perfectly the exciting wave. In the case of the telephone, the characteristics of the speech input are not faithfully reproduced in the sound output at the receiver. The difference in characteristics or quality is termed distortion.

In order to perform a useful function a resistance modulator must be provided with a transmitting system that will cause the exciting wave to act on the resistance unit, and this system always causes distortion. Again using the carbon transmitter to illustrate; distortion of the voice wave occurs in the air transmission leading up to the button diaphragm because of mechanical characteristics of an air column, in the diaphragm because of its own mechanical qualities combined with clamping difficulties and in the carbon because of its mechanical behavior and because carbon does not have a linear pressure-resistance relation. However, these causes of distortion are all subject to modifications which will diminish such distortion.

For the purpose of the study a perfect resistance modulator is assumed. The resistance element gives a change of resistance which is an exact copy of the exciting wave. Thus the assumed perfect modulator represents the limit in quality which improved materials and design may produce in such a device. But, a resistance modulator gives a current wave in the output circuit which in form is an inverse function of the exciting wave and not a true copy of it. The result is distortion, such distortion being inherent in the nature of the device. In order to analyze the effect quantitatively, an exciting wave following the sine law is assumed. By assumption it produces a corresponding wave of resistance modulation. Then the resistance of the modulator will be a constant quantity plus the modulating resistance and may be written:

$$R_0 + r \cos \omega t.$$

The distortion effect is graphically illustrated in Fig. 1.

The amount of distortion occurring in a transmitting circuit due to modulation is determined by both the modulator and circuit electrical constants. The study contemplates a critical analysis of the modulated current from the quality standpoint. The usual method of analysis by harmonic components has been employed.

SIMPLE SERIES RESISTANCE CIRCUIT

(First Development—Single Frequency)

Considering the most simple possible circuit, a battery, a resistance, and a modulator in series.

Equation,

$$i = E/(R_x + R_0 + r \cos \theta)$$

Let

$$R_x + R_0 = R \text{ and } r/R = K$$

By simple division:

$$i = E/R \{ 1 - K \cos \theta + K^2 \cos^2 \theta - K^3 \cos^3 \theta + K^4 \cos^4 \theta + \ . \ . \ . \}$$

The series converges—Proof

$$U_{n+1} < U_n \text{ and } L_{n \to \infty} U_n = 0$$

$$U_{n+1} = (r/R)^{n+1}; U_n = (r/R)^n; (r/R)^{n+1} < (r/R)_n \text{ if } r < R$$

Also

$$L_{n \to \infty} (r/R)^n = 0 \text{ if } r < R$$

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Note: Since this paper was written, the study has been extended to the series circuit containing resistance and inductance.

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which is true always since the modulating resistance must be less than the total resistance.

Expanding: Whole expansion to be multiplied by E/R

Constant: $1 + K^2/2 + 3\,K^4/8 + 5\,K^6/16 + 35\,K^8/128 + 63\,K^{10}/256 + \ldots$

Fundamental: $-\cos\theta\,(K + 3\,K^3/4 + 5\,K^5/8 + 35\,K^7/64 + 63\,K^9/128 + \ldots)$

2nd Harmonic: $\cos 2\,\theta\,(K^2/2 + K^4/2 + 15\,K^6/32 + 7\,K^8/16 + 105\,K^{10}/256 + \ldots)$

3rd Harmonic: $-\cos 3\,\theta\,(K^3/4 + 5\,K^5/16 + 21\,K^7/64 + 21\,K^9/64 + \ldots)$

4th Harmonic: $\cos 4\,\theta\,(K^4/8 + 3\,K^6/16 + 7\,K^8/32 + 15\,K^{10}/64 + \ldots)$

5th Harmonic: $-\cos 5\,\theta\,(K^5/16 + 7\,K^7/64 + 9\,K^9/64 + \ldots)$

6th Harmonic: $\cos 6\,\theta\,(K^6/32 + K^8/16 + 45\,K^{10}/512 + \ldots)$

7th Harmonic: $-\cos 7\,\theta\,(K^7/64 + 9\,K^9/256 + \ldots)$

8th Harmonic: $\cos 8\,\theta\,(K^8/128 + 5\,K^{10}/256 + \ldots)$

9th Harmonic: $-\cos 9\,\theta\,(K^9/256 + \ldots)$

10th Harmonic: $\cos 10\,\theta\,(K^{10}/512 + \ldots)$

Each harmonic becomes an infinite series; hence an approximation. The expansion of a power of the cosine results in a single term for every even or for every odd harmonic starting with the same numbered harmonic as the power developed and going on down to the fundamental if an odd power or to the constant term if an even power. Thus in the expansion given the series become more approximate by twos.

From this solution a very clear mental picture of the process of modulation may be gained. The fundamental or first harmonic is the result of the direct current flowing across the varying resistance of the modulator, the second harmonic, and the second constant term result from secondary modulation, that is, the fundamental flowing through the modulating resistance. Thus by reason of the eighth harmonic there arises the first term of the ninth series, the second term of the seventh, the third term of the fifth, the fourth term of the third, and the fifth term of the first series.

Simple Series Resistance Circuit

(First Development—Double Frequency)

Equation

$$i = E/(R_x + R_0 + r_1\cos\alpha + r_2\cos\theta)$$
$$= E/(R + r_1\cos\alpha + r_2\cos\theta)$$
$$= E/R\,[1/1 + K_1\cos\alpha + K_2\cos\theta]$$

Where

$$\alpha = 2\,\pi f_1\,t \text{ and } \theta = 2\,\pi f_2\,t$$

By division:

$$i = E/R\,[1 - (K_1\cos\alpha + K_2\cos\theta) + (K_1\cos\alpha + K_2\cos\theta)^2 - (K_1\cos\alpha + K_2\cos\theta)^3 + (K_1\cos\alpha + K_2\cos\theta)^4 + \ldots]$$

Writing in harmonic components: (Fourth order modulation)

The whole expansion must be multiplied by E/R.

$$1 + K_1^2/2 + K_2^2/2 + 3\,K_1^4/8 + 3\,K_2^4/8 + 3\,K_1^2\,K_2^2/2$$
$$- [K_1 + 3\,K_1^3/4 + 3\,K_1\,K_2^2/2]\cos\alpha - [K_2 + 3\,K_2^3/4 + 3\,K_1^2\,K_2/2]\cos\theta$$
$$+ [K_1^2/2 + K_1^4/2 + 3\,K_1^2\,K_2^2/2]\cos 2\,\alpha + [K_2^2/2 + K_2^4/2 + 3\,K_1^2\,K_2^2/2]\cos 2\,\theta$$
$$- [K_1^3/4]\cos 3\,\alpha - [K_2^3/4]\cos 3\,\theta + [K_1^4/8]\cos 4\,\alpha + [K_2^4/8]\cos 4\,\theta$$
$$+ [K_1\,K_2 + 3\,K_1^3\,K_2/2 + 3\,K_1\,K_2^3/2]\cos(\alpha + \theta) + [K_1\,K_2 + 3\,K_1^3\,K_2/2 + 3\,K_1\,K_2^3/2]\cos(\alpha - \theta)$$
$$- [3\,K_1^2\,K_2/4]\cos(2\,\alpha + \theta) - [3\,K_1^2\,K_2/4]\cos(2\,\alpha - \theta)$$
$$- [3\,K_1\,K_2^2/4]\cos(2\,\theta + \alpha) - [3\,K_1\,K_2^2/4]\cos(2\,\theta - \alpha)$$
$$+ [K_1^3\,K_2/2]\cos(3\,\alpha + \theta) + [K_1^3\,K_2/2]\cos(3\,\alpha - \theta)$$
$$+ [K_1\,K_2^3/2]\cos(3\,\theta + \alpha) + [K_1\,K_2^3/2]\cos(3\,\theta - \alpha)$$
$$+ [3\,K_1^2\,K_2^2/4]\cos(2\,\alpha + 2\,\theta) + [3\,K_1^2\,K_2^2/4]\cos(2\,\alpha - 2\,\theta)$$

This development is the explanation for sub-frequencies and odd multiple frequencies in the output circuit of a resistance modulator even though the instrument be acted upon by pure waves. These odd frequency components of current may become quite large. Assuming constants of $E = 12$, $r_1 = 80$, $r_2 = 20$, $R_0 = 100$, and $R_x = 100$ where r_1 is associated with the frequency f_1, giving the time angle α and r_2 is associated with the time angle θ, a current i will flow equal to

$$65.82 - 24.32\cos\alpha - 7.49\cos\theta + 5.71\cos 2\,\alpha + 0.45\cos 2\,\theta + 2.98\cos(\alpha + \theta) + 2.98\cos(\alpha - \theta) - 0.72\cos(2\,\alpha + \theta) - 0.72\cos(2\,\alpha - \theta) + \ldots$$

where the values are in milliamperes.

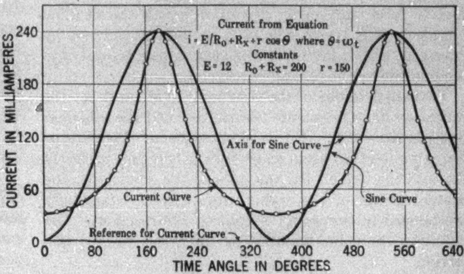

Fig. 1

A particularly valuable feature of mixed frequency modulation analysis is that it offers the possibility of measuring distortion in an actual instrument. By exciting an instrument with two pure waves of known magnitudes and with frequencies prime to each other and measuring one of the combination frequencies in the output and comparing with the calculated value a definite knowledge of what the instrument has done may be gained.

SIMPLE SERIES RESISTANCE CIRCUIT

(Second Development)

Equation

$$i = E/(R_0 + R_x + r\cos\theta)$$

$$= E/R\,[1/(1 + K\cos\theta)] \quad \text{where } r/R = K$$

Examining for the constant term of a harmonic series:

Constant

$$= E/R \cdot \frac{1}{2\pi}\int_0^{2\pi} d\,\theta/(1 + K\cos\theta) = E/R\,[1/\sqrt{1-K^2}]$$

Examining for the coefficient of the nth harmonic:

$$A_n = E/R \cdot \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\cos n\theta}{1 + K\cos\theta} \cdot d\,\theta$$

$$= E/R\,\frac{2(-1)^n}{\sqrt{1-K^2}}\left[\frac{1-\sqrt{1-K^2}}{K}\right]^n$$

Writing the series:

$$i = E/R\left\{(1/\sqrt{1-K^2} - 2/\sqrt{1-K^2}\left[\frac{1-\sqrt{1-K^2}}{K}\right]\cos\theta\right.$$

$$+ \frac{2}{\sqrt{1-K^2}}\left[\frac{1-\sqrt{1-K^2}}{K}\right]^2\cos 2\theta$$

$$- \frac{2}{\sqrt{1-K^2}}\left[\frac{1-\sqrt{1-K^2}}{K}\right]^3\cos 3\theta$$

$$\left. + \;.\;.\;.\; \frac{2(-1)^n}{\sqrt{1-K^2}}\left[\frac{1-\sqrt{1-K^2}}{K}\right]^n\cos n\theta\right\}$$

Sample calculation:

Assumed constants:

R = 200 ohms, r = 80 ohms, and E = 12 volts.

Writing current in milliamperes:

$$i = 65.4666 - 27.364788\cos\theta + 5.719110\cos 2\theta - 1.195278\cos 3\theta + .249818\cos 4\theta - .052216\cos 5\theta + .010912\cos 6\theta - .002281\cos 7\theta + .0004767\cos 8\theta - .00009964\cos 9\theta + .00002082\cos 10\theta + \ldots$$

Dropping the constant term and writing the harmonics in per cent of the fundamental:

$$i = -100\cos\theta + 20.90\cos 2\theta - 4.37\cos 3\theta + .914\cos 4\theta - .191\cos 5\theta + .0399\cos 6\theta - .00834\cos 7\theta + \ldots$$

Fig. 2 illustrates the manner in which the first three distorting harmonics change relative to the fundamental when modulator and circuit constants remain the same but the degree of modulation, *i. e.*, r, changes. Calculations were based upon the same assumed constants as in the example above with r changing in value from zero to 190 ohms. The latter value represents nearly complete modulation of all resistance in the circuit. Complete modulation is the limit which may be approached. It is interesting to note that if complete modulation could be realized all harmonics would be present and equal in value.

Fig. 3 is similar to Fig. 2. in that it shows the manner of variation of the first three distorting harmonics relative to the fundamental but with R_x, the resistance of the circuit external to the modulator, as the variable and the modulating resistance r held constant at 100.

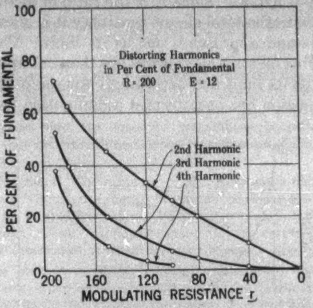

FIG. 2

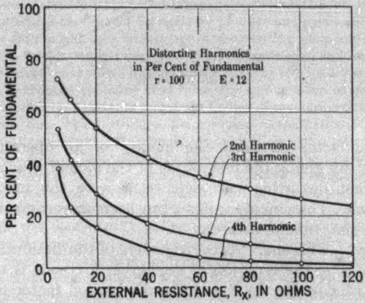

FIG. 3

R_0 is held at 100. As R is the sum of R_0 and R_x it must change. It may be noted that the division of the total circuit resistance between the modulator and the external circuit is of no consequence except that r is limited by R_0.

DIMINUTION FACTOR IN SIMPLE SERIES CIRCUITS

The equation of the current was found to be:

$$i = E/R\left\{1/\sqrt{1-K^2} - 2/\sqrt{1-K^2}\left[\frac{1-\sqrt{1-K^2}}{K}\right]\cos\theta\right.$$

$$+ \frac{2}{\sqrt{1-K^2}}\left[\frac{1-\sqrt{1-K^2}}{K}\right]^2\cos 2\theta$$

$$- \frac{2}{\sqrt{1-K^2}}\left[\frac{1-\sqrt{1-K^2}}{K}\right]^3\cos 3\theta$$

$$+ \ . \ . \ . \ \frac{2\,(-1)^n}{\sqrt{1-K^2}} \left[\frac{1-\sqrt{1-K^2}}{K}\right]^n \cos n\,\theta$$

From an examination of the equation it is seen that each succeeding harmonic differs in magnitude from the preceding harmonic by the multiplying factor $[1-\sqrt{1-K^2}]/K$. Since K is the ratio of modulating resistance to the total circuit resistance it is always less than one and approaches one as its limit. Thus the multiplying factor is also always less than one approaching one as its limit and accordingly for the purpose of this discussion has been termed a diminution factor.

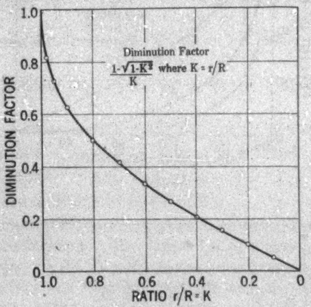

FIG. 4

The factor describes the quality of an operating circuit by giving the magnitude of the second which is the first distorting harmonic relative to the fundamental. The sum of the distorting harmonics compares with the fundamental as $(C + C^2 + C^3 + \ . \ . \ . \ C_n)$ compares with one. As the argument of the diminution factor is K the criterion of quality in a circuit is the ratio of r to R. In Fig. 4 the diminution factor has been plotted against the ratio.

TEST CIRCUIT

A circuit of special interest for test purposes, because the direct and alternating components of current are separated, is one consisting of a battery, a heavy inductance unit, and a modulator all in series and with a circuit consisting of a large condenser connected in series with a resistance unit in parallel with the modulator. The circuit is shown in Fig. 5. In order to effect an analysis an assumption is made, that no alternating current can flow in the battery circuit because of a preponderately heavy inductance and that no direct current can flow in the receiving circuit because of the condenser. The condenser is made so large that it imposes no appreciable impedance to alternating currents of frequencies dealt with. Although not rigorously true the assumption can be closely approached in a test circuit.

The voltage across the modulator may be written:

$$V_0 + v = (I_0 + i)\,(R_0 + r\cos\theta)$$

From Kirchoff's law $v = -\,i\,R_x$
Substituting and solving:

$$i = \frac{V_0 - I_0\,(R_0 + r\cos\theta)}{R_0 + R_x + r\cos\theta}$$

The mean voltage across the modulator may be arrived at in terms of the steady current flowing and the circuit constants. Solving the first equation for v:

$$v = R_x\,\frac{(V_0 - I_0\,R_0) - I_0\,r\cos\theta}{R_x + R_0 + r\cos\theta}$$

Summing up v over a complete cycle, setting the result equal to zero, and solving for V_0 there results:

$$V_0 = I_0\,\{\,\sqrt{(R_0 + R_x)^2 - r^2} - R_x)\,\}$$

The equation for current may be simplified by transformation to:

$$i = \frac{\dfrac{V_0 + I_0\,R_x}{R_0 + R_x}}{1 + (r/R_0 + R_x)\cos\theta} - I_0$$

$$= \frac{A}{1 + B\cos\theta} - I_0$$

The form of the equation is the same as that for the series resistance circuit except for a subtracting constant. The same methods may be used to expand it into harmonic series with the same characteristic results as were used for that circuit. Similar relations exist between harmonics. Using the second method the equation for current is:

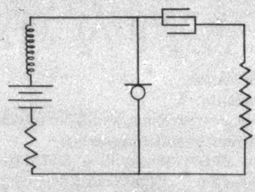

FIG. 5

$$i = \frac{A}{\sqrt{1-B^2}} - I_0 - \frac{2\,A}{\sqrt{1-B^2}}\,\frac{(1-\sqrt{1-B^2})}{B}\,\cos\theta$$

$$+ \frac{2\,A}{\sqrt{1-B^2}}\,\frac{(1-\sqrt{1-B^2})^2}{B^2}\,\cos 2\,\theta + \ . \ . \ .$$

TABLE OF SYMBOLS

R_0 = Average value of modulator resistance or value about which modulation occurs.
R_x = Resistance of receiver circuit.
$R = R_0 + R_x$.
r = Modulating resistance.
$\theta = \omega\,t = 2\,\pi\,f\,t$ = Time angle in terms of frequency and time.

E = Battery electromotive force.
i = Current in receiving circuit.
$\alpha = \omega t = 2\pi f t$ = Time angle with $f \neq$ to f in θ.
$K = r/R$.
V_0 = Constant component of modulator voltage.
v = Variable component of modulator voltage.
I_0 = Direct current.
C = Diminution factor $= [1 - \sqrt{1 - K^2}]/K$.

Discussion

R. M. Kerchner: It seems to me that some of Mr. Grandy's statements, while not incorrect, may be a little misleading, while some assumptions are made that exaggerate the poor qualities of the transmitter.

The statement that the current output wave is an inverse function of the exciting wave, while I think it is true, is, however, likely to give a wrong physical conception. Physically, the qualitative results of the exciting wave are similar to those of the output wave, that is, when the exciting wave has a high value the output wave has also a high value and similarly with low values. From his statement we might get the idea that if the output wave is an inverse function of the exciting wave, the output wave is small when the exciting wave is large, and vice versa. Technically, however, Mr. Grandy's statement, I think, is correct.

I disagree, however, with the statement that distortion is due to the fact that the pressure-resistance relation of the carbon-granule transmitter is not linear. It is by assuming that this relation is linear that the results tend to magnify the distortion due the output current being an inverse function of the exciting wave. As a matter of fact, if the pressure-resistance relation were a rectangular hyperbola and there were no resistance outside of that in the transmitter, the output wave would be a true image of the exciting wave, mechanical distortion neglected. This is shown as follows.

Let p be the pressure on the diaphragm at any time.

Let p_o be the pressure about which the pressure variation occurs.

According to the assumption $r\,p = c$

$$r = \frac{c}{p}$$

Assuming a sinusoidal exciting wave the pressure would be $p = P_o + p_o \sin\theta$ where p_o is the modulating pressure.

Hence

$$r = \frac{C}{p} = \frac{C}{P_o + p_o \sin\theta}$$

and

$$I = \frac{E}{r} = \frac{E\,(P_o + p_o \sin\theta)}{C} = \frac{E\,P_o}{C} + \frac{E}{C}\,p_o \sin\theta$$

which shows the output wave to be of the same form as the exciting wave if the pressure-resistance relation is a rectangular hyperbola, hence not linear. That the relation is actually a curve of the same general trend as a rectangular hyperbola, namely, concave upward, can be seen by a consideration of the limits of the resistance as the pressure is O and become infinite.

Actually, however, distortion will also exist with such a relation since the whole curve is shifted upward by an amount depending on the constant resistance in the circuit of the transmitter. Under these conditions it can be shown that the current wave due to a sinusoidal exciting wave would be

$$I = \frac{E\,(P_o + p_o \sin\theta)}{R_x\,P_o + R_x\,p_o \sin\theta + C}$$

which on account of the second term in the denominator will not give a pure sine wave.

From this it can be seen that the ideal condition would be to have the pressure-resistance relation of the transmitter follow the law of a rectangular hyperbola, with no resistance in the exterior circuit instead of a linear relation as Mr. Grandy implies. This, I believe, explains in part the fact that the distortion actually realized is not as great as might be suspected from simply an analysis of the expression $\dfrac{E}{R_x + R_o + r\cos\theta}$ given in Mr. Grandy's paper.

The conclusion that might be inferred from this discussion is that a transmitter that varies the e. m. f. in the circuit would lend itself more readily to production of distortionless output waves. This suggests that the magneto transmitter operates on a principle that underlies distortionless reproduction of diaphragm variations.

L. S. Grandy: Commenting on Mr. Kerchner's discussion; the original statement, that the current wave in the output circuit is an inverse function of the exciting wave, is misleading if not wrong. It was intended to state that the current wave is an inverse function of the modulating resistance.

A linear pressure-resistance relation was assumed to be ideal because any other relation would mean the introduction of harmonics into the variations of the modulating resistance so that resistance change would not be a copy of the exciting wave. Mr. Kerchner has shown that if a rectangular hyperbolic pressure-resistance relation could be realized, an instrument, in an hypothetical case, could be made to produce an output current wave that would be a true image of the exciting wave and that such a pressure-resistance law would permit the building of instruments that would presumably materially reduce distortion.

It is thought probable that for every combination of circuit and modulator constants there exists a law of pressure-resistance change that would produce distortionless reproduction of the exciting cause or at least, permit a reproduction with a minimum of distortion. Upon the linear assumption a diminution factor was defined as that factor which gives the relative magnitude of each succeeding harmonic relative to the preceding. Distortionless reproduction would be attained if the diminution factor was made zero. Referring to Fig. 4, such a situation would be approached if R, the resistance of the output circuit, was made very large in proportion to the modulating resistance r. This is the condition under which the carbon radio microphone operates as its output is impressed upon a vacuum-tube circuit of very high impedance. Thus the linear pressure-resistance relation becomes the perfect relation for that particular combination of modulator and circuit constants. As the carbon microphone gives very excellent results from the quality standpoint and as such results depend upon a linear relation it is thought that the pressure-resistance law for carbon must not depart very far from the linear form. It is true as Mr. Kerchner points out that at limiting conditions the law for carbon must depart from the linear form, but for pressures actually used, limiting conditions are not likely to be encountered. The pressures involved are of a very low order of magnitude. The writer has had other evidence that, while not at all conclusive, was strongly suggestive of the linear relation for carbon.

A magneto type, or a condenser type of modulator is very desirable from the quality standpoint. As pointed out in the first paragraph of the paper, such an instrument modulates current supplied to it by voltage changes in the circuit making the modulated product a direct function of the modulating change. However, it is impossible to get enough energy from the exciting cause into such an instrument to produce modulated currents of a magnitude that can be used in most applications. On the other hand the carbon instrument acts as a modulator and a very large amplifier at the same time.

Cab Signals for Railway Signaling

BY T. S. STEVENS[1]

Synopsis.*—There is described in this paper a system of continuous signals operating within the cab of a railway locomotive. This system was developed in an effort to provide more reliable and safer signals than the common semaphore or light signals. The particular type of system is called the three-speed, continuous-control signal system. It has been adopted quite extensively by the Atchinson, Topeka, and Santa Fe Railway System.*

* * * * *

PROBABLY every signal engineer has worried about the difficulty of placing wayside signals on a railroad so that they can be seen and the indication properly read at a sufficient distance for proper control of the train. There are two forms of wayside signals, the semaphore and daylight light signal. The first displays its indications by a movable arm attached to a mast. The second displays its indications by colored lights which, by virtue of special lenses, can be seen in ordinary day light at sufficient distance. Particularly with semaphores, the question of background is very essential. It is not quite so essential in connection with light signals, but even with these devices it is often difficult to place signals on a crooked piece of track so that there will be sufficient view of them. Fog is another agent which interferes to a great extent with a view of semaphores and with a view more or less of light signals; although in a fog, a light signal can be seen much the farther.

Then again any form of wayside signal is only intermittent in its indication. A train arrives at a signal which is displaying a certain indication. It passes it and the indication is lost except in the memory of the the engineman.

Thus, many years ago, we dreamed of the possibility of having a signal right ahead of the engine which would change under changing conditions or continuously display the same indication if the condition did not change. Much research work was accomplished and ultimately it was found possible, through means of practically wireless circuits, to provide a signal in the cab of the engine which would keep the engineman informed continuously of the condition ahead regardless of whether he was moving or standing still.

The type of cab signaling adopted by the Santa Fe is generally called *Three-speed continuous control.* It employs no wayside signals and all information is obtained from the signals in the engine cab.

Fig. 1 indicates a train proceeding along the track receiving a high-speed indication from the track ahead. Directly at the rear of the train, a zone of low-speed control is set up and just behind that, a zone of medium-speed control, so that a train following will receive a medium-speed indication and then a low-speed indication over sufficient length of track so that it can easily be brought to rest before getting within dangerously close proximity of the first train.

The three-speed, continuous control system necessarily requires the establishment of three different electrical conditions in order to provide the three controls. These three conditions are brought about by the proper control of a-c. circuits in the rails which establish magnetic fields around the rails and affect receiving apparatus mounted at the front and rear of the locomotive. The energy picked up by these receivers is amplified in two stages and used to operate the engine relay on the locomotive.

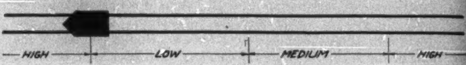

FIG. 1—AUTOMATIC TRAIN CONTROL

The railroad tracks are divided into track sections, or "blocks," by insulated joints located approximately 4000 ft. apart. Each section is equipped with an individual set of controlled circuits, two separate circuits being used as follows:

Fig. 2 shows the circuits mentioned above. *A* is the "Track Circuit" in which a small transformer feeds current into the rails, and this is received at a track relay at the other end of the section. The current in the track circuit flows through a limiting impedance coil to the track, down one rail, through the track relay or the axles of any train which may be in that particular section, and back through the other rail. Incidentally,

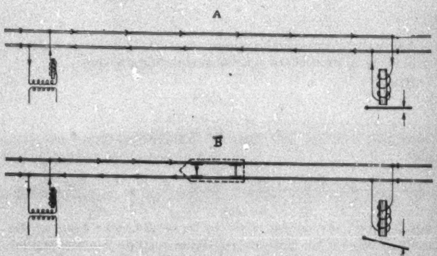

FIG. 2—ALTERNATING CURRENT TRACK CIRCUIT

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[1] Signal Engineer, Atchinson, Topeka, & Santa Fe Railway System.

Presented at the Regional Meeting of District No. 7 of the A. I. E. E., at Kansas City, Mo., March 17 and 18, 1927.

quite a large part of the total current may leak from rail to rail through the ground.

B shows the same track circuit occupied by a train moving from right to left, and it will be noted that the current is shunted away from the relay by the axles of the train so that the armature of the relay drops by gravity on the back contacts instead of the front contacts. These contacts are used to control circuits in such a way as to indicate the presence or non-presence of a train on the particular track section concerned.

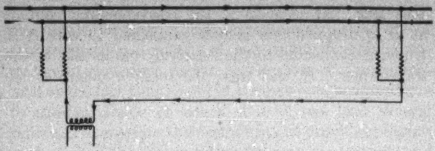

Fig. 3—Loop Circuit

Fig. 3 is the "Loop Circuit" in which current travels down both rails in the same direction. It leaves the loop transformer at one end of the circuit, divides through two resistance coils and flows down both rails in both directions under the train. At the end of the track section it is again brought together through two resistance coils and returns over a wire installed on the pole line.

Fig. 4 shows in a diagrammatic way the engine receiver, of which one is mounted in front of the first pair of wheels on the locomotive and another at the rear of the tender with a vertical clearance of about six in. above the rail. This is a structure of laminated iron with coils mounted thereon in such a manner as to pick up energy from the magnetic field around the rails. The "Track Receiver" being at the front of the locomotive picks up track energy before it is shunted by the wheels. The coils on the track receiver are connected in such a manner that the voltages induced in them are additive when the current is passing through the two rails in a direction opposite from that indicated for the track circuit in Fig. 1.

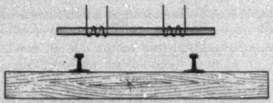

Fig. 4—Engine Receiver

The "Loop Receiver" being mounted on the rear end of the tender is out of the zone of track circuit current because this has been shunted through the wheels of the locomotive. The coils of the loop receiver are connected in such a manner that the voltages induced in them are additive for currents passing through both rails in the same direction. Thus they pick up energy from the loop circuits which is shown in Fig. 2.

The "High-Speed" indication is established when the track is unoccupied for a specified distance ahead. It is brought about by energization of both the track circuit and the loop circuit with the normal direction of current flowing in each circuit.

The "Medium Speed" indication is established when the track is occupied at a specified distance ahead. This is brought about by energization of both the track and the loop circuits with the current in the latter circuit reversed.

The "Slow Speed" indication is established when the track is occupied at a specified shorter distance ahead. This is brought about by de-energization of either the track circuit or the loop circuit.

The method of controlling the track circuit and loop circuit automatically by the presence of a train on the track will be explained in detail a little later.

The train control engine relay is a two element affair, one element of which is energized by induction from current in the track circuit shown in Fig. 1 except when this current is shunted away from it by the wheels and axles of a train. The other element is energized by induction from current in the loop circuit shown in Fig. 2. The operating part of the relay is a vane somewhat similar to that used in a watthour meter. In order to obtain three indications, it is necessary to provide a means for this disk to be operated in two directions in order to close different contacts. It must also be actuated by gravity to a point where it opens both of these contacts and closes another. In order to accomplish this, suitable means are provided to reverse the relative polarity of the current in the loop circuit.

Fig. 5 shows the circuit used on the locomotive. Directly under the train control relay is shown the contacts which bring about the display of the different lights in the cab signal. The letter in the indicator used on the Santa Fe means *low*, *medium* and *high* describing the speeds which are authorized. In this same connection, the relay controls two electropneumatic valves designated as "Control Magnets," which, in turn, set up conditions through which the speed of the train is actually controlled automatically regardless of the action of the engineman.

The author does not intend to give a full description of this automatic control, but it is interesting to a mechanical engineer and the author will be glad to arrange for answers to any questions which may be asked.

Fig. 6 illustrates in greater detail than Figs. 1 and 2, the circuits used on the roadway. In this connection, it will be noted that a three-phase line is used to energize the system. This line is very nearly balanced by using one of the three phases for a five-mi. stretch, the next phase for the next five miles and the third phase, for for the next five miles. It will be noted also that the secondaries of the line transformers are divided into two circuits, each of 110 volts, in order to provide a convenient way for reversal of current polarity in the loop circuit without a complicated pole changing device.

Now, as to the circuits on the roadside which bring about the different indications: In the following discussion, the current which is permanently applied to the rails will be designated as the "track circuit current" and that which is controlled through contacts will be called "loop current."

In order to provide a high-speed indication, both of these currents must be present, and the loop current must be of the proper relative polarity. To provide a medium-speed indication, both of these currents must be present and the relative polarity of the loop current must be reversed. There are two ways of providing the slow-speed indication: One is shown between locations 1 and 2, where the current is shunted away from the relay by the train. Under these conditions, if another train enters this section, there will be no track circuit current available. The other is shown at location 2. At this point contact *A* is open and therefore no energy is applied to the primary of loop transformer *B*. In either case, one element of the engine relay is de-energized and its rotor drops by gravity to the low speed contact.

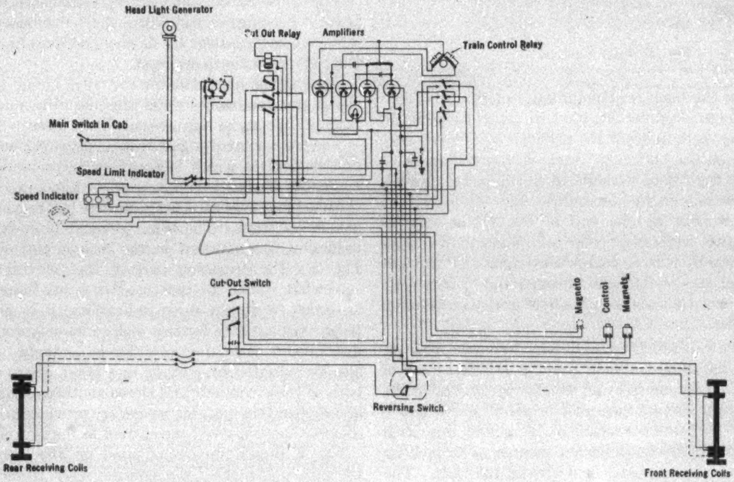
Head Light Generator
Cut Out Relay
Amplifiers
Train Control Relay
Main Switch in Cab
Speed Limit Indicator
Speed Indicator
Cut-Out Switch
Magneto
Control
Magnets
Reversing Switch
Rear Receiving Coils
Front Receiving Coils

FIG. 5—CIRCUIT DIAGRAM AUTOMATIC TRAIN CONTROL

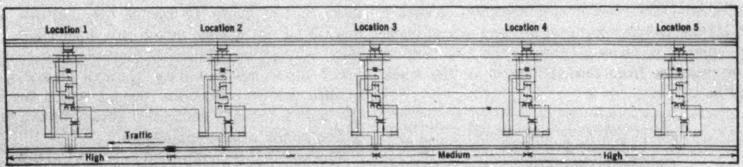
Location 1
Location 2
Location 3
Location 4
Location 5
Traffic
High
Medium
High

FIG. 6—TYPICAL TRAIN CONTROL ROADWAY CIRCUITS

The method of providing a medium-speed indication is shown at location 2. It will be noted at location 1 that contact *C* is connected to wire *B X* which, in turn, is connected to the line transformer on the relatively plus side. At location 2, the corresponding contact at *D* is dropped and is now connected to wire *N X* which is connected to the relatively minus side of the transformer. In this way, the relative polarities between the track circuit current and the loop current have been reversed, which in turn reverses the engine relay.

The trade name of the particular scheme as described is "Continuous Control" because of its continuous indications. It seems easy as here described, but a great many years were spent in research work before a simple description of this kind became possible.

Various new problems confronted us when we had the opportunity to study the problem in connection with an actual installation. Very interesting studies have been made about the various paths which a current will take when a conductor is grounded as all steam railroad rails must be grounded more or less through the ties and ballast. The problem of providing safe operation, with due consideration for the inductive effects of adjacent power lines of the same frequency was very interesting. The real reason for all the above was to provide a means of controlling a train automatically regardless of the carelessness of the engineman or the fact that he might become incapacitated due to sudden illness or death, but it has lead several railroads to consider the desirability of providing signals in the cab of the engine rather than on the wayside and in this sense has had a good effect.

Whether the automatic control of a train will ultimately be successful is debatable, but at present it looks as if it is a factor in railway operation which has come to stay.

There are various other schemes now in use which are very interesting from a technical standpoint, but all of these involve intermittent controls. None of them provides an adequate means for the use of cab signals without the addition of wayside devices. They simply provide for a check on the engineman should he disobey a signal indication. They are not self checking, as is the device described above, and therefore as a means of adding to the efficiency as well as safety of train movements they seem incomplete. Only time will show whether this idea is correct or not.

Discussion

F. E. Snell: Though continued development of the semaphore and light type of signals resulted in the production of a highly efficient unit, it yet has some serious drawbacks as pointed out by Mr. Stevens. These unfavorable conditions, attendant on wayside signals, which could not be remedied by electrical means, naturally resulted in the development of a cab signal. The cab type of signal may also offer a solution to one of the problems with which we are confronted in Cleveland, and which, no doubt, affect other operators of rapid transit lines in metropolitan areas, where a portion of the right-of-way is through the less polite residential sections. I refer to the malicious destruction of signal equipment by trespassers, particularly small boys who throw stones, using the wayside signal light lenses as targets.

It would be interesting to hear, not only of the problems presented by the inductive effects of adjacent power lines of the same frequency as the signal current, but also something regarding the ability of the engine equipment to function properly under severe conditions to which it is no doubt subjected. In other words, do vibrations, bumps, etc., cause excess relay or lamp failures when compared with the wayside type of signal?

Wishing to give the traveling public the greatest possible protection, signal engineers have experimented with, and developed, various forms of automatic train control, the ultimate success of which, Mr. Stevens states, is debatable. Our company has had no occasion, of course, to consider such a system, but from the wonderful progress made in the signal field in a comparatively short time, as well as in other branches of the electrical industry, I can see no reason why continued experimenting and future developments will not soon result in more efficient and reliable methods of signaling and automatic control of trains.

O. S. Major: As presented by Mr. Stevens, the cab-signal system appears to be comparatively simple, but when one digs into the problem a little deeper and considers various traffic conditions to be contended with and traffic reversal as used on the Santa Fe, situations sometimes arise which are extremely complicated.

Other general types of automatic train control not taken up by Mr. Stevens include the intermittent inductive, intermittent contact (ramp type), continuous two-speed, and continuous stop; they are all intended to accomplish the same general purpose, namely, enforcing of obedience to, or cognizance of, signal indications.

In the intermittent contact or ramp type, ramps are placed at intervals along the roadway which make contact with a shoe on the locomotive. The ramp is energized or deenergized, depending upon track conditions ahead or wayside signal indication; the shoe, on passing over the ramp, completes electrical circuits on the locomotive which in turn control relays and valves, causing a reduction of brake pipe pressure under certain conditions.

In the intermittent inductive types, several methods are used to impart an electrical impulse to the locomotive circuits through an air-gap to a receiver mounted on the locomotive which in turn controls electropneumatic valves causing a reduction of brake-pipe pressure under certain conditions of track and signal indications.

The track element, or inductor, in one device consists of permanent magnets and electromagnets, placed adjacent to one another in such a manner that when energy is applied to the electromagnets, the field of the permanent magnet is suppressed or flattened out to the extent that the locomotive receiver or valve in passing over it will not receive an impulse, which would be the condition set up for a clear block. The energy supplying the electromagnets is selected through certain relays and contacts in such a manner that if the track ahead is *not* clear, this circuit will be open, the field of the permanent magnet will not be suppressed and the locomotive receiver or valve on passing through this field is subjected to the effects of these lines of force. The action of the locomotive receiver or valve is purely magnetic, embodying the principles of north and south poles bucking and helping one another. An air valve with pressure on one side working against a magnet on the other side vents pressure to atmosphere when the magnetic effect on the valve is diminished or neutralized on account of passing over the extended field of the track element.

Another type of intermittent inductive train control uses an inert track element or inductor which imparts an impulse to a receiver mounted on the locomotive when the receiver passes over it. The inductor consists of a laminated iron core around which wire is wound. The receiver consists of a primary and secondary winding on a laminated U-shaped core. The primary is constantly energized from the headlight generator, thereby forming an electromagnet. If the leads to the track element or inductor are closed, as in clear block conditions, the engine circuits are not materially disturbed when the receiver passes over the inductors. However, if the winding on the inductor is open, as in the caution or stop block condition, the reactance of the inductor is changed as well as the reluctance of the magnetic circuit, between receiver, air, and inductor, and a current is induced in the secondary circuit of the receiver, opposite in polarity to and larger than the normal holding current of engine-control relay, which drops this relay and causes an electropneumatic valve to function resulting in a reduction of pressure. Acknowledging and forestalling devices can be provided on most of the devices which enable the engineman to forestall an automatic application of the brakes, providing he is sufficiently alert to perform a certain duty in a specified length of time at a certain place.

The Chicago & Northwestern has installed a system of continuous control which is rather unique in many respects, in that the operating mechanism controlling the speed of the train in a yellow or caution block is a function of both speed and distance. The mechanism is driven from the axle. A lever or arm, working on a sliding governor, opens and closes certain contacts at various speeds and when a yellow block is entered, another arm or lever engages a worm gear which operates the arm to open and close certain other contacts at predetermined intervals of space or distance, the worm gear being also axle-driven. Another set of arms or levers combines the speed and distance features and operates still another set of contacts. The combination speed and distance contacts provide a means for compelling the engineman to control the speed of his train in conformity with a predetermined tapered speed curve, and require that the speed of his train be below a certain low speed at a particular point. If the speed of this train is not below the low-speed limit at a particular point or place, he will be penalized with an automatic application of the brakes which he cannot release until his train has come to a stop. Cab signals are also provided in this installation, but I believe the old wayside signals, which are of the disk type, are still in service.

In the systems of continuous control, I find that some roads reverse the polarity of the loop circuit and some reverse the track circuit in order to get the normal, neutral, and reversed positions of the engine train-control relay.

The device Mr. Stevens refers to as the "Coder Device" which he states the Santa Fe is testing should fill a long-felt need if it proves satisfactory. It eliminates the loop circuit and renders the locomotive apparatus immune to picking up stray alternating currents of commercial frequencies.

I am rather inclined to agree with Mr. Stevens that in the last analysis continuous control is the end to which we should work as it paves the way for the ultimate elimination of wayside signals and provides a continuous check on the integrity and continuity of the track as well as the engineman.

An automatic train-control device using radio frequencies has been under development and test for some time on the P. M. Railway with encouraging results, but I am not advised as to the present status of this device. Considering the rapid progress and development in radio in the past few years, it is not entirely impossible to conceive of automatic continuous radio train control.

A. Herz: Mr. Stevens has made it clear that the locomotive signals depend on the magnetic field impulse transmitted through several inches of space, usually six in. or more over each rail, making a total air-gap of over one ft. The train control signal current used by the railroad is weak, in fact, not over one ampere. The power interests transmit large quantities of current over wires often located adjacent to railroad tracks, since the conditions of supply and demand make this necessary. These power wires are located at a distance making an air-gap many times greater than that used by the railroad interests in sending the train control impulse through space from rail to locomotive. However, it is quite impossible to transmit these large currents without setting up the usual magnetic fields around the conductors, which will in many instances cut the railroad tracks and thus create a very weak current flow along these tracks quite similar to that made use of by the railroad for this train control. This is especially pertinent and has a great bearing on the whole problem, since in some instances the railroad makes use of 60-cycle current and since the receiving devices and the whole set-up is such as to respond to the impulses from a 60-cycle magnetic field, so that the normal magnetic field liberated by the power wires may also influence the signals in the same manner, and very often in an undesirable manner.

That this is actually the case has been demonstrated in various localities. Some recent experimental work on a parallel between a continuous inductive type of train control installation and a power line along the edge of the right-of-way demonstrated that a comparatively small current, I believe it was on the order of seven amperes, carried on the power lighting circuit, produced enough current in the rails actually to set up a false clear signal on the locomotive. The almost obvious reason for such a failure, of most dangerous nature, is the simple fact that the field of the power line had the same frequency as the field used in the track circuit to transmit the signals.

I want to draw attention to the obvious danger of having these devices set up so that they are responsive to the normal stray fields of the commercial power systems which are today almost universal. If the magnetic field made use of by the train control signal employs a materially different frequency, such interference can be absolutely prevented. This has been proven and is practically self-evident. As a matter of fact, some train control systems now being installed and also some now in use operate at 100 cycles, instead of 60 cycles. Some will go so far as to use 100 cycles plus mechanical tuning, that is, to employ an interrupted 100-cycle current, the rate of interruption being such that a mechanically resonant relay will respond to it and none other. This will provide a double element of safety or protection against possible interference from commercial power systems. However, there is probably a good deal of experimental work which must be done before the continuous inductive train control system is really perfected for general adoption.

T. S. Stevens: The relays for controlling automatic block signals on electrically operated railroads are designed so that they can be operated not only on direct-current operated railroads by alternating currents but on alternating-current propulsion railroads by the use of relays of a different frequency, centrifugal relay, sensitive to a different frequency from that used for propulsion. So the complete track circuit is used in that way.

Mr. Snell spoke of operating cab signals on electrically operated railways and it is rather interesting because this is probably the most interesting electrical development of the age.

I don't want to start a discussion about frequency because it is too deep. The effects of the use of any frequency, whether it happens to be a different frequency from that of the power companies or the same, are not thoroughly known. What corrections can be made with the same frequency and still provide safe operation, are still doubtful. Whether the 100 cycles will not lead us into other difficulties is not known. A device of some kind which involves only very, very low frequency may be developed in the future, but we are pioneering.

I want to say in favor of Mr. Herz that he has been and we also have been working along amicably toward a settlement of this problem and no doubt will get to the solution of it some time.

Carrier-Current Selector Supervisory Equipment

BY C. E. STEWART[1]
Applicant for Membership

and C. F. WHITNEY[1]
Non-Member

Synopsis.—*The rapid advances made in the application of automatic principles to the control of electrical energy have produced many new and novel installations. This paper describes a supervisory system which centralizes the control and indication of all power used in operating a modern interurban railroad. Supervision is accomplished over a single circuit consisting of one insulated wire with ground return. A single frequency carrier current is used for transmitting all signals over the circuit connecting all stations. Considerable attention has been given to make the equipment simple in design, rugged in construction, easy to install, and convenient to maintain.*

* * * * *

THE application of automatic substations for supplying energy to suburban and interurban electric traction systems has undoubtedly added a very vital chapter in the economic life of these systems. From an operating viewpoint, however, the automatic station equipment has entirely eliminated the human intelligence at the station so essential to the former method of operation. In its place we have left a number of inanimate devices, arranged in electrical circuits to respond to certain definite electrical and mechanical laws. Thus, we obtain operations in a predetermined definite sequence, dependent upon the changes of the master devices within the substation.

Such a system operates very satisfactorily and would meet all requirements were it not for the occasional unusual occurrence which is foreign to any predetermined set-up, but which is nevertheless vital to unit operation. For example, to meet emergency conditions in this class of service, it is essential that means be provided for quickly opening all feeders supplying a particular trolley section. In many installations where load conditions permit, it has also been found profitable and advisable to shut down and lock out certain automatic substations during light load periods.

There is a demand, therefore, for some means of supervising these unattended automatic substations from a central point or dispatcher's station. A new class of equipment was developed several years ago, and is known as "automatic supervisory equipment." This equipment provides a dispatcher with a means of selectively controlling devices in the substations and automatically gives him a visual indication of the substation apparatus by means of standard indicating lamps located in cabinets at his office.

One of the most interesting and unique applications of automatic supervisory equipment to interurban railway service is that installed on the Chicago South Shore and South Bend R. R. which runs between Chicago and South Bend, Indiana. This gives supervision over a 1500-volt, d-c. electrification with converter and mercury arc rectifier substations. There are eight substations located between Hammond, Indiana, and South Bend, a distance of about 65 mi. The stations named in order along the right of way are as follows: Hammond, Gary, Wycliff, Furnessville, Michigan City, Tee Lake, New Carlisle, and Grandview. Columbia Avenue and Michigan City substations are manually operated but have automatic reclosing d-c. feeder equipment.

The power dispatcher is located in the Hammond Substation. He has direct supervision over the high-tension incoming line breakers and machine equipments. An indication is also given him of the d-c. feeder breakers in all eight substations.

The train dispatcher is located near the Michigan City substation. He has supervision over the d-c. feeder breakers in all eight substations and also indication of the machine equipments.

The supervisory equipment is of the carrier-current selector type and operates over one line wire and ground. A single No. 6 B and S copper wire is strung on standard 2300-volt insulators for the entire distance, For about five miles at one end of the system this wire is carried on the same pole as the 33,000-volt power circuits.

With the above picture in mind, it is interesting to note the following special conditions which effected, to a large extent, the choice of this particular class of equipment.

1. A problem was presented due to the severe sleet and ice conditions peculiar to this section of the country. Line wires as ordinarily constructed for telephony are subject to many breakages during the winter months. An attempt was made to increase the reliability of operation by installing larger wire and using distribution insulators. The choice of the No. 6, B and S line wire was due, therefore, to its mechanical qualities rather than its electrical characteristics, although the latter are highly desirable.

2. On account of the increase in cost of the specially constructed circuits, it was necessary, therefore, to use a minimum number of line wires, running between stations.

3. Since the supervisory circuits were exposed to high tension disturbances, it was essential to provide protection to connected apparatus. Where line wires are connected metallically to sending and receiving equipments we have a very serious problem of obtaining proper protection. Protection of equipment so con-

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[1] Both of the General Electric Co., Schenectady, N. Y.

Presented at the Regional Meeting of District No. 7, of the A. I. E. E., Kansas City, Mo., March 17-18, 1927.

nected presents an entirely different problem from that experienced in telephony. It is impossible to use line insulating transformers and drainage coils are undesirable on account of the resultant losses. The use of insulating transformers is not possible where the operation of the system is dependent upon d-c. lock-out or holding circuits.

The supervisory equipment line wires can be run, in many instances, in lead sheath cable in order to obtain proper protection to the equipment. The cost of such construction becomes prohibitive on a right-of-way of this mileage.

4. The carrier-current system provides a satisfactory solution to the protection problem. The coupling between the station's equipment and the external circuit is readily made by means of high-voltage condensers which, from the standpoint of protection, serve, in effect, the same purpose as the insulating transformers. Low-frequency drainage units serve to remove the electrostatic induction from the line wires, and standard protective apparatus, consisting of gaps to ground, fuses, arresters, and choke coils, can be employed to protect against unusual disturbances.

Carrier-current equipment as designed will satisfactorily operate over any circuit which will offer low impedance to the high-frequency carrier current. The equipment can be coupled, therefore, through suitable condensers to the high-tension power lines. In the installation referred to, the transmission lines did not offer a satisfactory circuit because of their interconnections and possible separation along the right-of-way. The cost of the necessary by-pass equipment to secure a satisfactory carrier-current circuit was prohibitive.

FIG. 1—DISPATCHER'S KEY CABINET (COLUMBIA AVE. SUBSTATION, HAMMOND, IND.)

A very important feature in connection with the design of this apparatus is that all devices are panel mounted. This applies to all equipment with the exception of the dispatcher's office key and lamp cabinet. The equipment is mounted on standard 90-in. switchboard panels. The panels are completely wired and all external circuit connections are made to standard connection blocks on the back of panel. Installation costs are reduced, therefore, to a minimum as it is only necessary to interconnect between major apparatus in the stations. All installation wiring can be of the standard control size ordinarily used on power switchboards.

The carrier-current system uses the same equipment which has been used so successfully for a number of years in the selector system. A carrier-current panel is added to the standard selector equipment at each station.

In the 90-in. carrier-current panel, a cover over the back affords protection to vacuum tubes, condensers, and variometers. On this panel are mounted the selectors and polarized relays which set up the indication circuits on the key and lamp cabinet in accordance with coded signals received from the substations.

Fig. 1 shows a key and lamp cabinet. For each substation unit supervised this cabinet contains two manually-operated, spring-driven code-sending keys and two indicating lamps. It also contains the master operating key, checking keys, and common indicating lamps.

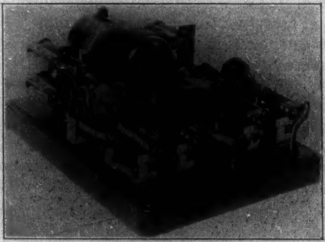

FIG. 2—SUBSTATION MOTOR SENDING KEY

On a typical substation panel, the selectors in connection with an associated carrier-current panel respond to coded control signals from the dispatcher.

Mounted on this panel are the motor-driven sending keys (Fig. 2) which originate the code signals for each supervised substation unit. Associated with each motor sending key is a checking relay. This relay provides a means whereby the dispatcher can check the position of supervised equipment without operating it. This feature is important from a maintenance viewpoint because it provides a check on both the control and indication operation of the equipment.

All operating potentials are obtained from storage batteries. The supervisory system, therefore, is not affected by the momentary loss of the high-tension power circuits and renders reliable service under emergency conditions.

The selection of a carrier-current frequency is, in general, dependent upon the impedance characteristics of the circuit over which the equipment must operate. On an installation involving a number of stations it is desirable to obtain this from actual field tests at the time the equipment is placed in service. Such tests show that there are one or more frequencies to which the entire system will respond most effectively. The carrier-current selector supervisory equipment, therefore, is designed so that frequency adjustments can be made over a wide range. Once the frequency is chosen for an installation, the tuning adjustments of all transmitters and receivers are locked in position and require no further attention.

The carrier-current circuits may be divided into two distinct divisions which may be designated as the transmitter and the receiver. The function of the transmitter is to transform the d-c. impulses originated by the code sending keys into a high-frequency form so that they may be impressed upon the carrier-current line. The function of the receiver is to take the high-frequency impulses from the line and, by means of a vacuum tube rectifier, reduce them to such a form that they may be used for operating the usual supervisory relays. Both the transmitter and the receiver depend upon the three-element vacuum tube or pliotron for their action.

The transmitter utilizes three vacuum tubes of the 50-watt size. One tube is used in the master oscillator circuit which generates the high-frequency oscillations. The other tubes form a part of the power amplifier circuit the function of which is to increase the output of the element. The master oscillator uses the Colpitts circuit for generating the high frequency. A 1000-volt, d-c. source is required between the plate and filament of the tube and a 10-volt, a-c. source heats the filament. The inductance of the oscillator circuit is in the form of a variometer having a movable rotor. Depending on the position of this rotor, the master oscillator can be made to generate any frequency between 20 and 80 kilocycles.

The elements of the two power amplifier tubes are connected in parallel and their plate and filament voltages are taken from the 1000-volt, d-c. source and 10-volt, a-c. source, respectively. The high-frequency voltage taken across one of the condensers in the oscillator circuit is applied between the grids and filaments of the power amplifier tubes. This high-frequency voltage applied to the grids results in a variation in the d-c. plate current to the power amplifiers of the same frequency. The primary of an output transformer is connected in the plate circuit of the power amplifiers and the high-frequency output of these tubes is utilized by connecting to the secondary of this transformer. The secondary is provided with taps so that the desired voltage may be obtained, the correct voltage being dependent on the line impedance at the frequency used. One side of the secondary of the output transformer is connected to ground and the other side through a coupling condenser to the carrier transmission line. A second variometer and a thermoammeter are in series with the coupling condenser. A resonant circuit is formed by the output transformer secondary, the output variometer, the coupling condenser, and the impedance of the line to ground. This circuit is tuned to resonance for the frequency generated by the master oscillator by varying the position of the rotor in the output variometer until maximum current is indicated by the output ammeter. The maximum input of power to the line at this frequency is then obtained. Two tubes are used in parallel in the power amplifier circuit in order that the desired output may be obtained from the transmitter without overheating the tube elements. The maximum output of the transmitter is 150 watts.

The high-frequency output from the transmitter is controlled by means of a relay called the transmitter keying relay. When the coil of the relay is energized, one pair of contacts completes the circuit from the 1000-volt, d-c. source to the plates of the transmitter tubes, causing the high frequency to be generated. A second pair of contacts connects the secondary of the output circuit to ground.

The 1000-volt, d-c. power for the plates of the transmitter tubes is derived from a 500-watt motor-generator set which is driven from a 120-volt storage battery. The motor is in reality a converter as it has slip-rings which supply 88 volts at 40 cycles. By means of a small transformer, this source is stepped down to 10 volts for lighting the filaments of the transmitter tubes. Two rheostats are mounted on the carrier-current panel, one for controlling the field current of the generator and the other is in series with the primary of the filament transformer for adjusting the filament voltage.

Test jacks and keys are provided on the panel to facilitate checking the operation of the transmitter. A jack, in series with the common grid connection to the tubes, allows a milliammeter to be connected in this circuit. If the master oscillator is not generating high frequency no reading will be obtained on this meter. If the master oscillator is functioning properly but the power amplifier is not, the meter will read about 80 milliamperes. If both master oscillator and power amplifier are functioning properly, a reading of approximately 150 milliamperes will be obtained. A jack across the filament circuit of the transmitter tubes provides for connecting an a-c. voltmeter in order to adjust the filament voltage. The output ammeter is normally shunted out of the circuit and is used for testing and maintenance purposes only. When it is necessary to start the transmitter for inspection purposes, a switch on the panel may be closed, operating the motor starting contactor. This switch also opens a circuit which delays the operation of the motor sending key until the inspection is completed. Another switch completes the circuit to the keying relay, causing the transmitter to generate and send out the carrier current. It is sometimes necessary for the maintainer to make inspections, on the back of the panel, which might cause him to come in contact with high potential circuits if the motor-generator were running. A switch is provided to open the circuit to the motor starting contactor, thus preventing the generating of high voltage until his inspection is completed. All of these test switches are plainly marked with individual nameplates on the front of the panel.

The receiver is connected to the carrier-current line by means of a coupler which consists essentially of two coils so mounted that their inductive relation to one another may be varied. One terminal of the primary coil is connected to ground and the other terminal is connected to the carrier-current line through the coupling condenser. A variable condenser is connected across the terminals of the secondary coil forming a resonant circuit which may be tuned to the carrier frequency by adjusting the condenser to the proper capacity. The voltage across this resonant circuit is applied to the grids of the detector tubes which are of the 7.5-watt type and are slightly larger than the average tube used in radio broadcast receiving sets. There are two of these tubes in the receiver with their elements connected in parallel. Plate voltage is supplied from the 120-volt station battery and the filaments are lighted from a 24-volt tap on the same battery. The grids are connected through the secondary of the coupler to the low potential end of the filament resistor. This makes the potential of the grid minus 18 volts with respect to the filaments when no high-frequency voltage exists across the secondary coil. This prevents any current from flowing to the plate and consequently the detector relay, which is operated by this plate current, is not energized. When the carrier frequency voltage exists between the carrier line and ground, a voltage of this frequency is superimposed upon the d-c. voltage applied to the grid of the detector tube. Due to the characteristics of the vacuum tube, this a-c. voltage superimposed upon the negative 18 volts of direct current causes the average voltage of the grids to be increased, allowing plate current to flow and energizing the detector relay. A jack is provided in the plate circuit by means of which a five-milliampere ammeter may be connected in this circuit. This provides a means of checking the operation of the receiver as well as a means of tuning the receiver to the desired frequency. When carrier current is being transmitted from some other station the capacity of the receiver tuning condenser may be varied until a maximum plate current is obtained. This indicates that the receiver is tuned to the proper frequency.

Two detector tubes are used in order to give a maximum reliability to the equipment since the operation of the receiver depends upon the proper functioning of the detector tube. The action of the receiver is equally as satisfactory with either tube alone as with both tubes. The failure of one tube, due to a broken filament or loss of emission, does not impair the operation of the equipment and the maintainer, during his regular inspection, will discover and replace an inoperative tube. The detector tube is of sturdy design and in this class of service has a life of approximately one year.

When the detector relay is energized it completes a circuit from the 24-volt source to the reversing relay which, in turn, operates the selector according to the coded carrier-current impulses. The detector relay also breaks a circuit which prevents the dispatcher, or the motor key as the case may be, from using the associated transmitter for a definite time after the last carrier-current impulse. In order to prevent the receiver from detecting the carrier voltage generated by the associated transmitter, the keying relay is provided with contacts which disconnect the receiver from the carrier-current line and open the plate circuit of the detector tubes. In the dispatcher's station, the detector relay completes a circuit to a pilot lamp on top of the key cabinet, causing it to flash in accordance with the incoming code. The equipment is interlocked to prevent the dispatcher from using the transmitter while the receiver is in operation.

The operation of the system may be divided into two distinct functions; namely, the control operation and the indication operation. The control operation is originated by one of the dispatchers and is for the purpose of starting or stopping one of the machines, or closing or tripping one of the breakers assigned to his supervision. The indication operation is originated by the starting or stopping of some machine or the closing or opening of some breaker in an outlying station. Either type of operation requires the use of only the transmitter at the originating station and the receivers in all of the rest of the stations. All of the operations of either type are transmitted on a single carrier-current frequency. In order that only one transmitter may be in operation at a time, the transmitters are interlocked with the receivers.

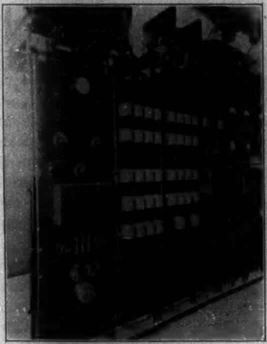

FIG. 3—DISPATCHER'S PANEL EQUIPMENT (COLUMBIA AVE. SUBSTATION, HAMMOND, IND.)

Before any station may transmit a coded signal it must send out a short impulse of carrier current called the locking impulse. This is detected by all receivers which immediately interlock their respective transmitters. A transmitter may function only in case its associated receiver has not detected carrier current on the line for a definite period of time. This allows for the completion of any code which might be in the process of transmission.

In order to demonstrate more clearly the working of the system, a complete operation of each type will be followed through.

Assume that the dispatcher at station No. 1 wishes to trip the a-c. incoming line breaker at station No. 4. The red lamp corresponding to this breaker indicates that the breaker is closed. If the white lamp on top of the key and lamp cabinet is not flashing, no signals are being received and the dispatcher can use the line. He turns the master key 90 deg. in a clockwise direction and releases it. The spring which is wound up by this turn revolves a contact wheel. While revolving, this wheel completes a circuit through auxiliary devices to start the motor-generator set. The generator lights the filaments of the transmitter tubes and supplies the 1000 volts d-c. for their plates. A timing relay is energized and at a definite later time, a circuit to the keying relay of the transmitter is closed for 0.2 of a second to send out the locking impulse. At the termination of this locking impulse the circuit is completed to a green lamp opposite the dispatcher's master key. This indicates to the dispatcher that his transmitter is now ready for operation and that he has control of the line. He now turns the operation key, which in this case would be labeled "Trip a-c. breaker, station No. 4." When released, this key keeps the motor-generator set running and operates the keying relay in accordance with the code setting on its impulse wheel. This causes impulses of carrier current to be sent out over the line. The detector relays in all of the other stations are operated in accordance with this code and the selectors at all these points are actuated. Only the selector in station No. 4, however, closes its contact and trips the a-c. incoming line breaker. When the key released by the dispatcher completes its rotation, circuits are opened which shut down the motor-generator set and extinguish the green lamp opposite the master key.

Whenever a supervised unit at an outlying station changes its position, a circuit is established to the motor driven sending key. A circuit is also made to start the motor-generator set and energize a timing relay. If there has been no carrier current on the line for a definite period of time, this timing relay completes a momentary circuit to the keying relay and sends out the locking impulse. This prevents any other station from obtaining the line during the transmission of the coded signal. As soon as the locking impulse has been sent, the code wheel associated with the supervised unit is released. As this revolves it operates the keying relay which transmits the coded group of carrier-current impulses.

These impulses of carrier current are received on the detector circuits in all other stations. Only one selector in the dispatcher's office notches up and completes its contact. Depending upon the code received, a circuit is established to light either the red or green indicating lamp on the dispatcher's key and lamp panel. In addition to any change made in the dispatcher's indicating lamps, there is also a bell which calls his attention to any change in the position of his supervised units.

When any station has completed the transmission of a signal the motor-generator set comes to rest and all associated transmitting circuits are de-energized. Only the receiving tubes in the entire equipment require current continuously.

It is expected that this class of supervisory equipment operating over a well constructed line wire will prove more dependable than a system operating over several wires between the respective stations. Experience indicates that more difficulties are encountered due to the line wires than with the actual selector equipment. Great care has been taken to obtain simplicity of circuits, well constructed and reliable devices as well as a design layout which invites and facilitates proper maintenance.

In conclusion, supervisory systems of reliable design have centralized in the person of the dispatcher, the human intelligence which was removed from the substations by the advent of the automatic station equipment. The dispatcher is able to operate his system more efficiently and is prepared to meet promptly any special problems which may arise. The supervisory and automatic equipments are waiting night and day to give immediate response to his orders.

Discussion

R. J. Wensley: There are many applications for remote control of power apparatus where the distance is such that control conductors become prohibitively expensive. Such applications are usually in connection with high-voltage systems where inductive interference would be severe should control circuits be constructed along the transmission right-of-way. The Alabama Power Company solved such a problem in a very ingenious manner. They have an extensive 110-kv., "H"-type wooden-pole transmission system. Their southern loop is to be sectionalized at intervals of about 15 mi. by motor-operated disconnecting switches. As there are several hundred miles of this loop, the construction of separate control circuits would have been very expensive.

The standard practise of the Alabama Power Company is to install two ground wires on the tops of the two poles of the "H" frame. These wires serve as static guards and as return circuits for the ground relays. In addition, these wires are now used for the supervisory control of the section switches.

To enable their use, insulators were installed and all direct ground connections were removed. Transformers were installed at each section switch with center taps on the primary windings, these taps being grounded. The control is effected by impulses of 500-cycle and 650-cycle energy.

The control equipment is an adaptation of the Westinghouse audible system. Motor-generator sets provide the two frequencies. The impulses are originated by an ordinary automatic telephone dial.

Selection is obtained by dialing impulses of one frequency. Answering signals are received by audible signals originated by a buzzer at the distant point which checks both the station called and the device selected. Operation is obtained by application of the second frequency by means of the control key on the dispatcher's cabinet.

The impulses of the two frequencies are received on tuned relays. These relays are tuned to resonance by condensers in series with their coils and will not operate unless the frequency is within about 50 cycles of the point of resonance. The selective equipment is operated by a local 12-volt battery. This same battery is used for the motor of the disconnecting switch.

The batteries are to be charged at certain central locations and distributed by trucks at regular intervals. An experimental installation for trickle charging from the drainage current in the ground connection of the transformer primary is now being tried. This involves a special transformer and a Rectox rectifier. If successful, this method will eliminate the handling of batteries. The receiving equipment illustrated is located in a substation, but at most of the locations there is no station and the equipment is housed in an outdoor steel switchhouse converted for the purpose.

All control connections to the ground wires are made through transformers. Spark-gaps on both primary and secondary guard against actual crosses between the ground wires and the 110-kv. lines. A series relay short circuits the transformer primary when the current exceeds its safe rating.

Mercury Arc Power Rectifiers
Their Applications and Characteristics

BY O. K. MARTI[1] and H. WINOGRAD[1]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Steel-enclosed mercury arc rectifiers, due to their advantages over rotary converters in efficiency, ease and reliability of operation, and ability to produce high d-c. voltages, are gradually replacing other forms of converters in all fields of application. There are at present 600,000 kw. of rectifier installations, distributed over different parts of the world. Statistical data are given showing the growth of installed rectifier capacity since 1911, the distribution of rectifiers over various fields of application, and their increasing use at higher voltages for railways. The high efficiency and reliability of rectifiers at high voltages will undoubtedly influence the selection of systems and voltages for main-line electrification. Comparative operating figures are given for rectifiers and motor-generator sets at 3000 volts d-c. Several types of Brown Boveri rectifiers and their load curves are shown. Due to the fact that the d-c. voltage of a rectifier consists of portions of sine waves, the voltage wave is somewhat undulated. The magnitude of the undulations depends on the number of phases and varies with the load. The effect of the undulations in the voltage wave on the shape of the current wave for various types of loads is discussed, and oscillograms of the voltage and current waves of a rectifier feeding a railway load under various conditions are shown. The effects of the undulations on different kinds of load—batteries, lighting, and power circuits—and on communication circuits paralleling the d-c. feeders are discussed briefly.*

Direct current, in spite of the many advantages of alternating current, has its own numerous and valuable characteristics and uses. Among these might be mentioned trolley and other city railway lines, interurban and main-line railroads, rolling mills, special drives requiring the facility of control made available only by the use of direct current, electrochemical applications, and so forth. The generation of d-c. power at ordinarily used voltages would be very uneconomical due to the small power involved for particular requirements. Furthermore, at the voltages at which it is at present generated and used, transmission of the d-c. power over long distances could be accomplished only with considerable losses. The only solution of this problem, therefore, is to generate alternating current, transmit it at high voltages to the site of its application, and there convert it by the best means available into the desired d-c. voltages. Rotating converters have been the only means commercially available for this purpose, until the comparatively recent advent of the steel-enclosed mercury arc power rectifier.

On account of its newness in the commercial field, there was at first a lack of confidence in the rectifier. This, however, has been dispelled by its advantages and successful operation in all parts of the world and in every field of application. Contrary to the rotating conversion apparatus, the electrical energy is not first changed into a mechanical form and then changed back again to the electrical form, but the conversion occurs directly with no intermediate stages. The losses and other disadvantages accompanying conversion by means of rotating machinery are either greatly reduced or eliminated entirely. In a rectifier there are no iron, windage, friction, or ventilation losses, and those which do occur (losses due to the voltage drop in the arc) do not vary as in the usual electrical machines and apparatus, as the square of the current, but only as a linear function and independent of the voltage. Two important properties of the rectifier are dependent on this last mentioned fact: the efficiency remains practically constant at all loads, and since the losses in the rectifier proper are practically constant at all operating voltages, the efficiency increases as the operating voltage is increased. This characteristic of the rectifier—a high efficiency at partial loads—is of particular importance in cases when the conversion machinery has to be operated under conditions which impose a low annual load factor, as in the supply of d-c. power to rolling mills, dredges, elevators, and especially for traction motors. For the last mentioned application, the simplicity and rapidity of the starting operation are also outstanding advantages. These advantages, together with others, have proved the rectifier to be superior to the rotary converter, and have contributed to the great popularity which rectifiers have gained during recent years.

As an example of the superiority of the mercury arc rectifier, it may be mentioned that the chief reason for not using d-c. voltages above 1500 volts for traction lies in the fact that this value is already close to the maximum which can reliably and safely be applied to one commutator of rotating converters. For higher d-c. voltages, two machines must be connected in series, which greatly reduces the efficiency, appreciably increases the initial cost of the installation, and introduces further operating difficulties. With rectifiers, this is not the case, as a single cylinder is capable of producing many times this voltage.

The fact that today there are in service throughout the world steel-enclosed rectifiers with a total capacity of more than 600,000 kw. is without doubt a proof of the soundness of the basic design of these devices. In Fig. 1 are shown the total installed capacities from year to year, and the fact that the steepness of the rise becomes greater year by year indicates the possibility that

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[1] Both of the American Brown Boveri Electric Corp., Camden, N. J.

Presented at the Regional Meetings of the A. I. E. E., Kansas City, Mo., March 18, 1927 and Bethlehem, Pa., April 21-23, 1927.

this device will soon largely replace all other forms of converters.

The field of application of the steel-enclosed mercury arc power rectifier is already very wide, as is clearly illustrated in Fig. 2. Mercury arc rectifiers are naturally used with greater advantage where their peculiar qualities meet the requirements of the service in question. To these classes of service belong installations subjected to large fluctuations in load and to heavy and short current peaks, such as main-line railways, street cars, subway and elevated railroads, rolling mills and the like. A comparison of the shaded areas in Fig. 2 shows that the use of rectifiers for street car and railroad service is twice as great as for all other purposes combined. The next largest field of application is for power and light. Then follow motors for rolling mills, special drives, shovels, dredges, elevators, and mining locomotives. The smallest application of rectifiers, because it is the newest, is for electrochemical purposes. Successful load tests have been carried out with rectifiers at d-c. voltages of 5000 volts and 8000 volts for special electrolytic purposes. Considerable study is being devoted to this field, and attention has naturally been given to assure safety of operation at the high voltages mentioned. These tests have shown that the limits of d-c. voltages for which rectifiers can be used are still unknown.

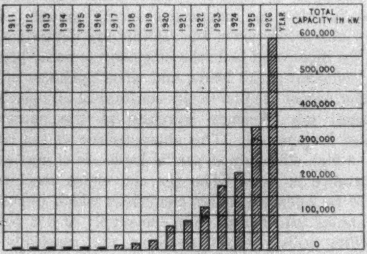

Fig. 1—Growth of Total Capacity in Kw. of Rectifiers Installed

Installations on a commercial scale were begun in Europe in the year 1912, and then only in capacities of 100 kw. One of the earliest installations is shown in Fig. 3, which is for a municipal lighting plant, the direct current being employed for power and lighting purposes. This plant consists of four 6-anode rectifiers, each rated for 150 kw. at 220 volts. The supply current is three-phase, 50-cycle, 5250-volt. This installation was made in 1914 with three cylinders, the fourth being added later on.

Attention may be called to the fact that the principal elements in the design of the Brown Boveri type of rectifier have changed little during the past twelve years. This is evident at once when Fig. 3 is compared with Fig. 4, showing the relative sizes of the different types of rectifiers made at present by the Brown Boveri companies. Inspection of Fig. 3 and Type C in Fig. 4 reveals that the relative dimensions and arrangement of the various elements are practically identical in the 1914 and the present designs. In spite of the fact that there has been no great change in the basic structure of the rectifier, many refinements have nevertheless been made in the design of such details as cooling, anodes, seals, etc., and in the development of more suitable material. Improvements have also been made in the auxiliaries, which, together with the improvements in the rectifier proper, allow a far better utilization of the device. These facts are mentioned merely to remove any possible impression which may exist that the mercury arc power rectifier is still an unfinished product in its first stages of development.

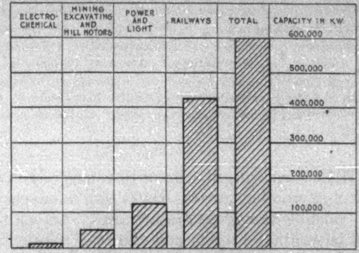

Fig. 2—Distribution of Rectifiers Installed According to Classes of Service

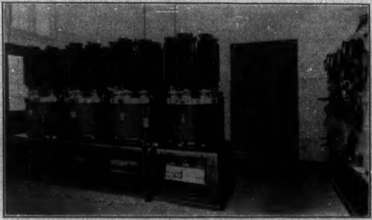

Fig. 3—One of the Earliest Rectifier Installations

Operating Features

The following figures might be of interest in connection with the operation of rectifiers.

The number of rectifiers operating in parallel with rotary converters and batteries, either in substations or over feeders, is about 900, distributed over approximately 450 installations, with a total rated capacity of more than 450,000 kw. Parallel operations of rectifiers with each other, with rotary converters, d-c. generators, or batteries, is governed by the same principles as govern the parallel operation of d-c. generators. In accordance with the fundamental principles on which the operation of the rectifier depends, energy cannot flow from the d-c. to the a-c. network, since the current can pass only from the anode to the cathode. This fact is of considerable importance because a satisfactory parallel operation of rectifiers connected to two independent a-c. networks is thereby possible. The two networks are not restricted, in that they must have the same frequency, nor does a change in frequency affect safe operation, because a change in the frequency in one network will not cause motoring or other troublesome and dangerous occurrences in the rectifier, as would be the case in synchronous converters should they be connected to independent networks.

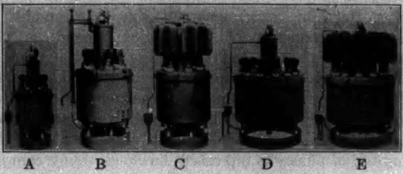

FIG. 4—RELATIVE SIZE OF A SERIES OF BROWN BOVERI TYPE RECTIFIERS

In regard to the adaptability of the rectifier to full automatic control, the fact that the total number of fully automatic substations reaches the appreciable figure of 100 may be of interest. Due to the simplicity and speed of starting and stopping a mercury arc rectifier, this device lends itself to automatic control much more readily than a synchronous converter.

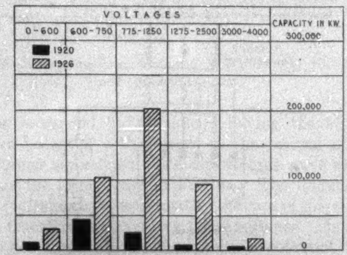

FIG. 5—DISTRIBUTION OF RECTIFIERS INSTALLED FOR RAILWAY SERVICE ACCORDING TO VOLTAGE

Another outstanding feature of the characteristics of the rectifier may be brought out here. As can be seen from Fig. 2, the application of rectifiers for railway service is far greater in respect to total capacity than for any other field; in fact, twice as much as for all other applications together. A most interesting fact in relation to railway service is illustrated in Fig. 5, in which the total capacity rating of rectifier installations for railway service—including city, interurban, and main-line—at different voltages is given, for the years 1920 (solid areas) and 1926 (shaded areas), respectively. The solid areas show that the voltages most frequently used in 1920 lay between 600 and 750 volts, while the shaded areas show that in 1926 the average choice lay between 775 and 1250 volts. It is unquestionably true that the influence of the outstanding rectifier characteristics, consisting of reliable and safe operation at voltages above 600 volts and increased efficiency at the higher voltages, accounts in great part for this fact. The authors firmly believe that an even more pronounced effect of these characteristics on the selection of voltages for the electrification of railroads will soon be noticeable, and thus the mercury arc power rectifiers will soon exert an influence on the question of d-c. versus a-c. systems.

In support of this, the following figures, which show the advantages of a 2000-kw. mercury arc rectifier as compared to a 2000-kw. motor-generator set at 3000 volts d-c., both at nominal rating, may be adduced. Assuming a load factor of 40 per cent, which is common in railway service (800 kw. for 24 hours), a total of 19,200 kw-hr. is obtained, and, assuming a load characteristic of two hrs. at 150 per cent load, eight hrs. at 50 per cent load, eight hrs., at 30 per cent load, four hours at five per cent load, and two hrs. at no-load, the table below will illustrate the large saving which can be effected by employing a rectifier in place of a motor-generator set for this particular load characteristic.

Time (hrs.)	Load kw.	Efficiencies		Losses in kw-hr.	
		Rectifier	M. G.	Rectifier	M. G.
2	3000	97.0	90.8	185	600
8	1000	96.8	86.4	264	1260
8	600	95.5	81.8	266	1070
4	100	82.5	46	82	470
2	0			38	232
				795	3632

Hence, the saving effected during 24 hrs. amounts to 2837 kw-hr., and per year to 1,035,000 kw-hr. Assuming the cost of power to be one cent per kw-hr., an annual saving of about $10,000.00 would be obtained, which would pay for the substation in a few years.

Additional savings in the annual costs would result from the use of a rectifier of the above rating in place of a motor-generator set on account of the lower initial cost of the rectifier, which is about 55 per cent of the cost of the motor-generator set, and on account of the lower cost of the substation, since the building required by the rectifier would be smaller, and would not need special foundations nor cooling ducts.

LOAD CHARACTERISTICS

The mercury arc rectifier is inherently a machine with a continuous rating, due to the very small masses which can absorb and store up the heat produced during its operation. Due principally to the absence of all rotating parts, however, it has a high momentary overload capacity and can respond very quickly to these overloads due to the absence of the inertia of a magnetic field. Standard types of Brown Boveri rectifiers are shown in Fig. 4; their current and kilowatt ratings at various voltages, up to 5000 volts, are given by the curves in Fig. 6. At present they are built for voltages from 220 volts to 5000 volts d-c., and in capacities from 220 kw. to 2700 kw. The ratings given in Fig. 6 are reduced somewhat if overloads of appreciable duration are required. In order to give a picture of the overloads which can be cared for by rectifiers, the following figures have been noted: Type D, shown in Fig. 4, has a continuous capacity of 1500 kw. at 1500 volts, with an overload capacity of 2250 kw. for 15 min., 3000 kw. for five min., and 4500 kw. for one minute. The significance of these figures may be better appreciated if it is realized that it would be possible to start and run a train of average size with one such unit, since it does not take more than 5 to 10 minutes to bring such a train up to speed.

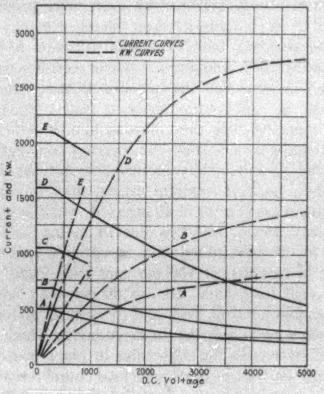

FIG. 6—LOAD CURVES OF BROWN BOVERI RECTIFIERS SHOWN IN FIG. 4

It can be seen from Fig. 6 that the output for a given cylinder increases with the voltage. In spite of this fact, rectifiers are at present rated on the basis adopted for the rating of electrical apparatus and machinery before the advent of the mercury arc power rectifier. The authors believe that, in view of the fact that the field of application of rectifiers is constantly enlarging, it would be justifiable to take steps to work out standards of rating for rectifiers based on their peculiar characteristics rather than on those of rotating machinery. The curves for Types D and E show the characteristic fact that with increasing voltage the kilowatt output increases in a straight line at first, but that the rate of rise decreases with further increases in the voltage.

The rectifier transformer serves the same purpose as the transformer for a synchronous converter; namely, to obtain the proper d-c. voltage and to split the primary power into the desired number of secondary phases. As is illustrated in Fig. 8, the d-c. voltage, during conversion, retains the caps of the sine wave of the a-c. supply voltage. It will be shown later that, in consequence of this, the amplitude and the frequency of the ripples depend on the number of phases, and that increasing the number of phases reduces the magnitude of the undulations. This is one of the reasons why the number of phases employed with rectifiers is relatively large: usually not less than six, and often as high as twelve. As the number of phases is increased, however, many complications enter into the design of the transformer, and its utilization decreases. This latter fact can easily be realized from a consideration of Fig. 8. The interval per cycle, during which the arc is maintained between the cathode and any one anode, decreases with a larger number of phases so that the time of utilization of each phase is shortened. During the other intervals, the phases are not under load and hence not fully utilized. Therefore, for a given d-c. output, the rating of the transformer will increase with the number of phases (see curve 3, Fig 9) which necessitates limiting the number of phases for economic reasons, at a reduction in the smoothness of the d-c. voltage wave. It is therefore necessary to take into account a certain amount of undulation in the voltage wave of all rectifiers, and accordingly we shall look into this question more thoroughly in the next part of this paper.

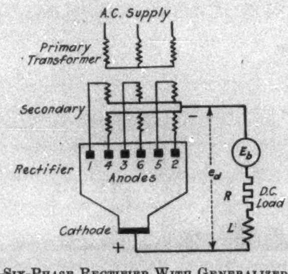

FIG. 7—SIX-PHASE RECTIFIER WITH GENERALIZED D-C. LOAD

The various applications and present status of steel-enclosed mercury arc rectifiers have been dealt with in the first part of this paper. We shall consider now the characteristics of the rectifier as affected by the character of the load and electrical conditions on the d-c. side, for the various applications.

VOLTAGE WAVE

In a preceding paper by one of the present authors[2], the current and voltage relations in circuits of polyphase rectifiers were derived with the assumption that the direct-current wave is a straight line. While this assumption leads to sufficiently accurate results, for all practical purposes, insofar as the relations of voltage, current and power on the d-c. and a-c. sides of the rectifier are concerned, and is entirely justified when there is a considerable amount of inductance on the d-c. side, yet in some cases the undulations in the d-c. current and voltage waves become a factor worth considering, as will be brought out later on.

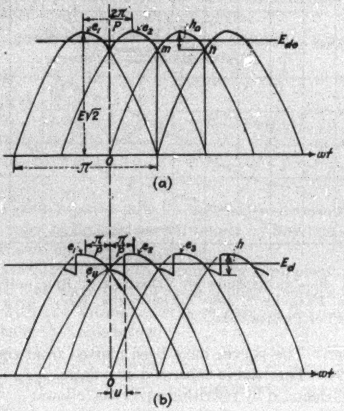

FIG. 8—RECTIFIER D-C. VOLTAGE WAVE AT (A) NO LOAD, AND (B) FULL LOAD

In a polyphase rectifier, the load current at any instant is carried by the anode having the highest positive potential with respect to the neutral of the transformer secondary. The d-c. voltage wave at no-load has the form shown in Fig. 8A. The undulation of the (voltage) wave is formed by the caps of sine waves of the transformer secondary phase voltages. As each phase assumes a maximum positive potential once during every cycle, the number of pulsations per cycle must be equal to the number of phases, and the frequency of pulsation or the number of pulsations per second must be equal to the product of the frequency of the a-c. supply and the number of secondary phases.

If the transformer, the a-c. line, and generator supplying the rectifier were free of reactance, each anode of a p-phase rectifier would carry the whole d-c. current during $\frac{2\pi}{p}$ part of a cycle only; thus, in Fig. 8, the whole load current would be transferred instantly from phase 2 to phase 3 at m and from phase 3 to phase 4 at n. Under such conditions, the d-c. voltage wave under load would have the same form as at no load.

Due to the unavoidable reactance present in the transformer, the current cannot die down nor build up instantly in any phase. As a result, there are intervals during which two adjoining phases carry current simultaneously, as the current in one phase is dying down and the current in the other is building up. This period of overlapping between two adjoining phases begins at the intersection of their respective sine waves and continues until the current in the leading phase becomes zero. The angle of overlap u is given by the expression

$$\cos u = 1 - \frac{I\,X}{E\,\sqrt{2}\sin\frac{\pi}{p}} \tag{1}$$

in which I is the direct current, X the reactance per phase of transformer secondary[3] at the primary frequency, E the effective value of phase voltage, and p the number of secondary phases.[4]

The d-c. voltage during the period of overlapping is equal to the mean of the overlapping phase voltages.

Using the point of intersection of phase voltages e_1 and e_2 (Fig. 8B) as the origin, these voltages may be expressed by

$$e_1 = E\sqrt{2}\cos\left(\omega\,t + \frac{\pi}{p}\right) \tag{2}$$

$$e_2 = E\sqrt{2}\cos\left(\omega\,t - \frac{\pi}{p}\right) \tag{3}$$

The d-c. voltage during the period of overlapping is

$$e_u = \frac{e_1 + e_2}{2} = E\sqrt{2}\cos\frac{\pi}{p}\cos\omega\,t \tag{4}$$

When the period of overlapping is over, the d-c. voltage is equal to the voltage of the working phase.

The average d-c. voltage E_d (including the constant drop in the arc), is given by the expression

$$E_d = \frac{E\sqrt{2}\sin\pi/p}{\pi/p} - \frac{I\,X}{2\,\pi/p} \tag{5}$$

The first term to the right of eq. (5) is the average d-c. voltage at no-load; the second term is the voltage drop at load current, I. The rectifier d-c. voltage wave under load is shown in Fig. 8B. Oscillograms Nos. 1 and 2, Fig. 13, show rectifier voltage waves at no-load and full load, respectively.

The magnitude of the angle of overlap, and therefore the shape of the d-c. voltage wave under load depend somewhat on the nature of the load. Eq. (1) was derived on the assumption that the current curve is a straight line. The angle u will be greater or less than that given by eq. (1) depending on whether the current during the period of overlap is greater or less than the average current. The difference, however, is negligible, and the voltage wave is assumed to be independent of the character of the load.

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[2] *Rectification of Alternating Currents*, by O. K. Marti, TRANS. A. I. E. E., 1926, Vol. 45, p. 668.

[3] Including equivalent reactance of transformer primary and line.

[4] For derivation of eqs. (1) and (5) see paper mentioned in footnote 2.

The total height, h, of the ripple in the voltage wave is equal to the difference between the maximum and minimum ordinates of the wave. From Fig. 8B, it is readily seen that for values of $u < \frac{\pi}{p}$, the maximum ordinate is equal to the amplitude of e_2, while the minimum ordinate is equal to the value of e_u for $\omega t = u$. Therefore,

$$h = E\sqrt{2} - E\sqrt{2}\cos\frac{\pi}{p}\cos u$$

$$= E\sqrt{2}\left(1 - \cos\frac{\pi}{p}\cos u\right) \tag{6}$$

Expressing h as a fraction a of the average d-c. voltage at no-load (see eq. (5)),

$$a = \frac{h}{E\sqrt{2}\sin\frac{\pi}{p}/\frac{\pi}{p}}$$

$$= \frac{E\sqrt{2}\left(1 - \cos\frac{\pi}{p}\cos u\right)}{E\sqrt{2}\sin\frac{\pi}{p}/\frac{\pi}{p}}$$

$$= \frac{1 - \cos\frac{\pi}{p}\cos u}{\sin\frac{\pi}{p}/\frac{\pi}{p}} \tag{7}$$

For values of $u > \frac{\pi}{p}$, the maximum ordinate is equal to the value of e_2 for $\omega t = u$, and the minimum ordinate to the value of e_u for $\omega t = u$. Therefore

$$h = E\sqrt{2}\cos\left(u - \frac{\pi}{p}\right) - E\sqrt{2}\cos\frac{\pi}{p}\cos u$$

$$= E\sqrt{2}\sin\frac{\pi}{p}\sin u\ . \tag{8}$$

$$a \quad \frac{E\sqrt{2}\sin\frac{\pi}{p}\sin u\ .}{E\sqrt{2}\sin\frac{\pi}{p}/\frac{\pi}{p}} = \frac{\pi}{p}\sin u \tag{9}$$

The variation of the ripple in the d-c. voltage wave with the number of phases, at no load, is shown by Curve 1 in Fig. 9. In the same figure are plotted the frequency of the main ripple and the ratio of the transformer rating to d-c. load, to show the effect of the number of phases used on these quantities. The magnitude of the ripple naturally decreases as the number of phases is increased; but to counterbalance that, the size of the transformer increases with the number of phases. Furthermore, the frequency of the ripple increases with the number of phases, which is often objectionable.

In Fig. 10 are shown the variations of the voltage ripple of a six-phase rectifier with the load on the rectifier. The curves have been plotted from eqs. (7) and (9). The load is expressed as a ratio, I/I_s. This ratio is deduced by rewriting eq. (1) as follows:

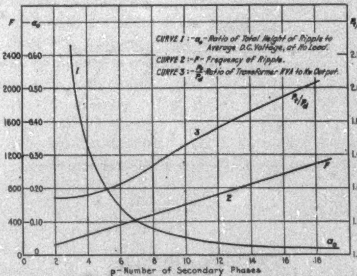

FIG. 9—CURVES SHOWING VARIATION IN RIPPLE HEIGHT, RIPPLE FREQUENCY, AND TRANSFORMER KV-A. WITH THE NUMBER OF PHASES USED.

$$\cos u = 1 - \frac{I}{\frac{E\sqrt{2}}{X}\sin\frac{\pi}{p}} = 1 - \frac{1}{\sin\frac{\pi}{p}}\cdot\frac{I}{I_s} \tag{10}$$

where $I_s = \frac{E\sqrt{2}}{X}$.

The point on the abscissa corresponding to full-load current of a rectifier is determined by the value of X, and therefore depends upon the design of the transformer. The smaller the value of X for a given transformer rating, the larger is I_s and therefore the smaller the ratio I/I_s at full load. The value of I/I_s corresponding to full load is approximately 0.05.

CURRENT WAVE

It was shown above that the form of the rectifier d-c. voltage wave depends on the number of phases used and on the design of the transformer, and that it varies with the magnitude of the load; but is practically independent of the nature of the load. The wave consists of a d-c. component equal to the average value of the voltage, on which is superimposed an alternating component made up of the upper portions of sinusoidal waves. The alternating component is irregular in shape and cannot be expressed by a continuous function. It may be resolved into harmonic components by means of a Fourier series. The first harmonic has a frequency equal to the product of the frequency of the a-c. supply and the number of phases used; it is therefore the *p*th harmonic with respect to the a-c. voltage supplied to the rectifier. The frequencies of the higher harmonics are multiples of the frequency of the first harmonic and since the positive and negative portions of the wave are not symmetrical, there are even multiples as well as odd. Thus, the d-c. voltage wave of a six-phase rectifier supplied by a 60-cycle system has an alternating component consisting of sinusoidal waves of frequencies 360, 720, 1080, etc., cycles.

The general equation of the d-c. voltage of a *p*-phase rectifier, expressed in a Fourier series, is

$$\begin{aligned} e_d = {} & E_d + A_{p1} \sin p\,\omega\,t + A_{p2} \sin 2\,p\,\omega\,t \\ & + A_{p3} \sin 3\,p\,\omega\,t + \ldots \\ & + A_{pn} \sin n\,p\,\omega\,t + \ldots + B_{p1} \cos p\,\omega\,t \\ & + B_{p2} \cos 2\,p\,\omega\,t + \\ & + B_{p3} \cos 3\,p\,\omega\,t + \ldots + B_{pn} \cos n\,p\,\omega\,t + \ldots \end{aligned} \qquad (11)$$

The voltage curve may be analyzed to determine the amplitudes of the various harmonics by any one of the well-known methods of analysis.

A typical analysis of the alternating component in the d-c. voltage wave of a 60-cycle 6-phase rectifier under load, with and without a series reactor on the d-c. side, is given in the table below:

Order of harmonic	Frequency in cycles per sec.	Per cent amplitude of harmonic to d-c. voltage	
		Without reactor	With reactor
First	360	8.46	2.36
Second	720	1.70	0.40
Third	1080	0.95	0.28
Fourth	1440	1.05	0.17
Fifth	1800	0.71	0.12
Sixth	2160	0.58	0.10
Seventh	2520	0.45	0.09
Eighth	2880	0.39	0.08

When the voltage wave with its d-c. and a-c. components is known, the shape of the current wave may readily be determined when the constants of the load are known.

A six-phase rectifier with a generalized d-c. load is shown in Fig. 7. The load may consist of one of the following combinations.

1. Resistance only (R).
2. Resistance and back-e. m. f. ($R + E_b$).
3. Resistance and inductance ($R + L$).
4. Resistance, inductance, and back-e. m. f. ($R + L + E_b$).

1. *Resistance only.* With a load consisting of resistance only, such as a lighting or heating load, the current wave has the same shape as the voltage wave; *i. e.*, the harmonic components in the ripple bear the same ratios to the average value of current as in the voltage wave.

2. *Resistance and back-e. m. f.* With a load consisting of resistance and a constant back-e. m. f., such as a battery, the shape of the current wave depends upon the relative magnitude of the average d-c. voltage, E_d, and the back-e. m. f., E_b. The conditions are shown in Fig. 11. The current is produced by the portion of the voltage wave lying above the line $b\,b'$, the average value of which is $E_d - E_b$. The ratio of the height h of the ripple to this voltage is greater than its ratio to E_d (eqs. **7** and **9**) in the proportion of $\dfrac{E_d}{E_d - E_b}$. The ripple in the current wave, therefore, has the same shape as that of the voltage wave; but the percentage of the current ripple is greater than that of the voltage ripple in the ratio, $\dfrac{E_d}{E_d - E_b}$.

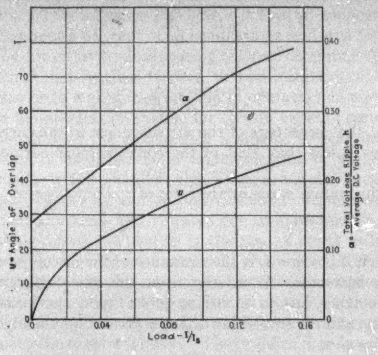

FIG. 10—CURVES SHOWING VARIATION OF THE ANGLE OF OVERLAP AND HEIGHT OF RIPPLE WITH THE LOAD

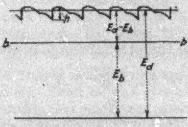

FIG. 11—VOLTAGE CONDITIONS OF RECTIFIER SUPPLYING A LOAD HAVING A BACK ELECTROMOTIVE FORCE

3. *Resistance and inductance.* With a load consisting of resistance and inductance, such as a lighting or heating load fed over a line having a certain amount of inductance, or with a series reactor connected into the d-c. circuit for smoothing out the wave, the average d-c. current is equal to the ratio of average d-c. voltage to the resistance: $I_d = E_d/R$. The amplitude of the *n*th

harmonic in the current ripple, however, is equal to the height of the corresponding harmonic in the voltage ripple divided by the impedance of the circuit to that harmonic:

$$I_n = E_n / \sqrt{R^2 + X_n^2}$$

From the above,

$$\frac{I_n}{I_d} = \frac{R}{\sqrt{R^2 + X_n^2}} \cdot \frac{E_n}{E_d} \tag{12}$$

$$= \frac{1}{\sqrt{1 + X_n^2/R^2}} \cdot \frac{E_n}{E_d},$$

i. e., the percentage of the *n*th harmonic in the current wave is less than that of the corresponding harmonic in the voltage wave by the ratio of $\frac{1}{\sqrt{1 + \frac{X_n^2}{R^2}}}$, in which $X_n = p\ n\ L$ is the reactance of the circuit to the *n*th harmonic. It is seen from the above that the inductance has a smoothing effect upon the current wave, and the smoothing action is greater for the higher harmonics.

4. *Resistance, inductance, and back-e. m. f.* This type of load is by far the most common load supplied by rectifiers, as it is characteristic of all d-c. motors. While starting, when the speed of the motor is zero, the back e. m. f. is also zero, and the load conditions are as given under 3. When the motor is running, a back e. m. f. is generated, in opposition to the applied e. m. f.; the voltage conditions are then as shown in Fig. 11. The current is produced by the portion of the voltage curve lying above line $b\ b'$, as for load 2. The load here, however, is inductive and the current wave is consequently smoothed. The average d-c. current,

$$I_d = \frac{E_d - E_b}{R}$$

The amplitude of the *n*th harmonic in the current wave,

$$I_n = E_n / \sqrt{R^2 + X_n^2}$$

$$\frac{I_n}{I_d} = \frac{R}{\sqrt{R^2 + X_n^2}} \cdot \frac{E_n}{E_d - E_b}$$

$$= \frac{1}{\sqrt{1 + \frac{X_n^2}{R^2} \cdot \left(1 - \frac{E_b}{E_d}\right)}} \cdot \frac{E_n}{E_d} \tag{13}$$

From eq. (**13**) it is seen that the percentage of the *n*th harmonic in the current wave is smaller than the corresponding harmonic of the voltage wave in the proportion of

$$\frac{1}{\sqrt{1 + \frac{X_n^2}{R^2} \cdot \left(1 - \frac{E_b}{E_d}\right)}}$$

the symbols having the same meaning as in eq. (**12**).

The series d-c. motor, used for railways, hoists, etc., is the most common motor fed by rectifiers; in fact, it is the favorable characteristics of the series, d-c. motor for traction purposes which have brought about the present large scale conversion from a-c. to d-c. The series motor is also the most favorable load for smoothing out the ripples in the current wave, due to the inductance of the series field of the motor. In oscillogram No. 3, Fig. 13, are shown the voltage and current waves of a rectifier supplying a railway load. The oscillogram was taken on a 1500-volt rectifier at 200 per cent of the rated load when the voltage ripple is greater than at rated load, and shows the smoothing effect of the series motor on the current wave.

A further smoothing out of the current, and also of the voltage wave supplied to the line can be realized by connecting a reactor into the d-c. lead of the rectifier. The smoothing of the voltage wave is produced by the a-c. voltage drop across the reactor, due to the ripple in the current. The effect of the reactor on the current and voltage waves supplied to the line by a rectifier is shown in oscillogram No. 4, Fig. 13. The oscillogram was taken at approximately the same load and under the same conditions as oscillogram No. 3, except that a series reactor of approximately 3 millihenrys was connected into the circuit when oscillogram No. 4 was taken.

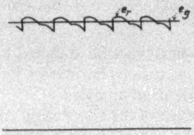

Fig. 12—Voltage Conditions of Rectifier Operating in Parallel With Rotary Converter or D-c. Generator

When a rectifier operates in parallel with a rotary converter or a d-c. generator which has a smoother voltage wave than the rectifier, the resultant line voltage and current waves are smoother than those of a rectifier alone. This condition is shown in Fig. 12. In this sketch, e_r is the voltage wave of the rectifier and e_g that of the rotary machine. For the sake of simplicity, the commutator ripples of the rotary machine are not shown. The smoothing of the voltage wave is produced by the interchange of a small alternating current between the rectifier and the rotary machine.

The interchange current is produced by the alternating component in the difference of the two voltage waves. The a-c. voltage drop in the reactance of the rectifier transformer, produced by the a-c. current component flowing between the rectifier and rotary machine, reduces the ripple in the voltage wave of the rectifier. In this respect, the rotary acts somewhat as a shunt condenser across the rectifier in that it absorbs an alternating component.

When a series reactor is connected into the d-c. lead of a rectifier operating in parallel with a rotary, the wave of line voltage is improved on account of the additional drop in the reactor due to alternating interchange current.

The above conditions are clearly shown in oscillograms 5 and 6, Fig. 13. Oscillogram No. 5 was taken with a rectifier operating in parallel with a rotary in the same station. Oscillogram No. 6 was taken at about the same current and under the same conditions as oscillogram No. 5, except that there was a series reactor in the rectifier circuit.

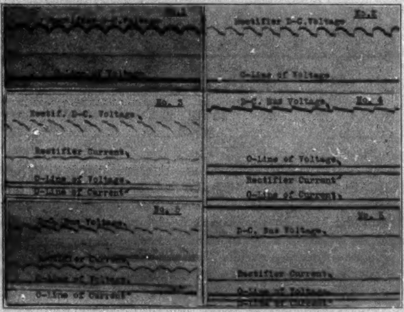

FIG. 13—OSCILLOGRAMS OF VOLTAGES AND CURRENTS OF A 1500-KW., 1500-VOLT RECTIFIER

No. 1. D-c. voltage wave at no load.
No. 2. D-c. voltage wave under load.
No. 3. Waves of voltage and current supplied to railway load, rectifier working alone.
No. 4. Waves of voltage and current supplied to railway load, rectifier working alone with a series reactor.
No. 5. Waves of voltage and current supplied to railway load, rectifier working in parallel with rotary converter in the same station.
No. 6. Waves of voltage and current supplied to railway load, rectifier working in parallel with rotary converter in the same station and with a series reactor in the rectifier circuit.

The effect of various load conditions on the wave form of rectifier voltage and current having been discussed, it might be of interest to mention here that extensive tests have been carried out to determine the effect of the undulations in the waves upon the load connected to a rectifier. For this purpose, batteries and shunt-wound and series-wound motors were connected, first to a battery (*i. e.*, a constant d-c. voltage supply), then to a rectifier fed, respectively, by a single-phase, three-phase, and six-phase a-c. supply. With six-phase operation, no difference in the efficiency as compared to operation on a battery could be noticed in the case of either motor; nor did the undulations have any effect upon commutation.

It can readily be seen that the undulations in the d-c. voltage will have no effect upon the usual lighting circuits, since the frequency of the undulations is high and their magnitude small as compared to the 60-cycle a-c. voltage used for lighting.

The presence of these ripples in the d-c. voltage wave of a rectifier, particularly the harmonics lying within the audible frequency band, has raised the question of their influence upon neighboring communication circuits. This influence has been detected in connection with about 5 per cent of all installations and was found to be due either to close spacing between communication circuits and rectifier feeders, to bad insulation conditions, or to a grounded method of operation on part of the communication system for certain types of service. The cause of the interference is similar to that which results in disturbances in communication or signaling circuits which parallel high-voltage supply lines at small separations and for long distances; *i. e.*, induced or leakage currents and voltages due to harmonics in the voltage wave. When, as in the case of street railway systems, one side is permanently grounded, the ripples may appear in the communication circuits, especially when the method of operation or interconnection involves grounded equipment, wet soil conditions aggravating the case materially. The problem may be solved by the elimination of the exposure, the employment of a non-grounded method of operation or interconnection, or the utilization of a so-called filter, which consists of a combination of inductance and capacitance, so arranged in the circuit as to smooth out the high-frequency ripples in the voltage wave of the rectifier. To the knowledge of the authors, such filters have been found necessary in but few cases. The subject is merely mentioned here, as a thorough discussion of the question would lead beyond the scope of this paper.

Discussion

PAPERS ON MERCURY ARC RECTIFIERS

(MARTI AND WINOGRAD[1], SHAND[2], BUTCHER[3])
KANSAS CITY, MO., MARCH 18, 1927
BETHLEHEM, PA., APRIL 21, 1927
(DISCUSSION AT KANSAS CITY)

B. Blasser: A limited study on the application of mercury arc rectifiers to a city street railway system brought out some interesting comparisons that might be of interest.

On the property of the Kansas City Public Service Company the daily load factor is almost constant for any week day, varying between 49 and 51 per cent. With this condition, it is a simple matter to construct typical load curves for any season from the average output over the period under consideration as compared to the output on the maximum day in the year.

For the purpose of this discussion the four curves in the accompanying Fig. 1 were constructed; (1) the maximum day in the year taken from hourly readings, (2) the typical winter day which is 81 per cent of the maximum day, (3) average day for the year, 70 per cent of the maximum day, and (4) a typical summer day, 61 per cent of the maximum day.

For a single-unit substation in an outlying district, the time of operation may be somewhat different from a centralized substation and with the application of a 750-kw. unit to the load of 1000 amperes for the maximum hour as shown by Curve No. 1 it is assumed that the station will be shut down whenever the load drops to 200 kw. This operation would give 20.5 hr. operation on the maximum day and 17.75 hr. per day in the summer time. If it should be necessary to operate this station through the "owl" period at less than 200 kw., the comparative results would be materially changed. The dotted line shows the approximate load that would be carried by the machine on the "owl" if this station had its proportionate share of the total load. The 24-hr. operation would give a station load factor of 51 per cent as compared to a load factor under proposed operation of 60.8 per cent on maximum day and 65.5 per cent load factor during proposed summer operation.

In comparing the operation of 750-kw. synchronous converters with 750 kw. rectifiers, the daily load curves shown were used to determine total loss for the day and the difference between the loss with converters and the loss with rectifiers (expressed as a percentage of total output) was plotted against the average load of the machine for the four typical loads shown, Fig. 3A. Positive values show rectifier more efficient and negative values show converter more efficient.

This set of curves shows the effect of load factor and more variation in performance between converters and rectifiers is obtained on 60-cycle systems than on 25-cycle systems.

A similar set of curves, Figs. 2 and 3B, was plotted for a 1500-kw. station using a single-unit converter and two 750-kw. rectifiers in parallel. It was assumed that the rectifiers were operated as a 1500-kw. unit and that one element could not be shut down during periods of light load. The loads were taken as double the values of the load curves shown for the 750-kw. station except that it is assumed that the station is located at an intermediate point between outlying district and downtown making it necessary to operate the station during the "owl" period. It is also assumed that this station will pick up load from surrounding districts so that it will carry a minimum load of 50 kw.

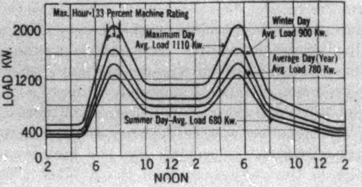

Fig. 1

Even with the additional period of light-load operation, the 25-cycle converter is almost as efficient as the rectifier for the year's operation and is more efficient during the winter period.

On 60-cycle systems, the rectifier is more efficient than converters although the margin is less with the double-unit station.

For the 1500-kw. station, another set of curves, Fig. 3C, was made up with the assumption that one of the 750-kw. rectifiers could be shut down for all loads under 750 kw. This eliminates the loss of one unit over a considerable period of time and results in an increase in efficiency for the rectifier. This operation gives about the same percentage of saving as was obtained with the 750-kw. single-unit station.

From the standpoint of station losses, I should conclude that the application of rectifiers on 25-cycle systems would be justified only after their satisfactory operation in service had been proved, but that the margin of saving when applied on 60-cycle systems is sufficient to justify the expense of trial installations. The tendency of 60-cycle converters to flash over is a further incentive to the trial application of rectifiers on that frequency.

Rectifiers have not been in service for sufficient time in this country to obtain maintenance costs that are truly comparative with other types of apparatus and until this information is available comparisons of total operating costs will have an unknown factor that might lead to erroneous conclusions.

R. G. McCurdy: I wish to discuss briefly the effect on inductive coordination between electric street railway circuits and telephone circuits induced by the use of mercury arc rectifiers. In his paper Mr. Shand refers to the importance of this problem and it is mentioned briefly in the paper by Messrs. Marti and Winograd. It may be of interest to consider the relative advantages of rotating machinery and mercury arc rectifiers in this respect, along with the relative advantages from the power standpoint. It would be unfortunate in this respect if such a new-development should result in a general increase in the a-c. components on street railway circuits in the range of frequencies employed in the telephone circuits, thus increasing the inductive influence of the street railway circuits.

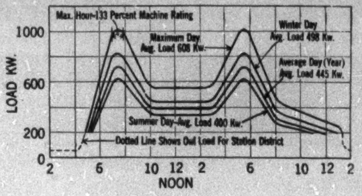

Fig. 2

In considering this problem, it should be remembered that the distribution system of the street railways is a system having one side connected to ground. For given magnitudes of voltages and currents of voice frequencies, the inductive effects are much more severe than with circuits which are connected to ground only at neutral points. Moreover, it is of course impossible to employ transpositions in the trolley circuits to reduce coupling with neighboring communication circuits.

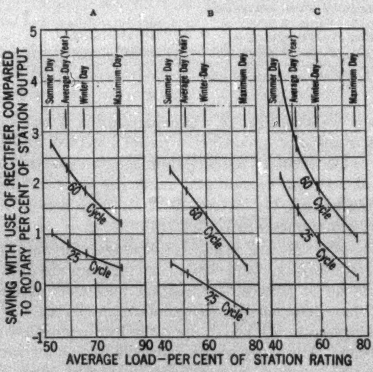

Fig. 3

As brought out in the paper by Messrs. Marti and Winograd, there are present in the voltage wave shape on the d-c. side of the rectifier, ripples whose magnitude and frequency depend upon the frequency of the fundamental a-c. supply and the number of phases. The fundamental frequency of the ripple is equal to the

CALCULATED DATA USED FOR CURVES USED IN DISCUSSION OF APPLICATION OF MERCURY ARC RECTIFIER

	Load, kw., max. hr.	Hours oper.	Avg. load, kw.	Load factor per cent	Machine factor	Saving with rectifier, per cent	
						60 cycle	25 cycle
750-Kw. Station—Outlying District							
Maximum day	1000	20.5	608	60.8	81	1.23	0.35
Average winter day	810	20.0	498	61.5	66.5	1.85	0.64
Average day for year	700	18.88	445	63.5	59.4	2.30	0.83
Average summer day	610	17.75	400	65.5	53.3	2.80	1.02
1500-Kw. Station—Intermediate District Rectifiers in Parallel at all Times							
Maximum day	2000	24	1110	55.5	76	0.32	−0.52
Average winter day	1620	24	900	55.5	60	1.30	−0.05
Average day for year	1400	24	780	55.5	52	1.80	0.20
Average summer day	1220	24	680	55.5	45.5	2.30	0.42
1500-Kw. Stations—Intermediate District Single Unit 750-Kw. Rectifier Up to 750-Kw. Load							
Maximum day	2000	24	1110	55.5	76	0.90	0.10
Average winter day	1620	24	900	55.5	60	1.90	0.89
Average day for year	1400	24	780	55.5	52	2.75	1.40
Average summer day	1220	24	680	55.5	45.5	4.20	2.15

product of the fundamental frequency of the a-c. supply and the number of secondary phases. The fundamental ripple contains both odd and even harmonics. As the number of phases increases, the magnitude of the fundamental ripple decreases, as is shown by Curve I of Fig. 9 of the paper. Thus in a 6-phase rectifier, supplied from a 60-cycle source, the fundamental frequency of the ripple is 360 cycles, and in a 12-phase rectifier, 720 cycles, the magnitudes of the 720-cycle component in the 12-phase rectifier being considerably less than that of the 360-cycle component in the 6-phase rectifier. If there were 24 phases, the fundamental frequency of the ripple would be 1440 cycles and its magnitude would be much less than that of the 720-cycle ripple present in the 12-phase device.

In the table on the seventh page of the paper by Messrs. Marti and Winograd are shown amplitudes of the harmonic components of the voltage from a 6-phase, 60-cycle rectifier including the harmonics up to the eighth. It will be noted that the magnitudes of these harmonics fall off with increasing frequency. The second harmonic, 720 cycles, is about one-fourth of the 360-cycle component while that of the 1440-cycle component is about one-eighth. It will be seen, therefore, that the 6-phase rectifier has all the harmonic components that the 12-phase rectifier contains and in addition contains 360, 1080 and other odd harmonic multiples of 360 cycles.

As brought out in the papers, there is usually considerable series inductance in the feeder circuits and load of street railway systems. The percentage harmonic in the current is usually less, therefore, than it is in the voltage, the percentage of current harmonic with respect to voltage decreasing with increasing order. In a particular instance which I have in mind, the impedance of the load for the harmonic frequencies was equivalent to an inductance of about one millihenry.

In the table on the seventh page of the paper, to which I previously referred, there is shown also the effect on the wave shape of a series reactor. The rectifier itself at voltages of about 600 has a low internal reactance as compared with the reactance of the feeders and the load. The rectifier acts, therefore, practically as a constant-potential generator of these frequencies. As the impedance of the load varies for these frequencies, the harmonic voltage impressed upon it does not change appreciably. If a series reactor is inserted, it is evident that the total harmonic current and the harmonic voltage impressed upon the load will be reduced in the ratio which the total impedance of the load and series reactor bears to the impedance of the load. In the case cited in the paper by Messrs. Marti and Winograd, the series reactor was about three millihenrys, the load impedance being about one millihenry. There is thus a reduction of about four to one in the harmonic voltages impressed upon the load. It is evident that if the load impedance were larger or the series reactor smaller, the reductions obtained would also be smaller.

It is known, of course, that there are also harmonic ripples present in the d-c. sides of rotating apparatus for converting from alternating to direct current. It will be of interest, therefore, to compare mercury arc rectifiers with rotating converting apparatus in this respect. To date in this country there has been a number of installations of mercury arc rectifiers on street railway systems from which experience has been obtained. These experiences have indicated that the inductive influence on telephone lines of street railway systems will be increased by a factor of between five and ten to one by the use of a mercury arc rectifier not equipped with auxiliary devices for correcting the wave shape, as compared to the influence when rotating converting apparatus is employed. The general experience with induction from street railway systems supplied by rotating converting equipment has been that the noise is largely due to commutator ripples from the car motors. When mercury arc rectifiers are used, the source of the noise has been due chiefly to the harmonics from the rectifier, the effects from the car motors being overshadowed.

The impedance of rotary converters to the harmonic frequencies is of the same order of magnitude as the internal impedance of the rectifier. When a rotary converter and a mercury arc rectifier are operated in parallel in the same substation through a reactor, the harmonic currents circulate locally and do not get out to the feeders. Under such conditions, the noise effects on the telephone lines will be very much reduced. When rectifiers and rotary converters are operated in parallel in different substations without a series reactor, harmonic currents are exchanged between the rectifier and the converter, which are determined by the magnitudes of the harmonic voltages from the rectifier and the impedance of the feeders between the two substations. The harmonic currents taken by car motors are small as compared to these currents. When these tie feeders are involved in telephone exposure, the effects are particularly severe, the noise being practically constant in magnitude and independent of load so long as the rotary converter and the rectifier are connected to the line.

All types of telephone circuits are affected by these induced disturbances, including interoffice trunk circuits used for private-branch exchanges and party-line subscribers' circuits. Individual subscriber circuits are also affected under some conditions. These circuits are affected even when they are carried in underground cables.

Reference is made in the paper to a small percentage of cases involving mercury arc rectifiers which have given telephone troubles, but, as I understand the matter, this figure is based upon European experience. So far, experience in this country has indicated that the percentage which will require remedial measurements will be very much higher than that. Our experience has been that where these mercury arc rectifiers are connected without auxiliary devices for modifying wave shape to street railway circuits involved in telephone exposures, severe noise is caused in the telephone circuits. In two or three instances it has been necessary to temporarily shut down the mercury arc rectifier until a remedy could be applied.

In cooperation with the manufacturers of these rectifiers, and with the street railway companies that are particularly involved, work is being carried on by the American Telephone and Telegraph Company and Associated Telephone Companies to determine remedial measures. Consideration is being given both to measures that may be applied to telephone circuits so that they may withstand a higher degree of inductive influence, and to corrective measures which may be applied to the mercury arc rectifiers to reduce their inductive influence.

In reference to the latter, consideration is being given to the use of filters. One particular arrangement which is being investigated consists of a series reactor having an inductance of about one millihenry, with three shunt branches, each consisting of inductance and capacity tuned for the 360-, 720-, and 1080-cycle components. This arrangement gives large reductions in the magnitudes of those particular harmonics and appreciable reductions in the magnitudes of the harmonics of 1440 cycles and above. With such a filter connected to the rectifier, its telephone interference factor will be reduced to that of a reasonably good rotary converter. Under such conditions it would seem to make no particular difference, so far as inductive coordination between the d-c. circuits and telephone circuits is concerned, whether the power were obtained from a mercury arc rectifier or from rotating converting equipment. At present, it is not possible to give any precise figures on what the cost of such a filter will be, but it does not seem that it will very materially change the cost comparisons between mercury arc rectifiers and rotary converters.

In addition to the effects involving the d-c. side of the rectifier, there is also a problem of wave-shape distortion on the a-c. side. The current input to the rectifiers from the a-c. supply contains harmonics of considerable magnitudes. The effects on the telephone interference factor of the voltage depends upon the relative ratings of the rectifier and the generating system and the impedances of the feeders to the harmonics. If the rectifier load is small as compared to the power capacity of the generating system and the feeder is short, the effect on the a-c. system is not large. If rectifiers form a large proportion of the load, such as might be the case with a main line railroad electrification, the effect on the a-c. voltage and current waves would be important and the difficulty of coordinating the a-c. supply circuits with neighboring telephone circuits would be materially increased. There is an important difference, however, in that these harmonics are in the balanced voltages and currents rather than in a circuit having ground as one side.

A. S. Biesecker: In connection with the first method of which you spoke—that is, the introduction of the reactance in order to smooth out the current curve—by what percentage is that going to affect the regulation?

R. G. McCurdy: The regulation of the rectifier is effected only by the d-c. resistance of the series reactor. The power losses in the shunt elements are entirely negligible. There seems to be no particular difficulty in designing the series reactor to have a voltage drop of 0.5 per cent or less of the rectifier voltage where this is 500 volts or above. With lower voltage rectifiers, of course, the loss in the series coil would be more serious.

G. E. King: What, in microfarads, is the capacity of the condenser you would have to put in there?

R. G. McCurdy: In the particular arrangement I have described, which is applicable to a 600-volt, 60-cycle, 6-phase rectifier, the series reactor is one millihenry and the reactor in each of the three shunt branches is two millihenrys; approximately 100 microfarads are required in the 360-cycle branch, 25 microfarads in the 720-cycle branch, and 10 microfarads in the 1080-cycle branch. A design involving more inductance and less capacitance would be more economical for higher d-c. voltages.

A. S. Biesecker: Does it require as much capacity as that for each installation?

R. G. McCurdy: That is one of the matters which must be determined by the cooperative studies by the manufacturers and street railway and telephone companies, to which I have referred. It is obvious, of course, that if the mercury arc rectifier were connected to a trolley system which is not involved in telephone exposures, no filter of any sort would be required. As a rule, however, wherever we find street railway systems, we are likely to find telephone systems in proximity so that some filtering equipment will be required. The total amount of capacity required will, of course, be determined by the reduction in harmonic voltages and currents which must be obtained.

O. S. Clark: Is the trouble caused by harmonics in the rectifier comparable with the trouble caused by harmonics in the rotary converter?

R. G. McCurdy: While there are occasionally encountered rotary converters which have fairly large harmonic components in the voltage wave, approaching those of the mercury arc rectifier, as a rule, rotating equipment has noise-producing harmonics amounting to not more than 10 per cent or so of a six-phase rectifier not equipped with correcting means. In a number of instances where rotating apparatus having these large harmonics has been encountered, these harmonics have been reduced by the application of auxiliary devices, such as resonant shunts. As a rule, there has been sufficient series impedance in the generator or converter to make it unnecessary to introduce a series reactor.

R. E. Curtis: The Luzerne County Gas and Electric Corporation has had experience with harmonic filters. We have installed a filter on a 25,000-kv-a. generator for this very trouble, and I wonder if the American Telephone and Telegraph Company is not in a position to stand the expense for any of these harmonic filters that are required.

R. G. McCurdy: I feel that this matter of the division of cost of coordinated measures between the power and telephone companies is somewhat beyond the scope of this discussion. It seems to me, however, that when any utility is considering the use of new apparatus or methods of operation resulting in operating economics to the particular utility employing it, a study should be made to determine what methods of inductive coordination would be required by the introduction of the new apparatus or methods. It seems obvious that over-all economies should be considered rather than the individual economy to any particular utility, and the introduction of any new devices or methods, which require for inductive coordination the expenditure of sums of money larger than the amount involved in the operating economies, is a step in the wrong direction. The whole matter of inductive coordination should be attacked from the standpoint of the best engineering solution using apparatus and methods in the plants of both utilities which will permit them both to meet their present and future service requirements effectively and at a minimum total cost.

Caesar Antoniono: We have been listening mostly to one side of the story, the manufacturers'. We have one of these rectifiers which was put in operation a year ago last June. At that time we had in Chicago the problem of handling the crowds of the Eucharistic Congress, which was one of the biggest tasks that any railroad ever undertook—to carry such a large number of people over such a distance in that time. This rectifier station was the last leg of the outfit. We had confidence in it. We had to install eight substations to take care of that load. About 18 mi. of double track were built to take care of that. So, this rectifier was very much depended upon.

Just to show you what we know about the rectifier and what we don't know— it seems there are many things we don't know about it yet. The theories presented here this morning I take with a grain of salt—some are true, or they appear to be true, so far as our experience goes.

At noon before the Eucharistic Congress we were still baking out the rectifiers, uncertain whether we were going to have them in service or not. It developed at noon that we had one anode leaking on one tank. Consideration showed that if we were going to take the tank apart again and try to re-seal the leaking anode, we would not be able to re-bake it in time to put it into service. That service was to begin at three o'clock the next morning, and this was noon.

It was decided, therefore, to take a change, inasmuch as by keeping the vacuum pump running, it would keep the vacuum up. We put it into service that afternoon. It carried a light load—until 3 a. m. From then on, the rectifiers stood up under momentary peaks of 300 per cent load without a flicker or any effect apparently on the rectifier. The only trouble we got into right off— when it began to pick up the load—was heating. It seems that the way we had arranged the cooling water was not proper to take care of this load.

This substation was installed in the country and the cooling water is pumped continously from a well. The first idea was that the well water was perhaps too cold to apply to the cathode and to the tanks and, therefore, it was considered the best policy to discharge the water back into the well case. Right away we warmed that too much. So, it was necessary to discharge the water outside and we went through that day without any hitch at all as far as the rectifier carrying the load was concerned.

It happened that a few weeks after that, the designer of that particular rectifier came on the job and we told him the anode was leaking. He said, "Well, if I had been on the job, it wouldn't have been put on the line." That anode was there until a month ago, when for some reason the factory decided to change it. It was considered that a perfect vacuum must be had to run. This rectifier had a leaky anode running for ten or eleven months without causing any trouble.

Now I am optimistic in believing the mercury rectifier has a field, and if some of the properties will make trial stations, I think that the troubles will be overcome pretty quickly.

We have been compelled by ordinances in some towns not to put up certain types of buildings in certain locations. That will ban the converters from those locations on account of the noise unless an expensive structure is designed to be sound proof.

There is another place where the rectifier adapts itself very readily to the load in certain conditions. This is at a railroad crossing. Our trains may approach a crossing at a speed of 75 mi. per hr. and then slow down to 15 mi. per hr. which is the ruling of the commission, in crossing the railroad. The result is when a train approaches the crossing, at a distance of about 2 mi. it shuts off the power and goes over the crossing without power. When it gets on the other side of the crossing power must be available. With the rotary converter station it is necessary to set the relays very high or slow, otherwise the station may shut down. It takes from 25 to 35 sec. to get the station back on the line. Rectifiers have proved to be better equipment for that class of service. The rectifier picks up the load right away when the trains get on the other side of the crossing, in about 10 sec. We haven't heard of any telephone interference. We have a private telephone cable for the railroad's use passing right by the station and there is no interference there. The telephone property is about ¾ mi. away and there is no interference at this time, although in that same vicinity we have had radio complaints on the rotary converters.

There are other things that appear to be in favor of the rectifier as against the converter. One thing we have to contend with in the rotary converter is flashing over and locking out. That seems to be in some cases quite serious.

Then, we have the commutator and brush troubles. Every one knows the troubles of the commutator of a high-speed machine. Also the a-c. brush is as much trouble to the converter man as the d-c. brush on account of serious dust. The dust spreads all over station equipment and causes troubles on contacts and insulation.

A question has been asked about the load division. This load division question brings us back again to what we don't know about the rectifier. We had a case where one starting anode stuck closed for over a week and would not draw an arc and pick up any load. During this time we assumed the two tanks were working though one was not. Without any apparent trouble one tank was doing the work both were supposed to do. We didn't know the difference.

As to repairs, as has been brought out, we have not had much experience on what repairs amount to, but the impression I got is that the repairs are not going to be anything as compared to those on rotary converters.

A. Herz: Mr. McCurdy mentioned something about telephone interference. On that subject Mr. McCurdy can tell you more than I can, but the audibility or sensitivity curve of the human ear is, I think very interesting. From analysis of the inductive influence liberated by these rectifiers, we know that a great percentage of induction is in the order of the 6th harmonic, that is, 360 cycles. Another large batch of it comes in at the 36th harmonic, that is, 2100 cycles. Between these there is a valley in this induction. This, of course, is a favorable circumstance and is a material help in the problem of inductive coordination.

It is an interesting fact that the inductive influence caused by the rectifiers is really in the valley of the sensitivity curve of the human ear. It may come in higher up again, as some claim that the human ear has two sensitive spots, one of which is much higher than the one I have mentioned.

Changing the subject, this matter of having substations 3¼ mi. apart on heavy electrification raises the question where are we going to stop? If we increase our traffic we shall have to put in substations 1½ mi. apart. assuming the use of 700 volts or less on the trolley. The rectifier now gives us an opportunity to increase the voltage. I believe it is very opportune that it does. When you have to put substations in at spacing of 1½ mi. it is nearly time to increase our trolley voltage. Certainly 1500 volts is practically out of the question in city electrification on the streets; 1500 volts or higher is more applicable to interurban or main-line electrification and electrification outside of the cities.

I believe the Europeans have gone a little deeper than we have into the transformer construction layout used in connection with the rectifiers. If the transformer is the answer to some of the troubles we have we should have the transformers made in Europe or else our manufacturers might be induced to change to the better way.

The subject of speed of operation is vital. The rectifier can be placed in service in a few seconds. You have to pay no attention to synchronizing. As a matter of fact you can get a rectifier on as quickly as your switches will operate. With rotaries that is out of the question, especially with 1500-volt installations where you have two rotaries in series. The question of polarity on the rotaries is very important, but not so with the rectifier—it comes up correctly every time.

As regards noise both within the substation and outside, we must acknowledge that the advantages are all with the rectifier.

Flashovers have been mentioned. Flashovers on rotaries are bad—we know it. Flashbacks occur in rectifiers, but I feel confident that they are less serious to the system and I am sure they are not as hard on the operators.

In the matter of cooling these rectifiers, we have had certain difficulties caused by electrolysis. Parts of these rectifiers float

1500 volts above ground and when a semi-conductive cooling medium is used there will be some current leakage. I wonder why the manufacturers don't go to oil cooling, using the same oil over and over again and cooling this with water through cooling coils. That would mean a change in apparatus but I am quite sure it is feasible. Attempts to overcome this difficulty by the use of distilled water have been unsuccessful; within two weeks it becomes contaminated to such an extent that it is worse than Lake Michigan water normally is. So, I think serious consideration should be given to cooling methods, making use of oil.

W. C. DuVall: It seems to me in this big subject of rectification, it is going to be a matter of economics and I am quite confident that we are going to do in the future the same as we have in the past—engineers are going to find ways and means of correcting such things as telephone interference and other factors that would be in the way of progress.

The biggest thing that the rectifier is doing is to help fight the battle of electrification of railroads, alternating current versus direct current, and no doubt, we are going to sift it down to a field where the rectifier will take its place at one end, and the converters at the other. Just where that division is going to be it is hard to say, but in any event, I am sure engineers will find a solution.

W. B. Anderson: In the latter part of his paper, Mr. Shand mentions a series of tests on one of the old rectifiers, Unit No. 56, which was built by the Westinghouse Company in 1916. These and other later tests were made by the writer, who is associated with Mr. Shand, to obtain a direct comparison of certain features in the old and new designs. The older unit was operated for approximately three months. The results were such as readily to convince anyone that successfully operating steel-tank rectifiers were made by American manufacturers previous to their present commercial activity.

The vacuum chamber of Unit No. 56 is a drawn steel tank 19 in. in diameter and 26 in. deep, this being supported by an outer sheet metal casing which also serves as a water jacket. The two anodes are made of drawn steel and constructed to permit circulation of water for cooling. Steel anode shields are used and the cathode is not insulated from the rectifier tank. The unit is also equipped with a Pirani type of vacuum gage, the same principle now being applied to what is more commonly known as the "hot-wire" vacuum gage. All of the seals are made with "vacuum cement" a sealing compound developed especially for mercury-arc rectifiers. This cement is still one of the best materials that can be found for vacuum tight seals for certain applications.

As Mr. Shand states, this rectifier had been out of service for more than eight years. The tank was opened, cleaned, and reassembled, the only changes being the replacement of an old solder valve with a modern diaphragm-type valve. A set of vacuum pumps and a McLeod gage was added to make the rectifier an operative unit.

The rectifier was operated at different voltages from 175 to 750 volts and on continuous loads as high as 700 amperes It was operated 24 hr. per day for a week on a motor-generator set load of 600 amperes without a single interruption. Overloads as high as 1200 amperes were thrown on the rectifier for 5 min. with no apparent distress. Higher momentary overloads tending toward short circuits merely opened the circuit breakers. Operation was resumed as soon as the breakers could be closed again. Several times the pressure was allowed to increase until an internal short circuit occurred. Following these, the tank could be pumped out again in three or four minutes and operation resumed. The internal design with the steel shields seemed to withstand rather abusive operating conditions.

Two methods of supplying cooling water to the anodes were tried. In the first method, a part of the tank discharge water was passed through the anodes. This meant that with normal full load operating temperatures, the anode intake water temperature was about 35 deg. cent. When starting, the anodes were at room temperature or lower. The other method, which is similar to the practise of ten years ago, was to recirculate the anode discharge water, adding just enough make-up water to maintain a predetermined temperature. Before placing the rectifier in service, the anode water was heated to permit starting with warm anodes. The latter method proved much more satisfactory. Operation at uniform and higher anode temperatures was possible and the necessity of starting with cold anodes was eliminated.

With such an anode-cooling system, it was possible to collect some interesting data and verify certain ideas relative to the proper temperatures at which anodes should be operated. Several mornings when starting up after the rectifier had been standing all night, full load and overloads were thrown on the rectifier. With cold anodes, the first few minutes of operation were very unstable and internal short circuits frequently occurred. By preheating the anode water, operation was made stable and there were no short circuits.

For continuous operation, the anode water intake and discharge temperatures were limited to values corresponding to the limiting temperature of the seals. On several occasions, these temperatures were allowed to increase approximately 50 per cent for periods of about 2 hrs. following continuous operation at the first mentioned temperatures. The result was improved vacuum and increased stability under abnormal operating conditions as compared to similar operation with normal anode water temperatures. These tests demonstrated very forcibly the desirability of relatively higher anode temperatures.

As already mentioned, the loss in the rectifier was dissipated either in the anode or the tank cooling systems. Measurements indicated that at full load, approximately 65 per cent of the loss was dissipated in the tank cooling water and 35 per cent in the anode cooling water. These values represent continuous operation at the most satisfactory discharge water temperatures for this particular design.

One interesting and very noticeable thing connected with the operation of this old rectifier was a decided improvement in vacuum when it was placed on load after standing idle a few hours. This was apparently due to a redistribution of pressure.

As Mr. Shand has pointed out, one of the objectives of the older rectifier development was the perfection of a rectifier normally requiring no vacuum pumps. The tests on Unit No. 56 indicated that rectifiers can be made to operate for long periods without pumping. Rectifiers with such a high degree of vacuum tightness are not required now. Vacuum pumping equipment that is simple, compact, and reliable is available. With a rotating oil vacuum pump and a mercury diffusion pump such as used on modern steel rectifiers, it is possible to evacuate a tank of 13-cu. ft. volume from atmospheric to 0.001 m. m. of mercury pressure in 60 min.

E. F. Sipher: (communicated after adjournment) Mr. Shand has very accurately given the steps in the development and I believe that the decision to operate a rectifier with a pump running continuously is the turning point in the development. Until the latter part of the development described by Mr. Shand, we attempted to secure exceptional vacuum-tight joints because the pumps which were available then were very expensive and could not maintain a high vacuum against the small leaks. With the advent of the diffusion pump, about the time work was discontinued, the possibilities of operating with the pump running continuously and building rectifiers with a less perfect seal were being seriously considered.

If the operators will accept steel-tank rectifiers, which must be pumped frequently or continuously in order to maintain a vacuum, I can see no reason why it should not be possible to furnish rectifiers of considerable capacity, and which will operate satisfactorily, as it will no longer be necessary to build seamless drawn steel tanks, nor to construct extremely tight vacuum joints, which are inherently expensive and hard to maintain.

(DISCUSSION AT BETHLEHEM ON MARTI AND WINOGRAD PAPER ONLY)

L. A. Doggett: Referring to the table, where the comparison is made on the efficiency basis between the rectifier and the motor generator set, it seems to me there ought to be another column in that table that would include the rotary converter, for it seems to me the rotary converter of a 3000-kw. size would have an efficiency of nearly 96 per cent which would make the efficiencies almost equal for the two rectifying devices.

A. J. Standing: There are three questions that probably have been covered in this paper but unfortunately I have not had an opportunity to read it. The first question is: Are rectifiers available in 25-cycle 6600-volt circuits? The next question is: Are rectifiers affected by changes in temperature? And the third question is: What is the maintenance on them and what troubles are they subject to?

J. T. Waugh: I would like to ask Mr. Marti what information he has on the reliability and maintenance of mercury-arc rectifiers, as found by experimentation or actual practise? Can he give us the relative cost between a rectifier with its transformer equipment and a motor-generator set of approximately 50 to 100 kw. and for say 220 volts d-c.

John Grotzinger: It is evident that the application of this rectifier to electric-traction substations is offering decided advantages.

Unfortunately this does not apply to industrial plants where d-c. at 240/120 volts is required for the operation of small and medium-sized variable-speed motors. I take it that for a three-wire system two rectifiers would be required, one on each side of the neutral, operating at 120 volts d-c.

At this low voltage there is no gain in efficiency over the motor-generator set and the cost of the rectifier would be excessive. The attractive feature in such an arrangement would be the freedom from the compounding trouble met with when operating several three-wire generators in parallel.

I would like to have Mr. Marti tell whether in operating several such units in parallel they divide the load uniformly, also what takes place in the case of a short circuit.

G. M. Kennedy: I want to know whether you use a three-phase transformer or a single-phase. Suppose we had a 300-kw. rectifier, what transformer capacity would we use with that?

W. H. Lesser: How does the attention required with these rectifiers compare with the attention that we need with an automatic motor-generator set or an automatic converter?

O. S. Clark: Fig. 5 shows increased tendency in Europe toward higher voltages. I wonder if this is being done to reduce distribution losses, or does it indicate a tendency to adopt the mercury-arc rectifier which operates more efficiently at the higher voltages.

D. C. Prince: On this question of telephone interference the General Electric Company has taken the position that we have to go halfway with the telephone company, and I believe all the rectifiers which we have put out are equipped with the reactance which Mr. Marti shows at the top of his figures.

On the question of filters it has been a matter of where the exposure has been bad, and in our installations in most cases the exposures have not been bad enough to require the filters. However, there seems to be still some hidden question regarding that, because one hears all sorts of stories about what interference this rectifier and that rectifier produces. So far, our rectifiers with the series line reactance have made no trouble in the places where they have been fed over heavy power lines, and it is observed that if the rectifier is supplied over a large power line, so that it is a relatively small part of a power load, the interference seems to be considerably less than where the rectifier is the only piece of apparatus fed over the entire line. A definite correction procedure will probably be developed as a matter of experience, and I don't believe the cost will be great. In most cases where we have had to put in the filters the cost has not been prohibitive.

Mention is made that a rectifier is naturally a continuous-rated machine. The rectifier itself is only a small part of the installation, and for that reason the characteristics of the rectifier are not necessarily imposed on the system as a whole; that is, if it were desirable to supply any particular 2-hr. rating, the fact that the rectifier might come to its final temperature inside of half an hour would not make it necessary to supply a complete installation on the basis of half an hour period overload; *i. e.*, the fact that the rectifier itself is a relatively small part of the whole makes it possible to discount its own particular characteristics by supplying a margin in rectifier capacity. We have installed in Chicago a rectifier with a capacity of 1500 kw., and this unit has repeatedly carried loads of over 9000 kw. with no ill effects whatever. We hope that the policy will be to install rectifiers of sufficient size to handle anything that can come, and then, as the device becomes more exactly known, it may be possible to make savings, but there again such savings will be only in the rectifier, which is but a small part of the whole installation.

Mr. Marti has also gone into this question of the rating of transformers. The transformer rating for a three-phase rectifier is somewhere around 130 per cent of the kw. capacity, and for a six-phase rectifier he has given the figures as somewhere around 150 per cent, and that leads to the question: Aren't we sacrificing something by going to polyphase operation; that is, to a higher number of phases? We have made an investigation to see if the advantage in wave form of the higher number of phases cannot be obtained with three-phase units, which have the advantage not only in that the transformer ratings are less, but the regulation also tends to be better and the telephone interference tends to be less. As a result, our standard rectifiers, instead of being six-phase or more, are all three-phase, connected through interphase transformers, either two or four units operating at an angular displacement to bring the total phases up to six or twelve.

Sidney Withington: (communicated after adjournment) There is no doubt that mercury arc rectifiers possess many material advantages as compared with rotary apparatus, either converters or motor-generators. They are relatively simple in their operation and, being static, are especially adapted to automatic substation operation; furthermore, they can, like transformers, be placed on the line instantaneously by the closing of a switch as required to take care of sudden unexpected loads. As Messrs. Marti and Winograd have pointed out, rectifiers are especially advantageous for high voltages, both from the point of view of their operating characteristics, and cost as compared to rotary apparatus. Then, too, as has been stated, they are very attractive as regards efficiency, especially at fractional loads. It should be borne in mind, however, that the relation between overload capacity and continous rating is relatively unfavorable as compared to rotary converters or motor-generator sets, especially for steam-railway electrification. Where the load increments are relatively large compared to the total load, and where, therefore, the peaks may be of but short duration, the overload capacities of rotary apparatus are of value, whereas the relatively small thermal capacities of the rectifiers are a limiting factor. In the case of urban traffic, where the load increments are a relatively small part of the total load, this feature is of less importance, and this is the field in which the greatest development of the rectifier will undoubtedly lie.

An important characteristic of mercury rectifiers is that if they are subjected to conditions which are too severe, they are not likely to be permanently damaged as are many other types of apparatus, but after a break-down due to overload may as a rule be immediately restored to service.

The question of overload characteristics is an important one, and it would appear to be opportune at this time to consider special ratings for mercury arc rectifiers which would recognize the relatively high continuous capacity as compared to their overload capacity. Such data might well be included in A. I. E. E. Standards. At present a "nominal" rating which means little, is assumed in order that standard overload specifications may be met.

A limitation of the mercury arc rectifier is in its voltage regulation. There is, of course, no possibility of "over-compounding," and therefore there is a drop in bus voltage at heavy loads. This feature, however, especially in the case of street-railway loads, may be offset by scattering automatic rectifier substations about the territory served, utilizing high-voltage a-c. distribution. Such a plan would, of course, mean a relatively low load factor on each substation and, therefore, a considerably greater installed capacity would be necessary than if the apparatus were concentrated. It is probable, however, that before long, on account of quantity production and amortization of development expense, mercury rectifiers will be available at considerably lower cost than rotary apparatus, even for 600-volt service, and the excess capacity will thus be justified by the obvious advantages of small scattered substations.

It is probably going a little too far to say that the mercury arc rectifier will soon exert an influence on the much-discussed question of direct current versus alternating current for heavy traction systems. The electrification system finally adopted as a standard in this country will necessarily be capable of handling all branches of operation economically and reliably, including suburban service and "through" traffic of all kinds, over the same tracks. National standardization in this respect is nearly as important as track-gage standardization, and should rest on considerations which are broader than the economics of any one individual problem.

The chief limitations thus far with d-c. systems are not in the generation of power so much as in the flexibility of its distribution and use on the rolling stock, and with the increasing voltages mentioned by Messrs. Marti and Winograd, even though they may be practicable at the substations, problems will necessarily be met on the cars or locomotives in main motors and auxiliary facilities. Although it is of course impossible to predict what the future holds in store for us, nevertheless it may be said that the fundamental advantage of alternating current as compared with direct current in the present state of development of the art of electrification, is in the flexibility of the a-c. system, wherein it is possible on a single distribution system to operate motor cars, either singly or in multiple-unit trains up to ten or twelve cars, as well as high-speed "through" passenger trains, and extremely heavy-tonnage freight trains with concentration of 15,000 or 20,000 hp. in a single train. This degree of flexibility has thus far not been reached with any d-c. installation yet designed, and it does not seem probable that the mercury arc rectifier can change this phase of the situation very much, especially as it is, of course, impracticable to regenerate power and feed back into a transmission system through this type of apparatus.

O. K. Marti and H. Winograd: Mr. McCurdy answered practically all the questions on interference which were raised during the discussion, and he brought out very clearly the problems involved in mercury-arc rectifier installations in connection with communication systems. I should only like to follow up one point a little further. As he states, there are also harmonic ripples present in the d-c. sides of synchronous converters, and I might point out that interference is sometimes caused by synchronous generators and condensers. Up to the present time, no definite rule has been established by technical societies with regard to the permissible volume of this interference for any kind of machine, and it is therefore difficult to arrive at a proper conclusion in regard to the permissible magnitude of the ripples. In connection with the machines which cause interference, the noise-meter and the telephone-interference-factor meter are used to obtain some quantitative measure of the interference effect. Assuming, now, that the influence of the rectifier ripples is measured by this meter, and that we allow about the same telephone interference factor for rectifiers as for other machines, a filter equipment which is not prohibitive as to cost and maintenance can probably be found, especially since the recent improvements in condensers, due to the greater demand for them for radio and for power-factor correction, have improved their quality and also made them lower in price.

In regard to radio interference, I should like to mention that even if the mercury arc should radiate waves of the frequencies used in radio communication, they would probably be shielded by the steel tank. However, it does not radiate such waves, and no difficulty has to be feared in that regard. It might, however, be mentioned that a great deal of investigating has been done in this connection in Europe, Canada, and in this country, without any influence on radio being observed.

I had intended to discuss Mr. Butcher's paper quite thoroughly, but Mr. Herz and Mr. Antoniono, whose companies have had rectifiers in operation for an appreciable period of time, gave a much better comparison of the advantages and disadvantages of mercury-arc rectifiers and synchronous converters than I would have been able to give.

I should, however, like to make some remarks regarding the first cost and the cost of maintenance, as brought out in this paper. The maintenance cost of rectifiers and their automatic equipment is less than for synchronous converters, as was proved by many comparative studies made abroad and in this country. It is unjust to base a comparison in regard to maintenance on the experiences obtained with the few early trial installations in this country, and the comparison will come to be more in favor of the rectifier as this equipment and its characteristics become better known.

The cost of an installation as indicated on the last page of Mr. Butcher's paper, giving the cost of the building and foundation as $5000 for either a rectifier or a rotary converter, cannot be correct for two reasons: First, the number of cubic feet of space required for a rectifier installation of a certain rating is much less than for a converter installation, and, since there are no heavy moving parts in a rectifier, the foundation costs practically nothing by comparison. According to our estimate, the cost of a rectifier substation would be about two-thirds the amount stated in Mr. Butcher's comparative table. Since the heat produced by the losses in a rectifier is carried away by the water, no extensive space is required above the rectifier, as is necessary for the proper cooling of a synchronous converter. The cost of making a well in case no running water is available can be circumvented by using a recooling system, the cost of which is considerably lower than the cost of making a well as given in Mr. Butcher's paper. If the comparison had been made upon such a basis, it would have shown that the first cost of a rectifier installation compared to a converter installation for 600 volts, direct current, would be the same, or even less. There is a tendency, and I do not see any reason why it cannot be done, to increase the trolley voltage to 750, and even to 800 volts. At such a voltage the first cost of the rectifier will be less and the saving effected due to lower losses in rectifier and feeder will be extremely favorable.

Answering Mr. Clark's question, I believe the tendency toward higher voltages in Europe is accounted for by a desire to secure a higher rectifier efficiency and to reduce the distribution losses as well. In one case 800 volts was adopted in order to do away entirely with all feeders, the rectifiers working directly on the trolley line at 800 volts. The spacing of the substations is very close, being only about half a mile. The rectifiers are mounted right on the station platform, there being no enclosure other than a screen, and the transformers are mounted close to the rectifiers. In this case, almost the entire cost of a substation building and feeders was eliminated.

In answer to Mr. Lesser's question in regard to the attendance required, I should think that the same practise as applied to rotary-converter stations should be adopted for rectifiers. Due to the fact, however, that a fully automatic rectifier installation is simpler and involves less automatic equipment, it requires less attention.

In this connection may be answered the last part of Mr. Standing's question about the troubles to which rectifiers are subject. The main trouble experienced with the rectifier itself has been back-fires, or arc-backs. By improvements in the design of the rectifier and its auxiliaries, these have been practically eliminated, and when they do occur, they have no serious consequences as the rectifier can be put back into service immediately. In automatic rectifier substations they are taken care of by automatically reclosing circuit breakers.

Mr. Grotzinger made a remark about the use of rectifiers for 240/120-volt, direct-current, three-wire systems for industrial plants. There are a number of such installations in service. Since the rectifier efficiency is considerably reduced at the lower voltages, on such systems the rectifier is usually connected to the outside wires, and a balancer, which may be a battery or a small motor-generator set, is used for obtaining the middle wire.

As for parallel operation, the rectifier is far better suited for such service than a rotary converter. Rectifiers will operate in parallel satisfactorily even if connected to two independent a-c. systems having different frequencies. Should the voltage be lowered, the rectifier cannot feed back, so that one rectifier cannot affect other rectifiers operating from the same or other a-c. networks. Frequently a power company would like to operate a rectifier from 25- and 60-cycle systems during certain periods. That, again, is an advantage of rectifiers, as they can operate equally well at 15, 25, 60, or even 100 cycles. The only factor which will be affected is the size of the transformer, which of course must be dimensioned for the lower frequency. This will cause a slight decrease in the efficiency of operation at lower frequencies.

In regard to the question of whether a rectifier transformer has to be rated for a higher capacity than a transformer for a synchronous converter, and to what extent the size is affected by the number of phases, see curve No. 3 in Fig. 9 of the paper. From this curve it is evident that a two-phase transformer, for instance, has a rating of 125 per cent of the d-c. output, and about 145 per cent of the d-c. output for six-phase rectification. As a rule, because of the simplicity of the interconnections between the phases and the lower cost, three-phase transformers are used with rectifiers.

If a rectifier is operated in a place where the temperature is high, its operation is entirely unaffected, as the heat generated by it is taken away by the cooling water and therefore the ambient temperature does not matter. This is another advantage of the rectifier; one can place it in a small space and does not have to bother with ventilators and ventilation ducts to remove the heated air from the room. Incidentally, the amount of noise generated by the vacuum-pump motor is so small that rectifiers may be located in places where rotary converters would be out of question. In fact, we have rectifiers operating in department stores, municipal buildings, hospitals, and other locations where the absence of noise is of prime importance.

In reply to Mr. Waugh's question regarding the relative costs of rectifiers and motor-generator sets for 50 and 100 kw., 220 volts, the price of rectifiers would be higher. For such capacities, rectifiers would be used only if other factors, such as noise, flexibility, or operation, etc., make their application advisable.

As to Mr. Doggett's remarks, I wish to say that the comparison of efficiencies in the paper concerns 3000-volt conversion, for which synchronous converters would not be used.

E. B. Shand: Mr. Marti has referred to the difficulties of the design of transformers when used with rectifiers,—a point which was also touched upon by Mr. Herz. I mentioned in my paper some extensive work on the investigation of rectifiers and circuits, and I believe that by adhering to the principles determined, these difficulties are not great; and in fact, they have not been found so. Where complex arrangements of the transformer windings are used, there naturally will be some undesirable complications. The transformer inductance determines, to a large extent, the voltage regulation of the rectifier, and it requires careful design to obtain the proper distribution of this inductance in the windings, particularly in the cases where complex schemes of connection are used. It may be said, therefore, that there is something to be gained in the choice of a simpler transformer arrangement.

Referring to Mr. Marti's remarks of the heat-storage characteristics of rectifiers as compared with rotating apparatus, I am not sure that I am in strict agreement. It must be remembered that water is the cooling medium of the rectifier and that water has a very great thermal capacity. Some calculations on this subject made some time ago indicated that the rectifier, with its cooling water, would have a slightly greater heat-storage capacity than a corresponding rotating machine. Of course, where the anodes are not equipped with water-filled radiators, the anodes will have a lower thermal capacity.

There is probably a certain amount of general misunderstanding on the subject of the meaning of the over-load capacity of normal-rated machines. Tests have shown that a rotating machine, such as a d-c. generator or synchronous converter, will reach very nearly its maximum temperature in 35 to 45 min., so that there is no basis for the supposition that the machine can carry its guaranteed overload on account of its thermal capacity. The limitation for the continuous rating is based rather on the limitation on the current-collecting parts; that is, if operated continuously on the overload rating, the commutation limit would be exceeded and the maintenance on both commutator and collector rings would be unnecessarily high. With our present state of information on the rectifier, it does not appear to have these two kinds of limitations to the same extent, so that its application may not be made to the best advantage by directly following converter practise.

In regard to telephone interference, I believe that Mr. McCurdy has brought out a number of interesting points. As he mentioned, when rectifiers are connected to a highly inductive load, such as presented by street-car motors, the harmonic currents are reduced to a point where they do not cause any appreciable trouble; but as is usual practise, where low-impedance feeders connect different substations, the differential voltage between two pieces of apparatus in different substations will produce much larger harmonics over the connecting feeder. An exposure of much less than 10 ampere-miles of such harmonic currents has been found to cause trouble in telephone circuits. It has been demonstrated both in the investigation referred to in my paper, and also in later work that telephone interference can be eliminated by installing special apparatus either in the telephone circuits or in the rectifier circuit. The practicability of the elimination of telephone interference comes down, therefore, to a question of economics, both in the first cost of apparatus, and, where installed with the rectifier, in the additional losses involved.

The comments of Mr. McCurdy on the relative advantages of 6-phase and 12-phase rectifiers are not borne out in the experimental values of telephone-interference-factor which he has obtained from these two types of apparatus. It may be noted that the wave shapes of rectifiers under load are such that the advantage of the 12-phase connection will be very much reduced on account of the effect of overlap which produces further variations of voltage fluctuation but which do not occur at no load.

C. A. Butcher: It is seldom required that converters of different frequencies be operated in parallel although this is being done very satisfactorily. On the Edison system in Chicago, 25-cycle and 60-cycle converters are operated in parallel on the d-c. bus with little or no apparent difficulty.

There is something to be said about rectifier development apparently having been on a more rapid scale in Europe than in America. A reason for this probably lies principally with the rates of development of the central-station industries in the two countries.

The central station industry had its birth in this country in the three-wire d-c. system. This was quickly followed by the a-c. system, which discouraged the installation of numerous small isolated plants. Following later, the development of the central station in Europe was principally at 50 cycles, whereas 25 cycles was the predominating frequency in this country. The 25-cycle converter was, therefore, not in the same demand in Europe as in America and, perhaps, has never reached the same stage of perfection. In the substitution of central-station service for the isolated plant, the rectifier in Europe has been the competitor of the motor generator and the motor converter rather than the synchronous converter.

The development of the 60-cycle converter followed later and those familiar with it know that its early troubles were many. However, these have been quite successfully overcome. In spite of the remarks made about flashovers, the 60-cycle converter is a satisfactory piece of conversion apparatus. The development of the high-speed breaker, and the high-reluctance commutating pole has done much to assist in its proper performance. The operators can do well to study those features in the application of a synchronous converter which will contribute to its more satisfactory performance. Perhaps 50 per cent or more of the responsibility for the flashovers of synchronous converters is with matters of application and operation and not in the limitations of design.

The increase of voltage above 600 brings in many other problems in the way of design of traction motors, control, and distribution insulation, also the problem of using efficiently the higher voltages in congested metropolitan areas. The automatic substation, since attendance is not required, probably lowers the scale of voltages on which d-c. railways may be operated economically. There are probably any number of installations which years ago would have been made at higher voltages had not the automatic substation been perfected to the point where a greater number of substations with smaller spacing might be used without excessive operating cost.

The transformer design, which enables a rectifier designer to gain low efficiency at light loads, is also possible with synchronous converters since it is merely a matter of proper ratio of iron to copper losses.

Power-factor correction by operating rectifiers and converters in parallel in the same station,—a question was brought up by Mr. Blasser—is probably something that has not been given a great deal of consideration. When one stops to think that the power factor of the rectifier, 93 per cent, is much better than that of the average industrial load, and that generators in our power stations are designed for 80 per cent power factor, I doubt if he will find very much to be gained by the parallel operation of converters and transformers in the same station for that purpose.

The question of reverse current at light loads merely means that the designs for interurban substations or metropolitan substations must be considered in conjunction with the a-c. supply in order to gain stability of synchronous apparatus and the system as a whole.

Operating third-rail systems with 60-cycle synchronous converters does not render the problem of flashovers more serious. If the installation is properly made, the flashovers are probably not so severe, on a third-rail system as on overhead, for the reason that the magnetic induction on the third rail on the average fault, causes the current to rise very slowly, and thus it may be interrupted by a breaker of ordinary speed before the load on the converter is such as to cause flashover.

Electricity in the Drilling of Oil Wells

BY L. J. MURPHY[1]

Non-Member

Synopsis.—As in other industries, electricity has taken the lead in the petroleum industry in putting the drilling of oil wells on an engineering basis and as a result of numerous installations it can be said that oil companies in general at the present time have a very receptive attitude toward electric drilling. Satisfied drillers, easy operation, low maintenance, low power bills, fewer shut downs, perfect motion, faster drilling, heavier pulling, no stand-by losses—all these factors have contributed to make electricity the coming accepted standard by which all other forms of oil-well drilling will be judged.

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CABLE-TOOL DRILLING

THERE are two forms of drilling practise in common use in the United States today, *i. e.*, cable-tool and rotary. The cable-tool or percussion system was the original method and is the one most extensively employed at the present time. It is used exclusively in the Pennsylvania, Ohio, Illinois, and Kentucky fields and to a very large extent in the mid-continent and western fields.

In drilling with this system, the drilling tools are suspended from a steel cable or manila rope and moved in an up-and-down motion with the speed of the walking beam, which supports the tools, corresponding to the natural period of the string of tools. Naturally this period becomes greater as the depth of the hole increases and, in order to obtain maximum drilling speed, the motor driving the rig must be capable of very delicate adjustment over quite a wide range of speed. It can be seen readily that this method is better adapted to hard solid formations than to soft formations on account of the fact that repeated jarring of the earth will loosen soft structures and cause cave-ins which necessitate fishing jobs and no end of trouble.

There are four principal operations in the drilling of oil wells by the cable-tool system, namely, spudding-in, drilling, hoisting tools, and bailing, each operation necessitating distinct requirements of the electrical equipment operating the rig.

The "spudding" operation consists of raising the drilling tools through a vertical distance of three or four ft. and allowing them to fall. The operation is accomplished by means of a jerk line one end of which is attached to the crank on the band wheel and the other end through a sliding shoe to the cable which supports the drilling tools. The tools are supported usually by a steel or manila cable which passes over a sheave wheel on top of the derrick and then is spooled on a hoisting drum or bull wheel. The tools are fed downward by releasing the brake on the bull wheel. Near the top of the hole the band wheel is operated at a speed of 42 to 45 rev. per min. to give most satisfactory operation and the speed must be very closely controllable. This operation requires approximately 30 to 40 hp.

After a 200- to 400-ft. length of hole has been drilled by the spudding operation the remainder of the drilling is done on the beam. Drilling on the beam requires that the speed of the driving motor or motors be very closely regulated and that the motors have relatively small fly-wheel effect in order to permit rapid variation in speed in response to variations in torque throughout the drilling cycle. The range of speed throughout the drilling of a well runs from 40 to 45 strokes per min. at the top of the hole down to as low as 14 to 20 strokes per min. at the bottom.

The average speed of the drilling equipment must be adjustable at all times to correspond exactly to the natural period of the drilling tools and drilling line, if maximum progress is to be obtained. If the speed of the drilling tool is set too low, the bit does not strike a sharp blow and the progress is slow, while if the speed is set too fast, excessively high stresses are set up in the drilling cable, drilling is slow, and fishing jobs are to be expected.

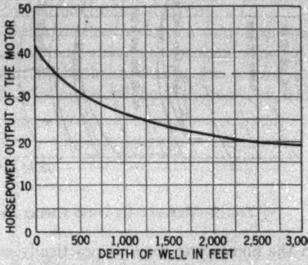

FIG. 1—POWER REQUIREMENTS FOR OIL WELL DRILLING BY THE CABLE-TOOL METHOD

The horse power required will be found to vary but in general it can be said to decrease somewhat as the depth increases. If the hole is free of water, the horse power required to swing the tools will be comparatively light, the beam will operate at a larger number of strokes, and the drilling will progress faster than if water in appreciable quantity is encountered in the hole.

Fig. 1 is a curve showing how the horsepower output of the motor varies with the depth of the well. As no two wells are exactly alike and as conditions will vary

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considerably throughout different parts of the country, the above mentioned curve can be considered as representative only of the general trend of the power requirements with increase in depth.

After every six-foot length of hole has been drilled, it is necessary to remove the cuttings from the hole and before this can be done the drilling tools must be hoisted. In this operation heavy ropes are slipped into a large grooved pulley on the bull wheel and into similar grooves on a tug rim bolted to the band wheel. Thus, the same motor equipment that is used for drilling is used for hoisting also. The pulley ratios between the motor equipment and the band wheel are such that a maximum speed of 80 to 90 rev. per min. is possible at the band wheel and approximately 90 to 100 rev. per min. at the bull wheel. The tools are accordingly hoisted out of the well at a rate of 350 to 700 ft. per min., the speed increasing as the bit nears the surface, due to the increasingly larger effective drum diameter. As the effective drum diameter is increasing, however, the weight to be lifted is decreasing, so that for a given condition the horsepower remains practically constant from bottom to top. At the start of the drilling, the hoisting of the tools is not exceptionally heavy duty, but as the hole becomes deeper this load is likely to become as high as 150 hp., especially if the rig is not very efficient.

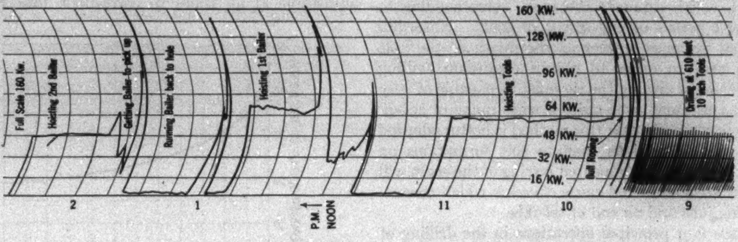

Fig. 2—Typical Graphic Chart Showing Various Operations in Cable-Tool Drilling

As soon as the drilling tools have been hoisted out of the hole, the bailer is lowered several times to remove the cuttings. The bailer, which consists of a piece of pipe with a dart valve at the bottom, is operated by a sand reel, friction-driven from the band wheel. The speed of hoisting the bailer is approximately 2 to 2½ times as fast as the speed of hoisting tools, but because of the lighter weight of the bailer the load on this operation is practically the same as that of hoisting tools.

Fig. 2 represents a section of a drilling chart, showing the various operations when drilling at a depth of approximately 600 ft., using a 15-in. bit. It will be noted that for a considerable portion of the drilling cycle, the load is approximately 16 kw. but that at the peak the meter registered 53 kw. with an average probably around 25 to 30 kw. The operation of hoisting the tools shows that considerable power is required to accelerate the load to full speed but that as soon as the motor has come up to full speed the load remains practically constant. Since the motor is already up to full speed at the time the bailing operation commences, the peaks at the start of this operation can be attributed to pulling the bailer loose from the mud at the bottom of the hole and accelerating it to full speed. It will be noted that the first bailer was evidently more fully loaded than the second.

In order to take care of all the operations incident to the drilling of an oil well by the cable-tool method, there are two equipments in common use in the United States at the present time, one involving two two-speed motors and the other involving one larger single-speed motor. The former method employs two standard two-speed, 15/35-hp., oil-well pumping motors belted to a common counter shaft whereby the drilling is accomplished on the low-speed, low-horsepower connections and the hoisting taken care of on the high-speed, high-horsepower connections. Each motor has nine points of speed control on each speed connection and by operating one controller on one point and the other on any one of its nine points, a total of 45 speeds on each speed connection is available. The latter method employs a standard 75-hp., single-speed motor in which the low drilling speeds are obtained by the insertion of slip resistance in the secondary circuit of the motor. A main controller takes care of acceleration and reversing and an auxiliary controller gives fine speed adjustment by subdividing one of the steps of resistance into a number of smaller steps.

In regard to power consumption, the kilowatt-hours per foot will vary considerably with the location and formation but in general it can be said to vary from 4 to 10 kw-hr. per ft. for a 2000-ft. well with an average of 7 and from 5 to 12 kw.-hr. per ft. for a 3000-ft. well with the average correspondingly increased. The two-motor scheme will be found, as a rule, to be the more economical of the two, due to the fact that during the major part of the drilling operation the two motors are operating at more nearly their synchronous speed and therefore little power is lost in secondary resistance. Fig. 3 shows an installation of this kind.

Fig. 4 shows a typical power curve for cable-tool drilling in the mid-continent field and indicates that the total power consumed tends to increase slightly faster than the increase in depth, due, no doubt, to the greater power consumption in the hoisting and bailing operations.

Rotary Drilling

In certain sections of the country where the formations are, for the most part, of unconsolidated material such as sand, gravel, and boulders, considerable difficulty was encountered in endeavoring to apply the cable-tool system as the jarring of the tools would loosen the earth and cause cave-ins with resulting fishing jobs. To handle these formations successfully, the rotary system of drilling was developed.

Fig. 3—Installation of two 15/35-Hp. Pumping Motors for Cable-Tool Drilling

In this system a hollow drill pipe with a suitable bit, usually shaped like a fish tail, fastened to the bottom of it, is rotated at a speed of 60 to 90 rev. per min. and gradually fed downward. At the same time plunger pumps force mud down through the drill pipe, through two small holes or eyes in the bit, and up on the outside of the drill pipe. The combination of the scraping action of the bit with the jetting action of the mud through the holes, actually drills the well, but in addition to assisting in drilling the mud functions also to carry the cuttings continuously to the surface. The high hydrostatic head forces this mud into the soft formations and the rotating pipe trowels it into place and thus the walls of the hole are built up rather solidly. The amount of casing required with this system is thereby reduced to a minimum.

The equipment involved in an electrically-driven rotary drilling rig consists essentially of a motor, a suitable single reduction gear unit, a draw works, a rotary table, and two motor-driven mud pumps. The draw works comprises a line shaft chain-driven from the gear unit and a drum shaft on which is mounted a hoisting drum so arranged with a number of sprockets and jaw clutches, chain-driven from the line shaft, that two or three hoisting speeds can be obtained. The rotary table is also chain-driven from the line shaft with a jaw clutch on the latter whereby the rotary table can be disengaged at any time.

The operation to be performed in rotary drilling may be segregated into three distinct classes: drilling, hoisting and lowering drill pipe, and circulating, each requiring certain fundamental features in the design of the electrical equipment. The drilling operation requires from 30 to 100 kw., depending on the depth, size of hole, and the formation, and as mentioned previously, the table speed ranges normally from 60 to 90 rev. per min., although under some conditions this speed may be as low as 30 rev. per min. Certain formations require as high as 100 kw. at a speed of 70 rev. per min. which is probably the worst condition encountered, while in other formations the most satisfactory progress of the bit will occur at 90 rev. per min. with loads as light as 30 kw. When special types of rock bits are employed, the speed of rotation of the drill pipe will be as low as 30 rev. per min. and the power drawn from the line will seldom exceed 30 kw. Thus it can be seen readily that the ranges of power and speed for satisfactory penetration of the bit vary considerably with the formations and the location.

There is still another feature to take into consideration and that is the ultimate torsional strength of the drill pipe. With the necessity for excessively high torques in hoisting, as will be described later, it is found highly desirable when drilling to insert a certain amount of permanent resistance in the secondary circuit of the motor which will not only limit the maximum strain that can be applied to the drill pipe but which will also give the motor a drooping speed-torque characteristic. This drooping characteristic offers a cushioning effect between the motor and the drill pipe, a feature particularly desirable when drilling through boulders or hard shells where the equipment is frequently subjected to very severe shocks.

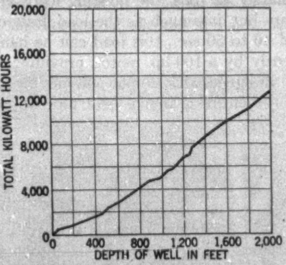

Fig. 4—Typical Curve of Power Consumption of a Well Drilled in Kansas by the Cable-Tool System

After drilling has progressed for four or five hours, the bit has usually become dull and out of gage and it is therefore necessary to replace it with a fresh bit. To do so means removing all the drill pipe from the hole and standing it up in the derrick in "fourbels" or approximately 85-ft. lengths. As the well becomes deeper this operation requires an ever increasing percentage of the total time and as a result it is highly desirable to have equipment which will take care of extremely high overloads for short periods of time in order that as much of the drill pipe as possible can be hoisted in high gear. Records show that when handling 3500 ft. of 6-in. diameter drill pipe, representing approximately 42 tons, the hoisting loads are as high as 300 hp. but inasmuch as the load lasts only for a period of 40 to 50 sec., this load can be handled very conveniently by a 100-hp. motor, providing the motor has been designed with ample pull-out torque. After hoisting the entire string of drill pipe approximately 85 ft., one stand or fourbel is uncoupled and stood in the derrick, the elevators are lowered, a new hold taken on the drill pipe, and the hoisting operation repeated. Thus there is an interval of about one minute when the motor is required to do little or no work. Each succeeding hoisting operation means less load on the motor until, when the last length of pipe is drawn out of the hole, the load is a minimum, amounting only to that required by friction, the weight of the blocks, and one stand of pipe.

FIG. 5—INSTALLATION OF 100-HP. ROTARY DRILLING EQUIPMENT SHOWING RELATIVE LOCATION OF MOTOR, CONTROL, GEAR UNIT AND DRAW WORKS

After the worn bit has been replaced, the pipe is put back in the hole, and during this operation very little work is required of the motor other than lifting the 85-ft. sections into place for coupling together and running the blocks up light to the top of the derrick.

Circulating is an operation in which the drill pipe is held off bottom and rotated very slowly at a speed seldom exceeding 15 rev. per min. The purpose of circulating is to allow the pump to force mud through the drill pipe and build up the walls of the hole. Since the load is exceptionally light and the speed extremely low, a considerable amount of secondary resistance is required to accomplish the desired results. Rotating the drill pipe at too high a speed in this operation will cause the bit to become out of gage.

For extremely shallow wells it has been found that a 15/35-hp., two-speed pumping motor has sufficient capacity to do the work, but for the average wells a 100-hp. or 125-hp. motor is required, with the possibility of utilizing a 75-hp. motor on moderately light wells in some territories where oil is reached at a depth not in excess of 3000 ft. It is interesting to note that the deepest well in the world, 8046 ft. deep, was recently completed in California and electric motors were used throughout, both for driving the drilling machinery and for operating the mud pumps.

Fig. 6 is a typical graphic chart showing the load on a 100-hp. drilling motor for the various operations.

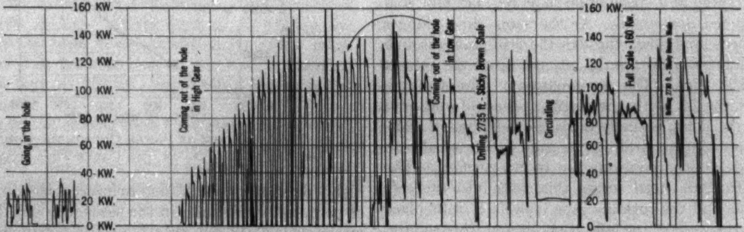

FIG. 6—GRAPHIC WATTMETER CHART OF 100-HP. ROTARY DRILLING EQUIPMENT OPERATING AT A DEPTH OF 2730 FT.

The power consumption per foot of hole will vary considerably with the location, with the ultimate depth, and with the diameter of the hole. In the Gulf Coast district for wells around 850 to 950 ft. deep, the kw-hr.-per-ft. range from 1.5 to 4 with an average around 2.5. Deeper wells in this same territory require from 8 to 14 kw-hr. per ft. with an average of 9.5, the great increase over the shallower wells being due primarily to the larger diameter hole, the harder formations at the greater depths, the high power consumption during hoisting, and the extra horse power to maintain mud circulation at the greater depths. In the California fields, for 3500- to 4000-ft. wells, the power consumption runs as high as 25 kw-hr. per ft., although, with the general adoption of more efficient bits, this figure has been reduced considerably. It is therefore apparent that no definite figures can be given as to the power consumption for various depths of hole as conditions vary in different fields and even in the same field the formation varies to a great extent.

AUTOMATIC ROTARY DRILLING

A large proportion of the difficulties in rotary drilling are fundamentally caused by improper feed of the drill. The feed is in the hands of an individual operator and the regulation of proper feed depends upon the personal equation of this individual, his judgment, his experience, his desire to do good work, etc. Errors of the individual are responsible for many such accidents as twist-offs, balled bits, and crooked holes. If the feed of the bit could be made scientifically proportional to the resistance it encounters, and if this feed could be cared for automatically and independent of any personal equation, many of the accidents and errors due to incorrect drill feed would be eliminated, the speed of drilling would be increased, delays would be decreased, and costs would be reduced. The Hild Differential Drive was developed with this idea of providing a scientifically regulated feed for the drill bit in the place of haphazard manual feed dictated by personal judgment and inclination.

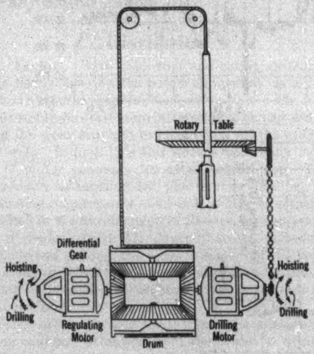

FIG. 7—SCHEMATIC DIAGRAM SHOWING PRINCIPLE OF OPERATION OF THE HILD DIFFERENTIAL DRIVE FOR AUTOMATIC ROTARY DRILLING

SPEED (REV. PER MIN.) OF DIFFERENT PARTS OF AUTOMATIC DRILLER

Operation	Reg. motor	Differential	Drilling motor
Drilling, no progress..........	1000 Forward	0	1000 Forward
Drilling progress.	990 Forward	5 Down	1000 Forward
Drilling progress.	990 Forward	2 Down	996 Forward
Retrieval........	990 Forward	45 Up	900 Forward
Hoisting........	1000 Forward	1000 Up	1000 Reverse

The fundamental principle of the Hild Differential Drive is that the downward feed of the drill pipe is varied according to the power required to revolve the bit on bottom. This relationship between the feed and the power required for drilling is not only adjustable but is fixed for any given adjustment due to the inherent characteristics of induction motors and the principle of the differential gear drive. In operation, if the formation changes so that the load on the drill pipe increases, the downward progress of the drill pipe is automatically retarded. Thus the device not only tends to keep the load on the drill pipe constant for any given setting but also tends to hold the pipe on bottom at all times and at a constant pressure. Furthermore, if the load on the drill pipe suddenly becomes excessive, as would be the case in encountering boulders, the drill pipe is raised until the bit is free of the obstruction, after which downward progress is again resumed.

Briefly, the equipment makes use of a differential gear unit and two motors. The drilling motor drives one-half of the differential, and also the rotary table, direct through gears and chains; while the regulating motor is connected to the other half of the differential. The hoist drum is connected to the central portion or floating part of the differential gear unit.

Referring to Fig. 7, it will be seen that if the two motors are rotating in opposite directions as indicated by the arrows, with the drilling motor having the slightly higher speed, there will be a slight downward feed of the bit. Now if the load on the drill pipe increases, the increase is reflected in a slowing down of the drilling motor and a resultant decrease in the rate of feed. If the speed of the drilling motor is reduced below the speed of the regulating motor, the feed reverses and the bit is raised off bottom. Thus the device tends to maintain a constant predetermined pressure of the bit on bottom, at the same time limiting the torsional stresses in the drill pipe. By reversing the drilling motor, the equipment is used for hoisting with the maximum power of both motors available. Fig. 8 shows an installation of this type of drive.

FIG. 8—INSTALLATION OF HILD DIFFERENTIAL DRIVE FOR AUTOMATIC ROTARY DRILLING

In the operation of this equipment, the drilling, the hoisting, and the other minor operations are all performed in the same manner as with the hand-fed equipment.

In Fig. 9 are shown two typical parallel graphic wattmeter charts representing the loads on the drilling and regulating motors when drilling at depths of 1460 to 1490 ft., using 6-in. diameter drill pipe and 14½-in. fish-tail bits. The total drilling load is represented by the sum of the loads on the two motors. During hoisting both motors are pulling together and the total hoisting load is divided evenly between the two machines. A comparison of the charts in Fig. 9 with those in Fig. 6 demonstrates conclusively the advantages of the automatic feeding of the bit over the hand-fed method, both from an economic and from an engineering standpoint. The absence of peaks during the drilling operation with the automatic feed lessens the strains in the equipment as well as in the drill pipe and the steady uniform load insures that the bit is on bottom the maximum possible portion of the time, resulting naturally in much faster drilling.

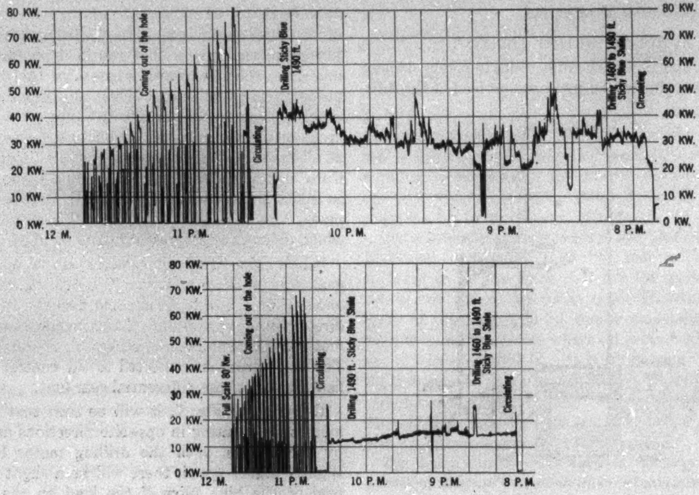

FIG. 9—PARALLEL GRAPHIC WATTMETER CHARTS OF LOAD ON DRILLING AND REGULATING MOTORS

Upper—Drilling motor
Lower—Regulating motor

Note the comparison between the loads for automatic drilling and hand fed drilling as shown in Fig. 6

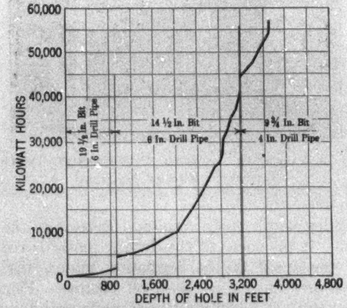

FIG. 10—TYPICAL CURVE OF POWER CONSUMPTION FOR A WELL DRILLED BY THE HILD DIFFERENTIAL DRIVE IN CALIFORNIA

Fig. 10 shows how the total power consumption varies with the depth of the hole in the drilling of a 3700-ft. well in the California fields with the Hild Differential Drive. It will be noticed that in this particular case the power consumption amounted to 15.5 kw-hr. per ft., while for other wells in the same field using the same drive the consumption has run as low as 6.6 kw-hr. per ft. The general shape of the curve appears to be a slight modification of a parabola which differs from the shape of the curve for wells drilled by the cable-tool method as shown in Fig. 4.

The past three years have demonstrated the success of the automatic drive and, in the opinion of quite a few operating men, this drive constitutes the greatest advance in the art of oil-well drilling since the advent of the rotary system.

Everywhere in the oil fields, the operator is in a very receptive mood towards electrification as a result of the success of the pumping and drilling drives. The possibilities are unlimited, especially when it is considered that the industry is at the present time less than 10 per cent electrified and the central stations have only just recently realized the desirability of the oil field load. The drilling load in itself is not so desirable but wherever a well is drilled electrically, it is pumped by the same power and the pumping load with practically a 100 per cent load factors represents the ideal load for any central station.

Discussion

W. G. Taylor: Both cable-tool and rotary drilling impose extremely heavy duty on the power equipment, as is indicated to some extent by the graphic charts submitted in Mr. Murphy's paper. It would be even more interesting if he had shown some records of the work required to handle and set the casing which lines the well. In fact, it is this work which really determines the size of motor necessary for cable-tool drilling. The work is fully as heavy as handling drill pipe in rotary drilling.

The exacting requirements of oil-well drilling by either method have resulted in the development of the special equipments and schemes of control which Mr. Murphy has described. In the case of cable-tool drilling, the relative success of the twin-motor and single-motor schemes has been determined by the practical rather than by the economic aspects, and some of these may well be mentioned here.

Experience has shown that the twin-motor equipment requires more than ordinary attention from the driller to avoid serious overloading of one motor, and that the average driller in the fields cannot be depended upon to handle this type of equipment without burning out a motor now and then. Belt maintenance has proved excessive, and it has been found difficult to keep the motors and countershaft in alinement. On the other hand, the single-motor equipment has been relatively free from troubles. These things explain why oil companies, especially those which have tried out both kinds of equipments on a large scale under similar field conditions and have thus obtained a fair comparison, have a decided preference for the single-motor cable-tool equipment.

It is of interest that a 75-hp. single-motor cable-tool rig very successfully drilled a well in Colorado to a depth of 7300 ft., with every indication that the motor could continue the drilling work indefinitely to greater depths without the least distress.

Automatic feed of the bit in rotary drilling as exemplified by the Hild differential drive is a recent and very interesting development. Two devices of this kind are on the market, both operating on the fundamental principle of the differential gear. One of these is the Hild drive described by Mr. Murphy, and the other is the Halliburton drive. Both accomplish similar results, but the latter is driven by a single motor, or it may be driven by an engine of almost any type. The motor or engine drives the ring gear of the differential, as an engine drives the differential on an automobile. The two shafts, corresponding to the rear-wheel axles of the automobile, drive the drilling bit and the draw-works hoisting drum through suitable chain or gear reductions. Thus the feed of the bit is balanced against the torque of the load and varies inversely as the load, and the bit is automatically retrieved when the load exceeds a certain amount. A number of the Halliburton drives are in successful operation, with both motor and steam-engine drive, and the future will very probably see many more of both types installed, but it seems likely that their application will be limited to deep drilling as long as their present high prices prevail.

B. T. McCormick: Fig. 4 in Mr. Murphy's paper shows that for a well of about 2000 ft. in depth 12,000 kw-hr. is required, which means about an average of 6 kw-hr. per ft.

Some of our recent experience in Pennsylvania field wells of about 2000-ft. depth drilled with gear units and also the old fashioned jack-shaft drive by Star Delta drilling motors indicates that we get an average of about 3 kw-hr. per ft. I do not know whether that difference is due to the difference in formation or not. I should imagine that difference might account for the very much lower kilowatt-hour draw in the Pennsylvania field.

I would like to ask just why the depth of the well influences the power requirements in the way it seems to. There seems to be a marked reduction in horse-power required as the well becomes deeper. That point has not been made entirely clear.

L. J. Murphy: In answer to the latter question, it might be stated that this reduction is due to several different causes—first, the decrease in diameter of the hole as the depth increases, second, a decrease in the size of drill stem usod and, third, the slower motion.

Mr. McCormick mentioned that in the Pennsylvania fields they have records which show that drilling could be accomplished to 2000 ft. with a power consumption as low as 3 kw-hr. per ft. In this connection I might state that I have seen installations in the Bradford territory where drilling was accomplished to a similar depth with a 30-hp. motor with a power consumption of 2.4 kw-hr. per ft. However, this equipment is such that high hoisting speeds are not possible, and I believe the same holds true of the equipment which Mr. McCormick mentions. In other words, the band-wheel speeds are not in excess of 50 rev. per min., and, hence, little power is used in the secondary resistance. This accounts for the low power consumption, but, in obtaining low power costs, the motor, during drilling, is operating at more nearly synchronous speed with the result that it has a rather stiff speed-torque characteristic not conducive to satisfactory drilling motion except with a manilla drilling line. The Bradford field is one of the few territories using this type of cable.

Application of Electricity in Cement Mills

BY W. E. NORTH[1]

Associate, A. I. E. E.

***Synopsis.**—The advantages of electric drive for cement mills are enumerated in this paper and general pointers on installing electrical equipment are given. The electrical installation recently made in a modern cement plant is described.*

NO single factor has contributed more to the present design and efficient operation of a modern cement mill than the application of electricity as its motive power.

The older cement plants were designed to operate on steam power and since this necessitated the use of long line shafts to accommodate the numerous pulleys required to drive the many small manufacturing units then in use, these plants were practically built around an engine room. For this reason it was not possible to arrange the machinery used for the manufacture of cement in such a way as to insure maximum efficiency, nor could the elevating and conveying systems be installed so as to give the best flow of materials through the mill.

The first application of electric motors in cement mills was the use of d-c. motors to drive auxiliary machinery requiring from 1 to 50 hp. It was, for example, most inconvenient to transmit power from the line shafts to elevator heads and overhead conveyors, and tests showed that from 50 to 90 per cent of the power was lost in transmission, due to speed reductions usually accomplished with long chain and sprocket drives. Electric motors in such places proved an immediate success. They not only cut the transmission losses but it was soon found possible to install an astonishing amount of connected load in motor horsepower, on a generator set of much less rated capacity. This was due to the fact that such drives are usually over-motored due to the high ratio of the maximum to the average power required by the individual motors. In one case known to the writer a total of 375 hp. in rated motor capacity was carried by generators rated at 150 kw. with only occasional interruptions in service due to opening of circuit breakers. This constituted such a radical and valuable change from the old line transmission practise that small generator units driven by special high-speed engines of from 100 to 500 hp. became a feature of every cement plant.

The electrification of the cement plants in the Lehigh Valley was started on a larger scale when the Lehigh Navigation Electric Company built its plant at Hauto and offered attractive power rates to the cement manufacturers, most of whom were operating with steam power plants that were either in poor condition or badly overloaded due to increased production demands. The work of changing over these mills consisted primarily of replacing the old line shaft drives by individual motors, and in most cases the general layout of the cement machinery was not changed to any great extent to get greater advantages of the use of electric motors. One of the plants installed 2200-volt and 220-volt induction motors, two plants used 550-volt induction motors, and one plant installed d-c. motors.

The advantages of the electrification were realized very soon. At the Coplay Cement Manufacturing Company's plant the production was increased from 2600 barrels per day to over 3000 barrels per day without the addition of a single grinding unit and since the meters on the various feeder circuits gave accurate records of power consumption, causes of trouble and faulty operation could be detected easily and the unit cost of manufacture was decreased.

The use of electrical machinery in a modern cement mill is necessary for the following reasons:

1. It makes it possible to design a plant to meet manufacturing conditions without being restricted by conditions imposed when using other forms of power.

2. Increasing cost of labor necessitates the use of labor saving devices that are not practical except when driven by electric motors.

3. Saving in operating efficiency on account of not running idle machinery.

4. Necessity of keeping accurate daily cost data which is greatly aided by proper use of electric meters.

5. Greater flexibility in making repairs and adjustments to various parts of mill without interfering with other operations.

6. General trend toward larger manufacturing units.

One of the most important points to consider in the operation of a cement mill is the continuous operation of the various departments according to a prearranged schedule. The schedule of operation depends mostly upon local conditions, for, although it is necessary to run the kilns without shut-down, it is sometimes advisable to shut down certain departments over the week-ends. The quarrying and packing operations are in many cases discontinued on Sundays except during periods of maximum shipping requirements.

The manufacturing departments of a dry process cement mill may be divided as follows:

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[1] Coplay Cement Mfg. Co., Coplay, Pa.

Presented at the Regional Meeting of District No. 2 of the A. I. E. E., Bethlehem, Pa., April 21-23, 1927.

Quarrying,
Stone Crushing and Drying,
Raw Material Grinding,
Kilns,
Coal Crushing and Grinding,
Clinker Grinding,
Stocking and Packing.

It is seen easily that if a mill is designed with sufficient storage capacities between these departments, any department except the kilns and possibly the coal department may be shut down for repairs or other reasons without interfering with the other operations.

In a cement plant as outlined above, the electrical feeders should be so arranged as to supply one department only. If this rule is followed, repairs and adjustments to electrical apparatus and mill machinery can be made without interfering seriously with the operation of the mill. Such a feeder layout will not always meet with approval from an electrical point of view since the power requirements of the various departments vary within wide limits, which means that the feeder panels and the distribution feeders will not be the same size, but the advantages gained from a manufacturing standpoint offset its disadvantages when considered simply as an electrical installation.

The advantage of flexibility of operation and the readiness with which repairs and adjustments may be made without interference have been explained. Another advantage is derived from a cost accounting standpoint. Compared with other industries, the ratio of the cost of power to the total value of the product manufactured is great, varying from 15 to 20 per cent of the total cost, depending upon cost of power and efficiency of operation. To keep accurate and reliable account of the power costs is therefore of utmost importance and if each manufacturing department is provided with its own feeder panel and necessary metering devices, accurate data as to power cost can be obtained daily. Since the power consumption is an indication of the efficiency of general operating conditions, other troubles are easily located and corrected before serious trouble is caused or costs increased. The safety factor is also improved as any department not in operation can be cut off entirely from the feeder system.

The plans under way for the installation of new 60-cycle motors and for remodeling the mills of the Coplay Cement Manufacturing Company at Coplay, Pa. are based on the above principles; that is, the electrical equipment simply supplies a means to drive the machinery and in no way influences the layout of the mill.

When the improvements in the mill are completed, sufficient storages will be supplied between all departments to allow for flexible and economical operation and the electrical system has been installed so as to meet all of the requirements of operation of the mill.

The power for the electrical machinery is purchased from the Pennsylvania Power and Light Company, 60 cycles at 66,000 volts. Due to dusty conditions, and to insure as far as possible freedom from interruptions in power service, all of the 66,000-volt equipment was installed indoors, and the installation was designed to combine the greatest possible protection affording safe and continuous operation with the greatest simplicity of arrangement of equipment.

The transformer house is built of concrete with a cement tile roof supported on steel trusses. The concrete work was erected by the use of sliding forms. The pitched roof (six in. per ft.) with ventilators was used to secure the maximum amount of ventilation for the self-cooled transformers without the use of fans. A tunnel extends through the building under the switchboard and contains cable racks, steam, water, and compressed air service pipes and a concrete tank of sufficient capacity to hold the transformer oil in case it is necessary to empty the transformer tanks for any reason.

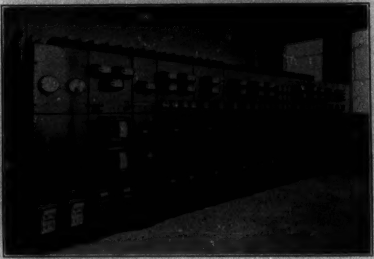

Fig. 1—Distribution Switchboard

The building was erected and all of the equipment installed by the construction forces of the company.

The transformer house and arrangement of the electrical machinery is shown in Figs. 1, 2, and 3, and the schematic wiring diagram of the whole system in Fig. 4.

The incoming feeders enter the building through 110,000-volt wall entrance bushings and the equipment is protected by oxide film lightning arresters. The main line oil circuit breakers have manually operated closing mechanism and are equipped with bushing type current transformers and have d-c. trip coils operated by induction type overload and reverse power relays.

The rupturing capacity of these switches is sufficient to interrupt the current due to a short circuit on any part of the system. The transformers are self-cooled, three-phase, 66,000-2200 volts, 5000-kv-a. capacity with four 2½-per cent taps below 66,000 and are equipped with conservator tanks, thermometers, and temperature indicators connected to coils in the windings. They are mounted on trucks and provision is made for a hoist beam for repairs.

The disconnecting switches between the oil circuit breakers and the bus are three-pole, gang-operated and are mechanically interlocked so that it is impossible to operate them when the oil breaker is closed. The bus tie switch and the transformer bank switches are three-pole, gang-operated air break switches and are used for breaking the parallel operation and the magnetizing current of the transformers.

It will be noted, Figs. 2-4, that when operating on one transformer bank, one side of the station can be entirely disconnected, making safe repairs and adjustments possible.

The operating switchboard is in a separate room, and consists of two transformer panels, two totalizing meter panels, feeder panels, and a bus tie panel.

Fig. 2—5000-Kv-a., 66,000/2200-Volt, Three-Phase Transformer with Air Brake Switch and Main Line Oil Switch

The bus-tie switch is operated by an instantaneous overload relay and is used to sectionalize the bus in case of a dead short circuit on one of the feeders to reduce the rupturing capacity required by the feeder circuit breakers which have d-c. trip coils operated by inverse time limit relays. This combination operated successfully on two occasions when short circuits occurred on feeder cables during the construction period.

Fig. 3—Transformer House

The totalizing panel is equipped with watthour meters, printometers, ammeter, voltmeter, curve drawing wattmeter, power-factor indicator, and wattless component indicator to be used with watthour meter in computing the average power factor.

The feeder panels are equipped with oil circuit breakers, disconnecting switches, ammeter, wattmeter and watthour meters, and with a complete set of testing studs on the front of the board for testing meters and relays.

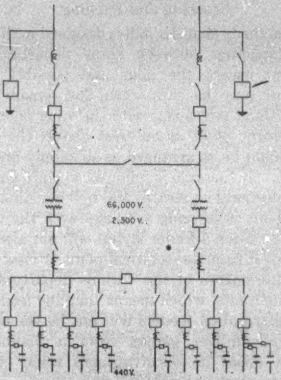

Fig. 4—Schematic Wiring Diagram of System

All of the feeder circuits to the various departments are of armored lead-covered varnish-cambric-insulated cable. These cables are run underground to the various departments and when located out of doors are buried about three ft. underground and spaced several inches apart. Boards are placed about six in. above the cables as a protection against injury by workmen making excavations.

Fig. 5—600-Hp., Three-Phase, 2200-Volt Supersynchronous Motor Driving Tube Mill

Where the cables are located in buildings having concrete floors, the ditches in which the cables are laid are filled with earth and covered with a 2-in. concrete slab marked to show location of cables and to allow the concrete to be broken out easily, if necessary.

The starting equipment for the various motors connected to a feeder circuit is in most cases arranged in a group at the termination of the feeder. Since 440-volt motors are used for all sizes under 50 horsepower, small distribution transformers are connected to each feeder to take care of these motors.

The 2300-volt induction motors with few exceptions are controlled by manually operated starting compensators and the 440-volt induction motors are controlled by magnetic starters except in case of variable speed motors that have drum controllers and resistors. These starting switches are mounted on panels containing disconnecting switches, fuse blocks, and testing jacks all mounted in a single sheet steel box with safety catches.

The motors for the main grinding units are 600- and 200-hp. super-synchronous motors controlled by automatic panels; see Fig. 5. Where necessary, all motors driving different units of an elevating and conveying system serving a main grinding machine are interlocked to prevent choking of materials in case of stoppage of one of the units. All automatic starters are equipped with but one starting station but many have several stop stations.

With the exception of gasoline locomotives operating in the quarries and on the railroad and two gasoline-engine-operated well drills used for prospecting at points distant from our feeder circuits, all power applications in the mill are motor driven.

The distribution of the motor load may be classified as follows:

Pumps	260 hp.
Air Compressors	170 hp.
Blowers	315 hp.
Well Drills	50 hp.
Electric Shovels (M-G Set)	240 hp.
Quarry Hoists	125 hp.
Bridge Crane (M-G Sets)	200 hp.
Elevators and Conveyors	1055 hp.
Crushers	615 hp.
Dryers	110 hp.
Kilns	240 hp.
Grinding Machinery	4950 hp.
Packing Machinery	140 hp.
Machine Tools and Miscellaneous Applications	288 hp.
Total	8758 hp.

The types of motors used are as follows:

D-c. Motors, 220-volt (Electric Shovels, Cranes, Etc.)	374 hp.
Squirrel-Cage Induction Motors, 440-volt	1886 hp.
Variable-Speed Induction Motors, 440-volt	280 hp.
Squirrel-Cage Induction Motors, 2300-volt	3100 hp.
Hoist Duty Induction Motors, 2300-volt	125 hp.
Synchronous Motors, 2300-volt	3340 hp.
Total	9105 hp.

With the exception of comparatively few motors with characteristics suitable for the operation of electric shovels, cranes, and other labor saving machinery, most of the motors used in the cement industry are of standard design and construction and since the general practise in the installation of this equipment is to make the starting operation as automatic as practicable, specialized mechanics are not required for the operation of the motors and few skilled men are required for their maintenance.

An installation such as described above would be expected to operate 24 hr. a day for 360 days per year, and would have a yearly load factor (ratio of average demand to maximum demand) of 80 per cent and a monthly load factor of 88 per cent with an average power factor of 90 per cent, making it a desirable load from a power generating standpoint.

The tendency in cement mill work is toward larger grinding units and the most efficient electrical apparatus obtainable, standardization in sizes and speeds of the general purpose motors, and distribution of the motor load into circuit so as to best meet the manufacturing requirements of the mill so as to be an aid to more efficient operation.

The recent development of synchronous motors of high starting torque, with either mechanical or magnetic clutches, has caused the installation of more direct-connected units, eliminating many expensive belts and pulleys and resulting in great saving in space and in efficiency and safety of operation.

In many instances, ball and roller bearings have been used in extremely dusty places with good results, but a modern cement mill can be made to be so free from dust that when proper attention is paid to the condition of the equipment, motors with standard babbited bearings can be operated with as little trouble as when installed on similar machinery in other industries.

The use of belts and chains on countershafts for speed reduction necessary to drive elevators and conveyors has been almost universally replaced by the use of gear reduction units direct connected to motors through flexible couplings mounted on common bases. Reducers of these types can be built for speed reductions ranging from 4 to 1 up to 8000 to 1 when power does not exceed 500 hp. and therefore cover the entire range of cement making machinery except in case of the larger grinding units. Rock crushers and heavy machinery subject to severe shock are still usually belt driven to provide flexibility and reduce the strains on the motor bearings and coils.

In addition to the above described motor applications, electricity is used in cement mills for magnetic separators, rivet heaters, arc and spot welders, pyrometers, and various other application of electricity, all of which have become most satisfactory features in operating cement plants, and as is the case in other industries, electric power has become one of the greatest factors in production, and from raw material to the finished product the responsibility of uninterrupted manufacture rests primarily upon the electric motor.

Discussion

E. B. Wagner: Mr. North mentions that the main substation was enclosed in a brick building in order to protect the equipment from dust. I should like to ask how he was able to keep the dust out of the building.

He told us that he had provided the super-synchronous motor with an automatic starter. I should like to know if this is a full automatic starter; that is, does it take care of automatically applying the brake on the revolving stator so as to slow that down and bring the rotor up to speed?

W. E. North: The high-tension transformers and switching equipment were housed in a building because the combination of cement dust and water causes a lot of trouble on 110,000-volt insulators that were used for the 66,000-volt service, and although it might have been satisfactory to build an entirely outdoor station, with a little less money, we didn't believe that it was wise to take that chance since the main thing we wanted was absolutely continuous operation.

A small amount of dust gets into the building but the dry dust doesn't have nearly the same disastrous effect on insulators as cement dust coked up with water.

The automatic starters we have on the synchronous motors do not automatically put on the brakes. That has been a later development. The motors are not thrown directly across the line but are started with an autotransformer with reduced-voltage taps. I don't know on which taps they were working at present but about 40 per cent increase over the normal operating current is required for starting and the brake is put on by hand after the starter automatically throws on the field. The men operating these motors have become fairly skillful in the operation of the brake, and we can't see any rise in the power curve when the mill is brought up to speed. We allow from 15 to 20 seconds for the acceleration of the motors and it is entirely possible to operate the brake by hand to get these results.

F. C. Caldwell: I want to ask if high-tension electric precipitation devices for abstracting the dust from the air are in use at all in cement mills; also, whether the general tendency to extend the use of roller and ball bearings that is going on these days is likely to result in their general adoption in the case of the cement-mill motors. Mr. Findlay of the Giant Portland Cement Company told me that his company has used nothing but ball or roller bearings for the last five years and they are thoroughly "sold" on the use of that type of bearing for cement-mill work; that they have had no necessity for the replacement of the roller or ball bearings, and that they would use no other kind.

W. E. North: From 1917 to 1919 we had a precipitation apparatus at our plant. It will collect the dust. The principal reason that we abandoned it is that in our particular case the gain we made was far offset by the cost of operating it. In some cases, in a new plant possibly, where the dust collector is built with the plant and designed to operate with that particular plant, you would probably have much better operating conditions. The mill in which we installed an electric precipitation dust collector was an old one and we had to put this dust collector where we had available space and it was not as efficient as it would have been if we had sufficient space to install a system of greater capacity.

In regard to the bearings, the best way, I believe, to protect motor bearings, or any other kind of bearings or any other machinery around a cement mill is not first to try to make the bearings dust-tight but to provide dust-collecting apparatus to remove the cause of the trouble. However, if that cannot be done without enormous expense, the next step is to protect the bearings. With regard to that, however, our mill is not a dustless mill and we have just installed some new 60-cycle motors there. The original installation was made in 1913 and 1914, and it was taken out the first of January this year, which is 13 years of operation.

As to the life of the bearings in these motors we would sometimes lose a bearing in 2 weeks at first but we soon remedied that. We have, for instance, one 75-hp. motor that ran 12 years without a change of bearings. Our tube mills were driven by 150-hp. squirrel-cage motors mounted in rather dusty places on the second floor because of lack of space on the first floor, and they were running with excessive belt tension. The bearings on these motors averaged 2 years. The average life of the smaller motor bearings; that is 25-, 15-, and 10-hp. motors driving elevators and conveyors which are usually mounted in fairly dusty places, was 1½ years. As to the vertical motors with ball and roller bearings driving Fuller mills in a coal mill, one of those never had the bearings changed. Our vertical motors with ball bearings averaged 4 years.

As to the effect of dust on the coils, we have 80 motors whose coils were never repaired, 70 motors were patched, and 10 of them were completely rewound in 13 years' service.

As to the ball and roller bearing applications on motors, I have personally no data on their life in the cement industry.

In a properly laid out plant, as a rule, the time required to replace babbitted bearings does not often hold up production very much because such repairs can be foreseen and may be made during regular repair periods.

D. M. Petty: There seems to be doubt in the minds of some people, particularly those who have not been in the steel industry to any great extent, as to why some of us in the steel industry are particularly interested in ball bearings. We know from experience in the steel industry that the babbitted bearings will be cheaper than ball bearings, so far as the cost of bearings is concerned. The reason we put ball bearings in our motors, however, is to eliminate the loss of windings due to the lubricating oil getting out of the bearings and into the windings. We found by keeping a very careful record in our own plant and by questionnaires sent out among all of the other larger steel plants where proper records were kept, that at least 75 per cent of all motor failures were due to the oil getting out of the bearings into the windings, and that is the big reason why I, personally, and the committee on which I served came to the conclusion that ball and roller bearings would be a good proposition in the steel industry. In the cement industry, motor windings are not subjected to all the hazards that we have in the steel industry. Cement itself is a fair insulator. If we could only get our motors filled with cement instead of smoke and gasses and carbons and other such things, I don't believe we would have to worry about ball bearings in steel mills. I don't see why ball bearings wouldn't work out in the cement industry, but as in the steel industry more for the sake of saving windings than for the bearings themselves.

G. M. Kennedy: I understand Mr. North to say that the 2300-volt squirrel-cage motors are manually operated and the 440-volt squirrel-cage motors are automatically controlled. I should like to ask why that is the case.

W. E. North: The 440-volt motors are thrown directly across the line simply by means of an automatic magnetic switch. The reason we have manual operation on the 2300-volt squirrel-cage motors is that quite a few of these motors were put on machines which would probably be replaced soon after we finished the reconstruction of the mill and we did not spend any more money than necessary on them.

A. J. Standing: I should like to ask Mr. North, if in the layout of a new mill today he would use 440 volts or would it be possible to go to 2200? What I am getting at is the safety of the men around the plant.

W. E. North: I believe I would put in 440 volts. That, of course, is a matter of personal opinion. I believe that 440-volt equipment with the proper apparatus can be made safe enough not to interfere with safe operation. For 14 years we operated at Coplay with 550 volts and we did not have any serious accidents to our operating force due to the voltage. We have had a few slight accidents due to flashover on opening switches. On the 440-volt apparatus that we have now, the starting equipment is totally enclosed; there is nothing exposed whatever on the whole layout. I think I would use 440 volts in preference to 220 volts for small motors on account of the feeder conditions, and 2200 volts for larger motors.

John Grotzinger: I am particularly interested in the application of the super-synchronous motor to tube-mill drives as the application of synchronous motors to heavy mill drives in the rubber industry, with which I am connected, has been a feature of the past two years.

Can Mr. North give us the torque characteristics of his super-synchronous motor, particularly the pull-out torque?

Is excitation furnished from a belted exciter or an external d-c. system and what method is used to apply the field at the proper time during the starting operation? Is a frequency relay used for this purpose or current lockout in combination with a definite time relay?

W. E. North: These 600-hp. super-synchronous motors I was speaking about drive tube mills which require a running load of about 425 to 450 kw. We have never made any tests on the starting torque or the pull-out torque on those mills. Since the load is applied by means of a brake the principal starting consideration seems to be the period of acceleration. After the motor is up to speed we begin to apply a manually operated brake and it is possible to hold the current constant at full-load running value, if sufficient care is exercised in tightening the brake. We never have had any trouble with starting and therefore have never made any tests on the starting torque of those motors.

The excitation is supplied for three of these motors by a motor generator or exciter set. I cannot give the exact details of the relays used but to the best of my recollection, current-lockout in combination with time-limit relays are used.

F. E. Fairman: In connection with the operation of these super-synchronous motors and also the other 2300-volt switching equipment, I should like to know just what experience was had with the operation of auxiliary devices and auxiliary contacts for the field contacts on the synchronous motors, first, with dust in the substations, and just what steps were found necessary to be taken in maintenance, cleaning, and the like.

H. H. Leh: We have noticed that when surges occur on the line feeding our plant the undervoltage release on our super-synchronous motor opens quite frequently, throwing the motor off the line, whereas the induction motor in the plant keeps in continuous operation without the undervoltage release opening. I should like to know, if possible, whether this is an inherent characteristic of the motor or if it is only due to the design of the undervoltage releases.

Aubrey Smith: Relays are available which can be used to prevent the disconnection of a synchronous motor from its source of power by voltage dips which are not too great in magnitude nor too long in duration. Ordinarily the lower the voltage dip, the shorter the time during which synchronous operation of the motor may be maintained and vice versa. The degree of continuity of service which can be secured in any given case by use of such relay equipment depends more upon the characteristics of the synchronous apparatus than anything else. General-purpose synchronous motors, for instance, are available which will carry their load operating as induction motors for several minutes, without over-heating, at a slightly reduced speed. On the return of voltage to its normal value, these machines can be made to come up again to synchronous speed and operate as synchronous motors. Of course, in such cases control equipment must be provided which will attend to the removal and re-application of the field when the proper rotating speed of the machine is reached.

Installations making use of the control principles and equipments mentioned have been made in various places and in various industries. Such relay equipment is effective in increasing the continuity of service, especially if line disturbances are not too violent in character. If it is known at the time of planning the new installation that the source of power will be subject to momentary voltage dips, plans should be made at the outset to equip the apparatus with relays designed to sustain synchronous operation as long as possible.

W. H. Lesser: I can give some experience about that, too. We have in operation at one of our collieries a new system of preparing coal known as the sand-flotation process. Each time there is a surge in the voltage or an interruption in the power, this apparatus stops and it takes about half an hour to agitate the sand again. We have sixteen motors there and the control boards are all in one room, and every time we have a surge on the line the whole plant shuts down. Mr. Lloyd looked over the proposition and we ordered a master control panel for this installation, with a time relay on it. We set the relay for 4 sec. which holds the motors on the line during the surges.

There is another point I should like to ask Mr. North about. Do you have any trouble in the operation of the super-synchronous motor; that is, with flashovers and things similar to that?

W. E. North: No, these motors have been running continuously since they have been put in. The only trouble we have had, has been on account of misalinement of a flexible coupling. That was not the fault of the motor and it was remedied within a few hours.

W. E. Lloyd: A few years ago when the interconnection system in this territory was being built and surges were comparatively new to the customers now on this system, the tripping off of apparatus, either synchronous or induction, due to the low-voltage release was very common. We advocated either a time delay on the low-voltage trip or removing the low-voltage trip entirely, that being dependent upon the particular operation. We went through the same conditions in our power stations; fans and pumps, and other auxiliary apparatus would trip off during these surges, which we could not afford because it meant an extended interruption of many minutes rather than one minute. So we have taken off the low-voltage releases in our power stations; many of our customers have taken them off.

I feel that the super-synchronous motor is a new piece of equipment. We, as well as the customers, are feeling our way, but actually I think the time will come when we shall treat the super-synchronous motor from a low-voltage-release standpoint just as we now treat the induction motor; that is, put on an adequate time delay to hold that motor on the line during surges, allowing it to trip clear if the voltage goes entirely off for, say, half a minute or a minute, or else take it off entirely in case the power fails and rely upon the operators to clear the switch and have it cleared before the power comes back on.

In connection with the dust problem Mr. North told how he housed his equipment in a building. We can't very well house our incoming transmission line in a building and we have to fight a problem of dust on the insulators. Of course, we might run the incoming line for the last half mile in a 60 kv-a. cable, but it has not been done to date. It was our practise several years ago to wipe off the insulators every week on the last few towers adjacent to each cement mill. This worked only fairly well. We had no definite plan for changing these insulators and after a protracted dry spell followed by a light rain or a fog, we invariably had a number of insulator failures caused by leakage over the surface of the insulator due to cement dust and collection of the moisture. If we were fortunate enough to get a real heavy rain following a dry period, that heavy rain usually washed off the insulator dust sufficiently so that no trouble was encountered. Our recent experience has been to change these insulators on the last three or four towers of the line going into the cement mill every six months. The insulators further back on that line will be changed probably in periods from two to five years, depending upon just how much precipitation of dust there is on the insulator. These insulators were taken down every six months, scraped, and then finally scrubbed with a weak solution of hydrochloric acid and water, after which they were tested and if good were put into service again. It was a rather expensive proposition to scrub and clean those insulators. In the last two or three years we have been taking hot paraffin and painting the insulators with a brush. It does not affect the insulating quality at all, and when they are taken down we simply dip them into a bucket of boiling water and it melts off the paraffin and along with it comes the cement dust, and the insulator is ready to go back into service again.

Mr. North speaks about the 66,000-volt transformers which he has installed at his mill. I want to mention the reliability of that transformer. We have at least 250,000 kv-a. of this type transformer in service on the system, either our own transformer or customers', and to date we have not experienced a single failure. When I say that, I want to qualify it in this way: We have had bushings fail, but that is an attachment which can be replaced promptly and stock units are always available. We have had winding failures, where a short circuit occurred on the low-voltage side and the switching equipment did not clear the trouble, but we don't consider that an inherent defect in the transformer. So that I believe Mr. North and every one using this particular transformer at this particular voltage is going to get excellent service from the unit.

Mr. North, in speaking of his load factor of 88 per cent, reminded me of a cement mill which is using rotary converters of 4000-kv-a. capacity. They run a load factor for the month as high as 96.8 per cent at unity power factor. It was very interesting to see the operators in that substation regulate their load. They would go to the lighting switchboard and open and close a knife-switch and by a series of flashes they would signal to a motor tender out in the mill and he would merely drop off one motor or two motors, and they would thus maintain a constant load of 4000 kv-a. hour after hour.

O. S. Clark: I shonder if Mr. North has found it to be an economical proposition to install lead distribution feeder cables underground rather than in conduit. Of course, it is cheaper to install a cable underground without the conduit, but it is also much more expensive to make repairs, and there must be a balance some place, depending upon the extent of the distribution system. I should like to hear from Mr. North on that.

W. E. North: Before putting in the armored lead cables, we got complete prices and cost data on what the cost would be for the installation of lead cables in conduit. Ours is an old mill and if we put in conduit we would have a great number of bends and in many places we would not have room to put in the conduit.

To reduce repairs we have been liberal in our selection of cables. We use cables with 5000-volt insulation for 2300-volt service and the capacities are ample. On lead cables buried in the ground we use 70 per cent of the underwriters' rating in calculating the capacity of the cable, because we believe that many failures in varnished-cambric-covered cable come from a gradual baking out of the varnish which settles to the lower part of the cable, and if the cable is never heated failures are not so frequent. I don't know what the economy would be in a plant where you had plenty of room to put in the cable ducts but in our particular case it would have been rather expensive to attempt to put in any kind of a duct system on account of the numerous manholes we would have to put in. We would have had 6 bends in one run of 400 ft.

Regarding lead cables and open or closed ditches, our experience has been just the reverse of the one mentioned, about overheating of the cables. Of course, we were fortunate enough not to have to run the main feeder lead cables near the kilns. All of the cables in the ditches are separated about 6 in. apart and clay or sand is put in to keep a cable, if it flashes over, from passing the trouble to the other ones. During our construction work some workmen doing concrete work near the cables built a fire to thaw a frozen water pipe, not knowing the cables were near. The fire was about 2 in. from a cable. In very short time the cable burned through and although there was another cable 6 in. away it was not damaged. This, I believe, justifies the use of sand between the cables.

J. T. Waugh: In discussing Mr. North's paper, I might say that the electrical distribution systems in a wet-process mill and in a dry-process, are identical.

One type of construction, for a low-tension transformer station, that has successfully coped with the dust problem is based on the elimination of exposed leakage surfaces.

Where radiant heat is exceptionally high, the use of ventilated cable trenches with the omission of sand has been found satisfactory.

Both air-break and oil-immersed starters have been used and the results are still open to controversy. In a coal mill, however, it is important to eliminate all air contacts due to the ever prevalent inflammable pulverized coal dust and its potential explosive properties, if the proper amount of oxygen is supplied by a sudden gust of wind, and proper care has not been exercised in keeping the place clean.

In further answer to the question of the gentlemen who asks how to take care of dust in a high-tension transformer station already built, I believe the permanent sealing of all windows and the use of louvers covered with muslin will greatly tend to minimize the dust menace. In such a transformer station in the heart of a cement mill, where the above ventilation is used, shut downs for the purpose of cleaning insulators have been practically eliminated.

F. A. Scheffler: I should like to disabuse the minds of some in regard to the statement that there is a possibility of frequent explosions in cement plants. That is altogether up to the cement operating force and is a question of house cleaning. If the cement companies keep their coal mills clean and do not abuse the pulverizing equipment, they will have no trouble whatever from explosions.

Recent Developments in Electric Drives for Rolling Mills

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***Synopsis.**—The application of electric power in the steel industry introduced many radical changes and improvements in rolling-mill layout and practise. The electric drives, of capacities larger than encountered elsewhere, are usually designed to fit individual cases. Special machines or special combinations of them are frequently used.*

Several representative cases are outlined, and some methods of solving the encountered problems are analyzed.

INTRODUCTION

THE iron and steel industry is the largest single consumer of electric power. In 1924 this industry used more than 6,000,000,000 kw-hr., which is about 20 per cent of the total power consumed by all industries in the United States. It is of interest to note that the combined output of all central stations in the country equalled 54,413,403,000 kw-hr. during the same year.

A modern steel plant, starting with an iron ore as a raw product, produces at its blast and open hearth furnaces and at the coke ovens a large amount of waste gas or heat. Electricity gives means of conveniently converting and transmitting this potential power to the centers of its consumption. This explains the rapid growth of power generating plants in the steel mills; one steel plant has an installed capacity of over 100,000 kw.; a number of plants have a demand in excess of 50,000 kw. In 1926 alone the steel industry purchased for its use a 30,000-kw. turbo generator and three others each rated at 20,000 kw., not counting many other units of 15,000-kw. capacity and less.

So great is the demand for power in the steel industry that even plants having their own blast furnaces often purchase additional power from public utilities. Many other plants, deprived of the use of blast furnace gas, run almost exclusively on purchased power. The latter amounted in 1924 to 39 per cent of the total power consumed.

The bulk of this vast amount of energy goes for the work of shaping the steel; the rolling mill drives are the principal outlets of the generated power. Here the electric drive predominates. Hardly any new mills are being equipped with anything but electric motors; older steam driven mills are being gradually electrified, for purely economic reasons.

Many electrical engineers, not connected directly with the steel industry, may not fully realize the profound, almost revolutionary changes which the electric drive brought about in the rolling mills. It is not merely the question of performing the operations in a better, more efficient, or more reliable manner than otherwise possible; but the point, which is sometimes lost sight of, is that many operations and processes, now in wide use, are practically impossible without the agency of electric power. Rolling mill designers have taken advantage of the possibilities of electric drives and have built mills on radically new principles, exceptionally advantageous for steel plants, but not practical, were it not for the presence of electrical motors. On the other hand, the electrical engineers have developed new machines, or new combinations of machines, primarily, if not exclusively, for rolling mill application. Thus the new rolling mill has become closely tied to its drive and is unthinkable without it; the influence between the electrical and mechanical equipments is now not only great—it is also mutual. Many new problems were brought up and were solved more or less successfully.

There will be outlined in this paper, in a necessarily short space, those solutions offered by electrical engineers for a few of these problems. A brief sketch of the types of new rolling mills will give the necessary background.

CONTINUOUS ROLLING AND CONTINUOUS MILLS

It has been generally recognized that for a large tonnage output a continuous rolling mill possesses decided advantages. Such a mill, see Fig. 1, consists of a number of two-high stands, arranged in tandem and conventionally driven through a line shaft and gears by a single motor or engine. The hot bloom or bar passes in succession through all stands, as indicated by the arrow. Each pair of rolls reduces the cross-section of the bar until the latter leaves the last stand as a finished product of the desired shape. The layout is compact; little heat is lost between stands; the metal is rolled at a high temperature and with a relatively low power consumption; the steel requires little, if any, handling; the labor costs per ton are reduced to a minimum.

The bulk of the country's steel output passes through a continuous mill of one kind or another.

To maintain the high tonnages and to keep the cost of handling down, the rolled bars are usually of considerable length; a finished length of several hundred feet is quite common. In order to save floor space the stands are located close to each other. This means that

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the metal is in several stands at the same time. It is obvious that with such an arrangement the speed of each consecutive pair of rolls is increased in proportion to the reduction of the cross-section area. For a given mill the speed relation between stands is fixed and is determined by ratio of the several gears; hence the reductions per pass, or the so-called drafts, are also more or less fixed. Thus, a continuous mill of the outlined type, capable of producing large tonnages of a certain class of sections, is not quite flexible when it comes to rolling of a diversified line of products.

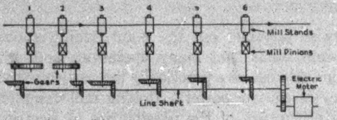

FIG. 1—ELEMENTARY DIAGRAM OF A CONTINUOUS MILL WITH A SINGLE DRIVE

Individual drives for several stands of a continuous mill give it the necessary flexibility, at the same time maintaining its inherent advantages.

For instance, the mill, Fig. 2, has its first three roughing stands driven by one motor, the next two stands by another motor, and the last three, or finishing, stands are each provided with a separate drive. If all motors, or several of them, are of the adjustable speed type, then the speed ratio between the stands may be readily changed. A wide variety of products may be then successfully rolled, each at its proper speed, each with the most suitable reductions at the several stands.

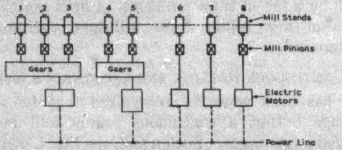

FIG. 2—ELEMENTARY DIAGRAM OF A MULTI-DRIVE CONTINUOUS MILL

Mills, designed and built on this principle, are springing up all over the country. Hot strip, rods, merchant, and certain structural shapes are being rolled on such mills. They are believed to be economical, flexible and tonnage producing. In many cases one mill of this type takes the place of two or three less modern mills.

Such layouts would be hardly feasible were it not for the application of the electric motor. We are usually accepting it as a matter of fact, and are apt to forget that there is no other device which can concentrate a large bulk of power in a limited space, which is efficient even in small units and which is capable of speed adjustment, yet will closely maintain its speed, once it has been adjusted.

It is outside the scope of the present paper and outside the competence of the writer to offer a thorough analysis of mill layouts from the standpoint of rolling mill operations. It was not intended to convey the idea that, for instance, a continuous mill with individual drives is the best combination or layout for all applications; such a mill was merely discussed in order to illustrate the profound influence of electricity on rolling mill engineering and practise.

TYPES OF ELECTRIC DRIVES

It will be shown presently how the electrical engineers are providing suitable drives for mills of the kind just described. While no radically new machine was invented nor introduced, some new combinations of machines were conceived and were successfully applied.

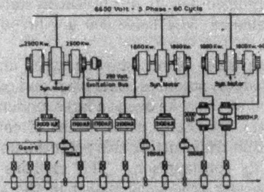

FIG. 3—LAYOUT OF A LARGE CONTINUOUS MILL DRIVEN BY D-C. MOTORS. THE GENERAL VIEW OF THE MOTOR ROOM IS SHOWN ON FIG. 4

D-c. Drives. When a mill requires a number of adjustable speed drives it is the simplest and, in many cases, the best way to make each drive a d-c. motor and to furnish power to them from motor-generator sets or from synchronous converters.

FIG. 4—GENERAL VIEW OF THE MOTOR ROOM AT THE 14-IN. MERCHANT MILL OF THE JONES AND LAUGHLIN STEEL CORPORATION, WOODLAWN, PA.

The elementary diagram of connection is shown on Fig. 3

Figs. 3 and 4 give the schematic layout and the general view of the motor room of one of the most modern mills of this type.

A 3000-hp., 200/360-rev. per min. motor drives the roughing train of three stands; two 1700-hp., 90/204-rev. per min. motors and two 2100-hp., 150/-460-rev. per min. motors are individually driving the next four stands; the two finishing stands are each driven by double-unit, 2000 hp. motors, consisting of two 1000-h. p. armatures which can be connected either in series or in multiple, and operating up to 800 rev. per min. Three smaller edging roll stands are also electrically driven.

All motors are 600-volt, d-c. machines and the power to them is furnished from three large synchronous motor-generator sets, aggregating 12,200 kw. (40 deg. cent. continuous capacity). Practically each motor has a corresponding generator, as is shown on the diagram. Ward-Leonard control is used for starting, and the combination of generator voltage and motor field control gives a very wide speed range (as wide as 4:1 and 5:1) to each drive.

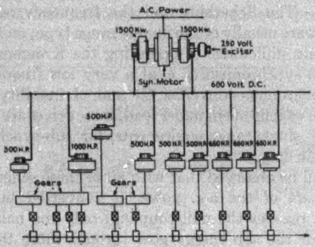

Fig. 5—Typical Arrangement of a Moderate Sized Continuous Mill Driven by D-c. Motors

Another interesting example of a modern mill with stands individually driven by d-c. motors is represented by the layout in Fig. 5. The capacity of each drive is indicated on the diagram. The power to the motors is supplied from a 3000-kw., 600-volt, three-unit motor-generator set. Ward-Leonard control is used for starting, and motor field control for speed adjustment.

D-c. Versus A-c. Drives. When a mill requires a number of adjustable speed drives, especially of the average or of less than the average capacity, then it is usually more economical to make them of the d-c. type, as just described. When a speed range larger than 2:1 is necessary, the use of direct current becomes almost imperative. The speed regulating control is quite simple, usually consisting of one or several field rheostats. The use of direct current may also reduce the cost of the high voltage switching equipment.

On the other hand, the necessity of converting the full amount of electrical power three times from the available a-c. line to the mill coupling, greatly reduces the over-all efficiency of the drive and increases the running light losses. Assuming an efficiency of a d-c. motor at 92 per cent and that of a motor-generator set at 88 per cent, the over-all efficiency of the drive at full load is only 81 per cent. When the d-c. machines are operating at reduced voltage (*i. e.*, when part of the speed range is covered by Ward-Leonard control) their efficiency goes down quite appreciably. The actual over-all efficiency and the power consumption (in terms of kilowatt-hours used per ton of rolled material) are still further unfavorably affected by the fact that the average mill load is usually much less than the rating of the drives.

Thus, much as a straight d-c. system may seem attractive, in many cases, from the operating standpoint, it would be a fallacy to consider it as a standard for any multi-drive mill.

With alternating current universally adopted in all steel mills for power generation and distribution, the engineers should always analyze whether the available a-c. power could not be more directly used for driving the mills. When large amounts of energy and large tonnages are involved, the possible improvement of 5 or 6 per cent, or more, in over-all efficiency, presents an attractive goal worth striving for. Say, a mill rolls 50,000 tons of steel per month, consuming approximately 40 kw-hr. per ton, or 2,000,000 kw-hr. per month; a saving of 5 per cent at, say, 0.9 cent per kw-hr. will net over $10,000 per year. Such economy alone would justify an additional investment as high as $50,000 if it were required. But, if it is obtainable without any additional outlay, or even with a lower first cost than with a d-c. drive, then the application of a-c. drives becomes vital and their possibilities should be most carefully studied.

A-c. Drives. The art of engineering thus far knows of but one way to build adjustable speed, a-c. drives, of such capacities as are involved in steel mill work. This is to use a slip-ring induction motor and to regulate its speed by acting on its secondary circuit in one or another well known manner. These methods were described in great detail, at various times, before this Institute or before other engineering societies, and the most representative of them are diagrammatically shown on Fig. 6.

Broadly speaking, all these methods have one thing in common. An induction motor, running at a subsynchronous speed, delivers at its shaft, as mechanical energy, only that portion of the power transmitted to the rotor which is proportional to the speed; the balance of this power, proportional to the slip, is available at the slip-rings and is usually called the slip energy; it is of a frequency and voltage proportional to the slip. This energy is either converted into mechanical power and is returned to the main motor shaft, see 6B, 6D, 6F, or is converted into electric power of the line frequency and voltage and is returned to the a-c. system; see Figs. 6A, 6C, 6E. In the first case the drive is of a "constant horse power" type, as approximately the same amount of power (neglecting conversion losses) is available at the motor coupling at all operating speeds; in other words, larger torque is available at the reduced speed. In the second case the drive is of "constant torque" type, *i. e.*, the power available at the coupling varies in proportion to the speed.

Figs. 6A and 6B represent the Scherbius system employing an a-c. polyphase commutator motor *R* to convert the slip energy into mechanical power at *its* shaft; then it is either returned electrically to the line through an induction or synchronous generator *K*, or is utilized directly on the drive shaft. The regulating machine *R* is shunt wound and is excited from the slip-rings of the main motor; by controlling the amount of excitation the speed of the drive may be adjusted suitably.

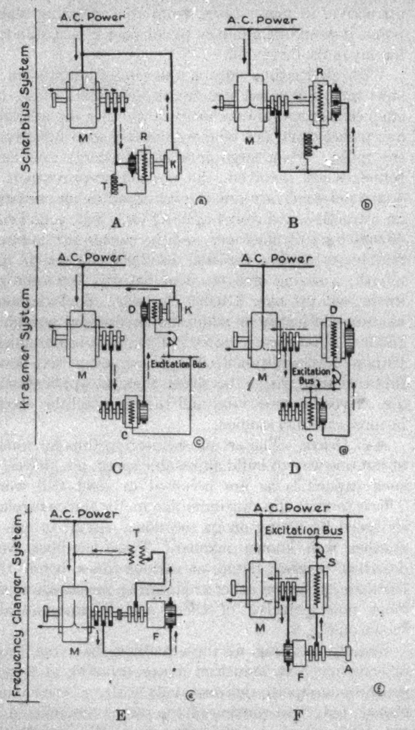

FIG. 6—ELEMENTARY DIAGRAMS OF SEVERAL A-C. ADJUSTABLE SPEED DRIVES

In the Kraemer system the slip energy is converted into d-c. power by means of a synchronous converter *C*. This d-c. power (of variable voltage) may be also either pumped back to the line through a motor-generator set *D–K*, Fig. 6C, or returned mechanically to the main motor shaft, Fig. 6D. In either case the speed of the drive is adjusted by controlling the excitation of the motor *D*.

Or, the slip energy may be transformed by means of a frequency converter, *F*, running at the motor speed, and a regulating transformer, *T*, Fig. 6E, into line frequency and voltage. Finally, it may be converted suitably by a frequency converter, *F*, Fig. 6F, separately driven by a small synchronous motor, *A*, and is then returned as mechanical power to the main shaft by means of a synchronous motor, *S*; its excitation provides the speed regulating means. Several other schemes, employing a frequency converter, are also conceivable.

Either scheme is capable of regulating the speed of the main motor not only below but also above synchronism, forming a so-called double range drive. Obviously the slip energy is then of a reverse direction; it flows from the regulating machines to the slip-rings, and not from them; the arrows, see Fig. 6, indicating, by dotted lines, the flow of power will have to be reversed. The Scherbius and the frequency converter systems are usually of the double range type; on account of certain difficulties of operating the Kraemer drives close to synchronism (*i. e.* at a very low frequency at the synchronous converter *C*) and of inability to go through synchronism under load, these drives are usually built as single range equipments, for sub-synchronous operation only.

It will be observed that with all of these schemes the main part of the a-c. power is converted but once until it reaches the mill coupling; only the balance of power or the slip energy goes through more than one transformation before it is utilized. Naturally, the over-all efficiency is higher than with the d-c. drives and is usually around 90 per cent at full load. The machines used for speed regulation should have a capacity depending on the size of the main motor and on the amount of speed range. Therefore, the greater the speed range, the more expensive becomes the a-c. speed regulating equipment, and the less becomes its advantage over a d-c. drive, both from the standpoint of first cost and efficiency. Obviously, with a double-range drive, the same speed regulating equipment is utilized to a greater extent than with a single range drive.

New Combination Drives. While some of the outlined a-c. systems were widely used during the last 10 or 15 years, their application for multi-drive mills, discussed in this paper, gives the engineers an occasional opportunity to get off the beaten track, and to group the well known machines in some new and more advantageous arrangement. The underlying principle of several new combination drives is this:

If the slip energy, contingent on speed regulation of one or several a-c. drives, need not be returned to the shafts of these drives, it may be made use of for furnishing, completely or in part, the power required to drive some other sections of the same mill, or some other mills.

This principle was applied for the first time in 1925 in connection with the equipment shown on Fig. 7, and, to the best of the author's knowledge, it had not been suggested nor applied previously.

This sketch represents a single line diagram of a large continuous rolling mill equipment recently put in operation in the Chicago district. For the sake of simplicity, the switching and control apparatus are not shown. The mill consists of several stands arranged in tandem. The roughing stands were to be driven by one motor *I M*—1 developing 3600 hp. at 290 rev. per min. and 1960 hp. at 156 rev. per min,; the intermediate train was to be driven by another motor, *IM*-2, developing 7500 hp. at 250 rev. per min. and 4040 hp. at 136 rev. per min. The three finishing stands were to be each driven by a 2000-hp. motor, developing this capacity at any speed from 85 to 165 rev. per min. A set of edging rolls required a 250-hp. drive. The electric power was available at 2200 volts, three-phase, 60-cycle.

The electrical engineers have solved the problem of selecting the drives for this mill in the following manner.

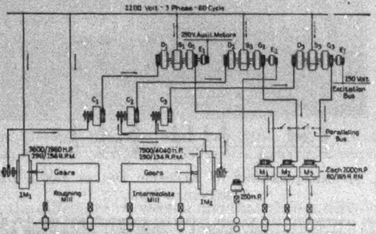

FIG. 7—ELECTRICAL ARRANGEMENT OF COMBINATION A-C. AND D-C. DRIVES FOR A LARGE CONTINUOUS MILL

The total continuous capacity of the equipment is 17,350 hp.

The 3600-hp. and 7500-hp. drives, being large units running at reasonably high speeds, could be economically designed as induction motors, with speed adjusted by the Kraemer method. The finishing mill drives, smaller in capacity and much lower in speed, could be more advantageously and more compactly built as 600-volt, d-c. motors, with speed adjustment by motor field control. The power to these motors is furnished from three 1700-kw. 600-rev. per min., d-c. generators, *G*1, *G*2, and *G*3, driven by synchronous motors, *S*1, *S*2, and *S*3. Low-speed, 60-cycle induction motors for driving the three finishing mills would be expensive machines with a rather poor power factor; the use of reduction gears would not be very advantageous, nor feasible, due to certain local conditions.

It will be observed that the 3600-hp. induction motor, *IM*-1, when running at 156 rev. per min. (*i. e.*, at 52 per cent synchronous speed), is required to develop only 1960 hp. as mechanical power at its shaft; the other 48 per cent or 1640 hp. are available as slip energy. The latter is converted by means of the synchronous converter, *C*1, into d-c. power and drives a d-c. machine, *D*1, as a motor. The excitation of the latter determines its voltage and, therefore, the speed of the drive *IM*-1.

Likewise, the 7500-hp. motor, running at 136 rev. per min. or 52 per cent synchronous speed, delivers to the mill 4040 hp., while the available slip energy amounts to 3460 hp. It is converted to direct current by means of two synchronous converters, *C*2 and *C*3, duplicates of *C*1, and is used for driving the d-c. machines, *D*2 and *D*3, as motors; by changing simultaneously the excitation of *D*2 and *D*3 the speed of the drive *IM*-2 is adjusted.

As the maximum amount of the slip energy to be handled by each of the machines *D*1, *D*2, and *D*3 is approximately 1750 hp., they are made exact duplicates of the 1700-kw. generators, *G*1, *G*2, and *G*3.

Arrows on Fig. 7 indicate the flow of power when the entire mill is in operation. It will be seen that the machines, *D*, are running as motors and are assisting the synchronous motors, *S*, in driving the generators, *G*. This assistance is the greater, of course, the lower the speed of the drives *I M*-1 and *I M*-2; if this speed is close to synchronism, or if the load on the drives *I M*-1 and *I M*-2 is relatively light, then the synchronous motors are more heavily loaded. In the extreme case they should be capable of furnishing the total power required by the generators and cover the friction and windage of the machines, *D*. In other words, the synchronous motors need be only 50 per cent of the capacity required for a three-unit motor-generator set of the same d-c. rating.

Ordinarily, the motors, *S*, would be running underloaded, providing an additional amount of leading kilovolt-amperes, and compensating for the reactive kilovolt-amperes of the large induction motors. The power factor of the whole installation is approximately 97 per cent leading at full load.

Thus the slip energy of the constant torque drives *I M*-1 and *I M*-2 is not returned mechanically to the shaft of these drives where it was not required in this case, nor is it returned electrically to the power bus. Instead of this, it is made use of in a more direct manner; namely, for driving the finishing mills. The over-all efficiency is improved and the required capacity of the regulating apparatus is reduced to a minimum.

The drive just described possesses a number of secondary advantageous points, although these are not directly connected with the new principle of utilizing the slip energy. For instance, in case of light loads, it is possible to operate the drive *I M*-2 with only one synchronous converter and one d-c. motor, say with *C* 2 and *D* 2, and to operate the three d-c. motors *M* 1, *M* 2, and *M* 3 from only two generators, say *G* 1 and *G* 2, by using the paralleling bus. This will permit the shutting down of one motor-generator set, thereby reducing the running light losses. Although it is hard to estimate with any degree of accuracy the resultant saving in power, it is evident that any such saving is a net gain. It may be truly said in this connection that in steel mill drives, which are usually liberally motored to take care of the maximum load conditions, the low running light losses are just as big a factor in conservation of power as the high efficiency.

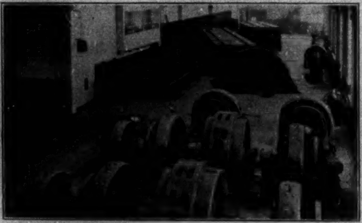

Fig. 8—General View of the Mill Motor Room Containing the Electrical Equipment Arranged as Shown on Fig. 7

Two out of three synchronous motor generator sets and two synchronous converters are seen in the foreground; three 2000-hp., d-c. motors are located next to the wall, with the 7500-hp. Kraemer drive seen to the right of them. The 3600-hp. Kraemer drive is located back of the gear cases and is not shown on the photograph

The motor room of the mill just described is illustrated by Figs. 8 and 9.

Another continuous mill, now being built for a large eastern steel manufacturer, will be equipped with electric drives embodying to a smaller extent the same principle; several new features of a different nature will make a brief review of this equipment rather interesting.

Fig. 9—Close-Up View Showing Three 2000-Hp., D-c. Motors, Each Driving a Finishing Stand of a Continuous Mill

See Figs. 7 and 8

In this case the mill will have a number of stands arranged in tandem, as shown diagrammatically on Fig. 10. The rolling requirements being different, the first several stands will be driven by constant speed motors. The power supply is 6600-volt, 25-cycle.

The roughing train will take a 4000-hp., 83.3-rev. per min. motor, *A*, the intermediate train will be driven by a 6500-hp., 187.5-rev. per min. motor, *B*. The following group of stands will be jointly driven through a train of gears by an adjustable speed equipment, *C*, developing 6700 hp. at 500 rev. per min. and 3350 hp. at 250 rev. per min.

The last finishing stand will take a separate direct-connected drive, *D*, with an output of 2600 hp. at a speed of 275 rev. per min.; constant horsepower output will be maintained for speeds above 275 rev. per min., and reduced output on constant torque basis, for speeds below this value.

These drives will never be required to start their respective mills with metal in the rolls. Mill friction on a cold winter day, after a prolonged shut-down, would be the most severe starting condition. Several tests have shown that a torque of about 25 or 30 per cent normal will start a continuous mill under most adverse conditions.

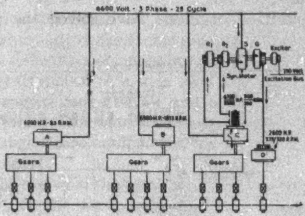

Fig. 10—Arrangement of Electric Drives for a Large Continuous Mill

Two synchronous motors, one Scherbius equipment and one d-c. motor aggregating approximately 20,000 hp. continuous capacity will be used for driving this mill

Actual experience with a 9000-hp., 107- rev. per min., synchronous motor, driving since the summer of 1926 a large continuous rolling mill at the McKinney Steel Company, Cleveland, Ohio, has proved conclusively that a synchronous drive is quite applicable for mills of this nature. This synchronous motor, shown on Fig. 11, is capable of developing a starting torque of 265 per cent normal if started on full voltage; it is usually started on a low voltage tap of an auto-transformer, developing the starting torque actually required with considerably less than normal line kilovolt-ampere input.

Under circumstances it has been decided to build the drives *A* and *B* as synchronous motors and to take advantage of their leading kilovolt-amperes for power factor correction of the steel plant.

The large adjustable speed drive, *C*, will consist of a 5000-hp., 375-rev. per min. slip-ring induction motor, the speed of which will be adjusted up to 33 per cent above, and up to 33 per cent below, synchronism (*i. e.*, from 500 rev. per min. to 250 rev. per min.) by means of the two Scherbius regulating machines *R* 1 and *R* 2. With this constant torque layout, the capacity of the drive will be 6700 hp. at 500 rev. per min. and 3300 hp. at 250 rev. per min. An a-c. drive of such capacity and speed can be built more economically and with a much higher efficiency than any combination of d-c. machines. The fact that the power supply was 25-cycle gave the Scherbius system an advantage over the Kraemer drive.

The last finishing mill drive, *D*, will have a wider speed range, is of smaller capacity, and runs at a lower speed than the drive *C*. While a Scherbius equipment for the drive, *D*, would be fully competitive in first cost, the difference between it and that of a d-c. drive was not as wide as in the case of the drive *C*. For the sake of greater flexibility of control it was decided to make the drive *D* of the d-c. type.

Fig. 11—9000-Hp.-107, Rev. Per Min. 6600-Volt, 25-Cycle, Synchronous Motor Driving a Large Continuous Rolling Mill

A 500-rev. per min. synchronous motor, *S*, will drive a 2300-kw., d-c. generator *G* (furnishing power to the motor *D*) and the two 650-kv-a. Scherbius speed regulating machines *R* 1 and *R* 2 used for adjusting the speed of the induction motor *C*. When the motor *C* runs below its synchronous speed, the slip energy flows to the machines *R* 1 and *R* 2; the latter run as motors and assist the synchronous motor *S* in driving the generator *G*. In other words, the slip energy does not have to be returned as electric power to the incoming line; instead of this, it may be used for driving, wholly or in part, the finishing mill *D*. The flow of power is indicated by arrows. This is another application of the same principle which was illustrated on Fig. 7.

When the drive *C* is running above synchronism, the slip energy becomes negative and arrows shown by the dotted lines, see Fig. 11, will be reversed. The machines *R* 1 and *R* 2 act then as generators, and derive their power from the synchronous motor *S*.

A direct-connected exciter provides the necessary 250-volt excitation to the synchronous motors *A*, *B*, and *S*, and to the d-c. machines *G* and *D*.

The use of two regulating machines *R* 1 and *R* 2 for controlling the speed of the motor *C* presents some interesting features. The maximum amount of the slip energy to be handled by the speed regulating equipment is 1700-hp.; it is not practicable to build an a-c. commutator machine of such capacity and to run at 500 rev. per min.; a lower speed like 375 rev. per min. or 300 rev. per min. would be required. With the proposed layout such reduced speed would considerably increase the cost of the d-c. generator *G* and of the motor *S*. It would be still more expensive to provide a separate low speed drive for the regulating machines *R* 1 and *R* 2, and to drive the generator *G* by another 500-rev. per min. motor. It was quite advantageous, therefore, to split the capacity of the regulating equipment in two units and to run them at 500 rev. per min.

The connections of the regulating machines to the secondary winding of the induction motor are shown on the Fig. 12. The 5000-hp. motor is equipped with six slip-rings, with both ends of each phase of the rotor brought out. Each set of three slip-rings is connected electrically to the commutator of the regulating machines *R* 1 and *R* 2, which thus forms the two Y-points of the secondary circuit. In other words, the two machines *R* 1 and *R* 2 act as if they were connected in series with each other, their e. m. fs. added together. The shunt fields *F* 1 and *F* 2 are adjusted simultaneously by a common speed control apparatus.

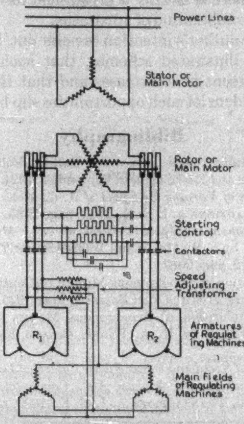
Power Lines
Stator or Main Motor
Rotor or Main Motor
Starting Control
Contactors
Speed Adjusting Transformer
R_1
R_2
Armatures of Regulating Machines
Main Fields of Regulating Machines

Fig. 12—Elementary Diagram of Electrical Connections of a Scherbius Adjustable Speed Drive with Two Regulating Machines Connected in Series

By disconnecting one regulating machine and by short-circuiting the corresponding set of slip-rings, it is still possible to operate the drive with the other regulating machine; full torque of the drive will be obtainable, but the speed range will be cut in half;

i. e., it will be in this case approximately 312/437 rev. per min.

CONCLUSION

None of the several electrical layouts described on the preceding pages should be considered as anything more than what they were originally intended for:—a good combination of electrical machines to fit a set of given requirements. Certain principles may be used again in some future drives; the whole combination may never be repeated. Electric drives for modern large rolling mills can hardly be standardized. They rather are and may rightly be called "custom made."

Many single mills require up to, or over, 20,000 hp. in electric drives; investment runs into several hundred thousands of dollars; the cost of power consumed in a year may approach the same figure. This alone justifies a thorough engineering study and a preparation of an individual layout for each case. Machines of special design need not necessarily be built for any new drive, but there is usually a broad field for working up a good new combination of apparatus.

It was the author's intention to point out, by means of the several illustrated schemes, that such an opportunity is present in most cases and that the electrical engineers seldom let such opportunities slip by.

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Discussion

D. M. Petty: In reading the introduction to this paper, I noted that there was just one rather important point which Mr. Umansky did not cover. I think it is a rather vital point and is becoming more vital every day. That is, the relation of the power generated in steel plants to that which is purchased. A number of steel plants are making it a point to buy a certain proportion of their power and with the advent of interconnections of large power companies, I think the interchanging of power between steel companies and power companies becomes increasingly important. The power companies themselves, naturally, having a wider range of load, are now able to absorb waste-gas and waste-heat energy better than they otherwise could have done had their system been limited to a small territory, it being understood, of course, that the steel plant has this power available as a by-product.

Immediately I imagine the problem arises in the minds of a good many of the central station men that most steel plants are 25-cycle and the power companies are 60-cycle. This has for a long time apparently offered an almost insurmountable barrier to an interchange of power agreement, but I find in talking to some of the power engineers, that as these super-power systems grow and their lines extend over greater distances, the use of synchronous condensers becomes a considerable factor in the regulation of voltages at different points.

It has occurred to me that possibly some of these synchronous condensers at different points could have a 25-cycle generator connected to the big, 60-cycle synchronous condensers. It might look, in a good many cases, like an exciter connected to the synchronous condenser. However, it might be big enough to provide a means of interchange of power in sufficient quantities to provide for the needs of the steel plant when it wanted to buy power and also to provide an outlet for a maximum amount of power which the steel plant might have available at any particular time in its operations.

The various rolling-mill layouts which Mr. Umansky showed, I think, are very typical of the modern trend of steel-mill design and rolling-mill design. There is just one thought that I should like to mention in this connection, for fear that possibly some one who is not a steel-mill engineer might be a little misled. The big thing in rolling-mill practise is to obtain tonnage from the mill, and, with the various Kraemer and Scherbius drives as illustrated, it must always be borne in mind that the first consideration is that the particular electrical drive to be hooked to the mill must enable the millman to deliver from the mill the maximum amount of tonnage at the lowest possible cost, and cost in a rolling mill hinges almost always around production more than any other factor. So that when you think of cost in a rolling mill, you are almost saying "reliability," since a drive that is not reliable, either because of complications or anything else, would immediately run the cost up by virtue of the fact that it would reduce the tonnage. The actual dollars and cents expended on labor and material in repairs might be negligible but the reduction in tonnage would be a serious matter. So that while we like to feel that we are going after all the efficiency we can get in mill drives, we should not hesitate to throw out 3 or 4 per cent of efficiency if there was a thought that a d-c. drive might produce more tonnage on a mill than either a Kraemer or a Scherbius or any other system of a-c. drive. In other words, production is the most important factor, especially where the product of the mill is not determined at the time it is laid out.

Of course, if it is known beforehand that the mill is never going to roll anything else but a particular product, it is comparatively easy to make the layout. So that while we want to do everything we can to make the drives more efficient, the big point is always reliability and flexibility, whenever flexibility means more tonnage. That, to my mind, is the big factor in the matter of drives.

I wish also to emphasize further the point that you cannot say that one drive, because it has worked out excellently in one mill, will always work out so well in the next mill. Each mill must necessarily be laid out for itself and for the purposes which the product of the mill demands. In some cases two mills may look alike in the number of stands, but the product may be different and consequently it may be necessary to have an entirely different type of drive. I feel that that particular point is one of the best points brought out by Mr. Umansky.

A. J. Standing: The mills should be so designed that the mill layout may not be the limiting factor; in other words, the mill shall be so laid out from the heating and the furnace end of it that steel can be delivered to the drive as fast as the drive can roll it, and that, at the other end of the drive, steel can be taken care of at the finishing and disposal end fast enough to take care of the drive. The reason I mention this is because oftentimes the mill is laid out for the production of one product, and later we find that the mill has undergone considerable change from the time when original product was got out. So that in putting a drive on a mill, the past experience has a great deal of weight due to the fact that information has been obtained as to the speed with which we can handle steel both before it enters the drive and especially after it leaves the mill. With that in view, I think the steel manufacturers each year are gaining more experience with various installations so that those who are installing electrical drives from now on will be able to profit by the actual applications and the improvements which have been made in the tonnages, especially from the standpoint of cost per ton.

R. H. Wright: Under present-day business conditions, it is essential that any organization, to be successful, must be quick to adopt new methods and equipment. Steel plant engineers and executives for years have been pioneers in the application of electrical equipment to heavy mechanical operations and, through continual expansion of their electrical power systems, they have effected great economies of operation.

One of the chief advantages of the electric motor drive in the steel industry, in addition to the inherent economy, is the extreme flexibility and the ease with which it can be applied to new processes and labor-saving devices. Through liberal use of electric motors and automatic control in the modern steel plant, the output per man has been increased many times and the accident hazard reduced to a minimum. That the advantages of electrification can be obtained without sacrificing plant production during installation of new equipment is illustrated by a project recently completed in the Philadelphia district. All the steam engines in this plant were replaced by electric drives operating on purchased power in less than three years with no appreciable loss in production during the change-over period. The saving in power cost and operating labor will, in three years, cover the cost of the installation.

To meet the peculiar needs in the industry, the electrical manufacturers have developed complete lines of equipment for steel mill service. One of the first main-roll drives was a reversing motor, installed twenty years ago to drive a plate mill. Since the initial installation reversing motors have been increasing in size until units rated 7000 hp. at 50 rev. per min. are now quite common and one motor has been built for 8000 hp. at 40 rev. per min.

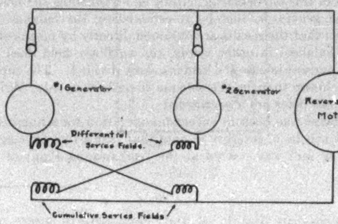
Up to a few years ago all reversing motors above 5000-hp. rating were built with two armatures on a common shaft. Direct-current power was supplied to the motor through a motor-generator set having two generators. In order to keep the voltage to ground from exceeding the potential of one generator, a sandwich scheme of electrical connections was employed in the circuit between the generator armatures and the double-armature motor. Motor-generator sets with two generators are used with the more recent equipments, but the double-unit armature construction of the motors has been abandoned, and one 8000-hp. and six 7000-hp. motors have been built with single-unit armatures by the Westinghouse Company.

It will be obvious that the single-unit motor has one-half the armature end windings of a double-unit motor of the same rating and diameter and that the number of cross connections for the commutating pole and compensating windings is also reduced to half that required for a double-unit motor. The copper loss of the single-unit motor is therefore about 25 per cent less than the copper loss of a double-unit motor of the same rating and the efficiency is higher.

When two generators are used in the motor-generator set they are connected in parallel. Equal division of the rapidly varying load to which such generators are subjected is obtained by means of special field connections. As shown in the accompanying figure, each generator has a differential series winding and a cumulative series winding. These windings are identical and under balanced load conditions, they neutralize each other. However, if No. 1 generator, for example, should tend to take more than its share of the load, the differential series field of the No. 1 generator would be stronger than the cumulative series field and the voltage; consequently, the load on the No. 1 generator would be reduced. At the same time, the excess of current from the No. 1 generator would strengthen the cumulative winding of the No. 2 generator, causing it to take more load. Any tendency for unequal division of load is therefore corrected very quickly.

W. E. Lloyd: I should like to include one question. How is it controlled? Do you have a dozen men or one man with a push button?

L. A. Umansky: This paper was not intended to cover the very interesting point just brought up by Mr. Petty; namely, the interchange of power between the steel plants and the public utilities. It is very fortunate, however, that this problem has been mentioned here as it is a very vital one and will undoubtedly grow in importance as time goes on. Whenever the steel plant operates at a frequency of 25 cycles, while the purchased power is available at 60 cycles, the two systems will be undoubtedly tied-in by means of special frequency-changer sets. Their purpose will be not only to convert the 60-cycle to 25-cycle power, or vice versa, but also to *control* the flow of power between the two systems. It means that while one unit, presumably at the 60-cycle end of the set, will be of a synchronous type, the 25-cycle unit is likely to be an induction motor with a speed-regulating equipment attached to it. The latter may be similar to the Scherbius, Kraemer, or similar systems outlined in this paper. The actual speed of the frequency-changer set is not changed as long as the frequencies remain fixed, but, by controlling the frequency applied to the secondary circuit of the induction machine, the latter may be given a tendency to operate either as a motor or as a generator; in this manner the interchange of power between the two systems may be readily controlled. The synchronous motor of the set, if suitably designed, may also act as a synchronous condenser on the 60-cycle line.



We all agree with Mr. Petty that the question of reliability and flexibility is just as important in selecting electric drives for rolling mills as is the question of efficiency. So many of both a-c. and d-c. drives are in successful operation for many years that their reliability certainly should be considered on a par.

No reversing drives were mentioned in my discussion as there were no radical changes made in this line for the last ten or fifteen years. Many details were undoubtedly improved, as pointed out by Mr. Wright, but the method of operation of the machines remains substantially unchanged. A single-unit armature of a, say, 7000-hp. reversing motor, is an improvement of size rather than of kind. When two generators are supplying power to a single-armature reversing motor, means should be provided, of course, to divide the load automatically and evenly between the two generators. The scheme mentioned by Mr. Wright is very effective for this purpose; as a matter of fact, the same scheme is used for a number of years on the double-unit, d-c. motors shown on Figs. 3 and 4 of my paper. It should be borne in mind, however, that if we equip each of the two machines with two series fields, one cumulative and one differential, then the load balance is maintained at the expense of crowding the main poles of the machine with a double amount of series field turns; the latter carry the full current but, in the ideal case, do not produce any flux. In other words, the balance is obtained magnetically and not electrically. By causing the current to circulate through two series fields, the copper losses are increased and the efficiency is, therefore, reduced.

The reversing drives usually give an excellent opportunity to apply a simpler but an equally effective scheme which, at the same time, is devoid of the above shortcoming. A relatively small potential winding may be mounted on the main poles of each generator and these auxiliary fields of the two machines are then connected in series with each other. They are so wired that, when one of them acts cumulatively with the main field of its generator, the other auxiliary winding acts differentially with the main field of the second generator. The free ends of the auxiliary fields are connected to terminals of the two generators of the same polarity. The current of each generator is carried separately to the reversing motor, where the two circuits are joined together. When the load is evenly divided between the two generators the IR drop in each circuit is equal and, therefore, no current is flowing through the auxiliary fields. If, for any reason, one generator carries less than its proper share of the load, the IR drops in the two circuits will differ and therefore a current will flow through the auxiliary fields, strengthening the main field of this particular generator and weakening the field of the second generator, thereby re-establishing the balance. It will be seen that the results are obtained directly by means of an electrical balance; in other words, the auxiliary field does not carry any current in case the load is evenly divided. The copper losses are lower than in the scheme described by Mr. Wright and the main poles are less crowded.

This scheme has been in successful operation for a number of years. It will be of interest to note that just a few days ago a large rolling mill was started at the Lackawanna Plant of the Bethlehem Steel Company. This mill includes five reversing drives, of which three are 7000-hp., single-armature motors, each furnished with power from two generators connected in multiple. The division of load is load is maintained in a perfect manner by means of the scheme just described.

Once the reversing drives are discussed, I wish to mention one other point which may eventually change in one respect the conventional form of these equipments. It has been commonly understood that any reversing drive requires a motor-generator set equipped with a heavy flywheel to equalize the load on the incoming line. This is still true for the larger reversing drives, but, as the power systems grow, the question of limiting the instantaneous peak load will acquire relatively less importance. Just recently, a reversing drive which will include a synchronous motor-generator set was ordered. The power in this case was purchased from a public utility and the question was put squarely before the power company: Did it prefer to take on its system a heavier instantaneous peak load, not smoothed out by any flywheel, or to reduce this peak load and contend with the lagging power factor of the induction-motor-driven set. The answer was in favor of the synchronous motor drive. While this particular reversing drive is of a moderate size, (4200-kv-a. synchronous motor), I believe that it is a forerunner of larger equipments provided with synchronous motor-generator sets. Maybe in ten or fifteen years from now, even the 7000-hp., reversing blooming-mill drives will lose one of their typical features,—the flywheel set.

Mr. Lloyd inquired as to how many operators usually control a large continuous mill with individual drives. The motors and the motor-generator sets are usually fully protected by automatic devices, and, strictly speaking, no attendant is required in the motor room. The actual starting speed adjustment and stopping of each drive is controlled by one man in the operating pulpit having within his easy reach the necessary master switches, push buttons, rheostats, etc. One man is usually sufficient to do this work; a second man would be comparatively useless.

Notes on the Use of a Radio Frequency Voltmeter

BY W. N. GOODWIN, Jr.[1]
Fellow, A. I. E. E.

Synopsis.—*A thermovoltmeter having a special form of shielding, for use in the accurate measurement of voltage at radio frequencies was described in 1924 in a paper by Mr. Leon T. Wilson. This instrument has since been developed into a commercial form, and this paper describes the technique for using it in a few of its practical applications.*

The voltmeter correctly indicates the voltage at its terminals for any frequency up to about 1500 kilocycles. For low frequencies the instrument is used exactly as an ordinary voltmeter. For high frequencies, it is necessary to pay especial attention to the external connections, in order that the voltage desired to be measured shall be applied unchanged to the voltmeter binding posts.

Methods are described for measuring the radio frequency resistance and inductance of coils, and for establishing a definite voltage across a circuit such as the input circuit of an amplifier.

* * * * *

At the Midwinter Convention held in Philadelphia, Feb. 1924, Mr. Leon T. Wilson described a specially shielded voltmeter designed for the measurement of voltages at radio frequencies. A detailed description of the instrument will not be repeated at this time, as this was quite fully covered by Mr. Wilson in his paper, except to state that it is a thermocouple type voltmeter enclosed in a metal shield formed in two halves, insulated from each other. The movement and couple are placed between the two shields, symmetrical with them, and the resistor is divided in two parts, symmetrically disposed, one on each side of the movement. The two terminals are connected directly to the shields, one to each.

The complete voltmeter is illustrated in Fig. 1, and the arrangement of parts in Fig. 2. Briefly, the principle of the shielding is as follows: The thermocouple circuit and d-c. movement, in which it is desired that no alternating current shall flow, are placed exactly midway between the two shields, so that the values of their electrostatic capacities to both shields are equal.

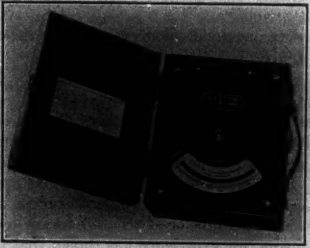


Fig. 1—Thermo-Voltmeter

The resistors, in two halves, also have equal electrostatic capacities with their corresponding parts of the shield. On account of this symmetry, therefore, there is no tendency for the alternating current to pass from the heating wire through the couple into the movable coil, which is the cause of errors in voltmeters not provided with this form of shield.

This type of voltmeter is used in a manner very similar to the ordinary low-frequency voltmeter, and since it requires current for its operation, is subject to

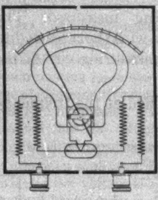


Fig. 2

the same limitations; that is, it will not give correct indications of voltage in a circuit if the current drawn from that circuit for the instrument appreciably affects the voltage to be measured. The current required varies from two to eight milliamperes, depending upon the range of the instrument. In addition to this operating current, there is also the charging current for the shields, which form the plates of a condenser in parallel to the instrument circuit. This current, at a frequency of 1000 kc., is of the same order of magnitude as the current in the instrument circuit, but it is in quadrature with it.

In using the voltmeter for direct current or alternating current of low frequencies, it is used in exactly the same manner as an ordinary voltmeter. For high-frequency measurements, however, while the principle of the measurement is the same as for low frequency, there is a definite technique which must be followed in order that the voltage at the binding posts of the instrument will be the value desired to be measured, and not the resultant of this voltage and those of mutual and self-inductance in the connections and the effects of capacity. A few examples will be given to illustrate the technique.

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[1] Chief Electrical Engineer, Weston Electrical Instrument Corp., Newark, N. J.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

TO MEASURE THE EFFECTIVE RESISTANCE AND INDUCTANCE OF A COIL

The method used is the usual voltmeter-ammeter method in which the resistance is measured by the drop across it produced by a measured current.

The connections are made as shown in Fig. 3. The coil *D*, the resistance of which is to be measured, is connected in series with a condenser C_2, having a negligible or known loss and a thermomilliammeter A_2. The circuit thus formed is connected across the non-inductive resistance *R* at the points $E_1 E_2$. The point E_2 should be carefully grounded and the shielded side of the condenser C_2 connected to the milliammeter, which is in the grounded side. The resistor *R* and connections should all be short, especially the connection between B_2 and the condenser C_2, as this is the only part of the circuit at relatively high potential. This connection should preferably be suspended, together with the coil, away from all dielectrics except air, in order to prevent any losses being added to those of the coil being measured. The voltmeter V_M must be connected with short leads directly at the points $E_1 E_2$ where the measuring circuit is connected to the resistance *R*.

Current is supplied at the frequency desired from an oscillator, either shielded or placed at a considerable distance from the measuring circuit and connected by leads placed close together, to avoid mutual inductance effects in the measuring circuit. The coil L_2 is loosely coupled to the oscillator coil L_1 and has a tuning condenser C_1 in series with it to adjust the current I_1 in the circuit. The frequency of the oscillator is adjusted by the use of the wavemeter *W*.

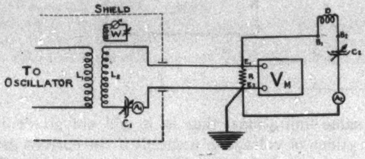

FIG. 3

The current I_1 should be 10 or more times greater than I_2, so that the tuning of the two circuits will not materially affect each other. The drop across *R* then becomes practically equivalent to a non-inductive source of high-frequency voltage for use in the measuring circuit.

To measure the resistance of a coil *D*, for example, such as is used as a tuning coil for ordinary broadcasting frequencies, the condenser C_2 should have a maximum capacity of about 500 $\mu.\ \mu.$ f., the milliammeter A_2 a range of about 100 milliamperes, the voltmeter V_M a full scale value of about three volts, and the resistor *R* a resistance of about 1.2 ohms. A very desirable resistor for the purpose may consist of about two inches of manganin, constantan, or other high-resistance wire about 0.0065 in. in diameter.

It is desirable to have a thermoammeter A_1 in the high current circuit, having a range of two or three amperes. An oscillator having two five-watt tubes in parallel will supply sufficient power for this purpose. The oscillator is adjusted to the proper frequency, and then the current I_1 adjusted to one or two amperes by means of the condenser C_1, to give a readable indication on the voltmeter scale. The condenser C_2 is then adjusted until the current I_2, indicated by the milliammeter A_2, is a maximum. It is desirable to use a vernier condenser or vernier attachment on condenser C_2, as the tuning is exceedingly sharp. Under this condition, all self-inductance in the measuring circuit has been neutralized and the resulting current is due solely to the voltage *V* indicated by the voltmeter and the resistance of the circuit, which is computed in the usual manner by Ohm's law. The coil resistance is then obtained by subtracting from the computed resistance of the entire circuit the known resistance of the milliammeter, and the resistance of the condenser if it is not negligible. It is obvious, of course, that the resistance *R* is not included in this circuit.

The same setting may be also used to measure the effective inductance of the coil, if the condenser C_2 is a calibrated one. The condenser capacity to give the maximum current I_2 is noted, together with the frequency. Since the inductive and capacity reactances are equal, the inductance is then

$$L = \frac{25.35 \times 10^6}{f^2 C} \text{ millihenrys,}$$

where f = frequency in kilocycles, and *C* is the capacity of C_2 in $\mu.\ \mu.$ f.

It has been found, in using the voltmeter for the measurement of resistance, that whereas the voltmeter measures the voltage at its terminals with high accuracy, the final accuracy of the resistance measurement almost wholly depends upon the set-up of the measuring circuit. It must not be concluded that this method or any other method for measuring resistances at radio frequencies is as simple or as accurate as low-frequency measurements, for experience has shown that the best arrangement of circuit is often difficult to determine, and considerable experimental work is necessary to obtain the best results.

The general rules referred to above may be followed but the actual circuit and location of measuring apparatus will depend in each case upon the particular apparatus used. It is advisable to make the "set-up" permanent when the best arrangement has been found. The chief advantage of the voltmeter method is the rapidity with which coils of various types can be compared.

ESTABLISHING A VOLTAGE

It is often desired to apply a definite measured high-frequency voltage to the grid circuit of an amplifier or to the primary of a transformer or to some other device. The voltmeter operates very satisfactory for this purpose by simply connecting it across the input circuit between the points where the voltage is to be established.

LIMITATIONS OF USE AND ACCURACY

The voltmeter is limited at high frequencies to measurements of voltage in continuous wave circuits and to frequencies up to about 1500 kc., and where the current required to operate the instrument and its electrostatic capacity do not materially alter the value of the voltage to be measured, which is similar to the limitation under which the ordinary low-frequency voltmeter operates.

It cannot be used, for example, to measure voltages from spark generators, or in continuous wave circuits having large harmonics greatly exceeding 1500 kc., nor can it be used to measure the output voltage of a radio frequency transformer, or coupling coil, or, which is often the same thing, the grid input voltage to an interstage tube of an amplifier, on account of the disturbing effects of operating current and the capacity of the instrument.

The error in the voltmeter due to frequency from 60 cycles to 1000 kc. averages about 0.5 per cent, and to 1500 kc., about one per cent.

Discussion

L. T. Wilson: It has been mentioned that this meter takes from 2 to 8 milliamperes, and that a displacement current flows between the shields of the same order of magnitude. It was not specified in that statement that charging current is of the same order of magnitude as this 2 to 8 milliamperes only at 1,000,000 cycles. Of course, at any frequency lower than that, the charging current is proportionately lower, and down at very low frequencies, it is entirely negligible.

P. A. Borden: Mr. Goodwin states that in using the instrument he has described, it becomes necessary to get the voltage at the terminals of the instrument to be the voltage which it is desired to measure; and in this regard its application does not differ from that of any other voltmeter. To obtain this object he suggests what he has styled a "point source" of voltage, which is very nicely worked out on high frequency circuits. Without having had the opportunity to fully study the circuit, my question in this regard is,—can that arrangement be applied to commercial frequencies? If so, it would be very useful in measuring small voltages, and potential differences across circuits where a very small amount of power is available. I refer to circuits where the power consumption of the instrument compared with the power available in the circuit is so great that the ordinary instrument would disturb the value of the quantity under measurement. For example, in the determination of the volt-ampere consumption of the voltage coil of a watthour meter, current and voltage being measured simultaneously, it would be very valuable if the "point source" of voltage could be applied in the measurement.

H. M. Turner: Mr. Goodwin mentioned the fact that it is desirable to have a fairly large ratio between the currents I_1 and I_2 of say 10 to 1. The advantages are that the current amplitudes may be easily controlled by adjusting C, (Fig. 3 of the paper) without changing the oscillator coupling and that when tuning the measuring circuit to resonance the reaction on the oscillator is usually negligible.

I have used this method in the laboratory and find it convenient and satisfactory for measuring circuit constants.

W. N. Goodwin, Jr: Mr. Borden asks whether the "point source" of electromotive force could not also be applied advantageously to low-frequency measurements. What I termed a "point source" of electromotive force was provided as a means for obtaining a source of electromotive force which for all practical purposes was free from inductance. To obtain this at radio frequencies, it was found that the voltage drop across a high-resistance wire of very small dimensions gave the desired results.

On low-frequency circuits a somewhat similar method is often used in practise in which a low voltage is obtained from a higher one by means of the well-known voltage divider. In this case, however, there is no advantage in using what I have termed a "point source," since the ratio of reactance to resistance can readily be made negligible without special reduction in dimensions.

The method described, either for low or for radio frequencies is not intended for the measurement of unknown voltages, but for establishing a voltage for use in measurements. The voltage can, of course, be adjusted to a definite value by knowing the value of the resistance and that of the current passing through it.

I wish to thank Mr. Wilson for calling attention to the fact that the value of the charging current given for the shields is for frequencies of the order of 1,000,000 cycles. This was inadvertently omitted from the text.

Substitution Method for the Determination of Resistance of Inductors and Capacitors at Radio Frequencies

BY C. T. BURKE[1]

Associate, A. I. E. E.

Synopsis.—*The use of a series resonant circuit for the measurement of the resistance of coils and capacitors at high frequencies is discussed. The circuit is tuned with the unknown in circuit and with it removed, introducing sufficient resistance to keep the current constant. From the condition of constant current, and the circuit constants, the constants of the unknown are derived. The derivation of the expression for the resistance of a capacitor involves certain assumptions which are shown to be justified for frequencies at which coils and capacitors are now generally used. Sources of error are pointed out and discussed. In the measurement of resistances of the order of two ohms and larger, the precision of the method is of the order of one per cent. A change of circuit resistance of approximately 0.02 ohm can be measured.*

* * * * *

THE measurement at radio frequency of the resistance of coils and capacities has presented some difficulty. A number of methods has been proposed, but most of them have been rather elaborate and some have involved quite formidable complications. The method to be described does not involve any new principle, and is, in fact, rather obvious. It possesses a simplicity which makes it compare most favorably with many of the schemes heretofore proposed, and renders it deserving of more attention than it has received. While it does not pretend to hair-splitting exactitude, its precision, if the system is constructed with reasonable care, is equal to the requirements of a vast number of factory and research requirements. It also possesses the advantage that the sources of error are known, and most of them can be reduced to almost any degree required for precision work if the necessary precautions are taken.

The measuring circuit consists of a simple series circuit containing inductance, capacitance, and resistance, and a current indicator. This circuit is coupled to the output coil of a radio frequency oscillator of suitable frequency range. Provision is made for connecting the inductor to be measured in series in this circuit. Capacitors under measurement are connected in parallel with the test capacitor. While the absolute value of the measurement may be questioned, the method is of great value in determining the relative efficiency of coils and condensers, which is in many cases of more interest than the actual resistance. Using the set-up described, changes in circuit resistance of the order of 0.02 ohm can be detected. This permits the measurement of coil resistance of the order of one ohm with fair accuracy, and of resistances of two ohms and over to one per cent.

High Frequency Measurement of Inductors

In the measurement of inductors, the unknown inductor is connected at L_x and the variable resistance set at zero. The measuring circuit is tuned to resonance with the oscillator, adjusting the coupling to give a convenient reading on the current-indicating device.

Then

$$I = \frac{j\,\omega\,M\,I_0}{Z}$$

L_x is removed, a short-circuiting plug substituted, and the circuit retuned to resonance. Adjusting R until the current is reduced to its former value, it follows that

$$Z_1 = Z_2$$

This assumes: (1) the current in the oscillator is constant; (2) the coupling between the circuits is not changed. The first of these conditions is readily obtainable. The realization of the second requires sufficient spacing between the coil under measurement and the oscillator to prevent appreciable coupling between them. Stray coupling between the oscillator and the load coil is of no consequence, as no change is made in this portion of the circuit during the measurement. All question on this point can be removed, where the precision required justifies it, by means of suitable shielding.

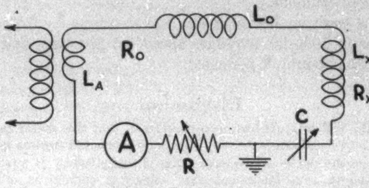

Fig. 1

$$Z_1 = R_0 + R_x + R_{c1} + j\,\omega\left(L_A + L_0 + L_x - \frac{1}{\omega^2\,C_1}\right)$$

$$Z_2 = R_0 + R + R_{c2} + j\,\omega\left(L_A + L_0 - \frac{1}{\omega^2\,C_2}\right)$$

At resonance

$$L_A + L_0 + L_x - \frac{1}{\omega^2\,C_1} = 0$$

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[1] General Radio Co., 30 State St., Cambridge, Mass.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

$$L_A + L_0 - \frac{1}{\omega^2 C_2} = 0$$

Hence

$$L_A + L_0 + L_x - \frac{1}{\omega^2 C_1} = L_A + L_0 - \frac{1}{\omega^2 C_2}$$

and

$$L_x = \frac{1}{\omega^2 C_1} - \frac{1}{\omega^2 C_2} = \frac{1}{\omega^2 C_1 C_2} (C_2 - C_1)$$

Likewise

$$R_0 + R_x + R_{c1} = R_0 + R + R_{c2}$$

and

$$R_x = R + R_{c2} - R_{c1}$$

This assumes the variable resistance to be non-reactive.

If the capacitor used for tuning the circuit to resonance has a series resistance which is appreciable in comparison with the coil resistance, a calibration of the equivalent series resistance as a function of the capacitance will be required. A capacitor designed so as to eliminate this complication should be used, however. This is readily accomplished, as the loading coil is so chosen that the capacitor is used near the maximum capacitance where the equivalent series resistance is small. It is possible by means of proper design to reduce the capacitor resistance to the vanishing point. Using a capacitor of such design, it is obvious that the change in resistance disappears.

Therefore:

$$R_x = R$$

Summary of Procedure for Inductance Measurements. With the unknown coil in the circuit, and the loading coil of such inductance as to give a capacitor setting near the maximum capacitance, the circuit is tuned to resonance as indicated by maximum galvanometer reading. The coupling coil is moved so as to give a convenient galvanometer deflection. Removing the unknown coil, the coil mounting is short-circuited and the circuit retuned to resonance and R adjusted until the current equals that obtained on the first reading. Then

$R_x = R$ (Equivalent series resistance = reading of standard resistance.)

and

$$L_x = \frac{1}{\omega^2 C_1 C_2} (C_2 - C_1)$$

The loading coil should be so chosen as to give a high capacitor reading without the test coil, for two reasons. Not only is the capacitor resistance less at high capacitance settings but the potential of the coil under test with respect to ground is determined principally by the inductance of the loading coil. As the potential of the coil with respect to earth affects its resistance somewhat, it is desirable to have the test coil as near ground potential as possible.

HIGH FREQUENCY MEASUREMENT OF CAPACITORS

In the measurement of capacitors, the unknown is connected in parallel with the capacitor of the test set. Sufficient inductance is connected in circuit to give resonance at a rather low value of capacitance. The procedure in measuring is the same as that employed with inductors. The circuit is adjusted to resonance with the unknown connected across the test capacitor, the unknown removed, and the circuit retuned, introducing sufficient resistance after the second tuning to bring the current to its former value.

Again the condition of constant current in the measuring circuit requires constant impedance in the circuit. As all other portions of the circuit are unchanged, it is necessary to consider only the measuring condenser, the unknown condenser, and the variable resistance.

When the test circuit is first adjusted to resonance, this portion of the circuit consists of the standard capacitor, shunted by the capacitor under measurement. After the second adjustment, the impedance consists of the standards capacitor in series with a resistance. Equating the impedances of these two circuits, the relation between the resistance added to reduce the current to its first value and the equivalent series resistance of the unknown capacitor may be derived.

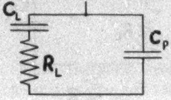

FIG. 2

In a properly designed capacitor, the dielectric is placed in a constant field, so that the dielectric loss does not vary with setting. It is convenient to represent such a capacitor as a fixed capacitance in series with a fixed resistance, the whole shunted by a perfect variable capacitor; see Fig. 2.

In this circuit, the customary equation for parallel circuits (resultant impedance equals the reciprocal of the sum of the reciprocals) reduces to:

$$Z = \frac{Z_L Z_P}{Z_L + Z_P}$$

where

Z = resultant impedance,
Z_L = impedance of "loss" circuit,
Z_P = impedance of perfect capacitor.

$$Z = \frac{\left(R_L + \frac{1}{j\,\omega\,C_L}\right)\left(\frac{1}{j\,\omega\,C_P}\right)}{R_L + \frac{1}{j\,\omega\,C_L} + \frac{1}{j\,\omega\,C_P}}$$

$$= \frac{-(1 + j\,\omega\,R_L\,C_L)}{R_L\,\omega^2\,C_P\,C_L - j\,\omega\,(C_P + C_L)}$$

Rationalizing and reducing:

$$Z = \frac{R_L\,\omega^2\,C_L^2 - j\,\omega\,[(C_P + C_L) + \omega^2\,R_L^2\,C_P\,C_L^2]}{(R_L\,\omega^2\,C_P\,C_L)^2 + \omega^2\,(C_P + C_L)^2}$$

The order of magnitude of the second term of the reactance under test conditions is 10^{-30}; that of the first term, 10^{-10}. The second term is therefore negligible for values of ω up to 10^9.

Taking out ω^2, the first term of the denominator is of the order of $10^{-40}\,\omega^2$, while the second is of the order of 10^{-20}. The first term of the denominator is therefore also negligible for values of ω less than 10^{-9}, and the expression reduces to

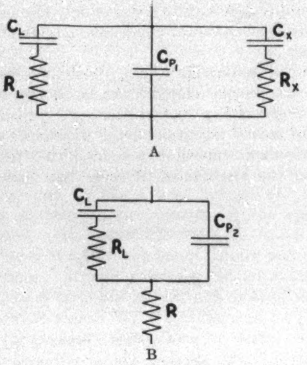

FIG. 3

$$Z = R_L\,\frac{C_L^2}{(C_P + C_L)^2} - \frac{j}{\omega\,(C_P + C_L)}$$

$(C_P + C_L)$ is the total capacitance of the capacitor; therefore, the system is equivalent to a capacitor of capacitance $C = (C_P + C_L)$, and equivalent series resistance $R = R_L\,\frac{C_L^2}{C^2}$.

Making use of this convention, Figs. 3A and 3B represent the variable portion of the test circuit at the first and second stages of the measurement. The solution will be simplified by considering the admittances rather than the impedances of these circuits. In the figure:

C_L = imperfect "loss" capacitance of the standard capacitor,
R_L = imperfect "loss" resistance of the standard capacitor,
C_{P1} and C_{P2} = settings of perfect capacitor,
C_x = unknown capacitance,
R_x = equivalent series resistance of the unknown capacitor.

Then

$$Z_L = \sqrt{R_L^2 + \frac{1}{\omega^2\,C_L^2}} \qquad \theta_L = \tan^{-1}\frac{1}{R\,\omega\,C_L}$$

$$Z_P = \frac{1}{\omega^2\,C_P} \qquad \theta_P = \tan^{-1}\infty$$

$$Z_x = \sqrt{R_x^2 + \frac{1}{\omega^2\,C_x^2}} \qquad \theta_x = \tan^{-1}\frac{1}{R\,\omega\,C_x}$$

$$\overline{Z}_L = Z_L\,\epsilon^{j\theta_L} \qquad \overline{Z}_P = Z_P\,\epsilon^{j\theta_P} \qquad \overline{Z}_x = Z_x\,\epsilon^{j\theta_x}$$

The resultant admittance of circuit A is

$$\overline{Y}_A = \frac{1}{Z_L}\,\epsilon^{-j\theta_L} + \frac{1}{Z_P}\,\epsilon^{-j\theta_P} + \frac{1}{Z_x}\,\epsilon^{-j\theta_x}$$

$$= \frac{Z_P\,Z_x\,\epsilon^{-j\theta_L} + Z_L\,Z_x\,\epsilon^{-j\theta_P} + Z_L\,Z_P\,\epsilon^{-j\theta_x}}{Z_L\,Z_P\,Z_x}$$

$$= \frac{1}{Z_L\,Z_P\,Z_x}\,[Z_P\,Z_x\,(\cos\theta_L - j\sin\theta_L) + Z_L\,Z_x\,(\cos\theta_P - j\sin\theta_P) + Z_L\,Z_P\,(\cos\theta_x - j\sin\theta_x)]$$

As the j terms disappear at resonance:

$$\overline{Y}_A = \frac{1}{Z_L\,Z_P\,Z_x}\left[Z_P\,Z_x\,\frac{R_L}{Z_L} + Z_L\,Z_P\,\frac{R_x}{Z_x}\right]$$

$$= \frac{R_L}{Z_L^2} + \frac{R_x}{Z_x^2}$$

$$= \frac{R_L}{R_L^2 + \frac{1}{\omega^2\,C_L^2}} + \frac{R_x}{R_x^2 + \frac{1}{\omega\,C_x^2}}$$

As the capacitances are of the order of 10^{-10}, and the resistance of the order of unity, the first term of the denominator may be neglected for values of ω up to the order of 10^9.

Therefore:

$$Y_A = R_L\,\omega^2\,C_L^2 + R_x\,\omega^2\,C_x^2$$

In the circuit of Fig. 3B,
for the parallel portion as previously developed:

$$Z = R_L\,\frac{C_L^2}{C_{s2}} - \frac{j}{\omega\,C_{s2}}$$

where

$C_{s2} = C_L + C_{P2}$ = total capacitance of standard capacitor.

For the complete circuit:

$$Z_B = R + R_L\,\frac{C_L^2}{C_{s2}^2} - \frac{j}{\omega\,C_{s2}}$$

$$= \frac{\omega\,C_{s2}\left[R + R_L\left(\frac{C_L}{C_{s2}}\right)^2\right] - j\,1}{\omega\,C_{s2}}$$

$$Y_B = \frac{1}{Z_B} = \frac{\omega C_{s2}}{\omega C_{s2}\left[R + R_L\left(\frac{C_L}{C_{s2}}\right)^2\right] - j1}$$

$$= \frac{\omega^2 C_{s2}^2\left[R + R_L\left(\frac{C_L}{C_{s2}}\right)^2 + j\,\omega C_{s2}\right]}{\omega^2 C_{s2}^2\left[R + R_L\left(\frac{C_L}{C_{s2}}\right)^2\right]^2 + 1}$$

The first term of the denominator is of the order of 10^{-6}, when ω is less than 10^9.
Therefore:

$$Y_B = \omega^2 C_{s2}^2\left[R + R_L\left(\frac{C_L}{C_{s2}}\right)^2\right] + j\,\omega C_{s2}$$

At resonance the j term disappears.

For constant current, equal admittances are required. Therefore

$$R_L\,\omega^2 C_L^2 + R_x\,\omega^2 C_x^2 = R\,\omega^2\,C_{s2}^2 + R_L\,\omega^2 C_L^2$$

or

$$R_x = R\frac{C_{s2}^2}{C_x^2}$$

The capacitance of the unknown is equal, of course, to the change in capacitance, *i. e.*,

$$C_x = C_{s2} - C_{s2}$$

It will be noted that the resistance measurement of capacitors involves the assumption that the field through the solid dielectric does not change with the position of the capacitor plates, a requirement met by any properly designed capacitor for laboratory use, and that the conductive resistance of the capacitor is negligible. The latter requirement may require special construction, especially as the higher frequencies are approached.

As the sensitivity of the measurement depends to a considerable degree upon the sharpness of resonance of the circuit, a current-indicating device of low resistance is required. A sensitive current indicator is of value as it permits the use of an oscillator of low power, and consequently reduced stray field. Since sensitive a-c. meters have a relatively high resistance, the arrangement on Fig. 4 was resorted to as a current indicator. The drop across a small coil is used to actuate a microammeter through a crystal detector. There is no difficulty in keeping the detector sensitivity constant for the short time required; in fact, the sensitivity of the fixed crystal used is constant over a long period of time.

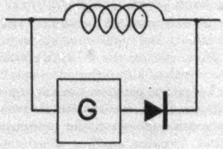

Fig. 4

The actual layout of apparatus used is shown in the reproduction of the photograph. Plug mountings for the loading coil and the unknown inductor were mounted on insulators to reduce the capacitance between the coils and earth. As it is desirable to keep the circuit resistance as low as possible, the jacks were mounted on hard rubber, and the dielectric directly between the jacks was cut away to lengthen the path. The mountings for the two coils were permanently secured about two feet apart. This distance seems sufficient to eliminate coupling between the coil under measurement and the loading coil or oscillator. The pick-up coil was mounted on a wooden block and connected to the rest of the circuit by means of flexible leads so that the coupling to the oscillator could be conveniently altered. The drop coil and detector were mounted on the side of the galvanometer. The galvanometer used had full scale deflection on 50 microamperes. The pick-up coil was so wound as to have little external field.

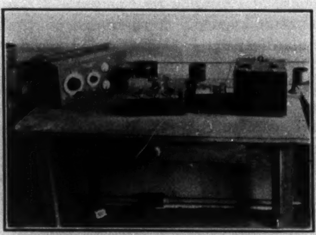

Fig. 5

The resistance used was one of the standard decade type, using the Ayrtron-Perry type of non-reactive winding. It is probable that this resistance has a slight reactance at the higher radio frequencies and would not be proper for extremely precise work. There is undoubtedly a change in the resistance of these units with frequency. Where the requirements of the work warrant it, the resistance standard should be calibrated at the frequency used.

A special type of capacitor was used. The capacitor was designed so as to permit a very fine adjustment of the capacitance and to have a very low equivalent series resistance.

The main scale in the capacitor is fitted with a vernier, making possible readings to one-tenth of the smallest scale division. A small uncalibrated capacitor is placed across the main capacitor in order to permit setting exactly to resonance. The error of calibration on the main scale due to this capacitor is ordinarily negligible.

A shield was used under the galvanometer and associated apparatus, the resistance, and the standard capacitor. The junction of the standard capacitor and the standard resistor was selected for grounding. It is desirable that one side of the standard and also of the unknown capacitor be grounded. If the junction of the coil and capacitor were grounded, the resistor and meter would be at high potential. As the potential drop through the resistance is small, the junction of the resistor and current indicator could have been grounded with practically identical results. The oscillator is of standard type, using a $U\ V$ 199 tube and plug-in coils to cover a wide frequency range.

Summary of Procedure for Condenser Measurements. The unknown capacitor is connected in parallel with the standard capacitor, loading coils being used in one or both mountings, and the circuit is tuned to resonance, adjusting the coupling for a convenient galvanometer reading. The unknown capacitor is removed and the circuit is again tuned to resonance, adding resistance to bring the current to its former value.

Then $C_x = C_2 - C_1$

$$R_x = R\,\frac{C_2^2}{(C_2 - C_1)^2} = R\,\frac{C_2^2}{C_x^2}$$

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Discussion

T. E. Shea: This paper offers an opportunity to call attention to certain equivalent networks, due to Dr. O. J. Zobel, which do not seem to have become generally absorbed into the literature to the extent that they deserve.

In the design of wave filters, of equalizers, or of balancing networks for simulating telephone lines, these equivalents have two values. In the first place, they simplify a great many formulas and give a better picture of what is going on in the circuit. In the second place, more desirable inductance, capacity and resistance values are often obtainable.

Now consider the network represented in Fig. 2 of the present paper. We are all familiar with the various relationships of equivalent T and π networks (for example, those made up of capacities) where there are three terminals for each network. Nobody would ever question, if he had one mesh and it were more convenient to substitute another in a circuit and get a better picture of what was going on or get a more easily calculated circuit, the validity of using the second mesh. If he obtained more desirable capacity values, or a more convenient arrangement of capacities, he might even substitute the one mesh physically in place of the other.

In the case of two-terminal networks, that is, impedances in which there is only one main current path, there is a large number of similar equivalents. In the case of Fig. 2, the corresponding equivalent two-terminal network is in the form of capacity in series with a parallel combination of resistance and capacity. Likewise there are equivalent two-terminal meshes using inductance and resistance, and inductance and capacity, respectively, and for any number of component impedance elements.

If in this present case, for example, one wishes to investigate the matter of selectivity—the change in sensitivity of the device as the frequency is slightly changed—I suggest that it can be done very much more readily by means of equivalent networks, although, of course, the formulas in the paper ultimately simplify down to give the same results.

Two-terminal equivalent networks are discussed in many places in the literature; for example, in Zobel's papers on "Wave Filters," R. S. Hoyt's paper on "Design of Networks to Simulate Smooth Lines," and R. M. Foster's paper on "A Reactance Theorem," and in K. S. Johnson's book "Transmission Circuits for Telephonic Communication."

J. G. Ferguson: I should like to point out the small modification necessary in a circuit of this type to transform it into a bridge network and thus eliminate the necessity of measuring the current in the circuit in order to determine the point of resonance.

In Fig. 1 of the paper, we have a resonant circuit consisting of the three inductances, the variable capacitance, and the variable resistance, all connected in series. In order to measure resistance, as described in the paper, it is necessary to adjust the circuit for resonance. The resistance is determined by adjusting R to give the same current reading with the test coil or condenser in and out of the circuit. The difference in the reading of R then gives the value of resistance required.

Now the e. m. f. applied to this circuit is induced in Coil L_A. If we place across this coil in series, two equal resistances, we do not change the circuit in any way except to introduce a shunt across the generator coil. By connecting a detector from the midpoint of the two added resistances to ground, we transform the circuit into a bridge network, having two equal resistance arms, a third arm containing the resistance R, and a fourth arm containing L_o, L_x, and C in series. If we balance this bridge by adjusting R and C to give zero current through the detector, then L_o and L_x in series must be in resonance with C, and the total resistance of this arm must be equal to R.

This method is practically identical with the method described, except that it is unnecessary to measure the current, or even to hold it constant. Consequently, we do not have the same requirement of stability in the oscillator.

The general method described is really a substitution one, the assumption being made that the change in the capacitance of C may be made without changing the loss in the circuit. The usual assumption made is that if the dielectric loss is known or negligible, there are no other losses to be considered. However, at radio frequencies this is by no means the case, as in certain designs of capacitors we may have considerable eddy-current loss. Consequently one of the chief limitations of the method is in the determination of the loss in the standard capacitor. Knowing the loss in the standard, the method consists simply of substituting the unknown for the known, keeping other conditions the same and reading the change in the setting of the variable resistor R.

Probably the only absolute way of determining the loss in a standard capacitor is the calorimeter method, and there are so many difficulties to be overcome in a measurement of that type, that a high degree of accuracy can be obtained only by use of extraordinary precautions.

In conclusion, there is just one other point which makes a bridge method preferable to an indicating method. If the detector is a heterodyne type or some type which allows us to discriminate between the frequency at which we wish to make the measurement and other frequencies, it is possible to balance the bridge for one frequency only, that is, for the fundamental. Any indicating method such as described in the paper which is dependent on a current measurement, measures the total current in the circuit including all of the harmonics, and the result obtained becomes a function of wave shape.

A. Nyman: I was rather amused on the question of nomenclature and particularly as applied to condensers. The company I am connected with is making millions of condensers every year, and I believe if we started changing the name to capacitor we would find ourselves in difficulty; people wouldn't know what we were talking about.

In connection with the measurement of condensers in Mr. Burke's paper, particularly at radio frequency, we worked out a method of measuring condensers at radio frequency which I believe is very accurate and quite simple.

The accompanying figure shows the construction of this test set. It consists of two independent oscillating circuits both with adjustable capacities. By arranging the capacities close to the same values, a beat note is formed between the two oscillating circuits, and this beat note can be brought very close to 1000 cycles by introducing in the circuit a 1000-cycle note from a tuning fork. Thus, on the left-hand side of the diagram, a standard condenser is adjusted alone until the 1000-cycle beat note tunes in with the 1000-cycle note from the tuning fork. Then a condenser to be measured is inserted in parallel with the standard and the standard readjusted to get the same condition.

It is of course necessary to watch that this beat note is always on the same side of the resonance curve. This will be the condition if the resonance is always approached from the same side, say, reducing the condenser in each case. Since the frequency of the circuit is now the same as it was before, the total capacity is the same as it was originally.

It has been found that it is possible to gage the frequency of the beat between the two 1000-cycle notes to within one cycle every two or three sec. With a million cycles on each oscillating circuit, this would give an accuracy of comparison of condensers of one in a million. Of course, this accuracy is a good deal higher than is required under any practical circumstances. Moreover, the actual accuracy is limited by the construction of the standard condenser. With a General Radio type 222 standard condenser, this accuracy is not more than one in 25,000.

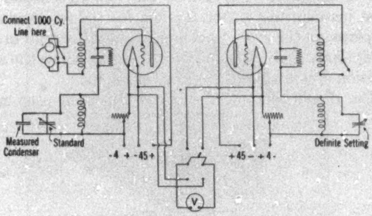

FIG. 1—PRECISION CAPACITY MEASUREMENT

This method of measurement has been found quite reliable and is not affected by any outside circumstances. Since the two measurements are made on the same oscillating circuit, and the measured condenser is generally considerably smaller than the standard, the load conditions on each tube are practically identical and there is no tendency to upset the frequency relations of the two circuits. Moreover, the coupling between these two circuits is so loose that any change in one circuit does not affect the oscillation in the other circuit until approximate resonance (beat note of about 25 to 60 cycles) is reached. Thus, at the 1000-cycle beat note, the effect of one circuit on the other is entirely negligible.

C. T. Burke: In regard to the points raised by Mr. Ferguson, the condenser is the weakest link in the circuit. We have found in fact that on very high frequencies of the order of 50,000 kilocycles or thereabouts, the use of a condenser of the soldered-plate type does show a noticeable improvement over the usual stacked-plate type.

In connection with the other question as to the effect of harmonics, since this circuit is tuned to the fundamental of the oscillator, the magnitude of harmonics in the circuit should be rather small, and as the actual value of current is not used in the computation but is only used to reset the circuit, harmonics in the circuit should not affect the accuracy of the method unless the magnitude of the harmonic differs between the two tunings, and there does not seem to be any reason for expecting this to occur.

As to the other point raised in the matter of nomenclature, the terms "capacitor" and "condenser" do refer to the same thing.

Condenser Shunt for Measurement of High-Frequency Currents of Large Magnitude

BY ALEXANDER NYMAN[1]
Member, A. I. E. E.

Synopsis.—*The necessity for an accurate ammeter for large high-frequency currents is pointed out. A new device consisting of a large condenser in parallel with a small condenser, and the latter carrying the current to a small thermocouple ammeter, is described.*

A device of this nature can be made very accurate; in fact, comparable in accuracy to any available standards.

The construction of the device includes provisions for restricting the electrostatic and electromagnetic field, due to large current, the reduction of distributed inductance and capacity, and a provision to prevent the resonance effect of high harmonics of the operating current. Provisions are also made for locating the measuring instrument at a distance from the circuit. Large ratings are possible by connecting a number of condenser blocks in parallel.

THE use of large broadcasting stations and other continuous-wave, high-power installations has created a demand for accurate means of measuring high-frequency current of large magnitude.

The methods so far in use are all limited in one particular or another. The use of the hot-wire expansion type instruments is not feasible for values above 10 amperes, as the size of heating element becomes excessively large and the skin effect does not allow the subdivision of hot wire into parallel elements.

The direct thermocouple type has been used with satisfactory results up to currents over 100 amperes, but the heating element of the higher ranges becomes bulky and expensive to build on account of the large-sized conductors and careful workmanship required. Also the skin effect becomes appreciable at the higher frequencies.

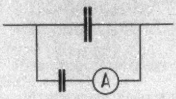

FIG. 1

An iron core transformer for reducing the high-frequency current so that it can be applied directly to a small instrument gives satisfactory results for frequencies up to 500 kc. For higher frequencies, the heating of the iron parts of the transformer becomes quite appreciable and is the greatest drawback. At 2000 kc. and above, it is difficult to use such a transformer; the heating of parts, the influence of stray fields, and the distributed capacity of windings become quite objectionable.

This article describes a novel arrangement which will permit the limits of operation to be extended as far as the present art of radio transmission requires.

The advantage of the new condenser type of ammeter for large currents lies in its accuracy and simplicity, combined with its comparatively low cost, even for the highest of frequencies.

Fig. 1 illustrates the method by which currents of large magnitude and high frequency can be satisfactorily measured. It consists in general of two condensers in parallel: a large one which carries the greater portion of the current to be measured, without appreciable voltage drop, and so constructed that it can pass large current at high frequency without appreciable losses; and one considerably smaller, designed to shunt off a predetermined fraction of the total current through a small ammeter, either of the hot-wire or the thermocouple type. In the latter case, the meter may be located at a distance from the main circuit, as illustrated in Fig. 2.

A device of this nature, if properly designed, will give satisfactory measurements of current at frequencies as high as 60,000 kc., which is practically the limit of the present-day operations of radio stations. It can be designed for higher frequencies. The error due to the resistance of the thermocouple element is practically negligible, even at the highest frequencies used. Thus, if the condenser has a capacity of 0.001 μf. and the thermocouple element has a resistance of 2.6 ohms, the error becomes one-half of one per cent at 6000 kc. (50-meter wavelength). For shorter waves, smaller capacity would be used.

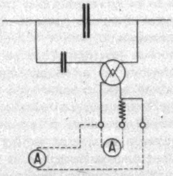

FIG. 2

The only source of error actually found in operation is due to the fact that the two condensers and the thermocouple form a closed circuit which has a resonant frequency that sometimes comes within the range of some harmonic of the frequency of operation of the instrument. The presence of such a condition becomes apparent in an obvious irregularity of the meter reading. Fig. 3 shows a very evident method to avoid this error at the resonant frequency. An auxiliary circuit which is tuned to the resonant frequency of the closed circuit referred to is connected to some point of the condenser shunt and actually absorbs the power of the harmonic from this circuit and in this way eliminates this error.

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Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

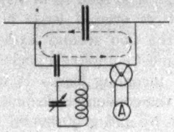

Fig. 3

Since this tuned circuit is connected only at one point, its effect at all other frequencies is entirely negligible.

It is worthy of notice that the accuracy of this instrument cannot really be checked by any available standards of high-frequency current. Probably the most accurate fundamental method of measuring high-frequency current is by means of the calorimeter ammeter in which the heating due to the current registers the value of that current in terms of the resistance of the heating element. Even this method, however, is subject to two errors which are difficult to eliminate. One is the actual value of resistance at the high frequency and the second is the distributed capacity of the heating elements and the calorimeter apparatus.

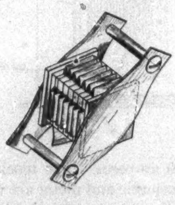

Fig. 4

If it is remembered that the capacity values used in condenser shunt are considerably in excess of any distributed capacity, and moreover, with a properly constructed mica condenser, these values are constant at all frequencies, and if it is further realized that the distributed inductance and resistance of leads are really negligible, the accuracy of this method becomes self-evident; it establishes a standard of large high-frequency current determined only by the accuracy of the meter element in series with the small condenser.

Fig. 4 illustrates the construction of the condenser element suitable for this apparatus. It shows a unitary structure with a powerful clamp and two capacity elements, both within this clamp. One element consisting of a number of metal foils in parallel gives the large capacity, while one extra foil brought out as a separate lead gives the small capacity. It is evident that the construction is made so symmetrical that there is no chance of one capacity changing relatively to the other. It will also be seen that the incoming and the outgoing leads of this condenser are on the same side of the clamp. There is no magnetic loop around the condenser element, therefore, and the losses are reduced to a minimum.

It is a well-known fact that for high-frequency operation, the presence of iron near the conductors results in considerable losses and consequent heating which may become prohibitive. In the improved construction of this element, although a steel clamp is used to hold together the condenser under a high pressure, the arrangement of conductors is such that with a current of 50 amperes the losses are negligible.

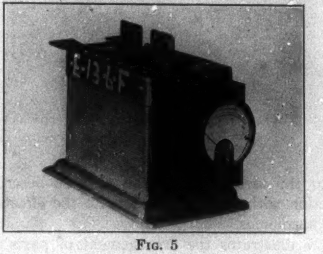

Fig. 5

A typical example for a condenser to operate at 50 amperes is as follows: The shunt element is 0.199 μ f. and the part constituting the small condenser is 0.001 μ f. The shunt element consists of a large number of metal foils, and therefore each foil will carry only a fraction of an ampere; also each one of these foils will be of a very short length. For this reason, the resistance losses in the foils will be quite small and it will carry a total current of 50 amperes, without appreciable heating.

At 6000 kc., the potential across this condenser would be only 6.6 volts. Although the clamp is of the same potential as the condenser, it is separated from it by a space of almost half an inch, which is sufficient to avoid the losses in the steel clamp due to the spreading electromagnetic and electrostatic field of the condenser element.

Fig. 5 is an illustration of a meter of this type constructed for operating with a current of 100 amperes. It will be seen that there are two leads coming out through the cover which can be connected in parallel or individually, depending on the current to be measured. There are two condenser elements corresponding to these leads. Only one of these condenser elements contains a small capacity in series with the meter, that is, the element closest to the meter end. If this element alone is used, the reading is 50 amperes. If the second element is also connected, the reading becomes 100 amperes, and the meter readings must be multiplied by 2 without affecting the calibration of the instrument.

Fig. 6 shows this condenser viewed from the bottom, with the base removed. The two condenser elements will be seen inside a perforated aluminum container with the thermocouple and instrument mounted on one side. A small adjustable condenser and spool next to the meter are for the purpose described in connection with Fig. 3, to eliminate the effect of the higher harmonics.

Fig. 6

Fig. 7 illustrates the arrangement of parts in this instrument. It shows a small resistance element *R* connected in the output leads of the thermocouple. This is a small spool of resistance wire which is equivalent in resistance to a certain length of lead so that when it is desirable to use the meter at a distance from the circuit this can be done without any change in the calibration by leaving out the spool and substituting the resistance of a standard length of leads. Fig. 7 also shows that the leads from the condenser element to the heater are arranged to have the minimum effect from stray fields and a minimum induction. These leads are seen to be made of two twisted wires, see Fig. 6, placed together closely, and brought to the thermocouple. Fig. 7 shows the terminals located at the end opposite to the meter, and when connected to the circuit, the leads would be going in a direction away from the meter.

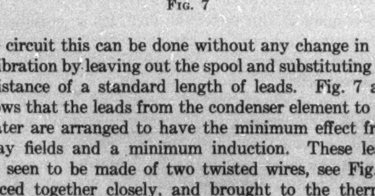

Fig. 7

If the meter is used at a distance from the thermocouple, the leads from the latter to the meter are brought through a metal shield casing which is grounded to prevent any radio frequency from affecting the circuit.

Fig. 8 shows the mounting of the condenser element inside the casing. It is seen to be held by two metal plates which act as ground leads of the condenser element. Thus the two ground leads are parallel to each other and almost completely enclose the condenser element and the incoming lead. It will be observed from Fig. 6 that the condenser element is connected to these metal plates by means of several thin, wide metal leads. It can be seen readily, by observing Fig. 8, that the electrostatic field is confined within the ground leads and its influence on the meter and the thermal element is thereby minimized.

For high current ratings, the principle of block additions of condenser elements is utilized. Thus for 50 amperes, one block is used; for 100 amperes, two are used; for 150 amperes, three blocks are used; etc. The same thermocouple and meter are used in each case and the scale need only be multiplied by the number of blocks connected.

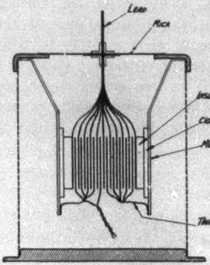

Fig. 8

With the arrangement described, the ratio of capacity is 200 to 1; therefore, the thermocouple will carry only a quarter of an ampere while 50 amperes are flowing in the radio frequency circuit. A quarter of an ampere is quite practical for a sensitive and accurate thermal instrument. With 100, 150, or 200 amperes, the thermocouple will receive the same current, but the ratio will have changed in accordance with the capacity, that is, the number of blocks.

A meter as described above has been in continual use for over two years on a testing set where the frequencies have ranged from 100 kc. to 6000 kc., and the current values have ranged up to 120 amperes. It has been found that the meter indications are consistent and reliable at all these values. In fact, its accuracy has been such that it was possible to measure the voltage in a circuit by connecting such an ammeter in series with a known condenser, and determining the voltage drop in the condenser by calculation.

For developing a special thermocouple ammeter used in connection with the condenser shunt, an acknowledgment is due the Weston Electrical Instrument Corporation of Newark, N. J.

Discussion

B. E. Lenehan: It is not apparent why the capacity of a condenser can be assumed to be constant up to frequencies as high as 6000 kilocycles. Two conductors spaced approximately 10 times their diameter will have an inductive drop at 50 amperes 6000 kc. of about 345 volts per ft. The mutual inductance between the meter and shunt circuit can hardly be disregarded as only 1 per cent coupling would introduce errors of around 50 per cent.

The method is a very good practical way of making high-frequency measurements after its accuracy has been established, but it can hardly be classed as a standard. The well known difficulties of shielding high-frequency circuits are evidently due to the high resistance of all materials. This effect accounts for the use of only moderately high frequencies in induction furnaces.

We must remember that no method of demonstrated accuracy exists for measurement of currents above 1000 kc. Resistances and inductances are all affected by the proximity of the return conductor of a circuit if any exists and the usual effects by which one makes measurements are complicated by the fact that the usual "constants" of a circuit are not constant.

Alexander Nyman: Concerning the statements of Mr. Lenehan, with regard to the voltage drop on leads at 6000 kc. and 50 amperes, it is not clear what size of lead he is contemplating, as of course the bigger the lead the less would be the drop per unit length. It is of course also true that the leads to condenser shunt are short from the junction point of the small condenser and these leads are so arranged as to confine the magnetic field to a small space and for this season the voltage drop is practically negligible.

The inductive effect of these leads is again reduced by their mutual shielding arrangement, and by the fact that the leads to the thermo-couple are very short. This inductive effect can of course be determined by connecting the small condenser element to the large, in which case it will be found that no current passes through the thermo-couple. In other words, the mutual inductance which Mr. Lenehan mentions is much less than 1 per cent and probably could not be readily determined.

With regard to the second paragraph, the writer is fully aware of the difficulties of shielding as this factor had been carefully studied in the design of the condenser shunt. The shortness of leads, the construction which encloses the incoming lead by outgoing leads, and a complete separation in a metallic container are the essential features which were found necessary for shielding. In other words, if the construction is such that there is no external field except in close proximity to the conductors, then the magnetic effect evidently is much easier to shield than than for an exposed conductor.

Methods for the Measurement of Radio Field Strengths

BY C. R. ENGLUND[1] and H. T. FRIIS[1]
Non-member Member, A. I. E. E.

***Synopsis.**—This paper presents a review of successful methods of measuring several classes of radio waves, (a) sinusoidal waves of frequencies below 1000 kilocycles and (b) sinusoidal waves of frequencies above 1000 kilocycles, (c) telegraph dots and dashes and (d) static.*

* * * * * *

FOR several reasons the radio art was well advanced before the most fundamental measurements, those of the signal strength in free space, were undertaken. Chief among these reasons, perhaps, was the fact that the damped spark transmitter was the most common generator. Such a radiator has so many variables that accuracy in measurement is difficult. The simultaneous advent of the vacuum tube generator and amplifier furnished the sinusoidal radiation and the sensitive receiving set, so that an exact knowledge of the field strength became a matter of greater engineering moment and at the same time more accurate and sensitive means of measurement became available.

The earliest transmission measurements were made by inserting sensitive a-c. ammeters in the receiving antenna. Later the coupled circuit with crystal rectifier and d-c. instrument was used. Here the coupling allowed a matching of circuits and the high sensitivity of the d-c. instrument made it a moderately sensitive method. Still later the telephone and audibility meter were added, increasing the sensitivity and making the measurement one differing only slightly from the normal operating technique, a desirable feature. The advent of the vacuum tube amplifier and detector enormously increased the intensity range of the signal which could be received and estimated as a unit of audibility.

There were serious deficiencies, however. The measuring unit was too far away electrically from the antenna, the step from free space to antenna was not bridged, and the threshold of audibility which furnished the basis of measurement was not an accurately reproducible standard. One of us, therefore, in 1917[2] proposed the idea of introducing a local measurable signal into the antenna and turning the measurement setting into a comparison between local and distant signals. The advantages would be that the type of receiving set would become immaterial, since this set serves merely as a comparator, and the comparison would become one of equal voltages impressed in the antenna, the only antenna constant required being the so called effective height. This method has now become standard for frequencies below 10^6 cycles,[3] (300 meters,) and while it has been found inapplicable at higher frequencies, due to technical difficulties,[4] some of its features are retained. Its chief disadvantages are the physical difficulties of generating and controlling the local signal and the increased cost of apparatus. The limiting signal which can be measured, in the absence of static, is fixed by the leakage through the comparison oscillator shielding and it is the control of this that has made the method feasible.

From the theoretical viewpoint, there is no question as to what should be measured, and granting that technical troubles do not intervene, the same holds for the practical viewpoint. This is to measure the electromagnetic condition at the receiving antenna, or rather the condition as it would be if the antenna were not there. Whether the unit should be that of energy density, electric field, or magnetic displacement may be debatable; it has been chosen as a practical matter, and by common consent, as the electric field expressed in microvolts per meter. This gives a convenient unit, the range of from 0.1 to 500,000 microvolts per meter representing the present reception levels. Since the energy density in free space of an electromagnetic disturbance is

$$W = \frac{E^2 + H^2}{8\pi}$$

and for pure radiation fields $|E| = |H|$, in absolute electrostatic and electromagnetic units respectively, it is immaterial which is chosen. For very short transmission distances, the second equation does not obtain as the fields are not pure radiation fields[5].

When a fine wire conductor, where eddy currents may be neglected, is exposed to an electromagnetic radiation, a total electromotive force is developed along the wire length which is equal to

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[1] Both of the Bell Telephone Laboratories, Inc., 463 West St., New York, N. Y.

[2] Englund, *Proc. I. R. E.*, **5**, p. 248, 1917, Discussion.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., at Pittsfield, Mass., May 25-28, 1927.

[3] Englund, *Proc. I. R. E.*, **11**, p. 26, 1923, "Note on the Measurement of Radio Signals"; Bown, Englund, & Friis, *Proc. I. R. E.*, **11**, p. 115, 1923, "Radio Transmission Measurements"; Jensen, *Proc. I. R. E.*, **14**, p. 333, 1926, "Portable Receiving Sets for Measuring Field Strengths at Broadcast Frequencies"; Jensen, *Phy. Rev.*, **26**, p. 118, 1925, "Potentiometer Arrangement for Measuring Micro-Voltages at Radio Frequencies."

[4] Friis & Bruce, *Proc. I. R. E.*, **14**, p. 507, 1926, "A Radio Field-Strength Measuring System for Frequencies up to 40 Megacycles."

[5] See Appendix I.

$$\int_a^b E_s\,d\,l$$

where E_s is the component of electric field parallel to the wire and a and b are the coordinates of the wire ends. If $a \neq b$ we have the ordinary "open" or "marconi" antenna; if $a = b$ we have the closed or "loop" antenna. In the latter case, the line integral around the closed curve always gives the same value as the surface integral of the rate of change of magnetic displacement through the closed curve. This is the origin of the inexact notion that the open antenna operates from the electric field and the closed antenna from the magnetic field.

When either of the antennas mentioned is exposed to electromagnetic fields, currents flow in it. In the general case the relation between current and driving field at any point on the conductor is a very complicated one and while an equivalent lumped electromotive force can always be found to replace the distributed electromotive force in a single given connection of input and indicating apparatus, the most desirable state of affairs is that when the integral of the wire component of the electric field can be replaced by an equal total lumped electromotive force. This condition can be met for an open antenna by making it short and adding large capacity areas at the end; for a loop antenna we need only make the dimensions small and wind to such an inductance that the tuning capacity greatly exceeds the natural or "shunt" capacity of the conductor. A comparison in the antenna itself of a locally generated electromotive force and the signal electromotive force, both of the same frequency, does not require an antenna tune. It is advisable to tune, however, because of the sensitivity and selectivity resulting. The free space field can then be determined in terms of what may be termed the "effective height" of the receiving antenna. For the open antenna, the effective height is the physical length and the resultant voltage in the antenna V_0 is related to the electric field E by the condition $V_0 = h_0 E$. For the closed antenna of N turns, the resultant voltage is

$$V_L = \frac{N}{c}\,\frac{\partial}{\partial t}\int\int B_s\,d\,s$$

which is to be evaluated for any given field and loop area. If the loop is in free space and is so small that the field is sensibly constant over its area A and if the field is sinusoidal of frequency $\frac{\omega}{2\pi}$ then

$$V_L = \frac{\omega N A H}{c}\,.$$

Now for pure radiation $|H| = |E|$ and accordingly an effective height or height of equivalent open antenna h_L can be defined as

$$V_L = E\,h_L = \frac{\omega N A E}{c} \text{ or } h_L = \frac{\omega A N}{c}$$

and is convenient as long as the transmission distance is great enough to assure only a radiation field. The tacit assumption is of course made that the fields are rectilinear and the antennas oriented for maximum pick-up.

As a practical fact, we have had little success in using an open antenna as an antenna for which the effective height is determined from physical dimensions. We have always had to rely on the loop as the absolute antenna type. In the last analysis, then, all transmission measurements and formulas are based on a computed loop effective height which it is impossible to check by direct measurement. We have had no indirect evidence to date showing that the effective height computed in this manner is incorrect.

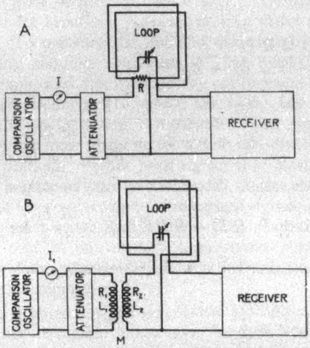

FIG. 1—MEASURING METHODS OF LONG WAVE FIELD STRENGTH

The method of introducing the sinusoidal comparison voltage into the antenna is at the disposal of the operator. Two which have been commonly used are the resistance drop and the mutual inductance voltage[6]. Figs. 1-A and 1-B are schematic of these.

If K be the current attenuation ratio, $(K > 1)$, then the voltage introduced into the antenna is, in the two cases,

$$\frac{R I_1}{K},\ \frac{i\,\omega M I_1}{K}$$

and the currents resulting in the antennas, (we actually indirectly compare voltages through the resulting currents), are

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[6] Vallauri, *Proc. I. R. E.*, **8**, p. 286, 1920; Lunnon, *Inst. El. Eng. Jl.*, **59**, 1921, p. 685; Beverage and Peterson, *Proc. I. R. E.*, **11**, 1923, p. 661; Hollingsworth, *Inst. El. Eng. Jl.*, **61**, 1923, p. 501.

$$\frac{R\,I_1}{K\,(R+Z)}, \quad \frac{i\,\omega\,M\,I_1}{K\,(i\,\omega\,L_2 + R_2 + Z)}$$

where Z is the antenna impedance beyond the local input terminals. Now the currents produced in the antennas by the radiation fields are

$$\frac{h\,E}{\dfrac{R\,r}{r+R} + Z}, \text{ and}$$

$$\frac{h\,E}{R_2 + i\,\omega\,L_2 + \dfrac{\omega^2\,M^2}{r + R_1 + i\,\omega\,L_1} + Z}$$

where r is the impedance of the attenuator looking backwards into it. The condition of equality means that

$$\frac{R\,I_1}{K\,(R+Z)} = \frac{h\,E}{\dfrac{R\,r}{r+R} + Z}$$

$$\frac{i\,\omega\,M\,I_1}{K\,[R_2 + i\,\omega\,L_2 + Z]} = \frac{h\,E}{R_2 + i\,\omega\,L_2 + \dfrac{\omega^2\,M^2}{R_1 + i\,\omega\,L_1 + r} + Z}$$

in the two cases. Now it is very easy to make $r >> R_1$ and, this done, $R\,I_1 = K\,h\,E$ or $E = \dfrac{R\,I_1}{K\,h}$ for the first equation. For the second, it is equally simple to make

$$\left|\frac{\omega^2\,M^2}{R_1 + i\,\omega\,L_1 + r}\right| << \left|R_2 + i\,\omega\,L_2 + Z\right|$$

and, this achieved,

$$K\,h\,E = i\,\omega\,M\,I_1 \text{ or } E = \frac{i\,\omega\,M\,I_1}{K\,h}$$

In case a loop is used, the last equation further simplifies. For substituting the effective height of the loop gives

$$|\,E\,| = \frac{\omega\,M\,|\,I_1\,|}{K\,h_L} = \frac{M\,c}{K\,A\,N}\,|\,I_1\,|$$

and the frequency has cancelled out which is an advantage. The relative availability of these two methods will depend somewhat on the occasion. A workable mutual inductance unit may be purchased in the open market while a resistance unit cannot be so bought. On the other hand, it is our experience that a resistance can be made without a-c. calibration which will function more accurately than a small mutual inductance can be calibrated, particularly when shielding is necessary to prevent antenna pick-up. Another point in favor of the resistance input is that the reactance of the unit can be as much as 14 per cent of the resistance before the impedance drop across it increases 1 per cent. Moreover, the constant impedance attenuation network of resistances is the first that comes to mind because of its independence of frequency and a resistance termination is then the logical one. No question of efficiency is involved since rejection and not conservation is aimed at.

We have used two types of resistance attenuator[7] and have found that below 1000 kc., safe resistance units can be made and used. Even if reactive units should result, a reasonable agreement of phase angle among them will maintain the high accuracy of the resulting attenuator and the utility of the resulting instrument exceeds that of similar apparatus made up of inductances or capacities. For example, the shielding of a resistance attenuator is very easy, a statement not valid for an inductance attenuator. At high frequencies, the unavoidable inductance of leads ruins the operation of a capacity attenuator.

The theory of the measurement by means of a comparison signal in the antenna has been sketched above; it was also stated that a different method was forced upon us at the higher frequencies by reason of the impossibility of constructing predeterminate attenuating networks. Several suggestions have been investigated but at these frequencies the dimensions of conductors having negligibly small impedances become unworkable and it is necessary to make a radical move. The resulting method[8], requiring a double detection type of receiver, may be sketched as follows.

The loop signal and beating oscillator voltages are applied to the first detector grid. (The beating oscillator input may also be introduced into the detector plate circuit.) The detector output passes through a controllable attenuation element and a selective intermediate frequency amplifier, the amplified intermediate frequency being applied to the last or low-frequency detector grid. A meter in this detector plate circuit serves as an indicator and by adjusting the attenuation unit a convenient deflection is obtained from the signal, Fig. 2A. Let this attenuation ratio be K_1. The beating oscillator is now disconnected and a comparison oscillator input introduced into the loop and increased until a readable deflection is obtained in a meter in the first detector plate circuit, Fig. 2B. This detector having been calibrated as a vacuum tube voltmeter, the voltage is read directly from the calibration curve, say V. Next the beating oscillator is reconnected and the attenuation increased until the output meter gives the original deflection. The attenuation is now K_2 and it is evident that the original signal voltage applied to the first detector grid was $\dfrac{K_1}{K_2}\,V$. Finally, the loop tune is short-circuited so that the unchanged comparison oscillator output is applied directly to the first detector grid. A reduction in the attenuation to a third setting K_3 brings the output up to its original value. The signal voltage picked up by the loop is thus,

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[7] Englund, loc. cit; Jensen, loc. cit.

[8] Friis and Bruce, loc. cit.

$$\frac{K_1 \ K_3}{K_2^2} V$$

and equating this to the product of field strength times loop effective height gives

$$E = \frac{K_1 \ K_3}{K_2^2} \cdot \frac{V}{h_L} .$$

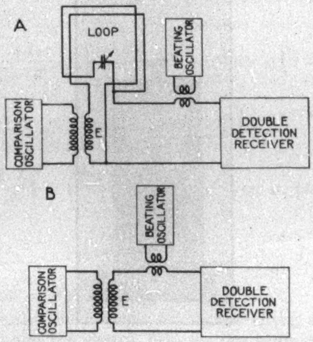

Fig. 2—Measuring Method of Short Wave Field Strength

As practical procedure, V is conveniently chosen as one volt. The necessary condition is that the intermediate frequency output of the first detector be proportional to the product of the two inputs over the wide range from signal voltage up to unit voltage, a condition which is found to be satisfied. The attenuation unit, being in the intermediate frequency circuit, operates accurately as in the direct use for lower frequencies. It may be added that when consideration is taken of the extraordinary variability of short wave transmission, both in amplitude and polarization angle, the accuracy of measurement required is not great and is amply met by the apparatus described above. In fact, we believe that the significant type of measurement for short wave signals is a continuous record over extended periods, such as would be obtained by an automatic recorder operating out of a receiving set. It will be necessary, at any rate, to record both the amplitude and the polarization angle.

Another type of signal comparison measurement, very easy of execution, is that of erecting a local transmitting antenna of low effective height and comparing its radiation with that of the signal. It is not possible to make this an accurate method as a little consideration will show. A method of determining the effective height of an antenna by a set of interconnected measurements on three antennas has been proposed[9] but it is difficult to see how it can compare with a loop measurement in accuracy. Naturally, an open antenna has a great effective height compared to that of a usable loop and the problems of comparison oscillator shielding, high current attenuations, and high amplification at a low noise level, in great measure fade out.

The discussion so far has assumed a simple sinusoidal signal but this is not the only type awaiting measurement. The major applications of radio are for communication purposes and such applications require a modulation of the simple sinusoid. To mention only two such, we have the broadcast transmitter and the radio telegraph sender. The former carries a dominant unmodulated carrier and measurements are simply made on this carrier frequency using a meter in the final detector plate circuit for comparison. Normal signal strengths for broadcast purposes make a meter reading simple. To measure the dot and dash amplitudes of telegraphy requires something of the nature of an oscillograph, and the cathode ray oscillograph using a neon tube discharger[10] to spread the beam has proved serviceable as a visual comparator. The signal intensities are here likely to be small and static interference marked. It has been shown that static interference is proportional to the width of the resonance band[11] of the receiver and for telegraph signals a sharp low-frequency tune will reduce the interference so greatly that it has usually been found possible to measure with the cathode ray tube if the signals can be read[12] in the phones.

Static is another signal type awaiting measurement. The rather intangible "noise value" of the static is what it is desired to measure in radio telephony and for rectifying systems such as telegraph recorders and printers the total static energy is of greatest importance. The first static measurements were obtained by measuring the intensity of the telegraph signal which was just masked or rendered unintelligible by the static. Consistent results can be obtained on continuous static by this method. For telephony, the same considerations demand a signal representative of speech and the so called "warbler"[13] was chosen for this. This signal is produced by varying the frequency of a continuous-wave oscillator, over the range covered by the transmission band of the receiver, at a slow rate—5 to 10 times per sec. But the great variability of static, in direction as well as in amplitude and type, makes isolated observations of limited value and an automatic static recorder appears to be the logical attack upon the static measurement problem. Because of the extreme ranges of received energy which occur, it is necessary to invert such a measurement and instead of recording the static itself, record the gain of the receiving set necessary to maintain the static output constant; and this has been the method chosen[14]. The particular output metering sytem which seems to have the greater number of advantages is one employing a "fluxmeter" or non-restoring type of deflection instrument to add up the energy received for a definite interval. This is not a "lagging" type of average such as a sluggish thermocouple would give and is yet immune from the wild fluctuations of set gain which result when a "rapid" thermocouple is used.

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[9] Pession, *Rad. Rev.*, **2**, 1921, p. 228.

[10] Bailey, *Phy. Rev.*, **25**, p. 585, 1925.

[11] Carson, *Bell System Tech. Jl.*, **4**, p. 265, 1925.

[12] Espenschied, Anderson, & Bailey, *Bell System Tech. Jl.*, **4**, p. 459, 1925.

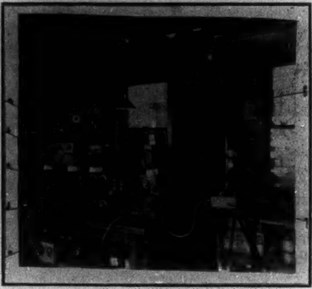

FIG. 3—FIELD STRENGTH MEASUREMENT APPARATUS, 15-500 KC.

a. Attenuator; b. Comparison oscillator; c. Intermediate frequency amplifier and low frequency detector; d. Intermediate frequency filter; e. Intermediate frequency detector and beating oscillator; f. Loop; g. Loop input resistance

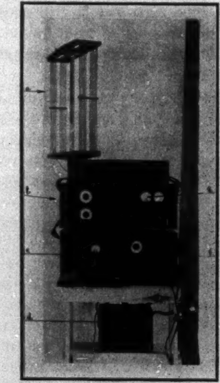

FIG. 4—FIELD STRENGTH MEASUREMENT APPARATUS, 500-3000 KC.

a. Loop; b. Double detection receiver; c. Attenuator; d. Batteries; e. Output meter; f. Comparison oscillator

The static measurement desired is naturally a measurement of the interfering effect or signal destructiveness of the static. This is exactly the property of greatest interest since the signal to static ratio thus defined is the engineering factor of safety for continuous transmission of the desired intelligence.

In closing this resume of radio field measurement methods of today, a few observations from the practical side may be welcome. We have had occasion to make measurements of field strengths from 15 kc. to 25 megacycles and from 0.1 to 500,000 microvolts per meter. It is a striking fact that for the entire frequency range a measuring set sensitivity of 0.1 microvolt per meter reaches the weakest usable signals. From a radio telephone point of view the most important frequency ranges at present are 40-70 kc., 550-1500 kc., and 3 to 25 megacycles. In Figs. 3, 4, and 5, reproductions of photographs of measuring apparatus for each of these ranges are given. This apparatus possesses enough flexibility to bridge the frequency gaps and cover the 15-kc. to 25-megacycle range already mentioned. The measuring sets of Figs. 3 and 4 measure by means of local comparison signals in the loops; the apparatus of Fig. 5 uses a calibrated intermediate frequency detector with an attenuator in its output circuit as already described. All three sets are double detection receivers and are intended to be portable.

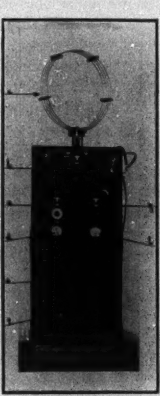

FIG. 5—FIELD STRENGTH MEASUREMENT APPARATUS, 1-40-MEGACYCLES

a. Loop; b. Antenna circuits; c. Comparison oscillator; d. Intermediate frequency detector meter; e. Attenuator; f. Intermediate frequency amplifier and low frequency detector; g. Intermediate frequency detector and beating oscillator; h. Output meter

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[13] Bown, Englund, and Friis, loc. cit.

[14] Friis, *Bell System Tech. Jl.*, 5, p. 282, 1926.

Appendix I

The "Hertzian doublet" is essentially a rectilinear antenna with such large capacity areas at its ends that the current in the linear portion is everywhere the same. If we now assume such an antenna in free space with a capacity area separation of h and carrying a current $I_0 \sin \omega t$, then at a distance d such that $d > > h$, the fields, in cylindrical coordinates, are given by the following expressions, where the center of the antenna has been chosen as the zero of the coordinate system and the antenna along the Z axis:

$$H_\theta = \frac{h I_0 r}{d^2} \left[\frac{\omega}{c} \cos \omega \left(t - \frac{d}{c} \right) + \frac{\sin \omega \left(t - \frac{d}{c} \right)}{d} \right]$$

$$E_r = \frac{h I_0 r z}{d^4} \left[\left(\frac{\omega d}{c} - \frac{3 c}{\omega d} \right) \cos \omega \left(t - \frac{d}{c} \right) + 3 \sin \omega \left(t - \frac{d}{c} \right) \right]$$

$$E_z = \frac{h I_0}{d^4} \left[- \left(\frac{\omega r^2 d}{c} + c \frac{2 z^2 - r^2}{\omega d} \right) \cos \omega \left(t - \frac{d}{c} \right) + (2 z^2 - r^2) \sin \omega \left(t - \frac{d}{c} \right) \right]$$

where $d = \sqrt{r^2 + z^2}$.

Obviously, the magnetic field lies in circles about the z axis and the electric field in the plane containing d and the z axis. As before, the units are absolute electromagnetic and electrostatic, respectively.

For distances where d is not great compared with h, these formulas must be integrated over the antenna. Thus, for the magnetic field,

$$H_\theta = r \omega \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{I_l \cos \omega \left(t - \frac{\sqrt{r^2 + (z' - l)^2}}{c} \right)}{r^2 + (z' - l)^2} \, d l + r \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{I_l \sin \omega \left(t - \frac{\sqrt{r^2 + (z' - l)^2}}{c} \right)}{[r^2 + (z' - l)^2]^{3/2}} \, d l$$

With similar expressions for the electric field, (r, z') are the coordinates of the point of observation and the symmetry makes a θ coordinate unnecessary. In the formula I_l, the current function is arbitrary and may be any given function of l. Until this is specified, the integrations cannot be carried out.

For the equatorial plane, $z = 0$, the simple doublet equations reduce to

$$H_\theta = h I_0 \left[\frac{\omega \cos \omega \left(t - \frac{r}{c} \right)}{c r} + \frac{\sin \omega \left(t - \frac{r}{c} \right)}{r^2} \right]$$

$$E_z = h I_0 \left[- \frac{\omega \cos \omega \left(t - \frac{r}{c} \right)}{c r} - \frac{\sin \omega \left(t - \frac{r}{c} \right)}{r^2} + \frac{c \cos \omega \left(t - \frac{r}{c} \right)}{\omega r^3} \right]$$

Notice that the first two terms in H_θ and E_z are separately equal. At great distances, all but the first terms in H_θ and E_z become negligible, these two terms being the radiation fields. It is easily seen that the radiation components $|E_z|$, $|H_\theta|$, and r form, in the order named, a right hand screw system.

To pass from the antenna in space to a grounded antenna we need only notice that if the grounded antenna is half the length of the free antenna, the fields in free space are the same. We assume, of course, that the ground conductivity is great enough to produce a true antenna image.

In amperes, volts, and meters the electric field in the meridian plane of the doublet becomes, at great distances,

$$| E | = \frac{60 \pi}{r \lambda} \cdot h I_0$$

Quantitative Determination of Radio Receiver Performance

BY H. D. OAKLEY[1]
Associate, A. I. E. E.

***Synopsis.**—The practise of making quantitative measurements on the individual units of radio receivers is quite general, but seldom are such measurements made on receivers as a whole because of certain difficulties encountered in this type of measurement. This paper describes apparatus developed to overcome these difficulties and to make possible a study of the performance of receivers as such. The over-all characteristics of receivers are classified and described, the method of tests for obtaining measurements on them explained, with some curves shown to illustrate the results obtained from these tests.*

* * * * *

INTRODUCTION

IN the development and design of a device consisting of more than a single unit, it is advantageous to subject it to a series of tests to determine its operating characteristics as a whole, as well as the characteristics of the individual units alone. A radio receiver is comprised of apparatus performing, as a rule, five main functions: (1) Selecting a voltage of a particular radio frequency from among several co-existing voltages, the frequency of each one being different from that of the desired voltage; (2) amplifying the selected voltage; (3) deriving from this amplified voltage an audio-frequency voltage; (4) amplifying the audio-frequency voltage; (5) converting it into sound energy. Present day receivers perform functions (1) and (2) simultaneously in the radio-frequency selector amplifier, function (3) in the detector, (4) in the audio frequency amplifier, and (5) in the loud speaker. Testing the apparatus performing any one of these five functions is relatively easy but to make measurements of the whole assemblage is rather difficult.

It is the purpose of this paper to describe briefly the test apparatus used by the General Electric Company, to explain the quantities measured, and the method of test, and to present some results obtained from over-all tests on receivers.

To obtain satisfactory results from over-all tests, three main conditions must be satisfied. 1. To the input terminals of the receiver must be supplied a radio-frequency voltage of a character and magnitude comparable with normal operating conditions. 2. The testing apparatus must be so constructed and arranged that the receiver is subjected to the signal in a known manner and is not affected by unknown stray signal effects. 3. The test conditions must be so controlled that the receiver is not influenced by electrical disturbances other than the test signal.

DESCRIPTION OF TEST APPARATUS

The apparatus developed to meet these conditions consists of a signal generator, a current-controlling and measuring device, a voltage attenuator, a dummy antenna, and an output voltmeter. The first two units are in a shielded room and in another are the last three, as is also the receiver to be tested. The signal generator is a miniature broadcast transmitter employing the Heising system of modulation, and is composed of five main units: (1) An audio-frequency oscillator with a range of 40 to 10,000 cycles; (2) a radio-frequency oscillator, having a range of from 550 to 1500 kilocycles; (3) a modulator, with its grid controlled by the audio-frequency oscillator and its plate controlling the plate voltage of the modulated radio-frequency amplifier; (4) a modulated radio-frequency amplifier the grid of which is excited by the radio-frequency oscillator and whose output is varied at an audio-frequency rate by the modulator; (5) a power amplifier (provided with a modulation indicator), excited by the radio-frequency amplifier. The power amplifier supplies current to the current controlling and measuring device. This is a variable mutual inductor with its primary connected to the signal generator and in its secondary circuit a tuning condenser, a thermoammeter, and a voltage attenuator. The value of the current flowing in the voltage attenuator is measured by the thermoammeter, and is controlled by varying the mutual inductance. The attenuator is a special form of self inductor, first proposed and used by Dr. N. H. Williams of the University of Michigan. It consists of a coaxial metal shell and rod, which terminate at one end in a metal plate perpendicular to their axes. At the point where the rod meets the plate it is connected to ground. The design of this inductor is such that its inductance per unit length is easily calculated. This particular inductor has three taps providing inductances of approximately 0.001, 0.0055, and 0.02 microhenrys. With these taps and an input current, range of from 0.5 to 350.0 milliamperes, a voltage range of from 1.5 to 22,000 microvolts may be obtained at the lower frequency end of the broadcast range, (550 to 1500 kilocycles), and three times these values at the higher end. The dummy antenna is a circuit so designed that its characteristics are representative of those of the average broadcast receiver antenna. An arbitrary value of ten meters is given as its effective height, because it is believed that this is the average value of antennas commonly used. The output voltmeter

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measures the effective value of audio-frequency voltage existing across the output of the receiver. It has a range of 0.2 to 250 volts and its indications are independent of frequency. The impedance (160,000 ohms) of the voltmeter is so great compared with that of the loud speaker that the extra load imposed upon the circuit by the voltmeter is negligible.

This concludes a description of the apparatus used in making the usual over-all characteristic tests. The quantitites measured will be explained now; then the test procedure for obtaining them.

QUANTITIES MEASURED

The characteristics commonly measured are sensitivity, selectivity, and quality. In some special cases, the radiation from the set is also measured. Sensitivity is defined as the degree to which a radio receiving set responds to signals of the frequency to which it is tuned. The output of a receiver is not directly proportional to the input field strength. Therefore, sensitivity cannot be expressed by a single figure, but takes the form of a curve. Sensitivity can be expressed by an input-output curve; *i. e.*, a curve showing the relation between the potential induced in the antenna circuit and the voltage existing across the loud speaker. It is convenient, however, to express sensitivity as the ratio of output voltage to input field strength at various output voltages. This method is somewhat analogous to transformer practise wherein relations are found between the transformer ratio and the voltage supplied to the load at various loads, rather than between impressed primary voltage and voltage supplied to the load. The ratio of output voltage to input field strength, expressed by a dimensional formula, reduces to a length. Since this is so and since the field strength is expressed as a certain potential per meter, the unit of sensitivity has been called the meter. A receiver is said to have a sensitivity of one meter when a field with strength of one volt per meter acting upon the antenna circuit causes a potential of one volt to exist across the loud speaker.

Selectivity is the degree to which a receiving set is capable of differentiating between signals of different frequencies. As measured, selectivity is a curve showing the input field strength required to maintain a constant signal voltage in the output as the frequency of the field is changed. Of course, the field strength is lowest at the frequency of the desired signal, and increases in value as the frequency becomes greater or less than that of the desired signal.

The definition of quality is the degree to which sound is faithfully reproduced. To be strictly in accord with this definition, quality measurements made on receivers should be such that the sound wave modulating the radio wave actuating the receiver can be compared with the sound wave generated by the loudspeaker of the receiver. Then the results of these measurements should be expressed in such a way that the nature and magnitude of the discrepancies between the original and the reproduced sound waves can be shown. The measurements as actually made are not so rigid. The assumption is made that the radio-frequency voltage induced in the antenna circuit is of such a nature that, were the receiver and sound reproducing apparatus perfect, the reproduced sound would be identical with that which acted upon the microphone of the transmitter. The measurements as made furnish data showing the relation between the receiver output voltage and the modulation frequency, as this frequency is varied from 40 to 10,000 cycles, without changing either the voltage induced in the antenna or the degree of modulation. A perfect receiver subjected to this test would maintain a constant value of voltage across its output and therefore its quality curve would be a straight line parallel to the modulation frequency axis. In order to compare the quality of different receivers, it has been the practise to plot these curves as output voltage in percentage of output voltage at some one frequency (usually 1000 cycles), against frequency. The more nearly the output voltage remains constant, the better is the quality.

The last receiver characteristic to be considered is radiation. Radiation is defined as the process of emitting electromagnetic waves into space. Obviously it is impossible to measure a process, so that when radiation measurements are discussed, the act in mind is not that of measuring the process but rather that of measuring the quantities causing the phenomenon of radiation. The relative distances to which transmitters can maintain communication depend upon their antenna height and the current flowing in the antenna. The antenna height is expressed ordinarily in meters and the current in amperes, so that it is customary to express the radiating power of a transmitter in meter amperes—the product of the antenna height and the current. Some receivers contain an oscillating tube and it is possible to so adjust the controls of certain others that one or more tubes will oscillate. In both cases disturbances are sent out which seriously interfere with the operation of neighboring receiver sets. Tests are made to determine both the magnitude and frequency of this disturbance. The results are expressed in meter amperes, since, in this case, the receiver is considered as a transmitter and the strength of its field at various distant points can be determined if the equivalent meter amperes and the distances are known.

METHOD OF TEST

Sensitivity, selectivity, and quality are more or less inter-related so that, in general, it is impossible to adjust a receiver to obtain a maximum of one of these quantities without a decrease in one or both of the other two. In making tests on a receiver an attempt is made to so adjust it that a satisfactory compromise among the three is reached. This condition is considered as the average normal operating one. Fig. 1 is a schematic layout of the test apparatus and an antenna type receiver which is undergoing test. An RCA model-100 loud speaker is used as the output load, and across it is measured the output voltage.

Sensitivity. To determine sensitivity, the signal generator is set to some desired carrier frequency and adjustments are made so that the current is modulated 50 per cent at 1000 cycles. The current flowing through the voltage attenuator is adjusted to some convenient value and the receiver antenna circuit is connected to one of the attenuator taps. The receiver is then tuned to the signal and other adjustments made so that the receiver is operating normally. The test is then begun by adjusting the input to the receiver, (input current to the attenuator or changing taps), so that some low value of output voltage is obtained. The input is increased in steps and at each step the corresponding output voltage is recorded. The test is continued until the grid of one of the receiver tubes becomes positive during a part of the cycle (this usually occurs first in the tube located in the last stage of the audio frequency amplifier). This point is considered the operating limit of the set, since any further increase in input will result in a distorted output. During this test the following data were recorded: carrier frequency, current flowing through attenuator, attenuator tap used, and output voltage. To plot sensitivity curves the field strength for the different output voltages must be known. It is found in this way: knowing the current flowing through the attenuator and its frequency, and the inductance corresponding with the tap used, the voltage induced in the antenna circuit can be calculated. This voltage, divided by the height of the antenna in meters, (in this case 10 meters), is considered as the field strength of the signal. The sensitivity of the receiver in meters is then found by dividing a given output expressed in microvolts by the corresponding field strength expressed in microvolts per meter.

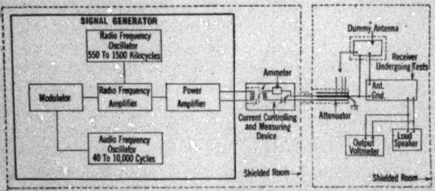
SIGNAL GENERATOR
Radio Frequency Oscillator 550 To 1500 Kilocycles
Modulator
Radio Frequency Amplifier
Power Amplifier
Audio Frequency Oscillator 40 To 10,000 Cycles
Ammeter
Current Controlling and Measuring Device
Shielded Room
Dummy Antenna
Receiver Undergoing Tests
Ant.
Gnd.
Attenuator
Output Voltmeter
Loud Speaker
Shielded Room

FIG. 1—SCHEMATIC LAYOUT OF RECEIVER TEST APPARATUS

The test just described is made with the receiver tuned to 560 kilocycles. The signal generator is then reset to give the same sort of signal as before except that the frequency is now 1000 kilocycles. The receiver is tuned to this new frequency and the test repeated. A third setting of the generator and retuning of the receiver to 1300 kilocycles completes the sensitivity tests. The complete test then shows not only how the sensitivity varies for different input field strengths but also gives an idea of how it varies for different carrier frequencies. Tests to determine the other receiver characteristics are also made at these same three carrier frequencies so the whole set of curves obtained furnishes a very good picture of the behavior of the receiver over the entire broadcast range.

Selectivity. With the receiver tuned to 560 kilocycles, the frequency of the signal generator is set to a value such that with the receiver connected to the highest tap of the attenuator and the maximum obtainable current flowing through it, no signal exists in the receiver output. The frequency is brought nearer and nearer to the tuning frequency of the receiver until a point is reached where some low value of signal (0.5 volt) exists in the output. From this point on, through resonance and beyond as far as it is possible to go, the frequency is changed in steps, and at each step, the current through the attenuator and the attenuator tap is chosen so that the voltage in the output remains constant. As the frequency of the signal generator is changed, its modulation frequency and degree of modulation remains unchanged. These data are recorded: frequency, value of current flowing through attenuator, attenuator tap used, and the value at which the output voltage was maintained. The field strength corresponding with the several frequencies is then calculated in the manner explained under sensitivity tests. The curve obtained by plotting field strength required to maintain a constant output voltage against carrier frequency is the selectivity curve for this particular tuning point. After this test, another is made with the receiver tuned to 1000 kilocycles and after that a third test is made with the tuning point at 1300 kilocycles.

Quality. A preliminary test is made to determine at what audio frequency the maximum output voltage is obtained. After finding this point the input to the receiver is adjusted so that at this frequency the grid of the last tube does not at any time become positive. The input is then maintained at this value while the modulation frequency is varied from 40 to 10,000 cycles and the degree of modulation is held constant at 50 per cent. At each frequency, the output voltage is recorded. A plot is then made of output voltage in per cent of output voltage at some one frequency (usually 1000 cycles), against modulation frequency and the quality can then be judged by the amount the curve deviates from a horizontal straight line—the greater the deviation, the poorer the quality.

Radiation. These measurements fall into two classes: (1) Measurement of radiation from loop or antenna alone; (2) measurement of radiation from receiver and loop or antenna combined. In the case of sets operating with an antenna, the amount of radiation from the set itself as compared with that from the antenna is usually small, so that on this type of set measurements are confined to radiation from the antenna itself. But in the case of sets operating with loops, the electrical dimensions of them and of the receiver circuits are comparable, so that quite a large part of the radiation may originate in the receiver. Thus, measurements on this type of set include both loop and set radiation.

The first mentioned type of measurement is made in this manner. The receiver to be tested is set up in the same room with the signal generator and in the other room is another receiver which will be designated as the indicator. The two current leads of the attenuator are disconnected from the current controlling and measuring unit and are connected to the antenna circuit of the receiver. The indicator is connected to the highest tap of the attenuator. Tests are made to be certain the receiver is tuned to some carrier frequency, and then if it is radiating it will be sending current through its antenna circuit, which consists of the dummy antenna and the attenuator. This current flowing through the attenuator will set up a voltage across the input of the indicator resulting in a certain value of output voltage. The current leads of the attenuator are then removed from the antenna circuit and returned to their normal connection. The indicator is connected to the lowest tap on the attenuator and the signal generator supplies current (of the same frequency as that generated by the receiver) the value of which is so adjusted that the output voltage of the indicator is the same as before. This current, multiplied by the ratio of the inductance of the lowest tap to that of the highest, gives the value of current that was in the receiver antenna circuit. This current, in amperes, multiplied by the height of the antenna in meters, (in this case 10 meters), expresses the radiation of the receiver in meter amperes.

To measure the combined radiation from a receiver and a loop, they are set up near the attenuator; and in the same room, but at a distance from them, is another receiver called the indicator. The receiver is properly tuned to receive some carrier frequency. If the receiver is radiating it will cause a voltage to exist in the indicator. The value of this voltage is noted, the receiver and its loop removed, and in their stead is set up a loop circuit whose effective height and resistance are known. With the aid of the attenuator and signal generator a current is set up in the loop circuit, of such a value that the voltage in the indicator is the same as before. Multiplying this current in amperes by the effective height of the loop in meters—(not the receiver loop)—the combined radiation of the receiver circuits and its loop is expressed in meter amperes.

EFFECTIVE HEIGHT OF RECEIVER LOOPS

The sensitivity, selectivity, and quality testsd escribed were for the antenna type of receiver. These tests are the same for loop receivers, but the method of determining field strength needs an explanation. When testing this type of receiver, the dummy antenna, of course, is not used but the attenuator is connected directly in the low side of the loop circuit. The connections are as shown in Fig. 2. The voltage induced in the loop circuit is found in exactly the same manner as it was in the case of the antenna circuit. The same dummy antenna is used for all antenna receivers and its height is always considered to be 10 meters; but when testing loop receivers, the loop belonging to the particular set undergoing tests is used. It is therefore necessary to determine the effective height of this loop in order to find the input field strength. The effective height of a loop is simply a figure which, multiplied by the field strength of the electric component of an electromagnetic wave, gives the value of the voltage induced in the loop. In other words, if a loop and an antenna were simultaneously subjected to the influence of a moving electromagnetic field, and the height of the antenna happened to be such that the voltage induced in the antenna was equal to that induced in the loop, then the effective height of the loop would be considered equal to the height of the antenna. It can be shown that the effective height of a loop is dependent upon the frequency of the field, the area of the plane enclosed by the loop winding, and the number of turns in the loop. The relation is:

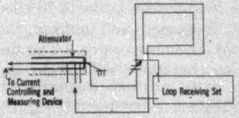

FIG. 2—DIAGRAM OF CONNECTIONS FOR TESTING A LOOP TYPE RECEIVER

$$H_m = 0.66\,N\,A\,F\,10^{-9}$$

where

H_m = Loop height in meters

N = Number of turns in loop

A = Area in sq. cm. enclosed by loop winding

F = Frequency of the field in kilocycles

CURVES OBTAINED FROM TEST DATA

A number of different types and makes of receivers have been tested, but for purposes of illustration, the curves plotted from data obtained from tests on one well-known make of receiver are shown here (see Figs. 3 to 6). These curves are typical of those obtained for other receivers and show up the faults of present day sets. The sensitivity is different at different carrier frequencies within the broadcast range. This is not a desirable characteristic, and receivers are now being designed so that the sensitivity will be more nearly uniform. Another bad feature is that a weak input signal produces an output voltage that is proportionately very much smaller than that produced by a stronger input signal. This condition is due to the type of detector in use today. There exist special forms of detectors which overcome this difficulty, but they are not generally used because of their lack of simplicity as compared with the usual form.

The selectivity of a receiver should be such that there is very little discrimination among frequencies over a range of from 10 kilocycles below to 10 kilocycles above the resonant frequency and outside the band the receiver should be unresponsive. The first condition is set down so that there will be no impairment in quality of the audio output of the receiver and the second condition so that undesired signals can not affect the receiver and thus also impair the quality. Fig. 5 shows the selectivity curves. At 560 kilocycles the selectivity is very good with respect to eliminating undesired signals, but is too selective for good quality. At 1000 and 1300 kilocycles the selectivity is not so good so far as eliminating interference is concerned but is good from the standpoint of quality. Selectivity is then another characteristic that may be improved upon, and the ideal to be reached is constant selectivity throughout the broadcast range, complete suppression of all undesired signals, and non-discrimination among frequencies over a limited range both sides of the resonant one.

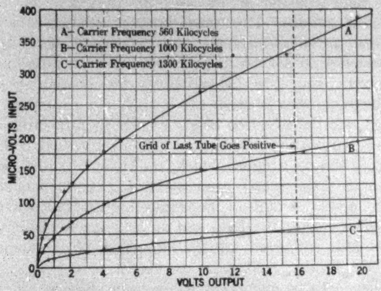

FIG. 3—CURVES SHOWING THE RELATION BETWEEN THE POTENTIAL INDUCED IN THE ANTENNA CIRCUIT AND THE VOLTAGE EXISTING ACROSS THE LOUD SPEAKER OF A RECEIVER

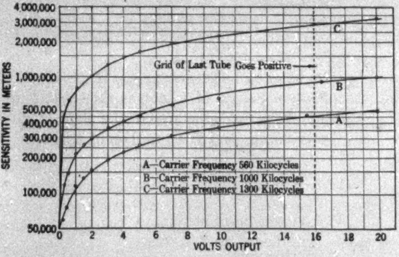

FIG. 4—CURVES SHOWING THE RELATION BETWEEN THE SENSITIVITY OF A RECEIVER AND THE VOLTAGE EXISTING ACROSS ITS LOUD SPEAKER

Fig. 6 is an example of how far the quality of modern receivers departs from the ideal. The falling off of the low frequency part of the characteristic can be attributed to faults in the audio-frequency amplifier and loud speaker circuits. The drooping of the high-frequency end is due partly to these faults, partly to too great selectivity, and a great deal to the characteristics of the detector.

SUMMARY

Over-all measurements supply the means for obtaining first, a picture of the performance of a receiver as a whole and, with such pictures, the performance of various receivers may be easily compared; and second, quantitative data which are particularly valuable in development work because they enable one to determine accurately the degree and manner in which improvements in individual units affect the characteristics of the receiver as a whole.

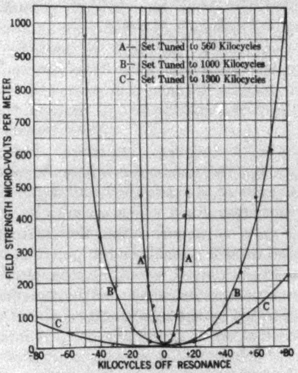

FIG. 5—CURVES SHOWING THE RELATION BETWEEN THE FREQUENCY AND THE STRENGTH OF THE FIELD FOR CONSTANT VOLTAGE ACROSS THE LOUD SPEAKER OF THE RECEIVER

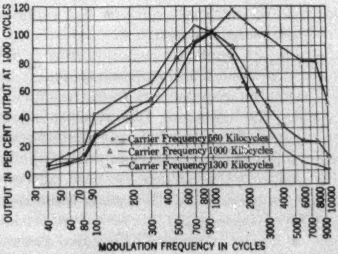

FIG. 6—CURVES SHOWING THE RELATION BETWEEN VOLTAGE ACROSS THE LOUD SPEAKER AND MODULATION FREQUENCY WHEN THE FIELD STRENGTH AND DEGREE OF MODULATION ARE HELD CONSTANT

Discussion

A. L. Cook: I think it is a decided indication of progress in the development of sets when the manufacturers begin to test their complete sets and publish results of these tests.

I wish to ask particularly about the curves on the fifth page of the paper showing the quality of reproduction of a certain receiver. I wish to ask does the method of testing used really tell whether or not the set has good quality of reproduction?

As I understand it, the method is based upon the measurement of voltage amplification at different audio frequencies and I wish to ask if this is a true measure of the quality? That is, if this were a straight line parallel to the horizontal axis, would this represent a perfect set?

I believe that most of the energy in a complex note is in the lower frequencies, and it would seem to me that the energy amplification would be more important than the voltage amplification. I should like to ask Mr. Oakley, therefore, if an indication of the relative energy amplification at different frequencies would not be a more exact representation of the quality than is the voltage amplification?

B. V. K. French: I should like to ask Mr. Oakley about the amount of error in the attenuator used. The attenuator was described, I believe, by A. W. Hull, in the *Physical Review* of 1925, and I wonder if any errors have been calculated?

Another question I should like to ask is, how is the percentage modulation measured on the oscillator defined, and how can we all reach the same agreement on the necessary percentage modulation to simulate broadcast reception conditions?

H. D. Oakley: The point about the quality curves of receivers is at present quite a disputed one. It has already been proposed to measure the power delivered to a loudspeaker rather than the voltage across it. But even in this case, the designers of loudspeakers are not prepared to say whether a definite relation exists between the power supplied to a loudspeaker and the sound received by the ear of the listener. Variations in room conditions cause variation in the sound received by the ear. If we could measure the sound pressure at the microphone of the transmitter and the sound pressure at the ear of the listener, then a true representation of quality for a particular set of conditions could be made. Because so many variables do exist, and because it is more convenient to measure voltage than power delivered to a loudspeaker, our quality curves are plotted in terms of voltage against frequency. We depend a great deal upon past experience in interpreting the picture presented to us by the quality curves.

The other point brought up was concerning the type of inductor used in making the measurements. Calculation of inductance involves a term which is the log of the ratio of the radius of the cylindrical shell to the radius of the rod inside the shell. Slight errors in determining the magnitudes of these two radii will cause a very slight error in the calculated value of inductance. There are two other sources of error. (1) The point at which the rod enters the end plate is grounded. Consequently there may be some current flowing from the point through ground and back into the circuit again. So calculations based upon the assumption that all the current flowing in the rod return through the shell may be slightly in error. And (2) the electric field about the shell will induce voltages in the receiver in addition to those obtained from the tap to which the receiver is connected. This error has been eliminated by enclosing the inductor in a shield. Measurements on the inductance show that it is within 4 per cent of calculated values.

The degree of modulation of the current supplied by the oscillator is expressed by the ratio

$$\frac{A - B}{A + B} \times 100$$

where A is the maximum and B the minimum amplitude of the modulated current. For 50 per cent modulation, A would be three times B. No single value of percentage modulation can simulate broadcast-reception conditions, since the percentage modulation of a broadcast transmitter is continually changing and may be any value from zero to distressing over-modulation. But the audio-frequency output of a properly designed receiver should vary directly with the degree of modulation and for measurement purposes a reasonable value should be satisfactory; 50 per cent modulation was used by us, but there is at least a tentative agreement now to use 30 per cent for all receiver measurements.

High-Frequency Measurements
of Communication Lines

BY H. A. AFFEL[1] and J. T. O'LEARY[1]
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.*—This paper describes briefly a testing technique which has been developed for making line measurements at high frequencies in connection with the application of carrier telephone and telegraph systems in the Bell System. The measurement of line attenuation, impedance, and cross-talk up to frequencies as high as 50,000 cycles is described, using a number of specially developed and standardized high-frequency apparatus units. These include an oscillator, a detector-amplifier, an impedance bridge, a thermo-milliammeter set, a variable attenuator, a cross-talk set, and a frequency meter. A brief description of the individual units is given, as well as their coordinated use in complete testing circuits.*

* * * * * *

PRIOR to the introduction of multiplex carrier telephone and telegraph systems, the operating telephone plant was concerned only with the transmission of frequencies in the voice range. The considerable application of carrier communication systems to long distance open-wire circuits, whereby a pair of such open wires carries several messages simultaneously by the employment of higher frequencies, has however greatly raised the upper limit of the frequency range used in the telephone plant. Among other things, this has required the development of a special technique suitable for the testing at these frequencies of the line circuits over which the various types of carrier systems are operated.

A brief description of this technique was included in a paper delivered before the Institute by Messrs. Colpitts and Blackwell.[2] It is the purpose of the present paper to describe these high-frequency measuring methods in greater detail and as they are now applied with recently improved apparatus.

The high-frequency line characteristics of chief interest are attenuation, impedance, and cross-talk for frequencies up to about 50,000 cycles. It is thought that the interest of others in the measuring methods employed may, perhaps, reside not so much in their novelty, but rather in that they represent a solution of the problem of standardizing the technique of high-frequency measurements for use under practical telephone plant conditions and of a relatively wide-spread application. The apparatus with which the measurements are made has been standardized and is in general use by the operating telephone companies of the Bell System.

Consideration may conveniently be given to the subject matter under two general headings: First, a brief description of the different apparatus units which serve as "tools" for the work; and secondly, the combination of these units in ways needed to effect particular measurements. Simple illustrations of the results attained in the work have been included.

APPARATUS UNITS EMPLOYED

The several individual types of apparatus which have been made available are as follows:

1. An *oscillator* providing a source of high-frequency current adjustable to any frequency between 100 and 50,000 cycles and having an output of a high degree of constancy and purity of wave form.
2. A *detector-amplifier* which provides a visual or, by heterodyne action, an audible indicator of high-frequency currents.
3. An *impedance bridge* suitable for measurements within the above mentioned frequency range.
4. A *thermomilliammeter* set which provides for the absolute measurement of relatively weak high-frequency currents and which also incorporates means for calibrating the thermocouples.
5. A *variable attenuator* or resistance artificial line.
6. A *cross-talk set* or variable attenuator of special type used for measuring cross-talk between mutually interfering circuits.
7. A *frequency meter* capable of measuring frequencies between 3000 and 50,000 cycles.

These units are all designed to meet relatively severe requirements of sturdiness, stability, and reasonable simplicity of operation in order that they may be employed generally in the field under conditions of use demanding portability.

Oscillator. Practically all measurements require a source of current variable in frequency, and a satisfactory generator has been made available to meet this need. This unit is a portable vacuum tube oscillator. It supplies current of frequencies variable from 100 to 50,000 cycles. The output is under the control of a potentiometer and provides a maximum power of from 0.4 to 0.7 watt, the exact value depending upon the frequency. The frequency is varied in the usual way by changing the constants of the feed-back circuit. Different values of inductance and resistance, as well as capacitance, are provided in order to insure that the instrument may have a good wave form and stability of frequency. The wave form is such that in the range from 3000 to 50,000 cycles the harmonics do not exceed

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[1] Both of the American Tel. & Tel. Co., 195 Broadway, New York.

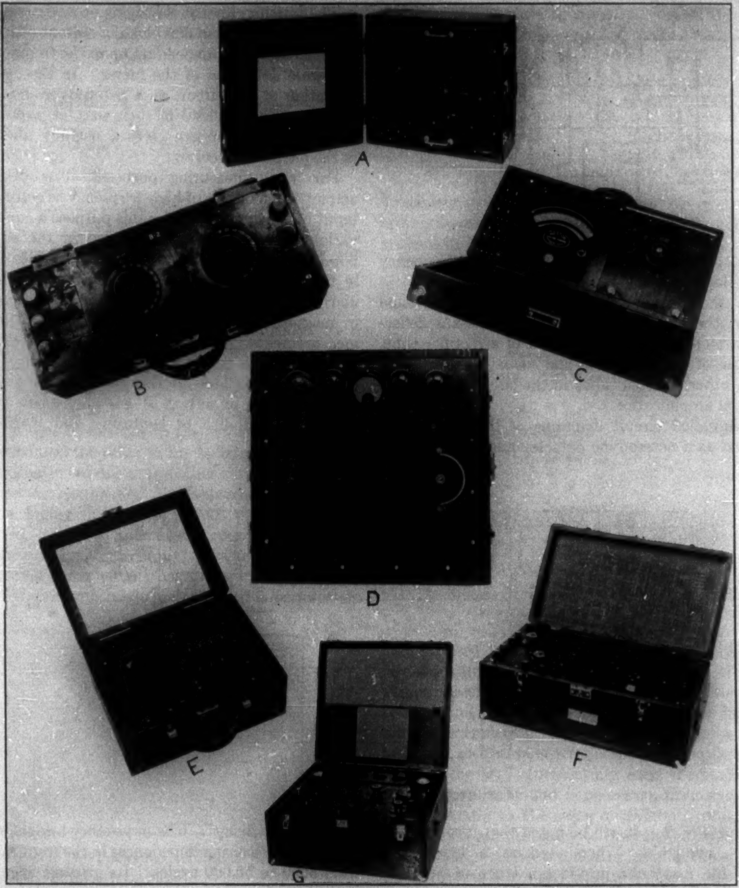
[2] See *Carrier Current Telephony and Telegraphy*, by E. H. Colpitts and O. B. Blackwell, A. I. E. E., TRANS., 1921. Vol. 40, p. 205.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

10 per cent. r. m. s. value as compared with the fundamental under the conditions of maximum output.

It is arranged to be supplied by power from the usual telephone office battery sources, plate potential at 130 volts and filament current at 24 volts. Its stability is such that for a temperature range of from 60 deg. to 80 deg. fahr., and battery voltage changes such as are ordinarily experienced the frequency variation does not exceed 0.5 per cent. The maximum variation occurs for the relatively high frequencies, and below 20,000 cycles its temperature stability is considerably better.

The simplified circuit of the oscillator is shown in Fig. 1. It will be noted that the circuit consists of an oscillating tube and two stages of amplification, the second stage of which is formed by the use of two tubes in parallel in order to meet the necessary output requirements. The various switches and controls will be noted on the face of the panel.



A. Frequency Meter; B. Attenuator; C. Thermomilliammeter Set; D. Oscillator; E. Crosstalk Set; F. Impedance Bridge; G. Detector Amplifier

Detector-Amplifier. The detector-amplifier is a sensitive device for detecting visually or by ear high-frequency currents from about 3000 to 50,000 cycles Since it is designed to be employed only in compariso.

measuring circuits or in null measurements, as in conjunction with the impedance bridge, only a limited degree of stability is necessary. When used for aural reception, the apparatus functions as a heterodyne detector, giving an audible note in the telephone receiver by beating the incoming current with a high-frequency current generated by an adjustable frequency local oscillator.

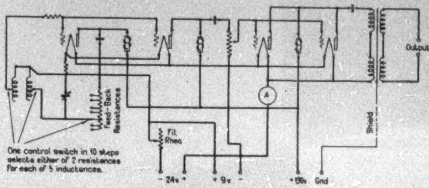

Output
A
Feed-Back Resistances
Fil. Rheo
Shield
One control switch in 10 steps selects either of 2 resistances for each of 5 inductances.
- 24v +
+ 9v -
+130v
Grd

FIG. 1—SCHEMATIC CIRCUIT OF OSCILLATOR

When the detector-amplifier serves as a visual indicator, the high-frequency current causes a deflection on a meter in the plate circuit of a rectifier tube. In the case of both the aural and visual indicator connections, other vacuum tubes in the circuit provide for substantial amplification.

The simplified circuit diagrams of the instrument, when used as a heterodyne detector and as a rectifier, are given in Fig. 2. It will be noted that four vacuum tubes are available. When used as a heterodyne detector, the first tube provides a stage of high-frequency amplification, the second tube operates as an oscillator to supply the local beating frequency, the third tube is the beat modulator or detector, and the fourth tube operates as an audio amplifier of the beat or tone frequency. The latter is usually adjusted to the order of 1000 cycles. A cam switch affords means for changing the circuit connections so that when employed as a rectifier for visual indication, the tubes provide in sequence two stages of high-frequency amplification and a final "rectifier" stage, using a three-element tube with high C potential. The fourth tube, otherwise employed as an oscillator in the heterodyne circuit, is not operated. The rectifier circuit provides for the indication on a milliammeter circuit noted at the extreme right of the diagram.

The circuit in its rectifying connection requires into its impedance of 600 ohms, a current of from 50 to 300 microamperes, depending upon the frequency, for a fair scale deflection of the meter. In the case of the operation of the circuit as a heterodyne detector, the amplification provided is such that an input of only about 0.5 microampere gives a usefully audible note in the telephone receiver.

For many measuring purposes, it is desirable to employ some degree of high-frequency selectivity at the input of the device, and for this purpose a continuously variable tuned circuit is included in the circuit. A potentiometer adjustment at the input is also embodied in the instrument. This has been found to be particularly useful where the input to the circuit may vary in magnitude over a large range, such as in bridge measurements. It is characteristic of devices providing such a high degree of amplification that unless the input is controllable in this manner, an overloading in the rectifier or output stages may result with considerable possibility of confusion and inaccuracy in measurement.

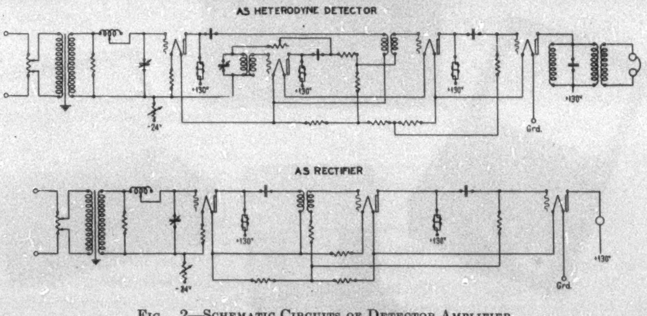


FIG. 2—SCHEMATIC CIRCUITS OF DETECTOR AMPLIFIER

Impedance Bridge. The impedance bridge provides a means for measuring impedances in the frequency range from 3000 to 50,000 cycles. Its greatest accuracy lies in the impedance range from approximately 50 to 10,000 ohms. It is of the balancing or differential coil type, or what, in telephone language, might be termed the "hybrid coil" type of bridge. The balancing coil is a four-terminal type of network, as noted in Fig. 3, having an input circuit to which is applied the source of high-frequency current, a detector circuit, and two balancing arms. When impedances applied to the two balancing arms are adjusted equally, the loss between the input and detector circuit terminals is infinite; that is, the bridge is balanced as in the case of the ordinary Wheatstone bridge. The balancing coil type of structure offers for this purpose a constructional symmetry and convenient possibility of shielding.

The rheostat arm or adjustable impedance provided consists of a resistance variable from 0 to 10,000 ohms in steps of one ohm, and a capacitance continuously variable from 300 micro-microfarads to 11 microfarads. Each of these known impedances is controlled by a separate switch so that it may be conveniently connected into different parts of the bridge circuit. Optional connections permit the variable resistance and capacitance to be connected in series in the balancing arm, or one in series with the line or apparatus to be measured and the other in the balancing arm, or the entire elimination of one or the other.

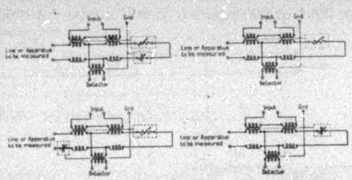


Fig. 3—Schematic Circuits of Impedance Bridge for Different Kinds of Unknown Impedances

Thermomilliammeter Set. In measuring the currents which are higher in frequency than the usual power frequencies, the dynamometer types of measuring instruments are not generally satisfactory, and the thermocouple meter circuit has found extensive application. The thermocouple meter circuit is perhaps too well known to warrant any detailed explanations. Such an arrangement using independent thermocouples and meters has been employed extensively in laboratory and field measurements of high-frequency currents. There are cases where greater accuracy is desired than can be insured by relying upon calibrations made at infrequent intervals. For this purpose the portable thermomilliammeter set shown on Fig. 4 was designed. In one unit it provides not only a sensitive meter and several thermocouples, but also a complete calibrating circuit consisting of d-c. power source, rheostat, and d-c. calibrating meter.

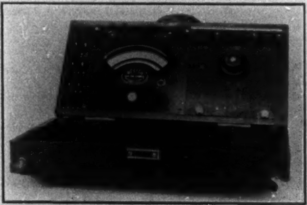


Fig. 4—Thermomilliammeter Set

The instrument provides for the optional use of one of three thermocouples of different characteristics. The thermocouples are constructed for insertion in bayonet-type sockets, and are thus easily replaced if damaged. The three available couples have the following characteristics:

	Approximate Heater Resistance	Approximate Useful Sensitivity Range
1.	600 ohms	0.2 to 2.0 milliamperes
2.	45 ohms	2.0 to 10.0 milliamperes
3.	5 ohms	10.0 to 50.0 milliamperes

These sensitivities and resistances cover the range ordinarily required in the measuring work on carrier systems in the field.

The instrument is so arranged that the meter, which in its function as a sensitive microammeter is used in the couple circuit of the thermocouple, may be switched and shunted to provide the calibrating milliammeter circuit. These calibrating and measuring circuits are shown schematically in Fig. 5. The various thermocouples and the meter which make up the instrument are under control of a number of switches which may be noted on the photograph showing the face appearance of the set. Incidentally each thermocouple has its own terminals, and if necessary they may be used independently in the same or different measuring circuits if it is not necessary to read them simultaneously.

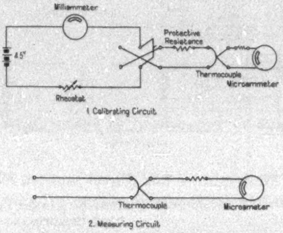


Fig. 5—Schematic Circuits of Thermomilliammeter Set for Calibrating and Measuring

Attenuator. The attenuator or resistance artificial line provides a simple type of network of calibrated loss and definite terminal impedances. Schematically, it comprises several sections of H type resistance networks as shown in Fig. 6. Since its application is largely concerned with the approximate simulation of open-wire line circuits over which carrier systems are operated and which have a characteristic impedance in the neighborhood of 600 ohms, the attenuator has been designed accordingly to have such terminal impedance.

It covers a range of attenuation extending from 0 to 75 transmission units[3] in steps of 0.5 T. U. The maximum of 75 T. U. is made up in three sections, two of these being controlled by dial switches, one graduated in 5 T. U. steps up to 50 T. U. and the other in 0.5 T. U. steps up to 5 T. U. The third section, consisting of a fixed attenuator of 20 T. U., is controlled by a single key switch. Each of the sections making up the attenuator is shielded to prevent mutual as well as external interference. The resistances making up the network are wound to a high degree of accuracy, and the attenuation derived is substantially independent of frequency in the range up to 50,000 cycles.

Cross-Talk Set. A schematic circuit of the cross-talk set is shown on Fig. 7. This is also a variable attenuator but it has certain circuit differences and includes several switches and provisions for terminating lines which adapt it particularly for cross-talk measurements. The variable attenuator feature consists of five fixed resistances associated with two slide wires which are mounted and controlled simultaneously by a single-handle and dial. Such an arrangement gives an approximate squared relation between input-output current ratio and dial motion which makes it possible to cover a wide range of attenuation with reasonable accuracy in reading the scales at all points. The scale associated with the dial is calibrated in transmission units as well as so-called cross-talk units; *i. e.*, millionths of the transmitted current. The setting of the slide wire affects the impedance slightly. For ordinary values of cross-talk the impedance is close to 600 ohms, and for extreme values it rises as high as 640 ohms. The attenuator proper is well balanced to ground. This is necessary since it may be connected directly to an open-wire pair.

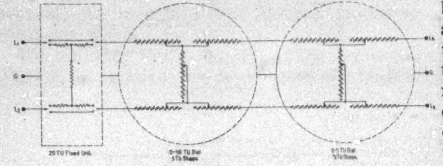


FIG. 6—SCHEMATIC CIRCUIT OF ATTENUATOR

Switches for connecting the oscillator, detector, and thermocouple meter to various parts of the testing circuits, as required by the different measurements, are included in the instrument.

Frequency Meter. For the purpose of checking the calibration of the oscillator circuit, and for other uses in the telephone plant in connection with adjusting the carrier frequencies of the various systems, a frequency meter has been designed. This is shown in Fig. 8. It provides a means for checking to an accuracy of about 0.1 per cent frequencies in the range from 3000 to about 50,000 cycles.

A null method is employed using a Wheatstone bridge type circuit as shown in Fig. 9. The unknown frequency source is applied to the input of the bridge circuit, and certain known variable elements are adjusted until a balance is obtained, as evidenced by the null deflection or silence in the detector circuit. The bridge includes equal ratio arms and two sets of variable arms. One of the variable arms includes series inductance and capacitance, the other a pure variable resistance. The null point indication is obviously the resonance condition of the series inductance-capacitance circuit, the variable resistance being employed to establish the resistance balance and to compensate for the variation of loss with frequency in the inductance and capacitance. The settings of the different variable elements are of course calibrated with respect to frequency and, in practical use, reference is made to the calibration chart furnished with the meter. The variable elements include two inductance coils of optional use depending upon the range of operation, the selection being made by means of a key switch. The condenser includes fixed mica condensers with switching means and a precision continuously variable condenser.

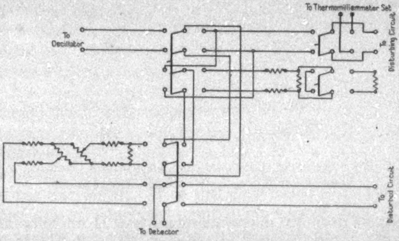


FIG. 7—SCHEMATIC CIRCUIT OF CROSS-TALK SET

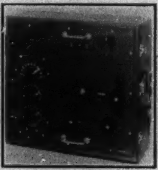


FIG. 8—FREQUENCY METER

MEASUREMENTS

The most important field use of these several apparatus units is in connection with the following measurement:

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3. *The Transmission Unit and Telephone Transmission Reference Systems*, W. H. Martin, TRANS., A. I. E. E., 1924, pp. 797-801

1. Line attenuation.
2. Line impedance.
3. Cross-talk between different lines or pairs.

Attenuation. The general magnitude of the high-frequency attenuation of a circuit for carrier transmission is, of course, predeterminable, but there often exist specific sources of loss which may not be evident until actual measurements of attenuation are made. In a practical case, the carrier system may be applied for operation over a line of perhaps 600 to 1000 mi. in length. Before the system is installed it is customary to measure the attenuation of this circuit for all frequencies which are employed in the carrier transmission, so that if unusual effects are present, they may be satisfactorily remedied and a minimum attenuation achieved. A long carrier circuit of course, is usually divided into sections joined by high-frequency repeaters or amplifiers, so that the currents as they are attenuated receive renewed energy at intervals. These intervals may range from 150 to 300 miles, depending upon the type of system employed and the frequencies involved. It is common practise to measure each repeater section separately, chiefly because the measurement of transmission over several repeater sections in tandem, by virtue of the very high attenuation, presents a more difficult problem.

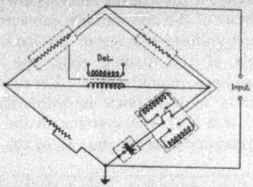

Fig. 9—Schematic Circuit of Frequency Meter

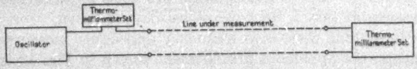

Fig. 10—Measurement of Attenuation by Current Transmitted—Received Method

The attenuation measurements are made by either of two methods; the first, a transmitted-received current comparison; and the second, a substitution method involving the use of a variable attenuator.

In the current transmitted-received method the apparatus connections employed are of the simplest type as noted in Fig. 10. At one end of the line, the oscillator supplies high-frequency currents to the pair of wires under test. The magnitude of this current is determined by the use of the thermomilliammeter set. It is common practise to transmit something of the order of 20 to 40 milliamperes and to employ the 5-ohm thermocouple. At the remote terminal of the line under test a thermomilliammeter set is similarly employed to measure received current. The impedance of the receiving thermocouple circuit is chosen to match closely the characteristic impedance of the line in order to simulate the conditions under which the carrier system apparatus is connected to the line circuit in practise. Ordinarily this condition is simply to attain to the required degree of accuracy, since the open-wire line circuits have a characteristic impedance which is close to 600 ohms with no appreciable reactance component. The 600-ohm thermocouple of the thermomilliammeter set used directly, or either of the other thermocouples in that set with the proper amount of resistance in series, provides this termination.

The results of a series of measurements using this type of circuit are usually tabulated or plotted in T. U. attenuation; *i. e.*, the current ratio is converted into the corresponding logarithmic or T. U. equivalent. From these data, it is simple, if desired, to compute the T. U. attenuation per unit length and a comparison can thus be made with the similar characteristics of other line circuits or theoretical data.

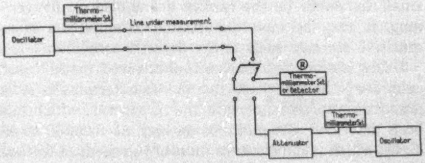

Fig. 11—Measurement of Attenuation by Substitution Method

The alternative or substitution method is perhaps more useful under conditions in which both the transmitting and the receiving terminal of the circuit under test are at the same point as, for example, in the case of a test involving the attenuation of a piece of apparatus such as a filter instead of a line circuit. It may also, however, be employed for the measurement of attenuation between two remote points, in which case a second oscillator is needed at the receiving terminal. A variable attenuator is also required. The method has an advantage over the transmitted-received current method, in that the attenuation of the line under measurement is determined by the reading of the variable attenuator and the results are available in transmission units directly for each reading without further computation.

The circuit arrangement is as shown in Fig. 11. It will be noted that the transmitting end involves an oscillator and thermocouple set for measuring the output to the line as in the case of the direct measurement noted above. At the receiving terminal, the oscillator provided is similarly connected with a thermocouple circuit to transmit measured current into a variable artificial line. A double-pole, double-throw switch is provided to connect another thermocouple circuit either to the second terminal of the variable artificial line or to the real line under test. Under conditions in which the current from the transmitting terminal oscillator is adjusted to be the same as that of the oscillator provided at the receiving terminal and transmitting into the artificial line, if the variable artificial line is so adjusted that the received current in the thermomilliammeter set, *R*, when connected to the artificial line is equal to that received when connected to the real line under test, the reading of the artificial line attenuation is obviously equal to that of the real line under test. Of course this is true only under conditions of exact similarity of impedance in the real line and artificial line circuits, and for most line measurements a simple circuit of this type suffices to measure the line attenuation with sufficient accuracy.

It is to be noted that this method requires no calibration of the final receiving thermomilliammeter set. For this reason, where the current received is extremely small, as would be the case in a line electrically very long, it may be more practicable to employ a more sensitive receiver such as the detector-amplifier.

Using the current transmitted-received method and calibrated thermocouples at the two terminals, it is practicable to measure with the apparatus, which has been described, attenuations as high as from 30 to 40 T. U., which is sufficient for most of the needs in the use of carrier systems in the Bell System. In the substitution method last described and the use of the detector-amplifier circuit as a detector, attenuations up to the limit of the attenuator, which is 75 T. U., may fairly readily be measured. In the latter case this means an energy ratio of about 30×10^6. The accuracy afforded by either of the methods is approximately ± 0.25 T. U. or abour 3 per cent in current ratio.

The scope of this paper does not permit an extensive discussion of the specific results obtained by the use of measuring methods of this type. It may be of interest, however, to present an example of the results of some attenuation measurements. Fig. 12 is a sample of the results of two sets of measurements on a particular line. In this case it will be noted that the attenuation of the circuit as measured initially presented two substantial absorption points which manifested themselves by "humps" of increased attenuation in the transmission frequency curve. Absorptions of the magnitude of those shown on Fig. 12 are rather extreme and are found only occasionally in practise. Remedial measures in the form of special transpositions made it possible later to remove these absorption humps entirely as shown by the subsequent measurement and thus to present a substantially smooth attenuation for the operation of a carrier telephone system.

Impedance. A line circuit of uniform constants and not subject to any substantial degree to the mutual effect of adjacent circuits such as, for example, occasionally results in absorption points as noted above, has a substantially uniform impedance. In particular, for frequencies above about 5000 cycles, this characteristic impedance is practically uniform resistance with little or no reactance component. This condition, of course, implies a theoretically infinite line or, what is its equivalent, a line terminated in its characteristic impedance. Where the line circuit loses this uniformity—for example, by the insertion of short sections of cable at intermediate points,—the impedance takes on certain characteristic irregularities. In the case of the loading of a cable circuit, if the loading sections are of irregular length by virtue of the non-uniform capacitance of the cable, the non-uniform spacing of the loading coils or the constants of the loading coils, the impedance obviously also loses its uniformity. For many reasons, such uniformity of impedance is ordinarily desirable in the application of carrier systems on the wires, and impedance measurements by the use of the bridge are commonly carried out in order that where the departures from uniform impedance are such as to exceed reasonable limits, remedial steps may be taken.

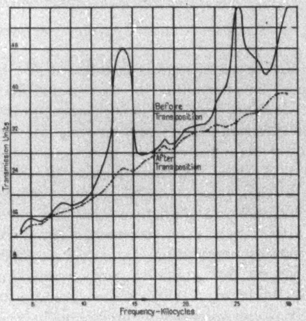

FIG. 12—ATTENUATION OF LINE CIRCUIT BEFORE AND AFTER SPECIAL TRANSPOSITION WORK

In making these high-frequency impedance measurements, the apparatus arrangement shown in Fig. 13 is employed. It involves ordinarily the oscillator, the impedance bridge, and the detector-amplifier. It is also common practise in most measurements to terminate the line circuit under measurement at its farther end in approximately its characteristic impedance. This would generally consist of a resistance of about 600 ohms.

In the operation of the bridge circuit, the manipulations required include first an adjustment of the frequency of the oscillator, then the adjustment of the detector-amplifier circuit for required sensitivity, selectivity, and beating frequency, if the heterodyne method is employed, and finally the adjustment of the balancing arms of the bridge. The heterodyne oscillator of the detector is ordinarily adjusted to provide a note o approximately 1000 cycles. The adjustment of the balancing condenser and resistance of the bridge results, if properly made, in entire extinction of the received detector note. Under these conditions of course the impedance of the circuit under test is made known.

In the description of the impedance bridge previously given, it has been noted that the connections of the variable resistance and capacitance permit measurements of impedance having either positive or negative reactance or resistance alone. Some experience is obviously required in the manipulation of the circuit, in particular in the case where the reactance component of the unknown circuit is positive instead of negative, in which case it is necessary to switch the variable capacitance to the "unknown" side of the balancing coil. This is not infrequently the situation when measuring a line over an extended frequency range. In obtaining a balance, it is found extremely important to adjust the detector-amplifier circuit in sensitivity so that it is not overloaded, in which case, under some conditions, an increase in input current may actually cause a decrease in the output tone or current. This gives a false impression that the bridge is approaching a point of balance when, as a matter of fact, it is doing just the opposite. The potentiometer must, therefore, be carefully adjusted so that the minimum gain in the apparatus, consistent with reasonable response in the output of the circuit, is required.

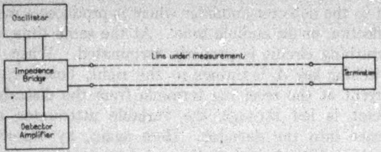

FIG. 13—MEASUREMENT OF IMPEDANCE

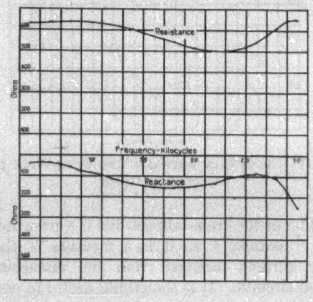

FIG. 14—IMPEDANCE OF CABLE CIRCUIT

The results of a typical set of impedance measurements are shown in Fig. 14. In this case, the measurements were made for the purpose of establishing the accuracy with which high-frequency loading[4] had been applied to a short section of cable. It will be noted that the measurements show considerable irregularity in impedance, illustrating the presence of perhaps an imperfectly spaced loading coil, a condition which would have to be remedied before the line would be considered entirely satisfactory for carrier service.

Cross-talk. Carrier systems of similar type are not infrequently operated over nearby pairs on the same pole line. Satisfactory operation of the systems in that case depends upon the degree to which the "cross-talk" between the pairs concerned has been reduced. The problem of rendering circuit combinations free or reasonably free from mutual cross-talk at the frequencies employed for carrier systems is one which presents many difficulties. Its solution involves the application of very refined methods of transposing the wires concerned in order to balance out the mutual inductive and capacitive effects. It is common practise, therefore, to make measurements before and after the application of the special transposing work designed to reduce the high-frequency cross-talk.

The measuring of cross-talk is obviously but a specialized form of attenuation measurement. In the case of line transmission, it is desired, of course, that the attenuation of the line circuit be as low as possible so as to permit the ready flow of energy from one terminal to the other. In the case of cross-talk, however, it is desired that the attenuation be as high as possible. The measurement of cross-talk, therefore, involves the reception of extremely weak currents. Whereas, in the case of line attenuation measurements, a thermocouple circuit may be employed at the receiving terminal, in the case of cross-talk measurements, the sensitivity required makes it necessary to provide relatively high gain at the receiving terminal or apparatus of the type of the detector-amplifier.

Measurements of cross-talk involve a transmitting or disturbing line and a paralleling receiving or disturbed line. Where cross-talk is heard on the disturbed circuit at the terminal at which the disturbing current enters its circuit, it is termed "near-end cross-talk." The cross-talk heard at the further end of the circuit is termed "far-end cross-talk." In most cases of carrier system operation, separate measurements are desired to determine the extent of the cross-talk of both types.

In general, cross-talk measuring methods follow in principle the attenuation measurements by the substitution method which has been previously described. For this reason, near-end cross-talk measurements are somewhat simpler than far-end cross-talk measurements. Fig. 15 shows the connections of the cross-talk set when employed for near-end cross-talk measurements. As noted previously, the cross-talk set, itself, provides not only an attenuator circuit but certain switching means indicated schematically by the pair of double-pole, double-throw switches *A*. The cross-talk set is employed in conjunction with an oscillator and a detector such as those previously described. For the right-hand position of switch *A*, the circuit provides for switching the oscillator output to the cross-talking or disturbing circuit. At the same time, the detector is switching to the disturbed circuit. Upon throwing the switch to the left, the disturbing and disturbed circuit connections are removed and there is substituted the attenuator portion of the cross-talk set. Carrying out a measurement involves the adjustment of the attenuator dial to such a position that when the switch is thrown alternately left and right, approximately equal deflections or sounds are obtained in the detector circuit. The reading of the cross-talk meter dial then indicates directly the attenuation or cross-talk between the circuits under test. This may be read in T. U. or in cross-talk units.

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[4] *Development and Application of Loading for Telephone Circuits*, T. Shaw and W. Fondiller, TRANS., A. I. E. E., 1926, p. 268.

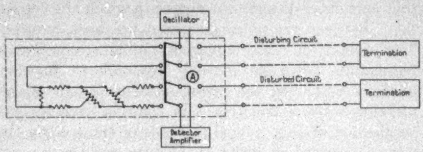

FIG. 15—MEASUREMENT OF NEAR-END CROSS-TALK

Where far-end cross-talk is being measured and the disturbing and disturbed terminals are at remote points, a more complex circuit arrangement is employed. If attenuation is also to be measured means, such as the thermomilliammeter set, are required for adjusting the output of the oscillator at the sending terminal and for noting the received current at the receiving terminal of the disturbing circuit. The circuit connections are shown in Fig. 16. At the sending end or disturbing end, the oscillator is adjusted to the frequency at which the measurement is desired and connected to the disturbing circuit through the thermomilliammeter set. The disturbed circuit at this point is terminated in its characteristic impedance. The amount of current flowing into the disturbing circuit is recorded. At the receiving end, through the double-pole, double-throw switch *B*, a similar thermocouple set is connected to the disturbing circuit and the current reading there is also noted. A comparison of the current readings at the two terminals obviously measured the attenuation of that circuit in a manner similar to that described under "Attenuation."

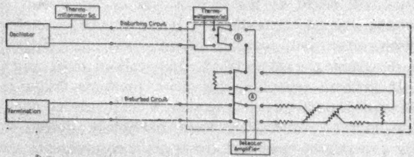

FIG. 16—MEASUREMENT OF FAR-END CROSS-TALK

The switching key *B* is then thrown to a position which disconnects the receiving circuit thermomilliammeter set and connects the receiving end of the disturbing circuit to the switching key *A*. Under these conditions, when the switching key *A* is thrown to the left, the cross-talk received in the disturbed circuit is led to the detector-amplifier where it produces a meter deflection or an audible tone. At the same time, the disturbing circuit is properly terminated. When the switching key *A* is thrown to the right, however, the current at the receiving terminal from the disturbing circuit is led through the variable attenuator and thence into the detector. Here again, by throwing the key *A* back and forth, an adjustment can be made in the attenuator circuit so that approximately equal deflections or tones are noted in the detector for the two positions of the key. Under these conditions, the reading of the cross-talk set dial is a measure of the far-end cross-talk; *i. e.*, it is a measure of the comparative current received from the disturbing and disturbed circuits. It is sometimes useful to obtain an equivalent comparison of received current as noted on the disturbed circuit with the transmitted current as it enters the disturbing circuit, and it is obviously necessary to multiply the measured cross-talk by a factor which is the attenuation ratio of the disturbing circuit as measured initially.

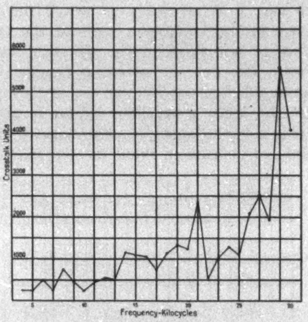

FIG. 17—CROSS-TALK BETWEEN TWO OPEN-WIRE LINES IN CLOSE PROXIMITY

The results of a typical set of cross-talk measurements at high frequencies are shown plotted in Fig. 17. The circuits involved pairs of wires in close proximity and these wires had been previously transposed for the distance of approximately 85 mi. in a special manner to reduce the cross-talk for carrier system operation. It will be noted that as might be expected, the cross-talk increased rapidly with frequency, and it presents irregularities with frequency which are the result of the phase and magnitude reactions in a relatively complex electrical circuit.

CONCLUSION

The various measurements which have been described give a representative picture of the testing work which may be done in preparing a wire circuit for the installation of a carrier system. Such tests are also occasionally made in clearing troubles which may appear from time to time.

The measuring methods and apparatus employed are the results of the coordinated efforts of numerous engineers in the Department of Development and Research of the American Telephone and Telegraph Company and the Bell Telephone Laboratories. Particular credit is due those at the last mentioned institution who are responsible for the design of the individual testing units.

Methods of Measuring the Insulation
of Telephone Lines at High Frequencies

BY E. I. GREEN[1]
Associate, A. I. E. E.

Synopsis—*This paper outlines the problem of measuring the insulation of open-wire telephone circuits in the frequency range from 3000 to 50,000 cycles, and discusses a method which has been used in the experimental study of insulator losses at these frequencies. The paper includes a description of a special line which has been constructed for the testing of insulators, an explanation of the essentials of the measuring technique, and a brief summary of the results which have been obtained.*

MEASUREMENTS involving the transmission of high-frequency currents over open-wire telephone lines began in the Bell System about 10 years ago, as a preliminary to the application of the first carrier telephone and telegraph systems.[2] Since that time, more or less continuous study has been given to the different problems involved in the transmission over line circuits of carrier frequencies ranging from about 3000 to 50,000 cycles. From the beginning it was apparent that the attenuation of open-wire line circuits at these higher frequencies is very much greater than in the voice range of frequencies, and undergoes wide variations due to changing weather conditions. Inasmuch as the attenuation is one of the most important factors in high-frequency transmission, its investigation has been very actively prosecuted along both theoretical and experimental lines.

The fundamental problem which originally presented itself was that of segregating the different losses which are experienced by the high-frequency energy transmitted over an open-wire circuit. It was obvious that a substantial part of the increased attenuation at high frequencies resulted from the increase in wire resistance due to skin effect, but it was equally obvious that other sources of loss were also contributing in large measure to the attenuation. It was known that radiation was a negligible factor in the line losses. It was known also that the "leakage" of the insulators increased rapidly with frequency, and that there was no direct relation between the high-frequency leakage and the d-c. leakage, which had previously been used as the principal criterion of the condition of the insulation of circuits, but information regarding the precise nature of the leakage losses at high frequencies was lacking.

The theory underlying computations of the skin effect resistance of conductors was well established at that time, and the effective resistance of the wires could be readily determined. In order to study the other losses properly, however, it was necessary to develop methods and apparatus for more accurately measuring their magnitude. These methods and apparatus, and their application in practise, form the subject of this paper.

THEORY OF LEAKAGE MEASUREMENT

Transmission over wires at high frequencies is accomplished in precisely the same manner as transmission at low frequencies, the wires acting as the guiding medium for the energy in both cases, and the same fundamental equations may be applied to both. The rate of attenuation for a uniform line circuit which is terminated so as to avoid reflection effects may, therefore, be determined by means of familiar transmission formulas. The attenuation constant α determines the decrease in magnitude of the voltage and current transmitted over the circuit, according to the equations

$$\frac{E_2}{E_1} = \frac{I_2}{I_1} = \epsilon^{-l\alpha} \qquad (1)$$

where E_1 and I_1 are the voltage and current at the sending end of a section of the circuit, and E_2 and I_2 are the voltage and current at a point distant l units from the sending end.

The value of the attenuation constant at any frequency may be derived from the so called "primary constants" of the circuit, which are as follows:

R = series resistance in ohms per unit length,
L = series inductance in henrys per unit length,
C = shunt capacitance in farads per unit length,
G = equivalent shunt leakage conductance in mhos per unit length.

These constants determine the value of the well-known expression

$$\gamma = \alpha + j\,\beta = \sqrt{(R + j\,\omega\,L)\ (G + j\,\omega\,C)} \qquad (2)$$

of which the attenuation constant is the real part.

The symbol G in the above equation represents the "equivalent" leakage conductance. It is convenient to make this equivalent value of G include all of the a-c. losses suffered by the energy transmitted over the pair except the actual $I^2\,R$ loss in the wires themselves.

An obvious method of finding the value of G under such conditions is that of measuring the attenuation of a section of line and computing G by means of the attenuation formula, using known values of R, L, and C. This method has been used quite extensively, and is of

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[1] American Tel. and Tel. Co., 195 Broadway, New York, N. Y.

[2] See *Carrier-Current Telephony and Telegraphy*, E. H. Colpitts and O. B. Blackwell, TRANSACTIONS A. I. E. E., Vol. XL, pp. 205-300, 1921.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-27, 1927.

considerable value. Unfortunately, however, its use requires, for accurate results, a section of line of the order of at least 100 miles in length. Any important changes in a line of such length are quite expensive, and this method is consequently not well adapted to the experimental study of the equivalent leakage conductance obtained for different line arrangements and different conditions of insulation.

A much more satisfactory method for this purpose is to measure the leakage conductance on a short line. If the line is short enough to make propagation effects negligible, its impedance measured with the far end open will be

$$Z_s = \frac{1}{G + j\,\omega\,C} \tag{3}$$

and the value of the leakage conductance may be obtained directly.

The determination of the maximum length of line for which propagation effects are negligible is comparatively simple. The problem is merely that of making the length of the test line a small enough fraction of a wavelength to produce only a small phase change in the current or voltage transmitted over the line. It can be shown that when the total phase shift on the line does not exceed five degrees it may be neglected without appreciable error. Now the phase shift for an open-wire pair is approximately two deg. per kc. per mi. Hence, if the phase shift which is allowed is not to exceed five deg., and the measurements are to cover the frequency range up to 50 kc., the maximum length of line which can be measured is evidently 0.05 mi., or about 250 ft. It has been found that the use of lines whose length does not exceed this value gives quite satisfactory results.

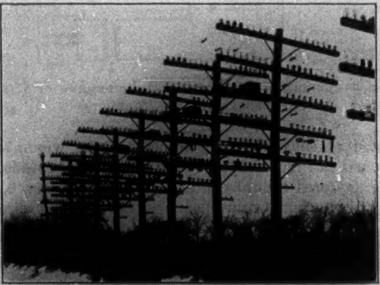

FIG. 1—INSULATOR TEST LINE AT PHOENIXVILLE, PA.

It should not escape attention that the use of a short line for measurements of the equivalent leakage conductance also involves the assumption that the short line provides all the sources of loss which are present on a longer line of, say, 100 mi. in length. The validity of this assumption has been tested by comparing the results obtained on a short line with those for a long line, and the comparison has shown that, whereas if the long line is infrequently transposed there may be some absorption of energy due to currents induced in the other circuits on the lead, the installation of transpositions required for minimizing high-frequency cross-talk ordinarily reduces such losses to a negligible value. This is only another way of saying that the shunt losses on a well transposed open-wire line occur almost entirely at the insulator points. This fact is one of outstanding importance, since it means that the insulation losses may be determined on a short line on which a considerable departure from the spacing of the insulators and the wires on a standard long line is permitted. The method of determining the leakage conductance on a short line is, therefore, extremely advantageous in the study of the effectiveness of different types of insulators at various frequencies and under various weather conditions.

INSULATOR TEST LINE

There is illustrated in Fig. 1 a short line which has been built near Phoenixville, Pennsylvania, for use in comparing the effectiveness of different types of insulators at high frequencies. This line includes about 25 poles spaced about seven ft. apart. A six-in. spacing between wires is used in order to make provision for the installation of a larger number of different types of insulators than could be obtained with standard wire spacing. About 40 different types of insulators are actually installed on the line in the picture.

In constructing this line, it was found convenient to make the length and spacing of the wires such that the value of the wire capacitance obtained was less than the minimum value desired for measuring purposes. With this arrangement, the capacitance may be increased to any desired value by shunting a condenser across the wires. The number of insulators installed on the wires was made slightly greater than the minimum value which was deemed essential for accurate measurements.

For a line only 175 ft. long, on which problems of external interference or interference between circuits do not exist, it might be supposed that the installation of transpositions would be useless. Upon investigation, however, it was found that an unbalanced relation between the two wires of a pair and the adjacent wires might produce appreciable loss at high frequencies. Accordingly, a simple transposition scheme for balancing the different capacitances between wires was devised and installed.

Owing to the comparatively small number of insulators employed on the line, it was necessary, in bringing the wires into the test station, to use every possible precaution in order to avoid having the entrance losses comparable in magnitude with some of the insulator losses which it was desired to measure. This difficulty has been obviated by the use of several interesting expedients which are illustrated in Figs. 2 and 3.

Fig. 2 shows the pole at the entrance to the test station, while Fig. 3 shows the entrance arrangements at close range. Each wire is brought into the test station through a glass tube. A special arrangement of glass shields or baffles is built over the entrance, the three narrow panes used for this purpose being barely discernible in the picture. In order to prevent rain from running down the wire to the glass tube, each wire is equipped with a drip washer. Springs are used to keep the wires taut.

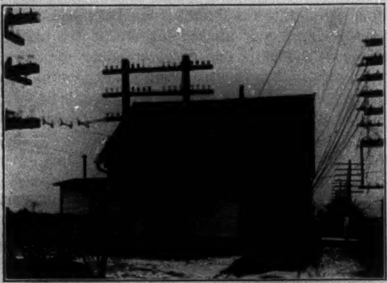

FIG. 2—ENTRANCE POLE ON INSULATOR TEST LINE

TESTING TECHNIQUE

For a test line short enough to avoid propagation effects, the impedance which is to be measured may be considered as a single conductance shunted by a given value of capacitance. This is equivalent, of course, to a leaky condenser, and a bridge method similar to those which have been employed for the determination of the loss angle or power factor of a condenser may be used in this case. Since the line wires have a large capacitance to ground, however, it is important that the bridge should be well balanced to ground.

FIG. 3—DETAILS OF ENTRANCE TO TEST STATION

The general arrangement of the bridge and associated apparatus used in a typical measurement of carrier-frequency leakage conductance on the test line at Phoenixville is shown in schematic form in Fig. 4. An illustration of the physical disposition of the apparatus is found in Fig. 5.

The salient feature of this bridge arrangement is a specially designed transformer consisting of three air-core coils mounted in a shielded container. In accordance with telephone parlance, this three-winding transformer is ordinarily termed a "hybrid coil." The hybrid coil type of bridge is well adapted for use in measuring impedances whose center is at ground potential since the capacitances of the windings to ground can be balanced. The details of the design of the air-cored hybrid coil are illustrated in Fig. 6, the most important feature being the use of bifilar wire for the two windings to which the oscillator terminals are connected.

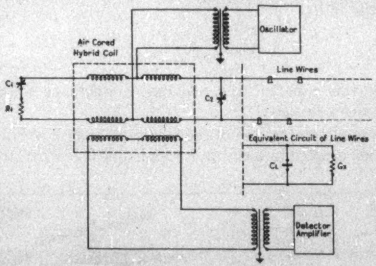

FIG. 4—SCHEMATIC CIRCUIT FOR MEASUREMENT OF CARRIER FREQUENCY LEAKAGE CONDUCTANCE

FIG. 5—ARRANGEMENT OF APPARATUS FOR MEASUREMENT OF CARRIER FREQUENCY LEAKAGE CONDUCTANCE

The source of high-frequency current for the bridge is a vacuum tube oscillator, and the detecting means is a detector-amplifier, both being of the types which are described in a parallel paper.[3] It is ordinarily assumed that the loss in the air condensers used in the measurement is negligible. In order to justify this assumption, considerable care must be exercised in the selection of the condensers.

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[3] *High-Frequency Measurements of Communication Lines*, H. A. Affel and J. T. O'Leary, TRANS., A. I. E. E., 1927. p. 504.

In the frequency range under consideration, the equivalent resistance of a small number of insulators in parallel is quite high and is therefore difficult to duplicate on the standard side of the bridge. It is well known, however, that a condenser having some dissipation of energy may be considered, at a given frequency, as equivalent to a hypothetical resistance either in series or in parallel with a perfect condenser. Hence the line impedance, which resembles that of a condenser and resistance in parallel, may be simulated on the standard side of the bridge by a resistance and condenser in series. The range of values of standard resistance required with this arrangement can be physically realized without difficulty.

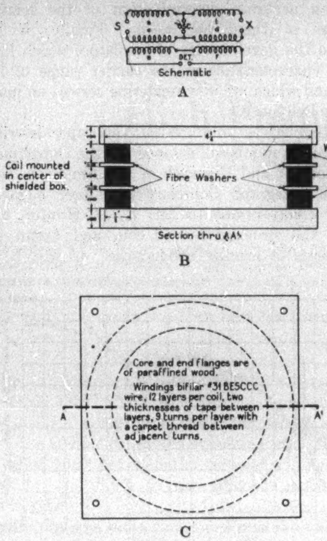
S
X
OSC.
DET.
Schematic
A
4¼"
Windings:
a,b
e,f
c,d
Coil mounted in center of shielded box.
Fibre Washers
1"
Section thru ℄AA'
B
Core and end flanges are of paraffined wood.
Windings bifilar #31 BESCCC wire, 12 layers per coil, two thicknesses of tape between layers, 9 turns per layer with a carpet thread between adjacent turns.
A
A'
C

FIG. 6—CONSTRUCTION OF AIR-CORED HYBRID COIL USED IN LEAKAGE TESTS AT PHOENIXVILLE, PA.

Instead of employing the obvious method of adjusting the capacitance on the standard side of the bridge to equal that on the line side, use is made of a method which is much more convenient for the computation of results. With this method the capacitance on the line side of the bridge is adjusted to the capacitance on the standard side. Thus condenser C_1 is kept at a fixed value, and the capacitance balance is obtained by varying the setting of condenser C_2 which is shunted across the line. The resistance balance is obtained by adjusting R_1. Only the frequency and the value of R_1 need be recorded for any measurement.

The theory by which the recorded values of R_1 and frequency in combination with the known value of C_1 may be converted into the desired value of leakage conductance is outlined in the Appendix, where it is shown that the unknown leakage conductance G_x is given by:

$$G_x = R_1\,\omega^2\,C_1^2 \tag{4}$$

This equation shows why it is unnecessary to read the value of condenser C_2 used on the unknown side of the bridge. It also indicates the dependence of the value of the balancing resistance R_1 upon the total capacitance on the unknown side of the bridge.

The balancing of the bridge is a rather delicate matter and extreme care is required in order to secure accurate results. The reason for this will be apparent when it is noted that the reactance represented by C_1 may be, in dry weather, several hundred times the value of the resistance R_1. The difficulty of attaining a high degree of accuracy in the reading of R_1 when the voltage across it is only a few thousandths of that applied to the bridge, scarcely needs to be pointed out.

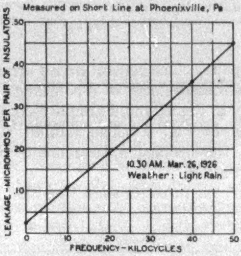

FIG. 7—TYPICAL CURVE OF MEASURED HIGH-FREQUENCY LEAKAGE CONDUCTANCE

In setting up the apparatus, every precaution is taken to avoid any stray pick-up of the oscillator output in the bridge or detector circuits. Such a mischance is ordinarily obviated by keeping the various pieces of apparatus well separated.

The results of a typical set of measurements of leakage conductance at different frequencies are plotted in Fig. 7. As indicated in the figure, it is ordinarily desirable to record in some detail the weather conditions which prevail at the time when the measurements are made. In the correlation of the measured values of leakage conductance with the weather variations, a recording rain gage has been found very useful for indicating the total precipitation and the rate of precipitation throughout any testing period.

The record of the a-c. leakage conductance is generally supplemented by a record of the d-c. leakage under the same conditions. This latter may be readily obtained with a source of direct voltage and a microammeter or a high-resistance voltmeter. It has also been found desirable to secure continuous records of the d-c. leakage over fairly long periods of time, and for this purpose a recording microammeter has been used with satisfactory results. By means of a multi-record instrument, simultaneous records may be obtained on as many as six pairs. The general circuit arrangement for a continuous measurement of d-c. leakage is indicated schematically in Fig. 8, and a sample record for a single pair is given in Fig. 9. The rainfall record corresponding to the leakage measurements of Fig. 9 is presented in Fig. 10.

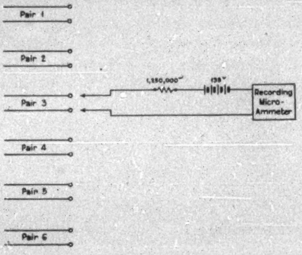

Fig. 8—Schematic Circuit for Continuous Measurement of D-C. Leakage

It has been recognized for some time that a method of obtaining a continuous record of the a-c. leakage conductance on one or more pairs would be extremely valuable, since it would provide a record throughout the night, when the test station is normally closed, and might economize on testing time during the day. The problem of developing such a method has been attacked from several angles, but no completely satisfactory result has as yet been attained.

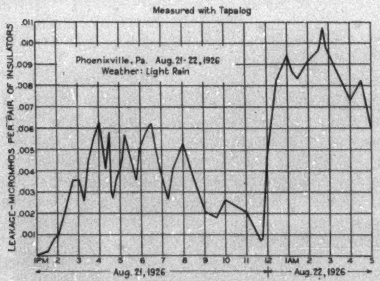

Fig. 9—Typical Curve of Measured D-C. Leakage

Conclusion

In conclusion it may be said that the general methods described above have been used at Phoenixville, Pennsylvania, in the study of the performance of various types of insulators over the entire frequency range up to 50,000 cycles. The work has served to illuminate the different phenomena involved in the leakage conductance of open-wire lines, and has made possible an accurate determination of the relative magnitudes of these phenomena. Finally, it has resulted in the development of insulators which have improved characteristics in the carrier range of frequencies and which are now rendering service on many lines of the Bell System.

The author wishes to state that this paper describes work in which a number of engineers of the Department of Development and Research of the American Telephone and Telegraph Company have been engaged. Particular credit is due to Mr. R. N. Hunter, who developed the air-cored hybrid coil, and began the measurements of leakage conductance, to Mr. L. T. Wilson, whose researches into the theory of insulator losses and their measurement have been of great value, and to Mr. F. A. Leibe, who has made important contributions to the measuring technique.

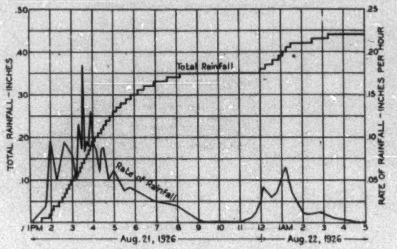

Fig. 10 —Typical Rain Gauge Record

Appendix

The theory by which the desired value of leakage may be derived from the known values of R_1, C_1 and frequency is as follows. The impedance of the standard side of the bridge is

$$Z_x = R_1 - \frac{j}{\omega C_1} \tag{5}$$

If the total capacitance on the unknown side of the bridge is represented by C_x, $(C_x = C_2 + C_L)$, and the leakage conductance by G, then the impedance on that side of the bridge is

$$Z_x = \frac{1}{G_x + j\,\omega\,C_x} \tag{6}$$

This equation may be reduced to

$$Z_x = \frac{\dfrac{1}{G_x}}{1 + \dfrac{\omega^2 C_x^2}{G_x^2}} - j\,\frac{\dfrac{1}{\omega C_x}}{\dfrac{G_x^2}{\omega^2 C_x^2} + 1} \qquad (7)$$

For the values of G which are commonly measured, the phase angle will be small enough so that

$$\tan \phi, \text{ which equals } \frac{G_x}{\omega C_x},$$

has a value of approximately 0.01 or less. Hence, no appreciable error is involved in simplifying equation (7) to

$$Z_x = \frac{G_x}{\omega^2 C_x^2} - \frac{j}{\omega C_x} \qquad (8)$$

It follows from (5) that

$$C_x = C_1 \qquad (9)$$

and

$$G_x = R_1\, \omega^2 C_x^2 = R_1\, \omega^2 C_1^2 \qquad (10)$$

High-Frequency Measurement
of Communication Apparatus

BY W. J. SHACKELTON[1] and J. G. FERGUSON[1]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*This paper describes precision high-frequency measurements of a fundamental type, special emphasis being placed on the measuring circuits rather than on the types of apparatus measured.*

Standards of frequency, resistance, capacitance, and inductance are discussed briefly. Bridge measurements are described for the measurement of frequency, inductance, effective resistance, capacitance, dielectric loss, capacitance balance, and inductance balance. Circuits for the measurement of other high-frequency characteristics such as attenuation, gain, cross-talk, and modulation are included.

INTRODUCTION

LONG DISTANCE electrical communication is now being effected by means of frequencies embracing the audible range and extending from there to the so called short wavelengths employed in radio transmission. According to the field of usefulness, this whole range has been subdivided into the audio, the carrier, and the radio ranges. From the viewpoint of the power engineer, all of the frequencies embraced in these ranges are high frequencies, but to the communication engineer, only those frequencies in the upper regions are considered high.

In this paper we will accept the power engineer's definition and will discuss methods of measurement and measuring instruments adapted to the measurement of communication apparatus over this complete range. Most of the measuring apparatus described, however, is designed particularly for use at audio and carrier frequencies. The measuring methods which are discussed are intended primarily for laboratory use in connection with the development and inspection of telephone apparatus prior to its application in the field.

Many of the transmission problems in the communication field involve the impedance characteristics of apparatus and circuits. In the manufacture of apparatus, impedance limits are used to a very great extent in inspection tests. Consequently, quantities of prime importance are those defining impedance characteristics; that is, inductance, capacitance, and resistance at specified conditions, of course, such as temperature, frequency, and current or voltage. Other characteristics, of a less fundamental nature but nevertheless of considerable importance, are attenuation, gain, inductance and capacitance balance, cross-talk, flutter, and modulation. Since the three impedance components mentioned above, together with frequency, are probably of more general interest, this paper will be devoted largely to a discussion of their measurement, only brief reference being made to the methods used for the measurement of the latter group of characteristics.

As in all measurement work, standards representing the quantity are required, and these are of two classes, prime standards and secondary or working standards. In our case, the prime standards are resistance and frequency. From these we derive inductance and capacitance. Working standards are stable types of inductance coils, air and mica condensers, adjustable resistances, and for frequency, resonance type meters and highly stable oscillators.

PRIME STANDARDS

Frequency. The standard frequency used is that described by Horton, Ricker, and Marrison[2].

Briefly, it comprises a special self-driven fork held at constant temperature and having all other conditions of operation so thoroughly controlled that a high degree of frequency stability is obtained. The exact frequency is measured by driving synchronously a

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[1] Both of the Bell Telephone Laboratories, Inc., New York, N. Y.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

[2] *Frequency Measurement in Electrical Communication,* Horton, Ricker, and Marrison, A. I. E. E. TRANS., 1923 p. 730.

phonic wheel for determining the number of cycles occurring in a given time interval. This time interval is usually a period of 24 hr. as measured by time signals received from Arlington. The average frequency of this fork is capable of being held constant and measured in this way with an accuracy of about 0.001 per cent.

The frequency of 100 cycles obtained from this fork is used to drive a 1000-cycle slave fork from which an equally constant 1000-cycle frequency is obtained. Having these frequencies, all other frequency measurements may be made with as high an accuracy as desired by direct comparison, using the cathode-ray tube as described in detail by Rasmussen[3].

Resistance. Resistance standards specially designed for use with direct currents and having a very high degree of stability may be readily purchased or constructed and calibrations to a high degree of accuracy may be obtained from the Bureau of Standards. These resistance standards are not suitable for precision measurements at high frequencies, usually being wound on metal spools, and the value of the phase angle receiving only secondary consideration. It is necessary, therefore, to use resistance standards of special construction depending upon the particular application to be made. In all cases, constancy of resistance with variations in atmospheric conditions, frequency, and time is imperative. Generally as small a phase angle as possible is also highly desirable, although for some uses a suitable degree of constancy may be sufficient provided that the angle is known, and not large enough to affect appreciably the magnitude of the impedance of the resistance over the frequency range used.

To obtain the highest degree of stability of both resistance and phase angle, it has been found desirable to wind the wire on a spool made of a material not affected appreciably by atmospheric conditions, for example, phenol fiber, and to immerse the complete resistance in a sufficient amount of a suitable sealing compound to exclude all moisture. Resistances meeting all of the requirements outlined have been constructed as described in a recent paper by one of the authors.[4] Coils such as described there, having a resistance of approximately 1000 ohms, may be constructed to have an effective inductance of less than five microhenrys, and this inductance is practically independent of frequency up to at least 100 kc. Coils having lower values down to about 10 ohms can be made with equally small phase angles. Below this value of resistance, it is more difficult to hold a low phase angle.

Coils constructed as described may be considered to have so small a change in resistance with frequency that a calibration with direct current may be used without appreciable error for all frequencies at which they are used. Both the variation in resistance with frequency and the phase angle may be most readily measured by comparison with some simple type of resistance of such geometrical form that the phase angle may be readily computed. Satisfactory resistances for this purpose are short lengths of fine wire of definite shape, sputtered metal films on glass or other insulating material, and carbon in the form of rod or film.

SECONDARY STANDARDS

Capacitance. The value of our capacitance standards is determined in terms of the prime standards of frequency and resistance. This determination may be made in several ways, the following bridge method being a simple and accurate one. The circuit, as shown in Fig. 1, consists of two equal resistance ratio arms, a resistance and capacitance in parallel in the third arm, and a resistance and capacitance in series in the fourth arm. When this bridge is balanced at any particular frequency, the relations between the impedance arms of the bridge are such that the value of each capacitance may be determined in terms of the frequency and the two resistances.

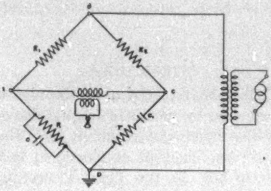

FIG. 1—BRIDGE CIRCUIT FOR MEASURING CAPACITANCE IN TERMS OF RESISTANCE AND FREQUENCY

The requirements for a capacitance standard are high constancy with variations in frequency, time, voltage, and atmospheric conditions, and a small phase difference. Mica has been found to be the best solid dielectric, used either alone or impregnated with a high quality wax such as paraffin. If mica alone is used, the condenser must be sealed to prevent the entrance of moisture.

Good mica condensers can be obtained with a temperature coefficient below 0.005 per cent per deg. cent., and having a variation of less than 0.1 per cent over a frequency range from 500 cycles to 100 kc. Variations in capacitance with voltage are also negligible provided voltages below 100 volts are used. It has been our experience that the paraffin impregnated condensers generally have a negative change of capacitance with temperature. This change is smaller than that of the unimpregnated type which has a positive change with temperature. The paraffin impregnated condensers, however, usually change more with time than the unimpregnated condensers.

Air condensers may be used as standards in small sizes. For the larger values, the air condensers become large and cumbersome and are not as stable as the mica condensers. Even in the smaller sizes, very special precautions must be taken to obtain air condensers which have appreciably smaller phase differences than the mica condensers, which may be made with phase differences considerably less than one minute.

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[3] *Frequency Measurements with the Cathode Ray Oscillograph,* F. J. Rasmussen, A. I. E. E. TRANS., 1926, p. 1256.

[4] *A Shielded Bridge for Inductive Impedance,* W. J. Shackelton, A. I. E. E., TRANS. Vol. XLV, 1926, p. 1266.

Inductance. Requirements for inductance standards are high constancy with variations in time, current, or saturation, atmospheric conditions, and frequency. It is also desirable that they be made with a small external field. Otherwise, very great care must be taken to avoid errors due to this cause.

In order to obtain stability with variations in saturation, it is usual to make inductance standards with air cores. This requires standards of large physical size if a time constant as large as the average iron core coil is desirable. This large size results in large capacitance distributed in the coil itself and from the coil to ground. These capacitances cause large variations in inductance with frequency and with the position of the coil with respect to ground. On account of this difficulty with air core coils, permalloy[5] as core material has been used with considerable success as described by one of the authors[4].

The calibration of these inductance standards may be made by comparison with any two of the quantities, capacitance, resistance, and frequency. Comparison with frequency and resistance may be made in a bridge circuit exactly similar to the one used for capacitance determination, substituting inductances for capacitances. A comparison with frequency and capacitance may be made by means of a resonant method, and comparison with capacitance and resistance may be made by means of the Owen bridge.[6] The resonant method is used generally except for those cases requiring large capacitance, in which cases the Owen bridge is used.

Frequency. As a secondary standard of frequency for use with the cathode ray tube, where practically only one standard frequency is required, a special 1000-cycle oscillator is used, designed particularly for high stability of frequency with ordinary variations in external conditions. This oscillator is shown in Fig. 2. It allows the use of a cathode ray tube for frequency measurements with a high degree of accuracy under conditions where the prime standard of frequency is not accessible.

Where a portable frequency standard is desirable, for instance, as a means of shop frequency checks, a resonance type of meter is used. This is shown in Fig. 3. It is essentially a resonance bridge circuit consisting of two equal resistance ratio arms, a third arm containing a resonant circuit, and a variable resistance as the fourth arm. The capacitance and resistance are variable over wide ranges by means of decade switches, and the capacitance is capable of fine variations by the use of a form of precision variable air condenser having provision for fine control. There are four air-core inductance coils which give, in conjunction with the variable capacitance, a frequency range of about 100 cycles to 150 kc.

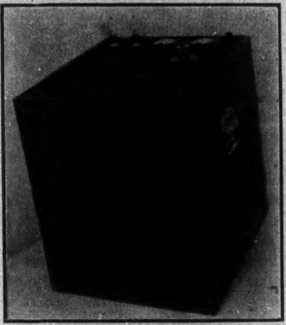

FIG. 2—SINGLE-FREQUENCY VACUUM TUBE OSCILLATOR USED AS SECONDARY STANDARD OF FREQUENCY

The meter is calibrated by balancing the circuit by means of the variable resistance and capacitance with a known frequency input, and recording the coil and condenser settings. It is used for checking frequencies by reversing the process, that is, connecting the source of unknown frequency to the bridge, balancing as before, and determining the frequency by reference to the calibration. There are no input or output transformers connected to this circuit and on this account certain precautions must be taken in connecting the output and

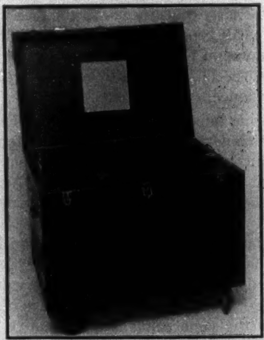

FIG. 3—RESONANCE-TYPE FREQUENCY METER

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5. H. D. Arnold and G. W. Elmen, *Franklin Institute Journal*, Vol. 195, 1923.

6. D. Owen, "A Bridge for the Measurement of Self-Inductance," *Proceedings of the Physical Society of London*, October, 1914.

input circuits to it; but it is a relatively low impedance circuit, and troubles due to this cause have not been found serious.

Resistance. A convenient secondary standard of resistance is a dial box having the resistance units designed to meet the same requirements as the prime standards. Commercial dial boxes are available, having satisfactory stability with variations in frequency and atmospheric conditions, and having sufficiently small phase angles for all frequencies but the highest radio frequencies.

A dial box, requiring as it does, a certain amount of wiring between dials, and having all of the dials connected permanently whether they are used or not, always has more capacitance and inductance associated with it than a single resistance of the same value. A certain amount of compensation between the capacitance and inductance may be effected by proper design, but it may be generally accepted that the inductance of the wiring makes the phase angle of the low resistance values comparatively high and the capacitance between dials and between units of each dial makes the phase angle of the high values comparatively high. This effect can only be overcome by a compact design using coils of small physical size. This sets a limitation on coils for use in dial boxes which is not present to such an extent in the case of single resistance units or single value prime standards.

METHODS OF MEASUREMENT

We have discussed already measurements of frequency and resistance in connection with the description of standards, and we will not discuss them further here. We are particularly concerned with the measurement of impedance of all types, it being understood that any resistance having a phase angle which is not negligible or which is of special interest is to be considered a special type of impedance.

In measuring impedances, we have found that those methods which determine the unknown in terms of circuit constants are superior to those requiring the measurement of current and voltage. Accordingly, bridge methods are used almost exclusively, and furthermore, the bridge type which is used wherever possible is the equal ratio arm bridge in which a direct comparison is made of the unknown impedance with a known impedance adjusted to that same value. This type of measurement has the disadvantage of requiring standards of the same value as the quantity measured over the whole range of impedances used, but it has the compensating advantages that, having standards whose value is known, this circuit is extremely simple, very easy to check at any time, and may be made extremely accurate.

Auxiliary Apparatus. Without going into details regarding the auxiliary apparatus used in connection with bridge measurements, we may state briefly that vacuum tube oscillators are used almost exclusively for furnishing all frequencies, and that the telephone receiver is used almost exclusively as a detector, due to its simplicity and the rapidity with which it may be used. For frequencies below 200 cycles, it is used with a chopper to give a tone of about 1000 cycles, and above 3000 cycles, it is used with a heterodyne detector to give a beat note of about 1000 cycles. In the audio frequency range, it is used alone or with an amplifier, if necessary.

While it is impossible to draw a distinct line between the methods of measurement of different types of impedances, certain bridge circuits have been designed primarily for certain types of measurements, and we will therefore classify them in this way, although in general they have a considerably wider sphere of usefulness than indicated.

Inductance. A simple shielded bridge for the measurement of inductance and resistance has been described by one of the authors[4] and is shown in schematic form in Fig. 4. It comprises two equal resistance ratio arms, an adjustable standard of self-inductance, an adjustable resistance standard, a thermocouple milliammeter, two reversing switches, two transformers, and two air condensers. This apparatus is grouped into three separate units, as shown in Fig. 5, one comprising the standards of inductance, one the resistance standard and one the remaining parts of the circuit. Each of these units is shielded electrostatically. The last assembly constitutes the balance element of the system, by means of which the unknown and standard impedances are compared. This unit may be used alone for the comparison of two impedances of any type since the only condition for balance is the exact equality of impedances in the two arms. Using in addition the standard inductance and resistance shown, it is adapted particularly for measuring inductance and effective resistance. The inductance standard may be made with a range from 10 henrys to a minimum of two millihenrys, using an inductometer having a minimum scale division of 0.1 millihenry, or the range may be any simple multiple of this. Values as low as one microhenry at frequencies as high as 150 kc. are measured in this way.

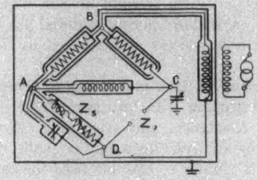

FIG. 4—SHIELDED IMPEDANCE BRIDGE CIRCUIT

By connecting the resistance in one arm of the bridge and a capacitance in series with an inductance in the other arm, we may use it to indicate resonance, and if we measure the frequency we may use this method for the comparison of capacitance with inductance. This is the method actually used for the calibration of the inductance standard used with the bridge. The bridge may be used for the comparison of capacitance. The bridge described later for the measurement of capacitance, however, has certain special features which make it peculiarly adapted to the measurement of capacitance and conductance.

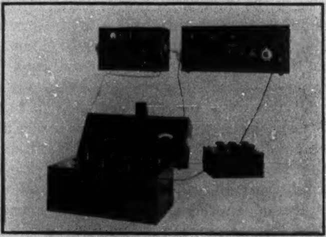

FIG. 5—SHIELDED IMPEDANCE BRIDGE AND STANDARDS, CONNECTED TO VACUUM TUBE OSCILLATOR AND HETERODYNE DETECTOR

Inductance with Superposed Direct Current. In telephone work, it is often of value to know the performance of apparatus, particularly of iron core impedances, when used at telephone frequencies while at the same time carrying direct current. The bridge shown in schematic form in Fig. 6 will measure the inductance of the coil at audio frequency with a direct current flowing through it. As shown in the figure, the direct current is kept out of all of the arms of the bridge except one ratio arm and the test arm, by means of condensers, and the alternating measuring current is separated from the direct current by means of a choke coil. None of these added features affect the bridge balance except the capacitance in the standard arm, and this is made large enough (26 μ f.) to have an impedance small compared with the impedance measured. In any case, a correction may be made by taking first a zero reading which will be slightly positive due to the inductance necessary to compensate for the capacitance in this circuit. This correction will vary with frequency but at 1800 cycles, for instance, with 26-μ f. capacitance, the correction is only about 0.3 millihenry and the inductances measured are usually considerably larger than this.

The circuit is extremely simple and convenient to use. The values of alternating current and direct current can each be measured separately outside of the bridge circuit and the inductance standards do not need to be constructed to carry the direct current. The only part of the bridge required to carry the direct current is one ratio arm and in consequence, it is a comparatively simple matter to construct such a bridge to carry several amperes of direct current. Where very high direct currents are required, the ratio arms may be reactances wound on a single core, instead of resistances, thus reducing the loss due to the passage of the direct current.

Flutter. In telephone circuits used for joint telephone and telegraph service, it is desirable to know the effect of the telegraph impulse on the telephone frequency inductance and effective resistance of the loading coils used on the lines. This effect, known as "flutter," with a method of measuring it, is described in detail by Fondiller and Martin[7]. The measuring circuit consists of a double bridge, the inner one consisting of two similar loading coils on which the flutter effect is to be measured and two other coils of comparatively high impedance approximately equal in value and which have negligible flutter effects, the four coils being connected to form a balanced bridge. The low frequency corresponding to the telegraph impulse is introduced at two diagonal corners and the other two corners, which are at a common potential with respect to the low frequency, are connected to the usual test terminals of an impedance bridge of the type already described. With no low frequency current passing through the coils, a continuous balance may be obtained on the main or high frequency bridge using an audio frequency input. From this, the normal effective resistance and inductance of the coils may be obtained.

When the low-frequency current passes through the coils, the inductance and effective resistance are different for every point of the low-frequency cycle. Thus, only an instantaneous balance of the outer bridge is possible. This instantaneous balance for any particular point in the low-frequency cycle may be made by the use of an electromagnetic oscillograph. By this means as described in the paper already mentioned, it is possible to obtain the curve of variation of inductance and effective resistance of the coil over one low-frequency cycle.

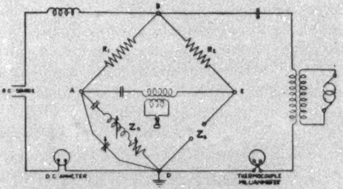

FIG. 6—BRIDGE CIRCUIT FOR MEASURING IMPEDANCES WITH SUPERPOSED DIRECT CURRENT

Another method used at the present time employs the same bridge circuit but an entirely different method of detecting the cyclic variation in the balance. This method of detection uses the cathode-ray oscillograph and is as follows. The low-frequency source is connected across a high resistance and condenser in series, the two having equal impedances. The potentials across the condenser and resistance are then placed respectively across the horizontal and vertical plates of the oscillograph. These two potentials, being equal in magnitude but 90 deg. apart in phase, give a circle on the screen. The output of the main bridge is now connected through a transformer whose secondary is connected in series with the oscillograph cathode potential. Due to the fact that the sensitivity of the tube to deflections by the plate potentials varies with the cathode potential, the radius of this circle produced by the low frequency is a function of the telephone frequency input from the bridge, and instead of a circle we get a band, the width of which is a measure of the degree of unbalance of the bridge. The point in the cycle at which the bridge is balanced, is indicated on the screen as the point where this band diminishes to a line, and the angular position of this point in the band determines the phase position of this balance with respect to the low-frequency cycle. It is possible in this way to balance the bridge for any angular position corresponding to any point in the low-frequency cycle, and by taking sufficient points, to obtain a curve of variation of the coil constants over a complete cycle. This method is found to be simpler and faster than the method using the mechanical oscillograph.

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[7] W. Fondiller and W. H. Martin, TRANSACTIONS of the A. I. E. E., 1921, Vol. 40, p. 553.

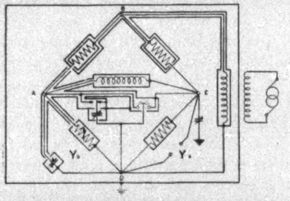

FIG. 7—SCHEMATIC CIRCUIT OF CAPACITANCE AND CONDUCTANCE BRIDGE

Inductance Balance. A simple form of bridge for measuring inductance balance of the two windings of a transformer or other coil uses the two windings of the transformer for two arms, the other two arms being resistances, one of which at least is variable. The balance is made by means of the variable resistance, the ratio of the two resistances at balance then giving the unbalance of the transformer. If one of these resistances is made 100 ohms, the variation of the other from 100 ohms at balance gives directly the percentage unbalance. Any unbalance in resistance is usually comparatively small and may be taken care of by low resistances in series with the transformer windings.

Ratio of Transformation. A similar bridge may be used for the measurement of ratio of transformation. There are many cases where the secondary of a step-up transformer has an inductance which is inconveniently large to measure directly, and the ratio of transformation circuit eliminates this necessity. The circuit used is practically the same as that already described for measuring inductance balance, the ratio of transformation being equal to the ratio of the resistance arms of the bridge at balance.

Capacitance. The direct comparison of capacitance is made in a special bridge known as the Campbell-[8] Colpitts[9] capacitance and conductance bridge. The ratio arms, input and output circuits, and the shielding, are similar to the impedance bridge already described. The unique feature of this bridge is the method of connecting the standard air condenser to eliminate the dielectric loss in the measurement of capacitance. The schematic diagram of the bridge is shown in Fig. 7. Instead of connecting the standard condensers in the arm *A D* as in the case of the impedance bridge already described, a special switch is used to switch these condensers from *A D* to *C D*, and in the case o fthe continuously variable condenser, the three-plate construction is used, causing a decrease in the capacitance in *C D* as the capacitance in *A D* is increased.

The method of construction of the unit air condensers is shown in Fig. 8. It may be seen from this figure that all capacitances which include dielectric material are permanently connected across *C D* or *A C* and so are not changed when the condenser is switched, or else they are switched so that capacitances across *A C*, which do not enter into the bridge balance, are short-circuited on switching. This scheme eliminates all dielectric loss in the standards when measuring condensers by comparison with them. It has the additional advantage that the capacitances in the bridge have twice the effect they would have if simply switched in and out of the circuit.

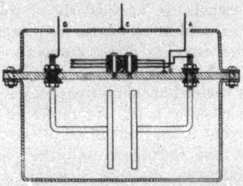

FIG. 8—AIR-CONDENSER CONSTRUCTION EMPLOYED IN THE CAPACITANCE AND CONDUCTANCE BRIDGE

By the use of this bridge, it is possible to measure capacitances up to the maximum limit of the range of the air condensers with a negligible loss in the standard condensers. This capacitance range is usually up to 0.01 μ f. and for condensers above this value the conductance is measured by comparison with that of the maximum value of the air condenser, assuming it to have negligible conductance. Of course this method of eliminating dielectric loss is not applicable to the use of mica condenser standards and if a range greater than 0.01 μ f. is desired, the mica condensers are simply connected in the usual way across *A D*.

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[8] G. A. Campbell, "The Shielded Balance," *Electrical World and Engineer*, April 2, 1904, p. 647.

[9] G. A. Campbell, "Measurement of Direct Capacities," *Bell System Technical Journal*, July, 1922, p. 18.

Another feature of this bridge is the method of measuring conductance. The connection of a variable resistance, either in series or in shunt, with the standard condenser for the measurement of loss in the test condenser has objections due to the wide range of resistance values required to cover the possible variations in losses. A compromise is effected in this bridge by connecting a 10,000-ohm shunt across each of the arms *C D* and *A D*. A slight difference in the losses in these two arms can then be measured by varying one of these resistances slightly. Since the standard condenser practically always will have lower losses than the condenser tested, it is usual to place a fixed 10,000-ohm resistance across *C D* and a resistance across *A D* variable in 0.01-ohm steps to 10,000 ohms. A change of one ohm in this resistance, when balancing a condenser, is equivalent to shunting it with a resistance of 100 megohms or 0.01 micromho. Accordingly, the conductance of a condenser may be measured in micromhos by simply dividing the resistance change in ohms by 100. This, of course, is only approximate in the case of large conductances, but is correct to 1 per cent for values up to one micromho.

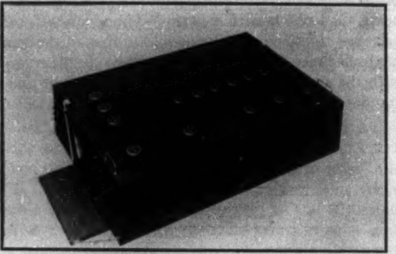

FIG. 9—CAPACITANCE AND CONDUCTANCE BRIDGE

Due to the condensers forming such an integral part of the bridge circuit, they are all built into the bridge. The complete bridge is shown in Figs. 9 and 10. Fig. 9 is a top view showing the capacitance and resistance dials for effecting a balance, and Fig. 10 is a view with the cover removed, showing the method of shielding the individual parts. The range of capacitance is from 0.1 μ μ f. up to three μ f., and the frequency range is from about 10 cycles up to about 150 kc., the only modifications required in the bridges to cover this whole frequency range being a change in input and output transformers, as it is not found practicable to design these transformers to give efficient operation over such a wide frequency range.

A comparison of this bridge with the impedance bridge already mentioned shows it to be essentially the same circuit, the capacitance bridge having conductance shunts not included in the impedance bridge which allow a conductance balance to be made more readily. It is obvious that any two impedances can be compared on this bridge. Inductances may be measured by parallel resonance by simply placing them in the *A D* arm in parallel with the standard condenser and effecting a balance with it. This method is used to some extent for the measurement of large inductances.

Capacitance Unbalance. In order to keep cross-talk low in long cable circuits, it is necessary to have a high degree of capacitance balance between the various conductors in the cable, more particularly between the four conductors of a phantom group. The unbalances of interest are the phantom to each side circuit and the side-to-side unbalances. These may be measured on a capacitance bridge by measuring all of the direct capacitances[9] associated with the group and computing the unbalances required. A special circuit, however, is generally used which measures directly the particular unbalances in which we are interested. It consists of an input and an output transformer, two equal resistance ratio arms, a variable air condenser of the three-plate type, four binding posts for connecting the four conductors of the quad, and switches for making the various connections. By means of the switches, the cable conductors are connected to the circuit in such a way that the reading of the air condenser when a balance is obtained indicates directly the unbalance, either side-to-side or phantom-to-side, according to the switch positions. This circuit is capable of measuring capacitance unbalance as low as 1 μ μ f.

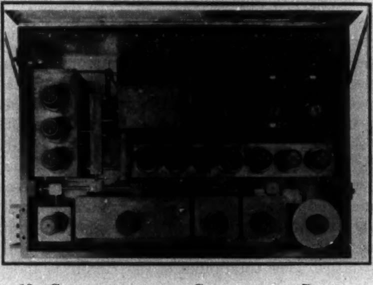

FIG. 10—CAPACITANCE AND CONDUCTANCE BRIDGE WITH COVER REMOVED

Showing method of assembly and shielding

Attenuation and Gain. So far, we have discussed the measurement of the fundamental impedance characteristics of apparatus. When the component parts have been found to meet their individual impedance requirements and are assembled to form the completed apparatus, it is desirable to have tests made of the overall performance of this apparatus. In a large number of cases, the requirement of greatest importance is the attenuation frequency characteristic. It is fairly obvious that this characteristic, of all apparatus used in telephone lines, is of interest, and this is particularly true of all types of filter circuits which are designed primarily for the purpose of furnishing definite attenuation frequency characteristics. These measurements are particularly required on apparatus used in carrier-current telephony and telegraphy.

From the very nature of the measurements, it is difficult to obtain a null method of measuring attenuation. The most direct method is to measure the input and the output of the apparatus under test simultaneously, from which the attenuation may be computed. The practical difficulty in doing this is to measure the extremely small outputs which are obtained from apparatus having high attenuations, where the characteristic must be obtained with the normal input, which is usually low. In general, it has been found necessary to use some form of amplifying device in the output circuit and it has not been found desirable to rely on the constancy of amplification of this device. Accordingly, the usual method used for the measurement of attenuation is a substitution one. The circuit is shown in Fig. 11A. There are two branches in this circuit, one of which includes the apparatus under test and the other containing a variable standard attenuator. The output of each branch is arranged to connect either to a detector of impedance Z_1 equal to the impedance of the standard attenuator or to a fixed impedance of the same value. If the apparatus under test has the same impedance as the standard attenuator, the input impedances Z_1 and Z_3 are made equal and the matching impedance Z_2 is omitted. Then the two branches of the circuit will be identical, provided the attenuation of the standard attenuator is equal to that of the apparatus under test. Accordingly, the method of measurement is to switch the detector to first one and then the other branch, adjusting the standard attenuator until an equal output is obtained for either switch position. The attenuator then reads directly the loss in the apparatus. The total input of the circuit is independent of the switch position, since the impedance conditions remain unchanged in switching.

If the apparatus under test has not the same impedance as the standard attenuator, the input impedance Z_3 and the matching network Z_2 are adjusted so that the circuit still reads directly.

The standard attenuator is a resistance network capable of variation in small steps, each step consisting of a network of the L, T, or H type, the resistance values being such as to give the desired attenuation between the output and input terminals. It is usually calibrated in 0.1-T. U. steps and may read as high as 100 T. U. corresponding to a ratio of power output to power input of ten billion to one or, if the impedances are the same, which is usually the case, corresponding to a current or voltage ratio of 100,000 to 1.

The calibration of these attenuators is based on the measurement of the individual resistances. Of course, sufficient measurements are made to determine that any capacitances which enter do not affect appreciably the accuracy of the attenuator at the maximum frequency used, which may be as high as 150 kc.

By modifying the circuit of Fig. 11A, we may use it to measure gain as shown in Fig. 11B. In this arrangement, the lower branch contains an impedance Z_4 that is adjusted to introduce a loss equal to that of the matching impedance Z_2 in the upper branch. In other words, with the amplifier under test out of the circuit and the standard attenuator set at zero, the detector will read the same for either position of the output switch. Then when the amplifier is introduced into the circuit, the attenuator is adjusted until the detector reads the same for either switch position, which means that the gain of the amplifier is just neutralized by the attenuator and the setting of the latter is read as gain.

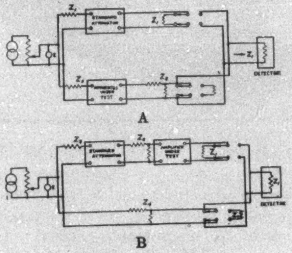

Fig. 11—Circuits for Measuring Attenuation and Gain

a. Arrangement for measuring loss

b. Arrangement for measuring gain

This circuit is used principally for the measurement of gain of audio frequency amplifiers, and is capable of measuring gain as high as 120 T.U. corresponding to a power output of 1,000,000,000,000 times the power input.

Cross-Talk. When there is an appreciable amount of coupling between two telephone circuits, any mutual interference which results is known as cross-talk. It is measured in cross-talk units, a cross-talk unit being defined as the relation existing between the two circuits when the current in the disturbed circuit is one millionth of the current in the disturbing circuit, the impedances of the two circuits being the same. Under these conditions, one cross-talk unit may be assumed the same as 120 T.U. Cross-talk in long toll cables is caused principally by unbalances in the cables and in the loading coils. The cross-talk due to the cable is measured simply by measuring the capacitance unbalance. The cross-talk due to loading coils is of a much more complex type, produced by a combination of capacitance, inductance, and resistance unbalances in the windings. Since the actual cross-talk caused by an unbalance in the coil is dependent upon all of the conditions of the circuit, it is necessary that any measurement of cross-talk made on the individual coils be made in a circuit as nearly as possible the equivalent of the line in which the coil is to be used. Consequently, all cross-talk circuits for the measurement of loading coil cross-talk consist of networks simulating the impedance of an ideal line of the type for which the loading coil is designed. The principle of the method is to apply to the disturbing circuit a definite input of a single frequency, usually 900 cycles, and to measure the cross-talk in the disturbed circuit at the desired point in it by comparing the tone heard in the telephone receiver connected at this point with the tone obtained from a cross-talk meter which is simply a device for obtaining a definite part of the input, and having a scale reading in millionths, that is, in cross-talk units. The measurement is made by switching from the cross-talk meter to the disturbed line and adjusting the cross-talk meter until the tone heard in each case is the same. The method is therefore not a null method and depends to some extent on the judgment of the operator, but results accurate to one or two cross-talk units may be obtained by this method. The coils as commercially produced after adjustment for this requirement are usually within 10 cross-talk units, representing an unbalance in the circuit due to the coil unbalance of less than one part in 100,000.

CONCLUSION

We have described in this paper a number of the more important high frequency methods of measurement and measuring circuits. It has been impossible to cover all of the different methods and circuits used, but we believe that the information given will be of value to those interested in this field of work.

We have not been able, in a paper of this type, to go into details concerning any specific circuits used, but we have referred to papers which describe in greater detail some of these methods and circuits, and it is expected that other papers will be published in the future covering other circuits which have received only brief mention here.

Discussion

J. R. Craighead: Will the authors kindly clarify the definition of a crosstalk unit? The definition of a unit as a "relation" is not easy to use for measurement purposes.

N. E. Bonn: Messrs. Shackelton and Ferguson have mentioned a vacuum-tube oscillator the frequency of which they can keep constant to within one part in 250,000 under ordinary variations in external conditions. If this is meant to include variations in plate voltage and filament current, the statement would appear to be at variance with universal experience which indicates that the frequency of vacuum-tube oscillators is profoundly affected by changes in tube impedance. In view of the importance of the subject will the authors please discuss the particular features of their circuit that make it so stable?

W. J. Shackelton: We are very glad to have had Mr. Craighead's comment on the matter of crosstalk. I will not attempt to redefine the crosstalk unit, but will simply try to explain what is meant by it, and then you can make your own definition.

If we have two circuits which, due to some undesired coupling, are so related that the current flowing in one of the circuits causes a current to flow in the second circuit, and if the current so induced (I am assuming that these circuits have the same impedance) is one-millionth part of the current in the circuit causing the disturbance, we say that the relation between the two circuits is such that one unit of crosstalk exists. If the induced current is twice as much the relation is such that we have two units of crosstalk. We don't have two relations, but you see we do have a different relation from that in the first case.

Mr. Bonn has raised some questions regarding the 1000-cycle oscillator. I think he assumed the statement that I made regarding the smallest division of the capacity element of the oscillating circuit to indicate the frequency stability of the oscillator as a whole. I said that the condenser could be set to one part in 250,000, but we do not claim that to represent the stability of the oscillator. We consider that to be, with respect to variation of filament current or plate potential, about one part in 100,000. I will ask Mr. Ferguson to explain in a general way how it is that we are able to obtain that degree of stability.

J. G. Ferguson: The frequency variations of the current in any oscillating circuit are due to a number of causes. One of the principal causes of variation would be variations of load. This is taken care of in this particular case by the use of one tube for an oscillator and a second tube as the amplifier. The load taken by the amplifier does not affect appreciably the oscillating circuit.

A second cause of variation is the variation in the level of oscillation in the oscillating circuit itself. In this case, we have to deal with only a single frequency, and it is possible to design the circuit so that the best conditions prevail for that frequency, that is, the degree of coupling controlling the level of oscillation is such that variations of plate potential, or of filament current, have the minimum effect on the frequency.

This is not so easily accomplished when we have an oscillator that covers a wide range of frequency, but in this case, for a single-frequency oscillator, it is comparatively easy to design the circuit so that the variations are very small due to changes in the level of oscillation.

Aside from these variables, the oscillating circuit itself, that is, the tuned circuit, is the greatest cause of variation, and this has been taken care of as described already by having the temperature variation due to the condenser equal and opposite to the temperature variation due to the coil and placing the whole tuned circuit in a separate assembly which is arranged so that any temperature changes take place very slowly, and the coil and condenser can be considered always to be at the same temperature.

Another way in which the characteristics are improved is to have a large capacity in the tuned circuit. In this particular case, the capacity is approximately ¼ microfarad. This means that variations in stray capacity and other variations which would cause frequency variations in the circuit if the circuit capacitance were small are reduced to a minimum.

The actual characteristics of this particular oscillator may be of interest. The frequency variations are less than 0.001 per cent with changes of B battery from 125 to 135 volts, the nominal voltage being 130 volts, and for current variations from 1.9 to 2.1 amperes, total current through the two-tube filaments in parallel.

The variation of the oscillator over a period of six months is less than 0.02 per cent. Such variations with respect to time, can be taken care of by recalibration. Over a period necessary to make a measurement or a series of measurements the stability is better than 0.001 per cent.

Impedance of a Non-Linear Circuit Element

BY EUGENE PETERSON[1]

Member, A. I. E. E.

Synopsis.—*Experimental determinations of the impedance of a variable element, such as an iron core coil worked at a high flux density, or a heavily loaded vacuum tube, are found to depend upon the impedance-frequency characteristics of the measuring circuit, as well as upon the complexity of the applied (measuring) potential wave. A physical picture of the action of a non-linear element in producing harmonics is built up, and it is shown that the flow of harmonic currents affects the measured impedance, at the fundamental frequency, of course, in two ways which are designated as the loading and reaction effects. The loading effect is that produced by the superposition of currents without any regard to the production of new frequencies, and the reaction effect, in accordance with energy conservation ideas, is that due to energy storage and dissipation at harmonic frequencies which appear as impedance reactions at the fundamental frequency.*

The physical picture set up in the above discussion appears to be capable of accounting for the experimental observations mentioned above in a qualitative way at least. In accordance with these ideas, it appears that even when the fundamental current is specified, there is no one definite value which can be assigned as the impedance of a variable element because of its dependence upon the circuit impedance-frequency characteristics. Several measuring methods are reviewed by which the impedance of the variable element may be determined under definite circuit conditions.

The above discussion is confined to the effects existing with a sinusoidal applied electromotive force, for simplicity, but the results with complex waves applied are analogous, as indicated by reference to the energy conservation principle.

IMPEDANCE determinations of non-linear elements, among which are included vacuum tubes and inductance coils with a more or less saturated iron core, involve factors additional to those present in the determination of impedances of ordinary invariable circuit elements. Such matters as impedance of the measuring circuit, impurity of the applied wave, and current or potential amplitude, which, except for extreme values leading to breakdown, do not ordinarily affect the impedance of the linear element, are found experimentally to be of considerable importance in the non-linear case.[2]

For our purposes we may assume a linear circuit element to be one distinguished by this property, that the relation between instantaneous current and instantaneous potential drop is a fixed and definite one expressed by a constant over the entire cycle. This is true not only of resistances but of reactances, in which the relation between current and potential drop is constant over the cycle, even though a constant phase angle is involved. A non-linear element, in contrast, is one in which the instantaneous current and potential are not related invariably throughout the cycle. Examples of this type of element which are of practical importance come readily to mind. For example, the two-electrode or the three-electrode vacuum tube will serve as an illustration of a variable resistance, and an iron core coil worked at high flux densities will serve as an illustration of both variable resistance and variable inductance. An important point in our distinction between linear and non-linear circuit elements has to do with the current-voltage relation over an entire cycle since it is possible in a non-linear element to have a relation between r. m. s. current and r. m. s. voltage as given by the usual type of a-c. meter, independent of amplitude. This particular condition arises in rectifiers in which the current wave for positive potentials has substantially the same wave form as the impressed potential wave, while for negative impressed potentials the current is zero. Here the relation between instantaneous current and instantaneous potential is specified by one of two values, depending upon the polarity of the applied potential wave. The relation between r. m. s. current and r. m. s. voltage, on the other hand, it is readily seen, is a perfectly linear one.

If now we attempt to determine the impedance of a non-linear element at a definite frequency and at a definite amplitude of fundamental current or potential by a-c. bridge measurements, in the usual manner, we should find the measured impedance to depend upon several factors which are not ordinarily considered in this connection. These are the presence of harmonics in the supply current, the magnitude of the ratio arm resistances, the impedances of the detector and of the current source at the fundamental frequency and at a number of harmonic frequencies, and finally the method used in attaining balance. To illustrate the last point, we might consider the non-linear inductance, with which it is observed that the measured resistance and reactance differ, when we balance the coil with a standard resistance and inductance, from the value obtained by resonating the coil with a standard capacity and balancing the resultant against a pure resistance.

The fact that in a non-linear element the relation between instantaneous current and potential is not constant over an entire cycle, and in which the impedance may be considered consequently as variable, means that the element serves as a generator of new frequencies which are ordinarily harmonics when but a single frequency is applied initially. It turns out that the effects mentioned above, which differ so markedly

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[1] Bell Telephone Laboratories, Inc., New York, N. Y.

[2] We shall not specifically consider those factors involved in impedance measurements which are common to linear and to non-linear elements, such as ground capacities; these are dealt with in the paper by Shackelton and Ferguson, p. 519.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

from our experience with impedance measurements of invariable elements, are to be explained by reference to the production of these harmonic frequencies in the circuit. To summarize the above account of the phenomena met with in measuring the impedance of a variable or non-linear element, we may say that the result depends upon the circuit to which the element is connected as well as upon the source of supply.

There are then two possibilities in general as to impedance measurements which are of practical importance. First, we may obtain a measurement characteristic of the non-linear element, and second, we may determine the impedance under specific circuit conditions. It is convenient to take the impedance of the element with harmonic currents suppressed as representative of the element, and this will be done in the following.

GENERATION OF HARMONIC FREQUENCIES WITH A SINGLE IMPRESSED FREQUENCY

Variable circuit elements may be definitely distinguished according to the manner in which the variations are produced; in one type, variations are produced by the current or the impressed potential; in the other, they are produced by means external to the circuit. In the first category belong the two variable elements discussed above; in the second belong the voice-actuated carbon transmitter and the mechanically driven generator. In general, when a circuit containing a variable element is subjected to a sinusoidal electromotive force, the reaction of the variable element causes a distortion of the current through it, or the potential across it, or both, depending upon circuit conditions to be discussed later, so that in consequence, harmonic currents and potentials exist in the circuit. These components may be determined in simple cases with the aid of Fourier's theorem from a detailed inspection of the wave form, or they may be measured experimentally with the aid of current or voltage analyzers as discussed in Mr. Horton's paper on that subject.

In order to provide a physical picture for the phenomena of harmonic production by a variable circuit element with a purely sinusoidal electromotive force impressed on the circuit, we may employ as a first approximation to the actual state of affairs a procedure which has proved fruitful and which is in agreement with mathematical analyses in the simple cases to which it has been applied. Here we assume the variable element replaced by a fixed impedance element together with a number of generators of the harmonic frequencies, each harmonic frequency being represented by an individual generator. The generators represent the driving potentials of harmonic frequency operative in the circuit, and to a first approximation the amplitudes of these potentials depend upon the fundamental current magnitudes and upon the properties of the non-linear element. This is a most important conception which serves as a basis for further discussion. From this it is evident that the amount of any one harmonic current flowing in the circuit depends upon the generated harmonic potential or upon the fundamental current, and upon the total circuit impedance at the harmonic frequency.

No simple law exists governing the dependence of the harmonic electromotive force upon the fundamental currents which is generally applicable, since the law varies with the type of element involved. Some of the harmonic electromotive forces may be zero at all fundamental amplitudes; thus, as is well known, the even harmonic potentials and consequently currents are substantially zero in unpolarized iron core coil circuits, just as the odd harmonic potentials and currents are substantially zero in the particular type of tube rectifier mentioned above. This is, of course, a particularly striking difference between the harmonic producing properties of different elements, which serves to emphasize the fact that the particular type of non-linearity in each case determines the connection between the harmonic driving electromotive force and the fundamental current which gives rise to it.

Although we cannot, in the light of the above examples, discuss the relations existing in non-linear elements in general, we can of course consider specific cases, and for illustrative purposes we propose to deal with the harmonic potentials developed in an iron core coil. In order to present data on harmonic production which shall be characteristic of the non-linear element and not of the circuit in which it happens to be connected, we shall discuss the equivalent generated or driving harmonic potentials. These, as we have previously suggested, may be thought of as depending solely upon the fundamental current in any specific type of non-linear element and are therefore independent of the impedance-frequency characteristics of the connecting circuit. Contrast this situation with that existing with regard to the harmonic currents. Here the total circuit impedance in magnitude and phase angle to each harmonic under consideration must be specified, since it is evident that the harmonic current will vary with the circuit impedance and has therefore no definite value characteristic of the core or the core material under investigation. If, however, the total circuit impedances are known, it is then possible to determine the driving voltages by multiplying them by the corresponding harmonic currents as determined by current analysis.

In the measurements to be described, the simpler procedure was adopted in measuring the harmonic driving potentials of providing a high circuit impedance to the harmonic frequencies and measuring the harmonic potential existing across the coil by a voltage analyzer. The arrangement is shown in schematic form in Fig. 1. This is equivalent to measuring the open circuit voltage of a generator so that an exact determination of the circuit impedances is unnecessary.

The results with a specific coil are given on Fig. 2 in which potentials across the coil are plotted as ordinates in terms of the current of fundamental frequency. The harmonic potentials are less at all fields than the fundamental potential drop and, at any field, the higher harmonics are of smaller amplitude than the lower ones. The lower harmonic potentials are roughly of the same order of magnitude as the fundamental and it becomes apparent that comparatively large harmonic currents may be made to flow in circuits which have low impedances to these harmonics; this would be the case if the impedance of the circuit external to the coil were capacitive and of the same value as the inductive reactance of the coil at the harmonic frequency. The ratio of harmonic to fundamental potential decreases rapidly as the fundamental field decreases, the rate of change being greater the higher the harmonic, so that at the low fields characteristic of high quality communication systems the ratio may be of the order of 1/1000 for the third harmonic. The higher fields are of course met with in magnetic frequency multipliers, for example, and in general in power work. The same sort of effect is observed to hold for other types of variable element—the low value of the ratio at small fundamental currents, the rapid increase of the ratio at high fundamentals. It may be found in certain cases that some of the harmonic potentials pass through a maximum. This last is observed in heavily loaded vacuum tubes, for example.

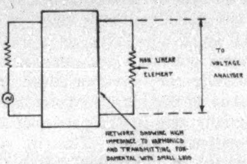
NON LINEAR
ELEMENT
TO
VOLTAGE
ANALYSER
NETWORK SHOWING HIGH IMPEDANCE TO HARMONICS AND TRANSMITTING FUNDAMENTAL WITH SMALL LOSS

Fig. 1

The problems that confront us here are those resulting from the complex form of the resultant current wave, a wave which includes the fundamental frequency together with those of its harmonics which are determined by the nature of the non-linear element and by the connecting circuit. Thus with a low external impedance to a harmonic, the harmonic potential across the circuit is small and the harmonic current relatively large. The situation is evidently reversed with a high circuit impedance to the harmonic under investigation. The effects produced by the flow of harmonic currents are two-fold: first, a different region of the non-linear current-voltage characteristic is traversed so that the impedance of the variable element is in general changed, and second, because of the relations between the harmonic current and the fundamental there is a further effect directly upon the fundamental current itself. To distinguish clearly between the two effects we shall find it advantageous to consider each effect independently.

Suppose now that by removing the effect of the introduction of new current components due to the non-linearity of the circuit with the aid of appropriate circuit impedances, we investigate the mutual effects of two independent currents of different frequency impressed on the circuit. In general, the impedance of the variable element to any current is influenced by the presence of other currents, although the effect decreases rapidly with the amplitudes of the currents in question. The presence of a small component has negligible effect upon the flow of a large component in the case of a two-component wave, although the impedance offered to the small component is greatly influenced by the large one. This is true of a vacuum tube, for example, while for saturated iron cores the effects depend not only upon the component amplitudes but also apparently upon their frequency ratio.[3] This effect, the alteration of impedance through the presence of a superposed current independently generated, we shall designate, in the following, as the loading effect.

Figs. 3A and 3B are reproduced from the paper referred to and illustrate the loading effect in a silicon steel core coil. Fig. 3A shows how the flux density of a definite frequency and corresponding to a definite magnetizing force depends upon the amplitude and frequency of a superposed magnetizing force; the ordinates are expressed in thousands of lines per cm.[2] Fig. 3B shows the corresponding effect of superposition upon energy loss expressed in thousands of ergs per cubic centimeter per cycle. It will be readily appreciated that the flux density and energy loss could be expressed in terms of inductance and resistance. For our present purposes it is sufficient to recognize the existence of the loading effect from these two figures.

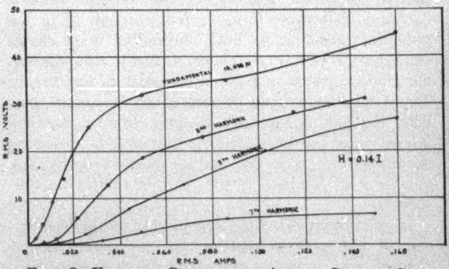

Fig. 2—Voltage Components Across Silicon Steel Core Coil as Function of Fundamental Magnetizing Current

We are now in position to investigate the second factor in determining the impedance of a variable element, which is quite distinct from the loading effect.

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[3] *Phys. Rev.*, Vol. 27, No. 3, pp. 323, 325.

For this purpose we consider a non-linear element in which the loading effect is not appreciable, such as the carbon button microphone externally agitated. Here the absence of what we have termed the loading effect is manifested by the fact that the button resistance over a certain range is independent of the current traversing it. The button develops a variable resistance, however, in view of the externally applied force, so that new frequencies are generated in the circuit when an electromotive force is applied. The currents produced by non-linearity dissipate and store energy, which must be supplied by the energy source. It is evident, therefore, that the circuit impedance at the impressed frequency depends upon the circuit conditions to the new frequencies developed. This effect is denoted in the following as the reaction. The energy dissipated or stored in this additional fundamental resistance and reactance term must then be sufficient to account for the energy dissipated or stored by the new frequencies, at least if we are to retain our ideas on energy conservation. The two effects of harmonic current flow, loading and reaction, may of course be present simultaneously, as in a vacuum tube circuit or in an iron-core coil circuit. The justification for the separation of the net effect into two factors lies in the fact that the two factors depend upon radically different circumstances, so that the separation permits us to follow the phenomena somewhat more closely.

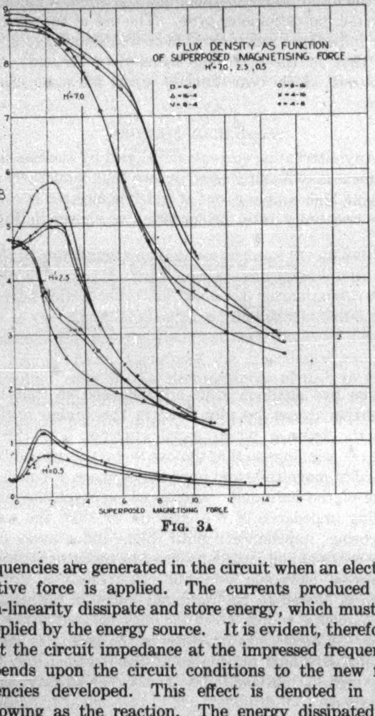

FIG. 3A

Another way of looking at the same effects is to consider the phenomena involved from the standpoint of the potentials developed, rather than from that of energy, as above. To do this, we may regard the non-linear circuit impedance as having harmonically variable components. Thus a vacuum tube would be regarded as having a constant resistance component together with other resistance components which vary at the fundamental and harmonic frequencies. Hence, when the fundamental current flows through the variable part of the tube resistance, an harmonic potential is produced which corresponds with the harmonic driving potential. The reaction of the harmonic flow on the fundamental is now to be obtained by considering the flow of a harmonic current through the variable element. It is found by a detailed consideration[4] that there is produced a voltage component of fundamental frequency across the variable element due to the flow of harmonic. Now assuming the harmonic resistance components involved to have fixed values, it becomes clear that the fundamental potential developed depends upon the harmonic current as well as upon the phase of that current, since a shift of 90 deg. converts a resistive component to a reactive one. Since the fundamental potential drop may be interpreted in terms of the fundamental impedance, the reaction of harmonic current flow is evident, quite apart from the loading effect.

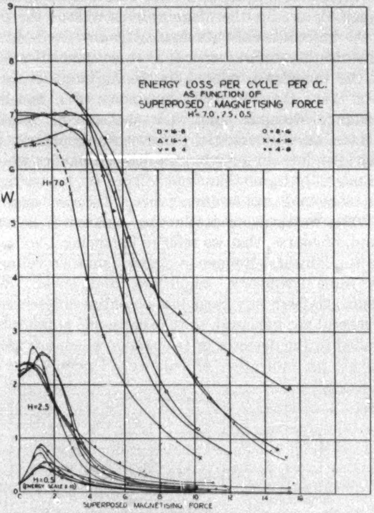

FIG. 3B

Consideration of the problem from the standpoint of harmonically varying impedances yields the same general results as those obtained above in which the presence of harmonic generators was assumed; in fact, the harmonic electromotive forces generated may be derived from a consideration of the variable impedance. The harmonic generator circuit, however, appears to offer a clearer picture of the manner in which harmonic currents are affected by circuit impedances, and has been developed to somewhat greater length for that reason. Fundamentally, of course, the two methods appear to be equally acceptable.

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[4] Appendix I.

With the effects of harmonic current flow in mind—we have designated them as loading and as reaction—it becomes possible to explain in at least a qualitative way the difficulties which were enumerated at the start. The influence of harmonics in the supply source will be recognized as primarily a loading effect of the same nature as those discussed in connection with Figs. 3A and 3B which applied to iron core coils. The magnitude of the ratio arm resistances in the bridge used for measurement, together with the impedances (invariable with time) of the detector and the generator source, determine the magnitude as well as the phase of the harmonic currents flowing at any fixed value of fundamental current, since it is these quantities which fix the impedance offered to the harmonics. In the same way, the measured impedance of a non-linear reactance depends upon the method of attaining balance, since by resonating the variable reactance we may introduce an impedance to the harmonics which is considerably lower than that offered by balancing the reactance by a standard reactance of the same sign.

When we speak of attaining a balance, it is understood, of course, that we refer to balancing a voltage of the fundamental frequency against another voltage of the same frequency, amplitude, and phase. When this is attained, any harmonic potential differences are in general not balanced, so that harmonic potentials are applied to the detector or telephones which are used to provide an indication of balance. Under these conditions, it is difficult, if not impossible, to determine the balance point since, generally speaking, it is obscured by the harmonics, and the values assigned to the impedance will depend upon the individual making the measurement. For this reason the detecting apparatus is provided with frequency selective apparatus which transmits the fundamental to the exclusion of the harmonics. This is, obviously, quite apart from any consideration of the influence of the detector impedance upon the actual value of the impedance of the element being measured.

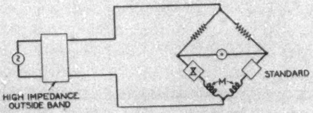

Fig. 4—Bridge for Measurement of Modulator Impedances

Impedance Measurements, Harmonics Suppressed

The complicating effect introduced by impurity of the impressed potential wave is removed without much difficulty by the introduction of tuned circuits or other frequency selective circuits in the generator supply circuit. The suppression of harmonics generated by the variable elements requires other means which will be discussed for two distinct circuits, one a modification of the usual bridge method, and the other, a method of the a-c. potentiometer type. The use of these circuits for impedance measurements is well known, of course, so that in the following we shall cover only those features which bear directly upon harmonic current flow.

Bridge Method

Any alternating currents developed by the non-linear element may be suppressed by two high inductance coils having high coupling, one of which is inserted in each of the two unity ratio bridge arms as shown in Fig. 4.

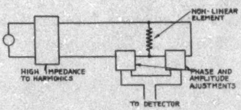

Fig. 5

These two auxiliary coils are balanced so that their insertion causes no alteration in the bridge settings. At the balance point, equal currents flow through the two windings so that the two fluxes are substantially equal in magnitude and opposite in phase. This is not true of the harmonic currents which meet the series-aiding impedance of the coils rather than the series-opposing impedance. With high inductance coils, this series-aiding impedance may be made so high as to suppress harmonic currents effectively. A parallel path is offered to the harmonic currents through the fundamental source which includes but one of the two coils. A network is provided in the generator circuit to oppose a high impedance to the harmonics so that the high-mutual coil may not be effectively short-circuited.

In practise, any lack of balance between the two inserted coils may be made up by corrections to the bridge settings determined by a separate balance when the two coils are balanced against one another. The null point is not as sharply defined with the two coils inserted because of the added impedance for departures from the balanced condition, so that it is well to keep the inductances down to the minimum value at which effective harmonic suppression is assured.

A-c. Potentiometer Method

Here harmonic suppression is attained entirely by a network which passes the fundamental freely while offering a high impedance to the harmonics generated by the element. The potential drop across the coil is changed in amplitude and in phase to balance the drop across a resistance in series with the non-linear element as indicated in Fig. 5. The phase and amplitude adjustments may be so arranged as to have a negligible effect upon the quantities being measured. The impedance of the element may be calculated from the constants of the circuit used to attain balance. The principles involved here were applied to the measurements represented in Figs. 3A and 3B.

IMPEDANCE MEASUREMENTS, HARMONICS FLOWING

The two methods discussed above were used to insure freedom from the effects of harmonic current flow and are fixed and definite and independent of the measuring means so long as the harmonics are suppressed. Under practical conditions of use, however, harmonic currents flow in the circuit and react on the fundamental impedance in the manner which has been discussed above. In this situation, therefore, the impedance can be determined only in the circumstances of use, so that the measuring circuit is required to leave the circuit impedances unaltered, or to duplicate them. For this purpose two measuring circuits are available, one a bridge modification, the other a potentiometer method, which are derived more or less obviously from the two methods previously considered.

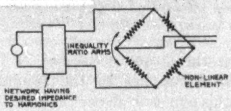

FIG. 6

INEQUALITY RATIO BRIDGE

The arrangement shown in Fig. 6 has been used to provide any desired impedance to the harmonics despite the fact that the harmonic circuit forms part of the bridge network. The bridge arms are made much higher than that of the element to be determined since they are connected in shunt to the standard and the unknown, and they are made unequal so that but a low series impedance, that of the standard, may be inserted in the harmonic path. The desired impedances to the harmonic frequencies are then inserted in the generator path. It is clear that when the ratio arm resistances are sufficiently high, the detector impedance may be neglected.

The circuit of Fig. 7 was set up for the impedance measurement of a small, two-element vacuum tube such as is used for low power rectification. It was desired to determine the impedance of the tube when the harmonics were permitted to flow through a series resonant circuit, the tuning of which was varied so as to cover the frequency range up to the fourth harmonic. The circuit details are given in Fig. 7, in which it is seen that the two ratio arms were made 10^5 and 10^4 ohms, respectively, so that the shunting resistance was about 100 times that of the element under measurement, while the series resistance to the harmonic was about one-tenth that of the tube. A direct current was supplied through a distinct circuit including a large choke coil so as to avoid shunting the alternating currents. The fundamental was supplied through a filter which at the same time maintained a high impedance to the harmonics, so that they were restricted to the tuned circuit provided for them, and so that the fundamental current maintained constant at 11.8 mils could be measured by a thermocouple as indicated, without error through the presence of harmonics.

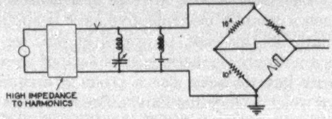

FIG. 7

The experimental results are set forth in Fig. 8 in which measured resistance and reactance are plotted as ordinates in terms of the capacity of the harmonic tuned circuit. Impedance changes are observed to occur when the tuned circuit approaches resonance for any harmonic. The nature of the impedance variation is observed to be in accord with the conclusions derived from the energy conservation argument, as well as with the general analytical conclusions of the appendix. These last are, in brief, that the fundamental resistance decreases as the harmonic impedance decreases, and that a reactance to a harmonic appears as a reactance of the same sign in the fundamental circuit. The dips in the resistance curve decrease in magnitude as the harmonic frequency increases, which is due partly to the fact that the harmonic driving electromotive forces decrease as the frequency increases.

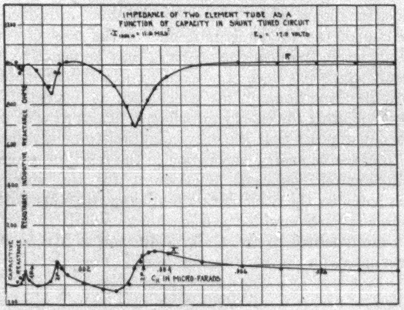

FIG. 8

POTENTIOMETER METHOD

With the preceding case in mind, the modification of the circuit of Fig. 5 to permit harmonics to flow is rather obvious and nothing further need be said about it beyond the statement that the desired end may be attained by shunting a harmonic current path around the input filter.

EXTENSION TO COMPLEX APPLIED ELECTROMOTIVE FORCES

The discussion up to this point has been confined for simplicity to the production of harmonics, with a sinusoidal electromotive force applied to a non-linear circuit. The same effects, at least in a general way, exist when a complex electromotive force is applied to the circuit. In this case, as is well known, we have developed not only the harmonics of each of the input frequencies but we have also a group of frequencies produced which include sums and differences of integral multiples of the input frequencies. Here we have the same reaction of the harmonics on the fundamentals which gave rise to them as we had in the sinusoidal input case, and in addition we have the reaction of the combination frequencies, which also affects those fundamentals from which the combination frequencies are derived. For example, if we suppose the combination frequency $m f_1 + n f_2$ to be developed in a non-linear circuit when the frequencies f_1 and f_2 are impressed on the circuit (m and n being integers), the impedances to both f_1 and f_2 are influenced by the flow of the combination frequency. From the energy standpoint, we can say that the energy dissipated and stored at the combination frequency is accounted for by the reflected reactance and resistance terms of the two fundamental frequencies. Some very interesting conclusions may be drawn from this relation, one of the most striking of which is that the resistance introduced to one of the fundamentals, due to the flow of the combination frequency $m f_1 - n f_2$, may be negative, a result which has been confirmed by experiment under suitable conditions. This conclusion, of course, far from violating the energy principle, is in accord with it, since it appears that the only condition to be satisfied is to have the energy absorption in the reaction to one fundamental, sufficient to account for the energy dissipated by the combination frequency, together with the energy delivered to the other fundamental.

I am indebted to Mr. C. R. Keith and to Mr. L. B. Arguimbau for the experimental results.

Appendix

Reaction of Harmonic Current Flow on Impedance to the Fundamental. If we have a potential of $E \cos p t$ impressed on a non-linear circuit in which the instantaneous resistance is expressed in the particularly simple form

$$r = R_k + R \cos p t \tag{1}$$

where R_k represents the circuit resistance at a frequency k, the current wave contains harmonics in addition to the fundamental. Considering the effect of but one of the harmonics for simplicity, the current wave may be expressed as

$$i = I_1 \cos p t + I_2 \cos 2 p t \tag{2}$$

Multiplying the instantaneous current and resistance together and setting the product equal to the impressed potential, we have

$$E \cos p t = \left(R_1 I_1 + \frac{R I_2}{2} \right) \cos p t + \left(\frac{R I_1}{2} + R_2 I_2 \right) \cos 2 p t \tag{3}$$

in which R_1 is the steady resistance at fundamental frequency and R_2 applies to the second harmonic. Inspection of this equation shows that the fundamental frequency term contains I_2, so that it depends upon the second harmonic current. Solving for I_2,

$$I_2 = - R I_1 / 2 R_2 \tag{4}$$

and the impedance at the fundamental frequency is given by the expression

$$R_1 - \frac{R^2}{4 R_2} \tag{5}$$

which is obtained by putting (4) in (3).

From this it appears that the condition for maximum fundamental power dissipation in the variable element requires a lower generator impedance than that required when no harmonic flows. The power dissipated by the second harmonic is

$$W = \frac{1}{2} I_2^2 R_2 \tag{6}$$

while the power expended in the resistance effectively introduced into the fundamental path is

$$\frac{1}{4} R I_1 I_2$$

which, with the aid of (4) reduces to (6).

The same general relations are found to hold when the circuit resistances are replaced by impedances, in that the energy stored and dissipated at the harmonic frequency is represented by a corresponding effective resistance and reactance at the fundamental frequency. Thus the fundamental impedance may be written

$$Z_1 - \frac{R^2}{4 Z_2}$$

from which it is seen that an inductive reactance in the harmonic path is reflected as such into the fundamental path.

The Empirical Analysis of Complex Electric Waves

BY J. W. HORTON[1]
Associate, A. I. E. E.

Synopsis.—*In many problems dealing with electric waves, particularly those encountered in electrical communication, it is necessary to examine with some care the several sinusoidal components present. Both qualitative and quantitative methods are in use.*

A convenient qualitative examination may be made by heterodyning the complex wave with a sinusoidal wave of variable frequency and providing means for detecting any low frequency beat currents which may be produced.

In practically all methods for the quantitative measurement of the individual components of a complex wave, means are provided for selecting a single component and for preventing other components from reaching the indicating apparatus. The general arrangement of the selecting and measuring elements depends upon the character of the information sought. If the problem involves an exploration of the frequency range of the complex wave, primarily for the purpose of determining what components may be present, automatic analyzers are available which cover the field point by point and plot its spectrum. There are also analyzers which permit the field to be covered by heterodyning with a variable frequency so as to cause the resulting wave to be swept across a fixed selective circuit.

For the precise measurement of any single component, particularly a component of small amplitude in the presence of others of larger amplitudes, analyzers have been developed which employ the well known heterodyne principle which utilizes selectivity at a frequency below that of the unknown wave, thus effecting an appreciable increase in the percentage separation between adjacent components.

* * * * *

IN the more usual measurements on electric waves it is sufficient to determine the magnitude of some quantity which is indicative of the amplitude of the wave. The most common example is, of course, the alternating-current ammeter or voltmeter the indications of which are proportional to the r. m. s. value of the current or voltage being measured. Similarly, certain devices are available which read directly the maximum amplitude attained by the wave in question.

For many purposes, the information furnished by measuring instruments of these types, together with some idea as to the shape of the wave, is all that need be known. The increasing refinement of electrical communication systems, however, makes it necessary to look for methods whereby the composition of a complex wave may be more accurately determined. This is particularly true when we come to systems in which the components constituting the normal signal wave are replaced by components lying upon some other portion of the frequency scale. In such systems it is necessary to employ modulators or similar devices which have a non-linear relation between the impressed wave and the resulting output wave. The characteristics of such devices, unlike those of selective networks or filters, cannot be determined adequately by direct comparisons between the input and output waves but must be studied through measurements upon the individual components present in each.

The value of information of this character has become very great through our increased knowledge of the relation between the steady state conditions in a system and its behavior with respect to transient phenomena. It is probably true that the exact statement of the requirements of a system for the electrical transmission of speech is one of the most difficult in the entire communication field, due to the lack of means by which the properties of the signal wave, either acoustic or electrical, may be quantitatively stated. It has been found possible, however, to specify certain steady state conditions, which must be fulfilled by a system in order that the inevitable modifications of such transient waves as are involved in communication shall be within the limits of tolerance. For this reason, measurements relating to steady state conditions become of the first order of importance and have resulted in the development of numerous methods for determining the individual components of complex sustained waves.

One of the earliest means for examining the composition of a current or voltage wave was that used by Lenz in 1849 and later developed to the point of satisfactory practical use by Joubert[2]. This employs a sliding contact driven in synchronism with the fundamental period of the wave and so arranged as to connect an electrostatic voltmeter or similar indicator to the circuit carrying the wave at any desired time during its fundamental cycle. By repeating the measurement at a series of points throughout the cycle, the shape of the wave may be plotted point by point. The more recent development of the oscillograph to its present perfection has practically supplanted the sliding contact as a means for securing a graphical picture of the shape of an electric wave. Having obtained a trace corresponding to the instantaneous time variation of amplitude, it is possible by mathematical means to resolve it into its several harmonic components, determining the phase and amplitude of each. The quantitative precision of this method is not great, however, being limited by the resolution of the wave tracing device and by the accuracy with which the analysis may be carried out.

That the oscillographic method is totally inadequate for many of the problems arising in communication work may be shown by using the multi-channel repeater of a carrier telephone system as an example. In such a repeater, the complex wave being amplified may contain components associated with three or more individual signals. As is well known, any distortion in amplifiers or other electrical circuits results in the development of wave components of frequencies other than those present in the impressed wave. These new components may have frequencies corresponding to multiples of the frequency of a single impressed component or to algebraic combinations of the frequencies of several impressed components. In the case of the multichannel carrier repeater, such developed components due to one or more of the impressed signal groups may well fall within the frequency range allotted to the group of components associated with some other signal. Having once been introduced into this portion of the frequency range, these extraneous components thereafter proceed with the proper signal components to the terminal equipment, and appear either as intelligible cross-talk or as noise superimposed upon the desired speech wave reproduced by the receiving equipment. The requirements of satisfactory speech transmission make it desirable in many cases to determine the amplitude of these extraneous components where their value is 0.1 per cent or less of the amplitude of the components properly associated with the signal. The detection of a component of this amplitude in the presence of the components producing it, is beyond the reach of the usual oscillographic method. The exact determination of its value by such a method is obviously quite impossible.

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[1] Bell Telephone Laboratories, Inc., New York, N. Y.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

[2] *Comptes Rendus*, Vol. XCI, p. 161, 1880.

As is usual in all analytical work, we are interested in both qualitative and quantitative determinations. For the detection of the presence of a component of very small amplitude in, say, a complex electric current, the heterodyne beat method, to mention one of several, has been found very practicable. The unknown wave and a sinusoidal wave of variable frequency are impressed jointly upon a detector circuit. This detector circuit may take the form of a three-element vacuum tube so biased, that no current flows in the plate circuit when no alternating electromotive force is set up in the grid circuit. When a wave is impressed on the grid circuit, however, there is present in the plate circuit a current containing components of zero frequency and of frequencies corresponding to the sums and differences of the frequencies of the components of the impressed waves. By the use of a suitable d-c. meter in the plate circuit, the amplitude of the d-c. component may be observed directly. Such a-c. components as are present are ordinarily of frequencies to which the moving parts of the meter are incapable of responding. When, however, the frequency of the sinusoidal current is brought close to the frequency of some component in the unknown wave, the component in the plate circuit of the detector which corresponds to their difference frequency will be of such low value that it may be followed by the needle, which will therefore vary periodically about the value corresponding to the d-c. component. By adjusting this difference frequency to the natural frequency of the meter, it is obvious that the beat due to a very minute component in the unknown complex wave may be detected. The amplitude of the beat does give some general idea, of course, as to the magnitude of this component although the method is not well suited to quantitative work. By arranging the detector tubes in a bridge circuit as shown in Fig. 1, the d-c. or zero frequency component may be balanced out, thus making it possible to use for the detection of the low frequency difference component, a meter of higher sensitivity than would be possible when this component appears as an increment superimposed on a zero frequency component of greater magnitude.

The complete quantitative determination of the components of a complex wave involves, of course, a determination of the amplitude and of the phase of each of the individual components. For most purposes, however, a knowledge of the amplitude alone suffices and many of the methods of analysis which have been developed are confined entirely to this determination.

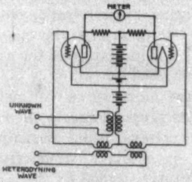

FIG. 1—CIRCUIT FOR THE DETECTION OF COMPONENTS OF A COMPLEX WAVE

The type of analyzer of necessity depends upon a number of factors among which are the resolution, the sensitivity, the frequency range, the precision, and the nature of the problem. Considering the last factor, it is apparent that an analyzer suitable for observing small variations in a single component, might be unsuited to a rapid determination of the number and frequency of the several components present in an unknown wave. In all cases, however, the system involves some selective device whereby the indicator may be rendered responsive to a single sinusoidal component and, at the same time, be unaffected by other components.

The possibilities of this general method were first recognized, about 1894, by Pupin,[3] who used it to investigate the harmonics generated by alternators. The application to waves of the magnitude of those encountered in communication was not possible, however, until the vacuum tube amplifier became available.

Practically all analyzers for waves of small amplitude are modifications of the elementary form in which a selective circuit couples a vacuum tube amplifier to the circuit containing the wave to be investigated. The output of the amplifier is thus determined largely by the amplitude of the particular component in the complex wave to which the selective circuit is tuned. In the examination of complex currents, a convenient form of coupling is to introduce a low resistance into the given circuit and to shunt this resistance by a series tuned circuit. As far as the original circuit goes, therefore, the effect is comparable to that of inserting an ordinary ammeter. For coupling to the vacuum tube amplifier, it is necessary simply to connect the grid of the first stage directly across the condenser or the inductance of the tuned circuit. By the proper choice of elements, the voltage thus resulting on the grid for the particular component to which the circuit is tuned will be many times the voltage drop across the resistance due to that component. The voltage step-up of the tuned circuit at resonance, it will be recalled, is given by the expression $\frac{\omega L}{R}$, or $\frac{1}{\omega C R}$, where R is the effective resistance in series with the inductance and the capacitance. In the frequency range between several hundred and several hundred thousand cycles per second the value of this quantity, which is frequently designated by Q, may well be of the order of 150 or more. In the case of carefully selected elements for use with radio frequencies, it may become, on occasion, as high as 300.

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[3] *Resonance Analysis of Alternating and Polyphase Currents*, TRANSACTIONS of A. I. E. E., Vol. 11, 1894 p. 523.

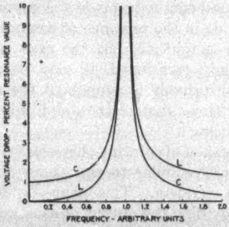

Fig. 2—Curves showing the Voltage Drop Across the Inductance and the Capacity of a Series Resonance Circuit for Impressed Voltages of Constant Amplitude and Varying Frequency

The choice between the inductance and the capacitance as the source of the voltage drop for connection to the grid depends upon whether most effective discrimination is required for components having higher or lower frequency than the component being measured. The curves of Fig. 2 show the voltages across the inductance and across the capacitance of a fixed tuned circuit, plotted as a function of frequency, for a constant voltage maintained across the entire series resonant circuit. Thus, greater discrimination may be secured against components of lower frequency than that to which the circuit is resonated, by using the voltage drop across the inductance, whereas for components of higher frequency greater discrimination may be secured by using the voltage drop across the condenser. This point is of significance, for example, in the measurement of the harmonics of a given fundamental. For a given voltage due to the second harmonic across either the inductance or the capacitance, the voltage across the inductance due to the fundamental will be only one-fourth of that across the capacitance. With higher harmonics the advantage of using the inductance will be even greater.

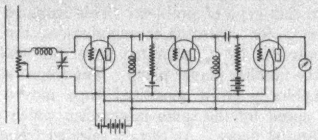

Fig. 3—Elementary Form of Current Analyzer Circuit

To analyze a voltage wave rather than a current wave, an anti-resonant circuit in series with a high resistance may be connected across the potential to be measured in the same fashion as the ordinary voltmeter. The properties of this circuit, with respect to the currents in the several members, are analogous to those of the previously described circuit with respect to the voltages across its members. In practise, it is more convenient to use an initial vacuum tube stage having a high input impedance and arranged to work into a current analyzer circuit.

Having secured as the output of the vacuum tube amplifier a voltage indicative of the amplitude of a single component of the complex wave, it is a simple matter, by suitable amplification, to obtain sufficient power to operate any desired indicating device. One of the most convenient methods is to use as the final stage a vacuum tube having sufficient negative bias on its grid to prevent the flow of space current when the a-c. input wave is zero. The arrangement of the elementary analyzer, therefore, is similar to that shown in Fig. 3. The determination of the absolute value of the amplitude of any component which may be selected requires simply that the sensitivity of the analyzer be determined as a function of the frequency over the range for which it is to be used.

A circuit similar to the elementary arrangement just described was constructed during the World War for the purpose of analyzing the currents present in various submarine sound detectors. Here the problem involved the exploration of a considerable range of frequencies to determine what components might be present, as well as a measurement of their relative magnitudes. The first tests were carried out by laboriously adjusting the capacitance of the tuned circuit to the values corresponding to a succession of different frequencies separated by small increments. For each capacitance setting, the deflection of the rectified current meter was noted. With this arrangement, the time required to cover the frequency range between 100 and 3000 cycles was several hours. Subsequent modifications in the apparatus to facilitate the manual labor reduced this somewhat but still left it so high as to make it impractical for most purposes. This led to the suggestion of employing some automatic method both for varying the tuning and for recording. A paper[4] presented by Wegel and Moore before the A. I. E. E. in 1924, describes the final form of an analyzer suited to this type of problem. This employs pneumatic relays for operating the condensers, the relays being controlled by a perforated paper roll similar to that of the familiar player piano. The record is made photographically on a sensitized tape moved at a uniform speed by the same mechanism which drives the perforated paper. At certain selected frequencies an auxiliary lamp, directly controlled by the perforated paper, traces lines across the photographic tape, identifying values along the frequency scale. An illustration of a record made by this analyzer is shown in Fig. 4. Its value in the qualitative exploration of an unknown wave is obvious. It is capable, furthermore, of considerable accuracy quantitatively.

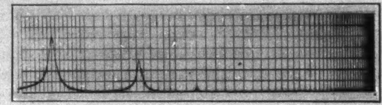

Fig. 4—Record made with Automatic Current Analyzer

The analyzer just described is admirably suited for the investigation of many questions arising in electrical communication, particularly where it is necessary both to discover what components may be present and to determine their relative amplitudes. There are occasions, however, when a more rapid survey of the frequency range is all that is required and for which the elaborate recording mechanism is not justified. To meet these conditions, an analyzer has been developed by Moore and Curtis[5] in which the entire frequency range is swept over by the manual adjustment of a single continuously variable condenser. To accomplish this result, the unknown wave is heterodyned by a variable frequency wave, whereby it is effectively transferred on the frequency scale in such a way, that the several components may be made to fall in succession within the field of a fixed selective measuring device. By properly choosing the frequencies of the heterodyning wave, it is possible to make the interval between the limits over which it must be varied, considered as a fraction of their absolute value, smaller than the corresponding interval between the limits of the original wave. The region over which the heterodyne wave must be adjusted is thus brought within the range of a single controlling element.

In replacing the unknown wave by a wave occupying a higher position on the frequency scale, in order to reduce the fractional separation between the limits, the fractional separation between adjacent components of necessity has been correspondingly decreased. In order to retain the required resolution in the analyzer as a whole, the fixed tuned circuit on which the wave components fall as the wave is moved along is made to have extremely high selectivity through the use of mechanical resonance. In fact, the high selectivity of the particular element chosen, as compared with the selectivity of the elementary analyzer previously described, more than compensates for the loss in selectivity resulting from the frequency translation. Means are provided, as described in detail in the published paper referred to, whereby the amplitude of any unknown current component may be determined. This type of analyzer is of particular value in examining the composition of sustained vocal or instrumental tones, or other periodic waves of comparatively short duration.

In many investigations, particularly those relating to modulation or to distortion occurring in non-linear circuit elements, it is desirable to examine the variations in a single component of a complex wave as conditions affecting it are varied. Here, the ability to explore the entire range of the wave need not be considered. Analyzers designed for this last mentioned type of problem, in general, require high selectivity in order to permit measurements to be made of components of very small amplitude in the presence of components of very much greater amplitude. In the case of the carrier repeater already mentioned, it may be necessary to examine quantitatively a component the amplitude of which is 10^{-5} times that of the amplitude of the components producing it.

A simple method of securing this selectivity is to use a number of units, similar to the elementary analyzer, connected in tandem. This may be accomplished readily by providing the output of each amplifier stage with a step-down transformer of high turns ratio, thus securing proper coupling between the output of the vacuum tube and the resistance element of the selective circuit. In tests made of modulation occurring in the Havana-Key West telephone cable, three such units were used in tandem, successfully. There are advantages in this arrangement. First, the discrimination against any unwanted component is the product of the discrimination of the several elements. Second, by alternating units containing amplification with units containing attenuation, the differences between the energy levels at any two parts of the system may be kept small. Third, the manipulation of the system is simplified because the tuning of one element may be carried out without affecting the adjustment of any of the others.

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[4] Transactions of the A. I. E. E., Vol. XLIII, 1924, p. 457.

[5] *Bell System Technical Journal*, April, 1927.

This relatively simple arrangement of multiple stages has recently been found inadequate for the exacting requirements which are to be met in the study of high frequency transmission apparatus. There has been developed, therefore, a heterodyne analyzer in which the frequency range to be examined is moved to a position on the frequency scale below that which it normally occupies. In this case, the translation is accompanied by a considerable increase in the fractional separation between the components. By varying the frequency of the heterodyning wave, the component produced by its interaction with any component of the original wave may be made to fall, as before, within the field of a fixed tuned circuit. Due to the increased fractional separation, a circuit of given selectivity is more effective at this lower frequency than at the frequency of the original component. Furthermore, inasmuch as the selective circuit need not be adjusted to any of a number of frequencies, a more elaborate design is possible, giving much greater selectivity than could be obtained with a circuit capable of convenient adjustment. These two effects working together make the discrimination of the heterodyne analyzer much greater than could be obtained by other means.

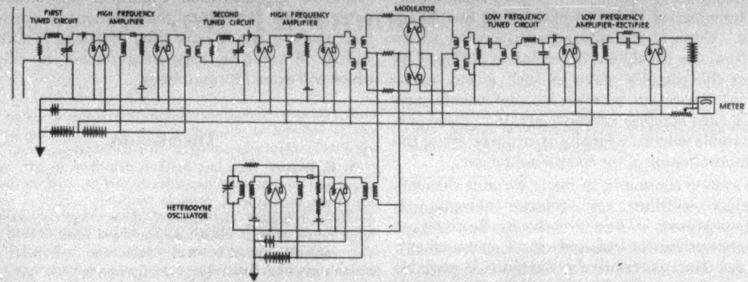

FIG. 5—SCHEMATIC DIAGRAM OF HETERODYNE CURRENT ANALYZER

It is apparent, since we are concerned with a particular component which has resulted from the transmission of one or more other components through a distorting device, that care must be taken to prevent these other components from reacting in the measuring equipment in such a way as to modify the amplitude of the component in question. This end may be secured in two different ways. First, the interacting components may be prevented from reaching the modulator in which the heterodyning is effected and, second, the design of the modulator itself may be such as to prevent components resulting from interactions between components present in the incoming wave from reaching the indicating device.

In the actual arrangement of the heterodyne analyzer, which is described in detail in a paper published elsewhere by A. G. Landeen,[6] both of these methods are employed. The modulator is preceded by two stages of simple selectivity and amplification so that any components other than the particular one to be measured are materially reduced in amplitude before reaching the modulator. The modulator is of the well-known balanced type in which any product of modulation between two waves impressed conjugately upon the input is delivered to the output circuit. Any modulation products resulting from interaction between two or more components of one of these conjugately impressed waves are differentially combined in the output with a subsequent reduction in their disturbing effect upon the reading. A schematic diagram of this

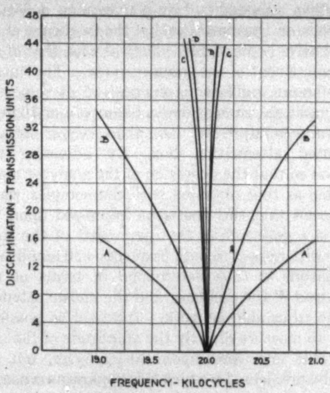

FIG. 6—CURVES SHOWING DISCRIMINATION OF VARIOUS CURRENT ANALYZER ELEMENTS

A—SINGLE SECTION OF ELEMENTARY ANALYZER

B—TWO SECTIONS OF ELEMENTARY ANALYZER IN TANDEM

C—DISCRIMINATION OF HETERODYNE ELEMENT OF HETERODYNE ANALYZER

D—TOTAL DISCRIMINATION OF COMPLETE HETERODYNE ANALYZER

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[6] *Bell System Technical Journal*, April, 1927.

analyzer is given in Fig. 5. The curves of Fig. 6 represent actual data taken with one system as adjusted for use. Curve *A* shows the discrimination, expressed in transmission units,[7] of the initial stage of this analyzer. It corresponds to the discrimination which might be secured with ordinary circuit elements arranged as in the elementary form of analyzer. Curve *B* shows the added discrimination resulting from using two stages in tandem. Curve *C*, as compared with Curve *A*, shows the gain from the standpoint of resolution obtained by translating the complex wave in such a way as to increase the separation between its components, and by using a fixed complex selective circuit in place of a simple variable circuit. Finally, Curve *D* gives the over-all discrimination of the complete analyzer.

With a system containing as many variable elements as that just described, any general calibration is practically worthless. The alternative is, effectively, to calibrate the system for each individual measurement. In practise, this is relatively simple. A variable frequency oscillator is provided, the output from which is substantially sinusoidal. After a particular component has been selected and the sensitivity of the analyzer adjusted to give a suitable deflection on the indicating meter, the complex wave is replaced by the output of this sinusoidal oscillator, the frequency of which is then adjusted to give a maximum deflection on the indicator, thus insuring that the frequency of the sinusoidal wave is practically identical with that of the selected component of the complex wave. The output of this reference oscillator is maintained at some convenient amplitude, such as one milliampere or 10 milliamperes, and is impressed upon the analyzer through a calibrated attenuation box. By adjusting this attenuation so that the deflection of the analyzer meter is the same as that obtained with the complex wave, it is apparent that the sine wave impressed upon the analyzer is a duplicate of the component of the complex wave with respect to both frequency and amplitude. The amplitude is thus determined in terms of the known output of the oscillator and the known attenuation of the adjustable network. It would be possible, of course, to measure directly the amplitude of the sine wave current impressed upon the analyzer, but for many of the problems for which this apparatus is used it would be necessary to employ measuring devices of such high sensitivity as to be far more cumbersome than the method described.

The several analyzers just considered are primarily intended, as has been apparent, for the examination of the complex waves occurring in electrical communication. The various types, nevertheless, are adapted to meet the requirements of such a wide range of problems that they may be applied generally in the investigation of electric waves. For other specific problems, however, it will undoubtedly be found desirable to work out still other arrangements, better suited to the particular conditions encountered. It is hoped that the several modifications of the elementary analyzer which have been discussed in this paper will be of value in suggesting further modifications which will extend its usefulness.

Discussion

N. E. Bonn: The last method described in Mr. Horton's paper and the one which he evidently prefers, involves the use of a sine-wave oscillator of variable frequency. Now a sine-wave oscillator, especially one whose frequency may be varied over a large range without disturbing the wave form, would make a very valuable adjunct to many a laboratory. Will Mr. Horton please state what method he uses to make certain of the purity of the wave form?

A method of wave analyis not mentioned by Mr. Horton, but somewhat similar to the one referred to above, was described in a well known French publication a little over a year ago by R. Thornton Coe (*Revue de l' Electricite*, XIX, 203-207, February, 1926). This method makes use of an astatic electrodynamometer. A sinusoidal current of adjustable frequency is passed through the stationary coil, while the moving coil carries the current whose wave form is sought. The deflection of the dynamometer at unity power factor is a direct measure of the amplitude of the particular harmonic, since the instrument will respond only to that component of the complex wave the frequency of which is equal to the frequency of the current passing through the fixed coil. As the method appears very simple, I should like to hear from Mr. Horton whether it received consideration and whether, in his opinion, it could be used at audio or carrier frequencies.

J. W. Horton: The first question concerns the purity of wave shape of the oscillator. I presume this refers to the oscillator used in the substitution method. As a matter of fact most measurements do not require an abnormally high degree of purity in this oscillator. For example, should the total harmonics aggregate, say, 5 per cent of the fundamental, the only error introduced is that the current used in the substitution method as measured by the thermocouple will appear to be about 5 per cent higher than the correct value. In other words, the absolute value of the component measured will be in error by the amount of the harmonics. These harmonics may, however, be determined by other measurements and a correction applied to take account of their presence.

In the actual oscillator used, the total amount of harmonics under normal conditions, that is, the square root of the sum of the squares of all components other than the fundamental, is less than 4 per cent of the total amplitude of the current as measured by a thermocouple. If the measurements demand that greater purity be obtained it is generally most economical to do it by using a low-pass filter in the output of the oscillator. There have been provided for certain measurements a series of such low-pass filters, the cut-off of one filter being approximately twice the cut-off of the other. With any filter of the series it is possible to obtain a current free from harmonics over the frequency range extending from just above one-half the cut-off frequency to just below the cut-off frequency. The several filters are so chosen that these ranges overlap.

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[7] For a discussion of this method of expressing current ratios, see *The Transmission Unit and Telephone Reference Systems*, by W. H. Martin, TRANSACTIONS A. I. E. E., Vol. XLIII, 1924, p. 797, also *Bell System Technical Journal*, July, 1924, Vol. III, No. 3 also "The Transmission Unit," R. V. L. Hartley, *Electrical Communication*, July, 1924, Vol. III, No. 1.

The second question refers to the dynamometer method for measuring individual components. I have had no experience with that method. From what little I know of it I should imagine it might possibly be somewhat cumbersome to apply in cases where the component to be measured is of the order of 0.0001 of the fundamental components in the circuit. I should also imagine that it might not be very accurate in cases where the components are so small that a considerable amplification is required to bring them to the point where they could be made to give a reliable reading on some indicating device, or where the frequencies are so high that distributed capacities within the dynamometer become effective.

A New Thermionic Instrument

BY S. C. HOARE[1]

Associate, A. I. E. E.

Synopsis.—*A new form of vacuum tube voltmeter that has certain merits not possessed by existing instruments is described by this paper. Advantage is taken of the properties of a bridge circuit, one of whose arms is the plate-filament impedance of a common three-element pliotron to permit the building of very sturdy instruments of unusual sensitivity and dependability.*

A number of schemes using standard three-element thermionic tubes has been used in various phases of electrical measurement work. The amplifying characteristics of these well-known devices immediately suggest themselves to those having unusual measurement problems. A number of voltmeters of the vacuum tube type have been described in the literature and some of these have reached the commercial stage. Unfortunately, however, most of these schemes have been quite complicated or required considerable auxiliary equipment and other measurement devices, to adjust the various currents and voltages of the circuits to a standard value. These complications have served as deterrents to the more general application of vacuum tubes to measurement work.

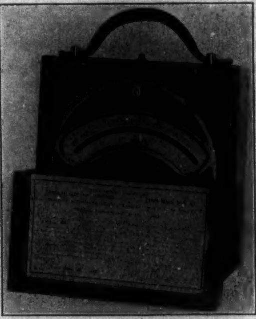

FIG. 1—TYPE DP-2 THERMIONIC VOLTMETER

Recent work has shown the practicality of ideas of using vacuum tubes in very simple circuits for measurements of reasonable precision where unusual electrical conditions must be met and where ordinary measuring instruments cannot be used or, if usable, are highly special and delicate. This new development permits the housing of all necessary circuit parts except the exciting battery in the case of a standard high torque d'Arsonval microammeter. There is but one adjustment necessary and this is made by a rheostat which is one of the parts incorporated in the case. The only battery required is one for the excitation of the filament. Due to the particular circuit used, the adjustment of the current from this battery by the rheostat adjusts the filament current, the grid bias, and plate voltage. This adjustment is made by turning the rheostat until the instrument pointer indicates zero. The instrument then becomes direct reading.

The circuit is essentially a Wheatstone bridge with three arms of fixed wire resistors and one arm, the plate-filament resistance or impedance of a three element vacuum tube. This impedance, as is well known, is a function of the grid potential and is radically changed by relatively slight changes of the grid potential. Thus, an e. m. f., either direct or alternating, applied to the grid, upsets the balance of the bridge. The degree of unbalance which is a measure

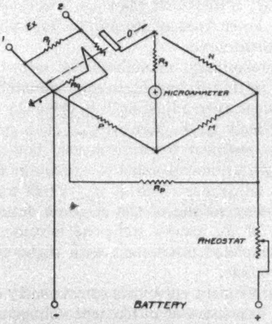

FIG. 2

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[1] Standardizing Laboratory, General Electric Co., West Lynn, Mass.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

of the applied voltage is read on the galvanometer or instrument element. A diagram of connections is given in Fig. 2. The resistances R_g and R_l serve to adjust the potential of the grid with respect to the filament to a definite value. These resistances are fixed in value, the adjustments being made at the time of calibration at the factory. The potential is determined by the $I\,R$ drop in R_p. R_l can be omitted when the test circuit forms a closed connection to the binding posts 1 and 2. R_l ordinarily lies between 50,000 ohms and 10 megohms, depending upon the proposed use of the instrument, type of tube used, etc.

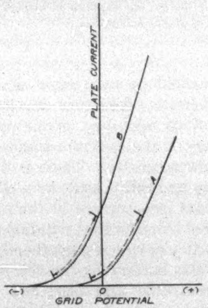

Fig. 3

The bridge arms, M, N, and P, are of manganin. Resistors R_g and R_p are also of manganin and serve the double purpose of limiting filament current and of supplying (by virtue of their $I\,R$ drops) the requisite grid and bridge (*viz.*, plate) potentials. The relation of R_g to R_p is such that the tube is worked at a point near the lower knee of its grid voltage-plate current characteristic curve.

The preliminary manufacturing adjustments are made by setting R_g and R_p to approximately predetermined values, then adjusting P to bring the instrument pointer closely to the zero point. R_p is next given a slight readjustment to cause normal flow of filament current, with nominal value of excitation voltage and with the adjustable rheostat in its half-way position. The half-way setting of the rheostat during this adjustment of R_p insures sufficient latitude of control in the completed instrument with slight variation in battery voltage.

Normal filament current is conveniently determined from the measurement of filament voltage as given by either a potentiometer or a very high resistance voltmeter connected across the filament of the tube. After making this adjustment it will be found that the bridge is still unbalanced as indicated by the movement of the microammeter pointer above or below zero. The network can now be balanced by readjusting P. During this work R_l is in position but with no external connections to the grid.

As previously mentioned, a voltage, either alternating or direct, impressed upon the grid when the bridge is in normal balance, causes an unbalance because the impedance of the tube is altered. The indications of the microammeter form a measure of the unbalance of the bridge and, hence, of the voltage applied across 1 and 2. With d-c. circuit, the terminal 2 is connected to the positive side of the line. This results in making the grid less negative, thus lowering the impedance of the tube and thereby causing an increase in the plate current. When an a-c. voltage is impressed across 1 and 2, the negative loops of the voltage wave are, in effect, suppressed, and thus the indications of the microammeter are somewhat less than half of those from a similar d-c. voltage application. The indications of the microammeter element are substantially proportional to average values. The resulting action of the complete device is somewhere between an "average value" and an "effective value" instrument. For most purposes the error resulting from distorted waves is not serious. The calibration is made by applying known values of sinusoidal voltage or current.

The current drawn from the lines by this instrument is determined by the grid bias voltage, which is controlled by the $I\,R$ drop across R_g in Fig. 2. With a six-volt excitation battery it is possible to build these instruments as alternating-current voltmeters with 10,000 ohms (effective) per volt. By making the grid bias more negative it is possible to reduce the current drawn by the grid to less than 1 microampere. This requires a higher excitation voltage. A number of instruments working from a 24-volt battery have been built. Some of these have required less than 0.1 microamperes as their operating current. The reason for the higher grid bias values is seen in Curve B of Fig. 3. This curve is the 24-volt characteristic of a tube and Curve A is the 6-volt characteristic. It is seen that with Curve B a much wider voltage range may be measured without having the grid positive. The grid does not draw current while negative.

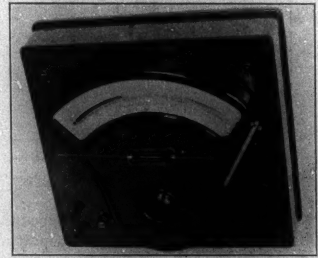

Fig. 4—Type DL-2Y Laboratory Standard Microammeter—Thermionic Type

The general arrangement of this circuit results in instruments with fairly uniform scales, as may be seen in Fig. 1. This is an unusual feature of thermionic instruments. The electrical and mechanical zeros are also coincident which is another feature not common in such devices.

These instruments have been built as low-range voltmeters, especially for alternating voltages, and as low capacity microammeters for alternating-current and direct-current. The microammeters are operated from the drop across a resistance in series with the line current to be measured. It has been found possible to construct such an instrument to have a full scale range of 0.1 microamperes direct-current with only 0.3 volt drop. This instrument is shown in Fig. 4. Its scale is 12 in. long.

If care is taken to use resistances of proper design for high frequency work, and if the parts are properly mounted and shielded, this scheme would serve effectively for measurements at high frequency. It is especially necessary that R_l be of proper construction and shielded if high frequency measurements are to be made. R_l can, of course, be replaced by a series or shunt condenser.

TABLE I

ALTERNATING-CURRENT VOLTMETERS

	Ordinary electro-dynamic	Thermo-couple type	Thermionic 6-volt battery	Thermionic 24-volt battery
Volts, full scale	5.0	5.0	5.0	5.0
Resistance between terminals (ohms)	20.0	700.	50,000.	infinity
Ohms-per-volt sensitivity	4.0	140.	10,000.	"
Current consumption at full scale (amperes)	0.25	0.0071	0.000100	nil
Torque, full scale (mm-g.)	1.5	0.05	0.9	0.9
Weight of moving element (grams)	4.8	0.3	1.8	1.8
Damping: Full scale to zero (seconds)	5.0	..	3.4	3.4

These data are representative of certain commercial instruments. Others with characteristics somowhat different might have been chosen. These, however, serve to show the possibilities of this type of thermionic circuit.

TABLE II

DIRECT-CURRENT VOLTMETERS

	Ordinary d'Arsonval	Special d'Arsonval	Extreme sensitivity d'Arsonval	Thermionic 6-volt battery	Thermionic 24-volt battery
Volts, full scale	5.0	5.0	5.0	5.0	5.0
Resistance between terminals (ohms)	500.0	5,000.0	100,000.	200,000.	infinity
Ohms-per-volt sensitivity	100.0	1,000.0	20,000.	40,000.	"
Current consumption at full scale (amperes)	0.010	0.001	0.000050	0.000025	nil
Torque, full scale (mm-g.)	5.0	3.2	0.3	0.9	0.9
Weight of moving element (grams)	1.8	2.2	1.8	1.8	1.8
Damping: Full scale to zero (seconds)	2.0	1.6	3.8	3.4	3.4

These instruments have some advantages over thermocouples, which are of necessity worked close to the burn-out point, because moderate overloads will not change the instrument and will not permanently impair the tube. The disadvantages of these instruments are the varying characteristics of tubes throughout their useful life, the need for an external excitation source, the need for accurately balancing the bridge before measurements are made, and the wave-form error when used on alternating-current. These disadvantageous features, however, are common to all thermionic instruments.

By careful selection of tubes for constancy of characteristics and by conservative selection of operating temperatures, it is possible to obtain quite long and useful lives for the tube. To some extent the circuit arrangements compensate for changing tube characteristics.

The external potential source can be a group of ordinary dry cells. It has been found that these, when in fresh condition, will require readjustment of the rheostat only infrequently.

Tests have been made to show the error resulting from inaccurate balancing of the bridge network. When the balance as indicated by the microammeter is within 0.010 in., (about three needle widths of the pointer), the calibration is within 0.5 per cent of full scale value.

The wave-form error is inherent in devices of this kind. For many measurements at high frequency where resonant circuits are used, the error will be quite inappreciable. For most measurements in the power frequency range the error will also be negligible. A test with a wave consistency of 30 per cent third harmonic and 10 per cent fifth gave but 4 per cent deviation from a sine of the same effective value. The fundamental in this case was 60 cycles.

As previously mentioned, this arrangement is one that is not especially susceptible to damage from moderate overloads. This is borne out by the data in the accompanying tabulation.

TABLE III

DIRECT-CURRENT MICROAMMETERS

	Laboratory standard thermionic 48-volt battery	Thermionic 24-volt battery
Microamperes, full scale	0.1	1.5
Resistance between terminals (ohms)	3,300,000.	750,000.
Drop at full scale (volts)	0.33	1.12
Torque, full scale (mm-g.)	0.9	0.9
Weight of moving element (grams)	2.8	1.8
Damping: Full scale to zero (seconds)	7.6	3.4

Some of the advantages of this general scheme are shown by the tabulations (Tables I and II) which compare ordinary instrument constants and the constants of some thermionic instruments and by Table III, which tabulates data of some microammeters of ranges well beyond the possibilities of ordinary d'Arsonval construction.

TEST DATA

Variation with full load self-heating (maximum, during a 30-minute period in terms of initial indication) (per cent of full scale)	−0.4
Variation per deg. cent. ambient rise (between 25 degrees and 50 degrees Centigrade) (per cent of point)	−0.05
Variation at 1000 cycles, sine wave (in terms of 60 cycle sine wave indication) (per cent of full scale)	−0.2
Variation with irregular wave form at 60 cycles (in terms of 60 cycle sine wave indication) peaked wave, form-factor = 1.13 (per cent of full scale)	−4.0
Variation due to inaccuracy of zero adjustment (pointer purposely set three thicknesses off true zero position) (±0.010 in.) (in terms of indication with correct zero adjustment) (per cent of full scale)	±0.5
Variation with application of overloads (without reactivation) in terms of indication prior to overloads	
2-minute grid overvoltage of 100 per cent (per cent of full scale)	+0.3
5-minute filament overvoltage of 10 per cent (per cent of full scale)	+1.0

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E. B. Moullin, "A Direct Reading Thermionic Voltmeter and its Applications" *Journal* of the Institution of Electrical Engineers, 1923.

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Discussion

N. E. Bonn: Mr. Hoare's instrument will undoubtedly be found very useful for certain special applications. For general laboratory work and as a detector in ordinary bridge and potentiometer circuits its value is very much limited by the exceptionally high internal impedance. True, it can be made to give a full-scale deflection while drawing only 0.1 microampere from the circuit, but it takes fully 0.3 volt to do so, and such high voltage is not available when one measures temperatures by means of thermocouples or in ordinary Wheatstone bridge and potentiometer circuits.

Throughout the paper the instrument is referred to a number of times as a microammeter. Now a microammeter is a current-measuring device the impedance of which must be much lower than the impedance of the circuit in which it is being used. To express the sensitivity of an instrument having an internal impedance in excess of 3 million ohms, in terms of microamperes is somewhat misleading. This instrument is strictly a voltmeter, but not a microammeter in the accepted meaning of the term.

B. W. St. Clair: (communicated after adjournment) Vacuum tube thermionic instruments are by no means new. There has been a number of different instruments on the market for several years. There are certain differences between the schemes proposed by Mr. Hoare and those that have been used heretofore and it is to the differences in these various instruments that this discussion is dedicated.

The biggest difference between this instrument and the more ordinary type of thermionic or vacuum-tube voltmeter is in the elimination of all auxiliary equipment with the exception of one exciting battery. Heretofore vacuum-tube instruments have required either B batteries or C batteries, or auxiliary ammeters or voltmeters in addition to the A battery required by this instrument. In several of the schemes the accuracy of the final measurement is determined by a setting of an auxiliary instrument.

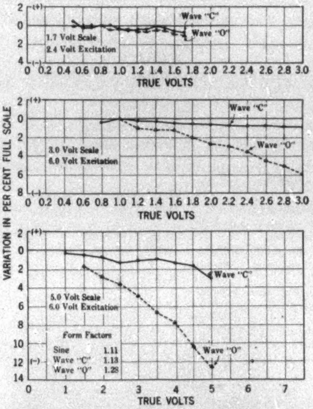

Fig. 1—Alternating-Current Thermionic Voltmeter Variation with Wave Form. Given in Terms of Sine-Wave Indications Tested at 60 Cycles

Another very real difference between this instrument and the older ones lies in the type of scale that results and also in the position of the electrical zero. In few of the previous schemes do the electrical and mechanical zeros coincide. Mr. Hoare's instrument is the only commercial one I know of that has coincident mechanical and electrical zeros. Most of the instruments proposed heretofore have scales that depart very seriously from a uniform law. The departure from a linear law of the scales of these instruments is not very serious. In fact, the scale can easily be read with about the same linear accuracy at any part of its length. This is not true in the older commercial instruments we are familiar with as they have scales that correspond more to reciprocal laws than to straight line laws. For many test purposes the uniform type of scale is very much to be desired.

S. C. Hoare: Under ordinary conditions, I doubt if many will object to the resistance of these thermionic microammeters since when measuring current values of the order of 20 to 1 microamperes or less, the resistance of the circuit is so many

	FOR DIRECT CURRENT			FOR ALTERNATING CURRENT	
	Thermionic Type	d'Arsonval Type	d'Arsonval Type	Thermo-couple Type	Electrodynamic Type
Ampere, full scale	0.0000001	0.0005	0.010	0.002	0.010
Resistance between terminals (ohms)	3,300,000	800.0	100.0	600.0	5000.0
Drop at full scale (volts)	0.33	0.4	1.0	1.2	2.8
Torque full scale (mm-g.)	0.9	2.2	1.9	——	2.8

times that of the instrument that 3 megohms would not be serious. A common use of these instruments is for photoelectric cell photometry where the circuit resistance is many times that of the instrument. In such work the instrument resistance can be considered almost insignificant.

Errors due to irregular wave-forms are inherent in those types of thermionic instruments in which the grid is worked at positive potentials. The errors become smaller with the reduction in positive grid potentials, and are quite negligible in those instruments in which the grid is kept always at a negative bias.

The accompanying characteristic curves illustrate the magnitude of wave-form errors with different conditions of grid-biasing. The upper set are typical of an instrument in which the grid is never permitted to become positive. In this type the current consumption in the input circuit is practically nil. The lower set represents the other extreme, wherein the grid is permitted to become appreciably positive. This type draws about 100 microamperes in the input circuit.

Waves *C* and *O* are badly distorted, *C* containing prominent fifth and seventh harmonics and *O* containing third and seventh harmonics.

I doubt if Mr. Bonn appreciates the magnitude of the voltage drop necessary in an instrument for the measurement of very small currents. Alternating-current instruments especially require quite an appreciable drop when the current range approaches milliamperes or microamperes. In the brief tabulation above are given some values for ordinary d-c. instruments and for a-c. instruments. It is there seen that the drop across the thermionic microammeters is not excessive when compared with some of the more ordinary types of test instruments.

The Oscilloscope: a Stabilized Cathode-Ray Oscillograph with Linear Time-Axis

BY FREDERICK BEDELL[1] and HERBERT J. REICH[1]
Fellow, A. I. E. E. Non-member

Synopsis.*—A method is described for using a cathode-ray oscillograph for the simultaneous observation of a number of variable quantities by means of a distributor. A linear time-axis, obtained by means of a gas-discharge lamp connected to a source of direct-current through a resistance or thermionic tube, is stabilized by introducing into this circuit a small e. m. f. derived from the same source that supplies the unknown quantities under observation. By making stabilization definitive rather than casual, distortion is avoided. The unknown quantities are thus shown in a form convenient for observation, appearing as stationary curves plotted with time as abscissa. The curves may be superposed about a common zero line, or displaced with reference to each other with separate zero lines. The name oscilloscope is applied to the apparatus here described, assembled as an instrument.*

THE oscillograph employing a vibrating cathode ray, first used by Braun and later so admirably developed by Ryan and others, has certain distinct advantages over the oscillograph of the Blondel or Duddell type in which a vibrating mirror is used. Foremost among these advantages is the fact that the cathode beam is free from inertia and can readily follow the variations of an electric or magnetic field even at high frequencies. The cathode-ray oscillograph, as hitherto commonly employed, has, however, the disadvantage that it has not been possible with it to show the variations in a number of quantities at the same time, nor to show these variations, as is done with the vibrating mirror oscillograph, as curves with time as abscissa in rectangular coordinates with which everyone is familiar.

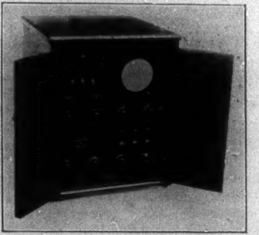

FIG. 1—STABILIZED OSCILLOSCOPE, FIELD TYPE
Made by R. C. Burt, 327 S. Michigan Ave., Pasadena, Calif.

While the cathode-ray oscillograph has proved a highly valuable tool for a wide range of engineering and scientific investigations, including both cyclic and transient phenomena, and, in certain respects, is superior to the oscillograph of the mirror type, it has suffered by the two limitations just described. These limitations, however, may be removed and the field of usefulness of the cathode-ray oscillograph so widened that it becomes practically a new instrument. As the instrument developed for this purpose, (shown in Fig. 1), is primarily intended for visual observation, we have given it the name "oscilloscope." Permanent record may be obtained, when desired, by a photograph in the usual way. On the other hand, an *oscillograph* of the Blondel or Duddell type, both in name and in fact, is primarily for graphical record.

Not being limited to a single cyclic phenomenon, the oscilloscope is polycyclic; furthermore, it is stabilized so that the wave or waves stand stationary for observation and this becomes particularly important when several waves are observed at one time. Recurrent transients, as well as more usual periodic phenomena, may be observed.

The cathode-ray oscillograph tube is so well known that it needs no description. The cathode beam, focused on a fluorescent screen or photographic plate, is deflected in one direction by one set of plates or coils, and in a perpendicular direction, by another set of plates or coils.

An admirable and full account of various types of cathode-ray oscillographs and their development from the beginning is given by A. B. Wood and others in the *Journal* I. E. E. (Nov., 1925), with 63 pages and 125 references. So complete is this account (with its bibliography), and so admirable is the presentation that further discussion here is unnecessary.

THE POLYCYCLIC DISTRIBUTOR

In using the cathode-ray oscillograph, it occurred to the writers that if one pair of deflecting plates or coils could be successively switched by a distributor from circuit to circuit in rapid succession, the cathode beam would follow each in turn, making possible the simultaneous observation of several unknown quantities. We have found this to be the case and that when switched at proper intervals, the curves appear to the eye as simultaneous and continuous, due to persistence of vision, and likewise so appear in a photographic record.

The development of a four-way experimental distributor for this purpose is shown in Fig. 2. A resistance is included in each of the four circuits as protection in case of short-circuit. A brush B bears on a continuous slip-ring to which are connected staggered quadrants. Each of the remaining four brushes comes in contact, in turn, with one of these quadrants. The terminals $Y_0\,Y$ are connected to one pair of deflecting plates of the oscilloscope, the terminals $Y_0\,Y_1$, $Y_0\,Y_2$,

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[1] Both of the Cornell University.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

$Y_0 Y_3$, $Y_0 Y_4$ being connected to the several circuits under test. It will be noted that Y_0 is a common terminal; if any of the circuits cannot be so connected, an insulating transformer should be interposed between such circuit and the distributor terminals. Avoiding the common terminal, a double distributor can be used in a special case when necessary.

LINEAR TIME-AXIS

A linear time-axis, desirable for the observation of a single quantity, becomes almost a necessity in order to make a satisfactory interpretation possible when several quantities are simultaneously studied.

The need for a linear time-axis with the cathode-ray oscillograph has long been recognized and various ingenious methods have been proposed for its accomplishment, often, however, with but limited application. Mechanical devices are limited in range of frequency; electrical devices, although not thus limited, are liable to be unstable, and stability, as has been pointed out, assumes prime importance when several curves are simultaneously observed, as any motion of the curves creates hopeless confusion. Even for a single curve, stability is needed if the curve is to be carefully studied and perhaps sketched, traced, or photographed. Furthermore, electric devices without proper precautions are liable to produce distortion.

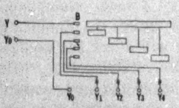

FIG. 2—POLYCYCLIC DISTRIBUTOR

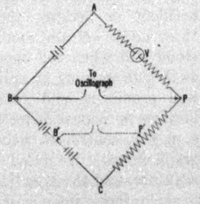

FIG. 3—BATTERY OPERATION

A linear time-axis may be obtained if we have available either a synchronous mechanical switch or a synchronous electric valve. Thus, if a condenser is charged at a uniform rate through a resistance, or otherwise, and is periodically discharged by a synchronous switch, the difference of potential across the condenser terminals will increase linearly with time while the switch is open, and drop suddenly to zero each time the switch is closed.

THE CORONA VALVE

A mechanical synchronous switch is cumbersome and can be operated only at low frequency. A synchronous electric valve that would automatically perform the same function at high or low frequency would evidently be better. Such a valve is found in the gas-discharge lamp, containing commonly neon or argon, the characteristics of which are well known. (See bibliography.) When subjected to an increasing voltage, no current will flow through such a lamp until a certain critical "ignition" voltage is reached. Current then flows (analogous to the closing of the switch) and continues to flow until a definite lower "extinction" voltage is reached. Current then stops, analogous to the opening of the switch. The difference between the ignition and extinction voltages depends upon the frequency and type of lamp.

Each gas-discharge lamp possesses a certain capacity, so that, when connected to a source of e. m. f. through a resistance (in excess of a certain "critical resistance"), the lamp will light and re-light at definite frequency. This frequency may be varied through a wide range by varying the resistance and capacitance of the circuit. The gas-discharge lamp acts as a synchronous electric valve, performing the functions of a synchronous mechanical switch. Due to its nature, it may be referred to as the "corona valve."

Many theoretical and experimental studies (see bibliography) have been made of this phenomenon. Suffice it to say that the oscillation frequency is under control by adjustment of resistance and capacitance, the maximum frequency being obtained by reducing capacitance—including the capacitance of all circuit connections—to a minimum. Capacitance may be obtained by means of condensers in parallel with the corona valve or in parallel with a series resistance.

While the oscillating circuit can be adjusted for audio, commercial, and lower frequencies, frequencies much higher than audio-frequencies are not readily obtainable with usual apparatus. A frequency of 95,000, however, is reported by Oschwald and Tarrant, using a resonant circuit; but resonance tends to produce a sine-wave oscillation rather than the straight saw-toothed wave required for a true linear time-axis, and is to be avoided in an oscilloscope in which accurate reproduction of wave-form is sought. On the other hand, low frequencies, of, say, one every minute or, indeed, every hour, are obtainable and there is no obvious reason why with sufficient capacitance, (and patience), one could not reduce the lower limit almost indefinitely, if there were any object in so doing.

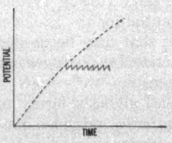

FIG. 4—POTENTIAL VARIATION

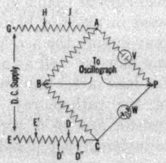

FIG. 5—RESISTOR OPERATION

OSCILLATING CIRCUIT

Circuit connections for obtaining a linear time-axis, are shown in Fig. 3. In the resistance circuit *A P C*, connected to the battery *A B C*, is inserted the corona valve *V*, which, either alone or with supplemental condensers in parallel with it, has a capacitance *c*, and a potential difference $v = q/c$. The corona valve passes no current, acting as an open switch, until the potential difference between its terminals, as charge accumulates, reaches the ignition voltage. Current then flows and the potential difference drops until extinction voltage is reached; the current then stops, the potential difference again builds up and the cycle is repeated.

Fig. 4 shows the variation in the difference of potential between a point P in the resistance circuit and a point of reference B of fixed potential. The curve of potential-rise is exponential, as shown by the dotted curve. By using only a short element of this curve, the rising part of the saw-toothed curve is sufficiently straight to give the desired linear time-variation, when P and B are connected directly or through an amplifier to the deflecting elements of the oscillograph. Other points, as P' and B', give a similar but smaller variation, the amplitude of the time variation being capable of adjustment in this manner. The short, falling part of the saw-tooth curve, corresponding to the brief interval during which current is flowing through the corona valve, is so rapid that the spot of light on the oscilloscope screen shows only a negligible trace as it sweeps back to repeat the cycle.

Fig. 5 shows the connections for operating the oscillating circuit when a thermionic tube W replaces part or all of the resistance in series with the corona valve. A constant current through W, when operated above saturation, gives a uniform increase in v during the rising part of the saw-tooth curve, and so assures a linear time-axis.

Connections are likewise shown in Fig. 5 for operating the oscillating circuit when resistors, supplied with direct current from a generator or battery eliminator, are used in place of batteries. The same resistor system may supply the accelerating potential for the cathode-ray tube as well as any voltages that may be needed for bias or for the vertical or horizontal displacement of the cathode-ray beam. Operation with such a resistor system is very convenient, giving a nicety of adjustment not possible with batteries, provided precautions are taken to design the circuits, (including the circuits of a battery eliminator, if used), so that the several adjustments are sufficiently independent. The use of more than one eliminator would, of course, obviate the difficulty, but a single eliminator, properly compensated and designed for the purpose, will suffice. After the use of such an eliminator, batteries seem cumbersome and are a source of error when they do not give just the proper voltage. When a generator supply is used, commutator ripples may be filtered out, if necessary.

STABILIZING

In order that the curves shown by the oscilloscope may be stationary, it is necessary first to synchronize and then to stabilize the oscillating circuit; that is, the frequency of the oscillating circuit is first so adjusted that the cathode beam sweeps back once every half cycle, or some multiple of it, of the varying quantity under observation, and is then locked in step and so stabilized. As an automobile engine is first synchronized and then thrown into a particular gear, the oscillating circuit is synchronized and then thrown into the desired gear, electrically, so the curve shown includes one or more cycles, or half cycles, of the quantity observed, as may be desired. Without being thus stabilized, the curves are liable to move and make observation difficult.

Stabilization may be effected in various ways, the simplest method being by introducing into the oscillating circuit a very small e. m. f. of the same frequency as the circuit under observation. This may occur in a way that is *casual* and uncontrolled (whether by accident or design), through leakage or induction; thus, under certain conditions, we found that curves stood still when the operator merely raised his hand as though warning an animate being. Such stabilizing was promising and fascinating. Casual stabilizing, however, produces distortion; for, unless the stabilizing e. m. f. is controlled, some of it will affect the oscillograph circuit. To avoid distortion, stabilizing must be *definitive*, and the e. m. f., whether introduced conductively or inductively, so localized and controlled as not to affect the oscillograph. Thus, without attempting to discuss all possible methods of stabilizing, it is obvious that the distortion produced by the introduction of an e. m. f. at $J\,D$, (Fig. 5), while appreciable, would be far less than if the e. m. f. were introduced at $P\,B$. The authors have found that, for the apparatus employed, it is possible to obtain a definitive stabilization without distortion and with an amount of energy so small that the disturbance to the circuit under test, even when it is very sensitive, is practically negligible.

The oscillating circuit may be brought to the proper frequency and then stabilized so as to show a single cycle or half cycle; or it may be brought to a lower frequency, (whereby more time is taken for the spot to sweep across the screen along the time-axis) so that several cycles or half cycles are shown. In observing 60-cycle phenomena, the oscillating frequency may, for example, be stabilized at 30, 15, 10, or 5 cycles, with, however, a decrease in stability. In this way, for the same phenomenon, different gear ratios may be used and curves as shown in Fig. 6 be obtained.

ZERO LINE

Curves are ordinarily superposed, as in Fig. 7, either with or without a zero line. A zero line is obtained by a short-circuiting connection between Y_0 and Y_1, Y_2, Y_3, or Y_4, in Fig. 2.

DISPLACEMENT

A curve may be displaced (raised or lowered) with respect to the others by interposing a battery, or other source of d-c. voltage, between it and the common terminal Y_0. Several curves may be so displaced, up or down, by varying amounts, depending upon the voltage and polarity of the battery.

A displaced zero line is similarly obtained by using a battery instead of a short-circuiting connection referred to above. Two zero lines may be used, one displaced with respect to the other, as in Fig. 8.

When a resistor system as shown in Fig. 5 is used for supplying accelerating potential to the cathode tube, displacement without batteries may be obtained by connecting the unknown not to Y_0 (and so to some point as D on the resistor) but to D' or D'' displaced therefrom.

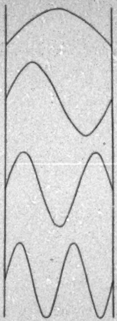

FIG. 6—STABILIZING WITH GEAR RATIOS ½, 1, 1½, AND 2

USES

The oscilloscope may be used not only in the varied fields of investigation in which the vibrating mirror or cathode-ray oscillograph is used but, on account of the characteristics here described, in additional fields as well. The stability of the linear time-axis, together with the multiple use of the oscilloscope by means of the polycyclic distributor, at once opens the way to many varied applications. With the oscillating circuit switched off, the instrument becomes available for all the uses of a cathode-ray oscillograph in the usual manner. The stabilized linear time-axis is an added feature extending its usefulness.

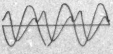

FIG. 7

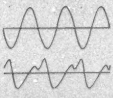

FIG. 8

FIG. 7—SIMULTANEOUS CURVES, SUPERPOSED, WITH OR WITHOUT ZERO LINE

FIG. 8—SIMULTANEOUS CURVES, DISPLACED, WITH OR WITHOUT ZERO LINES

On the other hand, the oscillating circuit may be used independently as a convenient source of current of controllable frequency. An ammeter in circuit may be used to indicate the frequency. A loud speaker connected through an amplifier becomes a source of sound of controllable, known pitch.

Although not limited to any one type of cathode-ray tube, a well-known low-voltage tube described by Johnson (*Journal* Opt. Soc., 6, 701, 1922) has been found well adapted for the oscilloscope and arrangements have been made for its use.

The principles of operation of the oscilloscope are simple. Practically, we have found that, in order to avoid error due to leakage or induction, many details though simple in principle are perplexing in execution, particularly when we are not seeking an elaborate laboratory equipment, limited in use on account of its scattered complexity, but an assembled, self-contained instrument, simple in operation and readily portable. Its availability adds materially to its usefulness. The senior author desires to express his appreciation of the assistance rendered by his colleague in the development and construction of the finished instrument.

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GAS DISCHARGE LAMPS

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Discussion

K. B. McEachron: It seems to me that the material in this paper will be of considerable value to all of us who are interested in the study of recurrent phenomena of a character that requires a measuring device which takes little or no energy from the circuit.

The paper describes a very interesting and ingenious scheme for securing two or three sets of measurements all on the screen at the same time by the use of the distributor. Also, a method is given for getting a linear time axis by the use of the so-called Schroeter valve which I believe is perhaps better known abroad than in this country.

I do want to say something, however, regarding the statement made in the first few paragraphs of this paper with reference to linear time axes of other investigators and to say something about the work which we have been doing ourselves along that line.

Some two years ago[1] Mr. Wade and myself showed a group of cathode-ray oscillograms having a linear time axis.

It is well known that for a few degrees either side of zero, the current of a sine wave will vary directly with the time and the linear time axis shown in the paper referred to was obtained by using such a portion of a sine wave of current in the oscillograph deflecting coils. Thus when the current is zero the electron stream impinges on the middle of the photographic film, the synchronous switch usually being so arranged that the unknown transient occurs at this moment.

For taking volt-ampere curves, it has been necessary to develop a special form of motion in which the spot moves at a uniform rate to the middle of the film where it undergoes deflection by reason of the transient being studied and then resumes its uniform motion, thus carrying the spot off the film.

For transients of very short duration it is necessary to use as a time scale a high-frequency wave which gives an approximately linear axis only near the middle of the film. With such a time scale either end is very much condensed.

It should be remembered that the oscillograph which is used by Professor Bedell and Mr. Reich is quite different from that used by ourselves, since the transients we study occur but once, while with the oscilloscope they must necessarily be recurrent to obtain satisfactory records. For our work it is necessary to place the photographic film within the vacuum chamber which is not necessary with the oscilloscope.

P. A. Borden: What I have to say in discussing Professor Bedell's paper on the oscilloscope may appear irrelevant; but I take this opportunity to press the plea that I have always put forward for better standardization in engineering nomenclature.

In this connection I want to call attention to the word "oscilloscope." There was a device produced in England a few years ago by a Mr. Elverson, designed for visualizing the movements of rapidly oscillating mechanisms. It is purely mechanical and optical in its nature and is now generally known as the Elverson *oscilloscope.* Now Professor Bedell shows us a new and valuable application of the cathode-ray oscillograph, which has been styled the "oscilloscope:" and while on a basis of technical exactitude, his right to use this term would appear to be quite as great as that of Mr. Elverson, I cannot but feel that this double use of the term does not serve to clarify our technical nomenclature. I am of the opinion that we need among our standards committees some "inter-technical" body which will endeavor to oversee the naming of new developments in different branches of science and thus prevent such overlapping as I have cited, without the formality of having terms registered in the Patent Office.

H. M. Turner: Professor Bedell explained how it is possible by means of a distributor to observe on the screen of the cathode-ray oscillograph several curves at the same time. I have obtained a similar result with considerable success by using a device called the transient visualizer which was described in the A. I. E. E. JOURNAL of June, 1924, which permits photographic records to be made of certain types of phenomena as well as visual observations on the cathode-ray oscillograph.

By means of the transient visualizer associated with the General Electric oscillograph we have taken as many as twelve separate exposures on a single film which makes it convenient for comparing curves taken under different conditions.

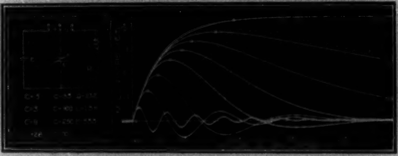

FIG. 1

F. Bedell: I wish to express my keen appreciation of the wonderful work done by Mr. McEachron and others in this country and by various workers in Europe in developing so successfully that type of cathode-ray oscillograph wherein a photographic plate is placed inside the cathode tube, so that the cathode rays impinge directly upon it. I admire the skill and patience of those who have done this work. In the hands of an expert, if expenditure of time and money be neglected, certain results can be thus obtained that can be obtained in no other way. Our object, however, has been to develop a simple instrument, fool-proof and portable, useful even for the man in the street with limited time and limited money, who wishes an instrument he can take where he will, as he would a voltmeter, observe the wave-form and get immediate results. He wants the results then and there, and does not want to wait until next week.

I am quite in sympathy with the desirability of uniformity in nomenclature. The oscilloscope is for viewing oscillations; the name is both descriptive and correct. If, as Mr. Borden points out, an instrument for viewing oscillations has already been termed an oscilloscope, all the more reason for retaining the name when a new instrument, even though operating on a different principle, is developed for performing this function.

Prof. Turner's remarks emphasize the desirability of visualizing several curves simultaneously.

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[1] A. I. E. E. TRANS., Vol. 44, 1925, p. 832.

Sensitivity Characteristics of a Low-Frequency Bridge Network for Locating Opens in Telephone Circuits

BY P. G. EDWARDS[1] and H. W. HERRINGTON[1]
Associate, A. I. E. E. Non-Member

Synopsis.—*The problem of locating opens in telephone cable conductors involves the determination of impedances. A study has been made of the degree of accuracy and sensitivity obtainable in impedance measurements with different frequencies of supply voltage. For long cables the input impedance is a hyperbolic rather than linear function of the characteristic impedance. The error in impedance measurement arising from this functional departure proves to be least for the lower frequencies. On the other hand, the bridge sensitivity is improved by somewhat higher frequencies.*

A mathematical and experimental analysis of the sensitivity of impedance measurements in cable fault location by means of a de Sauty bridge indicates the desirability of using frequencies of the order of four cycles. The sensitivity is further increased by controlling the phase of the field excitation of the bridge galvanometer.

This improved open location method and equipment are sufficiently accurate that in practically all cases a fault in a 60-mile length of cable may be located within a maximum variation of plus or minus one-half the length of a cable section (a section is the length of cable between splices—about 750 ft.), and therefore enables one to select, prior to the opening of the cable, one or the other of the two splices between which the fault lies. This degree of accuracy is very desirable for practical reasons.

* * * * *

An analysis of the variation of sensitivity with change in frequency has been coordinated with a study of the characteristic variation of errors with frequency changes. These studies were made primarily for the purpose of selecting a suitable frequency of testing potential for a more sensitive, accurate, and reliable method of impedance measurement for locating opens in the conductors of telephone cables. The characteristics of these errors and the nature of the impedance measurements are discussed only as an illustration of the method in which a frequency of testing potential was selected to give a suitable sensitivity as well as to reduce certain errors which vary with frequency.

An indication of the location of an open is given by the ratio between an impedance measurement made on the faulty wire and a similar impedance measurement made on a good wire of similar characteristics which follows the same route as the faulty wire. For short cables, the impedance measured to ground may be regarded as identical with the capacitance component of the impedance to ground, but as the length of cable increases, the input impedance can no longer be regarded as equivalent to the capacitance component of this impedance because the input impedance is not proportional to the capacitance but may be expressed as a hyperbolic function,

$$Z_L = Z_0 \coth P L \qquad (1)$$

where Z_0 is the characteristic impedance of the line, P is the propagation constant, and L is the length.

A calculation of this hyperbolic deviation or error for different frequencies of testing potential showed that this error decreased with frequency as indicated by the curve of Fig. 1. To minimize this error, the optimum value of frequency obviously should be zero. But the sensitivity of a capacitance bridge network is zero at zero frequency. The problem, then, was one of selecting a frequency which would be low enough to make the hyperbolic errors negligible for all types of conductors and at the same time provide a sensitivity which would be sufficient to permit an accurate balance of a bridge. From this standpoint, it may be observed that for a decreasing frequency of testing potential, the maximum rate of decrease of hyperbolic error is evident in the neighborhood of four cycles, as shown by Fig. 1. It was shown by a computation of the hyperbolic errors of measurements made on the maximum length of conductors that these errors at four cycles might be neglected as being within the required accuracy of the method. Hence, a computation of the frequency sensitivity characteristics of a bridge network appeared to be the next step in the selection of a suitable frequency of testing potential.

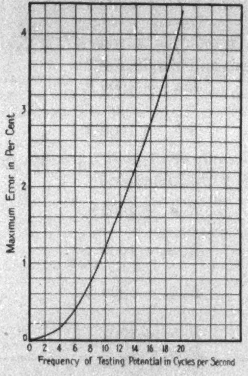

Fig. 1—The Maximum Hyperbolic Error for Various Frequencies of Testing Potential

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[1] Both of the American Tel. and Tel. Co., 195 Broadway, New York, N. Y.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

The sensitivities for several frequencies of testing potential were computed for a modified form of the de Sauty bridge. In this impedance bridge network, shown in Fig. 2, a very sensitive electrodynomameter, or a galvanometer equipped with an electromagnetic field, was used as a detector. The condition for balance in this impedance bridge network is

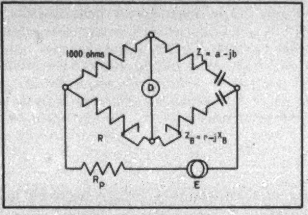

FIG. 2—BRIDGE USED IN THE LOCATION OF OPENS

E represents the low-frequency source, and R_p a protective resistance. The rheostat R and the 1000-ohm resistance may be regarded as the ratio arms of the bridge. The impedance of the line is represented by Z_L, a resistance and capacitance in series. The rheostat r is used in balancing the resistance component of the line impedance, so that the impedance angle of Z_B will equal the impedance angle of Z_L.

$$R\,Z_L = 1000\,Z_B$$

or

$$R\,(a - j\,b) = 1000\,(r - j\,X_B).$$

Collecting reals and imaginaries,

$$R\,a = 1000\,r$$

and

$$R\,b = 1000\,X_B$$

If measurements R_1, r_1 and R_2, r_2 are made on the bad and good wires, respectively,

$$R_1\,a_1 = 1000\,r_1$$

and

$$R_2\,a_2 = 1000\,r_2;$$

also

$$R_1\,b_1 = 1000\,X_B$$

and

$$R_2\,b_2 = 1000\,X_B,$$
$$R_1\,b_1 = R_2\,b_2,$$

or

$$\frac{R_1}{R_2} = \frac{b_2}{b_1}$$

Yet the design of a suitable bridge is not concerned alone with balanced condition, but rather with the sensitivity and ease of balance with slight unbalances present. An indication of the probable sensitivity is afforded by a complete solution of the bridge network to determine the phase and magnitude of the galvanometer unbalance current with respect to the impressed voltage. The following determination of the galvanometer unbalance current was obtained by an application of Kirchoff's law to the bridge network shown in Fig. 3.

$$i_2\,z_1 - i_3\,z_5 - (i_1 - i_2)\,z_4 = 0$$
$$(i_2 + i_3)\,z_2 - (i_1 - i_2 - i_3)\,z_3 + i_3\,z_5 = 0$$
$$(i_1 - i_2)\,z_4 + (i_1 - i_2 - i_3)\,z_3 + i_1\,z_6 = E$$

After collecting current coefficients determinants are used in obtaining an expression for the unbalance current i_3, which is given by equation (**2**).

Collecting coefficients:

$$-\,i_1\,z_4 + i_2\,(z_1 + z_4) - i_3\,z_5 = 0$$
$$-\,i_1\,z_3 + i_2\,(z_2 + z_3) + i_3\,(z_2 + z_3 + z_5) = 0$$
$$i_1\,(z_3 + z_4 + z_6) - i_2\,(z_3 + z_4) - i_3\,z_3 = E$$

Solving by determinants for i_3:

$$i_3 = \frac{\begin{vmatrix} O & (z_1 + z_4) & -\,z_4 \\ O & (z_2 + z_3) & -\,z_3 \\ E & -\,(z_3 + z_4) & (z_3 + z_4 + z_6) \end{vmatrix}}{\begin{vmatrix} -\,z_5 & (z_1 + z_4) & -\,z_4 \\ (z_2 + z_3 + z_5) & (z_2 + z_3) & -\,z_3 \\ -\,z_3 & -\,(z_3 + z_4) & (z_3 + z_4 + z_6) \end{vmatrix}}$$

$$i_3 = \frac{E\,(z_2\,z_4 - z_1\,z_3)}{\Delta} \qquad \textbf{(2)}$$

where Δ is the denominator of the above determinant.

Since the condition for balance is

$$z_1\,z_3 = z_2\,z_4$$

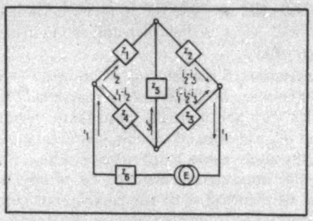

FIG. 3

we have, using the notation of Fig. 2,

$$1000\,(r - j\,X_B) = R\,(a - j\,b)$$

and, equating reals and imaginaries,

$$1000\,r = a\,R \qquad (3)$$
$$1000\,X_B = b\,R \qquad (4)$$

This impedance bridge can be balanced in two ways: by varying r and X_B, keeping R constant, or by varying R and r, keeping X_B constant; see Fig. 2. The effect is essentially the same in either case. When R is varied instead of X_B, the only difference is that the balance of the bridge is disturbed for both the real and imaginary components. This fact necessitates a correction of r each time R is changed in securing a balance of b against X_B. If X_B is varied, the balances of r against a and X_B against b are independent functions. In practise, it is easier to vary R and keep X_B

constant, but for the purpose of theoretical discussion, it lends clarity to consider X_B to be variable from the condition of balance.

From equation (**1**) above, the impedances of different lengths of line may be computed. A number of impedance values representing different lengths of 19-gage, non-loaded cable up to 60 mi., at three frequencies, *viz.*, 20, 8, and 4 cycles, was selected, and the condition of balance of the impedance bridge calculated for each case from equations (**3**) and (**4**). Curves of these impedances are shown in Fig. 4, where the reactances and resistances at the three chosen frequencies are plotted.

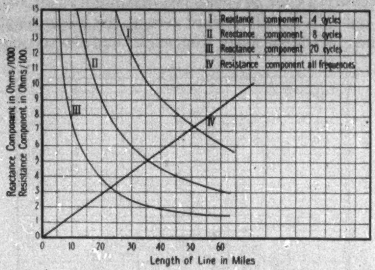

Fig. 4—Characteristics of the Resistance Component and Capacitance Component of the Impedance of Lengths 19-Gage, Non-Loaded Cable

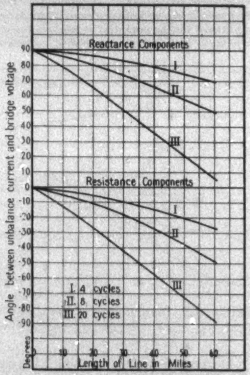

Fig. 5—The Variation of the Angle Between the Bridge Voltage and the Unbalance Current for the Reactance Component and Resistance Component of Non-Loaded 19-Gage Cable in Lengths Varying from Zero to Sixty Miles

The curves represent characteristics for frequencies of four, eight, and twenty cycles

Since the sensitivity of the bridge network should be a maximum when the unbalance is small, *i. e.*, when a balance is about to be secured, this condition is the one with which the sensitivity calculation is concerned. The capacitance X_B of the bridge network was assumed to vary 10 per cent from the condition of balance and the galvanometer unbalance current, i_3, was then calculated from equation (**2**) for each length of line at each frequency. Both the magnitude and phase angle were found. Similarly, the resistance r was assumed to vary 10 per cent from the condition of balance, X_B remaining balanced, and another set of galvanometer unbalance currents was determined. Such calculations are particularly tedious, involving successive additions and subtractions, multiplications and divisions of complex quantities. The results of these calculations are shown in Figs. 5, 6, and 7. Figure 5 represents the variation in phase angle of the current i_3 with respect to the bridge potential, Fig. 6 the magnitude i_3, and Fig.

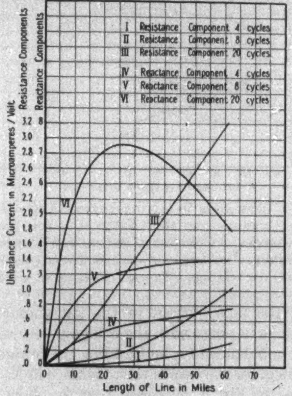

Fig. 6—Magnitude of the Unbalance Current for Measurements, on the Impedance of Non-Loaded 19-Gage Cable, at Frequencies of Four, Eight, and Twenty Cycles

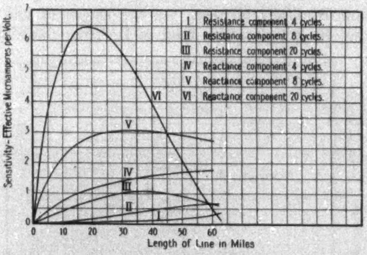

Fig. 7—Sensitivity to Unbalance for the Bridge Network for Impedance Measurements on Lenths of Non-Loaded 19-Gage Cable

These curves represent sensitivities for frequencies of testing protential of four, eight, and twenty cycles

7 the relative sensitivities obtained with different frequencies and different lengths of line. These sensitivities are proportional to i_3, Fig. 6, and to the cosine of the angle between i_3 and the field current of the a-c. galvanometer. In the calculation of the curves of Fig. 7, the field current of the galvanometer was assumed to be in phase with the bridge potential for the case of an unbalance in r, and leading the bridge potential by 90 deg. for the case of an unbalance in X_B.

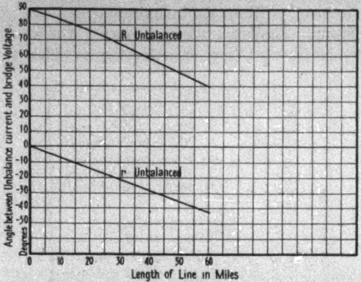

FIG. 8—CALCULATED VARIATION IN THE ANGLE BETWEEN UNBALANCE CURRENT AND BRIDGE VOLTAGE FOR UNBALANCE IN THE CAPACITANCE COMPONENT *R* AND FOR UNBALANCE IN THE RESISTANCE COMPONENT *r* OF THE IMPEDANCE OF NON-LOADED 19-GAGE CABLE. TESTING POTENTIAL OF 4-CYCLES

Referring to Fig. 5, it is seen that for short lengths of line the unbalance current caused by unbalancing *r* is almost in phase with the voltage, while the unbalance current due to unbalancing X_B leads the voltage by approximately 90 deg. As the length of line increases, the phase angles tend to lag from these positions, due to the effect of the convergent variation of the resistive and reactive components of the line impedance; that is, the resistance increases and the reactance decreases with increase in length of line. This lag is greater for the higher frequencies. The total variation for 60 mi. of cable measured at 20 cycles is practically 90 deg., which means that for a given field current the sensitivity using 20 cycles must approach zero with some length of line between zero and 60 mi. This condition is illustrated in Fig. 7 where the sensitivity for 20 cycles is a maximum at 20 to 30 mi., but falls rapidly toward zero at the longer lengths of line. With the lower frequencies and the setting of field current used, the general effect is an increase in sensitivity as the length of line increases, and a decrease in sensitivity with decrease in frequency. This decrease is due to the decrease in reactance with increase in length of line; see Fig. 4.

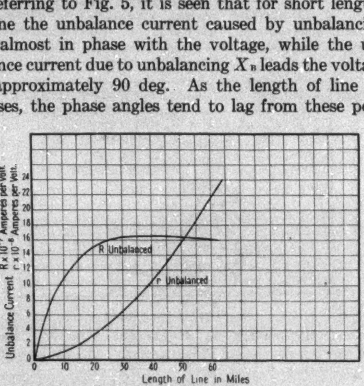

FIG. 9—CALCULATED MAGNITUDE OF THE UNBALANCE CURRENT FOR UNBALANCE IN THE CAPACITANCE COMPONENT *R* AND UNBALANCE IN THE RESISTANCE COMPONENT *r* OF THE IMPEDANCE OF NON-LOADED 19-GAGE CABLE. TESTING POTENTIAL 4-CYCLES

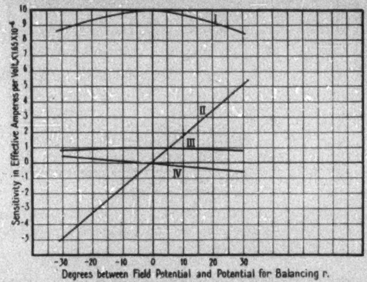

FIG. 10—CALCULATED SENSITIVITY OF THE BRIDGE AT FOUR CYCLES ON A 50-MILE LENGTH OF NON-LOADED, 19-GAGE CABLE

Sensitivities for unbalance in the component:

I. *R* with the phase of testing potential applied for balancing *K*
II. *R* with the phase of testing potential applied for balancing *r*
III. *r* with the phase of testing potential applied for balancing *r*
IV. *r* with the phase of testing potential applied for balancing *R*

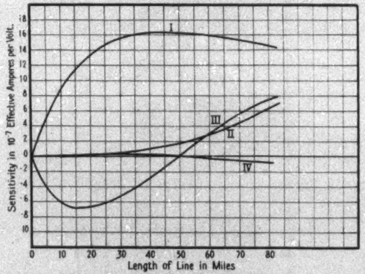

FIG. 11—CALCULATED SENSITIVITY FOR LENGTHS OF NON-LOADED 19-GAGE CONDUCTOR USING THE IMPEDANCE BRIDGE ARRANGEMENT WHICH GAVE THE MAXIMUM SENSITIVITY FOR CURVE I IN FIG. 11

Sensitivity for unbalance in the component:

I. *R* with the phase of testing potential applied for balancing *R*
II. *r* with the phase of testing potential applied for balancing *r*
III. *R* with the phase of testing potential applied for balancing *r*
IV. *r* with the phase of testing potential applied for balancing *R*

It would appear that provision should be made for shifting the phase of the field current through 90 deg., its two positions being respectively in phase with the bridge potential and 90 deg. leading. Thus the two components, r and X_B, could be balanced independently except for the shift in phase with different line lengths. This phase shift is small with a frequency of testing potential of four cycles.

A frequency of four cycles was selected as the optimum as regards the size of hyperbolic error discussed above, the sensitivity available, and the amount of phase shift with increase in length of line. The sensitivity at four cycles has the advantage of being sufficient but not excessive. To a large extent, the condition of phase shift, with increase in length of line, governs the ease of securing a balance over the range of line lengths.

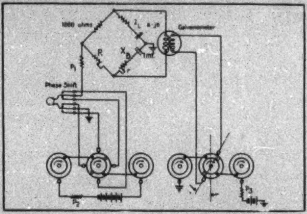

Fig. 12—Impedance Bridge Circuit Showing the Arrangement used in Applying 4-cycle Testing Potentials in Quadrature

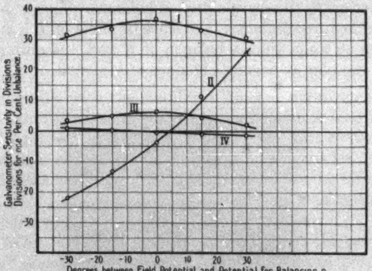

Fig. 13—Observed Galvanometer Sensitivities Which are Comparable to the Calulated Sensitivities of Fig. 11

Sensitivity for unbalance in the component:

I. R with the phase of testing potential applied for balancing R
II. R with the phase of testing potential applied for balancing r
III. r with the phase of testing potential applied for balancing r
IV. r with the phase of testing potential applied for balancing R

The ideal arrangement would be one in which the field current could always be placed in phase with the component of the bridge unbalance current it was desired to eliminate. Such a quality is not characteristic of the type of bridge used; however, a desirable approximation of such an arrangement is obtained when a four-cycle frequency of testing potential is used.

In order to check the assumption stated above, that the bridge could also be balanced by keeping X_B constant and varying R and r, without materially changing the conditions of balance, another set of galvanometer unbalance currents was calculated with X_B constant and R and r varied respectively 10 per cent from the condition of balance. The chosen frequency of four cycles was used in this calculation.

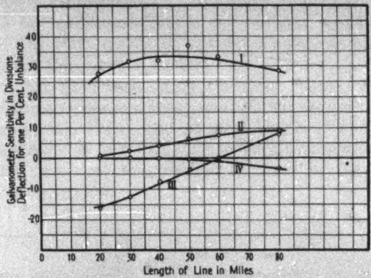

Fig. 14—Observed Galvanometer Sensitivities Which are Comparable to the Calculated Sensitivities of Fig. 17

Sensitivity for unbalance in the component:

I. R with the phase of testing potential applied for balancing R
II. r with the phase of testing potential applied for balancing r
III. R with the phase of testing potential applied for balancing r
IV. r with the phase of testing potential applied for balancing R

These curves of phase angle and current magnitude are shown in Figs. 8 and 9, and correspond to those shown in Figs. 5 and 6, calculated by the other method.

Since R increases with increase in length of line, the sensitivity per ohm change in R will be better throughout the range of lengths if the sensitivity for a given change in R is a maximum when R is a maximum.

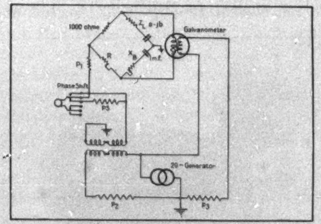

Fig. 15—Impedance Bridge Circuit Showing a Simplified Arrangement Employing 20-Cycles as a Source of Testing Potential with an Approximate Method of Shifting Phase

Taking 50 mi. as the average total length of line, and assuming two bridge potentials 90 deg. apart, a set of sensitivity curves was calculated for this length of line, the phase of the field potential being varied on either side of the bridge testing potential. The lag of the field current from the field voltage calculated from the field inductance and resistance was found to be about 40 deg. The resulting sensitivity curves are shown in Fig. 10, and these indicate a maximum sensitivity for R and r with the field potential in phase with the potential used to balance r.

With the assumed conditions as determined by this calculation, *viz.*, two bridge potentials 90 deg. apart and a field current lagging one bridge potential 40 deg., a complete set of sensitivity curves was calculated for different lengths of line from zero to 80 mi. These are shown in Fig. 11. It should be noted that the sensitivity for detecting an unbalance in R is a maximum at the desired length of 50 mi. The sensitivity for an unbalance in r is low at the shorter lengths of line, but increases as the length of line increases. It may be noted that in both Figs. 10 and 11 the sensitivity curve for changes in R passes through zero when the testing potential is applied for balancing r. Likewise, the sensitivity curve for changes in r passes through zero when the testing potential is applied for balancing R. This point is where the field current and galvanometer unbalance current are 90 deg. out of phase. Naturally, this point coincides with the point of maximum sensitivity for the normal potential arrangement. As stated above, it would be ideal if these "reverse sensitivities" could be zero and the "true sensitivities" could be maximum throughout the entire range of lengths. Reference to Fig. 11 will show that the reverse sensitivity for r is practically zero throughout the range of lengths. This fact in itself is significant. Assuming r to be set on zero, R could be varied to secure an approximate balance, using the proper testing potential. This balance would be fairly accurate since the reverse sensitivity for r is quite low throughout. Shifting bridge potentials 90 deg., r could be adjusted almost to the proper point since R is practically correct. The balance of R and r can then be refined as often as is necessary to secure a perfect balance. Since r is not used in any calculation, and since its effect on R is small once an approximate balance of R and r is obtained, the need for an accurate balance of r is small.

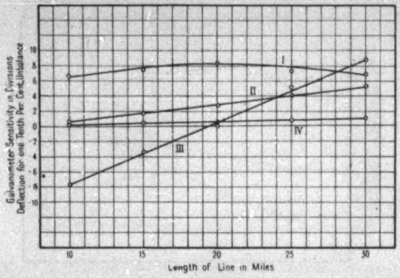

FIG. 16—OBSERVED GALVANOMETER SENSITIVITIES FOR A 20-CYCLE FREQUENCY OF TESTING POTENTIAL. THESE CURVES ARE COMPARABLE WITH THE 4-CYCLE CURVES OF FIG. 14

Sensitivities for unbalance in the component:

I. R with the phase of testing potential applied for balancing R
II. r with the phase of testing potential applied for balancing r
III. R with the phase of testing potential applied for balancing r
IV. r with the phase of testing potential applied for balancing R

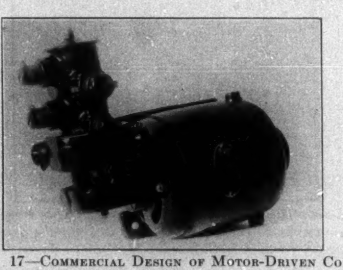

FIG. 17—COMMERCIAL DESIGN OF MOTOR-DRIVEN COMMUTATOR WHICH IS USED TO SUPPLY 4-CYCLE ALTERNATING POTENTIALS FOR THE IMPEDANCE BRIDGE

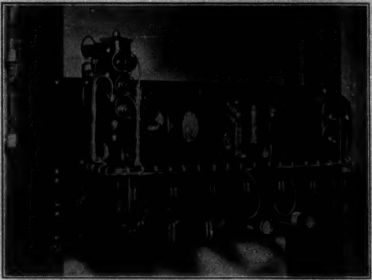

FIG. 18—4-CYCLE MOTOR-DRIVEN COMMUTATOR WITH ASSOCIATED EQUIPMENT

FIG. 19—PANEL ASSEMBLY OF THE IMPEDANCE BRIDGE

A series of observations made with experimental apparatus arranged as shown in Fig. 12 gave the sensitivity curves shown in Figs. 13 and 14. The observed sensitivity characteristics shown by these curves agree

A simplified arrangement for use in certain types of measurement not requiring the accuracy afforded by the four-cycle arrangement of Fig. 12 is shown in Fig. 15. A 20-cycle ringing generator is used as a source of single-phase testing potential. Phase shifting is accomplished by the insertion of a high resistance, *P S*, in series with the bridge supply. This affords a considerable shift of phase which is not, however, 90 deg., nor is it constant. A transformer is provided for changing the potential when desirable and to afford a non-grounded source of potential in the measurement of mutual capacitances. The 20-cycle generator is normally grounded.

Observed sensitivity curves for this arrangement of 20-cycle equipment are given in Fig. 16. It will be seen that these sensitivities are considerably higher in value than for the four-cycle arrangement because the reactance of a given length of line is less at 20 cycles than at 4 cycles. The range is less with 20 cycles due to the fact that the phase of the galvanometer moving coil current, with respect to the testing potential, changes much more rapidly at 20 cycles than at 4 cycles. Thus the range of line length is lower, unless provision is made for shifting the galvanometer field current. The maximum length of line is limited also by the allowable value of the hyperbolic error. The hyperbolic errors obtained with measurements made at 20 cycles on these short lengths of line are about the same as those obtained in the measurements made at 4 cycles on the longer lengths of line.

In general, the frequency of testing potential of four cycles affords a balance more quickly and easily than does the 20-cycle arrangement. This is because the phase shift with four cycles is exactly 90 deg. while that obtainable with the simplified 20-cycle arrangement is necessarily an approximation, and does not lend itself to regulation either in regard to phase shift or relative position of field current. With four cycles, these relations are more or less flexible and can be adjusted to afford characteristics tending toward an optimum condition.

After having completed an analysis of the problem and demonstrated the practicability of the proposed methods, it remained to develop applications of these methods for practical use. Equipment was designed to develop a low-frequency source of alternating potential by reversing a testing battery. The device developed for this purpose is a four-cycle, motor-driven commutator shown in Fig. 17. The assembly of the four-cycle commutator with associated apparatus is shown in Fig. 18.

The galvanometer and the equipment required in the modified form of the de Sauty bridge have been incorporated in a compact unit which is shown in Fig. 19. This bridge, while being particularly adapted to the four-cycle impedance measurements required for open location tests, is also applicable for d-c. bridge measurements. Assembly details for the bridge arrangement very favorably with the theoretical values shown by the curves of Figs. 10 and 11.
are shown in Fig. 20. The a-c. galvanometer is sufficiently sensitive to be directly actuated by any significant unbalance of the impedance bridge, so that it is unnecessary to use an amplifier, rectifier, or other converting apparatus which may be difficult of adjustment or maintenance.

A few additional features are outlined as some of the significant results of this development of an improved open location method.

It has been shown that the error caused by the deviation from the straight line relation between sending-end admittance and physical length of line, has been reduced to a value which may be neglected as being less than the required accuracy of the open location method.

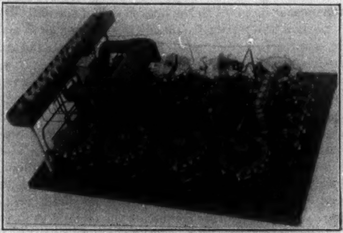

FIG. 20—INTERNAL ASSEMBLY OF THE IMPEDANCE BRIDGE

This impedance bridge arrangement employs several features which are distinct improvements on the methods previously utilized for this purpose. In order that the impedance angles of the impedance networks may be balanced, a variable resistance has been connected in that branch of the bridge which contains the comparison capacitance. This balance of the impedance angles of the impedance network was found to be important in obtaining a steady performance of the a-c. galvanometer. A system has been devised for separately balancing the resistance components as well as the capacitance components of the reactive networks of the impedance bridge. It was found that this arrangement gives a maximum sensitivity to each component and permits of a very rapid balance of the bridge.

Discussion

I. M. Stein: It seems to me that the method and apparatus described accomplish two things. First, the bridge circuit and the type of detector used permit of a great precision of balance; that is, the quantity being measured may be measured precisely enough to give the accuracy desired in locating the open circuit. Then, having a set-up which gives the desired precision, we have two options. One is to chart each long cable by putting "opens" at certain known places and making measurements; the other is to get rid of the non-linear characteristic so that no charting is necessary.

The second method is the one which has been chosen, and the desired result has been accomplished by lowering the frequency enough to give a linear relation between the measured quantity and the length of cable to the "open."

So that there are two steps. First, a precision of setting enabling an accurate measurement of the capacitance; and second, the use of a frequency low enough to eliminate certain errors, thus obtaining a linear relation between capacitance and distance.

I should like to ask if I have a correct understanding of the matter.

B. W. Kendall: The need for the development of this modification of Wheatstone-bridge methods arose in connection with the large growth of toll-cable plant within the last few years. The convenient distance between cable-testing stations is set by the use of telephone repeaters at about a 50-mi. spacing, and it was therefore desirable to be able to locate opens in the cable conductors over distances of this order. Because of the difficulty in getting access to the individual wires along the cable it was important that the location of the fault be as accurate as possible.

The terminal impedance as measured between a defective (open) wire and ground is a hyperbolic function of the distance from the measuring point to the broken end of the defective wire. In this method, the testing frequency is so chosen as to make this relation approach a reciprocal function so that the capacity measured is directly proportional to the distance. This choice is a matter of convenience in testing. It would be possible to use a higher frequency and then by means of tables of hyperbolic functions to determine the distance to the break. In the paper the deviation of the measured capacity from direct proportionality with the distance is spoken of as an "error." This is, of course, an error which would be made if a higher frequency were used for the measurement and it were then assumed that the measured capacity and the distance were in direct proportion. By using so low a measuring frequency the error that would thus be incurred is made negligible and the measuring technique is thereby simplified.

The other notable feature of this paper is the use of bridge and galvanometer currents of different phases for determining the settings of the two adjustable resistances of the bridge. It should be understood that the balance so obtained is correct for any phase of measuring current and that this method is applied simply to obtain a larger deflection of the galvanometer for a given unbalance of the bridge, thereby facilitating the measurement. This method was developed for measuring opens in cable circuits, which have certain definite and uniform characteristics, and I should like to ask Mr. Edwards or Mr. Herrington what experience they have had with the use of the same apparatus in locating faults in open wire in which the impedances would have different characteristics from those of cable circuits.

This paper shows what can be accomplished in the development of a special bridge method for readily making tests of circuits which are uniform in character. At other frequencies and in other arts where a large number of quantities which differ little among themselves are to be measured, the exercise of equal skill and ingenuity can develop methods to simplify and expedite the determinations.

S. P. Shackelton: An important phase of the work of an engineer involves the application to practical everyday problems of the results of pure scientific investigation and research. The contribution of Edwards and Herrington constitutes such a practical application to a problem which has been studied for many years. It occupied the minds of communication engineers before the advent of the telephone. The particular arrangements described do not disclose any new principles but rather represent the utilization of known characteristics of lines and networks to a useful purpose.

The bridge network used was described in 1891. An approximate utilization of transmission-line low-frequency characteristics has been in common use for open locations. Alternating-current galvanometers have been well known although their use as detectors in the balancing of bridge circuits has not been so common. The shifting of the phase in the bridge to assist in obtaining a balance had been treated prior to the work under discussion. The present work however, affords a convenient tool for the field man whether he be technical or not, to apply technical methods to his common problems of plant maintenance.

Specifically, the problem was one of adapting the technical methods to rather trying limitations imposed by plant conditions. A low-frequency measurement was desired with positive indications of balance and with a minimum adjustment of variables. A method was required which would afford a rather high degree of accuracy considering the precision of the apparatus and the possible errors outside the control of the operator. All this was combined in space limitations which necessitated rather careful detail design.

The authors have not specified any requirements as to wave shape although stressing the phase relations required in the different portions of the bridge network. The effect of phase angle on balance is noted and, of course, for any given length of line and frequency there is a best value of phase shift. It may be interesting to know the relative importance of this and its effect on other lengths of line.

C. R. Fischle: There is one phase of the subject presented by Messrs. Herrington and Edwards that it would seem warrants discussion, and that is the means employed to apply the principles brought out in the paper to practical telephone work.

In order to facilitate the application of these principles by the test-board forces, the Bell System has established routines outlining in a simple and orderly manner the procedures to be followed in determining the location of cable faults in toll cables. The engineering forces, as a rule, prepare in advance for each testing point, data such as temperature-correction curves and records giving information as to cable distances, loading, gage of conductors, cable make-up, etc., for the cables to be tested from the respective points. Work sheets are supplied to the test-board men on which the results of the electrical tests made at the time of a fault location are entered, together with values selected from the data mentioned above. By following the sequence of calculations set up on this work sheet, the fault location is quickly arrived at. These calculations are of a simple order and do not involve hyperbolics.

By administering the work in this manner the test-board men can expeditiously arrive at accurate fault locations, thus materially aiding in the dispatch of repair forces to the fault and in securing the restoration of defective circuits in the least possible time.

P. G. Edwards: One question which has arisen in connection with the low-frequency bridge is that of wave form. A square wave is used with results agreeing very closely with theoretical values based on calculations involving a sinusoidal wave shape. This is made possible by the fact that the bridge network employed is symmetrical. With a square wave and a network involving both inductance and capacitance sinusoidal methods of treatment probably would not be applicable.

A question has been brought up by Mr. Kendall—that of errors with frequencies other than 4 cycles. In earlier designs higher frequencies were used and a family of correction curves was plotted for each class of conductors, from which a correction was read for each capacitance ratio. This correction in per cent when applied to the capacitance ratio gave the true length ratio. On toll cables of average length and with a frequency of 8 cycles this correction reached values as high as 1 per cent.

In connection with the subject of correction curves, the question has arisen as to the practicability of plotting curves for individual conductors or classes of conductors. Mr. Stein's analysis of the situation in this respect is quite correct. Due to the large number of circuits involved, as well as classes of circuits, the number of combinations possible renders this procedure quite unwieldy and expensive. The problem has been treated rather by means of fundamental design. As pointed out by Fischle records of actual measurements, however, are filed and are accessible to the tester.

In regard to phase adjustment, as has been brought out in the paper, there is an optimum phase adjustment for each measurement. In the apparatus as outlined an adjustment has been selected which gives a desirable set of sensitivity characteristics for the type of work for which the apparatus was designed. The equipment arrangements are such that the phase relation of bridge potential and field potential can be adjusted where desirable. This is not done, however, in the normal routine of locating opens.

Mr. Kendall has brought up the question of open-wire impedances. The equipment as outlined has been tried out on exposed aerial lines with success. The chief departure from the problem of locating opens in cables is the presence of a leakance component in the line impedance. Toll cables are almost entirely free from this characteristic. This leakance component happily involves no modification in design, but is observed in the balancing of the bridge as an increase in the value of the resistance component of the condenser arm of the bridge and does not, over a considerable range of leakance, affect the ratio-arm setting, or in consequence, the actual fault location.

Microammeter Indication of High-Frequency Bridge Balance

BY H. M. TURNER[1]

Member, A. I. E. E.

Synopsis.—*A visual method of obtaining bridge balance by means of a d-c. microammeter in the plate circuit of an electron tube detector associated with one or more stage of amplification, which gives maximum reading for a state of bridge balance, thereby permitting use of a sensitive meter. A large bridge unbalance reduces the deflection to a low value and as the balance is approached the reading increases. This method overcomes the limitations of the aural method and renders a quantitative determination of the degree of unbalance.*

* * * * *

THE telephone receiver, due to its simplicity, convenience, and sensitivity, has been widely used for determining a-c. bridge balance and, under favorable conditions, is fairly satisfactory. The aural method, however, involving as it does the receiver associated with the ear, has two serious limitations namely, it can be used only where there is very method in a noisy laboratory, it is unnecessary to point little extraneous noise and for best operation the frequency range is restricted to a band of 200 to 2000 cycles unless a heterodyne scheme is adopted.

For those who have had experience with the aural out the difficulties encountered, other than to say that noise is always a source of annoyance and often makes measurements absolutely impossible.

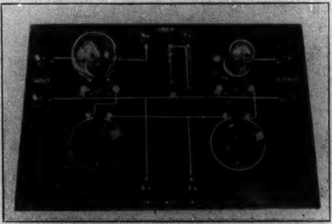

FIG. 1—AMPLIFIER AND DETECTOR UNIT

The combined limitations of the receiver and ear in connection with bridge measurements over a wide frequency range are not so generally appreciated[2]. While the limits of audibility are usually given as 16 to 40,000 cycles, the average person will be unable to hear above 15,000 and it will be impossible to determine even an approximate balance at these extreme frequencies. It is found that above 2000 or 3000 cycles, the sensitivity of the aural method is erratic and usually quite low. There is usually a marked difference in the sensitivity of the right ear and left ear of a given individual and it has been found frequently that the use of both ears is less effective than one alone. Different results are obtained on different days and, of course, there are wide variations among individuals. All of these factors point to the need of a more satisfactory method of determining bridge balance in order that results may be duplicated.

A visual method has been devised in our laboratory, using a d-c. microammeter in the plate circuit of a detector tube, associated with one or more stages of amplification, which has given very satisfactory results.

A reproduction of a photograph of the amplifier and detector unit is shown in Fig. 1 and other details including the plate and grid curves are given in Fig. 2.

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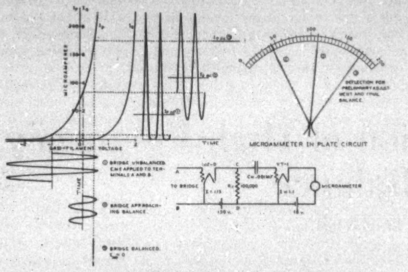
[1] Associate Professor of Electrical Engineering, Yale University.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

[2] Engineers of the Bell Telephone Laboratory are familiar with these matters but have not, so far as I know, published any papers calling specific attention to their bearing on bridge measurements.

Tubes with Gas

It will be observed that the grid-condenser form of detection is used, but without a grid-leak, and operates on the steep straight portion of the plate-current curve and at the sharp bend of the grid-current curve. Due to the current in the plate of the amplifier tube, there is a rather large difference of potential between *C* and *D*, that is, across the coupling resistance, *C* being negative with respect to *D*. This large negative voltage is impressed upon the grid of the detector tube through the condenser which would naturally reduce the plate current to a low value, but due to the presence of some gas in the tube, there is flow of current in the grid circuit in a counter clockwise direction which charges the condenser to a potential slightly in excess of that between *C* and *D* and, of course, in the opposite direction. The total voltage between grid and filament is about + 0.5 volt. This is the point where the grid current is zero and is changing from negative to positive as shown in Fig. 2. With an alternating electromotive force of constant value impressed upon *A B*, the charge of the grid condenser is reduced and the grid-filament potential becomes less than +0.5 and, for a large unbalanced bridge, electromotive force may be considerably negative, thus reducing the average plate current to a very small value. Regardless of the voltage impressed, the axis about which it varies moves to the left from the point where the grid current becomes zero, until the average charge on the condenser is constant, that is, until the discharge of the condenser, during the time the impressed electromotive force makes the grid more than + 0.5 volt, is equal to the charge during the time the grid is less than + 0.5 of a volt, as shown by the shaded areas Fig. 3. Since the negative grid current is very small as compared with the positive values, the grid potential is greater than + 0.5 for a very small part of the cycle as shown.



MICROAMPERES
TIME
GRID-FILAMENT VOLTAGE
MICROAMMETER IN PLATE CIRCUIT
DEFLECTION FOR PRELIMINARY ADJUSTMENT AND FINAL BALANCE
① BRIDGE UNBALANCED E.M.F. APPLIED TO TERMINALS A AND B.
② BRIDGE APPROACHING BALANCE.
③ BRIDGE BALANCED.
TO BRIDGE
MICROAMMETER
130 v.
16 v.

Fig. 2—Visual Bridge Balance Indicator

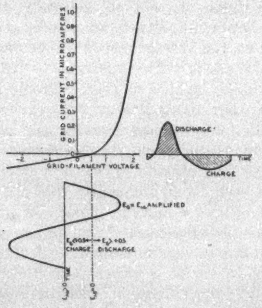
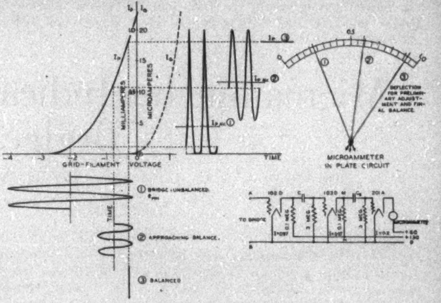
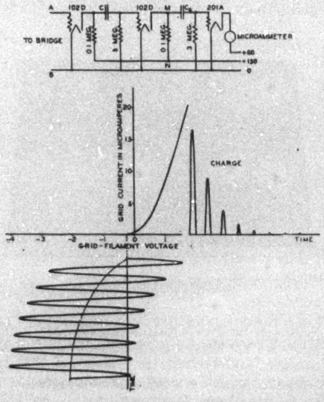


Fig. 3—Action of Grid Condenser with an Alternating E. M. F. of Constant Value Impressed upon Terminals *A* and *B*, Fig. 2



MILLIAMPERES
MICROAMPERES
GRID-FILAMENT VOLTAGE
TIME
MICROAMMETER IN PLATE CIRCUIT
DEFLECTION FOR PRELIMINARY ADJUSTMENT AND FINAL BALANCE
① BRIDGE UNBALANCED
② APPROACHING BALANCE.
③ BALANCED
TO BRIDGE
102 D
1020
201 A

Fig. 3a



A
102 D
102 D
201 A
TO BRIDGE
MICROAMMETER
0.1 MEG.
3 MEG.
+60
+130
B
0
GRID CURRENT IN MICROAMPERES
CHARGE
TIME
GRID-FILAMENT VOLTAGE

Fig. 3b

By this arrangement, the plate current of the detecting tube is a maximum when the alternating electromotive force impressed upon the terminals *A B* is zero, which corresponds to a balanced condition of the bridge, thereby permitting the use of a sensitive meter and at the same time making it fool-proof so far as the meter is concerned after the preliminary adjustment. The maximum current may be predetermined and is under the control of the operator. The preliminary adjustment is made by short-circuiting the terminals *A B*, equivalent to a bridge balance in that there is no impressed electromotive force, and increasing the filament current to give the desired reading which is usually near full scale as shown in Fig. 2. When the bridge is unbalanced, the amplified bridge electromotive force impressed upon the grid filament of the detecting tube results in a marked decrease in the average plate current as indicated by the microammeter, and as the balanced condition is approached the current gradually increases.

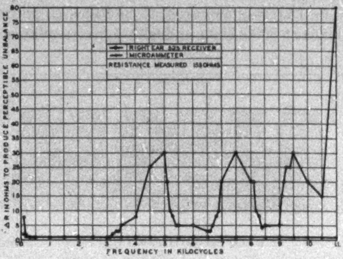


Fig. 4

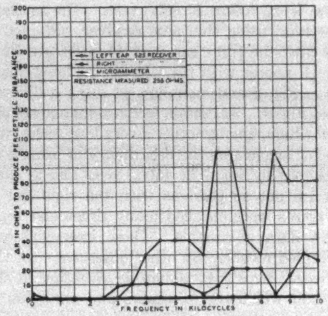


Fig. 5

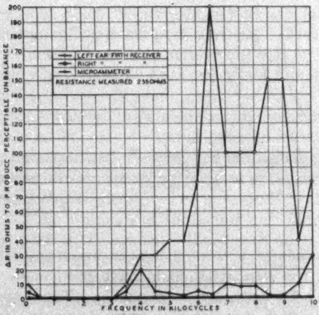


Fig. 6

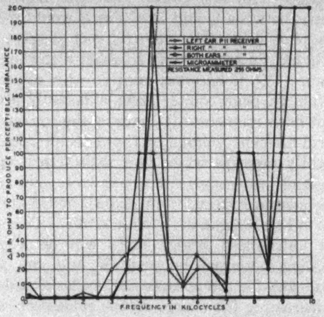


Fig. 7

At maximum deflection the meter is extremely sensitive, thus permitting a decidedly sharp balance to be obtained, a slight change in one of the circuit constants being sufficient to cause the indication to decrease several divisions on the scale. It is a simple matter to determine whether the balance is perfect without the necessity of remembering the initial maximum current. When the bridge is thought to be in balance, short-circuit the terminals *A B*, and if there is no change in the meter reading the bridge is balanced. In certain types of circuits, the presence of harmonics in the impressed electromotive force will make it impossible to fulfill

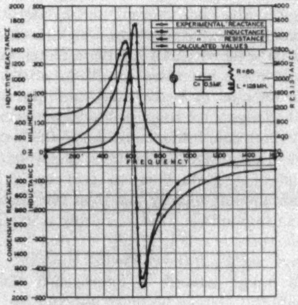


Fig. 8

the condition just referred to, in which case it may be desirable to use a multiple-band filter, as shown in Figs. 9 and 10, to eliminate this source of trouble.

HIGH VACUUM TUBES

Where a highly exhausted tube is used for the detector, such as the 201-A, a modification of the circuit is necessary for the reason that the high negative potential between C and D of Fig. 2 would reduce the steady plate current to zero and keep it there. However, by putting the battery in series with the coupling resistance *CD*, *C* becomes positive with respect to *D*, this voltage being quite large, a large positive grid current flows momentarily charging the condenser until the grid-filament voltage of the detector tube is zero or slightly negative which is a satisfactory operating point.

The modified circuit and characteristic curves are shown in Figs. 3A and 3B. With terminal *AB* connected to an unbalanced bridge there will be a constant alternating potential between *M* and the negative filament and for the positive half cycles a grid current flows increasing the charge on the condenser as shown by Fig. 3B. During the negative half cycle there can be no grid current so the charge is constant and the voltages add up in the usual way. At the beginning of the second cycle the operating point is to the left of the initial position due to the increased charge on the condenser. This action continues until the axis has been moved to the left by an amount almost equal to the maximum value of the alternating component. The average plate current is considerably reduced as shown by 1 Fig. 3A. Now as the bridge balance is approached the alternating e. m. f. component decreases but with a perfect grid condenser there would be no change in the plate current and in any event the change would be extremely slow. A leak resistance is provided in order to make the plate meter responsive to changes in the impressed e. m. f. in which case the condenser charge decreases and the average plate current increases until some new position such as 2 is reached. For any bridge unbalance the axis about which the alternating component of grid voltage varies is to the left of the point where the grid current becomes zero by an amount slightly less than the maximum of the alternating voltage, that is, during a small part of each cycle there is a flow of grid current to maintain the average charge on the condenser constant.

As the bridge balance is approached closely the alternating grid voltage becomes less and less and the excess charge on the condenser approaches zero as the charge leaks away and the average plate current is a maximum.

In order to illustrate the points that have been discussed, a number of curves that are more or less self-explanatory will be included. The simplest type of circuit is used in order to make the comparison more easily appreciated. Fig. 4 shows the change in resistance, ΔR, necessary to produce a perceptible unbalance as a function of the frequency. Here are compared the results obtained with the aural and visual methods. With the aural method, the sensitivity was entirely satisfactory from 200 to 3000 cycles but a change of 30 ohms in 155 was required to produce a perceptible unbalance at 5000, 7500, and 9500 cycles and 80 ohms at 11,000 cycles. The visual method gave a sensitivity of one ohm from 200 to 11,000 cycles. On another day, the observer, using the same receiver, obtained better results with the right ear than those shown in Fig. 4 but much poorer with the left ear. These curves are shown in Fig. 5 and a similar set using a different receiver in Fig. 6. At certain frequencies one ear shows a relatively high, and the other a very low, senstivity, and vice versa, suggesting the desirability of using a double receiver, but in Fig. 7 at 4500 cycles and at 9000 the results are not so good as with the right ear alone. These points were checked many times and represent the best data obtainable. These irregularities have been verified recently by other observers. In the different cases there is considerable variation in the frequencies at which the maximum and minimum points occur.

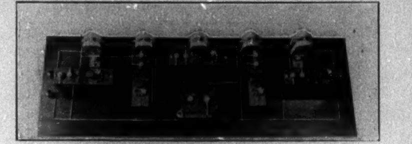


FIG. 9—MULTIPLE BAND FILTER

In all cases, the results obtained by the microammeter were satisfactory throughout the frequency range, although the sensitivity could have been made greater had it been necessary or desirable. There are limits beyond which it is not wise to go, however, for if the sensitivity is too great, the adjustment becomes difficult and unless the indicator set is well shielded, stray field may cause trouble.

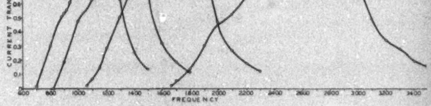


FIG. 10—MULTIPLE BAND PASS FILTER

In circuits having inductance and capacity in parallel, the apparent resistance and reactance change rapidly over a relatively few cycles near the resonant frequency and thus afford an opportunity to test the merit of the visual method. Both the experimental and calculated values are given by the curves in Fig. 8 and it will be observed that the results are in satisfactory agreement.

The visual method not only overcomes the limitations of the aural method but makes possible a quantitative determination of the degree of unbalance at any time, which is often a distinct advantage. Observers show a decided preference for the visual method.

Data for the various curves were obtained by the following graduate students in Communication Engineering, Yale University: Lt. Perry Wainer, W. H. Schlasman, and Captain A. M. Gurney.

Mechanical Forces between Electric Currents
and Saturated Magnetic Fields

BY VLADIMIR KARAPETOFF[1]
Fellow, A. I. E. E.

Synopsis.*—The general case considered is that of N independent electric circuits placed in a medium of variable permeability and subject to saturation, in parts or as a whole. The problem is to determine the component (in a given direction) of the mechanical force acting upon one of the electric circuits, upon a group of circuits, or upon a group of circuits with part of the magnetic medium rigidly attached to them. It is believed that the problem has not been solved in this general form heretofore.*

Use is made of the expression for the stored electromagnetic energy, W, of the system, assuming all the electric circuits to be originally open and then closed one by one. Such a treatment necessitates a number of partial saturation curves, giving the linkages with each individual electric circuit when some of the remaining circuits are closed and the rest are open. A virtual displacement, δs, is then given to the part of the system under consideration, keeping either the linkages or the currents constant, and the mechanical force, F, is determined from a comparison of the work done, $F.\ \delta s$, with the change in the stored energy, δW.

It is shown that the familiar reciprocal relationship for the mutual inductance, $M_{12} = M_{21}$, which holds true in a medium without saturation, can be generalized to a more involved integral expression for a saturated medium.

In order to connect the general treatment with the simpler cases previously solved in the literature of the subject, some intermediate cases of one and two circuits are considered, especially those of importance in applications. The substance of the general method used was presented before the American Physical Society, at the Philadelphia Meeting, in December, 1926.

INTRODUCTION

CONSIDER a system of stationary linear electric circuits in each of which a steady direct current is maintained by a suitable source of energy. Let these circuits be sufficiently close to each other to influence each other's magnetic fields. For the sake of generality, assume the medium to be of variable permeability; that is, let the permeability at a point be a scalar function of the position of the point. Moreover, let the medium be subject to saturation; that is, let the permeability be a function of the resultant flux density at that point.

Generally speaking, the system can be maintained in its given position only by some external mechanical forces or constraints, preventing the individual circuits from moving into a more stable position in the direction of maximum stored energy. The problem is to find the values of these mechanical forces for any individual circuit or part of the system. Since each circuit may require a force and a couple to hold it stationary, the problem may be formulated thus: To find the magnitude of the projection of the force (or of the turning couple) with which a given circuit tends to move along (or to rotate about) a given axis.

In its most general form, with variable permeability and saturation, the solution of the problem leads to quite complicated equations. Moreover, many less general cases are of greater importance in actual applications, and a physical interpretation of more general cases is facilitated by a previous study of simpler combinations. For these reasons, the treatment in this article follows the order "from specific to general," even though a treatment in the opposite order might have been somewhat shorter. The notation and the units are those used in the writer's "Magnetic Circuit," (McGraw-Hill Book Co.), and the references are to the pages of that book, unless stated otherwise.

IA. A SINGLE CIRCUIT IN A MEDIUM OF CONSTANT PERMEABILITY

Consider the coil shown on page 178. The electromagnetic energy stored in it is expressed by equation (**102a**), and from this expression, together with the definition

$$W = 0.5\, L\, i^2 \tag{1}$$

follow equations (**105**) and (**106**) for the coefficient of self-inductance. The same expressions hold true, in a medium of constant permeability, for an electric circuit of any form, not necessarily a coil.

Let a circuit, Fig. 1, be of such a shape that its movable part, *C*, tends to approach the stationary part, *H*, with a force, *F*, unless held in place by an external

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[1] Professor of Electrical Engineering, Cornell University, Ithaca, N. Y.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

force, $-F$. It is shown on page 251, equation (**182**), that

$$F = 0.5\, I^2\, \delta L / \delta s \qquad (\mathbf{2})$$

where s is the distance from an arbitrarily chosen fixed origin Q to some point D on the part C of the circuit. In other words, s is a coordinate which determines the position of C, and s increases when the two parts of the circuit come closer together[2].

If left to itself and not stopped by H, the part C of the circuit will finally come into a position in which $F = O$, and since in this position there is no force tending to move C any farther, the motion will stop. The condition $F = O$ means $\delta L / \delta s = O$, or $L = \max.$ Thus, *a circuit tends to assume a position of maximum inductance, or, with a constant current, that of maximum stored energy.*

With a non-linear deformable conductor, such as a bath of mercury or of molten metal, the mechanical forces tend to modify the shape of the body of the liquid for a maximum of stored electromagnetic energy, consistent with the shape of the container. The tendency is to increase the length and to contract the cross-section[3]. In some cases this tendency for maximum flux linkages results in a continuous motion, as in a homopolar motor. With liquid conductors, such a motion has been utilized for automatically stirring molten metal in an electric furnace[4]. The writer fully recognizes the value of the late Dr. Hering's ingenious experiments and of his work on electric furnaces; he only does not see the necessity for a new "law" to interpret the observed phenomena. As has been pointed out by several persons in the discussion of the above paper, the relationship expressed by eq. (**2**) is sufficient to interpret the observed forces and motions.

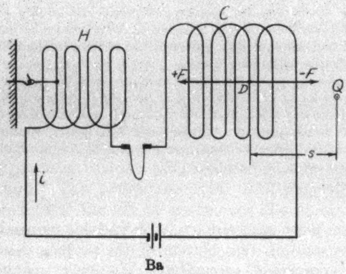

FIG. 1

IB. A SINGLE CIRCUIT IN A MEDIUM WHOSE PERMEABILITY VARIES FROM POINT TO POINT, BUT IS INDEPENDENT OF THE FLUX DENSITY

In such a medium, magnetic lines of force obey the law of refraction (page 119). With the same exciting circuit, the magnetic field has a shape different from that in a medium of constant permeability. Nevertheless, all the flux densities are proportional to the exciting m. m. fs., and the field retains its general shape as the m. m. f. is increased. Consequently, eqs. (**1**) and (**2**) still hold true, but values of L have to be determined for the actual elementary permeances as they enter in eq. (**105**) or (**106**).

As a simple example, consider the lifting magnet shown on page 243. Let the reluctances of all the iron parts be constant and let their sum be denoted by $\mathcal{R}_1$. Then, if the length of the air-gap is a, the total reluctance of the magnetic circuit is

$$\mathcal{R} = \mathcal{P}^{-1} = \mathcal{R}_1 + (a/\mu)\,(S_1^{-1} + S_2^{-1}) \qquad (\mathbf{3})$$

and

$$\begin{aligned} \delta \mathcal{P} / \delta s &= -\,\delta \mathcal{P} / \delta a = -\,\delta \mathcal{R}^{-1} / \delta a \\ &= \mathcal{R}^{-2}\,(S_1^{-1} + S_2^{-1}) / \mu \qquad (\mathbf{4}) \end{aligned}$$

Hence, disregarding the change in the partial linkages, eq. (**2**) gives

$$F = 0.5\, I^2\, n^2\, \mathcal{R}^{-2}\,(S_1^{-1} + S_2^{-1}) / \mu \qquad (\mathbf{5})$$

But $I\, n\, \mathcal{R}^{-1}$ is the total flux of complete linkages and $I\, n\, \mathcal{R}^{-1}\, S_1^{-1}$ is the corresponding flux density in the inner air-gap. Hence, eq. (**5**) may also be written in the form

$$F = 0.5\, B_1^2\, S_1 / \mu + 0.5\, B_2^2\, S_2 / \mu \qquad (\mathbf{6})$$

which agrees with eq. (**169**), and is the usual formula for the supporting force of a lifting magnet.

IC. A SINGLE CIRCUIT IN A COMPOSITE MEDIUM, PART OF WHICH IS SUBJECT TO SATURATION

Let now a coil or an electric circuit of any kind be placed in a position where the magnetic lines of force are partly in the air, partly in iron, the latter being somewhat saturated at the assumed values of electric current. To determine the mechanical force F between some two parts of the magnetic circuit, we shall assign to these parts a virtual relative displacement δs in the desired direction s, and let this displacement occur at a constant current.

If δs has a component in the direction of the longitudinal tension along the lines of force, the flux and the stored energy will be greater in the final than in the initial position. We then have for the final position:

$$e\, I\, \delta t = F\, \delta s + \delta W \qquad (\mathbf{7})$$

where $-e$ is the voltage induced in the circuit during the displacement, and $+e$ is the voltage applied from an external source to keep the current constant; δt is the interval of time during which the displacement occurs, and δW is the increase in the stored electromagnetic energy.

In Fig. 2, let $O\,A$ be the saturation curve of the circuit in the initial position and $O\,A'$ that in the final position. The current, i, is plotted as abscissas and the corresponding flux linkages, ϕ, as ordinates. By ϕ is meant the sum of the fluxes linking with the individual turns. For example, if the circuit consists of three turns, and the actual fluxes linking with these turns are 3, 2.5, and 2.7 kilolines respectively, then $\phi = 8.2$ kilolines. The actual value of the current at which the force F is to be determined is I and the corresponding sum of the linkages is Φ.

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[2] In this investigation, the variation sign δ is used to indicate a change with respect to s. The differential sign d is reserved to indicate a change with respect to the current.

[3] For some ingenious experiments illustrating spontaneous adaptation of various circuits to a shape of maximum energy storage, see Carl Hering, TRANS. A. I. E. E., 1923, Vol. 42, p. 321.

[4] *Ibid.*, Bibliography on p. 326.

Since the magnetic flux is a function of both the current i and the position s, it is necessary to distinguish between two increments of ϕ, which in Fig. 2 are denoted by $d\,\phi$ and $\delta\,\phi$ respectively. The increment $d\,\phi$ is the increase in the linkages, *at a constant s*, when the current increases from i to $i + d\,i$. The increment $\delta\,\phi$ is the increase in the linkages, *at a constant current i*, when s is changed to $s + \delta\,s$. If $d\,\phi$ be called the *differential* of ϕ, then $\delta\,\phi$ is the *variation* of ϕ. In equation (**7**)

$$e = \delta\,\Phi/\delta\,t \tag{8}$$

the symbol δ signifying that the current I is kept constant while s varies.

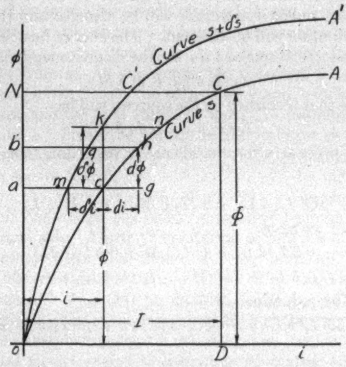

FIG. 2

To find an expression for the stored magnetic energy, it is necessary to compute the electric energy put into the circuit to establish the current I in the position s. During the process of building up the current, let the linkages increase by $d\,\phi$ when the current increases from i to $i + d\,i$. This is shown by the lengths $c\,g$ and $g\,h$ in Fig. 2. The induced voltage is $-d\,\phi/d\,t$ and the electric energy supplied by the external source is

$$i\,(d\,\phi/d\,t)\,d\,t = i\,d\,\phi \tag{9}$$

This energy is represented in Fig. 2 by the strip $a\,b\,h\,c$. The total stored energy in the position s is

$$W = \int_0^I i\,d\,\phi \tag{10}$$

and is proportional to the area $C\,N\,O\,c\,C$. The energy in the position $s + \delta\,s$, at the same current I, differs from W by the amount

$$\delta\,W = \int_0^I i\,\delta\,d\,\phi \tag{11}$$

which is obtained from eq. (**10**) by partial differentiation with respect to s.

It is permissible in this case to put the variation symbol δ under the integral sign, since the limits of integration are independent of ϕ or i.

Substituting now the expressions (**8**) and (**11**) in eq. (**7**), we get

$$I\,\delta\,\phi = F\,\delta\,s + \int_0^I i\,\delta\,d\,\phi \tag{12}$$

or, solving for F,

$$F = I\,(\delta\,\phi/\delta\,s) - \int_0^I i\,d\,(\delta\,\phi/\delta\,s) \tag{13}$$

In this expression, the symbols δ and d are interchanged under the integral sign, in accordance with the fundamental theorem of the Calculus of Variations; namely, that the variation of a differential is identically equal to the differential of the variation[5]. Using on the right hand side of eq. (**13**) the familiar transformation

$$\int x\,d\,y = x\,y - \int y\,d\,x \tag{13a}$$

we finally obtain

$$F = \int_0^I (\delta\,\phi/\delta\,s)\,d\,i \tag{14}$$

Equation (**14**) can also be proved by assuming the virtual displacement to occur at constant linkages, that is, under the condition

$$\delta\,\phi = 0 \tag{14a}$$

With this assumption there is no interchange of energy between the electric source and the magnetic circuit, so that the mechanical work is done at the expense of the stored magnetic energy. We therefore have

$$F\,\delta\,s + \delta\,W = 0$$

or

$$F\,\delta\,s + \int_0^I i\,d\,\delta\,\phi = 0 \tag{14b}$$

Integrating in parts, this becomes

$$F\,\delta\,s + I\,\delta\,\phi - \int_0^I \delta\,\phi\,d\,i = 0 \tag{14c}$$

According to eq. (**14a**), the middle term is equal to zero. Solving for F, eq. (**14**) is obtained[6].

The area of the strip $C'\,O\,C$ may be thought of as consisting of elementary parallelograms, such as $m\,q\,h\,c$. The area of this parallelogram is $-\,\delta\,i\,.\,d\,\phi$, the minus sign being necessary because $\delta\,i$ is a negative quantity. Hence, according to formula (**44**) in the Appendix, we have

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[5] In this particular case, the truth of this statement may be seen directly from the fact that $d^2\phi/di\,ds = d^2\phi/ds\,di$, the value of the second derivative being independent of the order of differentiation.

[6] See Appendix.

$$F = -\int_0^{\Phi} (\delta\, i/\delta\, s)\, d\,\phi \qquad \textbf{(14d)}$$

This is a useful modification of formula (**14**).

If ϕ is given as a function of both i and s, all the operations on the right hand side of eq. (**14**) may be performed, at least approximately, and the value of F determined. When the force is a simple tension in a small air-gap, as in Fig. 58, it is not necessary to use eq. (**14**). The force can be computed according to eq. (**169**) even though the core is saturated. Some inaccuracy is introduced, however, in those lines of force which are not normal to the faces of the air-gap.

When saturation is negligible, eq. (**14**) becomes identical with eq. (**2**) because in this case

$$\phi = i\, L \qquad \textbf{(15)}$$

so that

$$\delta\,\phi/\delta\, s = i\,\delta\, L/\delta\, s \qquad \textbf{(16)}$$

Thus eq. (**14**) becomes

$$F = \int_0^{I} (\delta\, L/\delta\, s)\, i\, d\, i\,.$$

Without saturation, L is independent of i, so that $\delta\, L/\delta\, s$ may be taken outside the integral sign. The result is

$$F = 0.5\, I^2\, (\delta\, L/\delta\, s) \qquad \textbf{(17)}$$

which is identical with eq. (**2**).

IIa. Two Circuits in a Composite Medium with Constant Permeabilities

Let there be two separate circuits, 1 and 2, each supplied with a constant direct current from a separate source. Let the medium be partly air and partly iron, the permeability of the latter being assumed to be constant for all flux densities, from zero to the highest for which the mechanical force is to be computed.

The first step is to determine the magnetic energy stored in the system. Let the circuit 2 be in place, excited with the final value of its current, I_2, and let the circuit 1 be initially open. When the switch of the circuit 1 is closed, let the voltage of the source 2 be so regulated that the current I_2 remains constant while the current in the circuit 1 rises from zero to its final value I_1. Let M_{12} be the sum of the flux linkages in circuit 2, due to one ampere in circuit 1. Then the instantaneous voltage induced in the circuit 2, when the current in circuit 1 increases from some value i_1 to $i_1 + d\, i_1$, is $- M_{12}\, d\, i_1/d\, t$. Hence, the total energy furnished by the source 2 to keep the current I_2 constant, is

$$W_{12} = I_2 \int_0^{I} M_{12}\, (d\, i_1/d\, t)\, d\, t = M_{12}\, I_1\, I_2 \qquad \textbf{(18)}$$

Therefore, the total stored energy of the system is

$$W = 0.5\, I_1^2\, L_1 + M\, I_1\, I_2 + 0.5\, I_2^2\, L_2 \qquad \textbf{(19)}$$

In this expression, the subscript of M has been omitted because $M_{12} = M_{21}$; that is, the coefficient of mutual inductance (or the sum of mutual magnetic linkages per unit current) is the same for both circuits. This follows from the symmetrical form of eq. (**19**). Had we started with the current I_1 at its full value and increased I_2 from zero to its final value, we would have obtained an identical formula, with M_{21} in place of M_{12}. But, without hysteresis, the phenomenon is reversible, and the total stored energy cannot depend on the manner in which the system has been established, so that W is the same in both cases; consequently $M_{12} = M_{21}$, and can be simply denoted by M.

Let now a part of the system be given a virtual displacement $\delta\, s$, while the rest of the system is kept stationary. Both currents are to be kept constant during the motion. The displaced part may include one of the coils and some iron, or only one of these. If a coil and an iron part are moved, they are supposed to be mechanically joined together, and the force F is that necessary to move the combination. Generally speaking, such a displacement will modify the values of L_1, L_2, and M, since the position of the coils with respect to the iron masses will be changed and the flux distribution will be different. The energy furnished by the electrical source 1 during the displacement will be

$$I_1\, (I_1\,\delta\, L_1 + I_2\,\delta\, M),$$

while that furnished by the source 2 will be

$$I_2\, (I_2\,\delta\, L_2 + I_1\,\delta\, M).$$

The increase in the stored energy, according to eq. (**19**), is

$$0.5\, I_1^2\,\delta\, L_1 + I_1\, I_2\,\delta\, M + 0.5\, I_2^2\,\delta\, L_2\,.$$

Hence, we have

$$F\,\delta\, s + (0.5\, I_1^2\,\delta\, L_1 + I_1\, I_2\,\delta\, M + 0.5\, I_2^2\,\delta\, L_2)$$
$$= I_1\, (I_1\,\delta\, L_1 + I_2\,\delta\, M) + I_2\, (I_2\,\delta\, L_2 + I_1\,\delta\, M) \qquad \textbf{(20)}$$

or, after reduction,

$$F = 0.5\, I_1^2\,\delta\, L_1/\delta\, s + I_1\, I_2\,\delta\, M/\delta\, s + 0.5\, I_2^2\,\delta\, L_2/\delta\, s \qquad \textbf{(21)}$$

This is a generalized form of Kelvin's law, to the effect that (with constant currents and constant permeabilities) the energy supplied by the electric sources during a displacement is divided into two equal parts. One half is converted into mechanical work; the other half increases the stored magnetic energy of the system. Conversely, if mechanical work is done on the system, pulling it apart, the energy returned to the sources is equal to twice the mechanical work done, the other half coming from the reduction in the stored magnetic energy.

When all media are of the same permeability, a relative motion of the coils does not alter their self-inductances, and the preceding equation is simplified to

$$F = I_1\, I_2\,\delta\, M/\delta\, s \qquad \textbf{(22)}$$

which is the one usually given in treatises on physics.[7]

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[7] See, for example, J. C. Maxwell, *Electricity and Magnetism*, Vol. II, p. 151; J. H. Jeans, *Electricity and Magnetism*, edition of 1920, p. 495; Alex. Russell, *Alternating Currents*, Vol. I, p. 40.

If the coil 1 is displaced without moving the iron parts, the last term in eq. (**21**) is equal to zero. The same is true if an iron part is so moved as to leave the value of L_2 unchanged.

IIB. TWO CIRCUITS; PART OF THE MEDIUM SUBJECT TO SATURATION

This is an extension of the case treated under Ic above. The equations of stored energy, corresponding to eq. (**19**), are

$$W = \int_0^{I_1} i_1\, d\, \phi_{12} + I_2 (\Phi_{21} - \Phi_{20}) + \int_0^{I_2} i_2\, d\, \phi_{20} \qquad \textbf{(23)}$$

$$W = \int_0^{I_2} i_2\, d\, \phi_{21} + I_1 (\Phi_{12} - \Phi_{10}) + \int_0^{I_1} i_1\, d\, \phi_{10} \qquad \textbf{(24)}$$

In eq. (**23**), it is assumed that in the circuit 2 the current is first brought up to its full value, I_2, while the circuit 1 is open. The last term on the right-hand side expresses the energy stored in the circuit 2 under these conditions, this term being identical with eq. (**10**). The subscript 20 (read two-o) means "linkages of the circuit 2 when the current in circuit 1 is zero." When the current in the circuit 1 is increased from zero to its final value, I_1, the flux linkages of the circuit 2 are changed from Φ_{20} to Φ_{21}, where the subscript 2 again indicates circuit 2, and the subscripts 0 and 1 indicate the initial and the final values (0 and I_1) of the current in circuit 1. Since the change in the linkages occurs at a constant current I_2, the amount of energy furnished by the source 2 is $I_2 (\Phi_{21} - \Phi_{20})$. This is the middle term on the right-hand side of eq. (**23**). The energy stored in the circuit 1 is given by the first term on the right-hand side. During the interval of time when the flux linking with this circuit is being built up, the current I_2 remains constant, as indicated by the double subscript 12. Eq. (**24**) is obtained from eq. (**23**) by interchanging the subscripts 1 and 2. On the supposition that the phenomenon is reversible and the value of the total stored energy is the same, no matter which circuit is closed first, the left-hand sides of both equations are denoted by the same symbol W.

Equating the right-hand sides of eqs. (**23**) and (**24**), we obtain the following "reciprocal" relationship:

$$\int_0^{I_1} i_1\, d\, \Delta\, \phi_1 + I_2\, \Delta\, \Phi_2 = \int_0^{I_2} i_2\, d\, \Delta\, \phi_2 + I_1\, \Delta\, \Phi_1 \qquad \textbf{(25)}$$

where

$$\Delta\, \phi_1 = \phi_{12} - \phi_{10} \qquad \textbf{(26a)}$$

and

$$\Delta\, \phi_2 = \phi_{21} - \phi_{20} \qquad \textbf{(26b)}$$

The symbol Δ in application to ϕ_1 stands for the increase in the linkages of circuit 1 due to an increase in the current i_2 from 0 to I_2, the current i_1 remaining constant. In application to ϕ_2 the symbol Δ stands for the increase in the linkages of circuit 2 at a constant current i_2, when i_1 is changed from 0 to I_1.

Applying the transformation (**13a**) to the two integrals in eq. (**25**), we obtain after reduction

$$\int_0^{I_1} \Delta\, \phi_1\, d\, i_1 = \int_0^{I_2} \Delta\, \phi_2\, d\, i_2 \qquad \textbf{(27)}$$

Eq. (**27**) is a generalized form of the relationship $M_{12} = M_{21}$, when saturation is to be taken into consideration. Four saturation curves must be thought of in connection with eqs. (**25**) and (**27**), namely,

between i_1 and ϕ_1 when $i_2 = 0$
between i_1 and ϕ_1 when $i_2 = I_2$
between i_2 and ϕ_2 when $i_1 = 0$
between i_2 and ϕ_2 when $i_1 = I_1$

Eq. (**25**) or (**27**) gives a necessary physical condition which these four curves satisfy.

Without saturation, $\Delta\, \phi_1 = M_{12}\, I_2$, and $\Delta\, \phi_2 = M_{21}\, I_1$. Eq. (**27**) simply becomes: $M_{12}\, I_2\, I_1 = M_{21}\, I_1\, I_2$, or $M_{21} = M_{12}$.

Using the value of $\delta\, W$ from eq. (**23**), the condition expressed by eqs. (**7**) and (**20**) can now be generalized as follows:

$$F\, \delta\, s + \int_0^{I_1} i_1\, \delta\, d\, \phi_{12} + I_2\, (\delta\, \Phi_{21} - \delta \Phi_{20}) + \int_0^{I_2} i_2\, \delta\, d\, \phi_{20} = I_1\, \delta\, \Phi_{12} + I_2\, \delta\, \Phi_{21} \qquad \textbf{(28)}$$

Cancelling $I_2\, \delta\, \Phi_{21}$ on both sides of this equation, and using the transformation (**13a**), we get

$$F = \int_0^{I_1} (\delta\, \phi_{12}/\delta\, s)\, d\, i_1 + \int_0^{I_2} (\delta\, \phi_{20}/\delta\, s)\, d\, i_2 \qquad \textbf{(29)}$$

In eq. (**28**), expression (**23**) for W is used. If eq. (**24**) be used instead, the subscripts 1 and 2 in eq. (**29**) would become interchanged, and we should get

$$F = \int_0^{I_2} (\delta\, \phi_{21}/\delta\, s)\, d\, i_2 + \int_0^{I_1} (\delta\, \phi_{10}/\delta\, s)\, d\, i_1 \qquad \textbf{(30)}$$

Eqs. (**29**) and (**30**) are the general expressions for the mechanical force between two electric circuits in a medium subject to saturation. Since eq. (**30**) may be obtained by combining eqs. (**29**) and (**27**), expressions (**29**) and (**30**) are identical.

As a special case, and as a check on these formulas, consider the condition of no saturation. Then

$$\phi_{12} = L_1\, i_1 + M\, I_2 \qquad \textbf{(31)}$$

and

$$\phi_{20} = i_2\, L_2\, . \qquad \textbf{(32)}$$

Hence,

$$\delta\, \phi_{12}/\delta\, s = i_1\, \delta\, L_1/\delta\, s + I_2\, \delta\, M/\delta\, s \qquad \textbf{(33)}$$

$$\delta\, \phi_{20}/\delta\, s = i_2\, \delta\, L_2/\delta\, s \qquad \textbf{(34)}$$

Substituting these values in eq. (**29**), and remembering that L_1, L_2, M, are independent of i_1 and i_2, we get

$$F = (\delta \cdot L_1/\delta\, s) \int_0^{I_1} i_1\, d\, i_1 + I_2\, (\delta\, M/\delta\, s) \int_0^{I_1} d\, i_1 + (\delta\, L_2/\delta\, s) \int_0^{I_2} i_2\, d\, i_2 \qquad \textbf{(35)}$$

After integration, this expression becomes identical with eq. (**21**).

In order to perform the operations indicated in eq. (**29**), ϕ_{12} and ϕ_{20} must be given as functions of the distance s and of the currents i_1 and i_2. Such saturation curves, for different values of s, can either be estimated by computation or obtained from test. The value of F can then be determined graphically to a desired degree of accuracy. Or else, ϕ_{12} and ϕ_{20} may be expressed as empirical functions of s, i_1, and i_2, and the integrations performed analytically.

IIIA. N SEPARATE CIRCUITS IN A COMPOSITE MEDIUM WITH CONSTANT PERMEABILITIES

This is a generalization of the Case IIA. The first step is to compute the total stored electromagnetic energy. Let the circuit 1 be closed first, then the circuit 2, etc. With two circuits, the total stored energy is expressed by eq. (**19**). When the circuit 3 is closed, its own stored energy, $0.5\, I_3^2\, L_3$, is added, and more energy must be furnished by the sources 1 and 2 in order to keep the corresponding currents constant during the transient period. These latter amounts of energy are equal to $I_1\, I_3\, M_{13}$ and $I_2\, I_3\, M_{23}$ respectively. Thus, the total stored energy of the three circuits is

$$W = 0.5\, I_1^2\, L_1 + 0.5\, I_2^2\, L_2 + 0.5\, I_3^2\, L_3 + M_{12}\, I_1\, I_2 + M_{23}\, I_2\, I_3 + M_{31}\, I_3\, I_1 \qquad (\mathbf{35a})$$

Extending the same process to N circuits, we may write

$$W = 0.5\, \Sigma\, I_k^2\, L_k + \Sigma\, I_k\, I_u\, M_{ku} \qquad (\mathbf{36})$$

In this expression, k and u have all the integer values from 1 to N inclusive, and in the second summation the subscripts correspond to *combinations*, and not to permutations. This means that if, for example, the values of $k = 2$ and $u = 5$ have been used in a term, the values $k = 5$ and $u = 2$ cannot be used any more.

Let now some of the circuits be combined into a subsystem, and be given a common virtual displacement, δs, with respect to the remaining circuits, all the currents being kept constant. For the sake of generality, assume that this displacement causes a change not only in the coefficients of mutual inductance, but in those of self-inductance as well. The energy furnished by the electric source in the kth circuit is

$$\delta W_k = I_k\, [I_k\, \delta L_k + \Sigma\, I_v\, \delta M_{kv}] \qquad (\mathbf{37})$$

Here v denotes any circuit except the kth; that is, v has all the integer values from 1 to $k - 1$ and from $k + 1$ to N. Eq. (**20**) becomes

$$F\, \delta s + 0.5\, \Sigma\, I_k^2\, \delta L_k + \Sigma\, I_k\, I_u\, \delta M_{ku} = \Sigma\, I_k^2\, \delta L_k + \Sigma\, \{I_k\, \Sigma\, I_v\, \delta M_{kv}\} \qquad (\mathbf{38})$$

The last summation on the right-hand side of this equation contains the same terms as the last summation on the left-hand side, only each term enters twice, because each source of voltage is here considered separately. Thus, we find that Kelvin's law holds true also in this case, and by analogy with eq. (**21**) we may write

$$F = 0.5\, \Sigma\, I_k^2\, \delta L_k / \delta s + \Sigma\, I_k\, I_u\, \delta M_{ku} / \delta s \qquad (\mathbf{39})$$

Depending on the particular circuit or circuits for which the force F is sought, the derivatives $\delta L / \delta s$ and $\delta M / \delta s$ have different values. Thus, it may be required to determine the mechanical force acting on a winding alone, or on a winding with the corresponding iron core, etc. In each case a virtual displacement must be assumed to take place for the part or parts under consideration, with respect to the rest of the system.

IIIB. N CIRCUITS; PART OF THE MEDIUM SUBJECT TO SATURATION

This is an extension of the case treated under IIB. The method is the same, only the subscripts become more numerous and involved. For this reason, it has been deemed sufficient to show the deduction of the final formula in application to three circuits only, since with N circuits each flux ϕ would have a subscript consisting of N numbers of different order. An extension of the reasoning given below to four or more circuits is quite evident, and the final formula for N circuits is written directly.

To compute the stored energy, we shall assume that the circuit 1 is closed first, then circuit 2, and finally circuit 3. By analogy with eq. (**24**), changing somewhat the order of the terms, we may write the following expression for the total stored energy corresponding to the final values of the currents:

$$W = \int_0^{I_1} i_1\, d\, \phi_{100} + \int_0^{I_2} i_2\, d\, \phi_{210} + \int_0^{I_3} i_3\, d\, \phi_{312} + I_1\, (\Phi_{123} - \Phi_{100}) + I_2\, (\Phi_{213} - \Phi_{210}) \qquad (\mathbf{40})$$

In this expression, the first integral represents the energy stored in circuit 1 when $i_2 = i_3 = O$. The subscript 100 (read one-o-o) means "flux linkages in circuit 1, when the currents in the other two circuits are equal to zero." The second integral represents the energy stored in circuit 2 when the current in the circuit 1 has already reached its full value, while that in circuit 3 is still equal to zero. This is indicated by the subscript 210. The third integral represents the energy stored in circuit 3, and the subscript 312 indicates that the currents i_1 and i_2 have reached their maximum values.

The term I_1 $(\Phi_{123} - \Phi_{100})$ represents the energy furnished by the source 1 in order to keep the current I_1 constant when the currents in the other two circuits are being increased from zero to their final values. Similarly, the last term gives the energy furnished by the source 2 when the circuit 3 is closed.

Let now a virtual displacement, δs, be allowed to occur in one part of the system with respect to the other, keeping all the currents constant. The energy furnished by the three sources is equal to

$$I_1\, \delta\, \Phi_{123} + I_2\, \delta\, \Phi_{213} + I_3\, \delta\, \Phi_{312}\, .$$

Writing an equation similar to eq. (**28**), and using the transformation (**13a**), we get, by analogy with eq. (**30**) with a reversed order of terms:

$$F = \int_0^{I_1} (\delta\, \phi_{100} / \delta s)\, d\, i_1 + \int_0^{I_2} (\delta\, \phi_{210} / \delta s)\, d\, i_2$$

I_3

$$+ \int_0 (\delta \phi_{312} / \delta s)\, d\, i_3 \qquad (41)$$

In order to determine F from this equation, the flux linkages ϕ_{100}, ϕ_{210}, and ϕ_{312} must be given as functions of s and of the corresponding currents. In practical cases, advantage may be taken of certain simplifications due to the arrangement of the circuits or to the particular shape of the saturation curves. Since the total stored energy is independent of the order in which the circuits are closed, certain "reciprocal" relationships must hold true. These may be deduced by analogy with eqs. (**25**) and (**27**).

Extending now the formula (**41**) to N circuits, we get

$$F = \Sigma \int_0^{I_k} (\delta \phi_q / \delta s)\, d\, i_k \qquad (42)$$

This expression consists of a sum of N integrals corresponding to the values of k from 1 to N. The subscript q is as follows:

$$q = k\,(k - 1) \;.\;.\;.\;.\; 21000 \;.\;.\;.\;. \qquad (43)$$

This means that for the kth circuit the saturation curve between i_k and ϕ_k, used in eq. (**42**), must be the one which obtains with the currents I_k, I_{k-1}, I_{k-2}, ... I_2, I_1 at their full values, while the currents i_{k+1}, i_{k+2}, ... , i_n, are all equal to zero.

Literature References. Comparatively little has been done on the general theory of mechanical forces in magnetic circuits, especially taking saturation into account. Some recent articles, of applied nature, are listed below. References to earlier contributions will be found in these articles.

Doherty and Park, *Mechanical Force between Electric Circuits*; A. I. E. E., TRANS., 1926, Vol. 45, p. 240.

Lehmann, *The Calculation of Magnetic Attraction*; Ibid., p. 383.

Hague, "Forces Acting on Conductors Near Iron;" *World Power*, 1926, Vol. 5, pp. 124 and 205.

Hak, "Calculation of Mechanical Stresses in Reactance Coils;" *Elek. u. Masch.*, 1924, Vol. 42, p. 17.

Liénard, *Revue Gén. de l'Elec.*, 1923, Vol. 14. p. 563.

Appendix

It is shown in connection with eq. (**10**) that the magnetic energy stored in a saturated circuit (Fig. 2) is represented by the area $C\,N\,O\,c\,C$. Similarly, the stored magnetic energy, after the displacement, $\delta\, s$, has taken place, is proportional to the area $C'\,N\,O\,m\,C'$. When this displacement occurs at constant linkages Φ, the mechanical work is done entirely at the expense of the stored magnetic energy. Consequently, the curved infinitesimal strip $C'\,O\,C$ represents the work done, $F\,\delta\,s$, so that

$$F = (\text{area of strip } C'\,O\,C)/\delta\, s \qquad (44)$$

This expression permits the visualization of the relations and also the solution of some special cases. Consider, for example, a saturated electromagnet with a small air-gap (a lifting magnet). Within a certain range of small values of air-gap, the lines of force in the gap may be assumed to be straight lines, normal to the iron surfaces, and the flux in the iron parts may be considered to follow the same paths and to have the same leakage, independent of the magnitude of the gap. In other words, within a certain range of gaps, the same saturation curve may be used for the iron, and only the exciting ampere-turns for the air-gap changed. With this limitation, the area of the strip $C'\,O\,C$ may be obtained from the air characteristic alone. For the air-gap we have

$$I\,T = (\Phi/T)\,(s_o - s)/(\mu\, A) \qquad (45)$$

where $I\,T$ are the exciting ampere-turns; s_o and s are some distances whose difference gives the length of the air-gap; A is the cross-section of the magnetic path in the air-gap, and μ the absolute permeability of the air. T being the number of turns, the linkages Φ divided by T give the actual flux. From eq. (**45**)

$$T\,\delta\, I = -\,(\Phi/T)\,\delta\, s/(\mu\, A) \qquad (46)$$

With the foregoing assumptions, the strip $C'\,O\,C$ becomes a triangle, so that

$$\text{area } C'\,O\,C = -\,0.5\,\delta\, I\,.\,\Phi\,. \qquad (47)$$

The minus sign is necessary because $\delta\, I$ is a negative quantity. Substituting this expression in eq. (**44**), and using for $\delta\, I$ its value from eq. (**46**), we get

$$F = 0.5\, A\, B^2/\mu \qquad (48)$$

where B is the flux density in the air-gap. Expression (**48**) is the usual formula for the lifting force of an electromagnet.

The same result may be obtained from eq. (**14d**). With the limitations stated above, the saturation curve for the whole electromagnet may be written in the form

$$T\,i = \psi\,(\phi) + (\phi/T)\,(s_o - s)/(\mu\, A) \qquad (49)$$

where the function $\psi\,(\phi)$ is the m. m. f. required for the iron parts. At a constant ϕ,

$$\delta\, i/\delta\, s = -\,(\phi/T^2)/(\mu\, A) \qquad (50)$$

Substituting in eq. (**14d**) and integrating, will give eq. (**48**).

Two Cases of Calculation of Mechanical Forces in Electric Circuits

BY H. B. DWIGHT[1]

Fellow, A. I. E. E.

***Synopsis.**—A formula is derived for the mechanical force in a circle of round wire, due to its own current. A formula,* $F = I^2 \operatorname{logh} \frac{a_1}{a_2}$, *is also derived for the longitudinal force exerted on a round conductor, due to its own current, where it changes its diameter. Where there is a constriction in a liquid conductor, this force acts in both directions away from the constriction, thus tending to accentuate it. It may be that this has more to do with the rupturing of a liquid conductor by heavy current, than the better known forces acting in a radial direction, which have been usually referred to under the name "pinch effect."*

THE measurement by a laboratory method of mechanical force in circular and rectangular circuits, described in a companion paper by Mr. J. W. Roper[2], lends interest to formulas for calculating such forces. In this paper, formulas are presented for the force acting in a circular circuit and also for the axial force acting in a straight cylindrical conductor where the size of the cross-section changes.

FORCE IN A WIRE CIRCLE

The force tending to stretch the wire in a circular circuit is calculated by the well-known method using the differential of the self-inductance of the circuit. The mutual inductance of two coaxial circular filaments is given with a great deal of precision by formulas involving elliptic integrals or by convergent series. Rayleigh and Niven have integrated the expression for this quantity over a circular cross-section, giving the self-inductance of a circle of round wire. Their formula is[3]

$$L = 4\,\pi\,a \left[\operatorname{logh} \frac{8\,a}{\rho} - \frac{7}{4} + \frac{\rho^2}{a^2} \left(\frac{1}{8} \operatorname{logh} \frac{8\,a}{\rho} + \frac{1}{24} \right) \ldots \right] \text{abhenrys} \quad (1)$$

where a is the radius of the circle and ρ is the radius of the wire. The expression logh denotes the hyperbolic or natural logarithm.

When current flows in a circle of wire, one half tends to repel the other half, and a tendency to stretch the wire is exerted at every part of the length of the wire. Let this force be F dynes and let s be the perimeter of the circle. If the current is turned on, and the force F stretches the wire a distance $\partial\,s$, the mechanical work done is $F\,\partial\,s$, since F acts in the direction of s. This can be equated to $\frac{1}{2}\,I^2\,\partial\,L$ where $\partial\,L$ is the change in the self-inductance of the circle due to its change in size.

This rather well-known expression can be derived as follows:

The rate of doing mechanical work is $F \frac{\partial\,s}{\partial\,t}$. Since the current I is kept constant and the inductance L is varying, a voltage is generated in the circle equal to $I \frac{\partial\,L}{\partial\,t}$ and the current I flowing against this voltage supplies energy at the rate $I^2 \frac{\partial\,L}{\partial\,t}$. This energy goes to supply the mechanical work and also to increase the stored energy of the magnetic field. The stored energy is $\frac{1}{2}\,L\,I^2$ for inductance[4] equal to L. The rate of change of the stored energy when I is constant and L varies is $\frac{1}{2}\,I^2 \frac{\partial\,L}{\partial\,t}$.

Therefore,

$$I^2 \frac{\partial\,L}{\partial\,t} = F \frac{\partial\,s}{\partial\,t} + \frac{1}{2}\,I^2 \frac{\partial\,L}{\partial\,t}$$

Then

$$F\,\partial\,s = \frac{1}{2}\,I^2\,\partial\,L$$

and

$$F = \frac{1}{2}\,I^2 \frac{\partial\,L}{\partial\,s} \quad (2)$$

This force is in dynes since absolute units are used throughout.

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[2] J. W. Roper, *Experimental Measurement of Mechanical Forces in Electric Circuits, Jl. A. I. E. E. Sept. 1927, p. 913.*

[3] Equation **63**, Scientific Paper No. 169 of the Bureau of Standards, 1911, and Rayleigh's collected papers, Vol. II, p. 15.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

[4] Principles of Alternating Currents, by R. R. Lawrence, p. 124, equation (**14**).

The dimension s is the circumference of the circle and is equal to $2\pi a$.

Therefore

$$F = \frac{1}{4\pi} I^2 \frac{\partial L}{\partial a} \qquad (3)$$

The differentiation of Rayleigh and Niven's formula (1) is quite straightforward and the following formula for force is obtained:

$$\frac{F}{I^2} = \operatorname{logh} \frac{8a}{\rho} - \frac{3}{4} - \frac{\rho^2}{a^2}\left(\frac{1}{8} \operatorname{logh} \frac{8a}{\rho} - \frac{1}{12}\right) \ldots \quad \text{dynes} \quad (4)$$

where I is the current in abamperes.

Example 1. The force in a wire of radius $\rho = 0.1245$ cm. bent into a circle of $a = 20.1$ cm. radius and carrying 100 amperes is

$$10^2 \left[\operatorname{logh} \frac{8 \times 20.1}{0.1245} - 0.75 - \frac{0.1245^2}{20.1^2}\left(\frac{1}{8} \operatorname{logh} \frac{8 \times 20.1}{0.1245} - \frac{1}{12}\right)\right]$$

$= 641$ dynes. (See the calculated curve of Fig. 2 of Mr. Roper's paper.)

LONGITUDINAL FORCE

The following short formula gives the amount of the longitudinal force; that is, the force acting in the direction of the axis, which is exerted on a round conductor where it changes its diameter:

$$F = I^2 \operatorname{logh} \frac{a_1}{a_2} \quad \text{dynes} \quad (5)$$

where a_1 and a_2 are the larger and smaller radii, respectively, and where I is the current in abamperes.

The force acts parallel to the axis and toward the larger section of conductor, irrespective of the direction of flow of current. The expression involves only the maximum and minimum radii and, as referred to later, the amount of the force is independent of the shape of the conductor while it is changing section, so long as it is round and centered on a straight axis.

Formula (5) is useful in calculating the force on a system of round wires in air where there are wires of more than one diameter. It is also of interest in connection with some kinds of electric furnaces where a trough of molten metal carries a heavy electric current. If there is a constriction in the section of the metal, this force produces a flow of metal away from the constriction and so accentuates the constriction. Such an action has been observed.

As is well known, electromagnetic forces also act on all elements of a liquid conductor at right angles to the direction of current flow; that is, in general, toward the axis. This produces greater hydrostatic pressure at the axis than in the outer parts, and, of course, hydrostatic pressure acts equally in all directions.

The hydrostatic pressure in a uniform round conductor, which acts in a radial direction, has been referred to for a number of years under the name, "pinch effect." It has been described as the cause of the phenomenon observed in electric furnaces, that at a certain heavy current a constriction would occur in the liquid conductor, and that this constriction would grow until the circuit was broken.

It is possible that the axial force described and calculated in this paper, which tends to separate the two tapered parts where a constriction occurs in a liquid conductor, is more directly connected with rupturing the conductor at the point of constriction than the radial "pinch effect" force.

The axial force is stronger in the outer parts of the tapered conductor than in the inner parts. In the case of a liquid conductor, unequal forces acting on different parts produce a flow of the liquid, sometimes in short return paths, like local eddies. The amount of this flow depends on the strength of the electric current and the shape and viscosity of the liquid conductor.

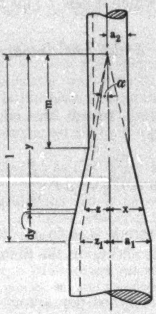

FIG. 1—CONDUCTOR WITH CHANGE IN SECTION

Equation (5) is derived by the method of finding the product of the strength of a current and the magnetic field in which it lies, to give the force on the conductor carrying the current.

Assume that the current density is uniform over any cross-section of the wire shown in Fig. 1. Slight departures from this assumption take place where the stream lines do not have abrupt changes in direction, but follow smooth curves. Skin effect also is not considered. The current density at distance y where the radius is x is

$$i = \frac{I}{\pi x^2}$$

The total current inside radius z is

$$\pi i z^2 = I \frac{z^2}{x^2}$$

The flux density at radius z is

$$\frac{2}{z}\;\frac{I\,z^2}{x^2} = \frac{2\,I}{z}\;\frac{z_1^2}{a_1^2}$$

The dotted line at radius z is always at the proportionate distance $\frac{z_1}{a_1}$ along the radius of the wire, and it is almost exactly a line of current flow. A short element of it has a length $\frac{d\,y}{\cos\alpha}$

where

$$\tan\alpha = \frac{z_1}{l} = \frac{z}{y} \tag{6}$$

A force acts on the short element of the filament carrying current at right angles to its length and proportionate to the current in the filament and the magnetic field in which it lies. This force is

$\frac{2\,I}{z}\;\frac{z_1^2}{a_1^2}\;\frac{d\,y}{\cos\alpha}$ dynes per abampere of current in the filament.

Multiply by $\sin\alpha$ to get the component of force parallel to the axis:

$$\frac{2\,I}{z}\;\frac{z_1^2}{a_1^2}\tan\alpha\;d\,y = 2\,I\,\frac{z_1^2}{a_1^2}\;\frac{d\,y}{y} \text{ from } (\mathbf{6})$$

Integrate this from $y = m$ to $y = l$. The total force parallel to the axis acting on the filament is

$2\,I\,\frac{z_1^2}{a_1^2}\operatorname{logh}\frac{l}{m}$ dynes per abampere of current in the filament. **(7)**

If the filament be considered to have a thickness $d\,z_1$ at the radius z_1 then the total area of all filaments at radius z_1 is

$$2\,\pi\,z_1\,d\,z_1$$

and the total current in them is

$$\frac{I}{\pi\,a_1^2}\;2\,\pi\,z_1\,d\,z_1 = \frac{2\,I}{a_1^2}\;z_1\,d\,z_1$$

The force parallel to the axis acting on the above current is, by (**7**),

$$2\,I\,\frac{z_1^2}{a_1^2}\left(\operatorname{logh}\frac{l}{m}\right)\frac{2\,I}{a_1^2}\;z_1\,d\,z_1$$

Integrate this from $z_1 = 0$ to $z_1 = a_1$.

The total force parallel to the axis is

$$I^2\operatorname{logh}\frac{l}{m}$$

$$= I^2\operatorname{logh}\frac{a_1}{a_2} \text{ which is equation } (\mathbf{5}).$$

If the wire tapers from a radius a_1 to a radius a_m, the axial force due to that part will be

$$I^2\operatorname{logh}\frac{a_1}{a_m}$$

If the wire then tapers at a different rate from radius a_m to radius a_2, the force due to that part will be

$$I^2\operatorname{logh}\frac{a_m}{a_2}$$

and the total force will be

$$I^2\operatorname{logh}\frac{a_1}{a_2}$$

This is the same as if the wire had tapered uniformly from radius a_1 to a_2, as in Fig. 1. The change in radius can therefore be made by means of a large number of tapers of different angles, and the total axial force will depend only on the initial and final radii according to eq. (**5**).

In the above calculation, the value of the flux density at a radius z is dependent on the total current I_1 within the circle of radius z. Since the wire is assumed to be straight and very long, and the return conductor so remote as to be negligible, the magnetic field lies in circles around the axis. The magnetomotive force around the circle of radius z is $4\,\pi\,I_1$. The length of the magnetic path is $2\,\pi\,z$ and the flux density at radius z is $\frac{2\,I_1}{z}$. This formula, which is applicable to isolated long, straight, round wires, is seen to be true also when there are changes in section, provided the wire is symmetrical around a straight axis.

In calculating the magnetic field due to a short length of round wire carrying a current, it is often assumed that the field is the same as if all the current were flowing in a small filament at the axis of the round wire. While this is very nearly true, it is exactly true only in the case of an infinitely long wire, and this fact is sometimes of importance in calculations of inductance and of electromagnetic force. The expression for the field at a given point due to a short length of round wire involves elliptic integrals or series equivalent to them.

A case where the current cannot be assumed to be concentrated in a filament at the axis of a conductor is in finding the force on a conductor bent into a quadrant of a circle, for such an assumption makes the calculated force infinitely great. An expression for the force on a quadrant conductor due to its own current must involve the dimensions of the cross-section of the conductor.

The writer desires to make acknowledgment of the assistance of Mr. S. P. Sawyer in preparing numerical examples, etc., in connection with the work of this paper.

Discussion

V. Karapetoff: Professor Dwight's equation (5) may be deduced directly from the general formula (2) for the mechanical force. Let the return conductor be a concentric cylinder of radius b. Let the virtual deformation of the given conductor consist of an axial lengthening of the portion of radius a_2 by an infinitesimal amount $d\,s$ and of a corresponding shortening of the portion of radius a_1 by the same amount $d\,s$. The tapered portion is supposed to move bodily without a change in shape. The inductance of a concentric cable, per unit length, is of the form,

$$L = 0.05 + 0.2 \operatorname{logh} (b/a) \qquad \textbf{(A)}$$

in perms per cm. length[1]

Therefore, the net increase in the inductance is

$$d\,l = 0.2 \,[\operatorname{logh} (b/a_2) - \operatorname{logh} (b/a_1)]\; d\,s \qquad \textbf{(B)}$$

Substituting this expression in Dwight's equation (2), his equation (5) is obtained. Therefore, equation (5) is valid so long as equation (**A**) for inductance is valid. The radius b of the return conductor does not enter into the result.

When considering the relationship between an electric current and its magnetic flux, it is safer to start with the universal relationship, which holds true at any point and is expressed by the familiar differential equation $u = \operatorname{curl} H$, where u is the current density.[1] The difference between a solid and a liquid conductor comes in the boundary conditions in the integration of this equation. For a solid conductor, the boundary surface is fixed and the component of the current density normal to it must be equal to zero. With a liquid conductor, the shape of the surface depends upon the magnitude of the total current and its distribution. Because of the pinch effect, the usual hydrostatic conditions must be satisfied at each point within the liquid.

This means that, for each infinitesimal volume taken within the conductor, the forces acting upon it must be in equilibrium. These forces are the resultant pressure exerted by the remainder of the liquid, the weight of the element, and the force between the current and the magnetic field.

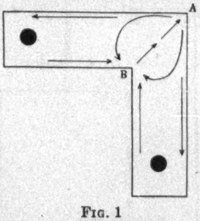

FIG. 1

E. H. Dodge: I have quite recently performed some experiments concerning forces acting on a liquid conductor with currents as high as 1500 amperes.

When a right-angle trough containing mercury has a current of 1000 amperes flowing, there is a movement in the liquid as shown in the accompanying Fig. 1. The direction of motion is from B to A and the level at A is about ¼ in. higher than the remaining liquid.

If a straight trough with a constriction is used, there is a pronounced flow away from the constriction, as shown in the accompanying Fig. 2. There are also eddies at the constriction. There is a swirling motion at the electrodes accompanied by a marked depression. As the current changes direction by 90 deg., the filaments carrying current stretch out and form this depression. Within the constriction, an interesting phenomenon occurs. With 1000 amperes, the mercury level rises at the center and falls at the edges, as if the mercury were trying to crowd together to form a circular cross-section. Just before the circuit was ruptured with 1450 amperes, the level decreased.

The rise in level would seem to be due to the hydrostatic pressure which acts radially, and has been given by Dr. E. F. Northrup as $p = \dfrac{I^2}{100\,\pi\,R^4}\,(R^2 - r^2)$ dynes per sq. cm. at any point r cm. from the axis of a conductor $2\,R$ cm. in diameter.

The decrease in level might be explained by the action of the longitudinal thrust or the taper force which tends to drain the mercury from the constriction and thus decrease the level.

The total thrust on a vertical plane omitting the effect of gravity has been given by Dr. Carl Hering as $T = \dfrac{I^2}{200}$ dynes.

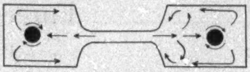

FIG. 2

This is obtained by integrating the above expression given by Dr. Northrup. This expression is independent of the size of the conductor: consequently, the thrust on a vertical plane due to the radial force is the same, regardless of the size of cross-section. With this in mind, it seems very reasonable that the rupturing of the circuit is caused by the taper force which has been given by Dr. Dwight as $F = \dfrac{I^2}{100} \operatorname{logh} \dfrac{R}{r}$ dynes.

A few simple calculations will bring out the differences in pressures. With a conductor carrying 1000 amperes and a cross-section with a radius which varies from 1 cm. to 6 cm., by Northrup's equation, the hydrostatic pressure at the center of the small section = 3180. dynes per sq. cm. which is equivalent to 0.0461 lb. per sq. in. and the pressure at the center of the large section = 89. dynes per sq. cm. which is equivalent to 0.00130 lb. per sq. in.

The total thrust on a vertical section by Hering's equation is $T = 5000$ dynes and equivalent to 0 0112 lb. which is the same for the large section and the small section.

Finally the taper force from Dr. Dwight is

$$F = 17900 \text{ dynes or } 0.0402 \text{ lb.}$$

Thus we see that the taper force is greater and seems to substantiate the assertion that it causes the circuit to rupture. The taper force becomes greater and greater as the small section becomes smaller before the rupture of the liquid conductor.

H. B. Dwight: In my presentation of this paper, I made the statement that equation (5) for a conductor consisting of two long uniform parts of different diameters, joined by a tapered part, can be derived by differentiating the well-known formula for the self-inductance of a long, uniform, round wire. This derivation shows that the two long parts do not need to have the same axis so long as they have parallel axes, and the shape of the tapered part does not matter.

Such a derivation, however, does not apply to the interesting case described by Mr. Dodge, where the two long parts of the conductor have the same diameter and are joined by a double taper, or contraction. Formula (A) of Professor Karapetoff's discussion cannot be applied to the minimum diameter of the contraction, but only to a long uniform conductor. Therefore, in order to show that equation (5) applies to each of the closely adjacent tapers of the contraction, some other derivation, such as the one given in my paper, is necessary.

It has not been shown, and it does not necessarily appear, that any use of the equation u = curl H gives greater safety in calculation than the methods followed in my paper.

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[1] See, for example G. W. Pierce, Electric Oscillations and Electric Waves. p. 370; M. Abraham, *Theorie der Elektrizitat*, 1912, p. 220.

While it has not been uncommon for mercury circuits of small cross-section to be broken by electric currents of the order of 100 amperes, the observation in detail of the progressive steps of the phenomenon and the description of the swirling of the liquid which carries an electric current have only rarely been made since Dr. Carl Hering described them. Such observations require a large section of liquid conductor and from 1000 to 1500 amperes. Accordingly, Mr. Dodge's experiments should be of interest to physicists and to designers of electric furnaces. In determining the swirling motion of the liquid, he used heavy floats which extended deep into the liquid, since surface tension prevented the motion from being shown by sprinkling dust on the mercury. When the circuit was broken, it continuously came together and broke at the rate of several times a second.

I agree with the statement in Professor Karapetoff's paper, that Dr. Hering's work on electric furnaces and liquid conductors was of great value, but that his experimental observations are in agreement with the usual methods of calculating the mechanical forces associated with electric currents, and do not require new methods of calculation.

Experimental Measurement of Mechanical Forces in Electric Circuits

BY J. WALTER ROPER[1]

Associate, A. I. E. E.

***Synopsis.**—This paper presents a simple laboratory method of measuring the mechanical forces exerted on the parts of a complete circuit due to current flowing in the circuit.*

Tests, using the method, show that the "classic" methods of computing such forces are reliable. Curves are included which show the comparison between the theoretical and measured forces. Tests were made on a rectangular circuit, representing a disconnecting switch, and on a circle of round wire.

CIRCULAR CIRCUITS

THE expression derived in the companion paper by Dr. H. B. Dwight for the tension acting in a circular circuit has been checked experimentally by the apparatus shown in Fig. 1 and described below. These measurements were made in the Electrical Engineering Department of the Massachusetts Institute of Technology.

A circle of copper wire 0.249 cm. (0.098 in.) in diameter was made, having a centerline diameter of 40.2 cm. (15.81 in.). The circle was divided into two equal parts the upper half of which was fixed by being fastened to a semicircle of wood. The current was led in and out of the circle by means of flexible leads which were clamped at *a*, about four inches from the terminals *T* of the circle. An elastic rubber band at *e*, one inch from *T*, provided a support for the end of the circle. The other joint of the circle was made at *b* by means of a slanting or tapered joint. A thin mercury film was used to insure good contact between the halves of the circle. The tapered joint was suggested by the late Dr. Carl Hering, and is a useful device. As shown later, a mercury cup gave approximately the same results as a tapered joint.

The lower half of the circle was braced by a wooden rod *d* in order to maintain the dimensions of the circuit when current was flowing and to provide a place to attach the measuring device. The motion of the semicircle was limited to 1/16 in. at the joint by a set of stops at *s*. The upper stop was fitted with a contact which touched a contact on the brace. These contacts were connected by means of very flexible leads with a dry cell and telephone headset. The force was measured by means of a lever arm and scale pan, connected to the brace by a copper rod a little over a yard in length. To offset the weight of the scale pan *B*, the auxiliary weight *W* was fastened to the brace as shown in Fig. 1.

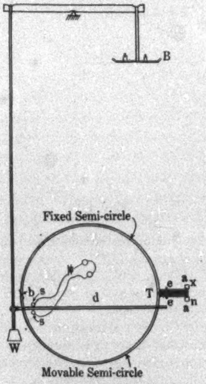

FIG. 1—THE CIRCULAR CIRCUIT

The vertical support above *W* was one inch horizontally from *b*. The moment of the force acting through this support about *e* was balanced by the moments of the electromagnetic forces at *b* and *T* about the same point *e*. Since the distances *W b* and *T e* were each equal to one inch, the force in the vertical support was taken equal to the force at *b*. If the leads *e a* were comparatively stiff so that the semicircle moved about *T* as a hinge, however, then the observed forces should be multiplied by the ratio of the lever arms which would be about 17 to 16. This may partly account for the fact that the observed forces are less than the calculated values in Fig. 2.

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[1] Massachusetts Institute of Technology.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

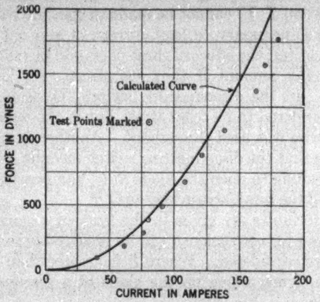

Fig. 2—Curve Showing the Relation Between Force and Current in the Circuit of Fig. 1

The initial position of the lower semicircle was determined by the upper stop which was visually adjusted so that the circuit was circular. In order to have an absolute measurement of the force exerted when current was flowing, it was necessary to determine the zero of the circuit, that is, the weight on the scale pan *B* which just held the contacts of the indicating circuit closed. This was done by placing weights on the scale pan *B* until the removal of a small weight, as carefully as possible, just closed the indicating circuit. If care was taken in placing and removing weights, this determination could be made to within 0.05 gram. The average of several trials gave a good determination of the zero position.

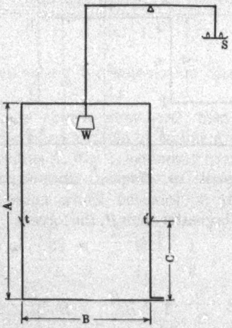

Fig. 3—Diagrammatic Sketch of a Rectangular Circuit

With the zero determined, points were obtained for a curve. The procedure was to add weights by tenths of grams to the scale pan *B*, and for each increment of weight to send current through the circle. As the current was increased the value of current at which the indicating circuit opened was noted, and as the current was reduced, the value at which it closed was also noted. The average of the two readings was taken as the one which eliminated errors due to friction in the joint and in the flexible leads. Several readings were taken at each point and the results averaged to give the current required for a given force. The curve of Fig. 2 shows the variation of force in dynes against averaged currents. The maximum value of current which could be used was 180 amperes, for with this current the mercury film blew out, probably due to heating.

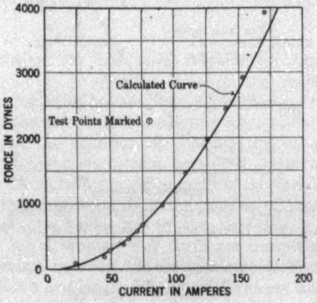

Fig. 4—Curve Showing Relation Between Force and Current in Rectangular Circuit

Length—89.6 cm.
Width—40.0 cm.
Diameter of Wire—0.249 cm.

The test results were compared, Fig. 2, with the curve calculated from equation (4) of the companion paper which for the dimensions used reduces to

$$F = 0.0641\, I^2 \text{ dynes} \qquad (1)$$

where I is the current in amperes. This comparison indicates an agreement of approximatly 10 per cent. The so called "pinch" effect of the mercury in the joint cannot be reasonably blamed for the divergence. If this were the case, the test curve should lie to the left of the calculated curve, since less current would be required for a given force if "pinch" effect aided the separation. It does not seem reasonable, either, to ascribe the difference to mercury tension or friction because these forces should remain constant over the range taken, and should give a constant error and consequently a continually decreasing percentage error, instead of a percentage error which is practically constant.

Rectangular Circuits

In connection with the tests concerned with the mechanical forces acting on a conductor due to current flowing in other parts of the same circuit, of which the tests on the circular circuit just described were a part, experiments with rectangular circuits were performed using the same method. A diagrammatic sketch of the set-up is shown in Fig. 3. In this case two separations, at t and t', were made and the force measured in the center of the top cross-piece, thus measuring the total force acting. Several different lengths of rectangle were tested and the results for the longest and the shortest are given. According to theory, it should make no difference where the cuts t and t' are made. In the case of the longest rectangle, 89.6 cm. long, the distance from the bottom of the rectangle to the cut was 40 cm. This is dimension C in Fig. 3. In the case of the shortest rectangle, 30.0 cms. long, the distance was 20 cm. At first it was deemed inadvisable to use mercury cups for the joint and the tests were made with the tapered joint previously described. A trial with mercury cups was made later and the results were found to agree quite closely when compared with those obtained by use of the tapered joint. This comparison is shown in Fig. 5.

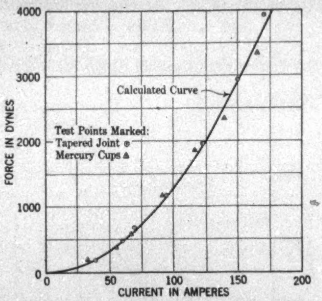

FIG. 5—CURVE SHOWING RELATION BETWEEN FORCE AND CURRENT IN RECTANGULAR CIRCUIT WITH AND WITHOUT MERCURY CUPS

Length—30.0 cm.
Width—40.0 cm.
Diameter of Wire—0.249 cm.

The results of the tests for the two sizes of rectangle are given in Fig. 4 and Fig. 5. They are compared with curves calculated from formulas (**20**) and (**21**) given in a paper, *The Calculation of the Magnetic Force on Disconnecting Switches*, by H. B. Dwight, TRANS. A. I. E. E., 1920, page 1337. This paper has been referred to frequently in several recent articles, but up to the present time there has been no experimental verification of the formulas derived in it. The formulas as given are for a disconnecting switch, of which a diagrammatic sketch is given in Fig. 6. This circuit reduces to a rectangle when S is made equal to A, so that the section $N' P'$ coinciding with $N P$ will cancel the effect of $N P$, as it should for a rectangle, since this part does not exist. The section $T' V'$ cannot be made to coincide with $T V$ since it is necessary to lead current in and out of the rectangle, but the effect of these parts will be reduced to a negligible amount if they are placed very close together. In the disconnecting switch the section $Q R$ was considered to be a flat strap, but in the tests made round wire was used throughout. Another point to be noted is that the disconnecting switch formula gives the force tending to open the blade at Q, and consequently only half the total force acting on $Q R$.

The formulas are given in the form of a convergent series and for convenience will be repeated here. Two cases are considered. Where B is greater than A, the force in dynes acting on the blade of a disconnecting switch is

$$i^2 \left[\operatorname{logh} \frac{2A}{r} - \frac{2}{3} - \frac{1}{2}\frac{A}{B} - \frac{1}{6}\frac{c^2}{A^2} + \frac{3}{20}\frac{r^2}{A^2} + \frac{1}{24}\frac{A^3}{B^3} + \frac{1}{24}\frac{A\,c^2}{B^3} + \frac{B}{S} \right] \qquad (2)$$

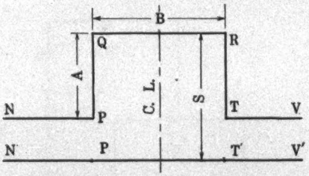

FIG. 6—DIAGRAM OF TYPICAL DISCONNECTING SWITCH

and where A is greater than B, the force is

$$i^2 \left[\operatorname{logh} \frac{B}{r} + \frac{1}{3} - \frac{B}{A} + \frac{1}{4}\frac{B^2}{A^2} + \frac{3\,r^2}{20\,A^2} - \frac{1}{32}\frac{B^4}{A^4} + \frac{B}{S} \right] \qquad (3)$$

where i is the current in abamperes, r is the radius of the section of wire $Q P$, and $R T$, $2c$ is the width of the blade and A, B, and S are the dimensions shown in Fig. 6.

Mechanical Forces in Transformers

BY J. E. CLEM[1]

Associate, A. I. E. E.

Synopsis—*In this paper a method of calculating the mechanical forces in transformers, based on mutual reactance between coils, is presented. A formula for the mutual inductance between coaxial solenoids is developed and from this expression the formula for the mechanical force between concentric cylindrical transformer coils is derived. The same method is followed to obtain the formula giving the mechanical force between individual coaxial coils. The method is checked by calculations of reactance of complicated arrangements of coils. Tables are given to facilitate calculations.*

* * * * *

IN the design of transformers it is essential that the mechanical forces set up on short circuit be predetermined so that the bracing structure may be proportioned properly. This feature becomes increasingly important as the size of the transformer and the extent of the power systems increase. In this paper there is developed a method, fundamental and analytical, by means of which the total axial force and the force on separate coils of a transformer having concentric cylindrical windings may be calculated easily and quickly. The method is simple, being based on the fact that the reactance of a transformer may be calculated from formulas of self- and mutual-inductance.

MUTUAL INDUCTANCE OF COAXIAL SOLENOIDS

This development is similar to other developments of the same problem but the result is given in a form that is more convenient for calculation than heretofore available.

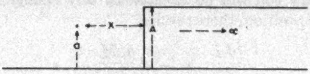

FIG. 1

It has been shown elsewhere[2] that the mutual inductance between a circle of radius a and a coaxial solenoid of radius A, Fig. 1, extending to infinity from a point X distant from the plane of the circle is given by an expression which becomes on transformation

$$M = 2\,\pi^2\,a^2\,N\left[1 - \frac{x}{r}\,F\right] \qquad (1)$$

$$F = F_0 + \frac{a^2}{r^2}\,F_2 + \frac{a^4}{r^4}\,F_4 + \frac{a^6}{r^6}\,F_6 + \text{etc.}$$

$$F_0 = 1$$

$$F_2 = \frac{3}{8}\;\frac{A^2}{r^2}$$

$$F_4 = \frac{5}{64}\;\frac{A^2}{r^2}\left(7\,\frac{A^2}{r^2} - 4\right)$$

$$F_6 = \frac{35}{1024}\;\frac{A^2}{r^2}\left(33\,\frac{A^4}{r^4} - 36\,\frac{A^2}{r^2} + 8\right)\text{ etc.}$$

$$r = \sqrt{A^2 + x^2}$$

Equation (1) has been transformed from those usually given to obtain a series for the quantity F in which the variables are always less than unity. This makes the calculations easier and extends the working range of the equation by keeping the value of the series for each component part of F down to a small figure throughout the entire range of the variable A^2/r^2.

In this expression M is the number of lines passing through the circle due to the semi-infinite solenoid which has a winding of N turns per cm. The mutual inductance between the finite solenoid, Fig. No. 2, of radius a and length S having n turns per cm. and the semi-infinite solenoid may be obtained by integrating the expression of (1) over the range from $x = x_1$ to $x = x_2$. This gives

$$M = 2\,\pi^2\,a^2\,n\,N\,[x_3 - r_3\,B^{III} - x_1 + r_1\,B^{I}] \qquad (2)$$

$$B = B_0 - \frac{a^2}{r^2}\,B_2 - \frac{a^4}{r^4}\,B_4 - \frac{a^6}{r^6}\,B_6 - \text{etc.}$$

$$B_0 = 1$$

$$B_2 = \frac{1}{8}\;\frac{A^2}{r^2}$$

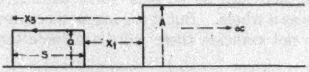

FIG. 2

$$B_4 = \frac{1}{64}\;\frac{A^2}{r^2}\left(5\,\frac{A^2}{r^2} - 4\right)$$

$$B_6 = \frac{5}{1024}\;\frac{A^2}{r^2}\left(21\,\frac{A^4}{r^4} - 28\,\frac{A^2}{r^2} + 8\right)\text{ etc.}$$

By reference to Fig. No. 3 it can be seen that the mutual inductance of the finite solenoid S and a second semi-infinite solenoid extending to infinity but starting P further away from the solenoid S than the the first

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[1] Central Station Engg. Dept., General Electric Co., Schenectady, N. Y.

[2] Bul. No. 169, Bureau of Standards.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

semi-infinite solenoid is given by a similar expression involving x_2 and x_4 in place of x_1 and x_3 as follows:

$$M = 2\pi^2 a^2 n N \left[x_4 - r_4 B^{IV} - x_2 + r_2 B^{II}\right] \qquad (3)$$

It follows naturally that the mutual inductance of coils S and P with centers x cm. apart as in Fig. No. 4 is the difference between (2) and (3), *i. e.*, (2) $\div$ (3) which gives as the final formula for the mutual inductance between two concentric coaxial solenoids

$$M = 2\pi^2 a^2 n N \left[r_1 B^{I} - r_2 B^{II} - r_3 B^{III} + r_4 B^{IV}\right] \qquad (4)$$

M = Mutual inductance in centimeters
a = Smaller radius of solenoids in centimeters
A = Larger radius of solenoids in centimeters
S = Length of a solenoid in centimeters
P = Length of A solenoid in centimeters
n = Turns per centimeter of a solenoid
N = Turns per centimeter of A solenoid
r = $\sqrt{x^2 + A^2}$ for each value of x
B = Function of the ratios A^2/r^2 and a^2/r^2 for each value of x as defined in equation (2). Values of B may be taken from Table 3.

$$x_1 = x - \frac{S+P}{2} \qquad x_3 = x + \frac{S-P}{2}$$

$$x_2 = x - \frac{S-P}{2} \qquad x_3 = x + \frac{S+P}{2}$$

SELF-INDUCTANCE OF SOLENOIDS

In order to calculate the net inductance of a pair of solenoids it is necessary to calculate the self-inductance of each. One of the most convenient methods of doing this is that given by Nagaoka as follows:

$$L = 4\pi^2 a^2 n^2 S K \qquad (5)$$

which gives the inductance in centimeters. The factor K is a function of the ratio of the diameter to the length of the coil and the values of K are available[3] in any book that treats on inductance.

AXIAL FORCE IN TWO COAXIAL SOLENOIDS

When the magnetic centers of two coaxial solenoids coincide there will be no axial force tending to move the coils as a whole. But if the magnetic centers of the coils do not coincide there will be a force tending to cause a still greater separation of the magnetic centers. This force will depend upon the stored magnetic energy and the rate at which the energy is changed by the differential motion of the coil, *i. e.*,

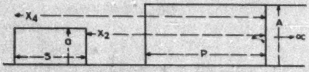

FIG. 3

$$f = -\frac{d\,W}{d\,x}$$

$$W = \frac{1}{2} L I^2$$

W = stored magnetic energy in joules
= 10^7 ergs or dyne-centimeters
L = inductance of circuit in henrys
I = maximum value of currents

$$f = -\frac{10^7}{2} I^2 \frac{d\,L}{d\,x} \text{ dynes} \qquad (6)$$

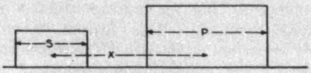

FIG. 4

In the pair of solenoids or transformer coils being considered

$$L = L_p + L_s - 2 M_{ps} \qquad (7)$$

L = Net inductance or leakage reactance of transformer
L_p = Self-inductance of primary
L_s = Self-inductance of secondary
M_{ps} = Mutual inductance between primary and secondary

Since the self-inductance of neither the primary nor secondary coil will be affected by any change in their relative position, there results

$$\frac{d\,L}{d\,x} = -2\frac{d\,M}{d\,x} \qquad (8)$$

Since M is given by equation (4), $\frac{d\,M}{d\,x}$ is given by the derivative of this equation. Rewriting (4) for inches and henrys,

$$M = 2.54 : \frac{2\pi^2 a^2 n N}{10^9} \sum r B$$

there results

$$\frac{d\,M}{d\,x} = 2.54 : \frac{2\pi^2 a^2 n N}{10^9} \sum \frac{x}{r} F$$

and

$$\frac{d\,L}{d\,x} = -2.54\,\frac{4\pi^2 a^2 n N}{10^9} \sum \frac{x}{r} F \qquad (9)$$

When (9) is substituted in (6), the force, after changing from dynes to pounds, is found to be

$$f = 444\, I^2 a^2 n N\, 10^{-9} \sum \frac{x}{r} F.$$

In this expression, I is the maximum value of current and to use the usual effective value we must write

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3. Bul. No. 169, Bureau of Standards.

$$f = 0.888\, I^2 a^2 n N 10^{-6} \sum \frac{x}{r} F \tag{10}$$

This expression gives the force in pounds when the dimensions are in inches and F is defined as in equation (**1**). The value of F may be taken from Table II.

Force on Individual Coils

To find an expression for the force between a solenoid and a single coil we proceed as above, *i. e.*, integrate to find the mutual inductance between the coil and the solenoid and then differentiate this expression to find the force. The mutual inductance between two circles, Fig. 5, is given by

$$M = 8\,\pi \sqrt{a}\, A \frac{F - E}{\sqrt{K}} \tag{11}$$

In this expression F and E are elliptic integrals, available from published data in works on inductance,[4] to the modulus K which is defined as

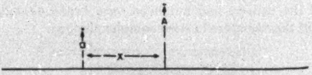

Fig. 5

$$K = \frac{4\, a\, A}{(r_2 + r_1)^2}$$

$$r_2^2 = (A + a)^2 + x^2$$

$$r_1^2 = (A - a)^2 + x^2$$

The mutual inductance between one of the circles and a solenoid is obtained by integrating (**11**) over the range S from $x = x_1$ to $x = x_2$ which gives

$$M = 8\,\pi\, n N \sqrt{a}\, A \int_{x_1}^{x_2} \frac{F - E}{\sqrt{K}}\, d\, x$$

$$= 8\,\pi\, n N \frac{\sqrt{a\, A}}{S} \left[\int D \right]_{x_1}^{x_2}$$

Upon differentiating this and substituting in the force equation there results

$$f = 0.698\, n N \frac{\sqrt{a\, A}}{S} \left[D'' - D' \right] \tag{12}$$

f = Force in pounds
n = Turns per inch of solenoidal coil
N = Turns in single coil
a = Radius of solenoid inches
A = Radius of coil inches
S = Length of solenoid inches

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[4] Bul. No. 169, Bureau of Standards.

TABLE I

COMPARISON OF CALCULATED AND MEASURED VOLTAGES FOR WINDINGS SHOWN IN FIG. 6

Connection	Fig. 6A		Fig. 6B	
	Test	Calc.	Test	Calc.
$P - S$	3350	3850	4023	3820
$P - T$	2130	1954	4530	4665
$P - Q$	660	665	1471	1318
$S - T$	5110	5344	4865	4770
$S - Q$	1520	1460	1464	1364
$\overline{P\,S} - T$	6048	5844	5264	5560
$\overline{P\,S} - Q$	1975	1725	1862	1725
$T - Q$	468	529	466	409
Ave.........	100	100.6	100	95.6

TABLE II

VALUES OF F

A^2/r^2	$a^2/r^2 = 1.$	0.9	0.8	0.7	0.6
1.	(2.2955)	1.9280	1.6833	1.5090	1.3803
0.95	1.7660	1.6280	1.5089	1.4076	1.3211
0.9	1.5369	1.4731	1.4058	1.3395	1.2767
0.85		1.3887	1.3405	1.2910	1.2416
0.8		1.3310	1.2921	1.2524	1.2131
0.7			1.2212	1.1944	1.1665
0.6				1.1497	1.1304
					1.1005
	$a^2/r^2 = 0.5$	0.4	0.3	0.2	0.1
1.	1.2816	1.2036	1.1397	1.0860	1.0400
0.95	1.2471	1.1835	1.1283	1.0802	1.0377
0.9	1.2187	1.1658	1.1179	1.0746	1.0354
0.85	1.1948	1.1500	1.1081	1.0692	1.0332
0.8	1.1739	1.1358	1.0990	1.0640	1.0310
0.7	1.1389	1.1106	1.0823	1.0542	1.0267
0.6	1.1102	1.0801	1.0673	1.0451	1.0226
0.5	1.0858	1.0702	1.0538	1.0365	1.0185
0.4	1.0646	1.0534	1.0413	1.0284	1.0146
0.3		1.0382	1.0299	1.0208	1.0108
0.2			1.0192	1.0135	1.0071
0.1				1.0071	1.0035

TABLE III

VALUES OF B

A^2/r^2	$a^2/r^2 = 1.$	0.9	0.8	0.7	0.6
1.	(0.8506)	0.8689	0.8861	0.9024	0.9179
0.9	0.8845	0.8956	0.9071	0.9187	0.9304
0.8		0.9149	0.9227	0.9317	0.9411
0.8					
0.7			0.9357	0.9430	0.9505
0.6				0.9530	0.9590
0.5					0.9669
	$a^2/r^2 = 0.5$	0.4	0.3	0.2	0.1
1.	0.9328	0.9471	0.9609	0.9743	0.9873
0.9	0.9422	0.9540	0.9657	0.9772	0.9887
0.8	0.9506	0.9603	0.9701	0.9800	0.9900
0.7	0.9582	0.9662	0.9744	0.9828	0.9913
0.6	0.9652	0.9717	0.9784	0.9854	0.9926
0.5	0.9718	0.9769	0.9823	0.9880	0.9939
0.4	0.9780	0.9819	0.9861	0.9905	0.9951
0.3		0.9867	0.9897	0.9930	0.9964
0.2			0.9933	0.9954	0.9976
0.1				0.9977	0.9988

Test Application

The method has been checked by the calculation of reactance for complicated arrangements of windings, on the basis that if the voltage can be calculated accurately then the force calculation as based on the differential of the voltage equation will be established as reliable. This was done because the voltages can be measured much more easily than forces.

In Figs. 6A and 6B are shown diagrams of two transformers for which voltage calculation was made. The agreement of calculated and measured voltage shown in Table I is reasonably close. These are high voltage transformers having extra insulated end turns so that the turns are not distributed uniformly over the high-voltage coils. This has an effect which is greater as the portion of the winding considered is less, but these calculations have been made as for a uniform distribution.

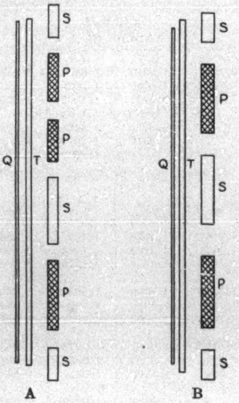

FIG. 6

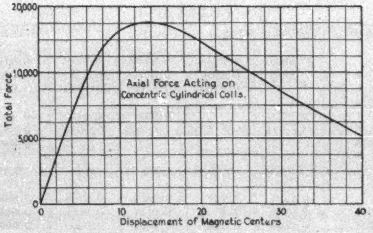

FIG. 7

Forces for a representative transformer have been calculated and the results are shown in Figs. 7 and 8. The coil forces in Fig. 8 are for a displacement of about 14 in. and the sum of the individual coil forces totals to the value on the curve in Fig. 7. In this case the forces are relatively low on account of the rather high reactance of the transformer for which these calculations are made. The displacement which gives the maximum is higher than would ever occur in a well designed transformer.

Discussion

W. S. Moody: The problem of taking care of mechanical forces in transformers became a really difficult one for the practical designing engineer some fifteen or twenty years ago. Previous to that, the power available in case of a short circuit and the size of transformers were not sufficient to require much more than good judgment on the part of the engineer to provide the necessary mechanical support. But with the increase in size, and more particularly with the increase in the available power, the necessity of accurate calculation of these forces arose. Ever since, therefore, we have welcomed most cordially the assistance of the ablest mathematicians and physicists who have shown interest in this difficult problem.

Time and again we have felt that the problem was fully solved, that one's design might be assuredly safe in every respect; but with the ever increasing complexity of transformer designs, now seldom consisting of two simple windings, but with three and even four complex windings, with many taps and the consequent inability to distribute equally the ampere-turns in all cases, new features have arisen showing that the previous study of the subject had not been on a broad enough gage to include all the variables in such complex designs.

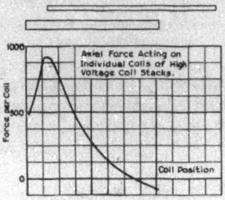

FIG. 8

So we still welcome such assistance as we have so generously received from our mathematical and physical friends.

After formulas have been developed to calculate accurately the mechanical forces resultant from all possible combinations of ampere-turns (if that time ever does come) we shall still have the interesting problem of making those formulas practical for use in "every-day" calculations. Such formulas as have been talked about in this paper are excellent for the general study of transformer design but when it comes to every day designs, which must be produced promptly and cheaply, one can see that these formulas are very burdensome and that there is necessity for short-cut methods giving equal accuracy. For this we must use special mathematical calculating machines, such as specially designed slide-rules and other forms of calculating machinery that make it possible to use the principles of these fundamental formulas in the daily routine of productive design.

F. H. Kierstead: There are often simplifications of difficult circuits that may be made in the calculation of the forces between these circuits. I wish to illustrate such a case.

Many times it is necessary to calculate the forces between two conductors (carrying currents) which have such a form that no standard formula applies. Usually time does not permit of deriving an accurate formula for the special case. Approximate calculations which are generally accurate enough can be made by replacing the actual circuits by equivalent ones to which the standard formula applies. As an example to illustrate this method let us take the case of the forces between a large and a small reactor with parallel but not co-axial axes.

The accompanying figure shows the position of the reactors relative to each other. From the standpoint of forces the reactors can be replaced by the circular filaments *W*, *X*, *Y*, and *Z* and the forces calculated between these filaments are the forces between the reactors. It is, however, difficult to calculate the force between two parallel unequal circles not coaxial. To facilitate this calculation the smaller circle can be replaced by the arcs of circles *G-F* and *H-I* (which are co-axial with the circular filaments of the larger reactor) and the radial lines *G-H* and *F-I*.

Standard formulas are available for calculating the force between co-axial circles and, therefore the forces between the filaments of the large reactor and circles of which arcs *G-F* and *H-I* form a part are easily calculated. The forces between the filaments of the large reactor and arcs *G-F* and *H-I* bear the same relation to the forces between the complete circles that the length of the arcs bear to length of the complete circles. Since that part of the equivalent circuit represented by *G-H* and *F-I* is radial to the filaments of the large reactor, there is no force between it and the filaments of the large reactor. Since the current in the arc *G-F* is in opposite direction to that in arc *F-I* the forces are in opposite directions.

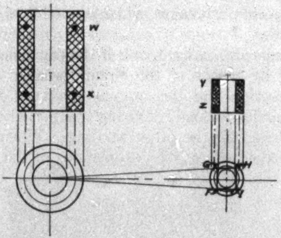

FIG. 1—DIAGRAM REPRESENTING THE EQUIVALENT FILAMENTS USED IN CALCULATING THE FORCES BETWEEN A LARGE AND A SMALL REACTOR

H. B. Dwight: Mr. Clem's paper is an interesting example of a case where it is desirable to have both an inductance formula and a force formula, not only because a knowledge of the inductance is useful, but because the inductance is more easily measured than the force.

Mr. Kierstead's trapezoid solution of the special problem is interesting to me personally. In 1917, I was asked to make an estimate of the force between two air-core reactors which were mounted side-by-side with parallel axes. As I had no formulas that I could use at that time for two circles, I used the device of representing one of the circles by a trapezoid, the same as Mr. Kierstead has described. I had not seen it published before, and perhaps Mr. Kierstead can say if there was a previous announcement of it. My publication of this was in the *Electrical World* of June 16, 1917.

Later, I was able to calculate the repulsion between circular coils with parallel axes, and in the TRANS. A. I. E. E., 1919, page 1678, Fig. 2, there is given a comparison between the circular-coil solution and the trapezoid solution. The curves lie very close together.

There is a formula in Gray's "Absolute Measurements" which may be of interest. If the axes of two coils meet in a point, a calculation is given by which the mutual inductance and repulsion can be obtained.

F. W. Grover: (communicated after adjournment) Mr. Clem has developed a new formula for the mutual inductance of coaxial solenoids, which offers some points of interest.

The Rosa formula for the mutual inductance of a solenoid and coaxial circular filament (*Bull.* Bureau of Standards 3, p. 209; 1907, formula (56) *Sci. Paper*, Bureau of Standards) is made the starting point. This formula, although very convergent for a wide range of cases, involves certain polynomials X_{2n} which, for long solenoids, may assume values so large as to cast doubt upon the degree of convergence. By a simple algebraic substitution Clem eliminates these polynomials, and obtains a transformation of the Rosa equation in which appear as variables only the ratios of the radii to the radius vector from the center of the circle to the circumference of the end turn of the solenoid. Since these variables lie in value between zero and unity as limits, this expression of Clem's is better adapted to tabulation than is that of Rosa. It is worthy of note that, making the proper changes to reduce to the same system of nomenclature, Clem's expression is seen to be identical with that derived by Lorenz, (*Wied. Ann.* 25, p. 1, 1885; also (53) *Sci. Paper* 169, Bureau of Standards).

The derivation of Clem's formula for the mutual inductance of coaxial solenoids from this formula for a solenoid and circle is straightforward and may readily be extended to obtain further terms of the series, if desired. The resulting new expression is readily used, and for practical calculations is much superior to the related formula of Gray, (*Abs. Meas.* 2, Part I, p. 274, or (40) *Sci. Paper* 169, Bureau of Standards) which has heretofore been the only formula available for this case, except the absolute complicated elliptic integral formulas. For example, the solution of Example 41, *Sci. Paper* 169, Bureau of Standards, by Clem's formula gives as a result 1086.2, the true value by the absolute formula being 1086.55. Using Gray's formula directly the result is 1092.3, and to obtain an accuracy equal to Clem's it is necessary to subdivide the coils and to sum the results for the different pairs of sections.

Although Clem's derivation is for the case where the coils are separated axially, it applies also to the case of overlapping coils. For the important case where the coils are *concentric*, only two main terms have to be calculated instead of four. For this case Clem's formula compares very well with the accurate formula of Searle and Airey, (*Lon. Electrician*, 56, p. 318, 1905, or (43) *Sci. Paper* Bur. of Stand.) which is the special form taken by Gray's general formula for the concentric case.

One precaution in using Clem's formula should be mentioned. When the coils are far apart, the main terms $(r_1 + r_4) - (r_2 + r_3)$ give the result as the relatively small difference of two larger numbers. However, these quantities may readily be calculated by a calculating machine to a sufficient number of places. Another obvious method is to expand them in series, but this is advantageous only for very distant coils.

Transformer Tap Changing Under Load

BY L. H. HILL[1]

Associate, A. I. E. E.

Synopsis.—*Changing the voltage ratio of transformers under load is now a recognized and established procedure. Methods of changing taps under load are discussed, illustrated, and compared. Equipment for obtaining smooth curve voltage control is discussed, as well as combination voltage and phase-angle control.*

* * * * *

INTRODUCTION

THE development of reliable equipment for changing the voltage ratio of transformers without disconnecting the load has, in effect, created a new type of apparatus ranking with the induction regulator or synchronous condenser in importance.

Transformers provided with equipment for tap changing under load, however, do not labor under the inherent disadvantages incident to the use of the other two kinds of apparatus. The rapid growth in popularity of this equipment has been due to the fact that for certain applications, a simpler, more compact, more reliable, sturdier, more effective, and cheaper piece of apparatus can be obtained by this than with the older forms of equipment.

The application of transformers provided with equipment for changing taps under load is very wide. Perhaps one of the most important is in connection with transformers used to tie together two large systems or parts of systems. Of next importance are units used for bulk voltage regulation of a secondary bus or to compensate for voltage drop in transmission circuits. Other interesting applications are those involved in the variation of voltage applied to rotary converters and furnace transformers.

METHODS OF TAP CHANGING

There have been numerous methods devised and, at least to a limited extent, used to change the voltage ratio of transformers under load. These may be divided into two classes—those changing the ratio in steps and those changing the ratio along a smooth curve.

CHANGING THE VOLTAGE RATIO IN STEPS

The majority of schemes proposed and used for changing the voltage ratio under load change the ratio in steps. Roughly, these may be divided into two general classes,—(1) those using duplicate paralleled windings in the transformer, each normally carrying one-half of the load, but adapted for carrying the entire load during the time that the taps on the other are being changed; and (2), those using a single winding with a preventive resistance, reactance, or auto-transformer across the taps involved in the transition.

THE PARALLEL WINDING METHOD

Fig. 1 indicates schematically a winding arranged to change taps under load by means of the parallel winding method.

Each of the parallel circuits contains a tap changer or ratio adjuster, usually located inside the transformer tank, and a circuit breaker, which is outside of the transformer tank.

When taps are changed, one of the paralleled circuits is opened by means of the circuit breaker in its respective section and the taps are changed while the winding carries no load. During this period the entire load is carried by the other winding. When the first circuit breaker closes, the two sections of the windings are paralleled with unequal taps and a circulating current exists. This is for a short time only, as the second breaker opens immediately, permitting the taps to be changed at no-load on the second winding, while the first carries the total load. After this, the tap changing operation is completed by closing the breaker on the second winding with the result that the two paralleled sections again operate on equal taps.

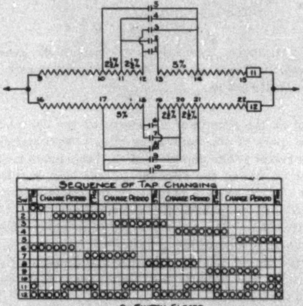

FIG. 1—SCHEMATIC DIAGRAM AND SEQUENCE CHART OF SWITCH OPERATIONS FOR THE PARALLEL WINDING METHOD

During the interval of tap changing, one of the paralleled windings carries double normal current. The windings are ordinarily designed with sufficient capacity to carry this abnormal current during the tap changing operation and differential protection between windings is provided to guard against accidental overloading for a longer period of time.

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[1] Transformer Engg. Dept., Westinghouse Elec. & Mfg. Co. Sharon, Pa.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

A transformer designed to change taps under load by means of the parallel winding method is illustrated in Fig. 2.

SINGLE WINDING METHOD

The other methods used in addition to the parallel winding method fall into the class which uses a single winding in the transformer with a preventive coil or some other device to limit the current during the transition period from one tap to the next.

Probably the oldest form of equipment for tap changing under load employed the Stillwell regulator principle, which uses a preventive resistance temporarily bridged across taps to limit the current during the transition period.

Equipments have also been made, to a limited extent, using a preventive reactance in the circuits instead of the resistance used with the original Stillwell scheme.

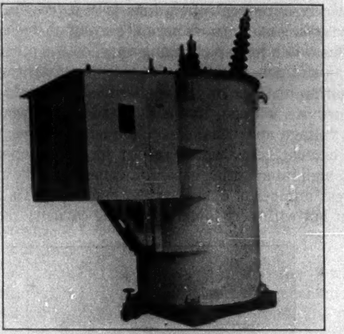


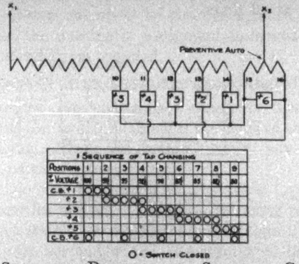
FIG. 2—20,000-KV-A., 66,000-VOLT, 60-CYCLE TRANSFORMERS USING PARALLEL WINDING METHOD

A much simpler scheme than either of these two consists of using a preventive auto-transformer with a mid-point tap, as schematically shown in Fig. 3. A great many units have been built employing this method.

To obtain the full winding of the transformer in the circuit, switches 1 and 6 are closed. The circuit is then through the full transformer winding and divides through the preventive auto-transformer, one-half being through one side of the auto-transformer and one-half being through the other half in the opposite direction. The voltage of the transformer is therefore the voltage induced in the entire winding. To change taps, switch 6 is opened and 2 is closed. This connects the auto-transformer across the two taps and, since the line lead is attached to the center of the preventive auto-transformer, the line voltage becomes the same as it would have been had the line lead been attached to a tap midway between the two actually brought out.

Similarly, to change taps still further, the process is repeated in this manner, (Fig. 3).

In the earlier installations built using this method, the switches and preventive auto-transformers were all mounted separately and apart from the main transformer tank. In later equipments, since the preventive auto-transformer has no moving parts and can be made entirely reliable, it is mounted inside the main tank and supported from the main transformer.



↓ SEQUENCE OF TAP CHANGING																		
POSITIONS	1		2		3		4		5		6		7		8		9	
% VOLTAGE	[illegible]		[illegible]		95		[illegible]		[illegible]		[illegible]		85		[illegible]		80	
C.B. #1	O	O	O															
#2			O	O	O	O	O											
#3							O	O	O	O	O							
#4											O	O	O	O	O			
#5															O	O	O	
C.B. #6	O				O				O				O				O	

O = SWITCH CLOSED

FIG. 3—SCHEMATIC DIAGRAM AND SEQUENCE CHART OF PREVENTIVE AUTO-TRANSFORMER METHOD

Fig. 4 illustrates an installation of transformers using this type of apparatus. The tap changing equipment is contained in the sheet iron house next to the transformer tank.

The tap leads are all brought through the side of the transformer tank, and are connected to the circuit breakers mounted in the upper portion of the steel house.

The circuit breakers are mechanically connected to the operating mechanism on the floor below.

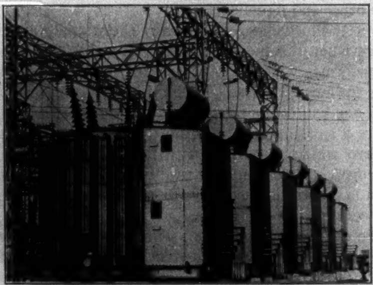
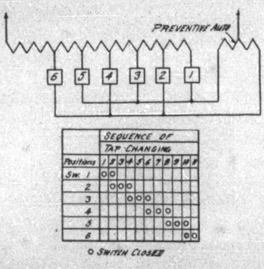


FIG. 4—INSTALLATION OF 6- 10-, 500-KV-A., 66,000-VOLT, 25-CYCLE TRANSFORMERS USING PREVENTIVE AUTO-TRANSFORMER METHOD

For a short time during the transition period, one-half of the auto-transformer winding carries all of the load current of the transformer, while the other half of the winding is open. The load current is then the magnetizing current, and the voltage across the preventive auto-transformer tends to rise somewhat above normal. To limit this voltage to a low value, the design of the auto-transformer is such that the core becomes saturated when the voltage reaches a value slightly above normal.

In changing from one voltage to another, two operations are required; namely, the opening of one circuit breaker and the closing of another. Since the circuit breakers are operated mechanically by means of cams on a drive shaft, the correct sequence of operation is assured at all times.



	SEQUENCE OF TAP CHANGING										
Positions	1	2	3	4	5	6	7	8	9	10	11
Sw. 1	○	○									
2		○	○	○							
3				○	○	○					
4						○	○	○			
5								○	○	○	
6										○	○

FIG. 5—SCHEMATIC DIAGRAM AND SEQUENCE CHART OF SINGLE-WINDING METHOD USING THE SIMPLIFIED PREVENTIVE AUTO-TRANSFORMER

SIMPLIFIED PREVENTIVE AUTO-TRANSFORMER METHOD

The preventive auto-transformer method, using a short-circuiting switch across the auto-transformer, has recently been simplified still further by merely the elimination of the short-circuiting switch and the use of an auto-transformer designed to carry the transformer full load current in either half of the winding with the other end disconnected.

Fig. 5 illustrates schematically the winding arrangement when this method is used. To obtain the entire transformer winding in the circuit, switch 1 is closed and the current passes through the transformer winding and one-half of the preventive auto-transformer. This gives a small impedance drop through the auto-transformer which is in series with the transformer. Since the drop is almost entirely reactive, its effect on regulation is practically negligible at power factors above 65 per cent.

To change taps one step, switch 2 is closed, placing the auto-transformer across the two taps, and giving a voltage on the mid-tap of the auto-transformer midway between the two actual tap voltages.

To change taps another step, switch 1 is opened and the conditions become as before, with the other half of the auto-transformer carrying the full current of the transformer.

To change taps still further, the process is continued in the same manner, as may be followed in detail from the sequence chart, Fig. 5.

It may be seen that, by this development, the process of tap changing has apparently reached its utmost simplicity for step-type tap changers with one switch operation to change taps, and in every other tap change, the switch closing instead of opening.

When full load current is passed through one-half of the auto-transformer with the other half open, the full-load current of the main transformer as in the case of the other auto-transformer method becomes the exciting current of the auto-transformer. Under this condition, there are no neutralizing ampere-turns from the other half, so that the transformer becomes a reactor. Air-gaps are provided in the core to give low impedance when operating in this manner which, of course, makes the exciting current, when operating as an auto-transformer across taps, higher than ordinary.

SWITCHES FOR TAP CHANGING SERVICE

Circuit-breaker devices for tap changing service have entirely different conditions to meet than the ordinary circuit breaker. The ordinary breaker is designed for relatively infrequent operations with a few operations interrupting many times normal current at line voltage.

The switch for tap changing duty, on the other hand, is called upon to merely transfer current from one circuit to another, and, while it must be insulated for the service voltage, it opens but a small fraction of line voltage and usually not more than twice normal line current. Instead of calling for a few operations with high interrupting capacity, it must be able to stand a great number of operations without losing its adjustment but with very low interrupting capacity required.

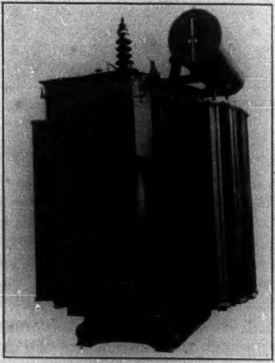


FIG. 6—12,000-KV-A., 132,000-VOLT TRANSFORMER USING THE SIMPLIFIED PREVENTIVE AUTO METHOD

A switch for tap changing service is never called upon to interrupt a short circuit except in the remote possibility of short circuit on the system occurring during a tap changing operation. Even in this case the voltage to be interrupted would be very low but the current would, of course, be considerably higher than normal. In fact, the service required of a switch for tap changing service, approaches that of a heavy duty contactor switch. Fig. 6 illustrates a transformer provided with equipment for tap changing under load, using a specially designed switch to meet these requirements, a very simple, compact, and sturdy mechanism. The single winding method, using the simplified preventive auto-transformer, was employed.

All mechanical equipment is isolated from the main transformer tank. The switches are contained in a separate oil-filled compartment on the side of the transformer case and the operating mechanism is contained in the housing below with a connecting tube enclosing the drive shaft which enters the upper compartment through an oil tight stuffing box in the bottom.

The general construction of the switch itself may be seen in Fig. 7. Condenser bushings through the side of the transformer tank support the stationary and movable contacts which are arranged to give the rolling action common with heavy-duty contactor switches. The rolling action is such that the arcing is taken at the tips so that the current carrying parts always remain in good condition. Opening and closing is definitely fixed in the proper sequence by the mechanical operation of the cams as in the case of the equipment using circuit breakers. The toggle mechanism assures quick opening, but in case of sticking or contact weld, the cams force the opening.

CONTROL OF TAP CHANGING EQUIPMENTS

Tap changing equipments are normally arranged for remote electrical control by the operator, with auxiliary arrangements for manual operation in case of failure of motor or control voltage.

The electrical control is such that after the operator has initiated a tap change, auxiliary mechanically operated switches on the equipment assure the completion of the tap changing operation irrespective of the action of the operator. Remote electrically operated position indicators of the dial type or of the indicating lamp type are generally used.

Tap changing equipments may also be built to operate under automatic control. The transformer illustrated in Fig. 6 was arranged to automatically control the voltage at a given point within predetermined limits.

The automatic control is initiated by a rise or fall in the low-voltage potential acting through a long time delay relay.

The use of automatic control with step type tap changers places unusual responsibility on the reliability of the apparatus. On account of the greater number of operations likely to be obtained with equipment responsive to the action of fluctuating line voltages, the time delay relay must be introduced to eliminate unnecessary operations—also to prevent the possibility of the tap changer operating during short circuits. Since a short circuit on the system tends to reduce the voltage, there would be a tendency for the tap changer to operate during the short circuit to raise the voltage, which in itself would be undesirable.

With automatic control of step type equipment it is necessary also to design the control equipment to free the motor-actuating circuits from the voltage-responsive circuits as soon as the motor-actuating circuits have become energized.

When transformers are operated as single-phase units in a bank with individual tap changing mechanisms or when two banks are operated in parallel, it is essential that out-of-step operation be guarded against. In all such cases, the automatic equipment is locked out of service and an alarm sounded.

COMPARISON OF STEP TYPE TAP CHANGING METHODS

Satisfactory tap changing equipments have been built using the two fundamental methods of step-type tap changing. There are certain inherent advantages, however, pertaining to each.

The single winding method requires fewer taps in the transformer for a given number of operating positions and gives a simpler transformer winding than the other method. On account of the less number of taps it is easier also to bring all operating parts outside of the transformer tank when this method is used. In addition, the fewer number of switch operations give the preventive auto method a decided advantage.

FIG. 7—SPECIAL SWITCH DESIGNED FOR TAP CHANGER SERVICE

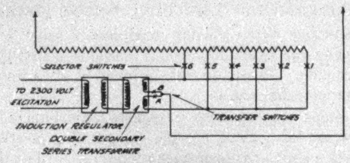
With the parallel winding method, however, when a wide range of taps in small steps is desired, a more compact equipment may usually be obtained by the use of the tap changer inside the main tank. This is particularly advantageous with small three-phase units where space limitations make it difficult to mount all equipments outside the main tank.

EQUIPMENT FOR CHANGING TRANSFORMER VOLTAGE RATIO IN SMOOTH TRANSITION

An interesting modification of the simplified preventive auto-transformer scheme of tap changing is obtained if the two halves of the preventive auto-transformer are replaced by the two sections of a series transformer—in combination with a small induction regulator.

Such a combination is called a step induction regulator and has been used principally for bus regulation and to apply variable voltage in a smooth curve to furnace transformers, testing transformers, and synchronous converters. It has been applied also to transformers used for interconnecting two systems. Referring to Fig. 8, the switches 1, 2, 3, 4, and 5 are called selector switches while *A* and *B* are called transfer switches. The induction regulator may be a standard feeder regulator with the addition of slip-rings to make the rotor suitable for continuous rotation.

Rotation of the induction regulator rotor through 180 deg. changes the voltage in its winding from a maximum in one direction to a maximum in the other direction. In the application to step induction equipment, the voltage of the regulator is added to or subtracted from a transformer tap to provide means for transferring from one tap position to the next and incidentally obtaining an infinite number of operating positions in between.



SEQUENCE OF OPERATION

SWITCHES	POSITION 1	2	3	4	5	6	
A	+		+		+		
B		+		+		+	
1	+						
2		+					
3			+				
4				+			
5					+		
6						+	

FIG. 8—SCHEMATIC DIAGRAM AND SEQUENCE CHART FOR STEP INDUCTION REGULATOR EQUIPMENT

Referring to Fig. 8, if the entire voltage of the transformer winding is desired, selector switch 1 and transfer switch *A* would be closed with the regulator rotor in the position of zero buck and boost.

To reduce the effective transformer coil voltage, the regulator is rotated to increase the voltage. At the position of maximum regulator voltage, the series transformer is designed so that the voltage of each half of the series winding is exactly the same as one-half the tap voltage.

At this point, switches 2 and *B* may be closed, since the half of the series winding connected to switch 1 reduces the effective coil voltage the same amount that the half connected to 2 adds to the voltage up to that point. Since the potential at the two points is the same, they may be connected. Continued rotation of the mechanism opens switches *A* and 1 and the voltage of the series transformer adds to the coil voltage of switch 2. As the regulator is rotated further, the voltage of the series transformer half decreases to zero when the line voltage becomes equal to the coil voltage up to tap 2. Continued rotation repeats the process to the next tap, as may be followed in detail from the sequence chart of switch operations Fig. 8.

Any of the infinite positions of the induction regulator become operating positions so that an infinite number of steps in voltage may be obtained between the extreme tap position.

It would be possible to eliminate the series transformer with this equipment, by building a special regulator with two sets of secondary coils. The use of the series transformer is desirable, however, not only because it eliminates the necessity of making a special regulator winding, but it isolates the induction regulator from the transformer circuits. The use of the relatively weaker induction regulator, therefore, does not reduce the inherent mechanical and electrical reliability of the main transformer.

When the range of voltage regulation is exceptionally large, it is economical to modify the above scheme by switching the induction regulator along an auxiliary winding, which, in turn, is switched along the main winding at less frequent intervals. The taps are changed on the auxiliary winding in the same manner as described above and the voltage of the auxiliary winding either added to or subtracted from the taps of the main winding.

Assuming that the voltage is to be increased, one of the auxiliary windings is connected to a tap such as tap 1 of the transformer, and the induction regulator switched along the auxiliary winding until the voltage of the double secondary windings is added to the auxiliary winding. The voltage added to tap 1 is then the same as the voltage of the winding between taps 1 and 2 minus the voltage of one winding of the series transformer. There will be no change, therefore, in voltage if the second double secondary winding is connected to tap 2 so that its voltage is subtracted from the tap. The voltage may be increased further by disconnecting the auxiliary winding from tap 1, and rotating the regulator rotor so that the voltage of the second auxiliary winding plus the voltage of one winding of the double

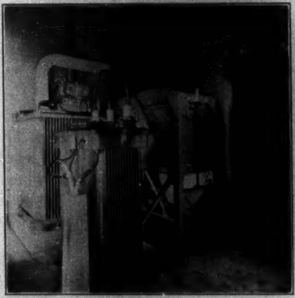


FIG. 9—2500-KV-A., STEP INDUCTION REGULATOR EQUIPMENT

secondary winding is added to tap 2. The connections are then changed as before, so as to subtract the voltage of one winding of the series transformer from tap 3. Further increases of voltage are obtained beyond tap 3 in a similar manner.

An interesting application of the step induction regulator principle is illustrated in Fig. 9, where the voltage applied to a 2500-kv-a., synchronous converter is varied in the ratio of 2 to 1, giving a voltage range of 50 per cent in smooth transitions. By the use of voltage regulating equipment of this kind it is possible to cover a wide voltage range without the use of booster type converters.

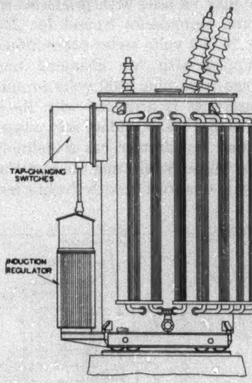


FIG. 10—SKETCH OF TRANSFORMER PROVIDED WITH STEP-INDUCTION REGULATOR EQUIPMENT BUILT INTEGRAL WITH IT

In Fig. 10 is illustrated the possible compact and simple arrangement of step induction regulator equipment with all moving parts external to the main unit but with the series transformer inside the main case. The selector and transfer switches are mechanically operated contactor switches driven by the regulator in the proper sequence.

COMPARISON OF STEP TYPE AND STEP INDUCTION REGULATOR METHODS

The step induction regulator is well adapted for use where bus or transmission circuit voltage is to be controlled, particularly where automatic control is desired. In this case, automatic control merely calls for a voltage actuated relay with time delay, giving a much more simple control equipment than is possible with the step type equipment. When automatic control is used, the number of tap changing operations is usually greater than otherwise so that the step induction regulator units are particularly adaptable on account of the fact that there is no burning of the contacts and therefore less maintenance required.

When the range in taps to be covered is not too large, and if small steps are not required, the step type equipment is more desirable on account of the somewhat simpler equipment.

SEPARATE REGULATING UNITS

The tap changing equipments heretofore considered have been applied directly to the transformer unit. On account of the fact that no additional transformer units are required, this generally gives the cheapest, most efficient, and most compact equipment to change the voltage.

There are applications, however, where modifications of this simple arrangement are desirable.

In general, tap changing mechanisms are so far practicable for direct use in circuits up to 33,000 volts or carrying not more than 1200 amperes. By using these units in the star connection of solidly grounded systems it is possible to apply them to transformers of much higher voltage by placing the tap changer in the grounded end.

In case of delta connections above 33,000 volts or ungrounded star windings of higher voltage, the use of a separate regulating unit becomes desirable at present.

The use of a separate regulating unit may be desirable also from other considerations. For example, if it is desired to obtain voltage control with transformers already installed, the separate regulating unit becomes very useful. In some cases the need for voltage regulation may not be permanent, so that a separate regulating unit may be used with the idea that it may later be transported to some other location.

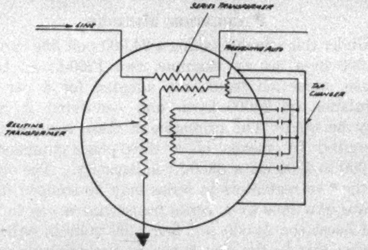


FIG. 11—SCHEMATIC DIAGRAM OF SEPARATE REGULATING UNIT

The use of separate regulating units, however, is accompanied by the requirement of more floor space, lower over-all efficiency on account of the extra transformers required, higher installation cost on account of the interconnections, etc., required between the regulating unit and the main unit.

Fig. 11 illustrates schematically the winding arrangement for a separate regulating unit. By suitably arranging the ratios of the respective series and exciting units, the tap changing equipment may be used to cover a wide range of applications. Both the series and exciting transformers may be mounted in the same case, to reduce the number of bushings and simplify the connections and installations.

In the case of equipment for tap changing under load designed to operate on high-voltage ungrounded units, the auxiliary series and exciting windings may be built into the same unit in such a way as to reduce the number of auxiliary windings and cores and increase the over-all efficiency. Assume, for example, a unit with tap changing under load required on a 120,000-volt delta winding. The series transformer as shown in Fig. 12 may be used but the exciting transformer in the separate regulating equipment may be replaced by a third winding in the transformer as shown.

COMBINATION VOLTAGE AND PHASE ANGLE CONTROL

When systems become larger and interconnections become more frequent, cases will arise where phase-angle control, as well as voltage control, may be required to obtain satisfactory results. This condition will exist where a substation is supplied from two sources with interconnecting lines of different impedances.

It appears that the control necessary will be of the order of plus or minus 10 per cent in-phase voltage with a phase-angle control of approximately plus or minus 6 deg. This means that an in-phase component of plus or minus 10 per cent, and a quadrature component of plus or minus 10 per cent, each independently adjustable, should be super-imposed upon one of the lines at some suitable point.

These problems may be worked out, using two induction regulators, two transformers, or by means of a single multi-winding transformer.

REGULATION METHODS

Under this scheme, taking a 24,000-volt line carrying 25,000 kv-a. as an example, two 1250-kv-a., three-phase, induction regulators, adapted for 5 per cent regulation on 25000 kv-a. and connected in series may be used. The primaries of these regulators are energized by means of a three-phase transformer, 24,000 to 4800 volts, 3300-kv-a. capacity. The output of the two regulators in series may be stepped up by means of a 2500-kv-a. series transformer so as to buck and boost the 24,000-volt line 2400 volts in either direction due to the two 5 per cent windings in series. Because of the fact that a three-phase regulator rotates the voltage vector—that is, it is fixed in magnitude but is variable in phase—the first regulator will add to the line voltage a voltage vector which is equal in magnitude to 5 per cent of the line voltage but whose phase angle with respect to the line voltage (depending upon the relative position of the rotor and stator of the regulator), may swing to any position in a complete circle pivoted upon the end of the line voltage vector. The regulator itself may be supplied with slip-rings so that it is capable of continuous rotation. The terminal voltage of the regulator is therefore represented by a circle of radius 5 per cent. To this, a second regulator is connected in series having an independent control, the terminal voltage of the second regulator describing a 5 per cent circle having its center located on the circle described by the first regulator, with the result that the terminal voltage of the second regulator may be adjusted at any point within a radius equal to 10 per cent and may therefore give a phase angle control of the line voltage of approximately 6 deg. and also a maximum in-phase buck or boost of 10 per cent either way.

TRANSFORMER METHOD

In the second method, two transformers of 2500-volt capacity each may be used with primaries wound for 24,000 volts and secondaries wound for 2400 volts, each rated for 24,000 volts series connection with the line and equipped with tap changing under load covering a range from plus 2400 volts to minus 2400. The first transformer is arranged with its primaries star connected so that the secondary is in phase with the line. The second transformer has a delta-connected primary and by properly selecting the secondary winding, a secondary in quadrature with the line is obtained. These two windings are placed in series with each other and with the line, and it is therefore possible to impose upon the line an in-phase voltage component of 10 per cent plus or minus and also a quadrature voltage component of 10 per cent plus or minus, each being controlled independently. By a manipulation of the two transformers in effect, it is evident that any degree of in-phase voltage regulation and any degree of phase-angle control within the limit of the apparatus may be obtained.

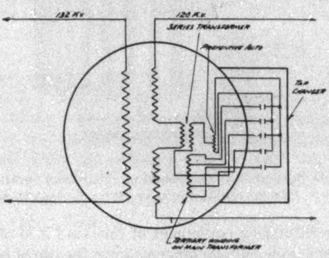


FIG. 12—SCHEMATIC DIAGRAM SHOWING APPLICATIONS OF LOW-VOLTAGE TAP-CHANGING EQUIPMENT TO HIGH-VOLTAGE TRANSFORMERS

The losses in the case of the regulator method are approximately double the transformer method.

MULTI-WINDING TRANSFORMER METHOD

By using a multi-winding transformer, the equipment may be still further simplified. Instead of using two transformers a single three-phase, three-winding transformer may be used. One winding is the exciting winding and the others are connected to give two voltages in series but displaced from each other by 60 deg. This would take the place of the two separate transformer windings in true quadrature. This arrangement results in a very decided decrease in the size of the transformer tank and also an increase in the over-all efficiency of the transformation, and results in a considerable reduction in cost.

In either of transformer equipments, any of the types of tap changing equipment may obviously be used.

CONCLUSION

The development of reliable sturdy apparatus for changing transformer voltage ratio under load has opened up a new field with enormous possibilities. While it can be seen that equipment of this nature is not inexpensive, yet it will generally be found that if voltage control of large capacity is desired, the use of transformers arranged for changing taps under load will be found economically desirable.

Discussion

For discussion of this paper see page 599.

Characteristics of Interconnected Power Systems
As Affected by Transformer Ratio Control

BY L. F. BLUME[1]
Associate, A. I. E. E.

***Synopsis.**—Operating characteristics of interconnected systems in which voltage is maintained constant by varying field of the generators is compared with operating characteristics, when, in addition to the control of generator field, transformers equipped with ratio control are employed. The use of transformers with variable ratio introduces a flexibility in operation which permits the division of wattless currents between generating stations independent of voltage held at the generator busses.*

A comparison is made of the use of synchronous condensers for the purpose of improving regulation as compared with the use of transformers equipped with ratio control.

The elementary conditions which govern the current distribution in a loop, L, are determined in terms of impedance characteristics of the net-work. The equipments necessary to control the current distribution and at the same time maintain good regulation in the loop are indicated.

THE steady and rapid growth of the use of variable ratio transformers in connection with interconnected central stations makes it of interest to state the fundamental characteristics of apparatus in systems which make these equipments desirable. In a paper[2] before the Institute two years ago, Mr. Albrecht indicated their field and compared their characteristics with induction regulators and synchronous condensers. The purpose of the present paper is to focus attention on a few of these characteristics in order to indicate how quantitative values may be obtained.

The various kinds of voltage control now being used on power systems are:

1. Voltage control by means of generator field current,
2. Synchronous condensers or phase modifiers by means of which the power factor in transmission lines is improved and thus better regulation obtained,
3. Voltage ratio control, either by means of transformers or induction regulators.

The above methods of voltage control will be considered in connection with specific, typical cases, although in the most practical instances it is admitted that the problem is more complicated than the assumptions of this paper imply. The principles as outlined here, however, are applicable, with proper modifications, to the more complicated ones. The four typical cases to be discussed are:

1. Where two generating systems are connected by means of transmission line and power may flow in either direction,
2. Where two generating systems are connected by means of transmission line but one-way flow of power only is required,
3. A generating system with a synchronous condenser floating on the end of the line,
4. Transmission line loop.

CASE I: TWO-WAY FLOW OF POWER OVER TRANSMISSION LINE CONNECTING TWO STATIONS

With two central stations connected together by means of an interconnecting transmission line (Fig. 1) the operation of the system involves the control of:

1. The division of energy between the two stations,
2. The division of wattless kilovolt-amperes,
3. The voltage at the two busses.

The controls available are (a) throttle control, (b) field control, (c) voltage ratio control. It is evident that the division of energy between the two stations is not determined by the electrical characteristics but by the characteristics of the prime mover and the throttle control. Field control at the two stations does not affect the energy flow, but determines for each load demand the voltages on the two busses, together with the division of reactive kilovolt-ampere between the stations. But field control cannot independently determine the bus voltages and division of reactive kilovolt-ampere. Either can be controlled, but not both simultaneously; thus the field control may be used to hold the bus voltages equal and constant for all loads, under which condition the division of reactive kilovolt-ampere between the stations is determinable but uncontrollable. Conversely, by means of field adjustments it is possible to control the division of reactive kilovolt-ampere between the stations or what amounts to the same thing, the power factor in the line, but when this is done, voltage control on the two busses is sacrificed. The voltage of one point on the system may be held constant, but the other bus voltage will vary through a range which is equal to the regulation drop between the two busses.

In order to control, independently of each other, the three characteristics, namely, *energy* division, *reactive* kilovolt-ampere division, and *voltage* at both busses, it is necessary to introduce a third independent control. This is readily accomplished by introducing variable voltage ratio between the two busses variable under load.

Thus, by means of the insertion of variable ratio between the two busses, it becomes possible to maintain

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[1] Transformer Engineering Dept., General Electric Co., Schenectady, N. Y.

[2] H. C. Albrecht, *Transformer Tap Changing Under Load*, A. I. E. E. TRANS., Vol. 44, 1925, p. 581.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

both bus voltages constant at all loads, which means that the regulation drop between the two busses is zero, and at the same time, to obtain any desired division of current between the two stations. With the flexibility of operation thus obtained, the bus voltage of the two stations may be maintained constant, and at the same time the division of current between stations may be adjusted so as to obtain either

A. Maximum electrical efficiency,

B. Maximum economy in operation,

C. Maximum utilization of apparatus.

Although all three of the above are desirable aims, it is rarely possible to obtain them simultaneously. For example operating for maximum electrical efficiency is simultaneous with operation for maximum economy only when the cost of energy delivered by station A to the load is equal to the cost of energy delivered by station B to the load. With equal energy cost, the division of current to obtain maximum efficiency depends entirely upon the relative losses in the two branches.

When the cost of energy in branch A and branch B, including the transmission line, differ materially, maximum economy is secured by shifting a portion of the load kilowatts from the station in which the cost of energy is greatest to the other station. This results in a reduction in efficiency but an increase in economy. As the division of reactive kilovolt-ampere for maximum economy is not affected to as great an extent by cost of energy, the currents flowing in the two branches for maximum economy are no longer in phase with the load.

Division of current, to obtain maximum economy in operation, is the desirable operating condition for fractional loads but when the total load demand is equal to, or approaches the full kilovolt-ampere of the system, the rating limitations of the apparatus and line may demand considerable departures from division of currents as determined by the consideration of maximum economy. In order to deliver a maximum output, the division of current must be such as to obtain the maximum utilization of the apparatus.

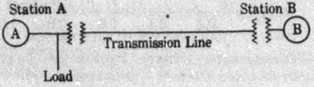

FIG. 1—ELEMENTARY DIAGRAM OF INTERCONNECTED SYSTEMS

It is a relatively simple matter to obtain a measure of the extent to which the use of ratio control increases the maximum load which a given system can deliver to a given point without sacrificing constant voltage at the two busses. This may be done by determining quantitatively the limitation which exists when the system is operated in which only throttle and generator field control are employed.

Voltage Relations. We shall assume that the load to be delivered may be concentrated on either bus of the two stations A and B, Fig. 1, and that it is desired to maintain voltages of busses A and B constant and independent of the load demand.

Assuming that the flow of power will sometimes be in one direction and sometimes reversed, the regulation drop in the transmission line and interconnecting transformers should be zero. Therefore, equating the expression for regulation to zero, we have approximately:

$$(\% I R) \cos \theta - (\% I R) \sin \theta = O$$

from which

$$\tan \theta = \frac{\% I R}{\% I R} \tag{1}$$

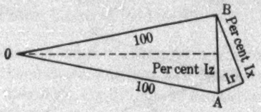

FIG. 2—VOLTAGE RELATIONS IN INTERCONNECTED SYSTEMS

O A—Equals voltage of bus near load,
O B—Equals voltage of bus far from load,
θ —Equals phase angle of current in transmission Line

From this formula it is evident that the maintenance of the two busses at constant voltage at all loads involves the operation of the transmission line at a leading power factor $\cos \theta$ the value of which is determined from the ratio of resistance and reactance of the line.

A more exact solution can be obtained graphically by plotting the vector diagram of the transmission line voltages, Fig. 2. The premises of the problem make the three voltages an isosceles triangle $O A B$ in which the impedance drop of the line is the base $A B$ and the sides are the bus voltages of the two stations. From this diagram we may write:

$$\sin \alpha = \frac{\% I Z}{200} \tag{2}$$

$$\tan (\theta - \alpha) = \frac{\% I R}{\% I X} \tag{3}$$

where

$O A$ = Voltage of bus near load,
$O B$ = Voltage of bus far from load,
2α = Phase angle between voltages $O A$ & $O B$,
θ = Phase angle of current in transmission line at load end,
$\% I R$ & $\% I X$ = Line constants including step-up and step-down transformers.

These equations are plotted in Fig. 4 by means of which it is possible to determine readily the phase angle θ of current in the interconnecting transmission line when the per cent impedance drop and the ratio of resistance to reactance is known.

Current Relations. The corresponding current relations can be easily derived from the vector diagram, Fig. 3, showing the current relations in terms of the power factor of the load being delivered and the power factor of the transmission line. In this it is assumed that the currents supplied by the two systems to the load are equal, under which condition the triangle of current $O\,B\,A$ is isosceles where $O\,A$ is the load current lagging an angle ϕ behind the voltage, $O\,B$ is the current delivered by the transmission line at the angle θ, and $B\,A$ is the current delivered by the local bus. A measure of the effectiveness of the transmission line and distant station in helping out the local station is determined by the ratio $O\,C\,/\,O\,B$, the point C being determined by making $C\,A$ and $B\,A$ equal to each other. This ratio, which may be called the transmission utility factor, can be expressed mathematically by the equation derived directly from Fig. 3.

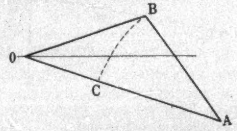

FIG. 3—CURRENT RELATIONS IN INTERCONNECTED SYSTEMS

$O\,B$—Equals current in transmission line,
$B\,A$—Equals current at Station "A",
$O\,A$—Equals load current
Load power factor $\cos\phi$ equals 95 per cent
Angle of lead in transmission line θ equals 20 deg.

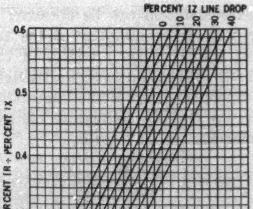

FIG. 4—RELATION BETWEEN TRANSMISSION LINE CHARACTERISTICS AND PHASE ANGLE FOR ZERO-LINE DROP

$$T_e = O\,C/O\,B = 2\cos(\theta + \phi) - 1 \qquad (4)$$

This equation is plotted in Fig. 5, from which for various values of phase angles of currents in the transmission line and for various power factors of load, the transmission utility factor can be determined.

By means of Figs. 4 and 5 it is possible to determine the resulting transmission utility factor, when the transmission line constants are known, for any power factor of load. The curves show exactly how much is sacrificed in order to obtain constant potential on the two busses.

Rating of Transmission Line Less than Generating Stations. In the preceding analysis it was assumed that the kilovolt-ampere rating of the transmission line was equal to the kilovolt-ampere rating of the generating stations, and for that reason the current in the transmission line and the current in the local generating station were made equal to each other. It may be, however, that the kilovolt-ampere rating of the transmission line is less than the rating of the stations, and for that reason, it becomes desirable to consider the case in which the division of load between the two stations is unequal. It is desirable, therefore, to have utility factor curves for

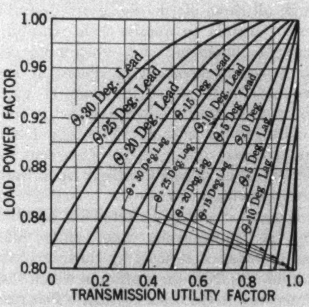

FIG. 5—TRANSMISSION UTILITY FACTOR OF TRANSMISSION SYSTEM AND DISTANT GENERATOR; f = 1.0

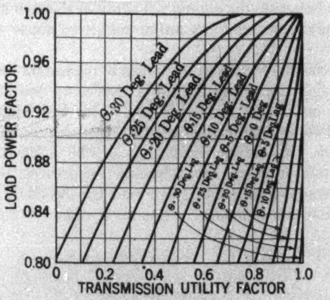

FIG. 6—TRANSMISSION UTILITY FACTOR OF TRANSMISSION SYSTEM AND DISTANT GENERATOR; f = 0.8

various ratios of current in transmission line to current in generator nearest to the load. Let

$$\frac{\text{Current in Transmission Line}}{\text{Current in Generator nearest to Load}} = F$$

The curves, Figs. 5, 6, and 7, correspond to values of $F = 1$, $F = 0.8$ and $F = 0.5$, respectively.

It is of interest to note that in all of these cases, Figs. 5, 6, and 7, that a transmission utility of 100 per cent is obtainable only when the currents in the transmission line and in the generators of both stations are all in phase with the load current, no matter what the power factor of the load may be, and thus it is evident that the maintenance of constant potential on the two busses cannot be obtained under the conditions assumed without sacrificing the utility factor.

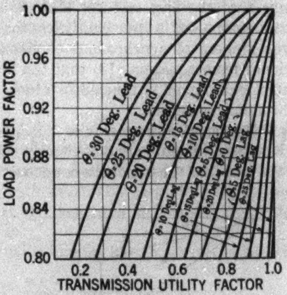

FIG. 7—TRANSMISSION UTILITY FACTOR OF TRANSMISSION SYSTEM AND DISTANT GENERATOR; $f = 0.5$

CASE II: ONE-WAY FLOW OF POWER—LINE DROP IN INTERCONNECTING LINE MAINTAINED CONSTANT

In case I, a two-way flow of power was assumed; consequently it was necessary to equate the line drop to zero. If the flow of power is in one direction only, it is sufficient that the line drop be maintained constant at all loads. A typical case is to assume 10 per cent line drop. For these assumptions, Fig. 8 has been determined giving the relations between the transmission line constants and phase angle of current in the transmission line. Fig. 8 is to be used in conjunction with transmission economy curves, Figs. 5, 6, and 7.

EXAMPLE FOR CASES I AND II

Transmission utility factors are determined in the following examples, in which constant voltage is maintained at two busses, by field control alone.[3]

Two stations, *A* and *B*, 14 miles apart, interconnected by a 66-kv. transmission line, have a rating of 80,000 kv-a. At this load, the line characteristics including step-up and step-down transformers are:

	Per Cent
Reactance drop	= 32.5
Resistance drop	= 7.5
Impedance drop	= 33.5
r/x	= 0.23
Power factor of load	= 85

To determine the value of using ratio control in connection with the above conditions, it is first necessary to examine the operating conditions or limitations which exist if the two station busses are maintained at constant potential by means of generator field control only, that is, without employing ratio control or other voltage regulating devices. These conditions impose upon the system the necessity of maintaining the current in the transmission line at a definite phase angle, the values of which, as determined from Figs. 4 and 5, are:

For two-way flow of power (Fig. 4) θ = 22-deg. leading

For one-way flow of power (Fig. 8) θ = 4-deg. leading

The values of transmission utility factor for various conditions of operation as determined from Figs. 5, 6, and 7 are given in column 4 of the accompanying table. This value is a measure of the usefulness of the distant station under the conditions assumed. It means that although in each case the distant generator and transmission line are loaded to the rated kilovolt-ampere of the line, they are effective only to the extent indicated by the percentages given in column 4.

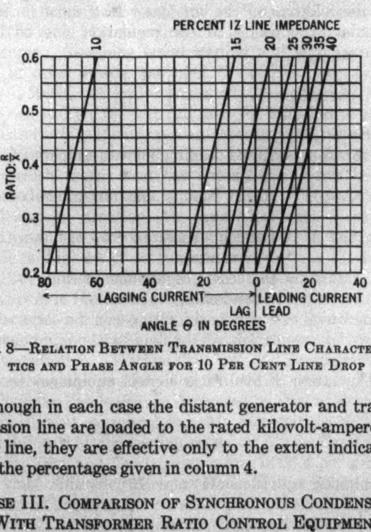

FIG. 8—RELATION BETWEEN TRANSMISSION LINE CHARACTERISTICS AND PHASE ANGLE FOR 10 PER CENT LINE DROP

CASE III. COMPARISON OF SYNCHRONOUS CONDENSERS WITH TRANSFORMER RATIO CONTROL EQUIPMENT

Comparison of Ratings. When used for the purpose of enabling the system to hold bus volt-

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[3] It should not be inferred that all of the assumed operating conditions in the above examples represent good practise. They are merely cited to show the inherent difficulty of operating with field control alone. In the first example where a transmission factor of 18 per cent is obtained, the power factor in the generator is so poor, about 15 per cent lagging, that it is doubtful whether the generator could deliver its full kilovolt-ampere.

CASE I: TWO-WAY FLOW OF POWER

Stations A and B Having Equal Ratings, Transmission Line Rating = 80,000 Kv-a.

1	2	3	4	
			Transmission Utility Factor	
	Rating station A and B	Ratio transmission line rating to station ratings	With field control only per cent	With ratio control only per cent
Fig. 5	80,000 kv-a.	1	18	100
Fig. 6	100,000 "	0.8	30	"
Fig. 7	160,000 "	0.5	42	"

CASE II: ONE-WAY FLOW OF POWER

Station B and Transmission Line Rating = 80,000 kv-a.

1	2	3	4	
			Transmission utility factor	
	Rating Station A	Ratio transmission line rating to station ratings	With field control only per cent	With ratio control only per cent
Fig. 5	80,000 kv-a.	1	62	100
Fig. 6	100,000 "	0.8	66	"
Fig. 7	160,000 "	0.5	72	"

ages at both ends of the line constant and independent of load changes, the synchronous condenser must compensate for the regulation of the line and interconnecting transformers. The condenser floating on the end of the line takes an additional wattless load the value of which, expressed in per cent of transmission line rating, is to be determined. The regulation due to the addition of the condenser load must be made equal and opposite to the regulation due to load. Hence

$$\left[\%\,(\text{Kv-a.})_c + \%\,(\text{Kv-a.})_c^1\right] \frac{\%\,I\,X}{100} = \%\,R \qquad \textbf{(11)}$$

where

$\%\,(\text{Kv-a.})_c$ = Rated leading kilovolt-ampere of condenser,

$\%\,(\text{Kv-a.})_c^1$ = Rated lagging kilovolt-ampere of condenser,

$\%\,I\,X$ = Per cent reactance drop of line without condenser,

$\%\,R$ = Per cent regulation of line without condenser,

from which the condenser rating can be determined when the regulation of the line, without condenser, is known.

The rating of load ratio control equipment for the same duty is:

$$\%\,(\text{Kv-a.})_{\text{RC}} = \frac{\%\,R}{2} \qquad \textbf{(12)}$$

Combining equations (**11**) and (**12**)

$$1/2\left[\%\,(\text{Kv-a.})_c + \%\,(\text{Kv-a.})_c^1\right] \frac{\%\,I\,X}{100} = \%\,(\text{Kv-a.})_{\text{RC}} \qquad \textbf{(13)}$$

which means that line reactance determines the relative size of condenser and ratio control apparatus for the same performance.

In equations (**12**) and (**13**) the fraction ½ is inserted assuming that the ratio control equipment has equal plus and minus ranges. This is generally, but not always, the case.

These equations determine the sum of the leading and lagging ratings of the condenser. They also show that as far as voltage control is concerned, it is immaterial how much of the sum is lagging and how much is leading.

Assuming that the first cost per kilovolt-ampere of ratio control equipment is appreciably less than the cost per kilovolt-ampere of synchronous condenser, it becomes evident from equation (**13**) that the first cost of ratio control equipment is inherently much less than the synchronous condenser.

Increased Output of System Due to Condenser. The costs of the *two* equipments are not directly comparable owing to the fact that the use of the condenser increases the output of the system. The amount can be determined from Fig. 9. As the maximum load is increased by the condenser current, the energy remaining constant, we can write:

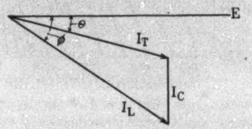

FIG. 9—CURRENT DIAGRAM SHOWING EFFECT OF SYNCHRONOUS CONDENSER ON TRANSMISSION LINE CURRENTS

$$(\text{Kv-a.})_{\text{L}} \cos \phi = (\text{Kv-a.})_{\text{T}} \cos \theta$$

or

$$(\text{Kv-a.})_{\text{L}} = (\text{Kv-a.})_{\text{T}} \frac{\cos \theta}{\cos \phi} \qquad \textbf{(14)}$$

where

$(\text{Kv-a.})_{\text{T}}$ = rating of generating plant,

$(\text{Kv-a.})_{\text{L}}$ = Kv-a. delivered to load,

which means that the ratio of the power factor of the load to the power factor of the line determines the increase in kilovolt-ampere of system on account of the condenser.

Perhaps the best way of obtaining an equable cost comparison between the use of a condenser and ratio control equipment is to determine the extra cost involved when ratio control is used to increase the system kilovolt-ampere by the ratio $\cos \theta / \cos \phi$. Ordinarily this merely involves providing the generators and interconnecting transformers with a correspondingly greater current carrying capacity. The increase in current in the overhead transmission line does not necessarily involve an increase in copper since the only practical effect of the increased current is greater line losses and increased regulation drop. An exception to this should be noted in the case of underground cables where the consideration of operating temperatures may make it undesirable to increase the maximum current. In this case the saving in cable equipment secured by employing the synchronous condensers may more than off-set its greater first cost.

Saving in Line Losses Obtained by Means of Synchronous Condenser. Consideration must also be given to the fact that the synchronous condenser, by improving the power factor of the line currents, decreases the copper losses in the line. The ratio of line copper losses with and without using the condenser is given by the expression:

$$\text{Loss ratio} = \left(\frac{\cos \phi}{\cos \theta}\right)^2 \qquad (\mathbf{15})$$

Improving, by condenser, the power factor from 80 per cent to 95 per cent, for example, reduces the line loss 25 per cent. Where poor power factor of load is combined with inherently large line losses, the saving in losses at full load due to the use of condensers is considerable.

The actual saving in loss, however, is less than may be inferred from the above, on account of the fact that under usual operating conditions the load factor is considerably less than unity. At fractional loads, the saving in losses resulting from the condenser is very much reduced, and at light loads, when the condenser is operating lagging, the line losses are actually increased by the condenser. It is reasonable to expect, therefore, that under normal conditions of load, the actual net saving in line losses is much less than the value obtained for full load conditions.

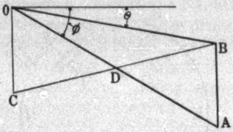

Fig. 10—Current Diagram for Fractional Loads showing Effect of Synchronous Condenser with Maximum Lagging Current Equal to Maximum Leading Current

From this it appears that the saving in line losses is very much affected by (a) the load factor and (b) the ratio of leading and lagging rating of the synchronous condenser.

Losses in Condenser vs. Losses in Ratio Control Apparatus. Furthermore, the net line losses saved by condenser are off-set partially by the fact that the transformer ratio control apparatus is inherently considerably more efficient than synchronous condensers, for several reasons:

1. On account of the smaller kilovolt-ampere rating of transformers,

2. On account of the inherently higher efficiency of transformers,

3. On account of the fact that in transformer ratio control, both the core and copper losses may be variable. The core loss varies from maximum to minimum when voltage ratio is shifted from maximum value to unity and the copper losses vary from maximum to minimum with changes in load.

To form a correct comparison between the two equipments, the actual values of the various losses just cited must be carefully estimated and capitalized.

EXAMPLE ILLUSTRATING CASE III

Assuming the following conditions:

	Per Cent
Line reactance drop	29
Line resistance drop	7
Power factor of load	86

it is desired to compare the use of ratio control equipment with a synchronous condenser when used to maintain voltage at both ends of line constant. Under these conditions the line regulation without the condenser is 22.7 per cent. By equation (**12**) the rating of ratio control equipment necessary to maintain voltage constant is 11.3 per cent of the kilovolt-ampere of the load.

By equation (**13**) the rating of the condenser is per cent $(\text{kv-a.})_c$ + per cent $(\text{kv-a.})_c^1$ = 70 per cent

The synchronous condenser performance depends upon the ratio of the leading and lagging rating of the condenser. Assuming equal leading and lagging ratings, in other words, per cent $(\text{kv-a.})_c$ = per cent $(\text{kv-a.})_c^1$ = 35 per cent, current relations are obtained as shown in Fig. 10. At full load the addition of the condenser brings the line power factor up to 99 per cent. The ratio of line power factor to load power factor is 1.14, which means that the system rating has been increased by 14 per cent, equation (**14**). The full load line losses by equation (**15**) are 77 per cent of what they would be without the condenser. The triangle *O B D* in Fig. 10 gives the current relations for full load; *O C* is the condenser current at no load; and the line *C D B* is the locus of the line current as the load increases from no-load to full load. At the intersect *D* corresponding to half load for Fig. 10, the condenser is not contributing. For less than half load the condenser is increasing the line losses, and for loads greater than half load the line losses are decreased by the condenser.

The current diagram, Fig. 11, corresponds to the assumption that the leading rating of the condenser is 46.5 per cent and the lagging rating is 23.5 per cent. The chief difference between Figs. 10 and 11 is the position of the intersect *D*, which now occurs at 33 per cent load. It is evident that operation, in accordance with the assumption upon which Fig. 11 is based, means a greater saving in line losses for a given load factor than operation in accordance with conditions assumed for Fig. 10.

CASE IV. TRANSMISSION LINE LOOP

Under cases I and II it was shown that by the help of one voltage control equipment constant voltage can be maintained at two points and at the same time any desired current relation obtained. The principle can be extended to any number of stations operating on a line so that with N stations and $N - 1$ voltage control equipments it is possible to maintain the bus voltage constant at N stations and at the same time secure any desired division of current in each portion of the line.

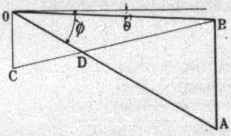

FIG. 11—CURRENT DIAGRAM FOR FRACTIONAL LOAD, SHOWING THE EFFECT OF SYNCHRONOUS CONDENSER WITH MAXIMUM LAGGING CURRENT 50 PER CENT OF MAXIMUM LEADING CURRENT

At each station voltage control may be inserted between high and low voltage of the power transformers as shown in Fig. 12, or the variable voltage may be inserted in series with the line as shown in Fig. 13. In the former case the over-all regulation of the line is unaffected, whereas in the latter case the variable voltage being inserted in the line itself, not only the station bus voltage but also the line voltage, is affected.

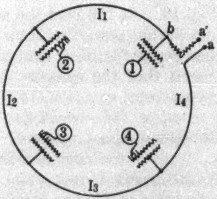

FIG. 12—TRANSMISSION LINE LOOP PROVIDED WITH IN-PHASE AND QUADRATURE VOLTAGE CONTROL

When the two ends of the lines a and b, in either Fig. 12 or 13, are connected together forming a loop, it is desirable to know whether complete current flexibility is still obtainable. It is evident for example, that if the currents in the four portions of the line, in Figs. 12 and 13, are to be independently variable, a variable voltage exists between a and b which is the resultant of the impedance drops in the four lines. This voltage (e) varies in value and in phase from time to time depending upon how much the currents are changing in the line. We may write for the voltage between a and b for Fig. 12

$$e = I_1 Z_1 + I_2 Z_2 + I_3 Z_3 + I_4 Z_4$$

where

$I_1 I_2 I_3 I_4$ are the currents desired in the lines;

$Z_1 Z_2 Z_3 Z_4$ are the corresponding line impedances.

For Fig. 13 the expression becomes:

$$e = I_1 Z_1 + I_2 Z_2 + I_3 Z_3 + I_4 Z_4 - e_2 - e_3 - e_4$$

where

e_2, e_3, e_4 are the control voltages inserted in the line at stations 2, 3, and 4.

If the loop is formed by connecting a and b without inserting any control equipment, the voltage (e), as given in the above equations, is short-circuited and a circulating current flows throughout the loop, the value of which is given by the expression,

$$I_c = e/(Z_1 + Z_2 + Z_3 + Z_4)$$

The line currents have now become

$$I_1 + I_c$$
$$I_2 + I_c$$
$$I_3 + I_c$$
$$I_4 + I_c$$

It is evident that the desired currents, I_1, I_2, I_3, I_4, can be obtained only in the remote coincidence that these currents happen to be of such values as to make the resultant voltage (e) zero. The alternative is to introduce into the line a voltage which is always equal and opposite to (e). Therefore, if the ends a and b are to be connected without interfering with current flexibility, it is necessary to bridge the points a and b with a voltage which is equal to, and in phase with, the vector sum of the impedance drops in the lines.

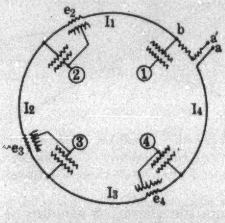

FIG. 13—TRANSMISSION LINE LOOP PROVIDED WITH IN-PHASE AND QUADRATURE VOLTAGE CONTROL

The insertion of such a low impedance voltage bridge between points a and b in which the voltage and also the phase angle are independently controllable can be accomplished by two ratio control equipments, one providing the proper in-phase voltage and the other providing the proper quadrature voltage. (See note following.)

In general, a transmission line loop in which a maximum current flexibility is desired together with N points maintained at constant potential, requires N voltage ratio controls and one quadrature control.

Although two independent controls, consisting of an in-phase and a quadrature control, are necessary to obtain complete flexibility, nevertheless, under particular conditions of load variation, one ratio control equipment is sufficient for loop control.[4]

For example, when the currents I_1, I_2, I_3, I_4 vary during the load cycle in such a manner that the resultant voltage (e) changes in value but not in phase, it is then only necessary to determine the phase angle of (e) with reference to the line voltage, and to design loop control equipment having this angle.

Loop control may be needed to prevent excessive current flowing in portions of circuit as, for example, when underground cables are used having a limited current rating, or it may be desired in order to be able at all loads to obtain a minimum of energy loss in the loop. In the latter case the exact setting of the control equipment can be calculated readily.

Current Distribution with Reference to Obtaining Minimum Energy Losses in Loop. At the end of this paper, equations are developed which give the condition for current distribution so as to obtain minimum energy losses in the loop. These equations show that the criterion for minimum losses is the condition that the vector sum of the resistance voltage drops around the loop should be zero. This criterion can always be met by means of two control equipments if they are so adjusted as to allow a circulating current to flow through the loop in accordance with

$$R\,I_c + \Sigma r\,I = 0$$

where

I_c is the circulating current,

R is the total resistance of the loop,

$\Sigma r\,I$ is the vector sum of the resistance drops (not including the drop due to circulating current).

To obtain this current distribution, the control equipments must introduce the following voltages into the line: An in-phase voltage equal to

$$e = j_1\,x_1 + j_2\,x_2 + j_3\,x_3 + \text{etc.}$$

and a quadrature voltage equal to

$$j\,e = i_1\,x_1 + i_2\,x_2 + i_3\,x_3 + \text{etc.}$$

An important particular case is when the ratio of resistance to reactance in each portion of the line is equal; that is, when

$$\frac{x_1}{r_1} = \frac{x_2}{r_2} = \frac{x_3}{r_3} = \frac{X}{R}$$

where R and X are the total resistance and reactance of the loop. Then

$$x_1 = \frac{r_1}{R}X \quad x_2 = \frac{r_2}{R}X \quad x_3 = \frac{r_3}{R}X$$

When these values are substituted in the equations for e and $j\,e$, they reduce to the following simple form:

$$j\,e = \sum r\,i\frac{X}{R}$$

$$e = \sum j\,r\frac{X}{R}$$

But by the conditions of the problem $\Sigma i\,r = 0$ and $\Sigma j\,r = 0$. It therefore follows that both e and $j\,e$ are zero. The important conclusion follows that when the ratios of resistance to reactance in each portion of the loop are all equal to each other, the currents distribute in the loop so as to obtain minimum copper loss, when the loop is closed without voltage inserted. Loop control equipment is therefore necessary only by virtue of the fact that the various portions of line have different ratios of resistance to reactance.

Acknowledgment is hereby made to Raymond Bailey, P. J. Walton, W. W. Lewis, H. O. Woods and M. B. Mallett for their assistance in the preparation of this paper.

An appendix with equations to find the current distribution in a loop with reference to obtaining minimum energy losses in a loop is included in the complete paper.

Appendix

To find the current distribution in a loop with reference to obtaining minimum energy losses in loop. Copper losses in loop for any current distribution such as shown in Fig. 12 can be written

$$L = r_1\,i_1^2 + r_2\,i_2^2 + r_3\,i_3^2 + \text{etc.}$$
$$+ r_1\,j_1^2 + r_2\,j_2^2 + r_3\,j_3^2 + \text{etc.}$$

With a circulating current $i_c + j_c$, the copper losses become

$$L = r_1\,(i_1 + i_c)^2 + r_2\,(i_2 + i_c)^2 + r_3\,(i_3 + i_c)^2 + \text{etc.}$$
$$+ r_1\,(j_1 + j_c)^2 + r_2\,(j_2 + j_c)^2 + r_3\,(j_3 + j_c)^2 + \text{etc.}$$

Expanding:

$$L = r_1\,i_1^2 + r_2\,i_2^2 + r_3\,i_3^2 + \text{etc.}$$
$$+ R\,i_c^2 + 2(r_1\,i_1 + r_2\,i_2 + r_3\,i_3 + \text{etc.})\,i_c$$
$$+ r_1\,j_1^2 + r_2\,j_2^2 + r_3\,j_3^2 + \text{etc.}$$
$$+ R\,j_c^2 + 2\,(r_1\,j_1 + r_2\,j_2 + r_3\,j_3 + \text{etc.})\,i_c$$

where R = total resistance of loop.

---

[4] Loop control can also be obtained by means of two polyphase inductive regulators, the series windings being mounted in the line and connected in series with each other. Each regulator inserts in the line a constant voltage with phase angle variable from zero to 360 deg. By adjusting their phase angles, the combined voltage introduced into the line can be made any desired value and any angle.

To find circulating current for minimum loss:

$$\frac{dL}{di_c} = 2Ri_c + 2(r_1 i_1 + r_2 i_2 + r_3 i_3 + \text{etc.}) = 0$$

$$\frac{dL}{dj_c} = 2Rj_c + 2(r_1 j_1 + r_2 j_2 + r_3 j_3 + \text{etc.}) = 0$$

Therefore

$$i_c = -\frac{r_1 i_1 + r_2 i_2 + r_3 i_3 + \text{etc.}}{R}$$

$$j_c = -\frac{r_1 j_1 + r_2 j_2 + r_3 j_3 + \text{etc.}}{R}$$

are the in-phase and quadrature components of circulating current which, added to currents $i_1 + j_1$, $i_2 + j_2$, $i_3 + j_3$, etc., result in a minimum loop loss. These equations may be written:

$$i_c = -\frac{\Sigma\, r\, i}{R} \qquad (1)$$

$$j_c = -\frac{\Sigma\, r\, j}{R} \qquad (2)$$

These equations mean that minimum losses are obtained in a loop when a circulating current is allowed to flow equal to the vector sum of the resistance drops due to the original currents, divided by the total resistance of the loop.

These equations may also be written:

$$R\, i_c + \Sigma\, r\, i = 0 \qquad (3)$$

$$R\, j_c + \Sigma\, r\, j = 0 \qquad (4)$$

or

$$\Sigma\, r\,(i + i_c) = 0 \qquad (5)$$

$$\Sigma\, r\,(j + j_c) = 0 \qquad (6)$$

From the last equation it is evident that the condition for minimum loss in a loop is a current distribution in which the vector sum of the resistance drop is zero. It is evident from these equations that in a d-c. loop or in an inductance less a-c. loop, the currents naturally divide so as to give the minimum copper loss.

In a loop with inductance, it is necessary to determine the summation of inductance drops under the conditions of minimum loss as given by equations (5) and (6). The summation of reactance volts when circulating current i_c flows is for the in-phase currents:

$$j\,e = (i_1 + i_c)\,X_1 + (i_2 + i_c)\,X_2 + (i_3 + i_c)\,X_3 + \text{etc.}$$

and for the quadrature currents

$$e = (j_1 + j_c)\,X_1 + (j_2 + j_c)\,X_2 + (j_3 + j_c)\,X_3 + \text{etc.}$$

Substituting equations (1) and (2)

$$j\,e = \left(i_1 - \frac{\Sigma\, i\, r}{R}\right) X_1 + \left(i_2 - \frac{\Sigma\, i\, r}{R}\right) X_2 + \left(i_3 - \frac{\Sigma\, i\, r}{R}\right) X_3 + \text{etc.} \qquad (7)$$

$$e = \left(j_1 - \frac{\Sigma\, j\, r}{R}\right) X_1 + \left(j_2 - \frac{\Sigma\, j\, r}{R}\right) X_2 + \left(j_3 - \frac{\Sigma\, j\, r}{R}\right) X_3 + \text{etc.} \qquad (8)$$

which gives the resultant or unneutralized voltage $(e + j\,e)$. In order to maintain this distribution of current, a voltage equal and opposite to $e + j\,e$ must be inserted in the loop.

In the particular case when

$$\frac{x_1}{r_1} = \frac{x_2}{r_2} = \frac{x_3}{r_3} = \frac{X}{R}$$

where

X = total inductance of loop,

R = total resistance of loop,

then

$$x_1 = \frac{r_1}{R}\,X$$

$$x_2 = \frac{r_2}{R}\,X$$

$$x_3 = \frac{r_3}{R}\,X \text{ etc.}$$

Substituting the above in equations (7) and (8), both e and $j\,e$ reduce to zero from which the important conclusion follows that if ratio inductance to resistance in all portions of the loop is equal, current distribution for minimum loss naturally occurs and therefore loop control is unnecessary.

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Discussion

CHANGING TAPS ON TRANSFORMERS UNDER LOAD

(Hill)

CHARACTERISTICS OF INTERCONNECTED POWER SYSTEMS AS AFFECTED BY TRANSFORMER RATIO CONTROL

(Blume)

E. F. Gehrkens: The subject of voltage control is an exceedingly important one and it was so recognized as far back as 1882, at which time an automatic generator voltage regulator for d-c. machines was developed.

When the a-c. system was introduced and large generating units were built, two types of *feeder* voltage regulators were developed,—the switch or step type and the induction type.

Each design has its advantages and disadvantages, and in order to eliminate the disadvantages so far as possible, as long ago as 1898, a combination of the two was designed and built for the control of electric furnaces.[3]

This combination is justified only in case large regulating units are required, and the delay in the development in this design was due partly to the lack of sufficient demand to warrant the expense and to difficulties of design; for although the theory and connection diagram are simple, the mechanical design is extremely difficult. Previous to 1898 induction regulators having capacities of 700 kw. were also built for the same purpose and while the latter was the more simple in design and operation, the former was less expensive and more efficient with a better power factor.

For lighting systems requiring relatively small regulators, the induction type is the recognized standard because of its simplicity and low cost, but for power and interconnecting lines requiring large regulators, the selection of the regulator depends upon the requirements and the cost. In design, the induction type is limited to approximately 13,000 volts, and for use on higher-voltage systems it requires an exciting and a series insulating transformer. This increases the cost and the losses. The switch type can be built for use in much higher voltage circuits and therefore has the advantage.

In the combination of the switch and induction types, the switch element may be used in a high-voltage circuit, but the induction element requires the insulating transformers or transformer windings. The choice between the switch and the combination type of regulator depends again upon the requirements. On long single lines, or even on long multiple interconnections, it would seem that the switch type should be satisfactory because of the high resistance and reactance of such lines, but for short lines or a combination of short and long lines, a more uniform voltage change may be required. In case the regulators are operated by hand or motor, a finer adjustment can be obtained by the combination type, but in case this type is operated automatically, about as close a regulation as can be obtained is one per cent either way from normal; that is, the combination type of regulator corresponds to the switch type having one per cent steps, and on this basis the deciding factor is probably the price.

The switch or step type is certainly the simpler in construction and has a higher efficiency and should therefore be used wherever conditions warrant.

Mention was made of the desirability of in-phase and out-of-phase control for parallel connecting lines. The results obtainable by the use of two polyphase induction regulators were fully illustrated and described in the London *Electrician* of Nov. 6, 1914, but the system has not been generally adopted by operating companies because of the cost and the difficulty of making this arrangement automatic. However, as the multiplying of lines increases and the necessity of controlling both the wattless and the power current in such lines is recognized, this arrangement will undoubtedly be more generally used.

F. F. Brand: Mr. Hill has outlined very well in his paper the different methods in use for changing transformer taps under load, and he has brought out the difference between the two principal schemes—the one of using parallel circuits, and the other of using the preventive reactance or auto-transformer.

I am inclined to take issue with him on the application of two different schemes, as I would consider that the scheme of using the preventive auto-transformer to derive mid-voltage between taps is more applicable to small transformers, and the other scheme of using parallel circuits is better adapted to large transformers.

For a 20,000-kv-a. transformer with taps 5 per cent apart, and with a preventive reactance used to derive the mid-point, the nominal size of the reactor is approximately 1000 kv-a., and the losses in the reactor would be of the order of 10 to 20 kilowatts. Furthermore, on this scheme in those positions in which the preventive auto-transformer is used to derive the mid-voltage, there exists a circulating current which is constant irrespective of load conditions, and the losses in the reactor and that portion of the winding through which the circulating current flows must be considered a part of the light-load transformer loss.

Also the use of the preventive auto-transformer to derive the mid-point does not give equal voltage steps, and a difference in the voltage derived from the true mid-point is greater the lower the power factor. For instance, at 80 per cent power factor and full load, the voltage actually derived is of the order of 1¾ per cent from one tap and 3¼ per cent from the other tap instead of 2½ per cent from each.

Better conditions of obtaining the true voltage required are obtained if the circulating current is increased, but this obviously increases the losses in the tap portion of the transformer, the size of the tap portion, and the size and losses of the auto-transformer.

It is quite true that the preventive auto-transformer scheme enables a slightly simpler winding to be used than parallel circuits, although it should be understood that it is not necessary in the parallel-circuit scheme to provide two complete parallel paths throughout the total winding. It is necessary to parallel only a portion of the winding to obtain sufficient reactance between these portions to limit the circulating current to a reasonable value in the transfer position. In fact, modern practise is to use a single-circuit winding including the tap portion and a parallel-circuit portion sufficient to obtain necessary reactance between these parallel portions. There are, therefore, only nine taps on a winding designed to give nine voltages, each tap being carried through two ratio adjusters respectively connected in circuit with the parallel portions.

When it is considered that in order to obtain the necessary reactance control, paralleling portions of the winding is undesirable, the preventive reactance, or auto-transformer, connected externally to the transformer winding, may be used. Ratio adjusters are used to select the taps, and the normal operating position is with both ratio adjusters on one tap. The current then divides in the halves of the preventive reactance, and the ampere-turns neutralize one another; therefore, the voltage drop is practically negligible. This scheme cuts the size of the preventive reactor to half that of when the reactor is used to derive the mid-voltage between taps. The losses are therefore less, and circulating current exists only at the time of transfer.

The question of whether or not it is desirable to reduce the number of operations depends principally upon the reliability of the apparatus to accomplish the tap changing. To my mind, the elimination of circulating-current losses at periods of light load, and the fact that one obtains an exactly equal division in voltages by the plan when the normal operating position is always at both ratio adjusters on one tap, far outweighs the slightly more complicated operating mechanism.

Referring to Mr. Blume's paper—it seems to me there is a good opportunity for the combination of load-ratio control transformers and synchronous condensers on transmission-line work. As he has pointed out, there are certain advantages in each type of control. The synchronous condenser has the advantage of controlling power factor, and therefore the possibility of reducing the losses. If used in combination with variable-ratio transformers, it is quite possible to reduce the losses under certain operating conditions; for instance, on the light-load condition, a synchronous condenser, if used to control the voltage draws lagging current, increases the line current and line losses, and the losses in the condenser of course increase as the current increases. When using a variable-ratio transformer, it should be quite possible to do away with lagging requirements of a condenser, except, perhaps, when it is necessary to hold down voltage on a long, lightly loaded, high-voltage line, when the voltage increases due to capacity charging current.

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[3] See *Electrical World*, January 7, 1899.

With a variable-ratio transformer it is possible to reduce leading kv-a. requirements of the condenser. That is, in the case of lines of very poor regulation it is not necessary to raise the power factor to unity or to leading in order to maintain proper voltage regulation. The combination of these two, that is condenser and variable-ratio transformer, may easily result in both cheaper equipment and very much lower losses over the average operating conditions.

H. O. Stephens: At first thought, one would be inclined to question why so many different methods of changing transformer voltage under load are used and if it would not be better and more economical in the long run to standardize upon one method? This is a perfectly logical question, but the very nature of the object to be accomplished makes it difficult to standardize upon one scheme that would be universally applicable. The variations in voltage and current are so great that one type of circuit-interrupting device can hardly be used throughout the range and if the voltage as well as the current to be interrupted is high, it is desirable to keep the number of circuit-interrupting devices as small as possible for the sake of economy and size.

Additional complications, such as auto-transformer connections, tertiary windings, etc., that may be introduced into the transformer, demand that the designer be given considerable latitude in his choice of equipment. However, it is very desirable to standardize upon as few methods as possible and considerable help in this direction can be given by the users of the apparatus by slightly modifying their requirements to meet the exigencies of design.

There is a tendency among certain operators to demand extremely small voltage steps on transformers equipped with load-ratio control which I believe, in most cases, is not justified. Even when automatic operation is specified, taps in steps less than 2½ per cent can hardly be considered necessary. It should be remembered that with 2½ per cent taps, the voltage can be adjusted to within 1¼ per cent of the desired value.

An induction regulator, which is generally considered to be a very satisfactory voltage-regulating device, requires at least a plus or minus variation of 1 per cent from the desired voltage for the contact-making voltmeters to function, and usually this difference is 1½ to 2 per cent, so that the transformer with 2½ per cent taps can be operated to give virtually as good regulation of voltage as an induction regulator. For this reason I believe the extreme refinement supposedly accomplished by the use of the so-called "step induction regulator" with a transformer with taps is an unnecessary refinement. When desired, a very small percentage in tap variation can readily be accomplished by the multiple winding scheme of transformer operation.

W. M. Dann: I think Mr. Stephens has hit the nail very squarely on the head when he talks about standardization of tap-changing-under-load equipment. Of course, this field of tap-changing under load is really in its infancy in spite of the pioneer equipments that have been in use for several years. The trend in the design has been very much toward simplification and standardization. I am very much in sympathy with Mr. Stephens' remarks about the demands of the industry itself. If we can get the operators to simplify some of those demands and make use of equipment that has already been designed and installed, it will help very materially towards that ideal of standardization.

K. A. Oplinger: It is unfortunate that no standard terminology exists upon the general subject of changing taps on transformers under load. Equipment for this purpose is usually called a tap changer but at other times the term "ratio adjustor" is used. The N. E. L. A. has started to collect data on equipment for controlling the voltage ratio of transformers under load and already a number of reports has been published covering these data. Although this equipment has developed very rapidly during the past two years, it is now fairly well standardized and a uniform terminology would be very desirable.

For example, the single-winding method, as described in the reports mentioned above, appears under the title of "multiple-switch method." This method has also been called the "preventive-auto-transformer method" because a preventive auto-transformer or protective reactance is switched along the transformer taps. This method was one of the first employed to change transformer taps under load.

The term "single-winding method" appears to be most appropriate for this particular scheme as it is comparable with the term "double-circuit method" which requires double windings on the transformer.

With reference to the automatic control of tap changers, an interesting application is under consideration at the present time in which the tap changer will automatically control the flow of wattless kv-a. between two generating stations in inverse proportion to the load on the two stations. A transformer with equipment for changing taps under load is to be used as a tie-in between the two stations and as the load on one station is increased the tap changer will automatically operate to transfer the wattless kv-a. to the other station. This same control might also be arranged to regulate the wattless kv-a. in direct proportion to the load of the two stations.

Other methods of automatic control may be arranged to have the tap changer limit the exchange of wattless kv-a. between stations or to have the tap changer responsive to changes in power factor.

J. S. Lennox: Mr. Hill brought out one point that I think is a very good one, and that is that with the parallel-winding method, the transformer must be protected against an accidental stoppage of the mechanism leaving a load on one-half of the winding. It is for that reason, I think, that the parallel-circuit method has developed into what Mr. Hill calls a single-winding method, but distinguished from the arrangement that Mr. Hill has described in his paper, by the fact that the circuits are broken in a separate device from the one that changes the tap connections, and by so doing, it is possible to place inside the transformer adjacent to the windings, a tap device of such simple and sturdy design that it involves no maintenance problem, and permits the use of small steps. Therefore, there is low deterioration of the contacts in the contact-making device placed outside the transformer tank and easily accessible for renewal of contacts.

L. H. Hill: Mr. Brand made a comparison of the two methods employed, the single-winding method and parallel-winding method, and he mentioned the increase in loss by the use of the preventive auto-transformer. It is obviously true that additional loss will be introduced by the introduction of additional apparatus. However, with the parallel-winding method, additional loss is also introduced in a more indirect manner. Reactance is required to effect the tap change regardless of the method used, so that if the parallel-winding method is used the windings must be so arranged and so interlaced that sufficient reactance will be obtained between them to limit the current to a reasonable value during the transition period. That ordinarily means making the core of the main transformer larger and naturally introduces additional loss, although as a general thing it is true that it is not as large as would be obtained by means of the preventive auto-transformer.

He also mentioned that in the single-winding method using the simplified preventive auto-transformer, unequal steps were obtained. Naturally, when the equipment is operating with one-half of the preventive auto-transformer in series with the line, it acts as a small series reactor. Since the drop is almost entirely reactive it is subtracted from the theoretical tap voltage at right angles, which at the higher power factors does not cause any appreciable unequality in tap voltages. The difference in reactance obtained on the two different operating positions of the preventive auto-transformer must be taken into account, however, under certain conditions of paralleling.

For example, assume a transformer with four per cent impedance, and with a two-one-half per cent tap, or five per cent in voltage across the whole winding of the preventive auto-transformer. The impedance volts at full load through one-half the auto-transformer in series with the line would be about one per cent or an increase in reactance from four to five per cent on alternate taps, which would not give good paralleling conditions with a transformer which did not use the preventive auto-transformer. However, with a transformer with eight per cent impedance, the change from eight to nine per cent impedance would be proportionally less, and can be compensated for by making the impedance of the main transformer winding midway between the two values, giving good paralleling conditions.

In cases where very close paralleling is required or even assuming conditions where the transformers are to be operated at very low power factor, when the impedance drops and unequality of steps might be of some importance, it becomes a simple matter to use the short-circuiting switch shown in Fig. 3 of my paper. In that case, the series impedance drop is eliminated entirely.

These things are merely a matter of detail design, and we feel the same as Mr. Lennox, that the two methods, single- or parallel-winding, could be changed from one to the other at will. The point is that the method using a preventive auto-transformer is fundamentally a single-winding method as distinguished from a parallel-winding method. A number of variations of the single-winding method is possible. For conditions of ordinary design, the range in taps to be obtained is generally not more than 20 or possibly 25 per cent in steps which ordinarily would not require more than 6 circuit breakers.

If it is desired to cover a very wide range, for example, 100 per cent range by this method, it is true it would not be desirable to add 5 or 6 more switches, as it would make a large and expensive mechanism. In this case, the switches in each tap lead may be contactors or some sort of ratio adjustors with auxiliary circuit breakers in the two end connections to the preventive auto-transformer to open and close the circuits. Even with this type of equipment the few taps required by the single-winding method make it possible to place all switches outside of the main tank, so that all moving parts are external to the main case.

In all the equipment which we have built with the exception of the early installation using the parallel-winding method, all switches were outside of the tank, and our policy in now building the single-winding method exclusively is based on experience with both types. We have built these 20,000-kv-a. single-phase units, which are large units, with the parallel-winding method, and we have built units up to 25,000 kv-a. with a single-winding method, and we are now building some 33,000 kv-a. single-phase units using the single-winding method.

A comparison of the parallel-winding method and single-winding method when used to obtain a typical range of eight, two one-half per cent taps may be of interest.

With the single-winding method five taps and five switches are required. With the parallel-winding method, nine taps are required in each winding or a total of 18, making almost four times as many taps and switches.

The measure of the inherent reliability of equipment for a certain performance is based on the amount of apparatus and moving parts required and the amount of operations of that equipment, and the amount of circuit interruptions of that equipment are a measure of the maintenance required and performance to be expected over a long period of time. To change taps by means of the parallel-winding method there are required at least six switch operations, whereas but one is required with the single-winding method, or in case the short-circuiting switch is used, there are but two.

In the case of the parallel-winding method there are two circuit interruptions per tap change. In the case of the single-winding method a circuit is closed instead of opened on every alternate tap change giving the equivalent of one-half a circuit interruption per tap change over the whole range.

With the parallel-winding method, it is not commercially practical to build each of the windings capable of carrying the whole load current continuously. Each winding is overloaded during the tap-changing operation and a protective system is needed to protect the windings in case of accidental stoppage of the equipment during a tap-changing operation. With the single-winding method no winding is overloaded during the tap-changing operation.

The few taps and switches needed with the single-winding method permit mounting all moving parts outside of the main tank, which is desirable from the standpoints of maintenance, operation, and safety.

The equipment developed for use with the single-winding method is very simple. Proper sequence of all switch operations is assured by positive mechanical drive using simple gears and linkages.

In the parallel-winding method, there is a great amount of mechanism needed inside the tank which must be connected to the operating mechanism outside, in order to assure proper sequence of operations between the internal switches and the external circuit breakers.

The statement has been made that the parallel-winding method is more applicable to large units. I have already mentioned some of the large units which have been built using the single-winding method. We recently built some three-winding units rated at 20,000 kv-a. water-cooled (25,000 kv-a. forced-cooled), single-phase on each winding. They were arranged to change taps under load on two windings, one 66-kv. and the other 132-kv. solidly grounded. To change taps with the single-winding method, five switches are used in the grounded end of each winding. The transformer has but few taps in it, a single winding, (no complicated interlacing of paralleled sections) and the two preventive auto-transformers are mounted at each end of the core, making a very simple arrangement for handling one of the largest and most complicated cases of tap changing under load likely to be encountered.

With the parallel-winding method a very complicated transformer with a great number of taps would result which would be very difficult to bring out. It would be necessary to interlace two 132,000-volt windings and two 66,000-volt windings, giving the equivalent of a 5-winding transformer, which is a simple 3-winding transformer when the single-winding method is used.

The separate regulating unit has been proposed as being desirable for such cases and that is undoubtedly true if the parallel-winding method is used, because it is very undesirable, in fact impracticable, to build such a transformer with the parallel-winding method while it is entirely practicable and simple to do it with the single-winding method. The single-winding method can be applied to any size and rating of transformer by the use of the series transformer for very high-voltage applications.

The separate regulating unit has certain definite fields of application, for example, where it is desired to use the voltage-control equipment on different parts of the system, or where the main units are already installed and additional control is desired. However, in the ordinary installation of single-phase units the tap changers can be put on each of these units including the spare, giving a very simple, reliable, and efficient form of installation.

It is true that when the spare transformer is used there are four separate mechanisms to operate with each bank. If a separate three-phase regulating transformer is used, as was mentioned, it is necessary to install underground or overhead structures to connect to the main units which, of course, increases the installation costs and space required and introduces far more losses than the preventive auto-transformer introduces with the single-winding method on the main transformers. The separate regulating unit generally has two cores, an exciting transformer and a series transformer, which obviously are much larger than in the auto-transformer used with the single-winding method, and naturally introduce a very considerable operating loss.

The regulating unit has the advantage that it does keep all the moving parts outside of the main transformer case, which is very desirable. But, the same thing is done with the single-winding method on the main units. The three-phase regulating unit has the additional drawback, that no spare tap-changing equipment is available as is the case when four single-phase transformers or seven single-phase units are used. Spare tap-changing equipment is becoming almost as important as spare capacity for the transformer.

There is one other place where the separate regulating unit might be desirable. In case the three-winding unit first mentioned had been arranged with an ungrounded neutral, so that it would be necessary to use the series transformer, in order to keep the tap-changer voltage down to a low value, then it would require two series transformers, in each winding. The series transformers are ordinarily placed inside the tank, along with the preventive auto-transformer and that would make quite a large number of transformers all in one tank. In the case of such a very special application it might be desirable to use a separate regulating unit, but even then, the single-phase regulating unit for the reasons just mentioned, has some advantage over the three-phase unit. Single-phase units of that type are now being built.

Mr. Stephens mentioned that on account of the steps required, it seems hardly necessary to use the step induction-regulator equipment. That is true perhaps in many cases from that point of view, although on account of the fact that no circuit interruptions are required for a tap change it is very good from a maintenance point of view, since it may be operated very frequently without any deterioration on the contacts or oil as would be the case with any step type of equipment; for this reason it is well suited for electric furnace operations or in synchronous-converter or certain automatic-substation applications.

L. F. Blume: In the paper by Mr. Hill, mention is made of the use of a preventive auto-transformer bridging across adjacent transformer taps. This auto-transformer is described as being physically of a relatively small size, but designed to operate at high-flux density. Operating with one contactor open, with the entire load current flowing in one winding of the auto-transformer, the saturation of the iron in the reactor prevents excessive voltage drop.

I am inclined to believe that this method of operation is open to the objection that the saturation of the iron in apparatus connected in series with a circuit introduces serious wave distortion in the line voltage.

Before coming to the conclusion that the arrangement having the fewer number of taps is to be preferred to the one using a greater number, consideration must be given to a number of advantages which are obtainable by the use of more taps. First, by increasing the number of taps, it becomes possible to reduce the voltage short-circuited and also makes it possible to avoid having a circulating current in an operating position, both of which materially add to the electrical efficiency.

Second, increasing the number of taps reduces the energy which must be ruptured, and on this account both the wear on contacts and the deterioration of oil are effectively reduced. The result of this is that contacts and oil must be changed less often when more taps are employed.

Although these arguments are negligible when ratio control is used for relatively small kv-a., in the larger equipment they become very appreciable and cannot be ignored.

P. H. Thomas: The tap-changing transformer is in many ways a substitute for the synchronous condenser. It is a simple, cheap, and effective method of accomplishing the same results under certain conditions. The great value of our synchronous type of apparatus, however, is its automatic quality. We can set it in a system with the proper regulators and then changes that have to be made due to changes in load and accidental conditions will occur automatically. In the most modern apparatus or in apparatus which is to be built hereafter, they occur sufficiently quickly to prevent synchronous apparatus dropping out of step.

This is an entirely different thing from adjusting conditions by hand to reduce power factor or to get a satisfactory voltage for customer circuits.

To have the tap-changing outfit the equivalent of the synchronous condenser for this sort of work, it is necessary that it be able to operate quickly enough to produce the necessary changes before the loss of synchronism and other things which we fear when sudden load conditions occur. That is one of the sides of the question that must be studied very carefully, the speed of action and the automatic control of the tap changes. It may be in response to voltage; it may be in response to power transmitted over some circuit; it may be in response to power factor; it may take account of positive and negative power factor; it may be dependent upon circuit-breaker operations, or many other conditions.—according to the particular place in which the tap changing is to be used.

Take, for example, a tie line between two large systems which must pass current backward and forward, according to the variations of load, perhaps at different times of the day. That is an exceedingly exacting duty on a transmission line and transformers and calls either for a wide change of power factor if the voltage is to be maintained constant at the two ends with a change in direction of the load, or a change of tap ratios.

If the large systems are to operate successfully in parallel, relying on the use of tap changers to control the interchange of power backward and forward, without disturbing voltages, they must act very rapidly, and they must act under the control of the necessary factor to produce such a result.

I don't know just how successful the present apparatus will be in that sort of thing, and I wonder if some of the authors could give us a little information as to what extent the tap-changing operations can be counted upon to meet accidental and automatically controlled changes in load.

L. H. Hill: Many types of automatic control to meet more or less steady-state conditions have been developed. The only installations in service that I know of are responsive only to voltage changes. The six 12,000-kv-a. units, illustrated in Fig. 6 of my paper, are operating under automatic control and maintain the bus pressure approximately constant. Under automatic control the tap-changing equipment is of course subjected to considerably more severe duty than would be met with under ordinary manual control. Under manual control, the tap-changing equipments in operation generally average not more than five operations a day, whereas under automatic control they may operate ten times as many.

To eliminate unnecessary operations and avoid hunting it is desirable to insert some time delay, the length depending on the application.

It is also perfectly feasible to work out automatic control to transfer the power at a constant power factor.

Mr. Oplinger mentioned a case where the tap changer will automatically control the flow of wattless kv-a. between two generating stations in inverse proportion to the load on the two stations. There is almost an infinite number of arrangements that can be used for automatic control under these more or less steady-state conditions. However, for the purpose of using tap-changing equipment to effect the stability of a system under transient conditions, the present equipment is hardly adequate, because the ordinary tap-changing operation requires a matter of seconds. With our equipment, it requires about four sec. to change one tap. It would be entirely practicable to increase this speed perhaps to one sec., which would even then be hardly short enough to do much good under transient conditions.

Mr. Blume in his later discussion mentioned that when the auto-transformer is connected in series with the line it might give trouble, due to harmonics. I think Mr. Blume is confusing some of the earlier auto-transformers which we used and which were provided with the short-circuiting switch. When the short-circuiting switch is used the auto-transformer can be used working high on a saturation curve, so that it saturates under double-load conditions but in that case it is short-circuited when in series with the line, so there is no voltage available to put harmonics on the line.

When the auto-transformer is used in this manner without a short-circuit switch, then as mentioned on the third page of my paper, the auto-transformer is provided with air-gaps in its core, so that the core does not saturate and a straight-line characteristic is obtained.

L. F. Blume: I wonder whether Mr. Thomas in his discussion did not have in mind more particularly the longer high-voltage lines. As far as I know, the use of tap-changing equipments has been applied for the most part to the shorter lines, by which stations not so very far apart are interconnected, where the power to be transmitted is large, and where the reactance between stations is not great, as it is in very long-distance transmission lines.

In long-distance transmission, the effect of capacitance and of line reactance introduces problems which were not considered in the paper. It was assumed that line capacitance could be neglected, and that the amount of line reactance was insufficient to introduce the question of stability.

In the longer lines, where the reactance is high, the paper shows that the higher reactance means that the kv-a. required for a synchronous condenser to maintain constant voltage is proportionally smaller than ratio-control equipments. For example, comparing a low-reactance line with a high-reactance line, the size of the synchronous condenser required to maintain constant voltage with increasing reactance does not increase in kv-a., but the size of the ratio-control apparatus is directly proportional to the reactance.

So, for the longer lines, ratio-control apparatus is not only relatively more costly, but also on account of other considerations, such as stability, is probably unsuitable.

Studies on Sparking in Air

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***Synopsis.**—Sparking voltages and gradients in air between spheres of unequal diameters and between two cylinders placed with their axes perpendicular to each other in space (called cross-cylinders) have been determined. Complete tables giving the coefficients of gradient corresponding to any given spacing and radii, have been tabulated, the formulas used being those of Kirchhoff and Russell. For cross-cylinders, an approximate formula has been developed.*

In the theoretical discussions, the more definite form of the theory of ionization by collision as given by Bergen Davis has been extended to the sphere-gap and a relation has been deduced to account for the variation of the sparking gradient as investigated in the first part of this work, which is in substantial agreement with those found by F. W. Peek, A. Russell, and others.

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INTRODUCTION

SPARKING voltages and gradients between needle-points and spheres have been measured by many investigators. It has been theoretically shown[1] that the relation between the sparking voltage and the spacing, when the spacing is so large that corona precedes a spark-over, should conform to a linear law. Such is, in fact, the case for the needle-gap, as shown by F. W. Peek[2] and others. For spheres, it has also been shown by Peek[3] and A. Russell[4] that for large spacings under twice the radius of the spheres, the results seem to indicate a constant sparking gradient for a given size of spheres when both of them are insulated and are at equal and opposite potentials, the middle of the high voltage winding of the transformer being connected to earth. As for the case when one of the spheres is grounded with the high potential applied to the other insulated sphere, all experimental data seem to contradict this view.[3-12] In this case, the sparking gradient decreases rapidly at first and then gradually increases with the spacing.

The existing theory to explain these phenomena is mainly due to Peek, who assumes that air has a constant dielectric strength, an assumption fairly well borne out by the corona measurements on concentric cylinders and parallel wires. Inasmuch as this theory was extended from the phenomenon of corona, it seems advisable to study the problem of sparking in air by using different kinds of electrodes, namely, two spheres of unequal diameters and two cylinders placed with their axes perpendicular to each other in space (called cross-cylinders, for brevity) to see if similar relations exist and to derive a law for sparking based on the more definite form of the theory of ionization as first formulated by J. S. Townsend and later extended by Bergen Davis to the corona measurements on concentric cylinders and parallel wires. It is thus seen that the present investigation falls into two parts: namely, the determination of the sparking voltages and gradients for the above types of electrodes, and the formulation of a theory to explain the more or less empirical relation of Peek and Russell based on the known laws of ionization by collision.

Part I

SPARKING VOLTAGES AND GRADIENTS BETWEEN UNEQUAL SPHERES AND BETWEEN CROSS-CYLINDERS

Formulas and Tables for the Calculation of the Maximum Gradients between Unequal Spheres and Cross-Cylinders. The case of two conducting spheres is of classical interest, being among the problems first successfully solved by Poisson by using his famous integral. Clerk Maxwell[13] also gave a solution but it was too complicated to be of any service in a numerical work. The first workable formula was given by Kirchhoff[14]. It was put into a manageable form by A. Russell[15], who proved the case for two equal spheres by the method of an infinite series of images, while G. R. Dean[16] solved the same problem by using hyperbolic functions and zonal harmonics. In the appendix to follow, it will be shown how Russell's reasoning can be extended to the general case of two spheres of unequal diameters. From the proof it will be seen that the gradient at the surface of the spheres on the line of centers may be calculated from the following relations:

$$G_1 = V_1 F_{11}/X - V_2 F_{21}/X$$

and

$$G_2 = V_2 F_{22}/X - V_1 F_{12}/X$$

in which X = spacing between the spheres, G_1 = the gradient at the surface of the smaller sphere, V_1 = the voltage on the smaller sphere, and G_2 and V_2, the corresponding values for the larger sphere. The different F's will be called *coefficients of gradient.* The double subscript adopted is similar to the notation used in electrostatics for the coefficients of capacity, or of induction, or of potential. By giving appropriate values to the different V's, these general formulas will be found to be applicable to the different arrangements that are investigated: namely, when $V_1 = - V_2 = V/2$, the case when both spheres are at equal and opposite potentials, V being the potential difference between them; and when $V_1 = O$ or when $V_2 = O$, the two cases when one of the spheres is grounded. Thus for the case when both spheres are insulated and

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1. For all references see Bibliography.

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are at equal and opposite potentials, the two coefficients of gradient may be taken as:

$$F_1 = (F_{11} + F_{21})/2, \text{ and } F_2 = (F_{22} + F_{12})/2,$$

respectively.

The various F's are expressed in convergent series, the arguments of which are functions of the ratios b to a, ratio of radii, and X to a, ratio of spacing to the smaller radius. The values for these when $b = a$, *i. e.*, when the spheres are equal, have been given by other investigators[17]. In the following tables only F_{11} and F_{21} are given; the values for F_{22} and F_{12}, except for the case when one sphere has infinite radius, can be made to depend on those of F_{11} and F_{21} by a reciprocal relation which will be discussed fully in the mathematical appendix.

For ready reference and interpolations, Figs. 1, 2, 3, and 4 giving values of F_{11}, F_{21}, F_{22}, and F_{12} are herein attached.

For the case of cross-cylinders, because of the lack of symmetry, it is not possible to obtain an exact solution of the problem by using known methods of mathematical analysis. In Appendix II, an attempt has been made at an approximate solution, the basic principle of which is quite similar to that employed by Dean[18] in deriving an approximate solution for two equal spheres. It will be seen that for the case in which both equal cylinders are insulated and are at equal and opposite potentials, the coefficients of gradient F are given by the following table of values, using which the values of the maximum gradient can be calculated from the relation:

$$G = \sqrt{2}\, V\, F/X$$

where G is the gradient, V the effective potential difference, and X the spacing.

Apparatus, Method of Measurement, Etc. Five sizes of spheres were used in the investigation, their diameters as measured by calipers being 25.0 cm., 20.2 cm., 15.0 cm., 10.0 cm., and 5.05 cm. They were made by soldering together two hemispherical bowls that were spun from copper. Their shape was quite satisfactory, as measurements of the curvature by means of a spherometer did not vary more than one per cent. The shanks were brass tubes, projecting 60 cm. from the spherical surface, each tube having a diameter about one-eighth of that of the sphere to which it was fitted. For the determination of sparking between a sphere and a plane, sphere of infinite radius, two sizes of plane surfaces were used, one being about 50 cm. in diameter and the other 150 cm. At short spacings, it was found that the determinations were practically the same, using either plate. Only when the spacing became large, about one-sixth of the diameter of the plate, did deviations begin to appear. From this, it was concluded that with the 150-cm. plate, the sparking voltages as measured might be assumed to be independent of the diameter of the disk for spacings smaller than 25 cm., which was the upper limit contemplated in the investigation. The small disk, 50 cm., was a cast iron plate with the surface carefully scraped. The larger circular plane, 150 cm., was made of galvanized sheet iron with a ¾-in. (2-cm.) lead tubing soldered all around the edge.

Three pairs of cross-cylinders were used: one pair with 25-cm. in diameter being 48 in. (122 cm.) long excluding the hemispherical ends, and the other two pairs, both 5 in. (12.7 cm.) in diameter, being in two different lengths, namely, 24 in. (61 cm.) and 48 in. (122 cm.), respectively. They were supported horizontally at the ends at about four feet (120 cm.) from ground.

The method of voltage measurement was by comparison with a pair of standard 25-cm. spheres, the sparking curves being those given by the A. I. E. E. standards. Two transformers, both having the center of the high voltage winding grounded, were first calibrated against the 25-cm. sphere-gap to determine their ratios of transformation. Knowing these, the sparking voltages on unequal spheres and cross-cylinders were determined from the readings on the low voltage side of the transformer. The voltage ranges of the transformers were approximately 150 kv. between terminals for the smaller unit and either 250 kv. or 500 kv. between terminals for the larger unit, according to whether the low voltage side was connected in series or in parallel. The wave shape of the low voltage supply was very nearly sinusoidal. The frequency was 60 cycles.

In all of the determinations, the atmospheric density was corrected for. The correction factor was taken to be simply $3.92B/(273 + t)$ to reduce all observations to the standard condition of 76 cm. Hg and 25 deg. cent., B being the barometric reading in cm. Hg and t the temperature in degrees centigrade.

Experimental Data and Results. In the following tables, the coefficients of gradient have not been entered, their values being interpolated from the curves constructed from the tables of values already given, Figs. 1, 2, 3, 4. Calculations for the gradients were made only for cases in which the spacing was under twice the radius of the smaller sphere, as beyond this spacing there was good reason to suspect the formation of corona before a spark-over so that similar computations would become meaningless.

Each value is the average of five consecutive readings. Unless otherwise indicated, deviation from this average is small and negligible.

The symbols at the head of each column are:

a and b = radii of spheres, in cm.,

X = spacing between the spheres, in cm.,

V = sparking voltage when both spheres are insulated and are at equal and opposite potentials, in kv. eff.,

V_1 = sparking voltage when small sphere is insulated and the large one grounded, in kv. eff.,

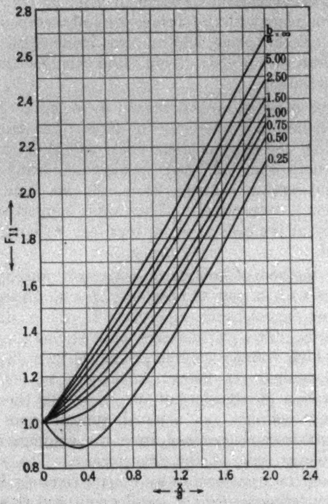

FIG. 1—COEFFICIENT OF GRADIENT FOR TWO SPHERES.

VALUES OF $F_{11}\left(\frac{x}{a}, \frac{b}{a}\right)$

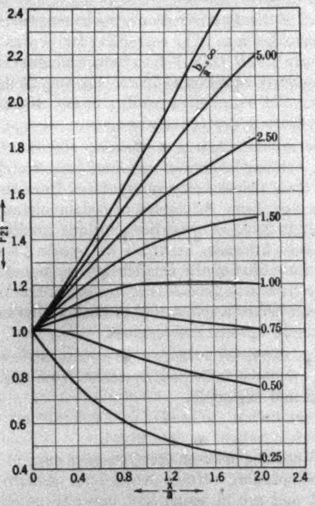

FIG. 2—COEFFICIENT OF GRADIENT FOR TWO SPHERES.

VALUES OF $F_{21}\left(\frac{x}{a}, \frac{b}{a}\right)$

V_2 = sparking voltage when large sphere is insulated and the small one grounded, in kv. eff.,

$G_1 = \sqrt{2}\, V F_1/X$; $G_{11} = \sqrt{2}\, V_1 F_{11}/X$; and $G_{21} = \sqrt{2}\, V_2 F_{21}/X$ are the maximum gradients at the surface of the small sphere corresponding to the three arrangements, in crest kv. per cm.,

$G_2 = \sqrt{2}\, V F_1/X$; $G_{12} = \sqrt{2}\, V_1 F_{12}/X$; and $G_{22} = \sqrt{2}\, V_2 F_{22}/X$ are the corresponding maximum gradients at the large sphere, in crest kv. per cm.

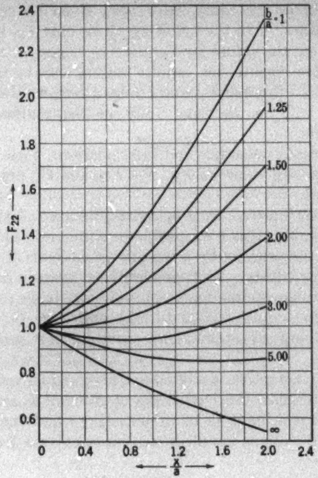

FIG. 3—COEFFICIENT OF GRADIENT FOR TWO SPHERES.

VALUE OF $F_{22}\left(\frac{x}{a}, \frac{b}{a}\right)$

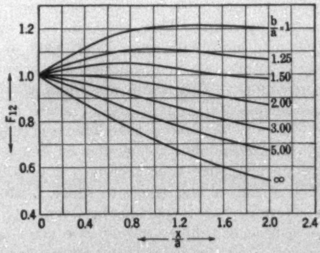

FIG. 4—COEFFICIENT OF GRADIENT FOR TWO SPHERES.

VALUES OF $F_{12}\left(\frac{x}{a}, \frac{b}{a}\right)$

Summary of Results. Part of the above data has been plotted into several sets of characteristic curves. From these (Figs. 5, 6, 7, 8) the following points may be inferred:

1. Unstable Part of the Curves. There is a part in each sparking curve where the sparking voltage is not definite. These parts have been shown as dotted lines. The irregularities at these portions are presumably due to the formation of excessive corona and begin at a separation somewhere around three or four times the diameter of the smaller sphere.

TABLE I

COEFFICIENTS OF GRADIENT FOR SPHERES

Values of F_{11}

b/a = ratio of radii; X = spacing

	b/a					
X/a	0.50	1.00	1.50	2.50	5.00	∞
0.0	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.01	1.07	1.09	1.11	1.12	1.14
0.5	1.08	1.20	1.25	1.29	1.32	1.36
0.8	1.25	1.38	1.44	1.49	1.54	1.60
1.1	1.46	1.59	1.66	1.72	1.78	1.86
1.4	1.70	1.83	1.90	1.97	2.04	2.12
1.7	1.96	2.08	2.15	2.22	2.30	2.40
2.0	2.23	2.34	2.41	2.48	2.57	2.68

TABLE II

COEFFICIENTS OF GRADIENT FOR SPHERES

Values of F_{21}

b/a = ratio of radii; X = spacing

	b/a					
X/a	0.50	1.00	1.50	2.50	5.00	∞
0.0	1.000	1.00	1.00	1.00	1.00	1.00
0.2	0.999	1.07	1.09	1.11	1.12	1.14
0.5	0.969	1.15	1.22	1.27	1.32	1.36
0.8	0.905	1.19	1.31	1.43	1.52	1.60
1.1	0.856	1.20	1.38	1.56	1.71	1.86
1.4	0.816	1.21	1.43	1.67	1.89	2.12
1.7	0.782	1.21	1.47	1.76	2.05	2.40
2.0	0.756	1.20	1.49	1.84	2.20	2.68

TABLE III

COEFFICIENTS OF GRADIENT FOR SPHERE AND PLANE

Values of $F_{22} = F_{12}$

X/a = spacing divided by radius of sphere

X/a	F_{22}	*X/a*	F_{22}
0.0	1.000	1.1	0.702
0.2	0.926	1.4	0.642
0.5	0.848	1.7	0.590
0.8	0.771	2.0	0.545

TABLE IV

COEFFICIENTS OF GRADIENT FOR CROSS-CYLINDERS

Both Electrodes Insulated

X = spacing; a = radius of cylinder

X/a	*F*	*X/a*	*F*
0.0	1.00	1.6	1.23
0.2	1.01	2.0	1.29
0.5	1.06	2.4	1.35
0.8	1.10	2.8	1.41
1.2	1.17	3.0	1.44

2. General Character of Curves of Series I, Both Spheres Insulated, and Series II, Small Sphere Insulated. These two series as shown by Figs. 5 and 6, show that the more nearly equal are the diameters of the spheres the higher is the sparking curve. Thus the highest curve is the one for equal spheres as interpolated from the data of the A. I. E. E. standards while the lowest curve is the one between a sphere and a plane.

3. General Character of Curves of Series III, Large Sphere Insulated and Small One Grounded. The same conclusions cannot be reached in this case as in the previous two cases, Fig. 7. When the diameters of the spheres do not differ greatly, the sparking curve bends over much more than the corresponding curves in the other two series. A probable reason for this will be

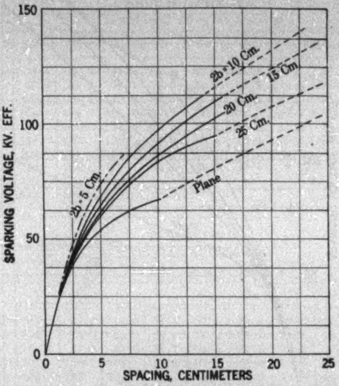

Fig. 5—Sparking Voltages V Between Unequal Spheres

Diameter of small sphere = 5.05 cm.
Diameter of large sphere = $2b$, as indicated
Both spheres insulated

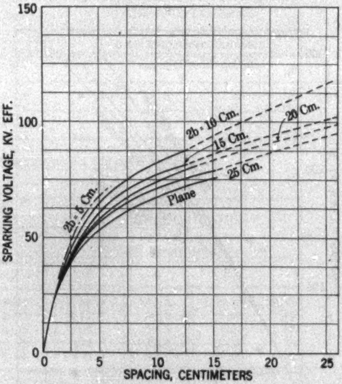

Fig. 6—Sparking Voltages V_1 Between Unequal Spheres

Diameter of small sphere = 5.05 cm.
Diameter of large sphere = $2b$, as indicated
Small sphere insulated; large one grounded

given when the sparking gradients are studied. It will also be seen that the sparking curve between a plane and a sphere is again lowest.

4. Sparking Gradients for Series I. From the tables, it is seen that the sparking gradients G_1 at the smaller sphere are very nearly constant for a given size of the smaller sphere within the range of the spacings investigated. The general relation is: *the smaller the radius, the higher the sparking gradient.*

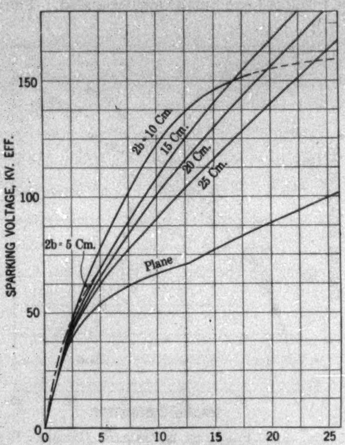

FIG. 7—SPARKING VOLTAGES V_2 BETWEEN UNEQUAL SPHERES

Diameter of small sphere = 5.05 cm.
Diameter of large sphere = 2 *b*, as indicated
Small sphere grounded; large one insulated

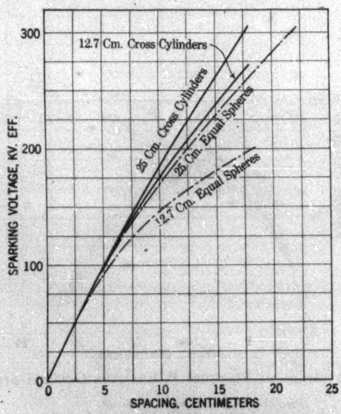

FIG. 8—SPARKING VOLTAGES BETWEEN EQUAL SPHERES AND BETWEEN EQUAL CROSS-CYLINDERS

Both electrodesinsulated

5. Sparking Gradients for Series II. The values for G_{11}, *i*. ., when the smaller sphere is insulated, will be seen to increase somewhat with the spacing. This increase seems to be greater as the ratio of the radii,

TABLE V

UNEQUAL SPHERES

2 *a* = 5.05 cm.; 2 *b* = 10.0 cm.; *b*/*a* = 1.98

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{21}	G_{22}
2.03	38.5	39.8	40.6	38.1	40.6	38.9	27.0	27.0	29.5
3.05	51.0	50.7	55.3	38.8	41.5	39.0	24.4	22.0	29.0
4.06	61.0	58.6	67.8	39.4	42.7	38.0	22.7	18.4	29.3
5.08	69.4	64.5	78.5	39.9	43.8	36.7	21.8	15.6	30.2
6.10†	76.2	71.6	89.0						
7.61	85.5	77.1	104						
9.65	95.8	82.0	121						
12.70	107	88.1	138						
17.75	125*	101*	152						
22.85	141*	108*	156*						

†From this point on, the gradient was not calculated on account of the formation of corona.

*Individual readings varied considerably from the average value here given.

TABLE VI

UNEQUAL SPHERES

2 *a* = 5.05 cm.; 2 *b* = 20.0 cm.; *b*/*a* = 4.00

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	36.7	36.8	37.3	38.7	39.2	38.7	22.4	22.4	23.2
3.05	47.2	46.4	48.2	39.0	39.7	38.3	18.5	17.4	19.5
4.06	55.4	53.3	58.0	39.4	40.4	38.7	15.9	13.9	17.8
5.08	62.0	59.0	65.8	39.8	41.5	38.3	14.2	12.6	17.18
6.60†	70.5	64.7	78.3						
7.62	76.5	70.1	84.4						
9.65	85.5	75.0	98.1						
12.70	94.8	80.3	118						
17.78	124	88.3	145						
22.86	134*	99.0*	171						

TABLE VII

UNEQUAL SPHERES

2 *a* = 5.05 cm.; 2 *b* = ∞; *b*/*a* = ∞

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	34.8	35.1	35.1	38.8	39.2	39.1	18.6	18.8	18.7
3.05	43.0	43.3	43.3	38.7	39.0	39.0	13.5	13.7	13.7
4.06	49.0	49.4	49.2	39.2	39.4	39.2	10.3	10.4	10.3
5.08	54.3	53.8	54.3	40.8	39.9	40.3	8.22	8.10	8.15
6.60†	60.0	59.4	60.0						
7.62	62.5	61.6	62.3						
10.16	67.2	66.7	67.5						
12.70	75*	72.4	71.3						
17.78	85*	81.5	85.2						
22.86	100*	93*	95.1						

TABLE VIII

UNEQUAL SPHERES

2 *a* = 10.0 cm.; 2 *b* = 15.0 cm.; *b*/*a* = 1.50

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	41.7	41.8	41.8	34.4	34.7	34.2	30.3	30.2	30.6
3.05	59.4	58.5	59.4	35.1	35.3	34.5	29.4	28.4	30.1
4.57	82.6	79.0	84.0	36.3	36.7	34.8	28.5	26.0	30.6
6.60	104	97.5	112	36.1	37.8	33.9	26.4	21.1	32.2
8.64	122	110	135	36.0	38.5	32.2	25.3	17.9	34.0
10.16	132	118	151	35.8	39.2	31.3	24.6	16.0	35.7
12.70†	148	128	168						
15.24	163	145	178						
20.32	187	152*	194						
25.40	206	145*	203						

b/a, is more nearly unity. Thus for equal spheres, this increase will be found to be greatest and for a plane and a sphere, it is almost entirely negligible.

6. Influence of G_2 and G_{12}. For the two series above discussed, the influence of the gradients at the surface of the larger sphere, *i. e.*, of G_2 and G_{12}, seems to be negligible since they vary between wide limits. It is probable that they do not have any primary effect in

TABLE IX

UNEQUAL SPHERES

$2a = 10.0$ cm.; $2b = 25.0$ cm.; $b/a = 2.50$

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	42.0	41.8	41.8	35.6	35.2	35.2	28.3	28.6	28.3
3.05	58.3	57.4	57.8	36.1	36.0	35.3	26.0	25.2	26.1
4.57	77.0	75.1	77.6	36.2	36.3	35.2	22.6	21.2	23.7
6.60	96.5	92.1	101	36.2	37.1	35.2	19.7	17.0	22.5
8.64	112	104	121	36.4	37.7	34.8	17.9	14.1	22.4
10.16	122	112	135	35.7	38.5	34.2	17.1	12.5	22.5
12.70†	137	122	157						
14.73	148	..	173						
18.29	..	136							
21.34	173	..							
25.40	184	142							

TABLE X

UNEQUAL SPHERES

$2a = 10.0$ cm.; $2b = \infty$; $b/a = \infty$

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	39.9	39.5	39.5	35.7	35.2	35.2	24.2	24.0	24.0
3.05	53.1	53.2	52.9	35.4	35.5	35.2	20.2	19.6	20.0
4.57	68.8	68.0	68.5	35.7	35.2	35.8	15.8	15.6	15.6
6.60	82.6	82.2	82.0	35.9	35.7	35.6	11.7	11.6	11.6
8.64	92.4	91.7	92.5	36.2	36.0	36.2	8.89	8.83	8.90
10.16	99.6	99.5	99.5	37.0	36.9	36.9	7.55	7.55	7.55
12.70†	108	107	108						
15.75	119*	116*	117						
19.30	122*	134*	128						
25.40	133*	138*	142						

TABLE XI

UNEQUAL SPHERES

$2a = 15.0$ cm.; $2b = 20.2$ cm.; $b/a = 1.345$

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	42.5	42.4	42.5	33.0	32.9	32.9	31.8	31.8	31.8
3.05	60.7	60.5	60.6	33.0	33.2	32.7	30.0	29.7	30.2
4.57	85.7	84.1	86.0	33.4	33.7	32.7	29.2	27.8	30.2
6.60	113	108	117	33.6	34.3	32.5	28.2	25.1	31.3
8.64	137	129	146	34.0	35.3	32.2	27.4	23.2	32.6
10.67	157	143	171	34.0	35.9	31.2	26.7	20.2	34.1
13.71	182	160	198	34.1	36.8	28.5	26.1	17.1	35.7
15.24	191	..		34.8	..		25.8	..	
16.76†	..	172							
19.30	218	182*							
22.86	..	199*							
25.40	248	205*							

TABLE XII

UNEQUAL SPHERES

$2a = 15.0$ cm.; $2b = \infty$; $b/a = \infty$

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	40.5	41.1	41.8	33.5	34.0	34.5	25.7	26.1	26.5
3.05	56.9	57.0	56.5	33.8	34.0	33.6	22.9	23.0	22.8
4.57	75.9	75.5	74.8	34.0	33.7	33.4	19.1	19.7	18.8
6.60	94.7	94.6	94.5	33.7	33.8	33.7	15.2	15.2	15.2
8.64	109.	109	110	34.1	34.1	34.3	12.2	12.2	12.4
10.67	120	121	122	34.2	34.2	34.7	10.0	10.1	10.2
13.71	135	133	137	35.0	34.4	36.0	7.85	7.72	7.94
16.76†	146	144	149						
19.30	155	153	160						
22.86	165	166*	172						
25.40	170	171*	180						

TABLE XIII

UNEQUAL SPHERES

$2a = 20.2$ cm.; $2b = \infty$; $b/a = \infty$

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	41.1	40.0	41.5	32.4	31.6	32.7	26.7	26.8	26.9
3.05	59.0	58.1	58.3	32.9	31.8	32.5	24.6	24.3	24.4
4.57	80.7	80.1	79.9	32.8	32.6	32.5	21.5	21.3	21.2
6.60	104	104	103	33.0	32.8	32.5	18.1	17.9	17.8
8.64	123	122	122	33.0	32.8	32.8	15.2	15.1	15.2
10.67	137	136	138	32.8	32.8	33.0	12.9	12.8	13.0
13.21	153	151	154	33.3	32.8	33.4	10.8	10.8	10.9
16.25	168	165	172	33.6	33.0	34.4	8.83	8.65	9.05
20.32†	183	180	190						
25.40	196	194	214						

TABLE XIV

UNEQUAL SPHERES

$2a = 25.0$ cm.; $2b = \infty$; $b/a = \infty$

X	V	V_1	V_2	G_1	G_{11}	G_{21}	G_2	G_{12}	G_{22}
2.03	41.0	41.0	41.6	31.5	31.5	31.5	27.0	27.0	27.1
3.05	58.6	58.4	58.2	31.6	31.4	31.4	25.1	25.0	24.8
4.57	82.2	81.3	81.9	31.7	31.5	31.6	22.5	22.2	22.4
6.60	107	106	108	31.4	31.4	31.6	19.0	19.0	19.4
9.15	131	133	135	31.0	31.5	31.9	15.9	16.1	16.3
11.18†	147	150	152	31.0	31.5	31.9	13.9	14.2	14.4
13.71	165	168	172	31.3	31.7	32.6	12.0	12.2	12.5
17.27	186	188	195	31.6	32.1	33.3	9.83	10.0	10.4
21.34	205			32.3			8.00		
25.40	221			32.8			6.72		

TABLE XV (A)

CROSS-CYLINDERS

Diameter = 12.7 cm.; net length = 61 cm.

Both Electrodes Insulated

X	V	G	X	V	G
2.23	45.7	30.3	7.14	134	30.4
3.17	64.3	30.1	9.65	169	30.0
4.32	88.5	30.5	11.15	188	29.6
5.86	113	30.4	12.50	192*	....

*Sparking across the ends; not reliable.

TABLE XV (B)

CROSS-CYLINDERS

Diameter = 12.7 cm.; net length = 122 cm.

Both Electrodes Insulated

X	V	G	X	V	G
2.74	55.8	30.3	9.75	171	30.3
3.83	77.7	30.3	11.86	195	30.1
4.95	96.7	30.5	14.00	221	29.7
6.43	122	30.7	15.40	242	30.2
7.95	145	30.5	16.50	255	30.3

TABLE XVI

CROSS-CYLINDERS

Diameter = 25.0 cm.; net length = 122 cm.

Both Electrodes Insulated

X	V	G	X	V	G
2.62	54.5	29.7	9.60	179	28.5
3.81	77.7	29.1	12.38	221	28.2
5.03	99.5	29.4	14.40	245	27.8
7.95	151	28.5	17.60	291*	....

*Sparking across the ends; not reliable.

causing the sparking, which, as already pointed out, seems to be mainly governed by the gradients G_1 and G_{11} at the smaller sphere.

7. Sparking Gradients for Series III. For the third series, namely, that in which the smaller sphere was grounded with the high voltage applied to the large sphere, the sparking gradient G_{31} at the surface of the smaller sphere sometimes increases and sometimes decreases with the spacing while the values of G_{22}, *i. e.,* the gradient at the larger sphere, are not always smaller than G_{21}. In fact, for the cases in which the sparking curves in the third series bend over and intersect the others as noted in (3) above, the gradient G_{22} increases and finally becomes larger than G_{21}; see Tables VIII and XI. In these cases it is probable that the law of sparking should be formulated to take account of the influence of both G_{22} and G_{21}. This is a probable explanation why the curves in this series bend over much more than the corresponding curves of the other two series.

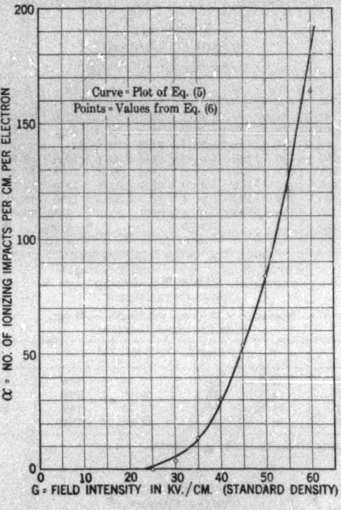

Fig. 9

8. Comparison of Curves for the Three Series. When the same pair of spheres is used, it will be seen that the sparking curve is highest when the smaller sphere is grounded and lowest when it is insulated, with the case when both spheres are insulated and are at equal and opposite potentials falling in between the two. For the arrangement when one sphere has infinite radius, i. e., a plane, all three curves are practically the same.

9. Data on Cross-Cylinders. The general relation that the smaller the radius, the higher the sparking gradient, has been found to be also true in the case of cross-cylinders. When the electrodes are at equal and opposite potentials, this gradient seems to be a constant for the spacings considered. As no mathematical calculation has been made to determine the sparking gradient when one cylinder is grounded, nothing can be inferred about this arrangement. Measurements of the sparking voltages in this latter arrangement showed, however, that the sparking voltages were not far different from those determined for the former case of both insulated electrodes. Because of the flux distribution, it will be noted that the sparking curve for cross-cylinders lies higher than the corresponding curve for spheres with the same radius, Fig. 8. It should also be noted, however, that the sparking gradient in the case of cross-cylinders is *smaller* than the corresponding value for equal spheres having the same radius.

From the data on the two lengths of the 12.7-cm. cylinders, it will be seen that when the spacing was below one-sixth of the net length of the cylinder, no appreciable difference in the sparking voltage was observed by using either pair of electrodes.

Part II

THEORETICAL CONSIDERATIONS

Before proceeding to a discussion of the Davis theory of the corona, from which it will be shown how to derive the relation between the sparking gradient and the radius of the sphere, it would be advisable to give a brief account of Peek's theory of the sphere-gap so as to afford a ready comparison of the two. Peek[19] assumes that air has a constant dielectric strength G_0. But to cause rupture, energy as well as gradient is necessary. Thus corona or sparking does not occur when the gradient at the surface of the electrode equals G_0, because energy has not yet been supplied sufficiently. It is only after the gradient at a certain distance away from the surface of the electrode equals G_0 and hence that at the electrode *exceeds* G_0, that the energy supplied is great enough to cause a rupture. Thus due to curvature of the electrodes, the sparking gradient at the electrode with the radius a should satisfy a relation:

$$G_a = G_0\,(1 + A/\sqrt{a}) \qquad (1)$$

in which A is a constant, G_0 the constant dielectric strength of air, and a the radius. From his results on concentric cylinders, parallel wires, and spheres, Peek finds that the values for G_0 vary somewhat for the three arrangements. He attributes this variation to the unbalancing of the field and claims that the dielectric strength of air is probably given by the value 31 kv. per cm. as obtained from the determinations on concentric cylinders where the field distribution is the same on all sides. In the case of parallel wires, there is some unbalancing so that G_0 becomes less, being only 29.8 kv. per cm., while for spheres, the unbalancing is greatest, in consequence of which the value of G_0 is also lowest, being only 27.2 kv. per cm.

Besides this difficulty and the rather indefinite magnitude of the rupturing energy required by Peek's theory, it is also necessary to account for the variation of the sparking gradients in the cases when one of the spheres is grounded. Although arguments have been advanced to explain this variation, they are not at all convincing[20]. As Bergen Davis has demonstrated how the theory of ionization by collision can be applied to the corona formation around cylindrical conductors, it appears best to extend this to the case of the sphere-gap since our present knowledge and confidence in the electron theory is far greater than at the time of the deductions of Davis.

In order to extend this theory, it is necessary to note the main assumptions as follows:[21]

1. On the Agency of Ionization. Under the conditions of sparking that are being dealt with, ionization of the gas molecules is primarily due to the collisions with electrons. The ionizations caused by positive ions and photoelectric effects are very small and may be neglected.

2. On the Energy Required for Ionization. When an electron collides with a molecule, ionization will follow only when the normal component of the kinetic energy of the electron equals or exceeds a certain least value, which value is a constant characteristic of the gas. Thus for a given density, the number of ionizing impacts depends on the field intensity. By statistical considerations, it can be shown how the number of ionizing impacts and the gradient are related. For practical purpose, a gradient at which ionization begins to be appreciable may be assumed to correspond with the value G_0 of Peek's formula (1), *but not equal to it.*

3. On the Physical Process of Sparking or Corona Formation in an Alternating Electric Field. Consider two concentric spheres for the sake of simplicity. Let the rupturing potential difference be applied. Suppose the inner sphere is passing through the crest value of the positive half of the voltage wave. Imagine a concentric sphere M to be the surface at which the gradient is G_0, *i. e.*, the value at which ionization by collision is appreciable under the given density conditions. As the inner sphere is at the positive half of the wave, free electrons will be moving towards it. When these free electrons pass inside of the region within the sphere M, ions and electrons multiply due to collision. Thus, if there are originally N_0 electrons at the surface of M, the number arriving at the inner sphere will be greater than N_0, say N_{01}. As the voltage wave decreases and finally becomes negative, some of these N_{01} electrons will move outward while a number of them will disappear, being taken up by the conductor as convection current. Those that move outward will again cause ionization by collision and will at the same time recombine with the positive ions that they overtake in their course. The net result of this is that at the next half-cycle, when the inner electrode is positive, the number of electrons at the surface of the imaginary sphere M is greater than N_0, the original number due to the natural ionization of the atmosphere. Call this N_1. These N_1 electrons will again produce ionization when they are within the sphere M and the number that reaches the inner electrode will be greater than N_1, say N_{11}. Of these, a portion will move outward as the field changes to negative value and causes, by the action already described, the number that moves inward in the following cycle to be still greater than N_1. Continuing the process in this way, it will be seen that there will be a cycle at which the number of electrons at the surface of the sphere M will be such a value, say n_0 per unit area, that moving under the field intensity and distribution, it will yield at the inner electrode an ionic density of n electrons per unit area. This density n is high enough to cause the layer immediately around the conductor to be conducting and hence the formation of corona or a progressive rupture. From this conception of the nature of the rupture, it will be noted that rupture does not occur immediately after the application of the rupturing voltage but only after the lapse of a few cycles. This conception seems to be in good agreement with observations on the time lag and transient sparking voltages of the different gaps.

The Law of the Sphere-Gap. In the above discussions, the spheres have been assumed to be concentric. If attention is confined to a small area around the line of centers on the surface of the smaller sphere, the same considerations will also be true for two spheres external to each other.

To find the relation between n and n_0, let us for simplicity again consider two concentric spheres. Denote by N_0 and N, respectively, the number that starts out from the sphere M and that arrives at the inner sphere. Let α be the number of electrons produced by one electron when it travels through a distance of one centimeter under an electric force of G kv. per cm. If there are N electrons passing through a surface,[22] then at a small distance $d\,y$, the number will be increased by

$$d\,N = \alpha\,N\,d\,y \tag{2}$$

If the field is uniform, *i. e.*, if the gradient is the same throughout, α is independent of y and the above can be at once integrated. Otherwise the variation of α with y must be taken into account. In any case (2) may be written as

$$\log N - \log N_0 = \int_a^{y_0} \alpha\,d\,y \tag{3}$$

Introducing the ionic densities n and n_0, equation (3) becomes

$$\log \frac{n\,a^2}{n_0\,y_0^2} = K = \int_a^{y_0} \alpha\,d\,y \tag{4}$$

where K has been written for the first member for brevity.

To integrate (**4**), it is necessary to express α in terms of y. This can be done if the relation between α and G is known, because, as will be shown in the Appendix, for small distances away from the surface of the sphere, the gradient G varies inversely as the distance from the center of the sphere. Thus, if G_0 is the gradient at the point on the line of centers whose distance from the center of the sphere is y_0, and G_a is the gradient at the surface of the sphere with radius a, then $G_a = G_0\, y_0^2/a^2$; and, in general, if y is the distance of a point on the line of centers from the center of the sphere, the gradient at y is $G = G_0\, y_0^2/y^2 = G_a\, a^2/y^2$, if $y - a$ is small.

Regarding the relation between α and G, Bergen Davis has developed a formula by considering the energy required for ionization as stated in (**2**) above. In terms of the ionization potential v and the mean free path L of the electrons, this relation takes the form:

$$\alpha = \frac{1}{L}\, e^{-\frac{v}{GL}} + \frac{v}{G\,L^2}\, E\,i\left(-\frac{v}{G\,L}\right) \qquad (5)$$

where $E\,i$ is the exponential integral. Using the value $v = 10.2$ volts as found by Bishop[23] and taking L as $4\sqrt{2}$ times the mean free path of the molecules in air[24], *i. e.*, $4\sqrt{2} \times 9.83 \times 10^{-6} = 5.55 \times 10^{-5}$ cm. for the free electrons in air, values of α can be calculated for different G's. A calculation of this sort has been made using the tables of Jahnke and Emde[25] for the values of the exponential integral. For values of G covered by the investigation, it is found that an empirical quadratic relation represents the theoretical relation (**5**) very satisfactorily. This relation is:

$$\alpha = C\,(G - G_0)^2 = 0.134\,(G - 25)^2 \qquad (6)$$

where G is in kv. per cm. The agreement between (**6**) and (**5**) is well shown by the accompanying curve, Fig. 9. As relation (**5**) can not be used for integration in finite terms, relation (**6**) will be used. Also as $G = G_0\, y_0^2/y^2$, (**6**) can be written as

$$\alpha = C\,G_0^2\left[\frac{y_0^2}{y^2} - 1\right]^2 \qquad (7)$$

Substituting this into (**4**) and performing the indicated integration, it is found that

$$\log\frac{n\,a^2}{n_0\,y_0^2} = K = 83.7\,a\left[\frac{8}{3}\sqrt{\frac{G_a}{G_0}} + \frac{1}{3}\left(\frac{G_a}{G_0}\right)^2 - \frac{2\,G_a}{G_0} - 1\right] \qquad (8)$$

In this form, the equation still contains a quantity n_0, of which nothing is known. If it is assumed for the present that the total number of electrons crossing any spherical surface completely enclosing one of the electrodes remains always the same, then the density of electrons on any such spherical surface will vary inversely as the square of the radius. In the equation above given, it is then permissible to put the quantity K equal to a constant. Granting such an assumption, the theory then leads to the result that the sparking gradient is a function of the radius of the smaller sphere only.

In his theory of the corona, Davis had to do with a similar quantity which he assumed to be constant and equal to 6 for concentric cylinders and to 4.3 for parallel wires. If K is taken as 4 for the sphere-gap, it will be found that equation (**8**) agrees fairly well with the data given by the Institute standards and those of the present investigation when only the case of symmetrical application of voltage is considered, *i. e.*, when both spheres are insulated and are at equal and opposite potentials. In fact, changing the square-root function to a quadratic function by the following empirical relation, which will be found to be accurate to within half of a per cent for values of X lying between 1 and 2, namely,

$$\sqrt{X} \simeq -\,0.071\,X^2 + 0.630\,X + 0.441 \qquad (9)$$

the relation (**8**) can be put in the form:

$$G_a = \left(1.105 + \frac{0.288\,\sqrt{K}}{\sqrt{a}}\right) G_0 \qquad (10)$$

With $K = 4$ and G_0 as 25, (**10**) at once becomes

$$G_a = 27.6\left(1 + \frac{0.52}{\sqrt{a}}\right) \qquad (11)$$

whose similarity to the relation of Peek, *viz.*,

$$G_a = 27.2\,(1 + 0.54/\sqrt{a})$$

or to that of Russell, *viz.*,

$$G_a = 27.4\,(1 + 0.515/\sqrt{a})$$

is very remarkable and striking.

CONCLUDING REMARKS

It should be noted that in the above work, the value of K has been assumed to be constant. It is the simplest assumption. On closer examination, it will be seen that n_0 cannot be as simply related to y_0 as an inverse square relation. For instance, as shown by Davis in his theory, when the spacing is very short, the electrons moving outward will be taken up and neutralized by the other sphere and the process of cumulative multiplication at the surface of the sphere M will not take place as readily. In fact, for short spacings, $n_0\, y_0^2$ decreases as the spacing (hence K increases) because of the disappearance of a number of electrons at the surface of the other electrode. It is therefore possible to explain the rapid increase of the sparking gradient as the spacing is decreased to very short values. Thus the simple relation of Peek and Russell, which may be derived from the theory of Davis, appears to be merely an approximation giving good results only for special cases, namely, when both spheres are insulated and are at equal and opposite potentials, in which case it is permissible to assume K to be a constant independent of the arrangement. As for the cases in which one of the spheres is grounded, the value of K is probably influenced both by the gradient at the surface of the larger sphere and the spacing between them, so that it is no longer a constant independent of the arrangement. On this account, it is thus also possible to explain why *the constancy of the sparking gradient is not of general validity.* A point of interest to note is that the smaller value of G_0, Peek's notation, for spheres, *viz.*, the value 27.6 as against 31 for concentric cylinders or 29.8 for parallel wires, is a natural consequence of the theory and does not require any explanation such as that advanced by Peek.

The writer is under great obligation to Prof. A. Wilmer Duff and Prof. Harold B. Smith for suggestions made and for facilities and encouragement in the preparation of the paper.

Appendix I

COEFFICIENTS OF GRADIENT FOR SPHERES

Given the radii a and b of two spheres and the distance of separation between their surfaces, X, it is required to find the field distribution when both spheres are charged and when one of them is charged and the other connected to ground. In particular, the values of the maximum gradients at the surfaces of the spheres are to be sought.

Let A_1 be the center of the sphere with radius a and B be that of the sphere with radius b. Suppose first that A is at potential V while B is grounded. Put a charge, $Q_1 = V_1 a$, at A_1. This will give to sphere A a uniform potential V_1. But due to it, the potential over sphere B is no longer zero. To reduce the potential over B to zero, put a second charge Q_1' at B_1, the point inverse to A_1 with respect to the sphere B, *i. e.*, $A_1 B \,.\, B_1 B = b^2$ and $Q_1/A_1 M + Q_1'/B_1 M = O$ where M is any point on sphere B. Due to Q_1' the potential over sphere A is now no longer uniformly equal to V_1. To restore it to the value V_1, another charge Q_2 may be put at the point inverse to B_1 with respect to the sphere A, namely, A_2, which satisfies the relation $A_1 A_2 \,.\, A_2 B_1 = a^2$. The magnitude of Q_2 must be such that $Q_2/A_2 N + Q_1'/B_1 N = O$ for any point N on the sphere A. Continuing the process, it is evident that the nth charge Q_n inside of sphere A is related to the nth charge Q_n' inside of B as follows:

$$Q_n/Q_n' = -M A_n/M B_n = -b/B B_n, \qquad (12)$$

where M is any point on sphere B, and A_n and B_n are two inverse points with respect to sphere B. Likewise, for any point N on sphere A, the relation must hold:

$$Q_{n+1}/Q_n' = -N A_{n+1}/N B_n = -A_1 A_{n+1}/a \,. \qquad (13)$$

Putting $B B_k = u_k'$ and $A_1 A_k = u_k$, the relation of two successive charges is $Q_{n+1}/Q_n = u_n' u_{n+1}/a b$ **(14)**

To find u_n' and u_{n+1} in terms of the radii a and b and the spacing X, first locate the two inverse points of the spheres, *i. e.*, the two points P and P' defined by:

$$A_1 P \,.\, A_1 P' = a^2 \text{ and } B P' \,.\, B P = b^2 \qquad (15)$$

Denoting $A_1 P/a = t$ and $B P'/b = t'$, it will be found that t and t' are given by:

$$2 a d t = d^2 + a^2 - b^2 - \sqrt{(d^2 + a^2 - b^2)^2 - 4 a^2 d^2}$$
$$\text{with } d = a + b + X \qquad (16)$$
$$2 b d t' = d^2 + b^2 - a^2 - \sqrt{(d^2 + b^2 - a^2)^2 - 4 b^2 d^2}$$

The general relation between u_n, u_n', and u_{n+1} is, by definition of inverse points, as follows:

$$(d - u_n) u_n' = b^2 \text{ and } (d - u_n') u_{n+1} = a^2, \qquad (17)$$

which gives, on the elimination of u_n',

$$u_n u_{n+1} - (d^2 - b^2) u_{n+1}/d - a^2/d + a^2 = o \qquad (18)$$

(18) can be put into the standard form of a difference equation by a change of the variable as follows:

$$u_n = v_{n+1}/v_n + (d^2 - b^2)/d \qquad (19)$$

giving

$$v_{n+2} + C v_{n+1} + D v_n = O, \qquad (20)$$

where

$$C = (d^2 - b^2 - a^2)/d \text{ and } D = a^2 b^2/d \qquad (21)$$

The solution of **(20)** is obtained by putting $v_n = z^n$, where z satisfies the quadratic

$$z^2 + C z + D = O \qquad (22)$$

Using the two roots of **(22)**, z_1 and z_2, the general solution of **(20)** is then $v_n = A z_1^n + B z_2^n$ **(23)**

where A and B are two arbitrary constants to be determined so as to satisfy the boundary conditions of the problem. Introducing for convenience the value of t from **(16)** into the solution, it is found that

$$v_n = A a^n (a/d - t)^n + B a^n (a/d - 1/t)^n \,. \qquad (24)$$

To determine the ratio of A to B, the initial condition that when $n = 1$, $u_1 = A_1 A_1 = O$ is used. This yields, from **(19)**,

$$v_2/v_1 = -(d^2 - b^2)/d = a\,(a/d - t - 1/t) \qquad (25)$$

together with simplification by using **(16)**. Equating the ratio v_2/v_1 found from **(24)** to that given by **(25)**, it is found that after reduction and simplification

$$B = -A t'^2 = -A s^4/t^2 \qquad (26)$$

where $s^2 = t t'$.

With this value for B, it is found that

$$u_n = \frac{a t (s^{4n-4} - 1)}{(t^2 s^{4n-4} - 1)}$$

and **(27)**

$$u_n' = \frac{b s^2 (t^2 s^{4n-4} - 1)}{t (s^{4n} - 1)} .$$

From **(14)**, the ratio of the various charges is then:

$$\frac{Q_{n+1}}{Q_n} = \frac{u_n' u_{n+1}}{a b} = \frac{s^2 (1 - t^2 s^{4n-4})}{(1 - t^2 s^{4n})} \qquad (28)$$

Since $Q_1 = V_1 a$,

$$Q_n = V_1 a \frac{(1 - t^2) s^{2n-2}}{(1 - t^2 s^{4n-4})}$$

and **(29)**

$$Q_n' = -Q_n u_n'/b = -\frac{V_1 a s^{2n} (1 - t^2)}{t (1 - s^{4n})}$$

On the lines of centers, we have then:

$$G_{11} = \frac{Q_1}{a^2} + \frac{Q_2}{(a - u_2)^2} + \ldots + \frac{Q_{n+1}}{(a - u_{n+1})^2} + \ldots$$

$$- \frac{Q_1'}{(d - u_1' - a)^2} - \ldots - \frac{Q_n'}{(d - u_n' - a)^2} - \ldots$$

for the gradient (or field intensity) at the surface of the small sphere due to the voltage V_1 on it. This can be reduced by using the values of Q and u already found, and is:

$$G_{11} = \frac{V_1 (1 + t)^2}{a (1 - t)} \left[\frac{1 - t}{(1 + t)^2} + \frac{s^2 (1 - t s^4)}{(1 + t s^4)^2} + \ldots \right] \tag{30}$$

or

$$G_{11} = \frac{V_1}{X} F_{11}$$

with

$$F_{11} = \frac{X (1 + t)^2}{a (1 - t)} \sum_{n = 0}^{\infty} \frac{s^{2n} (1 - t s^{4n})}{(1 + t s^{4n})^2} \tag{31}$$

In a similar way, the gradient at the surface of the larger sphere due to the voltage applied to the small sphere is

$$G_{12} = \frac{V_1 a (1 - t^2) (t + s^2)}{b^2 (t - s^2)^2} \sum_{n = 1}^{\infty} \frac{s^{2n} (1 - t s^{4n-2})}{(1 + t s^{4n-2})^2} \tag{32}$$

If now the large sphere is charged to the potential V_2 while the small one is kept at the potential zero, the gradient at the surface of the smaller sphere will be an expression similar to (**32**) with t changed to t' and b to a and a to b. It will be more convenient to express this analogous expression still in terms of t and then it will be found that this gradient is:

$$G_{21} = - V_2 F_{21}/X \text{ with}$$

$$F_{21} = \frac{X (1 + t)^2}{a (1 - t)} \sum_{0}^{\infty} \frac{s^{2n+4} (t - s^{4n+4})}{(t + s^{4n+4})^2} \tag{33}$$

The cases when sphere A is charged to the potential V_1 and the sphere B to potential V_2 will have for the gradients:

$$G_1 = G_{11} + G_{21} = V_1 F_{11}/X - V_2 F_{21}/X$$

and **(34)**

$$G_2 = G_{22} + G_{12} = V_2 F_{22}/X - V_1 F_{12}/X$$

the coefficients F_{22} and F_{12} being of the same form as those for F_{11} and F_{21} with all the a's and b's interchanged. Since it is immaterial which sphere is taken as the first sphere, it is evident that the values of F_{22} and F_{12} may be found to depend reciprocally on those of F_{11} and F_{21}. Thus it is always true that:

$$F_{11} (X/b, a/b) = F_{22} (X/a, b/a)$$

and

$$F_{12} (X/a, b/a) = F_{21} (X/b, a/b). \tag{35}$$

The tables that have been previously given for these coefficients were therefore calculated for F_{11} and F_{21} only but with values of b/a ranging from zero to infinity instead of merely from one to infinity.

When this reciprocal relation is applied to the case of a plane and a sphere, it will be found that the values become indeterminate. But going back to the original relation and allowing b to become infinite, it is found that the factor outside the summation sign in the expressions for F_{22} and F_{12} becomes simply $4 X/\sqrt{X + 2 a X}$ and the corresponding calculations can then be made. These values have been given in Table III.

The gradient at a short distance, h, away from the surface of the sphare with radius a on the line of centers is

$$G_{11} (a + h) = \frac{Q_1}{(a + h)^2} + \frac{Q_2}{(a + h - u_2)^2} + \ldots +$$

$$- \frac{Q_1'}{(d - a - h - u_1')^2} - \frac{Q_2'}{(d - a - h - u_2')^2} - \ldots$$

$$= \frac{Q_1}{a^2} \left(1 - \frac{2 h}{a}\right) + \frac{Q_2}{(a - u_2)^2} \left(1 - \frac{2 h}{(a - u_2)}\right) + \ldots$$

$$- \frac{Q_1'}{(d - a - u_1')^2} \left(1 + \frac{2 h}{d - a - u_1'}\right) - \ldots -$$

so that in the limit as $h \rightarrow 0$

$$\frac{d G}{d a} = - 2 \sum_{0}^{\infty} \frac{a + u_{n+1}}{a^2 (a - u_{n+1})^2} Q_{n+1} = - \frac{2 G}{a} \tag{36}$$

This then shows that for short distances away from the surface of the sphere, the variation of the gradient is inversely as the square of the distance from the center of the sphere, a relation that has been given in explicit form in the theoretical discussions.

Appendix II

COEFFICIENTS OF GRADIENTS FOR CROSS-CYLINDERS

Pass the plane of the figure, Fig. 10, through the axis of one of the cylinders and perpendicular to the other. Imagine line charges of q units per unit length to be placed at the positions A and B inside of the cylinders so that the cylindrical surfaces are approximately at the required potentials $V/2$ and $- V/2$ respectively. Let the distances be as shown in Fig. 10. At the point H, the gradient is greatest, its value being $G = 2 q [1/Z + 1/(Z + X)]$. The voltage at H is $V/2 = 2 q \log (Z + X)/Z$, so that on putting $X = n Z$ and eliminating q, the gradient is $G = V F/X$, where the coefficient F is given by

$$F = \frac{n (2 + n)}{2 (1 + n) \log (1 + n)} \tag{37}$$

To compute this, it is necessary to find n first. Consider the forces acting at a point P very near to H on the same plane. The resultant R is normal to the surface at P so that the tangential components of the two constituent forces R_1 and R_2, due to the two line charges at A and B, respectively, must be equal and opposite *i. e.*, $R_1 \sin a_1 = R_2 \sin a_2$. As R_1 and R_2 are inversely proportional to the distances r_1 and r_2, so $r_1/r_2 = (\sin a_1)/(\sin a_2)$. From the triangle ACP, $AP/AC = \sin a_2/\sin a_1$, *i. e.*,

$$r_1/(R - Z) = (\sin a_2)/(\sin a_1) = r_2/r_1 \qquad \textbf{(38)}$$

As P approaches H, r_2 approaches $Z + X$ and r_1 becomes Z, so that (**38**) becomes in the limit $Z/(R - Z) = (Z + X)/Z$ (**39**)

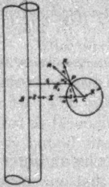

FIG. 10

With $X = n\,Z$, this can be reduced to a quadratic in n, the solution of which gives:

$$n = \frac{X}{2R} - \frac{1}{2} + \frac{1}{2}\sqrt{(X/R - 1)^2 + 8\,X/R} \qquad \textbf{(40)}$$

Thus, for given values of X/R, n can be computed from (**40**) and substituted into (**37**) to give the values as given in Table IV.

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Discussion

W. B. Kouwenhoven: In Mr. Sah's paper I find no reference to the effect of humidity. At the Johns Hopkins University we have been conducting an elaborate research upon the effect of humidity on the corona-forming voltage; this research is being carried out under our direction at the Bureau of Standards at Washington.

A few weeks ago I received a letter from E. S. Lee of the General Electric Company, saying that he had found that humidity affected his air condenser, and during the same week we obtained our first definite information that this was actually the case. The presence of humidity in the air has an effect on the corona-forming voltage. On perfectly clean surfaces the presence of moisture in the atmosphere raises the corona-forming voltage slightly, the maximum increase we have found is of the order of 1.2 per cent. In order to detect this effect it was necessary to refine our method of measuring and check everything very carefully. At the Bureau of Standards in the research to which I referred a moment ago, they are measuring the wave shape of the voltage directly in the high-voltage side of the circuit and have taken every precaution to insure accuracy. I feel that their results are accurate to at least one part of a 1000, if not better.

We have found also that humidity causes a lowering of the voltage when a corona forms on dirty or dusty surfaces. This lowering of the corona-forming voltage may amount to 5 per cent. In future work with the spark-gap it may be necessary to use correction tables for correcting for the amount of humidity in the air at the time of the test.

A. P. T. Sah: The influence of humidity on sparking voltages has been raised. Our experiments show that it is very small. It is true that humidity has a very pronounced effect on the sparking voltages when corona precedes a spark-over, for instance in the case of the needle gap. But for spherical and cylindrical electrodes, spaced at less than twice the radius of the smaller electrode, corona does not precede a spark-over. Thus in such cases the effect of humidity is negligible as our experiments show.

A Theory of Imperfect Solid Dielectrics

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Associate, A. I. E. E.

Synopsis.—*This paper covers Chapters VII, VIII, and IX of a thesis presented by the author to the Faculty of the Graduate School of Cornell University, for the degree of Doctor of Philosophy.*

In this thesis a summary is made of the experimental facts regarding the anomalous behavior of solid insulating materials under varying conditions of potential, time, temperature, frequency, humidity, ionizing radiations, and various other factors.

A bibliography containing about 400 articles dealing with experimental and theoretical research is appended to the thesis. These articles are chronologically arranged and numbered.

Five tables are given, including references to experimental research done on (a) dielectric resistivity, (b) dielectric charge and discharge, (c) dielectric constant, (d) dielectric strength, (e) dielectric energy loss.

Hypotheses are here established which account, in a general way, for the observed behavior of solid dielectrics. Definitions of the resistivity, permittivity, electric charge, and electric strength of solid dielectrics under both continuous and alternating potentials are submitted. Terms are introduced and defined: e. g., the "(i—t)-characteristic," the "electrization curve," and the "hystero-viscosity loop."

The various energy losses occurring in dielectrics are traced to their sources and subdivided into hysteresis, viscosity, and resistance losses. Methods are devised for separating the total dissipated energy into its three component parts.

Finally, the classical theory is shown to apply to imperfect solid dielectrics if the submitted definitions and terms be adopted.

* * * * *

Part I. General Considerations and Hypotheses

A. INTRODUCTION

THE fact that all theories so far propounded to explain the behavior of solid dielectrics have not stood experimental verification is an indication of the complexity of the phenomena which loom simultaneously whenever insulating materials are electrically tested.

Realizing the futility of a theory built on atomic, molecular, or inter-molecular considerations, the author proposes to abandon such a mode of procedure until our knowledge of the atomic and molecular structure of matter is more mature. This, however, does not prevent us from generalizing the laws governing the behavior of dielectrics nor does it presuppose a complete although temporary abandonment of the whole field of theoretical work. Indeed, if little is known about the physical nature of dielectrics, still less is known about that of magnetics. Yet the engineer, in his lust for the practical application of magnetic materials and their utilization to the highest degree of efficiency, has utterly forsaken any interpretation of the actual internal (molecular and atomic) mechanism of magnetism and has applied himself to the generalization of the gross facts in the form of "working laws."

While this is not the highway trod by the physicist and certainly not scientifically the surest and safest to pursue, yet the justification for the engineer's digression is to be found in the very rapid progress made through the application of such "working laws."

The author's convictions (a) that the electrical behavior of dielectrics is not only atomic but also molecular and inter-molecular; (b) that a complete knowledge of the atomic structure is a prerequisite to a clear understanding of the molecular and inter-molecular configuration; and finally (c) that our meager knowledge of the atomic structure furnishes but a slight encouragement to a prospective mastery of molecular structure, all lead him to pursue, gladly although reluctantly, the trodden path already followed by the engineer in the case of magnetism.

Such a course does not, however, prevent our formulating a picture of the internal workings of dielectrics. Indeed, a few general hypotheses will be next outlined, not with the view of building a mathematical theory of molecular behavior, based on them, but rather with the object of presenting a very sketchy interpretation of the internal phenomena which occur in a dielectric. The value of these hypotheses consists in their adaptability for explaining certain dielectric phenomena.

B. HYPOTHESES GOVERNING DIELECTRIC BEHAVIOR

1. We shall first postulate the existence of three types of electrons in solid dielectrics: free electrons, elastically bound electrons, and viscously bound electrons.

a. The free electrons are responsible for the process of electrical conduction. They actually traverse the body of a dielectric and form what is generally termed the "leaky current." The value of that current is a function of the number of free electrons present as well as the velocity with which they traverse the circuit. The existence of such electrons accounts for the phenomenon of dielectric conductivity.

b. The elastically bound electrons are not free to traverse the dielectric but are displaced under the action of an electric field, from their normal position. Their displacement is instantaneous. Moreover, with this type of electrons, action and reaction are reversible (see Part II-E). The well known phenomenon of dielectric polarization is due, in part, to the existence of this class of electrons.

c. The viscously bound electrons form a class which possesses characteristics that are intermediate between

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those of classes a and b. These electrons are neither free nor elastically bound. They creep rather sluggishly under the action of an electric field and, in the first stages of the field application, act as if they were free conduction electrons. But as time proceeds, their displacement from the position of equilibrium, in some cases, ceases to increase and they assume a new fixed position, thereby contributing to the polarization of the dielectric and resembling the elastically bound electrons in this respect. The existence of this type of electrons finds confirmation in the phenomenon of residual charge and in the well known shape of the "hystero-viscosity loop" (see Part II–E).

2. The foregoing three classes of electrons shall be arranged in three descending orders according to the elasticity of their bonds. The elastically bound electrons shall be considered as belonging to the highest or *first order*, the viscously bound electrons to the *second*, and the free electrons to the *third* order.

3. Although the electrons existing in a dielectric are thus arbitrarily subdivided according to the elasticity of their bonds, it must not be supposed that the elasticity of all electrons belonging to one class is the same. We shall therefore assume, as a separate hypothesis, the existence of various degrees of elasticity within every class of electrons. The two lines of demarcation between the three classes will consist of (1) the reversibility of action and reaction of the perfectly elastic electrons and (2) the absolute freedom of motion of the free electrons.

4. It will be next assumed that the three types of electrons, although distinct in their characteristic behavior, are physically identical. Therefore, an electron belonging to one order may easily assume the characteristic behavior of one belonging to a lower, or a higher order. Such an electron shall then be classified with that order. The external factors which determine the characteristic behavior of the various electrons are the magnitude of the potential applied, the temperature of the sample, the frequency of the e. m. f. source, ionizing radiations, the physical condition of the dielectric, time of potential application, humidity of the sample, and the chemical composition of the dielectric. Thus, with an increase of applied potential, some of the heretofore viscously bound electrons might become free electrons and may, in their turn, be replaced by some of the elastically bound electrons. Confirmation of this hypothesis is to be found, for example, in the increase of the dielectric conductivity γ with an increase in the potential gradient G.

5. This change of an electron or a group of electrons from a higher to a lower order or vice versa shall be assumed to be a discontinuous function of the factor causing the change. For example, within a certain range of potential the total number of electrons, in a given specimen, belonging to any one class, remains the same. A change in their number does not occur until the upper or lower limit of that range of potential shall have been exceeded. This hypothesis explains the results of the conductivity measurements made in (54–21),* where a distinct and abrupt change in the conductivity of calcite, quartz, rocksalt, and mica is noted at certain definite potential gradients, which vary from 51 to 10,000 volts per mm. depending upon the dielectric. Moreover, it is there shown that the same potential limits reveal themselves in the phenomenon of charge where a brisk change in the straight line relation of the $(Q - E)$ curve occurs. The existence of more than one potential limit is easily explained by the third hypothesis.

6. The fourth hypothesis presupposes the necessity of subdividing the various values that a factor (*e. g.*, temperature, potential, etc.) may assume, into several *ranges*, each having for its limits the critical values at which the dielectric behavior assumes an abrupt change. For example, the various values that the potential gradient may assume from zero to G, may be subdivided into various ranges $A, B, C \ldots N$ such that $G_A, G_B, G_C \ldots G_N$ are respectively the upper, and $O, G_A, G_B, G_C, \ldots G_{N-1}$ the lower limits of ranges $A, B, C, \ldots N$.

7. We shall further postulate that the transfers of electrons from one order to another (see the fourth hypothesis) are not necessarily superposable. To illustrate this hypothesis let G_A be the upper limit of the potential gradient for range A, and let N_1, N_2, and N_3, be the number of electrons in a certain specimen belonging to the first, second, and third order, respectively. Now, if the potential gradient be raised to G such that $G > G_A$ and then lowered again to G' such that $G' < G_A$, the number of electrons belonging to each order will then be N_1', N_2', and N_3', such that $N_1' \leqq N_1$, $N_2' \leqq N_2$, and $N_3' \geqq N_3$. The nature of the dielectric, its previous history, the magnitude of G, and the time allowed for the internal readjustments to ensue, are among the factors which determine the values N_1', N_2', and N_3'. This hypothesis explains the phenomena of potential hysteresis and temperature hysteresis referred to in (1/80, 13/94, 13-02, 38-08, 39-10, 4-16, 32-16, and 109-21).

8. Finally, it will be assumed that, for ranges exceeding the initial range A, action and reaction are not superposable. This at once limits Curie's law of superposition to just that range and further explains the phenomena revealed by the researches quoted in paragraph 7.

It is hoped that with the foregoing hypotheses a plausible explanation could be made of the various phenomena known as dielectric anomalies. As an illustration of the application of these hypotheses, the phenomenon of residual charge will be explained.

When an e. m. f. is impressed, for a time (t), on a slab of solid dielectric, placed between two metal plates,

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*All numbers (*eq.*, 54-21 or 1/80) occurring in this paper, refer to articles in the bibliography.

a current i flows through the connecting wires. This current may be divided into two components; the one i_r, leaks through the body of the dielectric, while the other, i_c, results in the accumulation of charge on the plates. The conduction current i_r is due to the existence of free electrons in the dielectric. The negative charge, which accumulates on one of the plates due to the charging current i_c, exerts a repulsive effect on the elastic and viscous electrons of the dielectric, thereby displacing them from their natural position of equilibrium. The actual molecular condition of the dielectric thus electrically stressed, is far from being known. The net result, however, may be visualized by imagining the negative charges on the plate as repelling the elastic and viscous electrons of the adjacent molecules of the dielectric and attracting their protons. This same phenomenon is communicated by the successive molecules to adjacent ones, thereby forming the state shown in Fig. 1 where the circles represent polarized adjacent molecules. The polarization is not altogether instantaneous because it takes the viscous electrons some time to attain their final positions of equilibrium. The negative and positive charges which appear on the surfaces adjacent to the positive and negative plates respectively act as counter e. m. fs. opposing the action of the impressed potential. The e. m. f. due to the polarization on each side shall be denoted by the letter E_p. Therefore, the total polarization e. m. f. is $2\,E_p$.

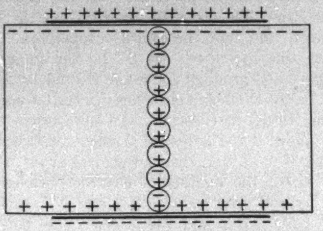

FIG. 1—POLARIZATION IN A SOLID DIELECTRIC

Now, as soon as the impressed e. m. f. is removed, the elastic electrons rebound instantaneously to their positions of equilibrium, thereby making available, for recombination upon short circuit, an equal number of electrons. The viscous electrons, however, being more or less sluggish, do not return instantaneously to their initial positions. The dielectric therefore persists in a state of polarization. Owing to this continued existence of polarization in the body of the dielectric, only a portion of the electrons accumulated on the negative plate of the condenser are available for recombination at the first discharge; the remainder are held by the polarized positive protons of the dielectric. If the condenser is allowed to rest after this first discharge, the internal polarization in the dielectric becomes less intense due to the return of some of the viscous electrons to their neutral position. Thus more electrons, on the negative plate, become available for recombination during the succeeding discharge. This process may be repeated ad infinitum or until all the viscous electrons have resumed their original neutral positions. The dielectric is then in an electrically neutral state and its polarization is nil.

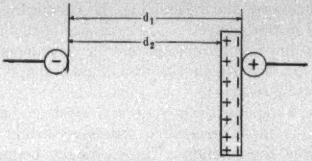

FIG. 2—DECREASE IN THE SPARKING DISTANCE OF A GAS DUE TO PLACING A SOLID DIELECTRIC NEAR THE ANODE

Interesting experiments supporting this view are described in (24-00, 12-02 and 23-18) where two knobs placed in air are subjected to a potential slightly lower than the sparking potential. When a glass rod is placed immediately touching the positive knob and lying in the space between the knobs, a spark passes, but no spark passes at all if the rod is placed anywhere else. The explanation of this phenomenon is that the polarization of the glass rod results in an effective decrease of the sparking distance, from d_1 to d_2 as shown in Fig. 2. That this phenomenon does not take place when the rod is placed next to the negative electrode can be easily explained because it is well known that a spark passes more easily when the point is negative and the plate positive than when the point and plate charges are otherwise.

Part II. Definitions of Terms

A. THE RESISTIVITY ρ OF SOLID DIELECTRICS

1. *Resistivity at Continuous Potential.*

The resistance R of a solid dielectric with a continuous applied potential will be defined as:

$$R = \underset{t \to t_r}{\text{Limit}}\ (E - 2\,E_p)/I_r \qquad (1)$$

The notations used in equation (1) may be visualized by referring to Fig. 4.

From equation (1) the resistivity ρ will be:

$$\rho = R\,a/d = [(E - 2\,E_p)/I_r]\ (a/d) \qquad (2)$$

For continuous potentials, and within the same potential range ρ will be constant. However, ρ will assume lower values as higher ranges of potential are reached until at the breakdown potential ρ becomes practically zero. The resistivity defined by equation (2) is easily measured because all the quantities on the right hand side are determinable. E_p is the only factor which seems difficult to measure. Its value, however, may be experimentally determined as was done in (53-15) and (2-25) by first impressing E for a time t_r, then suddenly decreasing E to such a value that i_r

becomes zero. This new value of E at which $i_r = 0$ is the value of $2E$, while (53-15) shows that if this is done, R (at least in the case of quartz and iceland spar) is not only independent of E, within certain limits, but it is also independent of the time t.

2. *Resistivity at Alternating Potentials.*

The value of ρ can thus be experimentally determined when continuous potentials are used. With alternating potentials, the conduction current cannot be experimentally separated from the charging current. The method generally adopted of measuring the power dissipated in a condenser and taking ρ of such a value that $W = I^2 R$, is physically erroneous, because of the fact that the energy dissipated is largely a hysteresis loss, and the $I^2 R$ loss forms but a very small percentage of the total dissipated energy (see 2/82, 94-12, and 17-15).

Two questions arise in regard to the resistivity when alternating potentials are used:

a. What conception must be formed of ρ with alternating potentials?

b. If ρ is a function of the potential range, what value of ρ must be adopted when alternating potentials assume instantaneous values which fall within more than one potential range?

The answer to the first question is that the concept of ρ with alternating potentials should not differ from that of ρ with continuous potentials.

The answer to the second question would lead to a definition of an effective resistance which, although having no physical basis when the potential gradient exceeds the initial range, leads to an approximate determination of the true $I^2 R$ loss when alternating potentials are used. Thus let $t_1, t_2, \ldots t_n$ be the times at which the sinusoidal voltage wave (Fig. 3) assumes instantaneous values $e_1, e_2 \ldots e_n$ such that $e_1 = E_A, e_2 = E_B, \ldots e_n = E_N$, where $E_A, E_B \ldots E_N$ are the upper potential limits of the potential ranges $A, B \ldots N$.

Then

$$\left.\begin{aligned} W_{r1} &= \int_0^{t_1} (i_{rA})^2 R_A \, d\,t = R_A \int_0^{t_1} (i_{rA})^2 \, d\,t \\ W_{r2} &= \int_{t_1}^{t_2} (i_{rB})^2 R_B \, d\,t = R_B \int_{t_1}^{t_2} (i_{rB})^2 \, d\,t \\ &\cdots\cdots\cdots \\ W_{rn} &= \int_{t(n-1)}^{t_n} (i_{rN})^2 R_N \, d\,t = R_N \int_{t(n-1)}^{t_n} (i_{rN})^2 \, d\,t \end{aligned}\right\} \quad (3)$$

where $i_{rA}, i_{rB}, \ldots i_{rN}$ are the conduction currents, $R_A, R_B, \ldots R_N$ are the resistances and $W_1, W_2 \ldots W_n$ are the energy losses that would be obtained at continuous potentials lying within the ranges $A, B \ldots N$.

If hypothesis 8 is correct, then for the initial range A, i_{rA} will be a sinusoidal function of time. Moreover, its effective value will be equal to the final steady value of the conduction current obtained with a continuous potential equal in magnitude to the effective value of an alternating potential whose amplitude is E_A.

The value of the first integrand in equation (**3**) will then be

$$W_{r1} = R_A \int_0^{t_1} [(I_{rA} \sqrt{2}) \sin \omega \, t]^2 \, d\,t$$

$$= (I_{rA}{}^2 R_A) \left[t_1 - \frac{\sin 2 \, \omega \, t_1}{2 \, \omega} \right] \quad \textbf{(4a)}$$

The other integrands in equation (**3**) cannot be so easily evaluated because, according to the eighth hypothesis, we cannot assert that the conduction currents $I_{rB} \ldots I_{rN}$ are sinusoidal. However, if we should follow the procedure adopted in obtaining equation (**4a**), the error will be only in the deviation of the current from the sinusoidal value. This error, for potential gradients lying well below the rupturing gradient G_s, cannot be very large. We shall therefore evaluate the remaining integrands as was done in the case of the first and obtain:

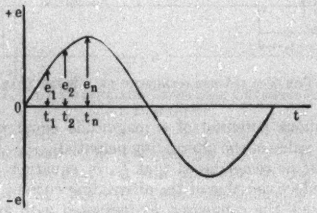

FIG. 3—A SINE WAVE OF VOLTAGE

$$W_{r2} = (I_{rB}{}^2 R_B) \left[(t_2 - t_1) - \frac{\sin 2 \, \omega t_2 - \sin 2 \, \omega t_1}{2 \, \omega} \right] \quad \textbf{(4b)}$$

$$\cdots\cdots\cdots$$

$$W_{rn} = (I_{rN}{}^2 R_N) \left[(t_n - t_{n1}) - \frac{\sin 2 \, \omega t_n - \sin 2 \, \omega t_{n-1}}{2 \, \omega} \right] \quad \textbf{(4n)}$$

Therefore the total energy dissipated per ¼ of a cycle is:

$$W_r = (W_{r1} + W_{r2} + \ldots + W_{rn}) \quad \textbf{(5)}$$

But with actual conducting materials the energy loss due to the passage of a current of effective value I_{rN} for a period of time $T/4$ is:

$$W_{rn} = I_{rN}{}^2 R_N \, T/4$$

from which

$$R_N = 4 \, W_{rn}/I_{rN}{}^2 \, T \quad \textbf{(6)}$$

The equivalent resistance of a dielectric at sinusoidal potentials, having instantaneous values which extend over more than one potential range, may be analogously defined as

$$R_{eq} = 4\,(W_{r1} + W_{r2} + \ .\ .\ .\ W_{rn})/I_{rN}^2\,T \qquad (7)$$

where W_{r1}, W_{r2} . . . W_{rn} are defined by equations (**4**)

At voltages whose amplitudes do not exceed the initial potential range, equation (**4a**) for ¼ of a cycle becomes:

$$W_r = I_{rA}^2\,R_A\,T/4 \qquad (8)$$

Therefore, the resistance of a slab of dielectric at such alternating potentials is equal to that obtained by using a continuous potential of a magnitude equal to the effective value of the alternating potential.

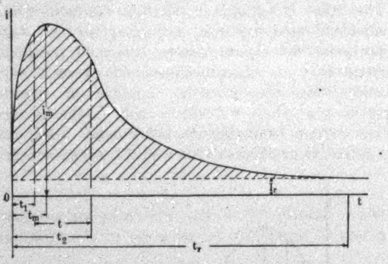

FIG. 4—THE $(i - t)$-CHARACTERISTIC OF A SOLID DIELECTRIC

It must be remembered that I_{rA} in equation (**8**) is *not* the effective value of the alternating current, flowing through the condenser, as measured with an a-c. ammeter. It actually corresponds to I_r in equation (**1**); *i. e.*, it is the true conduction current read at a time t_r on a d-c. instrument, when a continuous potential of a magnitude equal to the effective value of the alternating potential is applied to the condenser.

B. ELECTRIC CHARGE AND DISCHARGE OF SOLID DIELECTRICS

1. *The Charge with Continuous Potentials.*

a. Definition: If, for a definite temperature, humidity, and continuous potential, current readings be taken at various intervals of time, for a circuit with R, L, and C, then, upon plotting those readings, Fig. 4 will be obtained.

The charge acquired by the dielectric in the interval of time $t = (t_2 - t_1)$ will be defined thus:

$$Q = \int_{t_1}^{t_2} (i - I_r)\,d\,t = \int_{t_1}^{t_2} i_c\,d\,t \qquad (9)$$

In equation (**9**), i is the current measured at any instant and I_r is the leaky current represented by the constant value which it assumes after a time t_r; see equation (**1**).

b. Remarks on Fig. 4.

1. The value of the charge Q, expressed by equation (**9**), can be graphically represented by that part of the shaded area in Fig. 4, included between t_1 and t_2. It will be noted that Q is a function not only of the magnitude, $t = t_2 - t_1$, of the time interval, but also of the actual values of the limits t_1 and t_2. For the extreme case where $t_1 = t_r$ and $t_2 > t_r$, the charge is zero because $i - I_r = 0$.

2. The time t_m, corresponding to the maximum value i_m, is solely a function of the constants R, L, and C of the circuit. It can be reduced to a minimum by making $L \rightarrow O$.

3. The actual value i_m is a function of the dielectric material, the applied voltage gradient, temperature, and various other factors.

4. The character of that section of the curve which extends from t_m to t_r is an indication of the relative number of the three types of the electrons present in the dielectric as well as the degree of viscosity of the electrons belonging to the second order. Indeed, the curve is a function of the rate of growth of the polarization e. m. f. which forms on the boundary surface between the plates and the dielectric. This is best illustrated by Fig. 5, where curves are drawn for various materials possessing the same dielectric resistivity but having different $(i - t)$ - characteristics.

Curve a represents the $(i - t)$ - characteristic of an ideal condenser. The ascending and descending portions of the curve are identical. The descending part of the curve is a reproduction of the inverted ascending portion. Moreover, owing to the perfect elasticity of the ether, i_m is just a point on the curve. Again the final current becomes zero because a perfect condenser is devoid of any conductivity.

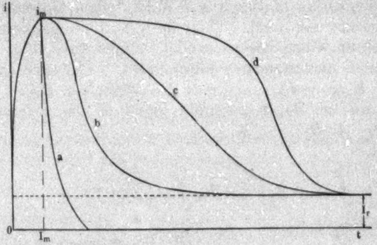

FIG. 5—VARIOUS SHAPES OF THE $(i - t)$-CHARACTERISTICS OF SOLID DIELECTRICS

Curve b stands for a condenser whose dielectric is rich in highly elastic electrons; the portion of the curve beyond t_m is very steep, showing that each of the viscous electrons possesses, to a more or less extent, the same degree of viscosity. That the value of i_m extends for a certain length along the curve, is an indication of the existence of viscous electrons, because it shows that the polarization took some time before approaching its steady final value.

Curve c is drawn for a condenser whose electrons have varying degrees of viscosity as indicated by the gentle slope of the curve and the time it takes for the polarization to be complete.

Curve d shows that although the electrons are very highly viscous, as indicated by the time required for the first point of inflection to occur, there exists very little difference in their degrees of viscosity. The very steep descent of the curve, as soon as the polarization attains its final value, is an indication of this latter fact.

5. While, in general, a dielectric which possesses the characteristic *Curve b* (Fig. 4) within a certain potential or temperature range, may be expected to exhibit a similar characteristic at other ranges, it must not be inferred that this always holds true. Indeed the behavior of a dielectric is a very complicated function of its internal structure. Therefore, it is very likely that the same dielectric will exhibit the various characteristics represented by *Curves b, c,* and *d,* at different potential ranges. This will be in strict conformity with the hypotheses laid down in Part I.

2. The Charge with Alternating Potentials.

a. Some Laws and Corollaries: The $(i-t)$-characteristic represented by Fig. 4 is true for steady continuous potentials. With alternating potentials the shape of the current-time curve is further complicated by the fact that e is a function of time.

Before formulating a definition of charge with alternating potentials we shall cite the laws of charge and discharge established in 2/89, taking the liberty to introduce such changes in them as have been necessitated by more recent researches.

Law 1. Within the initial potential range and for the same conditions of temperature, humidity, etc., the value of $i_c = (i - I_r)$ for a dielectric at one and the same instant of time (t), after impressing the potential, is proportional to the applied potential E. Thus, if i_c and i_c' are the charging currents measured at an instant t after impressing voltages E and E', respectively, and if E and E' are both less than the upper limit E_A of the initial potential range A, then

$$E/E' = i_c/i_c' \qquad (10)$$

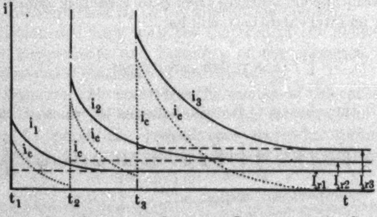

FIG. 6—THE RESULTANT CHARGING CURRENT DUE TO SUCCESSIVELY INCREASING POTENTIALS IMPRESSED UPON A DIELECTRIC

Law 2. The charging current i_c, due to successively increasing potentials impressed at successive instants on a condenser, is equal to the sum of the charging currents that would be produced by each potential acting alone, provided none of these potentials exceeds the upper limit E_A of the initial potential range. Thus, let $E_1, E_2, E_3 \ldots\ldots E_n$ be successively impressed on a condenser at times $t_1 < t_2 < t_3 \ldots\ldots < t_n$ and let $E_1 < E_2 < E_3 < \ldots\ldots < E_n < E_A$, then

$$i_{c1} + i_{c2} + i_{c3} \ldots\ldots + i_{cn} = i_c \qquad (11)$$

In order to visualize this law, imagine a circuit of such a small inductance that the time t_m, Fig. 4, may be neglected. The voltages $E_1, E_2 \ldots\ldots E_n$ applied at times $t_1, t_2 \ldots\ldots t_n$ will give rise to the currents $i_1, i_2 \ldots\ldots i_n$; Fig. 6. Then the charging currents $i_{c1}, i_{c2} \ldots\ldots i_{cn}$ will be:

$$\left.\begin{aligned} i_{c1} &= i_1 - I_{r1} \\ i_{c2} &= i_2 - I_{r2} \\ &\ldots\ldots\ldots \\ i_{cn} &= i_n - I_{rn} \end{aligned}\right\} \qquad (12)$$

and the instantaneous value of the total charging current $i_c = i_{c1} + i_{c2} + \ldots\ldots + i_{cn}$ will be represented by the dotted curve.

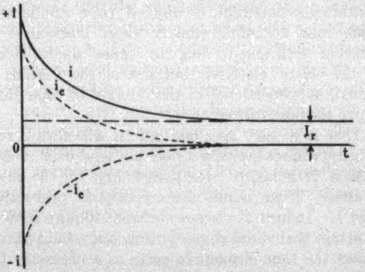

FIG. 7—THE CHARGE AND DISCHARGE CURRENTS OF A SOLID DIELECTRIC

Corollary 1. As is pointed out by Curie (2/89), one consequence of Law 2 is that the charging current curve is identical with that for the discharge current curve, but of opposite sign. The discharging of a condenser at short circuit is indeed equivalent to introducing into the circuit a voltage $(-E)$, equal in magnitude but opposite in direction, to that of the impressed voltage. If acting alone, this newly introduced voltage would cause a charging current identical in shape with, but opposite in sign to, the original charging current as is shown in Fig. 7.

Unfortunately this law does not apply to potentials higher than the upper limit of the initial potential range. (See references in hypothesis 7, Part I.) This accounts for the limitation introduced in formulating the law and for the statement made in the eighth hypothesis.

Corollary 2. Up to a certain time t the total charge acquired by a condenser, due to successively increasing potentials, is equal to the sum of the charges that would be acquired if each of the potentials were acting alone. Thus if $i_{c1}, i_{c2} \ldots . i_{cn}$ be the charging currents due to potentials $E_1 < E_2 < \ldots . < E_n \ldots . < E_A$, impressed at times $t_1 < t_2 < \ldots . < t_n$, then the total charge Q acquired by the condenser from t_1 to $t > t_n$ is $Q = Q_1 + Q_2 \ldots . + Q_n$ or

$$Q = \int_{t_1}^{t} i_{c1}\, d\, t + \int_{t_2}^{t} i_{c2}\, d\, t + \ldots . + \int_{t_n}^{t} i_{cn}\, d\, t \qquad \textbf{(13)}$$

Law 3. The value of the charging current i_c due to a continuous potential E applied to a condenser for more than an infinitesimally short interval (*i. e.* an interval sufficiently long to cause a displacement of the elastic electrons but not of the viscous electrons), is proportional to the interval of time through which the potential acts.

This is a very specious law; it will always remain as a hypothesis because it does not lend itself to experimental verification. Its plausibility will be evident, however, if we admit the hypotheses postulated in Part I. Indeed, if viscous electrons require some time to attain their final displacement, one would naturally expect the time element to enter as a necessary factor in determining the value of i_c.

b. The Equations for the Charging Current and Charge Acquired with Alternating Potentials. Let the potentials $E_1, < E_2, < \ . \ . \ . < E_n$ be comprised between the limits 0 and E_m, and let $t_1 < t_2 < t_3 \ . \ . \ . > t = T/4$ be the times at which these potentials are applied. Let E_m be the amplitude and T the period of the sinusoidal applied potential whose equation is

$$e = E_m \sin \omega\, t \qquad \textbf{(14)}$$

then

$$\left.\begin{array}{l} E_1 = E_m \sin \omega\, t_1 \\ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \\ E_n = E_m \sin \omega\, t_n \end{array}\right\} \qquad \textbf{(15)}$$

According to the first law expressed by equation (**10**), the value of the charging currents i_u and i_v measured at the same instant t after impressing E_u and E_v respectively, is

$$\begin{aligned} i_{cu}/i_{cv} = E_u/E_v &= [E_m \sin \omega\, t_u / E_m \sin \omega\, t_v] \\ &= \sin \omega\, t_u / \sin \omega\, t_v \end{aligned} \qquad \textbf{(16)}$$

and for the particular case where $t_v = T/4$ equation (**16**) becomes

$$i_{cu}/i_{cm} = E_u/E_m = \sin \omega\, t_u/1$$

whence

$$i_{cu} = (\sin \omega\, t_u)\, i_{cm} \qquad \textbf{(17)}$$

In what follows, two notations for time, t and τ, will be adopted in order to differentiate between time measured along the abscissa of the voltage curve (Fig. 8A), and that measured along the abscissa of the $(i - t)$ characteristic (Fig. 8B). Therefore t will be used for voltage variation and τ for charging-current variation with time.

Let

$$i_{cm} = f\left(\tau - \frac{T}{4}\right) \qquad \textbf{(18)}$$

be the function expressing the value of the charging current due to a continuous potential (of magnitude E_m equal to the maximum value of the alternating potential) which is impressed on the condenser at time $t = \dfrac{T}{4}$. Then the function expressing the value of the charging current for any other continuous potential of magnitude $E = E_m \sin \omega\, t$ impressed on the condenser at a time t and acting through an angle $d\ (\omega\, t)$ will be, by equation (**17**) and law 3,

$$d\, t_c = f\,(\tau - t)\ (\sin \omega\, t)\ d\ (\omega\, t) \qquad \textbf{(19)}$$

and the total current at any time $\tau \overset{=}{>} t$ due to all the instantaneous voltages from 0 to τ each of which acts for an interval $d\ (\omega\, t)$, will be:

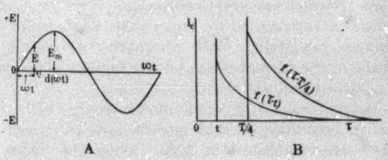

FIG. 8

A—A SINE WAVE OF POTENETIAL

B—THE CHARGING CURRENTS DUE TO TWO INSTANTANEOUS VALUES OF AN ALTERNATING POTENTIAL

$$i_c = \int_0^{\tau} f\,(\tau - t)\ (\sin \omega\, t)\ d\ (\omega\, t) \qquad \textbf{(20)}$$

and the charge Q acquired during ¼ of a cycle is

$$Q = \int_0^{T/4} i_c\, d\, \tau = \int_0^{T/4} \left[\int_0^{T/4} f\,(\tau - t)\ (\sin \omega\, t)\ d\ (\omega\, t) \right] d\, \tau \qquad \textbf{(21)}$$

The same reasoning can be extended to potentials with instantaneous values exceeding the initial potential range. Referring to Fig. 4, let $t_1, t_2, \ . \ . \ . \ t_n$ be the times at which the sinusoidal voltage assumes instantaneous values $e_1, e_2, \ . \ . \ . \ e_n$, so that $e_1 = E_A$, $e_2 = E_B, \ . \ . \ . \ e_n = E_N$, where $E_A, E_B, \ . \ . \ . \ E_N$ are the upper potential limits of the potential ranges $A, B, \ . \ . \ . \ N$, respectively. Let $f_A\,(\tau - t_1),\ f_B\,(\tau - t_2) \ . \ . \ . \ f_N\,(\tau - t_n)$ be the functions expressing the variations of $i_{c1}, i_{c2}, \ . \ . \ . \ i_{cn}$ with τ at continuous voltages $E_A, E_B, \ . \ . \ . \ E_N$. Then by analogy with equation (**21**), the value of i_c at any time τ, according to the corrections indicated, equation (**20a**) will become:

$$\left.\begin{array}{ll}
i_{cA} = \int_0^{\tau} f_A(\tau - t)(\sin \omega t)\, d(\omega t) & 0 \leqq \tau \leqq t_1 \\
\left.\begin{array}{l} i_{cB} = \int_0^{t_1} f_A(\tau - t)(\sin \omega t)\, d(\omega t) + \\ \qquad \int_{t_1}^{\tau} f_B(\tau - t)(\sin \omega t)\, d(\omega t)\,. \end{array}\right\} & t_1 \leqq \tau \leqq t_2 \\
\cdots\cdots\cdots\cdots & \\
\vdots & \\
\left.\begin{array}{l} i_{cN} = \int_0^{t_1} f_A(\tau - t)(\sin \omega t)\, d(\omega t) + \\ \qquad \int_{t_1}^{t_2} f_B(\tau - t)(\sin \omega t)\, d(\omega t) + \\ \qquad \cdots\cdots \\ \qquad \vdots \\ \quad + \int_{t(N-1)}^{t} f_N(\tau - t)(\sin \omega t)\, d(\omega t) \end{array}\right\} & t_{N-1} \leqq \tau \leqq t_n
\end{array}\right\} \qquad \textbf{(20a)}$$

The charge acquired by the condenser will be:

$$Q = \int_0^{T/4} i_{cN}\, d\tau \qquad \textbf{(21a)}$$

where i_{cN} is expressed by equation (**20a**).

c. Discussion of Equations (21) and (21a). In equation (**21**) the function $f(\tau - t)$ is that portion of the curve shown in Fig. 4 and extending from t_m to t_r. It should be determined at a continuous voltage equal to the amplitude E_m of the alternating potential. This function will vary with the material of the dielectric, the temperature and humidity of the specimen, and the physical condition of the dielectric.

Equation (**21**) comprises the essence of the cause for the decrease of charge with the frequency of the applied potential. Indeed, the mere inspection of equation (**21**) shows that the value of Q is a function of the upper limit $\frac{T}{4}$. The higher the frequency, the smaller is T and consequently the smaller the value of the integrand. In equation (**21a**), the values of i_{cA}, i_{cB} . . . i_{cN} substituted from equation (**20a**) contain $f_A(\tau - t)$, $f_B(\tau - t)$ . . . $f_N(\tau - t)$. These functions should be determined at continuous potentials of magnitudes equal to E_A, E_B, . . . E_N.

d. Discussion of Equations (20) and (20a). Equations (**20**) and (**20a**) give the instantaneous values of the charging current i_c for sinusoidal alternating potentials. The shape of this curve will certainly be distorted when instantaneous values of the applied potentials exceed the initial potential range. Whether distortion exists when the amplitude of the applied potential does not exceed the upper limit of the initial potential range, is a question which can be answered only when $f(\tau - t)$ is known. In general, with dielectrics that are rich in elastic electrons or whose viscous electrons have a high degree of elasticity, distortion may be negligible. However, with very viscous dielectrics, a certain degree of distortion in the sine form of the charging current wave may well be expected.

At all events, the current computed from equations (**20**) and (**20a**), whether sinusoidal or not, will have an effective value which can be computed. It must be remembered that the value of current determined by a current measuring instrument when alternating potentials are impressed on a condenser is the effective value not only of the charging current but of the leaky current, as well as of the current necessary to supply the viscosity and hysteresis losses discussed in part E of this paper. The value of equations (**20**) and (**20a**) consists, therefore, in affording a means of computing the true value of the charging current, thereby separating the current supplied to a condenser into its various components.

C. THE "DIELECTRIC PERMITTIVITY" K OF SOLID DIELECTRICS

Imagine two condensers No. I and No. II, of the same physical dimensions. Let the space between the plates of condenser No. I and condenser No. II be filled with a solid dielectric and vacuum respectively. Let the charges acquired by condensers No. I and No. II, when each is submitted to the same potential, be Q_I and Q_{II}. Then the dielectric permittivity will be:

$$K = Q_I / Q_{II} \qquad \textbf{(22)}$$

For continuous potentials:

$$K_1 = \frac{\int_0^{tr} i_c\, d\, t}{\int_0^{tr} i_c'\, d\, t} = \frac{\text{equation } \textbf{(9)}}{\int_0^{tr} i_c'\, d\, t} = \frac{\text{equation } \textbf{(9)}}{E\, K_v\, a/d} \qquad \textbf{(23)}$$

For alternating potentials where no instantaneous value exceeds the upper limit of the initial potential range:

$$K_2 = \frac{\text{equation } \textbf{(21)}}{\int_0^{T/4} i_c'\,(\cos \omega t)\, d\, t} = \frac{\text{eq. } \textbf{(21)}}{\text{eq. } \textbf{(45)}} \qquad \textbf{(24)}$$

and for alternating potentials where instantaneous values do exceed the upper limit of the initial potential range:

$$K_3 = \frac{\text{equation } \textbf{(21a)}}{\int_0^{T/4} i_c'\,(\text{Cos } \omega t)\, d\, t} = \frac{\text{eq. } \textbf{(21a)}}{\text{eq. } \textbf{(45)}} \qquad \textbf{(25)}$$

The denominators in equations (**23**), (**24**), and (**25**) can be very easily computed; see Part III. The numerators are defined by equation (**9**) with the limits changed from 0 to t_r, and by equations (**21**) and (**21a**) respectively.

The definitions given above take account of all the factors which influence K; see discussions of equations (**21**) and (**21a**). Moreover, the values of K, determined from these equations will be different for the same material if f and G be changed. The "dielectric constant" is thus a very inappropriate term. A better appellation, "dielectric coefficient", has been suggested. This new name is here replaced by the more descriptive and shorter one, **permittivity,** adopted by some text books. The writer regrets that he has never met with this term in reading the literature.

D. ELECTRIC STRENGTH OF SOLID DIELECTRICS

1. *Dielectric Strength with Continuous Potentials.*

a. **Definition.** The electric strength (G_s) of a solid dielectric at constant conditions of the factors enumerated in Part I hypothesis 4, and for continuous potentials is:

$$G_s = (E_s - 2\,E_p)/d \tag{26}$$

b. **Discussion of Equation (26):** The notations used in equation (**26**) will be defined as follows:

G_s = the voltage gradient at which rupture occurs. As indicated by equation (**26**), G_s is *not* obtained by dividing the applied potential by the dielectric thickness (d).

E_s = any voltage lying within the potential range S. This potential range comprises all values of voltage which, when applied to a specimen, will cause rupture.

$2\,E_p$ = the polarization potential which exists on the boundary surfaces between the plates and the dielectric of a condenser and makes its presence felt not only in the case of dielectric strength but also in all other phenomena connected with solid dielectrics.

For one and the same rupturing potential, E_p is a function of:

(1) The time of potential action.
(2) The method of applying the voltage.

(1) *E_p vs. time of Potential action*: If E_p is a polarization e. m. f., then it might be argued that since, in the case of viscous dielectrics, E_p increases with time, see Fig. 4, remark 4, then for G_s to remain constant, E_s must increase as the time of voltage application increases. Experiment shows that this is not the case. On the contrary, E_s decreases the longer the time of voltage application. This apparent contradiction to the definition proposed in equation (**26**) can be easily refuted if use is made of the fourth hypothesis established in Part I. At such high potentials, very radical changes occur in the characteristic behavior of the individual electrons belonging to the various orders. Thus many of the viscous electrons become conduction electrons and a large number of the elastic electrons becomes highly viscous. Therefore, as time proceeds, E_p decreases instead of increasing because of the continual depletion in the number of electrons which can and do take part in the polarization process. G_s thus increases as the time of voltage application increases.

(2). *E_p vs. Method of Applying the Voltage*: Experiment shows (53-19 and 87-25) that E_s is lower when the rupturing potential is attained by a slow rather than by a rapid increase of potential. This phenomenon is in strict conformity with the fourth hypothesis. Its explanation is the same as that given by the foregoing paragraph.

Besides accounting for the above observations, the introduction of E_p into the definition of G_s, explains the phenomenon of the decrease in the value of the rupturing gradient with an increase in the thickness of a dielectric. The polarization e. m. f. is in fact independent of the thickness. Moreover, there exists no reason for supposing that the rupturing gradient, (G_s), *within* the dielectric, would vary with (d). If equation (**26**) is converted into the form $G_s\,d + 2\,E_p = E_s$ and if E_s is plotted against d keeping G_s and E_p constant, a curve of the form shown in Fig. 9, will be obtained. The slope of this curve decreases as d increases, showing that a lower rupturing voltage gradient is required for large thicknesses.

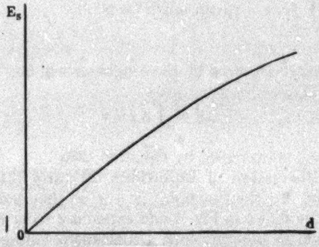

FIG. 9—VARIATION OF DIELECTRIC STRENGTH WITH THE THICKNESS OF A DIELECTRIC

This definition is further useful in ascertaining the value of E_p. Thus let E_{s1} and E_{s2} be the rupturing potentials for thicknesses d_1 and d_2, then by equation (**26**):

$$G_s = (E_{s1} - 2\,E_p)/d_1 = (E_{s2} - 2\,E_p)/d_2$$

from which

$$2\,E_p = (E_{s2}\,d_1 - E_{s1}\,d_2)/(d_1 - d_2) \tag{27}$$

In applying equation (**27**) care should be exercised that the ruptures in the two samples of dielectric occur under identical conditions as to time and method of applying the potential. Moreover, the samples must be identical in every respect except for thickness. Equation (**27**) does not hold for built-up insulation made of such materials as paraffined paper or cambric, because polarization potentials exist between the various layers of insulation, thus rendering the samples electrically different.

Besides furnishing a satisfactory explanation of the phenomena already cited, equation (**26**) accounts for the facts quoted in (9-04 and 89-25); *viz.*, that the majority of punctures occur along the edge rather than between the plates and that a lower potential will rupture a dielectric when the spark passes at the edge than when it occurs between the plates. Indeed these phenomena are due to nothing other than the presence of the polarization potential at the contact surfaces. Its effect is to reduce the voltage gradient in that portion of the dielectric which lies within the plates, thereby rendering rupture much easier at the edge where the voltage gradient is higher.

2. Dielectric Strength with Alternating Potentials.

a. **Definition:** The dielectric strength with alternating potentials may be defined similarly to equation (**26**) if it be remembered that E_s is a function of time and E_p is a very erratic quantity which varies not only with the amplitude but also with the frequency of the applied potential. The natural tendency is to substitute sine functions for E_s and E_p and to define dielectric strength at alternating potentials as:

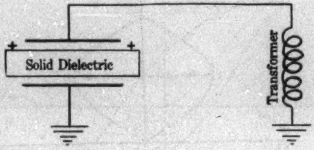

FIG. 10—UNIDIRECTIONAL POLARIZATION OF A DIELECTRIC UNDER STRONG ALTERNATING FIELDS

$$G_s \sin \omega t = [E_s \sin(\omega t + \theta) - 2 E_p \sin(\omega t + \phi)]/d \qquad (\mathbf{28})$$

where θ and ϕ are the respective angles between the E_s and the G_s vectors; and between the E_p and the G_s vectors.

Experiment shows, however, (70-20) that, at high alternating potentials, E_p is unidirectional. Moreover, if the connections are as shown in Fig. 10, then a positive charge will appear on the dielectric as indicated.

If this be the case when the plates touch the dielectric, then equation (**28**) becomes:

$$G_s \sin \omega t = (E_s \sin \omega t + E_p)/d \qquad (\mathbf{29})$$

As breakdown occurs more readily when the voltage gradient within the dielectric is highest, it is very natural to assume that rupture would take place when $\omega t = 90$ deg. We then have

$$G_s = (E_s + E_p)/d \qquad (\mathbf{30})$$

Equation (**30**) accounts for the fact that the breakdown occurs at a lower value with alternating than with continuous potentials. It contradicts, however, the known facts alluded to in the discussion of equation (**26**). Moreover, should the findings of (70-20) be utterly discarded, as not applicable to the case when the condenser plates do touch the dielectric, and equation (**28**) be adopted, we shall have three unknowns (θ, ϕ, and E_p) to determine for each dielectric. This leads to utterly fruitless results. Indeed, the writer has done this very thing. But, upon using experimental data to determine these constants, he obtained conflicting values. The length of the mathematical development of equation (**28**) and the fruitlessness of its results have led the writer to exclude it from this paper.

The absence of valuable data on the nature of dielectric breakdown with alternating potentials renders the formulation of a theory, or even a definition thereof, a matter of wild conjecture. In fact, various experimenters have endeavored to explain the phenomenon, that breakdown occurs at comparatively lower values with alternating than with continuous potentials, by maintaining that the heat generated in the sample due to dielectric hysteresis, with alternating potential, raises the temperature of the sample. Therefore, since G_s decreases as temperature increases, the voltage required to break down a sample is lower when alternating than when direct potentials are used. The writer begs to disagree with the propounders of the so called pyro-electric theory of breakdown (76-21, 104-22, 124-22, and 56-24) on the following grounds:

1. The energy dissipated in a dielectric in the period from the time of voltage application to the time of breakdown is indeed hardly sufficient to raise the temperature of that dielectric to such a degree as to account for the comparatively large decrease in the value of E_s when alternating potentials are used (106-23).

Undoubtedly, for very prolonged potential application, the phenomenon of heat does exert an effect on the rupturing voltage by raising the temperature of the sample. However, a theory built wholly on this effect is of necessity erroneous.

2. Actual temperature measurements (87-25) show that no temperature rise exceeding 10 deg. cent. does occur in glass before rupture.

A close analogy to breakdown with alternating potentials is furnished by the familiar process of breaking a piece of metal by repeated, alternate bending in two mutually opposite directions. The alternate compression and tension seem to be too much for the metal to stand. If an elastic piece of metal breaks down when bent with a certain force F acting *in one direction,* that same piece of metal will also break when a force $F' < F$ acts on the metal alternately in opposite directions. The phenomenon is ascribed to fatigue. An analogous reasoning may be applied to the rupturing potential which acts on dielectrics.

That fatigue does play a prominent part in the phenomenon of breakdown, is shown by (47-23 and 69-25).

In the absence of crucial research on the phenomenon of breakdown, we have to depend on tabulated data of the rupturing potentials. Such data, however, will be valuable only when accompanied with a detailed account of the various conditions under which the breakdown tests are made. Such conditions include: (a) mode of potential application, (b) frequency and wave form of the source, (c) temperature, (d) humidity, (e) shape of electrodes, (f) physical condition and thickness of the dielectric.

E. THE ENERGY DISSIPATED IN SOLID DIELECTRICS

1. *Causes of Energy Dissipation*:

The energy dissipated in solid dielectrics may be due to one or all of the following three clauses:

a. The resistance of a dielectric to the flow of current,

b. Dielectric hysteresis,

c. Dielectric viscosity.

No successful attempt has been made, so far, to separate the total energy loss into its three component parts. Moreover, the three sources of energy loss are not clearly defined. It will be our object, therefore, (1) to investigate the sources of these losses in the light of the established hypotheses; (2) to introduce definitions of the three types of energy loss; and (3) to devise means for the separation of the total energy dissipated into its three component parts.

2. *Sources of Energy Loss with Alternating Potentials.*

In view of the hypotheses established in Part I we shall consider separately the behavior of the three types of electrons in a dielectric subjected to alternating potentials.

a. **Perfectly Elastic Electrons.** When an alternating potential is applied to a dielectric, all the electrons are displaced from their neutral position of equilibrium. Those that are perfectly elastic will be displaced by a distance which may or may not be proportional, at every instant, to the instantaneous value of the impressed voltage. However, for this type of electrons, the same relation between the distance by which the electron is displaced and the instantaneous value of the applied potential exists no matter whether this distance be measured at increasing or decreasing values of potential. If δ is plotted as ordinate and $E_m \sin \omega t$ as abscissa, we get the curves shown in Fig. 11B.

Whether the curve is of the form (A) or (B) will depend entirely upon the characteristics of the individual electron. The interesting fact to remember is that the curve closes on itself and therefore no energy is lost.

b. **Perfectly Viscous Electrons.** The displacement of the viscous electrons gives a very different curve from the ones shown in Fig. 11B. Here the electron displays no elasticity; consequently, it continues to creep in the same direction irrespective of the instantaneous value of the impressed potential, provided that value maintains the same sign. As soon as the polarity of the applied voltage changes, the direction of creepage reverses. The distance covered by the creeping electron is at any instant directly proportional both to the instantaneous value of the applied potential and to the time through which that instantaneous value acts. We thus have for the displacement due to any potential $= E_m \sin \omega t$ and acting for an instant Δt:

$$\Delta \delta = h E_m \sin \omega t \, \Delta t$$

and for the total displacement attained during a time from 0 to t:

$$\delta_t = h E_m \int_0^t \sin \omega t \, d t = \frac{- h E_m}{\omega} \cos \omega t \qquad (31)$$

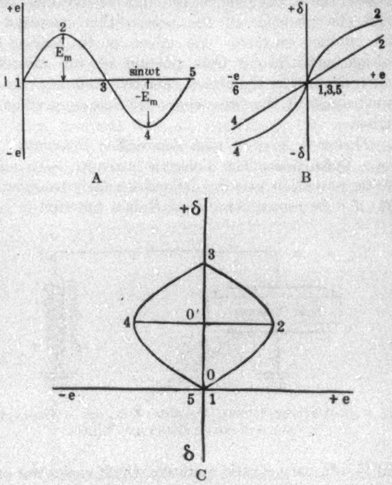

FIG. 11

A—A SINE WAVE OF VOLTAGE

B—VARIATION IN THE DISPLACEMENT OF A PERFECTLY ELASTIC ELECTRON WITH VOLTAGE

C—VARIATION IN THE DISPLACEMENT OF A PERFECTLY VISCOUS ELECTRON WITH VOLTAGE

This curve is plotted in Fig. 11C. It can be best visualized by shifting the origin O to the point O'. The existence of ω in the denominator of equation (31) shows that δ_t is a function of the period of the impressed potential.

c. **Slightly Elastic or Partially Viscous Electrons.** In the case of the partially viscous electrons, the elasticity of their bonds furnishes them with enough resilience to rebound whenever the impressed potential suffers a decrease. The distance by which they rebound is a function of their elasticity, the value of the impressed voltage, and the decrease suffered by that potential. The curve given in Fig. 12 is representative of one type of such a hysteresis loop. For other types of loops, the reader is referred to the very interesting work (54-21) wherein several loops are plotted with charge as abscissa against applied potential as ordinates.

d. **The Hystero-Viscosity Loop.** The hystero-viscosity loop (Fig. 12) may be determined for any dielectric as follows:

A continuous potential-gradient G_1 is impressed on the dielectric for a time t_{r1} and the charge Q_1 (see Fig. 4), acquired by the dielectric during the interval t_{r1} is measured. The charge density D_1 will then be:

$$D_1 = Q_1/a \qquad \textbf{(32)}$$

The gradient is next raised to G_2, the charge Q_2 acquired in the time 0 to t_{r2} is measured and D_2 computed. This process is repeated until the maximum gradient G_m, for which the loop is required, has been attained. We thus obtain the curve *0 a* (Fig. 12) which we shall call the "electrization curve" in analogy to the "magnetization curve."

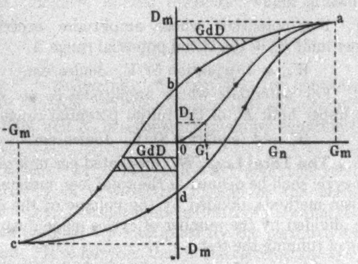

FIG. 12—A HYSTERO-VISCOSITY LOOP

Just as in the magnetization curve the permeability at any field intensity is $\mu = B/H$, so here the absolute permittivity K' of a dielectric at any voltage gradient G is:

$$K' = D/G \qquad \textbf{(33)}$$

The voltage gradient is next lowered to a value $G_n < G_m$. If G_n is low enough, *i. e.*, if $E_n < 2\,E_p$, a current will flow in the reverse direction for a time t_n at which time its value becomes zero. This reverse current is due to the fact that, the polarization potential being in excess of the new impressed potential, some of the elastic electrons rebound to their original position, thereby allowing a part of the charge on the plates to recombine. Moreover, the recombined charge in the dielectric is exactly equal to the released charge. If we measure this recombined charge and subtract its value from Q_m we obtain the charge Q_n which remains in the condenser at the gradient G_n. D_n will then be Q_n/a. By a similar process we may compute D_{n-1} at G_{n-1} . . . and D_0 at $G = 0$.

As the time element is of importance in determining this loop, ample time should be allowed for the viscous electrons to readjust themselves to each new state. Therefore, the time consumed in determining the curve *a b* should be at least equal to that spent in ascertaining the curve *0 a*. Moreover, the short circuit (at $G = 0$) should last long enough to allow all electrons which possess any degree of elasticity at all to rebound to their normal position. This time cannot be fixed for it depends upon the type of material experimented on.

The length of the line *0 b* represents the residual charge due to purely viscous electrons. These electrons, being utterly devoid of resilience, need a *coercive force* in the opposite direction to move them to their original neutral position.

The polarity of the applied potential is now reversed at the point *b*, and the density of charge D, corresponding to any gradient G, measured, as indicated above, until the negative maximum gradient ($-\,G_m$) is attained. We thus obtain the curve *b c*. The curve *c d* is obtained similarly to the curve *a b* by successively increasing the potential from ($-\,G_m$) to zero.

Finally the curve *d a* is plotted from data observed, as described above, by successively increasing the potential from zero to G_m.

e. **The physical nature of the hysteresis and viscosity losses.** The area of the loop will be shown in Part III to represent the maximum unrecoverable energy dissipated in a dielectric due to both viscosity and hysteresis at an infinitely low frequency. The question arises as to what part of the loss should be assigned to hysteresis and what part to viscosity. Before answering this question we shall enter into a discussion of the physical nature of both losses.

The hysteresis loss (W_h) will be ascribed to two causes: (1) the energy dissipated in the adjustment and readjustment of the electrons (both viscous and elastic) to each new variation in potential, and (2) the energy dissipated in the transfer of electrons from a higher to a lower order.

In order to form a picture of the first cause of hysteresis loss, imagine a crowd in a theater where a fire suddenly breaks out. The heretofore orderly populace at once assumes the characteristics of a mob. Each person will knock about in all directions and against all sorts of people before he gains an exit. A great deal of unrecoverable energy is liberated from the mob in the form of excitement, mental stress, noise, and bodily contacts and combats. The condition of affairs is the more frantic the greater the extent of the conflagration. Moreover, in the first stages of the disaster only the quick and energetic members of the mob take part in the excitement. All slow persons behave, for a time, as though nothing extraordinary had occurred until they awaken from their stupor to discover the peril confronting them. Indeed, if the manager of the theater has enough presence of mind and preparedness to suddenly stamp the fire and place matters under control, those dull, unfortunate (or perhaps fortunate) individuals would hardly suspect that anything had happened. They therefore do not take part in the excitement, and, except for the room they occupy, thereby obstructing the exit of the excited element, they may be considered as an utterly negligible factor in the commotion that takes place.

Such is the state of affairs when the mob of molecules in a solid dielectric is suddenly exposed to a potential gradient. Some unrecoverable energy is liberated in the form of molecular oscillation and molecular friction. Moreover, in the first stages of impressing the potential, practically all the energy dissipated is due to the more or less elastic electrons, the highly viscous ones supplying only a very small portion, in the form of obstruction. As time proceeds, however, they also supply their share of the total lost energy. This loss, however, shall be ascribed to viscosity.

That part of the hysteresis loss which is expended in changing the status of an electron from a higher to a lower order hardly needs comment. The energy of an electron belonging to a lower, is less than that of an electron connected with a higher order. Therefore, when a transfer does occur, the extra energy is naturally liberated in the form of heat.

The hysteresis loss, so defined, is independent of time and is a function only of the nature of the dielectric, its temperature, and the maximum gradient (G_m) attained. The true hysteresis loss per unit volume can be experimentally determined by impressing upon a dielectric potentials of the same amplitude but of different frequencies and noting the range of frequency at which the energy dissipated per cycle becomes constant. The loss per unit volume for frequencies lying within this range is the true hysteresis loss. Unfortunately this loss cannot be represented by a loop of the form shown in Fig. 12, because the element of time effects the shape and size of this loop. Therefore the only similarity that exists between magnetic and dielectric hysteresis is that of the physical nature of the two phenomena. Fortunately, at high fields, magnetic materials reveal no viscosity and the loss per cycle due to magnetic hysteresis may be actually represented for unit volume, by the area of the hysteresis loop. Such is not the case with dielectric materials. The loop in Fig. 12 thus represents the loss per unit volume due both to hysteresis and viscosity at an infinitely low frequency.

The Viscosity Loss (W_h) will also be ascribed to two causes: (1) the energy dissipated in the very slow adjustment and readjustment of the viscous electrons to each new variation of potential, and (2) the energy expended in supplying the work required by the electrons in creeping from their normal position through a distance δ and back again (see Fig. 11C). As both of these factors are increasing functions of time, the extent of the loss due to viscosity can be determined only indirectly as we shall see presently. The hystero-viscosity loop thus offers only an estimate of the magnitude of the combined loss, per cycle per unit volume, due to both hysteresis and viscosity, at infinitely low frequencies.

The above reasoning is strongly supported by (103-22, 9-23, 107-24, 63-10, 12-12, 85-12, and 55-14) which show that the energy loss per cycle, per unit volume is either a decreasing function of the frequency or independent of it. As to the relation of loss to the potential gradient, this will largely depend upon the nature of the dielectric and its behavior under the various potential ranges. Thus no definite relation can be established between the hystero-viscosity loss and the impressed potential.

3. *Definitions and Separation of Dielectric Losses.*

a. **The Resistance Loss** (**W**$_r$). The loss per unit volume, per cycle due to the resistance of a dielectric to the passage of current, at alternating potentials, shall be defined as follows:

1. For potentials whose amplitudes exceed the upper limit E_A of the initial potential range A,

$$W_{r1} = 4 \text{ (equation 5)}/V \quad \text{Joules/cm.}^3 \qquad \textbf{(34a)}$$

2. For potentials whose amplitudes *do not* exceed the upper limit E_A of the initial potential range A,

$$W_r = 4 \text{ (equation 8)}/V \quad \text{Joules/cm.}^3 \qquad \textbf{(34b)}$$

b. **The Total Loss** (**W**$_D$) dissipated per unit volume per cycle shall be defined as the total loss, measured by known methods, divided by the volume of the dielectric, divided by the number of cycles made during the time of running the test.

$$W_D = \text{(Measured Loss per cycle)}/V \quad \text{Joules/cm.}^3 \qquad \textbf{(35)}$$

For the same amplitude of applied potential, W_D will decrease as the frequency is increased, until a frequency f' is reached beyond which the loss per cycle is constant and independent of the frequency. This particular value of W_D will be defined as:

$$W_{D_f'} = \text{(Measured constant loss per cycle)}/V \text{ Joules/cm.}^3 \qquad \textbf{(36)}$$

c. **The Loss due to Hysteresis** (**W**$_h$): The hysteresis loss per unit volume per cycle at any frequency shall be defined as follows:

1. For potentials whose amplitudes exceed the upper limit E_A of the initial potential range A,

$$W_h = \text{(equation } \textbf{(36)}\text{)} - \text{(equation } \textbf{(34a)}\text{)} \text{ Joules/cm.}^3 \qquad \textbf{(37a)}$$

2. For potentials whose amplitudes do not exceed the upper limit E_A of the initial potential range A,

$$W_h = \text{(equation } \textbf{(36)}\text{)} - \text{(equation } \textbf{(34b)}\text{)} \text{ Joules/cm.}^3 \qquad \textbf{(37b)}$$

d. The Losses due to Viscosity (W_v): Energy dissipated per unit volume per cycle due to viscosity shall be defined as:

$$W_v = \text{(equation (35))} - \text{(equation (36)) Joules/cm.}^3 \qquad \textbf{(38)}$$

It must be remembered that the above equations hold true only under constant conditions of voltage gradient, temperature, humidity, etc. The only factors here eliminated are time and frequency.

Part III. Applications of the Classical Theory of Electrostatics to Solid Dielectrics

The classical theory of electrostatics holds true only for vacuum, a non-anomalous, perfect dielectric. It will be our object to show how the formulas and deductions arrived at in this theory may be applied to solid, anomalous dielectrics.

A. The Concept of Capacitance (*Permittance*). Experiment shows that for one and the same dielectric the capacitance (permittance) of a plane condenser (permittor) similar to the one shown in Fig. 1, varies directly as the area exposed to the plates and inversely as the thickness of the dielectric. This gives the equation:

$$C = K\,K_v\,a/d \text{ farads} \qquad \textbf{(39)}$$

where K is defined by equation (**23**) for continuous, and by equation (**24**) or equation (**25**) for alternating potentials, and K_v is a constant given by equation (**42**) below.

Equations (**23**), (**24**), and (**25**) give the relative permittivity of a dielectric instead of the absolute permittivity. The absolute permittivity is equal to the relative permittivity multiplied by the permittivity of vacuum. It so happens that in the electrostatic system of units $K_v = 1$. Therefore, the absolute and relative permittivities in this system of units are identical. In Heaviside's system, however, $K_v \neq 1$; see equation (**42**). This accounts for introducing K_v as a factor in equation (**39**). Whenever the term permittivity is used it will be understood to mean relative permittivity. The symbol K' will be used for absolute permittivity; see equation (**33**).

The permittance of a vacuum permittor is similarly defined as:

$$C_v = K_v\,a/d \text{ farads} \qquad \textbf{(40)}$$

Moreover, since in the case of vacuum, Q is proportional to E for all potential ranges, we have:

$$Q_v = C_v\,E = K_v\,E\,a/d \text{ coulombs} \qquad \textbf{(41)}$$

The value of K_v in Heaviside's system of units can be shown to be:

$$\left.\begin{aligned} K_v &= 0.08242 \times 10^{-12} \text{ farads/cm. cube} \\ &= 0.2244 \times 10^{-12} \text{ farads/inch cube} \end{aligned}\right\} \qquad \textbf{(42)}$$

Therefore, Q_v for a vacuum condenser can be easily ascertained. Its value is identical with Q_{11} in equation (**23**).

The denominators of equations (**24**) and (**25**) can be computed by making use of the fact that the capacitive reactance of a vacuum condenser at a frequency f is:

$$X_c = 1/(2\,\pi\,f\,C) = d/(2\,\pi\,f\,K_v\,a) \text{ ohms} \qquad \textbf{(43)}$$

$$i_c' = E_m \cos \omega t / X_c = (2\,\pi\,f\,K_v\,a/d)\,E_m \cos \omega\,t \text{ amperes} \qquad \textbf{(44)}$$

and

$$\int_0^{T/4} (i_c' \cos \omega\,t)\,d\,t = 2\,K_v\,a\,E_m/d \text{ coulombs} \qquad \textbf{(45)}$$

Equation (**45**) expresses the value of the denominator in equation (**24**) and equation (**25**) in terms of the amplitude of the applied potential and the dimensions of the condenser.

B. The Concept of Elastance. As it is sometimes convenient for the purpose of computations to use the inverse of certain constants we shall use the term elastance S and define it as:

$$S = 1/C = (1/K)\,(1/K_v)\,(d/a) \qquad \textbf{(46)}$$

In equation (**46**) the term $(1/K)$ shall be known as the "elastivity" of a dielectric and shall be denoted by the letter σ thus:

$$\sigma = 1/K \qquad \textbf{(47)}$$

The value of σ will vary inversely with K and will depend upon (a) the amplitude and frequency of the alternating potential used, or (b) the magnitude of the applied continuous potential.

Similarly the elastivity of vacuum is:

$$\begin{aligned} \sigma_v = 1/K_v &= 11.3 \times 10^{12} \text{ darafs/cm. cube} \\ &= 4.45 \times 10^{12} \text{ darafs/inch cube} \end{aligned} \qquad \textbf{(48)}$$

Substituting equations (**47**) and (**48**) in (**46**), we have:

$$S = (\sigma_v)\,d/a \qquad \textbf{(46a)}$$

With these new definitions of permittance and elastance the well known relation:

$$Q = C\,E \qquad \textbf{(49)}$$

becomes true for all potential ranges because the variation between Q and E is accounted for by the variation in the value of the coefficient C.

With continuous potentials, the writer would suggest the use of the electrization curve (Fig. 12) to define the absolute permittivity K' of a dielectric at any potential gradient G and for any charge density D. Then the value of C in equation (**49**) will be:

$$C = K'\,a/d \qquad \textbf{(50)}$$

From equation (**49**) we have $Q = E/S$ or

$$E = Q\,S \qquad \textbf{(51)}$$

But $E = G\,d$, $Q = D\,a$, and $S = (\sigma\,\sigma_v)\,d/a$

Substituting these values in equation (**50**) and simplifying:

$$G = \sigma\,\sigma_v\,D \qquad \textbf{(52)}$$

from which

$$D = G/\sigma\,\sigma_v = G\,K\,K_v \qquad \textbf{(53)}$$

Equations (**51**) and (**52**) hold true only for constant potential gradients and constant charge densities. For variable values of these quantities integration should be resorted to. We thus have:

$$E = \int G\,d\,d \qquad \textbf{(54)}$$

and

$$Q = \int D\,d\,a \qquad \textbf{(55)}$$

C. Energy in Dielectrics. Consider a plane concondenser (Fig. 1) which has a plate area (a) and a thickness (d). If i is the instantaneous value of the current supplied by the source of e. m. f. of instantaneous value e, then the energy delivered by the source during any interval of time $d\,t$ will be

$$d\,W = e\,i\,d\,t = e\,(i_c + I_r)\,d\,t \qquad \textbf{(56)}$$

and the total energy supplied during an interval of time t_r is:

$$W = \int_0^{t_r} e\,i_c\,d\,t + \int_0^{t_r} e\,I_r\,d\,t$$

$$= \int_0^{t_r} e\,d\,q + \int_0^{t_r} e\,I_r\,d\,t\;.\;.\;.\text{ watt seconds} \qquad \textbf{(57)}$$

The energy per unit volume supplied to the dielectric is:

$$W' = W/a\,d = \int_0^{t_r} (e/d)\,(d\,q/a) + \int_0^{t_r} (e/d)\,(I_r/a)\,d\,t$$

$$= \int_0^{t_r} G\,d\,D + \int_0^{t_r} G\,U\,d\,t \text{ watt sec/cm.}^3 \qquad \textbf{(58)}$$

Forgetting for a moment the second integrand in equation (**58**) (because its value does not enter in the energy storage), we shall prove that the area of the hysteresis loop (Fig. 12) represents the energy dissipated per unit volume per cycle, on account of hysteresis and viscosity at an infinitely low frequency.

1. From the point a to the point b, each decrease (G), in the voltage gradient, results in a corresponding decrease $d\,D$ in the density of charge accumulated on the dielectric. Therefore, energy is delivered from the dielectric to the external circuit. The amount of energy per unit volume is, according to equation (**58**),

$$-\,W_{ab}' = -\int_a^b G\,d\,D = -\,(\text{Area } a\,b\,D_m) \qquad \textbf{(59)}$$

2. During the part of the cycle extending from b to c energy is supplied from the source of e. m. f. for the purpose of transporting the viscous electrons represented by the line $o\,b$ to their neutral position and for the further object of charging the condenser in the reverse direction. This energy may be represented by equation (**58**) as:

$$+\,W_{bc} = \int_b^c G\,d\,D = (\text{Area} - D_m\,b\,C) \qquad \textbf{(60)}$$

3. Similarly, from the point c to the point d, energy is delivered by the dielectric to the circuit of such amount that

$$W_{cd} = -\int_c^d G\,d\,D = -\,(\text{Area} - D_m\,C\,d) \qquad \textbf{(61)}$$

4. Finally, the energy supplied to the dielectric during the part of the cycle $d\,a$ is

$$W_{da} = \int_d^a G\,d\,D = (\text{Area } d\,a\,D_m) \qquad \textbf{(62)}$$

The net energy retained by the dielectric and dissipated in the form of hysteresis loss per cycle per unit volume is:

$$\begin{aligned} W_h + W_v &= (\text{equation } \textbf{(60)} + \text{equation } \textbf{(62)}\,) \\ &\quad - (\text{equation } \textbf{(59)} + \text{equation } \textbf{(61)}\,)\,) \\ &= \text{Area of the loop } a\,b\,c\,d\,a \end{aligned} \qquad \textbf{(63)}$$

NOTATIONS

Symbol	Quantity	Units
a	Area..........................	Cm^2.
c	Chemical composition of a dielectric	
C	Capacitance (Permittance)........	Farads
c	(Subscript) charge	
d	Thickness......................	Cm.
d	(Subscript) discharge	
D_c, D_d	Charge and discharge density.....	Coulombs/cm.^{2}
E	Voltage (Continuous or effective)	Volts
e	Voltage, instantaneous..........	
f	Frequency......................	Cycles/sec.
G	Voltage gradient................	Volts/cm.
H	Humidity....................	= Per cent
I	Current, continuous or effective.	Amperes
i	Current, instantaneous..........	
K	Dielectric constant (Permittance)..	Numeral
M	Material of electrodes	
N	Refractive index; any number	
p	Physical condition of a dielectric	
P	Mechanical pressure on a dielectric.	kg./cm.^{2}
p	(Subscript) Polarization thus E_p = polarization e. m. f.	
Q_c	Charge........................	Coulombs
Q_d	Discharge.....................	
R	Resistance.....................	Ohms
R	Ionizing radiations	
s	Shape of electrodes	
S	Elastance = i/C.................	Darafs.
s	(Subscript) strength thus G_s = dielectric strength	
t_r	Time..........................	Sec.
T	Temperature....................	Deg. Abs.
U	Current density.................	Amps./cm.^{2}
V	Volume........................	Cu. cm.
W	Energy........................	Watt-sec.
W'	Energy density.................	watt. sec/cu. cm.
ρ	Resistivity.....................	mho/cm.^{3}
γ	Conductivity....................	ohm/cm.^{3}
σ	Elasticity......................	Darafs.
θ ϕ	Angles........................	Deg.
ω	$2\,\pi\,f$ (Angular velocity)..........	radians/sec.
$A, B, \ldots S$	(Subscripts) Limits of ranges	
$A, B\,\ldots S$	(Not subscripts) Ranges.	

Appendix

TABLE SHOWING EXPERIMENTAL EVIDENCE RELATIVE TO THE VARIATION OF R, Q, K, G, AND W, WITH THE VARIOUS FACTORS INDICATED AS COLUMN-HEADINGS

(For interpretation of column headings see notations)

Property of dielectric	Mode of variation	Number of researches showing the mode of variation with:												
		G	t	f	T	P	H	B	s	M	a	d	c	p
Resistivity ρ	Direct	0	10	1	0	0	0	0	0	0	0	2	0	0
	Inverse	19	0	1	28	2	10	12	0	0	0	0	0	0
	Indefinite	0	1	1	3	0	0	0	0	3	0	0	2	15
	No	0	0	0	0	0	0	0	0	0	0	0	0	0
Charge Q	Direct	21	21	0	9	3	2	3	0	0	0	0	0	0
	Inverse	0	0	0	1	0	0	0	0	0	0	0	0	0
	Indefinite	0	0	0	0	0	0	0	1	0	0	1	0	2
	No	0	0	0	0	0	0	0	0	0	0	1	0	0
Permittivity K	Direct	3	7	0	20	4	1	5	0	0	0	0	0	0
	Inverse	1	0	23	3	5	0	0	0	0	0	0	0	0
	Indefinite	0	0	0	1	0	0	0	0	1	0	0	3	12
	No	4	1	3	1	1	3	0	0	0	0	0	0	0
Electric Strength G_s	Direct	0	0	0	2	1	0	0	0	0	0	0	0	0
	Inverse	0	8	7	7	0	5	0	0	0	4	20	0	0
	Indefinite	0	0	0	1	1	0	0	2	0	0	0	1	6
	No	0	0	0	1	1	0	0	0	0	0	2	0	0
Energy dissipated W	Direct	30	0	9	14	1	2	0	0	0	0	0	0	0
	Inverse	0	4	5	3	0	0	0	0	0	0	1	0	2
	Indefinite	0	0	3	1	0	0	0	0	0	0	0	1	0
	No	0	0	10	0	0	0	0	0	0	0	1	0	0

Note:—"Direct variation" signifies an increase while "Inverse variation" means a decrease in the property of a dielectric with an increase in the factors constituting the column-headings.

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Discussion

S. J. Rosch: In considering the total loss in a dielectric, Mr. Malti claims to have been able to separate that particular loss due to hysteresis and he gives an expression $W_h = W_d - I^2 R$. In justification of this expression he states that as the frequency of the test voltage is increased, the loss per cycle decreases until some value of frequency is reached after which the loss per cycle remains a constant.

In order to accept this equation, we must assume that the loss due to hysteresis is constant at the lower frequencies. That is, something we have not as yet been able to substantiate and until its accuracy has been deduced as the result of experimental work in connection with different dielectrics, I would counsel caution in the acceptance of this expression.

Nevertheless, as a working hypothesis, the point is well taken and it would certainly be a worth-while fact for some of the research engineers to establish as a step toward the better understanding of the behavior of an imperfect solid dielectric.

E. S. Lee: The mechanism of dielectric breakdown is still unknown to us. Mr. Malti suggests the occurrence of certain internal phenomena in the dielectric to account for external phenomena as observed. While these internal conceptions may possibly be helpful, he does not make it clear how they may be made useful for prediction of phenomena not as yet observed. If this might be done, his work would have greater value.

Mr. Malti indicates the absence of crucial research on the phenomenon of breakdown. This is the dielectric property of greatest interest. There surely ought to be recognition of the large amount of work of high caliber which has been carried on and published relative thereto, which, if it has not been crucial research, has nevertheless given us considerable insight into the phenomena involved and the magnitude of their extent.

F. M. Clark: In discussing Mr. Malti's paper I am thinking more of the oil-treated cellulose material than I am of the type such as mica and glass.

Mr. Malti is to be congratulated in attacking the electronic phase of dielectric failure. I think, however, that he has perhaps fallen out of the frying pan into the fire when he endows his electrons with a degree of mentality. He has three or four different types of electrons. Free electrons, we can dispense with by the statement that they are there. If we want to increase them we subject the insulation to ionizing radiation, but when we come to electrons of various degrees of viscosity ranging all the way from elastic to viscous electrons, I am rather doubtful as to the source of those electrons themselves.

In view of the fact that he admits as everyone does that the electrons are practically identical with regard to charge and mass, it is hard to see how two electrons under the same conditions can one of them choose to be elastic and the other one choose to be rather viscous.

I want to suggest an idea on which we are working in the laboratory of the General Electric Company; that is, the consideration of the dielectric problem from the standpoint of molecular moment. If we have a molecule in which the center of mass gravity corresponds to the center of electrical gravity, we say it has no moment. That molecule ought to be inert chemically and a very good dielectric. If, on the other hand, we have the center of mass gravity not corresponding with the center of electrical gravity, then we have a molecule which is endowed with electric moment. With this fundamental idea, one can explain most dielectric behavior very similarly to the way Mr. Malti has done in adopting different viscosities for electrons.

I can illustrate one point very simply in following out that idea. I hope we shall soon have our data ready to present before the Institute. Mr. Malti discusses polarization. If you adopt the polar molecule you will get the same sort of a diagram as Mr. Malti has obtained. But immediately you raise the question that in some molecules, preferably with paraffin material, the electric moment is almost negligible. In that case if there is no electric moment we are compelled to admit then that we do not get orientation in the electric field.

Therefore, we have approached pretty close to what we might consider as a perfect dielectric. But even then we should have to explain the perfect dielectric as being not only of no electrical moment, but of having infinite affinity for its electrons, because as we raise the voltage we should get electron displacement and eventually should get electrons knocked off with resulting ionization.

It must be remembered, too, that in the ordinary type of solid insulation—and I mentioned oil-treated cellulose—the material is colloidal in character. If it is not a true colloid, it is colloid-like and subject to colloidal laws. Colloids in the electrical field, of course, will assume a charge, and for all intents and purposes we have then a polar molecule, even though inherently it is of no moment.

From that definition then, we would conclude that the perfect dielectric would be a molecule of no electrical moment and with infinite affinity for its electrons.

I shall not try to apply that idea to all of Mr. Malti's work, but I want to say that with such an idea, d-c. breakdown is subject to entirely different fundamental laws from a-c. breakdown. Only in a case of what we have defined as a perfect dielectric would the d-c. and a-c. breakdowns be subject to the same characteristics or laws.

Passing over to d-c. breakdown, Mr. Malti mentions the fact that d-c. breakdown is subject to time, is a function of insulation thickness and is therefore subjected to the mode of voltage application, and logically following these conclusions, is subject to a fatigue effect.

We have done considerable work in fatigue effect under a-c. potential, and we have been able to trace it out pretty well.

However, we have never been able, under carefully controlled conditions, to trace out a fatigue effect under d-c. potential. If the experiment is carefully controlled, the d-c. breakdown appears almost independent of the thickness. The rate of voltage application is not markedly effective. The time-voltage curve is almost flat. Those ideas fit in very nicely with the suggestion of molecular moment which I am offering.

Mr. Malti is led to the conclusion that the pyroelectric theory is to be rejected. That conclusion I think, should be carefully considered before it is generally accepted. For example, Mr. Malti's two reasons which he gives for this conclusion are of interest. We recognize that there are two types of breakdown perhaps—the instantaneous type and the so-called long-time type. To my idea the weakness of the pyroelectric theory has not been that it is wrong. It is apparently true, as far as I can see, that insulation failure is a heat phenomenon if you exclude the instantaneous type of breakdown. However, the pyroelectric theory begs the question almost entirely on the fundamental cause of that heat and it is hard to see why Mr. Malti can adopt his own ideas of viscous electrons and still retain the pyroelectric theory. If the heat theory is to be rejected on his ideas, what would he say is the direct cause of insulation failure? The motion of electrons is bound to create heat, and heat itself, from the second paragraph of his first reason, does exert an effect.

Mr. Malti also rejects the idea of pyroelectric theory because of measurements on glass. This is one place where I want to emphasize what Mr. Lee has said. It is the present tendency to give up the study of oiled solids and pass over to the study of what are thought to be perfect dielectrics such as glass and salt crystals. In doing that you must remember that as we pass from cellulose materials to salt crystals, we are passing from the organic to the inorganic realm. Whatever the molecular attraction may be in the organic realm, there seems to be considerable doubt that it is electrostatic. In the inorganic series, it is pretty certain that the attraction between atoms in the molecule is electrostatic.

In going from the organic to the inorganic, from cellulose materials to glass, we are passing to a material which has all the appearances of being an electrolytic conductor. With salts such as silver sulphide, it has been shown conclusively that the conductivity is partly electrolytic and partly metallic in character.

The dielectric characteristics of an inorganic insulator are probably affected by laws which do not apply to an organic molecule. I do not see how we can make any marked progress in obtaining a theory of insulation failure (by insulation meaning oiled organic material), by going over into a crystaline structure where in most cases we are dealing with inorganic materials showing either electrolytic or metallic conduction, or both.

Herman Halperin: (by telegram) It would obviously be of very great assistance to the art if Mr. Malti would conduct a series of experiments to verify some of his theories and statements especially in regard to Fig. 9 showing the variation of dielectric strength with the thickness of dielectric and to his disagreement with the pyroelectric theory. In connection with the pyroelectric theory, it has been found in a series of tests at various voltages on several hundred samples of impregnated paper insulated cables of various voltages and sizes that about 75 per cent of the failures in accelerated life tests occur at points along the cable sheath which were considerably warmer previous to the failure than the adjacent sheath. This percentage increases with the thicker insulations. Further data on this point are given in the paper on the quality rating of high-tension cables by Mr. Roper and myself.[1]

M. G. Malti: In answer to Mr. Halperin's telegram and to the various speakers as to my attitude on the pyroelectric theory of breakdown, I should like to refer to the following statement on the tenth page of my paper:

"Undoubtedly, for very prolonged potential application, the phenomenon of heat does exert an effect on the rupturing voltage by raising the temperature of the sample. However, a theory built *wholly* on this effect is of necessity erroneous."

Again, referring to the appendix (Column T) it will be noted that I am aware of at least seven references which confirm the view that the dielectric strength decreases with an increase in temperature. The question is really much deeper than would appear. It is this: *For very short time of potential application* is heat the *result* or the *cause* of breakdown? My answer is that for short time intervals heat is both a result and one of the weakest contributing factors of breakdown.

The reason I assert that heat is a *result*, in the case of instantaneous breakdowns, is that the phenomenon of breakdown is nothing more than the tearing up of the electrons from their orbits. The energy dissipated due to the consequent electronic oscillations, vibrations and friction appears in the sample as heat.

Answering Mr. Lee, I fully recognize the large amount of work of high caliber which has been carried on and published relative to dielectric breakdown. However, I beg to repeat that none of it appears to be crucial. If he finds opportunity to do some research I would suggest the following: take a group of samples of the same insulation all of the same thickness and all made by the same process of manufacture and as uniform in quality as can be had. Let these samples be tested under the following conditions:

1. Continuous potential (time of potential application S_1 seconds, breakdown potential applied in one step).

    a. Flat plates of the same material (plates not touching the insulation),

    b. Flat plates of the same material (plates touching the insulation),

    c. Flat plates of the same material (plate forming intimate contact with insulation),

    d. Increase and decrease size of plates and repeat tests a, b, c,

    e. Use spheres of varying diameters and repeat tests a, b, c,

    f. Use needle points and repeat tests a, b, c,

    g. Use a, b, c, d, e, and f in various combinations,

    h. Change material of plates and repeat tests a to g,

    i. Use plates of two different materials under various combinations and repeat tests a to g,

    j. Repeat tests a to i with various sources of continuous potential (*e. g.*, kenetron tube, d-c. generators, induction machines, etc.).

2. *Alternating potentials* (*time of potential application* S_1, *seconds*, full potential applied in one step): Repeat all tests listed for continuous potentials with alternating potentials of pure *sine waves* or of waves whose form is definitely known. Use ranges of frequency varying from 1 cycle per sec. to as high as laboratory facilities permit.

3. Repeat tests 1 and 2 for a breakdown of $S_2, S_3 \ldots S_n$ seconds.

4. Repeat tests 1, 2, and 3 with potential gradually increased to breakdown. This series of tests should be made under the same conditions of temperature, humidity, etc.

5. Change the thickness of the dielectric and repeat tests 1 to 4.

6. Change the temperature of the sample and repeat tests 1 to 4 for various thicknesses.

7. This suggested research should give some *crucial* results as regards only that one material. Therefore, repeat it for other materials.

8. Give a very detailed description of the physical and chemical properties of the samples used.

I know of no published work that has strictly followed this procedure. If this research can be made with the required material and *high-class labor*, the results will shed a bright light on the mechanism of breakdown.

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[1] A. I. E. E. TRANSACTIONS, 1926, Vol. 45, p. 528.

I do not know that phenomena Mr. Lee refers to when he says "prediction of phenomena not yet observed." Each of the phenomena mentioned in my paper is an entity. They are all well known and have been observed for ages.

Mr. Rosch takes exception to my definition of hysteresis loss appearing in Eqs. (**37a** and **37b**) on the ground that, these expressions would be correct if, and only if, the hysteresis loss remained constant at various frequencies. He further suggests that some research engineers should establish this fact.

Unfortunately both hysteresis and viscosity are so intimately connected together that they cannot be experimentally separated. In order to affect their separation we have to discover a dielectric that possesses one but not the other property. Paraffin ozokerite closely approaches this ideal. It would be indeed well worth while if an experimental research engineer would establish or refute my equations. I wish to thank Mr. Rosch for this suggestion.

Mr. Clark seems to infer, from the simile I give between the electrons and a crowd in a theater, that I endow the electrons with a degree of mentality. This simile is drawn only to help one's imagination as to what goes on when a potential is impressed on a dielectric. He cannot see how, if all the electrons are of the same nature, some of them can be viscous, others elastic and still others free. I beg to refer him to part I section B of my paper and to state that the terminology used there might help him. I refer there to these electrons as *free elastically bound* and *viscously bound.*

According to the modern electron theory of matter electrons are assumed to revolve in orbits of various diameters and various eccentricities about the proton. The picture is similar to our solar system with the proton corresponding to the sum and the electrons corresponding to the various planets.

Now from the fundamental laws of electrostatics, $F = \frac{q_1 q_2}{r^2}$

where q_1 and q_2 correspond to the charges on the proton and the electron and r to the distance and F is the force of attraction between the electron and proton[2].

It can be easily seen from this equation that the greater the distance between an electron and a proton, the less the force of attraction. Therefore, we may introduce, on this basis, the following definitions of the three types of electrons: 1. Free electrons are those which lie in the outermost orbit or orbits,

2. Viscously bound electrons are those lying in orbits nearer to the proton than those of the free electrons, and

3. Elastically bound electrons are those which occupy the innermost orbits.

Mr. Clark refers to the old Maxwellian conception of molecular moments. However, in the light of modern developments the old Maxwellian views are known and have been proven to be very crude and incorrect.

In regard to fatigue with d-c. potential Mr. Clark would probably be interested in the experiments of Professor Langsdorf and others. They have found fatigue. I shall be glad to supply a complete bibliography on the subject of fatigue which will not only be of interest to him but will probably suggest different modes of procedure from those he has been following in an effort to determine insulation fatigue.

As to the cause of insulation failure, I beg to refer Mr. Clark to my answer of Mr. Halperin's and Mr. Lee's discussions.

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[2] This definition is known not to be true for atomic structure but it is approximately true to illustrate my point.

High-Voltage Measurements on Cables and Insulators

BY C. L. KASSON[1]

Member, A. I. E. E.

Synopsis.—*This paper presents some of the results of a series of high-voltage tests on cables and insulators, extending over a period of eight years, to determine the electrical characteristics of the insulation. Leakage current, insulation resistance, and watt input tests were made with direct and alternating current. Paper-insulated and rubber-insulated cables and a 27,000-volt porcelain insulator were tested.*

From these tests several conclusions were drawn, the principal ones being as follows:

1. Insulation resistance of paper and rubber-insulated power cables increases to a maximum with increasing applied d-c. voltage, the characteristic depending upon the temperature.

2. It is necessary to use shields as well as guards in making tests to determine the electrical characteristics of cable insulation under d-c. voltage stresses above the ionization point.

3. The watt input to the insulation of a paper cable under d-c. stress, at a given temperature, depends upon the character of the voltage wave; the greater the ripple, the greater the watt input.

4. It is necessary to use shields as well as guards in making tests on short cable samples to determine the a-c. electrical characteristics, such as dielectric loss of cable insulation under voltage stresses above the ionization point.

5. The ionization point is very variable depending upon the physical circuit, together with the atmospheric conditions, and represents, in reality, local air breakdown.

6. The better (i. e., the more uniform) the dielectric, the greater is the tendency for the material to break down in its entirety rather than at a single point. Practically, of course, no dielectric is perfectly homogeneous, so that failure will be restricted to the weakest spot or spots.

TESTS ON PAPER-INSULATED CABLES

THE first series of tests described in the paper is that on paper-insulated, lead-covered cables. These were tested with direct current from a kenotron set and a high-voltage battery and with 60-cycle alternating current.

The results of the tests were plotted to show the variation of leakage current, insulation resistance, and watt input with applied voltage at given temperatures. These curves show the insulation-resistance-voltage stress and watt input voltage stress characteristics of the cables. The former indicates the resistance reaction of the insulation to d-c. voltage stress and the latter, the power input or dielectric loss. These latter characteristic curves are important because over a given period of time they are a measure of the energy input to the insulation, with resultant heating.

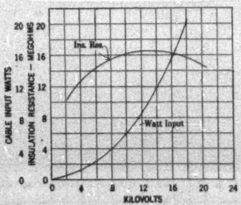

FIG. 1—VARIATION OF D-C. CHARACTERISTICS (INSULATION RESISTANCE AND WATT INPUT WITH VOLTAGE) OF OLD 13,800-VOLT PAPER-INSULATED CABLE LINE

Tests made on Line 5-97, consisting of rosin-oil impregnated, three-conductor, 4/0 copper cable, 18,283 ft. long. Conductor insulation, 7/32 in., outer belt, 7/32 in.

Voltage applied from kenotron and low voltage battery. Corrections made for temperature changes.

Insulation Resistance Varies with Voltage Applied. The first conclusion, namely, that insulation resistance increases to a maximum with increasing applied d-c. voltage, is supported by the tests the results of which are shown in Figs. 1 to 6 and 10 to 14, inclusive.

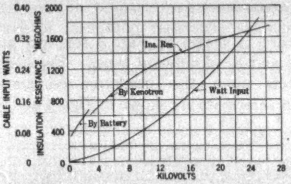

FIG. 2—VARIATION OF D-C. CHARACTERISTICS OF NEW 15,000-VOLT PAPER-INSULATED CABLE LINE

Tests made on Line 38-118, consisting of petrolatum impregnated, three-conductor, 300,000 cir. mils cable, 22,079 feet long. Conductor insulation 7/32 in., belt, 3/32 in.

Voltage applied from kenotron and low voltage battery.

An inspection of these curves indicates that both old and new cables show this characteristic in varying degree, depending upon the temperature. At the lower temperatures the insulation-resistance rise is very pronounced, but at the higher temperatures the rise is slight.

The first series of tests was made on an old (line 597) and a new (line 38-118) 15,000-volt cable. These cables were respectively 18,283 ft. and 22,079 ft. long. The results are shown in Figs. 1 and 2.

The old cable (Fig. 1) shows a maximum insulation resistance at the given temperature, under a stress of only 13 kv. d-c. The new type cable (Fig. 2) shows a

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[1] Edison Electric Illuminating Co., Boston, Mass.

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characteristic still rising at 26 kv. From the character of these curves and values of insulation resistance it might be possible to draw partial conclusions as to the general condition of the insulation of these paper cables. On this basis the condition of line 5-97 appears to be poor and that of line 38-118 good.

Effect of Temperature on Variation of Insulation Resistance with Applied D-c. Voltage. The effect of temperature is shown in Fig. 3 for two new paper cables (*A* and *B*) of different types. Further temperature effects upon the insulation-resistance-voltage-stress characteristics are shown in Figs. 4 and 5, for new *C* and old *D* paper cable, respectively. On the old cable, the effect of temperature is very marked. At 40-kv. direct current, the 23-deg. cent. characteristic is falling very fast, which probably indicates impending failure. The 60-deg. cent. curve would show a corresponding drop if plotted to suitable scale.

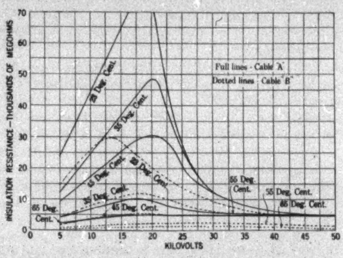

Fig. 3—Variation of Insulation Resistance with D-c. Voltage, at Several Temperatures, of Unshielded Reels of New Paper-Insulated Cable

Tests on 15,000-volt, three conductor, 350,000-cir. mil cable lengths. Conductor insulation 7/32 in., belt 3/32 in. Cable A, 644 feet long; petrolatum-impregnated; cable B, 645 feet long, impregnated with petrolatum and rosin-oil.

Voltage applied from kenotron.

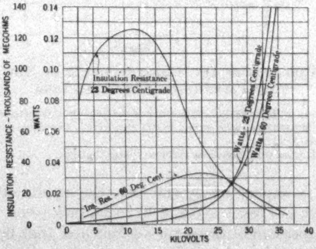

Fig. 4—Variation of D-c. Characteristics at Two Temperatures of Guarded, Unshielded Reel of New Paper-Insulated Cable

Tests made on cable C, 15,000-volt, three-conductor, 300,000 cir. mils, petrolatum-impregnated, 800 feet long. Conductor insulation 7/32 in., belt 3/32 in.

Voltage applied from kenotron.

Necessity for Using Shields. In making tests to determine the electrical characteristics of reel lengths of cables it became evident that complete shielding is very important. Figs. 3, 4, 5, and 6 illustrate cable characteristics measured without shields. In these figures most of the insulation-resistance curves against voltage stress rise to maximum and then decline. This decline is apparently due to ionization of the air, either within or outside the cables. The effect of the ionization of the air at the ends of the cable becomes proportionally less as the leakage current through the insulation increases; in fact, in case of very high leakage through the insulation, it may be a negligible percentage of the total measured current.

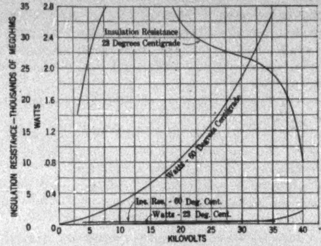

Fig. 5—Variation of D-c. Characteristics at Two Temperatures of Unshielded Reel of Old Paper-Insulated Cable

Tests made on cable D, 15,000-volt, three-conductor, 2/0, rosin-oil impregnated, 101 feet long. Conductor insulation 9/32 in., belt 9/32 in.

Voltage applied from kenotron.

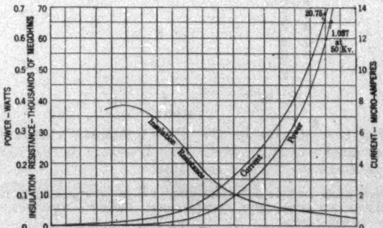

Fig. 6—Variation of D-c. Characteristics (Leakage Current, Insulation Resistance, and Watt Input with Voltage) of Unshielded Reel of New Paper-Insulated Cable

Tests on cable C (see Fig. 4). Voltage applied from high-voltage battery. Temperature 26-29 deg. cent.

Where the leakage current is low, however, the end effects may equal or completely swamp the cable insulation effect at stresses above the ionization point. In these cases, whether the cable is long or short, it is necessary to use shields and guards to obtain the true leakage current values through the insulation, from which values the cable characteristics are derived.

Further tests were made, therefore, on cable *C* using shields and the results are given in Figs. 10, 11, 13, and 14. These results support the second conclusions, that it is necessary to use shields as well as guards in making tests to determine the electrical characteristics of cable insulation under d-c. voltage stresses above the ionization point.

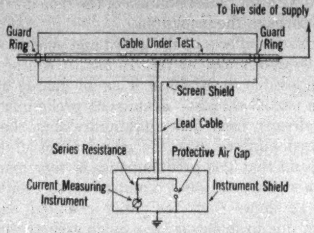

Fig. 7—Diagram of Connections for Tests with Cable and Instruments Shielded

The term "shielded guarded" means that a complete system of shielding was used in making these tests in addition to the ordinary guards or guard rings. This system of shields, in the case of the paper cable, is diagrammatically outlined in Fig. 7. The actual cable and instrument shields are shown in Figs. 8 and 9, respectively. From Fig. 7, it will be noted that the cable, measuring instruments, and connections are completely shielded.

Fig. 8—Shield for Reel of Cable

Figs. 10 and 11 were obtained with shields and guards, and No. 6 without them on an 800-ft. length of 15,000-volt paper cable *C*. It will be noted that the shielded guarded values are only a fraction of the unshielded unguarded values at the higher stresses. For instance, the shielded guarded values of insulation resistance, leakage current, and input watts at 50 kv. are 104,000 megohms, 0.48 microamperes, and 0.024 watts, respectively.

For comparison, the unshielded unguarded values at 50 kv. are 2400 megohms, 20.8 microamperes, and 1.04 watts, respectively. It will be observed that the leakage current and watt input at 50 kv., unshielded, unguarded are 43 times the shielded guarded, or true values.

Subtracting one value from the other indicates that the leakage current and watt input, in the surrounding atmosphere, amount to 20.3 microamperes and 1.02 watts, respectively, at 50 kv. Of course, these values depend upon the physical arrangement of test circuits and the atmospheric conditions.

Fig. 9—Shield for Measuring Instruments

Fig. 12 shows the comparisons between the characteristic curves of insulation resistance against voltage stress with and without shields and guards. From an inspection of Fig. 12, it appears that the curves of insulation resistance under the two conditions diverge at stresses far below the usual so called ionization point. There is a marked divergence at 10 kv. d-c. It will be noted that the shielded guarded curve continually rises and it might be inferred that it must reach a maximum and decline previous to failure due perhaps to the ionization of the air within the insulation.

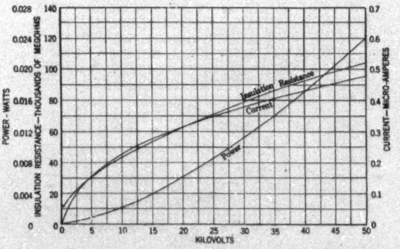

Fig. 10—Variation of D-c. (Battery) Characteristics on Shielded Reel of New Paper-Insulated Cable

Tests on cable C (see Fig. 4). Voltage applied from high-voltage battery. Temperature 26-29 deg. cent.

Fig. 16 shows the total end loss in watts, which is the difference between the unshielded unguarded and shielded guarded results, plotted against voltage stress. This curve evidently represents the power dissipated into the air at the various d-c. stresses.

It is apparently possible to dissipate 0.03 watt at 25 kv., 0.17 watt at 35 kv., 0.31 watt at 40 kv., and 1.02 watts at 50 kv. into the atmosphere under the given test conditions with a battery source of d-c. stress as shown by Fig. 16. It is obvious that the value of watts will increase rapidly with the higher voltages and is subject to the local atmospheric conditions at the ends of the cable.

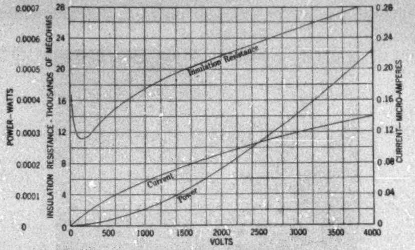

Fig. 11—Variation of D-c. (Battery) Characteristics of Shielded Reel of New Paper-Insulated Cable

Results of Fig. 10 plotted to larger scale

These results lead the author to believe that nearly all laboratory tests on cables and other insulations under d-c. stresses must be made with shields as well as guards in order to secure true and accurate results. Further, it is believed that such shields are necessary at stresses far below the ordinary ionization point.

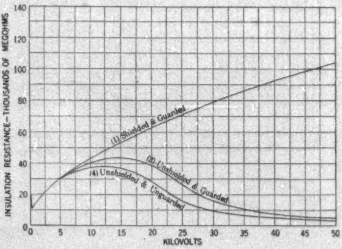

Fig. 12—Curves Showing Effect of Shielding and Guarding on Measured Insulation Resistance of Reel of New Paper-Insulated Cable

Tests on cable C (see Fig. 4). Voltage applied from high voltage battery. Temperature 26-29 deg. cent. (Includes data of Figs. 6 and 10.)

The need for the shield is due to the fact that the end effects are composed of surface leakage and leakage through the air. The guard takes care of the surface leakage and the shield takes care of the leakage through the air at the end of the cable.

The use of guards or guard rings is an old practise and some observers have used shields for instruments, but the author believes that the use of shields on the cable or dielectric under test is new and also very vital. These shields and guards are necessary in all cases where the end effects are of sufficient magnitude to interfere with the determination of the true d-c. leakage current through the insulation.

The relative effects of the guard and shield have not been fully determined as yet. In fact, it will be practically impossible to establish it as all guards are in a measure partial shields. From such preliminary work as has already been done, it appears that the shield effect predominates over the guard effect under d-c. stresses and that this predominance is very marked at the higher d-c. stresses. Comparison of the curves in Fig. 12 shows that, for paper-insulated cable C, the effect due to shields is very much greater than that due to the guards, the difference becoming more marked at the higher voltages.

An inspection of Figs. 10, 11, 13, and 14 indicates that with both battery and kenotron d-c. stress, the insulation resistance curves show a slight drop at the start before the general rising characteristic begins. It is thought that this results from either a residual electrification or a change within the insulation. This reverse action is perhaps important as indicating a complete change of conditions within the dielectric. It is perhaps related to the particle action and the initial state of the dielectric. This aspect of these tests is worthy of further study from the standpoint of dielectric action under stress.

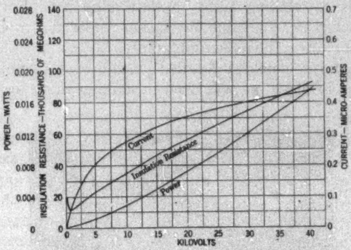

Fig. 13—Variation of D-c. (Kenotron) Characteristics of Guarded Shielded Reel of New Paper-Insulated Cable

Tests on cable C (see Fig. 4). Voltage applied from kenotron. Temperature 26 deg. cent.

The various curves presented show that the insulation resistance of modern paper cables under shielded conditions is exceedingly high and increases with increasing applied d-c. stress up to certain limits. From the d-c. standpoint a good paper cable dielectric is a much better insulator than has been previously apparent. In fact, a modern cable is a first class condenser of exceedingly low leakage. From the operating standpoint, perhaps it may be worth while to have a certain amount of leakage in a long cable to act as a safety valve when the line is subjected to a high transient voltage. In other words, perhaps a certain amount of leakage acts as a stabilizer provided that in obtaining this leakage there is no unnecessary sacrifice of dielectric strength.

Comparison of Battery and Kenotron Tests. Shielded guarded Figs. 10 and 11 obtained by use of the Cruft high-voltage battery may be compared with Figs. 13 and 14 by kenotron, for the 800-ft. length of 15,000-volt paper cable *C*. The results are quite similar, but the watt input with the kenotron is a little higher than that with the battery source. This is shown by Fig. 15. It is believed that this is due to the ripple of the kenotron wave.

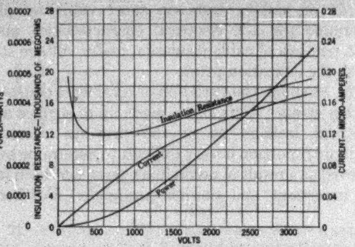

FIG. 14—VARIATION OF D-C. (KENOTRON) CHARACTERISTICS OF GUARDED, SHIELDED REEL OF NEW PAPER-INSULATED CABLE

Results of Fig. 13 plotted to larger scale

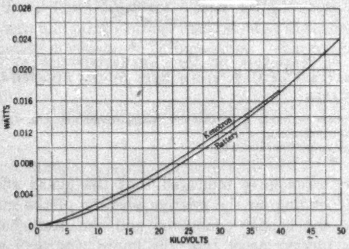

FIG. 15—COMPARISON OF WATT INPUTS WITH KENOTRON AND BATTERY ON SHIELDED REEL OF NEW PAPER-INSULATED CABLE

Data of Figs. 10 and 13

From this the third conclusion is drawn, that the watt input to the insulation of a paper cable under d-c. stress, at a given temperature, depends upon the character of the voltage wave; the greater the ripple, the greater the watt input. This would agree with the accepted fact that the watt loss in a dielectric increases with frequency. In other words, the watt loss in a dielectric at given ambient temperature ranges from a minimum on direct current with wave form better than that supplied by battery, to a maximum at high frequency.

A-c. Dielectric Loss Tests, Shielded and Unshielded. The failure of insulation is largely a matter of heat and its relative distribution in the material. Under d-c. stress the watt input and the consequent heating is very slight in comparison with that under so called equivalent a-c. stresses. On cable *C* the a-c. watt input at 24,000 volts, 60 cycles, was 131.8 and the d-c. watt input at 50 kv. was 0.024 watts, at room temperature.

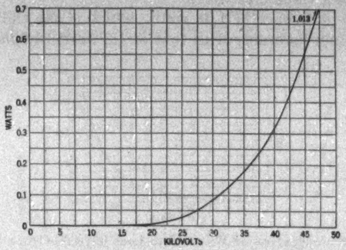

FIG. 16—VARIATION OF WATTS END LOSS WITH D-C. VOLTAGE (BATTERY) ON REEL OF NEW PAPER-INSULATED CABLE

Tests on cable C (see Fig. 4). Voltage applied from high-voltage battery. Temperatures 26-29 deg. cent. Data from Figs. 6 and 10

The shielded guarded values on cable *C* at 24,000-volts a-c. were 131.8 watts as against an unshielded unguarded value of 133 watts. This would indicate that shielding is not necessary in making a-c. dielectric loss tests on long cables, 800 or more feet in length, unless the dielectric losses are exceedingly low.

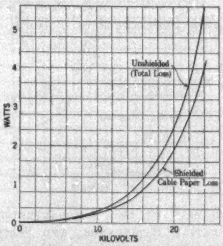

FIG. 17—VARIATION OF WATT INPUTS WITH 60-CYCLE VOLTAGE ON SHORT LENGTHS OF NEW PAPER-INSULATED CABLE, SHIELDED AND UNSHIELDED

Tests made on cable G, 15,000-volt, three conductor, 300,000 cir mils, petrolatum-impregnated, 24 ft. long. Conductor insulation 7/32 in., belt 3/32 in.

Voltage applied from 60-cycle source. Temperature 25 deg. cent.

Fig. 17 shows the curves of a-c., 60-cycle watt input (dielectric loss) against voltages stress for a 24-ft. length of 15,000-volt paper cable *G* with and without shields and guards. On such a short cable the a-c., 60-cycle end losses are not negligible and shields and guards are necessary to insure proper accuracy of dielectric loss measurements at stresses above the ionization point. The shielded and guarded value at 24,000 volts was 4.05 watts and the unshielded unguarded value 5.43 watts.

The difference of 1.38 watts, which is the end loss, compares favorably with that of 1.2 watts obtained in the previous case of the 800-ft. cable. The term end loss as used is the total end loss or difference between the unshielded unguarded value and the shielded guarded one as previously outlined in the case of the d-c. tests. The relative effect of shield and guard is very difficult to obtain because all guards act as shields. Leakage taking place 1/1000 of an in. above the surface of the paper is leakage through the air. Any guard ring intercepts this as well as the true surface leakage. All tests made so far have shown different values with and without shields even if guard rings were left on during the latter condition.

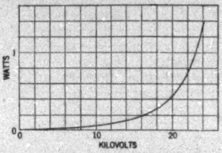

FIG. 18—VARIATION OF WATTS END LOSS WITH 60-CYCLE VOLTAGE ON SHORT LENGTH OF NEW PAPER-INSULATED CABLE

From data of Fig. 17

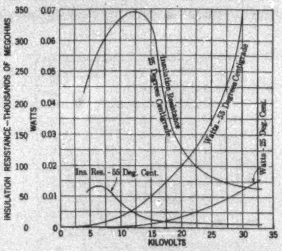

FIG. 19—VARIATION OF D-C. CHARACTERISTICS (LEAKAGE CURRENT, INSULATION RESISTANCE, AND WATT INPUT WITH VOLTAGE) AT TWO TEMPERATURES OF UNSHIELDED REEL OF NEW RUBBER-INSULATED CABLE

Tests made on cable E, 10,000-volt, single conductor, 4/0, 7/32 in. insulation, 505 feet long
Voltage applied from kenotron

The error of 34 per cent in the unshielded unguarded measurements indicates the necessity for using shields and guards in making measurements of a few watts loss under stresses around 24,000-volts a-c. From this, it may be deduced that unshielded unguarded watt loss measurements on cable samples of 10 ft. or less under a-c., 60-cycle, 24,000-volt stresses might be 100 per cent in error. The author believes that this is one of the reasons why dielectric loss measurements taken at different laboratories and more especially those on short lengths of cables as against long lengths, have sometimes failed to agree in the past.

From the foregoing results the fourth conclusion has been drawn that it is necessary to use shields as well as guards in making tests on short cable samples to determine the a-c. electrical characteristics, such as dielectric loss of cable insulation under voltage stresses above the ionization point.

Fig. 18 shows the variation of total watt end loss with voltage stress. This loss is made up of surface and air electrical leakage as in the previous cases. The relative proportion is very difficult to determine but it is believed as a result of preliminary tests that both factors are important and neither negligible. Further preliminary tests have indicated that the guard effects vary depending on the relative position of the guard ring to the copper conductor and the lead sheath. This is further proof that the guard ring acts as a partial shield.

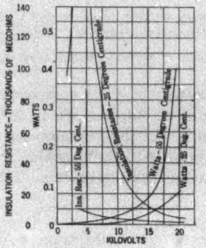

FIG. 20—VARIATION OF D-C. CHARACTERISTICS AT TWO TEMPERATURES OF UNSHIELDED REEL OF OLD RUBBER-INSULATED CABLE

Tests made on cable F, 10,000-volt, single-conductor, 4/0, 7/32 in. insulation, 319 feet long.
Voltage applied from kenotron

Ionization Point is Indeterminate. It will be observed that the difference between the shielded guarded and unshielded unguarded measurements steadily increases with increasing stress. The difference probably represents the increasing air loss at the ends of the cable. It will be noted that these curves (Fig. 17) diverge at even the lower stresses, as in the case of the previous d-c. tests shown in Fig. 12.

From this and the previous results, the fifth conclusion is drawn: that the so called ionization point is very variable depending upon the physical circuit together with the atmospheric conditions and represents, in reality, local air breakdown. This is supported by the tests with direct current on paper and rubber cables as well as by the a-c. tests on paper cable. In all cases the curves with and without shields and guards diverge at the lower values of stress below the commonly accepted ionization point.

From this, it may be deduced that the air conducts at all stresses both direct current and alternating current and that ionization points represent local air breakdown due to the local physical circuit, atmospheric, and stress conditions.

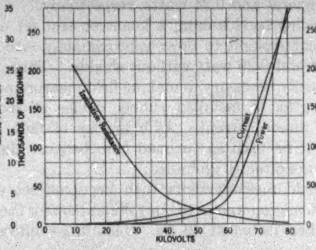

Fig. 21—Variation of D-c. Characteristics of Unshielded Length of New Rubber-Insulated Cable

Tests made on cable H, 10,000-volt, single-conductor, No. 6, 7/32 in. insulation, 89 feet long
Voltage applied from kenotron

TESTS ON RUBBER-INSULATED CABLE

Tests similar to those on the paper-insulated cables were made also on rubber-insulated cables.

Insulation Resistance Varies with Voltage Applied. A new and an old rubber cable showed the same general insulation-resistance-voltage-stress characteristic at 25 deg. cent. as did the paper-insulated cables. Figs. 19 and 20, giving results of tests on cables *E* and *F*, respectively, illustrate this. The new rubber cable *E*, Fig. 19, also shows the characteristic at 55 deg. cent., but the old cable *F*, Fig. 20, shows only a falling characteristic at this temperature. It is probable that if the test range had been extended, the curve of insulation resistance would have shown a rising characteristic at the lower voltages and a rapidly falling one at the higher voltages approaching the breakdown point.

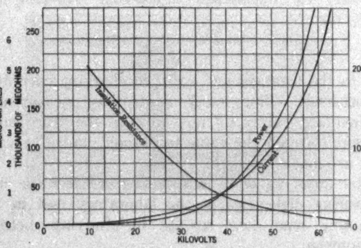

Fig. 22—Variation of D-c. Characteristics of Unshielded Length of New Rubber-Insulated Cable

Data of Fig. 21 plotted to larger voltage scale.

A series of d-c. tests by kenotron was made on an 89-ft. length of new 10,000-volt rubber cable *H*, both with and without shields. In these tests, the stresses were carried up to the breakdown point. The results are shown by Figs. 21, 22, 23, 24, and 25. The same shields and test set-ups were used as in the case of the paper cable.

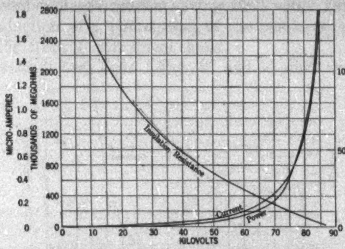

Fig. 23—Variation of D-c. Characteristics of Shielded Length of New Rubber-Insulated Cable

Tests made on cable H (See Fig. 21). Voltage applied from kenotron

From the curves, it will be noted that the insulation-resistance-voltage-stress curve, at room temperature, continually falls from the 10-kv. point. It is probable that at stresses below 10 kv., the curve would show a rising characteristic.

It is apparent that the leakage current and input watts rise very rapidly after the critical point of 60 kv. is reached under the unshielded condition. This is probably due to the ionization of the external air surrounding the ends of the cable.

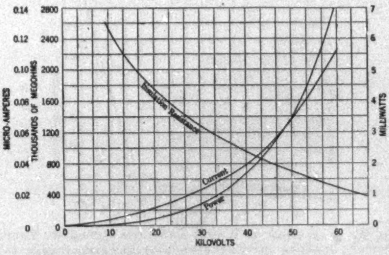

Fig. 24—Variation of D-c. Characteristics of Shielded Length of New Rubber-Insulated Cable

Data of Fig. 23 plotted to larger voltage scale

On the other hand, the abrupt rise under shielded conditions does not occur until 70 kv. is reached. This must be due to the ionization of the air entrapped in the dielectric itself. It would thus appear that the ionization of the external air around the cable ends takes place before the ionization of the air entrapped in the insulation, unless the test results are seriously modified by the action of surface leakages. Perhaps this is a question of air pressure and movement.

Effect of Shields. The effect of shielding and guarding is very marked in these tests as in the case of the paper cable. The true or shielded guarded watt input at 80 kv. d-c. was 0.064 watt and the unshielded unguarded value 2.80 watts, making the end loss 2.74 watts. It will be observed that the true loss is only 1/44 of the apparent or unshielded unguarded loss at the given stress. The relation is shown graphically by Fig. 25.

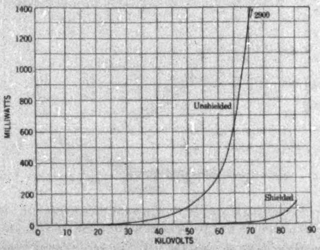

FIG. 25—COMPARISON OF CURVES OF WATT INPUT WITH D-C. VOLTAGE ON NEW RUBBER-INSULATED CABLE, SHIELDED AND UNSHIELDED

Tests on cable H (see Fig. 21). Data of Figs. 21 and 23

The use of shields as well as guards is thus absolutely necessary in making d-c. measurements on short rubber cables at high stresses in order not only to insure the accuracy of results but to determine the true characteristics of the cable insulation.

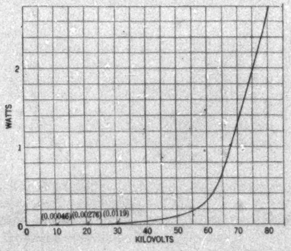

FIG. 26—VARIATION OF WATTS END LOSS WITH D-C. VOLTAGE ON LENGTH OF NEW RUBBER-INSULATED CABLE

Tests on cable H (see Fig. 21). Data from Fig. 25

If the cable in question had been 10 ft. long, the measuring error at 80 kv. d-c. might have been of the order of magnitude of 39,000 per cent, without shields. On the other hand, if the cable had been 18,000 ft. long the error might have been 21 per cent.

From the data it may be deduced that in testing an installed rubber cable of 18,000-ft. length, by kenotron at stress of 40 kv., the error in measuring the watts loss, due to end leakage, might be 13 per cent. The shielded leakage current and watt input at 40 kv. d-c. for the above cable would be approximately 8.1 microamperes and 0.34 watts, respectively.

According to this data, a kenotron test set used on such a cable ought to show approximately 9.16 microamperes and 0.39 watt without shields. Thus, shielding is hardly necessary in making field measurements on long lengths of installed rubber cables at moderate stresses direct current. As a matter of fact, shields could not be used as outlined here because the cable sheaths are directly grounded.

The field measurements with kenotron set usually show 100 microamperes to one milliampere for the above cable when tested at 40 kv. This large difference is due to the leakages and rectified charging currents of the set itself. Thus, many kenotron test measurements on installed cables do not indicate the true leakage current through the cable insulation, and give values that are much too high.

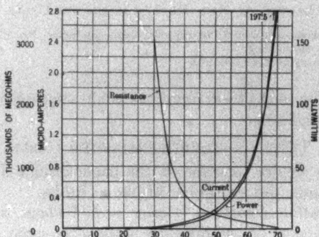

FIG. 27—VARIATION OF D-C. CHARACTERISTICS (LEAKAGE CURRENT, INSULATION RESISTANCE, AND WATT INPUT WITH VOLTAGE) OF UNSHIELDED 27,000-VOLT PIN-TYPE LINE INSULATOR

Voltage applied from kenotron

The great value of shields lies in their use when measuring dielectrics in the laboratory and in thus obtaining true data to use in studying dielectric phenomena.

Fig. 25 shows the comparison between the shielded and unshielded watt input to the rubber cable. The great difference between the two will be noted. Obviously, the shielded curve, if extended through the breakdown point, would rise nearly perpendicular to the abscissa (stress) becoming an asymptote. After breakdown, the current would be very great under stresses of 85 kv. and upward. Many very interesting speculations outside the scope of this paper can be drawn from this fact.

The end loss is given by Fig. 26. The general discussion on the paper cable end losses applies also to the rubber cable end losses.

Insulation Failure. Fig. 23 shows the true shielded guarded insulation-resistance-voltage-stress curve from 10 to 85 kv. Under the shielded condition, at approximately 84 kv., the curves of leakage and watt input are rising very rapidly and are nearly perpendicular to the abscissas (voltage stress). This, of course, indicates that the dielectric is about to fail. As a matter of fact, failure did take place at 85 kv. Upon examination, this new rubber cable was found to be literally riddled with incipient faults. After the first failure, a number of these other faults were developed by applying a comparatively low voltage. The rubber insulation had been stressed far above its safe or electrical elastic limit, and a general deterioration had set in.

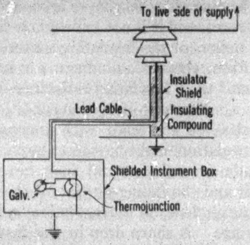

Fig. 28—Diagram of Connections for Measurements on Shielded Line Insulator

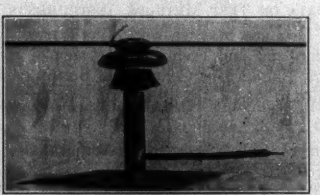

Fig. 29—Shield for Pin-Type Line Insulator

This leads to the sixth conclusion, that the better, *i. e.*, the more uniform the dielectric, the greater the tendency for the material to break down in its entirety rather than at a single point. Practically, of course, no dielectric is perfectly homogeneous, so that failure will take place at the weakest spot or spots. From this it might be deduced reasonably that the better the insulation the more care should be taken not to over-stress it. Such over-stressing, while not producing actual failure, must deteriorate and weaken the whole insulation. It might be further deduced from this that there must be an electrical elastic limit in dielectrics analogous to the physical elastic limit in metals. If the stress exceeds this limit a strain and eventual failure is the result.

Tests on a Line Insulator

Besides the tests on paper and rubber cables a similar series has been made on a 27,000-volt porcelain line insulator, both with and without shields. Fig. 28 shows the diagrammatic arrangement, Fig. 29 is a reproduction of a photograph of the insulator shield, and Fig. 9 of the instrument shield.

Results of D-c. Tests. Figs. 27 and 30 show the d-c. unshielded and shielded test results. It will be noted that the shielded insulation-resistance-voltage-stress curve has a slowly falling characteristic from 20 kv. upward. It is believed that this curve would have been nearly flat if the shield had been brought closer to the porcelain.

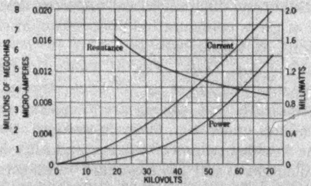

Fig. 30—Variation of D-c. Characteristics of Shielded 27,000-Volt, Pin-Type Line Insulator

Same insulator as in Fig. 27. Voltage applied from kenotron

At 70 kv. d-c. the shielded values are 0.019 micro-ampere, 1.32 milliwatts, and 3.5 millions of megohms. The corresponding unshielded values are 2.8 micro-amperes, 197.5 milliwatts, and 25,000 megohms. The relations shielded to unshielded are: leakage current and watt input 1/150 and resistance 150/1.

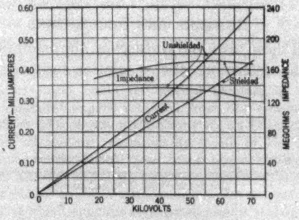

Fig. 31—Variation of Current and Impedance with 60-Cycle Voltage on 27,000-Volt, Pin-Type Line Insulator, Shielded and Unshielded

Same insulator as in Fig. 27. Voltage applied from 60-cycle source

From these curves, it will be noted that the shielded value of insulation resistance falls rather slowly and that the watt input is not increasing very rapidly at 70 kv., which is a low stress for the insulator. The porcelain apparently presents a more nearly constant resistance to electrical stress than impregnated paper and rubber insulation.

The unshielded curves show the effect of the electrical leakage through the air around the outside of the insulator. At 60 kv. the leakage and watt input curves rise very abruptly as in the case of the rubber cable. This is probably due to the ionization of the external surrounding air. The shielded curves (Fig. 30) of leakage current and watt input do not even show a rapidly rising characteristic at 70 kv. d-c. This leads to interesting speculation on the general question of ionization and dissipation of d-c. electric energy from a body of given configuration under given atmospheric conditions, at various stresses.

Results of A-c. Tests. Fig. 31 shows the results for the same insulator under a-c., 60-cycle stresses. These results indicate that shields are necessary to obtain true readings under a-c., 60-cycle stresses. At 70-kv. a. c. the shielded values of current and impedance are 0.42 milliamperes and 166 megohms. The corresponding values unshielded are 0.57 milliamperes, and 122 megohms, respectively. Shielding thus established the true value of 0.42 as against the apparent value of 0.57, a difference or error of 36 per cent.

It will be noted that the impedance voltage-stress curves are nearly flat, perhaps indicating that the insulator is well within its safe electrical stress limits. If, in future tests, these curves can be carried through the lower and higher ranges, undoubtedly new and valuable data can be obtained.

No a-c. watt measurements were made on this insulator but it is believed that these are very important and will throw further light on the situation. It is hoped to do more work along this line in the future.

REMARKS AND SUGGESTIONS

In all these tests and researches, the attempt has been made to determine the insulation-resistance-voltage-stress curve and the watt-input voltage-stress curve. It is believed that if such curves can be obtained for various insulations under electrical stresses, both direct current and alternating current of various frequencies from zero to breakdown at several temperatures, then the way will be opened for the practical study and determination of dielectric action under electrical stress.

The work covered by this paper involved for the most part d-c. stresses and it is not known how far the results reflect the action of the insulating material under a-c. stresses. From the d-c. standpoint, it might well be argued that the insulation-resistance-voltage-stress curves are a measure of insulation condition. As long as the resistance increases with increasing voltage stress the insulation might be said to be on the safe side.

The maximum value would then be the electrical elastic limit and the falling portion of the curve would represent the tendency of the insulation to deteriorate or electrically age. A sharp drop in the insulation resistance curve would indicate impending breakdown.

If this d-c. picture represents also the a-c. situation then a new means is provided for studying insulation under electrical stress. A large amount of a-c. research, however, must be conducted and matched against the direct current before these conclusions are justified.

The problem is further complicated because the d-c. insulation-resistance-voltage-stress curves vary widely with temperature, and the application of the higher a-c. stresses produces a temperature change in the insulation.

The author believes that continuation of this line of investigation will open the way for a real practical study of dielectric phenomena.

DATA ON TEST METHODS, INSTRUMENTS, ETC.

All tests and research work were carried on by the Standardizing and Testing Department of The Edison Electric Illuminating Company of Boston except those recorded in Figs. 3, 4, 5, 19, and 20, which were made by

TABLE I

TEST CONNECTIONS, TIME OF CHARGE, AND INSTRUMENTS USED IN MAKING MEASUREMENTS

Fig. No.	Test connection	Time of charge, min.	Instruments for measuring current and power*
1	Conductor 1 vs. 2, 3, and sheath	5	Rawson 10-microampere Type 501 galvanometer. Leeds & Northrop high-sensibility portable galvanometer
2	Same as Fig. 1	10	Same as Fig. 1
3	Same as Fig. 1	5 or 45	Rawson 2-microampere and 10-microampere galvanometers, Type 501
4, 5, 19 and 20	Conductor 1 vs. sheath. Conductors 2 and 3 floating in Figs. 4 and 5	5 or 45	Rawson Type 501 galvanometer
6,10,11,12,15 and 16	Three conductors vs. sheath	8	Rawson 1-microampere Type 501 microammeter. Leeds & Northrop Type H galvanometer
13, 14	Same as Fig. 6	8	Leeds & Northrup Type H galvanometer
17 and 18	Same as Fig. 6	..	G. E. Type ALi Astatic reflecting dynamometer
21, 22, 23, 24, 25 and 26	Conductor vs. sheath	6 or 30	Leeds & Northrup Type H galvanometer with series resistance and shunted protective gap
27, 30, and 31	....	..	For d-c., L. & N. Type H galvanometer. For a-c., Rawson 2-milliampere thermojunction with L. & N. Type H galvanometer

*D-c. voltages measured by voltage kenotron or electrostatic voltmeter.
A-c. voltages, by potential transformer and voltmeter.

graduate students of the Massachusetts Institute of Technology at the suggestion of the Standardizing and Testing Department.

The tests and research work were performed at the substations and laboratory of the above company, at the Massachusetts Institute of Technology, and at the Cruft Laboratory of Harvard University.

The tests recorded in Fig. 3 were part of a student thesis prepared by T. M. Burkholder and D. E. Replogle. Figs. 4, 5, 19, and 20 show some of the results obtained from tests made in connection with a student thesis written by E. E. Piepho and J. E. Handy.

The kenotron tests were made with a single-tube 50,000-volt and a two-tube, 100,000-volt kenotron test set of the Standardizing and Testing Department of The Edison Electric Illuminating Company of Boston. The high-voltage battery tests were made with the 100,000-volt battery of the Cruft Laboratory at Harvard University.

The test connections, time of electrification, and measuring equipment used in the various tests are given in the foregoing table. As the current flow on the d-c. tests varied with time of electrification, it was necessary to select arbitrarily a reasonable time of charge. In each case a value was taken at which the current had become fairly constant.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Professor G. W. Pierce and Professor E. L. Chaffee, of Harvard University, for the use of the 100,000-volt storage battery, and to Dr. V. Bush of the Massachusetts Institute of Technology for his cooperation and assistance in the matter of thesis tests.

The author thanks Messrs. T. M. Burkholder, D. E. Replogle, E. E. Piepho, and J. E. Handy for their theses, and appreciates the work of the members of the staff of the Standardizing and Testing Department of The Edison Electric Illuminating Company of Boston, particularly Messrs. H. C. Hamilton, R. W. Chadbourn, C. H. Smith, and W. B. Elmer.

Discussion

W. B. Kouwenhoven: I am very glad that Mr. Kasson has pointed out again the necessity of using shields and guards in making high-voltage measurements on cables. I pointed out at the Mid-Winter Convention of 1926 that unless shields and guards are employed the results may be widely in error.

It is not only important to shield specimens properly from all electrostatic fields but in case of the a-c. measurements of loss, it is also necessary that the voltage of a shield be the same as the voltage of the conductor it protects. It is therefore essential to place in series between the shield and ground an impedance that is the same as the impedance of the measuring instrument used. Unless this is done the difference in potential between the shield and its shielded conductor will cause an error in results. This point, I am afraid, is often overlooked.

Mr. Kasson's curves in Fig. 3, showing the effect of temperature on d-c. insulation resistance, do not conform with the law that we have found for some of our cable samples. This law states that the logarithm of the conductivity is equal to a constant divided by the temperature, plus another constant. When the logarithm of the conductivity is plotted against the temperature the curve is a straight line, although, some of our specimens give this straight line relation, there are others that are exceptions to the rule.

There is one other point which I would like to mention in regard to Mr. Kasson's work and that is his method of procedure. The time of charge must be considerable to obtain accurately the conductivity of specimens. It may take several hours before the final leakage current is reached, owing to the presence of the absorbed charge, and unless the test is continued over a long period of time you cannot be sure that the current measured is actually the final leakage current, corresponding to the conductivity of the sample.

One method that we have used at Johns Hopkins in this work, and which will shorten the time somewhat, is to take a charging run for about 45 min. and then immediately throw the cable in discharge and measure the absorbed charge coming out for the same period of time. The difference between these two curves represents the final leakage current.

Another difficulty that arises when measuring the conductivity or leakage current at several different voltages is the superposition of the curves. If, for example, you apply 5000 volts for a certain length of time and take a reading of the conductivity, and then raise the voltage to 10,000 volts the second curve is superimposed on top of the first and you cannot be sure of the results. If possible, it is best to discharge the cable between each run.

I should like to ask Mr. Kasson what method of measuring he used in determining his a-c. losses.

If, as pointed out by Mr. Kasson, it is possible to tell from the shape of the d-c. resistance curve plotted between resistance and voltage whether a cable is getting old and deteriorating, or whether it is in good condition, we have a very important aid with which the operating companies can determine the condition of their cables. We have been endeavoring to determine some such relation in our research work at Johns Hopkins but to date have not been able to find any definite relation.

Herman Halperin: (by telegram) In Mr. Kasson's paper, Fig. 1, shows, for cable with 7/16 in. insulation to sheath, the maximum insulation resistance occurs at 13 kv., which using the d-c. to a-c. ratio of 2.4 corresponds to about 5-kv. a-c. This indicates electrical action in the cable at a potential of only about half the operating voltage of this old cable. A-c. ionization tests made on several samples of old 12-kv. three conductor cables, removed from the system of the Commonwealth Edison Company have shown practically flat power-factor voltage characteristics up to 10- or 15-kv. three-phase. As the insulation on these cables was considerably less than on Mr. Kasson's cable, the stresses at which ionization took place in our cables would be approximately three times as much as the stress at which a change occurred in his d-c. tests. Perhaps by means of these tests he has discovered a new characteristic of the insulation in connection with the effect of shielding. In Fig. 8 he shows a metal box around the entire reel of cable. If the shielding is to take care of ionization near the end of the cable, I am wondering whether a metal box over each end of the length of cable would not be sufficient. Referring to the paragraph regarding the nonuniformity of cables, our testing has shown that cables with poor quality of insulation are liable to be more irregular in their quality than the cables of high quality which checks Mr. Kasson's statement. For instance poor cables would develop several hot shots in accelerated life tests and fail in rather short times. In some accelerated life tests at 2.5 rated voltage we took several 50-ft. samples from various sections of one make of a given size of high-tension cable which was giving trouble in service and shown to be irregular in factory tests. One sample failed, after a total of 26 hr., while another sample developed hot spots without failure after 405 hr. and a third sample withstood the test for 596 hours without any failure or hot spots.

S. J. Rosch: Several theories have been brought forward in explanation of dielectric phenomena, and I believe that the probabilities are that each theory may be correct as relating to the behavior of the particular dielectric under consideration. Our error in the past has been chiefly, in trying to make one theory or one set of laws govern the behavior of all dielectrics. That is why we have failed up to the present in solving the dielectric problem and in my estimation if we commence to study each dielectric individually, we shall come nearer to obtaining a truer picture of the laws governing its behavior.

Even in a general dielectric such as a paper cable, Dr. Kouwenhoven brought out the point that he has only been able to substantiate some of the results obtained by Mr. Kasson on some samples of cable, but not on others. Undoubtedly the observations made by Dr. Kouwenhoven that were not in accordance with those by Mr. Kasson, were made on samples of cable which although equally good as far as quality was concerned, nevertheless as dielectrics possessed characteristics obeying entirely different laws. We must establish the accuracy of this fact before we can hope to go further in solving the laws of dielectrics.

Mr. Halperin in his discussion, has stated that if we took the results of d-c. measurements as established by Mr. Kasson and converted them to the equivalent a-c. voltage, by using as a divisor the factor 2.4, the values thus obtained would be much lower than those obtained by Mr. Halperin. Here again we must question why we should use 2.4 when the work of Hayden, Eddy, and Delon has definitely proven that the ratios of d-c. to a-c. vary all the way from unity to 2.6 depending again upon the type of dielectric used. It may be that in the case of the cables tested by Mr. Kasson the ratio for that particular dielectric should have been about 1.4 in which case the results would have been comparable with those obtained by Mr. Halperin.

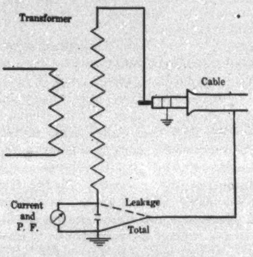

FIG. 1

I believe that before rejecting any of the theories propounded in the past, we ought first to establish definitely whether or not they are applicable in the case of some particular class of dielectrics.

E. S. Lee: Mr. Kasson has shown with great certainty the need for proper shielding to prevent the end loss of cable samples from being included with the measured loss. Wherever the end loss is high compared with the measured loss, then more perfect shielding becomes necessary. This is particularly the case with measurements on cables made with high direct voltages.

The variation of insulation resistance with voltage has been obtained at various times as shown in Figs. 3 to 6, though it has never been possible to justify with certainty the rapid decrease of the resistance with increase in voltage after passing the maximum point. It is quite gratifying to have Mr. Kasson point out that this variation is not true but has been obtained because of a faulty measurement.

It is pertinent that the end loss is also present in a-c. measurements of dielectric power loss, and it behooves us to investigate these particular measurements as they are being variously made, to be assured that the end losses are not included with the measured loss to vitiate the results.

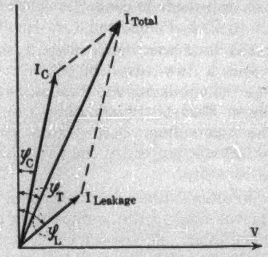

FIG. 2

P. T. Graff: With reference to the shielding of cables in making tests on cables for dielectric loss, we found that on short samples the power factors showed up rather high, and a method of test was devised which gave results on short samples, without resorting to shielding, that compared very well with the values for long lengths of cables.

The method used is shown in Fig. 1 herewith. Current and power-factor values for leakage and leakage plus cable losses are read. By the vector subtraction shown in Fig. 2 herewith the cable current and its phase angle with respect to the applied voltage are obtained. That figure shows relatively what is typical of actual cases. The effect of a small leakage current can be considerable in taking the apparent cable power factor.

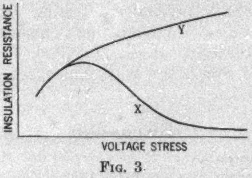

FIG. 3

Because hot spots are found where cables eventually break down on voltage tests, it cannot be construed that Mr. Malti is mistaken in not accepting the pyroelectric theory. That is based on weak microscopic filaments whereas hot spots may be due to imperfect "imperfect dielectric." No cable is uniform in its entirety and it is rather hard to apply a theory to such a dielectric. The electrophysicists are better able to handle such theories, and as engineers we should concentrate on making perfect "imperfect dielectrics."

C. L. Kasson: In regard to Mr. Halperin's comment on the difference between his results and those we obtained in Boston, I might say that perhaps the cable we used was different from the cable he used, and this point has been brought out by several other speakers. Fig. 1, to which Mr. Halperin referred, gives the results for very old rosin-oil cable that has failed repeatedly in service.

I believe that shields must be tried by a good many others before we can be sure of the results. I tried to bring out here simply the work of one group of engineers which must be checked by others before it can be accepted.

In regard to Professor Kouwenhoven's remarks, absorption curves were made in all cases and carried out until they became practically flat. The values of leakage current used were those taken from the flat portion of the curve. It usually required about 15 min. for the curve to become reasonably flat.

Professor Kouwenhoven asked in respect to the instrument used to measure the a-c. losses. A dynamometer with series resistance was used for this purpose.

In closing, I should like to show typical insulation-resistance voltage-stress curves. We plot insulation resistance and d-c. voltage stress on a short length of cable unshielded and obtain a curve something like X in Fig. 3 herewith. On the same cable, with shields, we obtain a curve like Y. Y is a very different curve from X, and it is apparently due to the fact that curve X is influenced by the ionization of the external air, and perhaps in some of the results in the past we have been talking about ionization of external air at the ends of the cable, rather than ionization of air within the cable.

In all nature we are interested in stress on one hand and in resistance on the other hand. To every stress there is resistance; to every resistance there is stress. Therefore, a resistance stress curve of a dielectric, *i. e.*, the resistance reaction of the dielectric against the stresses placed upon it, should be an important thing, and in order to obtain what we believe is a true curve, we must resort to shielding or at least some equivalent method.

Non-Harmonic Alternating Currents

BY FREDERICK BEDELL[1]
Fellow, A. I. E. E.

Synopsis.—*Certain principles are presented for solving non-harmonic a-c. problems. Graphical methods are already well known for solving practically any problem, when currents and electromotive forces are sinusoidal, by vector diagrams in a plane. In certain special cases such diagrams give correct results when currents and electromotive forces are non-sinusoidal, but in general diagrams in more than two dimensions are necessary. A wide range of material, some of which has been published before in scattered places, is brought into relation; no single general solution seems attainable. A bibliography with 61 references is appended.*

Scope and Limitations of Usual Sine-Wave Assumption. It is common, in discussing the theory of alternating currents, to assume that currents and electromotive forces are simple harmonic functions of the time; that is, they are plotted as sine waves. It has been found that, for most purposes[2], such a wave is superior to a wave that is irregular or complex, which, to ditinguish it from a simple harmonic wave, may be referred to as non-sine, or non-harmonic. The technique of alternator design has been highly developed with a view to attaining a sine wave of electromotive force as closely as possible, the subject being fully discussed in many texts and special articles, bibliography, 33. Methods have been developed for measuring and penalizing departure from a sine wave and much study has been given to the specification of allowable limits to such departure, bibliography, 9, 43, 44, 46, 47, 58, 61.

On the sine-wave assumption there has been developed a very complete analytical theory of alternating currents and, in parallel with it, a graphical method of representation and analysis that gives ready and accurate solution to practically any problem that arises. It has been found that harmonically varying quantities can be represented as vectors in a plane, and that these vectors, by showing the phase and amplitude of the several harmonic quantities, completely define them. Furthermore, any vector may be resolved into components and conversely, two or more vectors may be added or combined, as in the polygon of forces. Thus, a current may be resolved into two components at right angles to each other, one a power component in phase with electromotive force and the other a reactive component in quadrature thereto. Similarly, an electromotive force vector may be resolved into its power component in phase with current and a reactive component in quadrature, bibliography, 11.

Apparent power is the product of the electromotive force E and the current I, but real power is the product of E, I, and a power factor, $\cos \theta$, where θ is the angle of phase difference between E and I. Real power may, accordingly, be looked upon either as the product of the electromotive force and the power component of current or as the product of the current and the power component of electromotive force. The product $E I \sin \theta$ is a pulsating reactive power with average value zero, where $\sin \theta$ is the reactive factor and $I \sin \theta$ is the reactive component of current; $E \sin \theta$ is the reactive component of electromotive force.

There is scarcely a circuit problem that cannot be solved by methods based upon these principles; the whole treatment, however, is based upon the sine-wave assumption.

An exact sine wave, however, is practically never obtained from commercial apparatus. In practise, currents and electromotive forces deviate to a greater or less extent from a simple sine function of the time. It is true that in many cases, though there be this deviation from a sine wave, conclusions based on a sine-wave assumption are sufficiently accurate for practical purposes; indeed, in certain cases, conclusions thus obtained are strictly correct and the results are the same irrespective of whether we are dealing with sine or non-sine waves. In other cases, however, deviation from a sine wave causes error in results obtained on a sine-wave assumption, and this error may be large or small according to the conditions of the problem. A careful study of non-harmonic alternating currents, therefore, is desirable.

Complex Wave Represented by Fourier's Series. Any periodically varying quantity, such as we are here discussing, can be completely represented by a constant term and a series of sine terms, of definite amplitude, phase, and frequency. Cosine terms are unnecessary when the phase of each sine term is included. The series of sine terms used to represent an irregular a-c. wave includes a *fundamental*, or sine wave of fundamental frequency, and various *harmonics* with frequencies that are odd or even multiples of the fundamental frequency; a constant term is added in the general case. The introduction of factors of exponential form, that occur in transients and in cases of damping, will not be considered here, bibliography, 30.

For Usual Alternating Currents, Positive and Negative Areas are Equal and the Constant Term is Zero. The average value of any complex wave, expressed as in the

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[2] Although a sine wave-form is superior for general use, some other wave-form may, in certain respects, be better for a particular purpose; thus, as is well known, a peaked wave of electromotive force gives a flat wave of magnetic flux and by reducing the maximum flux, reduces the core loss in a transformer.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

preceding paragraph, is the constant term, the average value of the fundamental and the various harmonics, taking account of sign, being zero. The average value must be taken over a complete period. In an ordinary alternating current, with no predominating flow in one direction, the average value or constant term is zero; and, since the average of the positive ordinates is equal to the average of the negative, the areas of positive and negative half-waves are equal. Inequality of areas indicates a direct current combined with the alternating and this direct current would be represented by the constant term. This is a case that sometimes occurs.

In a generated electromotive force wave, the equality of positive and negative areas indicates that the total flux cut in a positive sense is equal to the total flux cut in a negative sense, as is the case in any dynamo-electric machine. It is seen, therefore, that in the wave for electromotive force, as well as in the wave for current, the constant term is zero.

When Positive and Negative Half-Waves are Alike, Odd Harmonics only can be Present. Omitting the constant term, any deviation from a simple harmonic function is represented by even and odd harmonics in addition to the fundamental. During the second half-period the fundamental wave and any odd harmonic have, respectively, the same values as during the first half, but with sign of each reversed. When only odd harmonics are present, each half-wave is, therefore, a repetition of the preceding half-wave with sign reversed; negative and positive half-waves, except for sign, are identical.

On the other hand, during the second half period of the fundamental, any even harmonic has the same sign as during the first half; hence, since the fundamental has reversed sign, an even harmonic that adds to the fundamental at any given point of the positive wave subtracts from the fundamental at the corresponding point of the negative wave. When even harmonics are added to the fundamental, the negative and positive waves are, therefore, unlike in form; that is, one is not the repetition of the other. This may be readily seen by sketching curves for a few typical cases.

A *Symmetrical Generator Produces only Odd Harmonics.* Any harmonics that exist, whether even or odd, must be produced either in the receiving circuit (as will be considered later) or in the generator. In practically all generators, with either revolving armature or revolving field, the generated wave of electromotive force is symmetrical, the negative half-wave being a repetition of the positive; hence, odd harmonics only are present[3]. Even harmonics in the electromotive force wave can be produced by a generator only when *both* field and armature are unsymmetrical, a condition that does not commonly exist, bibliography, 31.

Average Product Theorem. The average product of the instantaneous values of two harmonic quantities of different frequencies is zero. This is true irrespective of their relative phase positions and the ratio of their frequencies; the same is true of the product of any periodic quantities, each of which is represented by a series of harmonic terms.

This may be illustrated experimentally by passing currents of different frequencies through the two coils of a wattmeter and noting that the reading is zero. Formally, it may be shown by integration of the product of two sine functions, having different periods, over the least common multiple of the two periods, or over the longer period when one period is a multiple of the other. In case the two periods were nearly equal and their least common multiple very large, beats would occur, but the average value of the product over a sufficiently long time would still be zero, bibliography, 13, 14 (p. 391).

The average value of a constant, multiplied by a sine function, or by the sum of several sine functions, is likewise zero.

The results of these relations are far reaching; r.m.s. values and average power for non-harmonic waves both depend upon this average product theorem.

Root-mean-square Value of Non-harmonic Wave. A non-harmonic wave, as already explained, consists of a series of harmonic components of fundamental and higher frequencies. It will be shown that the r. m. s. value of the total wave is equal to the square root of the sum of the squares of the separate harmonic components. Thus, if the fundamental is A, and the several harmonics B, C, and D (r. m. s. values), the r. m. s. value of the total wave is $\sqrt{A^2 + B^2 + C^2 + D^2}$. This will be referred to later as the *square law.* It will be seen that the result is *independent of the phase and frequency* of the several components. The effect of each component is added as though it were in quadrature to each other component, bibliography, 13, 14.

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[3] With a symmetrical armature, irrespective of what the field may be, if a particular armature conductor cuts certain flux, another armature conductor connected in the opposite sense will cut this same flux just half a period later; hence, negative and positive half-waves are alike and odd harmonics only can be present. In a like manner, with a symmetrical field, irrespective of what the armature may be, if a particular armature conductor cuts certain flux, that same conductor will cut equal flux in an opposite sense just half a period later; hence, in this case also, negative and positive half-waves are alike and odd harmonics only can be present. Lack of symmetry of field may be produced by inequality of dimensions, spacing, or material; and, even when the structure is symmetrical, it can be produced by unequal excitation of various poles. Armature windings are commonly symmetrical, but may be otherwise, as in armatures with an odd number of slots.

It is here assumed that the speed at which the generator is driven and its field excitation are constant, as is usually the case. If the field is excited with alternating current and the armature is rotated synchronously, an electromotive force of double frequency is generated. If the field is excited with direct current and a superimposed alternating current, the electromotive force generated will, accordingly, consist of a fundamental and a second harmonic. It is thus possible to generate an even harmonic in a symmetrical armature. A development of this method may be employed for obtaining high frequencies, as in the Goldschmidt alternator.

This may be demonstrated experimentally by taking voltmeter readings of the separate and total voltages when two or more electromotive forces, not of the same frequency, are connected in series; or, more laboriously, by plotting curves for the instantaneous values of the components and determining r.m.s. values of their sum.

If, collectively, the harmonics B, C, and D are represented by H, where $H^2 = B^2 + C^2 + D^2$, the r. m. s. of the total wave is $\sqrt{F^2 + H^2}$, where F is the fundamental. It is seen that the collective harmonics add as a component in quadrature to the fundamental.

The proof of the foregoing is readily seen. The non-harmonic wave may be represented by a series of sine terms of different frequencies. One of these terms may be a constant, representing a d-c. component, if desired. When this series is squared, to get the r. m. s. value, every term is multiplied by itself (so that any term, as A, becomes A^2) and by every other term. But the cross-products, obtained by multiplying together terms of different frequencies, all vanish, for their average product, by a theorem already stated, is zero. The only terms remaining are the square terms, as A^2, B^2, etc., the r. m. s. value of which is $\sqrt{A^2 + B^2 + C^2 + D^2}$.

One of the terms in the preceding expression may be a constant, representing a d-c. component. It is thus seen that a direct current, when superimposed on an alternating current, adds vectorially in quadrature, in the same manner as an alternating current of a different frequency; the resultant is the square root of the sum of the squares of the component currents.

Principles of Independent Superposition of Harmonic Alternating Currents of Different Frequencies. When a pure sine electromotive force is impressed upon a circuit in which the various resistances, inductances, and capacities are constant, the current that flows is a pure sine wave of the same frequency, with amplitude and phase dependent upon the constants of the circuit.

When the electromotive force wave contains not only the fundamental sine wave, but harmonics as well, the current will contain a fundamental and harmonics of these same frequencies. Furthermore, each harmonic component of the electromotive force will produce in the current a corresponding harmonic component of the same frequency, *exactly as though all other components were absent*, bibliography, 6, Chap. XI.

This holds true of any part of a complex circuit, as well as of the circuit as a whole. Thus, if the harmonic components of the electromotive force, either in the whole circuit or in a part, are E_1, E_3, E_5, the corresponding components of current are I_1, I_3, I_5, each component being determined independently as though it alone existed.

By the square law, the sum of the component electromotive forces is

$$E = \sqrt{E_1^2 + E_3^2 + E_5^2}.$$

The sum of the component currents is

$$I = \sqrt{I_1^2 + I_3^2 + I_5^2}.$$

The phase of each harmonic component, as well as its amplitude, is determined independently. Although these phase relations affect the wave-shape, they do not affect the r. m. s. value of the total current.

When a circuit contains a constant resistance only, the harmonics in the current wave will have the same relative amplitudes and phase positions as the several harmonics in the electromotive force wave; that is, the current and electromotive force are similar in wave-form. With reactance present, however, this will not be true, for the relative amplitudes and phase positions of the several harmonics in the current will vary with their several frequencies.

In the case of inductive reactance, the reactance of the circuit for the several harmonics increases in proportion to frequency, thus increasing the lag and decreasing the amplitude of the corresponding harmonics in the current wave. Inductance tends to choke out the higher harmonics and to make a smoother current curve. Capacity reactance, on the other hand, amplifies harmonics in the current and so increases, rather than smooths out, irregularities in its wave-form. With either inductive or capacity reactance present, the wave-forms of current and electromotive force, except in the case of pure sine waves, are no longer similar. The resultant wave-form of current, however, is built up from its separate harmonic components, for the principle of independent superposition holds, so long as resistance, inductance, and capacity are constant. When one function, as current, is related to another, as electromotive force, by a linear differential equation in which the coefficients are constant, as a matter of course, one function may be derived from the other by a so called "distributive" operation, and the principle of independent superposition holds.

Should resistance, inductance, or capacity be variable, the principle of independent superposition cannot be applied. In this case, as discussed later, a sine-wave electromotive force does not produce a sine-wave current; the separate harmonic components of a complex electromotive force, therefore, do not produce corresponding independent components in the current.

When a Current Comprises Components of Different Frequencies, Each Component has Its Independent Resistance Loss; the Total Resistance Loss, RI^2, is the Sum of the Losses due to the Separate Components. As already shown, when a current I comprises components I_a, I_b, I_c, etc., of different frequencies, the r. m. s. value of the total current is the square root of the sum of the squares of the separate components;

$$I = \sqrt{I_a^2 + I_b^2 + I_c^2} + \ldots$$

Hence,

$$RI^2 = RI_a^2 + RI_b^2 + RI_c^2 + \ldots$$

Each component has its own resistance loss as though it alone existed; the total loss is merely the sum of these separate losses. This is equally true when one component is direct current.

As an illustration, if two unlike currents, either direct and alternating, or alternating currents of different frequencies, of 10 amperes each, are superposed in a circuit having a resistance of one ohm, the total loss is 200 watts, the separate losses being 100 watts each. This is an interesting contrast with the case when two *like* currents, two direct currents, or alternating currents of same phase and frequency, of 10 amperes each, are superposed in the same circuit having resistance of one ohm; the total loss in this case is 400 watts, although the loss would be only 100 watts for either current flowing alone. (For references, see next paragraph.)

Copper Saving in Composite Systems of Power Transmission. It is obvious from the foregoing that a considerable copper economy could be effected were it possible to use the same lines for the simultaneous transmission of power by unlike currents, that is, by direct and alternating currents or by alternating currents having different frequencies.

Inasmuch as copper drop is proportional to copper loss, each current in such a composite system would have its own independent copper drop as well as copper loss; and, where the amount of copper is determined by allowable copper drop or loss, no more copper would be required to carry several currents for composite transmission than would be required to carry one current alone. The independence of copper drop is very striking. Thus, if two generators transmit unlike currents over the same lines to independent loads, one for lighting and the other for power, the regulation of the lighting load is in no way affected by most extreme variations in the power load; for example, an entire power load with line drop of, say, 50 per cent could be thrown off and on without producing even a flicker in the lamps constituting the lighting load. These results are confirmed by test.

Although several composite systems have been proposed, the complications involved have thus far prevented their general adoption, bibliography 16, 17, 19, 20, 21, 22.

Equivalent Sine Wave. Although a non-harmonic alternating quantity requires a series of harmonic terms for its complete representation, it can be represented, for many purposes, by a single sine wave having the same r. m. s. value and the same frequency. Such a wave is called an *equivalent* sine wave. It is equivalent, however, in a restricted sense only. Thus, inductance and capacitance can completely neutralize each other in case of true sine waves only, complete neturalization being impossible for non-sine waves or for their equivalent sine waves.

If a non-sine wave is plotted in polar coordinates, the equivalent sine wave is plotted as a circle with the same area. If a non-sine wave and its equivalent sine wave are plotted in rectangular coordinates, the curves formed by the squares of their ordinates enclose equal areas, thus giving equal mean-square and r. m. s. values.

Vector Representation of Equivalent Sine Wave. A non-harmonic alternating quantity can be represented by a vector corresponding to its equivalent sine wave. There seems to be no criterion, however, for satisfactorily defining the absolute phase location of an equivalent sine wave with respect to the irregular wave to which it is equivalent, although the phase displacement of one equivalent sine wave with respect to another, as a current with respect to an electromotive force, can be determined. As discussed later, this phase relation is, in general, definite between pairs of quantities only. Accordingly, except in special cases, only two equivalent sine waves can be represented as vectors in a plane and three such waves as vectors in space, if all their phase relations, as well as magnitudes, are to be correct. More vectors would require more dimensions, or would necessitate a lack of significance in the relative phase positions of some of the vectors. The sum of two equivalent sine waves can be vectorially represented, however, in the same plane as the two waves themselves, and the sum of three equivalent sine waves can be represented with them in space, as discussed later.

Equivalent Phase Difference. The equivalent phase difference between two non-sine waves of the same frequency is the phase difference between the two equivalent sine waves when so located with respect to each other that the mean product of their instantaneous values is the same as the mean product of the instantaneous values of the two non-sine waves of which they are the equivalent. In the case of a current and an electromotive force, this mean product is the average power, the equivalent phase difference being an angle whose cosine is equal to the power factor.

Lag or Lead of Non-Sinusoidal Current. In case of sine waves of current and electromotive force, inspection of the waves plotted with respect to time shows at once whether the current is lagging or leading in relation to the electromotive force. If the current is lagging, the current passes through zero and attains successive values, as 1/4, 1/2, 3/4 and full maximum value, a certain interval of time, depending upon the constants of the circuit, after the electromotive force attains its corresponding value. This time interval or lag is the same between zero as between maximum values of current and electromotive force and is a measure of the phase difference. Power and power factor are increased when a lagging current is advanced, or when a leading current is retarded, in phase.

In case of non-sine waves, however, these definite relations do not hold. Except in special cases, the time-lag measured between zero points of current and electromotive force is different from the time-lag measured between maximum values, and in neither case does its value correspond to the equivalent phase difference. Indeed, the current might lag behind the electromotive force in reaching its maximum value and be in advance of it in passing through zero, so that inspection does not show whether the current is lagging or leading, nor whether the angle of equivalent phase difference is negative or positive. This uncertainty, however, does not affect the numerical value of the power factor, for the cosine of an angle is independent of its sign.

Arbitrarily, a current is said to lag when its maximum value occurs later than the maximum value of the electromotive force, bibliography 61; but the time of passing through zero has also been used as a criterion, bibliography, 26. Although not used, a better criterion might depend upon power. Thus, current would be referred to as lagging when an increase in power and power factor is brought about by advancing it in phase, and as leading when increase in power and power factor is brought about by retarding it. This is discussed later under power factor.

Two Equivalent Sine Waves of the Same Frequency, and their Sum or Difference, can be Represented by Vectors in a Plane. Three vectors, 1, 2, and 3, representing non-harmonic quantities of the same frequency, do not in general lie in a plane. Thus, if the equivalent phase difference between 1 and 2 is 40 deg., and between 2 and 3 is 10 deg., the equivalent phase difference between 1 and 3 may be neither 30 deg. nor 50 deg. The representation of three non-harmonic quantities by vectors in a plane is correct, however, when one of the quantities is the sum or difference of the other two; that is, the vectors in this case correctly show the relative phase differences between the three quantities, as well as their magnitudes. Thus, let A and B be the r. m. s. values of two non-harmonic quantities and C the sum. If three vectors are drawn in a plane so that C is the vector sum of A and B, it will be found that the angle between any two vectors will be their equivalent phase difference, as already defined; if one vector represents an electromotive force and the other a current, the cosine of this angle gives the true power factor. Or, if two vectors A and B are drawn with an angle between them equal to their equivalent phase difference, their vector sum will give the r. m. s. value of the sum of the two non-harmonic quantities that they represent, bibliography, 13, 14 p. 391, 15, 39.

Again, if a is the equivalent phase difference between two non-harmonic quantities and if b is the phase difference between them when one of the quantities is reversed, the sum of the angles a and b is 180 deg.; the vector representing one of the quantities is reversed, as though the quantities represented by the vectors were harmonic.

The correctness of vector addition and subtraction of non-harmonic quantities may be rigorously proved, or may be verified either by experiment or graphical construction, bibliography, 13, 14.

The once famous three-voltmeter and three-ammeter methods are illustrations of the general theorem that vectors in a plane may represent two non-harmonic waves and their sum. In the three-voltmeter method, three voltmeter readings are drawn as vectors in a plane; E_1 measured across a load; E_2 measured across a non-inductive resistance in series with the load; and E, their sum, measured across the two. In this case, not only the three voltages but also the current I can be represented[4] as vectors in a plane, for $I = E_2 \div R$, where R is constant, and any representation of I is the same as the representation of E_2 with the scale changed. The equivalent phase difference between E_1 and I in the load is thus determined, giving power factor cos θ, and power. In the three-ammeter method there is a similar relationship, vectors in a plane being drawn to represent the electromotive force E as well as the currents, I_1 in a load, I_2 in a non-inductive parallel circuit, and I, their sum. The three-voltmeter and three-ammeter methods may be considered as proved by the general theorem; or, they may be considered as a proof of the theorem for these special cases, bibliography, 2, 7.

Otherwise stated, we may represent as vectors in a plane the various currents and voltages in a system consisting of one unknown load circuit of any character and a single pure resistance, either in series or in parallel with it. By extension, see preceding footnote, we may include in the system any number of pure resistances connected in any way whatsoever with the one unknown circuit; thus, some of the resistances may be in series and some may be in parallel wth it, provided in each one there is a linear relation, $e = R\,i$, between current and electromotive force. In the one unknown load circuit, however, there need be no simple relation between e and i; the current in it may be distorted, as by hysteresis, and have a very different wave-form from the electromotive force. A single voltage and a single current in the unknown load circuit are included in the vector representation in a plane. If the voltage or current in the unknown circuit is split up into parts or components, such parts or components cannot be represented in the plane.

Three Equivalent Sine Waves of the Same Frequency, and their Sum, can be Represented by Vectors in Space. In a like manner three non-harmonic alternating quantities, together with their sum, can be represented as vectors in space, the length of the vectors being equal to the r. m. s. values of the various quantities and the angle between any two vectors being equal to their equivalent phase difference. More quantities can be so represented only when they are linear functions of those already represented; thus, if three currents are represented in space, any voltage drops proportional thereto may also be represented. This follows as an extension of the preceding case, bibliography, 16, 18 to 23, 25, 29 pp. 217-9, 34, 42, 45, 53.

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[4] More generally stated: Where certain quantities are represented as vectors in a plane, any other quantities that are linear functions of these may likewise be so represented, bibliography, 15, 23. When $e = R\,i$, e is a linear function of i. When $e = R_1\,i_1 + R_1\,i_2$, e is a linear function of i_1 and i_2.

More than three vectors representing non-harmonic quantities cannot, in general, be drawn in space; but the resultant of two of these can be used, together with the other two, to construct a three-dimensional figure. In the same way, three vectors cannot, in general, be drawn in a plane; but the resultant of two of these can be drawn with the third in a plane. The three resultants of any number of quantities can thus be represented in space, or two resultants in a plane, bibliography, 34, but this is a cumbersome procedure of doubtful utility.

Variation of Resistance, Inductance, or Capacitance Causes Wave Distortion. In a circuit with constant resistance, inductance, and capacitance, a sine electromotive force causes a sine current to flow without distortion; in case of a complex electromotive force, each harmonic component of electromotive force produces a corresponding harmonic component of current, as already pointed out in the paragraph on independent superposition. With a sine electromotive force, the current has a phase position with respect to the electromotive force depending upon the frequency and the relative values of resistance, inductance, and capacitance. When there is inductance or capacitance alone in the circuit, the current lags behind, or is in advance of, the electromotive force by an angle of 90 deg. and represents no power, for the power factor, cos 90 deg., is zero; with resistance alone, current and electromotive force are in phase and power factor is unity.

Should the values of resistance, inductance, or capacitance vary during a cycle, the current will be distorted, and, with a sine electromotive force, the current wave will contain harmonics in addition to the fundamental. Furthermore, in case of a varying inductance or capacitance, where the variation always involves hysteretic loss, the phase difference between the fundamental current wave and the electromotive force will be *less* than 90 deg., and the power factor will no longer be zero. Similarly, with a varying resistance, the phase difference between fundamental current and electromotive force will not be zero and power factor can never be unity, as discussed in detail in subsequent paragraphs.

Iron Introduces Odd Harmonics and so Distorts the Current Wave on Account of Variable Permeability; It Advances the Fundamental of the Current Wave in Phase and makes Increasing and Decreasing Values of Current unlike on Account of Hysteresis Loss. The instantaneous value of an inductive electromotive force, by Faraday's law, is always proportional to the rate of change of magnetic flux. The inductive electromotive force wave is, accordingly, the differential of the flux wave, and the latter is the integral of the former; therefore, if either one of these waves is a sine wave, the other is necessarily a sine wave differing from it 90 deg. in phase. This is true whether the circuit contains iron or not.

In a circuit containing no iron, permeability and inductance are constant, and current and flux are proportional; a sine flux, which always necessitates a sine electromotive force, also necessitates a sine current. However, when permeability is not constant, as in a circuit embracing iron, the current and flux are no longer proportional, this lack of proportionality being shown by the well-known curve of magnetization or hysteresis loop; a sine flux and sine electromotive force, accordingly, *necessitate a non-sine current.*

With the usual symmetrical hysteresis loop, the negative part of the distorted current wave will be a repetition of the positive and hence, for reasons already given, the current *contains odd harmonics only.* The *S*-like shape of the hysteresis loop indicates that the third harmonic must be prominent and this is always found to be the case; the fifth and higher harmonics have considerably smaller, although sometimes significant, values. Thus, a third harmonic of 30 per cent might be accompanied by a fifth harmonic of, say, 8 or 10 per cent, higher harmonics than the third being usually much smaller.

On account of the distortion of the current wave, a vector diagram in three dimensions can be advantageously used, the current harmonics produced by distortion being represented collectively at right angles to the fundamental plane, bibliography 25. This construction may be used whenever there is current distortion due to pulsations of any of the constants of the circuit.

Inasmuch as the ascending and descending curves that form a hysteresis loop are not alike, *the distorted current wave is not symmetrical* but has a different form for increasing and decreasing values of current. It is found that the third harmonic has such a phase position as to subtract from the fundamental during most of the time when the current is increasing and to add to the fundamental when the current is decreasing, thus humping the right hand (descending) side of the current wave.

Furthermore, reference to the hysteresis loop shows that maximum current occurs simultaneously with maximum flux, but that *zero current occurs before zero flux.* Although the flux wave always lags 90 deg. behind the electromotive force, the fundamental of the current wave is thus caused to lag *less* than 90 deg. This gives a power factor greater than zero, which accounts for the hysteretic consumption of power. *All* the power due to iron loss is thus supplied by the fundamental, for the harmonics in the current, when the electromotive force is sinusoidal, represent no power, bibliography, 1, 4, 5, 9, 10, 12, 25, 28, 35, 37, 52, 57.

In an imaginary case in which varying permeability gave like ascending and descending curves of magnetization, enclosing no area, there would be no hysteresis loop and no hysteresis loss. The varying permeability would nevertheless distort the current wave by introducing harmonics, but the fundamental would in this case lag 90 deg. behind the electromotive force, as always when there is no consumption of power.

Frequency Multiplication by Iron Distortion. It was early pointed out (bibliography 4) that the production of harmonics by wave distortion due to iron might be employed as a means for frequency multiplication. This is accomplished by combining circuits in such a way that the fundamentals of two waves oppose or neutralize each other, leaving a residuum of harmonics, of which the third is usually predominant. Triple frequency is thus obtained; or double frequency, by use of cores unsymmetrically magnetized by superposed d-c. excitation. Although the efficiency is not high enough to warrant the use of these methods in power transmission, they have been used in radio communication.

Harmonics in Three-phase Circuits. Obviously, in 3-phase circuits, the harmonics of triple frequency in the three circuits, whether originating in the generator or in transformers due to iron distortion, are in phase with each other. This is true not only for the third harmonic but for its multiples as well. In star-connected circuits, these harmonics appear in the electromotive force wave between line and neutral, tending to cause current to flow out through the three lines in parallel, but no current flows unless there is a grounded neutral or return neutral circuit to form a path for the return current.

In high-voltage circuits supplied through transformers, the harmonics produced by the iron in the transformers may be large enough to seriously strain insulation by increasing the maximum value of the electromotive force wave, particularly when some harmonic becomes amplified by resonance. To avoid this, the *primary or secondary windings* of transformers on three-phase circuits, or, in some cases, tertiary windings, are *commonly connected in delta,* so as to provide a local circuit in which the third harmonic current and its multiples can flow.

These harmonic currents act as magnetizing currents and reduce the corresponding harmonic voltages to a small value, just sufficient to circulate the current through the impedance of the three windings in series. The additional heating due to these local currents in delta-connected transformer windings is small. The delta connection of one set of transformer windings prevents the rise of voltage in any of the windings, even though some of these are not delta-connected.

Harmonic voltages in a generator are usually small compared with those produced by iron distortion in transformers and can be reduced by design. Delta connection of generator windings is therefore unnecessary. *Generators are usually star-connected* in order to provide a neutral for grounding.

The harmonic current in a neutral or ground return or in delta-connected windings of generator or transformers, as already pointed out, consists of the third harmonic and its multiples, for these are in phase in the three parts of a three-phase circuit. Other harmonics, as well as the fundamental, appear as voltages from line to line and cause current to flow only in the load, bibliography, 57, with full bibliography.

Pyramidal Space Diagram. The three star voltages of a three-phase system, which lie in a plane when there is no wave distortion, have the neutral point raised from the plane by the third harmonic voltage and its multiples, so that the three star voltages become the edges of a pyramid. Similarly, a diagram with vectors in space is required for representing line and delta currents, when harmonic currents circulate in the delta without appearing in the line, bibliography, 29, pp. 217-9. Commonly this effect of harmonics is ignored and plane diagrams are used, being sufficiently correct for most purposes.

T-connected Transformers Produce Dissimilar Wave Forms. Although the third harmonic does not ordinarily appear from line to line in a three-phase system, it may so appear when a two-phase generator with a third harmonic in the electromotive force wave of each phase is connected through *T*-connected transformers to a three-phase line. In this case the three line voltages are distorted dissimilarly, for the third harmonic appears differently in each, bibliography, 41.

A Resistance that Pulsates during a Cycle Introduces Odd Harmonics into the Current Wave; the Current and Electromotive Force Pass Through Zero at the Same Instant, but Power Factor is Less than Unity. In a circuit containing a constant resistance only, the value of the current at any instant is proportional to the value of the electromotive force at that instant, for $e = R\,i$; a curve plotted between e and i is a straight line. The current wave and electromotive force wave are, accordingly, identical in shape and pass through zero at the same instant; furthermore, the power factor of the circuit is unity.

In some cases resistance varies with the instantaneous value of current, as in an incandescent lamp in which resistance is a function of temperature, bibliography, 27, 38. In a solid conductor this variation in resistance is small, usually less than one per cent, but the instantaneous values of current i are no longer strictly proportional to the instantaneous values of electromotive force e; the curve plotted between e and i being a loop, showing a lag or hysteresis in temperature changes. The wave form of current differs, therefore, from the wave form of electromotive force. A sine electromotive force thus produces a non-sine current; in other words, a periodic variation in resistance introduces harmonics into the current wave and these harmonics will be odd when the resistance has the same series of values for negative as for positive currents.

With resistance only in the circuit, the current and electromotive force pass through zero at the same instant, with a pulsating as with a constant resistance. Power factor, however, is less than unity, in solid conductors usually only a fraction of a per cent less; for, with a sine electromotive force, the harmonics introduced into the current wave by the pulsating resistance represent no power. There will be, accordingly, a small phase difference between the equivalent sine wave of current and the electromotive force corresponding to the power factor.

It can be shown, bibliography, 42, that, as in the case of distortion produced by iron, the distortion in current produced by pulsating resistance is accounted for chiefly by the introduction of a third harmonic, that the current wave is either flattened or peaked, and the fundamental of current is either slightly advanced or retarded in phase, according to whether the temperature coefficient is positive or negative. The current, like any distorted current, can be shown in a vector diagram of three dimensions, the harmonics due to distortion being shown as a single vector at right angles to the fundamental plane, bibliography, 42. But all these effects of distortion, large when produced by iron, are so minute as to be usually negligible when produced by the pulsations in resistance of a solid conductor.

In a gas, however, as in the case of an arc lamp or mercury arc, the current distortion and resulting harmonics are very pronounced, so that power factor is, in some cases, reduced to 0.85 or less, bibliography, 3, 24, 26. In an arc that is discriminating, as between a ball and point, even as well as odd harmonics are introduced in the current. In a transmission line, an arc to ground introduces harmonics which may cause a dangerous rise in voltage through resonance, bibliography, 32.

It may be noted that power factor is less than unity with a direct current, in case resistance is pulsating or there are variations in current without corresponding variations in electromotive force.

Variable Capacitance Causes Wave Distortion, which is Practically Negligible in a Condenser; Corona Causes Pronounced Wave Distortion. In a perfect condenser, charge Q is proportional to potential E; the capacitance C is constant in the expression $Q = C\,E$ and the plot between Q and E is a straight line. When this plot is not a straight line, but is a curve or loop, bibliography, 8, harmonics are introduced and the current wave is distorted, as in case of varying inductance or permeability. In case of usual condensers such effects are practically negligible. In case of corona, however, there is a well-defined hysteresis loop in the curve plotted between Q and E, and a pronounced distortion in the current wave.

An increase in charging current and in charge Q, due to corona, occurs when the voltage is above a certain critical value, *i. e.*, during the peak of the voltage wave. There is a pulsation, therefore, in the ratio of Q to E and so in the capacitance of the line. The pulsation in capacitance may be explained in part as due to a periodic change in the effective diameter of the wire, but this does not account for the whole effect.

This periodic change in capacitance produces a pronounced third harmonic, as already shown for periodic changes in resistance or inductance, and this appears as a triple-frequency current to ground in a three-phase system, when the neutral is grounded at both ends of the line, bibliography, 50, 51, 54, 55, 56.

Power. When alternating currents and electromotive forces are complex, containing components of fundamental and higher frequencies, each component has its own independent power and power factor. The total power is the sum of the power of the separate components; thus, the total average power is

$$E\,I\cos\theta = E_1\,I_1\cos\theta_1 + E_3\,I_3\cos\theta_3 + \ldots$$

The independence of the average power of the separate harmonic components results from the fact, already pointed out, that the average product of harmonic components of different frequencies is zero, so that, when complex currents and electromotive forces are multiplied together, only products of currents and electromotive forces of the same frequency remain. This relation is always true, irrespective of the nature or the amount of distortion in the electromotive force or current wave.

Apparent power is not, in general, the sum of the separate apparent powers. *The instantaneous power* for each component is a quantity of double frequency varying harmonically about its average value, bibliography, 36.

Power Factor. From the foregoing, it follows that power factor, in case of a complex wave, is

$$\cos\theta = (E_1\,I_1\cos\theta_1 + E_3\,I_3\cos\theta_3 + \ldots) \div E\,I,$$

θ being the equivalent phase difference between the complex E and I.

It will be seen that in determining the combined power factor, $\cos\theta$, the power factor of each component is given a weight in proportion to the ratio of the apparent power of the component to the total apparent power. There is, however, no simple general relation between the combined power factor and the power factor of the separate components.

A circuit with reactance increasing and power factor decreasing with frequency, as R and L in series or R and C in parallel, will have a lower power factor with an irregular electromotive force wave than with a sine wave. On the other hand, a circuit with reactance decreasing and power factor increasing with frequency, as R and C in series or R and L in parallel, will have a higher power factor. It is thus seen that harmonics in an electromotive force wave may decrease or increase power factor according to the character of the circuit.

Any change in the relative amplitudes of the separate components of electromotive force and corresponding components of current changes the total power factor and at the same time changes wave form. Any change in their relative phase positions, however, while changing wave form, has no effect on power factor.

An exception occurs when the power factors cos θ, cos θ_1, cos θ_3, etc., are all equal, so that

$$E\,I = E_1\,I_1 + E_3\,I_3 + \ldots$$

In this case total apparent power is the sum of the apparent powers of the separate components, and, furthermore, the value of the total and separate power factors is not affected by any change in the relative amplitudes of the component electromotive forces and corresponding changes in component currents. This occurs when power factor is unity and in one special case, as discussed later.

Unity Power Factor Occurs only when the Current Wave is Precisely Similar to the Electromotive Force and in Phase with it. Reactance must be Zero and Resistance Constant. As power factor is real power $R\,I^2$, divided by apparent power $E\,I$, it is obvious that, in order to have unity power factor, $E\,I$ must equal $R\,I^2$ and $E = R\,I$. This can be true only in case the reactance of the circuit is zero and there is no capacity or inductance present.[5]

Power factor is the ratio of resistance R to the impedance Z. This ratio is unity only when reactance is zero and the circuit contains resistance only. Furthermore, for unity power factor, the resistance must be constant, for it has already been shown that a pulsating resistance reduces power factor to less than unity.

Constant resistance means that the current wave is similar to the electromotive force wave; the current i at any instant is proportional to the electromotive force e at that instant; $e = R\,i$. In this case the current and electromotive force pass through zero simultaneously and their maximum values coincide. Under no other circumstances can power factor be unity. There is no limit, however, to the irregularity of the current and electromotive force waves with unity power factor, provided the two waves are similar.

As a numerical illustration of unity power factor in case of a complex electromotive force, let the electromotive force $E = 50$, comprising a fundamental $E_1 = 40$ and a third harmonic $E_3 = 30$, be connected to a circuit with $R = 10$. The current is similar to the electromotive force; thus, $I_1 = 4$; $I_3 = 3$; $I = 5$. Power $R\,I^2 = R\,I_1^2 + R\,I_3^2$; or $250 = 160 + 90$. Apparent power $E\,I = 50 \times 5 = 250$. Power factor $= 1$. In like manner, the electromotive force may contain any number of harmonics and these may have any phase positions, as well as any amplitudes, for the phase of one harmonic with respect to others or with respect to the fundamental has no effect on power factor. For a more formal proof, see bibliography 23, 34.

Power Factor Less than Unity is due either to Current Displacement or Current Distortion. It will be seen that power factor is less than unity in two cases: (*a*) When an alternating current is shifted in phase with reference to the electromotive force, so that the zero and maximum values of current occur earlier, or later, than the corresponding zero and maximum values of electromotive force, power factor is less than unity, no matter what the wave forms of current and electromotive force may be. This is the common case and usually comes to mind when the power factor of an alternating current is referred to as being less than unity; the angle θ in the expression, power factor equals cos θ, represents a true phase displacement. (*b*) When current and electromotive force are unlike in wave form, power factor is less than unity, no matter what may be the phase position of the current with respect to the electromotive force; power factor may be brought to a maximum value by shifting the current wave with respect to the electromotive force, but this maximum is always less than unity.

The sign of θ, in the expression power factor equals cos θ, is ambiguous and does not indicate whether the current is lagging or leading. In fact, when the reduction in power factor is due to distortion, there may be neither lag nor lead. Thus, power factor 0.85, for example, indicates a phase angle of 32 deg. between the vectors I and E, representing current and electromotive force. If there is no lag or lead, the vector I is apparently caused to swing out of the usual plane of reference, as already referred to in the discussion of distortion due to pulsating resistance, inductance, or capacity.

Either current displacement or current distortion may operate alone to reduce power factor, or the two causes may operate simultaneously. Customarily the two effects are lumped together in a single resulting power factor cos θ, which, in some cases, may be separated into two factors, cos α due to current displacement and cos β due to current distortion. The angle α then represents lag or lead in the usual sense in the plane of reference and the angle β the amount the current vector swings out of that plane due to distortion, power factor being the product cos α cos β. A single factor, however, is more convenient and is quite sufficient for ordinary purposes. The values of the two separate factors are not readily determined and, except in special cases, are not independent of each other. bibliography, 59.

Peculiar Special Case. When the current wave is not similar to the electromotive force, any change in amplitude of any harmonic component changes the complex power factor, except in one peculiar case, namely, when only one harmonic, the *n*th harmonic, is present and the lag of the current of this harmonic with respect to its electromotive force is exactly equal to the lead of the fundamental current with respect to the fundamental electromotive force. This occurs when, in addition to a resistance, the circuit has a capacity reactance $1/C\ \omega$ which, for the fundamental, is n times the inductive reactance $L\ \omega$, bibliography, 59.

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[5] In case of a pure sine wave, capacity and inductance may be present and just neutralize each other, so as to give zero reactance and unity power factor, but this is possible with a sine wave only.

In this case, and in this case only, the harmonic and fundamental have the same power factor, lagging in one case and leading in the other, and this is the power factor of the complex wave. Whether the complex current is lagging or leading with respect to the complex electromotive force is apparently indeterminate.

Suppose, with a suitable frequency mixer, bibliography, 60, the relative values of fundamental and harmonic can be varied, so that the total r. m. s. value of the complex electromotive force remains constant. The current taken by the circuit, its real and apparent power, and its power factor remain unchanged, as the wave form of the electromotive force is varied from 100 per cent fundamental with leading power factor, to 100 per cent harmonic with lagging power factor. When fundamental and harmonic are both present in any proportions, the current vector apparently swings around in space, maintaining a constant phase angle while changing from lagging to leading. In a somewhat similar way, current in a synchronous motor changes from lagging to leading, with increase of field excitation, without the power factor passing through unity, bibliography, 49.

Conclusions. This paper, which is in itself a summary, may be further summarized by surveying the headings of the various paragraphs. It has been pointed out that there are well-known methods for obtaining the general solution for practically all problems dealing with simple harmonic alternating currents. There seems, however, to be no similar method that can be applied in all cases to the solution of problems when the currents are non-harmonic. Such problems have to be solved more or less separately, without a general solution. The purpose of this paper has been to present certain principles simply and clearly, in order that use may be made of them wherever applicable.

Attempts to use some kind of distortion factor, bibliography, 44, for modifying the usual alternating current solutions, so that equations and graphical diagrams based on the sine assumption may be extended so as to apply to non-sine currents, have not borne fruit, bibliography, 34, 45, nor has a general analytical discussion of non-sine alternating currents given practical working solutions, bibliography, 48. Although certain general principles are established, they do not afford the immediate solution of every particular problem.

An outstanding fact is that graphical diagrams for precisely representing non-harmonic alternating currents and electromotive forces require, except in some special cases, more than two dimensions. The ordinary plane diagrams are at best only approximations.

It is a consolation to know, however, that the errors in plane diagrams are commonly not large enough to assume importance, bibliography, 40, p. 228, except when leading as well as lagging currents are involved, in other words, except in circuits containing capacity reactance, as in condensers and synchronous machinery.

In inductive circuits, harmonics are, to a certain extent, choked out, so that errors are limited and there is no tendency for them to become amplified. The errors may be likened to the errors in using a drawing accurately made on a rumpled paper, which is later ironed out flat. The flat diagram, although not precise, may be accurate enough for most purposes. Thus, transformer diagrams drawn in a plane give excellent working results, despite wave distortion. In a synchronous motor, on the other hand, certain discrepancies due to current distortion may be corrected or eliminated by use of a diagram in three dimensions, bibliography, 49. It is well known that the power factor of a synchronous motor does not become unity when current changes from lagging to leading, as the plane diagram and usual theory based on the sine-wave assumption would indicate. The three-dimensional diagram makes it possible for the current vector, in passing from lagging to leading, to swing around in space so that the power factor passes through a maximum less than unity.

Plane diagrams are subject to the greatest errors in case of circuits containing inductance and capacity, when resonance for a particular harmonic may greatly amplify the errors due to wave distortion.

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Discussion

A. Boyajian: Professor Bedell is a pioneer in the theory of alternating currents, and his early book is still a classic on the subject. A few years ago I listened with great fascination to Professor Bedell narrating his early experiences in the fundamental theory of alternating currents. He said in the course of his narrative that when he analyzed the flow of currents in condensers, resistances, and inductances due to sine-wave voltage, his mathematical analysis showed him that the current of the condenser should be 90 deg. ahead of the voltage, the current of the inductors 90 deg. behind, and in a resistance in phase with the voltage. Up to that time nobody knew anything about phase relationship and this was very puzzling. It was peculiar how in a condenser you could get current ahead of voltage, like having an effect precede the cause instead of following the cause. They made some tests, plotted oscillograms, verified the mathematical analysis, and everybody was happy.

What he had done at that time for sine-wave voltages he has later done for non-harmonic voltages, and his paper puts together a lot of developments since that time.

One feature, or one problem, discussed in his paper which interested me a great deal was the subject of losses due to mixed frequencies. He emphasizes very strongly the fact that if we have two currents of different frequencies, the losses do not correspond to the square of the arithmetical sum of the currents, but that the losses for each current are independent of the other with the implication that it might be possible to take advanatage of this fact and increase the capacity of conductors for transmission of power.

A few years ago I had occasion to examine the possibility of increasing the transmission capacity of a line by the superposition of different frequencies, and I was sorry to come to the conclusion that such a thing was impossible. For given kilowatts to be transmitted at a given r. m. s. voltage, the $I^2 R$ line losses are the same regardless of whether this is done at a single frequency or mixed frequencies. The truth of this statement may be more easily seen considering the fact that "mixed frequencies" is another name for a distorted voltage. Is there any fundamental copper economy in transmitting power at a distorted voltage instead of sine-wave voltage? In any scheme of superposed frequencies, it will be found that any $I^2 R$ loss economy is due to raising the r. m. s. voltage of the lines.

The only fundamental benefit to be gained by the use of mixed frequencies for transmission appears to be one of convenience. In some cases the advantage in convenience may be so great as to lead indirectly to an economy in copper. For instance, if one can transmit 40 telephone conversations over the same pair of wires simultaneously, instead of over 40 separate pairs of wires, there would be a distinct economy in copper. But this is not due to a fundamentally lower $I^2 R$ loss, but due to increased transmission voltage and more effective utilization of the circuit. If one telephone message were to stress the line insulation to its limit, it would be impossible to superpose 40 messages; but telephone voltages are very small, and insulation stress is not the limiting feature.

Frederick Bedell: As Mr. Boyajian has clearly pointed out, there is no copper economy in the joint transmission of currents of different frequencies when the two currents supply a common receiver. There may, however, be a saving in certain cases, as discussed in references 16-22, when the two currents, transmitted jointly, supply separate receivers.

Development of Automatic Switching
Equipments in the United States and Europe

BY A. H. de GOEDE[1]

Associate, A. I. E. E.

***Synopsis.**—This paper gives a brief outline of the history and general development of automatic switching equipments in the United States and Europe, as applied to power equipment.*

A general review is made of the advantages of automatic equipments, consisting not only of a saving in operating expenses, but also of better operating characteristics as compared with manually controlled stations. The various designing problems, adherent thereto, are pointed out.

A comparison based on personal observation is made between the conditions in the United States and Europe, which accounts for the less rapid development and less extensive applications of automatic equipments in Europe, and a brief review is made of the results obtained in the latter years.

Mention is made of the more recent applications, such as to mercury arc rectifiers. A brief description is also given of supervisory control systems, the development of which has progressed hand in hand with the automatic equipments.

The accompanying illustrations show some typical modern American and European installations.

* * * * *

INTRODUCTION

It is interesting to note that the development of certain lines of the electrical industry shows widespread differences in the United States and Europe. The automatic switching equipments for the various classes of electric service form a striking example.

The idea to cut down the operating expenses of electric railways by the use of automatic substations, eliminating the attendants and operating only when needed, found its conception in the United States about 14 years ago. These equipments reached a high degree of reliability within a comparatively short time. After the war period, and after the initial installations had proved their worth, the automatic switching equipments found application on a large scale, not only for electric railways but for practically all kinds of service. In Europe, however, the first trial installations were not made until 1921-1922, and they had not been taken into commercial use on any appreciable scale until about 1924. The tardiness of Europe in using automatic equipments may be explained by the difference in economic and operating conditions as compared with the United States.

DEVELOPMENT IN THE UNITED STATES

It was only natural that the high cost of labor in the United States should create a demand for unattended stations. The first installation of this kind was tried out in Detroit in 1912 where a synchronous converter for lighting service was remotely controlled from a substation a mile away. The success of this arrangement directed the attention of electric interurban railways to it in an effort to reduce their operating expenses. Owing to the greater distances between substations, as in the case of interurban lines, the remote-control scheme was less suitable and the development of full automatic substations had to be taken in hand. Automatic stations should be designed in such a way that they perform the following duties, which are ordinarily taken care of by an operator in a manually controlled station:

1. Start the machine on load demand,
2. Protect the machine during the starting period,
3. Connect the machine to the system,
4. Protect the machine, when running, and control its output,
5. Take the machine out of service when there is no further demand for it.

How well the early designers of these equipments realized the various problems associated with their functioning is proved by the fact that the first installations of 1914 are still in service.

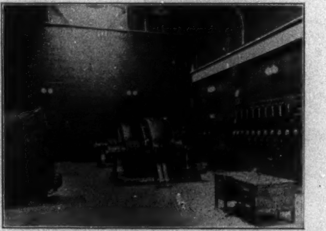

FIG. 1—AUTOMATIC SWITCHING EQUIPMENT FOR TWO 2000-KW., 600-VOLT, D-C., SYNCHRONOUS CONVERTERS FOR RAILWAY SERVICE, WITH FEEDERS, COMBINED WITH DISTRIBUTOR SUPERVISORY CONTROL. (OAK SQUARE SUBSTATION OF BOSTON ELEVATED RAILWAY, BRIGHTON, MASS.)

The success of the very first automatic switching equipments for interurban and city railway service was so striking that a demand developed soon for their application to other fields. After a temporary slackening during the actual war period, this demand became more urgent around 1920, and since that time they have been used extensively for all kinds of electric service, of which may be enumerated their widespread applica-

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tion to railway, hydroelectric, lighting, mining and industrial (in particular, steel mill) service.

Hand in hand with the development of automatic control equipments for machines came the design of automatic reclosing a-c. and d-c. feeders in order to derive the maximum benefit from these installations. By selecting the most appropriate machine and feeder control equipments, a most flexible installation can be secured.

The experience obtained with the first installations showed that the anticipated saving in operating expenses materialized. The main item was the saving in wages for two or three shifts of substation operators, against which stood a surprisingly small amount for periodic inspection. An appreciable item was also the reduction in power consumption, as the stations are only in operation when there exists a load demand on the system, thus saving the running light losses over considerable periods, especially in the case of interurban lines with infrequent service. Further savings resulted due to the fact that it was now economically possible to install a larger number of small substations throughout a certain territory instead of a few large substations. Not only did this give improvement of service owing to the better holding of constant voltage on the entire system but also a considerable saving in feeder copper was obtained which compensated to a certain extent for the higher initial cost of the automatic substation.

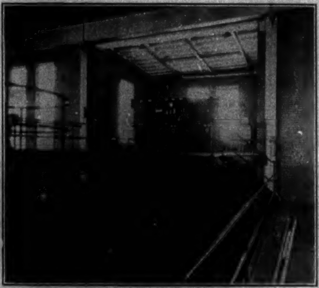

Fig. 2—Automatic Switching Equipment for Two 4600-Volt, A-c., 3000-Kv-a., Waterwheel Driven Generators. (Interior Sugar Island Station, St. Lawrence Valley Power Corporation, Potsdam, N. Y.)

It was soon discovered that not only a saving in operating expenses could be obtained by means of automatic stations but that these equipments had also many other important advantages. In order to secure proper operation, it is essential that an automatic station be provided with a complete set of the well-known protective features which should be designed and arranged in such a way that a certain predetermined function will take place for any anticipated emergency to prevent damage to the machines or attendant equipment. The ordinary manual station has only a few protective devices and depends for the rest on the experience and minute observation of the operator who will never be able, in case a certain trouble develops, to take such immediate positive action as a relay especially installed to perform a definite function in case of just that kind of trouble. All operations in an automatic station occur in a certain predetermined sequence, and each step in the sequence depends upon the proper completion of the previous one, so that faulty operation is excluded under all conditions. This will allow more reliable functioning than when the uncertain human element is present.

In this respect it will be clear that the success of automatic switching equipments depends upon the correct functioning of each individual device. This has been realized by the designers of these equipments since the beginning, and every endeavor has been made to make the devices as perfect as possible. A device must be not only electrically and mechanically strong but in many cases it is also essential that it be quite sensitive at the same time, which offers some unique problems. Furthermore, these devices should operate satisfactorily over a wide range of temperature, as automatic stations are not heated. Special devices have been designed as protective, checking, regulating and sequence relays especially for this kind of service, with due regard to their probable number of operations and duration of life. While in the beginning of automatic station operation there were a few cases of device failures, the art has progressed so rapidly that a very high degree of reliability has been reached and device failures rarely happen.

Fig. 3—Typical American Automatic Control Equipment for 275-Volt D-c., 300-Kw. Synchronous Motor Generator Set for Industrial Use

In this way a very high class of regulating equipment has been developed for use in automatic stations, which will allow these stations to operate temporarily at a much higher overload than would be considered safe in case of manual operation. The automatic protective relays will determine exactly when the load has to be reduced in order to prevent damage to the machine and will function to accomplish this. Usually, a d-c. machine will then be adjusted so as to deliver power up to its limit, to the system at a reduced voltage. As soon as the excessive load demand ceases, the voltage will be brought back to normal. In this respect the load limiting resistors in the circuit of synchronous converters may be mentioned, as well as rheostatic and counter-electromotive force control of the shunt fields of d-c. generators. These arrangements make it possible for an automatic station to deliver power up to the very limit of the machine so that service will be maintained as far as possible.

A further advantage accruing to the use of automatic switching equipments is the fact that smaller buildings of less elaborate design can be used to accommodate these equipments as no sanitary measures have to be taken for the attendant personnel as in manual stations. This makes it also possible to locate substation apparatus in places which would not be considered suitable for manual stations. For example, if the load on an Edison lighting system in the business district of a city increases to such an extent that a new substation becomes necessary, this may be located in the load center and installed in a basement, while in many cases with manual operation a more expensive site would have to be purchased, or the substation would have to be located at some distance from the load center, necessitating expensive cable runs. Another example is formed by the installation of substations with rotating apparatus in residential districts, to which many objections usually are voiced on account of the noise. When using automatic control, it is possible to use an entirely enclosed soundproof building. It will be clear that such "noiseless" substations are only feasible with unattended equipments.

FIG. 4—TWO VIEWS OF AUTOMATIC SWITCHBOARD TO CONTROL A 500-KW., 600-VOLT D-C. SYNCHRONOUS CONVERTER AT SEVRES SUBSTATION OF THE "SOCIETE DES TRANSPORTS EN COMMUN DE LA REGION PARISIENNE" AT PARIS, SHOWING THE EXTENSIVE USE OF AMERICAN-MADE DEVICES (REPRODUCED FROM *Le Genie Civil* OF DECEMBER 12, 1925)

The automatic switching equipments have also made possible the profitable development of small water power sites. It has been found in many instances that the building of small hydroelectric plants with automatic control was entirely feasible, while their development with manual control was not economically warranted, owing to excessive operating expenses. In some special cases it has been found that the building of several small plants along a river was cheaper than the development of a single high head plant of large capacity, due to the characteristics of the river-bed.

Summarizing the above, some of the most outstanding advantages of automatic switching equipments are the following:

1. No operators required,
   a. Saving in operating expenses,
   b. Freedom from labor trouble,
2. Stations operate only as needed,
   a. Saving in power,
   b. Less wear on apparatus,
3. Continuity of service and better regulation,
4. Reliability,
5. Constant protection with positive action,
6. Possibility of selecting the most economic location for a station,
   a. Saving in feeder copper,
   b. Less expensive sites,
7. Possibility of developing small water power sites.

It is not surprising that because of these paramount advantages of automatic switching equipment, their use has increased rapidly on a progressive scale. While in earlier years automatic equipments were only installed to reduce the operating expenses, it is a notable fact that during the last few years their use is considered in many instances solely based upon their more reliable service accomplishments as compared with manual control. They find more and more application in cases where continuity of service is of the utmost importance, such as in steel mills.

DEVELOPMENT IN EUROPE

While the development of automatic switching equipment went ahead with rapid strides in the United States, very little work was done in this line in Europe. During the war period the capital and men were lacking for experiments and investigations which were not directly useful for the progress of the war, so that the development in the electrical industry in general was virtually at a standstill. The principal work consisted of maintaining existing installations in operating condition, while extensions were practically impossible. At the end of the war the load on several systems had increased to such a value that they were operating without any reserve at all.

After the war the demand for electric service increased greatly and the available capital was applied to the extension of the systems. As this had to be done in as short a time as possible to satisfy the demand, and as the industry was still quite disorganized, it is apparent that the time was not very appropriate to try out radical changes in design, and only the more conventional equipments were installed.

Owing to cheaper labor in Europe and the less extensive use of interurban electric railways, the demand for automatic or unattended switching stations was less pronounced. The tremendous success of automatic installations in the United States, however, directed the attention of European manufacturers to this class of equipment, and the subject was given serious consideration, especially as the post-war depression necessitated the application of the most economical apparatus. In the years 1921-1922 several trial installations were built, practically all for synchronous converters for railway service. Two distinct designs can be noticed:

1. Those which were developed by European companies with American connections, and which followed largely the American design, sometimes to the extent of using several devices built in the United States,
2. Those which were developed independently.

Fig. 5—Automatic Switching Equipment for 1000-Kw., 600-Volt D-c. Mercury Arc Rectifier, Consisting of Two 6-Anode Tanks, for Railway Service, with Feeders, Combined with Selector Supervisory Control. (Chicago, North Shore, & Milwaukee Railroad Co.)

Even these latter equipments base their design upon the operating experience obtained in the American stations during several years. The only basic difference is that in many cases the synchronous converter is not self starting, which has never found great favor on the continent owing to the disturbances caused in the a-c. system, frequently of a small capacity. Instead, a special small starting motor is installed on one end of the converter shaft to bring the machine up to synchronous speed before connecting it to the a-c. system.

While it is the universal practise in the United States to raise the brushes from the commutator during the starting period, a few European manufacturers do not follow this procedure. It is claimed that excessive sparking on starting is eliminated by means of a special design of the commutating poles and of the field windings, so that there is no necessity for raising the brushes.

To insure correct polarity on starting, the converter field may be either separately excited or the polarity may be checked and, if necessary, automatically corrected by means of a polarized relay with permanent magnets. Both methods have found extensive use in the United States, and similarly both methods are being followed in European design. The only difference is that in American practise a special single-phase motor-generator set is used for separate excitation (field flashing), while it is the universal practise in Europe to mount an exciter on the converter shaft for this purpose.

After these trial installations had given a good account of themselves, the automatic control equipments for synchronous converters have found commercial application in the last three or four years, when their advantages became more fully appreciated. In the same manner as in the United States, they have been applied not only in order to secure operating economies but have also been used in cases where manual control was less suitable. In this regard mention may be made of a substation, Soho Square Substation of the Charing Cross Electricity Supply Commission, which is installed in the basement of a building in a thickly populated section of London. Incidentally, it may be noted that this substation will ultimately contain five 300-kw. synchronous converters for 210-volt, d-c. lighting service, which are arranged to start and stop automatically in a definite sequence depending upon load conditions of the d-c. system which is a large number of units compared with the usual station in the United States. This shows that faith exists in the reliability of these equipments. Nevertheless, automatic synchronous converter substations have found application only on a very limited scale in Europe when compared with their extensive use in the United States. The value of the load limiting resistors in the machine circuit does not seem to have been generally appreciated in Europe in the beginning. For railway work there have been installed equipments both with and without these resistors. This may be due to less rigid requirements for continuity of service, but still it can be noted that the most recent European installations make more general use of this method of load limiting.

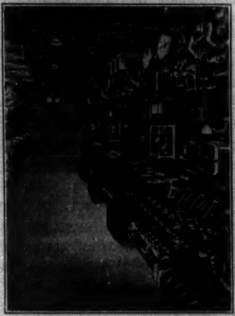
Fig. 6—Typical European Automatic Switchboard to Control four 300-Kw., 210-Volt, D-c. Synchronous Converters at Soho Square Substation of the Charing Cross Electricity Supply Commission at London. (Reproduced from the *Metropolitan-Vickers Gazette*, of December, 1926)

MERCURY ARC RECTIFIERS

At this point attention may be called to the steel enclosed mercury arc power rectifier. Being an American invention, it is remarkable that its development originally progressed even more rapidly in Europe than in the United States. Above a certain definite d-c. output voltage, this apparatus has a very decided attraction as the means of converting alternating current to direct current with the highest efficiency of any known conversion method.

These rectifiers have found widespread application in Europe during the last 8 or 10 years, and after proving their reliability, the step to make them automatically controlled was soon taken. Fundamentally, the automatic switching equipment for a rectifier is simpler than for a synchronous converter as no synchronizing with the a-c. system and no polarity check is necessary. Consequently, automatic control equipments for mercury arc rectifiers have found more general application and on a larger scale in Europe than the equipments for synchronous converters. Only in the last two years a comparatively small number of mercury arc rectifiers has been equipped with full automatic control equipments in the United States. Their application gives some complications which are not known in Europe, especially due to the fact that they are subjected to extreme changes in temperature in many sections of the United States, which may effect the correct operation of the rectifier. Automatic temperature control is therefore necessary, which is not required for European installations. For full automatic equipments the American practise requires also automatic vacuum control, which is not generally furnished with the European equipments. These complications have held back the application of automatic control equipments to mercury arc rectifiers a good deal, but still their use is gradually increasing.

FIG. 7—TYPICAL EUROPEAN INSTALLATION OF AUTOMATIC SWITCHING EQUIPMENT FOR THREE 1500-KW., 600-VOLT D-C. SYNCHRONOUS CONVERTERS AT BALHAM SUBSTATION OF THE LONDON UNDERGROUND RAILWAY, REMOTELY CONTROLLED. (REPRODUCED FROM THE *Metropolitan-Vickers Gazette* OF NOVEMBER, 1926)

SUPERVISORY CONTROL SYSTEMS

The development in the United States has long been concentrated on full automatic operation without any attendance whatsoever, except periodic inspection. In more recent years there seems to be a tendency to go back, not exactly to remote control, but rather to remote supervision. In this case the automatic station is allowed to function automatically by its own devices, but a dispatcher at a central point receives indications of the main switching functions, so that he is at all times fully informed by means of a system of lamps as to what happens in the remote automatic stations. Generally, the dispatcher has control of certain functions, so that he can start or stop a machine or open a feeder at will, regardless of the conditions on the system of which the substation forms a part. This feature is useful in certain emergencies. These supervisory systems have passed the experimental stage and have also reached a high degree of reliability. They allow the control and indication of a large number of separate functions over three or four common line wires, which under certain circumstances may also be used for telephony. The systems operate with a short time delay, which is only 5 or 10 sec. Simpler schemes rely on audible indication and are useful for systems where only a few functions have to be performed. These supervisory systems are based upon the use of devices which have proven their reliability in train signal, telegraph, and automatic telephone service, of course with the necessary modifications for power work. A further refinement of these methods of supervision is the remote metering. Several workable schemes have been developed to transmit various meter readings automatically over large distances from a remote automatic station to a dispatcher. While this supervisory equipment is fairly expensive, many operating companies consider it of the utmost importance, because the dispatcher is constantly informed about the actions of the automatic stations, which will explain why these

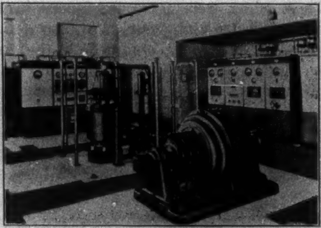

FIG. 8—TYPICAL EUROPEAN INSTALLATION OF AUTOMATIC SWITCHING EQUIPMENT FOR 300-KW., 800-VOLT D-C. SYNCHRONOUS CONVERTER AND 300-KW., 800-VOLT D-C. MERCURY ARC RECTIFIER AT ST. LEGIER, SWITZERLAND. (REPRODUCED FROM THE *Brown Boveri Review*, AUGUST 1926)

systems have found a widespread application in the United States during the last few years.

As automatic stations have not yet been in such extensive use in Europe, there has been very little demand for these supervisory systems. A few have been developed, based mainly on automatic telephone devices.

CONCLUSION

The leading manufacturers in the United States are at present prepared to furnish reliable automatic switching equipments suitable to meet all possible control operations for both a-c. and d-c. machines and for switching operations on a-c. and d-c. networks, in short, for all classes of electric service, with the only exception of generating stations driven by prime movers other than water wheels.

The application of automatic switching equipments in Europe has been on a much smaller scale and covers a smaller scope of work. Practically all installations are for railway and lighting service, with some isolated cases for the other classes of electric service.

Automatic switching equipments for practically every conceivable operating condition have been installed the world over and prove daily their paramount advantages. Their future is assured.

Discussion

Chester Lichtenberg: Since its development in 1914 the automatic station has been a very interesting topic to electrical engineers. The development has been very rapid. For example, in the United States today more than one-third of the entire switchboard output of the large manufacturing companies is automatic switchboard equipment for power stations and substations. Our records indicate that over 2,000,000 kw. of rotating apparatus are under automatic control and over 6,000,000 kw. of feeders and transformers are in automatic station service.

It has been thought that the development of the automatic station would make the attendants more mechanical. This has not been the case. Instead, the personnel of the operating companies who care for automatic stations have been required to use a greater degree of intelligence than those who attend the usual type of substation. It should not be thought, however, that the automatic station is an intricate or complicated collection of devices. It is not. It is a very simple combination of devices quite well known in the control art but so skilfully connected one to another that they perform functions heretofore delegated to human beings. When one considers the problem, this is as it should be because human beings are relatively slow to respond, while the changes in the electrical circuit take place with extreme rapidity. The ordinary human being responds to an impulse in about one half a second while the ordinary electrical device will respond to an electrical impulse in one-tenth or even one one-hundredth of this time.

Sometimes it is thought that the installation of automatic stations will require the engaging of skilled personnel for their maintenance and operation. Experience has indicated that this is not the case because as before stated the equipments consist of quite common control devices although there are more of them than in the usual manually operated station. Operating companies have demonstrated that they have in their employ many who can successfully install, maintain, and operate automatic stations.

Besides, the manufacturers have found that some of the operating companies have employees who can and do suggest improvements in the schemes of operation as a result of their practical experience.

Load-limiting or load-shifting resistors are almost always discussed when automatic stations are considered. These resistors have three functions;

1. To prevent the flow of excessive current between an on-coming machine and the bus.
2. To limit the drain on a machine for continuous service.
3. To shift to adjacent machines the load from a machine which might be overloaded.

Each of these functions is quite important. However, the importance varies with the application and with the type of transforming apparatus. Primarily, load-limiting or load-shifting resistors assist in maintaining service under unusual operating conditions and find here their most advantageous application.

H. S. Knowlton: I should like to ask Mr. Lichtenberg to say a few words about the training of men to maintain and inspect these automatic stations. Probably the designing engineers have provided very thorough instructions in printed form, but is there not a great gap between the ability of the designing engineer to comprehend the details of these automatic plants and that of the average operating man?

K. K. Paluff: It seems to me that the automatic substation has two very important characteristics. First, that there is no need of technical knowledge or training on the part of the operator; and second, that it makes small installations more economical.

The automatic substations will enable us to electrify small waterpower systems. I should like to add that in this country, particularly, the supply of peat was entirely neglected for the reason that the main problem connected with the development of peat was the impossibility of transportation. Peat made in such a manner as to withstand transportation becomes absolutely uneconomical. It seems to me, therefore, that the automatic stations may permit exploitation of the peat bogs in this country as well as in Russia to a far greater extent than at the present time.

E. de Mulinen: (by letter) Mr. de Goede mentions that the first trial installations of automatic switching equipment were not made in Europe until 1921-22. I should like to mention that an automatic converter substation in Basel (Switzerland) has been in successful operation since 1918. Mr. de Goede states that the development in Europe was forced by post-war conditions, but the actual development of automatic equipment was started during the war on account of shortage of labor.

Mr. de Goede mentions that load-limiting resistors are not much used in Europe. It might be said that a drooping characteristic is used in most of the European substations for traction purposes, resulting in a great flexibility in load distribution between the different substations. This scheme protects the converter from high overloads and assures continuity of service without destroying a considerable amount of energy in load-limiting resistors. In this country, about 90 per cent of the traction substations use compound-wound machines which require load-limiting resistors to give the system some flexibility, but when laying out a new system, it will always be found that a drooping characteristic will give the most economical results In this respect, it may be of interest to note that in one of the most recent automatic substations in Chicago—the Grimm

Avenue Substation—no load-shifting resistors are used, as the inherent voltage characteristics of the machines in this substation do not require such equipment.

In saying that automatic vacuum control is not generally furnished with the European automatic rectifier equipments, Mr. de Goede doubtless refers to the first trial installations. After considerable research, Brown, Boveri & Co. has succeeded in bringing out a most reliable vacuum control which has been in commercial use since 1923 and supplied with each automatic rectifier equipment. This type of automatic vacuum control is used with over a hundred automatic rectifiers in Europe and in eight automatic rectifier substations in this country.

A. H. de Goede: Mr. de Mulinen states some of the reasons why load-limiting resistors are in such little use in Europe. It might be said that the service conditions in the United States are usually much more severe, and in many instances it has been proved that during periods of heavy load it was not possible to keep a machine with drooping characteristics on the line, making it necessary to add load-limiting resistors to maintain service.

I was familiar with the fact that the Brown Boveri Company has equipped its automatic rectifier substations with automatic vacuum control during the last two years. However, as stated in my paper, this is still not generally furnished by most of the European manufacturers, while the American practise considers it an essential feature for full automatic operation.

The remarks of Mr. Lichtenberg supplement my paper on some interesting points. Many other points might have been treated more broadly, but in order to keep my paper within bounds, I have only endeavored to give a brief outline of the main advantages of automatic switching equipments, and of what has been accomplished along this line.

The Most Economical Power Factor

A Practical Design Formula for Distribution Circuits

BY H. S. LITCHFIELD[1]
Associate, A. I. E. E.

Synopsis.*—The use of power factor corrective devices on distribution circuits is justified, under certain conditions, by rather substantial savings in investment charges and by a reduction in the power losses of the system. The object of the present paper is to develop a practical working formula for calculating the most economical corrected power factor for a distribution circuit. Most economical conditions are assumed when the total of such annual circuit costs as are directly affected by a change in power factor, is a minimum.*

The usual methods for computing separately, the saving in $I^2 R$ losses and the decrease in investment charges due to power factor improvement, are inadequate. Particularly in the design of new circuits and extensions has the need for a more accurate method for calculating optimum power factor and conductor sizes been expressed.[2] Since these equations were originally worked out, two other solutions for the most economical corrected power-factor angle have been published, each having been obtained independently of the other. Menjelou[3] obtained his formula in the form:

$$\alpha \sin \theta = 1 - \beta \tan \theta$$

in which θ is the power-factor angle and α and β are constants computed from the circuit costs. Stevenson[4] obtained a similar expression:

$$\sin \theta = \delta - \eta \tan \theta$$

the difference lying in the constants to be evaluated.

The equation developed in this paper reduces to the simple form.

$$\sin \theta = \text{unit cost ratio}$$

That is, the sine of the most economical corrected power-factor angle is determined by the ratio of the annual cost of condenser capacity per reactive kilovolt-ampere of correction to the annual cost (fixed charges plus value of losses) per kilovolt-ampere delivered, of that portion of the supply circuit which is directly affected by the change in power factor. When the unit cost ratio is greater than the sine of the original power-factor angle, it is found that no investment in corrective equipment is economically justified.

The equation is set up in such a form that solutions are readily obtained for the most economical size of conductor as determined by the Kelvin law, and for the required kilovolt-ampere rating of the transformers and condensers. A method is suggested for including generating station costs with those of the individual circuit under consideration.

In evaluating circuit costs, the effects of load factor and the shape of the typical daily load curve upon capital investment and power losses have been worked out after the methods used by Gear and Williams[5] and by Reyneau and Seelye[6].

Equations for evaluating the circuit constants are included in the appendix. Several illustrative examples are worked out.

SO long as each community was served by its own generating station, the generator was the most convenient and, in some cases, the most economical source of magnetizing current. But with the elimination of small stations and with the growth of interconnection, the shunting of large blocks of reactive power from one point on the system to another has introduced serious operating complications. Low system power factor limits the availability of installed capacity, adds to the $I^2 R$ line losses, and increases the conductor sizes required to maintain proper voltage regulation. The kilowatt-hour losses on circuits operated at low power factor may run as high as 20 to 30 per cent of the annual input. For a switchboard cost of eight mils, a reduction in line losses from 25 to 15 per cent is nearly equivalent to a saving of one mil per kilowatt-hour at the stations. The economical utilization of the modern generating station is dependent not only upon a favorable base load but also upon the cost of distributing this lower cost power to the consumer. The full value of modern refinements in station design and operation cannot be realized unless there be a commensurate increase in the efficiency of the system of distribution. Certainly, industry cannot hope to obtain full advantage of low cost power made possible by the modern generating station so long as generators, bus structure, cables, substations, lines and transformers must be designed to carry excessive, inductive loads.

Considerable improvement in system power factor usually may be obtained through a more scientific application of the individual induction motor to its load. When loaded continuously between three-fourths and full load rating, a good induction motor should operate at a power factor of from 0.80 or 0.85 to 0.92 or 0.94 lagging, depending on size and speed. The prospective increase in high power-factor heating load will tend to raise the average of the system power factor.

For further improvement, rather substantial investment in corrective devices is necessary. Prices range from six to sixty dollars per reactive kilovolt-ampere of correction. If such large expenditures are to be made in unproductive equipment, it is necessary to know that the largest possible savings are to be effected. How much money can be invested profitably in such improvement? What is the most economical corrected

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[1] Electrical Distribution Engineer, 30 Beckwith St., Auburn, R. I.

[2] Serial Report No. 25-104, "Power Factor Improvement," p. 1; *Electrical Apparatus Committee of the National Electric Light Association.*

This reference contains a complete bibliography.

[3] *Rev. Generale de l'Electricite,* September 26, 1925. Article entitled "Rapid Calculation of Optimum Power Factor in Industrial Networks" by Rene Menjelou.

[4] *General Electric Review,* August, 1926, p. 574. Article entitled "The Economic Limit of Power Factor Correction," by A. R. Stevenson, Jr.

[5] Gear and Williams, "Electric Central Station Distribution Systems," Chap. XIII. Van Nostrand.

[6] Reyneau and Seelye, "Economics of Electrical Distribution," McGraw-Hill.

Presented at the Regional Meeting of District No. 1. of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

power factor? The term "most economical" is here used to indicate an economic balance in circuit design for which the total annual cost of supplying a standard power service is a minimum.

In the analysis that follows, a circuit cost equation is set up with θ, the corrected power-factor angle, as a variable. The minimum value is found by equating the first derivative to zero. The general equation includes only the variable circuit costs that are directly affected by a change in power factor. It does not include what may be termed fixed costs that are not reduced by an improvement in the power factor of the circuit. The mathematical correctness of the solution is not changed by the elimination of the fixed costs, since all constant terms drop out in the differentiation.

The equation is developed subject to the following conditions:

a. The value of generating station and outside plant capacity released through power factor correction depends upon its availability for carrying additional profitable kilowatt load. In the design of new circuits, smaller conductors and transformers may be used for a given kilowatt load.

b. The cost[7] of line losses per kilowatt-hour includes fuel cost at the generating station and demand charges prorated over the kilowatt-hour losses for the year.

c. Load is figured in terms of maximum kilowatt demand.

d. Power factor is measured at the time of maximum kilowatt demand at the point where the condenser is to be installed. It is defined as the ratio of kilowatt demand to coincident kilovolt-amperes. The term "power-factor angle" is used as a convenient mathematical convention rather than as a true instantaneous vector relationship.

e. It is assumed that the distribution voltage has been determined by conditions affecting the system as a whole. Existing standards usually determine in a general way the types and ratings of equipment.

f. Construction standards on overhead distribution systems are usually such that the cost of the supporting structure is the same for each of the several possible wire sizes which could be chosen for a given circuit.

g. The cost of spare condenser equipment is not included. This is offset by the reduction in spare transformer and line capacity required for emergency service.

h. Capacity rating of condensers determined by economical considerations: Synchronous condensers to be operated normally at full excitation to obtain maximum corrective effect during working hours, and shut-down during off-peak hours. Static condensers act as a constant capacity connected across the line with losses practically constant. When installed in accessible locations these are usually disconnected during off-peak hours. Annual condenser losses are determined by the number of hours connected.

i. Partial voltage control with capacity rating of synchronous equipment determined by economical formula: In this case condenser and circuit losses should be computed for the actual duty cycle of the condenser throughout the year.

j. Constant voltage control: The rating of the synchronous condenser in this case is determined by the constants of the circuit and by the generator and receiver voltages. The economical power-factor formula does not apply to this condition.

The equation is developed for an overhead line supplying power through a single bank of transformers. The power factor of the load is $\cos \theta_{or}$, lagging. A condenser is to be installed on the low voltage side of the distribution transformer. Kilowatt demand is known, as is also the required full load receiver voltage. It is assumed that the rating of the transformer will be just equal to the resultant kilovolt-ampere demand at the corrected power factor and that the size of the line conductors will be just sufficient to carry the resultant full load amperes at the most economical current density. The annual fixed charges on the investment and the cost of losses for the primary line conductors, transformers, and the condenser are to be set up in an algebraic equation in which the variable is θ, the corrected power-factor angle.

The following symbols are used:

Q_1	Annual fixed charges on the most economical line conductors per kilovolt-ampere delivered,
Q_2	Annual cost of line losses per kilovolt-ampere delivered,
Q_3	Annual fixed charges on transformers per kilovolt-ampere delivered,
Q_4	Annual cost of transformer iron losses per kilovolt-ampere,
Q_5	Annual cost of transformer copper losses per kilovolt-ampere,
K_1	Annual fixed charges on the condenser per reactive kilovolt-ampere of correction,
K_2	Annual cost of energy losses in the condenser per reactive kilovolt-ampere of correction,
P	Power in kilowatts delivered to the load at the time of maximum demand,
p	Kilowatt loss in the condenser at full load voltage and excitation,
$\cos \theta_{or}$	Power factor of the load, at the maximum demand,
$\cos \theta$	Corrected power factor at maximum load,
M	Cir. mils per ampere at full load for most economical line-current density in the conductor,
Y	Annual cost of the circuit, including the condenser; it includes only those costs which are directly affected by an improvement in power factor.

In the appendix will be found a list of additional symbols and a detailed mathematical development of the equations for evaluating the K and Q terms, to which reference is made in the following paragraphs.

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[7] See reference 6, Chap. IV, for a method of computing the cost of energy losses at any point on the system.

By the Kelvin[8] law, that wire size is assumed most economical for which the annual fixed charges on the investment in the conductors are just balanced by the yearly cost of $I^2 R$ energy losses. Assuming a current of one ampere, the most economical circular mils per ampere, designated by the symbol M, may be calculated by formula (**6**) in the appendix. The value of M depends on the cost of copper, the shape of the typical daily load curve, and the cost of energy. It is independent of the ampere load, voltage, phase, spacing, and the length of line. The size of the most economical conductor is then equal to M times the resultant amperes per wire. Multiplying by the length of line and by the unit weight of copper, and by the cost per pound, the cost of one conductor is obtained as in (**7**). The resultant current may be expressed in terms of kilowatt load, voltage, and power factor. Substituting for the current terms in (**7**) and multiplying through by the number of conductors and by the annual fixed charge rate yields expression (**8**). It represents the annual fixed charges on the most economical size of conductors expressed as a function of the resultant power-factor angle θ. The coefficient Q_1 is obtained in expression (**9**).

The energy loss in the most economical conductor at full load is $I^2 R$ watts. The resistance may be represented by the resistivity per cir. mil ft. times the length of the conductor divided by the area of the conductor in cir. mils. The cross-sectional area equals M times resultant amperes, as in the previous paragraph. Multiplying by the cost of energy and by the number of conductors, expression (**10**) gives the hourly cost of losses at constant full load.

The actual $I^2 R$ losses of a fluctuating load may be evaluated in terms of loss at full load by means of a special "loss factor" computed from typical daily load curves of the circuit under consideration. The loss factor is found by dividing the sum of the squares of the hourly ordinates of a typical daily load curve by 24 times the square of the annual peak. Descriptions of the method are given in the references[8]. The loss factor multiplied by 8760 hours gives the "equivalent hours" per year that it would be necessary for the annual peak load to continue in order to yield the same energy loss as that given by the actual fluctuating load throughout the year.

The annual cost of losses in the conductors is then obtained by multiplying expression (**10**) by the equivalent hours. The current term is expressed in terms of kilowatts, voltage, and power factor as before, yielding expression (**11**). The simplicity of the final result depends in part upon the fact that, by this method of setting up the equation, the cost of losses is expressed as a function of the power-factor angle to the first power rather than to the usual square.

Transformer costs apply to the nearest standard size required for the resultant kilovolt-ampere load. In cost analyses of this sort, where the unit cost varies inversely with the size, it is sometimes desirable to plot the cost per kilovolt-ampere against kilovolt-ampere rating and to obtain the equation of the line passing through the desired points. Under the conditions of the present problem, however, it is simpler to use the unit cost of the nearest standard size. A first approximation may be used and a second value substituted later, if necessary.

The evaluation of the transformer cost coefficients, Q_3, Q_4, and Q_5 is given in expressions (**13**) (**14**) and (**15**). Transformer copper losses are found by multiplying the loss at full load by the equivalent hours defined above. The iron losses are approximately constant and may be represented by loss at the manufacturer's rating times 8760 hr. in a year.

As in the case of the transformers, the unit cost of the condenser is that of the nearest standard size. For static condensers in groups of 30 kv-a., the unit cost does not vary greatly. In the larger sizes of synchronous condensers, the cost per kilovolt-ampere is fairly constant. For the smaller sizes of both types the unit price increases rapidly and it may be necessary to try one or two approximate values. The annual fixed charge rate includes interest, depreciation, and taxes. Where special attendance is required, as with synchronous equipment, the extra expense may be added to the fixed charge rate. Condenser losses are evaluated by multiplying the rated loss at full load voltage and excitation by the hours per year that the equipment is connected to the line. The evaluation of terms K_1 and K_2 is given in expressions (**16**) and (**17**).

THE CIRCUIT COST EQUATION

The kilovolt-ampere delivered by the transformers at the corrected power factor are

$$\frac{P + p}{\cos \theta} = (P + p) \sec \theta$$

The annual cost of line conductors and transformers at the resultant power factor is then:

$$(Q_1 + Q_2 + Q_3 + Q_4 + Q_5)\,(P + p) \sec \theta \qquad (1)$$

The condenser capacity required to raise the original load power factor from $\cos \theta'_{or}$ to $\cos \theta$ is shown graphically by the familiar condenser diagram, Fig. 1. The reactive kilovolt-amperes in leading quadrature to be carried by the condenser are given by the vertical line $D - F$. For static condensers and for synchronous condensers without mechanical load, the losses are comparatively small so that the quadrature difference is usually taken as the approximate condenser capacity required. This quadrature difference, or reactive

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8. For Kelvin law, "loss factor" and "equivalent hours," see reference 5; reference 6, Chaps, V. VI; also *Standard Handbook for Electrical Engineers*, Sec. 12, Par. 235-238; Section 13, Fig. 14.

8. loc. cit.

kilovolt-amperes of correction, may be expressed algebraically as

$$P \tan \theta_{or} - (P + p) \tan \theta$$

and the annual cost of condenser capacity as

$$(K_1 + K_2)\, P \tan \theta_{or} - (K_1 + K_2)\,(P + p) \tan \theta \qquad (2)$$

P is the maximum kilowatt load and is assumed constant. The energy loss, p, in static condensers is small, about 0.5 per cent, and is often neglected in calculating the power polygon. For synchronous condensers, losses will be about 1.7 to 3.5 per cent of rating for the larger machines and higher for the smaller units. If p is expressed as a function of θ, the equation becomes unnecessarily complicated. It is simpler, and sufficiently accurate within a limited range of the probable value of θ, to assume a fixed value[9] for p. The first approximation may be corrected later if necessary. θ_{or} is a constant, and θ the only variable.

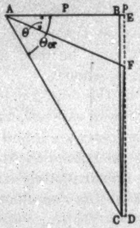

FIG. 1

P	Kilowatt load
p	Kilowatt loss in condenser
$A\ C$	Kv-a. at original load power factor
$A\ F$	Kv-a. at corrected power factor
$D\ F$	Reactive kv-a. correction

For 3 per cent condenser loss, the error in using $D\ F$ for $C\ F$ is less than 0.05 of one per cent

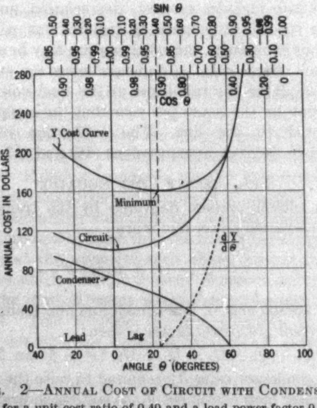

FIG. 2—ANNUAL COST OF CIRCUIT WITH CONDENSER

Drawn for a unit cost ratio of 0.40 and a load power factor 0.50. The minimum point on the Y curve occurs at $\sin \theta_{ec} = 0.40$. Corresponding to a corrected power factor $\cos \theta_{ec} = 0.916$

The annual cost of the circuit, including condenser, for any power factor, $\cos \theta$, is

$$Y = (Q_1 + Q_2 + Q_3 + Q_4 + Q_5)\,(P + p) \sec \theta$$
$$+ (K_1 + K_2)\, P \tan \theta_{or} - (K_1 + K_2)\,(P + p) \tan \theta \quad (3)$$

The minimum point on the cost curve is found by equating to zero the first derivative of Y with respect to θ, and solving

$$\frac{d\,Y}{d\,\theta} = 0 = (Q_1 + Q_2 + Q_3 + Q_4 + Q_5)(P + p) \tan \theta \sec \theta$$
$$- (K_1 + K_2)\,(P + p) \sec^2 \theta \qquad (4)$$

and

$$\sin \theta_{ec} = \frac{K_1 + K_2}{Q_1 + Q_2 + Q_3 + Q_4 + Q_5} \qquad (5)$$

The graphical representation of these cost functions in Fig. 2 indicates the variation in total cost as the condenser capacity is increased. The conditions for a minimum[10] are that the sum of the K terms be less than the sum of the Q terms; that the sum of the K terms be positive or equal to zero, and that the sum of the Q terms be positive and greater than zero.

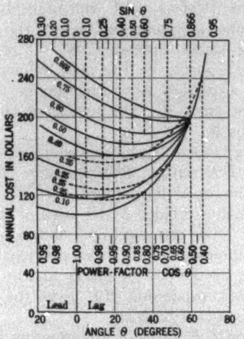

FIG. 3—COMPARISON OF Y-COST CURVES

The number on each curve is the unit cost ratio

If the sum of the K terms be zero, that is, if condenser capacity could be obtained at no extra expense, $\cos \theta_{ec} = 1.00$ and correction to unity power factor would be justified.

If the ratio of the K terms to the Q terms be equal to the sine of the original power-factor angle, then $\sin \theta_{ec} = \sin \theta_{or}$, and the original power factor is the most economical one. This is illustrated graphically by the upper curve in Fig. 3 marked 0.866. If the unit cost ratio is greater than $\sin \theta_{or}$, the mathematical minimum lies in the area of negative values of the condenser cost curve. The physical interpretation of the curves for values of the unit cost ratio greater than sin θ_{or} is that condenser correction is not economical. The lagging power-factor angle is taken as positive.

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[9] For percentage error in assuming p constant see appendix B.

[10] For test for minimum point see appendix B.

The conditions under which equation (**5**) may be used in practical problems are that the sum of the Q terms be positive and greater than zero, that the sum of the K terms be positive or equal to zero, and that the ratio of the K terms to the Q terms lie between zero and sin θ_{or}. Within these limits, the following conclusions are warranted.

A. The sine of the most economical corrected power-factor angle is determined by the ratio of unit annual condenser costs per reactive kilovolt-ampere of correction, to unit annual cost per kilovolt-ampere delivered, of that portion of the supply circuit directly benefited by the improvement.

B. Correction to unity is economically justified only if corrective capacity can be obtained at no additional expense.

C. If the unit cost ratio is found to be equal to, or greater than, the sine of the original load power-factor angle, investment in corrective equipment is not economically justified.

D. For a given unit cost ratio, the most economical power factor is the same, irrespective of the power factor of the load.

Conclusion D is illustrated by the dotted curves in Fig. 3 where each of the four total cost curves drawn at a unit cost ratio of 0.25, for load power factors 0.80, 0.65, 0.50, and 0.40, has its minimum value at sin $\theta_{ec} = 0.25$ corresponding to the corrected power factor, cos $\theta_{ec} = 0.986$.

Conclusion D is based on a constant unit cost ratio within the range of values considered. This condition is met in the case of a distribution substation with supply lines, built to a standard maximum current rating. Local power load is served, up to the limit of circuit capacity, by radiating branch feeders either at the same or at reduced voltage. The circuit cost per kilovolt-ampere of its capacity is known, and the average unit price of the static condensers, to be located at the load ends of the radiating branches, is fixed by the manufacturer's quotation. The unit cost ratio is therefore practically constant and a common most economical power factor for the circuit as a whole is determined by equation (**5**). Sufficient condenser capacity is installed at the end of each branch, whatever its original power factor, to raise the corrected power factor to the most economical value. This result, based on economic considerations, is in agreement with the best operating characteristics of the circuit. If all the condenser capacity be placed at the ends of the branches, but each corrected to a different power factor, circulating currents will be set up between the branches which tend to offset the desired savings.

GENERAL APPLICATION

In the design of a circuit to serve a definite kilowatt load, the unit cost ratio tends to decrease as the size of the condenser is increased, and the required line and transformer capacity becomes less. This is due to the variation in cost per kilovolt-ampere of condensers and transformers with a change in size. The correct relationship is given by the unit cost ratio at the minimum point on the total cost curve. In the usual case, standard sizes of equipment will not agree exactly with the most economical sizes of conductors, transformers, and condensers as calculated by the formula. The flatness of the cost curve near the minimum point, however, indicates that this value is not critical. With static condensers, it is quite practicable to install just the capacity required, since these are built up in small units.

The formula has been derived for the particular case of an overhead feeder. Similarly, it may be developed for distribution circuits in general, both overhead or underground, by introducing appropriate Q terms. The criterion for including a given cost is whether or not a saving can be effected in that particular item by improving power factor. The cost of the supporting structure should be included in the case of parallel circuits, when a reduction in the number of paralleled conductors can be effected after correction, due to the reduced ampere capacity required to supply the same kilowatt load. This applies also to the cost of ducts in an underground system, when fewer cables will be required after correction. The cost of stringing wire or running cable should be included if the labor charge for the larger size is materially higher than for the smaller. The unit annual costs per kilovolt-ampere of circuit capacity applying to cables, switches, bus structure, and substation electrical equipment in general should be evaluated as Q terms if an actual saving can be made. In including cost at the sending end, such as step-up transformers, the cost per kilovolt-ampere should be increased by a factor covering per cent line loss between that point and the condensers.

The sine relationship between unit condenser and circuit costs may be used also in making power factor improvement calculations in industrial plants. The method of computing optimum rating of static condensers within the consumer's plant is the same as for outside feeders. Where over-excited synchronous[11] motors or internal corrective motors of the synchronous-induction[12] type[13] are used, the power, p, consumed by the motor is included as a part of the useful load, P. The cost[3] properly chargeable to power factor improvement is the difference between the price of the synchronous motor and that of a lower priced induction motor of equivalent rating. The correction in reactive kilovolt-ampere is the sum of the power component in leading quadrature of the synchronous motor and the lagging component of the equivalent induction motor. The cost per reactive kilovolt-ampere of correction, term K_1 in the equation, is thus determined. Further correction may be justified when the rate schedule contains a special power factor clause.

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[11] *Electric Journal*, March 1926, p. 99.

[12] H. Weichsel, *A New Alternating-Current General-Purpose Motor*, presented at the Midwinter Convention of the A. I. E. E. New York, February, 1925.

[13] Underhill, "Power Factor Wastes," McGraw-Hill. JOURNAL of the A. I. E. E., October 1926, p. 949.

In making a basic cost study of the power factor problem on a utility system preparatory to working out new power rate schedules, the use of the sine formula is suggested as a means for determining the optimum consumer power factor upon which the schedule of penalties or bonus is based. In this case the cost of correction at the motor is credited with savings on the central station system back to the generator, in addition to savings within the industrial plant. Direct comparison may be made between the over-all savings obtained by consumer correction and those resulting from the use of condensers on the lines of the central station.

GENERATING STATION COSTS

Excess generating station costs chargeable to low lagging power factor include (a) fixed charges on generating capacity which could be released for profitable kilowatt load, (b) excess electrical losses due to increased excitation requirements and the reactive current component in station bus and transformers, and (c) lower steam economy due to the necessity for operating turbines below normal power rating or for floating spare units on the line as condensers. Figures for determining excitation losses are usually obtainable from the manufacturer's design data. Other losses must be worked out for each individual station from daily operating data and special tests.

The evaluation of station costs is not within the scope of the present paper. The following method is outlined for segregating that portion of the station costs directly affecting the distribution circuits.

Case I: Generating capacity insufficient to supply kilowatt demand at existing power factor, necessitating the purchase of a new generator or additional power from an outside source. It may be assumed that it would be cheaper to release existing generator capacity up to the limit of the horse power rating of the turbines by installing a large synchronous condenser at the station. The larger units have the lowest cost per kilovolt-ampere. Let the annual fixed charges, including routine maintenance, be K_3 and the cost of losses K_4 dollars per reactive kilovolt-ampere per year.

The problem is to determine how much of this corrective capacity could be installed more profitably at the load end of the distribution circuit. The annual unit cost of the load end condenser has been evaluated as $(K_1 + K_2)$ dollars per reactive kilovolt-ampere per year and is greater than $(K_3 + K_4)$. Due to load diversity between feeders, additional capacity is required at the load end than at the station. Let D = the ratio of coincident reactive kilovolt-ampere demand at the station peak to the sum of the reactive demands of the individual feeders. One reactive kilovolt-ampere of condenser correction out on the line saves $D\,(K_3 + K_4)$ dollars per year in station condenser costs, at the same time releasing generating capacity for profitable kilowatt load. The difference in the annual unit costs of the two condensers, representing the additional cost of load end correction, applies to the distribution circuit. Equation (**5**) in its general form is then:

$$\sin \theta_{ec} = \frac{(K_1 + K_2) - D\,(K_3 + K_4)}{Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + \ldots} \tag{5a}$$

the additional Q terms representing cost of cables, substations, et cetera, as previously noted.

Case II: Spare generator capacity available, station economy reduced by inefficient operation at low power factor. It is assumed that the expense of a station condenser is not justified. Term K_3 is not included since no immediate saving in annual fixed charges on generating equipment will be made. The excess cost of operating the station at low power factor, compared with the cost of generating the same kilowatt-hours in salable energy at unity power factor, represents the cost of using station generators as condensers. The excess cost throughout the year divided by the reactive power component at the station peak demand gives K_4, the annual cost per reactive kilovolt-ampere of correction at the station. The numerator of equation (**5a**) is then $(K_1 + K_2) - D\,K_4$.

The method does not apply to distant hydroelectric or steam stations operating at leading power factor. The sine formula does not apply to transmission lines[14] of such length and voltage that charging current, corona loss, conductor material, tower spacing, and other special factors enter into the problem. The correct rating of condensers for long lines is usually determined by the requirements for voltage control and system stability.

The sine formula may be used in the design of the individual distribution circuits of the network supplied by a transmission line, as in Case I for the generators. Thus, if it is found economical to correct the power factor at the load ends of the individual circuits of the distribution network from 0.70 to 0.90, the size of the synchronous condenser required to control transmission line voltage is considerably reduced.

The annual saving at the most economical power factor depends upon the power factor of the load and the unit cost ratio. In Figs. 2 and 3, the difference in the height of the ordinate to the total cost curve at the original power factor, and the ordinate at the minimum point indicates the saving effected. For a load power factor of 0.50 and a unit cost ratio 0.4, the saving at the economical power factor, 0.916, is about 20 per cent, but would be only about 15 per cent if corrected to unity. For a unit cost ratio of 0.5, there would be a saving of 14 per cent if corrected to 0.866, but only 7 per cent at unity power factor. The percentage annual saving at the most economical corrected power factor has been plotted against unit cost ratio in Fig. 4. These curves show in a general way the conditions under which substantial savings may be expected.

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[14] Kirsten and Loew, *The Line of Maximum Economy*, presented at the Pacific Coast Convention of the A. I. E. E., September, 1925.

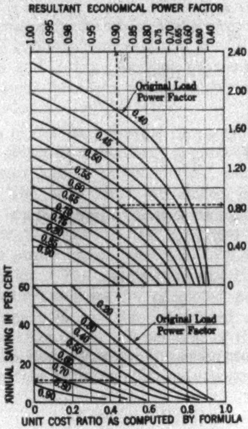

Fig. 4—Working Diagram for Use in Preliminary Calculations

Calculate unit cost ratio from K and Q terms. The dotted projection lines indicate use of chart. Final figures should be worked out accurately for the actual prices and ratings of equipment used.

The upper set of curves in Fig. 4 gives the required correction in reactive kilovolt-amperes for any power factor.

Conclusion

An annual cost equation has been developed for an overhead distribution circuit supplying a lagging power factor load partially corrected by a condenser. An economic balance in circuit design is obtained when the sine of the corrected power factor angle equals the ratio of the unit annual cost of correction to the unit annual cost of that portion of the supply circuit in which an actual saving is effected by power factor improvement. For this value of θ_{ec}, the total annual cost of the circuit is a minimum, and cos θ_{ec} is the most economical corrected power factor. Comparison is made on the basis of the most economical conductor sizes and transformer ratings equal to the resultant load. If the unit cost ratio is equal to, or greater than, the original load power-factor angle, correction is not economically justified.

Application of the formula may be extended to distribution circuits in general, both overhead and underground. A method has been outlined for working out the comparative advantages of power factor correction at the generating station and at substations, feeder load ends, and the motors. This requires a careful study of load diversity and of the effect of each type of condenser upon the shape of the load current curve and therefore upon $I^2 R$ losses.

The equations and methods here given have the definite advantage that they may be applied equally as well to the design of new circuits and extensions as to the selection of optimum condenser ratings for existing circuits.

Although the present discussion has been limited to the cost element of the power-factor problem, it should be understood throughout that applications to specific cases should always be made in accord with practical operating requirements, and with those more intangible attributes connoted by the term "electric service."

Appendix A

Supplementary List of Symbols and Working Formula. In addition to the symbols already defined, the following list is given for use in evaluating the K and Q terms. It represents circuit design data ordinarily available to the distribution engineer.

C_{cu} Cost of conductor in dollars per lb. of actual copper, actual price of bare wire. For insulated wire, multiply charging out price by ratio of weight per 1000 ft. insulated wire to weight 1000 ft. bare wire of same size A. W. G.

C_e Cost in dollars per kilowatt-hour of generating and distributing energy to supply losses.

C_{con} Cost of condenser equipment in dollars per kilovolt-ampere.

C_T Cost of nearest standard size of transformer to carry the resultant load, in dollars per kilovolt-ampere.

g Rate of annual fixed charges; includes interest, depreciation, insurance, and taxes.

U_{TFe} Decimal ratio of watts iron loss in transformer to the volt-ampere rating.

U_{TCu} Decimal ratio of watts copper loss in transformer at full load to the volt-ampere rating.

U_{con} Decimal ratio of kilowatts required to operate the condenser at full load to the kilovolt-ampere rating.

l Length of line in feet, one way.

N Number of conductors with respect to relative size.

N' Number of conductors carrying full current.
For 3-phase, 3-wire . . . $N = 3, N' = 3$

w Weight of conductor in lb. per 1000 cir. mil ft.
For copper, $w = 0.00303$ lb.

ρ Resistivity of conductor per cir. mil ft.

H 8760 hr. in a year.

f Load factor.

$f H$ Hours use of demand, yearly.

F Loss factor[8], computed from typical daily load curve

$$F = \frac{\text{sum of squares of hourly ordinates}}{24 \times (\text{peak load})^2}$$

$F\,H$ Equivalent hours[8] per year at constant full load to produce the same kilowatt-hours in $I^2\,R$ line losses as occur with the actual fluctuating load. Analogous to "hours use of demand."

h Hours per year that condenser is connected to the line.

E Receiver voltage, phase voltage at full load.

I_{ec} Amperes per wire at full load at the resultant economical power factor, for a 3-phase line,

$$I_{ec} = \frac{1000\,(P + p)}{\sqrt{3}\,E \cos \theta_{ec}}$$

$$\frac{l\,w\,I_{ec}\,M}{1000 \text{ ft.}} = \text{Weight of one conductor.}$$

$$\frac{\rho\,l}{I_{ec}\,M} = \text{Resistance of one conductor.}$$

The K and Q terms are set up for a 3-phase, delta-connected line. Similar expressions may be worked out for 3-phase, 4-wire and 2-phase, 3- or 4-wire circuits. The economical circular mils for one ampere are calculated from the Kelvin law.[8]

$$M = 5500 \sqrt{\frac{F\,C_e}{g\,C_{cu}}} \tag{6}$$

The area in circular mils of the economical size of conductor is $I_{ec}\,M$. If the line drop for this size proves too great, calculate M from the allowable regulation.

$$\text{The cost of one conductor} = \frac{C_{cu}\,l\,w\,M\,I_{ec}}{1000} \tag{7}$$

The annual fixed charges on N conductors, substituting for I_{ec},

$$= \frac{g\,C_{cu}\,N\,l\,w\,M}{1000} \times \frac{1000\,(P + p)}{\sqrt{3}\,E \cos \theta_{ec}}$$

$$= \frac{g\,C_{cu}\,N\,l\,w\,M\,(P + p) \sec \theta_{ec}}{\sqrt{3}\,E} \tag{8}$$

and

$$Q_1 = \frac{g\,C_{cu}\,N\,l\,w\,M}{\sqrt{3}\,E} = \frac{0.00525\,g\,C_{cu}\,l\,M}{E} \tag{9}$$

$$Q_1 \text{ per mile} = \frac{27.72\,g\,C_{cu}\,l\,M}{E} \text{ dollars per mile per year} \tag{9a}$$

The $I^2\,R$ losses in kilowatt at full load, for one conductor

$$= I_{ec}^2 \frac{\rho\,l}{I_{ec}\,M} \times \frac{1}{1000}.$$

The cost of losses in N' conductors at C_e dollars per kilowatt-hour

$$= \frac{C_e\,\rho\,l\,N'}{1000\,M} \times I_{ec} \tag{10}$$

dollars per hour at constant full load. When the load fluctuates, the $I^2\,R$ losses vary as the square of the current. The annual cost of loss for a fluctuating load may be approximated by multiplying loss at full load by the "equivalent hours," $F\,H$, defined above. The annual cost of line losses

$$= \frac{C_e\,\rho\,l\,N'\,F\,H}{1000\,M} \times I_{ec} = \frac{C_e\,\rho\,l\,N'\,F\,H}{1000\,M} \times \frac{1000\,(P + p)}{\sqrt{3}\,E \cos \theta_{ec}}$$

$$= \frac{C_e\,\rho\,l\,N'\,F\,H}{\sqrt{3}\,E\,M}\,(P + p) \sec \theta_{ec} \tag{11}$$

and

$$Q_2 = \frac{C_e\,\rho\,l\,N'\,F\,H}{\sqrt{3}\,E\,M} = \frac{160826\,C_e\,l\,F}{E\,M} \tag{12}$$

$$Q_2 \text{ per mile} = \frac{849.16\,C_e\,F\,10^6}{E\,M} \text{ dollars per mile per year} \tag{12a}$$

Transformer prices and losses are those of the nearest standard capacity actually required at the resultant power factor

$$g\,C_T\,(P + p) \sec \theta_{ec} \tag{13}$$

and

$$Q_3 = g\,C_T$$

Transformer iron losses may be assumed as practically constant through the year. Transformer iron loss

$$= U_{TFe}\,C_e\,H\,(P + p) \sec \theta_{ec} \tag{14}$$

$$Q_4 = 8760\,U_{TFe}\,C_e$$

Transformer copper losses vary as the square of the current. Annual losses in kilowatt-hours may be approximated by multiplying loss at full load by the "equivalent hours," $F\,H$. The annual cost of copper losses in the transformer

$$= U_{TCu}\,C_e\,F\,H\,(P + p) \sec \theta_{ec} \tag{15}$$

and

$$Q_5 = 8760\,U_{TCu}\,C_e\,F$$

Annual condenser costs are divided into fixed charges and cost of losses. The unit fixed charges are

$$K_1 = g\,C_{con} \tag{16}$$

For synchronous condensers, an estimated percentage should be added to the annual fixed charges, g, to cover the cost of routine maintenance.

The yearly losses may be approximated by multiplying loss at full load excitation by the hours use $= U_{con}\,h$ per kv-a. The unit annual cost of losses will be

$$K_2 = U_{con}\,C_e\,h \tag{17}$$

Appendix B

Test for Minimum. If a root of the first derivative of Y with respect to θ yields a positive result when substituted in the equation of the second derivative, a minimum is indicated at that point.

Let K = sum of K terms and Q = sum of Q terms. Taking the second derivative of (3) and substituting the value of θ from (**5**)

$$\frac{d^2 Y}{d\,\theta^2} = \frac{Q^2}{\sqrt{Q^2 - K^2}}$$

The expression is real and positive when Q is positive and > 0, K is positive or $= 0$, and $Q > K$. Negative values of K and Q terms have no physical significance in the present problem. Lagging power factor is taken as positive.

Accuracy of Results. For the purpose of calculating the power polygon in equations (**3**) and (**4**), the value of p, kilowatt loss in the condenser, was assumed a constant. The error thus introduced is practically negligible. In a typical numerical example, for a load of 1000 kw., the loss in the condenser required to correct the power factor from 0.50 to 0.85 was 3 per cent, or 33 kw. The assumption that the condenser loss would be the same for corrected power factors ranging from 0.75 to 0.95 introduced a maximum error of less than 0.9 of one per cent of the total kilovolt-amperes. Since plus errors in calculating condenser kilovolt-amperes offset minus errors in required transformer kilovolt-amperes, even these small errors tend to cancel out in evaluating the total cost, Y.

In equations (**1**), (**2**), and (**3**) and in Figs. 2 and 3, a constant unit cost per kilovolt-ampere is assumed for transformers and condensers. In the usual case, however, the unit price of the larger sizes tends to decrease with increase in rating. If necessary, this condition could be expressed mathematically by plotting cost per kilovolt-ampere against kilovolt-ampere rating and writing the equation of a line passing through the required points. Introducing these expressions into equation (**3**) would tend to complicate the problem unnecessarily. The location of the minimum point of the Y cost curve is all that is required, and this can be determined accurately from the simple formula in one or two trials. No error is involved when the conditions of formula (**5**) are satisfied.

In the application of the formula to practical problems, it may be desirable to design the circuit for a corrected power factor slightly above or below the calculated value in order to make the most advantageous use of standard sizes of wire, cable, transformers, and condensers. In the design of new circuits, probable growth in load may dictate the use of larger wires and transformers than are immediately required. Where static condensers are used, provision may be made for adding additional units as required, thus maintaining economical operation during the period of growth and increasing the ultimate kilowatt capacity of the circuit.

Instability in Transformer Banks

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Associate, A. I. E. E.

***Synopsis.**—This paper considers the instability which sometimes occurs in banks of transformers supplying a capacity load when certain harmonics in the primary current are suppressed, either by the type of transformer connections or by a resonant circuit in series with the primary of the transformer, and the similarity between the several unstable circuits is pointed out.*

Curves showing the triple-frequency voltage distortion as a function of the capacity load have been included for two of the unstable circuits. For one case, oscillograms taken during the instability are shown.

An explanation, substantiated by actual analyses, has been brought forward for the simplest unstable circuit, consisting of three branches connected in Y across a three-phase line with balanced, sinusoidal line voltages, with the neutral unconnected, each branch of the Y consisting of an iron-cored reactance in parallel with a capacity. This explanation is extended to the other cases, two with experimental evidence as justification, and the third by analogy only.

INTRODUCTION

IN 1915 Mr. L. N. Robinson published a paper[2] concerning the unstable condition which sometimes occurs when a Y-connected capacity load is supplied by a Y–Y-connected transformer bank with the secondary neutral closed but the primary neutral open. The phenomenon was evidently closely associated with the voltage distortion which occurs with this and similar transformer connections, and which has been studied by many investigators[3]. No very satisfactory explanation of this unstable condition, however, has ever been advanced so far as the author knows, and the principal object of this paper is to advance an explanation substantiated by considerable analytical proof and experimental observation.

Mr. Robinson suggested that the instability might be due to a "reversing transformer leg," but this theory has been disproved by oscillograms which show that the line voltages and currents are balanced during some of the unstable conditions, and by hysteresis loops observed during instability, which were found to remain symmetrical with respect to the two axes.

Mr. R. P. Shaw investigated this phenomenon[4] and took oscillograms and very complete data throughout both the stable and the unstable ranges of line voltage and capacity load. He also investigated a quite similar case of voltage distortion and instability which occurs when a capacity is inserted in the delta of a Y-delta-connected bank of transformers, primary neutral open. Two of his curves showing the third harmonic induced voltage as a function of the capacity load have been included in this paper.

EXPERIMENTAL WORK

Three identical transformers, each of 1½-kv-a. rating at 100 volts, 60 cycles, the ratio of transformation being unity, were connected Y–Y to supply a Y-connected, balanced capacity load of 12.5 μf per phase. The secondary neutral was closed while the primary neutral was open. The transformers were supplied by a 5-kv-a., 60-cycle alternator of very good wave form for balanced loads, direct-connected to an 8-hp., d-c. motor. Under these conditions, the transformers, emitted "grunts" or "beats" which sounded like a solid body, such as a wooden mallet, striking the laminations, and all the meters with the exception of the line voltmeter oscillated badly. The line voltage was about 200 volts, as indicated by a dynamometer type meter, and oscillated but slightly, as the change in load over a beat had but little effect upon the terminal voltage of the alternator.

This instability persisted with a wide range of capacity loads. Mr. Shaw, who worked with the same apparatus, records instability with a capacity load of as high as 40 μf. per phase. With any given line voltage, stability would occur with either a very high or a very low value of capacity, the maximum values of capacity which would produce instability being increased as the voltage was increased.

The instability was not due to the alternator supplying the transformers, as was proved by Mr. Shaw by connecting them to a large alternator, and also by taking oscillograms of the alternator field current, which was found to be perfectly regular. Mr. Robinson had also noted this instability under conditions differing widely enough to indicate that it was not due to the power source.

Fig. 1, taken from Shaw's work, shows the variation of third-harmonic induced voltage with the size of the capacity load, throughout the lower, stable range of capacity and part of the unstable range, where measurements were possible. These voltage readings were taken with a thermocouple heater element, in series with a high resistance, inserted in one corner of a delta formed with an auxiliary winding on each transformer, as shown in Fig. 2. Multiples of the third harmonic appeared in this voltage also, but they were small, as indicated by analyses of the wave form.

Mr. Shaw also experimented with a quite similar unstable condition produced by connecting the pri-

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[1] Research Division, Dept. of Elect. Engg., Mass. Inst. of Tech., Cambridge, Mass.

[2] TRANS. A. I. E. E., 1915, p. 2183.

[3] L. F. Blume, TRANS. A. I. E. E., 1914, p. 735. G. Faccioli, JOUR. A. I. E. E., May, 1922, p. 351. O. G. C. Dahl, TRANS. A. I. E. E., 1925, p. 792.

[4] M. I. T. thesis, 1924, Investigation of the Triple Frequency Distortion in Three-Phase Transformer Banks.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

maries of the transformers as before, in Y with the neutral unconnected, and the secondaries in delta, with a capacity inserted in one corner of this delta as shown in Fig. 3. The third-harmonic induced voltage was measured across one corner of an auxiliary delta as before. As was to be expected, the capacity necessary to produce instability in this latter case was about one-third that in the Y—Y connection, as the third-harmonic voltages induced in the secondary windings add up directly to make the voltage across the capacity three times the third-harmonic phase voltage. Fig. 4, due to Mr. Shaw, shows the third-harmonic phase voltage as a function of the capacity inserted in the delta, and is similar to Fig. 1 except that the capacity has been reduced to about one-third its previous value. Mr. Shaw records that the instability commenced in the Y-delta-delta case with a capacity of from 2 to 4 $\mu f.$, which corresponds to from 6 to 12 $\mu f.$ per phase in the Y—Y—delta connection. That the Y—Y connection and the Y-delta connections are essentially the same so far as the third harmonic and its multiples are concerned is evident from a comparison of Figs. 5 and 6, which represent the same circuit except that in Fig. 6 the leakage reactances of the transformer windings have been neglected.

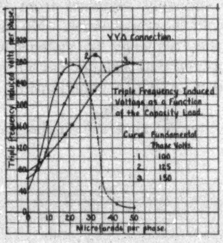

FIG. 1

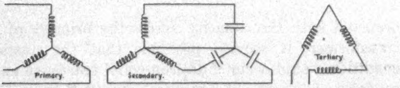

FIG. 2

As the removal of the primary neutral connection produced instability in an otherwise stable circuit, it seemed evident that the instability must be due to the third harmonic and its multiples, introduced by the varying permeability of the iron cores of the transformers. The existence of the Y-delta instability confirms this hypothesis.

As it was well established that the unstable condition which sometimes occurred in some star-connected transformer banks was due to the suppression of the third harmonic and its multiples in the primary current, it was conceived that by means of a series filter, the partial suppression of the harmonics in the primary current of a single-phase transformer supplying a capacity load might produce an unstable condition.

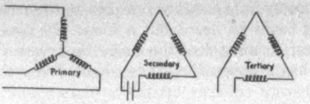

FIG. 3

Accordingly a circuit was set up as shown in Fig. 7, the transformer being one of those used in the three-phase case. The filter circuit consisted of a capacity in series with two identical air-core inductances which were mounted so that the mutual inductance between them could be varied. This filter circuit was adjusted for resonance at 60 cycles, the combined impedance being 12 ohms, while the capacity and the inductance each had an impedance of 203 ohms at 60 cycles.

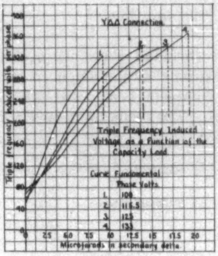

FIG. 4

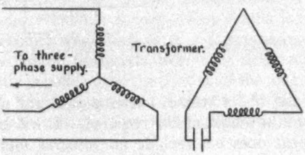

FIG. 5

With this arrangement, with 110 volts at 60 cycles impressed upon the circuit and with a capacity of 13.5 $\mu f.$ placed across the secondary of the transformer, the circuit was found to be distinctly unstable, even worse than the three-phase case, although the two instabilities were quite similar. This single-phase instability occurred over a considerable range of capacity either side of 13.5 $\mu f.$, but the beats seemed most violent at about this value of capacity. A great enough change of capacity in either direction would produce stability, the voltages and currents at the stable condition with high values of capacity being low, as in the three-phase case. By changing conditions, such as the voltage, frequency, or capacity load, the beats could be varied continuously from less than one per sec. to so many that nothing could be heard but a hum. In general, the beats were more violent the longer they were.

The beats were adjusted to about five per sec., so that each one included about 12 cycles, and simultaneous oscillograms long enough to show the wave form over a complete beat were taken. These are shown in Fig. 8. The oscillations were not very violent in this case, but a beat of longer period was not used because of the length of film required. It will be noted that a beat does not include an integral number of cycles.

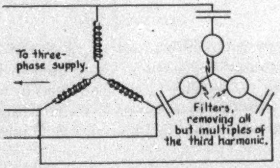

FIG. 6

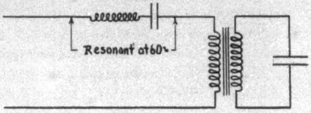

FIG. 7

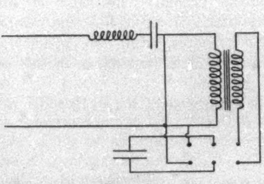

FIG. 7-A

By means of the two-dimensional oscillograph[5], an examination of the hysteresis loops of the transformer during instability was made. The hysteresis loops could be thrown upon a ground glass screen and viewed very well indeed. An attempt was made to photograph the series of loops during the unstable condition, but with no success, as the point of light traversed a given path but once, and was too faint to record this path. The loops thrown upon the screen brought out one fact—that the loops increase and decrease in size, but they always remain symmetrical with respect to the two axes.

It seemed possible that the unstable condition might not depend upon leakage reactance of the transformer windings, so a switch was arranged to shift the capacity load from the secondary to the primary, as shown in Fig. 7A. The unstable condition was established, and the capacity suddenly switched from the secondary to the primary of the transformer. No change could be detected in the frequency or violence of the beats. Thus it was proved that leakage reactance of the transformer windings is not a contributing cause of the instability.

In view of the fact that the single-phase instability occurred with the capacity across the primary of the transformer, it seemed probable that the unstable condition would occur if an iron-cored reactance and a capacity of the proper size were connected in parallel, and three such branches were connected in Y across a three-phase line of the proper voltage, the neutral being unconnected. Accordingly, the three transformers used before were connected in Y, the primary neutral open, and with a capacity of about 12.5 $\mu f.$ in parallel with each primary, as shown in Fig. 9. This combination was connected across a three-phase supply with a line voltage of 200 volts, at 60 cycles, and distinct beats occurred in each transformer, just as in the case with the capacity load across the secondary of the transformer.

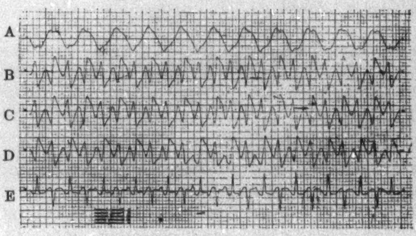

FIG. 8

A. Primary current
B. Primary voltage
C. Induced voltage
D. Secondary voltage
E. Secondary current.

EXPLANATION OF INSTABILITY

Of the circuits found to be unstable, the one shown in Fig. 9 is most easily analyzed, as nothing but odd multiples of the third harmonic can appear in the phase voltage under stable conditions with sinusoidal, balanced line voltages, and the third harmonic and its multiples in the exciting current must be equal in magnitude and opposite in phase to the respective harmonic currents through the capacity. All multiples of the third harmonic above the third itself seemed, from actual analyses, to be comparatively small and hence were neglected. The effect of the third harmonic in the phase voltage upon the third-harmonic exciting current was determined as follows.

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[5] E. L. Bowles, Discussion, TRANS. A. I. E. E. 1923, p. 346.

A fundamental flux density of 10.2 kilogausses was assumed, corresponding to a fundamental (60-cycle) phase voltage of 100 volts. Various amounts of third harmonic were introduced "in phase" with the fundamental; that is, so that the resultant flux wave was of the form, $A \sin w\,t + B \sin 3\,w\,t$, and the resultant flux wave was drawn. This was done with the third-harmonic flux from 12 per cent to 60 per cent of the fundamental. From Mr. P. A. Blackwell's series of hysteresis loops[6] for one of the transformers, the magnetization curve was drawn as shown in Fig. 11. From this curve, the exciting current corresponding to each resultant flux wave was determined, and each of these was analyzed for the third-harmonic current.

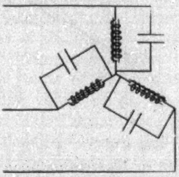

FIG. 9

When the third-harmonic flux was "in phase" with the fundamental flux, the resultant flux wave was, of course, always symmetrical with respect to the 90-deg. (fundamental scale) ordinate. As there is only one value of current corresponding to each value of flux density, using the magnetization curve, the exciting current was symmetrical, also, with respect to the 90-deg. ordinate. Thus no cosine component of the third harmonic could appear in the exciting current; in other words the third-harmonic current is in phase with the third-harmonic flux. From Fig. 10, it will be seen that this is a condition which must be fulfilled if the third-harmonic exciting current, I_3, is to be in phase opposition to the current through the capacity. In Fig. 10, ϕ_3 is the third-harmonic flux which produces the induced voltage E_3, while V_3 is the third-harmonic impressed voltage which causes the current I_{3c} to flow through the capacity. The resistance and leakage reactance of the transformer winding are neglected.

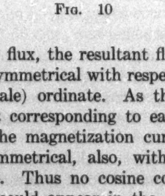

FIG. 10

Thus it will be seen that the phase relation of the third-harmonic exciting current remains correct if the magnetization curve is used. Moreover, the third-harmonic exciting current increases more rapidly than the third-harmonic current through the capacity, after the maximum flux density becomes high, so that the two will become equal in magnitude. Hence the saturation curve cannot produce instability by making it impossible for the two third-harmonic currents to become equal in magnitude and opposite in phase.

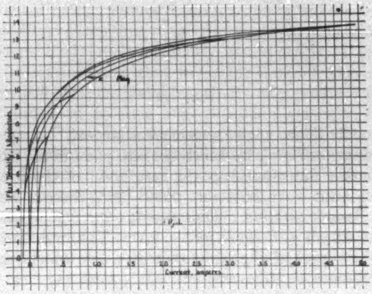

FIG. 11

As the use of the saturation curve would not indicate instability, the exciting current was determined from the hysteresis loops shown in Fig. 11 and taken by Mr. P. A. Blackwell for one of the transformers. For any desired maximum flux density, a loop similar in shape to the ones shown was interpolated It was assumed that each secondary hysteresis loop was thin enough to practically coincide with the portion of the main hysteresis loop where it started. Loops taken by Mr. W. M. Gilman[7] indicate that this assumption introduces but little error.

The third-harmonic flux density was varied, in steps of 12 per cent, from zero to 60 per cent of the fundamental flux density, as before, but each value of third harmonic flux density was introduced at 20-deg. (fundamental scale) intervals. The resulting exciting current waves were analyzed for the third harmonic by means of the Woodbury analyzer[8], which was quite satisfactory for waves with a cyclic length as long as was used, 12 inches. Fig. 12 shows a sample wave, the third-harmonic flux being 48 per cent of the fundamental, and introduced so that the resultant flux wave was of the form, $[A \sin w\,t + 0.48\,A \sin 3\,(\,w\,t + 20°)]$.

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[6] M. I. T. thesis, 1924, Unstable Effect in Three-Phase Transformer Bank with Capacity Load.

[7] M. I. T. thesis, 1925, Quantitative Analysis of Transformer Harmonics.

The magnitude of the third-harmonic exciting current, I_3, was plotted against the third-harmonic flux, ϕ_3, for given angular displacements between the fundamental and the third-harmonic fluxes, as shown in Fig. 13. This angle, which we shall call $\theta_{3\phi}$, is the fundamental angle by which the zero point of the third-harmonic flux wave lags the zero point of the fundamental flux wave, both zero points being those at which the slope is positive. Thus the flux wave is expressed as $A \sin w\,t + B \sin 3\,(\,w\,t + \theta_{3\phi})$. The intersections of the curves of I_3 against ϕ_3 with a line which represented I_3 against ϕ_3 for a capacity of 6.5 $\mu f.$ gave values of ϕ_3 and $\theta_{3\phi}$ at which the magnitude of the third-harmonic exciting current was correct for this value of capacity. These values of ϕ_3 and $\theta_{3\phi}$ were plotted as shown in Fig. 15, curve I.

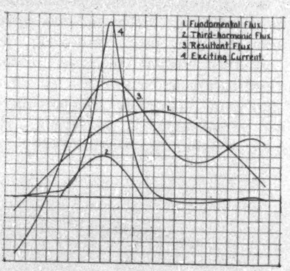

FIG. 12

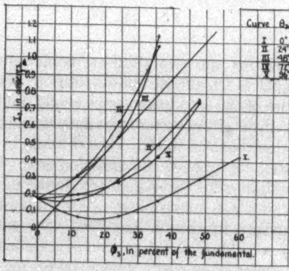

FIG. 13

The angle, θ_{3I}, by which the third-harmonic current lags the fundamental flux (that is, the fundamental angle by which zero point of the third-harmonic current lags the zero point of the fundamental flux, both zero points being those at which the slope is positive), for constant values of third-harmonic flux, was plotted against the lag of the third-harmonic flux behind the fundamental flux shown in Fig. 14. As this third-harmonic current must be in phase with the third-harmonic flux in order to be in phase opposition to the third-harmonic current through the capacity, the angles θ_{3I} and $\theta_{3\phi}$ must be the same so that the intersections of the curves of Fig. 14 with a straight line through the origin and with a slope of unity gave values of ϕ_3 and $\theta_{3\phi}$ at which the phase of the third-harmonic exciting current was correct. The locus of the point at which the phase was correct is shown in Fig. 15, Curve 2.

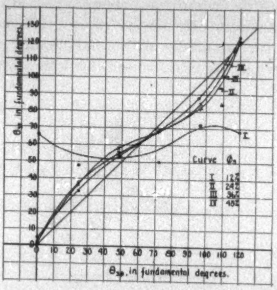

FIG. 14

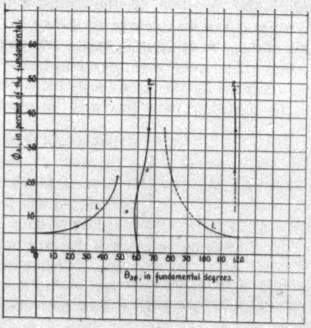

FIG. 15

It should be noted that the part of Curve 2 which lies at $\theta_{3\phi}$, equal to about 117 deg., is actually discontinuous downward, stopping at a value of ϕ_3 somewhere between 12 per cent and 24 per cent. The portion between these two points is shown dotted, as it is not known exactly where the curve stops. For each value of ϕ_3 below 12 per cent, there is only one value of $\theta_{3\phi}$ at which the phase of I_3 is correct. Also, it should be noted that the "phase correct" curve at $\theta_{3\phi}$ equal to about 60 deg. approaches 61 deg. as a limit, $\theta_{3\phi}$ at ϕ_3 equal to zero being indeterminate.

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[8] F. S. Dellenbaugh, Jr., A. I. E. E. JOUR., Jan. 1923. p. 58.

It will be seen from Fig. 13 that the magnitude of the third-harmonic exciting current is never correct for values of $\theta_{3\phi}$ between 48 deg. and 72 deg., as all the $I_3 - \phi_3$ curves for this range of $\theta_{3\phi}$ lie entirely above the straight line which is drawn for the 6.5-μf. condenser. As this straight line is tangent to the curve for $\theta_{3\phi}$ equal to 48 deg., the left-hand "magnitude correct" curve becomes vertical at this value of $\theta_{3\phi}$, and for increasing values of ϕ_3, $\theta_{3\phi}$ will decrease. Similarly, the right-hand "magnitude correct" curve becomes vertical at some value of $\theta_{3\phi}$ greater than 72 deg., and for increasing values of ϕ_3, $\theta_{3\phi}$ increases. The dotted line added to the "magnitude correct" curve is to show that it is definitely known that the curve never crosses a vertical line erected at $\theta_{3\phi}$ equal to 72 deg.

Thus, in Fig. 15, there is no intersection between the "magnitude correct" and the "phase correct" curves, which means that with a fundamental phase voltage of 100 volts and a capacity of 6.5 μf. per phase, the magnitude and phase of the third-harmonic exciting current never become correct at the same time, at least over the range of third-harmonic voltage considered; that is, up to about 150 per cent of the fundamental. This seems a reasonable cause of the instability, as under stable conditions the third-harmonic exciting current and the third-harmonic current through the capacity must be equal in magnitude and opposite in phase.

It would seem probable that with higher values of third-harmonic voltage, no intersection of the "magnitude correct" and "phase correct" curves would occur. In Fig. 13, it looks as though the straight line for 6.5-μf. capacity would not intersect the $I_3 - \phi_3$ curves for $\theta_{3\phi}$ near 120 deg., which would mean that no intersection would occur along the "phase correct" curve near $\theta_{3\phi}$ equal to 120 deg. Moreover, analyses of oscillograms and Mr. Shaw's measurements have not shown a third-harmonic voltage so great as 150 per cent of the fundamental for this value of capacity.

If, in Fig. 13, the straight line is given a greater slope —that is, the capacity is increased far enough—this line will intersect the $I_3 - \phi_3$ curves corresponding to $\theta_{3\phi}$ in the range about 60 deg., and hence an intersection of the "magnitude correct" and the "phase correct" curves would occur along the "phase correct" curve near $\theta_{3\phi}$ equal to 60 deg. Moreover, no matter how much more the capacity is increased, there will always be an intersection in Fig. 15, with $\theta_{3\phi}$ near 60 deg., and hence always a stable point.

If the capacity is decreased far enough, the $I_3 - \phi_3$ line for the capacity, in Fig. 13, will intersect the $I_3 - \phi_3$ curves for $\theta_{3\phi}$ near 117 deg., at values of ϕ_3 great enough to produce an intersection of the "phase correct" and "magnitude correct" curves near $\theta_{3\phi}$ equal to 120 deg. At very low values of capacity, the straight line, Fig. 13, becomes tangent to the curves for $\theta_{3\phi}$ equal to 120 deg., and of course for smaller capacities than this, the magnitude never becomes correct, according to the curves. For such a small capacity, however, higher multiples of the third harmonic may have appreciable effect upon the third-harmonic exciting current, or there may be instability which, due to the small current through the capacity, is unnoticeable.

According to the analysis described above, there appears to be a wide range of capacity at which the unstable condition occurs, although an insufficient number of $I_3 - \phi_3$ curves have been plotted to determine the limits of this range of capacity. Experimentally it was found that 8.5 μf. per phase would produce instability, the fundamental phase voltage being 100 volts at 60 cycles, while with four μf. per phase no instability could be detected. Lack of suitable capacity prevented more accurate determination of the range of instability. According to the curves of Figs. 13 and 15, it seems likely that instability would occur at four μf., which indicates that the lower limit of capacity which will produce instability is actually greater than that indicated by this analysis, due possibly to the stabilizing effect which is possible by the introduction of odd multiples of the third harmonic in the phase voltage.

As the circuit analyzed is equivalent to a Y–Y-connected transformer bank supplying a Y-connected capacity load, with the secondary neutral closed but with the primary neutral open, neglecting the leakage reactances of the transformer windings, and as experiment has shown that the instability is essentially unchanged by switching the capacity load from the primary to the secondary (taking due account of the ratio of transformation), the above explanation should apply to this latter type of circuit also. Moreover, as the circuit analyzed is equivalent, so far as the third harmonic and its multiples are concerned, to a Y-delta-connected bank of transformers with a capacity inserted in one corner of the delta, and with no primary neutral, again neglecting the leakage reactances of the transformer windings, the above explanation seems valid in this case also. Figs. 5 and 6 illustrate the similarity of the circuit analyzed and the above *y*-delta connection.

RESULTS AND CONCLUSIONS

1. An unstable condition, quite similar to the one which may occur in some star-connected transformer banks, may be produced with a single-phase transformer supplying a capacity load, by partial suppression of all higher harmonics in the primary current by means of a filter circuit. Stability will be produced if any of the conditions, as voltage, frequency, or size of the capacity load, are changed far enough in either direction.

2. Hysteresis loops traced upon a ground glass screen by the two-dimensional oscillograph during this unstable condition increased and decreased in size, but always remained symmetrical with respect to the two axes.

3. Both the single-phase and the three-phase instability existed with the capacity load connected across the primary. Thus leakage reactance cannot be a contributing cause of the instability.

4. Under the cause of the three-phase instability developed herein, the saturation curve cannot alone produce instability. Hysteresis, as well as non-linearity of the magnetization curve, is necessary.

5. There is considerable evidence that the three-phase instability (Y-connected branches of capacity and iron-cored inductance in parallel, with neutral unconnected) is due to the inability of the third harmonic and its multiples in the phase voltage to adjust themselves so as to make the third-harmonic exciting current and its multiples equal and opposite to the respective harmonics in the current through the capacity. As these conditions must be fulfilled during steady-state operation, this failure constitutes a reasonable cause of the instability. Hysteresis is necessary to produce instability, as mentioned above; non-linearity of the magnetization curve alone is not sufficient.

It has been shown, by drawing the exciting current wave from the hysteresis loops and the voltage waves, that when the iron-cored inductance and a certain capacity are connected in parallel, with three such branches connected in Y and with no neutral connection, across a three-phase supply with a certain value of balanced, sinusoidal line voltage, the third-harmonic phase voltage cannot adjust itself to make the third-harmonic current through the inductance equal and opposite to that through the capacity. This analysis neglects all multiples of the third harmonic above the third itself.

The lower limit of capacity which will produce instability, as indicated by this analysis, is somewhat less than the actual limit as indicated by experiment. This would seem to be due to the stabilizing effect produced by the introduction of higher odd multiples of the third harmonic in the phase voltage, these harmonics appearing only in order to produce stability.

6. The above explanation covers the instability which occurs when the capacity is connected across the secondaries of the transformers, Y—Y, with no primary neutral connection, as the two circuits are equivalent, neglecting leakage reactance of the transformer windings. It also covers the case of Y-delta-connected banks, with the primary neutral unconnected, when a capacity is inserted in the secondary delta, as this circuit is similar, in so far as the third harmonic is concerned, and neglecting leakage reactances, to the circuit for which the analysis was made.

7. It seems probable, although no proof has been advanced, that the single-phase instability is due to the inability of all the harmonics in the voltage impressed upon the inductance and capacity in parallel, to adjust themselves so as to make each harmonic in the exciting current equal and opposite to the corresponding harmonic current through the capacity, assuming that the filter circuit is perfect. The cause of this instability then becomes quite analogous to that of the three-phase case.

Discussion

V. M. Montsinger: I was very much interested in Mr. Gould's paper because it gives a possible explanation of the same kind of phenomena that I observed on some single-phase transformers a few years ago.

In 1914, I presented discussion for a series of papers on transformer connections, etc., and in this discussion, showed how condenser capacity, connected either across an opening in the delta of a Y-delta connected bank, or across the legs of the delta, intensified the harmonic voltages. Fig. 4, as given in Part I of the A. I. E. E. TRANSACTIONS, for 1914 p. 782, shows that at about 45 or 50 kilolines per sq. in. core density, the harmonic voltages suddenly increases to 200 per cent of the fundamental voltage. Upon further increasing the density, the harmonic voltage decreases and then increases again; in fact, as the core density increased, there were three points at which the harmonic voltages were intensified.

Fig. 6, shown on p. 783, gives results of similar tests made on a three-phase core-type transformer, and these curves demonstrate that no intensification of the harmonic voltages occurred at any point as the core density was increased—the maximum intensification being in the order of 15 to 20 per cent of the fundamental.

One point I wish particularly to emphasize is that while we have these dangerous harmonic voltages in single-phase transformers, we do not have them in three-phase core-type transformers. The reason for this obviously is due to the fact that in single-phase transformers, the third harmonic voltages in the three legs, which are in phase and flowing towards the neutral of the "Y" connection, have a return path through the iron core leg external to the windings; while in three-phase core-type transformers, the harmonic voltage flux must return through the air.

I should like very much to have Mr. Gould's comments on how this difference in phenomena between single-phase and three-phase core-type transformers lines up with the conclusions that the contributing cause of this intensification is leakage reactance.

K. E. Gould: I have done no work with the three-phase transformer, and the present paper considers only three single-phase transformers star-connected. The statement that the leakage reactance of the transformer windings is not a contributing factor in the instability is based on the fact that in changing the capacity from the secondary to the primary, there was practically no difference in the instability which seemed to me conclusive evidence that the leakage reactance is not a contributing cause.

This statement that leakage reactance does not cause instability, was made particularly in consideration of work that Mr. Shaw did at M. I. T. in 1924. He gave a very good explanation of the instability, showing that there were two conditions—two points at which the transformers could operate—due to the interaction of the capacity, and the leakage reactances of the transformer windings, particularly in the Y-delta case, where the capacity is inserted in one corner of the delta. However, as soon as it was discovered that the instability still existed with only the iron-core reactor in parallel with the capacity, it seemed evident that the instability was not due to what Mr. Shaw called a resonance effect between the capacity and the leakage reactance of the transformer windings.

An Instrument for Measuring Short-Circuit Torque

BY G. W. PENNEY[1]

Associate, A. I. E. E.

Synopsis.—*The torque produced by a short circuit or other transient will produce a corresponding acceleration of the rotor. If the rotor is not connected to a load the acceleration of the rotor will be directly proportional to the torque. A small instrument is described which can be attached to the end of the shaft of the machine to be tested. This instrument records the instantaneous acceleration of the rotor, the corresponding torque being calculated. The acceleration is measured by two separate methods. The first method gives points on an acceleration—time curve and the second gives a continuous record of the torque. The acceleration is recorded on the oscillogram so that by using a six element oscillograph a simultaneous record can be obtained showing both the acceleration and the short-circuit currents. The mechanism for closing the short circuit at the desired point of the voltage wave and the method of checking the accuracy of the instrument are also described. A record from an actual short-circuit test is shown. The results of the tests will be discussed in a later paper. The instrument can also be used for measuring sudden shocks on motors and other rotating machinery.*

INTRODUCTION

In order to design machines to withstand all possible operating conditions and yet not waste material, it is necessary to know the magnitude of the greatest forces which may act on the machine under the worst possible conditions. At the instant of a short-circuit surge, or when synchronized out of phase, enormous forces may act on a machine. The end windings are inherently rather weak mechanically and usually these are the first parts to be injured by a short circuit. These failures are caused by local magnetic forces, but in a few instances, the resultant torque of the machine has caused failure. One of the first large vertical shaft generators of low reactance sheared off the foundation bolts and turned through a considerable angle. In another case, a 6000-kv-a. frequency changer set had the frame supporting foot broken off and the holding down bolts stretched when the set was connected to the line out of phase.

These failures show the enormous forces produced by short circuits and other transients. In order to make machines sufficiently strong to withstand these abuses to which they are frequently subjected and yet not waste material, the maximum torque which may be developed under abnormal conditions must be known. Because of its transient nature, this torque is very difficult to calculate or measure. Methods thus far developed for calculating short-circuit torque are rather questionable because the assumptions which must be made do not accurately represent the actual conditions.

So far as the writer is aware, the only previous attempts to actually measure this torque have been by recording the oscillations of the rotor, using the torsiograph or similar instrument giving a space—time curve of the movement of the rotor or by measuring the voltage generated by a small generator connected to the shaft of the machine short circuited[2]. The determination of torque from a space—time record is very uncertain because the torque is proportional to the second derivative, *i.e.*, curvature of the space—time record. As is discussed later, this is very unsatisfactory for this purpose. The measurement of the voltage generated by a small direct-connected generator is much better but this method requires taking the slope of the record which is not entirely satisfactory. This paper outlines an analysis of the general problem of measuring torque produced by short circuits and other transient phenomena and describes the instrument which was developed. The instrument was designed to be attached to the end of the shaft of a machine and gives a record on the oscillogram of the instantaneous acceleration of the rotor which is proportional to the instantaneous torque provided the machine is not connected to an external load. If it is desired to measure the torque developed when the machine is short circuited while running under load, the test must be arranged so that the torque developed by the machine short circuited can be determined from the acceleration of its rotor and the known characteristics of the connected load or driving motor. The instrument was designed for the one purpose of measuring the acceleration of the rotor produced by a sudden pulsating torque. It was intended to give an accurate record of the first torque cycle and a fairly accurate record of succeeding torque cycles. It was designed for torque frequencies not exceeding 120 cycles and accelerations of 500 radians per sec. and above. It can be readjusted for measuring lower accelerations provided the frequency is reduced. In addition to studying short circuits, it may be useful

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[1] Power Engineering Dept., Westinghouse Elec. & Mfg. Co.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

[2] The torque developed by a short circuit was measured by H. Rikli, (*Bulletin Association Suisse des Electriciens* No. 5, 1925). He measured the voltage generated by the exciter which was operated on open circuit with its field supplied by storage batteries. The torque was of course proportional to the rate of change of voltage. If the torque developed had a sinusoidal form, it would be fairly easy to determine the rate of change of voltage, but with the irregular torque developed by a short circuit, it is very difficult to measure the peak acceleration.

for studying shocks on rolling mill equipment and similar applications.

SELECTION OF TYPE OF INSTRUMENT

There are many methods which might be used to measure the torque produced by a short circuit. Perhaps the most obvious method would be to mount the stator on trunions and attempt to measure the torque. This would require large, clumsy, and expensive apparatus, and since the torque is of a pulsating nature, it would be extremely difficult, if not impossible, to measure this torque accurately, for any known method of measuring such a large pulsating force would allow some motion of the stator and any such motion absorbs force in the inertia reaction of the stator, so that the torque measured would not be the same as the actual torque developed.

Another method of attack is to measure the effect on the rotor. If the rotor is sufficiently rigid so that it can be regarded as a single mass, the acceleration of the rotor will be directly proportional to the torque applied (assuming that the rotor is not connected to any other device). This general scheme was adopted since it requires only a record of the acceleration of the rotor and therefore lends itself to a small instrument which may be readily attached to any machine.

The pulsating nature of the torque produced by a short circuit is the major difficulty in measuring the torque accurately. For instance, if an attempt were made to measure the actual force acting on either the stator or the rotor a very slight movement would absorb a large force in the inertia reaction, resulting in a large error in the recorded force. For example, the stator of a representative 20,000-kv-a., 60-cycle generator has a moment of inertia of $\frac{2,600,000}{g}$. Then if this stator were mounted on trunions and perfectly free to move and if a sinusoidal torque of 60-cycle frequency and having a peak value equal to the normal torque of the machine were acting on the stator, the amplitude of the resulting movement would be only 0.00002 radians. At an 80-in. radius, the total movement (double amplitude) would be only 0.0032 in. This movement of 0.0032 in. assumes a sinusoidal torque whose average value is zero so that it has no tendency to produce continuous rotation and with the stator mounted on trunions, the only effect is to produce this torsional oscillation of 0.00004 radians total movement which absorbs the full pulsating torque in the inertia reaction of the stator. This is merely given as an example of the enormous force required to produce a very small oscillation of a stator or rotor at 60-cycle frequency. If an attempt were made to measure the torque developed during short circuit by measuring the force exerted by a stator mounted on trunions, it is evident from the above discussion that the allowable movement of the stator is very small. The problem is further complicated by the probability of resonance[3]. The same general difficulty applies to other methods of measurement, so that in any method of measurement, the flexibility of the instrument must be carefully considered for any slight relative movement permitted may result in very large errors. It is much easier to control the natural frequency in a small instrument which measures the acceleration of the rotor than in a large device for measuring force, so that the small instrument should be more accurate and reliable as well as cheaper and more convenient.

There are several possible methods of measuring the acceleration of the rotor. One method would be to obtain a space—time curve showing the instantaneous position of the rotor as a function of time. The second derivative of this record (*i. e.*, the curvature) would then give the acceleration. But in this method any vibration of the recording mechanism would indicate a torque which did not exist. In general, the tendency is to exaggerate errors when a record must be differentiated. A record of instantaneous speed could be obtained, but this would have to be differentiated to get the acceleration so that excessive accuracy would be required in the speed—time record in order to obtain a reasonably accurate record of acceleration. Thus, to secure an accurate record as well as to save time in interpreting the record, the instrument should measure acceleration directly.

The acceleration could be measured electrically by generating a voltage proportional to the rotor speed and impressing this voltage across a condenser. The charging current would then be proportional to the acceleration of the rotor. This method is possible, but to secure sufficient current to give a reasonable deflection on the oscillograph, the apparatus must be rather large or amplification must be used. There is considerable chance for error due to e. m. fs. induced by stray fields at the time of short circuit and by the variation in drop across the brushes which must be used to collect the current. The most serious disadvantage is that it does not lend itself to a small instrument and the calibration is neither as convenient nor as accurate as in the device adopted.

The general scheme of measuring the force required to drive a small flywheel is believed to be the most accurate method available for measuring the acceleration of the rotor. It is very convenient since it can be incorporated in a small instrument which can be attached directly to the shaft of a machine, and since it can be calibrated statically by applying a known torque to the flywheel, the corresponding acceleration being calculated from the moment of inertia of the flywheel.

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[3] The torque transmitted to the foundation by a stator mounted with some flexibility is discussed by Mr. Soderberg in an article published in the April, 1924 issue of the *Electric Journal* (p. 160). This article covers the steady state conditions. For a transient such as a short circuit, the solution is much more complicated.

THE INSTRUMENT

The instrument decided on combines two separate devices for measuring the acceleration of the rotor. The first merely gives eight points on the acceleration-time curve and the second gives a continuous record of the acceleration. These devices are in a single, very rigid casting which can be attached to the end of the shaft of the machine to be tested. The acceleration is recorded on the oscillogram giving a simultaneous record of acceleration and of short-circuit current. Figs. 1 and 2 show two views of the instrument attached to a small 1200-rev. per min. alternator.

FIG. 1—ILLUSTRATION OF THE INSTRUMENT ATTACHED TO THE SHAFT OF A 100-KV-A. GENERATOR

The device which records points on the acceleration—time curve consists of several small disk flywheels mounted on and free to rotate on the instrument shaft. Fig. 3 is a cross-section of the instrument showing one of the disk flywheels and also shows the wiring diagram for a group of flywheels. Each disk flywheel (*a*) has a projection (*b*) on its periphery which engages with a stop (*c*) on the body of the instrument. This stop also serves as an electrical contact. The flywheel projection is held against this stop by a spring (*d*) as shown in Fig. 3. The projection on the flywheel is made of hard rubber with a brass insert for a contact, so that current is carried through the spring, brass insert, and stop, being entirely insulated from the flywheel disk.

FIG. 2—ILLUSTRATION OF THE INSTRUMENT

When the instrument is accelerated in a clockwise direction, the inertia reaction of the flywheel will tend to open the contact, but the spring will hold the contact closed until the acceleration force exceeds the spring force. As soon as this point is reached, the contacts will start to separate thus breaking the circuit and recording the time at which the acceleration reaches a value corresponding to the spring setting. By using several flywheels with different spring settings an acceleration—time curve can be plotted.

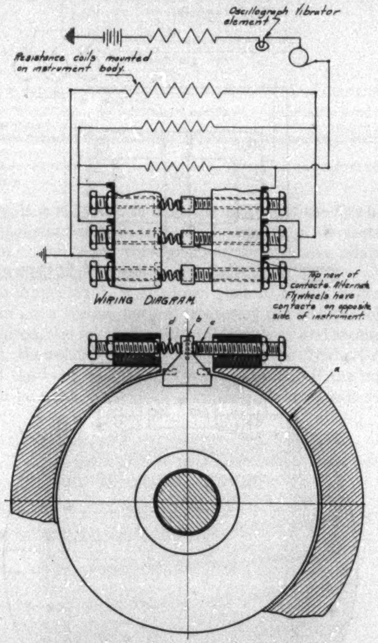
Oscillograph Vibrator element
Resistance coils mounted on instrument body.
WIRING DIAGRAM
Top view of contacts. Alternate Flywheels have contacts on opposite side of instrument.
d
b
c
a

FIG. 3—CROSS-SECTION SHOWING DISK FLYWHEEL AND CONTACT

This shows a disk flywheel and its contact. The time at which the contact opens shows when the acceleration reaches a certain value. The instrument includes eight of these flywheels. The wiring diagram shows the connection for recording the action of several contacts with one oscillograph vibrator element.

It is evident that at the instant when the contact pressure becomes zero, the contacts will start to separate very slowly at first and then more rapidly. If an appreciable separation of the contacts is required to break the circuit, a considerable time lag will be introduced. Fig. 5 shows the calculated rate of separation of contacts. It is evident that if a record of the instant when the pressure becomes zero, can be obtained, the instrument is accurate, but if a separation of one 1/1000 of an inch is required, a considerable error is introduced. A resistance is connected across the contact so that opening the contact will merely produce a slight decrease in current and by using a small current and low voltage, and making the circuit as nearly non-inductive as possible, the contact separation required is reduced to a minimum. As described later, a calibration test was made which showed that in this circuit a change in the current can be observed almost at the instant when the contact pressure becomes zero.

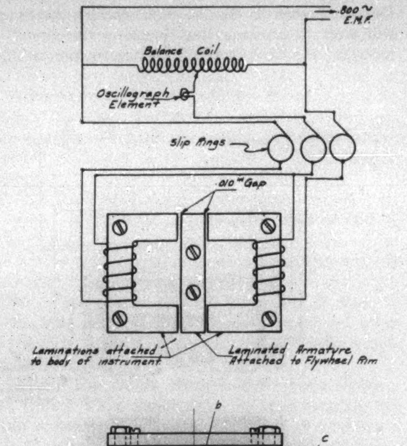
800 ~ E.M.F.
Balance Coil
Oscillograph Element
Slip Rings
.010" Gap
Laminations attached to body of instrument
Laminated Armature Attached to Flywheel Rim
b
c
c
d
a

Fig. 4—Cross-Section Showing Magnetic Accelerometer

The instrument combines two independent devices for measuring acceleration. This is a cross-section showing the device which gives a continuous record of the acceleration. The wiring diagram is shown above.

Eight of these flywheels are mounted on the instrument shaft. Each flywheel is 5¼ in. in diameter and ⅜ in. thick at the hub, so that the axial space required is only three inch. The flywheels are connected in groups as shown in the wiring diagram (Fig. 3). In this way, several contacts are connected in series, each contact being shunted by a resistance. When any contact opens, there will be a certain decrease in the current. The values of resistance are chosen such that if the contacts open in the normal order, the steps in the current record will be approximately equal, but if they open in any other order, the steps will be unequal and the contact which opens can be determined from the value of resistance shunting the contact. In this way, a group of flywheels requires only one slip-ring and one oscillograph vibrator element.

In this device there is considerable time lag in the closing of the contacts and severe chattering after they close so that the record is of no value after the first torque cycle, but in a short circuit the main point of interest is the first torque cycle, so that for this purpose this instrument is very satisfactory.

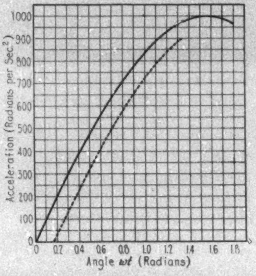

Fig. 5—Calculated Opening of Contacts

Full line shows the assumed sine-wave acceleration.

The dotted line shows the acceleration that would be recorded if a contact separation of 0.0001 in. were required to give a perceptible change in current. Actual tests showed a negligible time lag indicating that for the circuit used, the contact separation required to give a perceptible change in current was very much less than 0.0001 in.

The device which gives a continuous record of the torque consists of a flywheel rim mounted from a rigid hub with spokes which are flexible in a tangential direction. Then when the instrument is given an angular acceleration, the inertia reaction of the flywheel rim will deflect the spokes producing a relative tangential movement between the flywheel rim and body of the instrument. A magnetic device measures this relative movement which is proportional to the acceleration. Fig. 4 is a cross-section of the instrument showing this device for giving a continuous record of the acceleration. The flywheel rim (*a*) carries a laminated armature (*b*). Two sets of U-shaped laminations (*c*) are attached to the body of the instrument. Any relative tangential movement between the flywheel rim and body of the instrument will increase one air-gap and decrease the other. Each set of U-shaped laminations carries a coil. The two coils are connected in a bridge circuit with a balance coil and oscillograph vibrator element as shown in the wiring diagram (Fig. 4). An 800-cycle e. m. f. is impressed across the coils. The balance coil can be adjusted to give approximately zero current for the neutral position of the armature. Then any movement of the armature will decrease the inductance of one coil and increase that of the other, causing a current to flow in the oscillograph element. With negligible resistance, saturation, and leakage flux, the oscillograph current will be proportional to the movement of the armature. For the oscillograms shown in Fig. 9, the only attempt to balance the resistance was by constructing the resistances approximately equal. To reduce the initial current further, the resistance and reactance should be balanced separately.

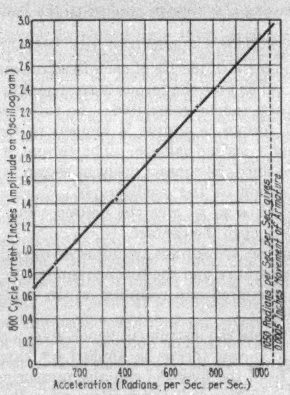

FIG. 6—CALIBRATION OF MAGNETIC ACCELEROMETER

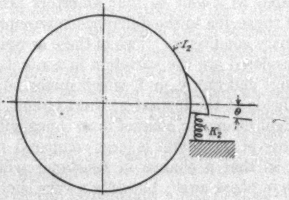

FIG. 7—SCHEME FOR CALIBRATION TEST

The instrument rotates at a known velocity and a projection on the flange strikes the spring "K_2" at $\theta = O$.

In an ideal instrument, the motion of the flywheel rim should be exactly the same as the motion of the body of the instrument so that the acceleration of the flywheel rim will be the acceleration which it is desired to measure. This would require the measurement of force without permitting relative movement. Practically, it is sufficient to have the motion of the flywheel rim substantially the same as the motion of the instrument. The error corresponding to a given relative movement is discussed in Appendix I.

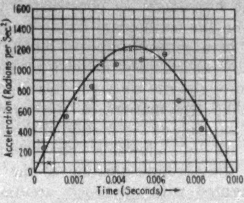

FIG. 8—RESULTS OF CALIBRATION TEST OF ACCELEROMETER

The full line shows the acceleration calculated from the initial speed and spring characteristics.

Points indicated by dot and circle (⊙) shows the acceleration as recorded by the magnetic device.

Points indicated by "x" are the points recorded by the disk flywheel contacts

FIG. 9—OSCILLOGRAM SHOWING CURRENTS AND TORQUE FOR A SINGLE-PHASE SHORT CIRCUIT

A—Terminal voltage
B—Current through a group of contacts
C—800-Cycle current—the magnetic record of acceleration.
D—Field current
E—Current in short-circuited phase

In Appendix I, it is shown that to obtain a given accuracy a certain natural frequency of the flywheel is required, and for a given natural frequency and acceleration, the relative movement between the flywheel rim and the body of the instrument is fixed. If the error is to be small, the relative movement allowable is very small. In this instrument, the natural frequency of the flywheel is 400 cycles per sec. so that for an acceleration of 1000 rad./sec.^{2} which is a fairly high value of acceleration, the corresponding relative movement between flywheel rim and instrument body is only one-half mil.

Since this one-half mil of relative movement is between two parts rotating at high velocity and subject to severe vibrations at the instant of short circuit, very careful design is required to insure an accurate measurement. The instrument as constructed gives a 2-in. amplitude on the oscillogram for an armature motion of ½ mil, or a magnification of 4000 times. This gives a very satisfactory method of measuring the acceleration of the rim since the relative movement is so small that the motion of the flywheel rim is practically the same as the motion of the shaft.

This ½ mil of allowable movement does not mean that the absolute position of the flywheel rim must be located with any such accuracy. It does require however, that there be no relative movement permitted between the flywheel rim and body of the instrument except the elastic tangential movement of the flywheel rim which is proportional to the acceleration. To accomplish this, the flywheel rim is carried on flexible spokes so that the flywheel and body of the instrument behave as one piece of metal having the proper flexibility to permit a slight relative tangential movement, but with sufficient rigidity in other directions so that all other relative movements are negligible. The magnetic instrument must then measure minute relative tangential movement with accuracy. To accomplish this, the air-gaps are set as nearly alike as possible and the final balance obtained with the balance coil adjustment. Then a very slight movement of the armature can give a very large oscillograph deflection.

A slight torsional vibration of the flywheel rim will sometimes occur, but with the high natural frequency of the flywheel (400 cycles) this can readily be distinguished from other effects. Serious trouble might occur if there was a tooth pulsation or other disturbance having a frequency the same as the natural frequency of the flywheel. For this reason, it is desirable to be able to change the natural frequency of the instrument. It is also desirable to have a means of changing the sensitivity of the instrument. Both of these objects are accomplished by the auxiliary springs (*d* in Fig. 4), which give additional stiffness to the flywheel system. Each spring is merely a piece of piano wire stretched between two parts of the instrument body and with its midpoint attached to the flywheel rim. It is so arranged that the size of wire can be readily changed. For the small movement required, this gives the stiffness required in a very light spring.

The amplitude of the current recorded by the oscillograph is a measure of the movement of the armature from its neutral position. If this current is adjusted to be zero in the neutral position, a movement in either direction will produce an increase in the amplitude of the recorded current, but since the direction of the first acceleration of a short circuit is known, this is satisfactory. With negligible resistance, saturation, and leakage flux, the amplitude of the current recorded should be proportional to the movement of the armature. The instrument used can be set to give a slight initial deflection and a straight line characteristic in one direction, but not quite a straight line characteristic in the reverse direction. In this case, the main point of interest is the first torque peak, so that this adjustment was used. Fig. 6, shows a calibration curve for the instrument. The magnification can be increased by increasing the voltage applied to the instrument.

METHOD OF CALIBRATING THE INSTRUMENTS

The moment of inertia of each flywheel can easily be determined so that the torque corresponding to a given acceleration can be calculated. To calibrate the disk flywheel springs, a torque is applied to the flywheel rim using a spring balance. The torque is gradually increased until the contact opens as shown by watching the deflection in the oscillograph. Each spring can then be adjusted to give the desired value of torque required to open the contact.

In practise, the springs are adjusted roughly to cover the desired range and the torque required to open each contact measured and recorded as the calibration for that contact.

To calibrate the instrument which gives a continuous record, a known torque is applied by hanging scale weights from the flywheel rim and measuring the corresponding deflection in the oscillograph. By doing this for several values of torque, an acceleration—deflection curve can be plotted. This calibration curve is shown in Fig. 6. The method of calibration is very simple, so that if desired it can be checked before and after a test to be sure that nothing is out of adjustment.

By calibrating the instrument in this way, the oscillograph and acceleration measuring instrument are calibrated as a unit so that all errors are eliminated except those due to the difference between rotating and stationary conditions. Two of these errors which must be considered are the variation in contact drop at the slip-rings and any e. m. f. which might be induced by stray fields at the instant of short circuit. The effect of a slight change in contact drop is negligible since the reactance of the circuit is large compared to the resistance, so that a change in resistance will result in a change in phase angle, but only a very slight change in impedance.

Any currents induced by stray fields will have a low frequency compared to the 800-cycle e. m. f. supplied to the instrument, so that an induced current will merely shift the zero line and not affect the amplitude of the 800-cycle record so that an induced current will not give an appreciable error. There is usually considerable lateral vibration at the instant of short circuit, but this does not produce angular acceleration in an accurately balanced flywheel.

As a check on the operation of the instrument in actually recording pulsating acceleration, a calibration test was made in which a sine-wave acceleration of known frequency and magnitude was imparted to the instrument by rotating the instrument and allowing a spring to engage with a projection on the periphery of the instrument. This method is discussed in Appendix II. Fig. 8 shows the agreement between the calculated acceleration and the acceleration as measured by the instrument. The agreement between the records was considered very close for measurements of this kind.

CLOSING THE SHORT CIRCUIT

The torque developed by a short circuit varies with the point of the voltage wave at which the short circuit occurs. In order to measure the maximum possible torque, it is necessary to close the circuit at the proper point of the voltage wave. To accomplish this, a switch is arranged to be tripped at a given position of the rotor. The switch is designed to close quickly (1/100 of a sec.) so that a small percentage of variation in the time required for the switch to close will give only a very small error in the time at which the switch closes. The position of the rotor at which the switch is tripped is adjustable. In making a test, one short circuit must be made with a known setting of the tripping mechanism. The point at which the switch is closed is noted on the oscillogram and the tripping device set ahead or back the required number of degrees to close the switch at the desired point of the voltage wave.

The switch shown in Fig. 1 was improvised using an ordinary knife switch. It is closed by very heavy springs and has a rubber bumper to absorb the shock at closing. The tripping device consists of a projection on the flange of the shaft coupling which engages with a very light phosphor bronze dog. This phosphor bronze dog can be moved in an arc of a circle to engage at the desired position of the rotor. This arc is graduated in degrees to facilitate setting the dog. This device can be seen at the bottom of Figs. 1 and 2. The dog is made as light as possible and is connected to the catch of the switch by a piece of small piano wire. These parts are very light, for if heavy, they would cause a serious jar of the instrument at the instant of short circuit which might destroy the accuracy of the record. A certain amount of flexibility is also essential to prevent breaking the parts due to the high speed at which they engage.

This device can be depended upon to close the circuit with less than 10 deg. variation from the desired position of the voltage wave, which is less than one-half thousandth of a second error. Prior to closing the short circuit, a piece of small fuse wire holds the phosphor bronze dog sidewise in a position where it will not engage with the rotating projection. A hand operated switch is used to start the oscillograph. This switch also connects 110-volts, d-c. power, directly across this small fuse wire, blowing the fuse and releasing the dog. Then as soon as the rotor reaches the proper position, the main switch will be tripped closing the circuit at the desired point of the voltage wave. In this way, closing the circuit and taking the record are controlled automatically so that all the operator has to do to take a record is to close a small instrument switch.

CONCLUSION

Some tests have been made and these have shown remarkable agreement between the two separate methods of recording the torque. This fact, together with the calibration test, shows quite conclusively that the instrument is very accurate. Fig. 9 shows an oscillogram taken of a short circuit, the circuit being closed slightly off zero voltage. The armature current and acceleration of the rotor are recorded simultaneously. Fig. 10 shows the torque developed by a single-phase short circuit. A discussion of the results of the tests will be given in a later paper.

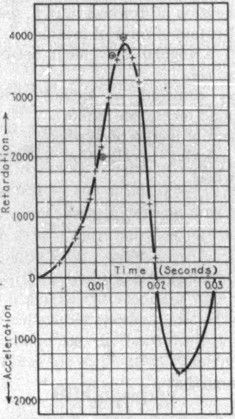

FIG. 10—TORQUE—TIME-CURVE FOR A SINGLE-PHASE SHORT CIRCUIT

Points shown by dot and circle (⊙) show acceleration as recorded by opening of contacts

Points indicated by cross "+" show acceleration recorded by magnetic instrument (peaks of 800-cycle current)

The means of measurement used are old devices, merely redesigned and combined for the particular purpose desired. The idea of measuring linear acceleration by a mass held against a contact by a spring has been used frequently, and its adaptation to angular acceleration was suggested by Mr. Soderberg several years ago.

The idea of measuring small movements by the variation in inductance resulting from the change in air-gap has been used in many instruments and experiments. The magnetic device used for measuring the movement of the flywheel rim in this instrument was copied from an instrument built four years ago by Mr. J. G. Ritter, for measuring railway track stresses.

The author is indebted to many men in the Westing-

house organization for valuable suggestions and ideas used in designing the instrument and wishes to take this opportunity to thank them, particularly Mr. C. R. Soderberg, Mr. J. G. Ritter, Mr. J. W. Legg, and Mr. C. J. Fechheimer.

Appendix I

In measuring the acceleration of a rotor by measuring the force required to drive a small flywheel, the flywheel should have exactly the same motion as the shaft, but since any known device for recording the instantaneous force allows a slight relative movement of the parts, the error due to this relative movement between flywheel and shaft must be considered.

Let

α_F = Acceleration of flywheel in rad./sec.^{2}

α_R = Acceleration of rotor of machine in rad./sec.^{2}

δ = Relative angular movement between rotor and flywheel.

K = Spring constant of device driving flywheel (pound-inches per radian).

I = Moment of inertia of flywheel.

$\alpha_R = \alpha_F$ + relative acceleration between rotor and flywheel.

$$\alpha_R = \alpha_F + \frac{d^2\,\delta}{d\,t^2} \qquad (1)$$

But

$$K\,\delta = I\,\alpha_F \qquad (2)$$

or

$$\delta = \frac{I}{K}\,\alpha_F$$

Then assume that

$$\alpha_R = A \sin \omega\, t$$

Then

$$\alpha_F + \frac{I}{K}\,\frac{d^2\,\alpha_F}{d\,t^2} = A \sin \omega\, t$$

or

$$\frac{d^2\,\alpha_F}{d\,t^2} + \omega_c^2\,\alpha_F = \omega_c^2\,A \sin \omega\, t$$

where

$$\omega_c = \sqrt{\frac{K}{I}}$$

This gives

$$\alpha_F = C_1 \cos \omega_c\, t + C_2 \sin \omega_c\, t + \frac{1}{1-\left(\frac{\omega}{\omega_c}\right)^2} A \sin \omega\, t \qquad (4)$$

In this solution $\left[A\,\frac{1}{1-\left(\frac{\omega}{\omega_c}\right)^2} \sin \omega\, t\right]$ represents the steady state condition and $C_1 \cos \omega_c\, t + C_2 \sin \omega_c\, t$ represents the transient condition which will be damped out by friction. Thus in the steady state condition, α_F, which is the acceleration measured, is equal to the actual acceleration of the rotor (α_R) multiplied by the factor $\left[\frac{1}{1-\left(\frac{\omega}{\omega_c}\right)^2}\right]$. So that $\left[\frac{1}{1-\left(\frac{\omega}{\omega_c}\right)^2}\right]$ represents a fundamental error in this method of measurement. The smaller the value of $\frac{\omega}{\omega_c}$ can be made, the smaller will be the error. If $\frac{\omega}{\omega_c} = 1/4$, the measured value will be 6½ per cent high.

Any mechanical device for continuously recording force can be regarded as having a certain flexibility and as measuring the deflection due to the action of a force on this flexibility. Then if the value of ω_c is fixed by the allowable error, and if a given acceleration must be measured, the corresponding relative movement between rotor and instrument flywheel is fixed.

For, from equation (2)

$$K\,\delta = I\,\alpha_F$$

or

$$\frac{K}{I} = \frac{\alpha_F}{\delta}$$

or

$$\omega_c = \sqrt{\frac{\alpha_F}{\delta}} \qquad (4)$$

or

$$\delta = \frac{\alpha_F}{\omega_c^2} \qquad (5)$$

In designing this instrument, a natural frequency of 400 was used and the instrument designed to measure an acceleration of 1000 rad./sec.^{2} with accuracy.

Using equation (5)

$$\delta = \frac{1000}{(400 \times 2\,\pi)^2} = 0.00016 \text{ radian}$$

The radius of the measuring device was approximately 3 in. so that 0.00026 multiplied by 3 = 0.00048 in., which the instrument must be able to measure with accuracy. This is a very small movement to measure accurately but it cannot be increased without increasing the size of the instrument or the fundamental error considered above.

The factor of error $\frac{1}{1-\left(\frac{\omega}{\omega_c}\right)^2}$ derived above applies only to measuring a steady sinusoidal acceleration. For measuring irregular accelerations, the factor will not be exactly the same, but the error will still be a function of the ratio of the acceleration frequency to the natural frequency of the instrument and the factor derived above will give a good approximation to the magnitude of the error to be expected.

Appendix II

As has been mentioned, the method used to calibrate the instrument was to rotate it at a known speed, but running free, and let a projection on the flange strike a very light, stiff spring. To rotate the instrument, it is mounted in a lathe but instead of driving it by a lathe dog a string is used having just sufficient strength to overcome friction. Then when the projection on the flange strikes the spring, the string breaks, allowing the instrument to move under the influence of the spring force and its own inertia. As shown below, this imparts a sinusoidal acceleration to the instrument.

Assumptions and symbols:
Friction negligible
Mass of spring negligible
Initial velocity $= \omega_0$
θ = Angular position of instrument
I_2 = Moment of inertia of instrument
K_2 = Spring constant (inch pounds per radian) of the spring used to stop the instrument
t = time/
At $t = 0$ the projection of the flange of the instrument strikes the spring.

Then

$$I_2 \frac{d^2\,\theta}{d\,t^2} + K_2\,\theta = 0 \qquad (5)$$

$$\theta = A \cos \sqrt{\frac{K_2}{I_2}}\,t + B \sin \sqrt{\frac{K_2}{I_2}}\,t \qquad (6)$$

Where A and B are constants of integration.

At $t = 0$, $\dfrac{d^2\,\theta}{d\,t^2} = 0$ so that $A = 0$

At $t = 0$, $\dfrac{d\,\theta}{d\,t} = \omega_0$

This gives

$$\omega_0 = B \sqrt{\frac{K_2}{I_2}}$$

or

$$B = \omega_0 \sqrt{\frac{I_2}{K_2}}$$

Then the acceleration of the instrument is

$$\frac{d^2\,\theta}{d\,t^2} = \omega_0 \sqrt{\frac{K_2}{I_2}} \sin \sqrt{\frac{K_2}{I_2}}\,t \qquad (7)$$

This equation holds only during the time when the spring is in contact with the projection on the instrument, which is from $t = 0$ to $\sqrt{\dfrac{K_2}{I_2}}\;t = \pi$. Or in other words the sinusoidal acceleration continues for only one-half a cycle.

From this it is evident that the acceleration imparted to the instrument in this manner has a frequency determined by the inertia of the instrument and the stiffness of the spring used. This frequency used for checking the instrument should be approximately the same as the frequency which the instrument is to measure. For any given frequency the magnitude of the acceleration is determined by the initial speed of rotation.

A piece of piano wire in tension was used for the spring. This gave a very light spring combined with the required high spring constant.

Fig. 7 shows the schematic arrangement for the test. Fig. 8 shows a plot of the results of this test as compared to the calculated acceleration. The agreement between calculated and test results is within the accuracy with which the oscillogram could be read.

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Reduction of Transformer Exciting Current
to Sine-Wave Basis

BY G. CAMILLI[1]
Associate, A. I. E. E.

Synopsis.—*As a sequel to an earlier investigation and development of a method for the reduction of core-loss measurement to sine-wave basis, this paper describes two methods developed for the reduction of exciting current to sine-wave basis.*

The first method consists of making two measurements at wave shapes as widely different as possible, setting the voltage in each case by means of the flux voltmeter. The current corresponding to sine-wave voltage is obtained by extrapolation from the observed values of currents and form factors. Although the method might be considered to some extent empirical, it is found to yield an accuracy within one per cent even under extremely unfavorable conditions.

The second method utilizes as before the flux voltmeter for setting the voltage but uses a "crest ammeter" (developed for this purpose) for reading the instantaneous maximum values of the corresponding currents. Measurements are made at 100 per cent, 86.6 per cent and 50 per cent voltages. These data determine the fundamental. third and fifth harmonics of the exciting current corresponding to sine-wave voltage and hence the exciting current itself, because these harmonics are the only important components in determining the effective value of the exciting current.

Theory of the crest ammeter is given, and its applicability (in conjunction with the flux voltmeter) to the determination of d-c. saturation curves by means of a-c. tests in magnetic investigations is indicated.

INTRODUCTION

It is well known that the no-load losses, that is, the iron loss and exciting current, of a transformer are dependent upon the wave shape of the excitation voltage. While the Institute rules provide that the efficiency rating of transformers must be based on sine-wave operation, it is known how difficult it is to obtain sine-wave voltage on a commercial scale for the testing of transformers. Some scheme that will reduce core loss and exciting current tests to a sine-wave basis is therefore a necessity, much more important now than it was some years ago, due primarily to the increased kv-a. capacity of transformers. This may be seen better by considering the fact that while the kv-a. capacity of transformer units has steadily increased, the kv-a. capacity of generating units used for testing them has not increased proportionately, and therefore the core-loss load on generators in testing departments is a much larger percentage of the generator capacity than was formerly the case, with the consequence that wave distortion is much larger.

In a paper presented to the Institute a year ago,[2] the writer described a new and accurate method for the reduction of transformer core-loss measurements to sine-wave basis, utilizing a flux voltmeter developed for that purpose by the writer. The accuracy of the meter and method was checked and endorsed by the Bureau of Standards,[3] and it is understood that a number of research laboratories besides the Bureau of Standards have already adopted the scheme.

Since the successful solution of the problem of the reduction of the core-loss component of the no-load loss measurements to sine-wave basis, the writer studied the problem of the reduction of the exciting current component of the no-load measurement to sine-wave basis.

Two different methods were developed for the reduction of exciting current measurements to sine-wave basis, as follows:

Method I. In core-loss measurements, setting the voltage by the flux voltmeter,[2] the maximum flux density and therefore the maximum value of the exciting current are those corresponding to sine-wave voltage regardless of the wave shape of the test voltage. The effective value of the exciting current, however, will be variable with the wave shape of the test voltage.

To apply a wave-shape correction to the observed effective value of the exciting current, it would be necessary to have some applicable measure of wave distortion. Now, form factor is one kind of a measure of wave-shape distortion and is given in a simple way by the flux voltmeter, and therefore it occurred to the writer that some simple relation might exist between form factor and effective value of the exciting current. Thus, indicating the values of form factors by F, and the values of the exciting current by Y, we may write as a general equation between these two variables:

$$Y = a + b\,F + c\,F^2 + d\,F^3 + \ldots + f\,F^n$$

Equations of this type are frequently used in engineering problems and are very convenient whenever it is found that the terms above the first or second power are negligible. Tests were therefore made to determine what approximation could be used, and it was found that all terms above the first power could safely be ignored; that is, the exciting current corresponding to sine-wave form factor may be extrapolated as a straight line function of the form factor.

$$\text{form factor} = 1.11 \times \frac{\text{A-c. Voltmeter reading}}{\text{Flux Voltmeter reading}}$$

In a dozen test cases, the error was not more than 1 per

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[1] General Transformer Engineering Department, General Electric Company, Pittsfield, Mass.

[2] See G. Camilli, *A Flux Voltmeter for Magnetic Tests*, JOURNAL A. I. E. E., October 1926.

[3] See Discussion by Mr. R. L. Sanford, JOURNAL A. I. E. E., October 1926, p. 1014.

Presented at the Regional Meeting of District No. 1 of the A. I. E. E., Pittsfield, Mass., May 25-28, 1927.

cent. With no correction applied, the error would have been up to 20 per cent, making the exciting current that much too high.

Method II. In the foregoing, it was mentioned that in using the flux voltmeter the maximum flux density and therefore the maximum value of the exciting current are determined. Consequently, if a transformer is tested at various voltages, observing the voltage on a flux voltmeter and the current on a crest ammeter (to be described below), points of the *B-H* curve of the transformer are obtained. Having the *B-H* curve, the effective current corresponding to sine-wave voltage can be calculated.

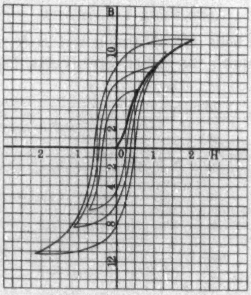

FIG. 1—H-B CURVE BY MEANS OF THE CREST AMMETER

It may appear at first as though this would be a very laborious method, but it is extremely simple. Three readings, *viz.*: one taken at full voltage, one at 86.6 per cent voltage, and one at 50 per cent voltage (by the flux voltmeter), enable us to determine the fundamental, third-harmonic and fifth-harmonic components of the exciting current corresponding to sine-wave voltage, and, since these are the only important harmonics, their resultant gives the total effective current for sine-wave voltage. In this method, the higher harmonics are not entirely neglected, because they appear partially in the first, third, and fifth harmonics by modifying their values. For greater accuracy, a larger number of readings and correspondingly larger number of harmonics may be included, but this appears to be hardly necessary. When a large number of points is taken, it becomes unnecessary to bring in the harmonics at all, as the r. m. s. value of the exciting current for sine-wave voltage may be calculated by taking points equi-distant in time. The harmonic idea is useful in obtaining a greater accuracy from a few points than would otherwise be possible.

The *B-H* curve obtained by observing simultaneous values of B_{max} and I_{max} will be recognized to be the locus of the tips of the symmetrical hysteresis loops for various densities, as shown by the heavy line in Fig. 1, and therefore, intermediate between the ascending and descending branches of the loop for maximum (normal density). The error which this introduces into the exciting current calculation is that of ignoring the power component of exciting current corresponding to hysteresis loss, which ordinarily may be neglected in the value of the exciting current. The exciting current thus obtained lacks, therefore, the hysteresis loss component; and therefore, if desired, this component may be added to it as determined by the core-loss test. This refinement, however, appears to be hardly necessary, because tests show that exciting currents obtained by this method err on the safe side; that is, results obtained by the three-point method are a little larger than those obtained by direct sine-wave tests.

Crest Ammeter. The principle underlying the functioning of the crest ammeter is as follows: In an air-core reactor, the maximum flux density, and consequently the maximum value of the current, is proportional to the arithmetical average value of the reactive voltage across it.

The proof of the theorem may be seen in the following way: Let *e* be the instantaneous voltage drop in the inductance of the circuit and *i* the current through it. By elementary theory:

$$e = L \frac{d\,i}{d\,t} \tag{1}$$

where *L* is the inductance of the circuit, a constant. From (**1**) it follows that

$$e\,d\,t = L\,d\,i \tag{2}$$

$$e \int d\,t = L \int d\,i \tag{3}$$

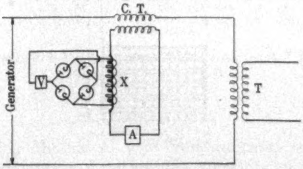

FIG. 2—USE OF THE CREST AMMETER

$$\int_{t1}^{t2} e\,d\,t = L \int_{I \text{ lowest value}}^{I \text{ highest value}} d\,i \tag{4}$$

But

$$\int_{t1}^{t2} e\,d\,t = \text{area of half-cycle} = e_{avg}\,(t_2 - t_1) \tag{5}$$

And

$$L \int_{I \text{ lowest value}}^{I \text{ highest value}} d\,i = L \text{ times maximum change in current.} \tag{6}$$

Therefore, substituting (**5**) and (**6**) in (**4**),

$e_{avg}\,(t_2 - t_1) = L$ times the maximum change in the current.

Since the maximum change in current is twice the maximum value of the current, e_{avg} is proportional to the maximum value of the current.

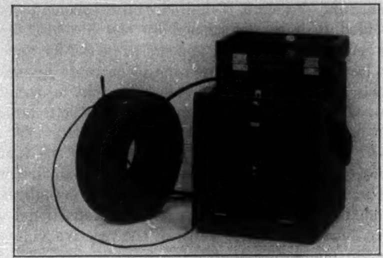

Fig. 3—Crest-Ammeter

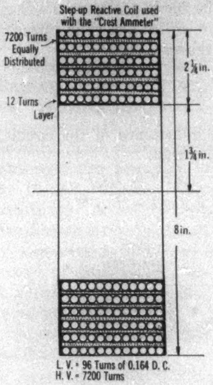
Step-up Reactive Coil used with the "Crest Ammeter"
7200 Turns Equally Distributed
12 Turns Layer
2 1/8 in.
1 3/4 in.
8 in.
L. V. = 96 Turns of 0.164 D. C.
H. V. = 7200 Turns

Fig. 4

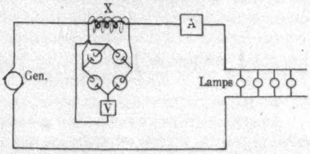

Fig. 5—Calibration of the Crest Ammeter

The arithmetical average voltage drop across the reactor can be measured by means of the flux voltmeter, so that the flux voltmeter shunted by a reactor becomes a crest ammeter.

The scheme of the apparatus and its connection to the circuit may be seen from Fig. 2 in which T is the transformer under test, $C\ T$ is an ordinary current transformer, A is an ordinary a-c. ammeter, X is the air-core reactor, and V is a flux voltmeter.

Fig. 3 shows a view of the outfit. Fig. 4 shows the details of the air-core reactor.

Fig. 5 shows how the calibration of the crest ammeter may be made. A sine-wave generator is used for this purpose connected to a non-inductive load.

The calibration is made by comparing the readings in the voltmeter V and the ammeter A. The calibration of this instrument corrects also for the slight error introduced into the readings by the fact that the shunt reactor cannot be of zero power factor required theoretically, but must have some little resistance.

B-H CURVES BY A-C. TESTS USING CREST AMMETER AND FLUX VOLTMETER

Usual methods for obtaining the *B-H* curve of a specimen require the use of direct current and laboratory instruments and methods unsuited for commercial test. It will be evident from the foregoing discussion of the crest ammeter that *B-H* curves (not hysteresis loops) can be obtained by very simple and convenient commercial a-c. tests by men with no particular laboratory training, using the crest ammeter and the flux voltmeter, two sturdy portable meters.

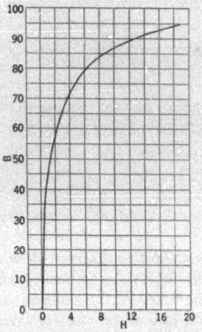

Fig. 6—D-c. Magnetization Curve Obtained by the Use of the Crest Ammeter and Flux-Voltmeter

Although, in a way, it may be considered a disadvantage that hysteresis loops cannot be obtained by these simple tests, on the other hand troubles from residuals, and need for proper demagnetization, etc., attendant on d-c. tests, are entirely avoided in the a-c. tests.

Fig. 6 shows the magnetization curve of a 2000-kv-a. transformer obtained by this method.

TEST DATA

Method I. In the tests tabulated below a given transformer was tested on two or more generators of widely different wave shapes, or on the same generator but connected to different voltage diagram, so as to obtain the exciting current for at least two different form factors.

The values so obtained were plotted in rectangular coordinates, and by extrapolation the exciting current for a sine-wave form factor was obtained.

As a check, the same transformer was tested with the best generator available and the two results were compared and are tabulated below. (See Table I.)

TABLE I

Rating	Form Factor	Exc. I	Remarks
H-60-2000-*C*-2300/18,400/32000 *Y*-2300/4600 Y	1.155	24.1	
	1.16	24.2	
	1.215	25.0	See Fig. 7a
	1.31	26.1	
	1.56	28.7	
	1.11	23.6	By extrapolation from
	1.11	23.4	Best wave test
H-60-1000-63,000 *Y*-6925-2400	1.20	20.19	See Fig. 7b
	1.12	18.76	Best wave test
	1.11	18.50	By extrapolation
H-60-1000-*C*-36,700/63,100 *Y*-6925/12,000 *Y*-2400	1.19	19.70	See Fig. 7c
	1.13	18	
	1.112	17.6	Best wave test
	1.11	17.5	By extrapolation
H-60-667-*C*-23,000-2300/4000 Y	1.19	8.91	
	1.13	8.31	
	1.112	8.16	Best wave test
	1.11	8.10	By extrapolation
H-50-2500-*C*-25,200-6300	1.19	19.9	
	1.27	20.8	
	1.11	19.1	By extrapolation
H-60-2000-*C*-34,700-2300/4000 *Y*	1.13	34.0	See Fig. 7d
	1.24	38.0	
	1.115	33.0	Best wave test
	1.110	33.2	By extrapolation

TABLE II

Rating	Form Factor	Exciting Current
H-60-667-*C*-13,200-2400	1.170	9.64
	1.121	9.40
	1.11 (By extra.)	9.36
W *C*-25-3333-45,100/78,000 *Y*-2200	1.155	47.6
	1.125	45.4
	1.11 (By extra.)	44.3
H-60-833-*C*-22,000-2300	1.21	29.3
	1.15	26.25
	1.11 (By extra.)	24.3
H-60-500-*C*-12,500-2300	1.155	7.90
	1.12	7.25
	1.11 (By extra.)	7.06
H-60-500-*C*-4000-2200	1.145	8.64
	1.115	8.34
	1.11 (By extra.)	8.24

The data in Table II are taken from the records of an earlier core-loss investigation[4] and illustrate the fact that the exciting current is a function of the form factor and increases with it.

Method II. In using this method, a given transformer is tested at three different voltages: *viz.*, 50 per cent, 86.6 per cent, and 100 per cent (set by the flux voltmeter), and value of the current corresponding to each voltage is measured by the crest ammeter. With the help of these data the effective value of the exciting current is calculated. (See Appendix B.)

As a check to the value of the current obtained by this method, six transformers were tested also with the best wave shape available, and the results are compared below.

ACKNOWLEDGMENT

The writer is indebted to Mr. Boyajian for his personal interest and consultation in this investigation.

APPENDIX A

In using Method I, instead of graphical extrapolation, calculation may be made by slide rule as follows:

Calling the value of current I_1 at form factor F_1 (for instance, the higher form factor), and I_2 at form factor F_2 (the smaller form factor), the value of exciting current I at sine wave may be expressed by:

$$I_{sine\ wave} = I_1 - \frac{(I_1 - I_2)\,(F_1 - 1.11)}{F_1 - F_2}$$

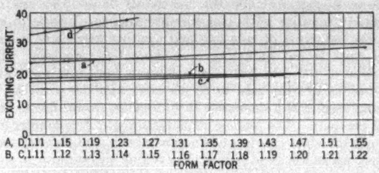

FIG. 7—METHOD NO. 1 FOR THE REDUCTION OF THE EXCITING CURRENT TO SINE-WAVE BASIS

APPENDIX B

In using Method II (the crest ammeter method), the effective value of the exciting current is found from the peak values of the exciting current corresponding to 50 per cent, 86.6 per cent, and 100 per cent rated voltage of the transformer. The following method gives in a simple way the fundamental, third and fifth harmonics by means of which it is possible to calculate the effective value of their resultant. Example:

Let A be the value of I_{max} corresponding to 100 per cent voltage.

Let B be the value of I_{max} corresponding to 50 per cent voltage.

Let C be the value of I_{max} corresponding to 86.6 per cent voltage.

Then

$$\text{3rd harmonic} = \frac{A - 2\,B}{3} = I_3$$

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[4] G. Camilli, *A Flux Voltmeter for Magnetic Tests*, JOURNAL A. I. E. E., Oct. 1926.

Item	Rating	Exciting current by crest ammeter	Exciting current by conventional method, a-c. ammeter	Exciting current at sine-wave voltage	Error of conventional method, per cent	Error of crest ammeter method, per cent
(1)	*H*-60-2000-*C*-73,000-2520	79	93	78.3	19	0.9
(2)	*H*-60-667-*C*-33,000-2300/4000 Y	7.97	8.9	8.03	11	0.85
(3)	*H*-60-3333-*C*-13,200 Y-2300/ 4000 *Y H* 480	61.8	65.1	61.5	6	0.5
(4)	*H*-60-2000-*C*-69,300-2400/ 4160*Y-K D H*	47.7	48.0	47.3	1.4	0.8
(5)	*H*-60-2000-*C*-2300-23,000	23.3	23.5	23.5	0	0.85
(6)	*H*-60-4000-*C*-23,000-4200/ 7260 Y	16	17.8	16	10	0

$$\text{5th harmonic} = \frac{A - (I_3 + 1.15\,C)}{2} = I_5$$

$$\text{1st} = A - (I_3 + I_5) = I_1$$

From which

$$I_{\textit{effective for sine wave}} = \sqrt{\frac{I_1^2 + I_3^2 + I_5^2}{2}}$$

Discussion

Aram Boyajian: For the benefit of those who are not intimately familiar with transformer problems I wish to say that the reduction of core loss and exciting current to a sine-wave basis is not quibbling over laboratory precision. The reduction of the core loss to a sine-wave basis involves wave-shape errors of from zero to 20 per cent, and the reduction of exciting current to a sine-wave basis involves errors of from zero to 50 per cent. The leading manufacturers of the country use methods to reduce core loss to sine-wave basis, and possibly some of them use methods to reduce the exciting current also to sine-wave basis. These methods greatly lessen but do not completely eliminate the possible errors mentioned, due to the fact that the bases of all the methods used in the past have been imperfect. Mr. Camilli therefore has undertaken to develop methods which will have a better basis and will be entirely reliable. I think he has succeeded surpassingly well.

K. K. Palueff: With the tests made by various manufacturers under different conditions, it is impossible to adjudge the comparative characteristics of different transformers, and it seems to me that the methods described by Mr. Camilli will give a very fine basis on which to decide whether the magnetizing current really is 2 per cent or 4 per cent.

I should like to take exception to Mr. Camilli's statement in the first paragraph of his paper wherein he says that the testing facilities, in regard to the capacity of the generators, are not increasing in proportion to the transformer kv-a. capacity. That isn't our case here in the General Electric Company. For the last five years, our testing capacity in generators has increased from 3000-kv-a. to 25,000-kv-a. units, and at the present time we are equipped to test the largest transformer we have yet built, or are going to build for several years to come, with a generator which will give very good agreement with that of the perfect sine-wave generator.

Research

Annual Report of the Committee on Research[*]

To the Board of Directors:

1. The Training of Research Workers

There is, perhaps no more important problem at this time than that of the training of research workers and engineers. The popular appreciation of research is increasing. This is good because the chief stimulus of research is a certain state of mind akin to, but more than, curiosity and inquisitiveness which without doubt can be developed in the proper atmosphere. It was a similar state of mind, a dissatisfaction, a desire to go where others had not gone and see what others had not seen that actuated our pioneer ancestors and resulted in America. It would thus seem that the right material should be available; but more than material and popular appreciation is required to create the necessary state of mind. Are our colleges doing their part? As was pointed out in the report of this Committee last year, indications are that they are not. As a gage on the research in electrical engineering at colleges, Dr. F. E. Terman, of Stanford University, has made a statistical study of research papers presented by college professors and their students. The following is quoted from his report which appeared in the April 22, 1927 of *Science:*

"The summary of this survey shows that the electrical engineering schools of our country produce about one-eighth of the electrical and radio research that is reported in the pages of the national engineering societies. This represents about eleven articles a year. Of these eleven articles coming from the colleges each year, approximately seven come from four universities. There is a total of several hundred. Apparently not over a dozen technical schools are making much effort, if any, in the way of research. Over half of the university research in electrical engineering is the work of eight men.

This is the situation, and it now remains to consider the consequences of this condition. University research in electrical engineering is primarily significant as an indication of the situation which exists today in the education of electrical engineers. The laboratories of the big electrical companies make technical progress assured even without university research, but the country's supply of technically trained young men can come only from the university."

These facts are disconcerting. In correcting this condition, it must be kept in mind that true research workers are more than mere readers of instruments and collectors of data and cannot be turned out mechanically.

It is not a mere matter of money and apparatus; atmosphere and inspiration are necessary. These must be supplied. There is, perhaps, something still to be done by the industry in further recognition of the research worker in a monetary way.

2. Active Research

This Committee serves as the Advisory Committee on Electrical Engineering to the National Research Council. At the request of that Council, the Committee on Research has been instrumental in establishing in the National Research Council, a Committee on Electrical Insulation. This has given work of a definite character to the Committee on Research and a small group of its members has been active in this connection.

The Committee on Insulation, largely made up of members of the Committee on Research, has already made two reports; the second of which is a comprehensive review of the literature and present information on the subject of dielectric absorption, and has suggested channels for further research in this field. As a result, a number of researches are now under way in different universities; one of these, in Johns Hopkins University, is of special interest on the present occasion, as it is being supported by a generous fund guaranteed by Engineering Foundation.

A third report of this Committee on Dielectric Strength of Solid and Liquid Dielectrics is being presented at this meeting.

Research on cable insulation, under the auspices of the N. E. L. A., is under way in several colleges. Part of this work—Influence of Residual Air and Moisture on Impregnated Paper Insulation—has already been presented to the Institute by Doctor Whitehead.

3. Stimulation of Research

All divisions of electrical engineering offer wide opportunity for experimental study, development, and research. It is the special duty of our Committee to encourage and stimulate research, to keep in touch with the results accomplished, and to see to it that the members of the Institute and others interested are informed.

To do this, the Committee asks its members to report on all matters of the following natures that come to their attention:

1. New experimental work about to be undertaken. Information assists in co-ordination and often prevents duplication.

2. Important results of completed research. Information is necessary for our annual report.

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[*] Committee on Research:

John B. Whitehead, Chairman
F. W. Peek, Acting Chairman
Edward Bennett,
V. Bush,
E. H. Colpitts,
E. E. F. Creighton,
W. F. Davidson,
W. A. Del Mar,
B. Gherardi,
V. Karapetoff,
A. E. Kennelly,
M. G. Lloyd,
C. E. Magnusson,
Harold Pender,
E. W. Rice, Jr.,
D. W. Roper,
Clayton H. Sharp,
C. E. Skinner,
Harold B. Smith,
R. W. Sorensen.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

3. Suggestions of important problems for research. The Committee has frequent opportunities for suggesting promising problems for experimental attack.

4. Any method or occasion which suggests itself for obtaining important research papers for the Institute.

President Chesney has done much to stimulate research in his inspiring talks at our various Sections in the United States and Canada. While he has pointed out clearly the necessity of pure research and adequate support of our colleges, he has not let us forget that the work cannot go on by research alone. Combined with this research spirit of adventure, there must be the spirit of cooperation and willingness of standardization at the proper time. Strange as it may seem, research and standardization must go together if the maximum prosperity is to be attained. It is well, in closing, to place the engineer by quoting from President Chesney's Chicago talk:

"The electrical engineer's hopes and aspirations turn him to the future. His spirit is essentially the spirit of progress, and, while he reveres the accomplishments of the past in which figures of men were more conspicuous than events, the changing conditions of modern day civilization still leave him the embodiment of progress. Complacent satisfaction with things as they are has ever been foreign to his nature; he is constantly striving for improvement."

F. W. PEEK, Acting Chairman.

Discussion

F. C. Caldwell: There are two types of minds that are particularly adapted to the training for research,—on the one hand, the student who is distinctively scientific, who has the real research spirit, the desire to know, the scientific curiosity,—on the other hand, the inventor, the man who has ideas that seem, to him at least, to be inventions of importance.

The man of the first type is interested only moderately in the money side of his work. The man of the second type usually has in mind that financial return his invention will bring to him and often vastly over-estimates its money value.

Such a student needs to have impressed upon him the real situation with regard to invention and development, the problems and the difficulties involved and the value of team work in this connection. He may thus come to appreciate the advantage of working cooperatively in an organization rather than trying to play a lone hand.

The other student, the one who is of the scientific research type, wants to know that he will earn a good living, assuming a reasonable degree of success, and that he will not have to worry too much about the cost of living, if he trains himself for and devotes himself to research.

Recently one of the large electrical organizations sent us a curve showing the average salaries which the graduates of our institution were earning at various intervals after leaving college It was a very interesting curve, and quite encouraging. It showed that for a man who was satisfied with a comfortable living, that was a good organization to join; little prospect of making a fortune, but an opportunity to live a comfortable life.

It would be interesting if we could have such a curve for research workers. A curve like this, if it shows such an upward slope as I believe it will, should be very encouraging to would-be research workers.

Another point,—often when a university teacher does develop some research ability, he is taken out of teaching work and absorbed by industry. That is a real difficulty, but we certainly would not want to eliminate it. The loss of such men from education is a hard problem to solve. Perhaps there is no solution for it, until the time comes, if it ever does, when teachers will be paid salaries commensurate with those that the same men get in industry.

R. W. Sorensen: The educational problem in our engineering colleges up to the present time has been largely with undergraduate students whose college life is limited to four years. A few engineering colleges are extending their courses to five and in some instances to six years. This seems to indicate that in the near future there will be available in the colleges graduate students who will have the required preparation and time to do research work.

Just at present the scientists are ahead of the engineering students in their ability to raise finances for pure research work as compared to applied research work or engineering research work.

In those colleges with which I am acquainted a pure scientist research teacher may have one course to teach, and usually that is just a lecture course, or perhaps he has only a few research men with whom he holds conference regarding their research problems. Engineering teachers for the most part have to devote about half their time to teaching; some of them also have to devote a large share of time to administrative work to keep a department going, in addition to being expected to try to get in a bit of personal research work and at the same time inspire students' work of a research type.

In engineering as well as in pure science we must find a way to make available a large number of fellowships which capable men may make use of while doing research work and studying for a Ph. D. degree in engineering. At the present time the few available fellowships of this nature make it necessary for engineering students who are not members of fairly well-to-do families to find all kinds of outside work in order that a few dollars may be earned to keep soul and body together.

Another factor is, only a few of the industries of today have expressed a willingness to pay for increased engineering training beyond that required to obtain a Bachelor's degree. I think it can well be said the few who have been liberal in paying men who have been worthy of doing graduate work have found the increased pay a profitable investment. Another factor is that engineering research men have been directed toward commercial problems because of larger pay frequently given to commercial men.

If colleges are to undertake engineering research problems they must also give considerable attention to the study of patents, how to obtain them, and the ownership of patents resulting from research work in college laboratories. It is my opinion that for the best results, patents thus developed should be the property of the educational institution where the work is done, but the institution must, on the other hand, do as the industries have done, recognize patentable features resulting from research work as worthy of being the basis for financial compensation to the workers who make it possible to obtain patents. Inasmuch as educational institutions have limited funds available for taking out patents, industries, who after all will get the return by manufacturing the goods, must be very liberal in assisting the educational institutions in developing and patenting new devices.

In fact, the whole problem is intricate and requires a great deal of study, but that study is inevitable because our technical problems are reaching a state where they can be analyzed and extended further only by engineers who have a thorough understanding of modern physics and mathematics; that is, the engineering profession has become so thoroughly a language of physics and mathematics that four years of training is insufficient for the man who wishes to be in the forefront of technical progress. To make it possible for colleges to turn out such men the industries and colleges must work out a new plan for financing men qualified to do graduate work. The colleges must see to it that they admit to graduate work only men peculiarly fitted for it.

Some criticism has been made regarding the number of papers coming from our college research laboratories. I am quite convinced, however, that the colleges are producing results commensurate with the amount of money spent in their laboratories equal to that in the industrial laboratories. Our own experience has been such as to indicate that often research problems useful to industries can be carried on by students in a college laboratory as well as by a man in an industrial laboratory for much less money than the industry would require should they use the same man for the same work in one of their own laboratories.

R. E. Hellmund: I think the subject of research in colleges is a very timely one to discuss. The reason why I believe so is that there are relatively few students coming from the colleges with the intention of taking up research work or design work which is similar to research. I have for many years interviewed most of the students coming to the Westinghouse Company to take up engineering work, and until recently the large majority of them showed a preference for application work and but very few came with the idea of going into design or research.

We, of course, have given this a good deal of thought, because a manufacturing company is primarily in need of design engineers. The usual idea that the compensation may be the reason for the existing condition can hardly apply because the salaries of research and design engineers are in general the same as those of application engineers. I believe the real reason is that the colleges do not instill into the students a desire to do this kind of work. I have found, for instance, that in some schools the students during their senior year do a number of design calculations for induction motors, generators, transformers, etc. They are given formulas for this purpose, and in using them make numerous mistakes. Because of their inexperience, they do not discover such mistakes until well toward the end of their calculations, and as a consequence they are obliged to carry through the slide-rule work many times. As a matter of course, they become thoroughly disgusted with the work and make up their minds that this is not the kind of thing they wish to do all their lives.

With the realization by the schools that it is impossible to give the students a complete knowledge of design, there has for several years been a tendency to eliminate design activities entirely and merely equip the students with a fundamental knowledge. This point of view has also found support with the representatives of the industries, who felt that if the student had a thorough knowledge of fundamentals, the industry could teach him the particular design knowledge required. This may be true, but it must not be overlooked that even the fundamental knowledge is merely a tool and that the real function of the research man and the designer is to create. In other words, all the fundamental knowledge is of no use if the design engineer does not have the desire and ability to create, and I therefore do not believe it is safe to keep a student in college for four years merely absorbing knowledge and without continually fostering in him the desire to create something himself. Unless this is done, the student is not properly developed for his future work, nor will he be interested in design or research work upon entering industry.

It is, of course, impossible to teach all varieties of design and research during the limited time of a college course. However, I consider it essential to keep the student engaged in some kind of design work throughout the entire course, without any attempt to cover the entire field. Starting with the simplest kind of problems, more difficult ones may follow later, and care should be taken at all times to utilize and apply in this connection the knowledge acquired in the fundamental studies.

I fully agree with the previous discussors that additional financial resources are desirable in connection with the research work of the schools, but I also believe that a great deal can be accomplished simply by keeping the student interested in design and research work and continually fostering in him a desire to exercise his creative ability.

W. A. Del Mar: There is one observation I should like to make about training of research men which I believe has never come up in the discussion between the engineers and educators; that is, the importance of training men in manual dexterity, both for making instruments and experimental "set-ups," and for their manipulation.

Success in research work depends very largely upon the dexterity of the worker in making, assembling, and using apparatus. The man who depends entirely upon ready-made commercial instruments or who has to wait for the commercial concerns to make his instruments will not get very far in research work.

There is one other point in connection with training of research men, and the development of research mentality. I do not think there is a single research mentality. We need two different types of men in the research laboratory, one having reliability in making measurements, and the other having extraordinary coordinating power, bringing together the results of tests and uniting them together into something new. I think those are two entirely distinct types and ought to be recognized as such by the educators.

J. Tykocinski-Tykociner: (communicated after adjournment) Research becomes more and more the result of cooperation instead of the effort of a single individual. With the growing complexity of engineering problems, the number of individuals participating in the solution of a particular problem will naturally increase. This development, conspicuous in the industrial laboratory, will have to be met also in the colleges. The development of theory, experiment, and equipment forms the three special divisions of research activities in which the individual research workers will have to specialize according to their natural aptitude. The type of student gifted in theoretical formulation of problems finds ample opportunities of preparing himself and developing for research work. Originality and knowledge of mathematical methods is all that is required for his future success.

However, the gifted experimenter, whose chief aim is to apply his creative faculties to the development of new methods of investigation, the design of new apparatus, and to the surmounting of innumerable difficulties presenting themselves in the course of research work, finds himself limited by the lack of talent to do things requiring skilled manual work. An inventor of new scientific methods and devices will rarely be sufficiently skilled in mechanics to produce the actual experimental apparatus and precise measuring instruments he requires. In fact he will find in most cases that there is no mechanic available at the college who could do all the work he needs. If he can get the mechanic to do work on his apparatus, he will find him a dutiful craftsman but without initiative or understanding of the real needs for which the apparatus is to be built. Colleges as well as industries must face the fact that the old generation of good mechanics adapted for the varied auxiliary research work is vanishing and that the present sources for supplying trained men are inadequate. Continued progress in research calls for the creation of an efficient professional body of mechanics especially trained for research work. The research mechanic of the immediate future will regard his vocation as a fine art and he will acquire all the knowledge, training, and skill in a special department of an engineering school to be created for this purpose. The research mechanic will actively cooperate with the theoretical investigator and with the experimenter. His aim will be to produce and to improve the equipment required for the solution of the various problems. All three must have an understanding for one another's part of the common work. The work of each must be regarded as an equivalent contribution. Each of them will participate with his particular ingenuities for a common achievement. Cooperation of this kind and the organization of special courses for education and training research mechanics seem to me to be the factors which will further research and which should be stimulated at the present stage of development.

Electrophysics

Annual Report of the Electrophysics Committee*

To the Board of Directors:

The general views of the chairman of the committee will be found in the editorial entitled *Relationship between Physics and Electrical Engineering*, in the JOURNAL for February, 1927. This editorial may be considered as part of this report. During the year, some of the members of the committee volunteered to watch new developments in the following topics: Ferro-magnetism, theory of mapping of fields, short-time phenomena, high-voltage research, insulation and dielectrics (solid, liquid, and gaseous), arcs and discharges, short-wave propagation, atomic physics, spectroscopy, quantum theory, and surges. This list will give an idea of the scope of interest in electrophysics. Several manuscripts submitted to the Institute were read and passed upon by the committee.

The committee has felt that a constant influx of new ideas from the field of pure physics to Institute membership should be carefully maintained, in order that the profession might promptly take advantage of new discoveries, methods of measurement, and theories. As a partial realization of this endeavor, the committee desires to report as follows:

1. It has obtained permission from your Board to invite two members of the American Physical Society to sit with the committee. It is hoped that this arrangement will actually go into effect after August first.
2. It has obtained permission from the editor of *Physical Review* to publish in our JOURNAL abridgments of any papers appearing in the *Review*, with the usual credit. This will make it possible to note important articles immediately after their publication.
3. It has arranged with Professor K. T. Compton of Princeton University to write a paper on *The Nature of the Electric Arc* for presentation at this convention. It is hoped that another prominent physicist may be secured to address our next Winter Convention and that such addresses may become a regular practise in the future, at least at Winter Conventions.

PAPERS PRESENTED

The following papers and articles, which appeared in the JOURNAL during the period covered by this report, will give an idea of the range of topics in electrophysics covered. While not all of these papers were presented under the auspices of the Electrophysics Committee, they all lie within the scope of the committee's interests. The pages refer to the JOURNAL.

1926

Temperature of a Contact and Related Current-Interruption Problems, J. Slepian, October, p. 930.

Measurement of Transients by the Lichtenberg Figures, K. B. McEachron, October, p. 934.

The Space Charge That Surrounds a Conductor in Corona at 60 Cycles, J. S. Carroll and H. J. Ryan, November, p. 1136.

Coolidge's Cathode Ray Tube, November, p. 1143.

Vacuum Switching Experiments, R. W. Sorensen and H. E. Mendenhall, December, p. 1203.

1927

Frequency Measurements with Cathode Ray Oscillograph, F. J. Rasmussen, January, p. 3.

Maxwell's Theory of Layer Dielectrics, F. D. Murnaghan, February, p. 132.

A New Electronic Rectifier, L. O. Grondahl and E. H. Geigher, March, p. 215.

Space Charge and Current in Alternating Corona, C. H. Willis, March, p. 272.

Oil Breakdown at Large Spacings, D. F. Miner, April, p. 336.

Graphical Determination of Magnetic Fields, A. R. Stevenson, Jr., and R. H. Park, Winter Convention. E. E. Johnson and C. H. Green, June, p. 583.

ELECTRICAL DISCHARGES IN GASES

Disruptive Discharges. Using a development of Townsend's theory of ionization by collision, it has been proved possible to design electrode shapes of high-voltage spark-gaps so as to obtain minimum electrode size. "No time-lag" electrodes so designed are much smaller than corresponding "no time-lag" spheres, (Rogowski and Rengier, *Archiv. f. Elektrotech.*). A study of time lag of spark-over of gaps has shown that in many cases the duration of the lag is a matter of chance and must be dealt with statistically, (McEachron, A. I. E. E. JOURNAL; Braunbek, *Zeitsch. f. Physik*). The influence of the state of electrode surfaces on time lag has been confirmed, (Burawoy, *Archiv. f. Elektrotech.*). The Townsend theory of ionization by collision seems to be inadequate for explaining the shortness of the time lag, (Rogowski, *Archiv. f. Elektrotech.*). Progress has been made in the theory of lightning, (Simpson, *Proc. Roy. Soc.*; Dorsey, *Frank. Inst. Jl.*); experimental work has been done aiming to discover the manner of striking of lightning, (Peek, *Frank. Inst. Jl.*).

Corona. Progress is being made toward a rational theory of corona, and the influence of space charge in the discharge is being taken into account, (Carroll and Ryan, Jl., A. I. E. E.).

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*Committee on Electrophysics:

V. Karapetoff, Chairman
Carl Kinsley, Secretary
Oliver E. Buckley,
V. Bush,
J. F. H. Douglas,
Charles Fortescue,
Carl Kinsley
W. B. Kouwenhoven,
K. B. McEachron,
R. A. Millikan,
J. H. Morecroft,
Chester W. Rice,
J. Slepian,
Harold B. Smith,
Irving B. Smith,
J. B. Whitehead.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

Arcs. An improved equation for the volt—ampere characteristic of an arc, with one constant of the equation directly related to the boiling point of the anode, has been developed, (Nottingham, *Phys. Rev.*). Experimental and theoretical evidence has been produced for the existence of cold cathode arcs, (Newman, *Phil. Mag.;* Slepian, *Phys. Rev., Frank. Inst. Jl.*).

Miscellaneous. Important formulas have been theoretically derived and experimentally confirmed for the properties of electrodes immersed in gaseous electrical discharges, (Langmuir and Mott-Smith, *Phys. Rev.*). Cathode rays due to very high voltages have been brought outside the vacuum tube, and have produced strange phenomena, (Coolidge, *Frank. Inst. Jl.*).

SHORT-TIME PHENOMENA

Much progress has been made in developing instruments for the determination of the characteristics of short-time electrical phenomena. Through the use of the modern cathode ray oscillograph of the Dufour type, it is now possible to determine the relationship between time, voltage, and current for any device operating under transient conditions, such as a lightning arrester.

Heretofore, it has only been possible to calculate wave fronts, and such calculations were of course limited to the assumptions made and frequently gave results considerably in error because of the presence of unsuspected oscillations. The cathode ray oscillograph has been used successfully for transients whose crest voltage was attained in one ten-millionth of a sec.

The propagation of waves over circuits and effects of reflection points, the breakdown of insulation, and other similar problems, are being actively studied with the help of this oscillograph. It is also being used to study the effects of lightning on transmission circuits, and gives for the first time an opportunity for the determination of the character and form of transients due to lightning.

For the purpose of making field studies of transients and transmission lines, surge recorders based on Lichtenberg figures have become of very great importance. The magnitude, polarity, and frequency of occurence of disturbances have been determined for many systems. These data have added greatly to our knowledge of phenomena for which there were only few quantitative data in the past. The measurements cover a very wide range of voltage, having been made on practically all ratings, from telephone circuits to 220-kv. transmission systems.

The surge recorder is also very useful in the laboratory as a device for measuring the potential of transients, without drawing appreciable energy from the test circuit. Many investigators, both here and abroad, are now studying the phenomena of lightning with renewed interest, and considerable benefit is certain to come from these studies.

FERROMAGNETISM

During the past year notable advances have been made in our understanding of ferromagnetism, principal among which may be noted:

1. Studies of single crystals of iron and of nickel have shown remarkable magnetic properties different from those obtained in multi-crystalline materials. The principal contributors in this field have been Honda, Webster, Gerlach, and Sucksmith.

2. Studies of magnetostriction in permalloy by McKeehan have indicated that this phenomenon plays a much more important role in ferromagnetism than had previously been appreciated, and that hysteresis is very definitely related to magnetostriction. Work by Wedensky and Simanow and by S. R. Williams confirms the existence of such a relation.

3. Studies have been made on the magnetic properties of iron and magnetite at radio frequencies by Wait. His results throw doubt on the results previously reported by Arkadiew and his collaborators.

4. Studies of the specific heat of ferromagnetic metals by Sucksmith and of the closely related magneto-caloric effect by Weiss throw some light on the relations between thermal and magnetic energy.

A very notable contribution to the literature of ferromagnetism which is of particular interest to electrical engineers is Thomas Spooner's book on the properties and testing of magnetic materials.

HIGH-VOLTAGE RESEARCH

In the realm of high-voltage research, steady progress has been made. Corona has been studied up to potentials of 1,000,000 volts. Spark-gap measurements have been extended to 2,000,000 volts. The past year saw the first 220-kv. transmission system in the East put into operation (in Pennsylvania).

The use of the ground wire seems to find increasing favor as a means of decreasing the number of serious impulses occurring on transmission systems during lightning storms. Laboratory tests have shown quantitatively the benefits to be derived from the use of the ground wire under various conditions. The shielding of buildings and other structures from the effects of lightning has also been studied.

Several high-voltage laboratories are now available for research and test purposes, the latest being the new 2,000,000-volt Ryan Laboratory at Stanford University.

GENERAL PROGRESS IN ELECTROPHYSICS

With the discovery of X-rays and radioactive substances, some 30 years ago, the progress in physics has been phenomenal, and it is not possible even to enumerate briefly the important contributions which have been made within the last year or two, especially in the domain of our knowledge of the fine structure of spectral lines. These contributions follow the trend of ideas, previously established, with respect to the individual electronic orbits which determine the atomic and molecular structure of matter and the ultimate nature of various forms of radiation and other forms of energy.

An authoritative and monumental work on modern physics is now appearing under the title, *Handbuch der Physik*, edited by Geiger and Scheel, in 24 large volumes (Springer, Berlin). Scores of prominent physicists are contributing to this work. Numerous special books are also available on branches of physics of interest to our profession, such as X-rays, spectroscopy, ionization, dielectrics, photoelectricity, molecular structure, radiations, chemical physics, etc.

Those who wish to get a bird's-eye view on modern developments in physics should read Darrow's "Introduction to Contemporary Physics" (Van Nostrand) and his serial articles in the current issues of the *Bell System Technical Journal*. See also R. A. Millikan, "The Last Fifteen Years of Physics," *Amer. Philos. Soc. Proc.*, 65.2, pp. 68-78, 1926.

By following certain portions of *Science Abstracts*, Section A, one may readily keep in touch with the progress of any particular branch of physics in which one is interested. For work done in this country, the abstracts of papers presented before the American Physical Society and published in its bulletin should be consulted. This bulletin appears separately and is also subsequently reprinted in the *Physical Review*. Publications of the National Academy of Sciences, Franklin Institute, American Philosophical Society, Bell System, etc., will also be found useful.

VLADIMIR KARAPETOFF, *Chairman*.

Discussion

R. W. Sorensen: The 1927 Pacific Coast Convention papers will include reports of work done in a study of the characteristics of lightning and devising means of protecting oil reservoirs against lightning strokes. One of the two papers to be presented at that time is the result of work done in the research laboratories of California Institute of Technology, a group of people who wish protection for oil-storage reservoirs providing funds for this definite piece of research work. These funds have been used up and certain protective plans for oil reservoirs prescribed, but there remains much to be done in making a comprehensive study of lightning and protection against it. Also this is one of the types of problems which, if properly financed, could well be carried out by one or more college staffs. Research on such a problem should not be limited to one or two laboratories and their respective groups of research men, but we should have a number of field observation crews and a number of laboratory groups making studies about lightning. It is my hope that a way will be found for men interested in the subject of lightning and provided with high-voltage equipment to do experimental work at our colleges in order that we may train students for this kind of work by permitting members of the faculty and student body to have a part in the work.

I am much encouraged by the work already done. Observers are coming more nearly into accord as to what lightning phenomena are and how to protect against the destructive nature of lightning; but may I, in closing, urge that a comprehensive program be financed in such a way as to enable several college laboratories, as well as the high-voltage laboratories maintained by industrial organizations, to have a part in carrying out these investigations.

A. I. E. E. Standards

Annual Report of the Standards Committee[1]

To the Board of Directors:

The Standards Committee has continued actively the revision of the Standards of the Institute, both as to form and content. In 1922, the Institute published its Standards in a volume of nearly 200 pages, divided into 26 chapters, with an appendix in which are reprinted the Rules for Electrical Machinery of the International Electrotechnical Commission. Soon after this edition was issued the Standards Committee was reorganized by the Board, and a revision was begun which involved the splitting up into separate sections the existing Standards, rearranging the material, and bringing together into separate complete compilations, the definitions, service conditions, rating, heating, dielectric tests, markings, and other requirements applicable to any particular type of apparatus or brand of the art.

Thirty sections have been definitely projected of which 23 have been issued; 7 are in preparation. Of the 23 issued 2 are reprinted without change from the 1922 edition of the Standards, but one of these sections is now in a revised report form for comment and criticism, and will shortly be issued. Five of the revised Standards have been approved as American Standards by the American Engineering Standards Committee, and other sections are now before that committee for consideration. A complete list of Standards adopted and in preparation is given below as an appendix to this report.

The revised form of the Standards has justified itself in the ease with which revisions may be made in the published sections. The sections are published in comparatively small editions, and revisions are readily made and a new edition printed without much expense or difficulty. Several of the Standards have been revised in this way. Monthly lists of the Standards available are published in the JOURNAL of the Institute.

The Standards Committee is charged with the coordination of all the standardization activities of the Institute. The Committee has made recommendation to the President and Board of Directors upon all cases of representation of the Institute upon Sectional Committees working in accordance with the procedure of the American Engineering Standards Committee; upon acceptance of and requests for sponsorship by the Institute; and has cooperated very closely with the United States Committee of the International Electrotechnical Commission. The cooperation of the committee with the technical committees is being made closer and more effective. The chairman of each technical committee, or a member of the committee designated by the chairman, is a member of the Standards Committee. Several of the Standards have been formulated by technical committees and accepted by the Standards Committee, and in other cases subcommittees of technical committees have been made the working committee on specific standardization projects.

By direction of the Board, certain Standards have been translated into the Spanish language under the very able supervision of Past President Mailloux. The translation of 19 of the standards has been completed and 13 have been published by the Bureau of Foreign and Domestic Commerce of the U. S. Department of Commerce. By July 1, it is expected that all of the translated sections will be in print. The Spanish edition is printed in the same style and in the same size as the English edition. The translation has been received with interest by engineers in Spanish-speaking countries of South America. The Institute is very much indebted to the Bureau of Foreign and Domestic Commerce for the excellent manner in which the Spanish text has been published, and for the fine cooperation that exists between the Bureau and the Standards Committee. American electrical manufacturers will no doubt find this Spanish edition of considerable value in business relationships in Spanish-speaking countries.

The committee has cooperated to the fullest extent with the American Engineering Standards Committee, the International Electrotechnical Commission, and the Standardization activities of other organizations. In the formulation of Institute Standards, the committee has endeavored to enlist the fullest cooperation of all other organizations interested in electrical standardization. Standardization work as now organized in the electrical field, is somewhat complex, and there is of necessity a certain amount of unavoidable overlapping. It is believed, however, that very good progress is being made as represented by the present Standards of the Institute.

Sections of A. I. E. E. Standards

No. 1 General Principles upon which Temperature Limits are Based in the Rating of Electrical Machinery.

5 Standards for Direct-Current Generators and Motors and Direct-Current Commutator Machines in General.

7 Standards for Alternators, Synchronous Motors and Synchronous Machines in General.

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[1] Committee on Standards

J. Franklin Meyer, Chairman, Bureau of Standards, Washington, D. C.
H. E. Farrer, Secretary, 33 W. 39th St., New York.
H. A. Kidder,
A. M. MacCutcheon,
F. D. Newbury,
H. S. Osborne,
F. L. Rhodes,
L. T. Robinson,
C. E. Skinner,
W. I. Slichter,
R. H. Tapscott.

Ex Officio

Chairmen of Working Committees.
Chairmen of delegations on other standardizing bodies.
President of U. S. National Committee of I. E. C.

*8 Standards for Synchronous Converters.
9 Standards for Induction Motors and Induction Machines in General.
10 Standards for Direct-Current and Alternating-Current Fractional Horse Power Motors.
11 Standards for Railway Motors.
13 Standards for Transformers, Induction Regulators and Reactors.
*14 Standards for Instrument Transformers.
*15 Standards for Industrial Control Apparatus.
16 Standards for Railway Control and Mine Locomotive Control Apparatus.
19 Standards for Oil Circuit Breakers.
22 Standards for Disconnecting and Horn Gap Switches.
30 Standards for Wires and Cables.
33 Standards for Electrical Measuring Instruments.
34 Standards for Telegraphy and Telephony.
36 Standards for Storage Batteries.
*37 Standards for Illumination.
38 Standards for Electric Arc Welding Apparatus.
39 Standards for Electric Resistance Welding Apparatus.
41 Standards for Insulators.
*42 Standard Symbols for Electrical Equipment of Buildings.
*46 Standards for Hard Drawn Aluminum Conductors.

Sections in Preparation

No. 2 Standard Definitions and Symbols.
4 Standards for the Measurement of Test Voltages in Dielectric Tests.
20 Standards for Air Circuit Breakers.
26 Automatic Substations
28 Standards for Lightning Arresters.
45 Recommended Practise for Electrical Installations on Shipboard (Marine Rules).

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*Approved by A. E. S. C. as American Standard.

High-Frequency Measurements

Report of Committee on Instruments and Measurements

To the Board of Directors:

Three branches of the field of electrical measurements in which the demands of industry have stimulated new, improved and more precise methods and means of measurement are:—electric power and energy, dielectrics, and high frequency. The first two items were covered in the report of the committee for 1925-26 and in the symposium on dielectric measurements conducted at the Niagara Falls Regional Meeting of the Northeastern District, May, 1926.

HIGH-FREQUENCY MEASUREMENTS

This year's activity has been focussed on the matter of measurement of high-frequency quantities arising principally in the field of carrier telephony and radio. The committee, functioning largely through a subcommittee consisting of Messrs. H. M. Turner, Chairman, E. D. Doyle, Melville Eastham, W. N. Goodwin, Jr., and B. W. St. Clair, arranged for the presentation of a series of papers at the Pittsfield Regional Meeting of the First District (May 25-28, 1927).

A list of these papers and a résumé of the information in these papers are included as part of this report under the heading "Symposium on High-Frequency Measurements."

ELECTRICAL MEASUREMENT OF PHYSICAL VALUES

The committee has also continued, through a subcommittee of one, (namely Mr. P. A. Borden), the extension of the bibliography of articles in other periodicals dealing with the application of electrical methods to the measurement of other than purely electrical quantities. This bibliography is submitted as part of this report.

REMOTE METERING

There has been formed this year a new subcommittee to survey the field of distant indications of electrical quantities. This committee consists of Messrs. E. I. Rutan, R. T. Pierce, and P. A. Borden, and it will report at a later date.

A. E. KNOWLTON, *Chairman.*

SYMPOSIUM ON HIGH-FREQUENCY MEASUREMENTS

The following article consists of a résumé of a series of fifteen papers dealing with measurements at high frequencies. The study of this subject, the preparation of these papers and their presentation, will constitute the major activity of the Committee on Instruments and Measurements during the year 1926-27. The papers are to be presented at the Regional Meeting in Pittsfield, Mass., May 25, 1927. Complete copies may be obtained from Institute headquarters. The papers are as follows:

1. *Notes on the Use of a Radio-Frequency Voltmeter*, by W. N. Goodwin, Jr.
2. *Substitution Method for the Determination of Resistance of Inductors and Capacitors at Radio Frequencies*, by C. T. Burke.
3. *Condenser Shunt for Measurement of High-Frequency Currents of Large Magnitude*, by Alexander Nyman.
4. *Radio-Frequency Current Transformers*, by Paul MacGahan.
5. *Methods for the Measurement of Radio Field Strengths*, by C. R. Englund and H. T. Friis.
6. *The Quantitative Determination of Radio Receiver Performance*, by H. D. Oakley.
7. *High-Frequency Measurements of Communication Lines*, by H. A. Affel and J. T. O'Leary.
8. *Methods of Measuring the Insulation of Telephone Lines at High Frequencies*, by E. I. Green.
9. *High-Frequency Measurement of Communication Apparatus*, by W. J. Shackleton and J. G. Ferguson.
10. *Impedance of a Non-Linear Circuit Element*, by E. Petersen.
11. *Empirical Analysis of Complex Electric Waves*, by J. W. Horton.
12. *A New Thermionic Voltmeter*, by S. C. Hoare.
13. *The Oscilloscope: A Stabilized Cathode Ray Oscillograph with Linear Tome Axis*, by Frederick Bedell and H. J. Reich.
14. *Sensitivity Characteristics of a Low-Frequency Bridge Network*, by P. G. Edwards and H. W. Herrington.
15. *Microammeter Indication of High-Frequency Bridge Balance*, by H. M. Turner.

The committee feels that these papers reflect the latest development in the methods of measurement of quantities associated with frequencies ranging from those just above power and ordinary telephone frequencies through those used in radio communication.

The instruments commonly employed in measurements at ordinary power frequency have very definite limitations when used at the higher frequencies. Also it is a matter of common knowledge that measurements of circuit properties under the higher frequencies cannot in general, be made satisfactorily by direct determination of current and voltage drop in a series arrangement. Much of the progress in the field of high-frequency measurements has been in the direction

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927

Committee on Instruments and Measurements.

A. E. Knowlton, Chairman,
E. D. Doyle, Vice-Chairman,

O. J. Bliss,
Perry A. Borden,
W. M. Bradshaw,
H. B. Brooks,
J. R. Craighead,
Melville Eastham,
W. M. Goodwin, Jr.,
C. F. Hanson,
W. B. Kouwenhoven,
P. M. Lincoln,
W. M. McConahey,
Wm. J. Mowbray,
H. A. Perkins,
R. T. Pierce,
L. T. Robinson,
E. J. Rutan,
B. W. St. Clair,
G. A. Sawin,
I. B. Smith,
H. M. Turner,
Roy Wilkins.

of bridge modifications, adaptation of the electron tube to measurement circuits, and also the improvement of thermocouple type instruments.

INSTRUMENTS FOR RADIO FREQUENCIES

A shielded thermocouple type voltmeter for radio frequencies was described by L. T. Wilson in the 1924 TRANSACTIONS. The current consumption varies from 2 to 8 milliamperes for the conductive circuit of the instrument and a quadrature component for shield-charging of the same order of magnitude. Subsequent investigation during the development of the instrument have shown that in order that the inherent precision of the instrument shall be realized, the effect of inductance and capacitance in the connections must be avoided by observance of careful technique. Thus in measuring the effective resistance of a reactor, the low-potential section of the measuring circuit should be carefully grounded and the high-potential section kept short and clear of solid dielectrics and consequent stray loss.

In measuring the R and L of a broadcast tuning coil, for example: It is connected in series with a 500-m. m. f. condenser, a 100-milliampere ammeter and a 1.2-ohm resistor (say 2 in. of 0.0065-in. manganin) the latter serving as conductive coupler with an oscillator 10-watt or larger. The drop across the resistor as indicated by the voltmeter will be of the order of three volts when the measuring circuit is tuned to sharp resonance. The effective resistance of the coil is less than E/I by the amount of milliammeter and condenser resistances. The inductance of the coil is computed from the resonance formula and the capacity setting of the condenser. The frequency error of this device is about 1 per cent for frequencies of the order of 1500 kilocycles. (See paper No. 1.)

Circuit constants of capacitors and inductors are, in one method, of a substitution type, found by resonating them in a series circuit and using as an indicator a crystal in series with a d-c. microammeter, the combination shunted across a small inductance in the series circuit.

The resistance of an inductor is found in the value of a non-reactive resistor substituted for the inductor in a circuit tuned in both cases to resonance by adjustment of a capacitor having negligible equivalent series resistance. The inductance is found in terms of the quotient of difference and product of the capacities required for resonance. Similarly, the capacitance and resistance of a capacitor are found in terms of the change in resistance and capacitance between resonance with and resonance without the capacitor in question. (See paper No. 2).

In the measurement of high-frequency current of more than 10 amperes the hot wire instrument is not feasible because the size and resistance tend to become prohibitive. The thermocouple ammeters for larger ranges than 100 amperes become very expensive on account of considerations of skin effect and size of heating element. Iron-cored current transformers are satisfactory up to 500 kilocycles but for much higher frequencies the difficulties in design increase.

A method has been developed which employs a hot-wire or thermocouple ammeter in series with a relatively small condenser and the combination in parallel with a large condenser shunt. The error due to the thermocouple resistance need not exceed 0.5 per cent with frequencies up to 6000 kilocycles and satisfactory commercial measurements are feasible up to 60,000 kilocycles. Unit assembly of the shunt condenser readily permits the provision of several current ranges,—say 50, 100, 200 amperes, the thermocouple instrument in each case having a 0.25-ampere rating. Care must be taken to avoid losses from resonance of the closed circuit at some harmonic of the fundamental. The absolute calibration of the arrangement presents difficulty even by means of a calorimeter ammeter because of the uncertainty about high-frequency resistance and effect of distributed capacity. The condenser shunt is apparently entitled to greater confidence than a direct thermal determination. (See paper No. 3).

For the measurement of large currents at high frequency, there are also available current transformers of the through-type with secondary rated at one ampere; the indicator is usually a thermocouple type ammeter. (See paper No. 4.)

RADIO FIELD-STRENGTH AND RECEIVING SETS

The vacuum tube used as detector, amplifier, and voltmeter is the basis of sensitive comparator methods for determining radio field strengths at frequencies below 1000 kc. in the customary unit of micro volts per meter. The loop-antenna is employed in preference to the open-antenna and no indirect evidence has appeared which places in doubt the value of equivalent effective height computed for the loops used. Both the $I\,R$ drop and mutual-inductance voltage methods are employed for introducing into the antenna the sinusoidal comparison voltage; the resistance method is preferred because its reactance is of less concern than the resistance of a mutual inductance and also it serves admirably as a terminal impedance for a constant impedance attenuation network. Shielding is easier with resistance coupling.

For frequencies higher than 1000 kc. the above method becomes unworkable and a double-detection type of receiver is used after calibration as a vacuum tube voltmeter. The received field strength is evaluated in terms of three measured attenuation factors, the received signal voltage, and the loop effective height. Static energy and static "noise value" are of interest; continuous static is readily measurable in terms of the telegraph signal strength masked by it. The enormous variability of usual static has prompted measuring it by noting the gain of the receiving set necessary to maintain constant static output. A non-restoring type of deflection instrument comparable to a fluxmeter has merit in summing the received energy over a definite interval. (See paper No. 5.)

The problems of measurement of the common electrical properties of the individual elements and of circuit units of radio receiving sets having been dealt with, there are remaining those factors of set performance which differentiate sets with respect to their selectivity, sensitivity, fidelity of reproduction, and reradiation. Each of these attributes of a completed set have been reduced to a quantitative definition and measuring method which evaluates them in terms of output voltage obtained on response to the input from a controlled signal generator. Thus sensitivity is determined as the ratio of output voltage to input field strength at various output voltages and input frequencies. Dimensional analysis of the expressed ratio results in reduction to length units; therefore, sensitivities are expressed in meters. Selectivity is determined in terms of the input field strength required to maintain a constant minimum value of output voltage for the requisite range of frequencies. Quality performance is expressed as the ratio of output voltage at the various modulation frequencies, the antenna voltage and degree of modulation being maintained constant. Radiation is expressed in meter-amperes, the meters being the antenna height and the amperes that value of current required to establish various output voltages in a detector of known sensitivity when the latter is supplied with the radiation output of the receiving set. (See paper No. 6.)

TELEPHONE CARRIER-FREQUENCY MEASUREMENTS

In the field of telephone carrier frequencies, the line characteristics of chief interest are attenuation, impedance, and cross-talk for frequencies up to about 50,000 cycles. Apparatus for field and laboratory measurement of these quantities has been developed and standardized on a unit basis. The units consist of oscillator, detector-amplifier, impedance-bridge, thermomilliammeter, variable attenuator, cross-talk set, and frequency meter. The oscillator is a vacuum-tube and tuning circuit giving 0.4 to 0.7 watts maximum at frequencies from 100 to 50,000 cycles and above 3000 cycles has no harmonics of more than 10 per cent of the fundamental amplitude. The detector-amplifier is adapted to both aural and visual balancing or indication. The impedance bridge is of the balancing or differential coil type. The thermomilliameter carries its own d-c. calibrating circuit and provides for the use of three thermocouples of a range of characteristics to cover a current range from 0.2 to 50 milliamperes. The attenuator is a network of known loss and terminal impedance and the cross-talk-set is a similar attenuator adapted to cross-talk measurements of the order of 10^{-6} times the transmitted currents. The frequency meter is a resonance bridge. Attenuation measurements made on the current-transmitted versus current-received method are possible for energy ratios up to 30×10^6 to an accuracy of about 3 per cent. Impedance measurements are of importance in connection with non-homogeneous lines and these are generally made on the line after terminating it in its characteristic impedance, usually a resistance of about 600 ohms; the results indicate the efficacy of loading to meet carrier-current operation. Avoidance of cross-talk with carrier-frequency operation presents many difficulties and necessitates a highly refined system of transpositions; the cross-talk measurements made to determine the effectiveness of the transpositions are a specialized form of attenuation measurements, *i. e.*, attenuation to cross-talk must be high and to line transmission, low.

It is by such a system of measurement that a telephone circuit is tested for its quality after the necessary modifications have been made in preparation for carrier-current operation. (See paper No. 7.)

A substantial part of the increased attenuation at carrier frequencies is due to skin effect of the conductors, and the leakage conductance of the insulators is found to increase rapidly with the frequency; radiation is a negligible factor. It is permissible to attribute to leakage conductance all losses except those of an $I^2 R$ nature in the metallic conductors; the leakage conductance, G, may of course, be derived from measurements of the attenuation but the line would have to be at least 100 mi. in length. A direct measurement of G on a line short enough (250 ft.) to avoid propagation effects and a phase shift of more than five degrees has been made on an experimental line with sufficient comparability to represent the shunt losses in long lines; the line contained 25 poles spaced 7 ft. apart and with 6-in. spacing of the insulators on the crossarms. A certain amount of transposition was resorted to, but the important precautions pertained to the manner of leading the conductors into the test station.

Each circuit is in effect a conductance shunted by a capacitance and thus the equivalent of the leaky condenser; the bridge for the conductance measurements is similar to those employed for the determination of the loss angle or power factor of dielectrics and condensers. The high resistance of a few insulators in parallel would appear to require a correspondingly high value of resistance in the standard side of the bridge but this is avoided by placing a condenser in series with a lesser value of resistance. A method of obtaining continuous record of d-c. leakage has been developed; a similar continuous record of the high-frequency leakage is greatly to be desired but as yet awaits solution. (See paper No. 8.)

The performance of communication apparatus depends principally upon its impedance and in the precision and routine measurement of resistance, inductance, and capacitance, standards of primary and secondary nature are necessary. The prime standards may well be resistance and frequency and the derived standards those of inductance and capacitance. Self-driven forks (calibrated by phonic wheel for 24-hr. period against Arlington time) can be maintained within 0.001 per cent of 100 cycles. Other frequencies can be compared with the standard by means of the cathode ray oscillograph. Resistance standards must have minimum and constant phase angle; 1000-ohm standards have been constructed with effective inductance not exceeding five microhenrys up to 100 kc.

Secondary standards of capacitance are made with mica dielectric impregnated with paraffin; such condensers can be obtained with temperature coefficient below 0.005 per cent per deg. cent. and with less than 0.1 per cent capacitance variation from 500 cycles to 100 kc. and phase angles less than one minute. Air condensers are feasible for the smaller capacitances. Secondary standards of inductance must be constant and preferably with small external field. Air-cored standards of large inductance involve considerable distributed capacitance; on this account cores of permalloy have been used with considerable success. Secondary standards of resistance in dial form inevitably involve more distributed capacitance than single primary standard resistances.

In the comparison of secondary standards against primary standards, methods which determinate the unknown in terms of circuit constants are preferable to those requiring the measurement of current and voltage. The bridges used must be carefully shielded and the equal ratio arm bridge is to be preferred wherever possible. The bridge circuits in use provide for impedance determinations when direct current is superposed on the high-frequency alternating current; these bridge methods also provide means of measuring flutter (telegraph impulse affecting the telephone frequency inductance of apparatus in the common circuit), transformer ratios, capacitance unbalance, attenuation and gain, and cross-talk. (See paper No. 9.)

The harmonic components of non-sinusoidal quantities create difficulties in the measurement of impedance of circuit-elements of a non-linear nature, where the ratio of instantaneous currents and potentials is not constant throughout the cycle. Vacuum tubes are non-linear as to resistance and iron-cored coils at high flux densities are non-linear as to reactance. In a-c. bridge measurements of such quantities it is found that the measured impedance depends on harmonic factors introduced from the source of supply, the magnitude of the resistance in the bridge ratio arms, the impedances of the detector and of the source of supply to the fundamental frequency and to the possible harmonic frequencies, and also upon the method used in attaining bridge balance. As for the last item, the measured non-linear impedance may well be different if balanced, in one case, against standards of resistance and inductance and, in the other case, balanced against a non-inductive resistance after establishing resonance with a standard capacity.

It is thus often essential to arrange the measuring circuit so that the impedance or other quantity measured shall be characteristic of the non-linear device and not of the bridge and supply network. The complicating effect introduced by a non-sinsuoidal impressed potential wave is readily removed by the use of a frequency-selective circuit between the source and the measuring network. The complicating effect of harmonics arising out of the non-linear reaction of the element under measurement may be suppressed in two ways,—one, a modification of the usual bridge method and the other, an a-c. potentiometer method.

In the modified bridge method, two balanced high-inductance coils with high-coupling are inserted in the 1:1 ratio bridge arms. The fundamental fluxes neutralize but the harmonic components of current encounter the series-aiding impedance and are effectually suppressed. In the a-c. potentiometer method the harmonics are suppressed by a filter of low impedance to the fundamental and high impedance to the harmonics developed in the non-linear element. Further modifications make possible the determination of the non-linear characteristics of the element without suppressing the harmonic current flow. (See paper No. 10.)

In the transmission of speech it is not only essential that the circuit possess prescribed reactions to steady state conditions but also that it fulfill certain other limitations upon transient conditions. The oscillograph is inadequate to the analysis of the couples waves encountered in, for example, the multi-channel repeater employed in carrier telephone systems and other means of analysis had to be devised. Any distortion by amplifiers or circuit elements results in the development of new frequencies that are multiples of the components of the impressed wave or are algebraic combinations of those components; these extraneous components may call for detection and measurement when their amplitude is even as low as 0.1 per cent or less of the true signal components. The heterodyne beat method is found useful in such detection and measurement; by a d-c. indicator in the plate circuit of a biased grid tube the amplitude of the d-c. component is directly determined. The same indicator will by relatively slow periodic change in deflection show by a beat method the presence of a minute component of a particular difference-frequency when the oscillator frequency is brought close to the frequency of the component. The method does not lend itself readily to a quantitative determination however.

Practically all analyzers for waves of small amplitude are modifications of the elementary form in which a selective circuit couples a vacuum tube amplifier to the circuit under investigation. Whether the voltage drop across L or C be chosen for application to the grid of the detector depends upon the frequency of the components sought; L for low, and C for high frequencies. The procedure is tedious and long if a wide range of frequencies is sought and there has been developed a device for automatically tuning over the desired range and automatically recording the amplitudes of discovered components. Means are also available for examining the variation of a single component of a complex wave as conditions affecting it are varied; the required selectivity is attained by employing several analyzers in tandem.

For more exacting requirements even the above method is inadequate and for such cases a heterodyne analyzer has been developed; thereby the frequency range to be examined is translated to a lower position of the frequency scale with the advantage of greater fractional separation between components. (See paper No. 11.)

INSTRUMENTS FOR MODERATELY HIGH FREQUENCIES

A vacuum tube voltmeter has been developed in which the plate impedance forms one arm of a Wheatstone bridge. With zero voltage impressed upon the grid-filament circuit of the tube the bridge is initially balanced by means of an adjustable resistance, the bridge indicator then reading zero. When an unknown voltage either alternating or direct is then impressed upon the grid-filament the plate-impedance changes, and the bridge balance is disturbed; the resulting deflection of the bridge indicator is a direct indication, after appropriate calibration, of the voltage impressed on the grid filament. (See paper No. 12.)

Professor Bedell describes a method for producing stationary curves on the screen of a cathode ray oscillograph and establishing a linear time axis which involves the use of an auxiliary circuit consisting of a source of constant voltage, a neon gas-filled lamp and an electron-tube arranged in the general form of a bridge. The voltage across a portion of this circuit, which varies directly with time, is connected across one pair of deflecting plates of the oscillograph tube and in this way establishes a linear time axis. By means of a motor-driven distributor, the other pair of deflecting plates is connected first to one part of a circuit and then to another, thus making it possible to study several phenomena simultaneously. (See paper No. 13.)

The employment of very low frequencies (say three or four cycles per second) involves in some respects as much difficulty as the higher frequencies. The problem of locating opens in telephone cable conductors involves the determination of impedances; a study has been made of the degree of accuracy and sensitivity obtainable in impedance measurement with different frequencies of supply voltage. For long cables the input impedance is a hyperbolic rather than linear function of the characteristic impedance; the error in impedance measurement arising from this functional departure proves to be least for the lower frequencies. On the other hand, the bridge sensitivity is improved by somewhat higher frequencies. A thorough mathematical and experimental analysis of the sensitivity of impedance measurement of cable fault locations up to 70 mi., by means of a de Sauty bridge, indicates the desirability of using frequencies of the order of four cycles. The sensitivity is further increased by controlling the phase of the field excitation of the bridge galvanometer. Use of such low frequencies as four cycles per second is not common and the generating apparatus, bridge, detector, and graphical treatment of errors and sensitivity of measurement of impedances at this frequency are of interest in a report on measurements under other than power frequencies. (See paper No. 14.)

The telephone receiver, due to its simplicity, sensitivity and convenience, has been widely used for determining a-c. bridge balance and under favorable conditions is quite satisfactory. The aural method, however, involving as it does the receiver associated with the ear, has two serious limitations; first, it can be used only where there is very little extraneous noise and second, the frequency range for best operation is restricted to a band of, say, from 200 to 2000 cycles unless a heterodyne scheme is adopted.

A visual method has been devised using a d-c. microammeter in the plate circuit of an electron tube rectifier, associated with one or more stages of amplification, which gives maximum reading for a state of balance, thereby permitting the use of a sensitive meter and at the same time making it fool proof. A large bridge unbalance reduces the deflection to nearly zero and as balance is approached it increases. No change in reading on short-circuiting the indicator terminals of the bridge, which would correspond to zero voltage, shows definitely a perfect balance. This method not only completely overcomes the limitations of the aural method, but also renders a quantitative determination of the degree of unbalance. (See paper No. 15.)

ELECTRICAL MEASUREMENT OF PHYSICAL VALUES

By PERRY A. BORDEN

(Supplementary Bibliography)

The following bibliography, prepared at the instance of the Committee on Instruments and Measurements, is supplementary to that accompanying the writer's paper on the above subject published in the TRANSACTIONS of the A. I. E. E. Vol. XLIV (1925) p. 238. While most of the articles referred to have appeared in the technical press during the current year, some are of earlier dates, and a few references are made to standard works on electrical measurement. The arrangement of headings has been retained as in the original paper, but the references are not numbered.

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Electrical Communication

Annual Report of Committee on Communication[*]

To the Board of Directors:

During the past Institute year, notable progress has been made in many branches of the art of electrical communication. The committee has selected for its report the items which were considered to be of most general interest and these are described under the various subheadings below.

TELEGRAPHY

In the field of printing telegraphy, there has been a considerable extension during the year in the use of the simplified tape printers on branch office circuits and in private offices of business houses.

Additional installations have been made of the automatic tape transmission system for telegraphic tickers mentioned in the 1926 report, and direct service and full market quotations have been extended through the southwest.

For trans-ocean traffic, a notable example among the permalloy-loaded cables laid during the year is the New York–Bay Roberts–Penzance cable. Multi-channel operation for direct traffic between New York and London is now in effect on this cable.

In the larger telegraph central offices, a system for automatically dispatching carriers in pneumatic tube lines is displacing manual dispatching. Space at the routing center is conserved, efficiency of operation is improved, and savings are effected in operating costs. In this system, a tube clerk drops a carrier containing telegrams for transmission to a branch office into the proper one of a group of open-end gravity tubes located in front of the working position. The gravity tubes lead to the floor below where the automatic sending inlets are located. The inlet contains a rotor which oscillates on a horizontal axis through an arc of about 70 deg. at the rate of about six times a min. The carrier enters a pocket in the rotor when it is in alinement with the gravity tube at one end of the rotor's travel. At the other end of the arc, the pocket containing the carrier is in alinement with the end of the outgoing tube. The rotors of a group of sending inlets are driven by one motor through reducing gears and crank mechanisms. Interlocking devices feed one carrier at a time into each rotor and a visible signal is provided to indicate failure of any inlet to perform its function. Automatic sending inlets act as spacing devices in the transmission of carriers and eliminate trouble from overloading which occurs with manual sending on busy tubes. The average transit time from the main to the branch office is therefore generally decreased.

The great development of message telegraph systems involves interesting traffic problems, including layout of wires, traffic routing, office layout, operator assignment, etc. These problems were discussed in an interesting paper entitled, *Telegraph Traffic Engineering,* by Messrs. H. Mason and C. J. Walbran, which was presented at the Winter Convention.

Another very important telegraph subject which was discussed at the Winter Convention is the measurement of telegraph transmission (*Measurement of Telegraph Transmission,* by Messrs. H. Nyquist, R. B. Shanck, and S. I. Cory.) This subject has been of growing importance for a number of years, partly because of the advent of telegraph circuits having a large number of sections and partly on account of the increasing importance of accurately determining the effect on telegraph transmission of various amounts of interfering currents and of changes in individual circuit elements.

DIAL TELEPHONY

The rapid application of dial telephone systems has continued. During the year about 500,000 dial telephone stations were installed, bringing the total in service in this country, as of the first of January 1927, to approximately 2,400,000.

A means has been developed for remotely operating small magneto telephone plants where this is desirable. The name applied to this means is the semi-automatic magneto exchange. In a system of this type the subscribers' stations are equipped with magneto telephones. Connections are switched remotely by means of automatic telephone apparatus under the control of an operator, who employs a dial trunk from a control center into the exchange area. Any number of such exchanges may be controlled from the operating center.

TOLL TELEPHONE SERVICE

Further progress has been made in the development and application of methods for increasing the speed of toll telephone service. The plan, the so called A-B method, of handling the large volume of messages between nearby points in a manner quite similar to that used in handling local traffic, has been considerably extended. The improvement of service has progressed to a point where even at the longer hauls it is now possible in many cases to complete a toll call while the calling subscriber remains at the telephone. An important factor in this improvement is the development of a method of operation combining the work of the line and recording operators so that if the number of the called telephone is given the recording operator, she can proceed at once with the handling of the call.

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[*] Committee on Communication:

H. P. Charlesworth, Chairman
F. L. Baer,
L. W. Chubb,
J. L. Clarke,
Charles E. Davies,
H. W. Drake,
Major P. W. Evans,
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Chester W. Rice,
J. K. Roosevelt,
H. A. Shepard,
J. F. Skirrow,
E. B. Tuttle,
K. L. Wilkinson,
F. A. Wolff,
C. A. Wright.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

A new type of toll switchboard has been made available for toll centers having sufficient traffic to require separate toll and local switchboards. The signaling equipment which was previously located in the cord circuits has been transferred to the line and trunk circuits and use has been made to a considerable extent of common positional equipment.

TELEPHONE TOLL CABLES

The year witnessed the opening on December 15, 1926, of a new long distance cable link between Chicago and St. Louis, insuring for the future the best possible storm protection for communication through from New York to St. Louis as well as between intermediate points. This cable forms a part of the network of cables connecting Chicago, Detroit, Toledo, Cleveland, Pittsburgh, and other cities with the cities on the Atlantic seaboard, and contains circuits for both very long-haul and short-haul telephone business. The longest circuits of the network are about 1500 mi. in length, but even this does not represent the maximum distance over which circuits of this type can be operated. Telephone repeaters on these circuits are spaced at intervals of approximately 50 mi. Echo suppressors are used to permit operating with volume efficiencies comparable with other long distance telephone circuits. Automatic regulators are employed to compensate for the effect of temperature in changing the attenuation of the cable conductors by suitable changes in the amplification of repeaters in the circuit.

The new link is 344 mi. in length. It provides more than 250 telephone circuits, and over 500 telegraph messages can also be sent simultaneously, making it the equivalent of 10 heavy pole lines of open wire.

The increasing development of toll cable networks is of extreme importance in the protection of telephone service from interruptions due to sleet storms. During the year 1926, about 2000 mi. of such cables were put in service in various parts of the country, adding more than 400,000 mi. to the telephone circuits of the nation.

CARRIER-CURRENT SYSTEMS

An interesting example of the rapidity with which new developments are finding their way into practical use is to be found in the extensive application of carrier-current telephony and telegraphy to the long distance open wire telephone circuits of the country. There are now of the order of 100,000 mi. of long distance telephone facilities provided by carrier methods and some 250,000 mi. of telegraph facilities so provided.

The recent growth in carrier-current circuits results from the progress which has been made in perfecting the performance of the apparatus itself and in the development and standardization of methods for coordinating a large number of carrier systems upon the wires of a pole line.

During 1926, an interesting application of carrier telephony was made on one of the two Catalina Island telephone cables. Because of the relatively short length of these cables and of their transmission stability, it has been possible to obtain in this way as many as six additional two-way circuits, making a total of seven telephone channels and one telegraph channel on a single-conductor cable. The apparatus employed in this system is similar to that used on open wire lines. By an arrangement of this character, the loss of one of the cables would temporarily reduce the number of telephone circuits only from eight to seven. This system was described in a paper by Mr. H. W. Hitchcock which was presented at the Pacific Coast Convention in September.

The use of carrier-current telephony for communication on power transmission lines has increased appreciably. The power circuits involved vary from those of 22 kv. to 220 kv. and the distances communicated over vary from a few miles to those in the order of 300 mi.

The coupling to the transmission lines which in the earlier stages of the art was effected in some cases by means of parallel wires is now being accomplished very largely by coupling condensers or capacitors which are now available in several types for voltages up to and including 220 kv. Considerable progress has been made in the development of by-pass apparatus, repeater stations, portable equipment, and various other supplementary pieces of apparatus. With the increased application of this form of communication, the demand is rapidly increasing for multiple communication channels, particularly in connection with the more extensive individual power systems where load dispatching is divided into districts and in the case of the rapidly growing number of transmission line interconnections between the large power systems.

TRANSCONTINENTAL TELEPHONY

An important event during the year was the completion of a new transcontinental telephone and and telegraph route. The new route connects Chicago with Seattle by way of Minneapolis, Bismarck, Billings, Helena, and Spokane. The through circuits comprise at the present time three telephone circuits and 14 superposed telegraph channels. The repeaters in use on these circuits represent the very latest development in this field.

There are now three transcontinental telephone routes in service, the others being routed, one through Omaha, Denver, Salt Lake City, and Sacramento, and one via St. Louis, Dallas, El Paso, Phoenix, and Los Angeles. A discussion of some of the more interesting transmission problems and other considerations which are important factors in determining the design of these facilities was presented at the Pacific Coast Convention at Salt Lake City last September in a paper by Messrs. H. H. Nance and O. B. Jacobs entitled *Transmission Features of Transcontinental Telephony.*

LOADING OF TELEPHONE CIRCUITS

An outstanding development in loading coil design which has been put into commercial use in the past year is the use of permalloy in compressed powdered form in the cores of some types of these coils. The application of this desirable magnetic material has resulted in material reductions in both the size and cost of loading coils. The lower cost of loading resulting from this and the other improvements in loading coils referred to in last year's report is, together with the large installations of toll cable in this country, bringing about a very large increase in the use of loading coils.

ELECTRICAL AMPLIFICATION

In the development and application of amplifiers, there is sometimes occasion for amplifying extremely weak signals. Considerable interest attaches to the question of what limitation, if any, is imposed on the strength of the signals that can be amplified. Recent researches have shown that the limit of amplification may be set not by noises coming from the vacuum tubes or the batteries supplying them, but instead from the internal characteristics of the electrical conductors comprising the circuit whose minute currents are to be amplified. In any electrical conductor, minute electromotive forces are continuously produced by the thermal agitation of the electrons and atoms. This is true whether or not external electromotive forces are connected to the conductor. When an electrical conductor is connected to the input of a carefully built amplifier of sufficiently high amplification, readily audible sound may be heard in a telephone receiver connected to the output of the amplifier. The fact that the noise does not come from the amplifier can readily be proved by cooling the conductor by means of liquid air, when the noise heard in the receiver immediately diminishes in intensity, the reduction in noise being due to the reduced thermal agitation in the conductor. The laws underlying this phenomenon were determined experimentally by Mr. J. B. Johnson and presented in a paper at the December 1926 meeting of the American Physical Society at Philadelphia. These laws were later deduced from thermodynamical considerations and presented by Mr. H. Nyquist at the February 1927 meeting of the American Physical Society at New York.

CHARACTERISTICS OF SPEECH

The continued researches in the characteristics of speech and hearing and the nature of vocal and musical transmission have continued to give results of great importance for the improvement of telephone service, and have also led to many noteworthy developments in allied fields. A very important development based on these researches is the combination of improved phonographic recording devices and high quality reproduction synchronized with motion pictures.

A paper by C. F. Sacia and C. J. Beck entitled "The Power of Fundamental Speech Sounds" published in *The Bell System Technical Journal* of July 1926 describes the continuing work in the study of speech power by means of the oscillograph. Sounds are considered individually on the basis of instantaneous and mean power. In earlier analyses, the principal emphasis was placed upon the power in speech as a whole.

RADIO TELEGRAPHY

Long distance radio telegraph communication is rapidly changing from long waves or low frequencies generated by alternators or Poulsen arcs to short waves or high frequencies generated by thermionic tubes. Within the last 18 months, transmitters up to 40-kw. capacity operating on frequencies of 10,000 to 20,000 kc., 30 to 15 meters, have been produced and put into service. These are replacing arc generators up to 500-kw. and alternators of 200-kw. capacity. Reliable continuous daylight communication has been obtained by using wave lengths around 15 meters, notably between New York and Buenos Aires. During hours of darkness, wavelengths from 25 to 75 meters have been in use in both transatlantic and transpacific services. The greater reliability of the short waves is the result of almost complete immunity to summer static, and the new system is much more economical because of the low power consumption compared to that used for long wave transmission.

The scope of international radio service was further extended during the year by the opening of direct radio circuits for duplex operation between the United States and Brazil.

An analytical study entitled *Behavior of Radio Receiving Systems to Signals and to Interference*, made by Professor L. J. Peters, was reported by him at a Regional Meeting of the Institute at Madison, Wisconsin, in May 1926. This paper discusses methods for studying transient effects of current in radio systems, the degree to which interference can be mitigated by frequency selection methods, and the factors determining the interference caused by transmitting stations of various types and by static.

TRANSATLANTIC RADIO TELEPHONY

An event of outstanding importance in the progress of international electrical communications occurred early in the present year with the opening of transatlantic telephone service between the United States and England. This first telephonic bond between America and Europe was opened to the public on January 7, 1927, following an exchange of brief greetings between Mr. W. S. Gifford, President of the American Telephone and Telegraph Company, and Sir Evelyn Murray, Secretary of the General Post Office of Great Britain.

Although the service was at first limited to the metropolitan areas of New York and London, during the months following, service was extended successively to greater areas until it has included most of the British Isles on the European end and the United States and Cuba on the American end. No attempt has been made to give 24-hr. service, but service has been available daily for the period which includes the overlapping portions of the business day at the two ends, and is being extended.

The principal features of the system added since the description given in last year's report of this committee are that arrangements were perfected whereby both the east-bound channel and the west-bound channel are transmitted in the same frequency band. Thus, the entire two-way system occupies only 3 kc. (58.5 to 61.5 kc.). One thing which contributed materially to this accomplishment is the employment at both terminals of voice-current operated switching devices which function to cut the transmission path to and fro from west-bound to east-bound automatically in accordance with the flow of conversation between the two speakers.

RADIO BROADCASTING

The results of investigations carried out during the last few years in ascertaining the service area for which broadcast transmitting stations are effective were summarized in a paper entitled *Radiobroadcast Coverage of City Areas* presented at the New York Regional Meeting last November by Mr. Lloyd Espenschied and printed in the January issue of the JOURNAL. This is a subject that is receiving considerable attention. These investigations raise questions regarding the desirable power levels to be used for radio broadcasting.

ELECTRICAL TRANSMISSION OF PICTURES

During the year the scope of the commercial telephotograph service which has been given for two years between New York, Chicago, and San Francisco was materially enlarged by the extension of the network to Boston, Cleveland, St. Louis, Los Angeles, and Atlanta. Several important developments have taken place during the year, notably the arrangement of the circuits for two-way operation, the installation of phase-correctors on the New York-Boston and Chicago-St. Louis telephone cables to make these circuits suitable for picture transmission, and the fitting of the southern transcontinental route to make it available for service to Los Angeles and San Francisco.

TELEVISION

On April 7, 1927, a successful demonstration was given of electrical television by wire circuit between Washington, D. C., and New York, and by radio from an experimental station at Whippany, N. J., to New York.

Television employs many of the principles and some of the apparatus of telephony. The object of television is to reproduce a scene with action, and to do this a series of essentially instantaneous views must be transmitted and reproduced at a rate, 15 or more per second, such that an observer will detect no discontinuity of action. In its present form, the sending apparatus is adapted to obtaining for one participant in a telephone conversation a continuous view of the face of the other participant. The receiving apparatus recreates this view on a picture plane about two by two and one-half in.; or, with a alternative form of apparatus, on a plane about two ft. sq. for observation by more than a single person.

At the sending end a narrow beam of light, or rather a rapid succession of beams scan the subject to be transmitted, illuminating at one time an area about a quarter of an inch square and sweeping over the entire scene in less than one-fifteenth of a sec. This scanning process is repeated continuously. A group of large photoelectric cells responds to each change in the reflected light.

At the receiving station, a glass tube filled with rarified neon gas and provided with electrodes responds with a brilliancy corresponding to the current received from the photoelectric cell. The high potential requisite to the operation of the neon tube is obtained by the use of vacuum tube amplifiers in the connecting circuit. All parts of the neon tube have the same brilliancy at any instant but the observer views only a small portion at a time, which is uncovered by the synchronizing apparatus provided to insure that the light shall appear to the observer at each instant in the same position on a picture plane as that occupied by the beam-illuminated spot of the distant scene.

In the production of the larger image, a very long neon tube is folded back and forth to form a grid. This tube is provided with 2500 electrodes along its length. Each electrode corresponds to a single elemental area of the picture plane which is scanned by the light beam of the transmitting apparatus. As the current corresponding to each area reaches the receiving station, it is distributed through contacts to the appropriate electrode and so causes a flash of light similar in location and intensity. The speed of operation causes the observer to see not a series of flashes but a picture as a whole.

NEW RECTIFIERS

A new type of rectifier suitable, among other uses, for charging batteries used in communication circuits was described in a paper by Messrs. L. O. Grondahl and P. H. Geiger presented at the Winter Convention. The rectifier consists of partially oxidized disks of copper. The rectification appears to take place at the junction between the copper and the oxide without observable physical or chemical change, and is similar in character to rectification by the hot cathode type of rectifiers.

MANUFACTURE OF COPPER WIRE

The developments of the past few years have led to great improvements in the methods of drawing copper wire, particularly in the speed of the process. Some of the outstanding features in these developments, together with a description of a copper rod and wire mill designed to meet the new requirements and a brief survey of the copper rolling and wire drawing art, are included in the paper entitled *Developments in the Manufacture of Copper Wire*, by J. R. Shea and Samuel McMullan which was presented at the Winter Convention of the Institute.

WOOD PRESERVATION

During the past year, a large amount of research work has been continued in improvements in methods of preserving wood poles, crossarms, and other timber. As a result of studies which have been made by the Western Union Telegraph Company, they are placing in service a treatment by which a solution of zinc and arsenic is forced into the poles. Their investigations indicate that on exposure of the treated wood to the atmosphere, chemical changes take place which deposit in the wood zinc arsenite, a toxic material which is practically insoluble and permanent, and that this will constitute a very effective method of preservation against decay.

FIRE-ALARM AND POLICE SIGNAL SYSTEMS

The past year has seen some further refinements in alarm signaling devices. These were chiefly along the lines of simplification and increased reliability of recording devices, improved insulation for street fire-alarm boxes, and improved protective devices for circuits entering buildings. The early fire-alarm devices were insulated against other signaling circuits only; circuits entering buildings had only the comparatively simple lightning arresters then known to the electrical telegraph art. Now, with fire-alarm circuits on the same pole lines with 2300- and 4300-volt circuits, a high grade of insulation of the associated apparatus has become necessary. Recent improvements in boxes have been made to provide this necessary protection.

There has been a considerable increase in the use of electric sirens as fire-alarms in smaller communities where the expense of maintaining normally closed telegraph circuits for this purpose would be felt as a burden. The sirens are generally operated on normally open circuits actuated from the public power supply.

In April, the seventy-fifth anniversary of the opening of the first electrical fire-alarm system in the United States and the first successful one anywhere was celebrated. An exhibition was held at the new fire-alarm office in the Fenway, Boston.

There has been a general tendency toward the adoption of the red-amber-green cycle of signals for the regulation of traffic; red to stop, amber to warn of change and permit clearing of intersections, and green to go. This three-light cycle gives opportunity for control or stopping of all wheeled traffic in congested sections while foot traffic is permitted to proceed, and for stopping all traffic during the passage of fire apparatus, police cars, ambulances, etc. There has also been a widespread tendency toward synchronizing signals along a street or throughout a district. In the town or small city, the business district is frequently concentrated along a main street. Hence it becomes comparatively easy to synchronize signals along this street so that traffic may move fairly continuously for certain intervals, during the long intervals along the main street, during shorter intervals across the main street, with still shorter intervals between these while the light shows amber to clear the traffic from intersections. In some places this control obtains only during the period of heavy traffic; at other times only flasher or caution lights are shown at certain intersections. In the larger cities, this synchronizing may cover a large congested section so that by due attention to the time intervals between signals points, traffic moving at an average rate may proceed with little or no stopping. The possible saving in traffic police and in consequent expense due to carefully planned signals of this type is evident.

H. P. CHARLESWORTH, *Chairman.*

Production and Application of Light

Report of Committee on Production and Application of Light[*]

To the Board of Directors:

In accordance with requirements of the by-laws, there is submitted herewith a review of the development during the past year of the art of lighting with electricity. This review constitutes the annual report of your Committee on Production and Application of Light. It has been prepared through the cooperation of members of the Committee.

The personnel of this Committee has been chosen with a view to insuring comprehensive consideration of the subjects lying within the purview of the Committee. Its members concur in the view expressed by the Committee to Review Technical Activities in a report dated June 26th, 1924, to the effect that this Committee should function in an "initiatory and determinative" capacity in matters pertaining to the production of light and in a "joint or reportorial" capacity in matters pertaining to the application of light. Accordingly, the Committee endeavors to maintain close touch with developments in the production of light by electricity and looks to organizations more specifically concerned in the application of light for information as to developments therein.

PRODUCTION OF LIGHT

No developments in the production of light from electricity which are new in principle or which constitute a radical improvement in the art have come to the Committee's attention during the past year. Progress in the development of illuminants described in earlier reports is reviewed briefly in the following paragraphs.

Incandescent Filament Lamps. In the following paragraphs there is presented a brief review of significant changes in manufacture and utilization of incandescent electric lamps.

The past year has witnessed the general introduction of incandescent lamps with bulbs frosted on the inner surface, announced in the report of this Committee last year. Eighty per cent of the demand for replaceable types is now being supplied by lamps of the new type.

Among tungsten filament incandescent lamps the gas-filled principle has been extended to lower wattages than heretofore employed in this type for general lighting purposes in this country, though not to such small wattages as those in which it has been sometimes employed in Europe. In the 50-watt 115-volt size the principle has been applied to what appears to be the minimum size for which its use is justifiable in the present state of the art.

Limited ability to withstand rough usage and vibration has always been a handicap of the tungsten filament lamp. Despite notable improvements by American lamp manufacturers, which have increased greatly the sturdiness of the filament, it remains true that after some hours of burning, the filament crystallizes and becomes less sturdy than the filament of a new lamp, and too fragile for some forms of service.

In an effort to meet requirements for rough service (for example, in garages, where lamps are used on portable cords), manufacturers have recently developed a "rough service" lamp. This lamp is available in the 50-watt, 115-volt range. It is of the vacuum type and has a bulb of the same size as the usual 25-watt lamp.

Where continuous, high-frequency vibration is encountered (as that due to high-speed machinery), the lamp manufacturers recommend, if small lamps must be used, the 50-watt coil filament vacuum lamp in the P-19 bulb. It is preferable, however, to use larger, more sturdy lamps, and if necessary, to employ vibration reducing devices.

Two lamps for decorative service have been made available during the year in new forms of flame-shaped bulbs. One is of 15 watts with a candelabra screw base. A similar lamp had previously been supplied in a bulb having spiral fluting. The other is a 25-watt lamp with a medium screw base. These lamps are regularly supplied with a flame tint coating to add to the effect suggested by their flame shape.

According to a recent report issued by the Lamp Committee of the National Electric Light Association, carbon lamps are being used to a surprisingly large extent in ordinary lighting service. Because of their relatively low efficiency, the cost of producing light with these lamps usually is greatly in excess of that applying in the case of tungsten filament lamps.

The trend of operating practise appears to favor 115 and 120 volts. The demand for 110-volt lamps is steadily decreasing and is now only 12 per cent of the total in the 100-130-volt range; 115 volts accounts for 48 per cent and 120 volts for 35 per cent, leaving only 5 per cent for all other voltages in this range. The total number of lamps supplied from 200 to 260 volts is now but 3 per cent of the number supplied in the 115-volt range and this percentage is gradually diminishing. The concentration of lamp demand upon the fewest practicable number of voltages is desirable as a means of eliminating needless and expensive complications in manufacture and distribution.

The 23- and 36-watt lamps for street railway headlight service are now made in the "A" shape bulb of clear glass. The change from the round bulb

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Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

(G-18½) in which they were formerly supplied, enables one standard light center length of 2 3/16 inches to replace 2 1/16 inches, 2 3/16 inches and 2¼ inches light center lengths in the old bulbs. This can be done because the shape of the A bulb permits a wider range of adjustment of lamp position in the headlight.

A special lamp has been developed for use in traffic signals. It is a 60-watt gas-filled tungsten filament lamp in a clear bulb of the shape and size used for the regular 40-watt lamp in the inside-frosted line. The filament is semi-concentrated to permit of more accurate light control by the signal lenses. The lamp has a light center length of 2 7/16 inches. It is designed for burning in either a horizontal or base down position.

To promote simplification in the line of lamps for series burning, it has been recommended that street lighting circuits now operating at 4, 5.5 and 7.5 amperes be changed over to 6.6 amperes, for which there is by far the greatest demand. The progress of standardization in this respect is slow.

There is still a demand for series burning lamps of 600 and 800 lumens (approximately 43 and 55 watts respectively) which, it is generally believed, could be replaced advantageously with lamps of at least 1000 lumens (approximately 65 watts).

Series Lamps Unsatisfactory on Multiple Circuits. Some use has been made of series lamps operated with auto transformers from multiple circuits. Investigation of this form of operation has shown that multiple lamps operated on multiple circuits are more economical and give more satisfactory performance than any of the series lamp auto transformer combinations. The series lamp is designed to burn at a constant current which means that the filament cross sectional area is very accurately determined while variations in manufacture are noted by changes in filament length. This manufacturing variation is particularly noticeable when a series lamp is burned on a constant voltage circuit. Furthermore, in the larger size series lamps, a large amount of filament material during the normal life of the lamp is evaporated and deposited on the bulb. This blackening causes a decrease in lamp candle power. When the series lamp is operated on a constant current circuit the decrease in filament area results in an increase of brightness which partially offsets the blackening of the bulb. A lamp burned at constant voltage however, suffers because of diminution of current as the lamp ages due to increased filament resistance. It is seen, therefore, that all the factors present in both multiple and series lamps which make for decrease in lamp output are combined when a series lamp is operated on a constant voltage circuit.

The lamp manufacturers recommend that multiple lamps be burned on multiple circuits and that series lamps be burned only on series circuits.

Luminous Arc Lamps. It is understood that there have been no material changes in the design of construction of the luminous arc lamp during the past year. The modern lamp, with its shorter casings and larger globes, as constructed for service in Washington and elsewhere, represents the latest development in this type of lamp which, as shown elsewhere, is employed rather extensively in street lighting service.

Ultra-Violet Radiation. Efficient production of ultra-violet radiation is accomplished by electric discharge through vapors, usually of a metallic nature. The conditions of use impose the further limitation that the source, particularly one of the arc type, be completely enclosed to prevent the egress of undesirable vapors in therapeutic work, or the ingress of inflammable vapors in chemical work. The mercury arc in quartz, being inherently an enclosed arc, is uniquely adapted to use as a source of ultra-violet radiation. Units of 450 and 900 watts capacity for operation on 110 and 220 volts respectively have met with increasing use during the past five years. The former are used largely in therapeutic work for the direct irradiation of patients in the treatment of rickets, bone tuberculosis, skin diseases, superficial infections, etc., while the latter are used for water sterilization, the testing of materials, irradiation of foodstuffs to produce antirachitic properties, the treating of leather, varnishes, etc.

The use of rare gases as an aid in the starting of discharge has recently permitted the design of a practical induction lamp having unique properties when made of quartz. It permits greater control of the relative ultra-violet energy distribution than before was possible and it is especially well adapted to the solution of many of the mechanical problems limiting the usefulness in photochemical processes of the older mercury arcs.

It is said to be possible to duplicate economically by means of these artificial ultra-violet sources practically any photochemical effects now secured through exposures to direct sunlight. This is important for sunlight, though inexpensive, is offset by the fact that it is of value only during the middle five hours of the day, if available at all.

As an example of a highly developed ultra-violet application, mention may be made of a recently designed apparatus in which, by means of a quartz mercury arc and filters, tests may be made of the light fastness of dyed textiles, inked papers, or painted woods in a much shorter time than ever has been possible by sunlight and with a quality of fading action directly comparable with that of sunlight.

APPLICATION OF LIGHT

In contrast with the relatively meager developments in the production of light, the past year witnessed a wealth of development in the application of light. Artificial light is now available at such a low cost that improvement in its utilization can be undertaken with greater freedom. In consequence the lighting art is advancing rapidly.

Residence Lighting Equipment. The past year has seen an expanding interest on the part of central station companies in residence lighting equipment. Several companies have conducted re-fixturing campaigns with marked success and many are planning such activities for the current year.

Interest in this project is evidenced by the preparation, beginning in the autumn of 1925, of tentative specifications for residence luminaires, prepared under the supervision of a committee of the Association of Edison Illuminating Companies, with which the Illuminating Engineering Society has cooperated. These specifications promote consideration of a luminaire from three principal standpoints:

1. Illuminating qualities
2. Mechanical and electrical construction
3. Aesthetic values

The application of these specifications results in a final figure of percentage which represents the over-all quality rating for each luminaire considered. Part of the determination of necessity must be arrived at by personal judgment and part by laboratory measurements. Very consistent results have been obtained by using averages of the personal judgments of several competent persons. An interesting by-product of this activity is the production of a practicable glare-gaging device, in the form used in the measurement of glare in residence luminaires.

The Home Lighting Committee of the National Electric Light Association has already prepared outlines of typical plans for such re-fixturing campaigns. Other information and material is being prepared and will be disseminated by that Committee throughout the year. The month of October has been chosen as the period of specialized activity in promoting improved residence lighting.

There is a noticeable trend in manufacturers' lines toward luminaires embodying provisions for the shading of lamps, although the development is far too slow to be considered as satisfactory. Several inexpensive devices are on the market to enable the use of shades on modern types of lamps.

Street Lighting. The attitude of the electrical industry toward street lighting seems to be undergoing a wholesome alteration. Street lighting is coming to be regarded as a phase of utility operation which is capable of becoming remunerative both to the community and to the utility. Accordingly, both the engineering and commercial aspects of street lighting are being given more intensive and forward-looking consideration than in the past. It is coming to be recognized that through comprehensive planning, coupled with general plans for city improvements and growth, economies may be had through standardization, reduced obsolescence, efficient energy distribution, and control.

It is generally recognized that street lighting deserves the accurate methods of the engineer since nationally we are confronted with the need for many extensive improvements. There is dawning a new era in street lighting, trailing the new era in vehicular traffic. Street lighting systems for the main streets of large cities, and some already in use, have been designed to deliver 50,000 to 150,000 lumens (2- to 6-kw. demand) per standard instead of 10,000 to 15,000 lumens (0.5- to 0.75-kw. demand) which was considered adequate in the past.

Progress has been made in the manufacture of equipment for remote control over existing commercial networks to widely scattered relays actuating switches to supply the street lights from existing distribution systems. Several new remote control switches, both solenoid and motor operated, have been developed to be controlled from a pilot wire.

These new equipments are suitable to supply the power for either series or multiple lamps and may be selectively arranged to disconnect alternate units during the late hours of the night when there is little traffic. These various control equipments are receiving more attention with the trend toward increased load density.

There has been considerable activity in the further development of enclosing glassware for street lamps. The tendency seems to be in the direction of a compromise between directional control and diffusion, the one intended to place the light where it is wanted, and the other to avoid excessive glare.

In overhead lighting equipments the trend is definitely toward the use of dust-proof units, to obtain a higher average efficiency between cleaning periods by avoiding the absorption of light otherwise due to the collection of dirt on the lamp bulb, and on enclosing accessories.

The Illuminating Engineering Society's Committee on Street Lighting is making progress in developing a method of appraising the qualities of street lighting to determine the relative illuminating merits of various street lighting installations.

The Street and Highway Lighting Committee, National Electric Light Association, has outlined a three-year program to cover thoroughly the sales and financing aspects of street lighting systems. It purposes issuing a manual on street lighting.

The lighting of interurban highways is slowly progressing with promise of greatly accelerated growth in the near future. Rural electrification is an interlinking factor, and nation-wide legislation providing enabling acts is the primary desideratum. There are many indications that as legal obstructions are removed, large growth in highway lighting will follow. The problem in its present status is one of legislation rather than of engineering. However, more engineering analysis and evaluation of the social economic aspects of highway lighting will hasten the required legislation.

Table I presents a partial list of intensive street lighting systems in the United States as of March 1st, 1927. Additional intensive installations are being made or are planned in several other cities. From this list it is evident that existing high intensity installations are divided between luminous arc and tungsten lamps, although present indications point to a trend toward tungsten installations in the immediate future.

PARTIAL LIST OF INTENSIVE STREET LIGHTING SYSTEMS. AS OF MARCH. 1927

	Lumens per linear foot of street*		
Chicago, State Street	2000	2-lamp	tungsten
Seattle, Metropolitan Avenue	1050	2 "	"
Jersey City, N. J.—Journal Square Plaza	857	1 "	"
Salt Lake City, Business Section	822	3 "	arc
Niagara Falls, Falls Street	761	2 "	"
San Francisco, Market Street	750	3 "	"
Schenectady, Erie Boulevard	700	2 "	"
Portland, Oregon—Business District	600	2 "	tungsten
Columbus, Ohio—Business District	600	2 "	"
Schenectady, State Street	585	2 "	arc
Los Angeles—Several streets	375 to 510	2 "	tungsten
Indianapolis, Business Section	520	2 "	"
Los Angeles, Broadway	510	2 "	arc
San Francisco, Triangle District	500	2 "	"
El Paso, Business Section	500	2 "	"
Cleveland, Superior Avenue	500	1 "	tungsten
Rochester, N. Y.—East Main and East Ave.	472	2 "	arc
Lynn, Mass.—Central Avenue	450	2 "	"
Augusta, Ga.—Broad Street	450	2 "	tungsten
Davenport, Iowa—Business Section	450	2 "	"
Syracuse, Business Section	425	2 "	arc
Boston—Several business streets	400	1 "	"
Boston—Massachusetts Avenue	400	1 "	tungsten
Lansing, Michigan—Business Section	400	2 "	"
Lawrence, Mass., Essex Street	400	2 "	"
Chicago, South State Street	400	1 "	"
Gary, Indiana—Business District	381	2 "	"
Lynn, Mass.—Business District	351	1 "	arc
Racine, Wisconsin—Business District	350	1 "	tungsten
Chattanooga Tenn.—Business District	340	1 "	"
Cleveland—Business District	333	1 "	"
Worcester, Mass.	325	1 "	arc
Utica—Business District	304	1 "	"
Saratoga, Broadway	300	2 "	tungsten
Lowell, Mass.	300	1 "	arc
Nashua, N. H.—Business Section	300	1 "	"
Providence, R. I.—Business Section	300	1 "	"

*As an index of grade of lighting "lumens per linear foot" is evidently inexact. No better basis of terse statement is, however, available for these installations.

Signal Lights for Traffic Control. The need of standardization of electric traffic signals is apparent. In some cities one may make a right or left turn on a red signal while in others such a movement is prohibited. There seems to be a divergence of opinion as to whether there should be three colors or two. A committee appointed by the governor of an eastern state has recently gone on record as approving a two-color system, while a similar committee in a neighboring state has also gone definitely on record as favoring a three-color system. Tourists traversing these two states are likely to encounter trouble.

There also seems to be a wide difference of opinion as to the proper location of signals. Some city officials prefer the pedestal type mounted in the roadway; others prefer the bracket type of suspension from messenger wire across the street. It is not unusual to hear of controversies between officials of the State Highway Department and of municipalities as to the type of signal. Most of the State Highway Departments will not permit the use of pedestal type signals on the roadway owing to the fact that if they fail to light they become a hazard.

The lack of standardization in the use of the colored lights in connection with the control of traffic has made it difficult for the police to enforce the regulations. It is needless to dwell further upon the chaotic conditions which exist at the present time. The traffic problem requires the cooperation of the architect, civil engineer, police officials, electrical engineer, illuminating engineer, transportation engineer, etc., in order that all the various phases of the problem may receive due consideration. All of these are concerning themselves with it, but often independently and without coordination.

In a recent number of the *Architectural Forum* there is an article by a nationally known architect which deals with the relation of the height of buildings and the density of pedestrian traffic upon the streets.

Out of this maelstrom of independent activity comes the announcement of the organization of a committee of the American Engineering Council, under the chairmanship of Dean Dexter S. Kimball, of Cornell University, which shall study the problem and prepare a standard code, so that when the automobilists from New York are driving in San Francisco, or vice versa, the signals will carry the same message. Such standardization now exists in the railroad industry where red to the railroad man means only one thing—danger. Yet the general public has been educated to regard the use of red light in a building as a safety exit in case of fire, or, on a street, a safety aisle. In navigation the use of red for port and green for starboard is standard the world over. Possibly after the standardized code for the control of traffic is available, action can be taken to replace the red lights in the interior of buildings as an indication of safe exits in case of fire.

Automobile Headlighting. The art of automobile headlighting is receiving a great deal more attention than many people, perhaps, know. While it cannot be said that any great developments have been recently consummated, it is still a fact that efforts by many agencies, along different lines, but all directed toward the same object, have considerably advanced the general knowledge of the subject and have brought an ultimate satisfactory situation just so much closer. Among such activities now in progress, are: recognition of the Uniform Vehicle Code, enforcement of headlight laws, activities productive of a better understanding of the nature and prevention of glare, the relation of automobile headlighting to street lighting, mechanical requirements for headlights, and recognition of the shortcomings of present equipment.

The most notable development in automobile lighting practise during the past year has been the very general adoption by manufacturers of the better grades of cars, of a changeable beam headlighting system. This has been approved by all the states as legal. A brief review of the history of this development may not be out of place here.

A number of years ago, the Illuminating Engineering Society and the Society of Automotive Engineers, working together, endeavored to improve the unsatisfactory condition of automobile headlighting by adopting a set of minimum and maximum limits covering a light distribution that would yield a maximum of good driving light and a minimum of objectionable glare. These limits were based on permanent and fixed adjustment and pointing of the headlights. In the nature of things, these limits were a compromise, but were so well worked out as to produce a marked improvement in road driving conditions after dark.

Since these limits were adopted, however, there has been a considerable change in many of the elements of the problem. The average height of the eyes of drivers above the road surface is considerably less than it was five years ago, due to the constant lowering of cars. Vehicle springs are now being made softer than they used to be. Cars therefore pitch through greater angles due to road inequalities. Furthermore, because the passenger load comes very largely on the rear springs, there is a big change in car angle under varying conditions of load. Having these changed conditions in mind, committees of the Society of Automotive Engineers and of the Illuminating Engineering Society jointly undertook to devise specifications covering the use of an alternative distribution of the headlight beam for use in passing another car. The driver is then expected to change from one type of distribution to the other as conditions demand. Experience has shown that drivers can be trusted to do this if they are provided with a reasonably good passing light.

By the use of the alternative system of distribution of the beam, two things are accomplished: first, a great amelioration of the glare difficulty is obtained by the use of the changed beam, and second, since the beam can be removed from other drivers' eyes, the adjustment of the headlamps may be such that the top of the beam is higher than with the fixed beam equipment, thus providing a better driving light where the absence of oncoming cars renders it practicable to keep the beam in its normal position.

At the present time, a joint committee, composed of members of the two societies, is conducting research to determine the proper limits to govern the new types of light distribution. Without waiting for the final results of this committee's work, many motor car companies are adopting systems in which the regular driving beam is lowered through an angle of two or three degrees in passing another car. A number of simple and economical ways are now available for accomplishing this result and the public approval of the change has been very marked

The old system of reducing the candle power by dimming is condemned by everyone and it is hoped that it will become obsolete in the near future.

Lighting of Exteriors. The advantages of flood lighting have been amply demonstrated, not only by the number of large installations during the past year, but by the diversified character of the installations. Illumination intensities have in general been higher, and larger numbers of projectors have been used on individual installations than have ever been used before.

Two state capitols, those of New Jersey and Texas, were added to the list of half a dozen or more that have been flood-lighted previously. Two large office buildings in Detroit were lighted; in one case over three hundred and the other over one hundred projectors were used. In Brooklyn over one hundred projectors were used on an office building, and in New York nearly five hundred projectors are employed to light the upper portion of a new theater. Kansas City has lighted its huge war memorial by means of searchlights; steam emerging from the top is illuminated by colored light from projectors concealed in the top.

Great interest is developing in lighting recreational areas. In order that children may be kept from the streets, school and public playgrounds have been lighted. Colleges in increasing numbers are lighting their stadiums for night football. Considerable impetus was given this movement by the success of the lighting of the stadium at the Philadelphia Sesqui-Centennial grounds.

Electric Signs for Daylight Use. Electric lighting display, having been highly developed in this country for night use, is being extended into the daylight hours.

Tubes of neon and other gases or vapors offering striking color contrast with daylight are entering into service in some sections of the country quite extensively. This follows a like development in Europe.

The diameter of the neon tubes which are ordinarily employed for signs varies from 7 to 32 millimeters (usually 11 to 15 millimeters). The characteristic orange-red color of neon predominates in displays erected to date but other colors are to be seen. These are derived from the admixture of helium, argon, etc. The characteristic radiation of mercury vapor is likewise to be seen in some signs.

The tubes vary in length from 10 to 40 feet. Starting voltages are approximately 200 volts per foot of the tube; alternating current 25 to 60 cycles is employed, rotary converters being used where the supply is direct current.

For typical tubes of 20-foot length and 15-millimeter diameter, the manufacturers state that the consumption is about 200 watts. This increases rapidly if the diameter of the tube is increased. The power factor of such a sign is stated to be approximately 50 per cent.

The usual filament-lamp electric signs are being adapted in some cases to daytime use. For this purpose an area of brightness is built up optically so that the entire surface of a letter is given the brightness of the filament itself when viewed from certain directions.

In obtaining a large area brightness, the light has been concentrated into a relatively narrow angle; hence, within this angle only is its effectiveness at a maximum. Such a sign is of greatest value when the traffic is massed within a relatively narrow viewing angle, and where people approach the sign nearly "head on" for a considerable distance. There are many such locations—atop the marquise projecting over the sidewalk; at a dead-end street; on a highway curve;—where the new type of sign may be effective.

Illuminated Bulletin Boards. The use of illuminated poster and bulletin boards is rapidly increasing and it is noted that more consideration is being given architectural features as well as the use of novel lighting, and mechanical effects.

The appearance of animation or action is sometimes accomplished through color by the absorption method. The advertisement is painted on the bulletin board with carefully selected oil colors. There are two lighting equipments, one for example, for red lighting and the other for blue-green lighting. Supply circuits are controlled by a two-circuit flasher so that by the alternate flashing of the red and blue-green lights, the fading out of certain words or images on the bulletin is accomplished. For example, the red light will apparently "absorb" the red painted images or words on the sign leaving visible only the darker colors which do not contain red. Equivalent effects are had with the blue-green.

A rather interesting mechanical bulletin board has made its appearance in the Middle West. The face of this board consists of a series of equilateral triangular members each of which operates on an axis so that the entire face of the sign revolves simultaneously. In this manner three separate advertisements can be painted on the respective faces of the triangles. An electric motor operates the mechanism so that the sign may be changed four times a minute. These displays are illuminated for night operation.

A rather effective plan has been worked out whereby the use of a modern show-window is combined with a billboard. The show-window is built flush with the face of the bulletin. The bulletin bears the usual advertising, while behind the plate glass of the show-window is arranged a display of merchandise. The face of the bulletin is lighted by means of angle reflectors in the usual method and the interior of the show-window is illuminated with show-window reflectors.

It is interesting to note that the electrical advertising industry has taken action to improve the appearance of bulletin boards and also to restrain members of their industry from installing posters and bulletin boards in places where they impair the beauty of the scenery. The unrestrained activity on the part of various poster advertising companies in locating their stands in places where they detract from the natural beauty of the landscape has caused very unfavorable public comment and it is logical that the industry should take action to remove the cause of the criticism.

Lighting for Aviation. The rapid advance of aviation, especially in the United States Air Mail Service, has given rise to demand for lighted air-ways. It is reported that 3700 miles of transcontinental route are now lighted and appropriations have been passed for lighting the following routes for 1927:

New York to Boston
St. Louis to Chicago
Dallas to Chicago
Salt Lake City to Los Angeles
Pasco to Elko
Chicago to Twin Cities
Cheyenne to Pueblo

Under the auspices of the Department of Commerce there has been developed a lighting system, employing usually 24-inch revolving beacons, equipped with 900 or 1000-watt tungsten lamps located at average intervals of ten miles along the air routes. Intermediate landing fields are located every 25 miles along the routes, each equipped with a beacon and 20 boundary lights. A green approach light and red lights on top of obstructions near the fields are used.

A typical airport has approximately 30 kw. of lighting load, involving from $5000 to $12,000 worth of lighting equipment. It comprehends:

1. Revolving beacon to guide the aviator to the airport.

2. Boundary lights (60-c. p. series) all around the field to show the limitation of the boundary area.

3. Red lamps on all obstructions near the field, such as radio towers, telegraph poles, etc.

4. An illuminated wind indicator to show the strength and direction of the wind.

5. A ceiling light (1000-watt, 18-inch searchlight) to show the height of the bottom of the clouds.

6. Flood lights on the roofs and sides of the hangars (200-watt lamps).

7. A high-intensity arc searchlight, or a couple of 10-kw. tungsten lamps to floodlight the landing field itself.

There are already nineteen lighted fields in the United States and forty others from which regular flying is being done on such schedules that lighting is required. Many cities are alive to the coming air commerce, and are appropriating funds to prepare lighted ports. It is a movement which is spreading very fast. Estimates indicate that by the end of next year there will be 2000 lighted fields in the country.

In 1926 the Post Office Department used on their fields in the Air Mail Service, 3710, 900- and 1000-watt lamps; 1920 200-watt lamps and 1440 600-lumen series lamps. The 1927 plans are for over three times as much air mail service as in 1926, with a corresponding increase in lighting.

Subterranean Lighting. The new vehicular tunnel from New York to Jersey City offers the outstanding installation of this class of electric lighting. It will probably be the most heavily traveled long tunnel in the world.

Although the main travel will be in one direction in each of the two tubes, the use of unidirectional lighting was impracticable because of the possibility of only one tube being used for travel in both directions when repairs are being made. The lighting is accomplished by incandescent lamps behind windows set at the joints between the side walls and ceiling, and arranged so that the units on one side illuminate the opposite half of the tunnel, and avoid glare in the eyes of drivers. About two foot-candles are provided, with an overlapping distribution to minimize shadows from high vehicles.

At each end additional light is provided for daytime use, to lessen the contrast with daylight.

Railway Lighting. Developments in lighting in the steam railroad field during the past year or so have largely kept pace with the general development in other fields of lighting. As a whole, railroads are appreciating the benefits of higher intensities of illumination, particularly in shops and offices where artificial illumination may be required a large percentage of the time, with the consequence that the average levels of illumination intensities for interior lighting throughout the railroad field are being considerably raised.

Considerable attention is being given to providing better illumination in passenger carrying cars. In the matter of intensities the best practise of today represents from 75 to 100 per cent higher average illumination intensities than the practises of eight or ten years ago. This has been made possible by the improvements in the efficiencies of train lighting lamps and the successful development of lamps of higher wattages, also the development of economical car lighting axle generator equipment and batteries of larger capacity.

The past year has seen very rapid growth in the general interest in the subject of flood lighting as applied to railroad yards, as well as the application of this system at a rapidly increasing rate. The proper and economical lighting of large railroad yards presents many problems, in which connection, until recently there has been available but comparatively little engineering data that would aid in laying out such lighting systems. There is also still considerable difference of opinion among railway engineers as to the system of flood lighting that will produce the most effective results. This subject is being actively studied by the Committee on Illumination of the Association of Railway Electrical Engineers and it is expected that by another year illuminating engineering practise in the application of flood lighting in this field will gradually crystallize along definite lines of procedure.

In view of the number of lighting problems that are more or less peculiar to the lighting field the Association of Railway Electrical Engineers has also prepared, with the assistance of the illuminating engineering staffs of the incandescent lamp manufacturers a "Manual of Lighting Practises for Railroads" which serves as a general code of lighting practise as applying to this field.

Illumination of Outdoor Substations. The illumination of outdoor substations is primarily intended to facilitate operation, but it has been found, in many cases, to have an advertising value as well. One installation recently described in the technical press emphasized the advertising value by employing a hot galvanized finish on all structural steel and two coats of aluminum paint on transformer cases, switch housings, and other exposed metal surfaces.

Lighted Ornaments. Artificial light has been employed thus far primarily for utilitarian purposes. Only occasionally, and to a very slight extent, has it been employed in residences for the illumination of ornaments. Evidently the potentialities of such employment of artificial light are very great. There are some indications that these potentialities are beginning to be realized, and in the not very distant future the employment of lighted objects of decoration solely for the purpose of ornament may assume considerable proportions.

RELATED TOPICS

Photometry. Progress in photometry during the past year has been principally in the application and use of the photoelectric cell in conjunction with suitable light filters. At the present time photoelectric photometers are largely used for routine measurements of incandescent lamps; this includes street series lamps, miniature lamps, colored bulb lamps, etc. The photoelectric cell equipment has also been adapted to distribution photometers, the spectro-photometer, and color temperature determinations.

The extreme sensitivity of the photoelectric cell equipment has permitted the establishment of light values to a much higher degree of accuracy than obtains with visual methods.

Effect of Illumination on Industrial Production. The Committee on Industrial Lighting of the Division of Engineering Research of the National Research Council has completed a three year study of the effect of illumination upon industrial conditions. A report covering this investigation will be published in the near future and will contain many points of interest to illuminating engineers and factory managers alike.

Lighting Service Manual. A manual for Lighting Service Departments, under preparation by a committee of the National Electric Light Association, is approaching completion. Part 1, which deals with the lighting field, organization activities, etc., has been finished. It will provide an excellent guide for central station lighting activities.

Schools of Lighting. Evidence of the increasing interest in illumination is shown by the demand for local lighting schools. These have become more numerous during the past year. As a rule they are promoted by individual central stations for the benefit of their employees engaged in lighting. In several instances they have included local electrical contractors and dealers and when this has been the case, the schools have been held under the auspices of a local electrical league or some similar body. The instruction in these schools has been conducted mainly by the incandescent lamp manufacturers.

Illumination Items in the Journal. In view of the fact that lighting programs are included in Institute meetings only occasionally, this Committee has found it expedient to endeavor to keep Institute members advised of lighting developments through the medium of brief articles which appeared from time to time in the columns of the JOURNAL. List of titles of articles which have appeared during the past year is as follows:

A Daylight Electric Sign.
Lighting Totaling 25,000,000 Candle Power Burned Nightly in Broadway Signs.
Europe Organizes Its Lighting Activities.
Inside Frosted Lamps.
Trend of Electric Lighting.
Must the Traveler Read Slowly?
Practical Color Photometry.
Industrial Lighting Activity of N. E. L. A.
Meet the Well Lighted Car.
European Lighting Progress Discussed at Rome.
Illuminating Engineering in Germany.
Home Lighting Contest in France.
British Lighting Contest begins with a Burst of Enthusiasm.
A Recent Lighting Demonstration in Holland.
Incandescent Lamp Ratings in France.
Artificial Lighting in Foundries.
British Investigate Light and Industrial Efficiency.
Carbon Lamps.

CONCLUSION

The Committee on Production and Application of Light, notes with satisfaction advances which are being made in the application of electricity in the field of illumination, and is gratified to observe a tendency of the related industries to organize for more effective achievement along these lines. The potentialities in this field the Committee believes to be great, both in prospective engineering achievement and in benefit for the public.

P. S. MILLAR, *Chairman.*

Discussion

E. A. Willford: (communicated after adjournment) I should like to augment the information given in this report on the production and application of ultra violet light for medicinal and industrial purposes.

The chief natural source of ultra violet light is the sun. There are, however, many artificial sources of ultra violet light, among them being the various forms of carbon arcs, the mercury vapor arc in quartz, and other metal arcs.

The emanations from the mercury arc are confined to certain bands of wavelengths, especially in the shorter wavelengths in the region of 2200 to 3200 Angstrom units, with very little continuity of the spectrum. Every different metal gives its own characteristic quality of radiation when its vapors are introduced into the arc stream. It is possible, therefore, to control the quality of the radiation by modifying the chemical composition of the electrodes; or, in the case of the carbon electrodes, the composition of the core. The following are typical instances:

Arcs between pure carbon electrodes give ultra violet light chiefly of wavelengths from 3600 to 4000 Angstrom units.

If the electrodes are of nickel or if carbon electrodes are impregnated with nickel, a large proportion of the ultra violet is in a band from 3400 to 3600 Angstrom units.

Similarly, aluminum gives much radiation in the region of 2950 to 3300 Angstrom units, while cobalt gives an arc rich in the very short wavelengths from 2200 to 2500 Angstrom units and again from 3300 to 3500 Angstrom units.

Iron gives a large amount of radiation through the entire ultra violet spectrum. Cerium and the other rare earths give ultra violet from 2900 Angstrom units to the visible spectrum, quantitatively and qualitatively very similar to the spectrum of sunlight. For this reason, carbon electrodes impregnated with these rare earths have been found by the Bureau of Standards to be the nearest in quality to natural sunlight of any known artificial light source.

The materials referred to above are not toxic. They can, therefore, be used as arc electrodes without enclosing globes and without danger of toxic poisoning. If required for special applications, the arcs can be isolated from the surrounding atmosphere by suitably ventilated housings constructed partly of quartz or some of the newer ultra violet transmitting quartz substitutes.

These metals, if used as pure electrodes, give satisfactory arcs on two or three amperes of direct current. If the metals are used to impregnate carbon electrodes, so as to make the so-called impregnated or flaming arcs, they can be operated satisfactorily on either alternating or direct current at amperages from 2 to 150. Because of this wide range of energy consumption possible, any desired quantity of the particular type of radiation required can be obtained with these arcs.

The known applications of these different types of radiation are as varied as the qualities of the arcs themselves. For instance, those arcs giving long-wave ultra violet light are especially valuable in the photographic, photo-engraving, and blue-printing industries.

The arcs which give light similar to sunlight are essential in dye-fading and paint-testing work where such materials are ordinarily to be used in sunlight itself. Artificial sunlight from these arcs also is utilized by physicians to augment natural sunlight or to substitute for it when natural sunlight is not available in the treatment of tuberculosis and rickets.

Other electrodes containing metal are used when it is not necessary to attempt to duplicate sunlight. Such cases are those where it is sought only to produce a tan or artificial sunburn. Those arcs giving very short-wave ultra violet radiation give large amounts of light having a powerful sterilizing or germicidal action.

From the foregoing, it is apparent that it is possible to make a selection of an artificial source of ultra violet light that will best accomplish almost any work which requires the use of ultra violet radiation.

Electrical Machinery

Annual Report of the Committee on Electrical Machinery[*]

To the Board of Directors:

This committee has carried on its work during the past year according to the general plan of organization which has been in force for the past three years. The membership of the committee has been materially increased over the number of last year in an endeavor to be prepared to handle the increasing amount of work naturally resulting from the rapid growth in quantity, size, variety, and quality of electrical machinery. Experience has shown that the work of a committee can be effectively carried on only when the members are able to get together and carry on a discussion across a table, following, perhaps, a preliminary exchange of views by letter. For this reason, the membership of the committee has been restricted to those living within a day's journey of New York or in the territory east of the Mississippi River. This territory embraces practically all of the manufacturers of electrical machinery, a large number of universities, and large users of machinery for power generation and distribution. It is not intended, however, to exclude any members who are in a position to, or willing to, assist in any way whatsoever. In this connection, your attention is directed to the general call for volunteers which appeared on page 1 of the Journal of January, 1927, over the name of the chairman of this committee.

The committee has held two general meetings, one in October and one in February at the time of the Winter Convention. In addition to these, the various subcommittees have held meetings in connection with the work that has been assigned to them. In general, the subcommittees have reported progress of their work and presented opportunities for general discussion at the meetings of the whole committee.

The organization of this committee comprises subcommittees on (1) Standards, (2) Papers, (3) Research, and (4) Education. It is probably not necessary to review here the functions of these subcommittees. Mr. E. C. Stone is chairman of the Standard Subcommittee, Prof. V. Karapetoff is chairman of the Research Subcommittee, Prof. C. A. Adams is chairman of the Education Subcommittee, while Mr. H. M. Hobart with the whole committee has acted as clearing house for receiving suggestions, obtaining and reviewing papers dealing with electrical machinery.

During the year, 15 papers have been presented under the auspices of this committee at the general meetings of the Institute. Turning to the developments in research as affecting the design of electrical machinery, probably the most important theoretical contributions have been in connection with the subject of the synchronizing power and stability characteristics of synchronous machines and the determination of the flux distribution in magnetic fields. In the field of design and manufacture, advances have been made in capacities of turbo generators, transformers, waterwheel-driven generators, and synchronous condensers, there have been improvements in construction looking toward reductions of losses, the knowledge of the cooling and ventilation of machinery has been increased and definite steps have been taken to raise the operating characteristics of a-c. fractional horsepower motors to a higher level. In standardization, this committee has taken up a larger volume of work than ever before in the revision of existing A. I. E. E. Standards and the preparation of new Standards to keep pace with the continual development of improvements and new types of machinery and the necessity of changes arising from a better knowledge and understanding of the art. All of these additions to our knowledge and the improvements in design are chapters in the great story of the engineer's untiring efforts for the betterment of our social and economic status.

The following review has been prepared with the assistance and collaboration of the members of the committee and an attempt has been made to include the more important articles that have appeared in domestic and foreign journals in the several bibliographies. Undoubtedly, articles of real merit have been overlooked and the committee will welcome having such omissions brought to their attention.

RESEARCH

Undoubtedly the most important phase of this committee's work is that which has to deal with the advancement of the art of design and manufacture of electrical machinery, through research. Without research, progress would be very slow. It is with a considerable degree of satisfaction that important contributions have been made during the year, both theoretical and experimental.

Effect of Altitude on the Dielectric Strength of Insulations. The committee has carried out some experiments to determine the relative puncture strengths of standard insulation at different atmospheric pressures. The results of these tests indicate that the reduction in the value of the voltage causing pucnture with decreased atmospheric pressure is so small that no change in the A. I. E. E. Standards is considered desirable so far as insulated windings are concerned. When air is depended upon for insulation, however, a correction should be made for that part of the insulation which consists of air in series with solid insulation.

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R. H. Tapscott,
R. B. Williamson,
H. L. Zabriskie.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

Surge Tests of Insulation. Work that has already been done indicates that the breakdown strength of solid insulation depends upon the kind and mode of test voltage and it does not seem feasible to specify for general use a test that will truly duplicate service conditions. A conventional test of the simplest kind will be just as satisfactory, provided the magnitude of the voltage is sufficiently high to provide a reasonable margin for the most unfavorable transient voltage liable to occur under actual service conditions. For example, if high-frequency oscillations are liable to take place and experimental data should show an ultimate strength of only 50 per cent as compared with 60-cycle voltage, then the magnitude of a 60-cycle test must be at least twice that of the maximum high-frequency oscillation to which the machine may be occasionally subjected in operation. In large machines designed to meet definite service conditions where the most unfavorable overvoltages are reasonably well known, the factor of safety may be chosen accordingly, but competitive considerations will largely govern the choice of insulation and its test for machines made in quantities for the general market.

It is recommended that designers study such data as are now available on this subject and that experimental assembled machines be tested by the different kinds of voltages for the purpose of obtaining data upon which may be based ratios for use in design and in specifications and guarantees.

Hot Spots in Cores of Turbo Alternators. Tests made by three manufacturers of typical American machines show that the temperature at the bottom of the slots at ends of the core is less than that between coil sides at the middle of the core and it is concluded that no change is needed in the present A. I. E. E. Standards as regards the location of temperature detectors.

Evaluation of Conventional Losses. Some work is being done to determine the feasibility of determining the stray load losses of alternators, synchronous motors, and condensers by a more convenient means than those described in the Standards.

Calorimetric Method of Determining Losses in Alternators. Apparently this method requires such close supervision of detail and conditions that it may be considered more of a laboratory method than one which can be used for general commercial purposes. It is recognized, however, that this method has inherent possibilities for experimental work and is worthy of continued investigation. A paper giving results of extensive tests has been presented to the Institute during the year.

Stability of Alternators. A paper, *Stability Characteristics of Alternators*, has been presented by Mr. O. E. Shirley which showed the relation between stability and the short-circuit ratio and the subject has been referred to the Standards Subcommittee for the consideration of the establishment of a standard.

Some years ago it was the practise to design synchronous machinery with good inherent voltage regulation. With the advent of the vibrating voltage regulator, this practise changed since it was more economical to design machines with lower inherent voltage regulation and to depend on the voltage regulator to maintain voltage. This principle formed the basis of machine design until, in recent years, the work done on system stability indicated that for machines which were to be used on those systems where stability is an important consideration, a reversion to the former practise of designing for good inherent voltage regulation was desirable.

Recently, in the engineering of certain large power projects and extensions to existing systems, it has been decided to employ machines having lower leakage reactance and higher short-circuit ratio than machines of normal design for the same rating would have. The purpose of this is to increase the stability of the system upon which they are to be used and, in particular, to reduce the probability of system disturbances causing loss of synchronism of the terminal apparatus with consequent interruption to service.

For system stability it is desirable that reactance be kept low, whether it be that of transmission lines, transformers, or generators. It is not feasible to reduce the reactance of the transmission lines appreciably except by building additional lines in parallel. Transformer reactance is a relatively minor part of the total and can be reduced below the normal values only at considerable cost. On this account, attention must be focused on the generators where it is economical to increase the cost to reduce the reactance below normal since such a reduction increases the capacity of the relatively much more expensive lines to carry load with less probability of service interruption due to system disturbances.

Another important consideration in maintaining synchronism is that of sustaining voltage throughout the system during a disturbance. This may be partially accomplished by the use of machines having high short-circuit ratio. Higher values than those corresponding to normal design have been decided upon for certain projects for the purpose of increasing system stability. Beyond a certain degree, it is more economical to employ quick response excitation systems which serve to accomplish the same object as increasing the short-circuit ratio, namely, that of sustaining the voltage during a disturbance.

Within the past year, the construction of machines embodying these special features for improving system stability has been undertaken for certain high head developments in California where the length of the transmission lines and the amounts of power involved are such as to cause the stability to be an important problem. Machines of special characteristics are also under construction for certain low head hydroelectric developments in the East and the South where the reactance of the slow-speed machines is of necessity relatively high and where the amounts of power to be transmitted are very large.

Relation between Dielectric Tests on New and Used Machines. This subject has been referred to the Standards Subcommittee and it is intended that a paper should be prepared for the purpose of setting forth the principal considerations and a suggested standard.

Characteristics of Synchronous Machines. Supplementing a study of the characteristics of synchronous machines by an extension of Blondel's theory of two reactions as mentioned in the report of last year, the second part of the series of papers by Doherty and Nickle should also have been mentioned, which treated the steady-state, power-angle characteristics. A further study has now been presented by these authors on the torque-angle characteristics under transient conditions and a further study of torque characteristics under short-circuit and transient conditions has been promised. Another contribution to this subject has been made by Mr. H. V. Putman in a paper presented to the Institute. Results of experimental studies of the transverse armature reaction in synchronous machines have been presented in a paper to the Institute by Prof. J. F. H. Douglas to show that the effect of transverse reaction can be most accurately estimated by the use of a m. m. f. diagram.

Stray Load Losses. Sources of stray losses and means of their reduction and elimination present a field of study which is of considerable importance in the never-ending endeavor to improve the efficiency of electrical machines. Papers have been presented to the Institute dealing with several phases of this subject. An analysis of the m. m. f. waves of polyphase windings of the fractional slot or irregular types shows the possibility of the existence of sub-synchronous harmonics having wavelengths greater than two-pole pitches which may induce currents in the damper windings of synchronous machines. Connection arrangements of this type of windings have been investigated for the purpose of preventing these losses. The existence of eddy current losses in the copper of armature windings has been a fertile field of investigation both as to a means of determining their magnitude and their reduction. Recent studies have resulted in simple conductor transpositions and arrangements that have been effective in almost entirely eliminating these losses. Investigations made by the calorimetric method in connection with several turbo generators gave results that appear to confirm the correctness of assuming the stray load losses being equal to the additional losses under sustained short-circuit conditions.

One of the colleges has undertaken a series of experiments on methods of determining load losses of synchronous machines which, it is said, gives promise of adding materially to the knowledge of this subject. It is hoped that the results of these experiments will be presented to the Institute during the coming year.

Synchronous Converters. A treatment of the theory of the converter has been presented that is based on the method of "harmonic analysis" by which any regularly repeating function may be represented, and presenting a conception of the internal voltages, currents, heating, and armature reactions as related to the physical structure of the simple converter and as related to the passage of time which may be called space and time relations.

Synchronous Motors. A theory for the calculation of the complete starting performance of synchronous motors has been presented which utilizes a system of negatively rotating vectors to take care of the unbalance in the damper winding which is not continuous.

Reactances for Direct Current. A direct method of design for the predetermination of the correct air-gap in reactances and transformers which are to be used with direct current has been offered. This subject is of particular importance in building rectifier filters for radio telephone work.

Magnetic Fields. The distribution of magnetic flux is a very important factor in the design of electrical apparatus and there is need of methods for determining the magnetic fields with a reasonable degree of accuracy. A rather complete treatment of this subject has been presented to the Institute in a group of papers covering the graphical method from the standpoint of the theoretical considerations, comparison between calculations and tests and the practical application to a particular type of machine.

Dielectric Tests on Windings of Large Alternators. The attention of the Subcommittee on Research has been called to the question of whether or not there should be a difference in the value of voltage applied in making dielectric tests on one phase to other phases connected to ground and from all phases connected together to ground. Information on this subject is desired.

Effect of Damper Windings in Alternators upon Single-Phase Short Circuits. It is customary in Europe to add dampers to the field structures of alternators to enable them to more effectively carry unbalanced loads and reduce peak voltages when single-phase short circuits occur. This subject is included in a research report by the Department of Electrical Engineering of the M. I. T. of June 1926, and information is desired as to whether it is advisable to follow this practise in America.

Evaluation of Conventional Losses. A discussion of the paragraph under this heading in this committee's report of last year has suggested that suitable commercial test methods of determining the internal voltage drop of an alternator winding which is due to leakage reactance should be devised so that the real core loss under load conditions could be taken into account in figuring the efficiency instead of using the no-load value of core loss which may be considerably lower. If such tests could be made, the value of the conventional efficiency would more nearly approach the real efficiency. The value of this drop in field ampere-turns might be taken from the test short-circuit impedance curve if the armature reaction in ampere-turns were known. This latter term can be quite accurately calculated from the armature winding data but the test codes of the A. I. E. E. Standards are based upon the principle that the characteristics must be determined from only those quantities which can be directly measured by test. Some reasonably accurate method of measuring the leakage reactance would be a valuable addition to the present standards.

Alternator Short Circuits. The work done previously on this subject has been to determine the amount of current that will flow for various conditions of short circuit. The torque produced during a short circuit is also of importance and an instrument has been developed which will give a record of the instantaneous values of torque during a short circuit or similar transient condition. It is also possible with this instrument to investigate the synchronizing power of a machine and tests are being made at the present time to study this synchronizing power as well as the short-circuit torque of a number of machines.

Within the past few years, the methods of calculating phase-to-ground and phase-to-phase short-circuit values as well as the currents flowing for any abnormally unbalanced operating condition have been greatly simplified by the use of the system of symmetrical coordinates developed by Mr. C. L. Fortescue. In order to apply this method of symmetrical coordinates, it is necessary to have a knowledge of the impedance of the rotating machines to zero and negative phase sequence voltages. A great deal of experimental work has been done to determine the proper method of determining these impedances. A series of rather simple shop tests has been devised to obtain the desired information and during the past year a number of machines has been tested to find the values of their impedance to zero and negative phase sequence voltages.

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STANDARDS

The following report has been made by Mr. E. C. Stone, Chairman of the Subcommittee on Standards.

General. Following the policy laid down by the Chairman of the Committee on Electrical Machinery, the Committee on Standards has kept in close touch with developments in the manufacturing and operating fields, and has attempted to sense the needs, as they have arisen, for modification or further development of the present standards and for the setting up of new standards.

By consistently following the policy of having subcommittees continuously at work on the revision and further development of standards for electrical machinery, it is hoped that the Institute standards in this field may be kept fully abreast of the times and adequate to meet the changing conditions under which electrical machinery is called upon to operate.

The various subcommittees have been actively at work during the past year, have initiated a number of proposals for modification of old and creation of new standards and have carefully worked up a number of definite recommendations which has been formally proposed to the Standards Committee of the Institute.

The preparation of standards covering two types of apparatus, mercury arc rectifiers and constant current transformers, not covered by present standards, has been started and definite recommendations may be expected within another year.

A number of changes in, and additions to, present standards on synchronous machines and on transformers, A. I. E. E. Standards Nos. 7 and 13, has been recommended. Additions of importance include the following:

Method for the calculation of natural frequency for synchronous motors driving reciprocating machinery.

Definitions for short-circuit ratio, per cent synchronous impedance and per cent transient reactance.

Definition for and method of rating grounding transformers.

Ability of transformers and reactors to withstand short-circuit current.

Method of measuring losses in transformers.

The operation of electrical machinery on a total temperature basis rather than on a temperature rise basis was actively discussed by the committee. The increasing demand under economic pressure for operation of electrical machinery in such manner as to get out of it the greatest possible capacity under all conditions, and of taking advantage of the greater capacity inherently present when a machine is operated in a temperature below that for which it was designed, was recognized, and it was agreed that the committee should give active attention to the subject. In this work the committee was instructed to cooperate with Working Committee No. 38 of the Standards Committee, Mr. Hobart, Chairman, whose function is "to revise the general principles upon which the rating of electrical machinery is based, with a view of presenting at a later date a document that will explain the connection and distinction between test specifications for rating and *operation under service conditions*, the purpose being to place before the various working committees a working basis on which service may be established for each line of apparatus."

The subject of standards for dielectric tests immediately after putting in service and periodically while in operation, was taken up actively. The investigations have shown that the problems involved are not well understood and that there is a wide variation both in opinion and practise with respect to them. Accordingly, Messrs. Gilt and Barns have promised to prepare a paper on the general subject of dielectric tests on equipment after installation, in which the principles, problems, and practises will be crystalized and an attempt made to develop a definite method of arriving at a satisfactory solution to meet the various conditions that are encountered.

In the field of fractional horsepower motors, little could be done because of the unsettled status of the negotiations now under way between the N. E. M. A., A. E. I. C., and N. E. L. A. with regard to the performance characteristics of this class of motor.

There is a growing tendency among power companies to place restrictions on the permissible efficiency power factor, and starting current of fractional horsepower motors. These characteristics are particularly important in the ¼-hp. motors used for domestic refrigerating and oil burning equipments, which at present have very poor characteristics in these respects. One company has already put out a rule requiring that all such motors connected to its lines must have an apparent efficiency of not less than 42 per cent and a starting current of not more than 15 amperes at 115 volts.

The negotiations referred to above have been carefully followed by the subcommittee. It appears that definite progress has been made towards reaching an agreement at which the performance of fractional horsepower motors will be substantially improved, with a result that the over-all cost to the operator on such motors, giving consideration both to the cost of the motor and of the power to operate it, will be reduced.

The following is a brief résumé of the activities of the various subcommittees.

Standards for Alternators, Synchronous Motors, and Synchronous Machines in General. W. J. Foster, Chairman. Revisions of the following paragraphs have been suggested:

Paragraph Number	
7-66	Definition of Open Machine.
7-67	Definition of Protected Machine.
7-457 (b)	Ventilating Blower Losses.
7-457 (c)	Other Auxiliary Apparatus Losses.
7-465	Determination of Losses (to include 7-472).
7-467 (b)	Friction and Windage Losses of Engine Type Alternators (to include Synchronous Motors).
7-470	Stray Load Losses.
7-551	Insulation Resistance—Minimum Values.

Additions to this pamphlet have been recommended to cover the following:

Method of Calculation of Natural Frequency of Synchronous Motors Driving Reciprocating Machinery.

Definition, Short-Circuit Ratio.

Definition, Per Cent Synchronous Impedance.

Definition, Per Cent Transient Reactance.

Standards for Transformers, Induction Regulators and Reactors. G. Faccioli, Chairman. The following new paragraphs have been recommended:

13-161	Grounding Transformers—Definition and Rating.
13-254	Grounding Transformers—Momentary Load Limitations.

Revisions of the following paragraphs have been recommended:

13-250	Short-Circuit Current of Transformers—Momentary Load Limitations.
13-252	Current Limiting Reactors—Momentary Load Limitations.
13-306	Measurement of Losses in Transformers.

The following subjects are suggested for attention during the next year:

Guaranteed Secondary Voltage of Step-Up Transformers under No-Load Conditions.

Cooling of Air-Blast Transformer Windings after Shut-Down.

Self-Protection of Transformers against Impulse Transient Voltages.

Operation of Transformers by Temperature.

Standards for Synchronous Converters. C. H. Sanderson, Chairman. The further study of the following subjects for the revision of Standards No. 8 is recommended:

Normal Rating.

Measurement of Cooling Air.

Quantity of Air Required for Cooling.

Short-Circuit Protection.

Standards for Mercury Arc Rectifiers. B. G. Jamieson, Chairman. This subcommittee was created through the request of the Standards Committee of the Institute at the January 1927 meeting. It is intended that it will develop as rapidly as possible definite standards for mercury arc rectifiers.

Standards for Constant Current Regulating Transformers. H. C. Louis, Chairman. This subcommittee was organized at the time of the Winter Convention and it is expected to develop definite standards for constant current regulating transformers.

Standards for Fractional Horsepower Motors. E. C. Stone, Chairman. On account of the unsettled status of fractional horsepower motors due to the negotiations now under way between the N. E. M. A., A. E. I. C., and the N. E. L. A., it has been impossible to formulate any definite recommendations on this subject.

It is recommended that this subcommittee be continued next year and that the A. I. E. E. Standard No. 10 covering this subject be reviewed and such revisions recommended as may be necessary to fit in with the revised practise set up.

VENTILATION

The ventilation of machinery is a very important consideration not only in regard to the cooling of the machine itself and the supply of an adequate amount of clean, cool air but also as a means of deadening noises inherent in the operation of large and high speed machines. In the endeavor to realize every possible economy for obtaining high efficiencies, attention is being paid to relatively small features of construction to obtain small resistances to the passage of cooling air, and much attention has been paid to forms of fans. The importance of paying attention to small details in improving efficiencies of turbo alternators has led to investigations in the laboratory by the use of a model in which changes in fingers, slot wedges and dimensions of slots can be readily made and the effects determined. Tests are also being made to study the flow of air through rotors.

The report of this committee for last year mentioned that closed ventilation systems had been adopted for hydroelectric plants. The question may well be asked as to what consideration would make it advisable or desirable to use a closed system in a water power plant. The many illustrations of water power plants which are published in the current literature show locations remote from sources of smoke and dust which are not associated in our minds with broad expanses of water and wilderness. There are often conditions surrounding the location of power houses that make it advisable to provide a closed system. Hydroelectric plants located on rivers in the midst of manufacturing plants where much coal is burned have as much need of this type of ventilation as the turbo generators in steam plants in the same locality. There are also conditions in some remote locations which make closed systems advisable, such as severe dust storms in barren districts. It often happens that the initial installation in a hydroelectric development is only a small part of the final capacity and there may be construction work going on over a long period after the first few units have been put into operation with considerable dust from masonry work in the air. In an instance of this kind, it was found that the generators had become sufficiently clogged with dust to raise their temperature 10 deg. One installation of eight 12,550-kv-a., 100-rev. per min. waterwheel-driven generators includes closed ventilation systems. Probably the largest waterwheel-driven generator to be provided with this system is a 30,000-kv-a., 300-rev. per min. vertical unit. In these installations, the air coolers are located in enclosures immediately behind the stator frames, and after passing through them, enter ducts leading to the pit underneath the generators. In the case of the 30,000-kv-a. generator, the air is returned to the space above the rotor also. The coolers are arranged so that they can be lifted vertically out of their pockets for repair or replacement without disturbing any part of the generator themselves.

Another unusual application of the closed system of ventilation is in connection with a German Diesel-engine-driven 13,000-kv-a., 94 rev. per min. alternator.

Much valuable data for future design work regarding hydrogen cooling have been obtained from thorough tests on a 6250-kv-a., 3600-rev. per min., 13,200-volt turbo generator. These tests were highly gratifying and indicated that practically the same benefits may be obtained as may be expected from theoretical considerations. There are indications that hydrogen cooling may eventually be used for large synchronous condensers and frequency converters. It has been suggested that helium could be used with advantage in the place of hydrogen and this possibility is being investigated. A seal has been developed to prevent the loss of hydrogen or the admission of air at the section where the shaft of the machine enters the end bell. Tests made over long periods of time on this type of seal have proved very satisfactory. Tests have also been made on heat-flow across laminations when surrounded by hydrogen.

The necessity of minimizing the noises given out by rotating machinery in substations located in business and residential areas has presented some difficult problems, especially in connection with d-c. machinery. It is a comparatively simple matter to enclose a synchronous motor so that the noises are quite effectively deadened but in the case of a d-c. generator or a rotary converter, the necessity of providing ready access to the commutator brushes presents serious difficulties and the accumulation of metallic and carbon dust becomes a serious contributing factor toward insulation failures.

An early attempt to enclose a rotary converter consisted of a housing in the form of a wired glass and steel framework closely shrouding the commutator and mounted directly on the arms of the brush spider. A metal housing covered the collector rings and pedestal with sufficient room to allow an operator to enter the housing and inspect the rings and brushes. Test showed, however, that this arrangement hindered the free ventilation of the commutator and from an operating point of view the commutator was too inaccessible. A later attempt which has proved successful has taken the form of a large semi-cylindrical steel housing of about the same size as the over-all projected dimensions of the converter and of slightly greater height. Doors are provided at each end while steps on the bearing pedestals with suitable screens, guard rails, and adequate lighting make the rings and commutator safely and easily accessible. Air is introduced through the pit at the d-c. end, a small part being admitted under the collector rings and passing axially through the machine; it is discharged into the exhaust ducts from the top of the collector end of the enclosure. This scheme has been successfully applied to 4200-kw., 12-phase converters having their two transformers mounted on the base plate. Blowers located in the basement beneath the converters are used to force the air through the housings. Some 2100-kw., six-phase converters have been similarly equipped.

During the past year extensive experimental tests have been made to determine the surface heat transfers in electrical machinery with air flowing at various velocities through radial and axial ducts. Investigations have been made to determine the influence of shape, size, cross-section, condition of the surface, mean temperature, and several other factors.

The most important fact brought out in these tests is that the rate of heat transfer is not constant along the path of air-flow but varies in value from point to point along the path. This variation in the rate of heat liberation for a constant air velocity in a given duct is caused by the changes in the turbulency of the air-flow along the duct. As a result of this change of turbulency, it is found that the rate of heat transfer near the entrance of a duct is about twice that at a point further along where stable flow conditions are found. This explains why electrical machines with short duct length have a capacity in proportion to their surface greater than those with similar ducts but with longer air flow paths. This also explains why the surface heat transfer coefficient of a duct, averaged over its total length, will be an inverse function of the length. It also explains why results as given by experiments on ducts of various length should vary.

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Water-Wheel Generators

Probably the most notable water-wheel-driven generators of the past year are the seven machines now under construction for the Conowingo Development. These alternators are notable not only because of their size but also because of the fact that they are to supply power for the first 220-kv. transmission line in the eastern part of the United States. They are rated 40,000-kv-a., 90 per cent power factor, 81.8 rev. per min., 60 cycles, 13,800 volts, and are the largest in physical dimensions of any electrical machines that have ever been built. They are being built by two manufacturers, and by a large degree of cooperation between them, it has been possible to obtain similarity in characteristics and appearance and interchangeability of some important mechanical parts. The outside diameter of the stator frame is 38 ft. One manufacturer has made use of steel plate construction almost exclusively for the mechanical parts of the stator frame and the rotor. The rotor rim will consist of heavy rolled plates in several layers overlapped so as to give the greater strength for the amount of material used and fastened together with bolts and dowels. The pole pieces will be fastened with bolts through the rim. The stator frame is of the welded steel plate construction and greater uniformity in shape has been obtained than is possible with castings. The largest capacity thrust bearings ever built will be required for these generators. Their capacity will be a total load of 750 tons.

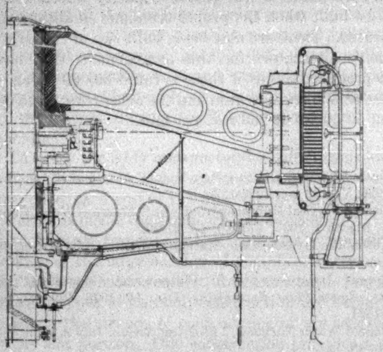

Fig. 1—Section of 27,500-Kv-a., 90-Rev. per min. Vertical-Shaft Alternator with Overhung Rotor

On account of the stability characteristics desirable for operation with the 220-kv. transmission line, the generators described above have a short-circuit ratio of 1.25 which is somewhat higher than has been found satisfactory for nearly all other hydroelectric developments. As a further aid in securing continuous parallel operation without hunting or dropping out of step during disturbances on the system, a scheme of high-speed excitation is being used. As in the case of the large generators at Niagara Falls, each main generator has direct-connected to its shaft a service generator which supplies power to a high-speed motor-generator set for providing excitation for the main generator. The service generator is provided with its own direct-connected exciter. The generator of the main exciter set will be separately excited by a suitable direct-connected exciter. To take full advantage of the high-speed excitation provided by this scheme, special voltage regulators are being used to control the main generator excitation through the control of the field current of the exciter generator. The fields of the exciter generator will be connected in two parallel circuits in order to obtain an effect equivalent to that of using double potential on the whole field connected in series. Generators having the same scheme of excitation have been installed in power houses on the Gatineau River in Quebec for supplying power to the 260-mile transmission from Ottawa to Toronto. One of these generators is rated 32,000 kv-a., 90 per cent power factor, 100 rev. per min., 6600 volts, 25 cycles, and the other is rated 22,500 kv-a., 88.3 rev. per min.

The largest vertical shaft water-wheel-driven alternators yet built in Europe are now under construction. They are rated 35,000 kv-a. at 337 rev. per min. and 375 rev. per min., 10,000 volts, 40 cycles. These machines are equipped with direct-connected main and auxiliary exciters. The rotors are made up of two wheels, each consisting of five cast steel disks bolted together. The pole pieces are dovetailed to the rotor spider.

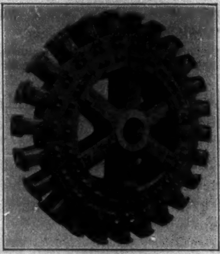

Fig. 2—Rotor of a Large Waterwheel-Driven Alternator of Recent Design using Steel Plate Construction

There are under construction three of the largest vertical shaft water-wheel generators of the "umbrella" type yet built. Two of them are rated 27,500 kv-a., 80 per cent power factor, 90 rev. per min., 13,800 volts and the other is rated 22,500 kv-a., 75 rev. per min., 60 cycles. With this construction, a common shaft is used for generator and water-wheel and the thrust and guide bearings are located beneath the rotor. There is no guide bearing above the rotor. The ventilation arrangement is noteworthy in that no air is taken from the wheel pit and no air currents pass over the bearing oil pans to draw oil vapour into the generator. A number of smaller generators of this type has now been put into operation and their operation has proved satisfactory.

European manufacturers of water-wheel equipment are apparently finding it more economical to offer geared units for low head installations. An installation on the Trent River in Ontario consists of a 1400-hp., 120-rev. per min. vertical shaft turbine geared to a 600-rev. per min. horizontal shaft alternator through a set of helical bevel gears which are claimed to have an efficiency of 98 per cent. Some recent German plants contain vertical shaft turbines driving vertical shaft alternators through double spiral gears.

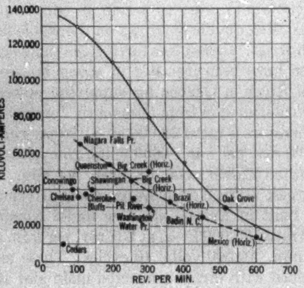

FIG. 3—COMPARISON OF RATINGS OF EXISTING SALIENT POLE ALTERNATORS WITH RATINGS WHICH MANUFACTURERS ARE PREPARED TO BUILD

The full line curve shows ratings which manufacturers are prepared to build as mentioned in the June 1926 report of this committee

During the year a very large automatic hydroelectric station has been put into operation. It consists of a 17,500-hp. vertical turbine driving a 13,333 kv-a., 6600-volt, 225-rev. per min. generator and is controlled from a station seven miles distant. Another large generator rated 9000 kv-a. arranged for full automatic control has been put into operation.

In the report of this committee last year, there was given a list of large ratings at different speeds which manufacturers were prepared to build. No doubt, if the necessity should arise, greater ratings could be constructed. A reader who was interested in knowing how close the ratings of machines already built came to these limits has plotted the list of maximum ratings and speeds in the form of a curve and added the more outstanding ratings that have been built. The curve and plot is reproduced to show what appears to be the limit line of hydroelectric unit capacities that have been considered economical up to the present time. It will be noticed that the highest ratings at different speeds of machines built or under construction fall on a well defined curve with the exception of two machines that are above the curve.

Recent developments in the design of Pelton wheel water turbines promise that larger horizontal shaft generators than heretofore built are possible requirements of the future. The standard arrangement of units with this type of turbine is to mount a runner on each end of the generator shaft. Units of this type have been built for capacities of 40,000 hp. and 56,000 hp. and studies have been made of still greater capacity wheels which may be built when the proper economic conditions are presented. There are now being built two of the largest capacity alternators for this arrangement that have been produced. One of these is rated 50,000 kv-a., 60 cycles, at 300 rev. per min. and the other is rated 45,000 kv-a., 50 cycles at 250 rev. per min.

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TURBO ALTERNATORS

The trend of development during the past year has been toward larger units. In connection with the manufacture, shipment, and installation of the large alternators now being produced, there has been an extension of the use of the skeleton frame construction for stators, and also in speeds of 1500 and 1800 rev. per min.

The old type of construction made a final unit which was necessarily heavy and usually too large for shipment on the railways. With the skeleton frame construction, the frame is strong enough to support itself during machining and assembly. Special bolted-on plates and trunnions for each frame facilitate handling during assembly at the works, loading and unloading during shipment, and final erection at the destination. The over-all dimensions of the frame, assembled for shipment, allow the completely assembled unit to be shipped direct to its destination. The ventilation requirements are cared for by an external superstructure of sheet metal applied at the destination. With this construction, 1800-rev. per min. turbine generators up to 75,000 kv-a. can be built and wound at the works and shipped complete. As a result of this, large turbine generators can be tested at the works, the freight charges are reduced, handling during assembly, shipment and erection is made easier, closer inspection is made possible during manufacture, and there is less confusion to the purchaser when shipment is made.

There has been considerable activity in developing new types of stator windings, involving transposition of strands in each conductor, and transpositions at the heads of the machine, or in the connections. The use of half-coils is becoming more common practise among the different manufacturers. It has been found that the use of continuous coils and the control of the eddy current loss factor by transposing the strands of each conductor at one end of the machine is satisfactory up to about 50,000 kv-a. In larger capacity machines, coil length and weight are so great that it is difficult to handle the coils during manufacture and assembly. This has favored the use of half-coils, especially in view of the facility with which damaged coils can be replaced in the machine.

To make the eddy current loss in half-coils as low as in continuous coils, either elaborate transposition of the strands at each end of the machine or transposition of the strands inside the armature core is necessary. Complete external transposition of half-coils is undesirable because of the large number of complicated connectors and the large space required for connection. For these reasons, types of half-coil construction are being used in which the strands of each conductor are transposed in the slot portion. One construction provides a complete transposition in the slot of all the strands in the conductor as one group while in another construction the strands are arranged in small groups and the strands of each group are transposed internally while the groups are transposed at the ends. These constructions permit the use of coils which are relatively easy to handle and assemble and yet make the eddy current losses as low as possible.

The past year has seen the realization of the predicted possibilities in large high-speed turbo generators which were mentioned in the report of this committee last year. There are under construction generators of the following ratings:

Single-Shaft Units.

100,000 kv-a., 90 per cent power factor, 1500 rev. per min.

75,000 kv-a., 80 per cent power factor, 1800 rev. per min.

Triple-Shaft Units.

Two—64,706 kv-a., 85 per cent power factor, 1800 rev. per min. and one 57,647 kv-a., 85 per cent power factor, 1800 rev. per min. with two 4286-kv-a. house service generators to form a 165,000-kw. unit.

Two—72,941 kv-a., 85 per cent power factor, 1800 rev. per min. and one 89,412 kv-a. 85 per cent power factor, 1800 rev. per min. with two 5333-kv-a. house service generators to form a 208,000 kw. unit.

The present tendency to very large generating units and ever increasing station capacities has made the switching problem, on account of the enormous currents involved, a serious one. This situation is being met in some instances by connecting step-up transformers directly to the generator terminals and doing all the switching on the high-tension side of the transformers. In other cases, the generators are being built to operate at voltages considerably higher than those previously employed. As examples of the latter practise, the two 100,000-kv-a. units mentioned above are being constructed for 16,500-volt operation; the main generators of the 208,000-kw. triple shaft unit above referred to will operate at 22,000 volts, and there is also under construction a single-shaft generator of 61,675-kv-a. capacity which will operate at 22,000 volts. All of these will have dielectric tests in accordance with the A. I. E. E. Standard, i. e., twice normal voltage plus 1000 volts.

During the year some large two-shaft units have been put into operation. One of these consists of two 40,000-kw. generators. The turbine and generators are operated as a unit, the two generators being solidly tied to each other and to an auto-transformer stepping up to 27,600 volts with switching on the high-tension side only. The neutral points of both generators and auto-transformers are solidly grounded. The unit is started with field on both generators, the low-pressure element starting as a motor. The two generators are identical and are each rated at 40,000 kw., 1800 rev. per min., 13,800 volts, 90 per cent power factor. The generators also have a one-hour overload rating of 80,000 kw. at 80 per cent power factor. A 250-volt exciter is direct-connected to the generator of the high-pressure element. Each generator has its own field rheostat but the two face plates are coupled together and operated as a unit by a single pilot motor. A closed cooling system is used for each generator.

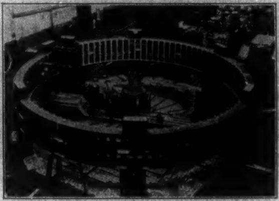

Fig. 4—Shop View of Stator Frame for 40,000-Kv-a. Vertical Shaft Alternator

For Conowingo Development

Another unit comprises a 64,700-kv-a., 85 per cent power factor, 1200-rev. per min. generator, a 38,825-kv-a., 85 per cent power factor, 1800-rev. per min. generator, and a 4666-kv-a., 75 per cent power factor, 1800-rev. per min. house service generator making a combined capacity of 91,500 kw.

Still another two-shaft unit which is under construction consists of an 88,200-kv-a., 85 per cent power factor, 1800-rev. per min. generator and a 100,000-kv-a., 95 per cent power factor, 1200-rev. per min. generator. Both generators are wound for 13,800 volts.

Two 62,500-kv-a., 1800-rev. per min. generators which have been put into service show by performance that the rating can be increased to 70,600 kv-a. without exceeding the original temperature guarantees and without change. They have a test efficiency of 98 per cent. Another manufacturer has completed a 59,000-kv-a., 12,000-volt, 1800-rev. per min. turbo generator which will soon go into operation. This single-shaft set is to be ventilated by two external blowers and the armature winding has transposed conductors.

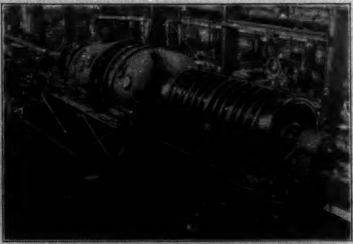

FIG. 5—SHOP VIEW OF 59,000-KV-A., 1800-REV. PER MIN. STEAM TURBINE-ALTERNATOR UNIT SHOWING SKELETON FRAME CONSTRUCTION

During the year, several of the European manufacturers have built 50-cycle turbo generators at 3000 rev. per min., in sizes from 30,000 kv-a. to 37,500 kv-a., but the experience of some of these companies in testing the generators before shipment has not been reassuring. In consequence, at least one of the principal manufacturers has decided to build 30,000 kv-a. hereafter, at 1500 rev. per min., instead of 3000.

FIG. 6—80,000-KW. CROSS-COMPOUND TURBO ALTERNATOR SET

In England there has been built a 31,250-kv-a., 80 per cent power factor generator which is remarkable for its voltage. It is wound for a terminal voltage of 33,000 volts and in order to avoid increasing the dimensions of the machine unduly, due to the thickness of the insulation, a special arrangement has been used to keep the voltage at a low value between adjacent conductors and ground. This high voltage would appear to be an innovation but it should be recalled that a 500-kv-a., 30,000-volt water-wheel-driven generator which was built for the city of Rome about 20 years ago, is reported to have operated all that time without breakdown.

Tests made to determine the additional losses in turbo alternators due to stray field have shown that the short-circuit losses can be reduced if non-magnetic rotor retaining end rings are used. Laminated stator end flanges and laminated magnetic shields attached to the stator flanges have been proved advantageous and a number of machines has been built with these features.

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TRANSFORMERS

The design of a transformer is often influenced by space limitations. To meet special limitations, individual radiators may be removed from the sides of the transformer tank and mounted in a single group. This arrangement has been used on an 89,000-kv-a. bank of self-cooled auto-transformers. The single-phase units which comprise this tank are the largest transformers on which radiators have been mounted apart from the transformer.

The development of the load ratio transformers which was reported last year has justified the claims for its growing importance. There has been a continual growth in the use of transformers arranged for load ratio for tying-in two operating systems for properly distributing the load over different portions of the same system, and for electrolytic and metallurgic processes in industrial service.

The voltage range for which these equipments were designed varied within wide limits, the maximum to date being 120 per cent range in voltage in 18 ratios, and the minimum 10 per cent voltage variation in nine ratios. The largest banks so operated are as follows:

Three-phase, water-cooled, 60-cycle regulating autotransformer capable of handling 60,000 kv-a. for use with three single-phase, 20,000-kv-a. units rated 132,000 grounded Y, 36,000 grounded Y, 12,000 volts.

Automatic control by means of a contact making voltmeter was provided for two three-phase, self-cooled, radiator type transformers of 10,000-kv-a., 60-cycle output. These machines transformed 11,000 volts to feed a grounded Y, 41,400-volt system. To adjust the line voltage in accordance with the changing load, the high-voltage windings have 11 taps of 2½ per cent each. To permit a change of taps without interrupting the load, a part of the 41,400-volt winding is made in two sections, operating normally in parallel and dividing the load equally. Each of these winding halves is connected to an 11-point ratio adjuster and the resulting circuits brought out of the transformer tank and led to two three-phase circuit breakers.

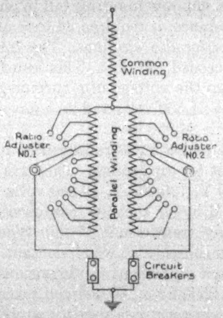

FIG. 7—DIAGRAM OF TYPICAL CIRCUIT ARRANGEMENT FOR AUTOMATIC LOAD RATIO CONTROL OF A POWER TRANSFORMER WITH THE TWO-WINDING ARRANGEMENT

It is thus possible during the tap-changing period to open-circuit one section in each phase, and change the voltage tap in this open-circuited section while the other section temporarily carries the entire load of the transformer. Copper of ample cross-section and the very short transition period make this possible. The same change is then made in the second half. The entire change from one voltage tap to the next requires only eight sec. For a brief period, less than 1½ sec., when both breakers are closed but the two ratio adjusters are one tap apart, an internal circulating current exists, which, however, is kept within predetermined limits by sufficient inherent reactance in the transformer windings.

A motor-operated mechanism mounted on the transformer truck insures a properly timed operation of the internal ratio adjusters and the external circuit breakers. For the proper execution of the tap-changing cycle, it is essential that the three corresponding ratio adjusters of the three phases move simultaneously from one tap to the next; therefore these ratio adjusters are mounted together on the same shaft with full phase insulation between them. The resulting two stacks of adjusters are arranged in a vertical position along the coil stacks of the transformer, and their two main shafts connect on top to a special internal intermittent gear. Turning the driving shaft of this gear train one complete revolution will first change one set of the three adjusters one step, then lock this set, and then turn the second set one step.

Contact making voltmeters relieve the operator of any manual starting of the mechanism. If the line voltage deviates from a set value for a period longer than a predetermined time value, the tap-changing mechanism is automatically put in motion in one direction or the other to bring the voltage back to normal. A positive but adjustable time delay is insured by a relay, driven by a small induction motor. Between the motor shaft and the contact making element, a train of gears with a gear shift mechanism is introduced, allowing adjustment from one sec. to 30 min.

Two contact making voltmeters are used on each of the two transformers, one adjusted to respond to a narrow range of voltage variations, while the other one is set for much wider differences in voltage. If the line voltage rises or drops only slightly, and if this condition persists long enough to bridge the introduced time delay, the transformer will shift to the next proper tap. If, on the other hand, a considerable rise or drop occurs, the second contact making voltmeter will respond and will cause immediate adjustment without any time delay. The necessary instruments, relays, timing devices, etc., are arranged on two switch panels.

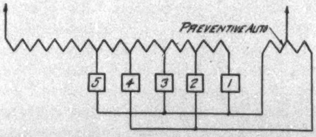

FIG. 8—DIAGRAM OF TYPICAL CIRCUIT ARRANGEMENT FOR LOAD RATIO CONTROL OF A POWER TRANSFORMER WITH THE SELF PROTECTING AUTO-TRANSFORMER AND SINGLE-WINDING ARRANGEMENT

A large number of transformers has been built during the past year with external auxiliary equipment for changing taps under load. A simplification of the problem of taps changing under load is claimed for a new development of the single-winding scheme. By use of a self-protecting preventive auto-transformer, only one switch operation is required to change the voltage ratio of the transformer under load and protective equipment is not required for the transformer windings as no winding is overloaded during the change in taps. This method of tap changing under load permits mounting all the switches external to the transformer tanks and requires a minimum number of switches.

The following is a description of the operation of the single-winding arrangement shown on the schematic diagram Fig. 8. On position No. 1 with switch No. 1 closed, one-half of the preventive auto-transformer carries the load current of the main transformer. The change from voltage position No. 1 to voltage position No. 2 is made by the single operation of closing switch No. 2. In this second position, each half of the preventive auto-transformer winding carries half of the load current of the main transformer and the voltage obtained is equivalent to a voltage midway between taps. The resultant current in the two halves of the auto-transformer will be the vector sum of the exciting current of the auto-transformer and one-half of the load current. The switching cycle as given above is repeated throughout the entire range of taps. As a switch is opened only upon each alternate voltage position, two voltage positions are obtained for each switching cycle, which is relatively light duty cycle for tap changing service. A slight inequality in voltage steps is found in changing from one voltage position to the next, due to the change in reactance on alternate positions. The reactance difference during this cycle is an invert function of the circulating current which is present when the auto-transformer is connected across adjacent taps and is controlled by the use of suitable air-gaps in the core of the auto-transformer. By this means, the reactance variation may be reduced to a minimum so the voltage difference is small and not objectionable.

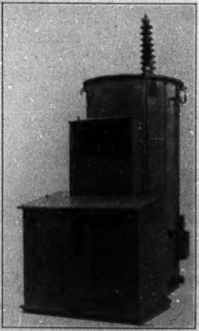

Fig. 9—20,000 Kv-a., 60-Cycle, Single-Phase, 13,200/12,000-Volt Power Transformer with Load Ratio Changer Placed in Grounded End of High Voltage Side

Two 36,000-kv-a. transformer banks have been installed using the transformer under load. The nominal voltage of the transformer banks is 132,000 to 11,500 volts with plus or minus 10 per cent voltage variation under load on the low-voltage side. The tap changing equipment used in this installation is the first equipment built with complete automatic control. As the low voltage raises and lowers with load variation, the tap changing mechanism automatically changes taps to compensate for the variation. If desired, the tap changer may also be operated by a remote control switch, or, in case of emergency, manual operation may be used.

During the last year, the largest artificially-cooled unit built was a 66,667-kv-a., 25-cycle auto-transformer. It is the largest transformer so far constructed in the United States, not only in rating, but in physical dimensions. For instance, it required 36 tons of steel and 17 tons of copper for the windings. The total weight including oil exceeded 120 tons. This transformer is utilized to step up the voltage of a turbine-generator line from 12,000 to 24,500 volts and its equivalent rating as a transformer is 34,000 kv-a.

Four single-phase auto-transformers of record size were also built for air cooling. They are rated 30,000 kv-a., 220,000 Y, 125,000 Y, 10,640 volts. These transformers have a larger capacity and exceed in physical dimensions any transformer of this type so far constructed. The conservator which contains 1300 gallons of oil is in itself equivalent in dimensions to the tank required for a 2500-kv-a., 60,000-volt transformer. The weight of the conservator is approximately five tons and the total weight of the transformer exceeds 130 tons.

In connection with the transmission of power from the Conowingo development, there will be required 580,000 kv-a. in 220-kv. transformers. The step-up service at the generating station will require 13 water-cooled transformers rated 26,667 kv-a., 220 kv. These will be connected in banks of 80,000 kv-a. In the step-down substation, there will be seven 33,333-kv. a., 220-kv. straight self-cooled, three-winding transformers to step the voltage down to 66 kv. These will be connected in banks of 100,000 kv-a. and will be arranged for ratio changing under load. These will be more than 50 per cent greater in rating than the largest self-cooled, single-phase transformers reported last year.

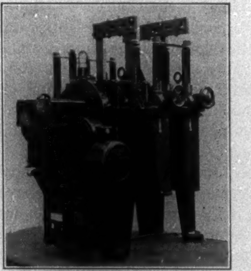

Fig. 10—Circuit Breakers and Operating Mechanism for Load Ratio Changer of 20,000-Kv-a. Transformer and Arranged for Motor or Hand Operation

The largest artificially cooled single-phase transformers were completed this year. They are rated at 31,400 kv-a., 60 cycles, 12,000 to 132,000 Y volts and are water-cooled. These transformers, while of greater rating, do not exceed in physical size the 28,000-kv-a., three-winding transformers shipped two years ago to Japan. These three-winding units, of American manufacture, have not been exceeded in physical size by any other water-cooled transformer. The present maximum capacity for two-winding transformers will be exceeded upon the completion of some 33,333-kv-a., two-winding transformers now under construction. The 33,333-kv-a. units are for 220,000-volt service, for which a great number of units has been constructed during the year. They are cooled by radiators with forced air cooling and are the largest size to which this method of cooling has been applied.

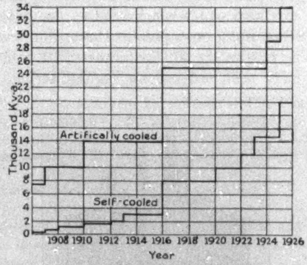

FIG. 11—GROWTH OF UNIT CAPACITY OF SELF-COOLED AND ARTIFICALLY-COOLED TRANSFORMERS

The largest totally self-cooled transformers were recently put in operation. They are rated at 25,000 kv-a., three-phase, 60 cycles, stepping-up from 13,000 to 120,000 volts. The windings are equipped with tap changers designed for future tap changing under load. An overload capacity giving 35,000 kv-a. is obtained by means of forced oil circulation through the cooler.

The development of means to increase the rating of self-cooled transformers by the method of blowing air on the cooling surface, which originated about five years ago, has been applied to a number of large installations during the past year. It has been found that in some cases where transformer efficiency is not of prime importance, the saving effected in the transformer itself is sufficient to offset the cost of the blower and air duct equipment.

For use in underground distribution systems, a new subway transformer tank has been developed which is suitable for larger capacities, higher voltages, and heavier currents. The junction boxes form an integral part of the tank and provide adequate high- and low-voltage terminal facilities.

Peculiar conditions in connection with the marketing of both 25-cycle and 60-cycle power from the new hydroelectric developments on the Gatineau River in Quebec have called for the use of some 14,000-kv-a. water-cooled transformers suitable for use on either 25 cycles or 60 cycles without any change whatever in the windings.

The outcome of the recent attempt to formulate standard voltages will have far-reaching effects upon the transformer industry. Standard voltages and capacities will result in economies both in manufacture and use.

Recognition of the advantages of a means of changing ratios under load has resulted in a large demand for this type of transformer in Europe, as well as in America.

In the report for last year, mention was made of some large single-phase transformers for 16 2/3-cycle railway work and arranged for three voltages. The claim is made that these are the largest transformers in the world. If a 50-cycle transformer were built with the same material, its rating would be 100,000 kv-a. The design of these transformers is of particular interest because it was necessary to divide the 15,000-volt winding between the 66,000- and 132,000-volt sides to secure the necessary mechanical strength for short circuits. To meet the various magnetic conditions for the different methods of operation, it was necessary to provide a special by-pass core for the magnetic leakage fluxes.

Among the large transformers built in Europe, it is of interest to mention six 3-phase, 5-leg, 44,000-kv-a., 50-cycle units for the Rummelsburg power station.

An interesting European development along the lines of high-voltage testing transformers are some 750,000-volt units which are constructed on an entirely new principle. It has been the practise to immerse the whole active parts of such a transformer in an oil tank but in this design only the windings are oil-immersed. This is accomplished by arranging the inner and outer insulating cylinders concentrically in such a way that they form a container for the oil. This allows dispensing with expensive insulating bushings. The following advantages are claimed for this construction:

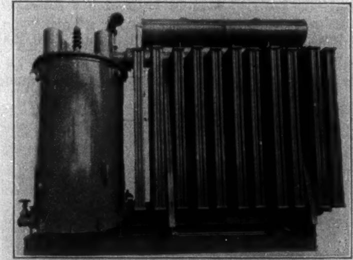

FIG. 13—29,667-KV-A. TRANSFORMER WITH RADIATORS MOUNTED APART FROM THE TANK

1. Low manufacturing cost, owing to the absence of bushings and tanks and less oil.

2. Small floor space as compared with testing plants using multiple stage cascade connections.
3. Increased reliability, owing to independence from atmospheric influences.
4. Ability to withstand heavy loads owing to the comparatively small leakage voltage.

Tests at 1,500,000 volts have been made with two of these transformers connected in series.

What are probably the highest capacity testing transformers were constructed during the year for use in cable testing. They are rated 400 kv-a., 2300/200,000 volts, 60 cycles, single-phase.

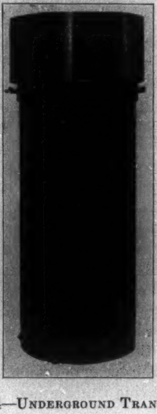

Fig. 14—Underground Transformer

This type of transformer may be buried in the ground

The highest voltage instrument potential transformers that have been considered for commercial use have recently been designed and will be built and installed during the coming year. These will be suitable for 220 kv.

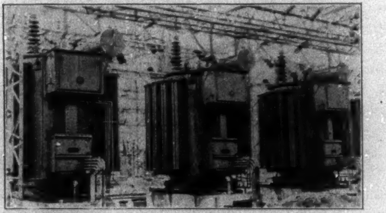

Fig. 15—36,000-Kv-a. Bank of Transformers with Complete Automatic Control of Ratio Changer under Load

Probably the largest electric furnace transformer ever built is now under construction in a Canadian factory for an aluminum plant. It is rated 15,000 kv-a., three phases, 60 cycles, 13,200-250 volts with taps on the high-tension winding to give full capacity in the low-tension winding over a range from 225 volts to 275 volts. The high current of 38,500 amperes at the 225-volt rating involves some very unusual problems in the arrangement of windings and terminals.

A year or two ago, the Power Commission of the city of Toronto was presented with the problem of dealing with objections raised by property owners in some of the better residence sections against mounting distribution transformers on poles. Arrangements were made with a manufacturer to build a few transformers that could be buried in the ground as an experiment. It was found that the ground is as effective in cooling as the air and practically the same heating was obtained from a given core and coil when buried in an underground tank as when hung on a pole in the ordinary out-door tank. The results are so satisfactory that other transformers have been ordered for regular service. These units are contained in welded tanks with an extension to the top to which is fitted a suitable man-hole cover. The whole may be buried in a lawn so that the man-hole cover is even with the surface of the ground and the primary and secondary leads carried underground to the nearest distribution pole. When it is desired to examine the transformer, the man-hole is removed to give access to the cover of the transformer tank proper. Later designs of these transformers have an arrangement whereby the taps may be changed by means of a key without removing the transformer cover. A surprising feature of the tests on the trial transformers was the fact that the snow did not melt appreciably more in the immediate vicinity of the transformer than in other parts of the lawn.

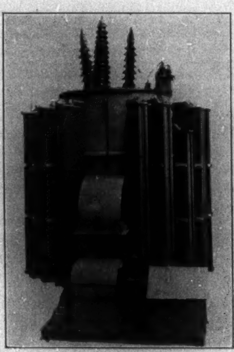

Fig. 16—33,330-Kv-a., 220,000-Volt Two-Winding Transformer Arranged with Forced Air Cooling Equipment

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INDUCTION MOTORS

The demand for squirrel-cage motors, suitable for starting at full voltage without compensator or other starting device, has very greatly increased during the past year. Where the service does not require very heavy starting torque, such as centrifugal pumps, blowers, motor-generator sets, etc., the starting current can be reduced sufficiently without reducing the starting torque beyond the requirements by simply increasing the reactance of the squirrel-cage winding. For other classes of service where a normal or even a high starting torque is required, the situation is met by adding a second higher resistance squirrel-cage to the regular working winding which still has high reactance. The high-resistance winding having a low reactance, this type of machine is commonly called a double squirrel-cage. Standard lines of motors up to 50 hp. of these types have been brought out by several different motor manufacturers. In general the characteristics of this type of machine are such that up to and including 30 hp., they are successfully made so that the static current is within the N. E. L. A. requirements, and the static and pull-in torque are substantially better than is possible with a single squirrel-cage winding. Generally the efficiency is practically the same as for the single squirrel-cage and the power factor is practically the same as for two-and four-pole machines but there is a reduction of a few points in the power factor of the six-pole machines and a correspondingly lower power factor as the number of poles increase.

The use of ball and roller bearings seems to be growing in favour for the number of motors with this type of bearings forms a greater part of the total sales each year. An advantage which is claimed is that motors may be shipped with the necessary lubrication ready to be put into operation and shields do not need to be changed to ceiling or wall mounting. Grease lubrication has been found satisfactory for motors up to two hp. at 8000 rev. per min.

The principle of directing jets of air on the surface of self-cooled transformers to accelerate the convection of heat has been applied to totally enclosed induction motors by equipping them with fans arranged to blow air over the outside surface. By this arrangement it is possible to build totally enclosed motors of somewhat larger capacity and especially in the larger sizes they are less costly than the straight enclosed motors without ventilation.

The growing tendency for system interconnection is resulting in the gradual elimination of odd frequencies so that the demand for motors is very largely for the standard 60-cycle frequency. It is probable that within a short time there will be no demand for 40-cycle motors because many of the 40-cycle systems are being changed over to 60-cycle.

For many years, 220 volts has been a standard voltage for small general purpose motors and power companies have provided suitable service for this voltage. Recently, due to the apparent economies of four-wire distribution for power and light services, an active demand has grown up for 200-volt motors, and it is probable that stocks of motors for this voltage will soon be regularly carried.

A very interesting development in single-phase, induction motor design was brought out by a German firm. The motor has no commutator, but is able to develop sufficient torque to make the motor suitable for railway work. This motor has two rotors on the same axis. The first rotor is synchronous and excited from a direct current source but does not transmit any torque to the shaft. The second one rotates inside the first driving the shaft and is an ordinary slip-ring motor. The synchronous rotor supplies the magnetization and it is possible to obtain unity power factor at different loads. It is also possible by increasing the excitation to compensate for the effect of voltage drop on the maximum running torque. A motor, rated 225 hp., 50 cycles, has been built.

An induction motor with Ward-Leonard control capable of carrying peak loads of 18,000 hp. deserves to be mentioned among large machines. This motor was recently built in Europe and was furnished for rolling mill drive.

The rapidly increasing use of fractional horsepower motors for domestic and other purposes has caused the power companies to become concerned about poor power factor conditions because there is no restriction about the performance characteristics of motors sold to the householder. The power companies have been seeking relief from this situation and recently an association of motor manufacturers, appliance manufacturers and power companies has been formed to study the situation. It has been agreed that motors for refrigerators and oil burners must have an apparent efficiency of 42 per cent and it is urged that improvements in design be made as rapidly as possible with the aim to obtain an apparent efficiency of 45½ per cent. It is possible that a single motor design will not be applicable to all domestic apparatus because of the wide range in duty.

A special application of a small motor to feeding the electrodes of an arc welding machine where it is required to operate close to the arc has led to the development of a water cooled motor. The motor has copper water tubes cast in the aluminum frame.

The desirability of domestic appliance motors being capable of operating without noise has lead to the provision of a spring support for some special application motors to eliminate the noise that is inherent in single-phase machines.

A new type of repulsion motor with a squirrel-cage in addition to the commutated winding has been developed in England for which advantages are claimed over the ordinary type.

The synchronous induction motor of the type using separate direct current excitation continues in favor in Europe. However, European literature indicates that the possibility of designing new types of self-excited machines is being actively investigated. Although the separately-excited type of motor was used commercially in America as long ago as 1911, it has not been so vigorously exploited as it has been in Europe. This year, however, a larger manufacturer has built motors of this type for driving tube mills where heavy starting torques are met and long periods of operation at full load make high power-factor motors highly desirable. This motor is the same in general construction as a wound rotor induction motor and has the same starting operation. It is provided with five collector rings for the rotor winding instead of the customary three rings. The exciter is wound for a low voltage and is connected permanently in the rotor circuit so that the starting operation is as simple as that of a wound rotor induction motor and requires no extra switching operation for the exciter. In order to limit the amount of excitation required these motors have deeper slots and considerable more copper for the rotor winding than the corresponding induction motor would have. Two motors of this type rated 900 hp., 180 rev. per min. have been put into operation this year and a 1500 hp. motor is being built.

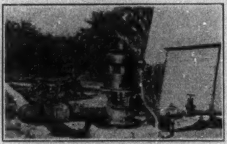

FIG. 17—VERTICAL INDUCTION MOTOR FOR PUMP DRIVE

Arranged with hollow shaft through which the pump shaft extends to clutch at the top.

A brush shifting, a-c., adjustable speed commutator type of motor is now available for operation with a continuous rating at low speed as well as at high speed.

The largest adjustable speed induction motor with Scherbius control to be built in America has been installed for driving a bar mill. It is rated 6700/5000/3320 hp. at 500/375/250 rev. per min.

During the past year, special induction motors have been designed and built for vertical irrigation pumps of the turbine or centrifugal type. These motors have hollow shafts through which the pump shafts project. The motor and pump shafts are connected by a clutch at the top of the motor. Important advantages are easy adjustment and alinement of the pump impeller, elimination of whipping action at the upper end of the pump shaft, or damage due to the momentary reversal of the motors during a power failure. Special windings obtain a low starting current when the motors are started at full line voltage. Relatively high starting and pull-out torques are obtained, with an efficiency and a power factor, at full load, approximately the same as those obtained with standard motors.

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SYNCHRONOUS MOTORS

In the report of last year, mention was made of the use of single-phase, synchronous motors as parts of motor-generator sets to be installed on railway locomotives for supplying d-c. power to the main traction motors. These sets have been made in three sizes,—400 kw., 1000 kw. and 2500 kw.,—the ratings of the synchronous motor being 500 kv-a., 1200 kv-a., and 3100 kv-a., respectively,—25 cycles, 2300 volts, 750 rev. per min. These sets are equipped with direct-connected exciters and single-phase commutator motors, which are used not only for starting up the sets but also for furnishing excitation to the main traction motors for regenerative braking. For the purpose of maintaining uniform voltage at the terminals of the synchronous motors, the single-phase transformers for stepping the voltage down from 11000 to 2300 volts are provided with automatic tap changers. The two smaller sets are constructed with two bearings in the end shields while the large set has four bearings with the synchronous motor at one end and driving the two 1250 kw. d-c. generators in tandem.

FIG. 18—DOUBLE PROPELLING MOTOR FOR DIESEL-ELECTRIC TUG BOAT

A new type of self-starting synchronous motor has been developed for use in time switches, traffic signals. and other small devices requiring very small amounts of power. It is similar to the ordinary induction disk motor which has a disk type rotor revolving between the poles of a stationary electromagnet. The synchronous characteristic is obtained by means of a series of small iron inserts pressed into the disk near the periphery, which locks into synchronism with the alternating flux. A motor of this type which is probably the smallest electric motor that was ever made for actual commercial use has been built. It is two inches high and weighs four ounces and its output is less than one-millionth hp.

As has been mentioned in the 1925 report of this committee, two types of synchronous motor construction, particularly suitable for starting heavy loads, have been developed. One makes use of a built-in magnetic clutch which allows the rotor to come up to synchronous speed without load, the other is arranged so that the armature will be brought up to synchronous speed without load. For tube mills in cement manufacturing plants, there has been an active demand for motors of this type. The largest clutch-type motors have been built during the year and are rated 900 hp., 1800 rev. per min., 100 per cent power factor. A new competitor in this field is the synchronous induction type, of which an important example is mentioned in the induction motor section of this report.

High-speed, synchronous motors of the turbo-alternator type have been successfully used for gas pressure boosters and blowers. Motors of 1800 rev. per min. and 3600 rev. per min. have been built for this service. The largest motor of this type that has been built was put into operation this year and is rated 2700 hp. at 180 rev. per min. It has successfully met the severe requirement of four starts in succession.

D-c. Machines

The application of Diesel electric power for the propulsion of boats, particularly tug boats and ferries which require a very flexible control for maneuvering, has swung strongly in favour of d-c. machines. For this purpose, some special types of propelling motors have been developed which are designed to fit the cramped space available for them. These motors are constructed with double armatures. Motors of this type rated 1250 hp. have been built for operation from a 1000-volt source. Normally the generating units are in pairs and their armatures are connected in series.

Fig. 19—8000-HP. Direct-Current Motor for Driving the Largest Blooming Mill in the World
This is the largest single armature direct current motor.

As in the case of induction motors a new line of totally enclosed fan-cooled, d-c. motors has been developed. These motors have fans, external to the parts that enclosed the windings and core, which draw air into the end shields and blow it over the surface. This arrangement makes it possible to obtain greater outputs from a given frame size than it is possible with the ordinary type of enclosure. For certain slow-speed ratings it is possible to increase the rating as much as 50 per cent.

Hydroelectric units with large size d-c., generators in vertical shaft settings are quite unusual, and for this reason, the installation in a European power house of four 4500-kw. machines should be noted. These will be capable of delivering 12,300 amperes at pressures ranging from 175 volts to 350 volts. The normal speed is 500 rev. per min.

Although synchronous and induction motor drives in steel mills have made great advances, d-c. motors are found to have indispensable characteristics for certain applications and there seems to be a decided tendency to go back to d-c. motors for drives where variable speed is required. During the year there has been built and put into operation, a blooming mill drive with Ilgner motor-generator set and reversing motor capable of handling peak loads of 15,000 hp. This apparatus embodies those features that have been found satisfactory in smaller units. During the past year work was completed on the 8000-hp. d-c. motor which is to drive the largest blooming mill in the world. This is the largest single armature d-c. motor ever built, both as to continuous horse-power and maximum torque. Other installations are a 3000-hp., 80/150-rev. per min., adjustable speed motor and a 7000-hp., 50/100-rev. per min., adjustable speed motor.

The first steel mill motor, using the series exciter scheme of connection for obtaining flat speed regulation, was put into service during the past year. This speed regulation is obtained by using two field windings on the mill motor. One of these fields is supplied from a constant potential source while the other has for its source a separate exciter, the field of which is in series with the armature of the main machine. The series field can therefore be made to compensate for the tendency of the motor to drop its speed with increase of load.

A recent contribution to the study of limits of large d-c. machines places the maximum commutator segment voltage at 30 volts and the maximum peripheral speed of the commutator at 360 ft. per sec., but at speeds above 100 ft. per sec., difficulties due to unreliable brush pressure are encountered.

Two 1400-kw., 125-volt, 11,200-ampere, 360-rev. per min. generators, built during the past year to electrolytic refining service are the largest 125-volt machines that have been built to date. These machines are notable for the extremely high current delivered at low voltage by a single commutator.

As mentioned in another part of this report, one of the means of increasing the transient stability of a transmission system is by increasing the speed of response of the exciter. This increase in the response of the d-c. machine can be accomplished by any or all of the following means:

1. Dividing the exciter field circuit into parallel paths.
2. Separately exciting the d-c. machines.
3. Increasing the rotational speed of the exciter by separate drive.

The advantages of this type of exciter in increasing system stability are now very generally recognized. Many large machines sold during the past year have been supplied with the quick response exciters.

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Mercury Arc Rectifiers

There have been no radical developments in mercury arc rectifiers during the past year. It has been the first year in which rectifiers have been in service in the United States in considerable numbers, and the first time that they have been used in steam railroad electrifications in this country. For the first time, such a length of main line has been fed from rectifiers that service of the road has been dependent upon their continuous operation. That this device is a dependable piece of apparatus is indicated by the fact that a 1500-kw., 1500-volt rectifier has carried peak loads of 9000 kw. during an emergency extending over a period of several weeks.

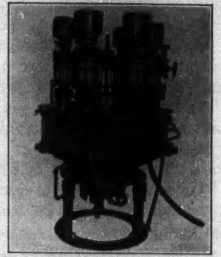

FIG. 20—STEEL-TANK MERCURY ARC RECTIFIER RATED 500 KW., 600 VOLTS, OR 750 KW., 1500 VOLTS

It has been observed that inductive interference may be experienced in some cases of mercury arc rectifier operation but this problem has been met and methods have been worked out to bring the performance within communication standards.

A new type of rectifier has been placed on the market which has a higher current capacity than any built heretofore. This machine has twelve anodes and has a continuous rating of 2000 amperes at 600 volts. At higher voltages the current rating is a little less. Further progress has been made in the development of rectifiers for high voltages. Successful load tests have been made on steel cased units at 8000 volts. The application of power rectifiers of such high voltage is, at present, confined to certain chemical processes. Its overall efficiency is approximately 99 per cent. The voltage of rectifiers suitable for railway work is limited at present only by the design of the motor equipment. The total capacity of rectifiers installed by one large European manufacturer during the past year amounts to about 200,000 kw. and 90 per cent of them are for railway work.

Probably the most outstanding installation of rectifiers in the United States is that for the electrification of the Illinois Central Railroad terminal and suburban service at Chicago, where 9000 kw. in rectifier capacity is in use. An indication of the European confidence in rectifiers is made manifest in the placing of orders for a total of 95 units having a total capacity of 114,000 kw. by the Metropolitan railroads of Berlin.

FIG. 21—VIEW OF MERCURY ARC RECTIFIER OF AMERICAN MANUFACTURE WITH COMPLETE EQUIPMENT

During the past two years the total capacity of rectifiers manufactured and installed the world over has doubled each year.

In 1924 the total capacity was approximately 150,000 kw. In 1925 it increased to 310,000 kw., and in 1926 to 600,000 kw.

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Diesel Engine Driven Alternators

During the year, the largest Diesel engine driven generator has been put into operation at Hamburg (Germany). The engine is rated at 15,000 hp. and is of the two-stroke cycle type with nine double acting cylinders. The direct-connected alternators are rated 13,000 kv-a., 94 rev. per min., 6000 volts, 50 cycles. The design of the shaft and coupling of a unit of this kind requires the consideration of torsional oscillations of the shaft.

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Frequency Converters

Fig. 22—Shop View of 30,000-Kv-a., 600-Rev. per min. Frequency Converter Set
Showing Arrangement of air intake and exhaust

Ties between power systems of the same frequency permitting control of the flow of power in either direction under conditions of varying voltage are made possible by the use of transformers having taps that may be changed under load and it is also possible to tie two systems of different frequency and control the flow of power under conditions of variations in frequencies by frequency converters of the induction type. This type of converter, consisting of a synchronous machine and an induction machine with Scherbius control, was mentioned in the report of this Committee in June 1925. A number of 5000-kw. sets of this description have been built. Another type of converter makes use of the cascade induction motor which depends upon a constant ratio of frequency. A number of 35,000- and 40,000-kw. sets of this type have been previously noted. During the past year there has been built a 15,000-kw. set of this description, and a 35,000-kw. synchronous frequency converter set has been arranged with suitable spring support for the generator stator so that it can be used for single-phase operation. This type of support minimizes the effect of the pulsating torque resulting from single-phase operation. The single-phase rating is 21,000 kv-a. The largest synchronous frequency converter set ever built has been put into operation. It is rated 50,000 kv-a., 300 rev. per min.

Two sets with unusually high speed, have been built in the past year; one, a 30,000-kv-a. frequency changer at 600 rev. per min., and the other, a 17,777-kv-a. set at 720 rev. per min. The 30,000-kv-a. set is a 50-cycle—60-cycle set. The 50-cycle machine has been designed particularly for transmission line stability and incorporated in it the features described under the heading *Machine Characteristics for Stability.*

In Europe there has been an increased demand for synchronous frequency converters principally for railway work.

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Synchronous Condensers

Because of the fact that synchronous condensers are in the class of non-revenue producing equipment, there has been a concerted effort to reduce costs and losses. This has been accomplished partly by designing for higher speeds. Sixty-cycle condensers have been constructed for operation at 900 rev. per min. up to a 10,000-kv-a. capacity and at 720 rev. per min. up to 20,000 kv-a. The largest condensers yet constructed are several that are for use for the regulation of the voltage at the receiving end of a 220-kv. transmission line in California. They are rated 50,000 kv-a., 600 rev. per min., 50 cycles, and the total calculated losses are only 1 2/3 per cent. They will be provided with a closed system of ventilation.

The largest condenser ever arranged for automatic control is under construction. It is rated 30,000 kv-a.

In Europe it is being found advantageous to employ as condensers asynchronous machines provided with phase advancers. A number of these have given complete success in service. They have been built in capacities up to 10,000 kv-a. Mention of these machines is made on p. 183 of the *Electrotechnische Zeitschrift* for February 10th, 1927.

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SINE-WAVE GENERATORS

The largest sine-wave testing generator has been completed during the year. It is rated 2000 kv-a., 2300 volts, 3-phase, 60 cycles, 1200 rev. per min., and is driven by a 600-kv-a., synchronous motor. This generator is capable of delivering 1200 kv-a., single-phase, and it is remarkable in that the voltage and current wave forms are sine-waves under all conditions of load. The characteristics are practically ideal. This set was built for use in testing high-voltage, underground cables.

HIGH-FREQUENCY MACHINES

During the past year a plan for the resonant control of street lights has been developed and this should give a new field for the application of high-frequency machines. This system of control requires a generating system which can supply both 440 and 660 cycles to the main lighting circuit. The entire system is described in the *Electric Journal* for February 1927.

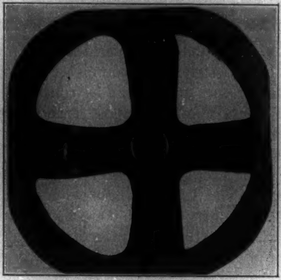

FIG. 23—ARRANGEMENT OF LOWER BEARING OF TAPERED ROLLER TYPE FOR INDUCTION VOLTAGE REGULATOR

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INDUCTION VOLTAGE REGULATORS

An improvement in the design of induction regulators for outdoor service has been made by bringing the driving spindle for the movable element out through one of the passages connecting the regulator tank and the expansion vessel, thus avoiding the use of all glands. An improvement in operation of induction voltage regulators as affecting vibration and noise has resulted from the use of roller bearings for the lower end of the shaft. A clearance in sleeve bearings will allow the movable element to vibrate and cause noise, yet tight bearings are liable to stick and interfere with the operation. Roller bearings can at once be made tight and free to turn. The use of all-welded, corrugated sheet-iron tanks has resulted in improved appearance, reduced weight, increased insurance against leakage, and greater ability to withstand shocks as compared with the corrugated cast-in construction.

SYNCHRONOUS CONVERTERS

During the year, some synchronous converters have been built which have an unusually wide range of direct current voltage. They are rated at 1500 amperes over a range of 87 to 175 volts. The method of obtaining this variation in voltage is unique in that the a-c. voltage at the collector rings is varied by means of tap changing on the transformer. It is necessary, of course, to change the field excitation to correspond to the different transformer taps. Voltage variation between taps is obtained by field control.

The twelve phase synchronous converter which was mentioned in the report of last year has been put into operation and two more have been built. The various advantages which were mentioned have been fully realized in actual practise and it is not improbable that for future production the number of 12-phase converters will exceed that of six phase.

A recent English development in synchronous converters is the socalled binary converter, which in effect is a device consisting of an induction motor and a special direct current generator having a common magnetic circuit of sufficient size to take care of both rotating and stationary fluxes. This machine consists of a uniform magnetic circuit and a rotating armature. Two windings are located in the same slots of the stationary member, one for producing the stationary current flux and the other for producing a rotating flux. By a combination of a two-pole a-c. winding and a six-pole d-c. winding the six-pole d-c. winding on the rotating armature acts as a short circuit or squirrel cage winding for the bipolar a-c. winding. The usual commutator and brush gear is used with the armature winding.

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B. L. BARNS, *Chairman* of Subcommittee
on Annual Report.

Progress in Power Generation

Annual Report of Committee on Power Generation*

To the Board of Directors:

A meeting was held shortly after the appointment of the present chairman in February, to formulate plans for the committee's activities during the remainder of the administrative year.

The question of taking up certain subjects for special study and investigation was given particular consideration, but it was not thought advisable to attempt any work of this kind, because of the limited time available in which to prepare the annual report. There were presented at the meeting three subjects which the committee therefore recommends for investigation by its successors; *viz.*, Interconnection—its relation to station and unit capacity, reliability, and economy; Oil Circuit Breakers of Large Capacity—with particular reference to reliability, economy of space, and costs; and Comparison of American and European Power Station Practise.

At the invitation of the Standards Committee a "contact officer" was appointed for prompt cooperation with that committee on any matter of standardization in which the Committee on Power Generation may be concerned. In this connection the Standards Committee has been asked to cooperate with the A. S. M. E. in the preparation of a section on "Measuring the Output of a Generator" for inclusion in the A. S. M. E. Test Code for Steam Turbines. A subcommittee of your Committee on Power Generation has been appointed to assist the Standards Committee in this work.

There have been secured by the committee for presentation at the Summer Convention in June, two papers dealing with the design and operation of Power Stations, one by F. A. Allner on *Holtwood Steam Plant, Design and Operation in Coordination with Water Power,* and one by J. W. Anderson and A. C. Monteith on *Auxiliary Power at Richmond Station.*

Many important developments in power generation during the past year have been extensively reviewed and the following résumé and bibliography are presented to show the progress made and the trend in the art.

Résumé of the Year's Progress in Power Generation

One of the outstanding features in power development is the tremendous increase in the so-called ultimate station capacity over that anticipated at the time of the initial installation. In some cases, the ultimate capacity will probably be more than double that originally expected. What has brought this about is the unprecedented increase in system loads and the introduction of units of much larger capacity than those first installed. In the eternal quest for greater and yet greater economies in power stations, notable technical achievements have been brought about in practically every line of the industry. Such developments during the past year have been so many and so varied that it is possible in a brief survey to touch on only a few of the most noticeable tendencies.

In this résumé, the committee does not attempt to predict the future of the art in power generation, but simply points out the trend as indicated by the important developments that have taken place or are now under construction.

Steam Turbine Generator Units

Capacity. Turbines and generators of unprecedented size have been designed and completed during the year and still larger ones are now under construction. Less than two years ago the largest turbine being built was under 100,000 kw. The largest turbine to date is the 208,000-kw. unit now under construction and one of the manufacturers has offered a machine in excess of 300,000 kw. The largest single-cylinder turbine yet built in this country is a 65,000-kw. normal pressure unit. Without extensive interconnection it would not be wise or economical to install such large generating units.

The motive for these huge machines is not so much better fuel economy as it is reduced plant investment and the desire to keep the total plant capacity in reasonably few units. Experience with large units has demonstrated that they can be operated almost as continuously and easily as smaller machines. We may expect therefore, that the construction of huge generating sets will continue, although to what final limit it is difficult to predict.

As an indication of the trend toward larger units, it has recently been decided that throughout Chicago and adjacent territory no turbine will be installed in the future of less than 50,000 kw. capacity.

Type. The combination of turbines and generators to make up a single generating unit is quite varied. Tandem compound and two- and three-element cross-compound formations offer numerous means of providing generating units in excess of 60,000 or 70,000 kw. capacity. The actual formation depends to a large extent on local conditions to be met and the ideas of the turbine manufacturer and the plant designer as to the best means of accomplishing the results desired. It is likely that turbines of the compound type will be used for the very large capacities. There appears however, to be a marked tendency on the part of many engineers toward still larger turbines of the single-cylinder design.

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Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

There are now under construction four outstanding turbine units each of a different type and the largest of its kind on record. The sizes and types of these units are: 208,000-kw. cross compound, three elements; 160,000-kw. cross compound, two elements; 94,000-kw. tandem compound; and 65,000-kw. single-cylinder.

Speed. The trend toward higher speed is evidenced by the fact that there are now on order a number of large turbines which are to operate at 1800 rev. per min. The prevailing turbine generator speeds are 1500 rev. per min. for 25 and 50 cycles and 1800 rev. per min. for 60 cycles. Smaller units up to as high as 10,000-kw. capacity have been made for operation at 3600 rev. per min. The largest 1800-rev. per min. generator designed to date is the 89,412-kv-a. machine for the high-pressure element of a triple-shaft unit now under construction, and the largest 1500 rev. per min. generators on record have a capacity of 100,000 kv-a. An indication of the rapidity of the development of the art is furnished by the fact that a little more than a year ago a 60,000-kw. generator was regarded as a conservative maximum limit of capacity in 1800-rev. per min. units.

It has been suggested by eminent authority on turbine design that some economies might be effected by a system of standardization, which would limit the number of sizes of machines built. The greatest economy in first cost is obtained by using the highest possible rotative speed, and the most profitable machine for builder and user alike is that of the greatest capacity at the given speed. It would be an advantage from the standpoint of investment to concentrate upon a relatively few standard capacities of large machines for 1800-rev. per min., 60-cycle service, and 1500-rev. per min., 25- and 50-cycle service. A larger number of standard capacities would be required in 3600-rev. per min. turbines of capacities of 10,000 kw. and lower for manufacturing requirements. In a growing power system having a number of interconnected stations, a new unit of large capacity added to the system may be operated within quite a range of its capacity without greatly affecting the efficiency of operation of the system, and therefore it would seem desirable to choose the largest standard machine to secure the advantage of lowest investment per kilowatt.

Pressure. The desire to secure high efficiencies in steam power plants has resulted in rapid increase in pressure and temperature to a maximum of 1400 pounds and 750 deg. fahr. in this country. In the United States, the boiler pressures in large steam power plants have been confined to three classes of approximately 1400 pounds, 600 pounds, and 400 pounds, the steam pressure at the turbine throttle being about 1200 pounds, 550 pounds, and 350 pounds respectively. These pressures represent about the average conditions, some may be 5 per cent or 10 per cent above or below. There appear to have developed three schools of engineering for the above pressures and each is apparently convinced that the pressure which it has adopted is the most economical one. From the turbine designer's standpoint no insurmountable problems have been presented up to the highest pressures so far considered.

The 1400- and 1200-pound boilers and turbines have continued to operate satisfactorily and the results in at least one station have justified the installation of additional high pressure units. For large stations, 500 pounds to 600 pounds has become more or less standard pressure. Stations with these pressures have given satisfactory service with no unusual difficulties. In fact, many of the expected serious difficulties with pipe joints and other details have so far failed to materialize. It may therefore be said that the way is clear for a general increase in the steam pressures of all new plants.

An investigation of the properties of steam was started about five years ago and it is expected that this work will be completed in the near future. These data will be of great value in designing high-pressure stations and will help us to analyze the results obtained in actual operation. As to the most economical pressure, some engineers maintain the following: "From our present knowledge it is not likely that steam pressures will very much exceed 400 pounds for all the main units of stations with average load factors and 650 pounds for all the main units of stations carrying what is commonly known as base load." A 400-pound steam pressure plant is probably the most economical installation for a small load factor or low price fuel. Furthermore, this pressure is very well adapted to the superimposition of a high-pressure steam cycle or a mercury cycle at a later date when warranted by an increased load factor or price of fuel. The experience at the Edgar Station of the Edison Electric Illuminating Company of Boston and the Lakeside Station of the Milwaukee Electric Railway and Light Company indicates that a boiler pressure of 1400 lb. per sq. in. is economically justified. Care must be taken, however, not to draw general conclusions from any particular installation for the local conditions are a big factor in determining what pressure is the most economical one for any contemplated station. A careful study must be made before a decision can be reached as to the proper boiler pressure for any new station. We must keep constantly in mind that the base load station of today will be the peak load station of tomorrow, and this fact should not be overlooked when figuring the investment warranted.

Temperature. The total temperature of steam has, in the last few years, been limited to 750 deg. fahr. for all pressures, while most of the plants are operating at temperatures varying from 700 deg. fahr. to 725 deg.

fahr. There probably will be some endeavor to resort to higher steam temperatures in the future, but whether the increase in temperature will be in decided steps by using special alloy steels, or whether there will be a gradual increase in steam temperature from year to year, cannot be predicted at present. Much research has been and is being conducted on the physical characteristics of materials at high temperatures, a feature being the tests carried out under continued stress and temperature. The general results will give opportunities for securing greater reliability under the maximum temperatures which obtain today in power plant operation in this country. It is reported that there are plants in Europe operating on the straight steam cycle at temperatures higher than 750 deg. fahr. but the general adoption of higher temperatures for this cycle will depend upon the results of the research work now in progress.

Bleeding and Reheating. Stage bleeding for heating the boiler feed water and evaporating the make-up water has become almost standard practise and in most of the large plants constructed recently, the bleeding is done in three or four, and in some cases five, separate stages.

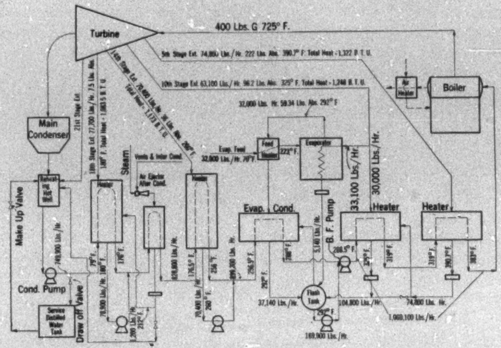
Turbine
400 Lbs. G 725° F.
5th Stage Ext. 74,800 Lbs./Hr. 222 Lbs. Abs. 390.7° F. Total Heat - 1,322 B. T. U.
10th Stage Ext. 63,100 Lbs./Hr. 96.2 Lbs. Abs. 325° F. Total Heat - 1,248 B. T. U.
14th Stage Ext. 70,400 Lbs./Hr. 36 Lbs. Abs. 260° F. Total Heat - 1,173 B. T. U.
18th Stage Ext. 77,700 Lbs./Hr. 7.5 Lbs. Abs. 180° F. Total Heat - 1,083.5 B. T. U.
21st Stage Ext.
Main Condenser
Air Heater
Boiler
32,000 Lbs. Hr. 58.34 Lbs. Abs. 292° F
Evap. Feed 32,800 Lbs./Hr. 70° F.
Feed Heater
223° F.
Evaporator
33,100 Lbs./Hr.
30,000 Lbs./Hr.
Steam
Vents & Inter Cond.
Air Ejector After Cond.
Heater
Heater
Evap. Cond.
Heater
Heater
B. F. Pump
Make Up Valve
Cond. Pump
Draw off Valve
Service Distilled Water Tank
749,900 Lbs./Hr.
79° F.
78,900 Lbs./Hr. 180° F.
176° F.
1,200 Lbs./Hr. 212° F.
828,800 Lbs./Hr.
176.5° F.
70,400 Lbs./Hr.
260° F.
256° F.
899,200 Lbs./Hr.
256.5° F.
292° F.
5,140 Lbs./Hr.
288.5° F.
325° F.
319° F.
319° F.
390.7° F.
383° F.
37,140 Lbs./Hr.
Flash Tank
104,800 Lbs./Hr.
74,800 Lbs./Hr.
1,060,100 Lbs./Hr.
292° F.
169,900 Lbs./Hr.

Fig. 1—Approximate Heat Diagram. Southern California Edison Co., Long Beach Steam Plant No. 3—No. 10 Unit

One method extensively used is bleeding in which the feed water heating is done entirely by steam extracted from the main units. Such a scheme, in which a four-stage extraction heating system contributes a final feed temperature of 383 deg. fahr., has been adopted for the 94,000-kw. turbine in Long Beach Power Station, and is shown in Fig. 1. The evaporator and its separate condenser are interposed between the 10th and 14th stage extractions, which is a thermally efficient disposal of this apparatus, by virtue of the fact that there is no external heat degradation from the 10th stage to the 14th stage. It is economically advisable in a set of this size to employ separate drip return pumps for the two low-pressure heaters and for the evaporator condenser. It will be noted that the drains from the 5th stage heater are passed to the shell of the 10th stage heater, from which point the combined drains of these two heaters enter the flash tank, which cooperates with the evaporator condenser. This generating set will have a very high use-factor so the heaters have been chosen with close terminal temperature differences. The evaporators and heaters are of Griscom-Russell design with removable tube bundles.

Reheating of the steam after it has completed a portion of its expansion, has proved satisfactory and successful where applied. Reheating in the boiler room however, requires large piping to and from the reheating boiler and also involves some loss in pressure which partially offsets the gains from reheating. This year has brought out a new development largely overcoming these objections and it is now proposed to reheat the steam at or near the turbine using high pressure live steam.

Voltage. With the increase in size of steam turbine-driven generators, which today have reached a maximum of 100,000 kv-a., an increase in the generator voltage is desirable. It is well known that this higher voltage will, by decreasing the current, affect a considerable reduction in the cost of busses, switches, cables, etc., and probably before very long many generators of 50,000 kv-a. and upward will be wound for higher voltages than have prevailed heretofore. As an illustration of this trend, there are now under construction a 100,000-kv-a. generator for 16,500-volt operation, the generators for a 208,000-kw. cross-compound three-element unit to be operated at 22,000 volts, and also a single-shaft generator of 61,765 kv-a. that will operate at 22,000 volts.

Generator Cooling and Fire Protection. It is almost universal practise to employ the closed system of air circulation with surface air coolers for machines above 6000 kw. and quite common practise on smaller machines. The possibility of using an inert gas continuously has been suggested, but the predicted advantages have not been considered sufficient to warrant its introduction. The use of hydrogen, because of its lower density and higher specific heat promises many advantages, and provision is being made for its use at a later date in some machines now under construction.

The introduction of the enclosed machine has resulted in the installation of a number of CO_2 fire extinguishing systems. Where this protection is used on generators and similar rotating electrical machines the CO_2 is injected in the intake air duct and a predetermined concentration is maintained until the machine comes to a stand still. Leakage during deceleration is compensated for by introducing additional gas at necessary intervals. This application of course is particularly adapted to closed recirculating ventilating systems.

The following are some of the outstanding turbine and generator installations recently completed or now under construction:

Single Cylinder Turbine Units. The outstanding single cylinder turbine unit installations during 1926 were the 60,000-kw. General Electric units at the Charles R. Huntley (formerly River) Power Station of the Buffalo General Electric Company and at the East River Power Station of the New York Edison Company. The machine installed at Buffalo is supplied with steam at 250 lb. gage and 656 deg. fahr. The speed is 1500 rev. per min. and the generator is rated at 66,667 kv-a. At the East River Station the two 60,000-kw. 100 per cent power-factor units are also General Electric machines which are supplied with steam at 375 lb. gage and 700 deg. fahr. The turbines are twenty-stage impulse type units operating at 1500 rev. per min. and are bled at three points for feed water heating.

The record size single cylinder installation of 1926, however, will be exceeded by the addition now being installed in the Edgar Station of the Edison Electric Illuminating Company of Boston. This installation will include a 65,000-kw. single-cylinder General Electric turbine, which will run at 1800 rev. per min. and drive a 75,000 kv-a. main generator and a 6250 kv-a. auxiliary generator. This turbine will operate on steam at 350 lb. gage and 725 deg. fahr. In addition to the 65,000-kw. normal pressure unit, a 10,000-kw. 1200-lb. unit will be installed which will exhaust through reheaters into the 350-lb. steam header. The feed water will be heated with steam bled from three points of the 65,000-kw. unit and also from the exhaust of the 10,000-kw. high-pressure unit.

Tandem-Compound Turbine Units. The largest tandem-compound (single-shaft) units in operation in 1926 are the two 50,000-kw., 20-stage, impulse type, General Electric units at the Richmond Station of the Philadelphia Electric Company, and the two 50,000-kw., reaction type, Westinghouse units at the Hudson Avenue Station of the Brooklyn Edison Company. The Richmond Station units operate at a steam pressure of 385 lb. and 675 deg. fahr. total temperature. The speed is 1800 rev. per min. and the generators are rated at 62,500 kv-a., furnishing power at 14,000 volts, 60 cycles. The generators are cooled by two motor-driven external fans, solidly connected to the generator leads through transformers. Recent tests indicate, however, that the Richmond Station units can be given a higher rating, which has been tentatively and probably will be finally fixed at 60,000 kw., 0.85 power factor, *i. e.*, 70,600 kv-a. The machines at the Hudson Avenue Station are supplied with steam at 265 lb. and 611 deg. fahr. total temperature. The speed is 1200 rev. per min. and the generator capacity is 62,500 kv-a.

The 50,000-kw., tandem-compound, Allis-Chalmers turbine generator unit recently placed in operation at the Waukegan Station of the Public Service Company of Northern Illinois is also a notable installation, in that the steam pressure will be 600 lb. gage and the total steam temperature 725 deg. fahr. The generator is rated at 58,800 kv-a., the speed being 1800 rev. per min. The exciter will be direct-connected to the shaft. The turbine will be provided with five extraction nozzles.

The Southern California Edison Company will install at its new Long Beach Power Station No. 3, a 94,000-kw., 1500-rev. per min. single-shaft, tandem-compound, turbine generator unit. This will be a 21-stage, impulse type, General Electric unit, and will be the largest single-shaft turbine on record. The turbine will have two cylinders with complete double flow of the steam in the low-pressure cylinder. The steam pressure at the turbine will be 400 lb. gage and 725 deg. fahr. total temperature, and there will be four extraction points for heating feed water to a total temperature of 383 deg. fahr. The turbine will be direct connected to a 100,000-kv-a., 16,500-volt, 50-cycle generator with a 5000-kv-a. auxiliary generator and 60-kw. exciter for the auxiliary generator on the same shaft.

Cross-Compound Turbine Units. The largest cross-compound turbine generator unit placed in operation in 1926 was the 80,000-kw. Westinghouse unit at the Hudson Avenue Station of the Brooklyn Edison Company. This consists of one high-pressure and one low-pressure cylinder operating at 1800 rev. per min., each driving a 45,000-kv-a. generator, with the exciter direct-connected to the high-pressure element. Steam is supplied at 375 lb. and 700 deg. fahr. total temperature. The high-pressure element is a combination impulse reaction turbine, while the low-pressure element is a double flow reaction turbine.

All previous records of turbine sizes will be exceeded by the remarkable cross-compound units now in course of construction—the 104,000-kw. Westinghouse unit for the Crawford Avenue Station of the Commonwealth Edison Company, the 108,700-kw. Westinghouse unit for the Hudson Avenue Station of the Brooklyn Edison Company, the 160,000-kw. American Brown Boveri unit for the Hell Gate Power Station of the United Electric Light and Power Company, the 165,000-kw. General Electric unit for the Philo Power Station of the Ohio Power Company and the 208,000-kw. General Electric unit for the State Line Power Station of the Commonwealth Edison system.

The 104,000-kw. Westinghouse unit (Fig. 2), for the Crawford Avenue Station is unique in that it was designed as a base load unit. The leaving loss of this unit is reduced to a minimum and is approximately 1 per cent at the economical load and 1¾ per cent at the maximum load, in both cases when steam is being bled at four extraction points for heating its own feed water. To secure this low leaving loss, the triple-flow, low-pressure principle is employed. The steam is supplied to the high pressure element at 550 lb. gage and 725 deg. fahr. and expanded to about 40 lb. absolute. It is then reheated, by means of a live steam reheater to 500 deg. fahr. the steam then entering the double flow intermediate element. One-third passes in one direction and is completely expanded, while two-thirds of it passes in the opposite direction expanding to 8 lb. absolute. It is then passed to the double-flow, low-pressure element which is in the same line of shafting as the intermediate element. By this arrangement, steam is exhausted to the six vertical condensers at three points, two condensers being at one end of the intermediate element and two condensers at each of the two ends of the double-flow, low-pressure element. Any one of the six condensers can be opened at the water end while the unit is in service with no other effect than a reduction of the condensing surface. The speed of each of the two lines of shafting will be 1800 rev. per min. and there will be two main generators.

The reheating of the exhaust steam from the high-pressure element by means of live steam from the high-pressure header is a new departure that is also being installed for the 91,500-kw. unit under construction for this station.

After the 104,000-kw. and 91,500-kw. units are installed, and including the 77,000-kw. unit recently placed in operation, the total installed generating capacity of the Crawford Avenue Station will be 432,500-kw.

The Brooklyn Edison Company has just closed an order for a Westinghouse 108,700-kw., 80 per cent power factor (136,000 kv-a.), 60-cycle, three-phase, 13,800-volt, cross-compound, two-element steam generating unit. The turbines are to be of the parallel-flow type and drive two 68000-kv-a., 1800-rev. per min. generators. The exciter is direct connected to the high pressure generator shaft. The machine will use a steam pressure at the throttle of 375 to 400 lb. per sq. in. at 700 deg. total temperature. It will heat the condensate in four stages of feed heating. Two interesting features of the design of the turbines are the five governor control valves in two steam chests and the stainless steel blading. The generators are arranged with internal fans, and generator air coolers are being supplied with the unit. The main generator fields will be alike so that they may be used on either stator, and the shaft ends will be alike for this purpose.

In deciding upon the size of the Hell Gate unit, the special problem arose of providing a turbine of the greatest possible output in the space available, which is

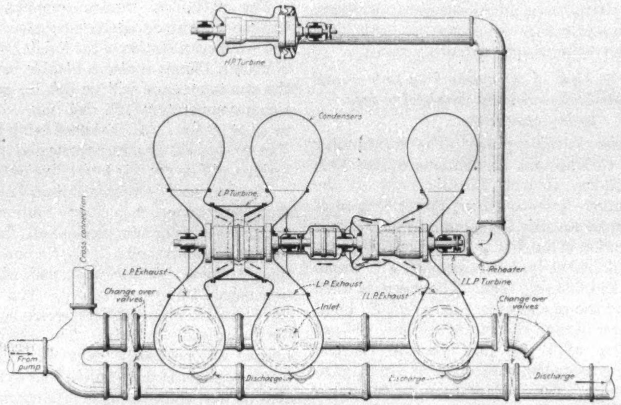

Fig. 2—Arrangement of Turbines for 104,000-Kw. Westinghouse Unit for Crawford Avenue Station of Commonwealth Edison Company

The circulating-water piping shown for three of the condensers is duplicated for the other three.

25½ ft. by 69 ft. Owing to the limited floor space, the turbine was designed as a two-cylinder machine with two lines of shafting (cross-compound) and it is a pure reaction turbine. The high-pressure element has a capacity of 75,000 kw. at 1800 rev. per min., and the low-pressure element can deliver 85,000 kw. at 1200 rev. per min. Steam will be supplied to the high-pressure element at 265 lb. and 600 deg. fahr. total temperature. There will be extraction at two stages for heating feed water, one at the low-pressure element inlet and one at about the middle of the low-pressure element. The alternators are built for a continuous output of 188,200 kv-a. at 13,800 volts and 60 cycles, the capacities of the generators driven by the high- and low-pressure turbines being 88,200 kv-a. and 100,000 kv-a., respectively. This unit will be required at present for a normal service of 50,000 to 100,000 kw., but in the event of one or more of the existing units failing, it must be able to take over 160,000 kw. continuously. In spite of the large overload, it was therefore necessary for the unit to have a high efficiency at a small load and it was designed with a flat efficiency curve.

The 165,000-kw. unit at the Philo Station will consist of one high-pressure and two low-pressure elements, each element having a speed of 1800 rev. per min. the high-pressure element having a capacity of 49,000 kw. and each low-pressure 55,000 kw. There will be in addition to the main generators, two 3000-kw. direct-connected auxiliary generators. Alternating current will be delivered at 11,000 volts, 60 cycles. The main generators will have a capacity of 57,647 kv-a. and 64,706 kv-a. each for the high- and low-pressure turbines respectively, and the auxiliary generators 4286 kv-a. each. Steam will be delivered to this machine at 600 lb. 725 deg. fahr. total temperature and there will be five extraction points for feed water heating, the highest point having a pressure of 360 lb. absolute, and the lowest 6.15 lb. absolute. The initial steam pressure for the low-pressure element will be 126 lb. absolute, 725 deg. fahr. total temperature after reheating.

The largest unit on record anywhere, is the 208,000-kw. three-element machine now under construction for the State Line Plant, a plan of which is shown in Fig. 3. It will consist of a 76,000-kw. high-pressure element and two 62,000-kw. low-pressure elements, all elements operating at 1800 rev. per min. Each of the two low-pressure turbines also operate a 4000-kw., direct-connected, auxiliary generator. The main generators will be wound for the remarkably high voltage of 22,000, the frequency being 60 cycles. The generator driven by the high-pressure turbine will have a capacity of 89,412 kv-a., the two generators driven by low-pressure turbines, 72,941 kv-a. each, and the auxiliary generators 5333 kv-a. each. The high pressure element will be supplied with steam at a pressure of 600 lb. and a total temperature of 730 deg. fahr., and the steam will be reheated between the high- and low-pressure elements to 500 deg. fahr., the pressure being 110 lb. absolute. The exhaust steam from the high-pressure element will be reheated with live steam from the high-pressure header. Eight Allis-Chalmers condensers having a surface of 22,000 sq. ft. each will be used with the 208,000-kw. unit. There will be five extraction points, including the cross-over, from a maximum of 380 lb. absolute to a minimum of 9.4 lb. absolute.

An interesting four-cylinder compound turbine generator unit has been reported in the technical press as being under construction by the Allgemeine Elektricitäts Gesellschaft for a new superpower station in the suburbs of Berlin. The unit will have a maximum rating of 85,000 kw. A high-pressure and an intermediate cylinder are arranged in tandem, driving a single generator and there are two low-pressure cylinders in tandem driving a second generator, the speed of both generators being 1500 rev. per min. The initial steam pressure and temperature are 500 lb. absolute and 750 deg. fahr., respectively. In the first stage a small Curtis wheel is installed and the remainder of the high pressure and intermediate-pressure bladings consist of 30 stages of simple pressure compounded, impulse type. The two low-pressure turbines have 24 stages of reaction blading in each, and the steam flows through the two casings in opposite directions in order to eliminate the axial thrust.

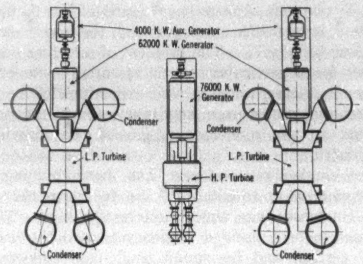

FIG. 3—ARRANGEMENT OF 208,000-KW. GENERAL ELECTRIC TURBINE GENERATOR UNIT FOR THE STATE LINE POWER STATION OF THE COMMONWEALTH EDISON SYSTEM

CONDENSERS

The factors governing the performance of a condenser are coming to be better understood. With increased knowledge of condenser performance and its relation to the cost of producing electrical energy, much attention has been paid in the last few years to increasing the efficiency of this apparatus. Condensers are now chosen on a capitalized basis, wherein initial cost, vacuum, power consumed by auxiliaries, and reliability are all given monetary value.

Condensers as measured by steam capacity have kept pace with the growth of turbine sizes. Although still larger units can be built, there is a tendency because of construction and installation difficulties, toward the use of divided units for very large sizes. As an indication of this trend, the largest turbine on record to date will have eight condensers which will handle a total of 1,600,000 lb. of steam per hr.

There has been a marked reduction in the ratio of square feet of condenser surface to kw. capacity of the turbine. This has been brought about because of a more intelligent arrangement of tubes in providing lanes that give a proper direction of the steam flow, bleeding of turbines and improved turbine water rates. An investigation regarding condenser installations in modern power plants shows that, among forty of the most prominent power stations completed within the last two years, only two have a ratio of condenser surface to kilowatt of normal turbine capacity under 1.0. This ratio for the different stations varies considerably, one being as high as 2.75, and the average for all stations being 1.396. This year however, has seen a further reduction in the amount of condensing surface per kilowatt of normal turbine capacity. Considering the condensers of appreciable size now under construction, 50 per cent of them have a ratio of less than 1.0, the four largest ones averaging 0.86, the lowest one of the four being 0.77. The problem of selecting a condenser for a particular unit is essentially one of an economic nature, and the ratio recommended by the different manufacturers may vary considerably according to the condenser design of the individual manufacturer.

The single-pass condenser has been gaining in favor whereas up to within the last two years the two-pass condenser was almost universally used. There appears to be a decided tendency toward the use of both vertical and horizontal single-pass condensers. The first single-pass vertical condensers to be built are now under construction for the Long Beach Station and will be installed early in 1928. There are also under construction, vertical bottom inlet, single-pass condensers for the 104,000-kw. and the 208,000-kw. units to be installed in the Chicago district.

A recent unique development in tube sheet construction which is claimed to eliminate leakage, will be applied to several condensers now being built. These condensers will have a floating tube sheet at one end with a rubber expansion joint between the tube sheet and the condenser shell. With this arrangement the tubes are expanded into both tube plates rigidly, the movement of expansion and contraction being taken up in the floating tube head.

Tube cleaning still presents a considerable field for investigation. During the past year experiments were conducted on a large condenser in one of the plants in the Chicago district operating on very foul circulating water, to show the effect of velocities on keeping tubes clean and decreasing the rapidity of slime formation. The results of approximately a month's run showed that the higher the velocity, the cleaner the average condition of the tubes. Also that flushing these tubes periodically at relatively higher velocities for short periods tended to return the efficiency of the tubes to their original condition at the start of the run.

Condensers built with divided water boxes and provided with two circulating pumps are becoming increasingly popular, as it permits cleaning one-half while the other half is in service.

Improvements have been made in the construction of hot-wells, so as to cause violent ebullition of the condensate before discharging into the suction of the pump. In one form of construction the condensate is exposed to a lower absolute pressure before discharging to the pump and in another form the drips from the bleeder heaters are led into the comparatively cool condensate. The resulting ebullition in each case has been found to be particularly successful in affecting deaeration. This construction may eliminate the necessity for deaerators.

Motor-driven auxiliaries are in the majority and there is a tendency toward the use of duplicate units. The prevailing practise seems to limit the number of circulating pumps to two per condenser, providing for each an independent source of power supply.

In a number of cases, condensers and piping have been arranged in such a manner as to permit the reversal of flow of the circulating water through the condenser tubes. This arrangement makes it possible to wash the trash out from the tube ends and water boxes, and it is particularly justified economically in cases where the water carries a considerable amount of trash most of the time.

An interesting development which has recently been applied to large power stations is a vertical screw impeller type of circulating water pump having high efficiency at low heads which is so designed that the pump may be located in a pit below the intake water level. It is therefore always primed and presents the advantage of being able to deliver water to the condenser without the necessity of priming suction and discharge lines.

The following are some of the outstanding condensers now under construction or recently placed in operation:

The 208,000-kw. turbine unit for the State Line Station of the Commonwealth Edison system will have eight Allis-Chalmers condensers of 22,000 sq. ft. cooling surface each or a total of 176,000 sq. ft., two condensers serving each of the low-pressure ends of the two double-flow, low-pressure turbines. The condensers will be of the vertical, single-pass type, the circulating water entering the lower water box, passing upward through the tubes, and discharging downward through two over-flow pipes contained in the condenser shell. With a circulating water rate of 360,000 gal. per min., it is capable of condensing 1,600,000 lb. of steam per hr. There will be four vertical circulating pumps placed in a crib house outside the generating station.

At the Crawford Avenue Station of the Commonwealth Edison Company, the 104,000-kw. unit will be served by a total of 90,000 sq. ft. of condensing surface composed of six 15,000-sq. ft. vertical Westinghouse condensers of the single-pass, radial flow design, which will be capable of condensing a total of 730,000 lb. of steam per hr. Circulating water will be sent up through a center pipe and down through the tubes at a rate of 180,000 gal. per min., two vertical pumps being used.

In the condensers for the 94,000-kw. unit for the Long Beach Station of the Southern California Edison Company, provision will be made for reversing the flow of circulating water in order to clean the tubes. There will be four Ingersoll-Rand condensers for this unit and they will be of the vertical, single-pass type having 20,000 sq. ft. of cooling surface each, or 80,000 sq. ft. in all. Two motor-driven pumps will supply approximately 150,000 gal. of circulating water per min.

An instance of the trend toward a decreasing ratio of condensing surface to turbine capacity is the 25,000-sq. ft., single-pass, Wheeler condenser for the 31,500-kw. unit to be installed at the Cabin Creek, W. Va., Power Station of the Appalachian Power and Light Company. The ratio of condensing surface to turbine capacity will be 0.793 sq. ft. per kw. Provision will be made for reversing the flow of circulating water and an external 1500-sq. ft. air cooler will be used.

The 50,000-sq. ft., single-pass, Wheeler condenser for the 55,000-kw. unit being constructed for the Pekin Power Station of the Super Power Company of Illinois will also have provision for reversing the flow of circulating water. Specially built-in valves in the water chambers will be provided for this purpose, and there will be a 5000-sq. ft. external air cooler.

The unique development in design of using a "floating" tube sheet will be applied in the case of the 25,200-sq. ft. single-pass, Wheeler condenser for the 30,000-kw. unit for the Virginia Electric and Power Company at Norfolk, Va. A storage hotwell will also be provided for this condenser and there will be a 2000-sq. ft. external air cooler.

Another instance of the trend of decreasing ratio of condenser surface to turbine capacity is the two pass, vertical, twin-type Worthington condenser for the 91,500-kw. unit being constructed for the Crawford Avenue Station of the Commonwealth Edison Company. The total condensing surface will be 70,520 sq. ft., or 0.77 sq. ft. per kw.

One of the largest condensers being built is that for the 41,250-kw. turbine for the Colfax Station of the Duquesne Light Company. It is a Westinghouse radial flow, two pass type with divided water boxes, having a tube surface of 62,500 sq. ft. Steam will be condensed at the rate of 412,000 lb. per hour when using 72,500 gallons of circulating water per minute.

BOILERS, SUPERHEATERS, AND ECONOMIZERS

Boiler development has been influenced by the trend toward larger units, higher pressures, higher operating ratings, new furnace designs exposing the maximum surface to the radiant heat of the fire, automatic combustion control, reduced plant investment, and operating cost.

The boiler will grow in size with the rest of the industry. One new station has been designed so that one boiler can readily generate all the steam needed to carry the main turbine, and a capacity of over 35,000 kw. has been developed with ease. It is reported that one boiler has already developed sufficient capacity to generate all the steam required for a 50,000-kw. unit. This has come about through operation at very high evaporative rates made possible by substituting water cooled walls for those refractory lined. The use of water-cooled furnace walls and bottoms, resulting in a large percentage of the heat absorption taking place in the furnace at heat transfer rates in the neighborhood of 60,000 B. t. u. per sq. ft. per hr., requires a readjustment of the boiler heating surface involving a reduction in that portion receiving heat solely by convection. This change caused higher boiler outlet gas temperatures which were reduced to very low values before entering the stack, by the extensive use of air preheaters and the adoption of water heating surface in the form of integral steaming economizers of relatively low cost as compared with water heating surface in the boiler proper. The use of economizers has been stimulated by the use of higher steam pressures permitting higher feed water temperatures so that in some cases both air heaters and economizers may prove economically justifiable.

That boiler designs are not only being modified and extended to huge proportions but also are being radically altered, is evidenced by the advent of the so-called Combustion Steam Generator. This equipment, utilizing pulverized fuel, is a recent product of the International Combustion Engineering Corporation and reflects the trend toward higher ratings, completely water-cooled furnaces, reduction of convection heating surface, and intense turbulence of the furnace gases. Twelve of these units have been contracted for, some of which are ready to go into service, and others are in course of erection. Considerable interest is being manifested in this development and the performance of the equipment will be closely watched.

The benefits of highly preheated air both for stokers and for powdered coal firing are becoming more generally realized. Developments in air preheaters have been rapid. It is probable that new stations will install air preheaters of such a capacity that the flue gases will be cooled to relatively low temperatures.

There have been no unusual developments in the design of superheaters during the past year except that manufacturers are ready to offer superheaters to furnish steam at a maximum of 900 deg. fahr. The preferential location of the superheater for high temperature seems to be in the inter-deck position, although a few installations have been made of the combination convection-radiant type. A novel arrangement in the Fordson Plant consists of placing the superheater tubes in the side walls of the furnace and behind a protecting screen of water tubes.

Owing to the rapid development in the utilization of high-pressure and high-temperature steam, it has frequently been found advisable, when constructing additions to existing stations, to select a higher steam pressure and temperature for the new part, than is used in the old part of a station. The benefits secured, from the standpoint of economy, generally outweigh the complications introduced when operating sections of a station at different pressures and temperatures. This is one satisfactory answer to the question, "What is to be done with old steam generating stations?" The steam connection necessary between the two sections of a plant so operated must contain, of course, reducing valves and desuperheating equipment. Reliable and economical operation has been aided by adapting automatic control equipment to regulate the flow of steam through this apparatus.

An installation embodying a desuperheater and reducing valves, operated automatically by a system using compressed air, is that at the Hudson Avenue Station, between its high- and low-pressure sections. The regulating valve of the desuperheater is controlled primarily by the flow of superheated steam through the desuperheater and secondarily by the temperature of the outgoing steam. The steam is desuperheated by passing through tubes surrounded by water at a controlled level. Inasmuch as the pressure of the water is such that the saturation temperatures of steam and water are not very different, it is not likely that the desuperheated steam will ever become wet.

Stokers and Furnaces

The inherent nature of the combustion problem necessarily obviates any spectacular accomplishments in the stoker field and limits the gains to what might be termed detailed refinements.

The insistent demand for constantly increasing steam output with high efficiency is characteristic of present day practise. So far, the boiler units have increased in horse-power rating at a greater ratio than in width. This has called for stokers of constantly increasing length with longer time-interval for the burning of the fuel and added complications in its distribution over such a length of grate surface.

Two factors have contributed in large measure, to the successful application of long stokers—means for the exact control of the movement of the fuel throughout the retort and regulation of the air to unit sections of the stoker according to the condition of the fuel bed at each individual section.

Up to the present time, boiler and stoker equipment has generally been selected without much reference to the heat exchanges of the plant as a whole. The development of such heat reclaiming devices as waterwalls, preheaters and economizers is gradually bringing about a tendency to consider the heat exchanges of the entire plant in the selection of the fuel burning equipment. There is also evidence indicating that joint selection of steam generating and fuel burning equipment is preferred to separate selection of the former without regard to its influence in the selection of the latter.

The recent developments in stokers for use with preheated air, have resulted in an appreciable reduction in furnace volume and an improvement in performance. There are under construction very large stokers which will be regularly operated with air preheated to about 400 deg. fahr. These stokers are being arranged for operation with air preheated to a maximum of approximately 600 deg. fahr., for the purpose of obtaining information as to their operating characteristics under such conditions. When air is preheated to high temperatures, it is necessary, because of its relatively large specific volume, that it pass through the fire bed at high velocities in order to maintain high rates of combustion per square foot of grate surface. There are some who maintain that there will be a considerable agitation of fuel on the grates with high air velocities and therefore the use of air preheated to a high temperature will result in reduced rates of combustion in stoker fired furnaces when air velocities are held down to rates that are not excessive.

As an indication of the trend toward the great increase in fuel burning capacity per foot of furnace width, stokers with 45 tuyeres that will underfeed coal for a distance of 16 ft. are being installed in furnaces 19 ft. 5-11/16 in. from the inside of front wall to the rear breaker apron of the clinker grinders, in extensions to the Edgar Station. Stokers of the same length will be installed in the Saginaw River Station of the Consumers Power Company in Michigan. Also in the Hudson Avenue Power Station there were installed, during the past year, stokers with 39 tuyeres underfeeding coal for a distance of 15 ft. A stoker for a furnace 19 ft. 1 in. from face of bridge wall to face of front wall has just been ordered by the Stamford (Conn.) Gas & Electric Company.

For the purpose of showing the recent marked improvement in underfeed stoker development, Fig. 4 is given which compares the performance of underfeed stokers only three years old, with that which is claimed by one manufacturer, for a unit on the present basis of design up to 700 per cent of boiler rating.

Probably the most radical change in any phase of central station design is in the furnace. It is significant that water-cooled walls with certain amounts of exposed refractory surface have been installed to an increasing extent during the past year, particularly in stoker installations such as at Richmond, Hudson

Avenue, and Edgar Stations. However, engineers have not yet reached any final conclusion in regard to the proper proportion of refractory surface to install in such furnace walls.

In the proposed addition to the boiler house at Kearny, to be stoker fired as is the original installation, all four walls will be water cooled by tubes with slight refractory material showing between tubes. Powdered fuel furnaces, such as are installed at Calumet, Fordson, Kipps Bay, East River, and Charles R. Huntley plants, apparently lend themselves more readily to complete water cooling than do stoker-fired furnaces.

The introduction of water walls has made possible the increasing of capacity of boilers to a point never even dreamed of three years ago. Before water walls were developed it was impossible to operate boilers continuously at high ratings owing to the limitations of refractory materials used. Now it is apparently only a question of the amount of fuel which can be burned within the furnace walls.

Furnace cooling by water walls has stimulated the use of air preheaters and it can be said these two pieces of equipment go hand in hand. The regenerative cycle with its high temperature feed water coming to the boiler room, minimizes the absorption of heat from the flue gases by an economizer unless it be of the steaming type. The ability of the preheater under these conditions to lower the flue gas temperature below the temperature possible to obtain with an economizer alone, has frequently dictated the installation of the preheater rather than an economizer.

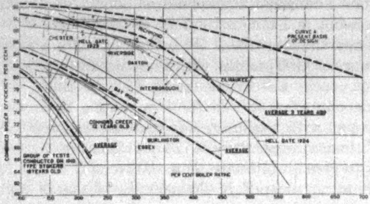

FIG. 4—DEVELOPMENT OF BOILER AND STOKER PERFORMANCES DURING THE PAST 17 YEARS

With brick walls it was impossible to take full advantage of the benefits of the air preheater, as higher furnace temperatures resulted in excessive maintenance of the refractory walls. With water cooled walls the limits to the degree of preheat are fixed by the character of the fuel when burned on stokers, the material used in the manufacture of the preheater, or the highest velocity of the air through the tuyeres, that will not blow the coal off the grates. In some cases it has been necessary to install an economizer before the air heater in order to hold down the temperature of the gases.

The size of pulverized fuel furnaces for a given amount of heat liberated is definitely on the decline. Turbulence accomplished in one way or another, to secure agitation and rapid mixing of air and carbon, greatly accelerating and improving combustion, has obviated the need for large combustion chambers formerly thought necessary to accomplish the same result. Preheated air and water cooled walls have also played a part in the reduction of furnace volume. All of these factors have either permitted or dictated that less excess air be introduced in the furnace thus allowing a smaller combustion space.

PULVERIZED FUEL

A striking feature in the expansion of pulverized fuel firing is the introduction of this form of combustion into a number of large outstanding steam generating stations during the past year. When it is recalled that the first major installation was made in connection with a 40,000-kw. plant in 1921, the development of burning pulverized fuel becomes impressive when it is considered that six years later it has been adopted for the power station which will have the largest generating unit on record.

In a comparatively short period of rapid growth it is perhaps natural that the best method of utilizing this system of combustion has not yet been defined generally. For example, turbulence is accomplished in several ways, each way requiring radically different furnaces; dryers of different types are installed in some cases and not in others. Further, quite a number of installations of unit pulverizers has been made in the last two years, but there is still a difference of opinion among engineers in regard to the question of unit mills as compared to the storage or central system. At the present time, each individual case must be studied at length, giving full consideration to operating conditions, price and kind of fuel, operating costs, and fixed charges. Results from unit system installations are becoming available and the characteristics of this type of firing show many peculiarities which should be carefully considered, particularly in central station application. These results no doubt will permit of a better comparison relative to lower combined operating costs and fixed charges when considering stokers or the storage system of pulverized fuel.

Considerable progress has been made in the last year in improvements of apparatus directly connected with pulverized fuel, and there seems to have been a radical departure from many previous methods of applying this system of firing to the steam boiler.

Four factors of primary importance are, the preliminary preparation of coal including drying, fineness of pulverization, turbulence in burning and furnace volume, all of which have been and are still being intensely studied. One of the most important single factors in the combustion of pulverized fuel is mixing of the air and coal streams. The intimate mixing of the secondary air with the primary air and coal immediately upon leaving the burners produces a turbulence which persists throughout the zone of flame activity, thereby completing combustion in a minimum of time and space. In order to keep the dimensions of the furnace within reasonable limits, it is likely that the tangential system of introducing the fuel will be most favored, as greater use of furnace volume can be secured. The importance of the intense turbulence is generally recognized and designers of coal burning equipment have endeavored to obtain this in various ways.

If a forecast can be made of developments, these will include turbulent firing, simplified storage systems, methods of drying of the fuel, and mills of greatly increased capacity. Through these developments still greater advantages will be realized by the use of pulverized fuel in central station practise. The opinion has been expressed that powdered fuel firing will probably be the standard method of the future as it will be impossible to burn sufficient coal per square foot on a stoker to develop the capacities which will be required. However no clear cut supremacy has as yet been demonstrated and the fact must not be lost sight of that stoker development has proceeded at a brisk pace with no indications of a diminution.

The rapid introduction of pulverized fuel into steam generating stations is indicated by the fact that at the present time 40 public utility plants in the United States either partially or fully operate with pulverized coal, the aggregate generator capacity so fired being 2,200,000 kw. This is exclusive of several installations in steam heating plants. In addition to this there are now under construction five new plants and extensions to two old plants, having together a total capacity of 440,000 kw., all of which will be operated with pulverized coal. The significance of these figures can better be appreciated from the following tabulation:

PULVERIZED COAL INSTALLATIONS

	No.	Capacity, kw.
Installations in operation	40	2,200,000
Installations under construction	7	440,000
Total	47	2,640,000
Installations in operation and under construction, entirely operated with pulverized coal	27	1,700,000
Plants with 40,000 kw. capacity or more, operated with pulverized coal	27	2,250,000

AUTOMATIC COMBUSTION CONTROL

Considerable progress has been made in the development and application of automatic combustion control equipment to stoker firing, pulverized fuel, and oil burning. Engineers are giving increased attention to this equipment which, judging from the number of installations in operation at the present time, is apparently well advanced from the development stage and should assume a major role in the process of converting the heat in the coal to heat in steam in the most economical manner possible.

Complete automatic combustion control has been in operation for some time in several of the outstanding power stations in the east, and the companies report satisfactory performance of the equipment. Results show that daily operating efficiencies are maintained within 2 per cent of test efficiencies.

The following are some of the outstanding installations of boiler room equipment recently completed, or under construction:

The second boiler in this country to generate steam at a pressure of more than 1000 pounds went into operation late in 1926 at the Lakeside Station. This is a single three drum Stirling boiler built for a working pressure of 1390 pounds and contains 28,532 sq. ft. of heating surface. The drums are forged steel 41 ft. 6 in. long, 40 in. inside diameter, and 5 in. thick. The walls of the furnace are formed by radiant heat superheaters on the sides designed to give an ultimate temperature of 720 deg. fahr., by a radiant heat resuperheater on the rear wall designed to reheat the steam from 447 deg. fahr. at 317 pounds pressure to approximately 720 deg. fahr., and by fin cooling tubes on the front wall. Pulverized coal equipment is used with a plate air heater of 20,160 sq. in., designed to preheat the air to 650 deg. fahr. at maximum load.

Four additional high-pressure boilers are planned, two being under construction, for the Edgar Station. The two units are B. & W. boilers built for a working pressure of 1400 pounds, with a heating surface of 15,090 sq. ft. each, the drums being 4⅝ in. thick, 48 in. inside diameter, and 38 ft. 4 in. long. Each unit is equipped with a primary superheater designed to give an ultimate temperature of 725 deg. fahr., and a reheater unit to give an ultimate temperature of 750 deg. fahr., both at an output of 200,000 lb. of steam per hr. The economizers are the wrought steel return bend type with 5596 sq. ft. of heating surface each, and the tubular air heaters each contain 33,032 sq. ft. The boilers are fired with Taylor underfeed stokers having 16 retorts and 45 tuyeres, the largest of their type ever built. These stokers, using preheated air, are capable of burning 37,700 pounds of coal per hour, and are provided with means for manually regulating the supply of air to various parts of the fire according to the condition of the fuel bed at each part. The furnace will be equipped with Bailey side and rear walls, and a ventilated Bigelow hung front wall.

An ultra high-pressure boiler and turbine installation is planned for the Northeast Power Station of the Kansas City Power and Light Company, but has not yet assumed definite enough form to warrant publication of any details.

European manufacturers are experimenting with ultra high pressure steam generation. It is reported that an installation of 18,000 kw. capacity is near completion and that the boiler in which water is evaporated by superheated steam will supply steam at a pressure of 1500 to 1700 pounds. The boiler setting contains only a superheater and economizer.

The redesigning and rebuilding of No. 2 boiler and furnace unit at the Fordson Plant, illustrates the tendency to secure the maximum capacity from a single unit. This unit is a Ladd boiler of originally 26,470 sq. ft. of heating surface with a furnace designed to burn pulverized coal, blast furnace gas, oil, tar, and coke oven gas, either singly or in combination. At the time of installation this boiler and three more of identical pattern in the Fordson Plant were the largest ever built. By adding 12 per cent of the total water-heating surface in the form of water screens in the bottom of the furnace, equipping the side walls with fin tubes and radiant superheaters, protecting the arches with water tubes besides doubling the number of burners and installing air preheaters the actual steaming capacity was increased 100 per cent. A peak output of 500,000 pounds of steam has been attained, which is said to be a record for a single unit.

There are being installed in the Stanton Power Station of the Pennsylvania Power & Light Company, six standard and two reheater B. & W. cross drum boilers built for a working pressure of 732 lb. and to be operated at approximately 650 lb. pressure. The standard boilers contain 17,962 sq. ft. of heating surface each and are equipped with superheaters designed to give an ultimate temperature of 750 deg. fahr. at an output of 150,000 lb. of steam per hour. The reheater units contain 5978 sq. ft. of heating surface each, and are equipped with primary superheaters to give 740 deg. fahr. at an output of 81,000 lb. of steam per hr., and with reheater elements designed to reheat the low pressure steam to 730-740 deg. fahr. The boilers will be fired by B. & W. chain grate stokers 24 ft. by 22 ft. burning anthracite slush with preheated air.

An experimental powdered coal installation has been in service in the Calumet Station since November 15, 1926. The heating surface of the unit is divided as follows:

Boiler Heating Surface	5,938 sq. ft.
Furnace Heating Surface	2,460 sq. ft.
Steaming Economizer Surface	8,365 sq. ft.
Total Water Surface	16,763 sq. ft.
Air Heater Surface	41,700 sq. ft.

This boiler has been operated for short intervals at a rate of 300,000 lb. of steam per hr., but this rate could not be maintained for longer periods because of the lack of pump capacity. This rate corresponds to an evaporation per sq. ft. of total water surface of 17.9 lb. or an evaporation of 35.7 lb. per sq. ft. on the basis of combined boiler and furnace heating surface.

The operation of this Calumet boiler equipment has proved to be so satisfactory that orders have been placed for five similar units for the State Line Generating Company, State Line, Ind. The entire arrangement of these units will be similar to the Calumet equipment as to boilers, superheaters, economizers, air heaters, Bailey furnaces, Calumet burners, and Fuller-Lehigh unit mill pulverized coal equipment, with the exception that the boilers will be built for a working pressure of 800 lb. to operate at about 600 lb. pressure. The individual units, however, will be much larger than the Calumet unit, the boiler drums being 52 in. in diameter, 3¼ in. thick and forged instead of riveted.

Satisfying the demand for still larger units, there is being installed in station "C", for the Pacific Gas & Electric Company, two B. & W. cross-drum boilers which are the largest of their type yet built. These boilers have a heating surface of 35,500 sq. ft. each, and are built for a working pressure of 460 lb. They are equipped with tubular air heaters of 51,232 sq. ft. each and superheaters designed to give 725 deg. fahr. ultimate temperature at an output of 350,000 lb. of steam per hr. The furnaces are to be oil fired and equipped with water-cooled walls. For the Long Beach station of the Southern California Edison Co., there are now under construction three cross drum units of the same type having 34,162 sq. ft. each and built for a working pressure of 450 lb. each with steam at 713 deg. fahr. The furnaces are designed to burn oil when the plant is first put into operation and pulverized coal at some future date, the furnace walls and floor being of water cooled construction.

A notable installation to go into service was the six (6) 1590 hp. Springfield boilers in the East River Plant of the New York Edison Company. The boilers are pulverized coal fired and furnish steam at 375 lb. pressure and 700 deg. fahr. Each boiler is capable, on continuous overload, of producing 250,000 lb. of steam per hr. No brickwork or refractory material is used, the furnace being completely enclosed by Murray fin tubes, backed up by plastic coating about 6 in. thick consisting of diatomaceous earth, cement, and a painted hard outside finish.

Another outstanding installation is the addition to the Hudson Avenue station, consisting of four 2292-hp. boilers furnishing steam at approximately 400 lb. per sq. in. and 700 deg. fahr. These units have the rear and side walls cooled by water tubes which are protected at the firing line by cast iron and carborundum blocks respectively. The front wall is lined with carborundum blocks. These boilers are fired by Westinghouse 14 retort, 39 tuyere, 18 ft. long underfeed stokers. The stokers use preheated air and provide an actual grate surface of 460 sq. ft. or 427 sq. ft. of projected area.

At the Kearny station a fifth row of three boilers will be added. They will be Springfield cross-drum units of 23,640 sq. ft. of heating surface, similar to the original units but with water-cooled rear, side, and front walls composed of tubes backed up by refractory tile, a layer of insulating material, and a finished casing of transite board. Cast iron blocks will be bolted to the wall cooling tubes just above the fire line to protect them and reduce heat absorption. Preheated air will also be employed in combustion. It is expected that with these two features of water cooled walls and preheated air, not possessed by the boilers in the original installation, much higher ratings will be secured. Riley superstokers of about the same huge dimensions as the original units in this station will be installed under these boilers.

The installation of four boilers, burning pulverized coal, at the Charles R. Huntley Station of the Buffialo General Electric Company is unique in several respects. These boilers have 12,515 sq. ft. of heating surface, well-type furnaces tangentially fired, and Bailey water walls on four sides. The wells in three of the furnaces are as wide as the furnace in each case and about two-thirds its length. Tap holes are provided for removing the ash as molten slag. The unit system of pulverizing is employed. These boilers can be operated at outputs of from 60,000 to 250,000 lb. of steam per hr.

Two Combustion Steam Generators furnishing steam at 825 lb. pressure and fired by unit pulverizers using preheated air, are being installed in the Syracuse, N. Y., plant of the Solvay Process Company. Another such unit is being installed in the Calumet Station, a brief description of which is as follows:

Water-heating surface		
Rear bank of tubes	3,637 sq. ft.	
Side Walls	1,710 sq. ft.	
Roof	254 sq. ft.	
Bottom Bank of tubes	1,171 sq. ft.	
Total Water-heating surface	6,772 sq. ft.	
Superheating surface	3,000 sq. ft.	(Approx.)
Economizer	5,250 sq. ft.	
Air heating surface	25,200 sq. ft.	
Steam pressure	360 lb. gage	
Effective Combustion space	5,000 cu. ft.	
Normal Capacity of unit	125,000 lbs. per hr.	
Peak Capacity of unit	150,000 lbs. per hr.	

Cheap water power, for industries using large amounts of steam for process work, has encouraged the development and use of electric steam boilers. At the beginning of 1927, an aggregate of 750,000 kw. of these units was installed in Canada and the U. S. Three such boilers having a capacity of 42,000 kw. each and operating at 6600 volts, were installed in 1926.

ULTRA-HIGH PRESSURE STEAM TURBINE GENERATOR INSTALLATIONS

Considerable progress has been made during the last few years in the development of turbines, boilers, and other equipment operating at the ultra-high pressures, from the pioneer stage into an important commercial development. At the Edgar Station of the Edison Illuminating Company of Boston, the original high-pressure installation in this country, a 3150-kw. unit has given remarkably satisfactory results for nearly two years. Upon the basis of this experience a 10,000-kw. 1200-lb. unit is now being installed and a second 10,000-kw. unit is contemplated. The Milwaukee Electric Railway and Light Company has also installed a 7000-kw., 1250-lb. unit in its Lakeside power station. It has been reported in the technical press that a third installation is contemplated for the Northeast Power Station of the Kansas City Power and Light Company. This installation will consist of a 1400-lb. boiler with a 10,000-kw. high-pressure turbine exhausting to the main steam header of the station.

The problem of the use of both high pressures and high temperatures is very difficult, particularly in the design of the boilers and superheaters, where the stresses in the tubes are increased by the temperature differences between the outside and inside surfaces. The difficulty of the use of both high temperatures and high pressures is due to the fact that at high temperatures the strength and stability of the materials normally used are seriously reduced. The question of materials for use in high pressure turbine construction is somewhat less troublesome than in boiler work, because of the fact that small high-speed units are used with temperatures fairly uniform at any section, and therefore the stresses can be controlled so as to prevent high unit-stresses in high temperature zones.

The advantage of using ultra-high pressure turbines in connection with normal pressure units is not only the increased fuel economy, but also the fact that the space required is nearly the same for the high and normal pressure installation combined as for the normal pressure alone. The increased capacity is therefore a net gain, which approximately balances the increased cost of the equipment, so that the improved thermal efficiency represents very nearly a corresponding economic gain.

The entirely satisfactory operating results and full realization of expected gains of the ultra-high-pressure installations now in operation in this country shows that they are of unquestionable commercial value and proves by actual test the advisability of improving the efficiency of existing "normal pressure" stations as well as new stations by the convenient addition of high pressure equipment instead of more low pressure apparatus. This is especially the case in stations having a low load factor where the equipment operating on the high pressure cycle can be installed sufficient to supply the base load only.

Edgar Station. The high-pressure plant now in operation at the Edgar Station consists of one high-pressure boiler and a 3150-kw. turbine. Based on its successful operation for nearly two years, an addition is now being constructed which includes two 15,090-sq. ft. cross-drum Babcock & Wilcox boilers, a 10,000-kw., 3600-rev. per min. General Electric high-pressure turbo generator together with the 65,000-kw. normal-pressure turbo generator. The boilers will generate steam at approximately 1400 lb. pressure and 725 deg. fahr. It will be expanded in the 10,000-kw. turbine, which has 16 impulse stages, to 375 lb. per sq. in. and returned to the reheating superheaters which form part of the new boilers. After being reheated to approximately 750 deg. fahr., the steam will be discharged into the main 350-lb. steam header and together with steam from the normal pressure boilers will supply the two existing 32,000-kw. turbo generators and the new 65,000-kw. turbo generator.

Each high-pressure boiler will be equipped with a 5596-sq. ft. economizer operating at approximately 1500 lb. water pressure and with a 33,032-sq. ft. air preheater, and will be fired by a 16 retort, 45 tuyere underfeed stoker. The side and rear furnace walls will consist of refractory-faced, cast-iron blocks, bolted to boiler tubes which will be connected to the boiler.

The next high pressure extension contemplated will include two additional 15,090-sq. ft. boilers and one 10,000-kw. turbine. At that time, the four high-pressure boilers will serve the two 10,000-kw. turbines and the steam from those two high pressure turbines will be sufficient to operate the 65,000-kw., 350-lb. pressure turbine.

Before entering the economizers, the feed water will be heated to 420 deg. fahr., by means of steam bled from three points of the 65,000-kw. turbine and from the exhaust of the 10,000-kw. turbine at a pressure of 375 lb. It is of interest to note that feed water has never before been heated to this high temperature by bled steam. This high-feed temperature is of particular interest in view of the fact that the feed water will pass through an economizer after leaving the high pressure heater and before entering the boiler.

Three boiler feed pumps of interesting design are being installed. Two will be motor-driven at 1800 rev. per min. and will be used for normal operation and the third will be turbine-driven at 3600 rev. per min. and will be used for emergency only. Each motor-driven pumping unit will consist of four pumps in series, one single and two five-stage volutes, and one six-stage turbine pump. The single stage volute and one five-stage volute will be driven by one motor and will discharge at 500 lb. per sq. in. through the high pressure feed water heaters to the suction of the second five-stage volute pump. The second five-stage volute pump and the six-stage turbine pump will be driven by one motor and will deliver the water to the boiler feed headers at a maximum pressure of 1600 lb. per sq. in. All motors will be adjustable speed and will be automatically regulated. The turbine-driven pump will also be a six-stage turbine pump and will develop the full 1600 lb. per sq. in, in the one casing. This pump is designed for automatic starting when the pressure in the 1600-lb. boiler feed header drops below a safe limit.

The coal consumption per kw-hr. at the Edgar Station is approximately 1.02 lb. when only the present 30,000-kw. normal pressure turbines are operating. When about one-third the output of the station is generated by steam from the high-pressure boilers and turbines, the coal rate is approximately 0.98 lb. per kw. hr., an improvement of 4 per cent. For a complete 1200-lb. installation it is estimated that the gain should be approximately three times this figure or 12 per cent.

Lakeside Station. Prior to the installation of the 1300-lb. boiler and the 1250-lb., 720 deg. fahr. turbine, the capacity of the Lakeside Station was 160,000 kw., made up of two 20,000-kw. and four 30,000-kw. machines. The boiler room capacity was 1,600,000 lb. of steam per hr. with a throttle pressure of 285 lb. per sq. in. and a temperature of 700 deg. fahr.

The new high pressure boiler is a Babcock and Wilcox–Stirling type boiler and is the largest of its kind. Its nominal rating is 2853 b. hp. and it is capable of delivering 240,000 lb. of steam per hr. Pulverized coal is burned in a 30,100 cu. ft. Lopulco type furnace. The high-pressure turbine unit is a 7000-kw. General Electric machine and its speed is 3600 rev. per min. It exhausts to the reheater at 310 lb., the temperature of the steam after reheating being approximately 720 deg. fahr.

The high pressure installation has been in service since October 1926. It was in continuous operation from January 29th to March 19th, 1927, a period of 50 days. During this period, the kw-hr. output of the high pressure turbine was about 7.5 per cent of the total station output, while the kw-hr. generated by both the high-pressure (1250-lb.) turbine unit and the normal pressure units from the steam originating in the high pressure boiler only, was about 34.3 per cent of the total station output. The load factor of the load (in this case equal to the capacity factor) on the high-pressure turbine was approximately 90 per cent, the load being less than maximum at times due to the fact that the total station load on Sundays is below the capacity of the high-pressure boiler.

Operating results showed a coal saving on the entire station of about 4 per cent due to the operation of the high-pressure cycle since this cycle was approximately 12 per cent more efficient than the 300-lb. cycle and furnished 34 per cent of the station output.

The high pressure boiler installation has shown several remarkable operating features. Ability to average 16½ per cent CO_2 over long periods without CO losses and with unusually low carbon losses has been obtained in the operation of the high-pressure installation. This CO_2 average represents use of 12 per cent excess air, and as such establishes a record in economy of fuel burning. Automatic and instantaneous stoppage of coal feed and by-passing of 1200-lb. steam to 300-lb. pressure has been utilized in service several times when the high-pressure turbine tripped from service. Not a safety valve opened under these conditions.

Mercury Vapor Installation

After some four years of experience with the mercury vapor installation in its Dutch Point Station, the Hartford Electric Light Company has ordered mercury vapor equipment, including a 10,000-kw. turbine, to be installed and to go into operation early in 1928 in its South Meadow Plant. This will be a strictly commercial application of the mercury-steam cycle and will be representative of the size and design of equipment to be placed on the market.

The commercial success of this process provides a means of effecting marked economies in power production, made possible by being able to go to higher temperatures of a working substance than when using steam alone. The very moderate pressures required permit using the higher temperatures with our present materials. In effect, the mercury is used to convey heat from the furnace to the steam boiler acting as a mercury condenser; before reaching the condenser some of the heat is developed into electrical energy by the mercury-turbine generator.

It is claimed that the remarkable record of 27 per cent efficiency, attained by the Columbia Plant, operating on a straight steam cycle, could be increased to 36 per cent in a similar plant arranged to operate on the mercury-steam cycle. The savings in fuel consumption for less efficient plants will be even greater. Of course, from a commercial viewpoint, the cost of plant must be studied with relation to capacity factor.

The original single-stage 1800-kw. unit operating at 35 lb. pressure, installed at Dutch Point in 1923, developed about 60 per cent of the available energy in the mercury. This was supplanted by a three-stage machine, developing 70 per cent of the available mercury energy, that went into operation late in 1926. The unit to go into the South Meadow Plant, estimated to develop 75 per cent of the available mercury energy, will be a five-stage 10,000-kw. machine receiving mercury vapor at about 70 lb. pressure and exhausting it at one lb. absolute to two mercury condensers. In these condensers 125,000 lb. of steam per hour will be generated at 250 to 350 lb. pressure and superheated by the mercury-boiler furnace flue gases to an ultimate temperature of 700 deg. fahr. It is expected that about 10,000 kw. will be obtained from the steam generated by the condensed mercury.

The present boiler at Dutch Point, of different design from the original one, generates mercury vapor at 70 lb. pressure and 884 deg. fahr. The new boiler consists of a group of drums, each carrying dead-ended tubes six ft. long, giving the unit the appearance of a huge coarse brush. There will be required for the entire installation 135,000 lb. of mercury, the cost of which will represent a substantial portion of the plant investment. In the process of generating energy the mercury will be circulated in the system eight or nine times an hour. An experimental boiler in the Schenectady G. E. Plant has been operated at 110 lb. pressure, generating vapor at 940 deg. fahr., at a rate more than twice that planned for the South Meadow unit with no difficulties whatsoever. It is expected that this Hartford unit will operate indefinitely without interruptions.

The approximate fuel saving, at an estimated figure of 11,000 B. t. u. per kw-hr. output developed from mercury and steam from the mercury condenser, based on a conservative use factor, is expected to be about $200,000 a year. While the maximum saving is obtained when carrying base load, under light load conditions the savings are material. The operating company reports that in its Dutch Point Station it has been able to obtain as good a coal economy on 5 per cent capacity factor as the entire station is capable of doing on a 60 per cent load factor.

As a means of increasing the capacity and the economy of existing stations or even planning new base load high-economy stations, this system of power generation is competitive with the ultra-high pressure generation and utilization of steam with its attendant steam reheating complications and relatively large auxiliary power consumption.

The supply of mercury is expected to be ample although price disturbances may occur until the industry adjusts itself to the increased demands to be ultimately made upon it.

HYDROELECTRIC DEVELOPMENT

While the increase in electrical energy generated during 1926 by steam plants of public utilities was only about 9 per cent over that generated in 1925, the increase for waterpower plants was approximately 17 per cent. Furthermore, the aggregate capacity of waterwheels and generators produced was greater than for the preceding year. However, except for the trend toward larger units, there have not been any radical changes in turbine types or general form or design, but certain details of design and special features have shown development or improvement.

During the past year a large number of power companies for whom hydroelectric units were installed, adopted electric drives for the governors. This form of drive is becoming increasingly popular for hydroelectric installations. The driving motors are of the induction type and operate in close synchronism with the frequency of the generator unit which it is required to regulate. This provides a simple and convenient drive and has been found to give extremely smooth and quiet operation free from operating troubles.

Another interesting development in hydraulic turbine design was the introduction of a water-lubricated guide bearing with a rubber lining. It may be of interest to note that bearings of this type have recently been adopted for use with four turbine installations in which the shaft diameters range from 9 to 24 in. The chief advantage of the rubber lined bearing is the great durability and the long life obtainable.

In the past year a number of hydraulic turbine units have been built, equipped with plate steel casings of the volute type. Engineers are becoming increasingly interested in the possibilities of welding instead of caulking the plate steel joints for these casings, and it may be of interest to note that the welding of these casing joints will actually be undertaken in connection with one or more of the largest and most recent turbine installations.

Hydroelectric plants automatically operated and controlled established another record during the past year. In one instance, two units of 28,500 hp. each have been installed in a station designed to operate automatically. New methods of applying automatic control to both a reaction and impulse type of unit are being developed. This equipment is of particular advantage in connection with steam-operated plants as supplementary sources of power.

In one of the outstanding major hydraulic developments under construction where the contracts for both the turbines and generators have been split between two companies, it was found economical to have one manufacturer build all of the oil pumping system. Also, in general, the design of all the main turbine parts which are subject to wear and replacement are made interchangeable. The generators are being built with the same degree of cooperation between the manufacturers, to insure interchangeability of some of the important mechanical parts.

There has been a tendency towards the closed circuit for air circulation with surface coolers, similar to the method of cooling commonly employed for steam turbine generators. This arrangement simplifies the construction of air ducts and permits the use of an inert fire extinguishing gas if desired. A number of machines have been constructed for the closed system of circulation.

Some of the Hydroelectric Developments of exceptional interest recently completed or now under construction are given below:

Conowingo Development. The outstanding hydraulic turbine development in 1926 was the seven 54,000-hp. 89-ft. head, 81.8-rev. per min. single-runner vertical-shaft hydraulic turbines for the Susquehanna Power Company's Conowingo Development. Three of the turbines are of I. P. Morris manufacture and four are Allis-Chalmers. The runners are made of cast steel in three sections and represent a very difficult problem in casting. The total weight of the runner will be approximately 200,000 lb. and outside diameter 179 in. The division of the runner into three parts was necessitated by shipping limitations which seem to be one of the principal factors now limiting the size of hydraulic equipment. The sections of the runner are bolted together by flange joints and in addition have steel bands mounted on the crown of the runner and the discharge ring. The spiral casing is made up of riveted steel plate sections and has an inlet diameter of 27 ft. The butterfly valve housing is joined to both the penstock and turbine casing by riveted connections. The feature of particular interest incorporated in the design of the butterfly valves is the installation of a rubber tube fitted into an annular recess in the valve body around the circumference of the gate when the valve is in closed position. This rubber tubing is designed to expand and hold tightly against the outer circumference of the gate when pressure is admitted to the inside of the tube when the gate is closed. This characteristic of the valve will insure unusual tightness against leakage. An innovation was used in the design of the pit ring which extends from the speed ring to the generator base in that this ring was built entirely of structural steel. After repeated tests, the hydraucone draft tube was accepted as the best design of tube offered for the conditions and as a result the hydraucone and the Moody spreading tube were used for the entire development. The draft tubes will be equipped with cast steel stay vanes at the lower ends, designed to carry the weight of the draft tube above, in addition to the weight of the unit and its portion of the station structure, this having been found to give greater economy in construction costs than to strengthen the concrete reinforcements which would otherwise be necessary. The center concrete cones for these tubes will extend all the way up to the turbine runner.

Each water-wheel unit will be direct connected to a 40,000 kv-a. 90 per cent power-factor, 81.8-rev. per min., 13,800-volt, 60-cycle generator, four of which will be of General Electric manufacture and three of which will be Westinghouse machines. These alternators are notable, not only because they are the largest in physical dimensions, of any electrical machines ever built, but also because of the fact that they are to supply power to the first 200-kv. transmission line in the eastern part of the United States. The outside diameter of the stator frame is 38 ft. The largest capacity thrust bearings ever built will be required for these generators, their capacity being the total load of 750 tons. Mounted upon each 40,000-kv-a. generator will be a 715-kv-a. auxiliary generator and above the auxiliary generator will be the 41-kw. exciter set.

By a large degree of cooperation between the manufacturers of turbines and generators, it has been possible to obtain similarity in characteristics and appearance and the interchangeability of some of the important mechanical parts.

The Conowingo units will be required to operate, in most cases, on the peak loads with unusual conditions, and will be shut down during the low load portions of the day in order to store water to the greatest possible extent. For this reason it is important to avoid leakage when the units are shut down and consequently the large pivot valves will be installed in the turbine casing rather than have head gates at the upper ends of the penstock, thus insuring quick closure and reducing the loss of water to a minimum. It is expected that this plant will show a world's record performance from the standpoint of efficiency and reliability of operation.

Automatic Hydroelectric Stations. During 1926, two 28,500-hp., I. P. Morris turbines driving 25,000-kv-a., 60-cycle, 11,000-volt Westinghouse generators at 300 rev. per min. were placed in operation in the Wallenpaupack Power Station of the Pennsylvania Water & Light Company. These units operate under a head of 300 ft. and the station is designed for carrier-current type of automatic control. These are the largest units on record that will be controlled automatically.

A 17,500-hp. Pelton Water Wheel driving a 13,333-kv-a., 60-cycle, G. E. generator is under construction for the Glines Canyon Power Station of the Northwestern Power & Light Company. This unit which will operate under a head of 190 ft. will be controlled by the operator of the Elwha River Plant, seven miles away by means of selector supervisory control. In addition to the main unit, the 62.5-kv-a. auxiliary water wheel and the motor driven oil pump will also be controlled by the automatic equipment.

The largest plant built up to date for full automatic control is the Louisville hydroelectric installation now under construction which will consist of eight 13,500 hp., Allis-Chalmers turbines. The generators are 12,500-kv-a., 14,000-volt, 100-rev. per min. vertical General Electric machines which will have a self-contained ventilating system because of the high air temperatures during the summer time when they will carry the heaviest loads.

High Head Impulse Wheels. The largest capacity impulse wheels ever constructed are the two 56,000-hp. turbines which are under construction for the Big Creek No. 2-A plant of the Southern California Edison Company. These machines will operate under a head of 2300 ft., one being an Allis-Chalmers machine and the other will be made by the Pelton Water Wheel Company. Both machines will be of the double overhung type, having separate governor control for each nozzle. The main shaft bearings of these units will be 30 in. in diameter and the total weight carried on each bearing will be 230,000 lb. The present speed of the units will be 250 rev. per min. for 50 cycles, but the machines are designed for 60-cycle operation at 300 rev. per min. The jet diameter for each overhung impulse wheel will be 8½ in. Each bucket will weigh 900 lb. and in case of the runaway of the unit, the combined forces on the bucket bolts will be approximately 300 tons per bucket.

The Westinghouse generators for these units will have a capacity of 45,000 kv-a. at 11,000 volts, 250 rev.per min. but will have a capacity of 50,000 kv-a. at 12,500 volts when operating at a speed of 300 rev. per min.

A 40,000 hp. Allis-Chalmers double overhung impulse wheel has been placed in operation at the Kings River Plant of the San Joaquin Light & Power Corporation for operation under a head of 2243 ft. The generator will be a General Electric 33,000-kv-a., 13,200-volt, 360-rev. per min. machine. Two exciters are provided, each of which is of sufficient capacity to excite the main generator. Each exciter is driven by a single-jet, single-overhung impulse wheel.

Two Pelton Water Wheels of 40,000 hp. capacity each have been installed at Santos, Brazil for operation under an effective head of 2450 ft. The General Electric alternators are of the horizontal type with a capacity of 33,000 kv-a. at 11,000 volts and 360 rev. per min.

The turbines to operate under the highest head up to the present time will be the units for the Bucks Creek Plant of the Feather River Power Company. These will be of the double overhung impulse type Pelton Wheels with a capacity of 30,000 hp. The head will be 2548 ft. and the turbines will drive 25,000-kv-a., 11,000-volt, 450-rev. per min. General Electric generators.

Propeller Type Turbines. There are being installed in the Great Falls Plant of the Manitoba Power Company, one 28,000-hp., Moody and one 31,500-hp. Bell type turbines driving General Electric generators. These units are the largest propeller type units now being installed and will operate under a head of 56 ft. at 138 rev. per min.

POWER STATION AUXILIARIES

The prevailing practise seems to be motor driven auxiliaries, with the added protection of having certain auxiliaries steam driven. The chief reasons for this are the extensive adoption of the regenerative cycle for feed water heating and the rapid development of motors suitable for auxiliary drive. The electric drive is very efficient and reliable and it is probable that in most cases a station using electrically driven auxiliaries will show on the average, a better thermal efficiency and a lower cost per unit of output including fixed charges, than if steam driven auxiliaries were used.

Steam drive, however, is still most favored for boiler feed pumps. It is interesting to note that a large station recently constructed in the East has adopted steam drive for all of its essential auxiliaries, but this is largely a local condition peculiar to its system. The refinements of design details, and improvements developed for large turbines have been extended to smaller capacities so that manufacturers are prepared to furnish turbines for auxiliary service having very much improved water rates.

There are still many opinions in regard to the best source of electric power for auxiliaries. On account of the higher efficiency of the main units, there has been a tendency to put all the station load on these machines. There are not so many house turbine generators being installed as in the past. The house generator in tandem with and being direct connected to the main generator shaft, appears to be gaining in favor and is extensively used.

POWER PRODUCTION ECONOMIES

A record for thermal efficiency was attained at the Columbia Station of the Columbia Power Company in Ohio. For a period of one month this station was operated on a heat consumption of approximately 12,462 B. t. u. per kw-hr. net output, which is the lowest figure obtained by any steam plant to date. The following figures, for two consecutive months, are of particular interest:

	December 1926	January 1927
Kw-hr. net output	44,998,400	42,547,900
Load factor	65.7%	60.7%
B. t. u. per kw-hr. net output	12,495	12,462
B. t. u. per lb. coal as fired	13,838	14,002
Coal factor, lb. per kw-hr. net Output	.903	.890
Station water rate	7.76	7.85
Standard Boilers;		
Average efficiency	87.06%	87.54%
Average Rating	216 %	233 %
Reheat Boilers:		
Average efficiency	90.7%	90.92%
Average Rating	408 %	422 %
Auxiliary power consumption	5.27%	5.41%

The results in many other stations put into operation during the last year have also been very reassuring and in some cases have exceeded the expectations of their designers. High pressures, high temperatures, water walls, regenerative cycle, reheat cycle, air preheaters, improved combustion due to better stokers or pulverized fuel, reduction in exit gas losses by economizers and preheaters, use of electrical drives for auxiliaries, and improvements in turbine and condenser design have all contributed to improve the thermal efficiency of power plants so that the reduction of operating costs have more than kept pace with the increased price of fuel and increased operating labor rates.

The marked improvement in utilization of fuel by public utility power plants is revealed by figures given in the Geological Survey Report, which shows that in 1926 the average large generating plant turned out a kw-hr. on 1.94 lb. of coal as compared with 2.07 in 1925. These figures include coal, oil, and gas fired plants and represent the equivalent coal consumption. It is interesting to note that since the World War, using 1919 as a basis, the equivalent coal consumption per kw-hr. has been reduced 40 per cent.

ACKNOWLEDGMENT

In attempting to present a résumé of the outstanding technical achievements brought about during the past year in a complex industry like that of power generation, every effort was made to obtain from many sources as complete and reliable information as possible of the existing conditions in the field. The committee is indebted to those engineers and manufacturers who so splendidly assisted in this work and wishes to express its appreciation of their cooperation.

W. S. GORSUCH, *Chairman.*

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Discussion

Philip Torchio: Electrical engineers should have particular interest in following the development of the size of the generators. The report states that there is a 208,000-kw. unit under construction. It should have said that it consists of three machines cross-compound. If I remember rightly, the largest single unit in operation in the New York Edison 60,000-kw. The Edison-United Companies have a large cross-compound 160,000-kw. unit on order and placed an order June 21 for a 165,000-kw. unit. The generators of the unit order last year are, one, 85,000-kw. and the other 75,000-kw. and, for the unit ordered yesterday, the generators will be 80,000-kw. each.

In pointing this out, I am not saying that the manufacturers are not ready to make larger generators, because, as a matter of fact, for another installation for which we are now securing bids, we already have offers of more than 100,000 kw. in one unit, and probably single units larger than 125,000-kw. will be obtained.

Another point that I wanted to bring out in this general review is that while we are making great progress in saving coal, it is nevertheless our duty not to let the public misunderstand how we accomplish it. The truth is that we are saving coal by spending more in investment in plants. It does not necessarily follow, once we consider the total cost, that we are making such radical improvements which might lead the public to assume that the cost of power is also rapidly being reduced. The cost of power is being reduced, but at a gradual, moderate rate. We are making great advances in saving coal, but we are putting more capital in our plant investment to secure those higher economies. An extreme illustration of this is the Dutch Point station with the mercury boilers and similar installations throughout the country.

F. A. Scheffler: My discussion of this report is more particularly an addition to the table of the pulverized-coal installations in the country.

The installations in operation last year consisted of an installed capacity of 2,200,000 kw. and those under construction, 440,000, making a total of 2,640,000-kw. capacity installed.

Assuming that these stations, of which there are about thirty-eight, average an annual load factor of 50 per cent, the output would be 9,414,810,000 kw-hr., and also assuming that the average coal consumption is 1½ lb. of coal per kw-hr., the total fuel used would be 7,061,100 short tons per year.

During the year 1927, the total coal consumption in public utility plants in the United States was 41,245,000 short tons.

The ratio of the total coal used in pulverized form to the total coal consumption, is therefore approximately 17 per cent.

In addition to the above, there are under construction new plants and additions to others as follows:

Station	Company	Kw. capacity installed
State Line..........	State Line Generating Co.	208,000
Toronto............	Ohio River Edison Co. (2-30,000 kw.)	60,000
Glenhead...........	Long Island Lighting Co.	25,000
Trenton Channel...	Detroit Edison Co.	50,000
Aurora, Ill..........	Western United Gas & Elec. Co.	10,000
Pekin, Ill...........	Super-Power	50,000
Montaup...........	Montaup Electric Co.	40,000
		443,000

With the exception of four or five on the above list of generating plants, all of the stations were new ones built during the last seven years.

There are five or six other utility plants using pulverized coal, which are not listed above, because they are not primarily power plants for generating current, but are used more for steam heating purposes. Some of these are as follows:

Puget Sound Traction, Light & Power Co., Seattle, Wash.—capacity about 8500 boiler hp.

Allegheny Heating Co., Pittsburgh, Pa.,—capacity 10,000 boiler hp.

Lockport Light, Heat & Power Co., Lockport, N. Y.—capacity 2500 boiler hp.

New York Steam Co., 36th St. & East River, New York—capacity 30,000 boiler hp.

Rochester Gas & Electric Corp., Lawn St. Station, Rochester, N. Y.—capacity 250,000 lb. steam per hr.

W. S. Gorsuch: It is important, as Mr. Torchio has pointed out, to state the type when referring to a certain size generator unit. This has been done in every case in the report, not in the brief outline of progress in the art of power generation, but in the description that follows, in which outline drawings are also given for exceptional designs. It will be noted that the plan of this report is first to give a general statement of the trend of the art in each class of power-station equipment and then under the heading "Outstanding Installations" a full description is given of the equipment referred to in the preceding general statement.

Reference has been made to the remarkable economies brought about in fuel burning stations and the additional investment cost to achieve these results. In this connection I believe because of the increasing number of interconnections a comprehensive study should be undertaken at this time to show the relative cost of power of hydroelectric and steam plants, and also the relative merits of the two systems.

Transmission and Distribution

Annual Report of Committee on Power Transmission and Distribution*

To the Board of Directors:

FOREWORD

In presenting the annual report of the Committee on Power Transmission and Distribution it has seemed best to largely confine its scope to a discussion of the progress that has been made in those branches of the art with which the Committee has been actively concerned during the year. Several of the members have collaborated in the preparation of the report and it represents the consensus of opinion of the Committee as a whole.

It is felt that the coordination of effort which has been secured through the medium of the Committee has resulted in a very substantial stimulus to the advancement of the state of knowledge regarding those problems which have been particularly studied and if this report in presenting a résumé of the progress made also succeeds in indicating profitable fields for further study and research its purpose will have been accomplished.

LIGHTNING ON TRANSMISSION LINES

With the growing tendency toward interconnection of large power systems and the increasing dependence on high-tension transmission lines for the service of large communities, the question of continuity of service for such lines has become very important.

Inasmuch as lightning has been responsible for most of the interruptions to service over transmission lines a subcommittee was appointed whose duties were to investigate lightning and its relation to transmission lines.

The work assigned to the sub-committee was of considerable magnitude and importance. The various lightning difficulties that had been encountered on transmission lines in the past two years were discussed, as well as the various data that had been gathered in connection with these difficulties.

The following are the subjects taken into consideration in the discussion of the general problem.

Klydonograph Tests. Inasmuch as the transients due to lightning are of extremely short duration it was very difficult to determine their characteristics until the advent of the klydonograph. By means of the klydonograph it has been possible to obtain information regarding the operation of transmission lines of voltages ranging from 6.6 kv. to 220 kv. The nature of the tests and the inherent variation of the physical conditions under which lightning occurs necessitated a great amount of data to determine the facts definitely. However, the data secured is sufficiently extensive to indicate the following important conclusions.

1. The most important voltage surges on overhead transmission lines are those produced by lightning.
2. On 120- to 140-kv. lines surges of 1200 to 1400 kv. and on 220-kv. lines surges of 1800 to 2000 kv. can be established.
3. Majority of lightning surges were positive.
4. Highest surges were negative, which would indicate that they were direct strokes and that the clouds causing these surges were negative.
5. While very high positive surges were recorded, they were few in number and only slightly over 1000 kv.
6. The number of surges per storm at a given point is not great.
7. When a surge above the flashover of the insulation is induced, the insulator flashover relieves the energy and limits further rise of voltage.
8. The steeper the rate of application, the higher is the voltage reached before flashover, and when no flashover occurs, the insulators must withstand the surge for its entire duration.
9. High-voltage surges do not travel far. Such surges are damped below the corona voltage in a relatively few miles, while low-voltage surges travel many miles.
10. Lightning strokes are unidirectional or at most highly damped oscillations.

Lightning Strokes. What is the amount of current that can be encountered in a lightning stroke?

It is fairly definitely known that in the case of a direct stroke, currents of the order of 10,000 to 100,000 amperes can be encountered, whereas in the case of an induced lightning stroke this current is probably of the order of 3000 amperes.

A point also to be taken into consideration is that when a lightning stroke does take place the number of insulator units that are likely to be spilled will depend to a large extent on the resistance of the principal unit paths, the resistance consisting of the tower and the ground resistances. To amplify: if the resistance of the lightning path at a particular string is of a high enough value, the effect of the current going through it will be to raise the potential of the conductor at that point above the flashover value of the string in question and a wave above the spillover value of the string will travel on to the next string to be spilled over and so on until it reaches the point where the flashover of a string is in excess of its crest in which case it will travel on until it is dissipated in some other manner and so disappears.

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This, of course, shows clearly that the ground resistance plays a considerable part in the determination of how the lightning flashover will act over the line. With a low value of ground resistance, it will take fewer strings to dissipate a particular impulse or stroke. On the other hand, with a low value of ground resistance, the amount of power current to be handled at any particular string will be greater and therefore the possible arc damage will also be greater.

There is another point to be taken into consideration in determining the value of ground resistance, and that is the question of short circuit current required for the successful operation of relays. It is essential that the relays controlling transmission lines operate very quickly under lightning disturbances in order to minimize the damage of the power arc to the insulator string and the conductor.

Some data were available as to what value of ground resistance was encountered in practise. Some companies had made measurements of ground resistance on towers and found that the average resistance was between 15 and 25 ohms, while some towers, however, showed a resistance as low as 2 ohms, others as high as 100 ohms, and the highest encountered was in the neighborhood of 300 ohms.

The question as to what is, therefore, a correct mean value of a ground resistance is one that is of considerable importance and it would be highly desirable to obtain some data both from a theoretical and experimental standpoint to establish that value definitely.

Lightning Discharges. The question of lightning discharges under power and the power arcs which follow brings with it another question and that is whether it is definitely known that in all cases a power arc follows the lightning discharge.

One or two cases were cited in which reports had been received from observers who claimed that they had seen a lightning discharge on a string of insulators with no accompanying power follow-up arc, but the Committee agreed that the amount of data available on this was meagre and in all probability what was available was highly unreliable. It is conceivable that a lightning discharge could take place and yet if the amount of lightning current were small enough, sufficient ionization of the atmosphere would not take place to render a power arc possible. However, just what is the amount of current necessary to do this is not known and it is a problem for further investigation.

It has been estimated that the duration of dangerous lightning impulses encountered on transmission lines is of the order of a fraction of a micro-second to ten micro-seconds.

Ground Wire. The value of the ground wire as a protection against lightning has been a much mooted question. In the past where the maximum operating voltages were in the neighborhood of 66 kv., the consensus of operating opinion has been that interruptions to service by lightning were as frequent on the transmission lines equipped with ground wire as those that were not equipped with ground wire. In addition a great deal of mechanical trouble was encountered with the ground wire which was often accompanied by interruptions to service.

Very little attention in the past has been paid to the erection of the ground wire both from a mechanical and electrical standpoint and it is believed that the chief objection to the ground wire on transmission lines has been as a result of this lack of attention. It is believed that the ground wire conductor should have a layer of non-magnetic conducting material. Some operating companies have used aluminum steel reinforced or copper clad conductors for ground wires and other companies are using copper and high-strength copper alloy conductors.

As to the value of ground wire for reducing the induced lightning voltage data obtained on a 132-kv. line operated without ground wire were studied, and charts prepared in which the various flashovers that occurred were plotted on a profile on which tower locations had been indicated by two different sets of coordinates. In one set, the coordinate consisted of distance from the station and average height above ground of the bottom conductor of the span on side of tower away from the station and in the other case, the coordinate consisted of distance from the station and elevations of the bottom conductors on the towers.

On this chart all the flashovers which occurred were spotted opposite each tower. A careful examination of the chart showed very markedly that the flashovers were concentrated on the high spots and were generally absent from the low spots on the line. It also indicated very clearly that the great majority of the trouble was on the top conductor.

There were three stations located on that line and while the line was literally peppered with lightning at other points, these stations were apparently free from trouble. The chart showed that the line was particularly low for a distance of two or three miles from the stations and this explains at least partially the absence of trouble at the stations.

The number of flashovers during the 1925 season was approximately 88. In the early spring of 1926 a ground wire was installed on this line and the corresponding number of flashovers during the lightning season of 1926 was only about 8 or 10. This with other evidence that is available clearly shows the value of the ground wire.

Effect of Tower Design. The question of the effect of the tower itself on the electro-static field surrounding a conductor and what effect this has on the flashover value of the insulator units, was considered.

It was the opinion of the Committee that any effect which the tower may have on the field is of relatively little importance in determining whether a flashover will or will not occur. The fact that some semi-flexible towers are known to have less lightning trouble than some of the other towers of the square type is undoubtedly due to the fact that in the first place they are very much lower lines, the spans being comparatively short and, second, they invariably are equipped with ground wires since the structures themselves are generally not self-supporting and the ground wire which is put in for mechanical reasons serves, of course, the additional function of giving them lightning protection at the same time.

It was considered advisable that all the points on which the committee was in agreement should be set down to serve in the nature of principles for guidance in the design of high-tension transmission lines.

It is the opinion of the Committee that these principles have sufficient theoretical background for their conclusions and generally also a considerable amount of actual experience to further back up the theory.

The following are the principles referred to:

1. Under any given set of conditions, the lightning voltage which can be picked up by a line is a function of the height of the line, being directly proportional to the height and is further a function of the ground wire arrangement but is independent of the power voltage.

2. The lightning voltage under any set of conditions on a line is limited by the insulator flashover for the particular wave in question. It should be pointed out, however, that this is only true where the ground resistance and the tower resistance are comparatively small. If this resistance is high, the total voltage may then be appreciably higher than that corresponding to the flashover of the insulator string.

3. In any line design, it is desirable first to hold down the lightning voltage. This can be done by

(a) Keeping the line as low as is economically feasible, and

(b) By the proper use of ground wires, again within economical limits. Failing to do this, the next best thing is to prevent the power arc following the lightning flashover. Unfortunately we are not at present in a position to state how this can be done.

4. The higher you go in transmission voltage, the more beneficial, as a rule, is the ground wire. This may be seen by the following simple example:

Assume two lines built with the same type of structure, the same conductor, with one built for operation at 66,000 volts, say the other for operation at 132,000 volts. Assume that the 66,000-volt line utilizes 5 ordinary suspension units and that the 132,000 volt line utilizes 10 suspension units. The lightning flashover of the first is in the neighborhood of 600,000 volts and of the second in the neighborhood of 1,200,000 volts. Assume that the average lightning voltage that you can get on the 66,000-volt line is 2,000,000 volts. Then if the ground wire has the effect of reducing that by 50 per cent, the lightning voltage with the ground wire will still be 1,000,000 volts which is 400,000 volts in excess of the flashover value of the 5 unit string. Therefore, under these conditions, a flashover will take place.

In the case of the 132-kv. line, the height, the conductor, and everything else being the same it follows that the voltage with or without the ground wire would be the same as before and therefore with the string having a lightning flashover of 1,200,000 volts no flashover will occur.

A similar example could be employed to show that a 220-kv. line where with 14 standard suspension units the lightning flashover of the insulators may be expected to reach values of the order of 1,800,000 volts would hold and not flash.

5. The design of substations should be co-ordinated with that of transmission lines as there is a great tendency to over insulate the line and thereby tend to transfer the trouble to substation equipment.

A great deal of attention is being paid at the present time in specifications for electrical apparatus to the impulse strength of apparatus.

The manufacturers are working on the problem of impulse strength of apparatus. There is considerable evidence to show that there is a fairly definite ratio between the impulse strength and the 60-cycle strength of the apparatus. If this is demonstrated to be true, it will be possible to design a complete system, including the transmission line and connected apparatus, with a relation of insulation values that will give the greatest efficiency and continuity of service consistent with minimum cost.

Future Work. For the lightning season of 1927, several power companies are arranging for future klydonograph tests which will give us more information on the value of the ground wire and on the amount of attenuation of traveling waves. We also hope to obtain data to determine the following points:

1. Wave fronts of lightning surges
2. Duration, of energy of lightning surges
3. Maximum voltages induced on continuous transmission lines
4. Maximum field gradients.

Arrangements are also being made to determine attenuation, potentials at stations, and at short distances therefrom, potentials on adjacent sections of line with and without ground wires, potential on both sides of choke coils, and discharge currents of lightning arresters.

LIGHTNING PROTECTION OF DISTRIBUTION CIRCUITS

For a number of years studies in lightning protection for 4000-volt, four-wire, three-phase circuits have been made in Chicago, and the results have been reported to the Institute.* During this period, several new types of arresters have been placed on the market, and the most promising of these arresters have been installed in Chicago.

In attempting to analyze the records obtained during the past few years on the various types of lightning arresters and determine their relative efficiency in protecting transformers from damage by lightning, some very discordant results were observed. The results obtained from the service records were not in accord with the theories, nor with the results of laboratory tests.

To make a more comprehensive study of the situation, the engineers of manufacturers whose lightning arresters were under investigation in Chicago accepted the invitation to join in a conference for determining the cause of these discrepancies. The investigations resulting from this conference will probably require at least a year before definite results can be secured. The indications are that, by giving more attention to the details of line construction and perhaps making some alterations, it will be possible to make a considerable further improvement in lightning protection on distribution circuits.

Many of the factors which affect lightning arrester performances are not constant and alike for all installations, and it appears from the data that a large share of the transformer failures, are due to limitations in protection which are imposed by conditions outside the arrester, of which good examples are, high ground resistance, currents from several circuits through ground connections of moderate resistance and entrance through the secondary. Before a conclusion is made that the arrester has failed to function properly, the variables must be thoroughly considered. A list of these variables is as follows:

1. Ground resistances
2. Transformer history
3. Primary exposure
4. Secondary exposure
5. Shielding
6. History of immediate territory
7. Lightning entrance.

These variables are quite well known to engineers who have studied lightning arrester performances. In attempting to determine their individual importance on each failure, they are studied somewhat as follows:

1. *Ground Resistance.* Ground resistances are measured on the nearest three arrester installations in each direction from the failure. This assists in determining the ability of the arresters to relieve the line. Resistance above 20 ohms should be considered inadequate, and lower values are in doubt, that is, the value of resistance to be considered as dangerous must be in relation to the amount of current which the ground connection may be expected to carry and the maximum strength of the insulation which is being protected.

2. *Transformer History.* The transformer history includes the make, size, age, connected load, previous failures, previous fuse failures, and the type of installation, whether it be power or light. Of these factors, the first two affect failures, inasmuch as some transformers may be more susceptible to damage than others. Windings of the older transformers are quite apt to be of a lower insulating value, transformers operating with an overload and previous fuse failures may also weaken this insulation. Transformers which have previously failed are repaired and re-installed on the lines. The rewinding of the transformer coils may result in the transformer being more susceptible to damage by lightning. Lightning transformer installations are grounded, and also one power transformer, of a power bank installation, is grounded. This factor—whether the transformer be grounded or not grounded—also may affect the failures.

3. *Primary Exposure.* The height, length, number of primary wires, number of arresters, and the underground cable connecting to the overhead conductors are features which are studied in connection with the primary conductors. The height of mains connected to the transformer failure are of importance, the charge induced increases as the distance from the earth, or the point of zero potential, increases. The length of these wires is important, since the increased exposure increases the charge on the line. The number of arresters discharging, and the length of the line relieved by each arrester, should be considered. Long lengths of underground cables connected to overhead conductors at distances of 100 feet or so from the failure aid in reducing the induced potential, and are a variable factor.

4. *Secondary Exposure.* The length and height of secondary phase wire and the length of secondary neutral exposure is of importance. The secondary neutral is grounded on some systems and ungrounded on others. The grounded secondary neutral furnishes shielding of considerable value.

5. *Shielding.* Structure shielding is quite low unless the structure is of steel adjacent to and higher than the line. Frame and brick structures, unless extending two or more stories above the line and adjacent to it, furnish little shielding. Trees extending above the line and adjacent to it, and telephone cables on the same pole as the primary conductors, furnish good shielding and must be considered.

6. *History of Immediate Territory.* Transformers burned out and fuses blown due to lightning in the

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*Studies in Lightning Protection on 4000-volt Circuits. TRANS. A. I. E. E., Vol. XXXIX, p. 1895; Vol. XXXV, p. 655.

adjacent areas of approximately one square mile should be considered to determine the severity of lightning within that area. Certain areas may be more susceptible to lightning disturbances than others; this susceptibility may be due to geographical conditions, or the character of the structures within the area.

7. *Lightning Entrance.* A study of the transformers burned out should be made and an attempt to determine the lightning entrance, whether it be over the phase or neutral primary, or over the secondary wires. Evidences of arcing on the case, bushings, and pole should be noted and the extent and nature of the burns on the windings of the transformer. Any damage to customer's equipment should be studied as evidence of lightning entrance over the secondary side of the transformer.

These are the factors which should be studied in connection with each case of failure, whether it be transformers, underground cables, or secondary equipment. After this study, one of the factors may be found to be almost entirely responsible for the failure, and with sufficient data covering a period of years, an attempt can be made to eliminate these factors as much as possible and better the protection.

The Commonwealth Edison Company now installs 2300-volt arresters on primary phase, and 300-volt arresters on the primary neutral; and as a result of their studies of the seven factors listed, the advisability of arrester installation on the secondary side of the transformer is being considered as a means of further reducing interruptions of service due to lightning burnouts.

VOLTAGE STANDARDIZATION

Several excellent papers dealing with transmission and distribution voltage standardization have been presented to the Institute during the past year and undoubtedly have contributed much toward clarifying the status of this problem. The ultimate solution, however, is still somewhat obscure. With the present tendency toward interconnection of systems the standardization of voltages becomes of increasing importance and it seems urgent to push the studies in this field.

STABILITY AND LOAD LIMITATIONS OF POWER SYSTEMS

During the past year, the most significant development has been the engineering studies of several of the power companies which have led them to adopt special means to improve operating conditions and minimize the probability of outage. The stability studies have led to the adoption of synchronous machines of low reactance and high short-circuit ratio provided with quick-response excitation. The first installation of this kind will be in California followed quickly by similar installations in Pennsylvania and Alabama. The inclusion of these features in specifications for these installations shows the importance that the engineers of large utilities are attaching to the subject of stability.

An important advancement in the transmission art is the development by Frank G. Baum of "A Transmission System." Broadly, the principle of this system consists in supplying to the line at each point the reactive kv-a. required for transmitting the power over the line at that point, irrespective of whether power is taken in or given out there and incorporating in the devices used for this purpose the necessary characteristics to enable them to supply the reactive power required for stability under all conditions of operation, Practically, this means the installation of synchronous condensers of proper characteristics at intermediate points of long distance transmission line, in order to increase the amount of power that may be transmitted over that line as compared with the same line without the condensers.

Sustained attention is being given to the design and construction of machines having characteristics appropriate for long-distance power transmission. It is recognized that the desirable characteristics in synchronous machines from the standpoint of stability are low leakage reactance and high short-circuit ratio. The better performance thus obtained may be utilized to increase the amount of power transmitted rather than the margin of stability. Generators of low reactance were decided upon by the Southern California Edison Company for its Big Creek 2-A power plant and for the motor end of the frequency changer set at Farmersville located at an intermediate point on their Big Creek transmission system. Maintenance of stability is essential not only for long-distance transmission lines but also for comparatively short systems of large capacity and high standard of service. For example, in the case of the Conowingo development of the Philadelphia Electric Company, it was found that the use of generators having special characteristics would increase the reliability of the system.

Quick-response excitation which has been introduced commercially during the past year, maintains a high average value of voltage in synchronous machines at time of changes in circuit or load conditions thus improving the stability of operation. In some cases it will be desirable to use quick-response excitation in addition to special characteristics in the synchronous machines. The speed of response of the excitation system is obtained by the use of multiple-connected field windings in the exciters and separate excitation. Motor-driven exciters have been employed instead of direct-connected exciters for low-speed generators, because quicker response can be obtained with exciters of higher rotational speed and smaller air gaps which are possible with motor-driven units. To obtain the advantages of the quick response excitation system it must be controlled by a suitable voltage regulator capable of acting promptly and keeping its contacts closed until the system voltage has approached normal value. In order to secure correct operation of the voltage regulator under all conditions of operation, two potential transformers with a positive phase sequence network are employed instead of the single potential transformers normally used. Quick-response excitation systems have been ordered for each of the plants described above, and in addition for the Lock 18 and Tallassee plants of the Alabama Power Company.

In last year's report mention was made of the theory of artificial stability which had been previously advanced and which had been substantiated by actual calculations and to some extent by experimental tests. Further experimental data not only confirms the laboratory data previously obtained but establishes the fact that artificial stability can be obtained on commercial power systems. While it is not expected that systems would normally be operated in the range of artificial stability it is undoubtedly desirable to take advantage of this increased limit as a margin. It should of course be appreciated that the real advantages of quick response excitation systems lie in the increased power limits under transient conditions.

Going further in the development of voltage sustaining devices, successful experiments with a small inherently compensated synchronous condenser have been carried out. The compensating current responds not only to a change in the magnitude of the load current but also to a change of the load power factor. It is necessary to keep in mind that this response to change of angle is quite important during transient conditions.

An important development during the past year has been the increased interest in the recording of systems data useful from the stability standpoint. The most suitable instruments for this purpose are of the oscillographic type arranged to operate automatically on the occurrence of system trouble. For instance, on the occurrence of a fault to ground, a ground relay places the automatic recording apparatus in operation and after the record had been obtained, auxiliary relays automatically disconnect the apparatus and prepare it for a subsequent operation. Particular mention should be made of the instantaneous watt elements, a sample of which was displayed to the Committee during the year. Such a watt element is particularly useful in stability studies.

These instruments should be very valuable to enable operating engineers to obtain data on the performance of their systems during transients. This data will be useful also in planning future extensions, ties, and interconnections.

Another point, mentioned in last year's report as being incomplete, is the matter of fault resistance. The effective value of the fault resistance at the time of a flashover cannot in general be measured directly. A new indirect method has been used successfully which consists in making a chart of calculated values of ground currents for various fault locations and for arbitrary values of fault resistance. Lines of constant resistance on this chart are used as parameters of reference and the actual fault current, as measured by the above mentioned oscillographic recording device, is plotted on this chart; its position with reference to the constant-resistance lines gives the actual fault resistance. In this way fault resistance, when comparable to the reactance of the system, can be obtained with a fair degree of accuracy, and since only the order of magnitude of fault resistance is important, this accuracy is adequate.

During the past year progress has been made in increasing our technical knowledge on the subject of transmission stability. One paper on this subject was presented by Mr. O. E. Shirley and another by Mr. H. V. Putman. Mention should also be made of the paper by Messrs. Doherty and Nickle on the *Theory of Synchronous Machine*, which gave consideration to the stability characteristics of machines. Noteworthy progress in the general understanding of the stability problem has been facilitated to a considerable degree by the use of the mechanical analogy due to Mr. S. B. Griscom, and presented before the Transmission Committee and several Section meetings.

UNDERGROUND CABLES

A pronounced drift toward the use of single-conductor cables and three-conductor metal-sheathed unbelted cables is making itself felt. More data are being secured as to the relative merits of these two types of cables as well as the ordinary belted three-conductor cable. Aside from any difference in dielectric strength the belted three-conductor cable is at a disadvantage due to the fact that faults frequently are from phase to phase instead of being confined to ground as is usually the case with the single-conductor or metal-sheathed three-conductor cables.

Where the duct size permits, three single-conductor cables may be installed in one duct thereby avoiding the necessity of having an individual duct for each cable which in many cases would make the subway cost prohibitive. This practise is being followed to a large extent on the system of the New York Edison-United Companies with very satisfactory results.

Progress is continually being made in the development of super-voltage cables. The 66-kv. Cleveland cable has been in service for about three years and the Philadelphia cable which was rated at 75 kv. but operated at 66 kv. has been in service for about one year. While some failures have occurred the operation in general has been satisfactory.

The Commonwealth Edison Company has in service two 75-kv. lines each consisting of three single conductor 750,000-cir. mil cables with 24/32-in. insulation and is installing additional circuits of this size and insulation which will go into service this Fall.

One of the encouraging features has been the highly successful operation of the oil-filled joints.

While short time tests indicate that the 24/32-in. insulation is liberal and that the cables might have a somewhat higher voltage rating, the deterioration rate of insulation stressed on this basis is not yet accurately determined.

Trial installations are being made on 66-kv. cables of joints which incorporate a means of insulating the intervening section of sheath. By suitably connecting the insulated sections the sheath currents may be minimized and the carrying capacity of the cables thereby increased.

The past year has seen considerable development in the use of oil reservoirs of various types on high-tension cable splices. The oil supplied by these reservoirs no doubt reduces or possibly eliminates voids, especially near the joints. This is of considerable importance as it has been found that the cable itself absorbs large quantities of oil for weeks or even months after installation. The indications are that the oil absorption by the line bears a direct relation to the temperature range through which the line is worked, the greater the temperature range the greater the oil absorption. Considerable attention should be given to the volume capacity of the oil reservoirs as the temperature changes also produce appreciable variations in the volume of the insulation for which it is desirable to compensate.

A clear distinction should be made between this type of cable and the 132-kv. cable now being installed in New York and Chicago. The latter has a hollow core filled with an oil which is fluid at all operating temperatures and this central space is connected at suitable intervals with large oil reservoirs capable of compensating for volumetric changes in both oil and cable whether due to temperature changes or other causes. No experience has yet been had with these 132-kv. installations but it is expected that their operating record during the next few years plus the further experience with the lines operating at 66 kv. will indicate whether or not the usual type of cable can be made to operate satisfactorily at over 75 kv. and also show whether the hollow core, oil filled type and its several accessories are good for 132 kv. or possibly more.

Voltage Surges on Underground Cable Systems. In the course of the studies which have been made of high voltage transients on underground cable systems klydonographs have been installed on 16 cable systems for the purpose of obtaining operating records of transients on these systems. The highest voltage recorded was 4.6 times normal. Nearly 99 per cent of all the surges recorded were under three times normal and 92 per cent of the total were unidirectional and therefore of brief duration. As the highest surges are of the same order as the commonly specified test voltage of the cable, but last only a small fraction of a second, it is probable that they have no effect on the cable insulation.

USE OF TEMPERATURE INDICATORS ON DISTRIBUTION TRANSFORMERS

Experience which has been obtained from the use of temperature indicators on distribution transformers in Boston has indicated that material savings may be accomplished due to a better loading of the transformers. Some operating companies, however, feel that temperature indicators are not a satisfactory substitute for load tests but in any case it is probable that under cool weather conditions where the ambient temperature is materially below 40 deg. cent. which usually corresponds to the peak-load season in Northern cities the transformers can be safely loaded to values considerably in excess of their rating. An important reduction in transformer investment may thereby result.

PHILIP TORCHIO, *Chairman.*

Discussion

D. W. Roper: This report calls attention to some of the lightning arresters and distribution circuits in Chicago. For fear that the readers might get a wrong impression, I want to add a little to what appears in the report. The lightning-arrester records of the several types appeared to indicate that the results obtained were not quite what were expected. Further investigation of that point has brought out an interesting feature, somewhat unlooked for, in that some of these burn-outs which have been recorded and which have affected our results were due to lightning entering via the secondary circuits. The lightning arresters we have are on the primary circuits. The primary distribution, in general, occupies the top arm and the secondary ordinarily the next lower arm. Sometimes it is on the same arm with the primary circuits. As the lightning potentials which appear on the line are in proportion to the height from the ground, it is seen that the secondary circuits have been getting almost the same lightning effects as the primary.

We have examined a few transformers—not very many—but as nearly as we can determine from the few which we have examined, something like one-third of our transformer burn-outs have been due to lightning which entered the secondary winding. We can hardly blame the lightning arresters which are connected to the primary circuits if some transformers burn out due to lightning entering on the secondary circuits.

S. J. Rosch: The report under the paragraph on Underground Cables, says, "There is a pronounced drift toward the use of single-conductor cables and three-conductor metal-sheathed unbelted cables." I believe it would be highly advisable in view of its importance, to include in this report some figures indicating the quantity of the latter type of cable now in use in this country. Undoubtedly, many operating engineers in making up their 1928 budget, will naturally look to this and similar reports, for an indication of what type of cable to purchase for their three-phase circuits, whether to use three-single conductors, the regular belted three-conductor cable, or the metal-sheathed unbelted type. It would also be of value to have some figures on the probable use in the near future, of the latter type of cable.

Alfred Herz: I have one question in mind in regard to the inter-bonding or grounding of cable sheaths. We all realize that considerable longitudinal voltage makes its appearance in these sheaths, especially in a sheath surrounding a single-conductor cable. What is the practise in taking care of, or rather in avoiding detrimental effect when you bond or ground such cable sheaths?

Philip Torchio: Answering Mr. Rosch's inquiry as to what per cent of three-conductor cable is now of the metal-sheathed unbelted type, I think there is a very small amount in use at the present time but several manufacturers are ready to make it and a considerable demand for it is anticipated.

Regarding the surge voltages on underground cables as indicated in the report the maximum recorded was 4.6 times normal. Now a cable which is operated at 40 to 50 volts per mil is tested at about 165 volts per mil so that the normal 5-min. test voltage is about four times the normal operating voltage. There remains still a large margin above the test voltage before actual breakdown is reached so that a surge of 4.6 times normal especially in view of its brief duration should not give deterioration. This is in further explanation of the Committee's intent in giving that view.

Replying to Mr. Herz's question about taking care of the induced voltages and currents in the sheaths of single-conductor cables, that can be done by providing insulating joints in the sheaths and cross connecting the insulated sheath sections in such a manner that the induced voltages are counterbalanced. As an alternative the insulated sheath sections may be grounded at one end giving a voltage normally of a few volts at the other end of the section which may under short-circuit conditions reach values of the order of 100 volts.

Herman Halperin: (communicated after adjournment) In the last part of the report, there is a discussion regarding the migration of oil from oil-filled joints into cable, and it is stated that the oil "no doubt reduces or possibly eliminates voids, especially near the joints." Then, at the end of the same paragraph is the following: "The use of such oil-filled joints makes it possible that satisfactory single-conductor cables of the usual type of construction may be obtained for operation at 110-kv., 3-phase."

Up to a few years ago, joints were the limiting feature for underground cables in going to higher voltages, but with the recent development in joints, this limit has been removed. In connection with the last quotation, if the cable as it leaves the factory is not of a quality to give satisfactory operation at 110-kv., 3-phase, then, according to experience with cable made in the past year, the addition of oil-filled joints will not make the cable operate satisfactorily. Apparently, this was not the intent of the quoted statement, but one might infer it from reading the report. Operating and laboratory data indicate that the principal factors necessary to obtain satisfactory 110-kv. cable are either to improve the quality of cable insulation furnished in cables of ordinary construction, or change the cable construction, or both.

In Chicago, No. 10 transformer oil has been used for filling about 150 three-conductor, 33-kv. joints and 750 single-conductor, 75-kv. joints. There have been occasions to examine the joints and cable adjacent in connection with cable failures, and the insulation next to the joint has been usually found well impregnated, partly due to the migration of oil from the joint into the cable insulation. However, considerable deterioration has been found in the cable insulation as close as 2 ft. from the joint.

In connection with laboratory tests on 75-ft. lengths of single-conductor, 75-kv. cable and also with lengths that had been in service several months, dissection has shown that the distance the oil traveled from potheads or joints, varied from a few feet next to the sheath to a maximum of about 50 ft. along the strands of the conductor, depending on the kind of impregnating compound in the cable insulation. The penetration of the oil readily into the insulation was for only a few layers, except for the few feet of cable immediately adjacent to the potheads or joints.

Operating experiences with underground lines and tests on cable samples have indicated great variations in the quality of insulation along the length of cable.

Apparently the effectiveness of oil in improving the quality of the insulation is practically limited to only a few feet, which is very short in comparison to the length of a section of cable between manholes that may be 400 to 700 ft. long.

W. A. Del Mar: (communicated after adjournment) The maintenance of impregnation is now recognized as an essential element of success in the operation of high-tension cables, and the report clearly calls attention to the distinction between the use of oil-filled joints with reservoirs and the hollow-core type used on the 132-kv. circuits at Chicago and New York.

Another distinction which might be made is between two variants of the former type, namely, cables in which the reservoirs are used merely to maintain the impregnation and those in which they are used to maintain a definite pressure within the cable. The former type is subject to limitations, especially where the cable is impregnated with a jelly compound, as the reservoir oil penetrates very slowly, and there is a tendency for the residual air to accumulate near the center of the section of cable. The latter type, *i. e.*, where the system is designed for pressure maintenance rather than penetration, assumes that air will be present and provides means for making it harmless. This is done subjecting the air to such pressure that it will not ionize at the existing dielectric stress.

An experimental installation of this kind has been made in Detroit, bellows reservoirs being used, which are kept under a pressure of approximately 0.4 atmosphere above normal by means of weighted lever. The desired pressure was predetermined as indicated in the discussion of my paper on *The Effect of Internal Vacua*, JOURNAL A. I. E. E., Oct. 1926, p. 1012, *i. e.*, the maximum dielectric stress in the cable was calculated and the air pressure determined at which thin air films begin to ionize at this stress. The reservoir pressure was set slightly above this point.

It is obvious that the success of this pressure system depends to some extent on the use of cable impregnating compound which is fairly soft or fluid at operating temperatures.

Protective Devices

Annual Report of Committee on Protective Devices*

To the Board of Directors:

This committee in its report last year gave a rather complete survey of the present state of the art in the field of protective devices for power systems. As many of the principal features of that report still describe the present practise in these various lines, the committee will report at this time more especially its activities during the past year.

The principal work of the committee this year has been, first, in the arranging for and the actual preparation of papers for presentation at meetings of the Institute, of which about 15 have been presented as listed in the reports of the subcommittees following, and second, in the work of standardization in connection with which during the year there were issued two reports on standards, one for lightning arresters and one for automatic stations.

The work of the committee has been carried on by subcommittees, each under the direction of its own chairman, and after the first organization meeting of the main committee, held at Chicago in October, the further meetings have been held by the subcommittees individually. The subjects covered and the chairmen in charge of the subcommittees are as follows:

Automatic Stations, W. H. Millan, Union Electric Light & Power Co., St. Louis, Mo.

Current Limiting Reactors, E. A. Hester, Duquesne Light Co., Pittsburgh, Pa.

Lightning Arresters, J. A. Johnson, Niagara Falls Power Co., Niagara Falls, N. Y.

Oil Circuit Breakers, J. M. Oliver, Alabama Power Co., Birmingham, Ala.

Protective Relays, H. P. Sleeper, Public Service Electric & Gas Co., Newark, N. J.

Reports of the individual subcommittees follow.

SUBCOMMITTEE ON AUTOMATIC STATIONS

Four papers have been arranged for by this subcommittee during the year:

Carrier-Current Selector Supervisory Equipment, by C. E. Stewart and C. F. Whitney.

Testing, Inspection, and Maintenance of Automatic Stations, by Chester Lichtenberg.

Automatic Substations, by D. W. Ellyson.

Development of Automatic Switching Equipments in United States and Europe, by A. H. de Goede.

In the matter of standardization, this subcommittee has prepared and circulated Report No. 26 on Standards for Automatic Stations. Criticism of this report is invited and it is hoped that many engineers to whom the report will be sent will respond with such suggestions as will permit adjustment of the substance of the report into a final set of standards.

In the matter of research work it has been suggested that the application of automatic control has a direct influence on the degree of service actually rendered, and that a study should be made of the subject, with a view to determining to what extent, if any, automatic control of station equipment has improved service.

SUBCOMMITTEE ON CURRENT LIMITING REACTORS

Since there has been no marked progress in design, and since no very unusual installations have been called to the attention of the subcommittee, the subject of development will be passed with just a word. The fact that practically all new reactors now being installed are of the insulated conductor type shows that it is to be preferred over the older type with bare conductors. The superiority of insulated conductors has also been rather definitely proved by exhaustive tests. There seems to be some hesitancy on the part of operating engineers to go to the use of reactors of a higher voltage than 33,000 volts, although there are some successful installations of higher voltages, and manufacturers express their confidence in being able to produce satisfactory high-voltage equipment.

In last year's report, certain recommendations were made covering subjects to be studied this year. These were for the most part problems which have been considered by previous subcommittees and to which no solution has yet been discovered. Chief among these is the question of the value of resistance shunted reactors. It was hoped that the extended use of the klydonograph and Dufour oscillograph would shed some light on this much mooted question, but nothing conclusive has been obtained.

Another suggestion was that some work be done in an effort to reduce the variety of reactors with respect to voltage, current, and reactance values. The idea was that they might be standardized, with respect to their various characteristics, in steps in much the same way as has been done on oil circuit breakers. This was discussed at one of the Main Committee meetings and a decision handed down that this problem properly belongs to the N. E. L. A. rather than to the A. I. E. E.

Further study of possible standardization for reactors is now under way.

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*Committee on Protective Devices:

F. L. Hunt, Chairman
H. R. Summerhayes, Vice-Chairman
E. A. Hester, Secretary
Raymond Bailey,
W. S. Edsall,
H. Halperin,
F. C. Hanker,
J. Allen Johnson,
M. G. Lloyd,
H. C. Louis,
W. B. Kirke,
K. B. McEachron,
W. H. Millan,
L. J. Moore,
J. M. Oliver,
E. J. Rutan,
H. P. Sleeper,
E. C. Stone,
A. H. Sweetnam,
A. Royal Wood.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

SUBCOMMITTEE ON LIGHTNING ARRESTERS

March 24, 1927

Papers and Research. Last year's report described in considerable detail two new tools which have become available for the study of lightning and other transient electric phenomena; namely, the klydonograph and the Dufour cathode ray oscillograph. That report also suggested three items of further work to be done; namely:

1. Standardization of technique for using lightning generators for testing lightning arresters,

2. Determination of voltage time characteristics of lightning arresters including rate of discharge, and the dielectric spark lag,

3. Statistical data of operating experience on high-voltage lines.

During the past year, substantial progress has been made along these lines by the use of the two devices above mentioned. This progress is recorded in the following papers presented before the Institute during the past year:

1. *Lightning and Other Experiences with 132-Kv. Steel Tower Transmission Lines,* by M. L. Sindeband and P. Sporn, JOURNAL, Vol. XLV, No. 7, p. 641.

2. *Measurement of Transients by the Lichtenberg Figures,* by K. B. McEachron, JOURNAL, Vol. XLV, No. 10, p. 934.

3. *Lightning—A Study of Lightning Rods and Cages with Special Reference to the Protection of Oil Tanks,* by F. W. Peek, jr., JOURNAL, Vol. XLV, No. 12, p. 1246.

4. *Measurement of Surge Voltages on Transmission Lines Due to Lightning,* by Everett S. Lee and C. M. Foust, JOURNAL, Vol. XLVI, No. 2, p. 149.

5. *Transmission Line Voltage Surges,* by J. H. Cox, JOURNAL, Vol. XLVI, No. 3, p. 263.

6. *Klydonograph Surge Investigation,* by J. H. Cox, P. H. McAuley, and L. Gale Huggins, JOURNAL, Vol. XLVI, No. 5, p. 459.

Since the progress in research during the year in general is summed up in the conclusions of these papers, it seems worthwhile to restate these conclusions here in so far as they throw light on the nature and magnitude of lightning surges and the characteristics of the devices being used to investigate them.

Mr. McEachron's paper concludes as follows:

"As a result of this investigation, it can be definitely stated that the size and appearance of both positive and negative Lichtenberg figures are dependent on the wave front as well as on the crest voltage.

Throughout the range of wave fronts probably found in service, the size of the positive figure is not much changed by a change in wave front only, except at voltages close to the upper limit of potential where a decrease in the size of figure is indicated with very abrupt fronts.

The positive figures may be divided into three type forms which are partly determined by wave front and partly by the value of the crest voltage. It is possible to gain some idea of the steepness of the front from the appearance of the positive figure.

The size and appearance of the negative figures are considerably affected by changes in wave front, the steepest waves always giving the largest figures. The percentage change with a constant crest voltage applied is greatest for the lower voltages. The change seems to be great enough so that it cannot be neglected. The negative figures change in appearance with increasing steepness of wave front, but the changes are so indefinite that it is only possible to state that a particular negative figure probably represents a fast wave or a slow wave."

The paper by Lee and Foust contains field klydonograph records showing surge voltages on a transmission line as high as 1500 to 2100 kv. In one case this was a highly damped oscillatory surge predominantly negative; in another case it was a unidirectional surge with positive polarity.

Practically all figures obtained on transmission lines were of the type II class (paper by McEachron) and may be placed, therefore, within the wide range of wave fronts which vary roughly from that of a slow 60-cycle wave to a surge which comes to its maximum value in a fraction of a microsecond.

The maximum surge voltages obtained compare favorably with the laboratory results of insulator flashover tests; the value 1800 kv. for the lightning sparkover of a 14-unit insulator string seems to be close to the upper limit of voltages actually measured on the line by means of recorders. The authors summarize this paper as follows:

"It has been shown that surge voltage recorders using the positive photographic Lichtenberg figures have given essentially the same calibration data under a variety of conditions; also that the accuracy of such an instrument is in the order of 25 per cent, with a somewhat better value possible for those measurements wherein several similar observations may be obtained.

"An extension of instrument design has been described wherein two recorders are used together, which allows the use of the positive figure as a voltage measure of all surge voltages, thus insuring greater certainty of result. A more comprehensive analysis of the figure characteristics is also possible, since both positive and negative figures are available.

"A means of connecting the surge voltage recorder to a transmission line of higher than instrument voltage has been described which has been proved in service to be simple, reliable, and easy to calibrate. Calibration data are presented to show that with such connection, reasonable accuracy may be obtained in recording voltages up to values in the order of 2000 kv. A specimen record of such voltages obtained in the field is shown.

"The records which can be obtained from surge voltage recorder instruments connected as desired along a transmission line will allow the facts regarding surge voltages on transmission lines to be determined with reasonable exactness."

Mr. Cox's paper concludes as follows with respect to lightning:

"1. Positive lightning strokes are frequent but weak. They are slow, of the order of 0.01 sec., and hence do not induce surges on transmission lines.

"2. Positive strokes, even though slow, may produce surges of importance on isolated low-voltage lines, such as communication lines.

"3. Negative lightning strokes are less frequent but more violent. They discharge in about *three microseconds* and hence produce high-voltage surges on transmission lines.

"4. The field gradient is often as high as 60 kv. per ft. and may reach 100 kv. per ft. Thus a surge of over 2000 kv. might be induced upon a line of ordinary height with sufficiently high insulation. *Eighteen hundred kv. has been recorded by the klydonograph.*

"5. The time lag of an insulator flashover is less than the time of discharge of a negative stroke and thus the impulse flashover voltage of the insulators limits the possible potential.

"6. The stroke of lightning itself is unidirectional. If an oscillatory surge due to lightning is recorded, it is a line oscillation resulting from a flashover."

The paper by Cox, McAuley, and Huggins contains the following conclusions with respect to lightning:

"1. Surge voltage due to lightning is unidirectional. The clouds which produce surges are of negative polarity, resulting in positive induced voltages and negative direct-stroke voltages.

"2. The maximum values, reached by lightning surges on transmission lines, are limited by the flashover of the insulators. It is believed that the flashover voltage of 220-kv. transmission line insulation, at the steepnesses of wave front of lightning surges, is comparable to the maximum potentials ordinarily induced by lightning.

"3. The flashover voltage of the average insulation of lines up to 140 kv. is about seven times normal for lightning impulses.

"4. Seldom more, and often less than two surges, comparable in magnitude to the insulator flashover voltage, appear at a given point of a line during a storm.

"5. The frequency of occurrence of the higher surges does not seem to be greater for low-voltage than for high-voltage lines.

"6. High-voltage surges are damped below the corona voltage in traversing a few miles of line. At low magnitudes they may travel long distances.

"7. The quantitative measurements with the klydonograph agree with the theories regarding induced voltages and the protection against these afforded by the ground wire.

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"13. Except for lightning surges and arcing grounds, no high-voltage disturbances of particular importance to the operating engineer appear on transmission lines.

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"18. In the investigations of the performance of lightning arresters in actual service, it was found that arresters in general give satisfactory operation, that is, they relieve all surge voltages above the standard test voltages for equipment insulation. Discharge currents up to 2500 amperes occur in practise. From these tests it is concluded that the field performances of arresters confirm predictions based on laboratory tests.

"19. Lightning arresters do not protect a line against flashovers at distant points."

It is desired to call particular attention to Fig. 12 of the paper by Lee and Foust which shows results of klydonograph calibrations reported by Messrs. Cox and Legg, Mr. McEachron, and Messrs. Lee and Foust. "These results show remarkable agreement for the work of the different observers in different laboratories, with different instruments and circuits, and give added weight and certainty to the calibrations of the Lichtenberg figures in regard to magnitude of voltage."

As for the determination of wave shape from Lichtenberg figures, the following excerpt from the paper by Messrs. Lee and Foust is significant:

"At the present time, the determination of wave shape from the Lichtenberg figure characteristics is not as definite or as certain as the determination of the magnitude from the figure size, and herein there is room for added study. Further study along these lines tending toward greater exactness in the interpretation of figure characteristics is desirable."

From the foregoing results of the researches of the several investigators, the following significant summary of present knowledge may be made:

1. Lightning strokes are unidirectional.
2. Positively charged clouds discharge in about one one-hundredth of a sec.; negatively charged, in about three microsec.
3. Surge voltages due to lightning are usually unidirectional. The clouds which produce surges are of negative polarity, resulting in positive induced voltages and negative direct-stroke voltages. Oscillatory surges are the result of flashovers and are highly damped.
4. The wave front steepness, or time required for a lightning surge to reach its crest, lies within the broad range between about one one-hundredth of a second and one microsec. The steepest waves probably reach their crest in a time of the same order as that required for the discharge of a negatively charged cloud, namely, about three microsec.
5. The maximum potential of lightning surges agrees with theory and laboratory tests and is limited by the flashover value of the line insulators.
6. Lightning arrester performance in service confirms laboratory tests.

With the foregoing facts reasonably well established, it would appear that the establishment of standards for lightning arresters and lightning arrester test apparatus and procedure may now be undertaken upon a rational scientific basis.

Standards. The progress which has been made during the past year is bringing nearer the time when rational standards for lightning arresters can be formulated. Since the lightning arrester is a device for dealing with transient voltages, the standardization of arrester characteristics and testing devices and procedure upon a rational basis demands the adoption of a *standard transient potential or lightning surge.* Such a standard transient or lightning surge should resemble as nearly as laboratory limitations will permit, the most destructive surges which natural lightning produces on transmission circuits. Sufficient evidence is now available to indicate that such natural surges reach their crest values in a time on the order of three to four microsec. It is believed that the demand of the art at the present time for the adoption of a standard transient for lightning arrester testing is sufficient to justify the adoption at this time of a tentative standard. Therefore, in the formulation of standards for lightning arresters, in which work the subcommittee is now actively engaged, it is proposed to establish a standard lightning surge for laboratory use which it is proposed to define as follows:

"The standard lightning surge shall be one which rises to its crest value in four microsec. and which does not decrease more than 2 per cent in the following 10 microsec."

For the purposes of lightning arrester standardization, it is proposed to fix the maximum value of the standard lightning surge at 100 kv. in order to limit the size of the necessary laboratory equipment.

It is felt that the accelerated progress which will result from the agreement upon a standard transient is sufficient justification for the adoption of such a standard at the present time, even though further experience may indicate that the exact form of the standard adopted may have to be changed. The Institute has a standard for cyclic voltages, namely, the sine wave. There would seem to be no reason why it should not likewise have a standard for transient voltages. Possibly more than one such standard may be required for different purposes.

Since the entire matter of standardization of test procedure for lightning arresters depends upon the adoption of a standard transient, the matter is mentioned here in order that the committee may have the benefit of open discussion of the matter in the work of formulating standards on this most difficult subject.

It is also desired to point out here that the cathode ray oscillograph is rapidly supplanting the use of sphere-gaps in determining the voltage and current characteristics of lightning arresters, and that consequently such terms as "equivalent sphere-gap," "discharge rate," and "dielectric spark lag" are rapidly being left behind, and are being replaced by actual voltage and current curves obtained with the cathode ray oscillograph. Such cathode ray oscillograms can be interpreted in terms of actual volts, amperes and times, even down to fractions of a microsecond, and consequently give far more comprehensive information regarding the performance of lightning arresters than ever was or ever could be possible from the use of sphere-gaps.

It is hoped that the standards now under preparation, in connection with which a considerable amount of research is also under way, may be sufficiently advanced for presentation sometime within the next few months.

SUBCOMMITTEE ON OIL CIRCUIT BREAKERS

There was presented at the Winter Convention, a paper entitled *Tests on High- and Low-Voltage Oil Circuit Breakers Conducted by the American Gas & Electric Company*, prepared by Philip Sporn and Harry P. St. Clair. This paper may properly be classed as research work, since it gives valuable information on the subject of rupturing capacity of oil circuit breakers and methods which may be used in determining what these capacities are. This is the most important problem in the matter of oil circuit breaker design, and needs much additional research work of this class. Several other companies are arranging for similar oil circuit tests, and most of these tests are being conducted according to the recommendation of uniform test procedure, which will insure comparative results and much valuable data.

Arrangements have also been made for and work is nor progressing on the preparation of a joint paper, *Rating and Selection of Oil Circuit Breakers*, which will bring up to date information presented some years ago in a paper of the same title by Messrs. Burnham, Hewlett, and Mahoney.

In the matter of standards, certain changes have been recommended, and are now under consideration by the Standards Committee, in Standards No. 19 and No. 22. Further work on standardization is necessary, in the opinion of the subcommittee, in connection with the temperature rating on switch and circuit breaker contacts and other parts. This work is being carried on as rapidly as possible with other interests that are involved.

We believe that further work in standardization can be accomplished by the study of factors which determine the interrupting duty on oil circuit breakers. This is recommended for future study.

SUBCOMMITTEE ON PROTECTIVE RELAYS

During the past year there have been presented under the auspices of this subcommittee, five papers, including:

Automatic Network Relays, by W. K. Bullard,

A-C. Network Relay Characteristics, by D. K. Blake,

Evolution of the Automatic Relay Unit, by J. S. Parsons,

Design and Application of Automatic A-C. Network Units, by G. G. Grissinger,

Ground Relay Protection of Transmission Systems by B. M. Jones and G. B. Dodds.

In studying the question of standardization, there has been prepared a report on current and potential transformer characteristics. The result of this study is presented herewith, and it is recommended that further consideration be given the subject, with a view to standardizing the limitations of use of current transformers of various characteristics.

Report of the Subcommittee on Current and Potential Transformer Characteristics

By H. M. Rankin, Chairman

1. The purpose of this subcommittee investigation was to determine the effect of very high currents on the characteristics of current and potential transformers, and to specify the nature and extent of information which is necessary to their application to protective relaying. It was the opinion of the members of the subcommittee that the characteristics of potential transformers were not sufficiently affected by high current conditions to warrant investigation from a relaying standpoint. This report, therefore, deals exclusively with current transformers, including both the "instrument type" with multiple primary turns and the "bushing type" with single turn primary.

2. Characteristic ratio curves for current transformers should have a lower limit of one ampere secondary current and an upper limit determined by any one of the following three conditions:

10,000-amperes primary current,
20 times normal rated current,
2 times nominal ratio.

3. Characteristic ratio curves should be furnished for both "instrument" and "bushing" type current transformers for inductive burdens, power factor 0.5, as follows:

15 Volt-amperes
25 " "
50 " "
100 " "
200 " "

All values of volt-amperes given are based on five amperes, 60 cycles. The various loads are also to be specified in ohms resistance and henries inductance.

4. Until further experience may demonstrate that more narrow limits may be adhered to, the manufacturers should furnish, for each type and ratio of current transformer, a characteristic curve which shall be correct within the following limits:

a. ± 2½ per cent deviation from standard curve up to 1.1 times nominal ratio.

b. ± 10 per cent deviation from standard curve at 2 times nominal ratio.

c. The deviations at points between 1.1 and 2 times nominal ratio shall be interpolated on a straight line basis.

The manufacturers will, in future, keep a cloe checks on current transformer tests to determine whether the above limits are reasonble or can be decreased. If greater accuracy than the above is required, pending the result of further investigation on the part of the manufacturers, it should be the subject of special request.

5. Change in phase angle under high-current conditions. Some change in phase angle undoubtedly does occur, especially with "bushing" type current transformers in connection with large non-inductive secondary burden. It is thought that under conditions normally met in operation with secondary burdens approximating 0.5 power factor, this change in phase angle will have no serious effect on relaying. Information is lacking, however, on this point and it is recommended that more complete tests be made.

6. Change in wave form under high-current conditions. The following oscillograms show plainly the wave distortion at high currents:

7. Comparison of relay test methods. Fig. 13, shows a comparison of the primary-secondary method to the shunt method of testing relays with "bushing" type current transformers. The curve marked "1-Turn Primary" represents, of course, the actual operating condition of the "bushing" type current transformer. For the curve marked "Shunt Method," the primary ampere-turns are calculated by multiply-

Fig. 1

Fig. 2

Fig. 3

Fig. 4

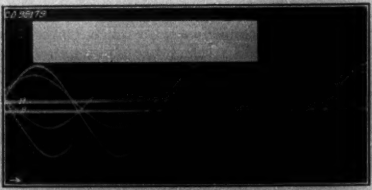

FIG. 5

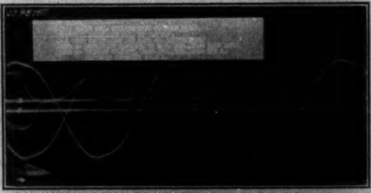

FIG. 6

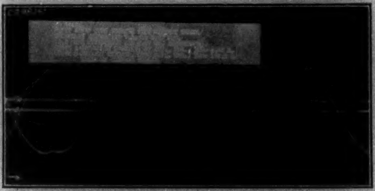

FIG. 7

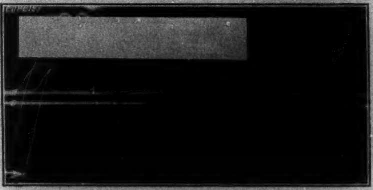

FIG. 8

FIG. 9

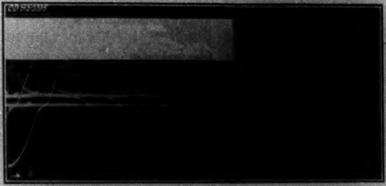

FIG. 10

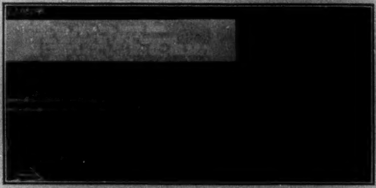

FIG. 11

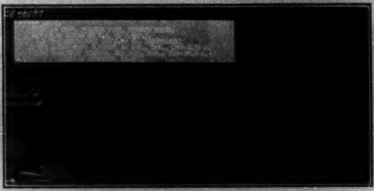

FIG. 12

Oscillogram number	Current transformer	Secondary burden		R. M. S. amperes final		Ratio
		Ohms resist.	M. H. induct.	Primary	Secondary	
C D-90371	A W-12 60/5 A	0.5		6,800	220	30.6:1
C D-90372	A W-12 60/5 A	1.4		6,800	145	49.0:1
C D-90296	B K-48 60/5 A	0.5		6,150	310	19.8:1
C D-90297	B K-48 60/5 A	1.4		6,150	175	35.1:1
C D-98179	57-turn bushing type	0.5	3.09	4,050	65	62:3.1
C D-98180	" " " "	2.5	3.09	4,442	66	67.3:1
C D-98181	" " " "	4.5	3.09	2,760	37	74.6:1
C D-98183	" " " "	6.5	3.09	4,260	34	125:1
C D-98189	" " " "	0.5	3.09	200	3.25	61.5:1
C D-98195	13-turn bushing type	4.5	3.09	11,000	16.7	660:1
C D-98196	" " " "	2.5	3.09	11,120	29.7	375:1
C D-98197	" " " "	0.5	3.09	556	40	13.9:1

The first four apply to "instrument" transformers and the remainder to "bushing" type transformers. It will be noted that though the distortion occurring under conditions of primary currents and secondary burdens within the range of ordinary operation may not be great enough to seriously affect relay performance, the imposition of excessive secondary burdens may have a decidedly bad effect.

ing the input current by the number of secondary turns on the "bushing" transformer. The "4-Turn Primary" curve shows the discrepancy which may be involved when testing with one-fourth of the primary current through four turns wound on the core. The "bushing" type current transformer chosen in this test was one having a very low ratio and with the secondary turns bunched in a small space on the core, in order to exaggerate the errors involved. With a higher ratio transformer in which the secondary turns would be more evenly distributed around the core, the discrepancies would probably be much reduced.

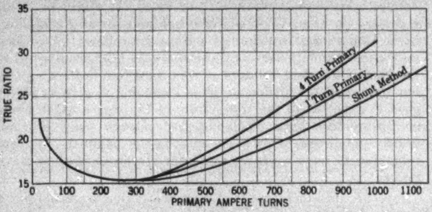

Fig. 13

8. Error due to eccentric location of primary conductor. Fig. 14, which is self-explanatory, shows the effect of eccentric location of the primary lead of a "bushing" type current transformer, combined with the effect of a bunched secondary winding. From a relaying standpoint, the discrepancy is so small as to be negligible.

The attention of the subcommittee for the past year has also been given to the matter of relay test specifications and standards. It has been found difficult to unify the varying practises of the many operating companies, as well as the test methods of the various manufacturers. It is not considered advisable at this time to undertake to offer a final and complete form, but the following data are given as the basis of tentative recommendations by this subcommittee.

1. Nameplate Data.
    - a. Descriptive name of relay.
    - c. Nominal operating current or voltage, or both.
    - c. Frequency.
    - d. Calibration curve.
    - e. Time setting chart.
    - f. Volt-ampere consumption and power factor or resistance and reactance of various coils.
    - g. Manufacturer's type or model designation.
    - h. Manufacturer's name or mark.
    - i. Interrupting capacity of tripping contacts.
    - j. Polarity of directional relays.
2. Allowable Temperature Rise.
    - a. Coils.
    - b. Contacts.
3. Insulation Resistance or Dielectric Strength Test.
    - a. Insulation resistance test made with a megger of either 500- or 1000-volt rating.
    - b. Dielectric strength test voltage, frequency, and duration of test.
4. Permissible Minimum Contact Separation.
5. Allowable Discrepancy from Nominal Value Given on Taps. (Current or voltage or both.)
6. Zero Torque Test on Zero Power Factor, Current Alone, Voltage Alone, Etc.
7. Chattering Test at High Current.
8. Vibration Tests.

It is hoped that the interested members will comment to the subcommittee on the above suggestion and that by another year, the report may be in the form of a recommended standard. The work of this committee should continue, therefore, for another 12-month period.

It is further recommended that the attention of this subcommittee be directed toward the establishment of other relay standards. This is a subject which deserves considerable attention as there are few phases of the art in general which are really standardized, and the need is great. Other suggested subjects are: Standardization of characteristic curves, standardization of descriptive nomenclature, standardization of relay symbols for single-line diagrams, standardization of relay symbols for wiring diagrams, and standardization of relay operation nomenclature.

It is recommended that a subcommittee be appointed on "Relay Handbook Revisions and Amendments." The book has now been published about two years, and it is believed that sufficient advances and improvements in the art have occurred to warrant their inclusion in the "Relay Handbook." It is the belief of this subcommittee that such action will be justified every two or three years. Otherwise, the value of the book will disappear in a few years' time by reason of obsolescence.

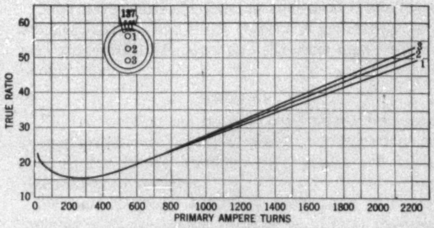

Fig. 14

In the matter of future papers, it is recommended that papers be prepared on the following subjects:

Operating Experience with Impedance Relaying

Operating Experience with Parallel Line Relay, Protection.

F. L. Hunt, *Chairman.*

Discussion

Alfred Herz: I want to mention a few things about the klydonograph. This device makes use of photographic emulsion coated on a base of celluloid or glass. The essential thing is the sensitivity of this emulsion.

A photographic emulsion after development will appear as a collection of grains of practically metallic silver. The size of these grains seems to have a direct bearing on the sensitivity of the emulsion. In general as the sensitivity is increased, the grains also increase in size. These grains appear under the miscroscope as separate islands, or clusters of islands with clear spaces between. Films usually supplied for cameras are coated with an emulsion of a fair degree of sensitivity. I believe such films are generally used in the klydonograph. It is possible to obtain emulsions which have practically no grain, such emulsions being usually made with albumin and not gelatine. They are abnormally slow for ordinary photographic work; but they make sensitive surfaces of such a fine grain that they can be used for miscroscopic photographs.

The usual photographic emulsions, made with gelatine, are quite hygroscopic, and I really believe that some caution and research should be carried on as to the effect of atmospheric conditions upon the results obtained with the klydonograph. I feel quite confident that some of the results are influenced by changes in the atmosphere. Therefore, if you want results that are really comparable, it is important first of all to make use of the same brand and speed of film or plates for all the tests contemplated. Furthermore, the experiments or tests should be carried on under some specific and similar conditions of the atmosphere.

I believe the Lichtenberg figures are produced by minute electric discharges between the grains mentioned, which will account for the ray-like images we obtain under certain conditions as well as for some of the figures resembling tree-like growth.

J. Allen Johnson: I wish to call attention to the matter of lightning-arrester standardization. It appears that by the use of the klydonograph (which you may call an approximately accurate instrument) and the Dufour oscillograph (which is probably a very accurate instrument) the fog which has for so many years surrounded the lightning-arrester question is gradually being dissipated. The difficulties in the way of the standardization of lightning arresters and lightning-arrester tests appear to be passing away. However, the lightning arrester is a device for dealing with transient voltages, for its protective value bears a relation to the transient voltage, not to the cyclic voltage which we generate on our lines. It therefore seems necessary, in order to standardize lightning-arrester characteristics, that we first standardize a transient voltage. That idea may sound revolutionary as we have always been accustomed to thinking of transient voltages as very uncertain in their nature, but the use of the klydonograph is beginning to give us pretty good evidence as to the true nature of the surges which occur on transmission lines. We find, for instance, that they are usually unidirectional, or, if not, highly damped. We are beginning to have evidence as to the steepness of their wave fronts.

It seems, therefore, that the time is nearly ripe to standardize a lightning surge for comparative tests of lightning arresters. This report suggests a definition of a standard surge.

I wish to announce that the lightning arrester subcommittee would be very glad to have any ideas on this subject so that the committee may have the benefit of them in working out this standardization. The committee has prepared a tentative form of standards for lightning arresters based upon—that is, starting from the basis of the report of the working committee in 1926. I have about twenty-five copies of this tentative form and I should be glad to give copies to any one sufficiently interested.

Iron and Steel Industry

Annual Report of Committee on Applications to Iron and Steel Production*

To the Board of Directors:

The importance of electricity to the production of iron and steel has reached such magnitude that this Committee believes that all engineers should be informed as to the situation so as to be prepared to apply it successfully in all its fields of application. To this end, the Committee would outline the extent to which electricity is being applied in this industry.

LIGHTING

Perhaps the first application was that to lighting. The arcs were replaced by incandescent lamps and now illumination is receiving much attention as to the proper lighting of various jobs and work spaces. Proper illumination is now credited with increase in both production and safety. Steel mills, however, are not yet lighted as they should be, and increased emphasis should be placed on this phase of their work by electrical engineers.

HEATING

The use of electricity for heating has increased with the installation during the past year of over 25 melting furnaces. These range from ¼ ton to 25 tons, and are of the arc type. The time of melt has been reduced considerably. The increase of production has also been greatly influenced by furnace design. The removable roof type of furnace appears to be the trend in design.

Resistor type furnaces are being used for annealing and for heat treating of alloy steels. Laboratory furnaces have been used in the steel mills for many years.

Several years ago electrical heating was applied to rolls in sheet and tin mills to increase production during the first turn after a shut down or a roll change. This use is apparently well grounded since a report by the A. I. & S. E. E. states that 168 roll heaters are in use in 19 plants. Electrical heating devices have been used in crane cabs and offices for many years.

MAIN ROLL DRIVES

During 1926 nearly 150 main roll drives were purchased, all of which were electrically operated. Of these only six were a-c., the rest were d-c., ranging from 230 to 900 volts. Of a special interest is the 8000-hp., 700-volt reversing, d-c. motor drive for the 54-in. blooming mill in a Pittsburgh mill. Large motors for such drives have become common. Individual motor drives on tandem strip mills are also of interest because of their increased use. While the use of d-c. motors has predominated, a few induction motors have been installed with speed control; also, a few large synchronous motors have been applied with apparent success on their particular mills. Undoubtedly, the use of synchronous motors for mill drive will increase.

To reduction of labor and the actual improvement in the steel produced is due the importance of the electrification of steel mills. Electric drives in steel mills permit the mill designer to produce a mill that will do things heretofore impossible.

AUXILIARY MILL DRIVES

In connection with the installation of many new main mill drives, auxiliary drives have come in for much attention, with the idea of giving closer control of these auxiliaries with fewer operators. Automatic control of screw-downs, tables, transfer cars, and drag-overs, together with furnace doors and pit covers, has increased with resultant efficiency, the ratio of steel production to men employed.

The A. I. & S. E. E. has completed its specifications for auxiliary and mill motors and one motor manufacturer has announced motors built according to these new specifications.

Alternating current is gaining some ground applied to auxiliary drives, but direct current is apparently very well grounded in the steel mill electrical man's scheme of operations.

WELDING

Electric welding can be mentioned briefly because it is used extensively for repair work, and indications are that in the future building construction will be influenced by this process.

Perhaps the most important application of electric welding is its use for the building up of large machines by welding plates. These welded structures are to replace castings. The ease with which complicated as well as simple structures can be made, together with their lightness and strength, is making this innovation one of importance and one which gives promise of rapidly increasing use.

SAFETY

Because of the wide-spread use of electricity in mills, the various electrical departments appear to be leaders in safety programs and the elimination of *all* hazards as well as those electrical. This may be because of the peculiar nature of electrical hazards, and the extensive steps taken by power companies and electricity users to eliminate these hazards and to take care of unfortunate victims. The prominent place of steel mill electrical men in the promotion of safety should be recognized.

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Presented at the Summer Convention of the A. I. E. E. at Detroit, Mich., June 20-24, 1927.

MEASUREMENTS AND INSTRUMENTS

The metering of electric power has been practised from the first, and its convenience has caused its more extensive use in the mills in order to determine not only the total power costs, but also the detailed operating costs down to individual machines and drives. Even auxiliary drive controllers are being specified to include permanent shunts for convenient metering. This permits proper distribution of costs for different processes.

Electricity also plays an important part in other than power measurements, such as tachometers and pyrometers for speed and temperature determinations. The metering of gases is done also readily with great convenience by electrical means.

By the increased use of electrical power measurements attention has been called to economies that are possible. These economies are watched by all departments and stimulate effort by department heads to make savings heretofore uncontemplated. Furthermore, these measurements and economies stimulate improvement of design to affect even greater economies.

CONCLUSION

In conclusion, it may be noted that there is considerable activity in the rebuilding of steel mills so as to produce more steel at a lower cost. The old steam drives are replaced by electric drives, most of which are for direct current.

The transmission of electric power at high voltages, together with the ease with which it can be converted for convenient application, has caused the use of electricity to drive out steam. Its use is now amply safeguarded and engineers and operators are more skilled in its application and use. The ease with which a few mill operators can control a large number of motors through remote-control devices further demonstrates the superiority of the electric drive This rapidly increasing use of electric power demands the closest attention of electrical engineers.

The improvement in engineering that is apparent today gives a certainty of predetermination of results which is not only gratifying to the engineer, but of greatest value to the executive.

Economic Aspects of Electricity in Mining Work

Annual Report of Committee on Applications to Mining*

To the Board of Directors:

The applications of electricity in the mining industry, especially in coal mining, have shown a marked increase during the past year, due principally to the fact that its use is an important element in the solution of the problem of high mining costs. These adverse economic conditions have forcibly brought to the attention of mine managers the necessity of replacing expensive labor by electrically operated mechanical devices.

Coal loading machinery operated by electricity has demonstrated its entire practicability and has shown a saving of 25 to 35 cents per ton in mining costs. Much study still remains to be made concerning the question of coordinating machine loading and mining methods. This new and concentrated use of power in a certain section of a mine means a complete reconstruction of the power system, in order to obtain a good voltage regulation for not only the new equipment but for the old as well.

An armoured cable capable of delivering 3000 kw. of power at 4000 volts and 80 per cent power factor has been recently installed in a metal mine shaft 5000 ft. deep. The lowering of a cable of this size into a vertical shaft and its proper clamping to the supporting timbers were problems solved in delivering a big block of power to the bottom of a deep shaft.

In gaseous coal mines where ventilating fans are electrically operated by power obtained from extensive high-tension systems, it is necessary to provide an emergency source of power to operate the fan in case of the failure of the normal power supply. A successful installation of this type was placed in service during the past year, and it consists of a gasoline-engine-driven generator set which will supply power to the emergency motor connected to the double extending fan shaft. After a failure of the normal source of power, 30 sec. are required to automatically start the gas engine set and restore normal fan service.

The use of storage battery power trucks with a capacity of about 150 kw-hr. for operating coal cutting machinery is increasing. Installations of this type result in an increase in the number of places cut and an improved load factor of the power system, providing good judgment is exercised in selecting the time at which the battery is charged. In gaseous mines, the use of battery power not only for cutting, but for pumping and hauling as well, introduces an element of safety heretofore unobtainable.

Automatic starting equipment has been successfully applied to pumping plants, converting apparatus and fans, and lately, air compressors have been operated without attendants. Successful applications of automatic starting equipment have been made to two- and three-speed induction motors, notwithstanding the complicated electrical layout which an installation of this type involves.

Improvements in the haulage systems are being effected by the use of gathering locomotives designed to operate at a slower speed than during the past. The converting equipment required for haulage locomotives is being placed nearer to the load centers, thereby improving the voltage regulation. In mines where a very large tonnage must be transported through a single outlet, belt conveyors have been installed which have demonstrated their value under the above mentioned special conditions.

Many installations of electrical shovels have been made in the metal mining industry, and more recently, the coal mining industry is using this type of shovel. The Ward Leonard control on the large shovels and a motor-generator set with d-c. motors on the smaller shovels show the trend in the electrical apparatus used on shovels.

The work of the United States Bureau of Mines in listing the permissible electrical equipment for use in gaseous mines is one which deserves the commendation and support of all those interested in the safe operation of coal mines. The list is growing rapidly and at the present time it is so complete that a mine manager may select permissible equipment for practically every application in mining. The equipment shows good design and a low maintenance cost. The fact that the Bureau of Mines has approved as much equipment during the past two years as was approved during the previous 10 years is proof sufficient that its work is being valued by the mining industry.

That electricity is being applied in the development of mine safety appliances is shown in the recently developed methane detecting device. Briefly, it consists of a platinum filament mounted on the end of a stick, a battery, and an indicator carried on the inspector's belt. Current from the battery is passed through the filament and it is heated to a constant temperature. The presence of methane or other hydrocarbon gas around the filament increases its temperature and this is indicated on a dial graduated to show the amount of methane in the air. Indicators may be permanently located in an airway and can be wired to a device in the mine office which will ring a bell when the methane rises above the point which is considered safe. Explosions are prevented by protecting the coil by gauze bonnets such as are used in the Davy safety lamp. The results obtained from this apparatus are very satisfactory.

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* Committee on Applications to Mining Work:

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L. C. Ilsley,
G. M. Kennedy,
R. L. Kingsland,
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Carl Lee,
John A. Malady,
Charles H. Matthews,
F. C. Nicholson,
H. F. Pigg,
L. L. Quigley,
Herbert S. Sands,
W. F. Schwedes,
E. D. Stewart,
F. L. Stone,
W. A. Thomas,
E. B. Wagner,
J. F. Wiggert,
C. D. Woodward.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

Further development of the miner's cap lamp has doubled the light available. Judging from the increased efficiency obtained from factory workers when the illumination is increased, there is no doubt that the increase in light furnished to the workers in the "darkest factory" in the world will result in an increase in safety, and efficiency as well.

When one considers that 80 per cent of the American mines are electrified, it seems reasonable to conclude that this accomplishment has been helpful in reducing operating costs.

Such a general adoption of electrical power by mines where the equipment is subjected to damp and gaseous mine air shows that the manufacturers have done their part in designing equipment to meet the conditions. The commercial power companies have also helped in that they are usually in a position to serve a mine with power even if it is located in an isolated section.

W. H. LESSER, *Chairman.*

Discussion

A. M. MacCutcheon: I should like to ask the committee if they can give us any more detail on the progress in getting apparatus approved with an unqualified approval of the Bureau of Mines. As I understand it, they give limited approval to certain types of apparatus. I have not yet learned that they give unqualified approval of types for use in gaseous mines. In talking to the people of the Bureau of Mines at Pittsburgh about two years ago, my conception was that they are approving apparatus because it is the best there is, but it is not thoroughly satisfactory. They said that if they used tests that they would be satisfied with, there was nothing on the market that would stand them. They said, "We are not giving unqualified approval. When we get the right kind of apparatus, we will give unqualified approval." I was wondering if apparatus now gets unqualified approval.

E. J. Gealy: Under the subject of permissible equipment, the industry is today getting much equipment which it has needed for a long time. Every piece of permissible equipment is given what the Bureau calls a "permissible approval plate." That is, it is approved only as long as it is kept in the condition in which the Bureau had it when it was inspected, so it is what you might call "limited."

Aside from that, there is being developed other equipment which is not strictly permissible, but is of a better type for mining service. That will probably result in what we might call "a semi-permissible type of apparatus."

W. H. Lesser: The only question to be answered was Mr. MacCutcheon's question, and Mr. Gealy answered that. I don't know now whether the Bureau of Mines will issue a plate showing unqualified approval.

Marine Work

Annual Report of Committee on Applications to Marine Work*

To the Board of Directors:

The committee in making its report this year does so with a feeling that its efforts to raise the standards of marine electrical installations and increase the utilization of this most efficient, flexible, and safe form of energy, appear more assured than at any time before.

The untiring efforts of the committee as a whole, and individually of the members, with the section of the industry they represent, have, by the dissemination of information, removed to a large extent the opposition and indifference to electric drives, which has been due to a lack of information, and experience in the past, on the part of the owners and operators.

An equally deciding factor, and one for which the committee hereby desires to acknowledge its appreciation, has been the recognition accorded by various bodies covering inspection, classification, and insurance; *viz.*, United States Government Departments, American Bureau of Shipping, National Board of Fire Underwriters, National Fire Protection Association, and similar associations, thus establishing complete harmony in the endeavor to increase the usefulness of electricity, surrounded by the necessary safeguards.

Electric drive is not only gaining headway in its application to deck machinery, engine room auxiliaries, and cargo pumps, but also as a means of propulsion for tugboats, ferryboats, and all classes of vessels where frequent starting and stopping is necessary, and where large torques are required at minimum speeds. This headway has been due to a careful study of the particular application by the designing and operating engineers and to the hearty cooperation of the electrical manufacturers to build special apparatus, where standards were not the most suitable and advantageous, and there is every evidence of a continuance of this spirit.

It will be recalled that in 1920, the Institute issued a volume, *Recommended Practise for Electrical Installations on Shipboard*, which was accepted and used by a great majority of the naval architects, marine engineers, ship owners and shipbuilders, and recognized by the Insurance and Classification Societies as having filled a long-felt want. The committee takes pleasure in announcing that after three years of close application to the study of revision, the Institute will at an early date issue the revised edition which represents the combined efforts of representatives from United States Navy Department, Classification and Insurance Societies, electrical manufacturers, and shipbuilders, for which sincere appreciation, by the Institute and the committee, is hereby acknowledged.

Arrangements are being made to give this edition wide circulation; first, that it may add to the simplification and standardization movement which is being so urgently advocated by the United States Government Department of Commerce, and secondly, that the Institute's standards may be universally adopted, which ultimately will result in better products of uniform manufacture at lower first costs and maintenance.

The economic feature of shipbuilding is at a low ebb and has not materially changed since the war; furthermore, there is little hope of any relief until some changes are made in our shipping laws.

Shipbuilders and owners of the United States cannot hope to compete with those of any other country, owing to material and labor prices. Our shipping laws provide protection only in the coastwise trade, and necessarily the advantages of electric drives which originated, and have been highly developed in this country, have opportunity for demonstration only in that class of vessels.

G. A. PIERCE, *Chairman.*

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*Committee on Applications to Marine Work:

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R. A. Beekman, Vice-Chairman
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Wm. Hetherington, Jr.,
H. L. Hibbard,
J. S. Jones,
A. Kennedy, Jr.,
Comdr. M. A. Libbey,
E. B. Merriam,
I. H. Osborne,
Wm. H. Reed,
H. M. Southgate,
W. E. Thau,
C. P. Turner,
A. E. Waller,
J. L. Wilson,
R. L. Witham.

Presented at the Summer Convention of the **A. I. E. E.**, *Detroit, Mich., June 20-24, 1927.*

Electrical Transportation

Annual Report of Committee on Transportation*

To the Board of Directors:

During the past year the application of electrical means to the various branches of transportation continued at an accelerated pace. The major division of the transportation industry, namely, steam railroads, added to its electrified lines and, although the mileage electrified is still a small percentage of the total, it is steadily increasing and is rapidly becoming an important factor in railroad operation. Into the field of city and suburban railways, improvements are constantly being introduced. On the water, electric drives are being adopted more and more, with either steam turbines or oil engines as prime movers. Bus transportation is adopting, extensively, gas-electric propulsion. The oil-electric locomotive and the gas-electric motor rail car are being introduced in certain phases of railroad operation.

Steam Railroad Electrification

The year 1926 has seen the completion of two major electrification projects, that of the suburban lines of the Illinois Central Railroad out of Chicago and the line of the Virginian Railway between Mullens, West Virginia, and Roanoke, Virginia. The Detroit & Ironton Railroad completed 17 mi. of electrification between Fordson and Flat Rock, Michigan.

The New York Central opened a new electrified section between High Bridge and Yonkers, New York.

Of the principal uncompleted projects, the Great Northern Railway is electrifying 80 continuous mi. of its line between Wenatchee and Skykomish, Washington. The Pennsylvania Railroad is extending electric suburban operation on its main line between Philadelphia and Wilmington, and also from Philadelphia to West Chester. The New York, Westchester, & Boston Railway is continuing its extension of electrified line between Larchmont and Port Chester, New York. The Long Island Railroad is installing freight electrification on its Bay Ridge division.

Illinois Central Railroad. The first step in the electrification of the Illinois Central out of Chicago was completed during July of last year when the suburban service was placed in electric operation over 28 mi. of the main line and 8.9 mi. on two branch lines. The ordinance under which this project was carried out calls for electrification of freight service within the city limits by 1935 and electrification of through-passenger service on both the Illinois Central and Michigan Central by 1940, provided a certain portion of the tenant roads then using the passenger station on East Roosevelt Boulevard are electrically operated at the time.

The 1500-volt, d-c. system with overhead contact wire was chosen since there is no immediate prospect for extension over main line divisions.

Power supply is secured at the railroad's right-of-way from substations owned and operated by outside power companies. The conversion from 60-cycle power to 1500-volt direct current is accomplished by means of synchronous converters and mercury arc rectifiers. One of the reasons for purchasing power rather than building a generating plant was the fact that the power companies can supply power from several plants over various routes and thus aid in securing continuity of service.

The distribution system of the railroad is so laid out that the wires over each track are separate electrically and can be sectionalized at substations and interlocking plants by automatic high-speed circuit breakers. Normally, the wires are tied together over all tracks. No feeders are required external to the catenary system. Trolley feeder switches in all substations and tie stations are operated from the railroad company's power supervisor's office by supervisory control. The power supervisor has electrical indication from each substation and tie station and, in case of trouble, he can cooperate with the train dispatcher who occupies a joint office.

The catenary system, which provides the entire current-carrying capacity, has an average conductivity over each track of about 790,000 cir. mils, copper equivalent. This figure takes into consideration average wear on the contact wire. The catenary system is completely non-ferrous, with a double contact wire. Chord construction is used on curves with the aid of shortened pole spacing. The rail bond, adopted as a final standard, is a U-type gas-weld bond consisting of two No. 1 A. w. g. flexible conductors.

The 260 multiple-unit cars for this service are built in two-car units. Two pantographs and four 250-hp. nominal-rating, self-ventilated, series railway motors are located on each motor car to which a trailer car is semi-permanently attached. Normally, only one pantograph on each motor car is in operation. Each pantograph exerts a pressure of about 20 lb. against the contact wires. Automatic couplers couple the cars together mechanically, electrically, and pneumatically with full automatic operation between the two-car units.

Virginian Railway. Heavy electric freight operation over the entire electrified zone of the Virginian Railway started in September, 1926. The electrification is now complete from Mullens, West Virginia, to Roanoke, Virginia, a route of 133 mi.

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*Committee on Transportation:

J. V. B. Duer, Chairman
E. R. Hill,
W. K. Howe,
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H. A. Kidder,
John Murphy,
W. S. Murray,
W. B. Potter,
N. W. Storer,
W. M. Vandersluis,
Richard H. Wheeler.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

An 11,000-volt, 25-cycle, single-phase, a-c. system is installed. The problem of taking a heavy tonnage train down a long grade was a large factor in deciding upon the a-c. system in order to use split-phase locomotives for regenerative braking.

The power plant, owned and operated by the railway, contains four 12,500-kv-a., 25-cycle, single-phase turbo generators and five water tube boilers each rated at 1521 b. hp. Pulverized fuel is used and has been found to be peculiarly well adapted to the rapidly fluctuating power load.

Electric power is transmitted from the plant at 88,000 volts over twin transmission lines. There are seven transformer substations along the right-of-way. They contain three-coil transformers in which the low-voltage winding is divided into two parts. One part of the winding supplies 11,000 volts between the trolley and rail, while the other part supplies 22,000 volts between a feeder and the rail. This gives a potential between trolley and feeder of 33,000 volts. Transformer windings are so constructed that reconnection for 22,000 volts from trolley to ground can be made when traffic demands require this to be done. Balancer stations containing auto-transformers are located between the main transformer stations in order to connect the feeder circuit to the trolley and rail.

The catenary system is completely non-ferrous with a copper and bronze composite messenger and a bronze contact wire. Inclined catenary is used in general, but on account of the great number and high degree of curves, pull-offs have also been employed.

The 12 road locomotives are each built in three units which are electrically identical. They receive power from the single-phase trolley through a transformer and phase converter, and are driven by three-phase traction motors at running speeds of 14 and 28 mi. per hr. Each complete locomotive weighs 637 tons.

Detroit & Ironton Railway. During 1926, the Detroit and Ironton Railway started electric operation on 17 mi. of line from Fordson to Flat Rock, Michigan.

Power is supplied at 44,000 volts, 25-cycle, single-phase, between feeder and contact wire, with 22,000 volts between contact wire and ground.

The catenary system is non-ferrous, composed of 4/0 bronze contact wire and a ½-in., seven-stand bronze messenger wire. The supports for the catenary are unique in that they are pre-cast reinforced concrete arches bolted together and placed on concrete foundations. Inclined catenary is used on curves. A ½-in. stranded copper feeder, together with auto-transformers, is used to secure three-wire feed.

The locomotives, two in number, are of the motor-generator type. In these locomotives, alternating current is stepped down to 1240 volts to drive a synchronous motor which, in turn, drives a 600-volt d-c. generator. Eight traction motors, of the d-c. type, rated at 225 hp., are mounted on each power unit which is articulated into two wheel bases of four axles each. The complete locomotive consists of the two power units. It has 32 driving wheels on which the total weight of 372 tons is carried.

Great Northern Railway. The line of the Great Northern Railway between Wenatchee and Skykomish, Washington, involving about 80 mi. of route, is being electrified with 11,000-volt, single-phase, alternating current. Twenty-six miles of the old line (from Cascade to Skykomish) is now in operation. A new tunnel 7¾ mi. long is being constructed to improve the route and to replace the old Cascade tunnel which was electrified in 1909 with 6600 volts, three-phase.

Two motor-generator type locomotives with two cabs each have been placed in service, which convert the 25-cycle power into 600-volt direct current to operate the traction motors. These two locomotives have a continuous rating of 3660 hp. at 15½ mi. per hr. with a tractive force of 88,500 lb.

Two single cab motor-generator type locomotives are now being built. They will convert the 25-cycle power to 1500-volt direct current to operate the motors with two motors in series. These locomotives will have a continuous rating of 3000 hp. at 18.6 mi. per hr. with a tractive force of 60,500 lb.

Pennsylvania Railroad. The Pennsylvania Railroad has under way the electrification of its main line for suburban service between Philadelphia and Wilmington, a distance of 27 mi. After this is completed, the suburban line between Philadelphia and West Chester will also be electrified. The design is laid out with due regard to the possibility of future extensions.

Electric power will be purchased, stepped up, and transmitted along the right-of-way at 132,000 volts, 25-cycle, single-phase, 66,000 volts to ground, over duplicate transmission lines to the transformer substations where it will be converted to 11,000 volts for the trolleys.

The catenary system will be completely non-ferrous. A bronze messenger wire with copper auxiliary wire and a single bronze contact wire will be used. The inclined type of catenary is to be installed on curves. The catenary supports are principally back guyed tubular poles with cross-span catenary to support the main catenary.

Multiple-unit cars of the type in the existing suburban electrification to Paoli and Chestnut Hill will be used.

New York, Westchester, & Boston Railway. The New York, Westchester, and Boston Railway has built an extension of its line to Harrison, New York, and will continue on to Port Chester. Much of this extension adjoins the trackage of the New York, New Haven, and Hartford Railroad. Multiple-unit suburban service is operated with an 11,000-volt, single-phase system.

Long Island Railroad. The Long Island Railroad is electrifying its freight line to Bay Ridge for 11,000 volts, single-phase, with overhead catenary construction. This involves about 100 mi. of trackage. Seven 150-ton locomotives for operation on this line have been delivered.

The extension of the third rail d-c. electrification over the West Hempstead passenger branch was completed last October.

New York Central Railroad. During 1926, the New York Central opened electric operation for multiple-unit service on the Putnam division, extending from Sedgwick Avenue Station, New York City, to Yonkers, New York, a distance of seven mi.

CITY AND SUBURBAN RAILWAYS

New car equipment being placed in service on electric railways is now confined, in most cases, to the light-weight type of car, in order to secure reduced operating costs.

Articulated train units are now in operation in street-car and subway service. Three-car articulated units for heavy subway and elevated service have been installed by the Brooklyn-Manhattan Transit Corporation. These units consist of three-car-bodies mounted on four trucks.

The most radical development is a car on which high-speed motors, entirely spring-supported, drive the axles through worm gears and a differential. Light weight and absence of noise are the outstanding characteristics of this car.

MARINE PROPULSION

Diesel-electric drive has been introduced on large tankers, suction and dipper dredges, ferries, and tug-boats. Double-end operation has been successfully introduced on tug-boats as well as on ferries.

Turbo-electric drive has been used on ferries in addition to its past application to large boats.

BUS TRANSPORTATION

Simplicity, ease of control, and durability are among the advantages which are causing the rapid introduction of gas-electric drive on buses.

RECENT DEVELOPMENTS

Diesel Electric Locomotive. The Diesel engine prime mover with electric drive is finding an increasing field in moderate sized locomotives up to 1000 hp., on account of its high efficiency, ease and flexibility of operation, and the absence of stand-by losses.

Gas-Electric Cars. Many of the gasoline-propelled rail cars now being placed in operation are equipped with electric drive.

Automatic Substations and Supervisory Control. The automatic substation is now finding its way into the electric railway field. The New York Central Railroad is installing three such substations to supply additional power for its New York Terminal electrification.

Supervisory control has been introduced by several of the recently completed railway electrifications, notably the suburban electrification of the Illinois Central, out of Chicago.

High-Speed D-C. Circuit Breaker. The high-speed d-c. circuit breaker, on account of the fact that it is opened by the rate of increase of the current rather than by the current value, has two distinct advantages in railroad electrification. First, the breaker will open before the current has reached a damaging value in any abnormal condition, such as motor flashovers, severe wheel sliping, or slight grounds. Second, the rapid rate of rise of current in a short circuit of any value makes the breaker more susceptible to short circuits than to heavy power loads and thus aids in securing selectivity between these two conditions.

Mercury Arc Rectifiers. The installation of mercury arc rectifiers for the Illinois Central is one of the first instances in which these rectifiers have been used in this country for a steam railroad electrification.

Motor-Generator Locomotives. Motor-generator locomotives are being placed in service by the Detroit and Ironton, Great Northern, and New York, New Haven, & Hartford Railroads. This type of locomotive can be built to give regenerative braking down to a very low speed. Speed control is flexible and the a-c. synchronous motor operates at a high power factor while the d-c. traction motors are developed to a high degree of efficiency.

Test Plant for Single-Phase Locomotives and Cars. In order to test electric locomotives and multiple-unit cars, the Pennsylvania Railroad, during 1925, equipped its Locomotive Test Plant at Altoona with a motor-generator set to convert 11,000-volt, three-phase, 60-cycle power into 11,000-volt, 25-cycle, single-phase power. A 204-ton freight and passenger locomotive was tested in 1925. During the year 1926, a complete test was made with a multiple-unit car.

New Single-Phase Induction Motor. A single-phase traction motor without a commutator has been built in Germany. It consists of two rotors on the same axis, one inside the other. The outer rotor is synchronous, excited by direct current. The inner rotor is a slip-ring motor which drives the shaft. According to the builders, power factor can be maintained at unity with this motor, and it is their hope that the motor will compete successfully with the a-c. commutator motor.

TECHNICAL PAPERS

The committee has been fortunate in securing some excellent papers for presentation at the Summer Convention in Detroit. They are as follows:

Current Collection from an Overhead Contact System Applied to Railroad Operation, S. M. Viele, Pennsylvania Railroad.

Catenary Design for Overhead Contact Systems, H. F. Brown, N. Y., N. H. & H. R. R.

Catenary Construction for Chicago Terminal Electrification of Illinois Central Railroad, J. S. Thorp, Illinois Central Railroad Co.

Collection of Current from Overhead Contact Wires, R. E. Wade and J. J. Linebaugh, General Electric Co.

Railway Inclined-Catenary Standardized Design, O. M. Jorstad, Westinghouse Electric & Manufacturing Co.

J. V. B. DUER, *Chairman.*

Electric Welding

Report of the Committee on Electric Welding*

To the Board of Directors:

It is the object of this report to give to the members of the Institute an idea of the commercial importance of Electric Arc Welding.

In such a report as this, the subject falls into two main divisions: Electric Arc Welding, in which the heat for doing the work comes from an arc drawn between the work and an electrode; and Resistance Welding, in which the heat comes from the electric resistance to the passage of a large alternating current across the abutting edges of the parts to be joined.

Electric Arc Welding is done by the *carbon arc* process and the *metallic arc* process. In the carbon arc process direct current is used and the arc is drawn from the work to a carbon electrode. In the metallic arc process either direct or alternating current may be used, but direct current is generally preferred and the arc is drawn between the work and a metallic rod, which melts into the work.

The carbon arc process was first used, on a commercial scale, fifteen or twenty years ago for the repair of steel castings. It has come to be accepted as the standard method of repair for minor defects in such castings. Practically all the steel foundries use this process and most of the 600,000 tons of steel castings made in this country each year have minor defects repaired by the carbon arc welding process.

LOCOMOTIVE BOILER REPAIRS

The metallic arc first came into commercial importance in 1914 in the repair of marine engine parts on some interned German ships. Since that time, this process has been used by the railroads to an increasing extent for the repair of machinery of all sorts.

At the present time, it is safe to say that the locomotive which drew the train that brought you to this Convention had all the flues welded in the works that built the locomotive in the first place, and that the boiler has had some repairs by the metallic welding process made on it in the repair shops of the road owning the engine. The fact that locomotive boiler repairs are so frequently made is very striking evidence of just what the process can do, for electric welds will stand up in a boiler of a locomotive carrying 200 to 250 lb. of steam and traveling over the rails at 60 mi. or more per hour. Electric welding is used for the repair of many parts of the locomotive beside the boiler. It is used for building up worn treads on driving wheels, worn guide bars, repairing broken frames, and for the repair of many other parts of the locomotive.

Mr. Wanamaker, one of the members of your Electric Welding Committee and Director of Welding for the Rock Island Systems has stated that the use of welding has enabled them to make repairs so much more promptly that they are able to get the same service with 20 locomotives less than would have been necessary with the old methods of repair.

To put it another way, welding has enabled the Rock Island System to reduce the investment in locomotives by over $1,000,000 without decreasing the number of engines available for service. This is in addition to the savings effected by the use of welding over the older methods of making the same repairs.

I quote from a letter from Mr. Wanamaker;

> "The latest estimate states that the railroads of this country now have invested approximately $4,000,000 in arc welding equipment, covering some 3500 welding equipments which are saving the railroads approximately $1,000,000 per month, or $12,000,000 per year. It is possible that the indirect savings will be greatly in excess of this figure. However, it is not the policy of railroads to assign a value to the indirect savings.
>
> "The use of electric arc welders for battered rail ends and special work rail is just now beginning to take strong root. I, personally, feel that the intelligent use of the electric arc welder has paid probably the greatest net return ever secured from any investment made in railway equipment, and I am somewhat at a loss as to why its use has not been more strongly furthered and fostered by the A. I. E. E."

Mr. Churchward, another member of your Committee, has a number of photographs showing the repair of an 8000-lb. bronze propeller by the addition of about 1000 lb. of bronze at a great saving in cost; he also has some photographs of the building up of the copper collector rings of a 1000-kw. rotary.

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*Committee on Electric Welding:

J. C. Lincoln, Chairman
C. A. Adams,
P. P. Alexander,
C. W. Bates,
Ernest Bauer,
A. M. Candy,
Alexander Churchward,
O. H. Eschholz,
F. M. Farmer,
H. M. Hobart,
C. J. Holslag,
C. L. Ipsen,
Ernest Lunn,
J. W. Owens,
William Spraragen,
H. W. Tobey,
Ernest Wanamaker.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

As important in repair and replacement work as is the use of electric welding, recent experience shows that its greatest field is as a new method of manufacture of new products.

It is clear that if a welded joint in steel can be made equal in strength to the plates to be joined, great economies in production can be obtained.

Following will be found a number of examples of construction on a large scale, in which welding has replaced riveting and the older methods of joining steel plates.

SHIPBUILDING

So far as we know, the first all-welded vessel was built in Ashtabula, Ohio, about 1914, for use on the Lakes. This was a boat 40 or 50 ft. long, used as a fishing boat. It has been in satisfactory service ever since it was built. On one occasion, a year or so after it was built, the plates near the waterline were bent by smashing through ice on the Lake, but no joint opened up.

Mr. James W. Owens, one of the members of your Committee and who, until his association with the Newport News shipbuilding & Dry Dock Company as its Director of Welding, was in charge of Welding Research and development for the U. S. Navy, says:

> "The Navy's investigations of welding at the Norfolk Yard have resulted in increased confidence of the Department in welding, with the result that it is now being used extensively in new construction and repair of naval ships. Its battle towing targets, which are 45-ton structures and of 3/8-in., 1/2-in. and 1-in. plate material, are now completely metal arc welded, 15 such targets being constructed to date. The shell deck and two water-tight bulk-heads of a pontoon for a 100-ton derrick have been completely welded. It is being extensively used in the construction of the light cruisers recently authorized by Congress, and an estimated saving of $250,000 is being secured by its use in the modernization of its capital ships.
>
> "The Coast Guard Cutter *Northern* which was recently launched at the Newport News shipyard, has its deck and superstructure very largely metal arc welded. Its rudder is completely welded and four seams at the waterline are welded in addition to being riveted in the usual manner, so as to enable the hull to more effectively resist ice pressure in Arctic service. Practically all oil, gas, and fresh and salt water tanks of ships being built at this yard, together with their piping systems, are being welded, and steps have been taken to completely weld all of the ventilating system ducts. During 1926 a total of 165 welded tanks was built and it is becoming standard practise to weld deck beams directly to bulkheads with or without the use of plate collars. Two completely welded condenser shells for merchant ships have recently been completed, together with the exhaust pipes for several ships."

WELDING STRUCTURAL STEEL FOR BUILDINGS

The steel frames for a number of buildings have been made by the arc welding process. In 1925 a three-story garage building, having a ground area of about 75 by 150 ft., was erected by the Wellman-Seaver-Morgan Company of Cleveland in Canton, Ohio. Although this was the first building arc welded by this firm, no difficulty was experienced in the erection, and they report a saving of 25 per cent in cost of erection, as compared with the cost of the same building riveted.

The largest building erected by arc welding to date has been that erected by the Westinghouse Electric & Manufacturing Company of Sharon, Pa. Mr. A. M. Candy, a member of your Welding Committee, was in general charge of the work and has written a very complete and valuable paper describing the work.

The building at the Sharon, Pa., Works of the Westinghouse Electric & Manufacturing Company is 70 by 220 ft., by 80 ft. high and required 790 tons of metal.

The following are some extracts from his paper:

> "That arc welded joints can be constructed in such manner as to develop fully the ultimate strength of the structural members connected.
>
> "That a steel I-beam of given section and length will sustain a far greater load if fixed at its ends by a suitably designed arc welded joint than if supported by standard riveted connections consisting of top and bottom angles. A 9-in. standard I-beam framed between rigid upright columns 8 ft. apart by means of specially designed welded connections sustained a load 25 per cent greater than a beam of the same size and length framed between columns by means of riveted top and bottom angles 1/2 in. thick.
>
> "A plate girder, assembled by welding and consisting of nothing but sheared plates has a far greater strength than a riveted plate and angle girder of the same weight, due to the better distribution of the steel in the cross section. A 15-in. plate girder, assembled by welding and simply supported on a 14-ft. span, developed more than 50 per cent greater strength than a riveted plate and angle girder of the same depth and the same weight.
>
> "The prevailing impression among the witnesses was that these tests demonstrated the superiority of welded connections to riveted connections in every case where direct comparisons were made, and brought out two general facts:
>
> "1. That complete continuity of lines of beams can be obtained in welded construction, whereas it is well known that this cannot be done in riveted construction.

"2. That in a welded building it will be possible to make every joint develop full strength of the main members, whereas in a riveted building many joints are weaker than the members due to the weakening effects of the rivet holes and the weakness of steel angles which have to be used for transmitting tension between two members at right angles to each other.

"It was proved that a welded plate girder was 50 per cent stronger than the riveted girder of relative depth, length, and weight.

"An Olsen testing machine, capable of applying 40,000 lb., was used."

In 1924 near Toronto, Canada, a highway bridge nearly 700 ft. long was erected by arc welding.

The American Welding Society has an active committee on Structural Steel Welding, the chairman of which is Mr. James H. Edwards, of the American Bridge Company. This committee is collecting data and making extensive tests to provide structural engineers with the fundamental information necessary to properly design an arc welded structure. The work of this committee is to provide the structural engineers with the information as to how long a weld and how thick a weld to use to give a strength of joint equal to the strength of the members being joined.

PIPE LINES

The carbon arc has been used for the manufacture of almost 90 mi. of pipe nearly six feet in diameter, for supplying water to Oakland, California, and the other Bay Cities in the vicinity.

Following are some quotations from a paper read by J. F. Lincoln, March 17th, 1927:

"As an illustration, The Mokelumne River pipe line which was made arc welded, is 90 mi. long and contains 78,000 tons of steel. If this same pipe line with the same strength of joint and the same ability to carry water had been made riveted, it would have required 128,000 tons of steel to accomplish the purpose, and the cost would be at least $3,000,000 more.

"The best illustration of the economic advantages of the above ideas are shown in the Mokelumne River Pipe Line, which brings the water supply a distance of 90 mi. to the East Bay Municipal Utility District in California. This line runs from the Mokelumne River to Oakland and supplies the water for all of the Bay Cities, with the exception of San Francisco. This job required, for the manufacture of the pipe, 78,000 tons of steel, the thickness of this steel being 3/8 in., 7/16 in., 1/2 in., 9/16 in., and 5/8 in., depending upon the part of the line in which the pipe was to be placed. It was manufactured in two lines—the first running from the San Juaquin River to Oakland and the second, from the Mokelumne River to the San Juaquin River.

"The first section of this, running from the San Juaquin River to Oakland, is at the present time completed and has been under pressure now for a considerable period. Because of the newness of arc welding, the engineers determined not to proceed with the second section of this pipe line until after the first section had been completed. They have, within the last sixty days, let the contracts for the second section, the specifications remaining identical in every particular with those of the first, thus showing the complete success of this method of construction.

"It is also interesting to note that, in spite of the fact that no such work as this had ever before been attempted, in so far as thickness of plate, diameter of pipe, etc., were concerned, the first 40 mi. were completed more than one month ahead of time, and this in the face of the fact that the building for the manufacture of this pipe had to be constructed, machines designed and built, and the whole plant put into operation.

"The second section of this line, which is to be done in the same time as the first, will undoubtedly beat its schedule by many months.

"The description of this line and of the problems involved, as concerns welding, can probably best be explained from the specifications which were drawn for it and under which the pipe was manufactured. The specifications covering the welding machines provided that the welding shall be done with an automatic electric welding machine, designed specifically for the work covered by those specifications. This machine comprised among other necessary parts, a traveling carriage for carrying the carbon electrode, arranged to move, at a controlled rate, along a tract located inside of the pipe above the bottom seam, and water-cooled mandrels located on the outside and inside of the pipe and extending along this seam throughout its length.

"Next, the method of testing was specified as follows:

"*Hydrostatic Test of Pipe Specimens.* After each section of the pipe has been welded, it shall be subjected to a hydrostatic test under internal pressure sufficient to develop a tensile stress of 20,250 lb. per sq. in. of plate, and while under this stress, shall be hammered vigorously on both sides of the weld with a 10-lb. hammer not more than once in every foot of pipe section. The pressure shall then be increased sufficiently to produce a tensile stress of 23,000 lb. per sq. in. of plate and so held until the efficiency of the seams can be determined by inspection. In the event of failure of the pipe section under this test, the contractor shall have the right to re-weld the pipe if practicable and re-submit it for test.

"*Rejection.* Any section of pipe that does not conform to these specifications may be rejected.

"It is claimed, in connection with this, that it is probably the most severe test that was ever put on any pipe made by any process, and it is only necessary to say that possibly no other method of manufacturing large pipe now known could pass the specifications for test as outlined in these specifications. As a matter of fact, there are a number of cases where the test pressures were carried up so high and so far beyond the statement in the specifications, that a permanent set was actually put in the steel of the pipe. In one case that came to the author's attention the diameter was increased by more than three inches because the pressure that was used in testing went considerably beyond the elastic limit of the steel."

USE OF WELDING TO REPLACE CASTINGS

Due to the fact that rolled steel has three or four times the tensile strength of cast iron, and at the same time costs from one-third to one-fourth as much per pound, it is possible to make many structures from rolled steel shapes by arc welding to replace cast iron at a very great saving in cost.

For instance, a certain cast iron bed plate weighed 560 lb., and at 5.5 cents per lb., cost $30.80. The cost of machining this was 90 cents, making a total cost of $31.70.

The corresponding base of angle iron welded up (with bosses welded in) weighed 233 lb., cost $6.38 for material and $1.07 for cutting off, welding, and drilling, or a total of $7.45. In addition, the welded base will stand any abuse that can be given to it, while cast iron bases break if not carefully handled

One of the first welded products we happened to know about was a compensator can. This can was formerly made of cast iron, very heavy, very expensive, and liable to leaks which made it necessary to reject part of those the foundry delivered. The arc welded can, for the same purpose, looked better, weighed less than 20 per cent of the old cast iron can, and cost less than 10 per cent.

The General Electric and Westinghouse Companies are using arc welding to an increasing extent in the manufacture of their product, though the process can be and no doubt will be used to a much greater extent in the future.

A paper by Mr. Warner, in the March issue of the American Welding Society *Journal*, gives an account of some of the work being done by the General Electric Company at the present time.

The Company with which your Chairman is connected has brought out a book "Arc-Welding, the New Age in Iron and Steel," in which there are hundreds of illustrations of structure of all kinds built by Electric Arc Welding.

This process makes possible the manufacture of many structures that are both better and cheaper than the same structures were when made of cast iron.

RESISTANCE WELDING

This process was invented by Elihu Thompson, in the early days of electric development, and at the present time is used for the production of some millions of feet of tubing every month. Practically all of the tubing used in automobiles and in the construction of bedsteads, is made by this process.

As you all known, a million dollars is a small sum when talking about automobile products, and as this process is used to a greater or less extent in the construction of all cars, we can be sure that the cost of the car you drive would be noticeably greater if it were not for the saving in cost of construction made possible by this process. The all-steel automobile body is coming into use and thousands of these bodies are made every year in which the seams are welded by this process.

The following is quoted from a letter from Mr. H. W. Tobey, one of the members of your Committee and in charge of work of this kind at the Pittsfield Works of the General Electric Company:

> "Our large spot welders are used in regular production for thicknesses of steel ranging from two pieces of 1/4 in., two pieces of 3/8 in., up to two pieces of 1/2 in., making a maximum total thickness of 1 in. This method has replaced to a very large degree the former practise of riveting.
>
> "Spot welding is also used throughout the plant for a variety of purposes where it is desired to join various parts of equipment together in a strong and effective manner at a reasonable cost.
>
> "Resistance line welding has been found to be of especial value wherever it can be applied in mass production to structures composed of sheet material of suitable shape and in thicknesses up to and including two pieces of 1/8 in. stock.
>
> "All factors entering into the several types of resistance welding, including butt welding, spot welding, and line welding, are under perfect control so that after correct settings have been obtained for current, pressure, speed, etc., the results can be duplicated with unerring regularity as long as the characteristics of metal remain the same. The material is brought to precisely the right temperature and strong, reliable welds uniform in character and appearance are assumed. The fact that no hood or eye protection through the use of colored glass is required, is also a distinct advantage."

J. C. LINCOLN, *Chairman*.

Discussion

J. D. Noyes: I should like to ask Mr. Lincoln if the committee has come to a conclusion as to the comparison or disadvantages between alternating current and direct current.

E. C. Crittenden: Recently there has come to my attention a field in which it appears that some companies are finding other methods better than electric welding. Mr. Wanamaker in the report, says, with regard to railroad practise, "The use of electric arc welders for battered rail ends and special work rail is just now beginning to take strong root." I understand also that the street railways are using arc welders in putting up crossings and other special work, but for the regular rails, the straight-away welding, they are finding other methods better.

It appears to me that probably the difficulty arises from the fact that they are using the old method of connecting rails with only a slight modification for welding. Two rails abutting, end to end, are held together by two fish-plates. The two plates are clamped on the sides of the rails and then welded around the edges. This method has the effect of stiffening that part of the rail and in service, the rails often break at the end of the welded and stiffened portion, presumably because the stresses are localized there. I believe some companies have abandoned arc welding of rails because they get such breakage. Is there available any electrical welding method which will avoid this difficulty?

F. W. Funk: Mention has been made in the report of several buildings that have been electrically welded. We have in Youngstown a welding company which erected a steel-frame building which was entirely welded; that is, there were no holes punched or drilled—no bolts used in temporary construction. It was a 100 per cent welded building. I think that is quite an advance because if we have to punch structural shapes, it offsets the saving.

Electric welding is coming into use for very heavy plate work and tank work. One concern which manufactures very large oil tanks for use in the oil fields is going into the electrical welding of heavy plate tanks. It is probable that the process will be an automatic welding process.

Another company is producing thousands of tons of complicated rolled shapes each year which are either seam- or lamp-welded, used principally, I think, in floor beams and roof members. I mention this concern particularly because it has done a great deal in the development of automatic welding machines.

I think Mr. Lincoln is entirely correct in saying that in new construction lies the greatest field for electric welding.

J. J. Shoemaker: I should like to ask Mr. Lincoln if this method of welding is approved by the insurance companies for boiler repairs?

J. V. B. Duer: I think some of these questions that have been discussed are important. The question of workmanship enters into electric welding more than most of us realize. I think if we could be assured that the workmanship on an electric weld was perfect, it would be a more generally accepted practise than it is at the present time.

I am also much interested in the question of the relative values of a-c. versus d-c. welding. I made a rather superficial investigation of the subject a few years ago and came to the general conclusion at that time that the principal objection to a-c. welding was that it involved a low power factor on the circuits to which the welders were connected. That was especially true of 60 cycles rather than 25 cycles. In a place where the welding load amounted to a large proportion of the total, such as it might in a roundhouse on a railroad or some place of that kind, the use of an a-c. welder might be objectionable whereas if a d-c. welder were used the high power factor due to the use of the welding might be obtained and some correction made for low power factor on the other circuits.

J. C. Lincoln: I may be prejudiced in favor of the direct current. It happens that the concern with which I am connected, manufactures d-c. apparatus. That fact has to be taken into consideration in weighing my reply. I believe myself that experience will show that it is easier to make a good weld with a good d-c. machine than it is with an a-c. machine. We feel this way about it; that the art would be advanced more rapidly by doing only good work. Therefore, only the best apparatus should be used. I believe, on the average, for metallic arc welding better work can be done with the direct current than with alternating current.

I think it is true that in some cases railroads which have formerly used the electric-arc method for joining their rails are using other methods at the present time. I have been very much interested in that particular part of the electric-welding art for a good many years. Notwithstanding the fact that there have been a number of failures, the amount of success with electric-welded joints has been sufficiently great so that they are used commercially to a great extent. In my opinion, the failures have been due partly to the cause as pointed out by Mr. Crittenden, but more largely to the fact that the welding methods themselves haven't been as good as we hope they will be some time.

In the electric welding of rails, in some cases at the bottom of the crater when the weld is finished there are hair cracks due to the contraction of the hot metal. Now, it is a fact, of course, that hot metal occupies a larger volume than the same metal when cold. Metal, when melted, occupies 107 per cent of the volume it does when cold. When the metal cools, one of two things happens—either the metal must stretch according to the amount of decrease in volume or hair cracks appear. If the metal is brittle enough, hair cracks appear. If the metal is of the proper quality, it will stretch and take up this decrease in volume.

It is my belief that most of the difficulty in the welding of rails has been due to the fact that the methods in use at the present time produce welds which are brittle so that these hair cracks appear.

Now, a hair crack at the end of the rail will start an incipient break. Take, for instance, a piece of glass. You know that if you start a crack in a piece of glass, the wind strains will finally work that crack further and further through the glass. If you will drill a hole at the end of the crack, you won't find any further trouble. The incipient crack doesn't progress. Something of the same nature occurs in the electric welding where the hair cracks appear. The remedy for that condition is to improve the quality of metal in the weld.

It is a fact that in welding of rails, processes are in use at the present time where these hair cracks are easily eliminated. If the work is properly done, satisfactory results are obtained. If it is not done properly, trouble results.

I believe experience will show that although, as pointed out, you have a weak spot at the welded point in the rail, when the welding is properly done, the weakness is not so great as to develop trouble.

The American Society of Mechanical Engineers has prepared a Boiler Code and this Boiler Code does not permit the use of electric welding to so great an extent as we who are interested in pushing electric welding would like. Electric welding is permitted on some types of pressure vessels but at the present time electric welding is not used on pressure vessels used in boilers.

The American Welding Society has a committee making extensive preparations to test a large number of pressure vessels at the Bureau of Standards to get complete information so that the Boiler Code Committee can feel justified in giving more liberal allowances for electric welding.

At the present time steam boilers are not allowed to be welded by the electric welding process. The queer part of it is that the railroads have been repairing locomotive boilers for years and these boiler repairs made on locomotives are on pressures of 200 to 250 lb. Years of experience in locomotive boilers have shown that method must be used. If you please, from a commercial standpoint, the railroads can't get along without it.

While there has been some trouble from improperly made welds on electric locomotives, the process is in use quite generally and I don't think there is any doubt that it will be used more generally in the future. But to answer the question specifically at the present time, pressure vessels—fired pressure vessels—are not allowed to be electrically welded.

Electrochemistry and Electrometallurgy

Annual Report of the Committee on Electrochemistry and Electrometallurgy*

To the Board of Directors:

The Committee on Electrochemistry and Electrometallurgy makes its annual report as follows:

About two years ago, this committee brought to the attention of the Standards Committee the desirability of revising the Institute Standards for storage batteries. A special working committee was then appointed to undertake this task and a tentative standard has been formulated defining the terms and conditions which characterize the rating and behavior of storage batteries. This report is now in the hands of the Standards Committee. It is believed that the work accomplished at the suggestion of this committee will be of value to the storage battery industry, which is an important unit in the electrochemical field.

Standards for the international electrical units are again receiving much attention at the various national standardizing laboratories. These fundamental standards which furnish a basis of measurement for both the physicist and the electrical engineer fall within the field of electrochemistry. Seventeen years have elapsed since the International Technical Committee met in Washington to carry out a joint investigation on the silver voltammeter and the standard cell. At the conclusion of its work the value 1.0183 international volts at 20 deg. cent. was adopted for the Weston Normal cell, and by international agreement, this value became effective on January 1, 1911. The various countries, therefore, started out on this date with identical values for the international volt, which, together with standards of resistance, served to fix the measurement of current, also. Since that time, the basis of reference has been carried forward by means of groups of standard cells and resistances. It is a matter of importance, therefore, that we should determine how accurately these standards have been maintained. Several new groups of standard cells have recently been prepared at the Bureau of Standards, which indicate a very close agreement with the Bureau's existing basis of reference. The results obtained, so far, cannot be considered as conclusive but indicate that our standard for voltage has been maintained over a long period within a few millionths of a volt. Comparisons have also been made with the standards of several other countries and reasonably close agreement has been found in most cases. The necessity still remains, however, for checking the standard for the international volt by means of the silver voltammeter and by the absolute current balance for the measurement of current. Progress is being made in absolute measurement of resistance. As pointed out in last year's report, it is apparent that a considerable difference exists between the international ohm and the absolute ohm.

In reviewing the progress during any particular year, it is always difficult to evaluate the definite advances within so limited a period. New methods, when first tried, are experimental and it is impossible to say whether or not they constitute an advance. Development in electrochemistry and electrometallurgy as well as in other lines is necessarily gradual. It is possible, however, to mention certain developments which appear to distinguish the past year as one of considerable progress in this field.

The introduction of the high-frequency furnace into electric steel foundries is a notable step. The high-frequency furnace has been regarded in the past as a laboratory instrument but now it is rapidly finding favor in the industry as a whole.

Arc furnaces have increased in size and power input largely as a result of the development of three-voltage switching. The principle of this development is based upon the fact that the cold charge within the furnace is capable of absorbing heat at a higher rate than after the charge has become molten. It is possible, therefore, to supply energy very rapidly at the beginning and decrease this as the charge is melted. In the final step, only enough energy is supplied to meet the furnace losses and whatever may be required for chemical reactions during the slagging period. It has been stated that nearly all of the large electric steel-melting installations made during the past year have utilized this principle.

The trend of electric furnace development, however, differs somewhat from that during the period of the World War. The economic limitations to its application are recognized, and the electric furnace is being used now in processes where quality of product is of importance. In this respect, it has been largely responsible for the rapid development in high grade alloys and tool steels. American manufacturers are still importing special steels and pure iron that should be produced here. According to recent reports, a great research campaign on steel is being instituted by one of the large manufacturers.

The advance in the application of electric furnaces in the steel and brass industries has been pointed out in papers presented at the 25th Anniversary Meeting of the Electrochemical Society. The electric steel industry began in this country about 1909, with a half-dozen furnaces producing 13,000 tons. Since then the number of furnaces has increased to over 500 and the product for 1925 was in excess of 600,000 tons. The proportion of electrically melted metal is much greater in the brass industry than in the steel industry. About 625 electric furnaces for brass melting are in use in the United States and Canada. It has been estimated that they used $3,000,000 worth of electrical energy in 1926. A notable saving of fuel compared with that which would have been used in fuel-fired furnaces was accomplished. A still greater economy was achieved by saving over 13,000 tons of metal valued at $2,500,000, which would have been lost in the fuel-fired furnaces.

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Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

Comprehensive studies of electric heating are being made by some power companies with a view to analyzing the load possibilities within surrounding territory. In connection with one of these surveys it is interesting to note that a rather definite line of demarcation between ovens and furnaces is made at the temperature of 1000 deg. fahr. Immersion types of electrical heating units are being utilized in existing fuel-fired, stereotype pots having capacities up to six tons.

The use of chromium, both as an electrodeposit and in certain alloys, again occupies a prominent part in the development of electrochemical industry. The previous predictions regarding the usefulness of chromium plating have been largely verified. At present, chromium plating is being applied successfully in the automobile industry, especially on such parts as radiator shells, where its hardness, high luster, and freedom from tarnish are advantageous. It is being used on gages because of its extreme hardness and resistance to abrasion. It is also being applied to plumbing supplies and to a great variety of metal products where good appearance and durability are desirable. The chief obstacle to the more general adoption of chromium plating is the poor "throwing power" that renders difficult the plating of recessed articles. Experience and mechanical ingenuity have frequently led, however, to the successful plating of rather irregular shapes.

A chromium surface affords a valuable protection to steel against corrosion as well as furnishing a high lustrous finish. It is unique in being particularly resistant to the action of sulphur compounds such as seriously impair silvered and other polished metal articles. Large molds for automobile tires, stills used in oil refining, and plates for engraving bank notes are all examples of articles whose life has been greatly extended by the use of chromium plating.

A method has been worked out for producing an exceedingly smooth chromium plating on mirror surfaces. The reflecting power of chromium is not as high as that of silver, but may perhaps be improved by plating with other metals. It is probable that a high average reflecting power of a chromium surface can be maintained over a long period of time, since the chromium surface is not subject to tarnish, discoloration, or scratching from cleaning operations. From the military standpoint, there is considerable advantage in a chromium plated mirror for search lights since such a mirror is free from shattering, if struck by a bullet, and it is better adapted to withstand the high temperature of the arc.

The use of chromium in iron and steel alloys has created widespread interest. There is a variety of these alloys and the properties depend upon the composition, as might readily be supposed. Up to 5 per cent of chromium, high strength, ductility, and hardness are obtained in the presence of at least one other element as, for example, nickel. Higher percentages of chromium impart notable resistance to oxidation, even at high temperatures. Above 20 per cent of chromium, the steels have in addition to oxidation resistance a marked resistance to the action of nitric acid and nitrates.

There have been few marked developments in the process of electrodepositing other metals than chromium, but there has been an increased interest in the theory and mechanism of making such deposits. Commercial processes are being conducted more and more on a scientific basis and it is interesting to find that the application of the hydrogen electrode to the control of refining and plating solutions is teaching the workers to understand the significance of the pH values of their solutions and how these may be used as a guide in the control of their product.

An industrial achievement during the past year has been the commercial development of the electrodeposition of rubber. Rubber is deposited anodically. Rubber deposits can be made quickly in forms corresponding to the shape of the anode. It is reported that wire can be insulated by passing it through the latex solution, the wire serving as the anode and the rubber being deposited on the wire as it passes through. The rubber on the wire then passes through a vulcanizer for the completion of the process.

The development of power devices for use with radio sets has been a matter of interest and concern to battery manufacturers. It seems probable that the inroads of such devices have been more in the field of storage batteries than in the field of dry cells. From the standpoint of the power company, it is interesting to estimate the electrical energy consumed by the use of some of these devices. Recent calculations have shown that a device operating a radio set of five tubes and charging a small storage battery when the set was not in use for a period of approximately 18 hr. per day, required one kilowatt-hour in each 24 hr. Charges for power in this particular case were estimated at $1.80 per month which, in view of the possible use of a large number of such devices, indicate a considerable revenue to be derived from them.

The development of small rectifiers has been stimulated by the growth of the radio industry. The life and efficiency of the aluminum rectifier has been materially increased. The use of tantalum has been extended not only to the radio field but to other important uses, as in railway signalling. So-called electronic rectifiers, including those having elements of copper oxide and copper sulphide, have become available commercially during the past few months. Such rectifiers are, however, subject to the limitation of a relatively low voltage across each element. They have not been in use for a sufficiently long time to obtain definite data as to their life but it seems likely that they will become of increasing importance. Alloys high in silicon are also being used as valve metals in electrolytic rectifiers.

The development of the Hoopes process for the preparation of a very pure aluminum, was described two years ago. This aluminum, which possesses somewhat different physical and chemical properties from those associated with the ordinary aluminum, is finding commercial use in collapsible tubes and also in metallurgical work where the aluminum may be studied unhampered by the presence of detrimental impurities. Remarkable results in heat-treated castings of the pure metal have been reported. New uses for this material are still being sought. There has been a large increase in the consumption of aluminum alloys, particularly that known as duralumin. Among the novel uses for these alloys may be mentioned the manufacture of aluminum furniture. The attractive finishes in browns, reds, and greens, together with the light weight, have been important factors in promoting the sale of this material.

In the field of rare metals the electrolytic production of pure beryllium may be noted. Some investigation has been made of alloys of aluminum and beryllium but as yet there is no definite information to show whether these will be commercially valuable. Pure thorium is also being made by electrolytic methods. The use of thorium has arisen from its exceptional thermionic properties. Amounts of vanadium sufficiently large for careful experimental investigation have also been produced.

The highly sensitive potassium photoelectric cell has largely displaced the selenium cell. The potassium cell possesses extreme sensitivity and responds instantly to a beam of light. Among the many possible applications for such a cell may be mentioned a recently developed method for television.

At the Anniversary Meeting of the American Electrochemical Society, held in Philadelphia at the close of the month of April, a number of interesting papers was presented reviewing the progress of electrochemistry during the 25 years since the society was founded. One session at this meeting was devoted to the electrochemistry of concentrated solutions. A new theory is rapidly setting aside the old theory of Arrhenius, and Professor Peter Debye, of the University of Zurich, one of the chief exponents of this new theory, was present to present his views. The interest in this matter is shown by the fact that a similar symposium was held at almost the same time by the Faraday Society in England. Industrial engineers found papers of interest in the session devoted to gaseous reduction of ores. Gaseous reduction of iron ores, followed by electric melting of sponge iron, may open up a further field for electrothermics.

The fall meeting of the Electrochemical Society is expected to take the form of a trip through the northwestern part of the United States with stops at numerous points of electrochemical and electrometallurgical interest. Such a trip should result in a better understanding of the work that is being done in this important region and at the same time focus the attention of electrochemists and electrical engineers on the power requirements and possibilities of this region.

The committee wishes to acknowledge with thanks the cooperation of Dr. William Blum, Dr. Colin G. Fink, and Dr. H. W. Gillett, members of the Electrochemical Society, who have furnished valuable information used in preparing this report.

GEORGE W. VINAL, *Chairman*

The Holtwood Steam Plant
Design and Operation in Coordination With Water Power

BY F. A. ALLNER[1]

Member, A. I. E. E.

Synopsis.—*The Holtwood Steam Plant is located in Lancaster County, Pa., on the Susquehanna River, about 24 miles above tidewater, adjacent to and closely coordinated with the 111,000-kw. hydroelectric plant of the Pennsylvania Water & Power Company, and through the latter company's customers, it is a part of a hydro-steam system including Baltimore, Md., and Lancaster, York, and Coatesville, Pa., with a total installed generator capacity of about 370,000 kw. and with high-tension connections to two other large systems. It is a pulverized fuel burning station, containing at present two 10,000-kw. generators and three 1400-hp. boilers. The plant is laid out for an ultimate capacity of at least 120,000 kw.*

The station went into operation in July 1925. The paper gives the reasons for building a steam plant at Holtwood, some of them being general advantages in such a location for a plant, which is supplementary to a run-of-river hydroelectric plant, and others being the particular advantage, in this instance. In general, during the low-flow period, the steam plant carries the base load or belt generation, and during high flow, when the hydroelectric plant is operated at maximum capacity whenever the load permits, the steam plant carries the peaks. This station was especially planned to suit such conditions of operation and was designed for mechanical sturdiness, reliability of service, quick starting, ability to float in at no load, and for maximum coördination with the hydroelectric plant rather than for maximum economy or minimum first cost. A number of special features incorporated to carry out this aim is described in the paper, among them being arrangements to facilitate quick starting and floating in, and a special governor so constructed that the speed regulation may be changed by remote control while the unit is running. A comparative analysis of starting times of hydroelectric and steam units at Holtwood is given, showing that in spite of special design, the steam unit requires a much longer time than the hydroelectric unit.

The pulverized fuel process was adopted in order to secure better sustained efficiency over a wide range of boiler rating, to reduce banking losses, and because it was desired to burn bituminous coal of various grades in the same furnaces with river bottom anthracite coal which is dredged from the upper end of the pond formed by the Holtwood dam. This has been successfully accomplished, approximately one-third of all the coal burned, so far, having been river anthracite. During short periods, anthracite alone has been burned, but most of the time the two kinds of coal are mixed before being dried and pulverized. Maintenance costs at the pulverizing plant when burning 100 per cent anthracite are very high.

Data are presented showing the first cost of the plant, fuel rates, costs of preparing river anthracite, outage time of the generating units, and other operating results. A brief summary is given also of minor difficulties.

On account of the unusual operating requirements as to loading, fuel supply, etc., the design and operation of this plant involved a number of interesting problems in design and operation, for the solution of which only a limited amount of experience was available. It is believed that nearly all of the major problems have been solved satisfactorily, but there is still need for further experimentation to secure the best possible results from the equipment installed.

BRIEF DESCRIPTION OF SYSTEM

THE Pennsylvania Water & Power Company is a generating and transmission company, selling power at wholesale to public utilities. Its system consists of a combined hydroelectric and steam power development at Holtwood, Pa., on the Susquehanna River about 10 miles (16.1 km) above the Pennsylvania-Maryland state line, with high-tension steel tower lines radiating like the spokes of a wheel (Fig. 1) to Baltimore, Md. 40 mil. (64.4 km.) distant; Lancaster, Pa., 14 mi. (22.5 km.); York, Pa., 23 mi. (37.0 km.); and Coatesville, Pa., 30 mi. (48.3 km.).

The Holtwood (formerly called the McCalls Ferry) hydroelectric development (Fig. 2) is a low-head run-of-river plant with a weighted average head under present conditions of about 52 ft. (15.8 m.). The first unit went into operation in October 1910 and after the initial five unit installation the plant was extended in several successive steps, reaching its present rated generator capacity of 111,000 kw. early in 1924. The rated capacity of the waterwheels varies from 13,500 hp. for the older type of double-runner turbines to 20,000 hp. for the latest single-runner type. The first eight units are 25-cycle, this frequency having been adopted to meet the requirements of Baltimore and Lancaster. The last two units installed in 1923-1924 are 60-cycle to meet the needs of York and Coatesville and what is now the larger and rapidly growing part of the Lancaster load, which is supplied over a 60-cycle line built in 1924. The 25- and 60-cycle systems can be tied together by a total capacity of 25,000 kw. in seven synchronous frequency changers, of which two 5000-kw. sets are located at the hydroelectric plant, and the balance of 15,000 kw. at Lancaster. The latter equipment was formerly used for supplying the 60-cycle load in that city when only two 25-cycle circuits were available from Holtwood.

The normal full-load discharge of the hydroelectric plant is now about 30,000 cubic ft. (850 cu. m.) per sec. with a maximum under favorable hydraulic conditions of 31,500 (892 cu. m. per sec.). The former flow is available in the average year 41 per cent of the time. The flow of the Susquehanna is unusually variable. The lowest 24 hr. flow recorded at Holtwood since the plant went into operation in 1910 (from which time more reliable flow measurements are available than had theretofore existed) was 3200 cu. ft. per sec. (90.7 cu. m. per sec.), and the lowest

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[1] General Superintendent, Pennsylvania Water & Power Co., Baltimore, Md.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

seven day average was 3600 cu. ft. per sec. (102 cu. m. per sec.); the maximum 24-hr. flow was 441,000 cu. ft. per sec. (12,470 cu. m. per sec.).

The pond or reservoir formed by the Holtwood dam has an area of about four sq. mi. and a drawdown capacity of approximately 800,000,000 cu. ft. (22,700,000 cu. m.). Under present conditions of system load demands this drawdown gives complete weekly pondage to effectually equalize differences in daily flow and shape of load curve up to an average weekly river stage of 4200 cu. ft. per sec. (119 cu. m. per sec.). The term "complete" pondage means that if the hydroelectric plant carries the entire system load above a certain base steam line, and the forebay on Friday evening is brought to a point not below the desired minimum level, the pond will just fill up again by Monday morning without spilling. In other words, the inflow of the river between Friday evening and Monday morning, minus whatever is used for a small amount of generation during that period, will be discharged during the period from Monday morning to Friday evening, in addition to the inflow of the river on these five days.

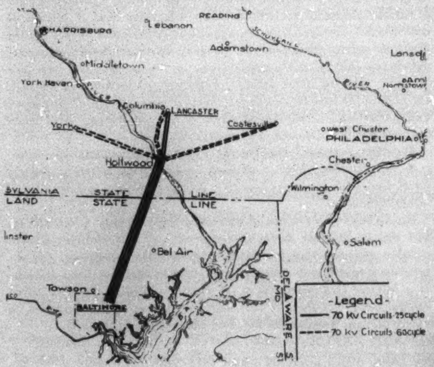

FIG. 1—MAP OF HOLTWOOD TRANSMISSION SYSTEM

With an average weekly flow of 4200 cu. ft. per sec. (119 cu. m. per sec.) the average five day draft can be increased by 1800 cu. ft. per sec. (51 cu. m. per sec.), to approximately 6000 cu. ft. per sec. (170 cu. m. per sec.); but with an average weekly flow of only 3600 cu. ft. per sec. (102 cu. m. per sec.), the effective five day average draft can be raised to only about 5200 cu. ft. per sec. (147 cu. m. per sec.), because the inflow at the rate of 3600 cu. ft. per sec. is not sufficient to completely refill the pond after maximum draw down as in the case of 4200 cu. ft per sec. inflow. For average weekly flows in excess of 4200 cu. ft. per sec. the increase in the daily discharge on the five week days will be limited to approximately 1800 cu. ft. per sec. on account of the minimum desired forebay level at the time of maximum drawdown, and the inflow over the weekend will be more than sufficient to completely refill the pond before Monday morning.

The installed capacity of the Baltimore steam plants of the Consolidated Gas, Electric Light & Power Company is about 220,000 kw. At Lancaster the Edison Electric Company has a standby steam plant with rated generator capacity of 7000 kw. At York, the Edison Light & Power Company has a steam generating station of 8500 kw. capacity and a tie-in connection of 7500 kw. with the large hydro-steam system of the Metropolitan Edison Company, from which it receives approximately one-half of its load requirements. At Coatesville, the Chester Valley Electric Company has a steam plant of a rated capacity slightly in excess of 3000 kw. and also a 5000-kw. tie-in connection with the Philadelphia Suburban Gas & Electric Company.

GENERAL SYSTEM OPERATION

The method of operating the system for maximum coordination of waterpower and steam has been explained in a previous paper[2] and will not be discussed in detail here. Generally speaking, during high or excess flow the aim is to deliver the maximum amount of energy from the river. As long as there is water wasted over the dam, hydraulic efficiency is of no importance. The hydroelectric units are operated at full load as long as possible and carry as much of the system load as is practicable when this is less than the hydroelectric capacity. The steam plants then operate "on the peaks" and carry all swings. In low or deficient flow, conditions are reversed. The steam plants are given the base load or belt generation and the hydroelectric plant operates "on the peaks," and takes care of load fluctuations during the hour, even though such operation under average low-flow conditions causes an energy loss of about 7 per cent, and at very low river stages as much as 15 per cent, compared with 100 per cent load factor,

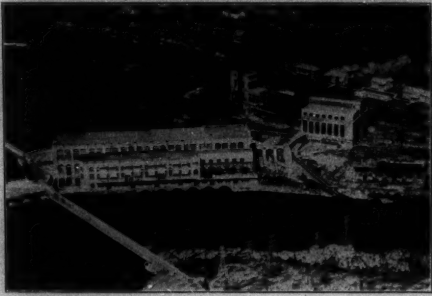

FIG. 2—VIEW OF HOLTWOOD WATER POWER PLANT AND STEAM STATION

belt load generation. These differences are due more to the large hydraulic gradient of the tailrace than to the carrying of swings and the uneconomical loading of individual units.

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[2] A. S. M. E. *Transactions*, 1925, p. 379.

Under normal conditions the pond is drawn upon only to the extent of compensating for daily and week-end fluctuations in inflow and load demand and restoration of maximum pond level is aimed at on the morning of every week day. Thus the maximum practical amount of pondage is held in reserve to meet major emergency conditions or temporary outages at the steam plants.

REASONS FOR BUILDING THE HOLTWOOD STEAM PLANT

Until recently the Power Company depended entirely upon its customers' steam plants for makeup steam generation. The company's contracts give it the right to call on any of the steam plants previously listed for generation under certain conditions In 1925, however, the company, through a subsidiary, the Holtwood Power Company, built a steam plant at Holtwood. Additional steam capacity was required for the system. The Consolidated Gas, Electric Light & Power Company of Baltimore was ready to proceed with its new Gould Street plant, but after careful joint study it was decided that Holtwood was the logical place for the next step in steam station construction and arrangements were made with the Baltimore company to temporarily postpone the Gould Street project. The principal reasons for this decision were as follows:

1. The 60-cycle load in Pennsylvania was increasing rapidly and additional capacity was essential. None of the Pennsylvania customers' steam plants is modern and well adapted for economical extension, and existing power agreements did not provide for the use of more than the capacity available at the time when Holtwood supply had started.

2. The transmission of primary power from Baltimore to the Pennsylvania 60-cycle system to take care of the growth of the Power Company's load commitments, would have required additional frequency changers at Holtwood, with consequent conversion losses in addition to increased transmission losses. A direct connection of the Baltimore 62½-cycle system with the 60-cycle system in Pennsylvania was not possible without extensive changes in Baltimore.

3. A plant at Holtwood, being at the hub of the wheel, so to speak, could offer the maximum protection to all of the radial lines. The transmission of power from any customer city to Holtwood and thence to another city requires a reversal of flow, with consequent complication of voltage regulation. From this point of view the logical place for a steam plant supplementary to hydro is at the water-power site.

4. Since the Lancaster dual frequency supply had grown to about 25,000 kw. it was desired to always maintain two sources of supply, which during very low river stages required an appreciable amount of 60-cycle hydroelectric generation during off-peak hours. Sixty-cycle supply from the Holtwood steam plant enabled the system to conserve the entire hydroelectric energy exclusively for peak service, thus indirectly gaining ineffective peak supply for the system appreciably more than the steam capacity proper.

5. There are large deposits of river bottom anthracite coal in the Holtwood pond. The location of a steam plant at Holtwood made it possible to use some of this coal and store same eventually in very large quantities, thus providing a more economical and effective emergency insurance of fuel supply than is practical with bituminous coal at the city stations.

6. The installation of a steam plant adjacent to a variable flow run-of-river hydroelectric plant makes it possible to effect considerable savings in operating expense through the coordination of operation and maintenance work. In such a system so far as possible, all maintenance work at the hydroelectric plant is done during low flow, whereas repairs at the steam plant are concentrated in the high-flow period. This makes it possible for the same force, at least in part, to be used at both plants.

BRIEF DESCRIPTION OF STEAM PLANT (FIGS. 3 & 4)

It is not the purpose of this paper to give a detailed description of the entire plant and an itemized list of equipment. However, the principal physical characteristics will be mentioned as a ground work for a discussion of special features and of the reasons for making certain decisions.

The generating equipment proper consists of two 10,000-kw., 80 per cent power factor, 1800-rev. per min., three-phase, 60-cycle, 13,200-volt generators driven by nine-stage turbines designed for 350 lb. (23.8 atm.) gage at the throttle. Initial operation is at 550 deg. fahr. (287.7 deg. cent.), but the turbines are guaranteed to operate successfully at 700 deg. fahr. (371.7 deg. cent.) if the superheat should be increased later. Each unit has a 75-kw., 250-volt shaft-end exciter. Steam is bled from the turbines at three stages—the third, fifth, and seventh. Each turbo generator is equipped with a 13,000 sq. ft. (1208 sq. m.) single pass condenser.

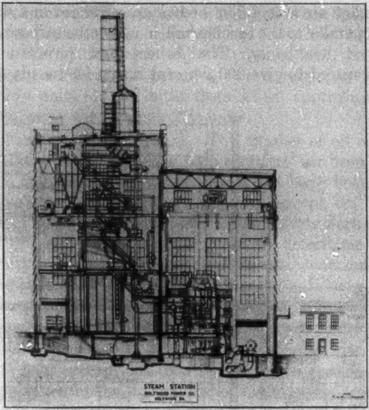


FIG. 3—CROSS SECTION THROUGH BOILER ROOM AND TURBINE HALL

The boiler installation consists of three cross drum vertically baffled three-pass B. & W. boilers, each with a heating surface of 14,056 sq. ft., (1305.7 sq. m.), exclusive of the water screen at the bottom and back of the furnace which has an additional surface of 775 sq. ft. (72.0 sq. m.). The working pressure is 385 lb.

(26.2 atm.) gage. Each boiler is equipped with a convection type superheater above the top row of water tubes, designed to give a steam temperature of 560 deg. fahr. (293.3 deg. cent.) at 250 per cent rating. Each boiler has its own stack with a height of 126 ft. (38.4 m.) above the burners. There are two induced draft fans above each boiler. Neither economizers nor air preheaters are installed, although provision is made to add such equipment later on, if found advisable. Makeup water is furnished by two horizontal single effect evaporators which are supplied with filtered river water.

The furnaces have air-cooled side and front walls. Each furnace has a volume of 11,500 cu. ft. (325 cu. m.) above the water screen. The vertical burners, of which there are eight to a boiler, are set under the high end of the boiler. Pulverized coal feeders are arranged in four motor-driven pairs per boiler. There are three primary air fans. The boilers are arranged in a single row parallel to the turbine hall, in which the turbines are placed lengthwise. The boilers and furnaces are guaranteed to give 300 per cent rating for twenty-four hr. or 400 per cent for twelve hr. when using bituminous coal, and 250 per cent for 24 hr. or 325 per cent for 12 hr. using river anthracite.

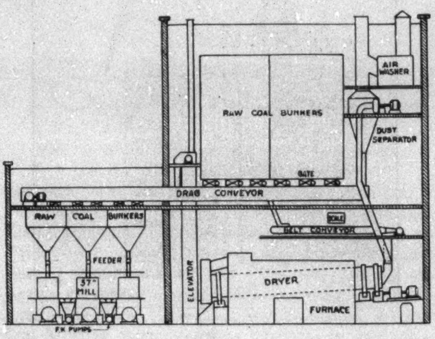


Fig. 4—Longitudinal Section Through Coal Preparation Plant

All auxiliaries are motor-driven with the exception of one steam turbine driven boiler feed pump. Each of the two circulating water pumps has a dual drive consisting of one 25-cycle and one 60-cycle motor. All other a-c. motors are 60-cycle. The 60-cycle hydroelectric units provide an independent source of power, which is considered of greater reliability than shaft-end auxiliary generators or house turbines. The boilers and furnaces are equipped with Bailey automatic combustion controls which operate the coal feeders, the induced draft fans, and the stack dampers.

The coal preparation plant occupies a separate building located upstream from the boiler house. At the top of the building there are two raw coal bins with a total capacity of 1000 tons to which coal is elevated by a skip hoist. From the raw coal bunkers coal is fed by a drag conveyer to a 6½-ft. by 50-ft. (1.98 m. by 15.2 m.) rotary dryer with a capacity of 25 net tons of bituminous or 24 net tons of river anthracite coal per hr. There are three dry coal bins each with a capacity of 20 net tons. The pulverizing equipment consists of three Fuller 57 in. (1.45 m.) screen type mills, each with a guaranteed capacity of 7½ net tons per hr. of bituminous or 4½ net tons of river coal when grinding to a fineness of 83 to 87 per cent through a 200 mesh screen. Pulverized coal is pumped to the boiler house by two Fuller-Kinyon pumps through two five-in. pipe lines with a horizontal length of about 350 ft. (106.7 m.) and a vertical rise of 85 ft. (25.9 m.).

Bituminous coal is received by railroad car and dumped into a track hopper mounted over a crusher. River coal is for the most part scowed to the forebay ramp where it is transferred to a railroad car and hauled to the track hopper.

At present the coal storage facilities consist of a drag scraper storage yard with a capacity of about 10,000 tons. Coal is also stocked out and reclaimed by locomotive crane from several areas adjacent to railroad sidings. With the increase in plant capacity the main storage will be gradually extended by filling in a large shallow area of the forebay.

Controlling Factors and Special Features of Design

The Holtwood plant was designed not so much for maximum economy or minimum first cost, but primarily for mechanical sturdiness, reliability of service, quick starting, ability to float in at no-load, and for facilitating the proper division of load between hydroelectric and steam plants. In carrying out these aims a number of interesting questions arose and several special features were incorporated which will be discussed below.

Choice of Site. The forebay of the hydroelectric plant, which is protected from floating ice and debris by a rock fill ramp, skimmer wall, and floating booms, offered an ideal source for circulating water. The exact location of the steam plant, however, presented an interesting problem. The site finally selected for the initial section of the power house was immediately below (*i. e.*, on the downstream side) and abutting on the wing wall from the river shore to the hydroelectric plant. Circulating water for the condensers is taken from the forebay and returned to the gate house of the hydroelectric plant, the tunnels having been cut through the wing wall.

Serious consideration was given to a site along the tailrace below the hydroelectric plant. The natural head from forebay to tailrace would then have been used for discharging water through the condensers, thus doing away with pumping equipment. This scheme looked particularly attractive from the point of view of saving a few minutes in the starting operation. The head, however, is considerably greater than that required to overcome the frictional resistance of the condensers and water conduits. Supplementary to this scheme, a plan was also considered of using the excess head to drive auxiliary hydraulic turbines, but this was an added complication.

As the heaviest steam generation and maximum draft of circulating water would be at times of very low flow, the use of water in this manner would reduce the hydroelectric plant output during these low river stages. During the high-flow period there would be a gain in the combined net output of the hydro and steam plants. If arrangements were made to utilize the excess head beyond the condensers, and if the hydroelectric plant operated 24 hr. a day as during moderate low flow, the reduction in hydroelectric plant output would be less than the equivalent amount of electrical energy consumed in the motors. However, in very low flow the hydroelectric plant shuts down at night and all the available hydroelectric energy is thrown in on the peaks. The diversion of water through the condensers during the 24 hr. of steam operation would still show a net gain in energy output, but would cause an appreciable reduction in the peak service rendered by the hydroelectric plant on account of the concentration of all the hydroelectric energy during the short period of peak generation. Maximum peak service in very low flow is of prime importance, particularly as there is a tendency for the peak service corresponding to a certain amount of hydroelectric energy to increase from year to year with the growth of the system load.

Perhaps the greatest advantage of the site selected was the greater amount of space available. The plant is laid out for six turbine bays and its ultimate capacity will be approximately 120,000 kw. The future extensions will be made to the north of the wing wall into a shallow portion of the forebay area where little cofferdamming will be required. The initial two-bay section of the power house was built entirely in the dry, the wing wall acting as a bulkhead.

Another advantage of the layout adopted is that the injection of warm condenser discharge into the end of the gate house is of value in reducing the chances of capacity reductions caused by frazil ice on the hydroelectric units at that end of the power house. It has been calculated that for the completed plant, enough heat would be discharged in the condenser water to raise the temperature of all the water entering the hydro units, if perfectly mixed with it, by nearly 0.2 deg. fahr. (0.11 deg. cent.) exclusive of the heat required to melt the ice. Experts on ice have declared that a rise of less than 0.01 deg. fahr. (0.0055 deg. cent.) may be enough to remove the "stickiness" of frazil ice.

Centralized Control. Electrically speaking, the steam plant is essentially a part of the 60-cycle section of the hydroelectric plant. The steam generator leads are carried through a cable duct to oil switches located in the hydroelectric plant switchroom. The control panels for the steam units are a part of the 60-cycle switchboard in the hydroelectric plant. The Tirrill element and the controls for the Keilholtz-Ricketts booster regulator used on the steam units are located on this switchboard.

Size and Type of Generating Units. From the point of view of low first cost and of economy of operation, under base load conditions, larger generating units would have been desirable. However, as previously stated, these considerations were subordinated to sturdiness, reliability of service, quick starting, and maximum coordination with the hydroelectric plant. Furthermore, under the existing conditions, these initial steam units must often operate at a comparatively small load, either when separated from the rest of the generating system or in high flow when operating on the peaks so as to "skim off" the top of the load and give the hydroelectric units maximum load. All of these considerations pointed toward relatively small sturdy units for the initial installation, approximating in capacity the hydroelectric units.

Auxiliary Power System. The number of oil circuit breakers on the auxiliary power system has been kept to a minimum. Magnetic contactors have been employed for all switching purposes, except on main feeders between distribution boards. Tests were made on different types of contactors to determine their ability to rupture the maximum expected short circuits. The control for contactors on essential auxiliaries is so arranged that either a contactor will not open on low-voltage, or will immediately reclose on restoration of voltage.

Special precautions to avoid explosion hazard have been taken in the installation of the electrical system; in the coal preparation plant, distribution switchboard and contactors for the main auxiliaries are located in a special switchroom, isolated from the main building by concrete fire walls. All push buttons, and such manual controllers as are necessary in the mill and dryer rooms, are of the Navy standard vaporproof type. All lighting fixtures are dust proof and vaporproof.

Quick Starting of Hydroelectric and Steam Units. In a hydroelectric system in high flow, when the system load is less than the hydroelectric capacity the steam plant should be shut down at night and theoretically should not operate in the morning until the load becomes equal to the hydroelectric capacity, when it should pick up that portion of the load over and above the hydroelectric capacity. Modern steam turbines designed with small clearances and operating at high temperatures require normally approximately one hour for starting. This, of course, entails the use of steam before it is really required. If it is desired always to have spare steam capacity available for immediate use in the event of breakdowns, it is necessary to operate normally a larger number of units than would be required for most economical generation.

Hydraulic turbines operating at low or medium heads or having only short penstocks are inherently quick starting machines. Temperature differences are practically nil, clearances are relatively large, and construction is sturdy. The starting time can be further reduced by special arrangement of control valves and instruments, by automatic starting of pumps supplying the pressure fluid of servo motors, etc., and by training the operators. At the Holtwood hydroelectric plant any of the 10 units, ranging in capacity from 13,500 to 20,000 hp. is commonly started from standstill without preliminary preparation and synchronized in less than one minute, and frequently in less than half a minute. Fig 5. gives an analysis of the few steps involved in this operation, requiring only two men—the governor man and the switchboard operator. The low-flow standby service of a low-head hydroelectric plant equipped with pondage has well been called "a-c. storage battery service," *i. e.*, its ability to synchronize the entire capacity in a few minutes and maintain full output for several hours or days, and thus tide over temporary emergencies at the steam plants. During the high-flow period this emergency service is not available at the hydroelectric plant, because all hydroelectric units are operating at maximum gate opening.

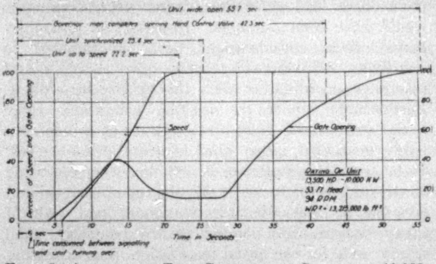
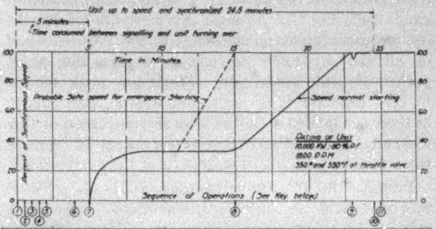


FIG. 5—ANALYSIS OF TIME CONSUMED IN STARTING A 10,000-KW. HYDRO UNIT

When going into the field of steam generation at Holtwood, the power company desired to extend so far as practicable this quick starting ability to its steam plant, thus giving to the hydroelectric steam combination a certain amount of quick emergency service also during the high-flow period. This was one of the reasons for selecting a particularly sturdy type of turbine of relatively small size. The specifications provide that "every effort is to be made in the construction of these machines to make them suitable for starting up in the minimum time." Under normal conditions a Holtwood steam unit is paralleled in 25 min. and in emergency the time can be reduced to 15 min. from the time the start signal is given. Fig. 6 gives an analysis of the operation. A total of four men are engaged in this operation and the time consumed and the number of steps required are still in striking contrast to hydroelectric starting.

The desire to facilitate quick starting was the chief reason for limiting the superheat in the initial installation to a total temperature of 550 degrees at the throttle. In future extensions it is planned to install larger units, using the first two for quick starting purposes. At that time radiant type superheaters may be installed, giving a total steam temperature of 725 deg. Various schemes have been proposed for maintaining the quick starting feature on the first units, if and when this step is taken, such as omitting the radiant superheater in one boiler, use of a Ruths accumulator, etc.

Floating In. As a practical matter, in a case such as the above mentioned building up of load, and whenever the load is at or near the hydroelectric capacity, it is necessary to have spare capacity on the bus. At such a time in high flow any load carried by a steam unit causes a net loss of the equivalent amount of water power. It is therefore highly desirable to have steam units able to float in or motor at no load. On the Holtwood units a by-pass is provided around the main steam control valves to admit enough steam to the turbine to keep it cool. The manufacturers' guarantee provides that with this by-pass connection open "it will be possible to motor the turbine by means of its generator with the control valves shut off a period of one hour without overheating any parts of the turbine."



No	Operations by Engineer	Condenserman	Oiler	Swbd Operator
1	Signals to condenser room to start condensate and circulating water pumps			Signals to engineer
2	Closes turbine drains and turns water and steam on shaft seals	Starts condensate pump	Opens valve to auxiliary steam line	
3	Starts steam jet air pump	Circulates condensate to tanks		
4	Starts auxiliary oil pump	Starts circulating pump		
5	Turns steam into turbine main			
6	Vents condenser water box and syphons water into tail pipe			
7	Opens throttle valve and brings turbine to ⅓ speed			
8	Brings turbine to speed and puts water on shaft seal, signals switchboard to close generator field circuit breaker			
9	Trips turbine by oil trip & reopens throttle valve			
10				Synchronizes generator
11	Adjusts seal leakoff, turns water on air & oil coolers	Turns condensate into suction header of boiler feed pump		

FIG. 6—ANALYSIS OF TIME CONSUMED IN STARTING A 10,000-KW. STEAM UNIT

A ¾-in. (1.9 cm.) orifice is at present installed in the by-pass. It has been found that this discharges enough steam to drive the unit at slightly more than synchronous speed under excitation. This is more than is necessary for cooling and prevents lowering of speed for synchronizing unless the speed is reduced by manual manipulation of the vacuum or the throttle. Tests are not yet completed to determine the safe minimum size of orifice.

Fig. 7 illustrates the division of load between a 12,000-kw. hydroelectric unit and one of the steam units over the noon hour drop when operating in parallel on the Lancaster 60-cycle load separated from the rest of the system. Until about 11:55 a. m. the hydroelectric unit is "wide open," with the steam turbine taking the momentary swings and the more gradual load changes. When the customer's load drops below the hydroelectric capacity, the hydroelectric governor begins to function or to "maintain the frequency" as the operators say, and the steam unit "floats in" or motors. Occasionally small amounts of power are momentarily supplied by the steam turbine, due to slight decreases in frequency which draw on the kinetic energy stored in the rotor and also in a few cases probably cause the steam governor to open a control valve momentarily. The graphic chart of steam turbine output does not show input when momentarily motoring because of a zero stop on the meter.

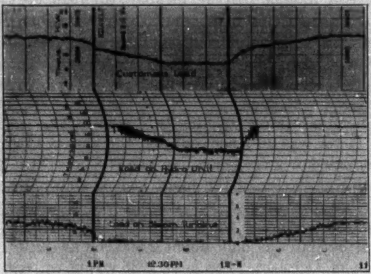


Fig. 7—Wattmeter Charts During Floating-In Period Over the Noon Hour

When the load has increased to the capacity of the hydroelectric unit the conditions prevailing before the noon hour are restored. The boiler operation is approximately as follows: a short time before noon the rate of coal feed is reduced so as to lower the steam sufficiently to avoid "blowing-off" as the load drops. The burner flames are extinguished just before the beginning of the motoring period, but are relighted occasionally as required to maintain the pressure within the desired range.

Special Governors. In order to change the per cent speed regulation on the standard type of governor it is usually necessary to shut down the steam turbine. In a hydroelectric steam system such as has been described, when the steam unit is carrying the base load it is desirable that its regulation should be relatively large, whereas when it is carrying the load fluctuations or peaks and the hydroelectric units are carrying the base load, the steam unit regulation should be relatively small. The Holtwood steam units operate also in parallel with a number of other steam units, where regulation cannot be adjusted while running. In order to control Holtwood steam generation in respect to these other steam stations, either towards assigning to the Holtwood units a steady load or a fluctuating load, it was thought desirable to be able to change from one condition to the other without shutting down the unit. Hence a special motor driven device was installed for changing the governor regulation. This device is controlled from the switchboard in the hydroelectric plant. The specifications provide for an adjustment range from 1 per cent to 5 per cent, although the guaranteed range is only 2 per cent to 5 per cent. Tests made on the governors at the factory showed a range from 1.6 per cent to 7.4 per cent.

The manufacturer's guarantees provide that the characteristics of the governors are such that there will not be any hunting between the steam turbines and the hydro units or between the two steam turbines under either of the two following operating conditions:

1. Steady continuous load carried on waterwheel generators, and fluctuating load carried on the steam turbines.
2. Steady continuous load carried on steam turbines and fluctuating load carried on waterwheel-driven generators.

The guarantees also provide that the speed regulation and the speed of operation of the governor and control mechanism of the turbine will be such that with full load thrown off the generator and the field circuit breaker opened simultaneously, the speed will not exceed that for which the emergency trip is set (the latter is to be not over 110 per cent).

In order to make it possible to restore normal frequency in case of a sudden overload due to the breakdown of a generating unit or other cause, by paralleling a steam unit without cutting off customers' load, a particularly wide range is provided in the governor control of the Holtwood units. The speed can be changed by remote control from 107 per cent or 64 cycles to 85 per cent or 51 cycles, but in the latter case the by-pass around the control valves must be throttled which, as stated previously, is at present too large.

Condensers. The heaviest duty on the steam units falls in the period of very low flow, which usually is also a warm weather period. Also in cold weather, as for example, in case of a frazil ice run, it may be desired to throw the maximum possible overload on a steam unit for a short time. This was the cause of conservatism in condenser design. For ordinary conditions probably a 9000-sq. ft. (836-sq. m.) single-pass condenser would have sufficed, but in line with the policy of insuring a safe margin of capacity and maximum coordination with the hydroelectric plant, even at the expense of first cost, a 13,000-sq. ft. (1208-sq. m.) single pass condenser was installed on each unit. Sustained gross generating capacity for highest water temperature in

December, 38 deg. fahr. (3.3 deg. cent.), was found to be 15,200 kw. limited by the turbine, and for highest water temperatures in September, 75 deg. fahr. (23.9 deg. cent.), 13,200 kw. at 100 per cent power factor, limited by the heating of the generator field.

Pulverized Fuel. The principal reasons for installing a pulverized coal burning plant were as follows:

1. It was believed that during the low-flow period when the steam units would carry heavy base loads as well as during the high-flow period with its fluctuating loads, the pulverized system would have an advantage over stokers in being able to maintain good efficiencies over a wider range of load with varying grades of fuel.
2. It was desired to reduce the starting and banking losses to a minimum. With a stoker-fired plant these would have been particularly great during the high-flow period when the steam plant carries peaks and fluctuations as outlined above.
3. Any known and tried stokers suitable for burning fine sizes of anthracite coal would not have been adapted to the use of eastern bituminous coal. It was believed that it would be possible by adopting the pulverized coal process to prepare and burn either or both types of coal with the same equipment. It will be shown later under operating results that this hope has been fulfilled. The high speed ball type of pulverizing mill was adopted because it is best suited for the preparation of anthracite as well as bituminous coal.
4. Economies could be effected in preparation of coal by utilizing surplus off-peak hydroelectric energy in high flow for operating the pulverizing plant.

CONSTRUCTION AND COST DATA

Decision to build the steam plant was made April 2, 1924. The turbo generators were ordered May 23, and excavation began June 24, 1924. The first generating unit went into regular commercial operation July 20, 1925, and the second on August 4, 1925.

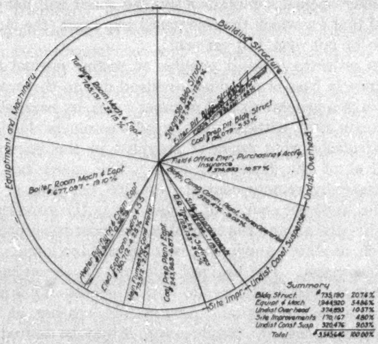


FIG. 8—ANALYSIS OF CONSTRUCTION COSTS

The design work was carried out by the Consolidated Gas, Electric Light & Power Company of Baltimore. The field engineering and construction work was handled by the power company's own forces.

Fig. 8 is a summary of the cost of the plant, exclusive of property, legal and corporate expenses, housing for employees, interest during construction, and exclusive of any contractors' fees. There are several factors which should be borne in mind in comparing these costs with those of other plants. The present boiler installation is larger than needed for the first two turbines, and surplus pulverizing capacity on a bituminous rating was installed to off-set partially the reduction in capacity when milling river coal. Switching equipment is installed for two more units and the cable tunnel to the hydroelectric plant will take care of the ultimate station. The intake and discharge tunnels are built for two more units of a capacity up to 35,000 kw. each. The width of the power house is sufficient to accommodate units up to 35,000-kw. capacity and larger boilers.

It may be of interest to note that the cost of the preparation plant per net ton of pulverized output per hour on a bituminous rating was $8,720 for structures and $10,830 for equipment, or a total of $19,550.

OPERATING RESULTS AND EXPERIENCE

General Results. In the calendar year 1926, the gross output of the steam plant was 60,870,000 kw-hr. The station uses, including coal handling and preparation, amounted to 4,200,000 kw-hr., or 6.9 per cent, giving a net generation of 56,670,000 kw-hr. The one-hour peak load was 26,600 kw. or 33 per cent more than the nominal rating of the two units. The annual load factor was 26.1 per cent and the annual capacity factor based on gross generation was 34.8 per cent. The variable character of the plant load is shown by the fact that the highest net output was a little over 10,000,000 kw-hr. in the month of July, whereas the lowest output was only about 2,500,000 kw-hr. in the month of October. The year 1926 was of better than average river flow, which accounts for the relatively low output of the steam plant.

The average economy for the year was 21,160 B. t. u. (5331 kg-cal.) per net kw-hr. The best weekly economy reported was 18,350 B. t. u. (4623 kg-cal.) per net kw-hr. burning 100 per cent bituminous, the best weekly economy burning 100 per cent anthracite was 19,800 B. t. u. (5010 kg-cal.) per net kw-hr. The average boiler and furnace efficiency for the year was 79.6 per cent without allowance for dryer fuel or for energy requirements of the boiler auxiliaries.

The boilers have operated up to 385 per cent rating for a few hours at a time. The average rating for the total steaming period was 201 per cent.

Of the total coal used, 35.4 per cent was river anthracite and 64.6 per cent bituminous.

Of the total coal used, 35.4 per cent was river anthracite and 64.6 per cent bituminous. The average quality of the two fuels was approximately as follows:

	Anthracite	Bituminous
Moisture	13.0 per cent	3.8 per cent
Ash, dry basis	16.7 per cent	11.1 per cent
B. t. u. wet basis (kg-cal.)	10,880 (6,044)	13,210 (7,339)

A great deal of the bituminous coal burned was of an inferior quality with an ash content as high as 20 per cent. Of the total dry ash in river anthracite, about 6½ per cent is intrinsic ash and the rest is foreign matter. Attempts have been made to evaluate various reductions in the latter item but the findings are not yet conclusive. Such reduction in ash means not only higher heating value per pound, decreased power consumption in grinding, and increase in effective plant output, but also a very appreciable reduction in maintenance cost of mills. Mill maintenance is affected particularly by sand, small pebbles, and stones. Several schemes of reducing the ash have been tried in the recovery process of river coal, but they are still in the experimental stage.

Use of River Anthracite. A detailed discussion of the use of river coal does not come within the scope of this paper, but a few outstanding phases of the subject will be mentioned.

Up to date roughly one-third of all the coal burned has been river anthracite. In a few periods aggregating about ten weeks 100 per cent anthracite has been burned. At certain other times the fuel was all bituminous. Between these two extremes various mixtures were tried out. The volatile in river anthracite is so low (averaging a little over 8 per cent) that it is somewhat difficult to ignite and the flame is not as stable as with bituminous. The anthracite coal is ignited by means of flexible hose oil torches of which there are two per boiler connected through pipes to an outside pressure tank. There has not been sufficient experience in starting up with river coal to train the operators thoroughly or to determine the best procedure and minimum amount of oil required, but experience to date has been as follows: When starting up after a weekday bank of approximately six hr. the torches were burned for 10 to 15 min. and consumed an average of about 20 gal. (75.7 l.) of oil per boiler. On individual days the consumption was as low as 10 gal. (37.8 l.). After a weekend bank of approximately 30 hr. the torches were used for 20 to 30 min. consuming about 80 gal. (303 l.) of oil. In starting up a cold boiler with straight river coal it is necessary to burn the torches for 35 to 45 min. A single measurement showed a consumption of about 125 gal. (473 l.).

One hundred per cent anthracite coal is not well adapted to high flow operation with many starts and stops and fluctuating loads. A scheme is under consideration to preheat the combustion air electrically during the high-flow period just before the morning peak, using otherwise wasted hydroelectric energy to shorten the starting period, to replace in part the large quantity of kerosene or fuel oil, and to save a considerable amount of now unburned fuel. Straight anthracite is also not suitable at Holtwood in very low flow when the steam plant is carrying the base load because of the reduced output of the mills. The latter objection could of course be overcome by installing more pulverizing capacity.

The river coal is abrasive and contains a varying amount of sand and small pebbles causing a large increase in the cost of maintenance of the preparation plant. Even small amounts of bituminous coal seem to act as a cushion or lubricant and reduce the wear on the mills.

The pulverizing mill balls are normally used until they have lost about 28.5 per cent of their initial weight, the latter being about 525 lb. (238 kg.) per ball. In milling straight anthracite, the average output is about 220 net tons per ball or 880 net tons for a set of balls. At rated capacity this would give an average life of about 195 hr. The material cost of balls is about 16.2 cents per net ton of coal ground.

The largest item in maintenance when using river coal is renewal of grinding rings. The life of a ring in tons is about 2060, and the cost of rings without labor is about 38 cents per net ton of coal. Operating and maintenance costs at the preparation plant, exclusive of power, during a five weeks run with anthracite, generating 3,284,000 kw-hr. under high-flow loading conditions, were as follows:

Operating labor	$.3435	per net ton
Supplies	.0175	" " "
Maintenance material	.8677	" " "
" labor	.0877	" " "
Total	$1.3164	" " "

During the above run the average ash content was 18.7 per cent dry basis, *i. e.*, 2.0 per cent higher than the average per cent for the year (16.7 per cent).

It is believed that due to a better understanding of the cause of yoke breakage, the cost of which amounted to 14 cents per net ton during the test period, this item could be greatly reduced.

A mixture of two kinds of coal is obtained by drawing bituminous and anthracite respectively from the two raw coal bunkers simultaneously upon the drag conveyor which discharges the coal into the dryer where it becomes thoroughly mixed. There has been no trouble due to separation of the two kinds of coal in the dryer, conveyor, mills, or transport system. A 50-50 mixture has been found entirely satisfactory for high flow fluctuating load operation.

The river coal dredging season lasts for only seven to nine months, depending on weather and river conditions. Fortunately this coal is particularly well adapted for storage purposes. The price of the raw coal is low, resulting in a minimum investment tied up in storage. The coal can be piled to any depth and does not deteriorate. In fact there is a certain improvement in that some of the moisture in the freshly delivered coal will drain off. For these reasons the fuel is particularly suited for use in a steam plant which is supplementary to a variable flow hydroelectric development, as it is not possible to make accurate forecasts of the coal requirements.

Much study has been devoted to the combustion problem when using anthracite and to the proper construction and adjustment of the burners, air ports, primary and secondary air combinations, etc., but there is still need for further investigation along these lines. The Holtwood plant has proved that this low grade fuel can be burned successfully in pulverized form. The proportionate amount of anthracite actually used has so far been controlled more by practical considerations of supply then by conditions of operation and it is hoped that in the future a still greater amount can be used.

In actual practise it has been found that the guaranteed degree of fineness of pulverizing quoted above is not necessary. The present operating standard is 70 per cent for bituminous and for mixtures and 75 per cent for anthracite alone.

Results of tests on the dryer when treating river anthracite were as follows:

	Maximum Output of dryer	Average operation
Moisture entering dryer	11.30 per cent	11.30 per cent
Moisture at dryer outlet	4.15 per cent	2.0 per cent
Moisture at mill inlet	2.41 per cent	1.0 per cent

Pulverized anthracite could not be burned successfully in the dryer and untreated river coal is now burned on hand-fired grates under the dryer. The coal fired per net ton of anthracite input to the dryer was 55.7 lb. (25.3 kg.). The heat units fired per pound of water evaporated in the dryer was 4100 B. t. u. (2277 kg-cal. per kg. of water).

Outage of Units. The excellent outage record of the Holtwood steam units shows that to a great extent the desire for maximum service reliability has been met. In about 21 months of operation of two units the total outage due to breakdowns or repairs of mechanical or electrical trouble has been 390 hr. or 1.26 per cent of the total unit hours in the period. This time could have been considerably reduced if necessary.

The only case of mechanical breakdown in the generating units and their auxiliaries was a shearing of two blades on the eighth stage wheel of one unit. In passing through the ninth or last stage these broken blades slightly damaged the ninth stage blading and then punctured four condenser tubes. As this trouble was not observed at the time it occurred, the unit was operated for several days until the damage to the tubes was discovered in the course of periodic tests on conductivity of condensate. It was then taken out at first for temporary repairs and a few weeks later for permanent replacement of the two damaged stages, but at the time the trouble was discovered the unit could have remained in service for a longer period if necessary.

The only major damage to the electrical equipment occurred during a severe local lightning storm, which caused a spillover between the high-tension and low-tension bushings on one of the 70-kv. step-up transformers, the low-tension side of which was connected to the same bus as the turbo-generator. The high voltage to which the generator in this way was exposed caused a puncture in the winding (2nd coil from the high voltage end out of 20), the flash taking place from winding to a projecting part of the supporting iron ring. The balanced relays tripped the oil circuit breaker and generator field immediately. The generator was energized 1½ minutes later and operated satisfactorily for six months, the puncture not being discovered until at the time of the annually scheduled routine inspection and disassembly of the unit. Only two armature coils had to be replaced requiring a total of 10 days of single shift work, the material being taken from stock. The fact that the unit could operate satisfactorily so long with a puncture in the armature winding is probably due to the fact that the system is operated with its neutral ungrounded.

The experience of other companies in respect to the capacity reducing effect of breakdowns may not have been as favorable with similar types of unit as the above described two accidents may indicate. But even if these two occurrences might have caused an immediate though temporary capacity outage at the Holtwood plant, the liberal provisions for spare equipment as practised for a number of years by the operating companies connected to the Holtwood system, would have permitted the bringing in of spare capacity, so that service should not have been affected by these breakdowns. Particularly on a system that does not depend wholly on steam power but draws a good portion of its requirements from a variable-flow hydroelectric plant equipped with pondage, such outages of a few hours or days can be readily taken care of during the low-flow period by the a-c. storage capacity, *i. e.*, the quick starting ability of the hydroelectric units referred to in the general description of the Holtwood system.

MINOR DIFFICULTIES

As might be expected when building a plant in which so many different requirements have to be met deviating from usual practise, a great many minor difficulties have been experienced before satisfactory performance could be obtained from every piece of equipment. A few brief statements on these experiences may be of interest.

Drag Conveyer. The coupling between motor and conveyer broke several times on account of overload and jamming, caused by the type of slide gates employed at the outlets of the raw coal bunkers. When the slide gates opened wide the coal flooded and clogged the conveyer, and when only partly opened the wet river coal had a tendency to arch. After skirt plates were installed between bunker outlets and conveyer and a shear pin was inserted in coupling, quite satisfactory operation was obtained.

Dryer. Attaching angles to the lifting blades of the drum greatly improved the performance of the dryer, although it has not yet fully met the guarantee, particularly in respect to river coal. The original plan of burning pulverized anthracite in the dryer had to be abandoned and hand-firing on pin hole grates, under forced draft, was finally adopted as the most satisfactory solution. The magnet in the discharge spout of the dryer did not remove all the tramp iron from the dried fuel before passing through the mills; this trouble was largely eliminated by placing three additional magnets along the spouts leading to the mills.

Explosions. So far only two explosions have been experienced, both of which can be traced back to excessive temperature in the dryer shell at high rating, aided by the presence of combustible foreign material. Damage was slight in one case but in the other the original explosion in the boot of the bucket elevator was followed by several smaller explosions, damaging the casing of the elevator. The vent stack at the top of the elevator casing was enlarged, steam lines were provided in the dryer, and an inspection window was cut in at the outlet end of the dryer. It is believed that these arrangements will greatly help to prevent explosions or to minimize their severity.

As a safe guard against explosions, dust is not allowed to accumulate in the mill house or in the dryer. The labor and expense of preventing such accumulations in the dryer have been reduced by installing soot blowers at points where dust might collect.

Mill Foundations. Regrouting of base rings and replacement of broken anchor bolts have been necessary at intervals, usually after a prolonged run of 100 per cent anthracite. The motor foundations as originally constructed developed serious cracks, but after they were enlarged no further trouble was experienced.

Mill House Lighting. Considerable difficulty was experienced at first with lighting in the mill house, due to breakage of filament from mill vibration. The lighting system was changed from 220 to 110 volts in order to take advantage of the greater mechanical strength of the lower-voltage filament. This only partially solved the problem. Special anti-vibration fixtures were then developed in conjunction with a fixture manufacturer. These now make possible normal lamp life.

Coal Transport Pumps. The pumps require a slightly larger amount of air than anticipated. They operate satisfactorily for either kind of fuel and for mixtures. The wear when pumping anthracite, though not excessive, is expected to be reduced by stelliting the periphery of the screw.

Fire in the Pulverized Coal Bunkers. On two occasions small smoldering fires started in the pulverized fuel bunkers. These were easily smothered by closing all vent openings and by slowly drawing down the stored fuel through the burners. These fires were probably started by some abnormal or temporary condition or quality of coal, as during a number of months two idle pulverized coal bins were normally kept full and coal has lain occasionally in a bin for several weeks without trouble.

Coal Feeders. Considerable wear was experienced on the coal feeders with anthracite. Hard metal studs were placed on the periphery of the screws, which compensated for the reduced diameter and maintained more equal coal feed to the burners.

Adjustment of Burners. In an effort to prevent secondary combustion within the boiler tubes, experiments were conducted with several devices for producing turbulency, a longer flame, and more thorough mixing of coal and air as discharged from the burners. One of these devices consisted of an aspirator discharging into the coal stream at the top of each burner. This caused the flames to impinge on the front wall and eroded them badly. The whirling type mixer now in use has proved fairly satisfactory for average ratings on both kinds of fuel. No experience has been obtained as yet over full range of rating.

Slagging of Bottom Tubes. In the early stages of operation with bituminous coal slagging of the two bottom rows of boiler tubes seriously interfered with ratings above 300 per cent. This difficulty was overcome by placing one soot blower at the front end between the two bottom rows of tubes, and a second one below the bottom tube at the rear entrance to the first pass, and by providing two additional lancing doors on each side of the boiler opposite the bottom tubes.

Dusting. Initially dusting was a problem of concern. The plates opposite the superheater and mud drum were replaced by an improved design. The dusting between side walls and end tube headers became serious due to the continuous motion between boiler casing and tubes, loosening the packing; this was largely overcome by installing holding plates of homemade design.

Secondary Air Dampers. This equipment does not function satisfactorily over the full range of rating for bituminous coal, anthracite, and various mixtures. Because of the wide variation in combustion conditions it is doubtful if these dampers can be made fully automatic.

Automatic Combustion Control. Control equipment of the furnace (coal feeders, stack dampers, induced draft fans) required considerable experimentation. Since the range of fan speed was extended automatic operation can now be relied upon over a range of loading sufficient for practical operation, but necessitating adjustment by hand for changes in the character of fuel.

Superheater. At the outset the superheat steam temperature was somewhat lower than specified; an exploration of the superheat space showed that a considerable part of the hot gases was short circuited around the end of the first pass baffle; the baffle was extended to the tubes with some increase in temperature; by further extending the baffle all the way through the superheater tubes the gases were forced to pass thoroughly through the superheat tubes before entering the second pass and the desired superheat was obtained at the expense of a slight increase in draft loss.

CONCLUSION

The relatively wide range of operating conditions at the Holtwood steam plant in respect to character of fuel, amount of generation, shape of load curve, coordination with hydroelectric plant, etc., involved a number of interesting problems in design and operation for which only a limited amount of experience was available from other installations. It is felt that nearly all these major problems have been solved satisfactorily, but there is still need for further experimentation to secure the best possible results from the equipment now installed.

Auxiliary Power at Richmond Station

Auxiliary Power System and Tests on House Turbine Generator

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Synopsis.—*This paper describes the auxiliary power system of the Richmond Generating Station of the Philadelphia Electric Company, mentioning briefly some of the factors which influenced the design of the system and discusses the starting of large motors at full voltage from an auxiliary turbine generator.*

* * * * *

It is the purpose of this paper to describe the auxiliary power system of the Richmond Generating Station of The Philadelphia Electric Company, mentioning briefly some of the factors which influenced the design of the system, and to discuss particularly a most important feature of this system; namely, the starting of large motors at full voltage from an auxiliary turbine generator. At the time this system was designed practically no data were available on this subject, and as there seems to be a general trend in power station design toward the use of auxiliary turbine generators or auxiliary generators direct-connected to main units, it is believed that a discussion of some of the factors involved and the results of tests made at Richmond Station after installation will be of value.

I. DESCRIPTION OF AUXILIARY POWER SYSTEM

General. Richmond Station is located in the northeastern section of Philadelphia along the Delaware River, about five miles from the central part of the city.

At the present time, the capacity of the station is 120,000 kw., made up of two turbine generators, each rated 60,000 kw., 0.85 power factor, 13,800-14,400 volts, three-phase, 60 cycles, 1800 rev. per min. Ultimately the plant will have a rating of at least 720,000 kw., and will consist of three separate building sections, each housing four turbo generators of at least 60,000 kw. capacity.

The first building section, which was completely constructed in the initial development, comprises three main parts; namely, boiler room, turbine hall, and switch house, with the boiler room nearest the river. Ultimately the boiler room will house 24 boilers. Twelve boilers have been installed initially, each rated at 1570 hp. and equipped with a superheater, an economizer, and an air preheater. Two induced-draft and one forced-draft fans are provided per boiler. Included in the turbine hall and its mezzanine galleries and basement are the main generating units (Fig. 1), condensers with their auxiliaries, heaters, evaporators, deaerators, boiler feed pumps, river and city water pumps, air compressors, auxiliary power generator and buses, generator and exciter field rheostats, main generator ventilating equipment, etc. In the switch house are installed duplicate 13,200-volt buses, oil circuit breakers, disconnecting switches, reactors, etc., all arranged for vertical phase isolation. A connecting building between the turbine hall and the switch house contains the d-c. power room, battery room, pipe room, and operating room. Transformers supplying all 2300-volt auxiliary power load and induction regulators for the three tie lines to Delaware Station are located outdoors between the turbine hall and the switch house.

FIG. 1—MAIN GENERATING UNITS

Auxiliary Power Supply. The auxiliary power system at Richmond Station is radically different from the systems at Delaware and Chester Stations, other major plants of the Philadelphia Electric system, due principally to differences in methods used for maintaining heat balance.

At Delaware and Chester, although electric drive is used extensively for auxiliaries, a number of the auxiliaries have steam drive; all boiler feed pumps are steam driven; one of the duplicate circulating water pumps provided for each unit has a dual drive by turbine and motor, the other circulating water pump

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being motor-driven and used only in emergencies; one of the two air pumps and one of the two condensate pumps of each unit are steam driven, the other air and condensate pumps being motor driven. Exhaust steam for feed water heating is obtained by running the proper number of steam-driven air and condensate pumps with the boiler feed pumps and dual drive circulating water pumps. Small adjustments to maintain the proper amount of exhaust steam are made automatically by varying the governor setting on the turbine of dual drive circulating water pumps. In view of the fact that steam drive is provided for those auxiliaries which are most essential to the continuous operation of these plants, a supply from the main buses of the station is entirely satisfactory for service to the electrically driven auxiliaries; consequently a layout wherein the entire alternating current auxiliary power supply is obtained from transformers connected to the main 13,200-volt buses was adopted for Delaware and Chester stations.

For Richmond station, investigations made subsequent to a study of many of the most modern plants in this country and abroad, indicated that for the conditions under which Richmond would operate, three-stage heating would be most economical. Accordingly, bleeding of the main turbines was adopted. Bleeder connections are provided at the tenth, twelfth, fifteenth, and eighteenth stages (the turbines are 20-stage, Curtis type). Tenth stage bleeder steam is used only to heat the building and to increase the capacity of evaporators in case of any excessive demands for make-up water. The three other stages are used in the normal operation of the heater system.

The adoption of stage bleeding for Richmond not only eliminated any need for steam driven auxiliaries as regularly operating units, but made such drive undesirable from the standpoint of operating economy. Electric drive was therefore a necessity for all auxiliaries except those installed for standby service only.

The requirement of electrically-driven auxiliaries carried with it the further requirement of a source of electrical power having the highest degree of reliability for service to motors driving the most essential auxiliaries, such as boiler feed pumps, circulating water pumps, air and condensate pumps, generator ventilating fans, etc. Consequently, a supply from the main buses of the station could not be used for these auxiliaries, since in case of bus trouble or severe system disturbances, this source might be so seriously affected as to cause the loss of these most important auxiliaries. However, a supply from the main buses was adopted for the less essential auxiliaries and for those auxiliaries, such as boiler fans, which it might be desirable to have shut down in case of a sudden loss of station load.

Three alternatives were considered in deciding on a source of power for the essential auxiliaries; namely, auxiliary turbine generators, auxiliary generators on main unit shafts, and transformers connected to main generator leads between the generator terminals and the main oil circuit breaker. Both first cost and operating cost of turbine generators eliminated them from consideration as regularly operating units, although an emergency non-condensing unit, suitable for starting large motors at full voltage as discussed elsewhere in this paper, is provided. Generators on the main unit shaft, although providing auxiliary power at low cost, added undesirable mechanical complications to the main unit, and as it was desired to have shaft-end exciters on the Richmond units because of their successful operation at other plants, this alternative was also eliminated from consideration. Accordingly, the use of transformers connected to main generator leads was adopted. This scheme has the advantages of high reliability, low first cost, and low operating cost, and although it introduces an electrical complication, the great reliability of modern transformers made this complication of but slight importance. The scheme is also open to the objection that should the unit be disconnected automatically while carrying a heavy load, either the overspeed device on the main turbine may operate causing the loss of auxiliaries, or, if the overspeed device does not operate, the auxiliary system will be subjected to a high overvoltage until the voltage regulator functions to bring the voltage back to normal. This objection can be overcome by careful adjustment of the overspeed device and by installing equipment designed to withstand the overvoltage. Equipment so designed has the advantage of high factor of safety under normal operating conditions.

As installed, the auxiliary power system at Richmond Station consists of five principal parts as follows:

1. A 2300-volt, three-phase, 60-cycle supply for boiler fan motors and large less essential auxiliaries.
2. A 2300-volt, three-phase, 60-cycle supply for essential auxiliaries.
3. A 230/115-volt, two-phase, 60-cycle supply for small auxiliaries, lighting, etc.
4. A 250-volt, direct-current supply for stoker motors, emergency excitation, etc.
5. A 250-volt, direct-current supply for control of oil circuit breakers, etc.

The 2300-volt supply for boiler fan motors and large less essential auxiliaries such as motor generator sets, fire and water pumps, is obtained from three (ultimately four) 3750-kv-a. oil-immersed, self-cooled transformer banks located between the switchhouse and turbine hall and supplied from the main 13,200-volt buses in the same manner as outgoing lines. These banks feed duplicate sectionalized 2300-volt buses located between turbine hall and the boiler room so that cable runs are of minimum length.

The 2300-volt supply for essential auxiliaries is laid out on the unit principle. A 2500-kv-a., oil-immersed, self-cooled transformer bank, also located outdoors between the switchhouse and turbine hall, is connected directly to the terminals of each generator and supplies a short bus installed on a gallery under the turbine hall floor. All essential auxiliaries associated with the generating unit are connected to this bus. To supply these auxiliaries while starting the unit, and also to act as reserve sources, each of these unit auxiliary buses is provided with two ties, one to the general 2300-volt system and one to another 2300-volt bus to which the auxiliary turbine generator may be connected. Thus each unit auxiliary bus has three separate sources of power. The breakers controlling these sources are interlocked so that normally no two of them can be paralleled, although provision is made to alter the interlocking by a synchronizing plug in case it is desired to parallel the auxiliary generator with the general 2300-volt system. In starting up a generator, the unit bus is usually supplied from the general 2300 volt system, and after the machine has been synchronized and connected to the station bus, the unit bus is connected to the transformer bank on the generator terminals. Suitable relays give an audible alarm in event of failure of voltage on the unit bus.

The 230/115-volt, two-phase supply is obtained from two (ultimately three) 1000-kv-a., oil-immersed, self-cooled, Scott-connected transformer banks supplied from the general 2300-volt system and located outdoors under the 230-volt power room at one end of the turbine hall. Adjacent to the 230-volt power room is an emergency valve control room from which all main electrically-operated valves can be closed.

The 250-volt direct-current system provided for the supply of stoker motors, emergency excitation, emergency lighting (automatic on failure of the alternating current supply), miscellaneous power, etc., is fed by three 200-kw. motor-generator sets and a 156 cell storage battery. The other 250-volt direct-current system supplies power for operating oil circuit breakers, valve motors, indicating lamps, etc., and is fed by two 20-kw. motor-generator sets and two 120 cell storage batteries. This system is entirely separate from the stoker and emergency excitation system, although an emergency tie is provided between the two. Control circuits of main and selector oil circuit breakers are operated on separate parts of the control system.

A diagram of the auxiliary power system at Richmond is shown in Fig. 2, which also indicates a typical normal method of operation. It will be noted that the system is so sectionalized that failure of any transformer bank, bus section, etc., will result in interruption to a minimum number of auxiliaries, and that such auxiliaries as may be affected can be either switched quickly to another source or replaced by duplicate units.

A list of the steam driven auxiliaries is also included in Fig. 2. All of these auxiliaries are intended for stand-by service only.

Motors. Table I lists the major auxiliary motors at Richmond, with their rating, type, method of starting, method of speed control, etc.

In general, all alternating-current motors above approximately 50 hp. are supplied at 2200 volts, three-phase, while those below are operated from the 220-volt, two-phase system. However, for special

TABLE I
MAJOR AUXILIARY MOTORS
RICHMOND STATION

Auxiliary	Motor rating	Type of motor	Starting	Speed control	Automatic restarting
Boiler feed pumps	550 hp.	Wound-rotor	Secondary resistance Push button	10 per cent range 18 points automatic	Yes
Circulating pumps	500 hp.	Squirrel-cage	Full voltage— Push button	....	Yes
Air pumps	50 hp.	Squirrel-cage	Full voltage— Push button	....	Yes
Condensate pumps	100 hp.	Squirrel-cage	Full voltage— Push button	....	Yes
Generator ventilating fans	150 hp.	Squirrel-cage	Full voltage— Push button	....	Yes
Forced draft fans	125 hp.	Wound-rotor	Secondary resistance Controller	50 per cent range 5 points	Yes
Induced draft fans	60 hp.	Wound-rotor	Secondary resistance Controller	50 per cent range 5 points	Yes
Stokers	15 hp.	D-c. shunt	Resistance— Controller	80 per cent range 22 points	No
Fire pump	200 hp.	Squirrel-cage	Compensator	....	No
River water pumps	275 hp.	Squirrel-cage	Full voltage automatic from float switch	....	Yes
Air compressors	190 hp.	Synchronous	Full voltage— Push button	....	No
Stoker supply motor generator sets	290 hp.	Synchronous	Full voltage— Push button	....	No

services such as those required for cranes or in the screen house, motors as large as 80 hp. are supplied at 220 volts.

In the design of the auxiliary power system, careful investigations were made as to the proper types of motors to be installed for the various auxiliaries. In the case of boiler fans, several types were considered including brush shifting a-c. motors, d-c. motors with variable field and armature control, as well as the standard wound-rotor motor which was finally adopted. For stoker service, both a-c. and d-c. motors of various types were considered before a decision was reached to install the variable-speed d-c. shunt motor.

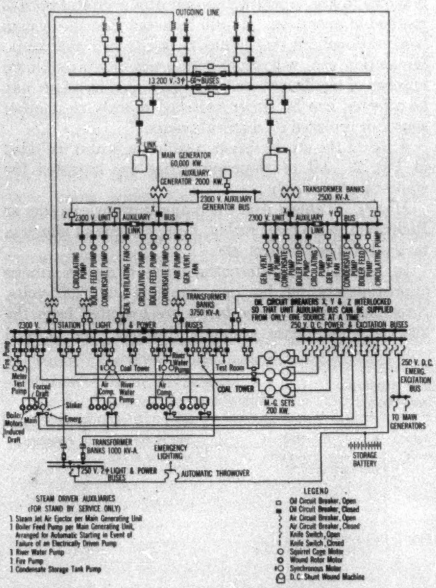
OUTGOING LINE
13,200 V.-3φ-60~BUSES
LINK
MAIN GENERATOR
60,000 KW.
AUXILIARY
GENERATOR 2000 KW.
2300 V. AUXILIARY
GENERATOR BUS
TRANSFORMER BANKS
2500 KV-A.
X
Y
Z
2300 V. UNIT AUXILIARY BUS
LINK
CIRCULATING PUMP
BOILER FEED PUMP
CONDENSATE PUMP
GEN. VENTILATING FAN
CIRCULATING PUMP
BOILER FEED PUMP
CONDENSATE PUMP
AIR PUMP
GEN. VENT. FAN
GEN. VENT. FAN
AIR PUMP
CONDENSATE PUMP
BOILER FEED PUMP
CIRCULATING PUMP
GEN. VENT. FAN
CONDENSATE PUMP
BOILER FEED PUMP
CIRCULATING PUMP
TRANSFORMER
BANKS
3750 KV-A.
OIL CIRCUIT BREAKERS X, Y & Z INTERLOCKED
SO THAT UNIT AUXILIARY BUS CAN BE SUPPLIED
FROM ONLY ONE SOURCE AT A TIME
2300 V. STATION LIGHT & POWER BUSES
250 V. D.C. POWER & EXCITATION BUSES
Fire Pump
Meter Test Pump
Forced Draft
Boiler Motors
Induced Draft
Main
Stoker
Emerg.
Coal Tower
Air Comp.
River Water Pump
Air Comp.
River Water Pump
Test Room
COAL TOWER
M.-G. SETS
200 KW.
250 V. D.C.
EMERG.
EXCITATION
BUS
TO MAIN
GENERATORS
TRANSFORMER
BANKS 1000 KV-A.
EMERGENCY
LIGHTING
250 V. 2φ LIGHT & POWER
BUSES
AUTOMATIC THROWOVER
STORAGE
BATTERY
STEAM DRIVEN AUXILIARIES
(FOR STAND BY SERVICE ONLY)
1 Steam Jet Air Ejector per Main Generating Unit
1 Boiler Feed Pump per Main Generating Unit, Arranged for Automatic Starting in Event of Failure of an Electrically Driven Pump
1 River Water Pump
1 Fire Pump
1 Condensate Storage Tank Pump
LEGEND
Oil Circuit Breaker, Open
Oil Circuit Breaker, Closed
Air Circuit Breaker, Open
Air Circuit Breaker, Closed
Knife Switch, Open
Knife Switch, Closed
Squirrel Cage Motor
Wound Rotor Motor
Synchronous Motor
D.C. Shunt Wound Machine

Fig. 2—Station Auxiliary Systems and Typical Normal Method of Operation

Motor Controls. Boiler feed pump motors are started and stopped from push button stations near them. In starting, a block of resistance is inserted in the rotor circuit and is short-circuited automatically when the motor is up to speed. Speed control may be obtained either manually from a push button station or automatically from a pressure regulator set to maintain the proper differential between feed water and steam pressures. A pilot motor controlled by the push button station or by the pressure regulator operates a drum controller which varies the amount of resistance in the rotor circuit of the motor. In case of loss of voltage, the starting resistance is automatically connected in the circuit, and upon restoration of voltage, the motor again comes up to the proper operating point.

Wound-rotor motors driving draft fans are controlled from manually operated controllers located at the boiler control panels. Control equipment is so arranged that the controller may be set at the desired operating point, and the motors will come up to this speed without additional manipulation of the controller. In event of loss of voltage, contactors operate to connect all resistance in the rotor circuit, and when voltage is restored, the motor comes back to the operating point for which the controller is set.

In connection with the control of constant speed motors, careful consideration was given to the various methods of starting that were applicable; namely, compensator starting, reactor starting, and starting at full voltage.

When the older stations were placed in service compensator starting was used extensively. However, as is well known, this type of starting has several serious objections, when used in large power stations, the outstanding one of which is the danger to life that may result from failure of the compensator. Other less important objections are the fact that the motor is subjected to two heavy current inrushes, one at the time of starting and one at the time of throwover from partial to full voltage, and the fact that automatic control with compensators is relatively complicated. As a result, reactors with shunt oil circuit breakers rather than compensators were used in several new installations, and on a number of existing installations compensator starting was replaced by reactor starting. However, although reactor starting greatly lessens danger to life, results in only one shock of current inrush to the motor, and is easily adapted to automatic control, it has the disadvantages of relatively high cost and large space requirements. In considering the type of starting to be used at Richmond Station, all of these features of compensator and reactor starting were taken into account and compared with starting at full voltage, with the result that full-voltage starting was adopted for all of the most important constant-speed auxiliaries except the fire pump, which had to be installed for construction purposes prior to the final design of the auxiliary system. Full-voltage starting not only eliminates the explosion hazard, but results in installations of minimum cost and control systems of the utmost simplicity wherein automatic restarting after an interruption is easily obtained. This last feature is particularly important, since the station operators can concentrate their attention elsewhere at times of trouble. This type of starting has, of course, the disadvantages of high current inrush during starting, possible damage to the motor because of the heavy mechanical forces to which the windings are subjected, and possible damage to the driven equipment because of the high starting torque developed. However, with the usual large amount of generating capacity in operation, the voltage dip resulting from the current inrush is negligible, and by proper mechanical design, the possibility of damage to the motor or to the driven equipment is very remote.

As installed, the synchronous motors at Richmond are controlled from push button stations. Pressing the starting button closes an oil-immersed contactor which connects the motor to the line. The field contactor closes automatically at the proper time. Protective equipment includes relays which open the field circuit in event of loss of a-c. voltage and which disconnect the motor from the line in case of loss of field, and also a time-delay low-voltage relay which on failure of a-c. voltage opens the oil-immersed contactor but which in case of short circuit in the armature winding, permits the clearing of the line oil circuit breaker before the contactor opens.

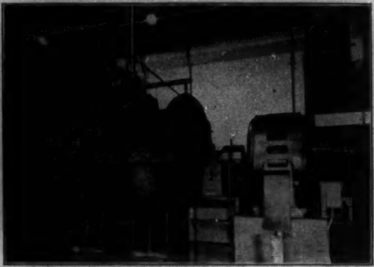

Fig. 3—Circulating Pump and Motor

Control equipment for squirrel-cage motors consists merely of an oil circuit breaker in the motor leads controlled from a twin pull button switch of the usual type located near the motor.

As shown in Table I, the largest constant speed motors started at full voltage are those driving circulating water pumps. These motors (Fig. 3) are rated at 500 hp., 2200 volts, three-phase, 60 cycles, 225 rev. per min. Full-load current is approximately 140 amperes, and starting current is around five times this value. Although these motors can be easily started under normal conditions, when they are supplied either from the main station busses or directly from the main generating units, the question of starting them from the auxiliary turbine generator was a problem of paramount importance, and was given very careful study both by the engineers of the Philadelphia Electric Company and by those of the Westinghouse Electric and Manufacturing Company, which furnished the auxiliary turbine generator.

As the auxiliary unit was for stand-by service only, it was desired, of course, to provide as small a unit as possible in order to keep investment costs at a minimum. However, in case of a complete interruption to the station, the unit must be able to start and carry sufficient auxiliaries to permit resumption of service. Since the circulating water pumps were the largest auxiliaries to be considered, the ability to start three such pumps in succession with valves closed was fixed as the duty to be met by the auxiliary unit. As a result of the study of this question, a unit of special design with a standard vibrating type voltage regulator was installed. The unit is rated 2000 kw., 3333 kv-a., 0.6 power factor, 2300 volts, three-phase, 60 cycles, 3600 rev. per min. (Fig. 4) and can be brought up to speed ready for load in fifteen seconds. Starting is accomplished by operating a motor-driven bearing oil pump; when the bearing oil has reached the proper pressure, a contact-making pressure gage opens an electrically-operated valve in the steam line to the turbine.

II. Full Voltage Starting of Large Motors

General. On account of the greater simplicity and reliability of the control equipment, and the ease with which automatic restarting can be obtained, the decided tendency has been towards the use of full-voltage starting on as many of the auxiliary motors as possible. A careful study of the problem of properly insulating motors for this class of service has been made, and the results of this study have been presented to the Institute.[3] As a result of these and subsequent tests, there seems to be no question that all squirrel-cage induction motor windings can be braced to withstand the mechanical effect due to full-voltage starting. A large number of the motors designed for low-voltage starting can be used when full-voltage starting is desired. The large high-speed motors, which have comparatively high starting currents and long coil extensions, require some additional bracing.

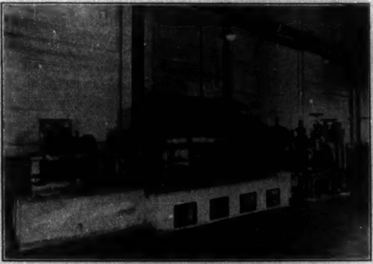

Fig. 4—Auxiliary Turbine Generator

In power stations, where the motors are started on the main auxiliary bus fed from the main unit, the starting of the motors on full voltage is usually a matter of mechanical considerations of the load. The drop in voltage at the motor terminals is due to the leakage reactive drop of the transformers and the drop in the cable and bus connecting the motor to the transformers. This is small as the transformer capacity is much greater than the largest motor to be started.

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[3] J. L. Rylander, *Effect of Full Voltage Starting on the Windings of Squirrel-Cage Induction Motors*, A. I. E. E. Trans., Vol. 44, 1925, p. 53.

When the auxiliaries are started from a small auxiliary unit not only the voltage drop due to the reactance of the generator and the drop in the leads, but also the effect of armature reaction must be considered. This may appear serious, when it is desired to start large motors, but there are several inherent properties in this particular application which make it possible to start such motors, provided good voltage regulation is not essential during the starting period.

Effect of Generator Characteristics. If calculations of the voltage drop are made assuming that both the generator reactance and armature reaction become effective instantly, and that the impedance of the load remains constant, a low value of voltage is arrived at. These are probably questionable assumptions but they are commonly used in making calculations. Since it takes time for the armature reaction to become fully effective, and also the effective motor impedance increases with speed, smaller voltage drops are actually obtained on test. When a three-phase short circuit is placed on a generator of this type, the flux will have reached a constant value in about 1.5 seconds. This is an average value for several machines. Increasing the external reactance considerably increases the time for the flux to become constant.

Low internal reactance is desirable as it reduces the instantaneous drop in voltage when a load is applied to the generator. Low reactance is inherent in the comparatively low-capacity high-speed turbine generators used for auxiliary units.

When hand regulation is used, high short-circuit ratio or good inherent voltage regulation is desirable, as the starting currents of squirrel-cage induction motors are of low power factor. This can be obtained either by using a generator of special design or a larger generator than necessary for the load or a combination of the two.

Voltage Regulation. A voltage regulator is essential when severe duty is placed on the auxiliary unit. It will not only keep the voltage constant on the auxiliary bus during normal operation but will maintain better voltage conditions when a motor is being started. High short-circuit ratio is not as essential when a voltage regulator is used. With the voltage regulator in operation the speed of response of the exciter has a decided bearing on the voltage drop obtained when a large motor is started. The higher the speed of response the nearer the effect of armature reaction is eliminated. A high speed of response is inherent in small high-speed exciters.

Effect of Load Characteristics. The characteristics of the load have a decided bearing on the size of motors that can be started with a given voltage drop. The motors driven from the auxiliary generator are usually the condensate, circulating water and boiler feed water pumps. All of these have torque characteristics which vary with speed. The torque on the circulating water pump motor varies approximately as the square of the speed. The others are nearly up to speed before the pump takes on load. The condensate pump motor is comparatively small, while the boiler feed pump is usually driven by a wound-rotor motor. The circulating water pump motor is in most cases the largest to be considered. At zero speed, the torque is only that necessary to overcome the static friction of the bearings. This allows considerable torque for acceleration, so that the motor will come up to speed very quickly and the low power factor starting current will decrease to normal full-load current of comparatively high power factor before the armature reaction has become fully effective. The voltage drop will, therefore, be less than if it took longer for the motor to reach full speed.

From the above it is seen that, if all factors are considered, it is difficult to calculate the voltage drop with any degree of accuracy. When Richmond station was being designed, calculations, using a step-by-step method, were made for several combinations of starting the circulating water pump motors. When the equipment was installed, tests which were made to determine the voltage drop, indicated better voltage conditions than calculated because pessimistic assumptions were used.

Tests at Richmond Station. The following is a brief description of the generator and the circulating water pump motors installed in the Richmond Station and the tests run.

The auxiliary generator is rated at 3333 kv-a., 0.6 power factor, three-phase, 60-cycle, 2300 volts at 3600 rev. per min. The leakage reactance is 5.6 per cent and the short-circuit ratio is about 1.7. No-load field current for 2300 volts is 68 amperes. The direct-connected self-excited exciter is rated at 33 kw., 250 volts and is a four-pole unit.

The 32-pole motors driving the circulating water pumps are rated at 500 hp., three-phase, 60 cycles and 2200 volts. These motors have a starting torque equal to full-load torque, and a pull-out torque of 2.8 times full-load torque. The slip at full load is 3.3 per cent of synchronous speed. The pump has a calculated torque characteristic which varies approximately as the square of the speed down to 70 rev. per min. Below 50 rev. per min. the torque increases with decrease of speed. The calculated torque with the valve closed is 65 per cent of full-load torque at full speed.

Since the auxiliary generator was required to start three circulating pumps in succession and only two were available for the test, an equivalent load, consisting of two generator ventilating fans driven by 150-hp., squirrel-cage motors and two boiler draft fans driven by 125-hp. wound-rotor motors, was used. The two pumps used were duplicate units on one condenser. In making the tests the generator and the above mentioned motors were isolated from the remainder of the station, so that complete tests could be made without affecting the operation of the station. Two six-element oscillographs with eight-foot films were used in obtaining records of the tests. Besides taking records of the auxiliary generator and exciter voltages and currents, one oscillograph element was connected to a special magneto, giving an indication of the speed of one pump at all times. The tests fall in four distinct groups.

FIG. 5—STARTING ONE MOTOR WITH FIXED GENERATOR EXCITATION

Group I. One circulating water pump motor was started with no-load excitation on the auxiliary generator. With the pump valve in the open and closed position tests were made with the generator under control of the vibrating type of regulator and also with fixed excitation.

Group II. With the equivalent of one motor running and with the voltage adjusted to normal, a second motor was started with the generator under the control of the automatic voltage regulator and also with fixed excitation The pump valves were open for these and subsequent tests.

Group III. With the equivalent of two motors running, and with the voltage adjusted to normal, a third motor was started with the generator under the control of the automatic voltage regulator and also with fixed excitation.

Group IV. With no-load excitation on the generator, the two ventilating fans, the two boiler draft fans, and one circulating pump motor were started together, and, after an interval, a second circulating pump motor was started. These tests were made with the generator under control of the automatic voltage regulator and with fixed excitation.

Several other miscellaneous tests which were made will be referred to when the results of the above tests are reviewed.

Tests Results. Sections of two of the oscillograms taken are shown. Fig. 5 shows the starting of a motor with fixed excitation on the generator. Fig.6 shows the starting of a motor with the generator under control of the automatic voltage regulator. Oscillograms taken simultaneously with those shown give the generator currents, exciter terminal voltage, and a timing wave. For the oscillograms shown the generator and motor currents were the same. In order to make the test results easier to interpret, the various values have been scaled off the oscillograms and plotted. The curves with broken lines indicate the results when the automatic voltage regulator was in operation, while those with solid lines indicate the results of the tests when fixed excitation was used on the generator. This same notation will be used throughout.

The results of the tests under Group I are shown in Fig. 7. With fixed excitation, the voltage dropped from 2350 volts to 1850 volts in about one second and the motor accelerated to 67 per cent of synchronous speed. From this point up to full-load speed the impedance of the motor increased rapidly, so that the current decreased. Assuming that the motor impedance is constant and that the effect of reactance and armature reaction becomes effective instantly, a value of 1400 volts is obtained with no-load excitation on the generator.

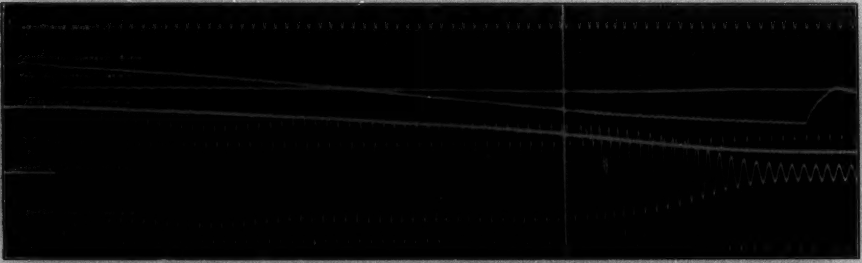
FIG. 6—STARTING ONE MOTOR WITH VOLTAGE REGULATOR IN OPERATION

Whether the pump valves are open or closed makes very little difference on the time to accelerate the motor to full speed. For either condition the current decreased to approximately 110 amperes, gradually increasing to 140 amperes when the valve was open. The motor came up to full speed in both cases in 1.8 seconds with fixed excitation on the generator.

With the regulator in operation, a minimum of 2060 volts was reached in 0.4 seconds. The regulator became effective very quickly, bringing the voltage back to normal in 1.1 seconds. In this case it took 1.4 seconds for the motor to come up to speed. The voltage was, therefore, normal before the motor reached full speed. The exciter terminal voltage built up from 110 to 220 volts in 1.1 seconds.

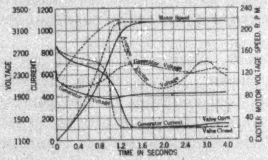

FIG. 7

The results of the tests under Group II are shown in Fig. 8. Without the voltage regulator in operation, the voltage dropped to 1920 volts in one second, and the motor reached full speed in 1.6 seconds. With the regulator in operation, the voltage dropped to 2080 volts in 0.4 seconds, and the motor was up to full speed in 1.4 seconds.

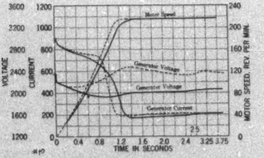

FIG. 8

The results of the tests under Group III are shown in Fig. 9. Without the voltage regulator in operation, the voltage was adjusted to 2540 volts on the generator with one motor running. The voltage dropped from 2540 to 2140 volts when the second motor was started, recovered to 2350 volts, and finally reached a minimum of 1600 volts when the third motor was started.

A record of the speed of the second motor was taken when the third motor was started. Although the voltage dropped considerably, the speed dropped gradually from 216 to 204 rev. per min. This drop in speed counteracted the voltage drop, so that the current taken by the motor only increased from 140 to 160 amperes. The inertia of the load already on the generator undoubtedly helped the conditions when the third motor was started. The tests show that the current taken by a motor actually decreases for a time. The effect of the inertia may be judged from two tests run. With the valve open and the condenser full of water, a test was made to see how long it would take the motor to stop. In four seconds the motor reversed, and at the end of nine seconds the motor had reached a maximum speed of 160 rev. per min. in this direction, due to the water flowing out of the condenser. With the valve closed, it took about 40 seconds for the pump to stop.

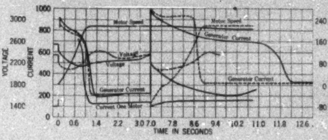

FIG. 9

With the voltage regulator in operation, the voltage dropped to 2080 when the second motor was started. The voltage had been restored to 2350 volts at the time of starting the third motor and dropped to 2070 in 0.4 seconds but was restored to normal in 1.7 seconds. In this case it took longer to bring the motor up to speed because it was running at 75 rev. per min. in a reverse direction when the voltage was applied. It took 0.6 second to stop the motor and a total of two seconds for the motor to reach full speed.

The results of the tests under Group IV are shown in Fig. 10. Without the voltage regulator in operation the voltage dropped to 1300 volts in 13 seconds, and when the last circulating pump motor was started, the voltage dropped to 900 volts and remained there. The second pump was rotating in a reverse direction and the voltage was not high enough to produce sufficient torque to stop the pump before the test was discontinued.

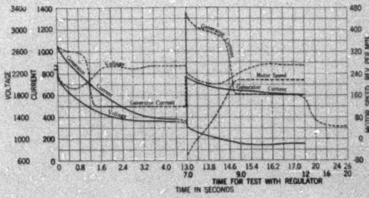

FIG. 10

Under regulator control the application of the first load caused the voltage to drop to 1920 volts in 0.6 second and it was restored to 2330 volts in two seconds.

When the second load was applied, the voltage dropped to 2000 volts in about one second and was restored to normal in 2.8 seconds. Again the pump was running at 70 rev. per min. in a reverse direction and it took a total of two seconds to start the motor.

The current, when the second load was applied, decreased rapidly to 600 amperes and remained at that value for a time, again falling to 315 amperes. The second drop no doubt indicates where some of the high inertia fans, started with the first load, approached full speed.

In no case did the average speed of the turbine fall more than 3 per cent. Although the currents were comparatively large the actual load was not excessive and came on gradually.

A test was made to determine the torque necessary to start the motor revolving. This was done by lowering the generator voltage and connecting the motor to the generator by closing the oil circuit breaker. It was found that with 750 volts on the generator, the motor revolved slowly. Considering that the starting torque varies as the square of the applied voltage, this indicates that it took approximately 11 per cent of full-load torque to start the motor revolving. This test was made after the motor had been run for a time.

It is seen from these tests that the actual voltage conditions are much better than calculated, such calculations being based on steady state conditions, assuming an instantaneous decrease of generator flux and constant motor impedance. The tests also show conclusively that an automatic voltage regulator has a decided stabilizing effect.

Recent Investigation of Transmission Line Operation

BY J. G. HEMSTREET[1]

Associate, A. I. E. E.

***Synopsis.**—This paper discusses transmission line operating experience on the 140,000-volt isolated neutral system of the Consumers Power Company, Michigan.*

Careful inspections and tests have been made to determine the conditions existing relative to insulator flashover and the findings and results are shown by tables and curves.

Ground resistance or soil conditions appear to have a very decided effect on the number of flashovers on the various lines. The necessity of providing suitable arc protection to the line conductors is shown, as is also the soundness of certain theories and recommendations for increasing the reliability of transmission lines, based upon laboratory experiments.

INTRODUCTION

THE design of an insulation system for high-voltage transmission lines that will be immune from failure during lightning storms or other abnormal conditions, is one of the most important problems confronting the transmission engineer at the present time. This is becoming increasingly so with the interconnection of large systems and the more exacting requirements of the consumers.

The experience of those who have been operating some of the larger systems should be of assistance in designing, in so far as possible, to guard against the difficulties that have been encountered, and it is with this thought in mind that the data in this paper are presented, as well as to add to the information on operating experience already available; also as a further check on certain theories and designs that have been and are proposed for the greater reliability of high-voltage transmission lines. This paper relates some experiences and the results of investigation of transmission line operation with particular reference to insulator flashover on the 140,000-volt system of the Consumers Power Company in Michigan. A map of this system is shown in Fig. 1.

HISTORY

The matter of insulator flashover became of some concern a few years ago as the system increased in size. The flashovers in some cases caused voltage disturbances or circuit breaker operation and in a few instances failure of the line due to burning of the conductor or hardware at the lower end of the insulator string. The earlier lines were not equipped with any form of arc protection but in 1920, standard 15-in. arcing horns, (shown in Fig. 2), were installed on the Argenta-Battle Creek section which at that time was added to the system. Later in the year, the Jackson-Battle Creek section was added and was also equipped with the standard arcing horn. The Edenville-Saginaw-Flint sections were added in 1924. At the time these last two sections were being designed, there was considerable discussion as to the nature of the transients, set up in high-voltage systems, that caused flashover, and based upon the theory that the trouble was due to rather sustained high frequency

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[1] Supt. of Operation, Consumers Power Co., Jackson, Mich.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

conditions[2], started by the original lightning discharge or some other cause, the insulator flux control was offered as a means of raising the flashover voltage of the insulator, as well as providing a horn as protection to the conductor. The Consumers Power system being delta-connected throughout, the possibility of the existence of severe high-frequency disturbances was thought to be more likely at the time than would be the case on other systems operating with the neutral grounded, and this was thought possibly to be the cause of some of the trouble. These last two sections of line were equipped with the flux control, as shown in Fig. 3. All of the arcing horns and flux controls were installed on the lower end of the insulator strings only.

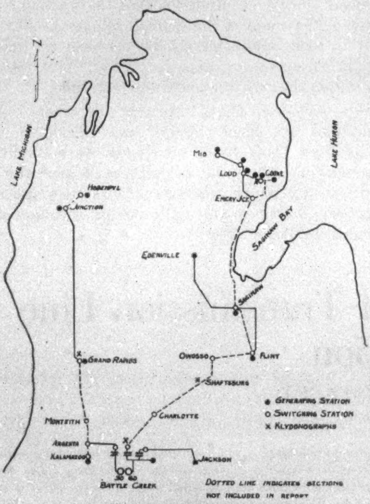
N
Lake Michigan
Lake Huron
Mio
Loud
Cooke
Emery Jct.
Hodenpyl
Junction
Saginaw Bay
Edenville
Saginaw
Owosso
Flint
Grand Rapids
Shaftsburg
Charlotte
Monteith
Argenta
Kalamazoo
Jackson
30
60
Battle Creek
● Generating Station
○ Switching Station
x Klydonographs
Dotted line indicates sections not included in report

Fig. 1—Map of 140,000-Volt System

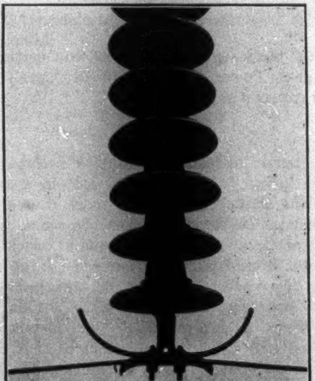

Fig. 2—Type of Arcing Horn Used

The development of the klydonograph[3] offered a means of determining the nature of the transients and four of these instruments were in service during 1925.

At the close of the 1925 lightning season, it was decided to make a careful inspection of the two sections of the transmission line added to the system during 1924. There had been no failures directly traceable to these sections but it was thought desirable to determine the exact conditions. The inspection was carried on with considerable care, an especially trained crew of men climbing each tower and carefully inspecting the insulators, conductors, and all parts of the tower tops, to locate any burns or evidence of insulator flashovers. Later, the inspection was extended to other parts of the system to obtain the comparative data, and also to locate and eliminate any weaknesses due to damaged conductors or other equipment. In only a very few cases could the damage be detected from the ground, nor had it been found by the regular patrol service.

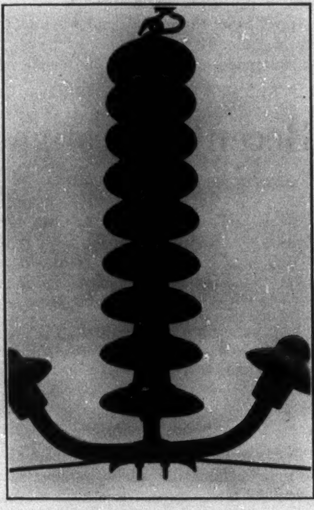

Fig. 3—Flux Control Assembly

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[2] A. O. Austin, "Insulation Systems," paper, Second International High-Tension Congress, Paris, 1923.

[3] J. F. Peters, "The Klydonograph," *Electrical World*, April 19, 1924.

Description of System

The system as shown on the map, Fig. 1, consisted of 431 mi. of line operated at 60 cycles in the eastern part of the state and 186 mi. operated at 30 cycles in the western part of the state. All 60-cycle lines are electrically connected through the busses at the station except the Jackson-Battle Creek section, this being isolated from the remainder of the system by transformers at Battle Creek. The 30-cycle lines are all connected. The two systems are interconnected through a 15,000-kv-a. frequency changer at Battle Creek. Sections shown by solid lines are those that were closely inspected and are referred to in the data in this paper. Those sections shown by broken line were put into operation prior to 1915 and were originally equipped with the older type cap and pin insulators, quite a large number of which have since been replaced. The shuffling of the insulators in replacing the defective ones made it impossible to obtain satisfactory comparative data on these sections. Capacities of generating equipment totaled approximately 225,000 kv-a. feeding into the 60-cycle system and 100,000 kv-a. into the 30-cycle system in 1925.

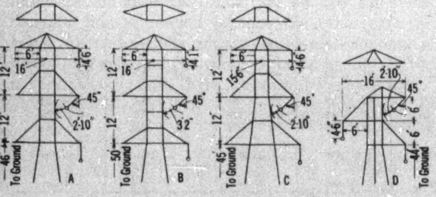

FIG. 4—TYPES OF TOWERS SHOWING DIMENSIONS AND CLEARANCES

Fig. 4 shows the types of construction conductor clearance and separation, height of tower, and other details of construction. There were no ground wires installed on any part of the system at that time, and one circuit was in place on the towers in all cases. The lines are equipped with Ohio Brass No. 25622 insulators.

Results of Inspection and Investigation

The inspection data cover a total of 305 mi. of line scattered throughout the state. The period of operation of the different sections varies from two to ten years. The country over which the lines are built is very flat, the maximum variation in height over any section being less than 200 ft. Therefore, the lines follow closely the contour of the ground.

Laboratory measurements by Peek[4] indicate that the voltage due to lightning storms in the delta-connected system such as this would be slightly less than in systems operating with the neutral grounded, all other factors, of course, being equal, and this is also borne out by comparison with the experiences on systems of the other type, so it would seem that these data should be fairly representative of results that might be obtained in other locations under similar conditions in so far as the number of flashovers are concerned.

Table I gives some additional construction details and a summary of the number of cases found where flashovers had occurred, their location on the tower, extent of damage to the line conductor, and other information. A study of this table shows some rather interesting conditions.

Rather wide variations in the results on different sections will be noted. The Junction-Grand Rapids, Mio-Loud and Loud-Emery Junction lines of very similar construction and not equipped with arc protection show a variation from 0.17 to 0.65 flashover per mile of line per year after 8, 10, and 9 years operation respectively.

The popular reason for this would be the variation in the severity and frequency of storms in the two localities. It is a fact that the Junction-Grand Rapids line runs north and south directly across the path of a great many storms as they pass inland off Lake Michigan, but on the other hand the Mio-Loud line follows very closely the bed of the Au Sable River, nearly east and west and lies parallel to the course of a great many storms which it is said by many have a tendency to follow the water courses. Possibly these factors may influence the results one way or another, but other conditions peculiar to the different lines may perhaps have a very decided influence on the operating results that have been obtained.

Height of Line Above Ground Soil Conditions, Ground Wire, and Other Factors

Measurements and tests made by Peek[5], have shown (a) that the voltage gradient between cloud and earth in the air under a storm cloud is approximately 100 kv. per ft. under severe conditions, and (b) that the voltage induced in the transmission line will vary with the height of the line above the ground, amounting to 30 to 50 kv. per ft. depending upon the closeness of the storm to the line and other factors. Also that placing a ground wire above the line reduces the lightning disturbances from 30 per cent to 50 per cent. The induced voltage, where the wires are placed in a vertical plane, should therefore be lowest in the conductor nearest the ground and correspondingly high in the other conductors.

The data showing the number and per cent of flashovers occurring on the top, middle, and lower conductors in Table I check quite closely with this law. There are some slight discrepancies, as on the Mio-Loud and Loud-Emery Junction lines, and in some cases flashovers

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[4] F. W. Peek, Jr., *Lightning and Other Transients on Transmission Lines*, A. I. E. E. TRANS., Vol. XLIII, p. 1212.

[5] F. W. Peek, Jr., *Lightning and Other Transients on Transmission Lines*, A. I. E. E. TRANS., Vol. XLIII, p. 1212.

TABLE I

SUMMARY OF FLASHOVER DATA

Section of line	Junction-Grand Rapids		Edenville-Saginaw		Saginaw-Flint		Argenta-Battle Creek		Battle Creek-Jackson		Mio-Loud		Loud-Emery Jct.	
Frequency of current, cycles	30		60		60		30		60		60		60	
Length of line—miles	101		40		44		26		43		32		19	
Conductor	110,000 cm. Copper		2/0 Copper		3/0 Copper		4/0 A. C. S. R.		2/0 Copper		110,000 cm. Copper		110,000 cm. Copper	
Number of years in operation	8		2		2		5		5		10		9	
Total number towers	1,057		348		352		270		438		301		188	
Nominal length of span—feet	530		660		660		530		530		530		530	
Average height lowest conductor—feet	31		34		34		32		32		28		28	
Number disks in insulator string—susp	10		9		9		10		10		10		10	
strain	12		12		12		12		12		12		12	
Arc protection	None		Flux control		Flux control		15 in. horns		15 in. horns		None		None	
Number suspension strings	2,811		1020		1047		843		1345		738		509	
Number strain strings	744		98		210		132		173		330		110	
Total number strings, insulators	3,555		1118		1257		975		1518		1,068		619	
Reference Fig. 4	A		B		B		C		D		D		C	
	No.	Per cent	No.	Per cent	No.	Per cent	No.	Per cent	No.	Per cent	No.	Per cent	No.	Per cent
Number suspension strings flashed over	486	17.3	34	3.3	56	5.3	73	8.7	75	5.6	43	5.8	27	5.3
Number strain strings flashed over	39	5.2	3	3.1	5	2.4	1	0.8	11	6.4	18	5.5	2	1.8
Total number strings flashed over	525	14.7	37	3.3	61	4.9	74	7.6	86	5.7	61	5.7	29	4.7
Number flashovers, top conductor	217	41	31	84	39	64	40	54	45	51	35	57	16	55
Number flashovers, middle conductor	161	31	5	13	13	21	20	27	25	29	11	18	6	21
Number flashover, lower conductor	147	28	1	3	9	15	14	19	16	20	15	25	7	24
Number cases of damaged conductor	345	66	2	5.3	7	11.5	2	3	4	5	23	38	7	24
Number flashovers per mile of line per year	0.650		0.461		0.691		0.570		0.405		0.180		0.170	
Failure of line due to burned wire or hardware	3		None		None		None		None		1		None	

occur on the middle and lower conductors and not on the top wire, but in general, agreement is noted. There is, however, quite a wide variation in the results in this respect between the individual lines. On the Edenville-Saginaw and Saginaw-Flint lines, equipped with flux controls, the percentages of flashovers on the middle and lower conductors are lower than on other sections of the line. This would tend to indicate that the flux control has raised the flashover voltage of the string to some extent, but not sufficiently to keep the lower conductor entirely free of flashovers. On the Edenville-Saginaw line, there was but one case of flashover on the lower conductor over the two-year period. The sphere-gap effect of the tubular horn used with the flux control may perhaps have some effect, together with the shielding effect of the control insulator.

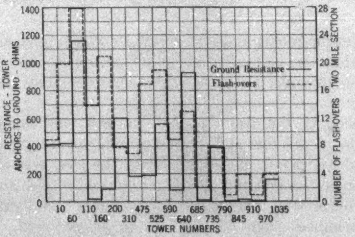

Fig. 5—Number Flashovers Compared to Ground Resistance, Junction-Grand Rapids Line

A comparison of the Edenville-Saginaw line with the Junction-Grand Rapids line shows a wide variation in this respect. The Junction-Grand Rapids line shows the smallest percentage of flashovers on the top phase, while the Edenville-Saginaw line shows the greatest with a wide variation over the three wires in spite of the higher nominal elevation of the conductors. In looking for other reasonable explanations for this difference, the possibility of soil conditions being somewhat responsible is suggested. The curves in Figs. 5 and 6 show a comparison of ground resistance measurements with the number of flashovers on the Junction-Grand Rapids and the Battle-Creek-Jackson lines.

The ground resistance was measured between the tower anchors (which are of the basket type) and ground rods driven into the ground approximately 24 ft. from the base of the tower. Complete resistance measurements of all the towers on the line were not available but at points approximately every five miles along the line, the ground resistance of several towers was measured. An average was calculated from each group, and from this data, the ground resistance curve was plotted. The broken line shows the number of flashovers occurring within one mile in each direction from the group of towers whose resistance was measured. That there is some relation between ground resistance and the number of flashovers seems apparent in the fact that both increase and decrease correspondingly with one or two exceptions which might be because of abnormal local conditions.

Peek[6] has called attention to the "water level" below the surface of the ground as being the effective ground level, and has stated that under equal conditions, an induced voltage is higher in dry sections because of the fact that the flux extends from cloud to water level, the effect being that of increasing the height of the line.

Referring to Fig. 5, it is noted that the ground resistance along the Junction-Grand Rapids line is very high, reaching a maximum of 1200 ohms. The soil along this line is very dry and sandy and in some small sections, it is almost devoid of vegetation. The country along this line is slightly more rolling in places than along some of the other lines, but not enough, apparently, to affect conditions to any extent. The high-resistance measurements were not confined to the higher points, but were found in all locations. The soil on the Edenville-Saginaw line represents perhaps the other extreme, being low and marshy with a considerable amount of heavy clay soil, and the water throughout this district is very salty; in fact, there are several chemical manufacturing plants obtaining their products from the brine pumped from the ground. A sufficient number of ground resistance measurements along the Edenville-Saginaw line were not available to plot a curve, but those taken indicate a resistance of not over four ohms.

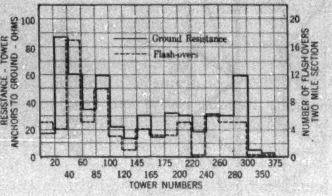

Fig. 6—Number Flashovers Compared to Ground Resistance, Battle Creek-Jackson Line

The conditions along the Battle Creek-Jackson line show lower ground resistance than the Junction-Grand Rapids line, varying between 10 and 100 ohms. The conductors being lower probably helped, but this was offset to some extent by the small arcing horns which probably increased the number of flashovers. The percentage of flashovers on the top conductor on this line is smaller than any other except Junction-Grand Rapids, and the other lines follow this trend, varying in proportion to the ground resistance. The location of the Mio-Loud line along the river probably helped to keep down the number of flashovers on account of the moist soil close under the line, also the triangular construction lowering the two top wires, undoubtedly helped to some extent.

With the increased effective height of the lines with higher ground resistance, all three of the conductors are well within the flashover voltage range and the effect of the difference in the height of each of the three conductors is proportionately smaller. On the Edenville-Saginaw line apparently the closer proximity of the lower wire to the effective ground level makes it almost immune to flashovers in spite of the higher nominal span. It also lowers the number of flashovers on the middle wire, but the height of the top wire raises it to within the range of the flashover voltage. The shorter strings of insulators on this line undoubtedly had some detrimental effect. Results of this kind suggest that the lines should be so constructed that all three conductors will be a minimum distance from ground or in a horizontal plane.

Comparison of the Junction-Grand Rapids and Edenville-Saginaw lines is of interest from the standpoint of the type of construction and possible effect of a ground wire. It is probable that if the Edenville-Saginaw line had been constructed with the horizontal configuration instead of its present form, this line would be very free from lightning trouble even without ground wires. On the other hand, the Junction-Grand Rapids line, with the same construction, would probably require one or more ground wires to produce equal results, and care would have to be taken that the ground wires were provided with a connection to water level or a good ground.

It has been supposed that flashovers might increase with the mileage of line, but the results on the Jackson-Battle Creek line, which is separated from the remainder of the system, does not bear out this theory and inasmuch as the lightning surges travel but a short distance, it is reasonable to believe that there would be no great difference.

Reports of operating results on lines in different sections of the country indicate quite a wide variation in results from the standpoint of trouble during lightning storms. In some cases, excellent results are obtained with but moderately insulated lines, and in other places, considerable trouble is experienced on much more highly insulated lines. It is apparent, however, that at least some of the conditions that cause these discrepancies are being cleared up, and before constructing lines in the future, the soil conditions and other factors not heretofore taken into consideration will be studied before the design is completed and more definite information obtained concerning the amount of insulation, number of ground wires, and other factors necessary to produce a line that will be as free as possible from these troubles.

Effectiveness of Arc Protection

Comparing the lines equipped with the flux controls and arcing horns with the others, the rather remarkable efficiency of the arc protection in reducing the damage to the conductor is noted. A decrease from 66 per cent, 38 per cent, and 24 per cent on the lines not equipped to

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[6] F. W. Peek, Jr., *Lightning and Other Transients on Transmission Lines*, A. I. E. E. Trans., Vol. XLIII, p. 1213.

3 per cent and 5 per cent on those equipped with arcing horns and 5.3 per cent and 11.5 per cent on those with flux controls is shown, the arcing horns being somewhat more effective than the flux controls. It will also be noted that there have been no failures due to burning of the conductor or insulator hardware on any line equipped with either type of horn which also emphasizes the advisability of providing some form of arc protection.

The conditions that were found on the Junction-Grand Rapids line with its 66 per cent of burns offer an excellent example of what may be expected of a line constructed in a locality of this kind without arc protection to the conductor or some means of holding down the lightning potentials.

It will be noted that the number of failures of the line due to the arc burning off the conductor, or some part of the insulator string at the time of the flashover, is rather small. There have been a total of only three in the eight years of operation on the Junction-Grand Rapids line and one on the Mio-Loud line. This, of course, is an excellent service record. However, of the 345 cases of damage found on the Junction-Grand Rapids line, a great many were so serious as to necessitate immediate repairs. Where the damage is not so great, there is, of course, some weakening of the conductor. This creates the hazard of failure at some future time when abnormal mechanical stresses are imposed upon the conductor, such as during severe sleet storms, and some failures of conductors on this line during sleet storms that have occurred in the past are now suspected of having been at least partly due to this weakened condition of the wire.

Aside from the damage to the conductor, the arcs do not seriously damage the other equipment. In some instances the top or lower disk is broken, but there has not been a sufficient amount of this to warrant the installation of protective equipment at the top of the string. In most cases there is no damage apparent from the ground. The current in the arc on the isolated system is smaller than if the neutral were grounded, which undoubtedly helps to a considerable extent.

The arcs shift around to a considerable degree over the surfaces upon which they are playing. The theory has been advanced that the arc causes a separate burn with each half cycle. The appearance of the spots tends to bear this out. The small round burns are found to be deeper and more severe on the 30-cycle than on the 60-cycle system due apparently to the greater duration in each location at the lower frequency. Using the number of spots found as an indication of the length of time the flash over exists it is thought that a great many of the flashovers are of very short duration, considerably less than one second, others, however, hold longer and evidently are the ones that cause the more serious damage.

The data also show that there are quite a number of flashovers on towers where the insulators are in the strain position. In most cases, however, the breakdown occurs between the jumper loop and the tower and it is believed in some instances that the loop is not always at a maximum distance from the tower due to wind conditions or distortion of the conductor due to other causes. Consideration in this case is being given to the use of methods of holding the loop away from the tower at the maximum distance under all conditions.

In Fig. 7 an effort has been made to show the conditions found relative to the travel of the arc and the cascading of the insulator strings. A typical tower top with the location of the burns that were found on lines with and without arc protection is shown. The figures opposite the insulator string and tower show the percentage of burns found on the different pieces of equipment with reference to the total number of flashovers. It is noted that in practically 50 per cent of the cases where no arc protection is used, the various caps of the insulators are burned. The number burned in each flashover varies, but the percentage of each is quite uniform over the total number of flashovers. The amount of cascading is not so great where horns are used varying from 19 per cent to 37 per cent on disks below the top one, but the amount of flashing to the tower basket is greater. This is particularly so with the flux controls and might indicate an effort on the part of the control to perform its duty of keeping the flashover away from the string. On the Junction-Grand Rapids line, about 15 per cent of the strings, where flashover had occurred, were found with all of the caps burned. On the other lines, however, this was of rare occurrence.

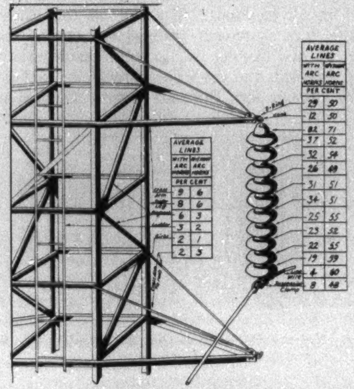

FIG. 7—RELATIVE LOCATION OF BURNS CAUSED BY FLASHOVERS

Fig. 7 shows again the very marked decrease in the amount of damage to the line conductor and clamp where arcing horns are used. These data are also of interest in showing that a very high percentage of the flashovers occur over the insulator string and does not take place between the conductor and tower independent of the insulator string as has been thought.

The cascading and burning of the caps, except at the end of the strings, apparently takes place at the time of the initial flashover, as tests that have been made on various types of arcing horns show that there is a strong tendency for the power arc to blow away from the string in almost all cases, regardless of the type of arc protection in use. During part of the 1926 lightning season, there was a new line between Argenta and Battle Creek on the opposite side of the tower from the old one which had not been put in service and had both ends grounded. An inspection showed that several flashovers had occurred on this dead line and the caps show burns similar to those on the live lines.

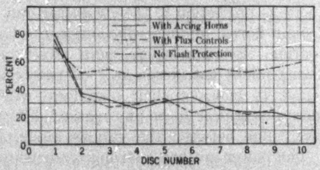

FIG. 8—CASCADING OF INSULATORS WITH AND WITHOUT ARC PROTECTION

The curves in Fig. 8 also show the amount of cascading quite clearly on the lines with and without arc protection. The curves show the effectiveness of these particular types of arc protection in reducing the amount of cascading.

MEASUREMENT OF SURGES

Four klydonographs were in service on the system during the year 1925 and as indicated in Fig. 1, instruments were placed at Cooke, Shaftsburg, and Battle Creek on the 60-cycle system and at Grand Rapids on the 30-cycle system. These instruments were in service for a period of about eight months, which included the lightning season.

In Table No. II is a summary of the record of the four installations and shows that a total of 567 surges of between 1.5 and 10 times normal voltage to ground were recorded. It is noted that only the lightning surges reach values in excess of five times normal and are undoubtedly the cause of the insulator flashover. This checks closely with operating records inasmuch as flashovers occur during lightning storms and in localities where the storms are known to be present. With the klydonographs so widely scattered, these registrations can only be taken as a partial record of the number and magnitude of the surges due to lightning, as this type of surge is of steep wave front and is quickly attenuated. In one case the klydonograph at Grand Rapids registered 10 times normal voltage on one phase and nine times on the other two. The maximum surge recorded was during a severe lightning storm directly over the Cooke station. At this time the potentiometer, which was of the ring type, supported on post type insulators, flashed over from the top ring to the bottom of the insulator column. The klydonograph flashed over between all three terminals and to ground and produced a very large image across the film. It is thought that about 15 times normal voltage would be required to flash over the potentiometer so it is evident that voltages in excess of this were present in the transmission line.

The insulator strings themselves are perhaps the best indicator of the magnitude of the surges. Peek[7] has shown that the lightning sparkover of a string of suspension units such as used on these lines is between 1,200,000 to 1,400,000 volts. The results indicate that voltages of this magnitude or greater are present on the lines and also show the correctness of the use of a potential gradient of from 30 to 50 kv. volts per ft. in calculating the voltage that may be present in lines close to lightning storms.

The surges due to switching and arcing grounds are much less severe than those caused by lightning and in so far as the line insulators are concerned, are not of

TABLE II

NUMBER AND VOLTAGE OF VARIOUS KINDS OF SURGES ON TOTAL SYSTEM AS RECORDED BY KLYDONOGRAPH

Number times normal crest voltage to ground		1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	8	9	10	Total
Switching surges	+	125	36	8	10	6	3	1	2								191
	−	1	1														2
	− +	10	4	3	1	1			3								22
Lightning surges	+	64	16	13	6	2	4	2		1	1		2				111
	−	1		1	1					1				1	2	1	8
	− +	2	12	1	8	2	1	3			2						31
Arcing ground surges	− +	11	25	35	25	8	8	4	2								118
Surges of unknown origin	+	53	11	5	3				1								73
	−	1															1
	− +	7	2	1													10
Total		275	107	67	54	19	16	10	8	2	3		2	1	2	1	567

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[7] F. W. Peek, Jr.. *Lightning and Other Transients on Transmission Lines*, A. I. E. E. TRANS., Vol. XLIII, p. 1215-16.

great concern. Undoubtedly the registrations made by them are nearer to their true value than is the case with lightning surges, as they travel greater distances along the line.

The records covering the klydonograph installations at the different locations did not vary to any great extent although the instrument at Grand Rapids recorded a greater number of more severe lightning surges than the other instruments which undoubtedly is consistent with the other flashover data, as this instrument was connected to one end of the Junction-Grand Rapids line where the most severe conditions are thought to exist.

Relay Protection

The relay scheme in use has not been shown. This has been developed along with other parts of the system over the period of years covered by these data, but in general it consisted of the usual standard relays.

In 1926 a system of ground relays operated by the unbalanced residual current in the line when a ground occurs was installed on the 60-cycle system and very good results have been obtained[8].

We have been unable to connect up all of the flashovers with surges or disturbances on the system, and it is thought that in many cases the arc extinguishes itself without starting a disturbance that would cause surging in voltage or circuit breaker operation. This is perhaps another peculiarity of the isolated system.

Insulating and Arcing Devices

Assuming that the lightning sparkover voltage of the insulator string varies with its length, it is apparent that there is an advantage in increasing the length of the string provided sufficient clearance is maintained from other parts of the structure. Insulators of rather close spacing are in use in a great many instances. One object is to reduce cascading as much as possible; also to reduce the stress across the individual units. The data on this system, however, indicate that even with a very closely spaced unit the cascading takes place. With the improvements that have been made in insulators during the past few years, the danger of puncture is much less than heretofore, so that it would appear that there is considerable gain in protection to the line from lightning flashovers as well as from the cost standpoint in using the wider spaced unit in obtaining a longer string.

In sections of the country removed from the sea coast and where trouble from fog or dust is not experienced, the use of wooden pole structures in the place of steel towers appears from the insulation standpoint to have some merit. Advantage is taken of the insulation of the wooden structure which apparently is considerable, and in certain locations like in the vicinity of the Edenville-Saginaw line, wood pole lines with the conductors carried in a horizontal plane at the minimum distance from the ground might provide a line that would be very free from lightning trouble, without the use of ground wires.

The choice of an arcing attachment for the insulator string is somewhat of a problem. Various kinds of horns and rings are offered. Some are said to be more efficient than others in not only protecting the conductor and insulators from the arc, but bettering conditions around the insulator string so that the flashover is not so apt to occur. Experience on this system has been confined to the types of horns shown in Figs. 2 and 3 and has demonstrated the high efficiency of the small arcing horn as protection to the conductor, but which, like all plain horns, undoubtedly lowers the flashover of the string. Systems where more severe arc conditions exist might not obtain as good protection and would have to consider their own local conditions. There does not appear to be any marked difference in protective value between the various kinds of plain horns. The power arc has a strong tendency to blow out away from the insulators in most cases, but devices that will prevent the cascading of the string at the time of the initial flashover might be of considerable value.

Remedial Measures

Ground wires are being installed on all tower lines now under construction on this system and are either being installed or contemplated for at least part of the older sections of the system.

On the Saginaw-Flint line, which has been completed for two years, a ground wire will be in place during the 1927 lightning season, and it is planned to secure data showing the effectiveness of this ground wire in reducing flashovers. There is a second line between these two places, shown on the map with a separation from two to eight miles, which has also been carefully inspected to determine its present condition in regard to flashovers. A number of surge recorders of the klydonograph type will be placed at frequent intervals along both of these lines and it is hoped to get a good comparison between the two lines with and without ground wire.

Ground resistance measurements are being made over a number of other lines with the idea of determining more definitely the effect of soil conditions and the location of the effective ground level and to determine the necessary protective equipment for future and also some of the present lines. It is planned to install arc protection of some type on all of the lines whether or not ground wires are installed, as it is evident there will still be a number of flashovers in spite of the equipment installed to reduce the disturbance caused by lightning.

Consideration is also being given in some locations to use of the wooden pole "H"-frame construction as a means of increasing the insulation of the line and placing the conductors in a horizontal plane as close as possible

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8. *Directional Ground Relay Protection of High-Tension Isolated Neutral Systems*, Breisky, North, and King, Summer Convention, A. I. E. E., Detroit, Mich., June 20-24, 1927.

to the ground. It would seem that this type of construction would give the maximum protection in so far as the lightning storms were concerned, provided some method is worked out to prevent the shattering of the poles and crossarms and the possible burning of the poles or crossarms in some localities due to leakage over the insulators.

Conductor clearance and separations have been increased on all new lines, and provisions made for the installation of longer insulator strings, if this seems desirable after the effectiveness of the ground wire has been determined.

On the strain type construction some means of holding the loop or jumper will be installed so that there will be no possibility of its coming close enough to the tower to cause flashovers.

CONCLUSIONS

1. Transmission lines in sections where lightning storms are prevalent are subjected to extremely high voltages, which will cause quite frequent flashovers of the insulators unless constructed in such a manner, or protected so as to hold down this voltage.

2. The severity of the conditions affecting the line varies considerably in different sections due to local surroundings and perhaps the severity of storms.

3. Unless equipped with some type of arc protection, damage is very apt to result to the line conductors, which, if it does not cause failure at the time, weakens the wire and creates the hazard of failure at some future time.

4. Cascading of the arc over the insulator strings will take place to some extent in nearly all flashovers unless the string is provided with suitable preventative equipment.

5. Surges due to switching or grounds in so far as the line insulators are concerned do not appear to be of serious consequence.

6. Careful inspection of lines is warranted on account of the defects found that are not apparent from the ground, particularly on a system operating with isolated neutral.

7. It seems desirable that a thorough study, including conditions affecting the height of line, as well as the frequency and severity of storms, be made before a line is constructed to determine the necessary protective features.

Discussion

G. H. Doan: The top wire of our 120-kv. lines is very much more susceptible to flashover and damage than any of the others. We had concrete evidence of that last year when a lightning stroke hit directly on a radio tower which was very close to the high-voltage lines. A klydonograph about a quarter of a mile away showed that there was about 5 times normal potential on the top wire, 3.3 on the middle, and 2.7 on the bottom. That, however, did not cause flashover, and did not, of course, cause opening of the oil circuit breaker.

This is further borne out by the position of broken insulators. The record for last year shows that 77 per cent of the insulator flashovers were on the top wire, 19.4 per cent on the middle, and only 2.8 per cent on the bottom wire.

We have also some very interesting data as to the good of ground wires. We started our 120-kv. system, and ran it during the season of 1925 without a ground wire on that portion which was working. During that season, we had 114 automatic switch openings. During the winter it was equipped with a ground wire, and the following year (in 1926) we had 7 openings, which is a reduction in switch openings of practically 94 per cent. The storms which passed over that area decreased 47 per cent, so that there is a possibility that the storms which we had were not so severe, and we certainly know we didn't have as many of them; but it seems that the ground wire had afforded quite a little protection.

Mr. Hemstreet points out that in dry sandy country he has experienced many more flashovers than in the lower districts. We also find that is true. In one particular section, where the land is rather high, rolling, and sandy, and the ground resistance rather high, we have experienced, I think, about 50 per cent of the flashovers of which I spoke.

R. L. McCoy: Mr. Hemstreet has pointed out the advantage of the longer-spaced suspension insulator. It has always been the policy of the Locke Insulator Corporation to recommend the use of a relatively long-spaced suspension unit.

A few years ago the major insulator problem was that of preventing puncture of the units by the voltages impressed upon them. That problem is now well in hand and the major problem is to prevent lightning flashovers as much as is possible and feasibly economical and to prevent the attending damage.

Researches which have been made by Mr. F. W. Peek have shown us the nature of the voltage impressed upon line insulators by lightning.

It is desirable that we study the action and characteristics of line insulators under these voltages. The lightning generator has given us an opportunity to make these studies. A very careful investigation has been made with lightning. We find that with all types of suspension insulators regardless of the spacing, cascading occurs. This fact is borne out by Mr. Hemstreet's experience.

The reason for this is found in the fact that the distribution of the voltage impressed on the various units of the insulator string is not uniform. The line unit bearing the brunt of the surge flashes over before the remaining insulators flash over. This, then, makes the second unit from the line at the same potential as the line and it flashes over. The flashover therefore is progressive, one unit at a time, with the arc striking to each insulator cap. The power arc follows the path of the lightning arc and starts as a pure cascade. This accounts for a large number of cases where we find cascading on all of the units of an insulator string when it was flashed over by lightning. Incidentally, we find this on insulators of all spacings.

Our problem then is to find a way of using the insulation in the insulator to the best advantage to produce high flashover values which will reduce number of flashovers, and prevent cascading of the lightning arc. Recent studies with lightning show no appreciable difference in cascading between insulators of minimum spacing and insulators of approximately 6-in. spacing for a 10-in. disk.

Flashover values are directly in proportion to the string length. The longer spacing is obviously advantageous from this point of view.

Mr. Hemstreet states further that the power arc has a strong tendency to blow away from the insulator if the initial cascade caused by lightning can be prevented. The grading shield offers a means of doing exactly this. The reason for this is that the grading shield reduces the voltage across the line unit and makes the voltage duty upon each unit more nearly uniform and when the flashover occurs it is a complete flashover from ring to ring, or, ring to horn, and the arc is started several inches away from the insulator. The chances of its doing damage therefore are much reduced.

Recent investigations show that the grading shield can be designed to prevent cascading of the lightning arc, and also because of better field conditions established, give an increase of from 15 to 20 per cent in the flashover value over that of the unprotected string. This is interesting, because on 132,000 volts or above we are near the point where the insulators have a flashover value equal to the voltage imposed upon them. An increase, therefore, of a few per cent in the flashover value may reduce very materially the number of flashovers.

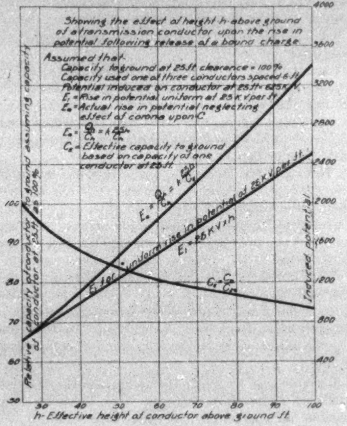

FIG. 1

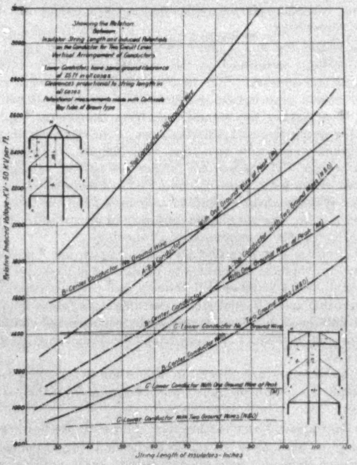

FIG. 2

A. O. Austin: In view of some of the points raised by Mr. Hemstreet's paper, it would seem that some recent theory and investigation should be of interest.

Flashovers on transmission lines are reduced by raising the effective flashover voltage of the insulator between conductor and ground, or by reducing the abnormal potential on the conductor. In addition to these two methods or factors, a new method has been developed which I have called the counterpotential method.

In this discussion I wish to call particular attention to the last two methods.

The new method eliminates the danger of burning and permits taking advantage of the very high flashover values for lightning possessed by wood structures, to which Mr. Hemstreet called attention.

Reduction in the effective height of the conductor and the installation of ground wires are two factors which can be used to advantage for increasing the ratio of flashover voltage to surge potential for steel tower lines. Recent improvement in the theory of the ground wire gives a working knowledge of the various factors governing its functioning so that it can be evaluated for any installation. In applying the ground wire, to obtain best results, it is necessary that we have a knowledge of just how it functions.

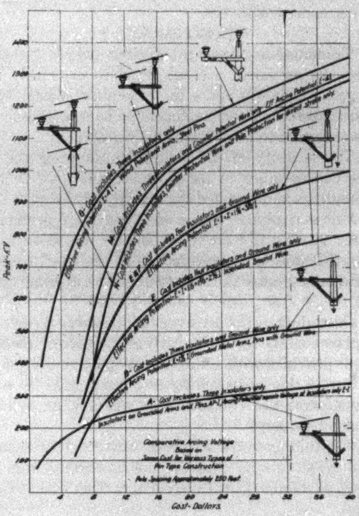

FIG. 3

A charged cloud over the power conductor will induce a charge on the conductor. This charge is a bound charge and will be the same for a conductor or a ground wire in the same position. Following the discharge of the cloud, the bound charge will be released and cause the potential of the conductor, if insulated, to rise. This rise in potential is of great importance and a study of the factors has been made. Before the cloud discharges, the conductor may be regarded as forming a condenser with the cloud. Following the discharge the conductor forms a condenser with ground or objects of lower potential such as a ground wire. In general, the potential on a conductor may be said to vary as Q/C where Q is the charge and C the effective capacitance of the conductor to ground. It therefore follows that factors which affect Q and C must be given consideration in determining the rise in potential for a given gradient.

The charge Q will increase directly with the height in a field of uniform gradient while the value of C is reduced with the height of the conductor. It therefore follows that the induced potential Q/C will increase at a more rapid rate for greater heights of conductor. This is shown in Fig. 1 herewith.

Bringing the power conductors nearer together reduces Q, but this advantage is partially offset since it also reduces C. Where a ground wire is used to increase C, it is evident, however, that close spacing reduces Q and increases C. Since the ground wire functions by increasing C rather than by reducing Q, its location and number should take this into account rather than its location for screening.

By the use of a cathode ray oscillograph of the Braun-tube type it is possible to measure induced voltages on model lines with ease.

Fig. 2 shows a study based on string length. In this study the induced voltage on the conductors of a two-circuit line is shown as measured by this method. It will be seen that the induced potential rises very rapidly with the increase in string length. Increasing the length of a string 2 ft. not only spreads the conductors but raises the top conductor at least 10 ft. Where the electrical gradient set up by the cloud is steep, it is seen that increasing the string length may result in more flashovers unless the height of the conductors is kept down or the factor C increased by placing ground wires at other points than at the peak of the tower.

In wood-pole structures the problem is different. The installation of a ground wire may eliminate insulation several times that of the insulator. It therefore follows that where a ground wire is usually an advantage on a steel-tower line, it almost invariably increases line trouble when applied to a wood-pole line which has effective insulation. The reasons for this are clearly shown in Figs. 3 and 4. It will also be noted that the insulated ground wire and the counter-potential wire have great advantages over the ground wire for preventing flashovers induced by lightning. While the insulated ground wire cannot be used where unbalanced leakage is likely to burn the pole, the counter-potential wire takes care of this and also facilitates the operation of relays.

The counter-potential wire may be very effective when applied to a steel tower having an insulating zone. In operation, the release of a bound charge raises the potential of the counter-potential wire and connected tower top or cross arm coincident with the rise in potential on the power conductor. Since the potential of the insulator support is raised, the stress tending to that flash the insulator or conductor will be reduced by the amount the tower top is raised by the counter-potential or charging wire.

By properly spacing the height of the counter-potential wire it is possible to control the potential of the support adjacent to the insulator so that the flashover voltage of the insulating zone of the tower can be added to that of the insulator, giving exceedingly high flashover values for what would otherwise be a low or moderate flashover voltage system. In addition to giving exceedingly high flashover values it makes it possible to effect very material economies due to the closer spacing of conductors.

Since the danger of burning of the insulated section is removed by the counter-potential wire and relay operation is insured, this new method of line construction has material possibilities which may be applied for improving old lines as well as being adapted for new construction where it is desired to reduce or eliminate flashovers.

It is also possible to use the construction in conjunction with a ground wire. It is, however, necessary to place the ground wire below the insulated zone or to insulate it from the upper section of the structure.

Mr. Hemstreet raises the question as to the effect of ground resistance on flashovers. From a consideration of the counter-potential wire it is apparent that if the effective height of the conductor has not been increased, a high ground resistance may permit the structure to be charged momentarily. This will tend to reduce the stress tending to flash the insulator so that the danger of arcing would really be less. If the nature of the ground, however, is such that the conductors have a very high effective height, this may more than offset the high ground resistance and result in increased flashovers.

Impact tests made recently on long strings with much larger amounts of energy than have heretofore been available, show that many insulator strings or arrangements which it has been assumed (from tests on short strings), would arc free, frequently cascade. These tests apparently give results similar to those shown in the examination carried out by Mr. Hemstreet. It is hoped that an oscillograph record of the flashovers on test will throw much light on the nature of some of the various types of flashovers noted on the system.

While the path of an arc may be determined at the expense of flashover voltage, once formed further control of the arc is lost, hence operation can best be improved by increasing the effective ratio of flashover voltage to the surge voltage. With a better understanding of the various factors it is now possible to consider the cost of the line per mile on a flashover basis as well as on a mechanical basis. When this is carried out, great improvement in the elimination of flashovers will result.

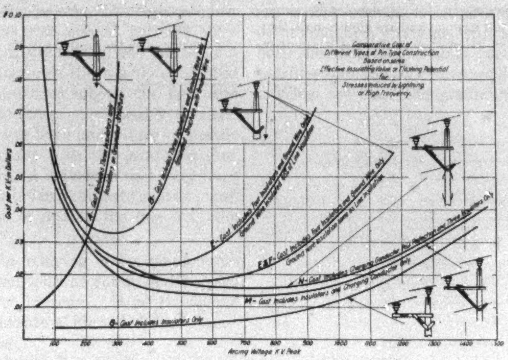

Fig. 4

W. L. Lloyd: I am glad to note that the operating results have checked so well Mr. Peek's laboratory tests, in that the lightning voltage is directly dependent upon the height of the phase wire above conducting soil.

With wet soils, the greatest number of flashovers should occur on the upper conductor. I understood Mr. Hemstreet to say that actually 85 per cent of the flashovers occurred on the upper conductor under such conditions, whereas the lower conductor was practically immune from lightning.

With dry soils, where the conducting level is perhaps far below the ground level, and the percentage difference between the effective heights of the three vertically spaced conductors is greatly diminished, there should be more flashovers; but a more even distribution in the number of flashovers should be expected. This is in accordance with the operating results reported by Mr. Hemstreet.

On important lines, we recommend the ground wire to reduce the number of flashovers and the grading shield to eliminate damage to the insulator string or line conductor in the case of those fewer number of flashovers which remain.

I am particularly interested in the klydonograph surge-recorder measurements on this system, and am glad to note that these measurements check so well with our estimates based upon tests in the laboratory with Mr. Peek's 2,000,000-volt artificial-lightning generator.

J. G. Hemstreet: It is gratifying and encouraging to know that the representatives of operating and manufacturing companies and those working in the laboratories on this same problem are in agreement on practically all of the various points that have been brought out.

It is evident that a great deal of the mystery surrounding the transient conditions existing in transmission systems is being cleared away and the knowledge that has been gained of these conditions from experience in the field and laboratory tests and investigations may enable us to provide high-voltage transmission lines that will be very free from trouble due to lightning.

Mr. Austin has introduced somewhat of an innovation in the counter-potential method of preventing insulator flashovers. Results of experience in the field will be awaited with interest to determine if this method is as effective as a ground wire. One of the benefits obtained from the use of ground wire is the protection to the station equipment by holding down the transient voltages in the transmission line. The counter-potential method evidently does not accomplish this purpose, but possibly a combination of the two using ground wire near the stations might accomplish the desired results.

Ground Relay Protection for Transmission Systems

BY B. M. JONES[1] and G. B. DODDS[1]
Member, A. I. E. E. Non-Member

***Synopsis.**—The relay problems of an electric power system are most important, and are very vital to the successful operation of the system. More and more consideration is being given to the relay engineers' point of view with resultant improvement in system operation. Several large, successful companies undertake no project having any bearing on the primary electric system without due regard to the relay engineer's recommendation.*

With the transmission networks becoming more and more interconnected, and with the injection into the problem the interconnections with other power companies, the absolute necessity of isolating a faulty line, (or a faulty piece of equipment), is essential and is becoming more clearly recognized by all large companies.

Since this article was written, other 66-kv., ground relay tests have been made with a resistor grounded neutral and with a solidly grounded neutral, and some very interesting and unexpected points were discovered. It is hoped these points will be brought out during the discussion of this paper.

* * * * *

THE question of ground protection has always been a serious one and also a very troublesome one. In the days when transmission systems were operated with a free or floating neutral and one leg of a line became grounded, this would throw a high potential on the other two legs and thus subject the lines and equipment to an unusual strain and possible damage. It would also endanger the lives and safety of people who might come in contact with the grounded line. To locate such a ground in a network was a very hard thing to do and often resulted in outages to a great many customers in the process of finding the line with the fault on it.

The subject of ground relaying is becoming more important, and is attracting more attention in the operation of light and power systems. It is considered absolutely necessary to isolate a grounded line immediately, for several reasons:

1. To reduce chance of injury to people who may come in contact with or approach a grounded line, thereby encountering dangerously high ground potentials,
2. To reduce the resultant damage to apparatus and lines by removing a ground as quickly as possible from the system.

On systems using overhead construction on wooden cross-arms, the ground currents encountered are relatively small and very sensitive ground relays are required to recognize these small currents.

As a result of the troubles encountered with an ungrounded system, a great many companies now make a practise of grounding their transmission system. The grounding of a system may be done in a number of ways, which are shown in Fig. 1. Where the star point of transformers or generators is brought out, this point can be grounded, either solidly or through a resistance. When there is no neutral point available, a grounding transformer may be installed. Grounding transformers are usually connected either star-delta, with the star point grounded, or zigzag, with the star point grounded, both methods having their particular advantages depending upon the particular application.

When the star-delta grounding transformer is used, the star point may be grounded either solidly or through a resistor. If desired, a load may be taken from the delta-connected secondary of the transformer bank. The advantage of the zigzag-connected grounding transformer bank is the relative cheapness of it, since with it no secondary winding is required and thus it is somewhat cheaper than the star-delta bank.

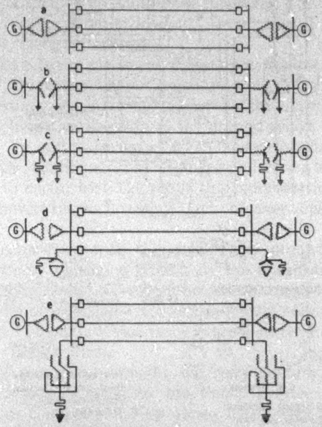

FIG. 1—METHODS OF GROUNDING TRANSMISSION SYSTEMS

a. Early ungrounded systems
b. Solidly grounded systems
c. Resistor grounded systems
d. Star-delta grounding transformer banks systems
e. Zigzag grounding transformer banks systems

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[1] Duquesne Light Company, Pittsburgh, Pa.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

Where the system is solidly grounded, the relaying for ground faults is comparatively simple if the fault current obtained is greater than the load current. In some cases, however, the current for ground faults is comparatively small, due either to long transmission lines or to contact resistance at the point of the fault. Where this condition exists, the phase relays will not protect the line for ground faults, and other means of protecting the lines for grounds must be used. It is interesting to note in this connection that some companies with very long high-voltage transmission lines protect these lines against ground faults only.

On systems which are grounded through a resistor in the transformer bank neutral, the relaying for ground faults is somewhat involved, due to the small ground currents obtained and, in the case of a network, to the distribution of this current through several sets of lines and relays.

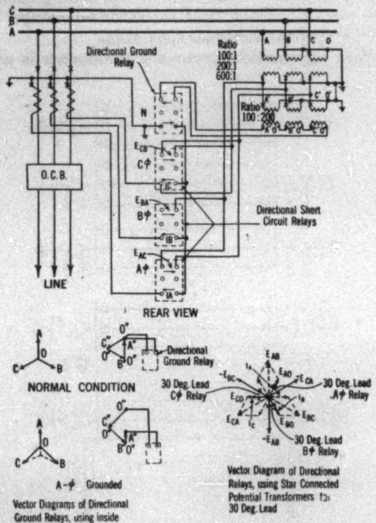
C
B
A
Directional Ground Relay
Ratio 100:1 200:1 600:1
A B C O
Ratio 100:200
A' B' C' O'
N
E_{CB}
Cϕ
IC
O.C.B.
E_{BA}
Bϕ
IB
Directional Short Circuit Relays
E_{AC}
Aϕ
IA
LINE
REAR VIEW
A
O
C
B
NORMAL CONDITION
O"
C"
A"
B"
Directional Ground Relay
A-ϕ Grounded
Vector Diagrams of Directional Ground Relays, using inside Delta Potential
E_{AB}
I_A
$-E_{BC}$
E_{AO}
$-E_{CA}$
30 Deg. Lead Cϕ Relay
30 Deg. Lead Aϕ Relay
E_{CO}
I_B
E_{CA}
I_C
E_{BO}
E_{BC}
$-E_{AB}$
30 Deg. Lead Bϕ Relay
Vector Diagram of Directional Relays, using Star Connected Potential Transformers to 30 Deg. Lead

Fig. 2—Directional Short-Circuit and Directional Ground Protection

Many different schemes for ground protection have been devised, some of which are indicated below:

1. Overcurrent non-directional relay in neutral of current transformer circuit,
2. Overcurrent non-directional relay interlocked with directional elements of phase relays,
3. Power relay with instantaneous time but interlocked with overcurrent non-directional relay for timing,
4. Power relay with self-contained timing element,
5. Impedance ground relay,
6. Power directional relay using inside delta voltage,
7. Balanced current ground relays for parallel lines.

The overcurrent non-directional ground relay is satisfactory for use on radial feeders or other locations where directional protection is not required, such as bus or transformer differential protection, or on radial lines. In connection with such uses, a relay with a low operating range and one which puts a small volt-ampere burden on the current transformers is very desirable and has been developed. This type of relay is very sensitive to small ground fault currents and, due to its low volt-ampere burden, does not overburden current transformers which have other apparatus than relays connected to them.

The overcurrent non-directional relay with its trip circuit interlocked with the directional element of the phase relays was used quite extensively and proved fairly satisfactory in some cases. This type of protection, however, has one main inherent defect which is impossible to overcome. This defect is that on a single-line loop having more than one looped station and having a feed from both ends of the loop, a fault may occur which, while providing enough current to operate the ground relay, will not be of sufficient magnitude to overbalance the load current, with the result that the directional contacts on the wrong set of relays will be held closed and the wrong breaker tripped out.

The power relay has not proved satisfactory for ground protection, due primarily to the fact that a sufficiently low range relay has not been used. With this scheme, the power relay obtained current from the neutral of the current transformers and voltage from the inside delta voltage of an auxiliary set of star-delta connected potential transformers. With this scheme of ground protection and with a fault occurring some distance out on a line away from the substation at which the relays are located, the resultant voltage distortion at the substation may be relatively small, even though the fault currents be comparatively large. Under these conditions of fairly large ground currents and small voltage impressed on the power relay, in conjunction with the poor power factor which sometimes exists during ground faults, the resultant watts in the power relay may be so low that the relay will not operate at all or will operate very slowly. Due to this condition, the possibility of clearing ground faults in a minimum time is decreased.

The low energy power directional ground relay using inside delta potential is one of the most satisfactory schemes. A diagram of connections is shown in Fig. 2. In this scheme the same potential is used as was used in the power relay, but it is used only to operate the directional contacts of the relay, the relay having a separate overcurrent element similar to that of the overcurrent relay. The overcurrent and directional contacts are connected in series as in other power directional relays. In this type of relay, the timing is obtained from the overcurrent element. By using this scheme, the directional elements may be made to operate on a very small number of watts, thereby making it sensitive to very small ground currents. An installation of this type of protection is shown in Figs. 3 and 4, which views were taken in a small substation on a consumer's property, there being one 22-kv. line looped through this station. Fig. 3 shows the front view of the relay panel and Fig. 4 shows the rear view of the panel with the auxiliary star-delta connected potential transformers, and also shows the individual relay test switches for each relay.

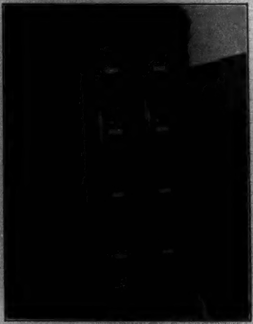

FIG. 3—FRONT VIEW OF CONSUMER'S RELAY INSTALLATION

The impedance type of relay has been used with reasonable satisfaction on systems which are solidly grounded. These relays may be self-contained, in which case one per phase, or three for a three-phase line, will be required, or the double-element phase and ground relay combined in one case may be used. These relays may obtain their potential direct from the line potential transformers or may use low-tension potential through compensators. The ground impedance relays have the same time characteristics as the phase impedance relays, thereby clearing faults close to the station in a short time and increasing the time for faults farther away from the station. These relays are not so satisfactory, however, for use on a system which is grounded through a resistor, since the voltage drop is usually not large enough to cause the impedance type ground relays to operate in a reasonably short time.

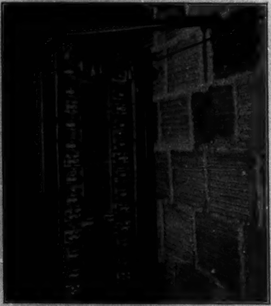

FIG. 4—REAR VIEW OF CONSUMER'S RELAY INSTALLATION

The balanced current type of protection is limited in its application to parallel lines. It furnishes a simple and fast method of clearing all ground faults and is an inexpensive method, for no potential transformers are required. The station must have two sources of feed, however, and when one of the pair of lines is out of service, an extra set of relays is sometimes required in order to permit increased time of operation to obtain selective action with other stations. This type of protection has been described in detail by Mr. H. P. Sleeper.[2]

There are several methods of checking the current and potential phase relations on the ground relays in order to determine whether the relay will operate correctly under fault conditions. Among these methods may be mentioned the following:

1. By using phantom load,
2. By using actual load,
3. By actually placing a ground upon the line.

The Duquesne Light Company, along with several other companies, is strongly in favor of the latter method of checking the direction of the ground relays, having found by sad experience, when using the first two methods mentioned, that mistakes and errors are very liable to occur, with the result that under actual fault conditions, improper relay and breaker operations result.

In connection with the checking of ground relays by actually placing a ground on the line, some very interesting results have been obtained, among which is the relation of ground current to phase voltage with different amounts of resistance in the high tension transformer bank neutral.

This point was brought out very forcibly during some tests conducted by the Public Service Electric and Gas Company of New Jersey which were made in July 1926. A tabulation of their test data is shown in Table I. An analysis of this data shows that with no resistance in the transformer bank neutral, a phase angle of around 90-deg. lag is obtained, and with a resistance of 75 ohms in the transformer bank neutral, the phase angle varies from 5-deg. lag to 15-deg. lag, while with a resistance of 300 ohms in the transformer bank neutral, the power factor ranges from unity to 18-deg. lead. As a result of the poor power factor obtained under conditions of the transformer bank neutral being solidly grounded, the resultant torque on the relay

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[2] H. P. Sleeper, A. I. E. E. TRANSACTIONS, Vol. 42, 1923, p. 513.

Discussion, A. I. E. E. JOURNAL, February, 1925, p. 182.

TABLE I

DIRECTIONAL GROUND RELAY TESTS

Test Data of High-Tension Line Ground Tests Made by the Public Service Electric and Gas Company

Test number	1	2	3	4	5	6	7	8	9
Date	7/11/26	7/11/26	7/11/26	7/11/26	7/11/26	7/11/26	7/11/26	7/11/26	7/11/26
Time	9:55 a.m.	10:30 a.m.	10:37 p.m.	12:15 a.m.	12:55 a.m.	2:45 p.m.	3:45 p.m.	4.00 p.m.	4:30 p.m.
Voltage of line tested	26 kv.	26 kv.	26 kv.	26 kv.	26 kv.	26 kv.	26 kv.	26 kv.	26 kv.
Bank kv-a	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Neutral resistance	0	300	300	300	300	75	75	75	0
Line grounded	Phase 1 of X-102	Phase 1 of X-102	Phase 1 of X-102	Phase 3 of S-357	Phase 3 of S-357	Phase 3 of S-357	Phase 1 of R-356	Phase 2 of N-196	Phase 2 of N-196
Location of ground	1 mi. from Essex	1 mi. from Essex	1 mi. from Essex	¼ mi. from Metuchen	¼ mi. from Metuchen	¼ mi. from Metuchen	1 mi. from Essex	1/8 mi. from Metuchen	1/8 mi. from Metuchen
No. of ground relays in series on C. T.	1	1	1	1	1	1	1	2	2
Ground relay current setting	0.4	0.53	0.2	0.53	0.53	0.53	0.53	0.5	0.5
High-tension ground amperes	240	64	49.6	90 +	102	178	99	160	420
Ground relay									
Volts	10	150 + (F. S.)	190	180	164	164	146	152	120
Watts	Meter reversed	120	98	240	245	450	230	280	2
Amperes	3.0	0.8	0.6	1.5 +	1.7	2.97	1.65	2.0	9.4
Phase angle	93-deg. lag.	7-deg. lead	10-deg. lead	0	18-deg. lead	10-deg. lag	5-deg. lag	15-deg. lag	90-deg. lag
L-t. star volts before test									
1-N	65.0	66.2	67	63	63	63	63	63	62
2-N	65.1	65.5	66	62.9	62.6	62.6	62.7	63	62
3-N	64.5	65.0	65.5	63	63	63	63	63.5	62
L-t. star volts during test									
1-N	64.5	23	23	106	101	99	24	93	64
2-N	68.8	88	87	98	98.5	97	84	20	33
3-N	63.0	106	106	20	18	18	97	92	82
L-t. delta volts before test									
1-N								110	106.5
2-N								110.5	107.8
3-N								109.2	109
L-t. delta volts during test									
1-N								109.5	103
2-N								111.5	97
3-N								109.2	94

disk is very small, even though the fault current be relatively large, and if the phase angle is greater than 90-deg. lag, the direction of operation of the relay will be reversed.

Similar tests were conducted by the Duquesne Light Company during May of 1926, in order to determine the following points:

1. To determine the minimum amount of ground current on which the 50-watt power relays would operate,
2. To make an actual check on the direction of operation of the power ground relays,
3. To determine the effects of putting more than one potential element of the ground relays in parallel on the same set of auxiliary potential transformers,
4. To determine the sensitivity of the low-energy power directional ground relays as compared with the 50-watt power relay.

The test data are shown in Table II and the conclusions arrived at are as follows:

1. The power relays set on a 50-watt tap would not operate satisfactorily with less than a 300-ampere ground, the ground being located at the substation at which the relays were installed. As the ground location would become farther away from the substation, the ground current necessary to operate the relay would become higher, due to less voltage being impressed on the power relay.

2. The direction of all ground relays tested was found to be correct.

3. The addition of an increased number of potential elements of ground relays to one set of auxiliary potential transformers did not result in any noticeable drop in voltage on the secondary of the auxiliary potential transformers. This indicated that the auxiliary potential transformers could be loaded up to their volt-ampere capacity.

4. It was found that the low-energy power directional ground relays would operate satisfactorily on a much lower ground current than the power relays.

Since the above tests were made, low energy power directional ground relays have been installed on the majority of the lines of the Duquesne Light Company 22-kv. system, and a large number of these relays has been tested by actually putting grounds on the lines which these relays protect.

The equipment used in making the line ground tests is shown in Fig. 5. Fig. 5A shows a two-ton trailer on which the necessary equipment for making high-tension line ground tests is mounted. Fig. 5B shows the water rheostat set up and connected to the line. Fig. 5C shows the water barrel emitting flame during line ground tests made at night. Night testing is sometimes necessary due to being unable to obtain the necessary outages during the day. The grounding is done by connecting a water rheostat between the line and a ground connection. The rheostat consists of a barrel inside of which is one fixed and one moveable brass plate. The ground current obtained can be varied by changing the distance between two brass plates im-

TABLE II

DIRECTIONAL GROUND RELAY TESTS

Test Data of High-Tension Line Ground Tests Made by Duquesne Light Company

Test no.	1-A	1-B	1-C	2-A	2-B	2-C	3-A	3-B	3-C	4-A	5-B	6-A
Date	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26	5–23–26
Time	9:48 a.m.	9:59 a.m.	10:11 a.m.	10:55 a.m.	10:44 a.m.	11:11 a.m.	12:11 a.m.	12:25 p.m.	12:36 p.m.	12:45 p.m.	2:39 p.m.	4:12 p.m.
Voltage of line tested	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.	22 kv.
Type of C. T.	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing	400/5 Bushing
Line grounded	Beaver F. Morado	Beaver F. Morado	Beaver F. Morado	Beaver F. Morado	Beaver F. Morado	Beaver F. Morado	Morado Koppel	Morado Koppel	Morado Koppel	Morado	Morado Jct. Pk.	Morado Jct. Pk.
Location of ground	Morado	Morado	Morado	Morado	Morado	Morado	Morado	Morado	Morado	Morado Koppel	MoradoNo.1	MoradoNo.2
Bank kv-a.	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
Current to ground	60	80–100	Off scale 400	70–80	160	500	400	480	440	440	408	200
Current in ground relay	Could not read	0.5	Off scale	0			4.0				3.5	2.0
Direction of operation of ground relays	Over current non-directional relays installed			Closed	Closed	Closed	Closed	Closed	Closed	Closed	Closed	Closed
Volts across pot. coil of relay		5–10	Off scale	10	15	52	47	54	46	44	58	35
L-t. star volts 1-N	62.5	61.5	62.4	62.5	62.5	62.5	49.5	Off	65.	65.	63.5	63.5
Before 2-N	63.0	61.5	61.5	61.5	61.5	61.5	49.5	Off	65.	65.	64.0	64.
Test 3-N	64.5	64.5	62.7	67.7	64.0	67.0	49.5	Off	66.	66.	65.0	64.
L-t. star volt 1-N	61.0	60.0		59.0	53.5	37.0	45.0		68.	42.	67.	67.
During 2-N	64.5	62.5		63.0	67.0	72.0	49.0	45.	52.	70	51	68
Test 3-N	64.0	64.0		63.0	64.0	67.0	45.0	35	75	66	72	68
Neutral resistor	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75	3.75
Phase angle					18-deg. lag	18-deg. lag						

mersed in the water. The ground lead is connected to the fixed brass plate and brought out through the side of the barrel. A current transformer is connected in this ground lead and is used for measuring the ground current.

This equipment is all mounted on the two-ton trailer which is hauled from place to place by a truck. The equipment consists of the following:

1 37-kv. single-pole oil circuit breaker,

1 30-cell, 60-volt battery,

1 Portable control panel with flexible leads 50 ft. in length,

2 Water barrels to be used as rheostats,

1 Water barrel for water supply,

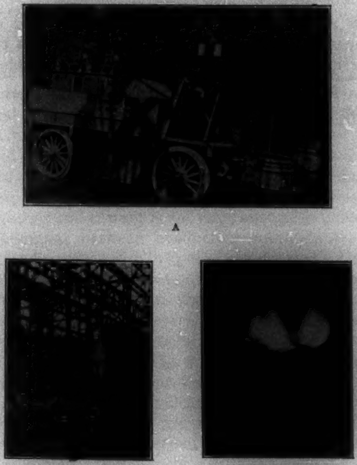

Fig. 5—Equipment Used for Making High-Tension Line Ground Tests

a. Trailer and equipment
b. Water rheostat set up for test
c. High-tension line ground tests made at night

2 Insulated stands on which to place the water barrel rheostats,

2 220-volt regulators,

3 Tool boxes for equipment,

3 Current transformers having the following ratios, 500 to 5, 400 to 5, 200 to 5,

3 12-volt flood lights,

Assorted leads of "00" flexible Tyrex single-conductor cable,

Assorted clamps of different types,

A number of pillar type insulators,

First aid box,

Fire fighting equipment,

Blocks and tackle.

The above equipment is all carried on the trailer and provides all of the equipment necessary for conducting grounding tests with the exception of portable meters which are not transported on the trailer but are obtained as required. An ammeter is mounted, however, on the portable control panel and is connected in the current transformer circuit of the grounding lead.

In connection with the testing of relay installations by actually placing faults on the lines, it may be remembered that a paper was presented at the A. I. E. E. Spring Convention held at Pittsburgh during April, 1923.[3] In this paper was described in detail the testing of the relay installations by putting faults on the 66-kv. line.

As a result of the experiences of this and other companies, it is believed that the inside delta power directional ground relaying is the most reliable and satisfactory type of ground protection that can be used on a system having a resistance in its neutral circuit. On systems which are solidly grounded, other types of ground protection are used, among which are the impedance ground relay and the same type of power directional relay which is used for phase protection, except in the case of extremely long lines where, if the fault current is less than load current, other means of protecting the lines must be found.

It has also been the experience of this and other companies that the most satisfactory method of checking the connections of ground relays has been by actually placing grounds upon the lines in question, since there is less chance of error in this method than in any other. It is further felt that the increased cost of this method of testing is entirely justified from the standpoint of the better relay operation obtained, and from the standpoint of reduction in number of interruptions, which is very important.

Discussion

For discussion of this paper see page 864.

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[3] H. P. Sleeper, A. I. E. E. TRANSACTIONS, Vol. 42, 1923, p. 513.

Directional Ground Relay Protection of
High-Tension Isolated Neutral Systems

BY J. V. BREISKY[1], J. R. NORTH[2], and G. W. KING[3]
Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.*—This paper discusses the problem of obtaining selective relay protection in case of accidental grounds on high-tension isolated neutral systems. A relay was developed whose overcurrent element operates on the residual charging current which exists when a ground occurs on such a system, and whose directional element is operated by residual charging current and residual voltage. Two schemes can be employed: One using high-tension potential transformers for energizing the voltage coil of the directional element; the other, which is more complicated but cheaper, making use of low-tension potential transformers. Tests were undertaken on the 140-kv. system of the Consumers Power Co. to determine the effectiveness of this relay system under conditions of arcing and solid grounds. These tests were successful and it was therefore decided to make general use of such relay equipment for the high-voltage isolated neutral system. The relay scheme was put into operation during March, 1926, and the operating records available adequately substantiate the test results.*

* * * * *

INTRODUCTION

SELECTIVE protection in case of accidental grounds on systems either solidly grounded or grounded through a resistance is a problem which has been solved for some time in a satisfactory manner. The paper which is being presented at this same convention by Mr. B. M. Jones and Mr. G. B. Dodds gives an outline covering the different relay systems which are commonly in use today. It was only for ungrounded systems, however, as well as systems grounded through an extremely high resistance, that no satisfactory solution had been found. It is true that a residual voltage relay has been used to indicate the presence of a ground on an ungrounded system, see scheme in Fig. 1, but such a relay can be used for indication only, as it will operate no matter at which point of the system the ground occurs.

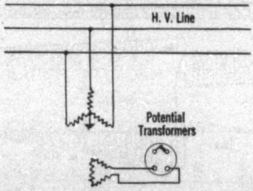

FIG. 1—RESIDUAL VOLTAGE RELAY FOR INDICATION OF ACCIDENTAL GROUNDS

In this paper a relay protective scheme is described which offers a solution of this problem. It makes use of the residual charging current for timing and of the residual charging current and residual voltage for directional discrimination. It can be used successfully on ungrounded systems operating at high voltages and having lines of considerable length.

THEORY AND DESIGN

General Theory. As is well known, the charging current obtaining on a three-phase system under normal conditions is caused by the capacity between wires as well as from each wire to ground. Since the capacity currents are equal in all three phases, the current in the neutral of three current transformers connected to the three phases of the system is normally zero. If a ground occurs on one of the phases, the balance is disturbed, however. While the charging current due to the capacity between wires is still the same, assuming that the ground current is not large enough to influence the potential between conductors considerably, the capacity from each wire to ground has changed. One of the phases is at ground potential, resulting in no charging current to ground, and the other two are now 1.73 times normal line to neutral voltage above ground. Therefore, a residual charging current results. This is shown schematically in Fig. 2, Fig. 2A showing normal conditions and Fig. 2B showing vector relations in case of a ground on phase *A*; charging currents due to the capacity between wires are not shown, as they stay balanced and do not contribute to the residual current. The vectors for residual voltage are also shown in Fig. 2. Under normal conditions, the secondary delta of the star-delta potential transformer is closed, giving zero residual voltage. The star side of the potential transformer must be grounded. In case of a ground on one of the phases, *e. g.*, phase *A* close to the station, E_a becomes zero, while E_b and E_c increase 1.73 times and also change in phase position. It is seen that the secondary delta is not closed any more and a residual voltage equal to three times the normal secondary delta voltage appears.

Studying the phase relations between currents and voltages, it will be noted that the normal charging current to ground in each phase leads the voltage between that phase and ground by 90 deg. and that under ground fault condition, the residual charging current also leads the residual voltage by 90 deg.

Graphic Representation of Residual Charging Current. In order to gain a clear conception of the operation of

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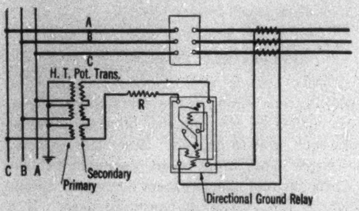
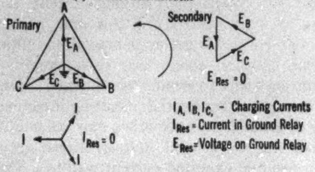
[1] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.
[2] Commonwealth Power Corp., Jackson, Mich.
[3] Consumers Power Co., Jackson, Mich.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

this relay system, the distribution of the residual charging current over the line must be studied. An excellent equivalent method of arriving at the residual currents has been published,[4] the gist of which is given here. With a solid ground at a certain point of the system, the potential between the grounded conductor and the ground is reduced to zero at that point, the normal potential between line and ground being indicated by E_n, Fig. 3. Instead of studying this problem now as a three-phase unbalance, let us assume that all constants of the system remain unchanged and that a single-phase potential whose value is $-E_n$ is superimposed over the normal potential at the point of accidental ground. The sum of these two potentials, E_n and $-E_n$, is zero, corresponding to a solid ground. The only difference between the system under normal conditions and in case of an accidental ground is now caused by this superimposed potential $-E_n$. Due to the effect of this superimposed potential, a ground current I_g is established which flows into the ground and divides itself up through the three parallel branches of the capacity to ground, C, of the polyphase system. It flows through these back to the ground connection, on the grounded phase directly and on the two ungrounded phases through the windings of the transformers; see Fig. 3B. In the grounded conductor, the ground current due to $-E_n$ neutralizes the normal capacity current. The latter is not shown. In the ungrounded conductors, it adds to the normal capacity current.


A
B
C
H. T. Pot. Trans.
R
C B A
Primary
Secondary
Directional Ground Relay

(a) Normal Phase Relations


Primary
A
E_A
E_C
E_B
C
B
Secondary
E_B
E_A
E_C
E_Res = 0
I
I
I
I_Res = 0
I_A, I_B, I_C, - Charging Currents
I_Res = Current in Ground Relay
E_Res = Voltage on Ground Relay

(b) Phase Relations in case of a Ground on Phase "A" ($E_A = 0$)

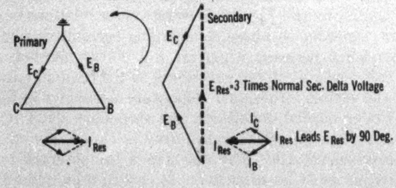

Primary
E_C
E_B
C
B
I_Res
Secondary
E_C
E_Res = 3 Times Normal Sec. Delta Voltage
E_B
I_C
I_Res
I_Res Leads E_Res by 90 Deg.
I_B

Fig. 2—Vector Relations on High- and Low-Voltage Side under Normal Conditions and in Case of Accidental Ground. Relay Scheme Employs High-Voltage Potential Transformers.

The charging currents enter the system equally distributed over the entire length so that the current in all three conductors increases lineally from the far end. An interruption of the linear relation exists in the grounded conductor at the point of ground. To the right of the ground point, the current flows in the same direction as in the ungrounded conductors; to the left, that part of the ground current which is caused by the grounded line itself flows through its own circuit to the point of ground, increasing in the opposite direction. Also, the two currents of the ungrounded conductors combine and return over this conductor to the ground point.

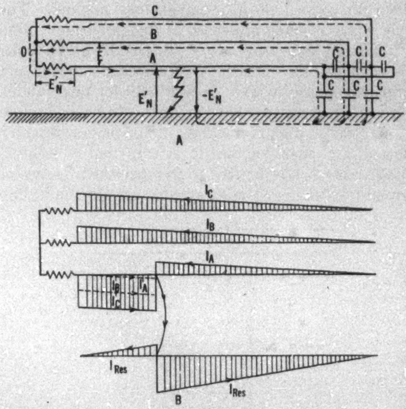

C
B
A
0
E'_N
E'_N
-E'_N
C
C
C
C
C
C
A
I_C
I_B
I_A
I_B
I_C
I_A
I_Res
I_Res
B

Fig. 3—Graphic Representation of Residual Charging Current

The distribution of the ground or residual current is obtained by adding the charging currents in each phase. This current is also shown in Fig. 3B. In the earth, one part of the current spreads out along the conductor path to the right and the other to the left. Both have a linear increase.

This current distribution, due to the accidental ground connection, superimposes itself over the normal current of the system, both the normal capacity and normal load currents, and produces a single-phase load and therefore dissymmetry in the potential and current of the system.

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[4] See R. Rüdenberg, "Electriscke Schaltvorgänge", pp. 150-152, Julius Springer, Berlin, 1923.

The total residual charging current depends on the constants of the line and can be calculated rather accurately.

Calculation of Residual Charging Current. The capacity to ground is given by the formula:

$$C = \frac{0.03882}{\log_{10} \frac{2\,h}{r_0}} \ (\mu \text{ f. per mile})^5,$$

where

h = height above ground in inches,

r_0 = equivalent radius of line conductors in inches.

The value of r_0 for different circuits may be computed as follows:

One circuit on pole:

$$r_0 = \sqrt[3]{r\,d^2}$$

Two circuits on same pole:

$$r_0 = \sqrt[6]{r\,d^5}$$

r = average physical radius of conductor in inches,

d = effective spacing of conductors in inches.

Capacity per mile, in farads:

$$C = \frac{0.03882}{\log_{10} \frac{2\,h}{r_0}} \times 10^{-6}$$

Admittance per mile, in mhos:

$$Y = 2\,\pi\,f\,C$$

The residual charging current in amperes per mile, in case of ground on one phase:

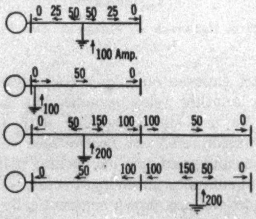

Fig. 4—Distribution of Residual Charge Current

$$I = E_n\,Y \ (E_n = \text{line to neutral voltage})$$

The series impedance, the impedance which the conductors offer to the flow of charging current, is negligible in comparison with the impedance to ground through capacity, and therefore has been neglected in the above calculations.

The residual charging current in a grounded polyphase system is three times as great as the normal capacity current to ground of each ungrounded conductor.

Behavior of Residual Charging Current. Referring back to the distribution of the residual charging current as shown in Fig. 3B, bottom, let us now note certain important facts in regard to the behavior of this current under different conditions of system connections and grounds.

a. The residual charging current is a maximum at the point of fault, and is zero at the far ends of the line.

b. The total value of residual charging current is practically constant for a given length of line, voltage, etc., no matter at which point of the line the ground occurs, neglecting the series impedance. This is illustrated in Fig. 4.

c. The value of the residual charging current is constant, regardless of the location of generating capacity, since no residual current can flow through ungrounded apparatus. The three line currents must add to zero in the apparatus; see Fig. 5.

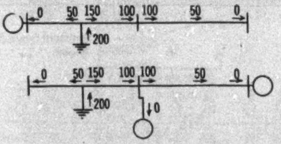

Fig. 5

Fig. 5—Distribution of Residual Charging Current

d. In the case of single lines, the value of residual current at a given distance from the end of a line, and for a given direction of current flow, is constant no matter at which point of the system the ground fault occurs; see Fig. 4.

Relay Protection. In order to utilize properly the characteristics of the residual charging current, a relay is used which has both a current and a directional element; see Fig. 2. The directional element utilizes the residual current and residual voltage and closes its contacts only if power flows in a given direction. The current element is energized by the residual current and can be used either on the definite minimum or inverse part of the time curve, since the relay can be set to close contacts in a given time with a certain current flowing. As mentioned before, the current through the relay is constant for a ground at any point on the system, for a given system connection and given direction of power flow.

Fig. 6 shows a typical radial single line system. The time settings of the relays for given currents are indicated, as well as the tripping direction of the directional element. Grounds are assumed at several points to illustrate the operation of the relay system.

Since there is no residual current at the extreme end of the line, charging current ground relays cannot be made to operate at that point. However, a residual voltage relay, as shown in Fig. 1, can be used at the far

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[5] Refer to Standard Handbook for Elec. Engrs., 5th edition, p. 82.

end of the line, set for a long time delay, so that it will not interfere with the time selectivity at other points.

Protection of Parallel Lines. The protection of parallel lines can be accomplished in two ways. The simplest solution is the use of differential current ground relays, operating on the unbalance of the residual currents in the parallel lines, with a ground on one of them. Another solution is the use of directional current relays as employed on single lines, except that here they must be used on the inverse portion of the time curve to obtain proper selectivity.

This problem is discussed in more detail in connection with the relay system used by the Consumers Power Company.

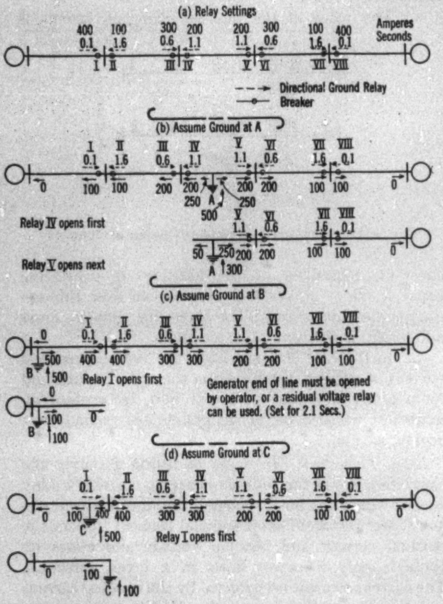
(a) Relay Settings
400 100 300 200 200 300 100 400 Amperes
0.1 1.6 0.6 1.1 1.1 0.6 1.6 0.1 Seconds
I II III IV V VI VII VIII
Directional Ground Relay
Breaker
(b) Assume Ground at A
Relay IV opens first
Relay V opens next
250 A 250
500
50 250 200 200 100 100 0
A 300
(c) Assume Ground at B
B 500
Relay I opens first
Generator end of line must be opened by operator, or a residual voltage relay can be used. (Set for 2.1 Secs.)
B 100
(d) Assume Ground at C
C 500
Relay I opens first
C 100

Fig. 6—Application of Directional Relay—Operating on Residual Charging Current—to a Radial Single Line System

Design of Relay. The design of this relay was a simple problem, after the behavior of the residual charging current and voltage had been studied. A standard low energy directional relay is used, except for the current phase-angle relations in the potential coil. As is evident from Fig. 2, the residual charge-current leads the residual voltage by 90 deg., neglecting a shift due to the series impedance of the line, which may shift the current 10 deg. to 15 deg.

The standard directional relay as used for short-circuit protection is designed to give maximum torque if the current in the current coil and the voltage in the voltage coil are in phase. This is achieved by lagging the current in the potential coil by almost 90 deg., since maximum torque in an induction element is obtained with the two fluxes, or two currents, 90 deg. out of phase.

Now, in the residual charging current relay, the current normally leads the residual voltage by almost 90 deg. Therefore, to obtain good torque conditions, the current in the potential coil is brought practically in phase with the potential, by adding a resistor R, Fig. 2, in series with the potential coil. The vector relations in the relay are shown in Fig. 7. It should be pointed out that the current and voltage in the relay are zero except in case of a ground, thus permitting the use of a sensitive device.

Scheme using Low-Tension Potential Transformers. The relay scheme discussed so far necessitates the use of high-tension potential transformers, since the unbalance of the neutral voltages on the high-tension side is not transmitted to the low-tension side through a power transformer bank if the high-tension side of the bank is ungrounded.

Naturally, high-tension potential transformers involve a great expense on high-voltage systems. For this reason, another relay scheme was worked out which can be used in connection with low-tension potential transformers.[6] It necessitates complication of connections and great care in checking polarities, but it will bring about material savings in cost.

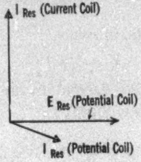
I_{Res} (Current Coil)
E_{Res} (Potential Coil)
I_{Res} (Potential Coil)

Fig. 7—Vector Relation of Current and Potential of Relay Coils

The connections are shown schematically in Fig. 8. Three directional relays are required. All three use the residual charging current but the potential element of each is connected to a different delta voltage. The wiring sketch shows the method of connections. When any two of the main relays close, either of the two auxiliary relays will pick up. If only one main relay closes contacts, neither of the auxiliary relays operates. The auxiliary relays are so connected that either will trip the breaker.

Vector diagram I shows the normal vector relations of a three-phase system. Vector II shows conditions with line A grounded, I_{AR} being the residual charging current. Diagram III is intended to show the currents in the relay current coils with grounds on the different phases. I_{AR}, I_{BR}, and I_{CR} are the residual charging currents for conditions of lines *A*, *B*, and *C* respectively grounded. Of course only one of these currents exists at a time. The currents in the potential coils are made to lag 30 deg. behind their respective voltages.

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[6] This scheme was developed in cooperation with Mr. H. A. Travers.

If we now consider the condition of a ground on line *A*, we will have the current I_{AR} flowing in the current coil of each relay. If the relays are adjusted so that with the current in the potential coil leading the charging current between 0 deg. and 180 deg., the contacts will close, while if the current in the potential coil is lagging 0 to 180 deg., the contacts will be held open. It is evident from an inspection of the diagram that relays carrying potential coil currents I_{BC} and I_{CA} will close their contacts, while the relay carrying potential coil current I_{AB} will remain open. Since two of the relay contacts close, the auxiliary relay will close and the breaker will be tripped. If, without changing anything else, the current I_{AR} flows in the opposite direction, it is clear that the reverse action would occur; *i. e.*, two relays would remain open and one would close, thus not operating the auxiliary relays and not tripping the breaker. With a ground on phase *B* or *C*, similar action takes place. Taking advantage of this fact, directional protection can be secured.

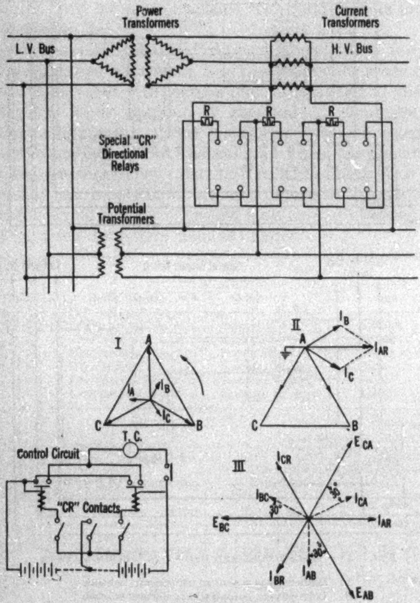
Power Transformers
Current Transformers
L. V. Bus
H. V. Bus
R
R
R
Special "CR" Directional Relays
Potential Transformers
I
A
B
C
I_A
I_B
I_C
II
I_B
A
I_{AR}
I_C
C
B
T. C.
Control Circuit
"CR" Contacts
III
E_{CA}
I_{CR}
I_{BC}
30°
30°
I_{CA}
E_{BC}
I_{AR}
30°
I_{BR}
I_{AB}
E_{AB}

Fig. 8—Relay Scheme Employing Low-Tension Potential Transformers

As mentioned before, the currents in the potential coils are made to lag 30 deg. behind their respective voltages. This scheme will function correctly if the position of I_{AR}, I_{BP}, and I_{CR} does not shift more than 30 deg. in the lead or lag direction, from the position shown in diagram III, which assumes that the residual charging currents lead their respective neutral voltages by 90 deg. In practical applications, it will be found that the residual charging current leads by somewhat less than 90 deg., on account of the series impedance of the line, making it more advantageous to somewhat increase the lag of the current in the potential coils, by changing the series resistors.

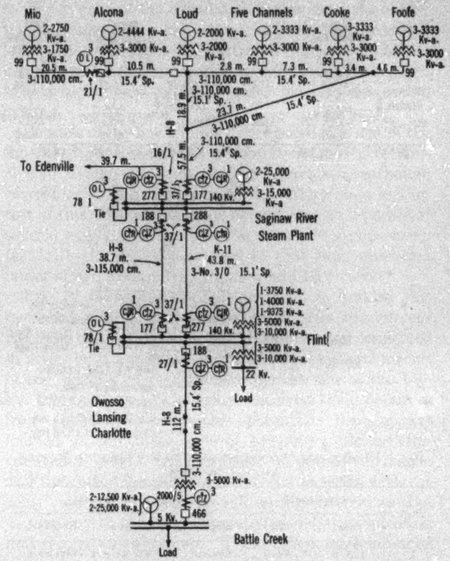
Mio
Alcona
Loud
Five Channels
Cooke
Foote
2-2750 Kv-a.
3-1750 Kv-a.
2-4444 Kv-a.
3-3000 Kv-a.
2-2000 Kv-a.
3-2000 Kv-a.
2-3333 Kv-a.
3-3000 Kv-a.
3-3333 Kv-a.
3-3000 Kv-a.
3-3333 Kv-a.
3-3000 Kv-a.
99
20.5 m.
3-110,000 cm.
21/1
10.5 m.
15.4' Sp.
2.8 m.
3-110,000 cm.
7.3 m.
15.4' Sp.
3.4 m.
4.6 m.
3-110,000 cm.
15.1' Sp.
18.9 m.
23.7 m.
3-110,000 cm.
15.4' Sp.
H-8
3-110,000 cm.
15.4' Sp.
57.5 m.
16/1
To Edenville
39.7 m.
OL
CJR
CTZ
37/1
277
177
140 Kv.
2-25,000 Kv-a
3-15,000 Kv-a
78 1
Tie
188
288
Saginaw River Steam Plant
H-8
38.7 m.
3-115,000 cm.
K-11
43.8 m.
3-No. 3/0 15.1' Sp
1-3750 Kv-a.
1-4000 Kv-a.
1-9375 Kv-a.
3-5000 Kv-a.
3-10,000 Kv-a.
Flint
78/1
3-5000 Kv-a.
3-10,000 Kv-a.
27/1
22 Kv.
Load
Owosso
Lansing
Charlotte
H-8
112 m.
15.4' Sp.
3-110,000 cm.
3-5000 Kv-a.
2-12,500 Kv-a.
2-25,000 Kv-a.
2000/5
466
5 Kv.
Load
Battle Creek

Fig. 9—System Connections of 140-Kv., 60-Cycle, H-8 System

Application

System Connections. The Consumers Power Company has a 140-kv., 60-cycle isolated neutral transmission system having a total connected length of approximately 380 mi. and extending from hydroelectric generating plants in the northern part of the state down along the eastern side through Saginaw, Flint, and across to Battle Creek. This is known as the "H-8" system and is shown in a simplified manner in Fig. 9. Power is supplied from hydroelectric generating stations on the Ausable River and the steam generating stations at Saginaw, Flint, and Battle Creek. Important loads are served from this transmission system at Saginaw, Edenville, Flint, Owosso, Lansing, Charlotte, and the Battle Creek stations.

Protective Requirements. The usual transmission-line faults may be classified as phase-to-phase short circuits or line-to-ground failures and any system of relay protection installed should effectively isolate the particular section of transmission line in trouble from the rest of the system with the least possible disturbance and under all operating conditions. An additional requirement of this particular system is that the relays must allow large amounts of power to be fed in either direction under normal conditions, since generating capacities and large loads are located at various points.

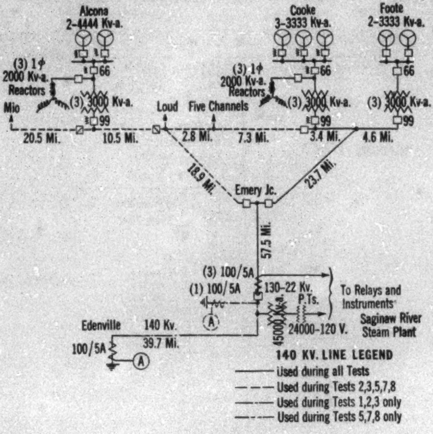
Alcona
2-4444 Kv-a.
Cooke
3-3333 Kv-a.
Foote
2-3333 Kv-a.
(3) 1φ
2000 Kv-a.
Reactors
66
(3) 3000 Kv-a.
99
Mio
Loud
Five Channels
20.5 Mi.
10.5 Mi.
2.8 Mi.
7.3 Mi.
3.4 Mi.
4.6 Mi.
18.9 Mi.
23.7 Mi.
Emery Jc.
57.5 Mi.
(3) 100/5A
(1) 100/5A
130-22 Kv.
P.Ts.
To Relays and Instruments
Saginaw River Steam Plant
4500 Kv-a.
24000-120 V.
A
Edenville
140 Kv.
39.7 Mi.
100/5A
140 KV. LINE LEGEND
Used during all Tests
Used during Tests 2,3,5,7,8
Used during Tests 1,2,3 only
Used during Tests 5,7,8 only

FIG. 10—SYSTEM CONNECTIONS USED FOR GROUND TESTS—AUGUST 16, 1925

Test No.	Line length		Generating capacity		
	Total	*Eff.	Station	Generators	Reactors
1	128.9	89.2	Cooke	3-3333 kv-a.	6000 kv-a.
			Foote	2-3333 kv-a.	. .
2, 3	188.9	149.2	Cooke	3-3333 kv-a.	6000 kv-a.
			Foote	2-3333 kv-a.	. .
			Alcona	2-4444 kv-a.	6000 kv-a.
4, 5, 6	149.2	149.2	Alcona	2-4444 kv-a.	6000 kv-a.

*Effective line length is the length of the connected line on opposite side of ground relays from ground fault. (Effective in producing residual charging current through relay.)

After studying the problem, type CZ directional impedance relays were recommended to take care of phase-to-phase short circuits since they would satisfactorily fulfill the requirements. Directional overcurrent relays could not be used as they would not allow large blocks of power to be fed in either direction and at the same time take care of minimum short-circuit conditions. Also, they would not give positive selective action for faults occurring on the Saginaw-Flint tie lines. Current balance relays could not be used on the tie lines because of the possibility of not operating them in parallel.

Protection in case of line-to-ground faults could not be obtained with the ordinary schemes of ground relaying since this is an isolated neutral system and the unbalance of the charging current to ground is the only definite indication of ground fault conditions; therefore the relaying scheme described in the first part of this paper was developed to take care of such fault conditions.

Field Tests. In connection with the application of this ground relaying scheme, it was decided that a field test should be made to check the theoretical calculations of the amount and distribution of residual charging current under ground fault conditions and also to observe the actual operation of the special directional ground relays which had been developed. Fig. 10 shows the system connections, generating capacity, test connections, and fault conditions during the tests. Power was supplied by hydroelectric generating stations on the Ausable River feeding through the 140-kv. lines to the Saginaw River Steam Plant and on to Edenville. The connected length of line and the generating capacity used during the tests are indicated.

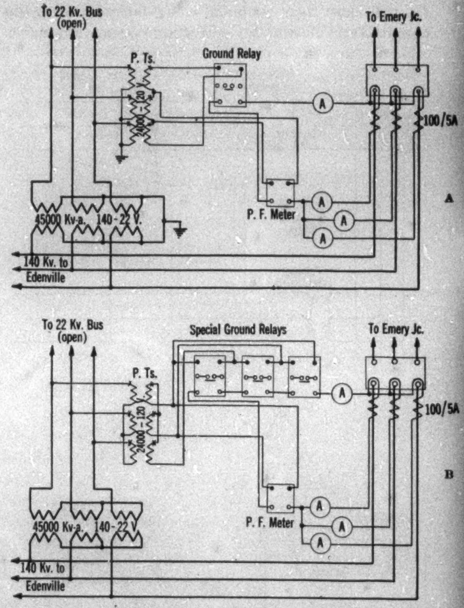
To 22 Kv. Bus (open)
To Emery Jc.
P. Ts.
Ground Relay
120
24000
A
100/5A
45000 Kv-a.
140-22 V.
P. F. Meter
140 Kv. to Edenville
A
To 22 Kv. Bus (open)
Special Ground Relays
To Emery Jc.
P. Ts.
120
24000
100/5A
45000 Kv-a.
140-22 V.
P. F. Meter
140 Kv. to Edenville
B

FIG. 11—RELAY SCHEMES USED FOR GROUND TESTS

A. High-voltage potential transformer scheme
B. Low-voltage potential transformer scheme

Both solid and arcing ground tests were made, the latter conditions being obtained by connecting a line wire to approximately 300 ft. of wire lying on the ground.

Two different schemes of ground protection had been developed, one using high-tension potential transformers and the other using low-tension potential transformers. Both schemes were tested out at this time and the detailed test connections used are shown in Fig. 11, A and B. The regular high-tension potential transformers were not available for these tests and therefore a large bank of *Y-Y*-connected, 140/22-kv. power transformers was used to step the voltage down from 140 kv. to 22 kv. The neutrals of this *Y-Y* transformer bank were grounded when testing scheme No. 1, but not when testing scheme No. 2. In each case, 24,000/120-volt potential transformers were used to supply potential for the relays and they were connected *Y-Y* for testing scheme No. 1 and *Y*-delta for scheme No. 2.

Four special type *C R* directional ground relays were provided, one for scheme No. 1 and three for scheme No. 2, together with various indicating meters at the Saginaw River Station. A standard 100/5-ampere, 15-kv. instrument current transformer was connected in the ground lead at the point of fault and a high speed graphic ammeter, having a roll speed of approximately nine in. per min., was used to give an indication of the actual amount of ground fault current obtained. The various tests were made as follows:

Test no.	Relay scheme	Ground	
		Location	Nature
1	Low tension (No. 2)	Edenville	Solid
2	Low tension (No. 2)	Edenville	Arcing
3	High tension (No. 1)	Edenville	Arcing
4	High tension (No. 1)	Saginaw River	Solid
5	Low tension (No. 2)	Saginaw River	Solid
6	Low tension (No. 2)	Saginaw River	Arcing

The results of the above tests were satisfactory and a summary of them is given in Table I.

Some line trouble in the nature of high-resistance intermittent grounds was experienced before and during the tests, which trouble was later found to be due to the line arcing into trees north of Saginaw. This somewhat affected the results, especially the accurate observation of current and voltage readings. It should be noted here that, judging from subsequent data on relay operations, during fault conditions, the actual ground currents obtained must have been within 10 to 20 per cent of the calculated values since the actual tripping time of the relays was very close to the calculated time. The test demonstrated that scheme No. 1, using high-tension potential transformers, will work satisfactorily with either a solid or with an arcing ground.

Because of the line trouble experienced, the test of the scheme using low-tension potential did not give conclusive evidence of satisfactory operation, although correct indications were observed. Therefore a further test of this scheme was made extending over a period of several weeks, this being in the nature of an operating test with the relays connected so that their operation under actual system conditions would be recorded.

Fig. 12 shows the connections used in making this operating test and it will be noted that the current elements of the three special *C R* relays were connected in series with a graphic ammeter in the neutral circuit of the bushing current transformers on one of the 140-kv. line breakers. The relays received their potential from delta-delta-connected potential transformers located on the 22-kv. side of 140/22-kv., delta-delta-connected power transformers. Graphic voltmeters and indicating lamps were connected in series with the relay contacts across a 125-volt storage battery so that in case any of the relays operated, a definite indication and record was obtained. These relays were intended

TABLE I
SUMMARY OF RESULTS OF GROUND TESTS

Test no.	Relay scheme	Test ground			Trans. line Length M.		Currents at relay					Total		P. F. %	Sec. Volts	Relay Operation		
							Line			Residual		Res.	Cur.					
		Location	Nature	ϕ	Total	Eff.*	X	Y	Z	Test	Calc.	Test	Calc.					
1	No. 2	Edenville	Solid	*Y*	128.9	89.2	80	190	..	45	58	104	84	62	X − Y 116	Open	Closed	Closed
							Reversed relay current coil connections								..	Closed	Open	Open
2	No. 2	"	Arcing	*Y*	188.9	149.2	90	240	96	50	82	120	122	80	98	Open	Closed	Closed
3	No. 1	"	"	*Y*	188.9	149.2	112	200	78	56	97	92	122	45	121	Opened strong		
							Reversed relay current coil connections								..	Closed strong		
4	No. 1	Saginaw	Solid	*Y*	149.2	149.2	120	200	140	40	97	101	97	30	Relay 212	Closed strong		
5	No. 2	"	Solid	*Y*	149.2	149.2	80	112	85	..	..	62	97	95	X − Z 110	Open	Closed	Closed
6	No. 2	"	Arcing	*Y*	149.2	149.2	75	112	85	..	..	Indef.	97	100	103	Indef.	Indef.	Indef.

*Note: Effective line length is the length of the connected line on opposite side of ground relays from ground fault. (Effective in producing residual charging current through relay.)

to operate as follows: When a line-to-ground fault occurred on the 140-kv. system, either one relay alone should operate, or two should operate and the other one not operate, depending on which side of the 140-kv. oil circuit breaker the fault occurred.

This test installation gave good results; for example, a ground fault occurred on the *Y*-phase of the 140-kv. line, approximately 80 mi. out on the Battle Creek line, and during this case of trouble, No. 2 indicating lamp flashed for approximately 15 sec. and No. 3 indicating lamp flashed for approximately 60 sec. This relay operation was checked and the vector diagrams shown in Fig. 12 indicate that the operation was correct.

Bushing Type Current Transformer Tests. At the time the application of these ground relays was being studied, there was little information available regarding the actual ratio and phase-angle characteristics of bushing type current transformers, especially under conditions of low or unbalanced primary currents and high secondary burdens. The amount of residual charging current available is small even under the best conditions and its phase relation is such that the phase-angle errors of the current transformers should be known, particularly for the low-tension relay scheme. The directional ground relays impose a considerable burden on the current transformers, it being in the nature of eight ohms or one volt-ampere per relay at the minimum tripping current. For these reasons, special ratio and phase-angle tests were made on several bushing type current transformers under conditions of primary currents and secondary burdens approximating those which were expected in service with the ground relays. The results of these and subsequent tests may be summarized as follows:

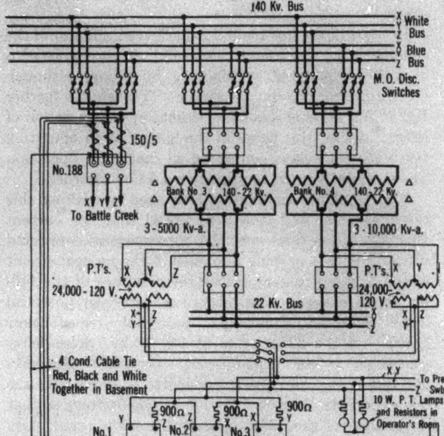
140 Kv. Bus
X Y Z White Bus
X Y Z Blue Bus
M. O. Disc. Switches
150/5
No. 188
X Y Z
To Battle Creek
Bank No. 3 140-22 Kv.
Bank No. 4 140-22 Kv.
3 - 5000 Kv-a.
3 - 10,000 Kv-a.
P.T's. 24,000 - 120 V.
P.T's. 24,000 - 120 V.
22 Kv. Bus
4 Cond. Cable Tie Red, Black and White Together in Basement
X Y Z To Pres. Swb'd.
10 W. P. T. Lamps and Resistors in Operator's Room
900Ω
No. 1 No. 2 No. 3
C. R. Relays
Lamp 1 2 3
120 V. D.C.
No. 1 Gr. Vm.
No. 2 Gr. Vm.
No. 3 Gr. Vm.
Gr. Am.
Indicating Lamps in Operator's Room
Normal Condition
Ground on Y Phase
E_ZX E_YZ E_XY I_1 I_2 I_3 I_Res
I_Res = Residual Charging Current
I_1 = Current in Pot. Element Relay No. 1
I_2 = Current in Pot. Element Relay No. 2
I_3 = Current in Pot. Element Relay No. 3

Fig. 12—Connections used for Ground Relay Operation Tests at Flint, Garfield Avenue Substation (Low-Tension Potential Transformer Scheme)

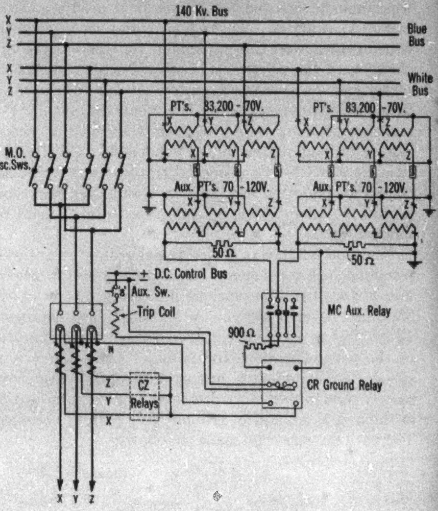
140 Kv. Bus
X Y Z Blue Bus
X Y Z White Bus
PT's. 83,200 - 70V.
Aux. PT's. 70 - 120V.
50 Ω
M.O. Disc. Sws.
± D.C. Control Bus
'a' Aux. Sw.
Trip Coil
900 Ω
MC Aux. Relay
CR Ground Relay
CZ Relays
N
X Y Z

Fig. 13—Directional Ground Relay Protection using High-Tension Potential Transformers

1. The true effective ratio of bushing current transformers of a given design depends on the primary current and the secondary burden. Under balanced conditions and with a constant secondary burden, the true ratio improves as the primary current is increased up to the saturation point of the core and then increases. At low values of primary current, such as the residual charging current, practically all the current is utilized for excitation and very small currents are obtained in the secondary circuit.

2. Under unbalanced conditions, such as line-to-ground faults, and with a heavy neutral burden, the current in the neutral circuit of such transformers is considerably less than the ordinary ratio characteristics would indicate. This is due to the other two current tranformers and their secondary circuits forming a shunt across the transformer in the grounded phase and part of the fault current flowing through them rather

than through the neutral circuit, thus giving a higher effective ratio for the neutral circuit.

3. The phase-angle error of bushing current transformers is quite large, varying in some cases from one-deg. lag to 15-deg. lead over a range of 50 to 200 per cent of rated primary current and at moderate burdens.

In view of their variable ratio characteristics, all bushing current transformers are tested after installation under conditions approximating actual operating conditions. In the case of transformers used with ground relays, they are now tested under both balanced and unbalanced conditions and the relay settings are based on the true ratio characteristics. Bushing current transformers cannot be used with the type of ground relays as applied here unless the minimum residual charging current is 80 amperes or more. In some installations it has been found desirable to use two transformers per phase, connected in series, where the available residual current is low. Such an arrangement materially improves the true ratio at low values of primary current.

Installations. One type *C R* directional ground current relay is used for ground protection on each of the 140-kv. lines at the Saginaw River Steam Plant and connected as shown in Fig. 13. The oil circuit breakers are equipped with three bushing type current transformers and the current coil of the ground relay is connected in the neutral circuit of the bushing current transformers. Two banks of high-tension potential transformers rated 83200/69.2 volts and connected *Y-Y* grounded are used, together with 70/120-volt auxiliary potential transformers connected *Y*-delta to furnish the potential for the ground relays. The auxiliary potential transformers are necessary because the high-tension potential transformers must be connected *Y-Y* and used for other relaying and synchronizing purposes. The potential coils of the ground relays are connected inside the delta circuit of the auxiliary potential transformers. One set of potential transformers is connected to each 140-kv. bus and auxiliary relays are provided to automatically transfer the potential circuits of the relays from one set to the other in case either one is de-energized.

TABLE II
TOTAL RESIDUAL CHARGING CURRENT

Transmission line section	Length	*I*
Ausable to Saginaw	149.2	97
Edenville to Saginaw	39.7	26
Saginaw to Flint (H-8)	41 (Av.)	27
Saginaw to Flint (K-11)	41 (Av.)	27
Flint to Battle Creek	111	72

A similar installation of directional ground relays and potential transformers is provided for the various 140-kv. lines at the Flint-Garfield Avenue Substation. The directional ground relays cannot be used at the end of a single line and therefore they are not installed at the present time at Edenville or Battle Creek. Also, no residual voltage relay is used at these stations since no high-tension potential transformers are available.

Ground Current Calculations. The amount and division of ground current flowing under various conditions of line-to-ground faults was calculated in order to determine the proper relay settings. These calculations were based on the line capacity and residual charging current formulas previously given. All the necessary circuit data are given in Fig. 9, except for the average height above ground, which is 50 ft. Therefore, for the lines on this particular system, $h = 600$ and r_0 (average) $= 18.2$. Substituting these values in the formulas:

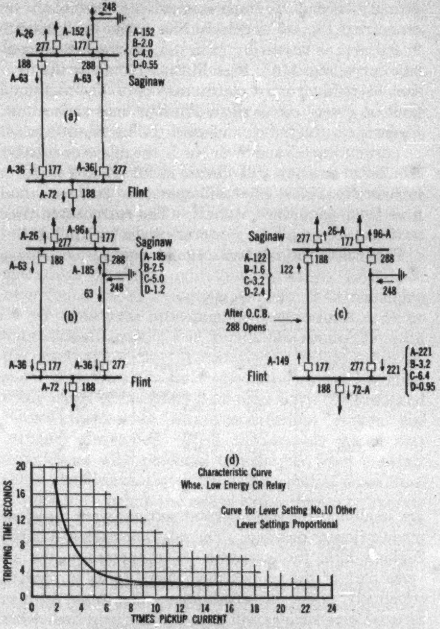
248
A-26
A-152
277
177
A-152
B-2.0
C-4.0
D-0.55
188
288
A-63
A-63
Saginaw
(a)
A-36
177
A-36
277
A-72
188
Flint
A-26
A-96
277
177
Saginaw
A-63
188
288
A-185
A-185
B-2.5
C-5.0
D-1.2
248
63
(b)
A-36
177
A-36
277
A-72
188
Flint
Saginaw
277
26-A
177
96-A
A-122
B-1.6
C-3.2
D-2.4
122
188
288
248
After O.C.B.
288 Opens
(c)
A-149
177
277
221
A-221
B-3.2
C-6.4
D-0.95
Flint
188
72-A
(d)
Characteristic Curve
Whse. Low Energy CR Relay
Curve for Lever Setting No. 10 Other
Lever Settings Proportional
TRIPPING TIME SECONDS
0
4
8
12
16
20
0
2
4
6
8
10
12
14
16
18
20
22
24
TIMES PICKUP CURRENT

Fig. 14—Division of Residual Charging Current Underground Fault Conditions

$$\text{Capacity } (C) = \frac{0.03882}{\log_{10} \dfrac{2\,h}{r_0}} \times 10^{-6} = 0.0214 \times 10^{-6}$$

farads per mi.

Admittance $(Y) = 2\,f\,C = 8.06 \times 10^{-6}$ mhos per mi.

Residual charging current $(I) = E_n\,Y = 0.65$ amperes per mi.

The total residual charging currents for the various sections of transmission lines are given in Table II.

As previously mentioned, the division of residual charging current depends on the transmission line lengths and connections. The calculated division of residual charging current and the relay operation for several assumed conditions of line-to-ground faults are given in Fig. 16, A, B, and C. In Fig. 14A, a line-to-ground fault is assumed on the *H*-8 north line at Saginaw and the total residual charging current at the fault is 248 amperes; of this, 152 amperes pass through breaker No. 177 and the rest flow in the opposite direction. The current through breaker No. 177 further subdivides as indicated. The 152 primary amperes give 2.0 secondary amperes and with the relays set on current tap 0.5 and lever No. 1, they will operate in 0.6 sec. as shown by their time-current characteristic curve, Fig. 14D. Figs. 14B and 16C show the division of residual currents in case of a line-to-ground fault on one of the Saginaw-Flint tie lines at Saginaw. Under such conditions and with the tie line relays set on current tap 0.5 and lever No. 3, the relays on breaker No. 288 at Saginaw will operate in 1.2 sec. and those on breaker No. 277 at Flint will operate in 2.15 sec., total time from occurrence of fault. The current and time settings used on all the relays are given in Table III.

The values of residual ground current obtainable are small and the relay current settings were therefore made as low as possible. The relays on each end of the Saginaw-Flint tie lines were set to operate on the inverse part of the time-current curve so as to operate selectively under all conditions. The time settings selected were such as will give a rather long time delay in order to allow minor flashovers to clear themselves before the relays operate. This helps to prevent disconnecting the lines unless serious trouble develops.

TABLE III
RELAY CURRENT AND TIME SETTINGS

Circuit breaker		Relay settings	
Station	Number	Current	Time
Saginaw	177	0.5 Amp.	Lever No. 1
"	277	0.5 "	" No. 1
"	188	0.5 "	" No. 3
"	288	0.5 "	" No. 3
Flint	177	0.5 "	" No. 3
"	277	0.5 "	" No. 3
"	188	0.5 "	" No. 1

Testing Directional Connections. Each directional ground relay must be connected so that it will operate in the proper direction when a ground fault occurs on its particular line. The residual charging current has a 90-deg. phase relation with the residual voltage and it would be desirable to be able to test these relays by actually putting a ground on the line but this cannot be conveniently done on this system and therefore the proper conditions have to be approximated by an artificial similation of grounded conditions using normal voltage and load currents.

De-energizing one phase of the auxiliary potential transformers gives a potential across the ground relay terminals which will have the same relative phase relation as the residual potential across the relay terminals when the *X*-phase of the line is grounded. The relay should thus operate properly with this potential and with a current leading it by 90 deg. If actual load currents are used, the direction of power flow must be ascertained in order to determine whether the directional contacts of the relay should open or close under the test conditions.

The actual procedure used in testing the directional setting of these ground relays is as follows: (refer to Fig. 13).

The fuse in the secondary circuit of the *X*-phase potential transformer is removed and the primary of the *X*-phase auxiliary potential transformer short-circuited.

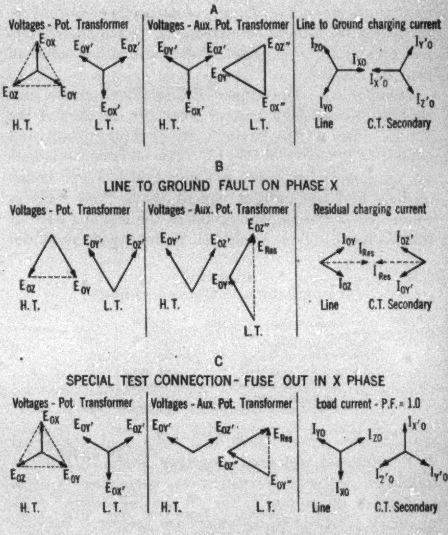
A
Voltages - Pot. Transformer
Voltages - Aux. Pot. Transformer
Line to Ground charging current
E_{OX}
E_{OZ}
E_{OY}
$E_{OY'}$
$E_{OZ'}$
$E_{OX'}$
$E_{OZ''}$
$E_{OY''}$
$E_{OX''}$
I_{ZO}
I_{XO}
I_{YO}
$I_{Y'O}$
$I_{X'O}$
$I_{Z'O}$
H. T.
L. T.
Line
C.T. Secondary
B
LINE TO GROUND FAULT ON PHASE X
Voltages - Pot. Transformer
Voltages - Aux. Pot. Transformer
Residual charging current
E_{Res}
I_{OY}
I_{OZ}
I_{Res}
$I_{OZ'}$
$I_{OY'}$
C
SPECIAL TEST CONNECTION - FUSE OUT IN X PHASE
Voltages - Pot. Transformer
Voltages - Aux. Pot. Transformer
Load current - P.F. = 1.0

FIG. 15—VECTOR DIAGRAMS, METHOD OF TESTING DIRECTIONAL SETTINGS

Fig. 15A shows the vector diagrams of voltage and line-to-ground charging current under normal balanced conditions. Fig. 15B shows the vector relations between voltage and residual charging current with the *X*-phase grounded, and Fig. 15C shows the vector relations between the voltage and load currents with the fuse in the *X*-phase of potential transformers removed and with a load of unity power factor. It will be noted that the potential across the relay terminals with the fuse removed has the same relative polarity and is of ⅓ the value of the potential with the *X*-phase grounded.

The phase relation of the normal voltage between X and neutral, low-tension side of high-tension potential transformers, and the load currents in phases X, Y, and Z are next determined by means of a phasemeter. The potential across the relay terminals is then compared with the normal voltage between X and neutral by checking with any one of the load currents previously used. These tests definitely fix the phase relation between the load currents, the normal potential between X and neutral and the test potential on the relay.

The actual direction of power flow in the line is next determined and the relays set to operate as follows:

a. With power flowing from the line into the bus, the relay should close its contacts with the current leading the voltage by 90 deg.

b. With power flowing from the bus out to the line, the relay should open its contacts with the current leading the voltage by 90 deg.

OPERATION AND RESULTS

Potential Transformer Secondary Fuses. In checking some relay operations soon after they were placed in service, it was discovered that trouble was being experienced due to the blowing of the fuses in the potential transformer secondary circuits. No particular attention was paid to this feature at the time of installation, it being felt that it could be treated the same as any other potential transformer installation and secondary fuses of three-ampere capacity were used. The blowing of these secondary fuses constituted a very serious condition for several reasons. These potential transformers are used for relaying, metering, and synchronizing; thus, with these fuses blown, the relays were helpless; there was no indication of potential and it was impossible to synchronize the lines.

The three-ampere fuses were replaced by ten-ampere ones and tests arranged to determine the cause of the high current flow. One feature of the situation was that the abnormal current did not depend upon line trouble in all cases but occurred very often during switching operations. The tests were made with one set of potential transformers connected to a long section of line and entirely isolated from the rest of the system. Various switching operations were performed with this section of line and the voltage and currents of the potential transformer circuits noted. The results of these tests were very interesting. It was discovered that the heavy current flowed in the transformer circuits when de-energizing the line rather than when energizing it, the current flow being of very short duration, less than one sec. Also, there was no noticeable voltage surge. The current observed at this time was of the order of 20 amperes but the duration was so short that the 10-ampere fuses were not blown. It was felt, therefore, that ten-ampere fuses were of ample capacity. These fuses, however, were later blown during line trouble and a graphic ammeter left in the circuit indicated a current of nearly 40 amperes on one occasion. Fuses of higher rating were installed, finally reaching 30 amperes, but still some trouble was experienced. In the meantime, the problem had been investigated more thoroughly and it was suggested to install a 50-ohm resistor in parallel with the ground relay potential coil. This suggestion was based on the following consideration. The potential transformers in question are the only pieces of apparatus on the system whose neutrals are grounded. When the transmission line is de-energized, the static charge on the line tries to dissipate itself through the primary windings of these transformers going to ground. In so doing, a high-frequency current of varying value is produced for a short time in the secondary circuit. It was felt that increasing the resistance of the circuit would cause the amplitude of the oscillations to decrease faster and, therefore, dampen out the disturbance more rapidly.

These resistors were installed the latter part of 1926 and since that time several cases of serious line trouble have occurred, as well as the usual switching operations, but no fuses have been blown.

Relay Operation. Approximately 75 per cent of the operations of these ground relays during 1926 have been found to be definitely correct. The remaining 25 per cent of the operations were questionable, as no indication of trouble was found. In connection with the questionable cases of operation, it should be understood that with the isolated neutral system, a great many cases of insulator flashovers have occurred and cleared themselves before any serious system disturbance developed. With the trouble which was experienced with potential transformer fuses blowing, there was always a question as to whether these fuses did not blow before the relays had a chance to operate in case of line trouble. Also, while these flashovers constitute a ground fault, the relays may operate before the damage to insulators or conductors is of sufficient magnitude to be noticeable on casual inspection.

Since the resistors were installed to prevent the fuses in the potential transformer secondary circuits from blowing, there has been no questionable operation. Several definite cases of ground faults have been isolated by these relays, and we feel confident that they will continue to operate in this satisfactory manner.

A point to be particularly mentioned in connection with these relays is the necessity of keeping the system connected at all times in order to obtain as much residual charging current as possible. Formerly, during times of storm, the system was split up in order to isolate any possible cases of trouble. With this method of relaying, however, it is essential to keep the system all connected together, and this is favorable to the present load conditions, as it is frequently impossible to split the system due to need for all of the generating equipment on the line.

Future Installations. Due to the very favorable operation of the relays which are installed at the Saginaw River Steam Plant and Garfield Avenue Substation, it is planned to increase the number of installations on the system. At large substations where it is planned to use high-tension potential transformers, the present relay scheme will be used. At the smaller stations having no high-tension potential transformer, the relay scheme utilizing the low-tension potential transformers will be used. With this second scheme, it is much more difficult to obtain the correct directional setting, but after the relays are installed and adjusted, they should work satisfactorily. Within the next year, these ground relays will probably be installed on the 140-kv. lines at our Owosso, Charlotte, and Blackstone substations.

Discussion

GROUND-RELAY PROTECTION FOR TRANSMISSION SYSTEMS

(Jones and Dodds)

DIRECTIONAL GROUND-RELAY PROTECTION

(Breisky, King, and North)

Detroit, Mich. June 21, 1927

H. P. Sleeper: I was not aware that we had progressed to the extent indicated by Mr. Breisky's paper, where we can actually place ground protection on a system where there is no ground current.

However, the method described by Mr. Jones in his paper is one which I believe indicates a distinct advance in the art. Perhaps five or six years ago it was a hard proposition to sell to operating engineers the idea of ground protection. At that time short-circuit protection was pretty much standardized, and they saw no necessity for complicating the situation.

However, today that is not the case, and I believe that ground protection is fully justified for two reasons in particular. In the first place, ground-relay protection will tend to minimize the damage at the point of fault. On the system with which I am identified, that of the Public Service Electric and Gas Company of New Jersey, we have several hundred installations of the type of ground-relay protection described by Mr. Jones. We are operating a 26-kv. system with 75 ohms in the neutral. This limits the current to ground to 200 amperes, high voltage. We find that its operation is extremely successful.

To illustrate the limiting of the damage at the point of fault, an example is a case which occurred recently when a ground was found at the joint of a 26-kv. cable. The line tripped out at both ends, and finally, by tests, the fault was located in the joint. The joint presented no external appearance of any abnormal condition within. The sheath was not enlarged nor blown apart, and there was no hole through it. The joint was opened, and there was a little hole through the compound, possibly as large as the end of one's little finger, showing where the current had gone to the sheath without burning through. On the conductor there was an extremely small amount of burning; so little that the joint was very easily re-made and re-insulated without the necessity of any additional line conductor being added.

The second effect, which really is an indirect one of the first result, is that of reducing the distress on the system. Prior to the installation of the 75-ohm resistor, we used the solid neutral on this system. A wire falling to the ground would give a severe voltage disturbance to the system, a 10 or 20 per cent dip being not uncommon during the trouble.

Now, a successful operation of the ground relays gives from 2 to 4 per cent voltage dip at the ends of the line which is not discernible on lights. I believe that this is very essential to the satisfaction of customers.

Another point that I should like to mention in connection with ground protection, is what I believe to be the necessity of making research faults on the system. I have made quite a few of such high-voltage grounds on the lines while investigating the efficiency of ground-relay protection, and in most cases to say the least I have found that the executives of the company were not heartily enthusiastic about the idea of making these faults. I believe that a certain amount of such testing is necessary as a research proposition.

That brings me to the third point which I should like to mention; namely, the methods of testing ground relays, to be certain they are properly connected and will operate correctly.

Mr. Jones has described the use of a phantom load, the use of the line load, and the actually making of faults on the high-voltage system, to check the ground relays. I have never been able to find a successful system of using phantom loads to do this. However, I find that the use of line load and line voltage can be used very successfully to check these relays.

We have devised a method of short-circuiting one current transformer while the line is carrying the load, and of opening one high-voltage disconnect switch without in any way disturbing the connections to the high-voltage system. With this method it is possible to obtain a set of phasing results on directional ground relays, which, to date, has given us no incorrect operations. I do not believe that the actual grounding of the high-voltage system is necessary.

I should like to say a word regarding Mr. Breisky's paper on the ground protection of isolated-neutral systems. As I understand the situation, the relay which has been developed to do this job is essentially the same relay that Mr. Jones has described wherein the system is actually grounded, the difference being that the relay on the isolated-neutral system uses charging current to operate it, and hence, different voltage relations result on the relay protection.

I should like to ask Mr. Breisky if it is not possible to use the same relay for both applications, thereby greatly simplifying a problem which is a difficult one for us right now. We have a system which has a neutral resistor at one station. If for any reason that one resistor must be removed from service, it becomes necessary or desirable to ground the system at some other point.

We have one system which has no duplicate resistor; therefore, the problem arose, shall we dead-ground the system or leave it ungrounded during the time the resistor is out? We tried dead-grounding the system but it was an unsatisfactory experience, for we found our ground-relay system worked backwards. I am referring to the ground relays which were tested to operate correctly with the 75-ohm resistor in the neutral. Hence our running orders are to leave the resistors isolated while the resistor is out. This we do not consider satisfactory. It occurs to me that possibly some method of external use of resistors and reactors would make it possible to use the same relay under all these conditions.

H. M. Trueblood: I have been interested in the two papers on ground relaying from a standpoint of which I think no mention has been made by the authors of either paper: I refer to the question of the control of inductive disturbances in exposed communication circuits at times of faults to ground in power systems. This in my judgment is one of the most important questions that we have in the field of inductive coordination.

Without desiring to enter upon any discussion of the relative merits and disadvantages of the various means that have been used or proposed for establishing and maintaining a relationship between a power system and the ground, I should like merely to say that it is an interesting thing that in the paper by Mr. Jones and Mr. Dodds, a reason is given for which a certain amount of resistance in the neutral ground connection is beneficial from the power operating standpoint. In the subsequent discussion of that paper, we have heard statements to the effect that from the standpoint of power operation, such resistance is advantageous for two other reasons, limitation of damage at the point of fault and lowering of stresses on equipment.

We can take it for granted, I think, that so far as inductive coordination is concerned, the limitation of ground-fault current is desirable, and it is very agreeable and satisfactory to note that good reasons also exist for which such limitation is desirable from the standpoint of power system operation.

It would be interesting to know whether data exist, and if so, what they show regarding the optimum value of resistance from the standpoint of ground-relay operation. It would seem from the paper by Mr. Jones and Mr. Dodds that beginning with zero neutral resistance, a rapid increase in the wattage available for relay operation will at first be realized as the neutral resistance is increased, and that as further increases are made in resistance, the benefit will accrue less rapidly. This has been pointed out to me by Mr. A. E. Bowen, who has also noted that if the resistance is increased beyond a certain value, the wattage for relaying would probably decrease. It seems probable that there will generally be a considerable range of magnitude of neutral resistance over which the watts available for relay operation would be favorable, and not greatly dependent upon the magnitude of the resistance.

E. E. George: Our operating experiences on the Tennessee Electric Power Company system confirm a good many of the recommendations suggested by the Duquesne Light Engineers, but there are several points of difference,—some of which are undoubtedly due to different operating conditions.

Our system operates with solidly grounded neutrals at all generating stations, all substations which can supply power outward, and at a few receiving stations. The size of grounding transformers on the 120,000-volt system ranges from 7500 kv-a. to 50,000 kv-a. and from 200 kv-a. to 25,000 kv-a. on the 44,000-volt system.

We are certain that directional ground relays are absolutely necessary on our high-voltage transmission system, more especially since our parallel balanced lines are very few, and since trunk tie lines with other companies without isolating transformers are becoming more common.

However, our scheme of directional ground relaying is different from the scheme outlined this morning, and is, we think, simpler, and more reliable, at least for systems which have transformers suitable for grounded-neutral operation at practically all switching points. We are using directional ground relays operated by two currents instead of by a current and potential. One current element of the relay is supplied from the neutral or residual of the bushing current transformers in the line to be protected, and the other element is supplied from a current transformer in the neutral ground lead of the grounding bank. We formerly used relays which had separate over-current and directional elements with the directional disk operated by the two currents and the overload disk operated by one current. We are now using relays which have the over-current and directional elements combined in one disk.

Very probably all reverse power or directional ground relays on our system installed hereafter will have only one common disk, as we have found that relays with separate over-current and directional elements are likely to trip incorrectly due to surges which will close the instantaneous directional element regardless of fault location after the overload element has closed.

We have no objection to the system of protection outlined by Mr. Jones and Mr. Dodds and expect to use it at any point where we are unable to secure grounding transformers, since the installation of star-delta high-voltage transformers will undoubtedly be cheaper than buying a grounding bank in most cases.

We have had very poor success with ground relays interlocked with reverse power relays. We have only one such installation and it will be removed this year. We have had some sad experiences with this scheme on tie lines where there is a possibility of surging of power or reactive kv-a. during trouble. We have had reverse power relays on both sides of a large load center set to trip outward, drop the load during trouble that should have operated neither relay. We have also had a large number of cases where the power load was heavier than the ground current and the reverse power relays could not operate. Even with heavy grounding transformers we find that ground currents are often less than load currents and ground protection independent of load protection is absolutely essential.

The application of reverse power relays on large transmission systems is apparently very limited. We find that the reversal of power is an infrequent indication of trouble and often occurs during abnormal operating conditions, and we have eliminated all reverse power relays on this system, except at two stations.

We have had very satisfactory operation with balanced ground relays after we learned to interlock them through "A" switches on the breakers so that the second line could not trip immediately after the first line, except by back-up over-current ground relay.

We are absolutely in accord with the Duquesne Light Company on the necessity of placing grounds on the system to test out important relay installations. We have not standardized the equipment or procedure of this as we have only made five such tests to date, but on four cases we have solidly grounded the line with reduced generation on the system and read the ground current at a station distant from the point of grounding. On a long line between the grounding transformer and the artificial ground, the ground current is limited so that it can be left on long enough to take meter readings without damaging power equipment or interfering with the public telephone and telegraph circuits.

H. A. P. Langstaff: One advantage of the ground relay which has not as yet been mentioned is the reduction in duty on oil circuit breakers. The West Penn System, like all others, had breakers which were unable to withstand the duty to which they would be subjected unless some modifications were made, and the ground relay along with the neutral resistor was of considerable advantage. When relays were originally applied, we used overhead ground wires on all 25-kv. circuits, and the majority of line failures were to ground, meaning a large percentage to ground-relay operation. The installation of higher capacity breakers and the removal of overhead ground wires has shifted the percentage of failures to phase relays rather than ground relays.

The clearing of faults by means of the ground relay materially reduced system voltage disturbance. We have replaced the watt-relay by the directional relay, having two separate elements,—namely, the current element and directional element,—thereby allowing all ground relays to have the same characteristics and simplified settings.

Another point which has not been mentioned is the protection supplied for reserve bus breakers. All operating companies experience line failures when lines are operating on reserve bus breakers, thereby removing from service the balanced relaying schemes. This, of course, subjects the system to greater disturbances due to higher settings, and I believe that this should receive serious consideration by all operating companies.

As to the checking of ground relays, we used originally a phantom load for the watt-relay. Then we started to apply grounds to the system to check the selectivity of the watt-relay against current relays. After changing to relays of similar characteristics, we continued the ground application scheme of checking, but found that the phantom-load scheme was satisfactory, especially when we use the scheme mentioned by Mr. Sleeper, namely, diverting phase currents having known direction through the ground relay, and opening one leg of the potential transformers. Our field testing crews are obtaining very reliable results from this scheme of testing and the only time we apply the ground is after an occasional faulty operation to check not only the ground relays but phase relays as well, in some cases.

G. H. Doan: I realize that there is a vast difference between the protection of a resistance-grounded system and that of a solidly grounded one. I should like to point out the scheme, however, that the Detroit Edison Company is using. In general, we have used for a number of years on all transmission lines—120-kv. and 24-kv.—a mechanically balanced differential relay. The operating experience with this for the past year has been in results, somewhere in the neighborhood of 98 and 99 per cent perfect.

This system is very advantageous, we think, in that it is almost instantaneous. Objection has been raised to the reduction of voltage at the time of fault. We realize that we do get voltage reduction at the time of fault, but we have found that if we can clear a fault quickly, we don't need to worry about the reduction. This relay has been doing it for some time.

Mention was made of low-energy relays. Again, we are different in that we believe that most of the relays furnished are not nearly sturdy enough and don't take nearly power enough, nor have sufficient force always to operate when you want them.

Manifestly, with a large system which is rather compact, it is rather difficult for us to see how we could go out and ground our lines for checking relays. As a matter of fact, I believe that our present check, made through the use of portable test transformers, (putting primary current through the current transformers as a current-transformer check and carefully tracing the potential-transformer circuits to the relays, added to a check on the relays as they are put into service to make sure that they are going in the right direction), has, so far, resulted in practically perfect connections. I cannot recall a single case of error of relay connections in the last two years. Unfortunately, we have had one or two cases of incorrect relay settings.

L. N. Crichton: The question has been raised as to the operation of ground relays on a system having its neutral dead-grounded.

The relay connection under consideration is the conventional one using a wattmeter type of relay, connected in the delta circuit of a bank of star-delta potential transformers. It happens that the voltage actually applied to the relay is the drop between the relay and the source of power, and consequently the phase relation between the current and voltage depends upon the characteristics of the power supply rather than the characteristics of the trouble.

When you take the resistance out of the neutral, the current lags far behind the voltage, and in addition, you get distorted phase relations due to unbalance in the voltage triangle, resulting in the current lagging more than 90 deg.; actual tests on a system showed 92 deg. Then, if you take into account other errors that are likely to creep in—unbalancing of potential transformers, and so on,—you will see that the relay has a poor chance of working correctly, if it is a true wattmeter. The trouble can be taken care of by making a relay which will work more efficiently on lagging current, and such a relay has recently been developed.

The question of removing the resistance from a number of systems is now under consideration and on these installations, the present relays can be kept in service by putting small phase-shifting devices on them.

The type of relay having its greatest torque with lagging current should be used, in general, on all systems, even those having a high resistance in the neutral, so that the installation will not require change if it should later be decided to remove the resistance.

J. Allen Johnson: (communicated after adjournment) I do not think that Messrs. Jones and Dodds intended to convey the impression that the use of "power relays with self-contained timing element" may not be entirely successful and satisfactory under some conditions, but such an impression may be created by their statement in the last paragraph of the first column on the second page of their paper.

The Niagara Falls Power Company has had in successful operation for several years on its 66-kv. circuits a system of ground protection using a specially designed low-energy power relay. The conditions here are very difficult as the neutral ground resistance may be as high as 750 ohms and the current transformers which supply current to the relays have a 600/5 ratio. Thus the ground-fault current may be as low as about 50 amperes, which may be divided over two parallel circuits, giving only about 25 amperes primary current in the current transformer or slightly over 0.2 ampere relay current.

The relays were designed to operate with 0.15 ampere with full voltage on the potential coil. Voltage for the relays is obtained from a potential transformer having a ratio of 34,500/115 connected directly across the 750-ohm neutral resistor. On account of the high neutral resistance, a very large proportion of the fault voltage always appears across the neutral resistance and potential transformer, and this resistance being practically non-inductive, the voltage is always nearly in phase with the fault current. This results in very positive relay action.

The excellent operating record of this installation during the 3½ years of its service is shown by the following record of operations.

	Ground relay operations on 66-kv. system			
	Correct	Desired	Incorrect	Undesired
1924	28	28	0	0
1925	28	28	0	0
1926	11	11	0	0
1927 (to June 25)	2	2	0	0
Totals	69	69	0	0

V. P. Brodsky: (communicated after adjournment) I should like to ask a question regarding the use and interpretation of the vector diagrams illustrating this paper. Subtractive-polarity current and potential transformers are used in the connection diagrams given in the paper. This may be seen from the polarity marks on these diagrams, and is also in accordance with the present standards.

In a subtractive-polarity transformer the primary (impressed) voltage is opposed in the internal circuit by the secondary (induced) e. m. f. This also applies to the currents.

However, in the external circuit, a subtractive-polarity transformer does not introduce a change in polarity, which remains the same as if the transformers were not existing. It is therefore the reversed secondary voltage or reversed secondary currrent delivered to the relays which should be made use of in these applications.

I notice that in the vector diagrams given in the paper, the primary current is opposed by the secondary current, and the primary voltage by the secondary voltage.

I should like to ask whether the writers have considered the internal circuit of the transformers or the external circuit.

B. M. Jones: Mr. Sleeper asked about the use of the same relay for solidly grounded or resistor-grounded systems. I understand that one manufacturer has actually developed internal additions to present ground relays to cover just that field. This worried us considerably, too, as we contemplated grounding solid.

We very strongly favor actually grounding the systems for testing the directional ground relays. We have just finished the program of applying 305 grounds on our 22-kv. system to check 484 relays and found 17 field errors which our best inspectors couldn't find by any phantom method.

We have also finished a program of grounding our 66-kv. system to test the ground relays and found six incorrectly connected. We start about midnight and finish about four or five in the morning; we make 35 or 40 grounds a night. We have the relay testers scattered over the system. It isn't very expensive and we feel that the expense is justified.

Mr. Johnson is right in assuming that the authors did not intend to convey the impression that the use of power relays with self-contained timing elements might not be entirely successful and satisfactory under some conditions. The statement made was that "the power relays have not proved satisfactory for ground protection due, primarily, to the fact that a sufficiently low-range relay has not been used."

The relay described by Mr. Johnson and used on the Niagara Falls Power Company System, operates on a very small number of watts and, as Mr. Johnson points out, the conditions on their system and their source of voltage for the relay are such that the voltage impressed on the relay under fault conditions is of a fairly high value; also, this voltage must be very nearly in phase with the fault current, thus insuring positive action of the relay.

The Niagara Falls Power Company's solution of its problem was very good, but this solution is not open to a great many power companies since many companies do not have ground connections at a large number of their stations.

As Mr. George of the Tennessee Electric Power Company points out, a great many of the points of difference are due to different operating conditions of various power companies. The scheme used by the Tennessee Company of getting its directional ground protection by the relative directions of two currents in the relay, one current being obtained from the line protected by the relay and the other from the grounded neutral of the power transformer bank, is a very cheap method of obtaining directional ground protection.

Again, however, this method is limited to those companies having their transmission systems grounded at their substations.

In regard to Mr. Brodsky's question as to whether the vector diagrams shown in the paper had reference to the internal circuit of the transformers or the external circuit, the vector diagrams, themselves, show only the phase relation existing between the current and potential on the relays, while the arrows in the relays indicate the instantaneous directions of the currents and voltages at a given time. For example, suppose we take the instant of time when a phase voltage is at a maximum, and assume that current is flowing out on the high-voltage line in the direction indicated by the arrows from the oil circuit breaker. Then, due to the subtractive polarity of the current transformer, the direction of current in the *A*-phase relay would be as indicated by the arrow. However, the voltage applied to this relay is the voltage from *A*-phase to *C*-phase, and since the instant of time was assumed when a phase voltage was at a maximum,, then the *C* voltage would be at about half maximum value, and *A*-phase, being at a higher potential than *C*-phase, the direction of potential would be from *A* to *C* in the external circuit.

The arrows and direction in the other two relays are also taken at the instant of time when that particular phase voltage is at its maximum value.

J. V. Breisky: Mr. Sleeper has asked whether the same relays could be used on systems grounded through resistance as on ungrounded systems? This can be done by somewhat changing the phase relations in the present design of relay, either by making changes internally or by means of external resistors and reactors as suggested by Mr. Sleeper. A relay could probably be developed which could be used interchangeably on these two types of systems since the limits of the phase position of the ground currents on both kinds of systems are quite well fixed. Of course this relay can be used on the ungrounded system only if the residual charging current is of sufficient magnitude to operate the relays as designed at present. It would be more satisfactory probably to dead-ground the system when it is necessary to remove the resistor and use a phase shifter to give correct operation (as mentioned by Mr. Crichton) with the present relay, which is designed to operate on systems grounded through resistance.

As pointed out by Mr. George, it is possible on systems grounded at every substation, through power or grounding transformers, to secure good ground protection by using relays operating on the product of the current through the transformer neutral and the line residual current. In this case, most of the ground current is supplied by the transformers adjacent to the grounded section and the relays may be operated on the inverse part of their curve to get good discrimination between the various sections. Such a relay is also directional, since the current through the transformer neutral always has the same direction, and it takes the place of voltage in serving as the reference quantity. Relays of this type have been used on the Pacific Coast with good success for several years and are being installed on several new jobs.

Certain types of over-current relays have slow-opening contacts. If, however, the fast-opening type is used with the directional contacts correctly spaced, I feel that the trouble mentioned by Mr. George, that of improper operations caused by closing of directional contact after over-current contacts have closed, will not be experienced. This could happen only after a breaker had opened, clearing a fault, and the flow of power had changed due to the distribution of load. If the over-current contacts are of the quick-opening type, they will be open by the time the directional elements close and the breaker will not operate. If for any reason the direction of power flow should change after a fault has occurred on a system and before it is removed, relays with a single element as well as those with separate over-current and directional elements would operate.

The trouble mentioned by Mr. George, of a load current of greater magnitude than the ground current preventing the reverse-power relay from operating correctly, can occur only when a non-directional ground relay is interlocked with directional elements operated by line current and voltage. The scheme outlined by Messrs. Jones and Dodds used when the system is solidly grounded with a suitable phase-shifting device as mentioned by Mr. Crichton, in which the relay is operated by residual current and voltage, is entirely independent of load currents.

Mr. George's statements that "the application of reverse-power relays on large transmission systems is apparently very limited and that reversal of power is an infrequent indication of trouble" are not quite clear to me. The reversal of power accompanied by an abnormal current is certainly an indication of trouble, and reversals unaccompanied by over-current will not cause operation of relays. There are thousands of relays of this type giving satisfactory service and in many cases it is the only type which will give the necessary protection.

The scheme of using mechanically balanced relays, as mentioned by Mr. Doan, is, in general, very satisfactory except that its use is limited to cases where there are enough lines in parallel so that there will always be at least two left under any operating condition. Such cases are not common except in large urban cable systems. Undoubtedly, as Mr. Doan points out, it would be desirable to use relays which would take more power to operate but which would be more positive in their operation. Unfortunately, however, the demand seems to be for a relay that will require less and less power to operate it except possibly on large cable systems where a large amount of power is concentrated into a relatively small area. So far as sturdiness is concerned, however, we seldom hear of the mechanical failure of a relay.

In answer to Mr. Brodsky's question regarding the vector diagrams, I should like to mention that it does not matter whether the secondary quantities are shown reversed or not, since this will affect both current and voltage similarly and the phase relations will remain unchanged. Although vector diagrams are not entirely standardized, I believe the line quantities and the relay quantities are usually drawn in the same direction when subtractive polarity transformers are used since this is in line with the idea that the transformers may be considered as straight-through connections.

The Electric Arc

BY K. T. COMPTON*

Non-member

Synopsis.—*DEFINITION OF ARC. An arc is a discharge of electricity, between electrodes in a gas or vapor, which has a voltage drop at the cathode of the order of the minimum ionizing or minimum exciting potential of the gas or vapor.*

ARC CHARACTERISTICS. The relation of arcs to glow discharges and coronas is illustrated by discussion of "generalized" curve of the gas discharge characteristic. Empirical equations for arc characteristics are interpreted, and a dependence on the boiling temperature of the anode is shown. Seeliger's experiments on the transition from glow to arc, accompanied by the development of a cathode spot, show that the mechanism of the current at the cathode is fundamentally different in the two types of discharge.

CATHODE SPOT. An analysis based on heat conduction in the cathode shows that the cathode spot has no sharp thermal definition, but does have a sharp boundary if defined by visual brightness or by thermionic emission. The phenomenon of moving cathode spots presents the problem of accounting for the observed temperatures.

THEORIES OF CATHODE FALL. Compton's theory is based on space charge considerations and the assumption that the thickness of the fall space is equal to the electronic mean free path. Langmuir's theory differs from Compton's in assuming this thickness to be considerably less than a free path. Considerations of energy balance at the cathode definitely support Langmuir's rather than Compton's theory.

ENERGY BALANCE AT CATHODE. Calorimetric measurements permit an estimate of the fraction of the current at the cathode which is carried by electrons. Though uncertain, the data are accurate enough definitely to support Langmuir's theory and to indicate that, in many cases, thermionic emission of electrons from the cathode is supplemented by a "pulling out" of electrons by the electric field which is concentrated at the cathode surface.

Factors which determine the anode drop and the potential fall and ionization in the negative glow and the positive column are briefly discussed.

IN the brief space at the writer's disposal, it is impossible to treat the subject in a comprehensive manner and there will therefore be discussed principally certain recent developments which have added much to our understanding of the processes involved in arc discharges.

DEFINITION OF ELECTRIC ARC

One is struck, in reading the literature of this subject, at finding no precise definition of an arc. This is due to the fact that, although we readily distinguish common forms of arcs from sparks, glow discharges, and coronas, yet there are gradations from one form to another so that the distinction is sometimes difficult to make. Child[1] describes an arc as "a continuous current of several amperes or more, passing through a gas and having a cathode drop which is comparatively small." Hagenbach[2] says, "In order to be able to define the arc, the cathode fall must be taken to be characteristic. As compared with the glow discharge, it is small . . . ."

The arc is characterized by a larger current and a lower voltage than any other type of gas discharge. It is generally obtained in gases or vapors whose density, at the cathode, corresponds to a pressure of the order of a millimeter of mercury, or more. Every arc has a region of luminous gas near the cathode. Whether or not there is another region of luminosity, the positive column, depends on the gas pressure, the distance from the cathode to the anode, the current, and the shape of the containing vessel if the arc is enclosed. High pressure, large distance, and constricted container favor the appearance of the positive column in an arc. The total voltage across the arc is the sum of (1) the cathode drop, which has a value characteristic of the gas; (2) the anode drop, which depends on the size and shape of the anode as well as the nature of the gas and its degree of ionization, and which may be positive or negative in sign; (3) the drop along the positive column, which is generally proportional to the length of the positive column and depends on the current and the nature and density of the gas; (4) a voltage drop, generally negative but usually small, between the region of the cathode fall and the beginning of the positive column. Of these parts it is only the cathode drop which appears to have a definite characteristic value; the other three may be altered by altering the current, the pressure, or the geometry of the arc path. Hence the total arc voltage is not particularly significant, although it may be considered as a characteristic parameter if the arc conditions are specified as, for example, an arc between plane parallel electrodes of large extent, with more than a minimal separation, and placed in a gas at a given pressure. The volt-ampere characteristics of an arc is generally negative, *i. e.*, the voltage across the arc falls as the current is increased. It may, however, be zero (voltage drop independent of current) as would be the case if the cathode drop constituted the entire voltage drop in the arc. Probably a slight positive characteristic could be obtained in an arc whose anode is of very small dimensions and is located within the region of negative glow just beyond the boundary of the cathode fall space.

In view of these considerations, which will be amplified later, the following definition of an arc is proposed: *An arc is a discharge of electricity, between electrodes in a gas or vapor, which has a voltage drop at the cathode of the order of the minimum ionizing or minimum exciting potential of the gas or vapor.*

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1. For all numbered references, see Bibliography.

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It must be remembered, however, that there are transition stages between typical arcs and typical glow discharges which cannot be defined as either. These transition stages are generally extremely unstable, so that they are seldom encountered in practise. Unless an extremely high series resistance and a correspondingly high source of e. m. f. are used, the transition from arc to glow or from arc to nothing is abrupt and these transition stages are not obtained.[3]

DISCHARGE CHARACTERISTICS AND TYPES OF DISCHARGE

The relation of the arc to other types of gas discharge is well illustrated by the generalized discharge characteristic[4]. The simplest gas discharge circuit consists of a source of e. m. f., E, a discharge between electrodes, D, and a series resistance, R, together with an ammeter and a voltmeter to measure the current i and the voltage drop V across the discharge. The external circuit characteristic is obviously

$$E = V + R\,i. \tag{1}$$

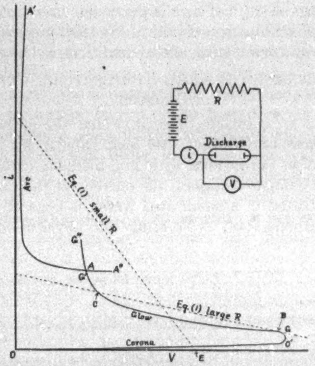

FIG. 1

The nature of the discharge apparatus itself is given by an internal characteristic

$$V = f(i)\,. \tag{2}$$

In Fig. 1 the external characteristic is represented by a straight line of slope $-1/R$ and voltage intercept E. The internal characteristic is represented by the curve $O\,O'\,G\,G'\,A\,A'$. The possible values of current and voltage are given by the intersection points of these curves (1) and (2).

If, keeping R constant, E is gradually increased from an initial value O, the current is first small and the volt-ampere characteristic is positive. This is the region of corona, or Townsend currents $O\,O'$. Beyond O' the current rapidly increases, the volt-ampere characteristic is negative and we have the region of the glow discharge $G\,G'$. The current jumps discontinuously from point B to point C on the curve. Obviously, the entire glow discharge may be skipped over if the resistance R is small, *i. e.*, the slope of the line $E\,B\,C$ large. In this case the discharge passes abruptly from the corona to the arc type. On the other hand, if the resistance R is very large and the line $E\,B\,C$ almost horizontal, the entire change from corona through glow to arc may be passed through continuously.

At $G'\,A$ there is a transition from glow to arc. Sometimes this transition is gradual and sometimes abrupt, in which cases the curve is rounded or sharp at the transition region $G'\,A$. If the transition is abrupt, there is evidence that the glow and arc characteristics intersect and may be prolonged as $G'\,G''$ and $A^\circ\,A$, and it is then possible to have either an arc or a glow discharge at the same voltage, or at the same current, and we have, within a small range, the anomalous situation of a glow discharge carrying larger currents than the arc at the same voltage.

In general, it is possible to determine the entire discharge characteristic of any given type of discharge apparatus by using sufficiently large ballast resistance, R, and correspondingly large e. m. f., E. Once this characteristic is known, the various changes in the discharge, which will be found when any variations of E or R are made in the circuit, may be predicted.

In this connection, mention only may be made of a very complete discussion of the question of *stability* of gas discharges given by Dallenbach[5] and summarized by Bär[6]. The fundamental condition for stability[7] is that $-\dfrac{d\,V}{d\,i} < R$, *i. e.*, that the slope of the internal characteristic curve (2) be greater than that of the external characteristic curve (1). In addition to this, inductance and capacity and inertia of ions must be taken into consideration.

EQUATIONS OF THE ARC CHARACTERISTIC

Several empirical equations have been proposed to describe the current-voltage characteristics of arcs. The best known of these are

Frolich[8] $$V = a + b\,l \tag{3}$$

Ayrton[9] $$V = a + b\,l + \frac{c + d\,l}{i} \tag{4}$$

Steinmetz[10] $$V = a + \frac{c\,(l + d)}{\sqrt{i}} \tag{5}$$

These may be interpreted as follows. The constant term a is the sum of the cathode and anode drops. The term $b\,l$ is the voltage drop in the positive column, whose length is taken to be equal to the total arc length l. The terms involving current i in the denominators of (**4**) and (**5**) take care of the negative characteristic feature of the arc. In (**4**) the term c/i involves the negative characteristic of the negative glow and perhaps of the anode drop, while the term $d\,l/i$ gives the negative characteristic of the positive column. These same features are described somewhat differently in (**5**). In any case, these equations are known to be only empirical approximations.

Recently Nottingham[11] has shown that a new equation

$$V = A + B/i^n \qquad (\mathbf{6})$$

is very accurate for all the large number of arcs tried, provided one of the electrodes, usually the anode, reaches a definite temperature which is usually fixed by its boiling point. In this case, n is found to depend on the boiling temperature T of the metal through the relation

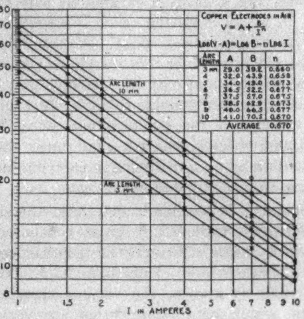

Fig. 2

$$n = 2.62\,(10)^{-4}\,T \qquad (\mathbf{7})$$

and A and B are constants for a given metal and arc length. The experimental determination of n is illustrated by the case of copper, Fig. 2. Here V and i are plotted logarithmically, so that n is given by the slope, which is seen to be the same for all arc lengths. The accuracy of relation (**7**) over the entire range of arc temperatures is shown by Fig. 3. Recent unpublished work has extended this curve to cadmium at $T = 1051$ deg. K. The significance of this dependence of n on the maximum electrode temperature is not yet understood, but the fact cannot be doubted.

FUNDAMENTAL IMPORTANCE OF PHENOMENA AT CATHODE

All lines of evidence indicate that the essential feature of an arc is the emission of electrons from the cathode which produces sufficient ionization of the surrounding gas to give a positive space charge just outside the cathode, thus facilitating ionization and permitting a large, generally saturation, electron emission at relatively low voltage. All other characteristics of arcs appear to be either consequences of this emission or prerequisites to it under the particular physical conditions in which the arc is produced. Thus it is possible to produce arcs in which the anode drop in potential is practically eliminated, in which the potential gradient in the gas is nearly zero or is reversed, in which there is no chemical action or consumption of the electrodes, or in which the gas or anode temperature is low. The cathode drop and its emission of electrons are indispensable, however. Theoretically *any* mechanism or process for supplying electrons from a cathode in sufficient numbers to produce, at low voltages of the order of the minimum critical potentials of the gas, enough ionization to give a positive space charge should suffice to maintain an arc. Actually, however, only two emission processes seem capable of supplying electron emission in sufficient amount: thermionic emission and the pulling of electrons from the cathode by the large field in the cathode fall space, or a combination of these two. J. J. Thomson[12] and Stark[13] first suggested the former theory and Langmuir[14] the latter one. The present evidence, some of which we shall now review, points to the truth of each in particular cases, and generally to a combination of both. We shall proceed, therefore, to an examination of the conditions at the cathode.

CATHODE SPOT: AREA, TEMPERATURE, CURRENT DENSITY

In all arcs, except those in which the cathode has small area and cannot lose heat rapidly by metallic conduction (as in arcs with hot filament cathodes as used in Tungar rectifiers) the current at the cathode is concentrated in a small area which is generally called the "cathode spot." To study the physical condition of the cathode, we must therefore examine this cathode spot. This is extremely difficult, however, owing to its small size, its frequent rapid motion, and the difficulty in defining it. Until very recently there were no measurements of the area of the cathode spot except in the case of carbon arcs, but recently measurements also have been made on several metallic arcs. These results are shown in Table I.

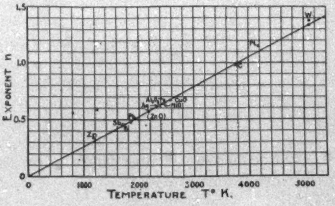

Fig. 3

In the case of carbon, the spot is stationary and the chief source of error is probably in the measurement of the photographic plate, owing to photographic broadening with overexposure and to failure to use reliable methods in correlating distribution of photographic density with intensity of light. It should be remarked, moreover, that different grades of carbon give different results, presumably due to the effect of alkaline impurities on the amount of thermionic emission. In the metal arcs, the spot usually wanders rapidly, so that Güntherschulze photographed it as a band after reflection from a revolving mirror. Seeliger[15] was unable to repeat Güntherschulze's work. Nottingham has used an accurate photometric method of measuring his photographic plates, but did not use a revolving mirror; the internal evidence in his work, however, justifies considerable confidence in its correctness.

TABLE I

Arc	Current density amp. per cm.^{2}	Observer	Reference
Carbon in air.....	210	Reich	*Phys. Zeits.* **7**, 73, 1906.
	318	Granquist	*Phys. Zeits.* **7**, 79, 1906.
	470	Guntherschulze	*Zeits. f. Phys.* **11**, 71, 1922.
Mercury vacuum.	4000	"	*Zeits. f. Phys.* **11**, 74, 1922.
Iron in air.........	7200	"	*Zeits. f. Phys.* **11**, 74, 1922.
Tungsten in air...	3200	Brauer	*Ann. d. Phys.* **60**, 95, 1919.
	700	Nottingham	To be published.
Cadmium in air...	5000	"	To be published.

Similarly, the temperature of the cathode spot is not very accurately known. The best determination for carbon arcs is probably that of Reich[16], who gives 3413 deg. *K*, although other observers give values from 2903 deg. *K*. to 3593 deg. *K*[17]. Hagenbach and Langbein[18] give for iron 2430 deg. *K*., nickel 2365 deg. *K*., tungsten 3000 deg. *K*., silver and copper below 1800 deg. *K*. Nottingham, however, has found fusion of tungsten in the cathode spot, which would prove its temperature to be at least 3643 deg. *K*. It is quite possible that the small size of cathode spots has led to an underestimate of their maximum temperature.

The cathode spot of a mercury arc has been estimated as between 2000 and 3000 deg. *K*. on account of a continuous spectrum emitted from it and ascribed by Stark[13] to local high temperature, in spite of the much lower boiling temperature of mercury, thus supporting his theory of the thermionic origin of the electron emission from the cathode. This spectrum, however, is not characteristic of so high a temperature and may be otherwise accounted for, and there is no certain evidence that the temperature is so high. Seeliger[15] applied Knudsen's equation to rate of evaporation as a function of temperature, using Güntherschulze's measurements of rate of evaporation[18], and calculated a lower limit of 673 deg. *K*. We shall present evidence below, however, indicating that the mercury loss measured by Güntherschulze was partly in the form of a spray rather than true evaporation, so that Seeliger's lower limit should be considerably less than 673 deg. *K*. Thus we really know very little regarding the temperature of the cathode spot in mercury arcs.

A very illuminating study of the theory of the cathode spot has been made by Seeliger[15]. Consider first a case where heat is liberated at a rate Q per cm.^{2} at a fixed circular area A on the plane surface of a metal block of indefinite extent. This corresponds roughly to the heated region of a cathode surface. Because of heat conduction in the metal, a temperature gradient is set up radially outward on the surface of the block as well as into its depth. The final steady surface temperature distribution is calculated by the known theory of heat conduction and is found to be of the form shown by the curve T, Fig. 4A. It is quite obvious that there is nothing in the nature of a sharply defined "hot" spot.

The spot is observed, however, by means of the light radiation from it, and it is well known that visual brightess L increases as a high power of the temperature T, being given approximately by the relation

$$T = \frac{11230}{5.367 - \log L} \qquad (8)$$

in the temperature range involved here. From this,

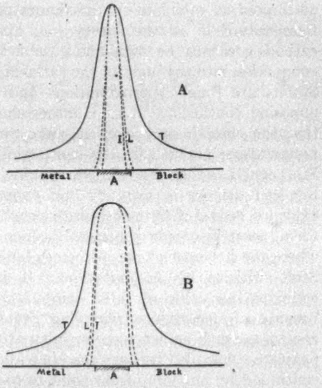

FIG. 4

and assuming that the maximum temperature in curve T, Fig. 4A, is 3300 deg. *K*., the visual brightness curve L is found to be as shown. This does limit quite a sharply defined region, which does not differ much in area from A. Thus, as seen by the eye, the cathode spot is sharply defined.

Electrically, however, it is neither the temperature nor the brightness, but the thermionic emission which is important, and this may be calculated as a function of temperature by Richardson's equation

$$I = A\,T^{1/2}\,\epsilon^{-b/T} \qquad (9)$$

Taking $b = 6\,(10)^4$ for carbon, this leads to curve I for the current density of thermionic emission from various regions of the spot. Here again the spot is quite sharply defined and has approximately the dimensions of A, although it is somewhat smaller than the "visual" spot.

It is certain that considerations such as these are involved in stationary cathode spots, especially if the electron emission is primarily of thermionic origin. The problem is further complicated, however, in cases where evaporation or sublimation tends to cool the cathode, and of course electron emission is itself a cooling process. Thus these cooling agencies, acting in addition to conduction through the body of the cathode, must tend to limit the temperature in the hottest regions of the spot, and thus to alter the distributions of Fig. 4A to something like the form of Fig. 4B, which helps to explain the fact that the area of the spot is so nearly directly proportional to the current.

In the case of metallic arcs whose spot wanders rapidly over the cathode surface, there is a real difficulty in explaining the high temperature of the spot, since the time available for heating is very short. In a mercury arc, for instance, the spot frequently wanders at a rate of at least 300 cm. per sec.[19,20] and may move 30 times this fast. Güntherschulze[21], Stolt[20], and Seeliger[15] have attempted to calculate the maximum possible rise in temperature if all the energy $i V_c$ liberated at the cathode goes into the metal and is carried away by heat conduction into the body of the cathode. Both calculations are rough approximations and they lead to opposing conclusions; *i. e.*, Güntherschulze concludes that the cathode spot even in mercury arcs rises to temperatures above 2000 deg. cent., while Seeliger and Stolt conclude that the temperature rises to only a few hundred degrees in mercury and copper arcs. The evidence is that Güntherschulze's conclusion is right, for at least in copper the metal is found to be fused where the hot spot passes, although both Seeliger and Stolt criticize his computations. It is difficult to estimate the value of these computations, not only because of uncertainty regarding the data and the constants (such as heat conductivity at elevated temperatures) but also because the spot may not wander continuously, but jump from point to point, remaining at each point long enough to heat it.

It is evident from this brief survey that, in spite of the attention which has been focussed on the cathode spot since its crucial importance in the theory of the arc has been realized, there is as yet no agreement as to whether the cathode spot *always* reaches such temperatures as to warrant a purely thermionic explanation of the electron emission from it.

THERMIONIC EMISSION FROM THE CATHODE

Table II gives thermionic emission values calculated from Richardson's equation (**9**). In comparing these values with current densities at the cathode spot in arcs, certain facts should be kept in mind. The emission values for carbon were given by Langmuir[22] for carbon as pure as could be obtained and with great care to avoid contamination. Such purity is utterly impossible in arc carbons, and the impurities which are known to be present are such as to increase the emission very considerably. An upper limit for arc carbons would be the values given for lime-impregnated carbon[23]. The actual thermionic emission from an arc carbon must lie between the values 26.7 and 4400 amperes per cm.^{2} Further than this we cannot say at present. But this makes it evident that much, and possibly practically all, of the arc current, see Table I, is simply thermionic emission of electrons from the cathode. Similarly, in the case of the tungsten arc in air, the thermionic emission at the temperature of the cathode spot is adequate to account for the arc current, if Nottingham's values are correct.

TABLE II

Carbon $A = 1.49\ (10)^{25}$ $b = 48,700$		Impregnated carbon $A = 3.3\ (10)^{26}$ $b = 42,000$		Tungsten $A = 1.55\ (10)^{26}$ $b = 52.500$	
T deg. K.	Amps. per cm.^{2}	T deg. K.	Amps. per cm.^{2}	T deg. K.	Amps. per cm.^{2}
2700	1.9	2700	500	2400	0.365
3000	13.2	3000	2390	2800	8.98
*3140	26.7	*3140	4400	3200	96.9
3300	54.7			3540	509
3500	127			*3640	977

The most accurate identification of arc current with thermionic emission from the cathode is obtained in arcs from a small non-vaporizing cathode, such as in Tungar rectifiers or Pointilite lamps in which there is no "hot" spot, but the entire cathode is at practically uniform temperature. In these cases, the temperature may be measured with an optical pyrometer and the thermionic emission current rather accurately estimated. In such cases, the arc current is generally found to be accounted for by thermionic emission[24], although in very intense arcs in gas at high pressure the arc current is somewhat larger than the calculated thermionic emission.

In the case of arcs from more easily volatilized cathodes the data, as we saw above, are too uncertain to support any very positive statement regarding the adequacy or inadequacy of thermionic emission in accounting for the arc currents. On the whole, the writer is inclined to the opinion that in these cases, as well as in the intense high pressure arcs above, the ordinary thermionic emission is increased by an effect of the intense electric field at the cathode in actually *pulling electrons away from the cathode* which would not otherwise be emitted. This theory, due principally to Langmuir, is discussed later. It is significant that some agency in addition to thermionic emission appears to be needed to account for arc currents in just those cases in which conditions for such a "pulling out" effect would be most anticipated.

DEVELOPMENT OF AN ARC

Seeliger[25] has recently made an instructive experiment on the development of an arc from a glow discharge, and the relation of this to the formation of the cathode spot. He used a very high resistance to stabilize the discharge and very pure electrodes, and followed through the variations in current density as the total discharge current was increased, beginning with a normal glow discharge covering only part of the cathode. The results are shown diagrammatically in Fig. 5.

In interval, *a b*, the current density was constant, the cathode fall of potential was constant at about 300 volts, and the glow did not completely cover the cathode. At *b*, the glow completely covered the cathode. Further increase in current was accompanied by increase in current density and large increase in the cathode drop, which may rise to several thousand volts. At *c* there was first observed a tendency for the cathode glow to concentrate into a hot spot, which tendency increased with further increase in current. Simultaneously the current density increased at an accelerated rate, while the cathode drop began again to diminish. At the point *d*, the cathode drop had fallen to a value less than the original normal cathode drop, and it was falling so fast and the current density was rising so fast that the series resistance was insufficient to stabilize the discharge and it passed abruptly to condition *e*, from which point on the discharge was a true arc, the glow was replaced entirely by the hot spot, and the cathode drop was in the neighborhood of 10 volts.

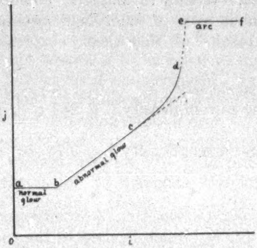

FIG. 5

This illustrates the fact that the mechanism of current transfer in the glow and arc discharges is quite different. In the glow discharge the current at the cathode is carried principally by positive ions, and the electron emission from the cathode is "secondary" emission due to positive ion bombardment and photoelectric action. In the arc discharge the current at the cathode is carried principally by electrons, which are probably liberated thermionically, assisted by the "pulling out" action of the field. The thickness of the cathode fall space in the arc is certainly thousands of times smaller than that in the normal glow discharge.

Qualitatively, the progressive stages in the development of an arc may be "explained" by the principle that *the potential distribution in a gas discharge adjusts itself so as to give maximum current, subject to the limitations imposed by Poisson's equation.* This principle appears to be new in the theory of gas discharges and has recently been applied with success to the theory of the normal glow discharge[26]. Further applications of it should be attempted.

COMPTON'S THEORY OF CURRENT AT CATHODE[27]

The existence of the cathode fall of potential is proof that the space charge near the cathode is positive, *i. e.*, that the concentration of positive ions exceeds that of electrons. Let i be the electron current density and $J + j$ be the positive ion current density. j is that part of the positive ion current which just neutralizes the space charge of the electrons, and J is the excess, which accounts for the positive space charge, whose density we shall call ρ. By Poisson's equation,

$$\frac{d^2 V}{d x^2} = -4 \pi \rho = -4 \pi \frac{J}{v}, \tag{10}$$

where v is the average velocity of advance of the positive ions in the field $- d V / d x$. If this field were uniform, and if the ion made numerous collisions with atoms, and if these collisions were either "head on" or inelastic, v would be given in terms of ionic charge e, mass M and mean free path L by [28]

$$v = \sqrt{\frac{\pi}{2}} \sqrt{L \frac{e}{M} \frac{d V}{d x}}. \tag{11}$$

If we consider the impacts to be elastic, not all "head on," but rather as if made by a sphere moving under constant force among similar spheres distributed at random, the factor $\sqrt{\pi / 2}$ changes to very nearly $.96 \sqrt{2}$[26], which is a little larger. If, however, the field is not constant, but is increasing toward the cathode, as is almost certainly the case, this factor is less, but cannot be less than $.96 \sqrt{2} / \sqrt{2}$ and is certainly much nearer the higher value if the positive ions make at least two or three collisions while traversing the cathode fall space. With these uncertainties in mind, we cannot be far wrong if we take equation (**11**), as was done in the original publication of the theory.

Substituting for v in equation (**10**) from equation (**11**), and integrating, we obtain

$$\left(\frac{d V}{d x}\right)^{3/2} = \frac{3}{2} B J x + C_1, \text{ where } B = 4 \pi / \sqrt{\frac{\pi}{2} \frac{e}{M} L}. \tag{12}$$

The integration constant C_1 is determined by the condition that $d V / d x = 0$ at the outer boundary of the cathode fall space. Taking $x = 0$ at the cathode surface and $x = c$ at the boundary of the fall space, we have

$$\frac{d V}{d x} = \left[\frac{3}{2} B J (c - x) \right]^{2/3}.$$

Integrating again, and putting $V = 0$ when $x = 0$, $V = V_c$ when $x = c$, and solving for the cathode fall of potential, we have

$$V_c = \frac{3}{5}\left(\frac{3}{2} B J\right)^{2/3} c^{5/3}.$$

Resubstituting the value of B from equation (**12**) gives

$$V_c = \frac{3}{5} \frac{(6\,\pi)^{2/3} J^{2/3} c^{5/3}}{\left(\frac{\pi}{2} \frac{e}{M} L\right)^{1/3}} \tag{13}$$

We are entirely without experimental evidence regarding the thickness of the cathode fall space in arcs, except for the knowledge that it is extremely small. It seems certain that it does not exceed the electronic mean free path l, since the electrons have their best chance to ionize at their first impact owing to the fact that electric intensity diminishes with distance from the cathode. In the present theory it is assumed that $c = l$, though it may be that this is an upper limit. Now the ionic free path L is $\sqrt{2}$ times the molecular free path λ, since the ions have a higher order of speed than do the molecules. Also, the electron free path l is $4\sqrt{2}$ times λ, and hence 4 times L, owing to the negligibly small dimensions of an electron. Thus, writing $c = l$ and $L = l/4$, and solving equation (**13**) for J, we find

$$J = \frac{1}{12}\left(\frac{\pi}{2} \frac{e}{M}\right)^{1/2} \left(\frac{5}{3} V_c\right)^{3/2} \frac{1}{l^2} \text{ in c. g. s. units.}$$

$$= 0.76\,(10)^{-7} \frac{V_c^{3/2}}{M^{1/2}\, l^2} \text{ in amperes per cm.}^2\text{, with } M \text{ in ordinary molecular weight units.} \tag{14}$$

If c in equation (**13**) should have been taken to be less than l, then J would be larger than calculated by equation (**14**).

Consider now that part of the positive ion current density j required to neutralize the electron space charge. An exact calculation of the relative times required for an electron and a positive ion to pass through the fall space appears impossible, but the following approximation is probably accurate enough for the present purpose. Assume first that the field in the fall space is uniform and hence equal to V_c/l. The time t required for an electron, starting from rest at the cathode, to traverse the fall space is given by

$$l = \frac{1}{2} at^2, \text{ where } a = \frac{e\,E}{m} = \frac{e\,V_c}{m\,l}, \text{ whence}$$

$$t^- = l\sqrt{\frac{2\,m}{e\,V_c}}\,. \tag{15}$$

The positive ions, on the other hand, move forward with average velocity given approximately by equation (**11**). Putting $d\,V/d\,x = V_c/l$, and $L = l/4$, and taking the time t^+ as distance l divided by mean velocity v, we have

$$t^+ = l\sqrt{\frac{8}{\pi} \frac{M}{e\,V_c}}\,. \tag{16}$$

Since $j = n^+ e\,v^+$ and $i = n^- e\,v^-$, and since $n^+ e = n^- e$ for exact compensation of space charge, we have

$$\frac{i}{j} = \frac{v^-}{v^+} = \frac{t^+}{t^-} = \sqrt{\frac{4}{\pi} \frac{M}{m}}\,. \tag{17}$$

In the actual case, however, the field is not constant but varies from a maximum value at $x = 0$ to zero at $x = l$. Thus the electrons always move faster and the positive ions slower than we have assumed, and the ratio i/j is larger than the value given by equation (**17**). A graphical integration, using the actual field distribution as given by equation (**12**) to find t^- and t^+, led to a value of i/j not much different from

$$\frac{i}{j} = 4\sqrt{2}\sqrt{\frac{M}{m}}, \tag{18}$$

which was the relation taken in the original statement of the theory[27], but derived there in a manner quite inconsistent with the actual physical conditions in the fall space. We shall use equation (**18**), therefore, in the belief that it is at least a fair approximation to the requirements of the theory.

Expressing currents in amperes, potential drop in volts and ionic mass M in ordinary atomic units, we have the results of this theory expressed by the equations:

$$\left.\begin{aligned} &\text{Total current density} && I = i + j + J \\ &\text{Neutralizing current density} && i = 242\,\sqrt{M}\,j \\ &\text{Space charge current density} && J = 0.76\,(10)^{-7} \frac{V_c^{3/2}}{M^{1/2}\, l^2} \end{aligned}\right\} \tag{19}$$

Applications: Carbon Arc. At atmospheric pressure and 3300 deg. *K.*, which is close to the cathode temperature $l = 0.66\,(10)^{-3}$ cm., V_c is given as about 8.6 volts[29], although no determination by a reliable method has ever been made, and the true value is probably several volts higher. Substitution in equation (**19**) gives $J = 1.6$ amperes per cm.^{2} Since the total current density I is of the order of 320 amperes per cm.^{2},[30] $J/I = 0.005$. Similarly $j/I = 0.001$. Thus altogether about 0.006 of the total current is carried by positive ions.

Mercury Arc. The vapor density at the cathode is of the order of an atmosphere[31] and its temperature is at least 400 deg. *K.*, and may reach 2000 deg. *K.*, although reasons are given later which weigh against this high value. We shall not be far wrong as to order of magnitude if we take 600 deg. *K.*, which gives $l = 0.000040$ cm. V_c lies between 5.5 and 10.3, and is probably about 8.6. This leads to $J = 162$ amperes per cm.^{2} Güntherschulze finds the current density I at the cathode to be 4000 amperes per cm.^{2}, whence $J/I = 0.040$. Similarly $j/I = 0.0003$. Thus about 0.04 of the total current is carried by positive ions.

Other cases agree in indicating that only a small fraction of the total current at the cathode is carried by positive ions.

A test of this theory is afforded by comparing these calculated values for the fraction of current carried by positive ions with the values calculated from considerations of thermal equilibrium at the cathode. Before doing this, however, we shall consider an alternative theory of the cathode fall space which has been proposed by Langmuir.

LANGMUIR'S THEORY OF CURRENT AT CATHODE[14]

On this theory, the cathode fall space is simply the positive ion sheath produced around the cathode by the incoming positive ions. If it is assumed that the positive ions traverse this fall space without colliding with gas molecules, *i. e.*, if $d < l$, the space charge equation of Child[32] and Langmuir[33] may be applied in the form,

$$J = \frac{\sqrt{2}}{9\,\pi}\sqrt{\frac{e}{M}}\,\frac{V_c^{3/2}}{d^2} \text{ in c. g. s. units}$$

$$= 0.543\,(10)^{-7}\,\frac{V_c^{3/2}}{M^{1/2}\,d^2} \text{ in ordinary electrical and molecular units.} \qquad (\mathbf{20})$$

This does not appear to differ much from equation (**19**) of Compton's theory, but in reality it may be quite different since it does not assume d to be equal to the electron free path l, but leaves d undetermined. In order to use this equation, information from some other source must be obtained regarding either J or d.

Two courses are open for finding independently the positive ion current density, $J + j$, in order to test Compton's theory or to complete the information necessary for Langmuir's theory. $J + j$ may perhaps be measured directly by Langmuir's exploring electrode method[34], although this has never been done near the cathode and presents experimental difficulties, or $J + j/I$ may be calculated from considerations of thermal equilibrium at the cathode, as follows:

ENERGY BALANCE AT CATHODE[35]

Let f be the fraction of the current at the cathode which is carried by electrons and $1 - f$ that carried by positive ions. Then, per ampere of current, we have the following rates of heat development, in watts:

A. *Heating of Cathode.* (1) By incoming positive ions, which fall through the cathode drop V_c, $(1 - f)$ $(V_c + \varphi_+)$, where φ_+ is the heat of neutralization of positive ions at the surface of the cathode; (2) by outgoing electrons, some of whose energy may be returned to the cathode, $[f(V_c - (1 - f)\,V_i]\,F$; here $f\,V_c$ is the energy gained by the electrons in the cathode fall space, $(1 - f)\,V_i$ is the energy of these electrons which is used in ionizing the gas whose ionizing potential is V_i, and F is the fraction of the remaining energy which returns to the cathode in the form of radiation, etc.; (3) by heat supplied by an external heating source, if there be one, H.

B. *Cooling of Cathode.* (1) By escape of electrons, $f\,\varphi_-$, where φ_- is the electron "work function," or heat of evaporation; (2) by conduction through the body of the cathode, C; (3) by gaseous conduction and convection, C'; (4) by radiation, R; (5) by evaporation of cathode material, E.

Grouping all these items, we find the equilibrium condition to be given by

$$f = \frac{V_c + \varphi_+ - F\,V_i + H - C - C' - R - E}{V_c + \varphi_+ + \varphi_- - F\,(V_c + V_i)} \qquad (\mathbf{21})$$

An experimental determination of the factors in this equation should therefore permit a calculation of the fraction f of the current at the cathode which is carried by electrons. Let us consider the various terms in this equation:

The cathode fall of potential V_c in various arcs has been measured with the following typical results:

TABLE III

Arc	V_c (volts)	Reference
Carbon in air (current I)	$7.6 + 13.6/I$	(24)
Carbon (impregnated) in air	8.5	(31)
Magnetite in air	13.9	(32)
Copper in air at reduced pressure	13.7	(32)
Mercury in vacuum	5.27	(33)
*Argon gas and heated non-vaporizing cathode	11.6	(34)
*Helium gas and heated non-vaporizing cathode	20.0	(35)
*Mercury vapor and heated non-vaporizing cathode	5.5	(35)

As all values except those marked * were obtained by the old probe method which is known to give incorrect results[34], they are only approximate and are probably several volts too low. More accurate values are greatly needed.

The heat of neutralization, or condensation, of positive ions φ_+ was formerly calculated from a theoretical relation $\varphi_+ = V_i + L - \varphi_-$ derived by Schottky and von Issendorff[41] and by Compton[35]. Recent experimental measurements[42] have shown that the true value is much less than this, and nearly zero. The discrepancy must be due to the fact that some of the energy liberated at the electrode surface during neutralization of an ion is radiated away and hence does not contribute to the heating of the electrode. Compton and Van Voorhis[43] give reasons for modifying the above equation to the form

$$\varphi_+ = r\,V_i + (L) - \varphi_-, \qquad (\mathbf{22})$$

where r is a "radiation factor" a little less than 0.5, and L is the latent heat of condensation of the neutralized ion on the electrode, in case the ion remains there deposited. If the material of the ion does not remain on the electrode after neutralization, L is to be omitted from equation (**22**).

In this connection, the writer would suggest that the luminosity of the cathode in mercury arcs, which has been taken to indicate high local temperatures exceeding 2000 deg. *K.*, may be simply this radiation accompanying ion neutralization at the cathode surface, and showing as a continuous spectrum because of the intense field at the surface. Evidence that electron emission in this case is not of ordinary thermionic origin will be presented below.

The ionizing potential V_i is accurately known for most gases and vapors. In the case of arcs in air between vaporizing electrodes, there is uncertainty regarding the type of gas which is being ionized.

The fraction F of the excess energy of the electrons which is returned to the cathode is unknown. It cannot exceed 0.5. It is probably much nearer 0.0, expecially in the case of a rapidly vaporizing cathode, where a blast of atoms would tend to carry away any high speed atoms which might have been indirectly accelerated by the electrons. Radiation of energy back to the electrode from the gas would be a small positive factor.

H, C, C', R, and E may all be measured or computed. In considering the cooling E by evaporation of cathode material, one must be cautious, however, since there is evidence in cases like the mercury arc that not all material lost is by true evaporation, but part of it is by mechanical loss as a "spray" which does not contribute to the cooling. Also, the radiation loss R may be partially compensated by radiation gain from the anode, which must be taken into account.

Evidently our present knowledge and our experimental technique are too limited to permit us to use equation (**21**) for accurate results. It may be used, however, to show orders of magnitude and to set certain upper and lower limits which permit us to draw some important conclusions.

Applications: Carbon Arc. Take, for a 10-ampere arc, $V_c = 9.0$ volts, $\varphi_- = 3.9$ volts, $\varphi_+ = 0$, $V_i = 16$ volts. A rough estimate of conductivity loss based on a hot spot area of 0.04 cm.^{2}, temperature gradient 2500 deg. per cm. and conductivity 0.01, gave $C = 0.04$ volt, though the data are uncertain and the writer is inclined to believe that the result is too low. Net loss by radiation, calculated as if cathode and anode hot spots were black body radiators at 3140 deg. *K*., and 3700 deg. *K*., respectively, gave $R = 0.75$ volt. E is relatively small, and so is C', provided the arc is not cooled by an air blast. With these values we find

$$f = 0.64, \text{ assuming } F = 0;$$
$$f = 0.63, \text{ assuming } F = 0.25.$$

f could be raised as high as 0.70 by neglecting *all* heat losses, $C + C' + R + E$, which is clearly an upper limit. No reasonable value of φ_+ differing from 0 would produce much change in f. The assumed value of V_c is probably several volts too small, but no reasonable increase would increase f greatly. φ_- could only be given a smaller value if the electrons were pulled out of the cathode by the field, rather than spontaneously emitted thermionically, and we have previously seen that the evidence proves certainly that no *large* effect of this kind can be important in the carbon arc. We thus seem forced from energy considerations to conclude that the fraction of current carried by electrons at the cathode is of the order of 60 to 70 per cent, rather than 99.4 per cent as predicted by Compton's theory. The fact that an earlier calculation[35] appeared to support Compton's theory was due, first, to the use of a value of φ_+ now known to be inadmissible[42] and second, to the use of an impossibly high value for F.

Mercury Arc. In this case, recent experiments by Güntherschulze[44] give apparently accurate data for most of the quantities involved, except for minor corrections pointed out by Seeliger[15] and included here. The data are, in watts (volts) per ampere of arc current, $C = 2.68$; $E = 2.8$ to 3.9, depending on the assumed temperature of the cathode spot; $R = 0.04$. Taking $\varphi_- = 3.9$, $V_i = 10.4$, $\varphi_+ = 0$, $V_c = 8.6$ volts (as a reasonable value owing to the fact that it must lie between lower and upper limits of 5.3 and 10.3 and probably nearer the upper value[43]) knowing C' to be negligible and H zero, and assuming $F = 0$, we find $f = 0.25$ to 0.16. If F is taken to be greater than zero, f becomes still smaller.

Even if cooling by radiation R and evaporation E is entirely neglected, which could only be justified if *all* mercury were lost from the cathode mechanically rather than by evaporation, and even if the cooling φ_- by electron emission were neglected, which would be justified if the emission were due entirely to the "pulling out" effect of the field, still equation (**21**) gives only $f = 0.70$. In any case, therefore, the fraction of current carried by electrons must be less than 70 per cent, whereas Compton's theory predicted 96 per cent.

CONCLUSIONS

From this consideration of energy balance at the cathode, therefore, it would appear that Compton's assumption that the thickness of the cathode fall space is equal to the electron mean free path is incorrect, and that this thickness is much smaller. If it is much smaller, the positive ions must move through it generally without colliding, and we have exactly the space charge condition leading to equation (**20**) of Langmuir's theory. We must therefore consider the evidence as strongly supporting Langmuir's theory.

Further than this, these energy considerations lead us to some conclusions regarding the mechanism of electron emission from the cathode of a mercury arc. Since almost certainly the cathode drop does not exceed the ionizing potential $V_i = 10.4$ volts, it is obvious that no electron can ionize more than once near the cathode. The fraction f cannot, therefore, be less than 0.5 and could only be that small in case the probability of ionization were unity, which cannot be so. From this consideration, f must *exceed* 0.5. An examination of equation (**21**) in connection with Güntherschulze's data shows that a value of $f > 0.5$ can only be obtained if $\varphi_- < 3.9$ and $E < 2.8$ by large margins. In other words, the field at the cathode surface acts to pull out electrons which would not otherwise be liberated, and some of the mercury is lost from the cathode mechanically, rather than by evaporation. The former of these possibilities was suggested by Langmuir, whose measurements of positive ion current densities led him to estimate the field at the cathode of a mercury arc to be of the order of 10^6 volts per cm. Such fields are known to pull electrons from metal surfaces in the presence of gases or vapors, and would probably be especially effective if the metal surface is heated, as in an arc, so that many electrons need only the additional assistance of the field to permit their escape[45].

CONDITIONS JUST BEYOND THE CATHODE FALL SPACE

This region, generally called the negative glow, is a region in which the concentration of ions is maximum. The electric field is of minimum strength and is often reversed in direction, the current being by diffusion of electrons in the direction of decreasing concentration[39,46]. Probably much of the radiation from this part of the arc is the result of recombination of ions and electrons[47].

CONDITIONS IN THE POSITIVE COLUMN

Here ionization occurs to just a sufficient extent to balance the loss of ions by recombination or diffusion to the walls, if the arc be enclosed. This ionization may be produced thermally, by electron impact, photoelectrically, or by a combination of these. There are reasons for ascribing much of it to high temperature in the carbon arc[35], while this certainly plays no role in the mercury arc, where the ionization is due to electron impacts, probably of a cumulative nature. The light from the positive column is almost certainly due to excitation rather than to recombination[47].

CONDITIONS AT THE ANODE

The anode drop in potential may be positive or negative according to conditions first explained by Langmuir and Mott-Smith[34] as follows: Surrounding the anode is an atmosphere of ions and electrons moving with more or less random motion. If, in this random motion, the excess of electrons over positive ions striking the anode would be greater than the total current in the circuit, then a negative, or reverse, anode drop is set up so as to hold back enough electrons to keep the current to the value demanded by the constants of the circuit. On the other hand, if the number naturally striking the anode is insufficient to carry the current, then a positive anode drop is set up so as to draw in more electrons. From these considerations, it is evident that anode drop decreases with increasing anode area and with increased ion concentration, as can be obtained by using a hollow anode or by promoting ionization near the anode.

The heating of the anode depends on three factors: (1) the heat of condensation of electrons φ_-; (2) the average energy $\overline{V}_-$ of the electrons in their initial random motion; (3) the anode drop V_a, if this be positive. Although this subject has been studied calorimetrically[48] and the order of magnitude of these predictions always verified, thus far only Van Voorhis[42] has measured all the quantities necessary to make an accurate quantitative test, which has exactly verified the above statements. A. W. Hull[49] reports that calorimetric work on high-power mercury arcs at the General Electric laboratory is also in quantitative agreement with these ideas.

From the preceding discussion it will be seen that much progress in the understanding of arc phenomena has been made during the past few years, and that there are at present numerous possibilities for further experimental research, guided by theoretical considerations.

ADDENDA

(1.) Complete discussions of earlier work on arcs, with bibliographies, may be found in

"The Electric Arc," Mrs. Ayrton (*The Electrician* 1902)

"The Electric Arc," Child (Van Nostrand 1913)

Lichtbogen, "Handbuch der Radiologie," Vol. IV, pp. 211-444, Hagenbach (Akademische Verlagsgesellschaft, Leipzig, 1917)

L'Arc Electrique, Leblanc Fils (*Journal de Physique* 1922)

Lichtbogen, "Handbuch der Physik," Vol. XIV, Hagenbach (Springer 1927)

(2) In terms of mechanism, the arc may be defined as a gas discharge in which the ionization near the cathode is produced by electrons which *have* fallen through the cathode fall of potential and thereby gained the energy necessary for ionization, whereas in the glow discharge the ionization is produced while the electrons are falling through the cathode fall space. In the glow discharge the ionization increases exponentially with distance from the cathode: in the arc there is no exponential building up of ionization. This definition is equivalent to the one already given.

(3) Different arc types are sometimes found under conditions in which transitions from one form to another may occur. It is suggested that the primarily "thermionic" arc and the primarily "pulling out of electrons" arc may be two such types. In Table I those arcs whose current densities are thousands of amperes are probably of the latter type and those with smaller current densities of the former type. Both types are shown for tungsten in Table I. There is some evidence of still another arc mechanism (Dr. Slepian, unpublished).

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Discussion

Joseph Slepian: There is hardly a scientist on whose work I lean more than Professor Compton's.

I remember as one of the best of Professor Compton's papers the one on the theory of the arc which he gave in the *Physical Review* in 1923, in which he so ably defended the thermionic theory which he has discussed tonight. Today, however, Professor Compton apparently feels that perhaps this theory may not always hold after all.

Remarkable also have been his papers on abnormally low-voltage arcs. The first deals with the theoretical difficulties, almost proving that an arc with a voltage less than the ionizing voltage of the gas is impossible. A later paper, however, demonstrates the existence of arcs with voltage less than the ionizing potential but greater than the resonance potential. Then followed a paper showing that where arcs with voltages lower than the resonance potential had apparently been obtained, oscillations had been present which momentarily would raise the voltage above the resonance potential. Then came still another paper showing that arcs with total voltage less than the resonance potential were possible under suitable conditions.

Another paper which I found exceedingly valuable, stimulating, and instructive, is that one on the Mobility of Ions in Discharges in which he very boldly sets out to calculate the ways in which ions will move in gases of considerable density under the action of the electrical fields, taking account of various kinds of collisions which electrons can have with molecules. This was an exceedingly difficult problem, and I marveled that anyone had the temerity to tackle it. Yet with a few skillful manipulations and ideas, Professor Compton derived equations which are quite easy to understand and exceedingly valuable.

There are some experiences which I have had in connection with my engineering work which I think will be interesting in relation to the theory of the arc. The various theories of the cathode of the arc mentioned by Professor Compton require that a considerable portion of the current be carried by electrons leaving the cathode. The question then arises as to how these electrons get out of the cathode, as ordinarily electrons will not pass from a metal into an adjoining gas. One agency which will assist electrons in escaping from a metal is heat. When its temperature is sufficiently high electrons can pass freely out of a cathode. This is essentially the thermionic theory of the cathode of the arc, which Professor Compton advocated a number of years ago.

Another possibility which Professor Compton has mentioned is that a very high electric gradient may develop at the cathode surface in the arc a gradient so high that the electrons are pulled out of the cathode even though it is not hot enough for thermionic emission.

At the time of the experiments which I am going to describe, I, along with almost everybody else, believed in the thermionic theory of the cathode; that is, that in an arc it was necessary to have a cathode hot enough for thermionic emission. If the cathode was not hot enough for this, an arc discharge would be impossible, and if any discharge was obtained it would have to be a glow or other high-voltage form. I tried to apply these ideas to the development of the arc which follows the breakdown of a spark-gap by application of high voltage.

Since the electrodes of the spark-gap are initially cold, it seemed necessary that the discharge should start as a glow and only after some point of the cathode reached a sufficiently high temperature should the discharge change into an arc. I tried to calculate the time for the heating up of the cathode spot, and therefore the time for the flow to change into an arc, using data for the watts input at the cathode of a glow on copper obtained from other experiments. I found it would take seconds before the copper would get to the melting point, let alone a temperature sufficient for thermionic emission. But the experiment showed that the arc struck almost at once. Immediately after the gap broke down, the voltage dropped to 20 volts, which is too low for a glow.

I had been of the opinion that the cathode had to be hot in order to maintain an arc; yet here, where the electrodes did not have time to get hot, I was getting a discharge with only 20 volts. More recently, this experiment has been repeated, using the DuFour Oscillograph, and it has been found that the time for the discharge to change from glow to arc is of the order of a microsecond.

Since the cathode couldn't have become hot in so short a time, this experiment made me feel that the thermionic theory of the cathode couldn't be correct; at least not all of the time.

Another experience in connection with my engineering work which made me believe that probably the cathode didn't have to be hot was in studying the operation of switches. I have seen switches in which the arc was blown rapidly along the arcing horns operate, and examined the horns afterward, finding stretches on the arcing horn absolutely free of burning. There might be some oxidation but no evidence of a very high temperature.

This seemed strange and would be hard to explain if the cathode had to be hot enough for thermionic emission. I looked into this a little more closely, and considered the hypothesis that perhaps the arc hopped from point to point, without passing over the intervening stretch, so that this stretch might not appear burned because the arc had not actually played on it.

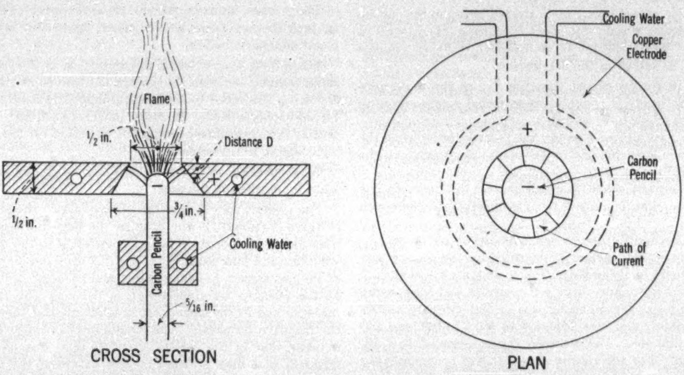

Fig. 1

I eliminated this possibility by bringing the electrodes very close together, and also took photographs with a high-speed camera. I found that even at those spots where the arc had played, as indicated by the photographs, there was no burning.

A German, H. Stolt, has also carried out similar experiments, published in the *Annalen der Physik.* Stolt caused an arc to move over a cathode so rapidly that apparently there was no heating of the cathode. The claims of Stolt were criticized by Güntherschulze, who is mentioned frequently in this paper of Professor Compton's; but Stolt replied quite well to the criticism of Güntherschulze, and I believe that Stolt's conclusions are fairly well established. Stolt did get a low-voltage discharge from copper and other metals, which moved so rapidly over the copper surface that no spot of the copper surface became hot enough to melt, let alone have thermionic emission. This seemed to me to disprove definitely the theory of the necessity for thermionic emission. I have carried experiments similar to Stolt's somewhat further, and have used currents as high as 20,000 amperes; that is, I have moved a 20,000-ampere arc, over a cathode surface so rapidly that there was no melting of the copper but merely a trace of oxidation. Incidentally, the current density in these experiments was of the order of 30,000 amperes per cm.^{2}

The application of the method of energy balance at the cathode which Professor Compton has used in his paper for estimating the fraction of the current carried by electrons is certainly very interesting, and the values $f = 0.25$ to $f = 0.16$ obtained for the mercury arc seem significant. If the ionization of the gas next to the cathode is primarily due to collisions from electrons coming from the cathode, f could not be less than 0.50.

Some time ago, I suggested in the *Physical Review* that perhaps no part of the current at the cathode was carried by electrons, but that all of the current was carried by positive ions coming from the highly ionized gas next to the cathode. The cause of the high state of ionization in the gas was to be sought in the very intense energy concentration there. The values of f which Compton finds indicate that this suggestion may be near the truth. Indeed, if a necessary correction to the energy balance equation is applied, the value of f comes even closer to zero in accordance with my suggestion. The correction is as follows. As item (1) under A, "Heating of the Cathode," Professor Compton has "By incoming positive ions, which fall through the cathode drop B_c, $(1 - f)(B_c + \phi_+)$." But all the positive ions do not fall through the cathode drop unimpeded. Some will collide with molecules and lose energy to the gas. Also, many positive ions will be reflected from the cathode thus increasing the chances of collision with molecules. Let α be the fraction of the energy acquired by falling through the cathode drop, which a positive ion, on the average gives up to the cathode, Then item (1), under A, becomes $(1 - f)(\alpha B_c + \phi_+)$.

Equation (*21*) then becomes

$$f = \frac{\alpha B_c + \phi_+ - F B_i + H - C - C' - R - E}{\alpha B_c + \phi_+ + \phi_- - F (B_c B_i)}$$

If we substitute the numerical values used by Compton we get

$$f = \frac{8.6\,\alpha - 5.5}{8.6\,\phi + 3.9} \text{ or } f = \frac{8.6\,\alpha - 6.6}{8.6\,\phi + 3.9}.$$

If we take $\alpha = I$ we get, of course, the values of Compton, $f = 0.25$ and $f = 0.16$. If, however, α is as low as 0.64 by the first formula, or 0.77 by the second formula, we get $f = 0$.

Now what is a reasonable estimate of the value of α? We may obtain some idea by determining the number of positive ions which cross the cathode space without colliding with a molecule. The electronic mean free path as given by Compton in this paper for the mercury-arc cathode is 4×10^{-6} cm. and therefore the ionic mean free path is 1.0×10^{-5} cm. On the other hand from equation (**20**) taking $-J = 4000$ as given by Güntherschulze, we find, for d, the cathode-fall space $d = 4.95 \times 10^{-6}$. The fraction of the positive ions which will have free paths greater than the cathode-fall space will therefore be

$$e^{-\frac{4.95\times10^{-6}}{1.0\times10^{-5}}} = e^{-0.495} = 0.61.$$

That is only 61 per cent of the positive ions crossing the cathode-fall space fail to collide with a gas molecule. Hence a value of α equal to 0.77 does not look altogether unreasonable.

J. C. Lincoln: At the plant of The Lincoln Electric Company, we have run across a new phenomenon which has to do with the nature of the arc and which has changed my notion of what happens in the arc. This phenomenon occurs in a device which we call an "electric torch."

The illustration herewith, shows the arrangement of the parts making up the torch. The copper electrode holder for the negative terminal is water-cooled as well as the copper positive terminal where the path of the cooling water is indicated. The copper anode has a tapered hole of the dimensions indicated on the sketch cut in it. The opening in the anode is round and the dimensions are those of a section through the center of the opening in the anode.

After the parts are set up an arc is started between the carbon cathode and copper anode by short circuiting them with a carbon pencil. While the arc is maintained, a flame projects from the anode as shown in the illustration.

Observation of the arc shows that the size of the flame projecting from the anode is roughly proportional to the amperes across the arc, as might be expected.

Furthermore, with a given current, the flame is larger when the distance *D* is smaller, the size of the flame decreasing as the distance *D* is increased.

A change of current causes a much greater difference in the size of the flame than is caused by a proportionate change in the distance *D*.

The flame is apparently due to very hot carbon particles.

If the flame is cooled, carbon is deposited on the cooling surface just as it would be if the flame from a wick were cooled.

A series of tests was made varying the current and the spacing *D*. The current was furnished by one of the company's 200-ampere, d-c. welders and was adjusted to 50, 75, 100, and 150 amperes across the arc. The distance *D* was adjusted to 3/16, 3/8, and 9/16 in. For the results shown in the accompanying table, the flame was above the torch, as shown in the illustration.

To determine the amount of heat in the flame, the amount of cooling water passing through the torch and its rise in temperature were measured. In the tests the initial temperature of the cooling water was 19.1 deg. cent. and 5.375 lb. per minute was used. From these measurements, the heat absorbed by the water was calculated. The rate at which this heat was absorbed was then expressed in watts. It was assumed that the watts input, minus the watts carried off by the water, equals the heat energy in the flame, and the table shows the percentages of heat in the water and the flame respectively.

The results indicate in general that the greater the current, the greater is the energy in the flame; also, that the smaller *D* is, the greater is the energy in the flame.

The table shows that from $^1/_2$ to $^2/_3$ of the heat appears in the flame, and I believe that if none of the heat developed in the flame was radiated and absorbed by the copper and water, an even larger proportion of the total heat in the arc would appear in the flame.

The direction of the flame can be affected by a magnet. By presenting the south-seeking pole of a bar magnet to the arc, the flame is pushed to one side. So far as I could judge the flame itself is not affected by the magnet. The direction of the flame is a function of the current from the carbon cathode to the anode. To put it another way, the magnet had no effect on the direction of the flame except when close to the arc between the carbon and copper anode.

TABLE I

D in sixteenths of an inch	Amperes	Volts	Watts input	Temp. rise of cooling water, deg. cent.*	Watts absorbed by water	Percentage of heat	
						In water	In flame
3	50	47	2350	5.9	1002	42.5	57.5
6	50	48	2400	6.9	1170	48.8	51.2
3	75	51	3820	8.9	1535	40.2	59.8
6	75	40	3750	9.9	1695	45.2	54.8
9	75	48	3600	11.9	2020	56.0	44.0
3	100	55	5500	10.9	1850	33.6	66.4
6	100	51	5100	13.9	2360	46.1	53.9
9	100	49.5	4950	16.2	2750	55.5	44.5
3	150	57	8560	18.4	3130	36.6	63.4
6	150	51	7670	20.9	3560	46.5	53.5
9	150	51	7670	23.9	4040	52.7	47.3

*5.375 lb. of water passed per min.

The current between the carbon and copper anode is effected in just the way one would expect from the laws governing electromagnetic action.

When there is no external magnetic field at the arc, the current flows radially between the carbon and the copper anode. When the arc is subjected to an external magnetic field, the current is forced to only a part of the radial path between carbon and copper and at the same time the flame is deflected so that it is more at right angles to the current.

What bearing do these results have on our conception of what takes place in the arc?

The present view is that the voltage across the arc is made up of three portions: (1) the drop at the negative terminal, which must be great enough to heat the terminal to the point where it will throw off ions readily, (2) the *I-R* drop due to the resistance of the gas stream between anode and cathode, and (3) the drop at the positive terminal which is fixed by the nature of the material and in the carbon arc is much greater than the drop at the negative terminal. The results of the measurements would indicate that in the carbon arc there is a drop at the positive terminal that may be fixed by the nature of the material, but that this drop is not nearly so great as has been supposed. The heat at the positive terminal in the ordinary carbon arc is the sum of the heat due to the inherent drop and the heat of the flame or blast from the negative terminal. The heat due to the flame or blast has been separated largely from the inherent anode drop in the electric torch and measured. The measurements indicate that the heat in the flame or blast is greater on the average than the sum of the anode drop, the cathode drop, and the *I-R* drop due to the resistance of the gaseous part of the arc.

I do not think it is far from the truth to say that two-thirds of the energy in the carbon arc appear as heat in the flame or blast from the cathode. The question naturally occurs—What is the nature of the flame? Two things can be said of it. First, particles of very hot carbon are shot off the end of the cathode and these draw the air with them so that the flame from a match is sucked downward through the opening in the anode when the apparatus is set up so that the flame is below the torch. The current of air was doubtless much stronger when the apparatus was turned over, for it was not possible to get the 9/16-in. reading with 50 amperes, for the arc would not persist long enough to permit of measurement of the heat.

I assumed that this was due to the stronger current of air through the opening in the anode when the apparatus was set up to take the measurements contained in the table.

The second thing is that the actinic value of the flame near the opening in the anode is very much greater than the value in most of the flame. When the flame was focused on the ground glass the image of the flame covered nearly the whole plate, but a short-time photograph showed a very small figure on the plate. The pictures were taken in 1/500 to 1/1000 sec. and at this speed, not more than 10 per cent of the flame that showed on the ground glass plate appeared on the photograph.

A picture of the flame was taken with the camera behind a piece of ⅛-in. thick pasteboard to see if the active part of the flame contained X-rays. The results were negative.

When the current was reversed in direction, the apparatus refused to work as a torch and the arc apparently tried to run up the carbon when it was made the positive terminal of the arc. This is a most noteworthy fact, for it depends on something beside the electromagnetic forces. In any piece of apparatus with which I am acquainted, the direction of motion is independent of the direction of current, for the reversal of current reverses the flux and with both flux and current reversed, the direction of motion is unchanged.

What is this blast or flame from the cathode? Apparently it is not a stream of electrons, for if it were, it would be affected by a magnetic field. At the same time it must be remembered that approximately two-thirds of the total energy in the arc appears in this flame. It is my opinion that the blast from the cathode in the carbon arc is due to vaporized carbon from the carbon pencil.

We do not know much about the latent heat of carbon, but it is possible and even probable that it is very high. If I am correct in the opinion that most of the energy of the carbon arc is expanded in vaporizing carbon from the carbon cathode and that most of the heat that appears at the positive anode is due to the solidification of the vaporized carbon at the anode, this would be evidence of a large amount of energy required as latent heat to vaporize carbon.

It is my belief that to get a more accurate conception of what occurs in the arc, we shall have to substitute the idea of the blast from the negative terminal as being the central and important thing which occurs in the arc, for the idea that there are inherent anode and cathode drops.

The tests described in this paper show that the flame is a phenomenon associated with the negative terminal.

The old way of looking at it would be to say that the inherent drop at the positive terminal was great enough to produce the heat that actually appears there. This old conception has, I think, been shown to be wrong by these tests.

The old idea was that the current passed across the arc in a solid stream and that a cross-section of the current in the arc would be a circle.

The experiment with the torch, as well as some others not described, indicate that the core of the arc is the blast from the negative terminal and that the current flows outside of the blast and that the section of the current across the arc would be an annulus and not a circle. In such a cross-section, the inner circle would be the cross-section of the blast from the negative terminal and the annulus outside of this inner circle would be the cross section of the current. There is no doubt that this is the true picture of the cross-section of the current in the case of the torch, and I believe it is the true picture in any carbon arc.

P. P. Alexander: I should like to ask Professor Compton to say a few words about the ionizing potentials of different gases. These are well known at ordinary temperatures, but at the temperature of the arc core, apparently, they are entirely different. For instance, the ionizing potential of nitrogen at ordinary temperature is something like 11 volts; at the temperature of the arc core, it appears to be several hundred times less.

I should like to ask Professor Compton if experiments have been made to determine the various ionizing potentials at high temperatures, because knowledge of these potentials is quite essential to the correct interpretation of arc phenomena.

V. Karapetoff: Dr. Compton's paper is mainly concerned with simple, steady arcs, and it is only right that an involved phenomenon should first be studied in its simplest form. In practical applications, we have mostly variable arcs, and our problem is two-fold: (1) To make an arc as steady as possible; for example, in arc furnaces, in electric welding, in arc lamps, rectifiers, etc.; or else, (2) to make an arc as unstable as possible so as to extinguish it quickly; for example, in switches, spark-gaps, relay contacts, flashovers, etc.

In either group of problems, it is of importance to know the factors which contribute both to the stability and instability of an arc, so as to intensify the desirable factors at will. This means that engineers will have to pay more and more attention to the physical nature of the arc, and Dr. Compton's paper, with its references to literature, should prove a valuable introduction to the subject as well as a guide to future investigators.

Dr. Compton quotes several empirical equations for the observed relationship between the voltage and the current in a steady arc. In a transient arc, or spark-over, both the current and the voltage are functions of time, and the apparent total resistance of the arc is variable. Dr. Max Toepler[1] has proposed the following function for this resistance:

$$R_t = k\,F/A_t \qquad \textbf{(a)}$$

Here k is an empirical constant, F, the length of the arc, and A_t the total quantity of electricity which has passed through the arc from the instant $t = 0$, when it was struck, to the instant t under consideration.

For a transient arc, there is some reason for Toepler's formula, in that the ionized state of the gas is established only gradually, and may be considered a function of the quantity of electricity which has passed through the arc, the conductance increasing with this quantity.

On the other hand, Toepler's formula has some serious defects; namely,

1. The resistance, according to formula (**a**), being infinite at the instant of striking, no finite voltage should be able to start an arc;
2. Should the arc continue over an indefinite period of time, its resistance, according to Toepler, should drop to zero;
3. The ratio of the voltage to the current is assumed to be proportional to the length of the arc; in reality there is a considerable and concentrated fall of potential at the cathode, and some drop at the anode.

It is proposed, therefore, to generalize Toepler's formula as follows:

$$R_t = (k\,F + k^1)/(A_t + q) + r \qquad \textbf{(b)}$$

In this expression, k^1, q, and r are additional constants, introduced for the purpose of correcting the above-mentioned defects of the original formula. When $Q_t = 0$, *i. e.*, at the beginning of the discharge, R_t is no more infinitely large, but has a high finite value, $R_0 = (k\,F + k^1)/q + r$. With a steady arc, when $Q_t = \infty$, the resistance is no more equal to zero, but has the limiting low value of $R_\infty = r$. Furthermore, the resistance is assumed to increase more slowly than the length F of the arc, there being a correction term k^1.

Dr. Otto Mayr has given a general theory of condenser discharge through a resistance and a sphere-gap, using Toepler's formula for the resistance of a transient arc[2]. He has also determined some values of k from the available experimental data. The next step should be to extend his theory on the basis of generalized formula (**b**), and to determine the numerical values of the constants which it contains.

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[1] *Archiv fur Elek.*, 1925, Vol. 14, p. 306.

[2] *Archiv fur Elek.*, 1926, Vol. 17, p. 53.

E. C. Starr: I should like to ask Dr. Compton and the gentlemen who have discussed his paper if they have any data on the order of magnitude of the transient resistance of an arc. I have reference to the type of arc that is initiated by a potential of several thousand volts between electrodes in air at normal pressure and temperature.

The size and shape of the electrodes, as well as the spacing, no doubt affect the resistance considerably. For example, the ionized path between the points of a needle-gap is not uniform in intensity of ionization and the effective cross-sectional area of the path is relatively small compared to the area of the path between large spheres or parallel disks. Hence it is to be expected that the resistance of an arc between the latter type of electrodes should have a lower value throughout the entire period of the transient than in the case of a needle-gap of the same spacing.

Dr. Slepian spoke of measuring the voltage transient of an arc. Perhaps he also recorded the current transient and could therefore readily determine the resistance characteristic.

The transient resistance equation suggested by Prof. Karapetoff should be of considerable value in the calculation of transients in circuits containing spark-gaps if the values of the constants can be determined.

R. W. Sorensen: (communicated after adjournment) I should like to supplement what has been said by telling some of the interesting things relating to arcs that Dr. Millikan and I have found, as we have endeavored to produce a non-arcing switch for use on electric circuits. At California Institute of Technology we have been interrupting high-voltage, high-power electric circuits by means of switches enclosed in a vacuum chamber. To date, we have been very successful in our attempts to do this, largely because the arc at the opening of the switch is very small, and apparently removes a negligible amount of material from the switch terminal when the arc is struck. We have some switches showing practically no burning or pitting of contacts after 4000 operations. Also, by means of relatively small contacts, currents of several thousand amperes at approximately 50,000 volts, have been successfully interrupted. In performing these interruptions, the switch terminals have not been unduly pitted and since there is no pitting of the metal, it is rather difficult to account for the energy dissipation at the switch during the time of opening.

Dr. Compton has defined an arc "as a discharge of electricity between electrodes in a gas or vapor, which has a negative or practically zero volt-ampere characteristic and a voltage drop at the cathodes of the order of the minimum ionizing or minimum exciting potential of the gas or vapor," all of which may be true but we have found from our many experiences that if gas or vapor is required to maintain an arc, the amount required is indeed very small. Perhaps if we could hypothecate a liquid or gas, which will not vaporize at arc temperature, it would still be possible, though it may appear improbable, to start an arc in such a liquid. I should like to ask Dr. Compton how it would affect his definition to leave out the words "gas or vapor" and have his definition read "an arc is a discharge of electricity between electrodes, which has a negative or practically zero volt-ampere characteristic and a voltage drop at the cathode of the minimum ionizing or minimum exciting potential of the material stripped from an electrode at the temperature of the arc." In other words, is it essential that there be a surrounding medium of gas or vapor in order that an arc may be struck by the electrodes.

If for the moment we assume a medium of gas or vapor not essential to the establishment of an arc, we must, of course, look for some other means of explaining the process by which an arc between electrodes is sustained even for a very short period of time. This presents a difficulty, which, however, may not be insurmountable. Contrary to public opinion, the best known vacuum is not a perfect insulator, in the sense that no electric current can be made to pass across such a vacuum, because we now know that electrons can be shot across vacuous spaces. When an arc is struck by the two separating electrodes, there must be present a host of ions or ionized particles, as well as free electrons which serve as the carrier for the arc. Is it not possible to picture a condition under which these carriers may be provided entirely from the electrodes, and not by a surrounding gas?

K. T. Compton: The work to which Dr. Slepian calls attention is, I think, some of the most interesting in connection with the theory of the arc. I made a reference to it in the paper, but I should like to call attention to just one thing for fear of being misunderstood.

There were two theories of Langmuir that have been discussed. One theory has no reference to the origin of the electrons. It would apply independently whether the electrons have thermionic or any other origin. It is merely a space-charge theory.

As to the other theory, *i. e.*, that electrons may be pulled out of the cathode by high electric fields, I think that we have there a possibility of two types of electric arcs. Of course there are often two or even more types of arcs. The character may change from one to the other, but both are recognized as arcs. It seems to me we have brought out in this discussion opportunity at least for two of these.

Dr. Slepian mentioned the case of a copper arc in which the cathode was not melted, and obviously didn't get to the melting temperature. On the other hand, we certainly do have copper arcs in which the copper does melt.

In Table I we notice two tungsten arcs, one with a current density of 3200 amperes per sq. cm., and the other with 700. In the latter case the thermionic emission can be calculated from the temperature that the arc reached. The tungsten melted, and at least reached the melting point of tungsten. In that case (see Table II) the thermionic emission, calculated from the constants of tungsten, comes as near 700 amperes per sq. cm. as the purity of the tungsten would justify. On the other hand, 3200 is clearly too high to be accounted for by thermionic emission. In tungsten we have these two different types, one evidently of thermionic origin and the other of different origin,—perhaps with electrons pulled out (Langmuir) or perhaps arising from intense ionization in front of the cathode (Slepian).

Nottingham used a special type of arc, especially designed to reduce heat conduction so that cathode temperature could rise. He got a large cathode spot. In the case of carbon also we have pretty good evidence of the large proportion of the emission thermionically, and in the case of Pointilite lamps and Tungar rectifiers where we can use a pyrometer to determine the temperature of the various parts of the cathodes, one gets pretty good agreement with the theory of thermionic emission.

In Table I, in the cases where we have current densities running into thousands of amperes per sq. cm. I think, with Dr. Slepian, that thermionic emission is not adequate to account for currents of that order. Another agency must be operative in such cases.

There are two ways in which the temperature may affect the ionizing potential. In the first place, the gas to be ionized may be dissociated from its molecular state into an atomic state as a result of high temperature. That such action is possible has been shown in experimental cases where it is possible to produce this dissociation under conditions that can be controlled, namely, in hydrogen, iodine, etc. In those cases the effect is always to reduce the ionizing potential. This action of temperature is indirect.

As regards any direct effect of temperature on the ionizing potential of the gas, this effect will probably be rather small because translating degrees centigrade into volts, about 8000 deg. cent. correspond to only one volt. There are no laboratory experiments that reach a temperature as high as 8000 deg. So the average energy imparted to electrons as the result of high temperature or to the molecules by high temperature would in general be only a fraction of a volt.

I don't know whether or not there are other ways in which the ionizing potential of the gas might be affected than these. The only ones I know that have been directly investigated have been dissociation of molecular gases into their constituents, and the direct thermal ionization of alkaline vapors in electric furnaces, as done at the Mt. Wilson Observatory.

With regard to the question by Mr. Starr, I am sorry that I cannot give the desired information because I have made no study of transient arc phenomena.

Professor Sorensen's suggestion that the ionization of materials stripped from the electrodes at the temperature of the arc be substituted for that of a surrounding gas or vapor appears to me to be quite permissible as including the interesting discharges which he described as true arcs. In fact such material is included in the term "vapor" in the sense that I have used. The important thing, as I see it, is the presence of some ionizable material in the space between the electrodes.

The great success of this current interrupter seems to be due to the fact that, at such low gas or vapor pressures, the mobility of the ions is so great that they effectively disappear from the arc space during the time of low voltage between voltage reversals. In high-pressure arcs as in oil-immersed circuit breakers, on the other hand, the ion mobility is so small that ions remain in in sufficient concentration to re-strike the arc after the voltage reversal.

In answer to Prof. Karapetoff I wish to say that I never discuss the question of an electric arc with anyone who has had any real practical experience with an electric arc without feeling how limited is the experience which we have in the laboratory. As I said, we physicists work with arcs on a small scale, and the attention of physicists has been devoted to arcs under the simplest conditions in order to find out something about the things going on in the arc. Unfortunately those aren't the arcs met with in engineering practise, where simplicity and even understanding of the phenomena are not the prime considerations. It may be, I am afraid, another generation of physicists which will be able to answer some of the questions which are uppermost in the minds of engineers.

Printing Telegraphs on Non-Loaded Ocean Cables

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Synopsis.—*This paper discusses the application of printing telegraphs to ocean cable operation. Reference is made to various improvements in apparatus and operation, tending toward increased output, and the effect of the application of these improvements from the laying of the first cable to the present time. Telegraph codes and their relation to speed and their applicability to printer operation are also discussed. Transmission methods, shaping of the signals, variable lag, and the effect of earth currents are also described in the paper. Manual, semi-automatic, and full automatic operation of long ocean cables are covered briefly. The characteristics and advantages of regenerative repeaters are pointed out and the operation of printers on cables described.*

* * * * *

HISTORY

SINCE the laying of the first successful ocean cable between Heart's Content, Newfoundland and Valentia, Ireland, cable engineers have worked to increase the output or speed by improving the terminal apparatus and by the use of more efficient operating methods.

The first cables were operated by the manipulation of two keys at the sending terminal of the cable, one used for a dot and the other for a dash of the cable Morse code. At the receiving end these signals were received on the well-known mirror galvanometer. The moving vane of this galvanometer carried a small mirror by means of which a beam of light was projected on a screen and deflected momentarily to the left or right, corresponding to the dot or dash of the Morse code, the center representing a space.

This form of reception was obviously a slow one and was later superseded by Lord Kelvin's siphon recorder. This was a great advance over the former method in that it gave a legible and permanent record of the Morse characters and increased the speed of reception and, therefore, the output of the cable. The next important improvement was the duplexing of the cables permitting simultaneous transmission in each direction which practically doubled the speed of operation.

Further increase in output was obtained by the development of automatic transmitting devices by which a cable could be operated at its maximum signaling speed, another step in advance over the original manual method. Still later came the introduction of magnifiers which so increased the visibility of the received signals that a further increase in the speed of operation was possible. The development of the cable printer permitted a further substantial increase of speed and in addition made the operation entirely automatic.

The progress made in cable operation since the early cables were laid will be more readily appreciated if we note the various steps of increased output of a given cable, laid approximately 50 years ago and which is still in service.

The original speed or output of this type cable when operated with the mirror galvanometer was about 70 letters per minute. By the use of the siphon recorder this speed was increased to 80 letters per minute or 13.3 words per minute. With the application of the duplex principle, by which two messages are sent at the same time, one in each direction, the output was practically doubled to 160 letters per minute. Then followed the introduction of the automatic transmitter raising it to about 220 letters per minute. The addition of the magnifier further increased the output to about 300 letters per minute. With printer operation the output of this cable has been increased to something like 375 letters per minute.

From this review, it will be seen that the original output of this cable of 70 letters per minute has been increased over 500 per cent, all of which has been brought about by the development of transmitting and receiving apparatus.

OBJECT OF PRINTER OPERATION ON OCEAN CABLES

There were at least two objects in view in applying a printing telegraph system to ocean cables, one being to further increase the output of the cable and the other to make the operation wholly automatic.

Up to the time of introduction of the cable printer, cable operation might be considered as semi-automatic, that is, while the transmission was automatic the reception was not, as it was necessary that the siphon recorder signals be manually translated by skilled operators, whereas with printer operation the signals are translated mechanically and printed on a page or tape as may be desired.

PROBLEM OF TRANSMISSION

Before discussing the actual work of the printing system it may be well to consider first the problem of transmission. This being fundamental it should be clearly understood in order to more fully appreciate what follows. In all systems of wire communication a necessary element is the code of signals or telegraphic alphabet to be used for transmitting the message.

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Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 21-24, 1927.

In this discussion the term *code* may be interpreted as a series of predetermined combinations of one or more electrical impulses or time units. These code combinations must be transmitted over the cable and translated into the letters of the alphabet at the receiving end. The selection of the code, for there are several kinds, determines the number of letters or words per minute which can be transmitted over a cable. The reason for this is that different codes have different time unit lengths. Not only do different codes vary in time unit length, but a particular code may in itself have combinations of variable time unit lengths.

For example, the international Morse code has an average length of about eight and one-half time units per combination or character, some of the combinations containing only four units and some twenty units. In the cable Morse code, which also has combinations of variable length, the average time unit length is 3.7 units. Such codes may be classed as non-uniform codes. Then there are the uniform codes in which each combination is of equal length such as the Baudot code of five time units and the Cook code of three units, so that we have at least four kinds of codes available for transmitting intelligence over the cable.

The next logical step is to determine the best code to use for the work to be undertaken. For this purpose reference will be made to the code used on the cable selected for this comparison and the speed at which the cable is operated. The speed of a cable is usually referred to in terms of the number of letters that can be transmitted and recorded in one minute. This is determined in practise by finding the highest rate of speed or frequency at which alternating current, or as termed in cable practise, reversals of current, can be transmitted over the cable and that will arrive at the receiving end with sufficient amplitude to properly operate or deflect the delicate recording instrument. This rate of speed is termed the working fundamental frequency. It does not, however, mean it to be the maximum frequency of the cable which is probably much greater.

With the code originally used on this particular cable as a basis, it can be determined which code should be used for printer operation, bearing in mind that the printer must necessarily yield an output as good as, or preferably better than the cable Morse code.

CODES

This particular cable was, previous to the installation of the printer, being operated at the rate of 150 letters per minute using the cable Morse code of 3.7 units. This means that the maximum number of time units that can in practise be transmitted over this cable is 150 × 3.7 = 555 units, which reduced to terms of fundamental frequency is $\frac{555}{60 \times 2}$ or 4.6 cycles per second. The output in letters per minute for any given code is found by dividing the known speed of a given cable in time units per minute, by the figure representing the average number of units of the code under consideration.

Applying this formula to the particular cable just mentioned, the speed of which was 555 time units per minute, the output in letters per minute for the different codes would be:

International (Two-current non-uniform code)	$\frac{555}{8.5}$	= 65.3 letters per minute
Cable Morse (Three-current non-uniform code)	$\frac{555}{3.7}$	= 150 letters per minute
Baudot (Two-current uniform code)	$\frac{555}{5}$	= 111 letters per minute
Cooke (Three-current uniform code)	$\frac{555}{3}$	= 185 letters per minute

Of the four codes available for printer operation, it is obvious that the International code gives too low an output and may therefore be eliminated for this reason, and also because a non-uniform code requires a more complex printer mechanism than does a uniform code. The Cable Morse or three-current code may also be eliminated because of its non-uniformity.

This leaves two codes for consideration, the Baudot five-unit and the Cooke three-unit. The five-unit code was chosen for the first experiment.

The question may arise as to why the longer code of the two was selected if increased output is to be a factor in the development of a printing system. The answer is that while both the Baudot five-unit code and the Cooke three-unit code are uniform, so far as their unit lengths are concerned, there is still another important difference. The Baudot is a two-current code while the Cooke is a three-current code. The two-current code is made up of positive and negative impulses and the three-current code of positive, negative, and zero impulses. The two-current code is more desirable for printer operation than the three-current code because it makes for greater simplicity of mechanism.

A further and important reason for selecting the two-current Baudot code of five units is that it is adaptable to a method of transmission, discussed later in the paper, which permits the actual doubling of the number of letters per minute shown in the above table of speeds. In this case we have the apparent paradox of the longer code giving a higher output than the shorter code. The three-current code is not adaptable to this method of transmission.

Notwithstanding these differences, however, both the Baudot and Cooke codes were tried out in actual service for purposes of comparison.

The first cable printer experiment with the five-unit code was tried over an ocean cable previous to the outbreak of the World War. The results of this experiment were considered satisfactory but a continuation of the test was interrupted due to the war's outbreak and the investigation was then confined to the laboratory. The second ocean cable experiment was tried out in 1916; this was with the three-unit code. The results of this experiment were also considered satisfactory and this system was operated under regular traffic conditions between Ireland and Newfoundland during 1919 and 1920. Later the investigation led back to the five-unit code which was also operated between Ireland and Newfoundland and has been in continuous operation for several years.

PRINCIPLE OF OCEAN CABLE PRINTING TELEGRAPH SYSTEM

A printing telegraph system applied to an ocean cable does not necessarily introduce new features with respect to the actual operation of the cable itself. It merely provides an organization of apparatus for transmitting the signals representing the letters of the alphabet and for causing them to be automatically translated into printed characters at the receiving end of the cable. Heretofore, this translation has been the work of the skilled operator.

BASIS OF THE SYSTEM

The basis of the cable printing telegraph system is a selective sending and receiving apparatus synchronously operated.

The sending apparatus includes a perforating machine resembling a typewriter keyboard which is used by the operator for preparing the message on a strip of paper tape, a constant-speed distributor or transmitter, driven by a tuning or driving fork, combined so as to select and transmit in proper sequence the code or letter combinations set up in the strip of tape.

The receiving apparatus includes a constant-speed distributor, also driven by a fork, and an automatic typewriter or printer, combined to select automatically the received signals or letters and cause them to operate the type bars of the automatic printer.

In Figure 1 is shown schematically a single channel printing system. In the transmitter at the upper left of the figure, T is the perforated strip of tape which feeds continuously through the transmitter at a constant speed.

The five reciprocating pins or rods $U\,P$ of the transmitter are operated seriatem by the five cams on the cam shaft $C\,S$. When a pin finds a hole in the tape it rises through the tape and rocks the pole-changer $P\,C$ to its marking contact $M\,C$. The absence of a hole causes it to be rocked to its spacing contact $S\,C$. The pins $U\,P$ which rise one after another are slightly staggered to compensate for the moving tape. The pole-changer operates a transmitting relay A, through circuit $C\,T$, which in turn operates the regular cable sending-on relays B that transmit into the cable through the sending condenser K. At the receiving end of the cable in the cross circuit of the duplex bridge are located the magnifier and cable relay. These instruments are comparatively sensitive to small currents, the magnifier requiring only five or six microamperes to operate it and the relay something in the order of 50 to 80 microamperes to give a good working signal. The contacts of the relay are shown connected to two local relays C which operate into the printer circuit.

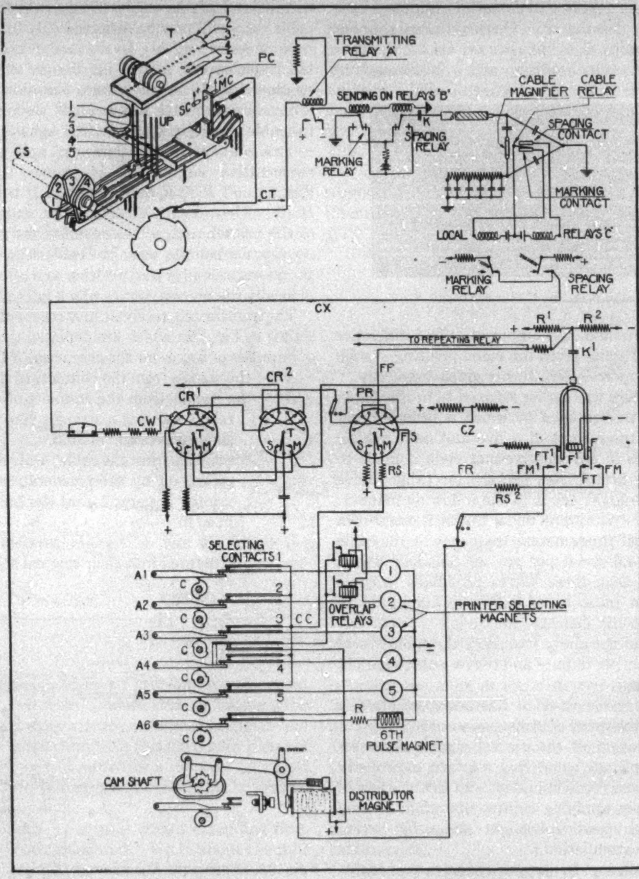
From here the circuit extends through $C\,X$ and $C\,W$ into the two control relays $C\,R^1$ and $C\,R^2$ and the printer relay $P\,R$ of the printer circuit, returning to the battery source at $C\,Z$.

The two control relays $C\,R^1$, $C\,R^2$ control the fork through magnet F^1 and keep it vibrating in synchronism and in phase with the distant transmitter. The driving fork in turn operates a step-by-step distributor through its contacts $F\,T^1$ and $F\,M^1$ and the distributor magnet. The magnet rotates a cam shaft containing a series of cams, C, which raise the levers $A1$ to $A6$ in sequence. These in turn close the selecting contacts 1 to 5 of the printer selecting magnets.

This arrangement of sending and receiving apparatus, operating synchronously, causes the five selecting contacts of the receiving apparatus to function in step with the five reciprocating pins of the transmitter, so that an impulse transmitted through the medium of any one pin of the transmitter will be received on the corresponding selecting contact of the receiving distributor. Mounted on the shaft with the five cams referred to is a sixth cam, the purpose of which is to cause the printer to function after the five selecting cams have completed their cycle.

To illustrate the actual transmission and reception of a signal the letter O, which we may assume has been prepared on the tape by the operator, will be followed through the transmitter. If we keep in mind the fact that the transmitter and receiving distributor are running in exact step, it will be quite simple to follow the train of events which occur to bring about the printing of any given character. We will assume that the tape is being fed into the transmitter and that the holes for the letter O are just passing over the transmitter pins. In this case the fourth and fifth pins become operative and rock the pole-changer to its marking contact $M\,C$ for a two-unit time length. The marking sending-on relay then operates and transmits negative or selecting current to the cable which in turn deflects the magnifier at the receiving end and correspondingly the cable relay to its marking contact for two units of time.

In this case the local marking relay, the control relay $C\,R^1$ and printer relay $P\,R$ are operated by the cable relay and therefore move their tongues to their marking contacts M. The control relay $C\,R^2$ moves its tongue to its spacing contact because its winding is in opposite direction to that of relay $C\,R^1$. Neglecting the operation of the intermediate control relays we pass on to the printer relay $P\,R$ which as stated was operated to its marking contact by the cable relay. From here the current path is through the selecting contacts 4 and 5 of the distributor, which correspondingly closed in step with the pins of the transmitter, causing the operation of selecting magnets 4 and 5 of the printer which control the *O* type bar.


T
2
3
4
5
PC
MC
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UP
1
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CS
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TRANSMITTING RELAY 'A'
SENDING ON RELAYS 'B'
K
SPACING RELAY
MARKING RELAY
CABLE MAGNIFIER
CABLE RELAY
SPACING CONTACT
MARKING CONTACT
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CX
R1
R2
TO REPEATING RELAY
K1
FP
CR1
CR2
PR
CW
CZ
T
S
M
FS
RS
FR
FM1
FT1
F1
F
FT
FM
RS2
SELECTING CONTACTS 1
A1
A2
A3
A4
A5
A6
C
2
3
4
5
C C
OVERLAP RELAYS
PRINTER SELECTING MAGNETS
R
6TH PULSE MAGNET
CAM SHAFT
DISTRIBUTOR MAGNET

FIG. 1—SCHEMATIC DIAGRAM OF CABLE PRINTER CIRCUITS

The current which operates the selecting magnets has its source from the middle point of the potentiometer $R^1 R^2$, condenser K^1 fork tine *F* and contact *F M* (when closed), resistance $R\ S^2$, tongue *T* of printer relay *P R*, contact *M*, resistance *R S*, to the common connection of the upper distributor contacts *C C*, thence through the corresponding lower springs of the fourth and fifth selecting contacts to their associated printer selector magnets 4 and 5.

While the above description discloses in simple form the operation of a one-channel printer system as applied to ocean cables, it does not attempt to describe the operation in its entirety.

There are at least two features in connection with this cable printing system which may be considered as outstanding, to which reference should be made.

One feature is the relatively low fundamental line frequency required for a given output in words per minute as compared with so-called five-unit single channel printing systems used in land line operation. The reason for this is what may be called "continuous transmission," that is, the five code impulses of the cable printing system are transmitted successively without any additional intervening impulses being sent between the fifth and first impulse. Up to the time that the cable printer was developed, the land line single channel systems transmitted seven impulses per revolution of the distributor. This lengthened the code to seven time units, five units being used for the code and two units for synchronizing and overlapping time between the fifth and first impulses. The difference between 5 and 7 units gives the cable printing system about 1.5 times more letters at any given frequency.

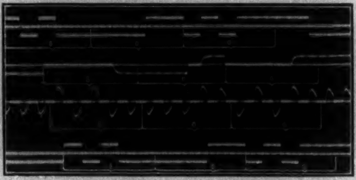

Fig. 2— Oscillogram of Cable Printer Signals

The other feature was earlier referred to in this paper as a method of transmission by which it becomes possible to double the output of a five-unit code printer system. That is to say the five-unit code is made to yield an output better than the shorter cable Morse code. This is brought about in the following manner:

Referring to the paragraph under Codes, it was shown that the practical fundamental frequency of the cable in question was 4.6 cycles per sec. or 555 half waves per min. Also that these waves or signals in cable Morse operation must be of sufficient amplitude to properly operate the delicate cable relay. Further it was shown that at the above frequency the cable Morse yielded 150 letters per minute and the five-unit code 111 letters per minute.

Heretofore, in cable work, it has been the practise only to increase the speed of transmission to such a point that the attenuation of the arrival signals does not decrease the amplitude below that required to properly operate the delicate receiving relay. In this method of transmission when applying printer operation to long cables, increased speed is brought about by taking advantage of the attenuation.[2]

This is accomplished by increasing the rate of transmission to practically twice the rate of the cable Morse speed. In this case the fundamental waves or alternations are attenuated to such an extent that they are practically undiscernable at the receiving end of the cable and, of course, cease to deflect the cable relay. To all intents and purposes these waves arrive as zero impulses similar to zero impulses as employed in cable Morse code operation. A wave of two-unit length, however, will now arrive with the same amplitude and duration as a single wave in cable Morse operation.

It will be understood from this then that a single wave or a series of alternating waves will arrive at the distant end of the cable as zero waves and that the cable relay will now be deflected only by a wave two or more units in length. In the case of the non-arrival of the single waves, a local impulse or wave producer is employed to fill in the missing impulses and to reconstruct or regenerate the received signals so that they resemble the original transmitted signal.[3]

This wave producer forms a part of the printer regenerative circuit. It is shown theoretically in Fig. 1 at *F*, *F T*, *F R*, *F S*, and *F P* to potentiometer R^1 R^2. This circuit is through an auxiliary winding of the printer relay which reverses the relay tongue to its opposite contact when the cable relay is deflected to the no-mans-land or zero position and automatically fills in locally the reversal waves which fail to arrive.

The transmitted, received, and regenerated signals are shown in Fig. 2 in which are depicted the combinations of impulses or waves for the characters *S O G*.

A = the signals from the contacts of the transmitter.
B = the signals from the contacts of the cable local relays showing a straight zero line where the single waves are missing.
C = the signals from the cable local relays (*B* signals) picked off by the regenerating action of the printer apparatus and the missing impulses filled in.
D = the *B* and *C* signals reassembled and regenerated into their original shape as at *A*.

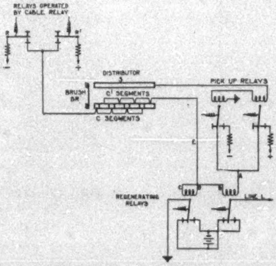

Fig. 2A—Schematic Diagram of Rotary Repeater for Cable Morse Code Operation

A cable printing system has now been shown, employing a five-unit two-current code, by which can be transmitted over this particular cable practically twice as many impulses as can be sent with the three-current cable Morse system; namely, 555 × 2 or 1110 impulses per minute. Correspondingly a greater number of letters per minute are transmitted *viz.* $\frac{1110}{5}$ or 222 letters, as compared with 150 letters obtained with the cable Morse code.

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[2] This principle was first suggested by K. Gulstad in 1898, and was applied to the operation of comparatively short sections of cable.

[3] In 1913 Walter Judd and Benjamin Davies of England invented a method for doing this and were the first to apply this principle to long ocean cables.

The Cooke three-current code and the cable Morse three-current code cannot be used in the manner just described because a zero current element forms part of these codes. Hence, there would be no way to differentiate between the zero forming part of the code and the zero resulting from the suppression of the single waves.

As already stated in the earlier part of this paper, the printing system was successfully operated over an ocean cable between Ireland and Newfoundland. This was not, however, the final solution of printer application to ocean cables. The next problem was to extend the system so that it would operate directly between the two terminals of a cable circuit, as between London and New York. In connection with this particular phase of the undertaking, we will revert for a moment to a brief history of the prevailing practise in cable operation between the terminal offices.

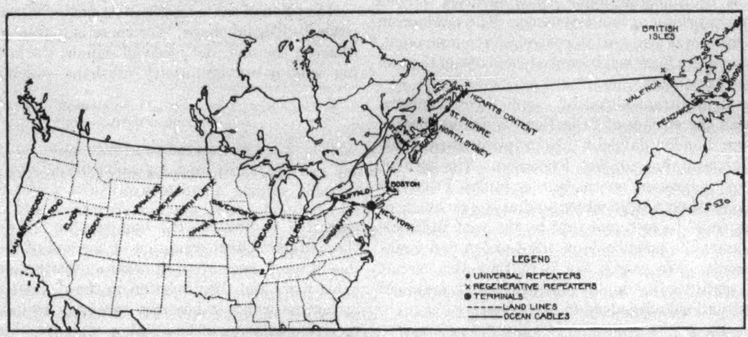

Fig. 3—Map of London—San Francisco Circuit

Fig. 3 is an outline of a cable circuit extending from London to New York or beyond. Up to about the time that the cable printer was conceived, practically all cable circuits were operated sectionally. For instance, in the case of a message whose destination was New York, London transmitted the message to Valentia, Ireland, through the repeating offices at Bristol and Penzance. The operators at Valentia translated the message and re-sent it to Heart's Content, Newfoundland; here the message was again translated and re-sent to North Sydney, Canada, where it was once more translated and resent to New York through the repeating offices at St. John, Canada, and Boston. Under these conditions the overall time consumed in passing messages between London and New York, or vice versa, was in some cases excessive. It is fair to state, however, that under certain conditions messages were regularly exchanged between these two telegraph centers within a minute. It is quite obvious that a semi-automatic system such as this could not be expected to meet the requirements of the ever increasing volume of traffic passing between Europe and America. The ultimate system must be wholly automatic, requiring no manual translation or retransmission between any two terminals of a long cable circuit. This problem was met, and the solution provided, by the development of the regenerative repeater. A regenerative repeater consists of apparatus which to all intents and purposes functions similarly to the printing system which we have just discussed.

It must be capable of mending, as it were, or reconstructing the more or less distorted signals received at the end of a land line or cable and retransmitting them into the next section, as new or regenerated signals. Distortion of signals is due to various causes, such as unbalanced duplex conditions, earth currents, characteristic cable effects, and other extraneous disturbances. Without the regenerative repeater it would be almost impossible to satisfactorily operate a long cable circuit.

There are at least two kinds of regenerative repeaters, one known as the rotary and the other as the fork repeater. These repeaters are arranged to repeat any of the codes which may be used in telegraph operation. Such a repeater consists of synchronous apparatus arranged to pick out the best portion of the arrival signal and reconstruct it so that it is identical with its original shape. Fig. 2 illustrates the progression of the operation of the repeater, the progression being in the order of B, C, D.

B = the received signals from the cable local relay contacts.
C = the picking out and filling in process.
D = the reconstructed signals to be transmitted into the next section.

This type of repeater permits direct operation between terminals separated by great distances. As an example of its efficiency, a direct circuit with printer operation was successfully worked between London and San Francisco. This circuit was about 7400 mi. in length including 2600 mi. of submarine cable and 4800 mi. of open line. There were twenty-one repeaters in the circuit, five being regenerators between New York and London and the remainder being standard land line repeaters. The occasion for this circuit was the Diamond Jubilee of the Birthday of San Francisco, September 5, 1925. As part of the program London and San Francisco exchanged greetings and other messages, while London had the honor of starting the grand march in the ballroom of the Civic Auditorium by transmitting signals which operated the bell signal in the printer at San Francisco. The mechanism of the printer was arranged to control at a remote point, the hammer of a huge bell borrowed from one of the San Francisco ferry boats.

These signals transmitted by London were synchronized with the strokes of "Big Ben" which chimed at 7:00 a. m. London daylight time, equivalent to 10:00 p. m. standard time at San Francisco. The printed characters as received on the printer in San Francisco were projected on a large screen so that the exchange of greetings could be seen and read by the vast audience. Fig. 3 shows the magnitude of the London-San Francisco circuit. We might say here that this circuit worked perfectly in duplex operation and messages were sent and acknowledged in one minute.

REGENERATIVE REPEATERS

Regenerative repeaters have been in use on the Western Union cable circuits for a number of years and practically all of its old type ocean cables are operated as direct through circuits between London and New York.

As before stated the repeaters are arranged to handle any of the telegraph codes which may be used. Some repeat the non-uniform cable code, others the straight uniform two-current code, and others the modified uniform two-current code. In some cases many of the signals arriving at the end of a circuit, whether it be cable or land line, are so distorted as to be unintelligible for sight reading to the skilled operator, yet the repeater through the medium of its regenerative action will neglect the bad portions of the signal and pick out enough of the good portion to reconstruct a whole new signal.

In addition to the type of regenerator disclosed in Fig. 1 in connection with the printer, another type of regenerator is shown in Fig. 2A. Here the regenerator is shown as adapted for cable code. The segments C pick up a portion of the received signal from the relays R and R^1 and transmit it into the biased pick-up relays via brush $B\ R$ and ring S. These relays in turn operate the biased regenerating relays via A, B, C, D, E, segments C^1, ring S and the windings of the pick-up relays. The regenerative relays transmit the new signal into line L.

RECEPTION OF SIGNALS OVER OCEAN CABLES

The quality or definition of the received signals has such a direct bearing on the operation of printing telegraphs over ocean cables that it necessarily becomes of primary importance. Much has been written about this particular subject and cable engineers have devoted considerable time towards improving the definition of cable signals. The recent new type of permalloy loaded cable developed by the Western Electric Company is a radical departure in cable design and has considerably increased the cable output above that of the best cables of the standard type.

Reception of cable signals in its final analysis simmers down to "signal shape," for there is a great difference between the sent and received signals, the latter arriving with a non-uniformity of shape which must be corrected to produce the best results. The cause for the non-permanent condition at the end of the cable is due in some measure to the characteristic effects of the cable itself and their relation to the delicate receiving apparatus which it becomes necessary to use. If the receiving instrument be a siphon recorder, these effects are not so detrimental, but for automatic or printer operation in which a cable relay is required, they must be counteracted so as to produce signals in order to function automatic apparatus.

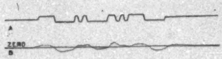

FIG. 4—SHOWING DIFFERENCE BETWEEN SENT AND RECEIVED SIGNALS

As an illustration of the relative difference between the shape of the transmitted and received signals, a group of signals is shown in Fig. 4, in which A equals the transmitted signals recorded from the contacts of the transmitter and B the received signals traced by the siphon recorder. It is this difference between the sent and received signals which must be corrected for automatic working.

In the discussion of the reception of cable signals, at least four characteristic effects met with in every day working are referred to, namely:

1. The "swaying effect" caused by earth currents.
2. The "falling away effect" of certain signals due to the charging up process of the cable and its associated condensers.
3. The "wandering zero effect."
4. The "variable lag effect."

The latter two effects are the result of a condition in the cable brought about by the transmission of impulses having variable lengths.

To illustrate the means for counteracting these different effects, a schematic diagram of a cable circuit as used for printer operation is represented in Fig. 5. In the figure, *M* is the magnifier and *C R* the cable relay.[4] Different types of magnifiers are used in cable operation but reference will be made to the magnifier used in connection with the printer experiments. This is known as the Heurtley hot wire magnifier. It has a moving coil to which is attached a pointer, the movement of the pointer functioning to unbalance the arms of a local Wheatstone bridge circuit. The insert in Fig. 5 shows, theoretically, this type of magnifier.

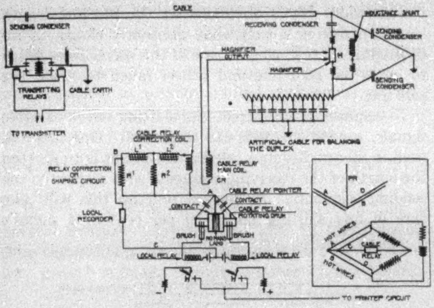
CABLE
SENDING CONDENSER
RECEIVING CONDENSER
INDUCTANCE SHUNT
SENDING CONDENSER
MAGNIFIER OUTPUT
M
MAGNIFIER
SENDING CONDENSER
TRANSMITTING RELAYS
CABLE EARTH
TO TRANSMITTER
ARTIFICIAL CABLE FOR BALANCING THE DUPLEX
CABLE RELAY CORRECTION COIL
L¹
L²
B
RELAY CORRECTION OR SHAPING CIRCUIT
R¹
R²
CABLE RELAY MAIN COIL
CR
A
CABLE RELAY POINTER
CONTACT
CONTACT
CABLE RELAY ROTATING DRUM
LOCAL RECORDER
BRUSH
BRUSH
C
LOCAL RELAY
LOCAL RELAY
TO PRINTER CIRCUIT
PR
A
C
D
B
HOT WIRES
CABLE RELAY
HOT WIRES

Fig. 5—Schematic Diagram of Cable Circuit

The pointer *P R* moves the two wires *A*, *B* so that *A* approaches a fixed wire *C*, while *B* recedes from a fixed wire *D* and vice versa. The four wires are heated with the local current in the bridge circuit and connected to form arms of the Wheatstone bridge. As the wire *A* approaches *C*, it is subjected to a greater amount of heat and *B* to a lesser amount; correspondingly the resistance of the wires change and unbalance the bridge. The output of the magnifier is taken from the cross wire of the bridge circuit.

In the figure it is shown that the cable is operated with unshunted sending and receiving condensers. The receiving end shows duplex connections, the sending end being shown in simplex for simplicity. The condensers serve the purpose of preventing to a certain extent the flow of the relatively low-frequency earth currents which are more or less prevalent in transatlantic cables. In some cases, however, earth currents reach considerable magnitude and may change or fluctuate so rapidly as to interfere with the operation of the cable.

The inductance shunt in Fig. 5 serves a two-fold purpose. It helps to improve the definition of the received signal and to counteract the effect of "wandering zero." Its action on the received signal is such that it offers a comparatively high impedance to the beginning of a signal, allowing the major portion of the received current to flow through the magnifier coil, then as the signal gradually falls away, the low ohmic resistance of the shunt comes into action and shunts the current from the magnifier coil, producing a signal with well defined slopes. The low ohmic resistance of the shunt also serves the purpose of a bypass for all low-frequency currents and within certain limits counteracts the effect of "wandering zero." The correction or shaping circuit in Fig. 5 is for counteracting the "falling away effect" of the arrival signals as they are passed into the cable relay and for transforming the curved waves into square topped signals.[5]

In Fig. 6 is shown a group of signals in which *A* represents the original transmitted signals recorded from the transmitter contacts which it is desired to reproduce at the distant end. *B* represents the received signals as they pass into the cable relay from the magnifier. For the purpose of illustrating the "wandering zero effect" and the "falling away effect" upon these signals, a zero line has been drawn through the curves to show how the zero portions of the curves have wandered from true zero at *X*, X^1.

Record *C* shows the signals from the contacts of the cable local relays before the correction has been applied. Record *D* shows the signals with the correction circuit applied and illustrates how it has held up the current waves and straightened out the zero waves.

The correction shown at *D* was brought about by applying a local current to the correction winding of the cable relay coil at a suitable moment when the signals began to fall away. This correction winding is shown in Fig. 5. The local current in this winding is regulated by the resistances R^1 R^2 in the shaping network and is timed by the inductances L^1, L^2 to take hold, as it were, at the proper moment. Actually what happens is this: The relay pointer, for instance, is deflected to a contact presumably to record a wave of several units in length, say, any one of the current waves shown in record *B*, Fig. 6. At some moment during the charging

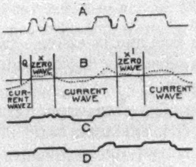
A
B
Q
X
ZERO WAVE
X¹
ZERO WAVE
CUR-RENT WAVE
Z
CURRENT WAVE
CURRENT WAVE
C
D

Fig. 6—Showing Uncorrected and Corrected Signals

process of the cable and condensers the current wave *Z* begins to fall away and eventually the slope of the wave drifts over to the opposite side of zero as at *Q* in Fig. 6.

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[4] There are at least three types of cable relays; the Drum relay of S. G. Brown, England, Gold Wire Relay of A. Muirhead, England, Antenna Relay of Eastern Telegraph Company, England.

[5] This method of signal shaping was invented by S. G. Brown of England.

The local correction circuit comes into action at the moment the pointer touches its contact at which time a local circuit is formed through the correction coil via the network, *A, B, C,* Fig. 5. Through the medium of the slow time constant of these inductances a minute current flows and gradually increases in strength in opposite direction to the falling signal and automatically counteracts the gradual "falling away effect" and holds the signal up for its full length as illustrated in record *D*. In the case of printer operation these signals shown at *D* are in turn regenerated by means of the regenerative feature of the printer apparatus in which case the single unit impulses arriving as zero waves are filled in and reconstructed to their original shape as shown at *A*.

VARIABLE LAG

Variable lag is only troublesome in connection with the reception of cable signals in their relation to synchronous automatic operation. In ordinary siphon recorder operation it is not apparent. Its effect is to shift or displace the arrived signal so that it does not arrive in the exact location assigned to it on the receiving distributor. That is to say, the synchronous receiving distributor, the face plate of which is cut into equal receiving segments, assumes the successive impulses or waves of each code combination will arrive so that a definite portion of each wave will fall upon a definite point on the distributor. As a matter of fact this is not the case and in extreme cases it has been found that the signals are displaced about 35 per cent of their unit length.

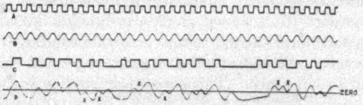

FIG. 7—SHOWING SIGNALS OF EQUAL AND UNEQUAL LENGTH

METHODS FOR COUNTERACTING THE EFFECT OF VARIABLE LAG

Variable lag is the result of irregular or non-uniform transmission. A regular or uniform transmission is one in which continuous alternations or reversals of current of equal amplitude and duration are sent into the cable, and if the alternate half waves be free from bias or other defects the electrical condition of the cable remains constant and the waves arrive in exact phase with the corresponding segments of the receiving distributor. Any deviation from this kind of transmission produces a variable condition in the cable and the effects of shifting lag become evident, resulting in the wave crests appearing at indefinite points on the receiving distributor.

The difference between arrival signals composed of alternating or equal length signals and unequal length signals, is shown in the groups of signals in Fig. 7 in which *A* and *B* represent the sent and received alternating or equal length signals and *C* and *D* the sent and received signals of unequal length. For this illustration the shaping circuits in the cross-circuit of the duplex bridge were disconnected in order to record raw or unshaped signals for the purpose of portraying more clearly the action of the cable upon these two kinds of transmission. It is obvious that the unequal length signals arrive in a somewhat disorderly shape, as indicated in *D* at *X*, where some of the waves have failed to cross the zero line and others failed to reach the zero line.

To counteract or correct the shifting effect of cable signals, several methods are employed. One method, used in all synchronous telegraph systems is to shorten the length of the receiving segments with respect to the sending segments. In general practise this will take care of any shifting up to 50 per cent of a signal's unit length. In addition to this the receiving distributor is arranged so that the segments can be rotated to a suitable position to compensate for the average degree of shifting which may be encountered. To illustrate this method of lag correction, two developed distributor rings are shown in Fig. 8. In this case a single transmitted time unit or half wave occupies a segment of 72 degrees of the sending distributor face plate which is shown cut into five equal segments. From the figure it is obvious that on account of the receiving segments being shortened, which in this case are one-eighth the length of the sending segments, the wave *A* must shift forwards or backwards approximately 50 per cent of its length before it will encroach upon the receiving segments $R\,S^2$ or $R\,S^4$. Such an amount of shifting is abnormal and is rarely met with in every day working. All the lesser degrees of shifting occur within the safety zones and are less harmful. While this counteracts the effect of the variable lag, it does not eliminate it. It must be dealt with at the transmission end of the circuit.

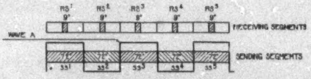

FIG. 8—THEORETICAL DIAGRAM SHOWING THE BEST POSITION OF THE RECEIVING SEGMENTS WITH RESPECT TO THE TRANSMITTED WAVE

With this in mind there have been devised various ways for transmitting cable signals, the object being to approach as nearly as possible an alternating current form of transmission. For practical purposes it has not yet been possible to transmit telegraph codes as used for ocean cable operation in the form of continuous alternating waves. A modification of this method of transmission was developed, however, known as the "suppressed transmission," whereby the code characters are transmitted in the form of broken alternating waves or reversals.

This is by far the best form of cable transmission yet presented and keeps the cable in an even state and counteracts or reduces as far as possible the variable lag. It was developed during the early stages of the printer experiments, and some outstanding results were obtained with it. It is not being used at present on account of its apparent limitations as to speed of operation. Further investigations, however, are being made with it because by taking into account its simplified receiving circuit and the fact that the duplex balancing of the cable is made easier as compared with other forms of transmission, it may be possible to increase the magnification, at present limited on account of cable unbalance, so that the suppressed transmission might be worked up to a relatively high rate of speed.

OPERATION OF SUPPRESSED TRANSMISSION—EARTHING METHOD

In Fig. 9 is shown a schematic diagram of one of several methods used for this kind of transmission. The transmitting circuit is arranged so that if two or any greater number of unit impulses of like polarity come in succession, the first unit only is sent into the cable and the cable is then earthed for the remainder of the successive units. For example, if two code combinations are to be transmitted, one containing five negative units and the other five positive units, the order of transmission will be one negative unit, four earthing units, one positive unit, and four earthing units. In the figure it is assumed that the tape is feeding towards the left until hole 1 is presented to oscillating pin *A*.

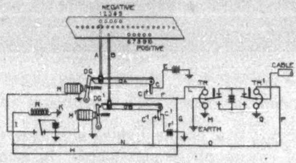

FIG. 9—TRANSMITTING CONNECTIONS FOR SUPPRESSED TRANSMISSION. EARTHING METHOD

As pin *A* rises through the tape, contacts *C, C* are closed energizing relay *R* via *E, C, C, F, G, H, I, R, K,* in a right direction. The closing of contacts *C, C* causes the latch magnet *M* to be deenergized through the contacts of relay *R*. This causes the latch *D G* of latch magnet *M* to fall into the path of the oscillating lever *O A* connected to pin *A*. As oscillating pin *A* is withdrawn from the hole number 1, it is prevented from further rising by the engagement of the oscillating lever *O A* with the latch *D G* as shown.

At the moment contacts, *C, C* closed, transmitting relay *T R* closed via *E, C, C, F, L, M* and transmitted one time unit of negative current to the cable, then when contacts *C, C* opened, transmitting relay *T R* returned to its upper contact and earthed the cable for the remaining units, 2, 3, 4, 5. The action of oscillating pin *B* is identically the same as that of *A*; in this case, however, one positive unit is transmitted by transmitting relay *T R*^{1} via *F*^{1}, *C*^{1}, *C*^{1}, *O, P, Q* and the cable earthed for the remaining units 7, 8, 9, 10.

Fig. 10 illustrates the suppressed earthing method of transmission in which *A* represents signals transmitted as blocks and *B* the signals with all but the first unit suppressed and the cable earthed during the time of suppression. While this kind of transmission greatly reduces the cause of variable lag, some lag still exists, for as already stated the cable can only be kept in an even electrical state when continuous or unbroken alternating current waves are transmitted, whereas in this case the alternate waves are transmitted in a non-continuous or broken form separated by variable length zero or earthing waves which still displace the signals to some extent.

The results obtained at the receiving end of the cable with this kind of transmission are really outstanding. The received signals being in the form of alternate current waves of unit length, arrive with extremely clear definition requiring the simplest form of shaping circuit. An important feature regarding the operation of a cable circuit with this kind of transmission, which will be appreciated by cable engineers, is the relatively small amount of time required to get the circuit going. At cable stations it is not uncommon when starting up a new cable circuit to spend one or more days adjusting the shape of the variable length cable signals, whereas with the suppressed transmission the shaping can be accomplished in a few minutes.

As an example of this we may refer to the first trial or experiment made with the "suppressed transmission" over an actual cable circuit. This cable was about 1400 nautical miles long having a total resistance of 4200 ohms and a total capacity of 476 microfarads. The time consumed in getting working results was well within an hour, which, in the opinion of cable engineers, is a very good performance. This included setting the terminal transmitting apparatus at a suitable speed for operating this particular cable, adjusting or shaping the signals at the other end of the cable, and repeating the signals, without the aid of a regenerator, back over the same cable to the terminal station where they were again reshaped and turned into the printer apparatus. It is fair to add that the attendants at the cable station had no previous experience in shaping this type of signal.

SUPPRESSED TRANSMISSION-INSULATING METHOD

Another form of transmission tried out to counteract the variable lag effect is a "suppressed transmission" in which the cable is insulated instead of being earthed[6], all else remaining the same as in the other method. Considerable time was devoted to this method of working and trials were made over one of the long Atlantic cables for a period of several months. It was found that the characteristic effects of the cable using the circuit arrangement shown in Fig. 5 were just as dominant with this kind of transmission as with the standard method.

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[6] First tried by Mr. Pierre Picard in 1898.

Fig. 11 depicts a simplified form of the transmitting circuit used for this kind of transmission. In the figure *K* represents a key which may be the contacts of a transmitter. Depressing *K* operates relay R^1 and raising *K* operates relay R^2. The condenser *C* is adjusted to hold either relay closed for the duration of one time unit only, then when the relay tongue returns to its contact *S* the cable is insulated for the remaining length of the signal. Switching arrangements were provided for changing from this kind of transmission to the regular type and as far as could be determined the regular method gave the more permanent results. The method, however, is used on shorter lengths of cable of approximately 400 miles or so and apparently gives satisfactory results. Not having the constants of these cables and the speeds at which they are worked it is not possible to state here just how efficiently they are operated. As far as is known, they are operated under simplex conditions, which operation is hardly comparable with that of long duplexed cables.

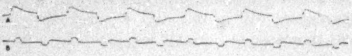

FIG. 10—SHOWING DIFFERENCE BETWEEN STANDARD AND SUPPRESSED TRANSMISSION. EARTHING METHOD

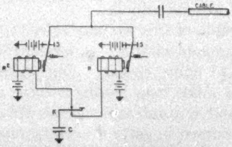

FIG. 11—TRANSMITTING CONNECTIONS FOR SUPPRESSED TRANSMISSION. INSULATING METHOD

Fig. 12 illustrates the suppressed insulating method of transmission in which *A* represents the standard block signals and *B* the suppressed signals with all but the first unit suppressed and the cable insulated during the time of suppression. At first glance the signals shown at *A* and *B* appear to be almost alike but if a zero line be drawn through the curves from *X* to X^1, it will be apparent that the prolongation of the current in *B* is less than in *A*.

Another form of transmission suggested is the unbroken or continuous alternating current transmission[7] employing three amplitudes, a large amplitude being used for a dash signal, a medium amplitude for the dot signal, and the smallest amplitude for the zero signal. Much consideration was given to the study of this type of cable transmission and experiments were carried out to determine its characteristics.

CURBED TRANSMISSION

Curbed transmission is old and has been in operation since automatic transmission was first developed. It is a method whereby the sending battery is impressed upon the cable for only a portion of a time unit, the cable being earthed for the remainder of the time unit. Other methods of curbing have been employed whereby a transmission time unit is divided into three elements instead of two as above mentioned. In some cases, perhaps 70 per cent of the time unit is transmitted with the maximum voltage, 20 per cent with a reversed lower voltage, and 10 per cent with earthing. By varying the curbing voltage and changing the degrees of curbing, various shapes of signals can be produced at the receiving end. Curbed transmission improves the shape of the received signals and correspondingly counteracts the effect of the variable lag. An illustration of curbed transmission is shown in Fig. 13 in which three-element block signals shown at *A* are transmitted as at *B*. The zero portion of the waves represents the time the cable is earthed between each element.

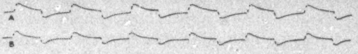

FIG. 12—SHOWING DIFFERENCE BETWEEN STANDARD AND SUPPRESSED TRANSMISSION. INSULATING METHOD

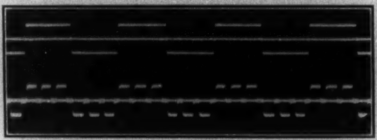

FIG. 13—OSCILLOGRAM OF STANDARD AND CURBED TRANSMISSION

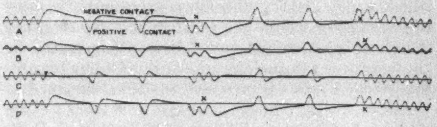

FIG. 13—SHOWING RECEIVED SIGNALS PRODUCED BY DIFFERENT METHODS OF TRANSMISSION

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[7] This method of transmission was suggested by General G. O. Squier.

Up to this point we have investigated four methods of transmission suggested for improving the reception of cable signals, namely:

1. Suppressed transmission earthing method.
2. Suppressed transmission insulating method.
3. Three amplitude transmission.
4. Curbed transmission.

To illustrate the characteristic cable effects on the received signals produced by some of the transmitting methods four groups of received signals are shown in Fig. 14, in which *A* represents received signals from block transmission, *B* curbed transmission, *C* suppressed transmission earthing method, and *D* suppressed transmission insulating method. The zero line running through the signals shows in the cases of *A*, *B*, and *D* that some of the wave crests which should have arrived on a negative contact have actually drifted to a positive contact and vice versa as indicated at *X*. In the case of *C*, all of the wave crests arrived on their respective contacts.

SHUNTED CONDENSERS

In discussing the various methods of transmission and the reception of cable signals, the cable circuit represented in Fig. 5 shows the cable to be operated with unshunted sending and receiving condensers. Until recently this has been the standard method employed by the Western Union Telegraph Company for operating transatlantic cables, and the major part of the printer experiments were carried out under these conditions. Experiments were made with shunted condensers and further investigation is being made along these lines. The reason, as already stated, for using unshunted condensers is to prevent the flow of the low-frequency earth currents, more or less prevalent in transatlantic cables. So far as signal shape is concerned, it seems to be generally accepted that signal shape is better when the condensers are shunted.

Discussion

A. F. Connery: The method of cable signaling, suggested by Gulstad in 1898 and which is used in the operation of the printing telegraph system described by Mr. Angel, appears to be the only method available if the 5-unit 2-element code is to attempt to compete with the widely used 3-element unequal letter cable code on long cables. The application of this signaling method to the old-style duplex cables which span the Atlantic is no mean achievement, and many difficult and trying problems, of which little mention is made in the paper, must have been solved.

I note Mr. Angel states that the speed of a certain cable was increased from 300 to 375 letters per minute. I wonder whether net increase is meant. My impression has been that the signaling described in this paper, whereby the single-element impulses are attenuated in the cable and are generated locally in the receiver, requires a high grade of duplex balance which is difficult to secure and maintain. It would seem that the time lost in adjusting and regulating would be greater than for the older methods of transmission.

A large proportion of the cable traffic at the present time is in code, and each word usually has the same number of letters. In a number of codes, each group consists of ten letters. One advantage of the well-known 3-element cable code is the ease in which errors may be detected by the transcribing operator. A loss or gain of an impulse due to a change in balance, lightning kick, improper adjustment of apparatus, etc., at some relay station, will either make an unintelligible signal combination or will cause a 10-letter word, for example, to become a 9- or 11-letter word. In any case, the error is usually apparent to the transcribing operator who can have the doubtful part of the message repeated.

The 5-unit code does not possess this advantage, and the loss or gain of a pulse will cause a wrong letter to be printed in place of the proper letter. Each word will have the proper number of letters and the receiving operator will be unable to detect any irregularity.

For plain English traffic the printer on slow-speed cables is attractive, but for code traffic it appears to be at a disadvantage as compared to the widely used cable code.

The foregoing remarks do not apply to the new high-speed inductively loaded cables which must be multiplexed to secure satisfactory traffic distribution and have no duplex balances with which to contend.

W. A. Houghtaling: Mr. Angel's paper describes printer operation on ocean cables in its simple form, that is, single-channel operation or one printer and transmitter at each terminal of the cable. It may not be clearly understood, however, that the printer operation is not limited to a single-channel, as by the use of a rotary distributor, several channels are possible. In fact, two other cables are being operated with four and five channels respectively, and still another is operating with two channels.

The cable which is operating as a two-channel circuit is of the same type as that referred to in the paper, while the two which provide four and five channels are the loaded type.

One of the points brought out in Mr. Angel's paper that seems to me to be quite interesting is the way he has shown the various steps of increased output of a type of cable laid fifty years ago, obtained by the use of improved terminal apparatus and improved operating methods. It would seem that the cable engineers and cable manufacturers of half a century ago built better than they realized. This type of cable has been able to meet the ever increasing traffic requirements through all these years by means of the terminal improvements mentioned until it is now carrying more than five times the amount of traffic it could handle originally.

W. C. Peterman: The securing of the maximum message-carrying capacity of a cable which is the primary object of the cable engineer depends both on sending as many cycles per second of signaling current as possible over the cable and getting the greatest amount of intelligence out of every cycle of current. It was pointed out in the paper that as the transmitting frequency is increased, the attenuation of the signals becomes greater, and at the receiving end of the cable more and more amplification of the signals must be used in order to get current strong enough to operate the mechanical relays. Of course with vacuum-tube amplifiers the amount of amplification possible has been increased vastly over what was previously obtainable, but unfortunately it has been found that all of the amplification thus possible cannot be used in practise because of the presence of electrical disturbances. These disturbances may be either man-made, such as interference from nearby power lines or other cable circuits, or from the duplex balance of the cable itself; or they may be electrical disturbances from apparently natural sources of unknown origin.

The difficulty of receiving weak signals from distant radio broadcasting stations when any considerable amount of static is present, no matter how much amplification is available, is familiar to everyone. In fact, greater amplification does not help at all after a certain point is reached; it only adds to the confusion. We have just about the same problem with receiving weak signals over a long cable. It thus becomes necessary to secure the greatest possible amount of information, as it were, from each received impulse, since we cannot increase without limit the number of impulses transmitted per second.

I just wish to emphasize, therefore, in the system described by Mr. Angel, and shown in his Fig. 1, that only the impulses of one-half the fundamental frequency must be received with sufficient strength to be well above the interference level, although we are getting the full amount of information out of the impulses transmitted at the fundamental frequency which impulses, by their very absence, because of greater attenuation, are caused to be filled in automatically at the receiving end. Thus we get the benefit of having to receive through the interference only, the comparatively stronger signals of one-half the working frequency while getting the full message-carrying advantage of the highest frequency impulses.

Herbert Angel: I should like to say, in reply to the comments by Mr. Connery, that the excellent method for increasing the speed of cables suggested by Mr. Gulstad in 1898, while quite successful for operating comparatively short lengths of cable, had never been found practicable to apply to long cables. This was pointed out by Messrs. Judd and Davies, of England, in their British patent of 1913, in which, for the first time, a method was described for filling in attenuated impulses for a synchronous system on long cables. The Western Union printing system does not use the Gulstad vibrating method for filling in the impulses.

Replying to Mr. Connery's query about the increased speed with printer operation, the output of 375 letters mentioned in the paper is a net output.

In reply to Mr. Connery's comment about the high-grade balance required for cable-printer operation, I should like to point out that in getting the 375-letter output which is an increase over the output obtained with cable Morse code, the frequency of the double received impulses is even then less than the maximum fundamental frequency when using cable Morse code. Therefore the received signaling impulses are larger and are better able to stand any slightly greater balance disturbance. Further, it has been found by repeated check, that under these conditions the cable printer is as accurate as the cable Morse code.

A Non-Rotary Regenerative Telegraph Repeater

BY A. F. CONNERY[1]

Associate, A. I. E. E.

***Synopsis.**—Rotary regenerative repeaters have made multiplex printing telegraphy possible over long distances. This paper gives a brief description of some types of rotary repeaters and then proceeds to describe in more detail a non-rotary regenerative repeater.*

INTRODUCTION

THE use of multiple-channel printing telegraph systems in the United States and Canada has resulted in many problems in connection with their operation over long circuits.

Practically all circuits are duplexed and the usual practise is to repeater them at intervals of 250 to 350 mi. A printer circuit operated between New York and San Francisco may have as many as 12 repeaters.

An ordinary relay telegraph repeater can never repeat signals as perfectly formed as those sent out by the originating transmitter since the signals always arrive somewhat out of shape and the repeating apparatus itself contributes a further modification.

Most overhead telegraph circuits use a ground return and some distortion of the signals is inevitable in addition to the distortion caused by the repeaters and imperfections in the duplex balance.

To operate a transcontinental multiplex printing telegraph circuit equipped throughout with ordinary relay repeaters, a very high grade of line and repeater maintenance is required. Accurate duplex balances are essential at repeater and terminal stations. The most direct routes must be used and line conductors chosen which have a favored location on the pole line so as to reduce the cross-fire from other conductors.

Even with high grade maintenance, the signal received at the terminal station is somewhat distorted and much time is consumed in balancing and adjusting. It is apparent that even a slight distortion of the signal in each line section results, in the aggregate, in an extremely distorted signal being received at the terminal station. The speed of operation is very slow as compared with the shorter circuits and the traffic capacity is low.

By the use of a regenerative type of repeater, the received distorted signal in passing through the repeater is regenerated and sent on to the next line section in perfect shape. The invention of the regenerative repeater is old and is attributed to Baudot of France.

A regenerative repeater for duplex working usually consists of electrically driven tuning forks controlling synchronous motors, rotary distributors driven by the synchronous motors, a plurality of storing relays together with the line relays and relays used in the synchronizing circuit. The synchronizing circuit maintains the distributor in step with the received signals.

Fig. 1 indicates a simple form of regenerative repeater. The sent, received, and regenerated signals are shown in addition to the two rings, $R1$ and $R2$, of the rotary distributor. The received signals operate line relay $L\,R$. Transmitting relay $T\,R$ can only be operated by $L\,R$ during the period that brush $B\,R$ has joined $R1$ to a segment of $R2$. The only part of the received signal utilized for retransmission is that marked S. Any distortion occurring outside of S will have no effect on the retransmitted signal. With this type of repeater, a slight distortion may be present in the retransmitted signals if the received signals are so badly distorted that they infringe upon S. Reducing the length of the segments $R2$ increases the ability of the repeater to regenerate badly distorted signals, but, if the segments are too short, the operating impulse to transmitting relay $T\,R$ may become too short to properly actuate the relay. The brushes $B\,R$ must be driven at such a speed that the time required for the brush to pass from the start of one segment to the start of the next segment is the same as the time occupied by the shortest signal element.

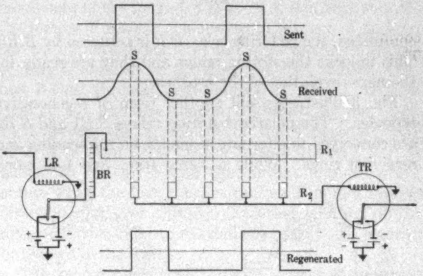

FIG. 1

Fig. 2 indicates a form of regenerative repeater which was invented by P. M. Rainey. Only a portion of the distributor face is shown. Line relay $L\,R$ is actuated by signals received over line 1. Brush $B\,R1$ connects successively the receiving segments of $R2$ to $R1$. The storing relays, $S\,R1$ to $S\,R5$, are operated whenever the line relay tongue is touching its marking contact at

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Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

the same instant that the brush makes contact with the respective segments of $R2$. Each storing relay, when operated, locks up through the left contact and tongue and the right contact applies the proper polarity of battery to the transmitting segments $R4$. Transmitting brush $B\,R2$, which is angularly displaced with respect to $B\,R1$, connects line 2 to the successive transmitting segments. When the transmission is completed, the restoring relay $R\,R$ is actuated by $B\,R3$. This unlocks the storing relays and they are ready for the next revolution of the brushes.

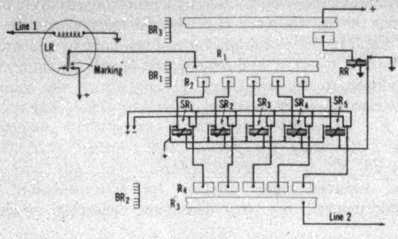

FIG. 2

Fig. 3 illustrates still another form of regenerative repeater. The polarized storing relays $S\,R1$ and $S\,R2$ are connected to alternate segments on the sending and receiving rings. While a signal from line 1 is being stored in one relay, the stored signal in the other relay is being transmitted to line 2.

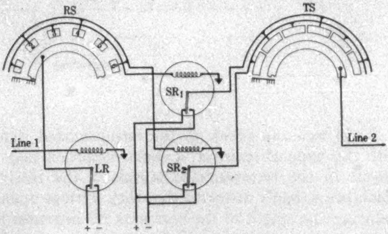

FIG. 3

There are two main types of synchronizing systems used to maintain phase relation between the distributor brush arms and the received signals. One type, which has been termed "shift the hands" correction, shifts the brush arms to maintain synchronism while the other type maintains synchronism by altering the natural period of the driving fork.

In one example of the "shift the hands" correction, the rotary distributor is arranged to run slightly faster and a mechanism controlled by the received signals steps back or "corrects" the distributor brushes when they gain a certain amount over the brushes of the sending distributor. Several opportunities for correction are available during each revolution. While this correcting scheme has been used to a considerable extent on rotary regenerative repeaters, it is open to some objections. The many opportunities for correction in each revolution make it possible for a few badly distorted signals to considerably alter the phase relations and several seconds are required for the distributor to work around to the proper phase. When several repeaters of this type are used in tandem on a long circuit, difficulty may be experienced in holding synchronism.

In the writer's opinion, it is preferable to use, in a regenerative repeater, a correction which alters the natural period of the driving fork because the correcting action is gradual and an occasional badly distorted signal will not seriously alter the phase relations.

NON-ROTARY REGENERATIVE REPEATER

The main object of this paper is to describe a new type of regenerative repeater which has no rotating parts. It should be realized in this connection that the rotating feature of the repeaters previously described in this paper has for its object the control of electrical contacts at uniform time intervals which agree with the rate at which signals are being sent over the line circuit. An electrically operated tuning fork adjusted to vibrate at a uniform rate and controlling electrical contacts, can be used, therefore, for timing purposes for regenerative telegraph repeaters and rotating members eliminated. A repeater for one-way repetition consists essentially, therefore, of an electrically operated tuning fork, a line relay, a locking relay, a transmitting relay and a correcting or synchronizing circuit with relays. It has been found possible in practise to make use of the same type of tuning fork as is now used with multiplex printing telegraph terminal sets.

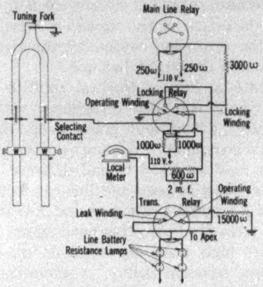

FIG. 4

SELECTING CIRCUIT

Fig. 4 shows the selecting and locking circuits. The locking relay is normally under the influence of the main line relay which controls the direction of the current through the operating winding. The current through the locking winding of the locking relay is stronger than the current in the operating winding and as long as the fork is touching its selecting contact the armature of the locking relay is prevented from moving. During the period that the fork is not touching the selecting contact, no current flows through the locking winding of the locking relay, and it therefore is under the influence of the tongue of the main line relay. The polarity of the signal transmitted by the transmitting relay depends upon the position of the tongue of the locking relay at the moment the fork engages its selecting contact. A long duration of engagement of the fork with the selecting contact does not injure the repeated signals. The locking up of the locking relay while the transmitting relay is operated prevents any clipping of the signals sent out from the transmitting relay. The shunted condenser permits a quick rise and fall of current in the locking winding. A leak current through one of the windings of the transmitting relay tends to hold the relay tongue to whichever contact it may be touching. The current in the leak winding is small and does not interfere with the operation of the relay. The local meter is used as a guide in checking speed and detecting breakups.

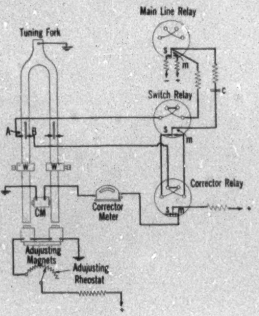

Fig. 5

Correction Circuit

The means used to hold the fork in synchronism with received signals is an adaptation of the Picard system. Fig. 5 shows the correction circuit.

By means of the weights *W* and the adjusting rheostat, the rate of vibration of the fork is set so that, with no current flowing through the corrector magnet *C M*, it makes slightly less than a complete cycle of vibration for each unit length of line signal. With current flowing through the corrector magnet, the fork speed increases so that it makes slightly more than a complete cycle of vibration for each unit length of line signal. In actual operation, the current through the corrector magnet occurs at irregular intervals due to the action of the correction circuit and the fork is held in step with the received signals. When the fork falls behind the signals, current flows through the corrector magnet and causes the fork speed to increase. When the fork gains on the signals, the current is cut off the corrector magnet and the fork speed is reduced.

Assume, for example, that the speed has been matched and the repeater is in operation. If the tongue of the main line relay moves from one of its contacts to the other at the instant the fork is touching contact *A*, Fig. 5, the tongue of the switch relay *S R* is moved to correspond with the position of the tongue of relay *M L R*. When the fork makes contact with *B*, a short impulse of current flows through condenser *C* and one winding of the corrector relay and the tongue of relay *C R* is moved to the marking contact and current flows through the corrector magnet and increases the speed of the fork. As long as the movements of the tongue of the main line relay *M L R* occur at the instant that the fork is touching contact *A*, the relay *C R* remains in the marking position. If, for example, the movement of the tongue of relay *M L R* is from *S* to *M*, the tongue of relay *S R* is moved to contact *M* and the current impulse through the condenser and the windings of relay *C R* which occurs when the fork touches contact B goes through one winding of relay *C R* from right to left and the tongue is thrown to the right. If, however, the movement of the tongue of relay *M L R* is from *M* to *S*, the tongue of relay *S R* is moved to contact *S* and the current impulse which occurs when the fork touches contact *B* is in a reverse direction, but goes through the other winding of the relay *C R* from right to left and the tongue still tends to be thrown to the right.

The tongue of relay *C R* being on contact *M*, a current flows, through the corrector magnet which increases the speed of the fork. In a short time the movements of the tongue of relay *M L R* occur when the fork is on contact *B*. The impulse through the condenser occurs just as soon as the tongue of relay *M L R* moves and as the relay *S R* is not affected since the fork is on contact *B*, the impulse through the condenser moves the tongue of relay *C R* to the left and the current is cut off the corrector magnet.

In reviewing the action of the correction circuit, it is apparent that if the operation of the tongue of relay *M L R* occurs when the fork is touching contact *A*, the switch relay tongue is moved and the impulse through the condenser which takes place when the fork moves to contact *B* throws the tongue of relay *C R* to the right and the fork speed increases. If the operation of the tongue of relay *M L R* occurs when the fork is touching contact *B*, the impulse through the condenser occurs immediately and since relay *S R* has not been operated, the impulse moves the tongue of the relay *C R* to the spacing or left-hand position. This breaks the current through the corrector magnet and the fork speed will reduce again. In actual operation, the fork corrector contact is moving from *A* to *B* or about to move when the tongue of relay *M L R* operates.

Regenerative Action

The correction circuit, as explained, holds the tuning fork in step with the line signals. The fork makes a complete cycle of vibration during the time of the shortest signal element. If, for example, the signals over the line are the equivalent of alternating current at a frequency of 25 cycles or 50 signal units per sec., the fork vibrates at 50 cycles.

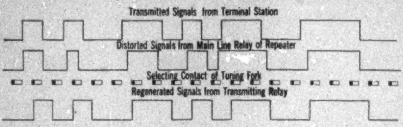
Transmitted Signals from Terminal Station
Distorted Signals from Main Line Relay of Repeater
Selecting Contact of Tuning Fork
Regenerated Signals from Transmitting Relay

Fig. 6

The received signals operate the main line relay which in turn operates the locking relay. The fork engages the selecting contact once for each unit length of received signal. The instant that the signal reverses on the transmitting relay is defined by the vibration of the fork into a contact with the selecting contact. The tuning fork maintains a constant rate of vibration and the repeated signals reverse at properly timed intervals.

Fig. 6 shows in a graphic form the transmitted, received, and regenerated signals. The comparatively long time during which the fork engages the selecting contact insures that the transmitting relay will be reliably operated and will tend to minimize chatter. Fig. 7 is a tracing from an oscillogram of some distorted received signals and their reconstruction by the repeater. The received signals were in the form of continuous reversals or alternating current. The variation in the distance between the points marked *X* would be the amount of distortion retransmitted by an ordinary relay repeater. The initial part of each regenerated signal shows a slight bounce or chatter of the transmitting relay contacts. Improved relays are now available, the use of which will reduce the chatter effect.

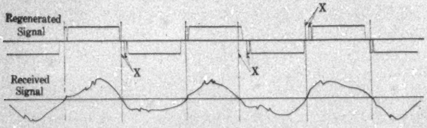

Fig. 7

The range of speed of the repeater using a standard tuning fork is from 15 to 30 cycles line frequency or 30 to 60 words per min. per channel of a two-channel, five-unit code printer circuit.

In the early development of this regenerative repeater, means were provided for varying the selecting or pickup point, thus giving the equivalent of orienting the contact segments of a rotary distributor. The correcting system which was adopted, however, proved to be very stable and it was found that the orienting feature could be dispensed with. Within the 15- to 30-cycle line frequency range, the point of selecting under all conditions is as close to the central portion of the signal as could be desired. In this connection it should be pointed out that an orienting feature is of less value at a regenerative repeater station where no printer record is available than on a printer terminal set. Valuable time is often lost in making futile adjustments of the orientation when another remedy is required.

Alternating Current for Balancing

One advantage of regenerative repeaters is the possibility of reducing the time required for balancing. If, for example, a long line circuit is equipped with a regenerative repeater at a central point, a readjustment or lineup of the balances of the relay repeaters on one side of the regenerative station may be conducted at the same time as the lineup on the other side. The regenerative repeater, however, must be equipped with a means of transmitting alternating current to either line, and it should preferably be of nearly the same frequency as that at which the working signals are transmitted.

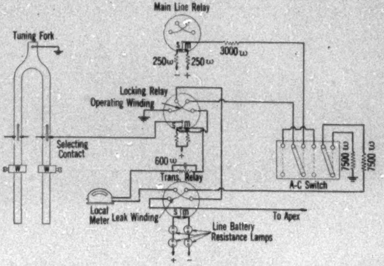

Fig. 8

Fig. 8 shows how this is accomplished on the non-rotary repeater without the need for additional equipment with the exception of the a-c. switch.

In the normal right-hand position of the a-c. switch, the operating winding of the locking relay is connected to the tongue of the main line relay and the leak winding of the transmitting relay goes to ground through 15,000 ohms. When the levers of the a-c. switch are thrown to the left, the operating winding of the locking relay is connected through the leak winding of the transmitting relay to the tongue of the same relay, through a 7500-ohm resistance. This leak current through the operating winding of the locking relay normally holds the tongue of the locking relay in a position opposite to that of the transmitting relay. When the fork engages the selecting contact, the tongue of the transmitting relay moves in the usual manner to a position corresponding with that of the locking relay. When the tongue of the transmitting relay operates, the current through the operating winding of the locking relay reverses but the locking relay is not operated immediately because the stronger current in its locking winding prevents its operation. When the fork moves away from the selecting contact, the locking current ceases to flow and the locking relay tongue, under the influence of the leak current from the transmitting relay, moves to the opposite contact.

From the foregoing, it is apparent that when the fork engages its selecting contact, the transmitting relay tongue moves to a similar position to that of the locking relay and when the fork disengages the selecting contact, the locking relay tongue moves to a dissimilar position to that of the transmitting relay. The alternating current generated in this manner is practically the same frequency as the alternating current from the terminal multiplex distributor.

CONVERSION TO PLAIN REPEATER

The repeater is equipped with cords and jacks by the use of which the regenerative action can be cut out and the set will function as a non-regenerative repeater.

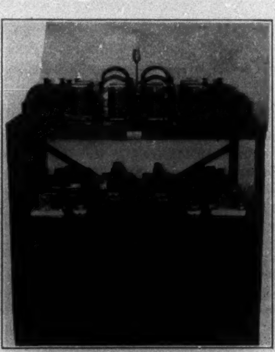

FIG. 9

FLOOR SPACE AND POWER REQUIREMENTS

The complete duplex repeater is shown in Fig. 9.

The repeater table is shipped completely wired for duplex operation. It is merely necessary to connect to power, lines, and ground and attach the forks and relays, etc., to the table. The floor space required is 43 in. by 27 in. The height of the table is 42 in.

The tuning fork, which is similar to those used on the terminal multiplex sets, is shown in Fig. 10.

The local power may be either 110 or 160 volts. The local current required for the duplex table is approximately one-half ampere and it has been found possible to install these repeaters in most telegraph offices without adding to the generator plant.

OPERATING EXPERIENCE IN POSTAL TELEGRAPH-CABLE SYSTEM

More than 50 of these repeaters are in use.

No difficulty has been experienced in operating several repeaters in one printer circuit.

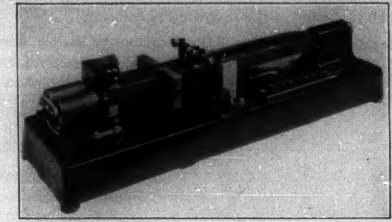

FIG. 10

Before regenerative repeaters were used, it was necessary to assign the best line wires to the overland circuits so that the overall distortion of the signal would be kept at a minimum. The present practise is to use a sufficient number of regenerative repeaters so that almost any line wire, provided it is electrically intact and free from swings, may be used in the overland circuits.

In several instances, wire routes which were unsuitable for use in overland circuits because of the long distances between repeater stations and which had insufficient wires to justify opening up additional repeater stations, have been made suitable for printer operation by the installation of regenerative repeaters adjacent to or near the long sections.

Fig. 11 shows the layout of two typical overland circuits. The speed of operation is approximately 50 words per min. per channel and the spacing between the regenerative repeaters is short enough to reduce lineups to minimum.

One or more regenerative repeaters are used in practically every printer circuit over 800 mi. in length with consequent reduction in lost circuit time and increase in speed of operation.

CONCLUSION

This regenerative repeater was developed to provide a simple and compact form of repeater which would give equivalent results to those obtained from the rotary regenerator without the expense of the rotary distributors and their synchronous motors.

The design eliminated the necessity for special types of distributors which otherwise would have had to be built to regenerate the multiplex circuits in use on the lines of the Postal Telegraph Cable Company.

The completed non-rotary regenerative repeaters have certain economies and improvements over the rotary forms considered and among these were:

The first cost is less because the expensive distributors and synchronous motors with accessories are eliminated.

The maintenance cost is lower because there are no distributor parts to require attention and renewal and less local current is needed.

The required amount of floor space is reduced.

The regeneration of the signals being accomplished directly by the fork eliminates the loss in margin caused by the tendency of the distributors to hunt.

A short interruption of the local power supply does not necessitate the attention of a repeater attendant. The tuning forks are self-starting and when the power supply is resumed the repeater starts up automatically.

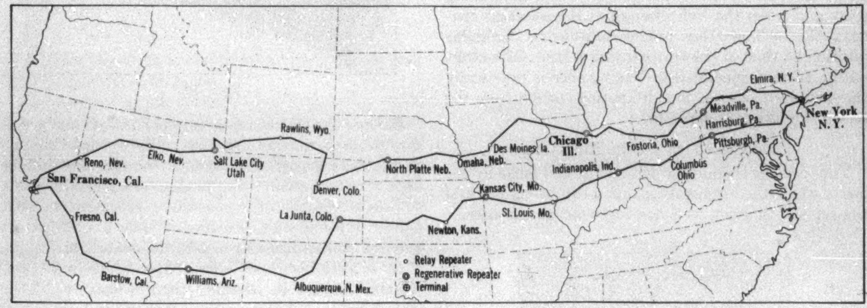

FIG. 11

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Discussion

W. C. Peterman: I should like to point out that a type of rotary repeater differing from those shown in Figs. 1, 2, and 3 in Mr. Connery's paper is shown in Fig. 2 of the paper by Herbert Angel[1].

In the type of rotary shown by Mr. Angel, which has been used very successfully by the Western Union on their long ocean cable circuits, the pick-up circuit is separate from the locking and sending-on relay circuit although they are both connected to segments on the same segmented rings. This regenerator gives complete regeneration; that is to say, if an impulse is received of sufficient duration to be locked up, the regenerated signal is of a definite duration.

In this respect it is the equivalent of the rotary repeater shown in Mr. Connery's Fig. 3, although it uses but one segmented ring and two unbiased relays when regenerating two-current signals, which tends for simplification. The repeater as shown in Fig. 2, however, is wired for cable Morse three-current signals and, therefore, uses four biased relays.

Herbert Angel: Considerable work has been done by the Western Union Telegraph Company with regenerative repeaters of both the rotary and non-rotary types, the latter being known as the fork regenerative repeater. Opinions differ as to which is the more advantageous type to use; each probably has its special field.

In a great many cases it has been found more desirable to use the rotary type with segmented rings for leaking off printers. Aside from this there is the question of brush transmission in one case and relay contact transmission in the other. It is the more general opinion that brush transmission is superior to relay contact transmission. For higher speeds the rotary type should be more reliable than the fork type as now developed.

A. F. Connery: Answering Mr. Peterman's comments, the diagrams of the rotary repeaters shown were of course made as simple as possible, and on some of them it would be necessary to add accessories to make practical repeaters such for example, as locking up the polarized transmitting relays to make them less subject to vibration and things like that.

We do not make claim for the non-rotary repeater that it will send out a better signal than a rotary repeater, but the advantage is that it is simpler and easier to maintain, and we think that in the hands of the average attendants, the net results will be better.

There are some purposes for which the rotary repeater is better; for instance, where we want drop-channel and others circuits like them. When we want to get into complicated circuits it is better simply to add segmented rings on the rotary mechanism, but on a straightforward regenerative repeater I think there are some advantages for the non-rotary repeater.

Mr. Angel's comments upon the advantages of brush transmission over relay transmission are well founded. In fact until recently there was a very decided advantage of brush transmission over relay, but relays have recently been developed with very low transit time; that is, the relays travel from one contact to the other at very high speed and means of reducing chatter have been developed so that now I think brush transmission has very little if any advantage over relay transmission and of course there is less maintenance of the relay contacts.

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[1] *Printing Telegraphs on Non-Loaded Ocean Cables*, A.I.E.E. Summer Convention, Detroit, Mich., June 23, 1927.

Electrical Reproduction from Phonograph Records

BY EDWARD W. KELLOGG[1]

Associate, A. I. E. E.

***Synopsis.**—A new and improved tool generally means new or improved accomplishments. Great improvements in sound recording and reproduction have been made possible by the thermionic amplifier.*

Electrical reproduction may be considered in three steps, (1) generation of a voltage by the vibrations of the needle, (2) amplification, (3) conversion of electrical power into sound. The first of these steps involves some interesting mechanical and electrical problems, and it is with these that the paper primarily deals, the problems of amplification and loud speaker design having been discussed in earlier publications to which references are given. Several types of phonograph "pick up" are possible; electrostatic, piezoelectric, electromagnetic, and variable resistance or microphonic. The electromagnetic principle is used in the device now manufactured. Since the moving armature cannot be actually at the needle tip, the little generator must function by transmitting the vibrations from the needle tip through a more or less flexible structure to the armature. Vibrations are inevitably transmitted, but when the requirements of freedom from appreciable distortion and maximum possible output are added, extreme care in design becomes necessary. An analysis is given of the mechanical behavior of the present model of reproducer.

MANY interesting improvements have been made recently in methods for recording and reproducing sound, which have resulted in truer and more pleasing reproduction. These have involved the use of electrical means in recording, while both electrical and direct or mechanical systems are being used with good results for reproduction. A discussion of some of the problems arising in an electrical system of reproduction, seems warranted in view of the widespread interest in such questions and of the fundamental nature of some of the mechanical problems involved.

Mechanical and Electrical Systems Compared. The inherent advantage of the electrical method of phonographic recording and reproduction, as compared with the older direct methods, lies in the fact that the electrical methods can make use of amplifiers. In the old system of recording, the cutting tool was mechanically connected to a diaphragm which was actuated by sound waves. The power available to give the necessary vibrations to the tool was thus limited to what could be collected from the original sound. In the case of electrical recording the power for vibrating the tool may be made as great as needed. This does not necessarily mean louder records or greater amplitude of vibrations. In acoustic apparatus extreme sensitivity is generally purchased at the expense of quality. In order to get sufficient amplitudes of cutting tool vibration, a horn was used to concentrate the sound waves, and a resonant diaphragm was employed. Both of these introduce distortion. In electrical recording, a sound pick up or transmitter without a horn is used. Its electrical output is small, but can be amplified without appreciable distortion, and relatively large forces can be applied magnetically to the cutting tool, which may now be heavily damped, thereby reducing its tendency to respond more to certain frequencies in the musical scale than to others.

When we come to the problem of reproduction from the record, the possibility of amplification does not give so great an advantage to the electrical system, as in the recording, because the reproducing needle, unlike the sound waves with which we started, can deliver considerable power, or apply large forces to the object which it is required to vibrate. This power at the reproducing needle is, of course, derived from the rotation of the record. In fact, cutting a record and then playing it may be regarded as a method of power amplification. In the old method of cutting and reproduction the power output in sound from the phonograph is normally many times the power collected by the horn used in recording.

Both the mechanical and electrical systems can be so designed as to give a very high order of quality in reproduction. The advantage of the mechanical system is its simplicity. The advantages of the electrical system are its flexibility, ease of adjusting loudness, and the possibility of obtaining greater volume of sound, where this is desired and the apparatus is designed accordingly.

Electrical reproduction may be considered in three steps, (1) the vibration of the needle must be made to generate a voltage whose wave form corresponds to the wave in the groove, (2) this voltage is amplified, and (3) an electrical loud speaker converts electrical power back into sound. The design of amplifiers and loud speakers has been discussed in earlier papers[2]. The present paper will, therefore, deal principally with the device in which the vibration of the needle generates the voltage which is to be supplied to the amplifier.

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[2] *Design of Non-Distorting Power Amplifiers*, Edward W. Kellogg, A. I. E. E. TRANS., Vol. 44, 1925, p. 302.

A New Type of Hornless Loud Speaker, by Chester W. Rice and Edward W. Kellogg, A. I. E. E., TRANS. Vol. 44, 1925, p. 461.

See also bibliography on amplifiers and loud speakers at end of paper.

POSSIBLE TYPES OF REPRODUCER

The same principles by which diaphragm vibrations in a transmitter are made to produce voice currents, may be applied to the case of the phonograph reproducer. Many types of transmitters have been used, among which are the magnetophone (Bell's first transmitter as well as receiver) the variable resistance transmitter such as in general use in telephony, the condenser transmitter[3] which is used in broadcasting stations, and a transmitter making use of a piezoelectric[4] crystal. These transmitters may be classified as of two types: (1) those in which the output voltage is proportional to the deflection, and (2) those in which the output voltage depends on the velocity of diaphragm movement. The carbon microphone, condenser, and piezoelectric transmitters as generally used are of the first type, while the magnetophone belongs to the second or velocity type. The condenser transmitter, for example, is kept charged through a very high resistance so that the charge upon the plates cannot change appreciably during an audio cycle. The voltage across the condenser then varies inversely as the capacity, or directly as the plate spacing. The voltage thus obtained which is applied to the grid of an amplifier tube is proportional to diaphragm deflection, independent of frequency except for frequencies so low that the condenser reactance is considerable compared with the leak resistance (for practical purposes we may say equal to the leak resistance). The same considerations apply to the piezoelectric crystal which may be regarded as a condenser of constant capacity but variable charge, the charge depending on the mechanical force applied to the crystal. On the other hand, both the condenser transmitter and the piezoelectric crystal may be made to act as velocity devices by using leaks whose resistance is low compared with the capacity reactance of the condenser throughout the essential frequency range. In this case the charge flows freely back and forth through the resistance, and the current (or voltage across the leak resistance) is proportional to the rate of change of charge which in turn is proportional to the diaphragm velocity. The magnetic transmitter works on the principle of changing the magnetic flux through a coil. Its open circuit voltage is proportional to the rate of change of the flux, and this again depends on velocity of movement. In terms of sine waves, the deflection devices (type No. 1) give a voltage proportional to amplitude independent of frequency, while the velocity devices (type No. 2) give a voltage proportional to amplitude multiplied by frequency.

Which of these types of device should we choose for a phonograph reproducer? If the system by which the record is cut is so designed that the deflection of the cutting point is proportional to the original sound wave pressure, then a deflection type reproducer is required. If the record is cut by a tool whose instantaneous velocity is proportional to the sound wave pressure, we shall require a velocity type reproducer. Both systems are equally correct from the standpoint that if their conditions are complied with, distortionless reproduction will result. The choice can, therefore, rest on such considerations as scratch noise ratio, wearing qualities of record, interchangeability of records, and designing a practical device which performs in accordance with the theoretical requirements. By interchangeability of records is meant that it is desirable that the electrical reproducing system shall not only give good results with records that are especially cut for it, but so far as possible give pleasing results with records cut by the old process, and the records which are correctly cut for electrical reproduction should sound well when played on a horn type machine. This requirement is most nearly met by the velocity system of cutting and reproducing. When a diaphragm is placed at the end of a long pipe it produces sound wave pressures in the pipe proportional to the diaphragm velocity. If, instead of a pipe, the diaphragm works into a horn of the usual (approximately exponential) shape, the same relation holds, very nearly, over most of the frequency range, the difference being that below a certain frequency the sound radiation from the horn drops to almost nothing[5]. If the needle motion is properly imparted to the diaphragm, a phonograph of the horn type may be regarded as a device giving output sound pressure proportional to needle velocity, except that its response is limited to frequencies above a certain value. In some of the new designs employing long slowly expanding horns, the range of response has been greatly extended in the direction of response to lower tones, this change plus the reduction of resonances in the system makes of the horn phonograph a machine which holds very closely to the relation—output sound pressure proportional to needle velocity. It is, therefore, clear that if the electrical reproducing system is to be such that records may be satisfactorily interchanged, it must work on the velocity principle. It is not apparent that in respect to wear of records and scratch ratio a system in which output depended on deflection rather than velocity would have any advantages to offset the disadvantage of not having the records interchangeable.

Of the possible devices giving voltage proportional to needle velocity, the magnetic and the piezoelectric have both been proved practicable, and the condenser is unquestionably a possibility. A very satisfactory design employing a piezoelectric crystal was worked out by Mr. Chester W. Rice of the General Electric Company. The magnetic reproducer gave substantially the same results, and was chosen on account of manufacturing considerations. It is the mechanical problems involved in the design of this device which are of especial interest.

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[3] See papers on the Condenser Transmitter by E. C. Wente, *Phys. Rev.*, *X-1*, p. 39 XI, p. 450 XIX.

[4] H. and P. Curie, *Compt. Rend.*, 91, pages 294 and 383, 1880.

J. Valasek "The Piezo-electric Activity of Rochelle Salt," *Phys. Rev.*, Vol. 19, p. 478, 1922.

A. Mc L. Nicolson, *The Piezoelectric Effect in the Composite Rochelle Salt Crystal*, A. I. E. E., TRANS., Vol. 37, 1919, p. 1315.

W. G. Cady, *I. R. E.*, Apr. 1922.

[5] *Function and Design of Horns for Loud Speakers*, by C. R. Hanna and J. Slepian, A. I. E. E., TRANS. Vol. 43, 1924, p 393.

The Performance and Theory of Loud Speaker Horns, by A. N. Goldsmith and John P. Minton, *I. R. E. Proceedings* Aug. 1925.

THE MAGNETIC REPRODUCING DEVICE OR PHONOGRAPH PICK-UP

Fig. 1 shows in outline several possible forms of magnetic reproducer for use with records having laterally cut grooves (as distinguished from grooves of varying depth). In each of these a movement of the needle causes a change in the flux through the coil or coils, and the voltage induced depends on the rate at which this flux changes, or on the velocity of movement of the iron armature. If there are no short-circuited turns, if the winding is electrically unloaded, and if the iron parts are of low reluctance at all frequencies compared with the air-gaps, the faithfulness of reproduction depends entirely on the similarity of motion of the armature and needle point. If the structures were rigid this similarity would be perfect and distortion would be nil.

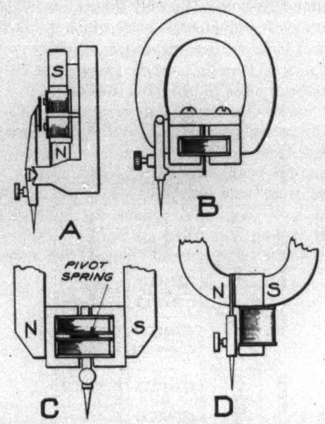
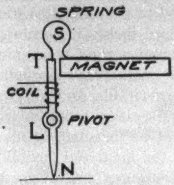


FIG. 1—SEVERAL POSSIBLE TYPES OF MAGNETIC PHONOGRAPH PICK-UP

Very little power is absorbed by the moving iron armature, for the winding is virtually unloaded, and even if the winding were loaded through a resistance, the power absorption would not be sufficient to produce much damping. There is, however, considerable stiffness in the mounting of the armature, necessary to resist the magnetic pull.

The mechanical problem may then be stated as follows: Given a certain movement of the needle tip, a motion of identical form must be imparted to a small magnetic element at an appreciable distance from the needle tip, displacement of the magnetic element from its main position being resisted by a spring or its equivalent. There are three types of structure by which the vibratory motion might be transmitted and the requirement of identical wave form be fulfilled: (1) a rigid structure, (2) a spring potentiometer, (3) a filter type or wave transmission structure. Fig. 2 shows a magnetic reproducing device or pick-up in schematic form. The only way in which the motion of the end *T* can differ in wave form from that at *N* is by bending of the lever or give at the pivot. In other words, making the system rigid will prevent distortion. But freedom from distortion is also compatible with flexibility in the lever and needle. If the needle point is pushed to one side of its normal position the yield is partly in the lever and partly in the spring *S*, the amount of motion at *T* depending on the relative stiffness of the two. I have called this a "spring potentiometer," by analogy with the electrical potentiometer. If two springs are connected in series as indicated in Fig. 3 the motion of the junction point *T* is a certain fraction of the motion at *N*. The ratio is only constant, however, if mass plays no appreciable part. If a mass is located at *T* the motion of the point *T* will be practically the same as though there were no mass present, provided the natural frequency determined by the mass in conjunction with the two springs is well above the highest frequency applied at *N*. The third type of structure by which the motion at *N* can be accurately reproduced at *T* is one in which both flexibility and mass are distributed. The distribution need not necessarily be uniform but the masses and flexible elements must have certain relations which depend on the range of frequencies to be transmitted. In this case there is definite wave motion



FIF. 2—ESSENTIAL FEATURES OF A MAGNETIC PICK-UP

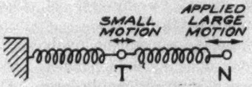


FIG. 3—SPRING POTENTIOMETER

and progressive phase differences between the motions of the successive parts of the structure. Because the masses and flexible elements are lumped rather than uniformly distributed, the wave motion structure has the properties of a filter, and has been called the "filter type structure" by Messrs. Maxfield and Harrison in their discussion of recording and direct or mechanical phonograph reproduction[6]. If the distance through which it is desired to transmit acoustic vibrations is of the order of several inches it becomes practically impossible to construct a rigid mechanical transmission, or one in which mass plays negligible part. Wave motion becomes inevitable, and to secure distortionless transmission it is necessary to so design the system that the waves will not be reflected. This means a proper proportioning of the mass and flexibility of each part, and the final absorption of the wave energy in a mechanical resistance of the correct value. If these conditions are met there is practically no limit to the distance to which the vibrations may be accurately transmitted. In the case of mechanical reproduction from a laterally cut phonograph record the necessary distance from needle point to diaphragm is too great for a truly rigid connection, while the mechanical resistance necessary for proper wave transmission is obtainable from the reaction of air on the diaphragm. The wave or filter system was, therefore, the logical choice.

For the electromagnetic reproducer on the other hand there appeared to be a possibility of making the distance short enough and the parts light enough so that a good approximation to a rigid structure could be obtained. If this should prove possible, the design would be much simplified, the required exactness of duplication would be reduced, and the necessity of obtaining a pure mechanical resistance would be avoided.

In considering the design of a magnetic reproducer it is necessary to choose between several possible types, such as moving-coil as against moving-magnetic armature, center-pivoted or full rocker as against end-pivoted or half rocker, windings on poles as against winding on armature. Several of these possible types are illustrated in Fig. 4.

The moving coil arrangement involves a long air-gap and, therefore, a heavy field magnet. Moreover the mass of the coil is objectionably large. Devices with iron armatures which move toward and away from the poles of a magnet have better possibility of producing a large change of flux interlinkage with a small movement of a small mass.

Again, there is decided choice between the various magnetic armature arrangements. In the first place it is better to place the windings around the armature than around the poles for much of the flux change in the armature involves only a slight shift of the flux from the pole pieces and does not cut all the turns of a coil wound on the pole. Hence we may limit our choice to column III.

Next is the question of single acting vs. double acting or "push pull" arrangements. Comparing *k* with *j* of Fig. 4, for example, we may say that adding the two poles on the left hand side of the armature has doubled the magnetic effect which results from a given motion of the armature, for there are just twice as many air-gaps whose reluctance is varied. Moreover, by placing opposite poles on the two sides we have reduced the steady flux which the armature has to carry, leaving only the residual or alternating flux. Hence the armature may actually be lighter in *k* than in *j*. Again, the rocker type armature has an advantage over the translation type armature in which both ends move in the same direction. It is only the motion of the ends of the armature opposite the poles which is effective to produce flux change. In the translation type all parts of the armature move equally, whereas in the rocker type, the middle has only slight motion for a given motion at the ends.

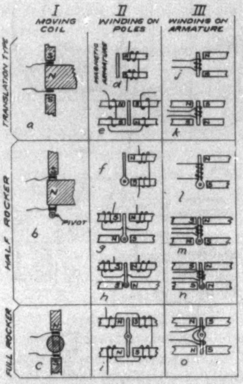


FIG. 4—MAGNETIC SYSTEMS AVAILABLE FOR REPRODUCING DEVICES

We have now reduced the choice to the half rocker as compared with the full rocker. The middle of the armature of the full rocker is a point of constant magnetic potential. If we imagine the armature cut in two and only the upper end moved, we should still get as much flux change through the upper end of the armature as we did in the full rocker, provided we could keep the lower end of the moving portion at constant magnetic potential. In other words, we might say that in the full rocker type the motion of the upper half gives rise to the flux change and that the motion of the lower end is required to keep the middle at constant magnetic potential. In the half rocker the pivot end of the armature can be kept at *nearly* constant magnetic potential by making the reluctance of the air-gaps at this end low compared with that of the gaps at the moving end. For example if the reluctance of the gaps at the pivot end is equal to that of the moving end gaps the flux variation will be half as much as in the full rocker, while if the pivot end gaps have one-fourth the reluctance of the moving end gaps, the flux variation in the armature will be eight-tenths as much as in the full rocker. This assumes negligible magnetic potential consumed in the armature and pole pieces, compared with that used in the gaps. It appears then that the half rocker is not necessarily at great disadvantage compared with the full rocker from the magnetic standpoint, and we shall see that it lends itself better to meeting the mechanical design requirements. The foregoing comparison of magnetic systems does not take into account the possible power output of the winding, nor is the elastic stiffness required to hold the armature in its mean position allowed to weigh in the choice. The comparison is wholly from the standpoint of obtaining the maximum open-circuit voltage with the minimum effective inertia of moving parts.

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[6] *High Quality Recording and Reproducing of Music and Speech.* J. P. Maxfield and H. C. Harrison, A. I. E. E., Feb. 1926, *Bell System Tech. Jour.*, July, 1926.

It is desirable that the magnetic reproducer shall utilize needles of about the size already in use. This means that the needle clamping screw must be within about ⅝ in. of the record. The screw is a potential source of trouble, first because it adds mass to the moving system and secondly because it has its own natural frequency of vibration which reacts on the motion of the armature producing both an anti-resonance and a resonance. If the screw is very short and stiff its natural frequency may be high enough to avoid detrimental effect, but the problem may then be to make it conveniently accessible. The types of device shown in Fig. 1 depend on using a short stiff screw. In designs of the type shown in Fig. 1B the set screw was accessible but these devices were found to depart too far from rigid structures, and high-frequency resonances resulted with almost complete loss of the frequencies above the resonance. In the type of reproducer shown in Fig. 1C the set screw is difficult to reach owing to the presence of the coil and pole pieces. Alternatives to the set screw were considered, but a more satisfactory solution appeared to be the location of the screw in the axis of rotation. In this position it can be made as long as desired and no effect of screw resonance has been observed. Placing the screw in the axis of rotation practically necessitates the half rocker rather than the full rocker construction, since the length of standard needles is such that the armature cannot extend far enough below the set screw to afford room for a coil and pole pieces.

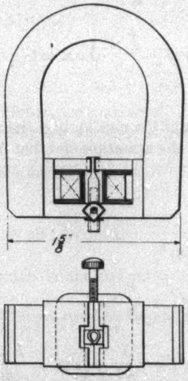

FIG. 5—MAGNETIC PHONOGRAPH PICK-UP AS MANUFACTURED

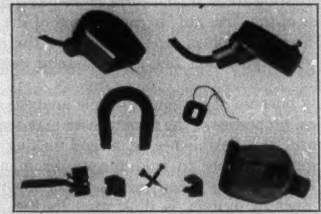

FIG. 6—MAGNETIC PHONOGRAPH REPRODUCER AND PARTS

Fig. 5 and 6 show the construction of a successful design of magnetic reproducer of the half rocker type. The armature is designed to have the smallest possible moment of inertia consistent with an adequate magnetic section and moderately large amplitude of movement of the upper end. In order that the axis of rotation may coincide with the screw and pivot axis at all frequencies, the armature is designed to have the center of gravity of the armature and needle coincide approximately with the pivot. The method of pivoting is unusual. In order that rotation might take place about the screw axis a journal type of bearing was desired, but all rubbing friction must be avoided in acoustic devices if distortion is to be obviated. Hence instead of having the shaft rotate within a journal with sliding friction, a film of rubber is interposed. This permits small rotation in either direction without friction but with a slight energy loss, more nearly resembling viscous damping, which is desirable. Objection might be made that such a pivoting system would permit translation as well as rotation. The objection may be answered in two ways. In the first place the yield of a sheet of rubber to direct compression is very slight. Rubber yields very readily in shear, and this permits rotation. It also yields easily in compression when it can expand freely in a lateral direction, but when a thin sheet of rubber completely fills the space between two surfaces whose dimensions are large compared with the thickness of the rubber, compression becomes very difficult. In the second place a slight translation would do no appreciable harm, having the effect simply of shifting the center of rotation by a small amount, which does not materially reduce the output. Experience so far with this method of pivoting has shown it very satisfactory.

While the rubber packed journal provides a restoring force, whose magnitude depends on the shape of the post and thickness of the rubber, it is not sufficient to hold the armature in neutral position when a strong magnet is used. It was necessary to provide a supplementary elastic restoring force in the form of rubber plugs on either side of the "fish tail" or moving end of the armature. The rubber plugs provide not only the needed stiffness or stabilization but a very useful degree of damping.

For the purpose of making a simple analysis of the mechanical properties of the structure we may treat the arrangement as a flexible needle, a rigid rocking beam, with a spring tending to hold the rocker in normal position and an energy absorber in conjunction with the spring. Such a system is illustrated in Fig. 7. A certain motion is assumed to be imparted to the needle tip A, and we are concerned with the motion of the end T of the armature.

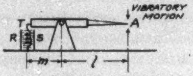

Fig. 7—Reference Figure for Discussion of Mechanical Behavior

Let I = movement of inertia of the armature

n = stiffness of needle, that is the force at A required to deflect end of needle a given amount, (dynes per centimeter)

l = distance from pivot to needle point (centimeters)

m = distance from pivot to end T of armature

s = stiffness of restoring spring at T (dynes per centimeter)

R = resistance factor at T, (dynes per unit velocity)

f = frequency

$\omega = 2\,\pi\,f$

A = amplitude of motion at needle tip

a = instantaneous deflection at the needle tip

X = amplitude of motion at end T

x = instantaneous deflection at point T

θ = angle of deflection of armature from mean direction.

θ is always small so that

$$\frac{x}{m} = \tan\theta = \sin\theta = \theta$$

P = maximum force applied at A

p = instantaneous force applied at A.

The driving moment is $p\,l$

The resisting moment is $m\,(s\,m\,\theta) + m\left(R\,m\,\frac{d\,\theta}{d\,t}\right)$

The difference between these two moments is expended in accelerating the armature

$$I\,\frac{d_2\,\theta}{d\,t^2} = p\,l - s\,m^2\,\theta - R\,m^2\,\frac{d\,\theta}{d\,t} \qquad (1)$$

The problem is most easily solved if we assume a sinusoidal motion at T and find the force and motion at A required to produce it.

Assume $x = X\cos 2\pi\,f\,t = X\cos\omega\,t$ (2)

$$\theta = \frac{x}{m} = \frac{X}{m}\cos\omega\,t \qquad (3)$$

$$\frac{d\,\theta}{d\,t} = -\,\omega\,\frac{X}{m}\sin\omega\,t \qquad (4)$$

$$\frac{d_2\,\theta}{d\,t^2} = -\,\omega^2\,\frac{X}{m}\cos\omega\,t \qquad (5)$$

Substituting (3), (4), and (5) in (1) gives

$$-I\,\omega^2\frac{X}{m}\cos\omega\,t = p\,l - s\,m\,X\cos\omega\,t + R\,m\,\omega\,X\sin\omega\,t$$

$$p\,l = X\left(s\,m - I\,\frac{\omega^2}{m}\right)\cos\omega\,t - X\,R\,m\,\omega\sin\omega\,t \qquad (6)$$

$$p = \frac{X}{l}\left\{\left(s\,m - \frac{I\,\omega^2}{m}\right)\cos\omega\,t - R\,m\,\omega\sin\omega\,t\right\} \qquad (7)$$

The deflection at the needle tip is the sum of that due to the tipping of the armature and that due to the bending of the needle.

$$a = l\,\theta + \frac{p}{n} \qquad (8)$$

or from (3) and (8)

$$a = \frac{X}{m}\,l\cos\omega\,t + \frac{p}{n} \qquad (9)$$

$$a = \frac{X}{m}\,l\cos\omega\,t + \frac{X}{n\,l}\left\{\left(s\,m - \frac{I\,\omega^2}{m}\right)\cos\omega\,t - R\,m\,\omega\sin\omega\,t\right\}$$

$$a = X\left\{\left(\frac{l}{m} + \frac{s\,m}{n\,l} - \frac{I\,\omega^2}{n\,l\,m}\right)\cos\omega\,t - \frac{R\,m\,\omega}{n\,l}\sin\omega\,t\right\} \qquad (10)$$

Or considering amplitudes only

$$\frac{A}{X} = \sqrt{\left(\frac{l}{m} + \frac{s\,m}{n\,l} - \frac{I\,\omega^2}{n\,l\,m}\right)^2 + \left(\frac{R\,m\,\omega}{n\,l}\right)^2} \tag{11}$$

$$\frac{X}{A} = \frac{1}{\sqrt{\left(\frac{l}{m} + \frac{s\,m}{n\,l} - \frac{I\,\omega^2}{n\,l\,m}\right)^2 + \left(\frac{R\,m\,\omega}{n\,l}\right)^2}} \tag{12}$$

Equation (**12**) shows the manner in which the response will vary with frequency. For uniform response $\frac{X}{A}$ should be constant or independent of ω. As was stated in an earlier paragraph this condition will be realized (1) if the structure is rigid or (2) if the structure is what we might call a spring potentiometer. A rigid structure would mean making the needle stiffness n infinite. This would make equation (**12**) become

$$\frac{X}{A} = \frac{1}{\left(\sqrt{\frac{2}{m}}\right)^2} = \frac{m}{l} \tag{13}$$

or the motions at the two ends are proportional to the lever arm lengths.

Again if the inertia and damping are zero we obtain a constant ratio for $\frac{X}{A}$ for equation (**12**) becomes

$$\frac{X}{A} = \frac{1}{\frac{l}{m} + \frac{s\,m}{n\,l}} \tag{14}$$

in which all the factors are constants.

In equation (**12**) we see that at a certain frequency or value of ω, the factor $\frac{l}{m} + \frac{s}{n\,l} - \frac{I\,\omega^2}{n\,l\,m}$ will become zero. At this frequency $\frac{X}{A}$ will have a maximum value which is limited only by the damping. Setting $\frac{l}{m} - \frac{s}{n\,l} - \frac{I\,\omega^2}{n\,l\,m}$ equal to zero in equation (**12**) gives

$$\frac{X}{A} = \frac{1}{\frac{R\,m\,\omega}{n\,l}} = \frac{n\,l}{R\,m\,\omega} \tag{15}$$

In general damping does not play an important part except near the resonant frequency. The inertia becomes a minor factor when the frequency is well below that at which resonance occurs, so that in the lower part of the frequency range the ratio $\frac{X}{A}$ approximates the constant value shown in equation (**14**). The construction of a reproducer in which distortion is reduced to a negligible quantity, therefore depends on making the resonance occur at a frequency so high that the most important part of the acoustic range is included below the resonance. It is furthermore necessary that there be sufficient damping so that the response at the resonance frequency is not excessive. It is to be noted that the resonance frequency is determined by the inertia of the armature and the restoring force not of the spring or cushions alone, but the combined stiffness of the springs and the needle. The natural frequency of the armature with the needle free has practically nothing to do with the response characteristic of the reproducer. The resonance corresponds to the natural frequency of the armature when the needle tip is held stationary. The action at resonance might be described as a whipping, such that a small movement at the needle tip causes large motion of the armature. Below resonance we may regard the vibrations as entirely forced by the cut in the record both as to amplitude and frequency.

To obtain a high resonance frequency, the first requisite is to make the moment of inertia of the armature small. At the same time the moving end of the armature must be far enough from the center of rotation to vibrate with considerable amplitude, and from the standpoint of magnetic design the cross-section of the armature must be sufficient to have low reluctance and to avoid any possibility of saturation. In practise it was found that the last mentioned conditions were met when the armature was made heavy enough for mechanical sturdiness and long enough to provide reasonable winding space. Special care has been taken to minimize the mass of those parts of the armature which have greatest motion, metal which is close to the axis of rotation having little influence on the moment of inertia.

The second requisite for high natural frequency is stiffness either in the spring or the needle or both. Reference to equation (**14**) which throughout most of the frequency range is a measure of the response, shows that if we increase the spring stiffness s, we decrease the sensitivity, while increasing n, the needle stiffness increases sensitivity up to a certain limit. It might seem, therefore, that the stiffest needle obtainable should be used, and the spring stiffness should be only what is required to hold the armature in neutral position. But the effect on sensitivity is small so long as the fraction $\frac{s\,m}{n\,l}$ is small compared with $\frac{l}{m}$, and damping is best when a "soft" needle is used, for the following reason. The needle is a highly resilient spring, whereas the spring S which in the present design consists in a pair of rubber plugs, has a large damping factor. In fact the damping R, and stiffness s, go together. With a given value of s

and R, the stiffer the needle the greater the resonance response, as shown in equation (**15**). Hence we work with the largest ratio of cushion stiffness to needle stiffness which we can use without serious loss of sensitivity. A factor which helps make it possible to get the desired damping without excessive stiffness at s, is the presence of the magnet, which tends to pull the armature away from neutral position and thereby reduces the net stiffness to considerably less than the value it has when the magnet is removed.

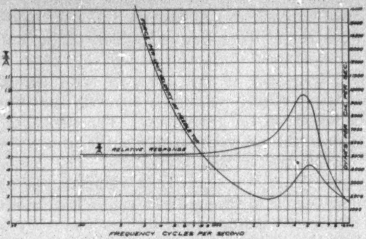

Fig. 8—Calculated Characteristics of Magnetic Reproducer

Measurements on a sample magnetic reproducer gave the following values of the principal constants.

Mass of armature with Victor medium needle 1.9 grams.

Moment of inertia of armature and needle $I = 0.28$ gm. cm.^{2}

Length, center of rotation to moving end of armature, $m = 1.1$ cm.

Length, center of rotation to tip of needle (Victor medium) $l = 1.5$ cm.

Stiffness of armature mounting (assumed concentrated at end) $s = 70{,}000{,}000$ dynes per cm.

Same with magnet removed. 90,000,000 dynes per cm.

Stiffness of needle (Victor medium, clamped for $\frac{3}{8}$ in.) $n = 86{,}000{,}000$ dynes per cm.

Damping factor as calculated from observed increase in response at resonance, $R = 3720$ dynes per cm. per sec.

Fig. 8 shows characteristics calculated from the above constants. It will be noticed that the required driving force at the needle tip has a minimum value at 2800 cycles. This is the natural frequency of the armature when the needle is free. It is also to be observed that the armature resonance at 5000 cycles is accompanied by an increase in the force reaction. Hence damping to keep down the resonance not only improves the response characteristic, but reduces the wear on the record.

Fig. 9 shows response curves of several factory samples of reproducers, taken by means of an oscillograph. The oscillograph vibrator is supplied from the output stage of a resistance coupled amplifier. The film is run slowly and the width of the envelope of the vibrations is a measure of the voltage applied by the reproducer to the grid of the first tube. The record used for this test is one cut by a special process so as to give an amplitude of cut varying inversely as the frequency. In other words a constant velocity cut is used. The approximate frequencies are indicated on each film. A slight increase in voltage is noticed toward the upper end of the frequency scale, followed by a drop to almost zero, for in all cases the output falls off very rapidly above the resonance frequency.

Scratch Control Circuit. Any exaggeration of the high frequencies produces a disagreeable increase in "surface noise" or "scratch." It has appeared desirable in fact to partially suppress the higher frequencies in order to reduce scratch. This has been accomplished by connecting a coil and condenser in series across the reproducer winding.[7] This shunt circuit tunes at about 4500 cycles but has a decided effect at 3500 cycles and above. The width of the frequency band affected may be controlled by varying the ratio of capacity to inductance, while the degree of suppression for the frequency at which the reactance is minimum, is determined by the resistance of the coil. The suppression of high frequencies is at the cost of some articulation in speech but on the whole gives a more pleasing result, particularly with musical numbers.

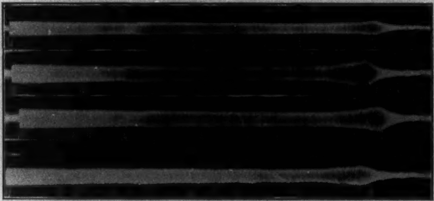

Fig. 9—Relation Between Voltage and Frequency for Magnetic Reproducers

Tone Arm Vibrations. The foregoing analysis of the action of the reproducer is based on the assumption that the device as a whole remains stationary with respect to the axis of the groove in the record. This condition, however, does not necessarily obtain. The net stiffness at the needle tip which is $1/(l^2/m^2 s + 1/n)$ is sufficient to resonate with the mass of the entire reproducer at a frequency of the order of 150 cycles. Since the reproducer is fairly rigidly mounted on the tone arm, the inertia, flexibility, and mechanical damping in the latter play a part in this type of resonance. Flimsy construction of the tone arm or its mounting gives rise to an irregularity in the response at low frequency, which while not an extreme resonance nor especially noticeable in listening is nevertheless a defect. Rigid construction and especially some energy loss at the pivot on which the arm swings, appear to be a practically complete cure for low frequency resonance of the type just described, and with such a satisfactory carriage the response becomes practically uniform from below 100 cycles to above 4000 cycles.

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[7] The "scratch control" circuit which is being used was a contribution of Mr. Julius Weinberger of the Technical and Test Department of the Radio Corporation of America. He also built the first magnetic reproducer of the bottom-pivoted half rocker type. This model which showed excellent characteristics, served as the basis for the design described in this paper.

ACKNOWLEDGMENT

The practical success of any device depends in large measure on the engineering skill with which it is placed on a manufacturing basis. In the present case some unusual problems presented themselves and much credit is due Messrs. F. C. Barton and J. E. Albright of the Radio Manufacturing Department, of the General Electric Co., and to Mr. R. Hillner, for the solution of the manufacturing problems and for turning out a final product equal in quality to the original laboratory samples and better in regard to output and reliability. Early experiments by Mr. Barton on magnetic pick-ups showed the possibilities of the system, and interested others in the development.

Acknowledgment should also be made of the important contributions of Mr. Julius Weinberger of the Technical and Test Department of the Radio Corporation of America.[7]

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Discussion

C. R. Hanna: In the paper by Mr. Kellogg no mention has been made of factors which determine the sensitivity of the pickup device other than that mass has something to do with it.

This statement is made: "The foregoing comparison of magnetic systems does not take into account the possible power output of the winding, nor is the elastic stiffness required to hold the armature in its mean position allowed to weigh in the choice. The comparison is wholly from the standpoint of obtaining the maximum open-circuit voltage with the minimum effective inertia of moving parts."

This discussion has to do with the factors which affect the sensitivity. The first of these is elastic stiffness which is not taken into account in the paper. In Fig. 8 of the paper, the curve given for the reaction force on the record against frequency, shows that the pickup device requires greater driving force in the low range of frequencies for a given velocity, and therefore, may cause excessive wear on the record. The stiffer the restoring member the steeper this curve will be. The stiffness of the restoring member, which in the design described by Mr. Kellogg is principally in the upper two rubber plugs, must be sufficiently great to prevent the armature from being pulled over to one pole or the other. The presence of the magnetic poles thus produces an effect which is known as magnetic reduction of stiffness and this reduction of stiffness must never be greater than the mechanical stiffness of the mechanical restoring member. In manufacture, it is never possible for the magnetic reduction of stiffness to be greater than some 50 per cent of the mechanical stiffness, leaving about 50 per cent as net stiffness. To prevent excessive wear to the record the net stiffness must be small, and since the magnetic reduction of stiffness bears a constant ratio to the mechanical stiffness, the magnetic stiffness cannot be greater than a certain amount. This necessitates either a weak magnet, a long air-gap, or fairly small pole faces, all of which make for low senstivity.

The other factor that weighs in sensitivity is the inductance. If more turns are wound on the coil, or a higher-ratio step-up transformer is used, to get greater output voltage, the inductive impedance of the device may become too high for the circuit into which it is to operate; namely, the grid of the vacuum-tube amplifier. Thus, the effective inductance of the device has a definite upper limit.

Now, as a general proposition, it can be shown that if magnetic saturation and leakage are negligible, the sensitivity expressed in volts per unit velocity at the needle is dependent upon two factors only: (1) the magnetic reduction of stiffness, which is limited because of wear on the record at low frequencies; and (2) the inductance of the device, which is limited because of the impedance of the vacuum tube into which the device is to operate. The relation[2] is $\text{Volts}/(\text{cm./sec.}) = \sqrt{10^{-7} \times \text{Inductance} \times \text{Magnetic Stiffness}}$. When these two factors are held constant, all of the arrangements which were shown in the paper will have the same sensitivity characteristic, provided magnetic saturation or magnetic leakage do not come into play.

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[2] C. R. Hanna, Design of Telephone Receivers for Loud-Speaking Purposes, *Proceedings*, I. R. E., August 1925. The relation given in this paper is for the force factor and must be divided by 10^7 to obtain generated voltage per unit velocity. While the proof of the relation is given for a particular structure, it can be shown that it applies to all devices in which the inductance varies inversely with a linear function of the displacement of the moving iron member.

And so the advantage of one construction over another, magnetically, in any case will be due to less leakage or less saturation, or both. Of the various arrangements having low saturation and leakage, the above relation shows that the one which is most suitable for mechanical considerations may be chosen without sacrifice of sensitivity.

E. W. Kellogg: Mr. Hanna has called attention to the fact that in making my choice from the numerous possible magnetic arrangements, I have limited my consideration to certain criteria; namely, securing the maximum volts per turn for a given magnitude of stored energy in the inertia of the vibrating armature. He introduces a different point of view, in which the stiffness of the armature mounting is to be limited in order not to cause excessive wear on the record, and he reaches the interesting conclusion that if well designed, almost any of the magnetic systems is as good as any other. This conclusion is itself a vindication of the point of view adopted in the paper. With armature stiffness as the controlling factor, there is no choice; but when we take small inertia as the desideratum, the choice is narrowed down to one or two arrangements. The reduction of inertia of the vibrating parts is desirable from the standpoint of wear on records, but is especially important for high-quality reproduction. The lighter the moving parts, the higher will be the frequency at which whip resonance occurs, and therefore the greater the range of frequencies which can be reproduced. The fact that the force on the needle tip is greater at low frequency where stiffness is the controlling factor than at high frequency where inertia predominates, may be taken as an indication that the effort to keep the inertia low was successful. The stiffness is not materially greater than that of mechanical reproducing systems, and if experience shows that greater flexibility is important, it will have to be secured, as Mr. Hanna's analysis shows, at the cost of lower sensitivity. In this connection I should like to commend Mr. Hanna's I. R. E. paper, for it is instructive and interesting. But the case considered there is a loud-speaker design and the factor which is of primary importance in one problem is not necessarily so in the other. The fundamental difference is that in the loud-speaker drive the current supplied results in a certain force being applied to the diaphragm and the amplitude is determined by the ease of movement of the diaphragm, while in the phonograph reproducer, the amplitude is practically fixed by the groove in the record.

The question might be raised why voltage per turn instead of power output was taken as the measure of efficiency of the magnetic arrangement. When the reproducer was first designed, it was expected that it would be connected directly to the grid of a tube, without a transformer. In this case, the minimum size of wire and available winding space being practically fixed, the useful output would be measured in terms of the voltage per turn developed by a given needle velocity. If the reproducer is used with a transformer, the possible power or volt-ampere output appears to be a more logical basis for comparison of different structures. I should like to add a brief discussion of this case.

If the transformer has a step-up ratio N, and if the capacity to ground of the tube grid and wiring is C, the effect of the transformer is to load the reproducer winding with an effective capacity $N^2 C$. But this must not resonate with the winding inductance within the working range of frequencies, for such resonance would impair quality of reproduction. The lower the winding inductance the greater the step-up ratio which may be employed. Hence, if we can lower the inductance by raising the reluctance of the magnetic circuit without reducing the flux change which results through the winding from a given needle movement, we make it possible to use a higher step-up ratio and apply more voltage to the grid. A consideration of the several magnetic systems illustrated in my Fig. 4 will show that we have no reason for revising our choice of double-acting or push-pull systems in preference to single-acting systems, or of rocking rather than translation type armatures.

There is, however, a different aspect to the comparison of the half-rocker and full-rocker armatures. It was argued in the paper that if the reluctance of the magnetic joints at the pivot of the half-rocker is kept low, the voltage developed would be nearly as much as with a full-rocker. For simplicity, let us take the ideal case in which this pivot joint reluctance is zero. The voltage per turn will be the same as with a full-rocker, while the inductance would be twice that of the full-rocker device. This would necessitate reducing the transformer step-up in the ratio $1/\sqrt{2}$ or 0.707, and the voltage at the grid would be reduced in the same ratio, which corresponds to cutting the power in half.

Were it not for practical difficulties having to do with clamping the needle, the full-rocker armature would perhaps have been chosen. But these difficulties were serious, and the advantage of the full-rocker is not so great as the two-to-one power ratio would make it appear. In the first place, this power ratio is based upon the assumption that both ends of the full-rocker move as much as the one moving end of the half-rocker. This would mean more metal to be moved, and if the whip frequency is to be kept high, there would have to be a compensating reduction in amplitude. Again the full-rocker, if proportioned to give twice the power output of the half-rocker, would require twice as stiff a mounting to stabilize the armature in the airgaps, and this, as Mr. Hanna has pointed out, is objectionable from the standpoint of record wear. By the time these factors are compensated by changing the ratio of armature to needle movement, practically all the advantage of the full-rocker armature has disappeared.

Television

BY HERBERT E. IVES*

Non-Member

Synopsis.*—The chief problems presented in the accomplishment of television are discussed. These are, the resolution of the scene into a series of electrical signals of adequate intensity for transmission; the provision of a transmission channel capable of transmitting a wide band of frequencies without distortion; means for utilizing the transmitted signals to re-create the image in a form suitable for viewing by one or more observers; arrangements for the accurate synchronization of the apparatus at the two ends of the transmission channel.*

* * * * *

INTRODUCTION

THIS paper is to serve as an introduction to the group of papers following, which describe the apparatus and methods used in the recent experimental demonstration of television over communication channels of the Bell System. In that demonstration television was shown both by wire and by radio. The wire demonstration consisted in the transmission of images from Washington, D. C. to the auditorium of the Bell Telephone Laboratories in New York, a distance of over 250 miles by wire. In the radio demonstration, images were transmitted from the Bell Laboratories experimental station at Whippany, New Jersey, to New York City, a distance of 22 miles. Reception was by two forms of apparatus. In one, a small image approximately 2 in. by 2½ in. was produced, suitable for viewing by a single person, in the other a large image, approximately 2 ft. by 2½ ft., was produced, for viewing by an audience of considerable size. The smaller form of apparatus was primarily intended as an adjunct to the telephone, and by its means individuals in New York were enabled to see their friends in Washington with whom they carried on telephone conversations. The larger form of receiving apparatus was designed to serve as a visual adjunct to a public address system. Images of speakers in Washington addressing remarks intended for an entire audience, and of singers and other entertainers at Whippany, were seen by its use, simultaneously with the reproduction of their voices by loud speaking equipment.

CHARACTERISTIC PROBLEMS OF TELEVISION

The problem of television in its broad outlines is that of converting light signals into electrical signals, transmitting these signals to a distance, and then converting the electrical signals back into light signals. Given means for accomplishing these three essential tasks, the problem becomes that of developing these means to the requisite degree of sensitiveness, speed, efficiency, and accuracy, in order to re-create a changing scene at a distant point, without appreciable lapse of time, in a form satisfactory to the eye.

A convenient starting point for the discussion of television is the human eye itself. In this an image is formed upon the retina, a sensitive screen, consisting of a multitude of individual light-sensitive elements. Each of these elements is the termination of a nerve fibre which goes directly to the brain, the entire group of many million fibres constituting the optic nerve. A theoretically possible television system could be made by copying the eye. Thus a large number of photosensitive elements could be connected each with an individual transmission channel leading to a distant point, and signals could be sent simultaneously from each of the sensitive elements to be simultaneously used for the re-creation of the image at the distant point. The number of wires or other communication channels demanded in a television system of this sort would be impractically large. For practical purposes, reduction of the number of transmission channels is made possible by the fact that, while in vision all parts of the image on the retina are simultaneously and continuously acting to send nerve impulses, the inertia of the visual system is such that a sensation of continuity is obtained from discontinuous signals, provided these succeed each other rapidly enough. Due to the phenomenon of persistence of vision, it is immaterial to the eye whether the whole view be presented simultaneously or whether its various elements be viewed in succession, provided the entire image be traversed in a sufficiently brief interval, which for purposes of discussion may be taken as 1/16th of a second or less[1]. We thus have available in television the same artifice which is used in the much less exacting problem of transmission of pictures over a telephone line, that is, of *scanning*, or running over the elements of the image in sequence, instead of endeavoring to transmit all of the elementary signals simultaneously. The development of a television system therefore necessitates, at an early stage, the design of some scanning system by which the image to be transmitted may be broken up into sequences of signals. In the simplest case, where one transmission channel is to be used, the whole image will be resolved into a single series of signals; if more than one transmission channel is to be utilized, the resolution may, by parallel scanning schemes, or their equivalent, be broken up into several series for simultaneous transmission.

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[1] This figure of 1/16th of a second, commonly quoted in discussions of this sort, is a convenient one, although the frequency of image repetition necessary to extinguish "flicker" is actually proportional to the logarithm of the field brightness. A somewhat higher rate of image repetition was used in the final television apparatus.

Like the eye, an artificial television system must have some light-sensitive element or elements by means of which the light from the object shall produce signals of the sort which can be transmitted by the transmission system to be used. For a television system to operate over electrical transmission lines this means some photoelectric device. It is obvious that this photoelectric device must be extremely rapid in its response, since the number of elements of any image to be transmitted must be some large multiple of the fundamental image repetition frequency, that is 16 per second. The response should, of course, be proportional to the intensity of the light, and finally, the device must be sufficiently sensitive so that it will give an electrical signal of manageable size with the amount of light available through the scanning system. This latter requirement, that of sensitiveness, is one which, it was realized from studies made with our earlier apparatus for the transmission of still pictures over wires[2], would be extremely difficult to meet. In the picture transmission system a very intense beam of light from a small aperture is projected through a transparent film and on to a photoelectric cell. In practical television, the system must be arranged to handle light reflected from a natural object, under an illumination which would not be harmful or uncomfortable to a human being. Actual experiment showed that the greatest amount of light which could be collected from an image, formed by a large aperture photographic lens on the small scanning aperture of the picture transmission apparatus was less by a factor of several thousand times than the light projected through it for still picture transmission purposes. Assuming the same kind of photoelectric cell to be used, the additional amplification required over that used in the picture transmission system, taking into account also the higher speed of response demanded, would bring us at once into the region where amplifier tube noise and other sources of interference would seriously affect the result. This indicated clearly that some more efficient method of gathering light from the object than the commonly assumed one of image formation by a lens was required, unless some much more sensitive type of photoelectric cell should be found.

Assuming that means could be developed for producing an electrical signal proportional to the intensity of the light, of sufficient quickness to follow a rapid scanning device, and of sufficient strength either as directly delivered from a photosensitive device or as amplified, the next problem is that of its transmission over an electrical communication system. We may quickly arrive at an understanding of certain of the transmission problems by reviewing the requirements for the transmission of photographs. In the system of still picture transmission now in use by the American Telephone & Telegraph Company, a picture 5 in. by 7 in. in size, divided into the equivalent of 10,000 elements per square inch or 350,000 elements, is transmitted in approximately seven minutes. This requires the transmission of a frequency band of about 400 cycles per second on each side of the carrier frequency. If we plan, in the transmission of television, to transmit images of the same fineness of grain, it would mean that what is now transmitted in seven minutes would have to be transmitted in a 16th of a second, which in turn means that the transmission frequency range would have to be nearly 7000 times as great. That is, a band approximately 3,000,000 cycles wide would be required. Bearing in mind that wire circuits are ordinarily not designed to utilize frequencies higher than 40,000 cycles per second, and that with radio systems uniform transmission of wide signal bands becomes extremely difficult, it is seen at once that either an image of considerably less detail than that which we have been considering must suffice, or else some means for splitting up the image so that it may be sent by a large number of channels is indicated.

A further theoretical requirement must also be given consideration. This is that the complete television signal will consist of all frequencies up to the highest above discussed, and down to zero, that is, an essential part of the signal is the direct current component, furnished by those parts of the scene which do not change. The problem of handling the very low frequency components, presents difficulties both in the vacuum tube amplifier system adjacent to the photosensitive device, and in ordinarily available transmission channels.

In any case certain fundamental transmission requirements must be met. These are that the attenuation of the signals must be uniform over the whole frequency range and that the speed of transmission of all frequencies must be the same. Also, as in the transmission of sound signals, the amount of interference or noise must be kept down sufficiently not to impair the quality of the signal or picture.

Assuming the undistorted transmission of the signals to a distant point, the next fundamental problem of television is the reconstruction of the image, or the translation of the electrical signal back into light of varying intensity. Just as at the sending end we have seen that the production of a useful electrical signal with the amount of light available from a naturally illuminated object is a major problem, so at the receiving end the converse problem, that of securing an adequately bright light from the electrical signal, presents great difficulty. The nature of the problem may be understood by assuming that it is to be done by projecting the received image on a screen similar to an optical lantern projection screen. If the spot of light which is to build up this image scans the whole area in the same way that the object is scanned, we find that the amount of light which can be concentrated into a small elementary spot will, when distributed by the scanning operation over the whole screen, reduce the brightness of the screen in the ratio of the relative areas of the elementary spot and the whole screen. The amount of this reduction will, of course, depend upon the number of elements into which the picture is divided, but will in any event be a factor of several thousand times. It is doubtful whether any light source exists of sufficient intensity such that an image projected by it can be spread out by a scanning operation over a large screen and give an average screen brightness which would be at all adequate. It is possible to imagine optical systems by which such a thing as the crater of an arc could be projected upon the screen, but the motion of this image and its variation in intensity would involve the extremely rapid motion of lenses, mirrors, and apertures of a size such as to render the operation mechanically impracticable. It appears from these considerations, that the only promising means of reconstructing the image would be those in which a light source, whose intensity can be controlled with great rapidity, is directly viewed.

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[2] "Transmission of Pictures over Telephone Lines," Ives, Horton, Parker, and Clark. *Bell System Technical Journal*, Vol. IV, No. 2, April, 1925.

Another element of a television system upon whose solution success depends as much as any other is that of synchronization; the reconstruction of the image, postulated in the last paragraph, is only possible if the reconstructed elements fall in exactly the right positions at the right times, to correspond with the signals as generated at the analyzing end. The criterion for satisfactory synchronization will be expressed in terms of variation from identity of speed by figures which will depend on the fineness of grain of the image which it is planned to send. No element of the image must, of course, be out of place by a considerable fraction of the size of the element.

GENERAL OUTLINE OF MEANS EMPLOYED IN THE PRESENT TELEVISION SYSTEM

It has been pointed out above that if the goal which we set in television is the transmission of extended scenes, with a large amount of detail and hence made up of an exceedingly large number of elementary areas, we meet with the necessity for transmission channels of a character which are not now available. In the present development it was decided at the start to restrict our experiments to a size and grain of picture which, if the scanning and re-creating means were developed, would be capable of transmission over practical transmission channels, either wire or radio. This restriction fortunately leaves us with the possibility of meeting what was felt to be the typical problem of a Telephone Company, namely the transmission of a human face in a television system used as an adjunct to a telephone system. Taking as a criterion of acceptable quality, reproduction by the halftone engraving process, it is known that the human face can be satisfactorily reproduced by a 50 line screen. Assuming equal definition in both directions, 50 lines means 2500 elementary areas in all; 2500 elements transmitted in 1/16 second is 40,000 elements per second. The frequency range necessary to transmit this number of elements per second with a fidelity satisfactory for television cannot be calculated with assurance in advance. An approximate value can however be arrived at from a study of the results obtained in still picture transmission. In pictures transmitted by the system already referred to, individual faces contained in a square space ½ inch on a side are quite recognizable[3]. Taking the ratio of this area to the area of the whole picture, and using the frequency range figure already deduced, for a complete 5 in. by 7 in. picture, it appears that a band of 20,000 cycles would be sufficient to transmit such an image in 1/16 second[4]. These considerations led to the choice of a 50 line (2500 element) image as one which would be both satisfactory as to detail rendering, for our purposes, and as calling for frequency transmission requirements sufficiently low to give a good margin of safety in existing single communication channels.

As a method of scanning, the method which is probably mechanically simplest, namely that of rotating disks with spirally arranged holes, proposed by Plotnow[5] in 1884, was chosen. In accordance with the choice of grain above indicated, the disks were perforated with 50 apertures.

For the second element of the problem, the light-sensitive means, the alkali metal photoelectric cell was chosen as possessing the qualities of proportionality of response and quickness of reaction. The currents produced by it are at best quite small, but they lend themselves to the process of amplification by the three-electrode vacuum tube amplifier.

The problem of securing a large enough signal, which is intimately associated with that of securing enough light from the object, was, in our development work, postponed in the earlier stages, our first experimental work having been done by concentrating light through photographic transparencies[6]. The solution of the problem of securing adequate light was subsequently attained by reversing the light path and projecting a narrow beam of light through the scanning disk upon the object. By this means only the element of the object which was being scanned was illuminated at any one time, thereby reducing the average illumination enormously, and the problem of increasing the signal strength could be attacked by increasing the amount of photo-sensitive surface as well as by increasing the brightness of the scanning light[7].

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[3] c. f. Fig. 18 of the paper referred to (Reference 2).

[4] A factor which this analogy does not cover is that if the image is moving so that it falls on several discrete scanning elements in rapid succession a very material apparent increase in the fineness of the image structure results. This effect is similar to that by which the relatively coarse grained individual images in a motion picture film fuse to give smooth appearing pictures.

[5] Plotnow, D. R. P. 30105, 6.1, 1884.

[6] As one step in the development work moving picture film, projected by a commercial projector in synchronism with the scanning disks, was successfully transmitted.

The problem of amplifying the photoelectric currents to sufficient value for transmission was solved by a practical compromise which at the same time met one of the transmission difficulties. This compromise consisted in amplifying and transmitting only the fluctuating or alternating current components of the signal, leaving the direct current component, which determines the general tone value of the image, for empirical reintroduction at the receiving end. By this scheme, stable amplifier constructions were made available, and the transmission channels, particularly the wire channels, could be utilized in their normal working form.

At the receiving end, the problem of securing a sufficiently bright image was solved, as indicated earlier, by the use of self-luminous surfaces of much higher intrinsic brightness than it is possible to secure by illumination of a surface by any light source which can be rapidly controlled as to its intensity. The self-luminous surfaces employed were glow lamps containing neon gas, the brightness of which changes with sufficient rapidity to follow the incoming signals.

The problem of synchronization was postponed in our earlier development work by mounting the scanning and receiving disks upon the same axle. It was later solved for the demonstration apparatus by the utilization of synchronous motors controlled by two frequencies, a low frequency, that of the image repetition period and a high frequency, chosen of such a value that the fraction of the cycle through which hunting occurred amounted in angular displacement to less than half the angular extent of a single disk aperture. The synchronization control therefore called for the transmission of additional currents for synchronization purposes over and above the picture current.

In order to transmit and synchronize the image signals it is necessary to transmit three different frequency bands, one for the image, and two for the high and low frequency synchronization controls. In the demonstration of April 7, 1927, the images were sent in the wire demonstration over a high quality open wire line. The synchronization control was sent over two separate carrier channels of a second telephone line. In addition to these lines, another line was used for conveying the telephone conversation. In the radio demonstration two different wavelengths were used respectively for the image signals and for the synchronization signals which were, as in the wire demonstration, carried on two different carrier frequencies. A third channel was used for the voice. In the case of both wire and radio transmission, it is quite possible to put all of the different signals upon the same transmission channel, using different carrier frequencies.

It will aid toward a clear understanding of the reasons for the success of the system of television described in the following papers if we summarize at this point the chief novel features to which that success is due. They may be listed as follows:

1. Choice of image size and structure such that the resultant signals fall within the transmission frequency range of available transmission channels.[8]
2. Scanning by means of a projected moving beam of light.
3. Transmission only of alternating current components of image.
4. Use of self-luminous surfaces of high intrinsic brilliancy for re-construction of the image.
5. High frequency synchronization.

Applications and Future Developments

It is not easy at this early date to predict with any confidence what will be the first or the chief uses for television, or the exact lines that future development may take. It must be clearly understood that television will always be a more expensive service than telephony, for the fundamental reason that it demands many times the transmission channel capacity necessary for voice transmission. This expense will inevitably increase in proportion to the size and quality of the transmitted image.

The kinds of service which are naturally thought of upon consideration of the services now rendered in connection with sound transmission are: first, service from individual to individual, parallel in character to telephone service, and as an adjunct thereto; second, public address service, by which the face of a speaker at a distant point could be viewed by an audience while his voice was transmitted by loud speaker; third, the broadcasting of scenic events of public interest, such as athletic contests, theatrical performances, and the like.

The first two types of service just mentioned lie within the range of physical practicability, with apparatus of the general type already developed. The third type, because of the uncontrolled conditions of illumination, and the much finer picture structure which would be necessary for satisfactory results, will require a very considerable advance in the sensitiveness and the efficiency of the apparatus, to say nothing of the greatly increased transmission facilities. For all three types of service, wire or radio transmission channels could be utilized, for while the problems incident to securing distortionless transmission over wide frequency bands, or multiple transmission channels, are different in detail in the two cases, they appear to be equally capable of solution by either means. However, the very serious degradation of image quality produced by the fading phenomena characteristic of radio indicates the practical restriction of radio television to fields where the much more reliable wire facilities are not available.

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[7] A still further advantage is obtained by limiting the scanning light to the region of the spectrum to which the photoelectric cells are sensitive (blue and violet). This is unnecessary where one way transmission only is used but is of value where in two way transmission a transmitted image is to be viewed by a person being scanned.

[8] As the succeeding papers show, the margin between the frequency range required by the scanning apparatus and that which could be made available was quite liberal. It appears in the light of our experience that apparatus with 60 or 70 scanning holes instead of 50 might be used with the transmission facilities which were at our disposal.

The Production and Utilization of Television Signals

BY FRANK GRAY*, Non-Member
J. W. HORTON*, Member, A. I. E. E.
and R. C. MATHES*, Associate, A. I. E. E.

Synopsis.*—The design of a television system, once the fundamental principles are understood, involves a detailed consideration of the methods by which the several important functions are to be performed.*

(1) *In the present system the initial signal wave is obtained by sweeping a spot of light over the subject in parallel lines completely scanning it once every 18th of a second. The light reflected is collected by large photoelectric cells which control the transmitted current. At the receiving station the picture current controls the brightness of a neon lamp from which the received image is built up by means of a small aperture moving in synchronism with the spot of light at the transmitting station. For presentation to a large audience television images may be produced by a neon lamp in the form of a grid having a large number of separate electrodes. A high frequency excitation controlled by the picture current is distributed to the successive electrodes in synchronism with the spot of light at the transmitting station.*

(2) *Space and time variations in the reflecting power of the subject are translated into time variations in signal strength. For design purposes these time variations are represented by component frequencies, a minimum band of which must be properly transmitted to insure an adequate reproduction of the image. Within this band there must be maintained a certain degree of uniformity in the efficiency of transmission of the separate components. Also, their phases must not be permitted to shift unduly in relation to each other.*

(3) *The design of the terminal amplifiers is based on the quantitatively determined characteristics of the photoelectric cells and of the neon lamps as well as on the limits imposed by the transmission study and by the characteristics of available transmission media, whether telephone line or radio system. The circuits employed at the transmitting station furnish an amplification such that the power delivered to the transmission medium is 10^{15} times the power received from the photoelectric cells.*

SECTION I

Apparatus for the Analysis and Synthesis of the Image

THE introductory paper to this series of articles on television explained principles along which any television system must operate to transmit an image over a single pair of wires or other channel of communication. As the first step in such a transmission, the space variations in brightness from point to point in the view must be translated into time variations in an electrical current that can be sent over the channel of communication. This translation may be accomplished by a scanning process that operates on the view to produce the same effect as if the view were cut up into a single long strip and passed rapidly in front of a light-sensitive cell to generate an electrical current varying with the brightness along the strip. To eliminate flicker in the reconstructed image and also to follow moving subjects in a view, the scanning process must be repeated and a new picture transmitted at least every sixteenth of a second.

Many purely theoretical methods could be, and have been, devised to accomplish such a scanning process and to translate a view into electrical currents or signals. Unfortunately, however, a practical system of television must operate with materials and conditions as they exist, and these practical limitations constitute the serious problems of television.

The high speeds and relatively large amplitudes with which any television scanning mechanism must move, and the necessity for synchronizing the transmitting and receiving apparatus lead to the use of synchronously rotating machines as apparently the only practical solution of the scanning and receiving problems. Consequently, the present television system has been designed to operate with continuously rotating mechanical parts.

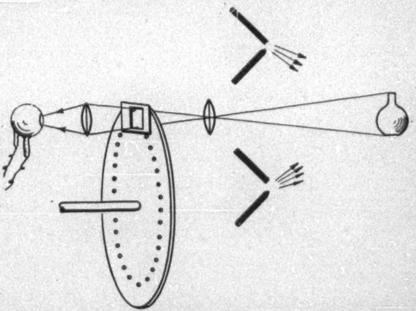

FIG. 1—SEVERAL LIGHT SOURCES ILLUMINATE THE SUBJECT; A LENS FORMS AN IMAGE WHICH IS SCANNED BY A SPIRAL OF APERTURES, THROUGH WHICH THE LIGHT FALLS ON A SINGLE PHOTOELECTRIC CELL

The efficiency that must be secured in the optical part of any scanning method is fixed by the three following factors—the amount of picture detail that is to be transmitted, the efficiency of the light sensitive cell, and the practical limit to amplifier systems. The first of

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*All of Bell Telephone Laboratories, Incorporated, New York, N. Y.

Presented at the Summer Convention of the A. I. E. E. Detroit, Mich., June 20-24, 1927.

these factors decides the area from which light can be collected at any one instant. In the present case this was fixed in an initial survey of the entire television problem when it was decided to confine the first attempt to the transmission of pictures as if they were made up of 2500 small elemental areas; that is, to scan the view in a series of fifty parallel lines. The second factor is determined by the sensitivity of the potassium hydride photoelectric cell. This cell is, at the present time, the most efficient light-sensitive cell that can follow the rapid variations in light intensity without a time lag. The third factor, the limitation of amplifier systems, results from the extraneous currents that are present in metallic conductors and amplifier tubes. The thermal agitation of the electrons in any input resistance generates such currents; and rapid variations in the number of electrons emitted from the hot filament of an amplifier tube also generate disturbing voltages. For successful amplification, the initial photoelectric current must be considerably larger than these extraneous currents. Consequently, the optical arrangement must be such that at any one instant it collects enough light from an elemental area of the view to generate this minimum permissible output current from the photoelectric cell.

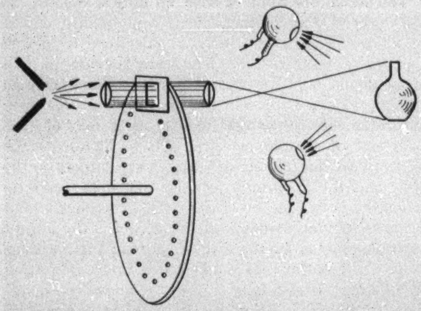

Fig. 2—Light from a Single Source is Projected as a Small Moving Spot on the Subject; the Reflected Light is Received by Several Photoelectric Cells

The operation and advantages of the scanning method actually used in the present process for transmitting television images may be better understood by first considering a simple and analogous method illustrated by Fig. 1. The subject is illuminated by lights placed in front of it as shown. A lens forms an image of the subject on the rotating disk. This disk is pierced with a series of small holes or apertures arranged in the form of a spiral; and, as the disk rotates, the apertures trace across the image one after the other in a series of parallel lines. The frame limits the size of the image and prevents more than one aperture being in the image at one time. Light, passing through an aperture as it travels across the image, falls in the light-sensitive cell and generates a picture current proportional to the brightness of the image from point to point along strips taken one after the other across the image.

In any system such as that outlined above, which depends upon scanning an image of the view as formed by a lens, the efficiency of the system is ultimately limited, for any given size of image that can be scanned, by the ratio of aperture to focal length of the best lens that can be secured. Experiments show that, with the best lens available to form a one-inch-square image, it would be necessary to illuminate a subject with a 16,000 candle power arc at a distance of about four feet in order to secure an image bright enough for a photoelectric cell to give an output current above the noise level in an amplifier system. In other words, television would apparently be extremely inconvenient to the subject if it were to be carried out from an image formed by a lens.

In the system actually used for television transmission, this apparent limitation has been evaded by reversing the entire optical system of Fig. 1 and arranging it as shown diagramatically in Fig. 2. Instead of scanning an image of the subject, the actual subject is scanned directly by a rapidly moving spot of light. An illustrative laboratory set-up, Fig. 3, shows the arrangement of parts in such a transmitting station. A fifteen-inch disk rotating approximately eighteen times per second carries a series of fifty small apertures arranged in the form of a spiral. A beam of light is condensed by a lens from a 40-ampere Sperry arc to intensely illuminate a limited area in the path of the moving apertures; and a slender, intense beam of light passes through each aperture as it moves across the illuminated area. A frame in front of the disk permits

Fig. 3—Illustrative Transmitting Apparatus

Light from the arc lamp is condensed on the disk, which is driven by a high frequency synchronous motor. The disk carries a spiral of pin hole apertures, each of which in turn projects a moving spot of light on the subject. Reflected light is collected by three large photoelectric cells light to emerge from only one aperture at a time and the lens in front of the disk focuses an image of this moving aperture on the subject. As a result of this arrangement the subject is completely scanned in a series of successive, parallel lines by a rapidly moving spot of light, once for each revolution of the disk; and on account of the transient nature of the illumination the subject is scarcely aware that he is being exposed to it.

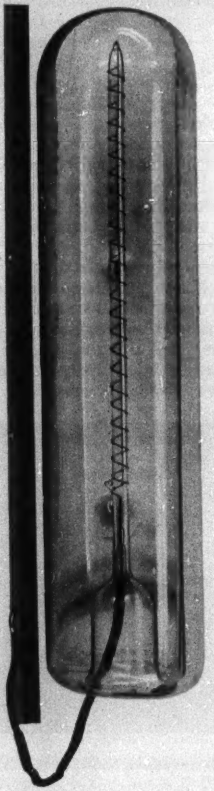

FIG. 4—LARGE PHOTOELECTRIC CELL

The cell presents forty square inches of photo-sensitive surface to receive light reflected from a subject

As the spot of light traces across the subject, light is diffusely reflected or scattered from the subject in all directions, and some of the light that is reflected forward passes into three large photoelectric cells placed just in front of the person who is being viewed. The current outputs from the three photoelectric cells operate in parallel into a common amplifier system. As the beam of light passes, for instance, across a person's eyebrow less light is reflected to the photoelectric cells, and as the beam passes across his forehead more light is reflected. Since the current output from the photoelectric cells is proportional to the received light, the current follows accurately the brightness of the various elemental areas of the subject's features as he is traced over by the scanning beam. This fluctuating current is unidirectional.

The actual operation of such an optical system, its influence on the lighting effects and quality of the reproduced image, may best be understood by noting that optically the system is identically the same as if all of the rays of light were reversed in direction to give an optical system equivalent to Fig. 1. The television apparatus sees the subject exactly as if rays of light came out of the photoelectric cells to illuminate the subject; the lens formed an image of the subject on the disk; and the apparatus scanned this image and reproduced it at the receiving end. The lights and shadows seen in the image are the same as if the subject were illuminated by three large lights in the positions of the photoelectric cells and looked at from the position of the lens. It also follows from the above considerations that, within its range of resolving power, this scanning method will not only reproduce a plane subject, such as a drawing, but that it will also faithfully reproduce three-dimensional figures with sharp edges and elevations and depressions, just as well as they could be reproduced in a photograph.

In addition, because the light passes in an approximately parallel beam through a disk aperture, the slender beams of light sweeping across the region in front of the transmitter just barely overlap each other even at a considerable distance from the apparatus. Consequently, it is not necessary that the subject be at the exact positions at which the small apertures are sharply focussed; and within wide limits no confusion results as the subject moves toward or away from the apparatus. The brightness as well as the size of the received image decreases as the subject moves away from the photoelectric cells; and for good transmission of the human features, which reflect very little blue light to which the photoelectric cells are sensitive, a person should not be more than a few feet away from the cells.

This method of scanning permits two very large gains to be made in the amount of light available for producing photoelectric currents. The transient nature of the light permits a very intense illumination to be used without inconvenience to the subject. Furthermore, the optical efficiency of the system is not limited by the apertures of available lenses; but can be increased by using large photoelectric cells and more than one cell connected in parallel.

The photoelectric cells of the potassium hydride, gas-filled type used in the transmitting stations, were specially constructed for the purpose and are probably the largest photoelectric cells that have ever been made, Fig. 4. Three of these cells present an aperture of 120 square inches to collect the reflected light.

With this large collecting area and the strong light intensity that can be used for the transient illumination, the cells give an electrical output that, though still extremely small, is safely above the noise level of an amplifier system.

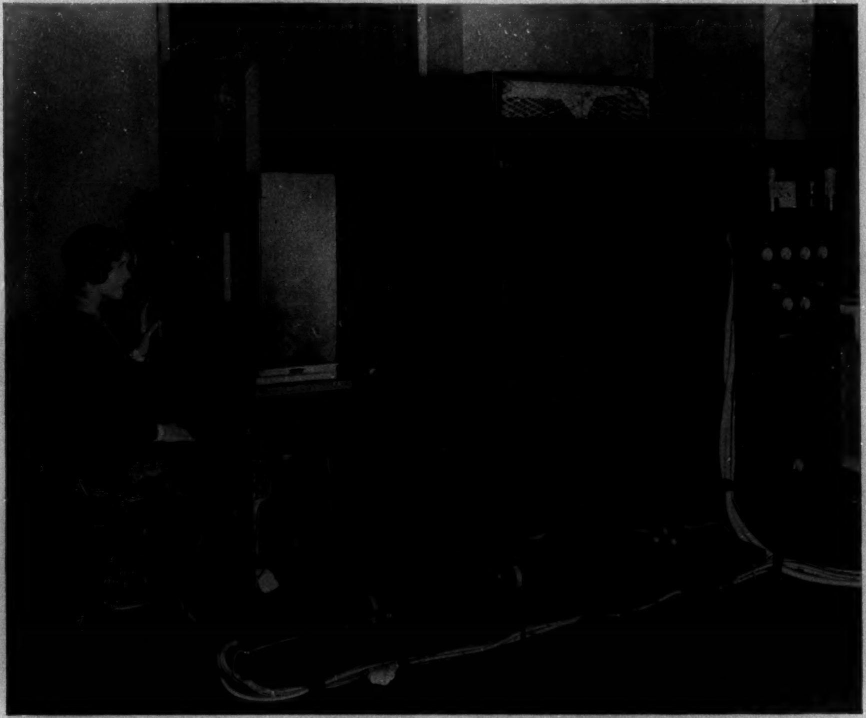

FIG. 5—TELEVISION TRANSMITTING APPARATUS

Sweeping beams of light pass out through the tunnel-like opening in the photoelectric cell case; light reflected from the subject is collected by three large photoelectric cells behind the screened openings

A photograph, Fig. 5, shows the details of a television transmitting station as it is operated in the field. The arc, rotating disk, and photoelectric cells are contained in separate cabinets and alined as shown in the photographs. The three photoelectric cells and first stages of amplification are mounted in a shielded, soundproof case. The slender, sweeping beam of light coming from the disk cabinet passes through the tunnel-like opening in the photoelectric cell case and scans the subject seated in front of it. The apparatus sees the person from light reflected back into the three large cells located just behind the screened openings in the case.

The variations of the feeble picture currents delivered from these photoelectric cells are highly amplified and transmitted over a wire or radio channel of communication by circuits described elsewhere in this series of articles. At the receiving station this current shape is re-amplified, impressed on a direct current, and finally produces an image in the receiving apparatus.

A photograph, Fig. 6, shows an illustrative arrangement of the parts in one type of television receiver. An essential part of this type of receiver is a disk similar to the one at the transmitting station and also provided with fifty small apertures arranged in the form of a spiral. The driving motor rotates the disk in exact

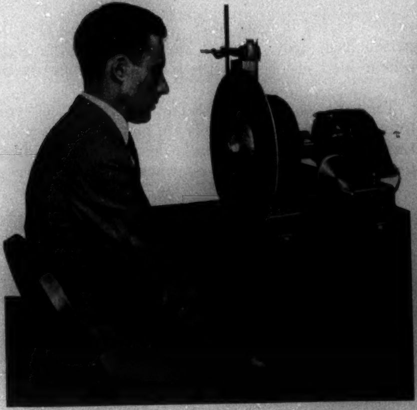

FIG. 6—ILLUSTRATIVE RECEIVING APPARATUS

A neon lamp operated from the picture current illuminates a series of small apertures as they pass across the field of view; the observer sees an image reproduced in the frame

FIG. 8—DISK RECEIVING APPARATUS

The observer looks through the shielding window at a picture on the 36 inch disk.

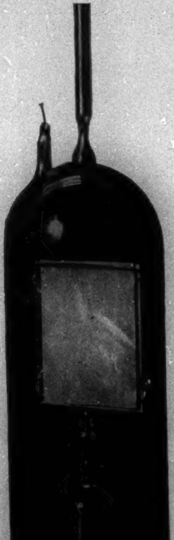

FIG. 7—NEON RECEIVING LAMP

The rectangular cathode is covered by a uniform layer of glow slightly larger than the field of view on a television disk

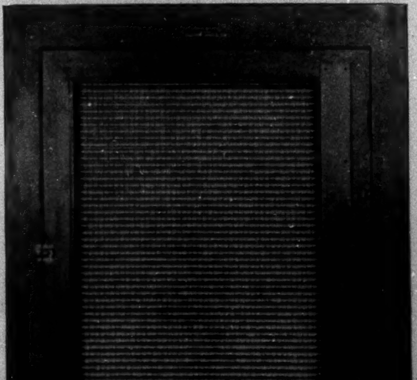

FIG. 9—LARGE GRID

The large grid is a neon lamp with 2500 electrodes on a tube bent back and forth to form a luminous screen that is visible throughout a large auditorium synchronism with the one at the transmitting station. The observer looks at a small rectangular opening or frame in front of the disk. This frame is of such dimensions that only one aperture can appear in the field of view at a time. As the disk rotates, the apertures pass across the frame one after the other in a series of parallel lines, each displaced a little from the preceding one until in one revolution of the disk the entire field has been covered. Beyond the disk is a special form of neon glow lamp shown in detail by Fig. 7. In this lamp, the cathode is a flat metal plate of a shape and area sufficient to fill entirely the field defined by the frame in front of the disk. The anode of the glow lamp is a similar metal plate separated from the cathode by only a very small space (about one millimeter). At the proper gas pressure this space between the plates is within the "cathode dark space" where no discharge can pass. As a consequence, the glow discharge develops on the outer surface of the cathode, where it shows as a perfectly uniform, thin, brightly glowing layer.

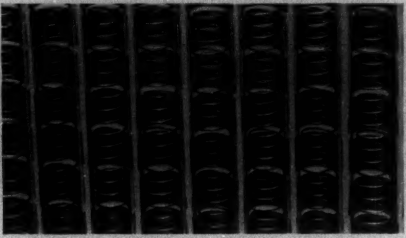

Fig. 10—Detailed Structure of the Grid

The exterior electrodes are pieces of metal foil cemented to the outside of the tube. The interior electrode is a long spiral of wire

As an aperture in the disk moves across the field, the observer, looking through at the neon lamp behind the disk, sees the aperture as a bright point. When the disk is rotating at high speed, the observer, owing to the persistency of vision, sees a uniformly illuminated area in the frame, provided that a constant current is flowing through the lamp. (The line structure that would otherwise appear in the field is largely eliminated by using apertures that slightly overlap in their paths across the field.)

The brightness of the neon lamp is directly proportional to the current flowing through it; and when a picture is being received, the lamp is operated directly from the received picture current. As a result of the system just described, there is at any instant, in the field of view at the receiving station, a small aperture illuminated proportionally to the brightness of a corresponding spot on the distant subject. Consequently, the observer sees an image of the distant subject reproduced in the frame at the receiving station.

Fig. 8 shows the external appearance of the disk type of receiver in which the images appear. The disk rotates inside of a rectangular cabinet and the observer views the image through the shielding window. The largest disk, three feet in diameter, gives a 2 in. by 2½ in. rectangular image. Each television receiver is also equipped with a telephone receiver and transmitter; and it is possible for the observer to both see and converse with a distant person at the same time.

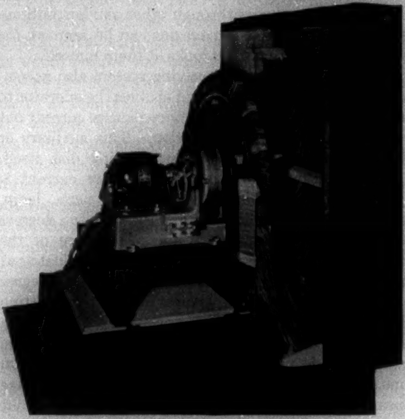

Fig. 11—Distributor and Wiring

High frequency current is distributed by 2500 wires to successive electrodes of the grid from 2500 bars on a high speed distributor

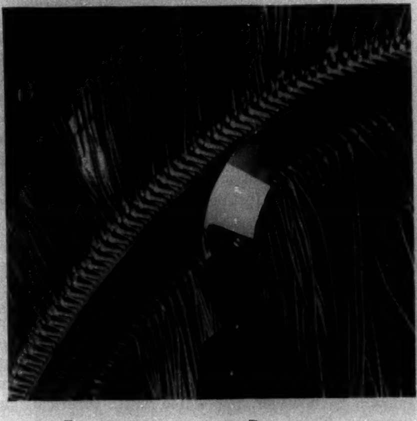

Fig. 12—Details of the Distributor

The bars are arranged in four rows each displaced with respect to the other three. The sliding brush is a strip of thin sheet metal

Considering the limited number of picture elements, a surprising amount of detail can be transmitted with this television system. A distant person can be seen and easily recognized and his motions can be plainly followed as he talks into a transmitter, turns the pages of a magazine, and goes through other similar motions. Large sized pictures in a magazine can be seen as the subject turns the pages and looks at them himself.

An auxiliary television receiving system also accompanies each transmitting set and enables the operator to see that he is sending a satisfactory picture current out over the channel of communication. This auxiliary or pilot picture is formed on the scanning disk itself. A small fraction of the outgoing picture current is tapped off and amplified to operate a neon lamp, which is placed behind the disk ninety degrees around from the scanning beam. An image of the subject may thus be seen on the scanning disk just as at a receiving station. To correct for the ninety degree phase shift, the spiral of apertures on the transmitting disk is continued by additional apertures a quarter of a turn beyond the starting point. The first turn alone of the spiral is used for scanning; and the last turn alone, to form the pilot image; consequently, this image appears exactly in frame. A small mirror on the front of the motor cabinet reflects this image to the operator and enables him to see the character of the picture that he is sending out over the channel of communication.

When it is desirable to present television images to a large audience a special grid type of receiver is used. The grid has the appearance of an illuminated screen and can be seen throughout a large auditorium. The image is not projected on the screen from a lantern like a moving picture; such optical projection would be inefficient and demand the electrical control of an impractical amount of light. The picture current itself is distributed by a commutator to successive elemental areas of a large neon lamp. This lamp, as shown in Fig. 9, consists of a single, long, neon-filled tube bent back and forth to give a series of fifty parallel sections of tubing. The tube has one interior electrode and 2500 exterior electrodes cemented along the back side of the glass tubing, Fig. 10. A high frequency voltage applied to the interior electrode and any one of the exterior electrodes will cause the tube to glow in front of that particular electrode. The glow discharge actually passes to the inside wall of the glass tubing and the high frequency current flows by a capacity effect out through the glass wall to the exterior electrode. The high frequency voltage is commutated to the electrodes in succession from 2500 bars on a distributor, Fig. 11, with a brush, Fig. 12, rotating synchronously with the disk at a transmitting station. Consequently, a spot of light moves rapidly and repeatedly across the grid in a series of parallel lines one after the other and in synchronism with the scanning beam at the transmitting station. With a constant exciting voltage the grid appears as a uniformly illuminated screen; but, when the high frequency voltage is modulated by the received picture current, an image of the distant subject is produced on the screen and his motions can be followed just as in the smaller images formed on a disk.

This method of presenting television images to a large audience permits a very efficient use of the available energy to reproduce a picture. The modulated current produces a glow discharge that exactly covers an elemental area of the picture on the screen and is viewed directly by the audience; consequently, there is absolutely no loss of energy after the picture current has been converted into light. In addition, each illuminated area of the screen responds to the picture current in the same manner; the exterior electrodes are exactly alike, and the use of a single tube assures the same pressure and purity of neon throughout the grid.

Fig. 9 shows such a screen set up for demonstration in an auditorium. A loud speaker is mounted just below the screen and it is thus possible for a large audience to both see and listen to a distant person at the same time.

SECTION II

The Television Signal Wave

So far it has been assumed that the electrical signal wave is perfectly transmitted between the conversion devices which transform the light variations into electrical variations and back again. Perfect transmission is, however, impossible with practical apparatus. There are certain requirements placed upon the generated signal wave by the characteristics of practical communication channels, and reciprocally certain demands are made upon a transmission system by the inherent nature of an adequate television signal. In addition to exploring these mutual requirements experimentally it is desirable to analyze them in such a way that, as far as possible, quantitative expression may be given to them. This expression in the case of the signal wave is best made by the methods of the Fourier analysis; considering the signal as made up of many sine wave components of various frequencies. The requirement on the signal may then be described in terms of these components and the requirements on the connecting transmission system in terms of attenuation and phase characteristics over a band of frequencies. These requirements will now be discussed as a basis for the subject matter of the succeeding section of this paper and of the following companion papers of this group on *Wire Transmission Systems for Television* and *Radio Transmission Systems for Television*.

The problems to be discussed may be conveniently considered under three headings:

(a) The Character of the Television Signal.

(b) Requirements upon the Signal Wave Set by the Characteristics of Available Transmission Channels.

(c) Requirements which the Transmission Channels must meet in order to carry Television Signals.

(a) *The Character of the Television Signal.* As we have seen, the voltage produced across the resistance in series with the photoelectric cell is a fluctuating unidirectional potential. The generated signal therefore has frequency components beginning at and including zero frequency. The value of the voltage at any instant is roughly proportional to the average reflected illumination at that instant from an illuminated spot whose size depends upon the apertures in the scanning disk. At any point where there is a sudden change in the tone value of the subject there will also be a sharp change in the generated voltage. It will, therefore, be seen that but for the limits of speed of action of the photoelectric cell and its connected circuits the generated signals would tend to include components over the whole frequency range up to infinity. Since it is possible to effectively transmit but a limited range of these components, the width and location of the frequency band necessary for the acceptable reproduction of a given size and structure of image must be determined. It is convenient to consider first the low frequency end of the band.

In the early experimental work it was soon found that in attempting to amplify the lower frequencies by the use of direct current amplifiers, unstable conditions of operation were reached before sufficient amplification was obtained to operate the receiving apparatus. Experiments were then made with resistance-condenser coupled amplifiers which showed that, if the efficiency of such an amplifier at the frequency equal to the number of pictures sent per second, was not more than about two T U below its average efficiency for the transmitted range, acceptable reproduction of the picture was secured together with stable operation of the amplifiers. When the low frequency cut-off of the amplifier was set much above this, spurious shadows were introduced into the picture. That there will be a critical lower frequency for the transmission of an unchanging scene is obvious since the Fourier series into which the signal may be analyzed starts with a constant term and the sine wave terms begin with the picture frequency and include a vast number of its harmonics. If the constant component (d-c.) is removed, the lowest frequency which remains to be transmitted is therefore the picture frequency.

The effect of removing the d-c. component of the signal can be qualitatively traced in a simple manner. Imagine three types of still pictures or scenes to be transmitted by the system. Let the first be quite dark in general effect and require fluctuations in the signal current of a certain average amount for its transmission. Such a picture would have a low direct current component. Let the second picture consist largely of medium grays and require about the same fluctuations in signal intensity for its delineation. Such a picture will have a medium direct current component. Let the third picture be very light in general effect with such difference in light and shadow as would require the same fluctuations in signal intensity as the other two pictures. Such a picture would have a relatively high direct current component. In passing through a resistance-condenser coupled amplifier, the signals for all three types of pictures would be changed from fluctuations superimposed upon direct current to alternating currents, all of about the same average value.

At the receiving end of the circuit the direct current component may be reinserted by superimposing the alternating current fluctuations upon a fixed value of direct current such as the steady state current in the last amplifier tube. This direct current component would give the best average results if it corresponded to that suitable for the gray picture, which would, of course, then be most nearly correctly reproduced. However, most of the detail of the dark and light scenes would also be reproduced though the tone values would be distributed about a medium gray. Fortunately a change in character of this kind has proved for the most part unimportant. Where it is important it can be taken care of very simply by providing, at the receiving end, means, either manual or automatic, for changing, in accordance with the type of scene being transmitted, the magnitude of the unidirectional current upon which the received alternating current is superposed, which amounts simply to the restoration of the direct current.

In the case of scenes which are changing, however, frequencies lower than picture frequency will in general be generated and their suppression may be expected to affect to some degree the perfection of the picture. In effect, these frequencies are analogous to changes in tone values in the case of still pictures and their elimination results in fluctuations in the apparent brightness of the image. This effect is not disturbing with many types of subjects, as for example in the reproduction of the face.

One remarkable result of not transmitting the direct current component of the signal in the case of the reflected beam method of scanning is that the television transmitting apparatus can be located and operated in a well-lighted room, for if this general illumination is constant it simply increases the direct current component of the signal. Similarly if the scene itself contains a source of steady light, this will be visible only in so far as it reflects the scanning beam.

Turning now to the upper part of the frequency range, experimental data on the highest necessary components were obtained by the use of circuits with low pass instead of high pass characteristics. With the television terminal apparatus operating at 17.7 pictures per second, it was found that a filter whose phase distortion had been corrected over practically all of its pass band of 15,000 cycles produced a degradation in image quality which was just detectable when the human countenance was being transmitted. Since the electrical terminal apparatus without the filters would efficiently transmit frequencies higher than this, the experiment showed either that frequencies higher than this were not present in the generated signal, that they were not effectively reproduced, or that they contribute little to the appearance of the image. This upper limit to the useful frequency range for this apparatus is rather lower than was anticipated from the initial survey but because of psychological factors (decreased discrimination of tone values for fine details, apparent improved resolution when the subject is moving, etc.) it proves satisfactory for television purposes.

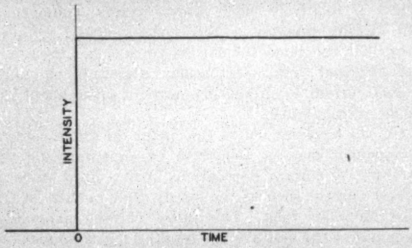

Fig. 13—Elementary Signal Change

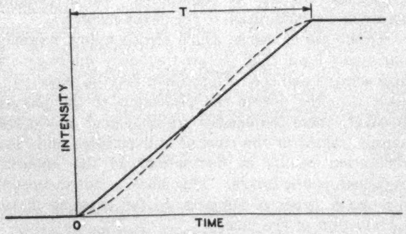

Fig. 14—Elementary Signal Change as Distorted by a Square Aperture

It is of importance, however, to know where the limitation in frequency range occurs in the apparatus and how it might be modified. Considerable information on this point is obtained by studying the nature of the distortion introduced by the aperture in the optical system and that introduced by frequency limitation in the electrical part of the system. It is convenient to consider them together as the type of distortion turns out to be similar for the two cases. This distortion may be considered most simply in relation to the type of signal corresponding to a sudden unit change in tone value at some point in the subject. With an ideal television system in which the instantaneous values of signal current are at all times proportional to the tone values of the points being scanned, the resulting signal would be represented by the graph of Fig. 13. Such a consideration involves no real loss in generality as any signal shape may be considered as the result of infinitesimal abrupt changes in intensity.

It is readily seen that if a square aperture passes with uniform velocity over a part of the picture having an abrupt change from dark to light the result is that we get a signal from the photoelectric cell which, instead of building up instantaneously, builds up linearly during a time, *T*, Fig. 14, which is the time required for the aperture to pass a given point.[1] The net effect is an apparent sluggishness in the response of the system. The dotted curve of Fig. 14 shows the integrated illumination passing through a circular aperture of a diameter corresponding to the same time, *T*, for the condition of Fig. 13. Due to the simpler analysis the discussion will be carried out in terms of the square aperture though the sluggishness due to the round one is seen to be slightly less.

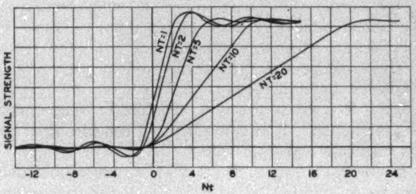

Fig. 15—Elementary Signal Change as Distorted by a Square Aperture and by Ideal Frequency Restriction

Now this kind of sluggishness in response is quite similar to that introduced in the electrical part of the system when the upper frequencies are cut out or not transmitted as efficiently as the lower ones. The effect of frequency limitation can be investigated theoretically in a fairly simple fashion if we make the ideal assumption that all frequencies are transmitted without distortion up to a cut-off frequency, f_c, and extinguished beyond it. In Appendix I, it is shown how the signal of Fig. 14 is affected by a frequency limitation of this type. We can then plot a set of curves as shown on Fig. 15 from which we can measure the total time of rise due to both the aperture and frequency limitation. The abscissa is the product of $N = 2\pi f_c$ and the time, t. Any one curve serves for a wide range of values of N and T as long as their product is the same. Call the new time of rise τ. Then we can plot a relation as on Fig. 16 between $N\tau$ and $N\,T$ from which we can draw conclusions as to the relative effects of aperture and frequency distortion.

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[1] This effect of aperture distortion was pointed out in the paper "Transmission of Pictures over Telephone Lines" by Ives, Horton, Parker, and Clark, *B. S. T. J.*, April, 1925.

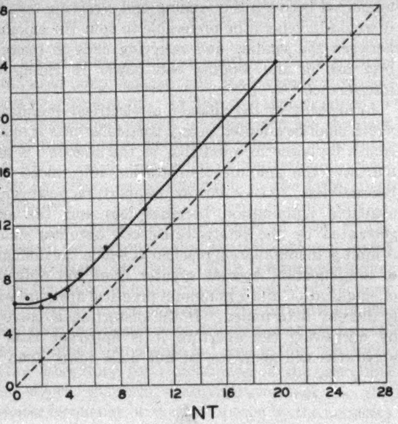

Fig. 16—Sluggishness Due to Distortion as a Function of the Aperture Width and Frequency Restriction

Below the knee of this curve we have approximately

$$N\tau = 2\pi$$

$$\tau = \frac{1}{f_c}$$ and the frequency cut-off determines the whole distortion.

Similarly above the knee

$$N\tau = N\,T + \pi$$

$$\tau = T + \frac{1}{2 f_c}$$ and the controlling influence is that of the aperture.

Unless one effect is much more easily remedied than the other, the knee of the curve appears a reasonable point to select for operation. At the knee $N\,T_k = 2\pi f_c T_k = \pi$ and $T_k = 1/2 f_c$. At this point the total lag is not much greater than that due to the frequency restriction alone and is $\frac{1}{f_c}$ or twice T_k. That is, at this point, the additional lag in the time of rise of signal due to the restricted frequency range is equal to that due originally to the aperture, though the additional lag due to the aperture is not much greater than that due to the frequency restriction alone. For a square aperture in a square picture of 2500 elements sent 16 times a second $T = \dfrac{1}{40{,}000}$ of a second, and $f_c = 20{,}000$ cycles at the knee of the curve. The point on the curve where the effect of frequency restriction introduces a sluggishness in following light changes comparable to that introduced by a square aperture is the same frequency as that arrived at as the upper limit to useful frequencies by considerations from still picture transmission, in the introductory paper by Mr. Ives. Its value is equal to one-half the number of picture elements.

It has furthermore been found possible to determine ideal electrical transmission characteristics or equivalent transfer admittances of circuits which produce exactly the same distortions as various types of apertures. While it appears impossible at present to construct a physical circuit which will produce such characteristics over the whole frequency range, the problem is not difficult if we limit ourselves to the most important frequency band. This is of interest as it points out the possibility of compensating for the effect of the aperture by putting in an electrical network with frequency transmission characteristics the inverse of those so determined. Within the range of important frequencies it turns out that the effect of the aperture is the same as that of a network which changes merely the relative amplitudes of the frequencies into which the picture signal may be analyzed. Neglecting constant multiplying factors, the relative variation over the frequency range for a square aperture is given by the factor $\dfrac{\sin T\,\omega/2}{\omega}$ and for a round aperture by $\dfrac{J_1\,(T\,\omega/2)}{\omega}$ where, as before, T is the maximum time for the aperture to pass a given point and J_1 is the Bessel's function of the first order. The derivation of these factors is given in Appendix II. On Fig. 17, Curve 1

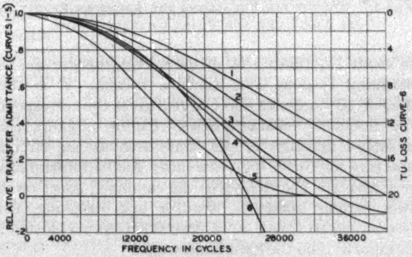

Fig. 17—Equivalent Transfer Admittance of Various Apertures

gives the relative values of the equivalent transfer admittance for the square aperture and Curve 2 for an inscribed circular aperture, both in case of a 50 line scanned picture which is square and sent 16 times per second. T then is equal to $\frac{1}{40{,}000}$ sec.

In the system as set up for demonstration the image is rectangular with the vertical and horizontal dimensions in about the ratio 5 to 4. The circular aperture is about 1¼ times 1/50 of the vertical height and the scanning is done 17.7 times a second. T is then 3.53×10^{-5} seconds and Curve 3 gives the corresponding frequency characteristics. Curve 4 shows that a square aperture of the same area as the circular aperture for Curve 3 gives a fairly good approximation to Curve 3. Curve 5 gives the combined effect of the two circular apertures, sending and receiving, corresponding to Curve 3. Curve 6 is Curve 5 plotted in terms of T U on the right hand scale.

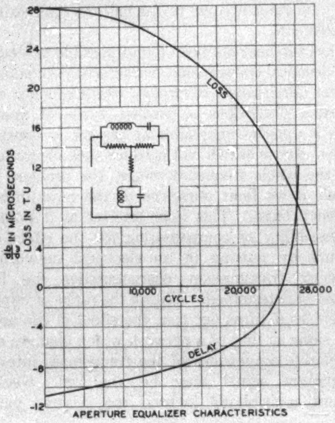

FIG. 18—CIRCUIT FOR EQUALIZING THE APERTURE EFFECT AND ITS AMPLITUDE AND PHASE CHARACTERISTICS

An inspection of this last curve indicates that this frequency attenuation characteristic of the aperture introduces a considerable loss at 15,000 cycles and leaves little of the signal components above 20,000 cycles. To see if an electrical circuit of characteristics inverse to those of the aperture would materially improve the resolution of the image, the circuit[2], which, together with its frequency characteristics, is shown in Fig. 18, was inserted between the sending and receiving amplifiers. It was designed to compensate for most of the aperture distortion and its phase distortion was made small below 20,000 cycles. On the fan-shaped test pattern of Fig. 19 a noticeable improvement was observed, the black and white angles being resolved closer to the tip of the pattern. In the case of faces the improvement appeared to be very little but could be detected in the slightly better definition of sharp narrow lines such as the frames of horn rimmed spectacles. When a system of considerable attenuation is employed between the sending and receiving terminals it would in general be preferable to split the equalizing between the sending and receiving ends to make the best use of the sending end power in riding over interference.

In arriving at the amount of electrical equalization which shall be adopted in any particular case it must of course be borne in mind that as the aperture is made narrower the amount of distortion introduced by it becomes less. As we narrow the aperture, however, the available illumination becomes less and the signal generated by the photoelectric cell becomes smaller. A limit is therefore soon reached at which the difficulties of amplification become greater than the difficulties of equalization and a minimum practical aperture width is thereby determined. If the distortion is corrected by narrowing the aperture it is apparent that the apparatus will generate, at but little lower than the correct relative efficiency, frequencies much higher than those thought necessary from the more general considerations in Mr. Ives' introductory paper. Decision as to the desirable frequency transmission band for the connecting communication channel would be no different for either method of reducing the distortion due to the aperture.

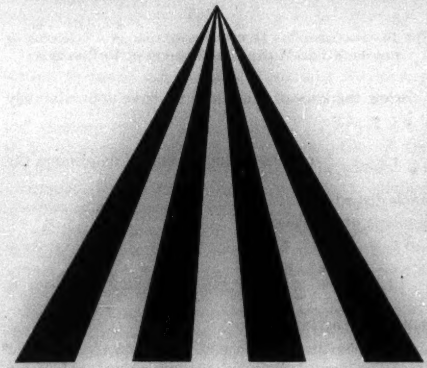

FIG. 19

In summary then, we may say that experiment and theory show that the lowest frequency essential to satisfactory results is the picture frequency, and the highest frequency required is approximately one-half the number of picture elements scanned per second.

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[2] This is a constant resistance type of corrective network or equalizer. See Chap. XVIII, "Transmission Circuits for Telephonic Communication," K. S. Johnson.

(b) *Requirements Upon the Signal Wave Set by the Characteristics of Available Transmission Channels:* The limitations upon the signal wave set by present available communication channels are:

1. The magnitude of the signal necessary to override the interference to which such channels are subject.

2. The frequency range which such channels can transmit.

The first of these is self-explanatory. It determines the required amplification and load capacity of the transmitting apparatus. In the companion paper on *Wire Transmission Systems for Television* are the data on interference and on permissible signal to noise ratio which were used in the design of the terminal transmitting amplifiers to be described in the latter part of this paper.

In considering the frequency range of lines, it was apparent in the beginning that the wire channel might include sections of cable. With exsisting loading systems for such cables a frequency range of not over 40,000 cycles appeared available. The terminal apparatus was therefore designed to deliver a generated signal whose essential components lay well within this limit, and the laboratory tests mentioned in the preceding section showed that this requirement was met.

A lower frequency limit was imposed by the necessity of a transformer for joining the transmission line to the terminal equipment. Fortunately it proved possible to design transformers as described in the final part of this paper in which this limit was at or below the essential low frequency limit found in the preceding discussion of the signal wave.

(c) *Requirements Which Transmission Channels Must Meet in Order to Carry Television Signals.* We have shown that a certain band width of frequency components is essential to the adequate reproduction of the image. This sets the frequency limits of the transmission channel which must be provided. It is essential, however, that within these transmission limits the channel should present a reasonably uniform attenuation, and that the phase relations should be fairly accurately maintained. The problem as presented to the transmission engineers of wire, radio, and terminal equipment for the recent demonstration was to meet the following requirements:

First, transmission must be provided for frequencies between about 10 cycles and 20,000 cycles.

Second, the amplitude frequency characteristics within this range should be uniform to about ± 2 T U.

Third, the phase shift through the range should be maintained so that the slope of its characteristic as a function of frequency is constant to ± 10 or 20 microseconds over all but the lowest part of the frequency range. There, about 50 times this limit was considered the maximum permissible.

These requirements were arrived at by considerations based on theory and experiments on television and analogy to similar requirements in telephotography. The first requirement follows directly from the discussion of the essential frequencies in the signal. The following paragraphs are intended to illustrate the significance of the remaining requirements.

As we have as yet no quantitative measure of the goodness of reproduction of the image, the matter of the second and third transmission requirements on received amplitude and phase characteristics over the frequency scale is one which had to be decided largely on the basis of the experimental results and judgment based on general considerations. We have already seen that the removal of the very lowest frequencies simply changes the tone value of the whole picture. It may be similarly reasoned that departures from the average efficiency of transmission in the lower part of the frequency range would result in the appearance of diffuse shadows or high lights. Likewise, it may be concluded that broad deviations from the average efficiency of transmission in the uppermost part of the signal frequency range would result in the accentuation or the fading out of the finer detail of the scene. Steep slopes in the amplitude-frequency curve would result in the superposition of oscillations upon signals representing sudden changes in intensity. To reduce these effects every reasonable effort was made to keep the variations in the amplitude characteristic with frequency as slight as possible, aiming to hold these characteristics for the separate parts of the demonstration system to within ± 2 T U or better.

In addition to transmitting the component frequencies with the same relative efficiency as regards amplitude, it is also particularly essential in television to send them through the system with small relative phase shifts; that is, with constant velocity or what is equivalent, a phase shift proportional to frequency. It has long been known in optical theory that the envelope of a group of waves of nearly the same wave length and nearly the same frequency may travel along with a "group velocity" somewhat different from the phase velocities of the component elements. If the system has but small departures from a flat amplitude-frequency characteristic and from a linear phase shift frequency characteristic, it can be shown that the time of group transmission or "envelope delay" is given by $d\,b/d\,\omega$,[2] the slope of the curve obtained by plotting the phase shift, b, for the system, against the angular velocity, $\omega = 2\,\pi f$. The time of transmission of a crest for any sine wave component of frequency $\frac{\omega}{2\,\pi}$ is, of course, given by $\frac{b}{\omega}$. If $b = c\,\omega$, $\frac{b}{\omega} = c$

and $\frac{d\,b}{d\,\omega} = c$. Then the phase and envelope times of transmission are equal and all frequencies as well as their group envelopes get over in the same time. If b is given in radians, $\frac{d\,b}{d\,\omega}$ is given in seconds. In general a knowledge of b as a function of ω is necessary and sufficient to determine the phase distortion. A knowledge of $\frac{d\,b}{d\,\omega}$ as a function of ω is not sufficient to determine all factors in signal distortion. It is, however, often easier to measure with the needed accuracy and in transmission systems such as have been used for still pictures and television has proven a useful index of phase characteristics.

After a preliminary estimate from experience with still pictures that the limit on $\frac{d\,b}{d\,\omega}$ should be ± 10 microseconds, an electrical network consisting of five sections of a simple lattice structure was used for testing the effect of phase distortion with television apparatus. This network introduced negligible amplitude distortion and a drift in the value of $\frac{d\,b}{d\,\omega}$ of 50 microseconds over the frequency range of 0 to 20,000 cycles. Its effect was perceptible in blurring the image of a face and it decidedly affected a sharp pattern of two parallel lines of such width and spacing as to be just within the resolving power of the apparatus. This variation of $\frac{d\,b}{d\,\omega}$ was about 2½ times greater than that postulated. Hence ± 10 microseconds was agreed on as a desirable limit for $\frac{d\,b}{d\,\omega}$, though it was felt that this limit might be exceeded by a factor of two in restricted parts of the frequency band.

When this network was combined with a filter the slope of whose envelope delay curve was in the opposite direction so that over the greater part of the frequency range the combined delay of the two circuits was constant and equal to 140 microseconds, this time delay effect was very graphically brought out. Every time the combined circuit was cut in, the undistorted received image jumped to a new position a little over 10 per cent of the width of the picture to one side in the direction of scanning.

To see why $\frac{d\,b}{d\,\omega}$ should be maintained at a constant value, consider two sharply defined details near together in the picture which would produce a variation in signal intensity with time as indicated in Fig. 20. Imagine each to be cyclically continued so that the small detail defines a frequency f_1 and the other defines a frequency f_2. It is then known from Fourier analysis that the frequency spectra of the two details are chiefly concentrated around the frequencies f_1 and f_2. If $\frac{d\,b}{d\,\omega}$ is appreciably different at the frequencies f_1 and f_2 for any part of the system, the two details will be displaced relatively to each other along the line of scanning and, in most cases, if this shift is appreciable, some change in the shape of the signal wave defining each detail results with further increase in the distortion. The same relative shift would occur if the narrow detail were located upon the broader one in which case, such a shift would be more apparent. It would seem reasonable to expect then that differences in the envelope time of transmission comparable to a whole picture element (about 28 microseconds in the demonstration apparatus) would be noticeable.

In most images very few details will have signal shapes, as in this special case, in which the frequency components are concentrated in narrow frequency bands. An abrupt change in signal strength, for instance, is represented by components distributed over the whole frequency range. We can imagine these frequencies divided into any arbitrary number of groups, each of which determines a wave form. When these wave forms are added together they will reproduce the original abrupt change in signal strength. If, however, they are sent through a system in which the envelope delays for the different groups are unequal, the individual wave forms will be relatively displaced and will no longer combine correctly. As a result the image is blurred. For some types of phase distortion the effect appears as an oscillatory transient following sudden changes in intensity.

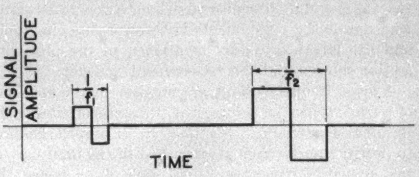

Fig. 20—Signal Details of Concentrated Frequency Spectrum for Illustrating the Effect of Envelope Delay

It was furthermore found by experiment that the limit of ± 10 microseconds was not necessary for the lower frequencies. Reference to the delay characteristics of the transformers described in the latter part of this paper shows that in the lower part of the frequency scale deviations from the nearly uniform value of delay at the upper frequencies appear of magnitude greater than 100 microseconds. When the signal was sent through these transformers, however, there was no observable distortion of the image. The requirements are therefore much more lenient at the low frequencies.

In the terminal apparatus the problem of meeting the above outlined phase transmission requirements was not a very serious one. The circuits involved are such that when a flat amplitude-frequency characteristic had been secured the phase distortion was also negligible.

SECTION III

Terminal Circuits for Sending and Receiving Television Signals

The preceding sections have discussed the methods by which an object, the image of which is to be transmitted, is made to control the time variations in a light, thus giving a luminous signal wave, and the means by which the image may be reconstructed with the aid of an electric signal wave corresponding to this initial luminous wave in its relative instantaneous amplitudes. Certain important relations between the characteristics of the signal wave and the resulting image have been pointed out. There remains the question of obtaining an electric signal wave suitable for long distance transmission and of providing for the control of the illumination at the receiving terminal by the electric signal wave as delivered by the transmission medium.

In the use of wire lines for television it is fortunately true that a suitably prepared open-wire circuit possesses a frequency range sufficient for the transmission of all the essential components of the signal wave. Details regarding the characteristics of the wire circuits are given in a companion paper by Messrs. Gannett and Green, from whose work are obtained data essential to the design of the terminal equipment. These data fix the power level at which the signal should be delivered to the line and the power level which will be available at the receiving end. When the transmission is by radio it is, of course, necessary to affect a frequency translation in order to secure a wave suitable for radiation and transmission through the ether. In this case, however, the radio system, which is described in a paper by Mr. E. L. Nelson, when considered as a whole may be conveniently taken as a system capable of the transmission of a signal wave occupying the same frequency range as that supplied to the wire circuits. In fact the design of the radio system is such that it may be used interchangeably with the wire line in so far as the remaining electrical terminal equipment is concerned.

The terminal circuits, then, fall into two groups: first, those used at the transmitting terminal for building up the wave controlled by the time variations in light to the power level required by the line and second, those used at the receiving terminal to bring the wave delivered by the line to the proper form for controlling the luminous sources from which the received picture is built up.

TRANSMITTING CIRCUITS

Starting with the photoelectric cell in which the initial luminous signal wave is converted to an electric signal wave, we are interested in the magnitude of various pertinent constants. The cell may be considered for our purposes as an impedance, the value of which is determined by the quantity of light reaching it. With no illumination at all this impedance is almost entirely a capacitance of the order of 10 m. m. f. When the cell is illuminated this capacitance becomes effectively shunted by a very small conductance which is roughly proportional to the square of the voltage between the electrodes. For a fixed potential the magnitude of this conductance is nearly a linear function of the illumination. With a suitable potential in series with the cell, then, there is obtained a current the amplitude of which is proportional to the quantity of light reaching the cell.

In order to connect the photoelectric cell to the amplifier there is introduced in series with the cell and its polarizing battery a pure resistance the voltage drop across which is used to control the grid potential of the first tube. It is desirable, of course, to make this resistance high in order to have available as much voltage as possible. Its value is, however, limited by two considerations. The added series conductance must not be so low that it appreciably disturbs the linear relation between the illumination and the total conductance of the circuit. The voltage drop must also be so small, in comparison with the total potential in the circuit, that the photoelectric cell operates at an approximately constant polarizing potential.

In view of the extremely small voltage of the electric signal wave as delivered by the photoelectric cell circuit, it is essential that great care be taken to prevent such interference as may enter the initial amplifier stages from approaching a comparable magnitude. The most troublesome sources of interference are electrostatic induction, electromagnetic induction, mechanical vibration, and acoustic vibration. By mechanical vibration is meant disturbances transmitted through the supports as the result of building vibrations and similar phenomena. By acoustic vibrations are meant impulses transmitted through the air which strike the several elements of the amplifier and cause motion which results in variations in their electrical constants. Electrical disturbances are reduced to a minimum by placing the amplifier as close as possible to the photoelectric cells, thereby keeping the leads short, which avoids electrostatic pick-up and also prevents the formation of closed loops of any appreciable size, thus avoiding electromagnetic induction. The amplifier is provided with a very complete electrical shield and both the shielded amplifier and the photoelectric cells are placed in a carefully shielded cabinet.

The tubes used, namely the so-called "peanut" tubes, are under ordinary conditions, remarkably free from any microphonic action. At the very low signal levels used, however, certain extra precautions have to be taken against this effect. In addition to lining the amplifier box with sound absorbing material the tubes themselves have been wrapped in felt and placed within a heavy lead case. This prevents such acoustic disturbances as reach the interior of the amplifier container from having any noticeable effect on the tube. The lead container is supported entirely by an elastic suspension and thus serves a dual function, as the heavy mass, supported in this way, is capable of little response to such mechanical vibrations as may be transmitted through the cabinet and the walls of the amplifier shield. With these precautions it has been found possible to make the effect of all external disturbances of about the magnitude of the thermal disturbances referred to in the first part of the paper.

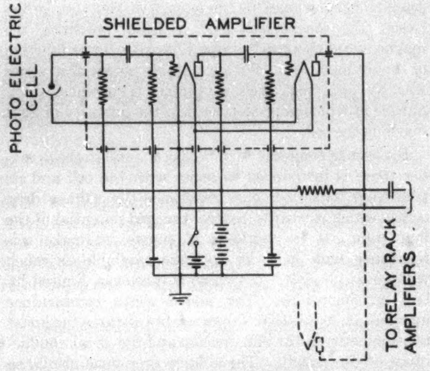

Fig. 21—Schematic of Vacuum Tube Amplifier Used with Photoelectric Cells

A schematic diagram of the amplifier tubes directly associated with the photoelectric cell is given in Fig. 21. Attention has already been called to the fact that the initial signal, that is, the time variation of the light reflected from the scanned object, contains a direct current component. The amplification of this direct current component is, as has been stated, out of the question in any amplifier intended for continued operation over long periods of time. The requirements as to the range of frequencies to be transmitted, as discussed in the preceding section, make it necessary to provide a circuit having practically uniform efficiency from 10 cycles to above 20 kilocycles. The relative phase shift of the several components must also be kept very small. In view of the large amplification and consequent large number of stages necessary, it has been thought impracticable to use transformer coupling between all stages as the aggregate frequency and phase distortion might well be greater than could be tolerated. The so-called resistance capacitance coupling has therefore been used.

The arrangement of the several photoelectric cells in their cabinet, as shown in Fig. 3, is such that one amplifier can be connected directly to two of the cells leaving the third to operate a second amplifier. The outputs of these two amplifiers are then connected in parallel to the common battery supply equipment shown at the bottom of the two vertical cells.

By the use two stages of amplification in the photoelectric cell amplifier, the signal is brought to such a level that it may be carried by suitably shielded leads to other amplifiers outside the photoelectric cell cabinet. This permits using the convenient relay rack form of mounting. The signal level is, however, still low and may be adequately handled in amplifier units which differ but little from those used with the photoelectric cell.

The remaining requirements placed on the amplifiers at the transmitting terminal are those set by the telephone line. One of primary importance is that which determines the amount of energy needed. In order that the signal wave shall be of such magnitude that any interference present in the line may be negligible in comparison, it is desired that the alternating current delivered by the final amplifier stage shall be at least 4 milliamperes into an impedance of 600 ohms. The energy to be supplied is, therefore, approximately 0.01 watts, which determines the choice of the last amplifier stage. To build up the signal to a value sufficient to operate this output tube it has been found that eight stages of the small sized tubes and one stage of greater load-carrying capacity must be used. The total amplification given by these ten stages is approximately 130 T U. It is through this known gain of the amplifiers that we get our only accurate quantitative data as to the magnitude of the initial signal wave. This comes out to be about 10^{-15} watts or, with a 100,000 ohm resistance in series with the photoelectric cell, the potential available at the first tube is roughly 10 microvolts.

The characteristics of the line also determine the means by which it shall be coupled to the final amplifier stage. In order to secure the proper impedance matching and to prevent the line from being unbalanced with respect to ground, it was felt desirable to use transformers if possible rather than to attempt the design of a tube circuit capable of meeting the requirements directly. The problem included both output and input transformers, and specified an amplitude-frequency characteristic constant to within ± 0.5 T U from 10 cycles to 25,000 cycles. The input coils intended for use at the receiving terminal had the additional requirement that a minimum of interference current should be induced in the secondary due to potentials between the line and ground. The success with which this problem has been solved is shown by the curves of Fig. 22. Curve 1 is the transmission characteristic of the output transformer which is designed to work between impedances of 2000 ohms and 600 ohms when connected between generator and load circuits having these values. Curves 2 and 3 show the effective transmission gain of transformers having voltage step-ups of 6.5 and 2.5 respectively, when used to connect the first stage of the vacuum tube amplifier to a 600 ohm generator impedance. The envelope delay curves for the output transformer and for the high ratio input transformer are given in Fig. 23. Photographs of the coils are given in Fig. 24. A large factor in being able to get coils of this type lay in the availability of permalloy for the core material. The output transformer is connected to the amplifier through a blocking condenser in order to avoid possible saturation in the core due to the passage of direct current.

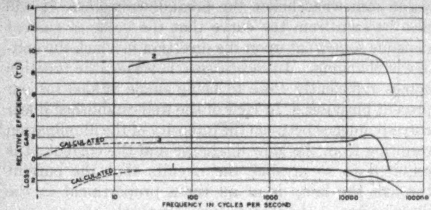

Fig. 22—Transmission Characteristics of Iron Core Transformers

1. Output transformer connected between impedances of 2000 ohms and 600 ohms

2. Input transformer having voltage step-up of 6.5 connected between 600-ohm line and vacuum tube.

3. Input transformer having voltage step-up of 2.5 connected between 600-ohm line and vacuum tube

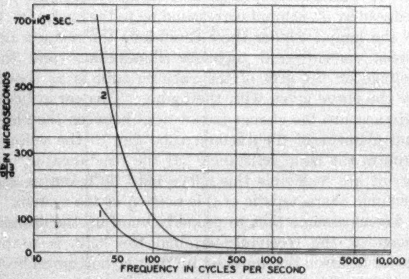

Fig. 23—Impulse Delay Characteristics of Transformers

1. Output transformer
2. High ratio input transformer

Measurements made on the several elements of the amplifier system have shown that its overall frequency characteristic is constant to within ± 2 T U from 10 to 20,000 cycles.

In an amplifier having as much gain as that just described it is apparent that a slight change in the potential of the power supply will cause a considerable change in the overall efficiency. Moreover variations in the intensity of the light source used with the scanning system will cause corresponding changes in the intensity of the initial luminous signal wave. To insure that the energy level supplied to the line is at all times of the proper magnitude a level indicator has been provided to permit continuous observations of the output of the amplifier. This consists of an amplifier-rectifier circuit so arranged that the space current of the last tube is a function of the alternating current voltage impressed on the first, being roughly proportional to the square of its amplitude. By means of a direct current milliammeter, therefore, it is possible to keep a very accurate check on the amplitude of the signal delivered to the line.

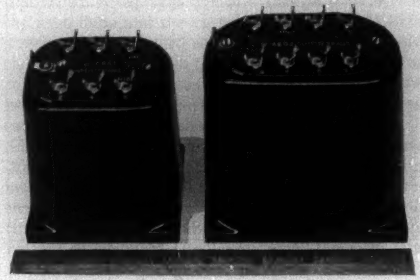

Fig. 24—Transformers Used for Coupling Amplifier Circuits to Long Distance Telephone Lines

Receiving Circuit

Coming now to the receiving terminal equipment we find that the signal wave which was delivered to the line at a power level of 10 milliwatts may, under some conditions, be reduced to a level 50 T U below this, or to 0.1 microwatt. It is, therefore, necessary first of all to provide amplification to bring the signal to a level where it may operate the circuits controlling the illumination from which the image is to be reconstructed. In view of the fact that several types of receiving equipment are to be operated and also since the signal may be derived from any of several sources, either wire line, radio, or local transmitting station, it is desirable to fix some one energy level as a reference point and to bring all signals to this value so that they may be supplied interchangeably to the several receiving systems. A convenient reference level is that already set as the proper input to a telephone line, namely 10 milliwatts. At the receiving terminal, therefore, amplifiers have been provided which are similar to the final stages used at the transmitting terminal. These include units containing the small sized tubes and terminate in units identical with that supplying current to the line except that the output transformer is omitted. The first stage is, as mentioned in the preceding section, connected to the line through an input transformer. The amplifiers associated with the several incoming signals are each provided with a level indicator of the type already described. These terminal amplifiers and the several receiving circuits are all terminated in jacks, exactly like telephone circuits, and it is possible, therefore, to connect any receiving machine to any desired transmitting station simply by patching the proper jacks together, exactly as telephone circuits are connected at the central office.

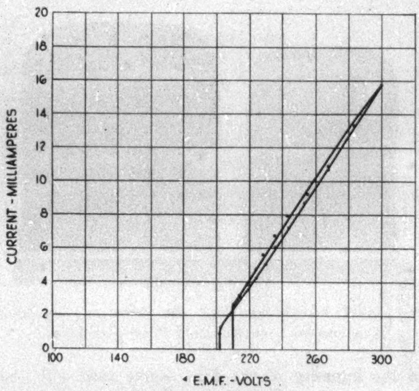

FIG. 25—CURRENT VOLTAGE CHARACTERISTIC OF TYPICAL NEON LAMP

Before describing the final stages of the amplifier circuits it is necessary first to examine the properties of the light source which is to be controlled. In the case of the disk receiving machines described in the first section of this paper it is recalled that a single neon lamp is used having a rectangular electrode the entire area of which glows at each instant with an intensity proportional to the intensity of the initial luminous signal. The current voltage characteristic of a typical neon lamp is given in Fig. 25. It will be seen that no current flows until the voltage across the lamp reaches the breakdown potential which, in the example shown, is about 210 volts. From this point on the current increases linearly with respect to voltages in excess of a value somewhat below the breakdown point. It will also be seen from the curve that the value of current depends somewhat upon the direction in which the voltage is changing. In most cases, however, the function comes sufficiently close to being single valued for our present purposes. In view of the well established linear correspondence between the intensity of the illumination resulting from the glow discharge and the current, it is required to so arrange the circuits that the current through the lamp is at all times proportional to the illumination at the transmitting terminal.

It will be recalled that the electric signal wave as transmitted through the various amplifier circuits differs fundamentally from the initial luminous wave in that the direct current component has been eliminated. It is necessary, therefore, to restore this component before the changes in light intensity at the receiving terminal will follow those at the transmitting terminal. The several factors entering at this point may perhaps best be examined in terms of an elementary circuit such as given in Fig. 26. In this case the neon lamp is connected in series with the plate circuit of a vacuum tube and its polarizing battery. The circuit may be considered for the present as equivalent to one in which the neon tube is replaced by an ohmic resistance and in which the potential of the polarizing battery is reduced by an amount corresponding to the back e. m. f. of the lamp. Under these conditions the relation between current—and therefore illumination—and the voltage on the grid of the vacuum tube is as shown by the curve given with the figure. This curve takes into account the change in potential between the plate and filament of the vacuum tube due to the voltage drop in the lamp resistance. If the reactances in the circuit are negligible this curve may be taken as the dynamic characteristic of this portion of the system.

Let us assume that to properly build up the desired image at the receiving terminal the light is to be varied between the limits set by the two horizontal lines *a* and *b*. It is apparent that two adjustments are necessary in the grid circuit. The amplitude of the impressed alternating current must be such that the difference between its positive and negative maxima is equal to the difference between the grid voltages corresponding to these currents. This is taken care of by suitable adjustments of the amplification. It is further necessary that the bias introduced by the grid polarizing battery be such that the positive and negative peaks coincide with these same values of grid voltage. Under these conditions the grid battery must be looked upon as supplying two absolutely distinct biases, one the bias for the tube and the other the bias for the signal. For example, if the signal wave as delivered to the grid circuit contained the original d-c. component properly amplified, it would be necessary to adjust the system so that zero current would be obtained with no impressed signal. To accomplish this the tube would require the negative grid bias E_{G3}. Variations in signal voltage would then be considered as taking place about this value of grid potential as the origin. Thus E_{G3} is the operating bias of the tube. To properly locate the signal wave, however, it is necessary to add the positive bias E_{G2}. It will be seen from the curve that this bias corresponds exactly to the direct current component which is to be restored to the signal. The sum of these two biases, obviously, gives the actual bias, E_{G1}, with which the tube is operated.

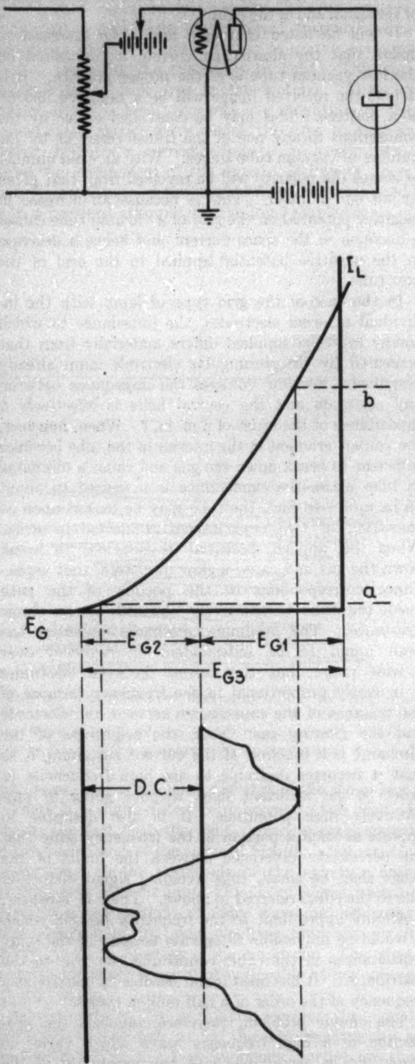

FIG. 26—CIRCUIT SCHEMATIC AND OPERATING CHARACTERISTIC OF NEON LAMP AMPLIFIER

In the circuit as shown the well-known curvature of the vacuum tube prevents us from obtaining a linear relation between the current through the neon lamp and the signal voltage. This condition may be overcome by a number of circuit modifications of which that shown in Fig. 27 is typical. Instead of connecting the neon lamp and the vacuum tube directly in series a resistance is provided across which is set up a potential, E_X, proportional to the current through it. Across this resistance is shunted the neon lamp and a biasing battery, E_B. The adjustment of this circuit is indicated by the curves shown. Curve *A* expresses the relation between the grid potential of the vacuum tube and its plate current. Curve *B* shows the relation between this same plate current and the voltage across the external resistance. When no current is flowing through the vacuum tube the potential of the biasing battery is insufficient to break down the neon lamp and no current flows through the circuit containing the neon lamp and the plate circuit resistance. As the current through the vacuum tube is increased from zero the total current flowing is that through the resistance branch. When, however, the potential drop across this resistance reaches such a magnitude that, together with the potential of the biasing battery, it is sufficient to break down the neon lamp, the latter will begin to draw current which thereafter increases linearly with further increases in the voltage, E_X, across the external resistance. The voltage across the neon lamp itself differs from that across the resistance by the amount of the battery E_B. The relation between the neon lamp current and the voltage across it, as given by Curve *C*, may therefore be plotted directly above the characteristic just discussed by displacing the vertical axis an amount corresponding to E_B. This amount is shown as E_{L1}. Here again we have two separate biases controlled by a single adjustment. The potential E_{L2} is fixed by the minimum plate current which can be taken from the tube without departing too seriously from the linear portion of the tube characteristic. It is, therefore, an operating bias of the circuit which is unaffected by any characteristic of the neon lamp. The latter, however, must be operated with a bias E_{L3} corresponding to its effective back e. m. f. As in the case of the grid circuit bias just considered, the bias E_{L1} actually introduced into the circuit is the difference between these two independently determined biases.

By projecting values of lamp current horizontally and plotting their intersections with vertical projections through the corresponding grid potentials on the vacuum tube characteristic we obtain Curve *D*, which expresses the relation between the instantaneous value of the signal and of the current in the neon lamp as derived from the characteristics of the several elements of the circuit. Inasmuch as the intensity of the illumination is proportional to the lamp current it will be seen that we have approached the desired linear correspondence between the instantaneous values of the signal and of the light.

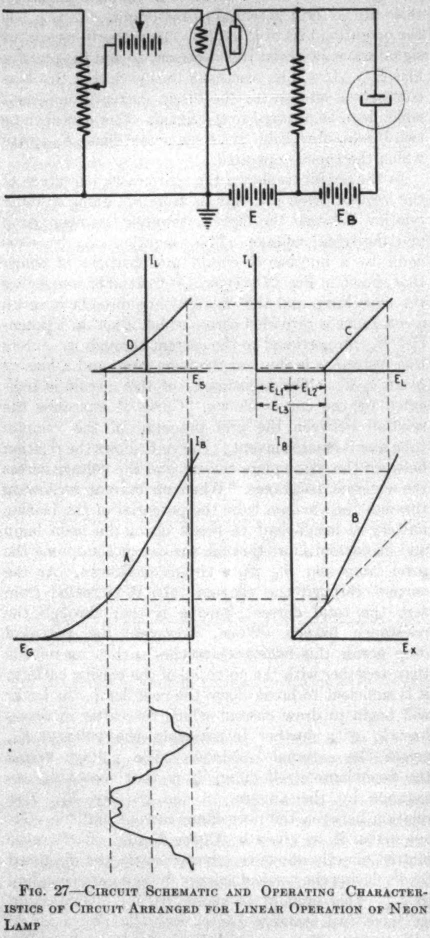

FIG. 27—CIRCUIT SCHEMATIC AND OPERATING CHARACTERISTICS OF CIRCUIT ARRANGED FOR LINEAR OPERATION OF NEON LAMP

It will be noted that care has to be exercised to insure that the alternating current as impressed on the last vacuum tube is of the proper polarity. If it is not, the received image will be a negative instead of a positive. This may be controlled either by the connections to any one of the transformers or by the number of vacuum tube stages. With an even number of stages the polarity will be reversed from that given by an odd number. This is because an increase in negative potential on the grid of a vacuum tube causes a decrease in the space current and hence a decrease in the negative potential applied to the grid of the next tube.

In the case of the grid type of lamp with the individual external electrodes, the impedance to which energy must be supplied differs materially from that presented by the rectangular electrode lamp already described. For low voltages the impedance between any electrode and the central helix is effectively a capacitance of the order of 6 m. m. f. When, however, the voltage gradient in the interior of the tube becomes sufficient to break down the gas and cause a discharge to take place, the capacitance is increased to about 15 m. m. f. In fact, the tube may be looked upon as consisting of two capacitances connected in series. When the applied potential is sufficient to break down the gas and cause a glow discharge, that capacitance corresponding to the portion of the path inside the tube is effectively shunted by an ohmic resistance. The minimum discharge potential has been found to be independent of frequency over a wide range, but the current between electrodes is inversely proportional to the frequency because of the presence of the capacitance between the electrode and the glowing gas. Now, the brightness of the discharge is a function of the current sustaining it so that it becomes desirable to use high frequencies in order to get sufficient light without going to prohibitively high potentials. It is also desirable to operate at such a portion of the frequency scale that the percentage difference between the limits of the range shall be small, thus avoiding signal distortion due to the effect referred to above. There is, however, a definite upper limit to the frequency beyond which it would be impossible to operate because of the stray capacitances in the cable connecting the grid to the distributor. It has been found feasible to operate at a frequency of the order of a half million cycles.

The circuit problem, therefore, involves the production of a high frequency wave which varies in amplitude in accordance with the amplitude of the received picture signal. The solution has been conveniently obtained by using a radio broadcast transmitter the voice frequency circuits of which have been so modified that the exteneded range of frequencies required might be handled with minimum distortion.

The envelope of the 500 kilocycle wave modulated by the picture signal, as shown in Fig. 28, is proportional to the signal amplitude plus a direct current biasing component of such magnitude that when the envelope reaches 160 volts the tube fails to light. This corresponds to a black area in the picture. When no picture signal is being received the amplitude of the unmodulated carrier wave causes the tube to light at average brightness, corresponding to the locally introduced d-c. component of the signal. It follows, then, that the amplitude of the unmodulated carrier is fixed, as in a previous example, by the joint requirements of two biases, that of the lamp and that of the signal bias.

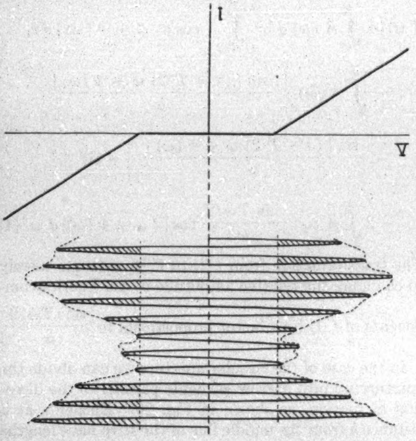

FIG. 28—DIAGRAMMATIC REPRESENTATION OF RELATION BETWEEN MODULATED HIGH FREQUENCY WAVE IMPRESSED ON GRID TYPE NEON LAMP AND LAMP CHARACTERISTICS. INTENSITY OF GLOW IS PROPORTIONAL TO SHADED AREA

There is a slight distortion inherent in this method due to the fact that the light, which is proportional to the shaded area of the curve of Fig. 28, is not strictly proportional to the amplitude of the envelope with respect to the 160 volt limit. This is, of course, because these peaks are portions of a sine wave and hence the time variation of the glow resulting from any given carrier cycle is a function of its amplitude. The effect is small, however, being most noticeable at low values of illumination.

In the case of the grid-lamp receiver the signal amplitude is adjusted, as for the disk receiver, by a potentiometer in the low frequency portion of the circuit. The carrier amplitude, however, is adjusted by varying the plate potential applied to the oscillating tube. The coupling to the lamp is made by connecting the central helix and the distributor brush across a portion of the condenser of the oscillating circuit.

The frequency-amplitude relation of the envelope has been made practically constant by employing resistance capacitance coupling in the signal input amplifiers, by providing extremely high inductance retard coils for the modulator—which is of the Heising type—and by inserting resistance in the oscillating circuit to provide sufficient damping. The relations between the original picture signal and the envelope of the high frequency wave, with respect to both amplitude and phase shift, were observed over the signal frequency range by means of a Braun tube and found to be satisfactory. The impedance of the connecting leads to the commutator was also measured and found to have a negligible effect on the frequency and damping of the oscillating circuit.

It has been found that there may be a lag between the time when the potential is applied to an electrode and the time when the gas breaks down. This is especially true following an interval during which there has been no discharge within the tube. Because of this those electrodes which are the first to be connected in any one of the parallel portions of the tube may fail to light. To overcome this effect a small pilot electrode is kept glowing at the left-hand end of each tube, thus irradiating the branch in such a way that the illumination of all electrodes follows immediately upon the application of potential. These pilot electrodes, which are obscured from view of the audience by the frame of the grid, are supplied by means of an auxiliary connection to the oscillator with a potential somewhat lower than that ordinarily impressed upon the picture segments.

Appendix I

The signal of Fig. 13 in the body of the paper may be represented as follows:

$$\left.\begin{aligned} f(t) &= 0 && \text{for } t < 0 \\ &= \frac{t}{T} && \text{for } 0 < t < T \\ &= 1 && \text{for } t > T \end{aligned}\right\} \qquad (1)$$

or by a Fourier integral in the form

$$f(t) = \frac{1}{\pi}\int_0^\infty d\,\omega \int_{-\infty}^{\infty} f(\lambda) \cos \omega\,(t-\lambda)\,d\,\lambda, \qquad (2)$$

where λ is an auxiliary variable of integration and ω is $2\,\pi$ times the frequency. To get the effect of sending this signal through a system which transmits all frefrequencies without phase or amplitude distortion up to a cut-off frequency f_c it is only necessary to replace the upper limit of the first integral sign by N where $N = 2\,\pi f_c$. Thus:

$$F(t) = \frac{1}{\pi}\int_0^N d\,\omega \int_{-\infty}^{\infty} f(\lambda) \cos \omega\,(t-\lambda)\,d\,\lambda.$$

Then from (1):

$$F(t) = \frac{1}{\pi}\int_0^N d\,\omega \int_0^T \frac{\lambda}{T}\cos\omega\,(t-\lambda)\,d\,\lambda$$

$$+\frac{1}{\pi}\int_0^N d\,\omega\int_T^\infty \cos\omega\,(t-\lambda)\,d\,\lambda$$

$$=\frac{1}{\pi\,N\,T}\{\cos N\,t-\cos N\,(t-T)$$

$$+N\,t\,[S\,i\,(N\,t)-S\,i\,(N\,t-N\,T)]\}$$

$$+\frac{1}{\pi}\Big[\,\pi+S\,i\,(N\,t-N\,T)\Big].$$

If we write $N\,t = x$, $N\,T = z$, and $\pi\,F\,(t) = y\,(x)$, then

$$y\,(x) = \frac{1}{z}\,\{\cos x-\cos(x-z)+x\,[S\,i\,(x)$$

$$-S\,i\,(x-z)\,]\,\}$$

$$+\frac{\pi}{2}+S\,i\,(x-z)\,,$$

where

$$S\,i\,(x)=\int_0^x\frac{\sin x}{x}\,d\,x.$$

A series of graphs of $y\,(x)$ for different values of the product $N\,T$ is given in Fig. 15 in the body of the paper. These are generalized curves, the time scale depending on the particular value of cut-off frequency used. From these curves we can get the additional lag in the time, τ, in the rise of these curves over the original time T in Fig. 14.

Appendix II

Let $f\,(t)$ be the instantaneous intensity of the picture, and let it be represented by a Fourier integral;

$$f\,(t)=\int_0^\infty A\,(\omega)\cos[\,t\,\omega+\Phi\,(\omega)\,]\,d\,\omega. \qquad (1)$$

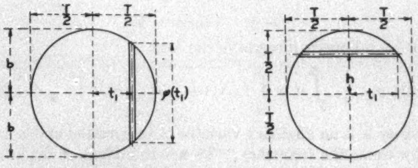

Fig 29 Fig 30

Analysis of the Aperture

Let T = time required for the aperture to pass a given point, Fig. 29.

Let $\varphi\,(t_1)$ be height of aperture at distance t_1 from its center.

The instantaneous amount of light passing through the aperture is

$$F\,(t)=\int_{t-T/2}^{t+T/2}\varphi\,(t_1)\,f\,(t_1)\,d\,t_1$$

$$=\int_{t-T/2}^{t+T/2}\varphi\,(t_1)\,d\,t_1\int_0^\infty A\,(\omega)\cos[t_1\,\omega+\Phi\,(\omega)]\,d\,\omega$$

$$=\int_0^\infty A\,(\omega)\,d\,\omega\int_{t-T/2}^{t+T/2}\varphi\,(t_1)\cos[t_1\,\omega+\Phi\,(\omega)]\,d\,t_1. \qquad (2)$$

In the case of the rectangular aperture

$$\varphi\,(t_1)=\text{a constant} \qquad (3)$$

and, except for a negligible constant factor,

$$F\,(t)=\int_0^\infty A\,(\omega)\,d\,\omega\int_{t-T/2}^{t+T/2}\cos[t_1\,\omega+\Phi\,(\omega)\,]\,d\,t_1$$

$$=\int_0^\infty A\,(\omega)\left\{\frac{\sin[\,(\,t+T/2)\,\omega+\Phi\,(\omega)\,]}{\omega}\right.$$

$$\left.-\frac{\sin[\,(\,t-T/2)\,\omega+\Phi\,(\omega)\,]}{\omega}\right\}d\,\omega$$

$$=2\int_0^\infty A\,(\omega)\,\frac{\sin T\,\omega/2}{\omega}\cos[\,t\,\omega+\Phi\,(\omega)]\,d\,\omega. \qquad (4)$$

The transformation from $f\,(t)$ to $F\,(t)$ amounts merely to changing the relative amplitude of the Fourier components of $f\,(t)$ by a factor proportional to $\dfrac{\sin\ T\,\omega/2}{\omega}$.

In the case of the circular aperture we can divide the aperture up into narrow elements parallel to the direction of motion, as shown in Fig. 30. Elements at a distance h from the middle line of the strip have lengths

$$2\,t_1=2\,\sqrt{T^2/4-h^2}. \qquad (5)$$

Each element considered as an independent *rectangular* aperture has the frequency characteristic

$$\frac{\sin t_1\,\omega}{\omega}=\frac{\sin\omega\,\sqrt{T^2/4-h^2}}{\omega}\,.$$

The mean of all of these elementary frequency characteristics is

$$\frac{1}{T}\int_{-T/2}^{T/2}\frac{1}{\omega}\sin\Big[\ \omega\ \sqrt{T^2/4-h^2}\ \Big]\,d\,h$$

$$=\frac{2}{T\,\omega}\int_0^{T/2}\sin\Big[\ \omega\ \sqrt{T^2/4-h^2}\ \Big]\,d\,h$$

$$=\frac{1}{\omega}\int_0^{T/2}\sin\left[\,\omega\,T/2\sqrt{1-\frac{4\,h^2}{T^2}}\right]\frac{2\,d\,h}{T}$$

$$=\frac{1}{\omega}\int_0^1\sin\Big[\ T\ \ \omega/2\ \sqrt{1-x^2}\ \Big]\,d\,x$$

$$= \frac{\pi}{2\,\omega} J_1\,(T\,\omega/2). \qquad (6)$$

where J_1 indicates a Bessel function of the first order. In place of the amplitude variation function $\dfrac{\sin\,(T\,\omega/2)}{\omega}$ for the square aperture, we have $\dfrac{J_1\,(T\omega/2)}{\omega}$ as such a factor. From the very nature of the physical processes under consideration it follows that this average value of the elementary frequency characteristics is effectively the frequency characteristic of the aperture as a whole.

Synchronization af Television

BY H. M. STOLLER* and E. R. MORTON*
Member, A. I. E. E. Non-member

***Synopsis.**—Synchronization of Television is the problem of holding two scanning disks so that their phase displacement is always less than four and one third minutes of arc. A 240 pole synchronous motor of the variable reluctance type is used as a basis. Coupled to it a direct curent motor carries the steady component of the load. Hunting is eliminated by a condenser in series with the two synchronous motors whose capacitance is slightly less than that required to tune the circuit.*

As the motor might lock into step in any of 120 possible angular positions, only one of which would give the proper phase relations, a two-pole motor, with only one locking position, was provided by tapping the armature of the direct current motor at two points and bringing out the leads to slip rings. This was used for synchronizing while the 240 pole motor, connected subsequently, held the close synchronism required. The disks rotate at 1062.5 rev. per min. which gives 17.7 cycles on the two-pole and 2125 cycles on the 240-pole motor.

For transmission the synchronizing current is attenuated to a level of 0.6 milliwatt and amplified at the receiving end. The 17.7-cycle current is an undesirably low frequency for transmission over telephone cables and so is used to modulate a 760-cycle current through a polarized relay. This is demodulated at the receiving end, where a polarized relay by interrupting a local battery current gives a rectangular wave which acts through vacuum tubes on the field of the direct-current motor.

* * * * * *

THE problem of synchronization involved in television transmitting and receiving equipment is similar in principle to any synchronous motor problem but the requirements are of such a special nature that it is necessary to employ unusual features of motor design and control circuits to secure the required results.

GENERAL REQUIREMENTS

At the transmitting end a scanning disk is employed containing 50 holes spirally spaced around the periphery of the disk rotating at a speed of 1060 rev. per min.[1] It is desired to rotate a similar scanning disk at the receiving end so that the hole through which the observer is looking at a neon lamp will be in a position corresponding to the hole which is transmitting light at the same instant at the transmitting end. Since there are 50 holes in each disk the holes will be spaced apart 7.2 degrees, thus 7.2 degrees of arc correspond at the receiving end to the width of the picture. Since the horizontal resolving power is approximately the same as the vertical (0.02 of the picture dimension) the arc occupied by a picture element is 0.02 X 7.2 or 0.144 degree. In order not to appreciably impair the quality of the picture, it is necessary to hold the synchronization within approximately ½ of the width of one element. This gives 0.144 degree divided by 2 or 0.07 degree as the requirement within which synchronization should be held. By way of comparison it might be mentioned that the angular twist in a length of 6 ft. of 1-in. steel shafting operated at rated load is of about the same order of magnitude.

An ordinary four-pole synchronous motor when operating at full load, unity power factor, has an angular phase displacement of about 20 electrical degrees between the impressed and back e. m. f. This corresponds to 10 mechanical degrees since the motor has two pairs of poles. If this motor is operated at constant load and the line voltage is varied the phase angle will decrease with increasing voltage or when the voltage is held constant and the load is varied, the phase angle will increase with increasing load. It is at once apparent therefore that the ordinary type of synchronous motor will not even approach the degree of precision required for the reason that any minute change in line voltage or load will cause variations in its phase angle of lag with respect to the impressed frequency of a far greater amount than 0.07 degree. Consider, however, a motor having 120 pairs of poles. Allowing 20 electrical degrees as the normal full load phase displacement this would be equivalent to 20 divided by 120 or 1/6 degree mechanical phase displacement. Even this amount is over twice the required permissible displacement of 0.07 degree. Since the variation of the phase displacement is the important factor and not the absolute amount of displacement, it is evident that if the line voltage and load are held reasonably constant a synchronous motor with 120 pairs of poles should be sufficiently precise.

Another requirement in addition to close phase synchronization is regulation of the acceleration or deceleration of the generator at the transmitting end. Such regulation is required due to the fact that an appreciable time is taken for the transmission of the synchronizing current a distance of 220 miles (circuit length) between New York and Washington. The velocity of propagation over the cable was approximately 19,000 miles per second while that of the picture on the open wire of 285 miles circuit length was about 175,000 miles per second, the corresponding times of transmission being 0.0116 second and 0.0016 second leaving a difference of 0.01 second approximately. Since the total permissible error in synchronization is

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[1] This speed was determined by transmission considerations and is discussed in the companion paper by Messrs. Gannett and Green.

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0.07 degree it is reasonable to allow 0.02 degree as error due to acceleration regulation. Let a be the acceleration in degrees per second per second. Substituting in the formula $s = \frac{1}{2} a t^2$ gives $0.02 = \frac{1}{2} a (.01)^2$ or $a = 400$ degrees per second per second or a little over one revolution per second per second. For comparison consider a ¼ hp. unregulated shunt motor. If the line voltage increases 10 per cent it will cause an increase in speed from 4 per cent to 8 per cent depending on the magnetic saturation in its field circuit. This increase in speed will take place in a half second or more depending upon the moment of inertia of the load. Thus the acceleration in the case of a 1060 rev. per min. speed would be much greater than one revolution per second per second.

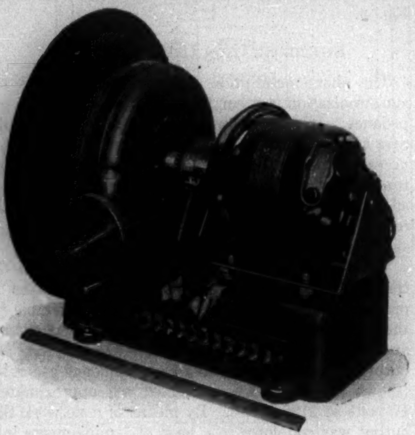

Fig. 1—Assembled Motor

Since this problem of speed regulation is a separate one from that of the synchronization the description of the regulating circuit is taken up later on.

Motor Design

In accordance with the phase displacement requirement as explained previously it was decided to build the synchronous motors with 120 pairs of poles, thus giving a frequency of 2125 cycles at 1062.5 rev. per min. which was the exact speed finally employed. For the sake of mechanical simplicity these machines were made of the variable reluctance type which gives one cycle per rotor tooth thus requiring 120 teeth. The variable reluctance construction also simplifies the coil arrangement, the machine having only eight armature coils instead of a separate coil for each tooth. Fig. 1 shows a photograph of the assembled motor and Fig. 2 an inside view of the stator and rotor.

In the preliminary experimental work two of these machines were directly connected (Fig. 3) permitting either machine to act as a synchronous motor loading down the other machine. Each machine was driven by a shunt d-c. motor having inherently poor regulation, the d-c. motors furnishing the power and the a-c. machines transferring the variations from one d-c. machine to the other to hold synchronism in a completely two-way system. As was to be expected, it was found that the motors hunted badly at a frequency of about four cycles per second. In other words, instead of holding within a fixed electrical phase angle of 20 degrees the receiving motor oscillated throughout a phase angle of about ± 20 electrical degrees. This, of course, made the picture wabble back and forth across the aperture and was therefore unsatisfactory.

The ordinary method of preventing hunting by means of copper bars embedded in the pole faces was not practical on account of the large number of poles and limited space. The hunting trouble was cured by employing a series condenser between the motors using a value of capacity somewhat less than that required to tune the circuit. A rigid analytical treatment of this anti-hunting circuit is beyond the scope of this paper but its operation depends in general upon the curvature of the tuning curve due to the variation of the inductance of the machine with phase displacement. Since the condenser operates on the total inductance of the circuit, it is desirable to make the natural periods of oscillation of the two motors different. Otherwise a decrease in the inductance of one machine may be accompanied by a simultaneous and equal increase in the inductance of the other thus leaving the total inductance unchanged and preventing the condenser from functioning. This was done by making one disk substantially heavier than the other.

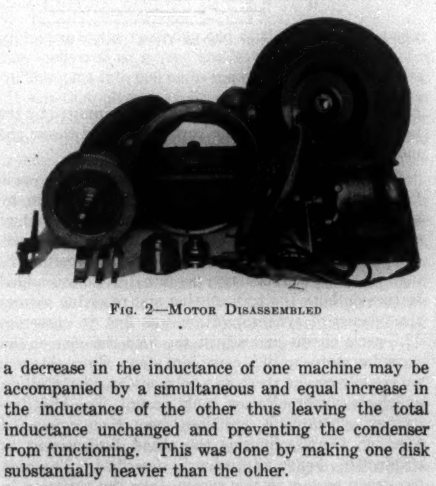

Fig. 2—Motor Disassembled

The series condenser also neutralizes the greater part of the internal reactance of the motors thereby increasing the steady state torque.

FRAMING OF PICTURE

There was still one unsatisfactory feature in this system in that the motor at the receiving end could interlock in any one of 120 different angular positions whereas in order to get proper framing of the picture it must be synchronized at a particular angular position. For example, if the disk at the receiving end is exactly 180 degrees out with respect to the disk at the transmitting end the observer will see the lower half of the picture on top; a dark space representing the dividing line between pictures and the upper half of the picture at the bottom. Similarly, if the disk is 90 degrees out at the receiving end the lower quarter of the picture will appear on the top and the upper three-quarters of the picture on the bottom. The disk at the receiving end may be brought into correct angular position by providing means for turning the entire motor through the necessary angle. It was found, however, that the rate at which the motor can be turned was limited by the fact that if it were rapidly turned it would throw the motor out of step.

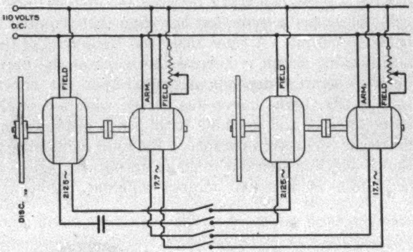

FIG. 3—SYNCHRONIZATION SYSTEM OVER SHORT WIRE LINE

As an aid to framing, therefore, a second two-pole low frequency interlock was added to the system by providing the d-c. motors on each end with a pair of slip rings tapped to two opposite commutator bars. The d-c. shunt motors thus acted as converters furnishing 17.7 cycles at 1062.5 rev. per min. With this added feature on both the transmitting and receiving motors the process of synchronization was first to close the 17.7 cycle circuit and adjust the field rheostat of the receiving motor until it came into step. Since this was a two-pole circuit there was only one angular position at which synchronization could occur. The high frequency synchronous machines were then connected together, thereby limiting the phase displacement to within 0.07 degree, as previously described. The high frequency motors in this system take the variation in load while the low frequency motor takes care of the steady constant component of load. Incidentally the addition of the low frequency synchronous motors greatly facilitated the synchronization of the high frequency motors inasmuch as it insured the proper initial speed. When the high frequency switch was closed there was merely a slight shift in phase angle to bring the receiving motor into step. The schematic circuit of the system thus far described is shown in Fig. 3.

SYNCHRONIZATION OVER LONG LINES

The above description explains the action of the synchronization system over lines of negligible impedance. In order, however, to secure similar results over a long distance telephone line or radio channel it is necessary to first attenuate the high and low frequencies to a power which can be safely applied to the transmitting end of the line and then amplify the power at the receiving end to restore it to the proper level. Fig. 4 shows the complete system employed.

While the high and low frequency machines on the transmitting end could have been designed so as to produce exactly the right power level it was desirable, for the sake of interchangeability, to build the transmitting and receiving motor equipment of the same size. The output from the transmitting high frequency generator (shown in Fig. 2) when untuned was approximately 17 volts at 2125 cycles. By means of a network this output was cut down to a level of 1 milliampere into 600 ohms impedance, the output impedance also being 600 ohms. This is a satisfactory level at which to transmit the high frequency, without inducing noise in adjacent wires in the telephone cables.

In the case of the low frequency interlock it was undesirable to attempt to transmit 17.7 cycles over a long distance line. The 17.7 cycles was therefore used to operate a polarized relay, the contacts of which modulated the ouput of a 760-cycle electro-mechanical oscillator[2] as shown in Fig. 6. In other words, the relay short-circuited the output of the oscillator alternate half cycles before application to the telephone line. Instead of using separate telephone pairs for the 2125-cycle and the modulated 760-cycle current, the two were combined by passing them through the line filter (shown in Fig. 7) thereby requiring only one pair for transmission of both frequencies. An identical network was employed for the radio channel. The problem of transmission of the synchronizing current is covered in the paper by Messrs. Gannett and Green and in the case of radio transmission in the paper by Mr. Nelson.

RECEIVING AND AMPLIFYING CIRCUITS

Passing over this part of the problem, therefore, assume that the synchronizing currents have been

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[2] Described in the Bell Laboratories *Record*, March 1927.

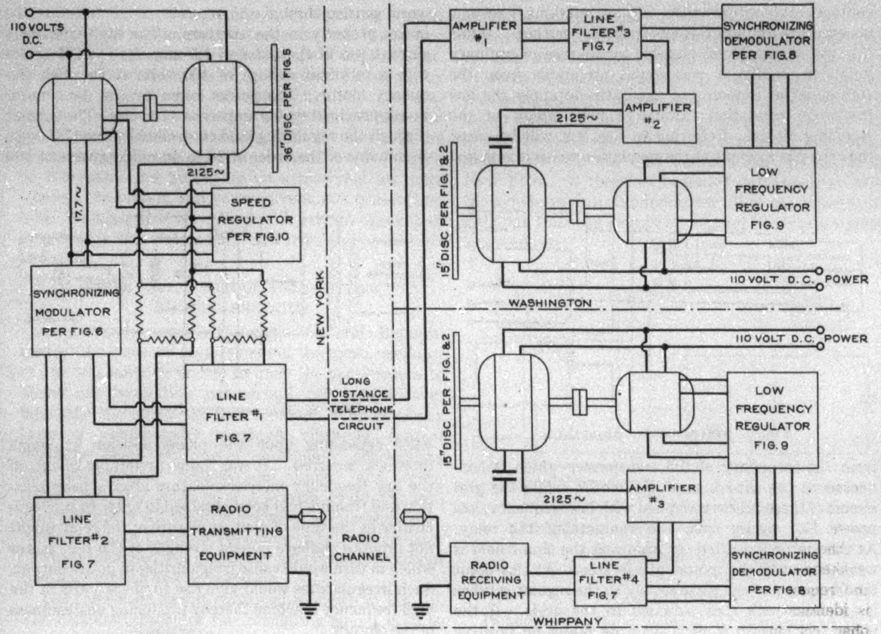
110 VOLTS D.C.
36" DISC PER FIG.5
2125~
17.7~
SPEED REGULATOR PER FIG.10
SYNCHRONIZING MODULATOR PER FIG.6
NEW YORK
LINE FILTER #1 FIG.7
LONG DISTANCE TELEPHONE CIRCUIT
LINE FILTER #2 FIG.7
RADIO TRANSMITTING EQUIPMENT
RADIO CHANNEL
AMPLIFIER #1
LINE FILTER #3 FIG.7
SYNCHRONIZING DEMODULATOR PER FIG.8
AMPLIFIER #2
2125~
15" DISC PER FIG.1 & 2
LOW FREQUENCY REGULATOR FIG.9
110 VOLT D.C. POWER
WASHINGTON
110 VOLT D.C. POWER
15" DISC PER FIG.1 & 2
LOW FREQUENCY REGULATOR FIG.9
AMPLIFIER #3
2125~
RADIO RECEIVING EQUIPMENT
LINE FILTER #4 FIG.7
SYNCHRONIZING DEMODULATOR PER FIG.8
WHIPPANY

Fig. 4—Complete Circuit of Synchronizing System

obtained at the receiving end of the line. This power was delivered at a very low level being about 0.3 of a milliampere into 600 ohms impedance, or 50 microwatts. It was then given a preliminary stage of amplification (Amplifier No. 1 Fig. 4), passed through the line filter No. 3 (Fig. 7) and separated into 2125 cycles and 760 cycles modulated at 17.7 cycles. The 2125 cycle component was then amplified by two stages of amplification (amplifier No. 2) ending in push-pull 50 watt tubes and applied to the high frequency motor. These amplifiers being of the standard type are not described. The terminal voltage on the output coil of the amplifier was made greater than that of the high frequency motor so that the power flow was normally from the amplifier to the motor. The anti-hunting condenser was retained between the amplifier and the motor.

In the case of the low frequency circuit the output from line filter No. 3 was received in the form of 760 cycles modulated at 17.7 cycles. This was passed through the demodulator (Fig. 8) which operated a polarized relay whose armature opened and closed its contacts at 17.7 cycles per second. The contacts of the relay provided square-wave low frequency current by interrupting power from a local battery source. On account of the limited power output which the vibrating

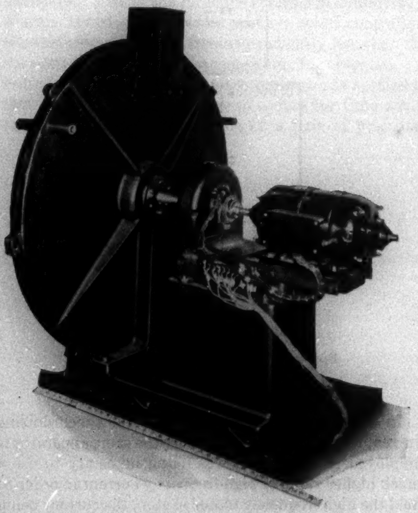

Fig. 5—Large Scanning Disk Motor

contacts could safely handle without sparking it became necessary to amplify this low frequency output. While this would have been possible by the use of ordinary amplifier circuits it was found preferable from the standpoint of economy of apparatus to apply the low frequency regulation through a field circuit of the receiving motor. Referring to Fig. 9 it will be noted that the plate circuit of the regulating tubes is supplied from the secondary of the transformer which is connected to the slip rings of the motor, while the grid circuit of these tubes is supplied with low frequency, low power 17.7 cycles from the contacts of the relay. As the motor is started up from rest the shunt field is weakened until the motor falls in step. At this point the frequency of the plate supply to the regulator tubes is identical with that supplied to the grids. If the phase relationship is such that the plates go positive at the same time that the grids are positive then the space current of the tubes is increased and the regulating field (which is an aiding auxiliary field) is strengthened, thereby preventing a further rise in the speed of the motor. In other words, for each combination of load and line voltage there is an equilibrium phase position between the plate and grid voltages at which the corresponding regulating field current maintains the speed at the desired value.

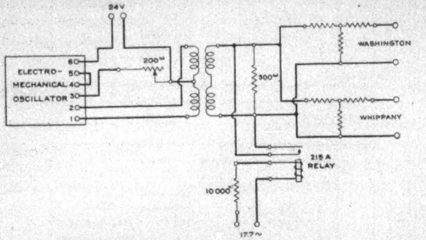

FIG. 6—SYNCHRONIZING MODULATOR

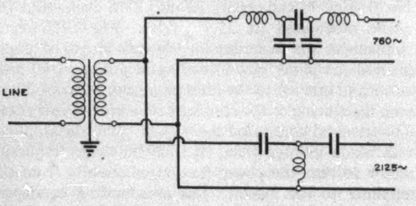

FIG. 7—LINE FILTERS FOR SYNCHRONIZING FREQUENCIES

MOTOR OPERATION

In actual operation the procedure was to first synchronize on the low frequency, and then on the high frequency circuit. The precise framing of the picture was then adjusted by rotating the motor by means of worm gearing through the necessary angle to center the image properly in the aperture. The high frequency current was of the order of 1.5 amperes at 2125 cycles with a terminal voltage of 100 volts at the high frequency motor. The power taken by the d-c. motor was approximately 0.8 ampere at 110 volts. The current through the regulating field controlled by the 17.7 cycle circuit was of the order of 20 to 40 milliamperes at 100 volts depending upon the phase position at which interlock occurred. It was found preferable to cut off the low frequency interlock feature after synchronization and framing had been obtained in order that irregularities in the time of contact closure of the relay might not produce changes in field strength of the d-c. motor which in turn would cause irregularities in power output. Such irregularities would give rise to phase shifts in the high frequency machine thereby producing unsteadiness of the picture.

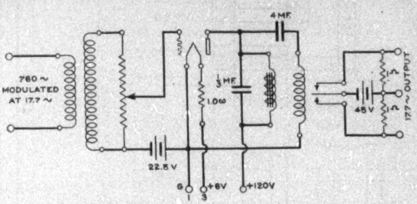

FIG. 8—SYNCHRONIZING DEMODULATOR

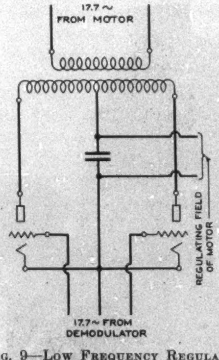

FIG. 9—LOW FREQUENCY REGULATOR

OPERATION ON RADIO CHANNEL

In the case of transmission of the synchronizing current by radio instead of by wire the same apparatus is employed except that it was found necessary to use a much higher value of high frequency current in order to hold the high frequency motor in step, the current being approximately 4 amperes as compared to 1.5 amperes in the case of the other motors. This greater current was found to be necessary in order to hold the motor in step within the necessary phase angle of displacement, in spite of various types of interference picked up by the radio receiver, and associated circuits. This was mainly inductive interference from the picture and speech transmission sets arising from the fact that the synchronizing current was transmitted from New York to Whippany and picked up on a receiving set there whereas the picture and voice current was transmitted from Whippany to New York. A certain amount of interference was also encountered from ship spark sets and static.

SPEED REGULATION OF TRANSMITTING MOTOR-GENERATOR

As previously explained under "General Requirements" the essential requirement of the speed regulator at the transmitting end is to limit the acceleration to about one revolution per second per second, over intervals as small as 0.01 second. The ordinary type of centrifugally operated vibrating contact regulator keeps the motor continually accelerating and decelerating between an upper and lower speed limit and while such a system could theoretically be employed if the flywheel were made large enough, it was obviously preferable to employ a type of regulator in which the speed was inherently held constant without such acceleration and deceleration.

The regulating circuit employed is shown in Fig. 10. The complete theory of this regulating circuit is to be covered in another paper to be presented before the Institute. Briefly the principle consists in employing a sharply tuned circuit as the primary speed controlling element resonating at a frequency slightly less than the frequency at which the machine is operated. A voltage from the high frequency generator is applied to this tuned circuit and thence to a detector tube which in turn operates on the grids of a pair of push-pull regulator tubes; these tubes controlling an auxiliary regulating field winding on the motor. The circuit also contains anti-hunting means, the theory of which will be given in the later paper. Instead of applying this regulating circuit to the small 15 in. scanning disk motor shown in Fig. 3, it was decided on account of its greater flywheel effect to use the large 36 in. disk shown in Fig. 5 which was used for receiving the picture at New York. It therefore became the transmitter from the synchronizing standpoint for all of the other units although from the picture standpoint the big disk acted as a receiver.

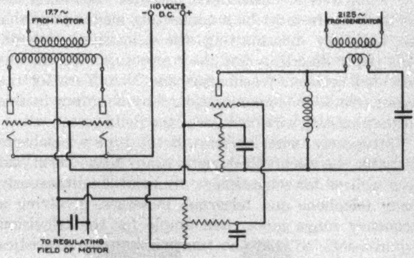

FIG. 10—SPEED REGULATOR

LOCAL STATIONS

In addition to the stations at Washington and Whippany there were three local stations in New York employing similar high and low frequency synchronous motors with 15 in. disks. These were controlled in the same manner except that first stage of amplification and the line filters were omitted. One station was employed for monitoring purposes, another operated a local transmitter, while the third operated the big grid receiver seen by the entire audience.

Wire Transmission System for Television

BY D. K. GANNETT* and E. I. GREEN*
Associate, A. I. E. E
Associate, A. I. E. E.

***Synopsis.**—This paper deals with the transmission problems which were met and solved in connection with providing wire circuits from Washington to New York for thet elevision demonstrations which took place on April 7, 1927 and following. For transmission of the television images a single transmission channel was set up combining the frequency ranges usually assigned to telegraph, telephone, an dcertain carrier channels. The special line requirements were met so successfully that the television images transmitted from Washington were indistinguishable from those transmitted locally.*

INTRODUCTION

A SYSTEM of television, to be worthy of the real meaning of the name, must be capable of operation over a considerable distance. Spanning this distance, there must be a connecting medium suitable for faithfully transmitting the television currents. This paper describes how the connecting medium was provided between Washington and New York for the recent television demonstrations,[1] by adapting to this purpose existing wire facilities of the Bell System.

Fortunately, wire facilities of the type which were available between Washington and New York had been utilized for some time to transmit simultaneously many telephone and telegraph messages, involving a frequency range more than ample for the television requirements, so that the transmission characteristics of the lines throughout the necessary range of frequencies were well known. The matter of providing a suitable channel to carry the television currents consisted, therefore, in throwing together the frequency ranges which had heretofore been utilized for providing a number of separate telephone and telegraph channels. In addition to providing this very wide band communication channel it was necessary to apply special distortion-correcting networks so that the overall channel would possess proper characteristics and also to take care to avoid introducing disturbances due to such things as line irregularities, noise, etc.

Due to the perfection of the transmission methods which were utilized, it was found that when the circuit was first established, in accordance with the requirements which had been deduced, the television images transmitted from Washington were indistinguishable in quality from those transmitted locally, this result being secured without any deviation from the adjustments which had been worked out in the original design.

REQUIREMENTS

General. The ideal requirement for a transmission line for television, or for that matter any other purpose, is, of course, that it introduce no distortion whatsoever, in which case there could be no question but that the television images obtained in the receiving apparatus after transmission over the long distance line would be identical with the image obtained with the transmission only over a distance of a few feet. Practical transmission lines, however, tend to introduce a certain amount of distortion and the less the allowable distortion which is specified the greater will be the cost of providing a proper line. Before going ahead with the matter of engineering the line required to transmit the television currents from Washington to New York it was, therefore, first necessary that the requirements be set. The requirements were made more severe than strictly necessary in cases where they were easy to meet.

Frequency Range. In any system for the electrical transmission of intelligence, the required frequency range is, in general, proportional to the speed of transmission. In the case of picture transmission or television, the speed of transmission may be expressed in terms of the number of picture elements which must be transmitted per second, where a picture element is the smallest unit area which it is intended to be able to distinguish in the received picture from its neighboring unit areas.

When the picture currents are transmitted in the most efficient manner the frequency range necessary is approximately equal to half the number of picture elements which must be transmitted per second. A simple way of seeing this is to realize that as the picture elements are transmitted in sequence, the greatest possible rate of variation of detail is obtained when alternate picture elements are black and white. A complete cycle corresponds in this case, therefore, to the time interval required to transmit two picture elements.

According to this relationship this particular television system in which about 40,000 picture elements per second are transmitted should require a frequency range of approximately 20,000 cycles. As a matter of fact it was found by a laboratory test that due to certain characteristics of the apparatus a frequency range as great as this was ample, just detectable distortion being introduced in the reproduction of the human face when the range was narrowed to about 14,000 cycles. In providing the line circuit, however, extending the frequency range to 20,000 cycles involved so little

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*Both of the American Telephone and Telegraph Company, New York, N. Y.

[1] *Television*, H. E. Ives; *The Production and Utilization of Television Signals*, F. Gray, J. W. Horton, and R. C. Mathes; *Synchronization in Television*, H. M. Stoller and E. R. Morton.

Presented at the Summer Convention of the A. I. E. E., at Detroit, Mich., June 20-24, 1927.

difficulty that it was decided to provide this very liberal frequency range.

In the particular television system which has been described the very low frequencies (below about 10 cycles) are suppressed. It was, therefore, not necessary that the line transmit these very low frequencies. The frequency range which the line should transmit was accordingly set as 10 cycles to 20,000 cycles.

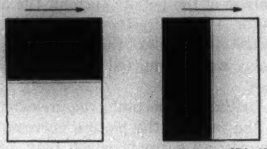
Attenuation. Referring to still picture transmission, it has been found that variations of attenuation with frequency of several transmission units do not appreciably impair the quality of the picture. Since no great difficulty was anticipated in meeting closer limits, however, it was decided to set the limits for the variation of attenuation with frequency at ± 2 T U within the frequency range of 10 to 20,000 cycles.

Phase Characteristics. A characteristic of wire lines, whose importance has been increasingly realized in recent years, is their phase characteristic. In speech transmission, transients due to unequal velocity of the different frequency components have been found to be an important consideration on some types of lines. In picture transmission and television, also, it is important that this phase distortion be controlled, as otherwise the image might be blurred due to the arrival of the various frequency components at different times. The type of transient which has been found to impair the quality of pictures is the type which is relatively rapid and the aim has been to make the phase characteristics such that those transients would be small.

The requirement with respect to phase for distortionless transmission is that $\frac{\beta}{\omega}$ be a constant where β is the phase change in radians for the entire circuit, and ω is equal to 2π times the frequency. $\frac{\beta}{\omega}$ is known as the "phase delay" or the steady-state time of transmission. $\frac{d\beta}{d\omega}$ is the time required for the transmission of the envelope of a wave whose components center closely about the frequency $\frac{\omega}{2\pi}$ and it will be referred to as the "envelope delay." Since it is more convenient to measure the envelope delay the requirements were set up in terms of this quantity. When $\frac{\beta}{\omega}$ is constant it is evident that $\frac{d\beta}{d\omega}$ is also constant. While the converse of this is not in general true, the conditions as actually encountered were such as to permit its use as a measure of the small variations involved.

The envelope delay characteristics of a number of circuits, which have been found to give varying degrees of transient on still pictures, have been measured. Also data were available from tests of picture transmission through filters and other networks whose delay characteristics were known. From these various data, the permissible deviations of the delay characteristic for still picture transmission were determined, and dividing these figures by 50, the ratio of the rate of transmission in picture elements per second in the two cases, the limits for the television circuits were obtained. In this way it was decided to attempt to keep within ± 10 microseconds, if possible, with outside limits of ± 20 microseconds. Check tests of these limits were made with the television apparatus in the laboratory by transmitting the currents through various known networks, and noting the effect on the received image.

Unlike the attenuation requirements, the delay requirements for television are not the same over the entire frequency range, but are much more lenient in the lower frequency range, as was shown by experiments in the laboratory. A physical picture of the reason for this may be obtained by reference to Fig. 1.



ARROWS SHOW DIRECTION OF MOTION OF LIGHT SPOT

A B

FIG. 1

Fig. 1A shows a picture placed in position before the sending machine, consisting of a piece of cardboard the same size as the image-area which can be transmitted, the upper half of the cardboard being colored black, while the lower half is white. As has been explained in the paper by Messrs. Gray, Horton, and Mathes, the picture is scanned by a spot of light which moves from left to right in successive lines, tracing 50 horizontal lines across the picture in one-sixteenth of a second. The first 25 of the lines lie on the black and the remaining 25 on the light part of the picture. The process is repeated 16 times per second, each repetition of 50 lines giving one complete cycle of black and white. The frequency components in this case are multiples of 16 cycles. A transient which blurs the picture outline over a given number, n, of picture elements (downwards) corresponds to a time interval equal to the time of tracing n lines, *i. e.*, $\frac{n}{800}$ second.

Now consider Fig. 1B. Here the picture has been rotated 90 degrees. In this position, a complete cycle of black and white is obtained with each line instead of with each 50 lines. The frequency components in this case are multiples of 800 cycles and bear the same relations to 800 cycles as the components spoken of above bear to 16 cycles. A transient which blurs the picture outline *n* picture elements (horizontally, this time) corresponds to a time interval of *n* forty-thousandths of a second. Evidently the delay requirements are 50 times more lenient in the former case than in the latter so that the delay requirement at the highest frequencies, which determine the fine detail in the direction of scanning, is 50 times as severe as at low frequencies, which determine the fine detail in a direction perpendicular to the direction of scanning.

In the still pictures referred to, the transients extended in the direction of travel of the light spot and there were no transients analogous to those discussed here in connection with Fig. 1A. For this reason the delay limits determined from still picture transmission are the ones which apply to the higher frequencies. For the lower frequencies the requirements are obtained by multiplying the high-frequency requirements by 50. For these reasons, together with the result of a Fourier analysis of the picture current, the limits were set at ± 10 or ± 20 microseconds from 400 to 20,000 cycles. Below 400 cycles, the departures from the constant delay were permitted to be ± 500 or ± 1000 microseconds.

Noise. Another important requirement is that relating to the ratio of the picture currents to the extraneous interfering currents which may arise in the line from power induction and other sources. Early experience with the television apparatus showed that considerably more noise was permissible in the case of television than in the case of still picture transmission so that in this case comparison with the still picture transmission would result in an unduly severe requirement. This is thought to be explained by the fact that in the case of television the pictures are flashed before the eye 16 times per second and the effects of the extraneous currents occur on successive flashes in different positions, so that defects of one flash are corrected on the next.

A set of experiments was performed from which it was determined that if the ratio of average picture currents to average noise currents exceeded about 10 the results were satisfactory. In order to assure considerable margin above this figure, it was decided to make the average television current to be transmitted into the line 4 milliamperes.

Echoes. If two paths exist by which the currents may travel from the sending point to the receiving point, the length of the two paths being different, a double image will be produced on the received picture, forming what may be termed visual echo. In the case of telephone lines, the echoes may exist on account of reflections between impedance irregularities in the circuit so that the currents arrive at the receiving point both by way of the direct transmission path and by way of a transmission path which includes an extra loop between two irregularities. If the echo is not greatly attenuated with respect to the main transmission, the result may be quite disturbing on the received picture. It has been found by experiment that the echo is too weak to be seen if it is more than 25 T U weaker than the main current and, accordingly, care was taken in setting up the New York-Washington circuit to avoid introducing echo paths of lower equivalent than this.

GENERAL CHOICE OF METHOD

Two general methods are possible for transmitting the currents over the line circuits. One method is to transmit the currents directly without change of frequency. This method involves the transmission of the currents of the frequency range determined upon above, namely, from about 10 cycles to about 20,000 cycles per second.

The other general method is the carrier method, in which the television currents modulate a carrier current of suitable frequency and are thereby moved to another portion of the frequency spectrum prior to transmission over the line. At the receiving end of the line the carrier currents are then restored to the original frequencies of the television currents.

Several different schemes of carrier transmission are possible. The simplest is to modulate a carrier with the television currents and to transmit both side bands. This has the disadvantage of requiring the transmission of twice as wide a frequency range as that occupied by the original television currents. Another scheme is to transmit a single side band. A third possible scheme is to transmit both side bands for the lower frequencies and only one side band for the higher frequencies.

One advantage to be secured by the carrier method is that it lessens the severity of some of the line problems through avoiding the transmission of very low frequencies over the line circuit. At these frequencies the amount of noise found on lines is usually considerably greater than at the higher frequencies.

After weighing the relative merits of the carrier and direct transmission methods it was decided to make use of the latter because of its simplicity. An important factor in this decision was the successful development, for use in connecting the apparatus to the lines, of transformers providing adequate transmission of the entire frequency range from 10 cycles to 20,000 cycles.

ARRANGEMENTS FOR TELEVISION CIRCUITS

Line Layout Between New York and Washington. The layout of the wires between New York and Washington is shown in Fig. 2. The circuit over which the waves actually carrying the pictures were transmitted (marked Picture Circuit) consisted principally of a pair of copper wires 165 mils in diameter. At a number of places on the route the circuits were carried in cable as indicated in the figure. The total length of the television circuits was about 285 miles, of which 8 miles consisted of cables and the remainder of open wire.

Transpositions. As the circuits employed were originally designed for voice-frequency operation only, except for a section at the New York end, it was necessary to add transpositions to them to prevent interaction with adjacent circuits at the high frequencies involved in the television transmission. The high-frequency currents were thus prevented from passing over into the adjacent circuits which would have resulted in irregularities in the attenuation, line impedance, and phase shift characteristics of the circuit.

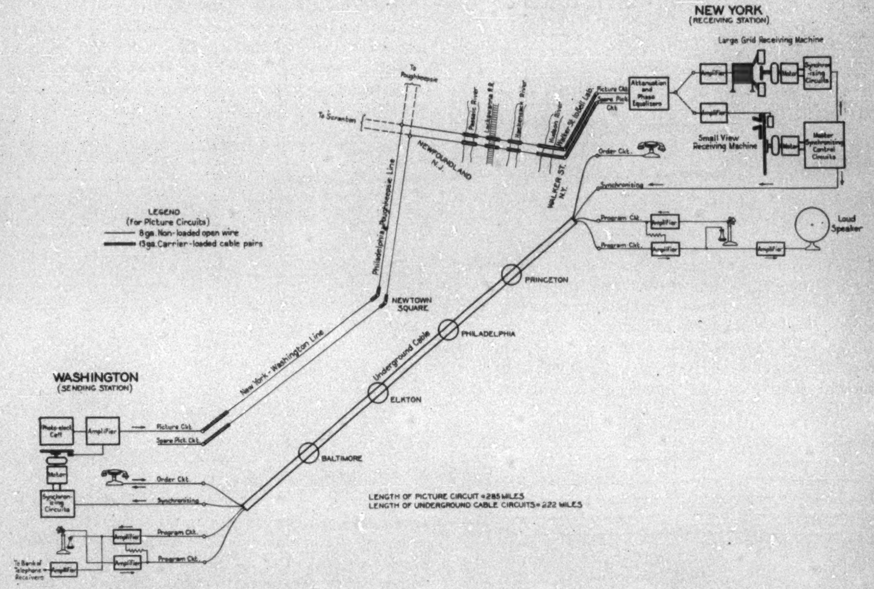

NEW YORK
(RECEIVING STATION)
Large Grid Receiving Machine
Amplifier
Motor
Synchronizing Circuits
Attenuation and Phase Equalizers
Picture Ckt.
Spare Pict. Ckt.
Amplifier
Small View Receiving Machine
Motor
Master Synchronizing Control Circuits
To Poughkeepsie
To Scranton
Passaic River
Lackawanna R.R.
Hackensack River
Hudson River
Walker St. to Bell Lab.
NEWFOUNDLAND N.J.
WALKER ST. N.Y.
Order Ckt.
Synchronizing
Program Ckt.
Amplifier
Program Ckt.
Amplifier
Amplifier
Loud Speaker
LEGEND
(for Picture Circuits)
8 ga. Non-loaded open wire
13 ga. Carrier-loaded cable pairs
Philadelphia-Poughkeepsie Line
NEWTOWN SQUARE
PRINCETON
PHILADELPHIA
Underground Cable
New York-Washington Line
ELKTON
BALTIMORE
WASHINGTON
(SENDING STATION)
Photo-elect. Cell
Amplifier
Picture Ckt.
Spare Pict. Ckt.
Motor
Order Ckt.
Synchronizing Circuits
Synchronizing
Program Ckt.
Program Ckt.
Amplifier
Amplifier
To Bank of Telephone Receivers
Amplifier
LENGTH OF PICTURE CIRCUIT = 285 MILES
LENGTH OF UNDERGROUND CABLE CIRCUITS = 222 MILES

FIG. 2—SCHEMATIC DIAGRAM OF CIRCUITS FOR TELEVISION DEMONSTRATION

Incidental Cables—Loading. Any appreciable length of non-loaded cable included in an open-wire television circuit has certain very objectionable effects. The impedance irregularities introduced by the cable destroy the uniformity of the line attenuation, impedance, and phase shift characteristics as a function of frequency, and tend to produce echoes as described above. Types of loading developed for use on incidental cables occurring in circuits employed for carrier telephone and carrier telegraphy operation[2] were employed to reduce these effects to a minimum. This carrier loading is designed so that when used on No. 13 A. W. G. cable circuits it provides an impedance which approximates very closely that of the open wire. With a spacing of about 930 feet between loading coils, this loading has a nominal cutoff of about 45,000 cycles, which corresponds to an effective transmission range extending up to about 36,000 cycles. In order to obtain a close match between the impedances of the open-wire and the cable pairs, thereby avoiding impedance irregularities, 13-gage pairs were selected for the television circuits in all of the cables.

The length of the submarine cable under the Hackensack River (about 1100 feet) was too great to permit the use of regular carrier loading, and a special loading arrangement having a slightly lower cutoff was, therefore, designed for this cable.

EQUALIZATION

Requirements. The requirements for the lines were stated earlier. In order to meet these overall requirements it was necessary to apply special forms of distortion-correcting networks.

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[2] *Development and Application of Loading for Telephone Circuits,* T. Shaw and W. Fondiller, JOURNAL A. I. E. E., Vol. XLV, pages 253-263, March, 1926.

Weather Changes. The above requirements applied, of course, to all of the various weather conditions to which an open-wire circuit is subject. Due to the changes in the leakage conductance occurring at the insulators, the attenuation of an open-wire circuit varies with changing weather conditions. This change is particularly important at the higher frequencies. At 20,000 cycles, for example, the attenuation of a 165-mil open-wire pair may vary as much as 40 per cent for a change from dry weather to extra wet weather. For the circuit between Washington and New York this represents a possible attenuation change of about 10 T U, or a change of 10 to 1 in the magnitude of the received power. At 1000 cycles, the effect of wet weather is comparatively small, so that the net effect of the weather variations is to change the requirements for the attenuation equalizers. The phase shift introduced by an open-wire pair likewise varies to some extent with changes of weather, although the percentage variation is much smaller than in the case of the attenuation. In view of these variations in the line characteristics it was decided to provide basic networks which would equalize for dry weather conditions, and to make available, in addition, several steps of equalization which would compensate for changes in the direction of wet weather.

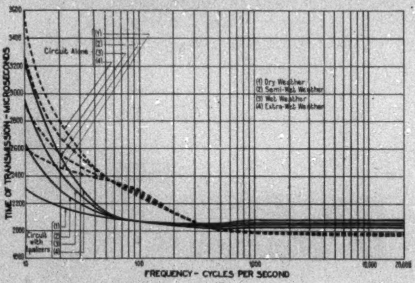

FIG. 3—COMPUTED PHASE DELAY (β/ω) OF TELEVISION CIRCUIT WITH AND WITHOUT EQUALIZERS

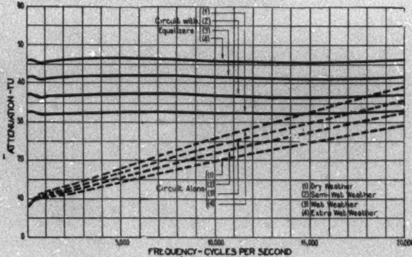

FIG. 4—COMPUTED ATTENUATION CHARACTERISTICS OF TELEVISION CIRCUIT WITH AND WITHOUT EQUALIZERS

Low-Frequency Network. Computed curves of attenuation and phase delay for the overall Washington-New York circuit without correcting networks are shown in Figs. 3 and 4, respectively. The form of the dry weather attenuation curve suggested the use of two correcting networks, one for low frequencies, the other for high frequencies. The network which was designed to equalize the attenuation at the lower frequencies is illustrated in Fig. 5. This network, in addition to equalizing the low-frequency attenuation, was made to provide sufficient correction for the low-frequency phase characteristic. It also proved satisfactory for all weather conditions.

High-Frequency Network for Dry Weather. The complete network for the correction at high frequencies under dry weather conditions was designed in two parts, an attenuation equalizer and a phase corrector. These two structures are illustrated in Fig. 6. The computed dry weather attenuation and phase delay resulting with the use of the combined low-frequency and high-frequency networks are illustrated in the curves of Figs. 3 and 4. It will be noted that the corrected attenuation curve is constant to within approximately ± 0.3 T U, while the corrected time of transmission falls well within the prescribed limits.

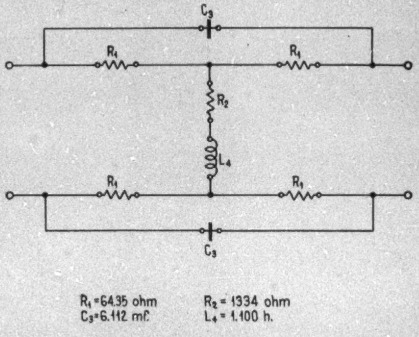

FIG. 5—LOW FREQUENCY EQUALIZING NETWORK (DRY AND WET WEATHER)

Weather Change Networks. Correction for the additional distortion introduced by changes from dry to wet weather was provided by three additional networks which were, for convenience, of identical design. The results obtained by using one, two, or three of these networks were made to correspond, respectively, to three assumed weather conditions which may be designated semi-wet, wet, and extra-wet. These three conditions were determined upon the basis of the range of leakage conditions which exist on open-wire lines under different weather conditions.

The attenuation equalizing and phase correcting networks for one of these steps are illustrated in Fig. 7, while the computed attenuation and time of transmission obtained by the use of the three different steps of weather correction are shown in Figs. 3 and 4.

The networks described above are of the "constant-resistance" type, whose characteristic impedance is a pure resistance at all frequencies.[3] These networks are designed to be connected in series. The methods used in the design of the networks involve a large amount of mathematical theory, a discussion of which is not necessary for the purposes of this paper.

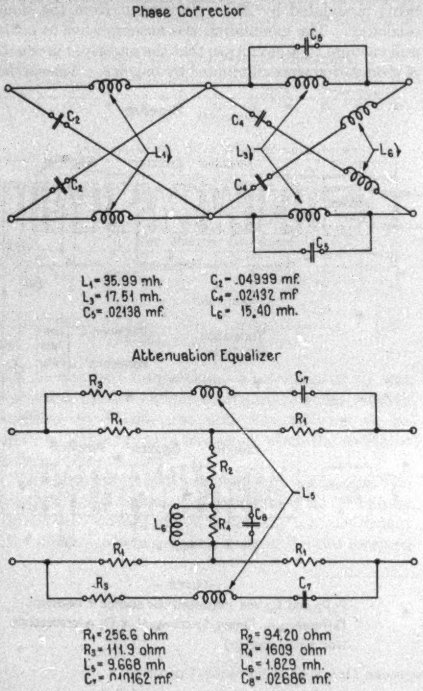
Phase Corrector
C5
C2
C2
C4
C4
L1
L3
L6
C5
L1= 35.99 mh.
L3= 17.51 mh.
C5= .02138 mf.
C2= .04999 mf.
C4= .02432 mf
L6= 15.40 mh.
Attenuation Equalizer
R3
C7
R1
R1
R2
L5
L6
R4
C8
R1
R1
R3
C7
R1= 256.6 ohm
R3= 111.9 ohm
L5= 9.668 mh
C7= .010162 mf.
R2= 94.20 ohm
R4= 1609 ohm
L6= 1.829 mh.
C8= .02686 mf.

FIG. 6—HIGH FREQUENCY EQUALIZING NETWORKS (DRY WEATHER)

SYNCHRONIZING AND VOICE CIRCUITS

So far the discussion has dealt only with the problem of transmitting the television currents. In addition to this, there is required the transmission of voice currents and of synchronizing currents. It is entirely feasible to transmit these currents together with the television currents over a single circuit. However, for the purpose of simplification, separate facilities were employed in the television experiments for picture, voice, and synchronizing currents.

The diagram in Fig. 2 shows the circuits which were actually provided for the demonstrations. It will be seen that in addition to the two picture or television circuits, there were provided a synchronizing circuit, a four-wire "program" circuit, and an order circuit.

The method of synchronizing the sending and the receiving machines has already been described in the paper by Mr. Stoller. It requires two currents, one having a frequency of about 18 cycles and the other about 2125 cycles. In order that an ordinary telephone circuit might be used for this purpose, the lower frequency was made to modulate by means of a telegraph relay, a carrier current having a frequency of about 750 cycles per second. An amplifier-detector at the receiving end of the synchronizing system demodulated the 750-cycle current, delivering 18 cycles to the television apparatus.

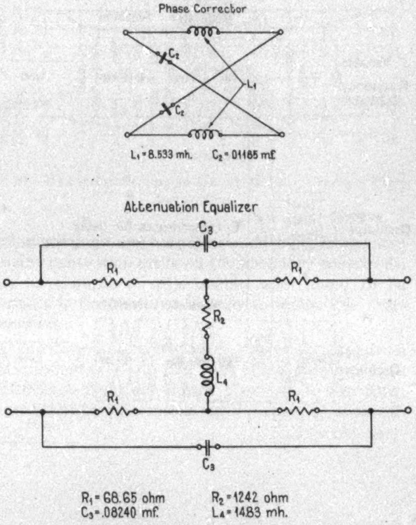
Phase Corrector
C2
L1
C2
L1= 8.533 mh.
C2= .01185 mf.
Attenuation Equalizer
C3
R1
R1
R2
L4
R1
R1
C3
R1= 68.65 ohm
C3= .08240 mf.
R2= 1242 ohm
L4= 1483 mh.

FIG. 7—WEATHER CHANGE EQUALIZING NETWORKS

The requirements for the synchronizing circuit were that it must transmit a narrow range near 750 cycles, and the single frequency of 2125 cycles. These synchronizing frequencies are determined by the speed of the motors, which was chosen so that the frequencies would be suitable for transmission over two channels of a voice-frequency carrier telegraph system,[4] but later it was found more convenient to use a separate telephone circuit.

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[3] Partially described in U. S. Patent No. 1,603,305 to O. J. Zobel.

The circuits labeled "program" provided telephonic communication between the observer at New York and the person being viewed at Washington. A loud speaker was also connected to this circuit at New York to transmit the voice to the audience when the large grid receiving arrangement was employed. A special by-passing connection was provided between the amplifiers at the terminals of the circuit so that speech from the local microphone could be heard as well as speech from the distant city.

The order circuit was for the purpose of providing communication between the engineers operating the television apparatus.

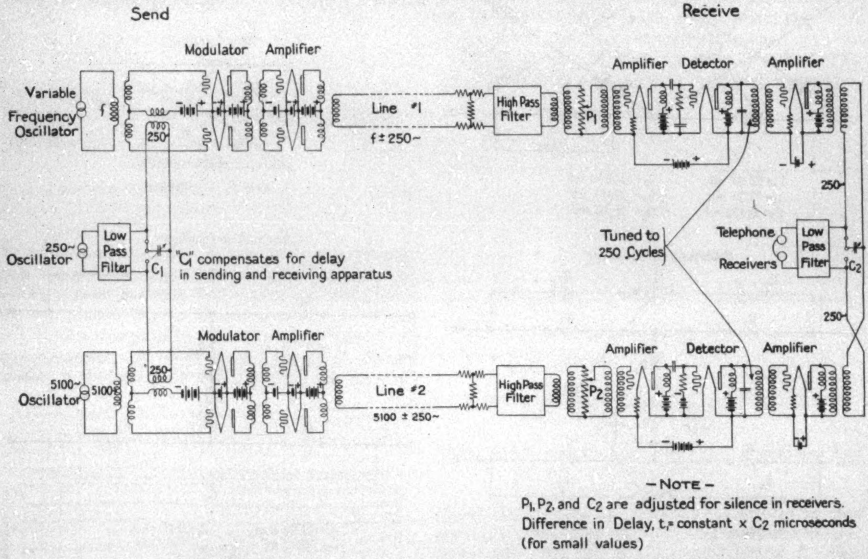
Send
Receive
Modulator
Amplifier
Variable Frequency Oscillator
f
250
Line #1
f ± 250~
High Pass Filter
Amplifier
Detector
Amplifier
P1
250~
250~ Oscillator
Low Pass Filter
C1
"C1" compensates for delay in sending and receiving apparatus
Tuned to 250 Cycles
Telephone Receivers
Low Pass Filter
C2
250~
Modulator
Amplifier
5100~ Oscillator
5100
250
Line #2
5100 ± 250~
High Pass Filter
Amplifier
Detector
Amplifier
P2
– NOTE –
P1, P2, and C2 are adjusted for silence in receivers.
Difference in Delay, t, = constant × C2 microseconds
(for small values)

FIG. 8—ARRANGEMENTS FOR MEASURING ENVELOPE DELAY OF TELEVISION CIRCUITS

LINE MEASUREMENTS

In order to determine that the circuits set up as outlined above were satisfactory, their overall characteristics were measured. Certain matters of interest in this work are noted below.

Measurements of Envelope Delay. In order to measure the envelope delay to an accuracy comparable to the requirements for the lines, it was necessary to develop special apparatus. Fig. 8 shows in schematic form the circuits of the apparatus designed for this purpose. The apparatus measures not the absolute envelope delay of a circuit, but the relative delay of one circuit at any frequency from about 600 cycles to 20,000 cycles or more with respect to the delay on the other circuit at a fixed frequency.

The functioning of the apparatus may be briefly described as follows: Simultaneously into each line there was transmitted a carrier current, each carrier being modulated by 250-cycle current from the same oscillator. The modulation was accomplished in push-pull vacuum tube circuits so that the undesired products of modulation were eliminated by balance. The carrier on the line under measurement was adjusted to the frequency at which a measurement was desired, and the carrier on the other circuit, used for reference, was kept at a fixed frequency of 5100 cycles.

At the receiving point identical circuits were provided for amplifying and demodulating the received currents from the two circuits. The 250-cycle outputs from the two sets of receiving apparatus were connected in opposition to a pair of telephone receivers through a low-pass filter. Potentiometers P_1 and P_2 were provided for adjusting the relative intensities of the two 250-cycle output voltages and a condenser C_2 was arranged so that it could be used to change the phase of either of the 250-cycle voltages. It is evident, then, that by making suitable adjustments the two voltages could be adjusted to exactly the same intensity and opposite phase so that no sound is heard in the telephone receivers. As long as the value of C_2 is small, the envelope delay of one line at the carrier frequency with respect to the delay of the other line at 5100 cycles is proportional to the value of C_2.

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[4] *Voice-Frequency Carrier Telegraph System for Cables,* B. P. Hamilton, H. Nyquist, M. B. Long, and W. A. Phelps, JOURNAL A. I. E. E., Vol. XLIV, pages 213-218, March, 1925.

The condenser C_1 shown at the sending station is for the purpose of introducing a phase shift in the 250-cycle current of either channel relative to the other in order to compensate for the differences in delay of the apparatus itself at the two frequencies. The value of C_1 was determined by experiment before moving the sending apparatus to Washington and was adjusted to its calibrated value for each frequency when the oscillator frequency was adjusted.

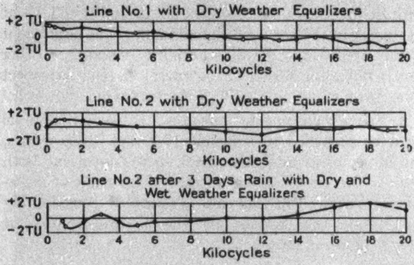

FIG. 9—MEASURED ATTENUATION CHARACTERISTICS OF LINE CIRCUITS PLUS EQUALIZERS

The measurement of the phase shift of the 250-cycle current, which is transmitted by means of a carrier over a circuit as described above, is actually a measurement of the difference between the phases of the two received side-band currents situated 250 cycles either side of the carrier. The envelope delay is equal to $\frac{\Delta \beta}{\Delta \omega}$ where $\Delta \omega$ equals 2π times 500, and $\Delta \beta$ equals the measured difference in phase of the two side bands in radians.

Measurements and Performance. How well the requirements which were set up earlier were met by the lines and the distortion-correcting networks is shown in Figs. 9 and 10. The attenuation characteristics are well within the established limits, and the phase characteristics show only a single slight departure for one circuit in a very narrow range of frequency. It is of interest, in view of the fact that the distortion-correcting networks were designed and built before any measurements were made on the lines they were to fit, that no changes or adjustments were found to be necessary in the networks, in order to obtain these characteristics.

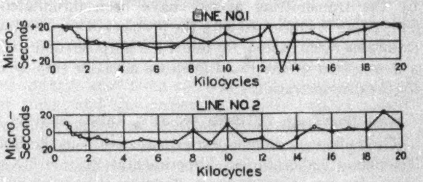

FIG. 10—MEASURED ENVELOPE DELAY OF LINE CIRCUITS PLUS EQUALIZERS

Comparison of the television images obtained from transmission over the line with those obtained from transmission from one side of the room to the other, showed that no difference in quality could be observed.

Radio Transmission System for Television

BY EDWARD L. NELSON*

Member, A. I. E. E.

Synopsis.*—Starting from the general requirements imposed on the transmitting medium, this paper discusses the engineering of a radio system for television purposes and describes the radio facilities actually employed for the recent Bell System demonstration. The tests to which the system was submitted to determine its suitability are outlined and the measured frequency-response characteristics are shown. An interesting phenomena due to multi-path transmission, the production of positive and negative secondary images, is reported. A brief series of experiments concerned with the transmission of both voice and image "on a single wavelength" is also described.*

* * * * *

IN other papers of this symposium, the general nature of the television problem has been discussed, the scope of the recent Bell System demonstration has been outlined, terminal apparatus for television has been described, and the general requirements to be met by the transmitting agency have been formulated. This paper is concerned with the problem of engineering a suitable radio system for television purposes and with a description of the radio facilities actually employed for the demonstration.

REQUIREMENTS IMPOSED ON THE RADIO SYSTEM

The radio experiments were conducted from the Bell Telephone Laboratories' Experimental Station 3XN at Whippany, New Jersey. Between this point and the main laboratories' building at 463 West Street, New York City, some 22 miles distant, three separate communication channels were required—one for the picture, a second for synchronizing, and a third for speech and music. The demonstration being of a three-cornered nature involving New York, Washington, and Whippany, it was deemed to be highly advantageous to transmit the necessary synchronising currents for both the wire and radio systems from a master generating set located in the auditorium of the West Street building. Hence the synchronizing channel was required to operate from New York to Whippany, while the picture and speech channels necessarily transmitted in the reverse direction.

From the radio standpoint, the problem presented for solution may be described as follows:

1. There is given television transmitting and receiving apparatus designed to work into and out of specified impedances at stated signal energy levels. Signal components ranging in frequency from 10 to 20,000 cycles must be transmitted with as little discrimination with respect to either amplitude or phase as reasonable design practises will permit. It is required that a suitable radio system be designed to afford satisfactory transmission between terminals when operated under prevailing conditions with respect to static, other radio traffic, and local electrical disturbances. The maximum allowable "noise" level is probably somewhat arbitrary but it has been found that if the ratio of signal to interference current is 10:1 the results are satisfactory. The variation of amplitude with frequency should probably not exceed ± 2 T U[1] at any point in the required signal band. The equivalent of the circuit must be substantially constant; in other words, no fading effects can be tolerated. In this respect a variation of perhaps 3 T U is the maximum allowable.[1]

2. For synchronizing purposes, a second channel must be provided to transmit 17.7 and 2125 cycles, the impedances and the signal energy levels at both ends of the circuit being known. The grade of transmission required in this case is probably considerably lower than that needed for the picture circuit but stable operation must be assured.

3. Arrangements must also be made for a high quality telephone channel to transmit speech and music for loud speaker reproduction.

4. All of these channels must, of course, be capable of operating simultaneously without mutual interference and without effect on established radio services.

PRELIMINARY SURVEY

In the vicinity of New York, an assignment of this type is surrounded with unusual difficulty due to the serious congestion which exists in the ether. Operations were started, therefore, by undertaking a survey of available frequency bands at periods of the day during which transmission might be required.

The pioneering nature of the project and the character of the apparatus available led to an early decision to base the system on the transmission of the carrier and both sidebands. Since the upper limit for the signal was specified as 20,000 cycles, an interference free band somewhat greater than 40,000 cycles in width was, therefore, required. The unusual width of this band indicated the desirability of fixing upon a relatively high carrier frequency. No readily available substitute for the ordinary type of tuned circuit was at hand and such circuits discriminate seriously against side frequencies differing by more than a few per cent from the frequency to which they are adjusted.

The results of the survey disclosed two bands somewhat wider than that required centering approxi-

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*Bell Telephone Laboratories Incorporated, New York, N. Y.

Presented at the Summer Convention of the A. I. E. E., Detroit, Michigan, June 20-24, 1927.

[1] Definitely agreed on limits were essential to proper coordination of the various development activities and figures of the order mentioned were assumed for design purposes.

mately about 1575 and 1750 kilocycles. It was also conclusively demonstrated that the operation of the synchronizing channel at a frequency above the broadcasting band was entirely out of the question. With two broadcasting stations located in the immediate neighborhood, one producing a field strength of perhaps 50 millivolts per meter and the other several volts per meter, the operation of a third transmitter on an adjoining frequency with the maximum obtainable separation between antennae, resulted in an almost continuous interference spectrum. It was decided, therefore, to transfer the synchronizing channel to a frequency of the order of 185 kilocycles, which would be sufficiently remote to remove interference from this source, and to make further studies in the regions about 1575 and 1750 kilocycles based on transmission from Whippany. No difficulty was anticipated in making suitable arrangements for the speech channel on account of the narrower band required and the well-known nature of the problem.

FIG. 1—GENERAL VIEW OF THE WHIPPANY STATION, 3 X N

THE WHIPPANY STATION, 3XN

A general view of the station site at Whippany is shown in Fig. 1. The property consists of some 47 acres. The main laboratory building, which is located near its center, is a two-story structure affording approximately 18,000 square feet of floor space. The principal antenna system involves two 250-foot steel towers with a suitable buried ground system, which is placed some 500 feet out in front of the building in order that the latter may be clear of the denser portion of the electric field. This antenna was assigned to the picture channel. For the voice channel, a separate structure located 500 feet in the rear of the laboratory building or approximately 1000 feet from the other was employed. The original supports in this second case were 60-foot wooden masts but subsequently metal topmasts were added, bringing the total height to 100 ft. Both antennae were energized by means of radio-frequency transmission lines. The antenna tuning and coupling apparatus was housed in small buildings placed under the center of each antenna, that for the larger structure having a copper roof which was securely connected to the ground network.

This type of installation is thought to afford a number of advantages. By separating the building and the antenna it becomes a much simpler matter to control the electrical factors which enter into the design of the latter. Removing the building from the field tends toward reduced dielectric and eddy current losses and consequently toward higher antenna efficiency. The resulting improvement may be expected to more than compensate for the slight loss in the line, which should not exceed 3 per cent. Removing the field from the building is equally advantageous in that it simplifies the precautions which normally have to be taken to prevent the radio-frequency energy from affecting the performance of audio amplifiers and other supplementary vacuum tube apparatus. The most serious disadvantages arise from the fact that the antenna must be tuned and the current in it measured at a point remote from the transmitting apparatus proper.

In spite of the fact that the station building was not directly under either antenna, some difficulty was anticipated from radio-frequency fields produced within the transmitting equipment due to the relatively high amplification employed with the photoelectric cells. In order to minimize trouble of this nature a special shielded studio was constructed in one of the wings of the building to house the television terminal apparatus. Walls, ceiling, and floor were completely covered with No. 24 gage sheet copper lapped about one inch and carefully soldered. The windows were covered with fine copper gauze. The door was covered with sheet copper which was carried around the edges so that in closing it made a firm wiping contact with the surrounding frame. Circuits for lighting and miscellaneous power service were led in through two specially constructed transformers fitted with grounded copper shields between the primary and secondary windings.

FIG. 2—OPERATING ROOM AT 3 X N. TRANSMITTER FOR TELEVISION CHANNEL ON THE RIGHT. POWER SUPPLY UNIT AND RADIO TRANSMITTER FOR THE SPEECH CHANNEL IN THE CENTER AND ON THE LEFT, RESPECTIVELY

The picture circuits leading to the radio transmitter, the microphone circuits, and the necessary studio signal and control circuits were run in lead cable and in most cases were brought into the room through suitable radio-frequency filters enclosed in metal boxes attached to the copper sheathing. In order to avoid the possibility of the heavy current leads to the arc bringing in radio-frequency energy, and to eliminate the noise and heat from the arc, provision was made for mounting the latter in its metal cabinet outside of the room. The circular opening through which the light beam was projected into the room was protected by the lamp cabinet which was also grounded to the sheathing. Satisfactory acoustic conditions within the studio were obtained by applying celotex wall board over the copper and by the use of suitable floor coverings.

Fig. 3—Television Transmitting Apparatus in the Studio at Whippany

Transmitting and Receiving Apparatus

For the television channel, arrangements were made to install a standard Western Electric 5-B Radio Broadcasting Transmitter and to modify it for the purpose. This transmitter is a 5 kilowatt unit (carrier output without modulation) designed for high quality telephone transmission in the 500-1500 kilocycle band. It will transmit signal components ranging from 50 to 5000 cycles without noteworthy discrimination. At 30 cycles and at 10,000 cycles there is some loss in efficiency and beyond these points the characteristic curve falls rapidly. The necessary changes, therefore, involved both the radio and audio circuits, the latter phase of the problem being perhaps the more difficult.

The schematic circuit of the modified transmitter is shown in Fig. 4. The revised radio frequency circuits were very similar to the standard arrangement, the changes mainly affecting the magnitudes of various coils and condensers. The output circuits were, of course, redesigned to meet the conditions imposed by the transmission line. The circuit was of the master oscillator-modulating amplifier-power amplifier type. The master oscillator employed a 50-watt tube operating in a circuit designed to afford a high degree of stability. This was connected to the input of the modulating amplifier through two radio-frequency stages, also employing 50-watt tubes. These two stages precluded the possibility of the oscillator frequency being appreciably altered by effects due to modulation. The modulating amplifier employed two 250-watt tubes in parallel and operated on the Heising system. In the standard equipment, the audio stages involve one 50-watt tube and two 250-watt tubes in parallel. To meet the more rigorous requirements of television with an ample factor of safety, this portion of the transmitter was removed from service and a specially constructed three-stage amplifier was substituted. As shown in the drawing, the latter consisted of two 50-watt resistance-coupled stages and a final power stage based on a 5-kilowatt water-cooled tube which raised the signal currents to a power level of approximately one-half kilowatt.

In order that it might be possible to check the performance of the radio transmitter under all operating conditions, a suitable monitoring rectifier was constructed and coupled to the output circuit of the radio-frequency power amplifier. A circuit was run back to suitable switches on the television control panel so that either the output of the photoelectric cell amplifiers or the rectified output of the radio transmitter could be impressed on the pilot lamp of the television transmitter. By comparing the two images, it thus became a relatively simple matter to detect any serious maladjustment in the radio apparatus.

The problem of providing a suitable transmitter for the speech channel was rendered quite simple by the fact that at the time there was in process of development at Whippany a 50-kilowatt equipment intended for broadcasting applications. The detailed description of this transmitter is beyond the scope of the present paper. It may be said, however, that it consists of a piezo-electrically controlled master oscillator employing a 50-watt tube directly followed by a 50-watt modulating amplifier. Modulation is by the Heising system, employing one 50-watt and one 250-watt tube in the audio stages. The output of the modulating amplifier is amplified by three push-pull, neutralized, radio-frequency stages the last of which employs six water-cooled tubes at approximately 17,000 volts. This set is capable of delivering 50 kilowatts (unmodulated carrier) to the antenna and during modulation instantaneous peaks approaching 200 kilowatts are attained.

The radio receiver employed at Whippany for the reception of the synchronizing signals at 185 kilocycles presents no features of unusual interest. A double-tuned input circuit was used followed by three stages of radio-frequency amplification, a detector, and two audio stages of conventional design employing transformer coupling. No serious difficulty was encountered in obtaining ample selectivity to insure satisfactory operation in the face of the strong local signals but care was necessary in locating the receiver and in laying out the antenna in order to avoid the inductive type of interference which is almost always experienced in the immediate vicinity of a large radio station. The receiving antenna was located approximately 700 feet from the two transmitting radiating systems.

The receiver employed at the New York terminus of the television channel presented a somewhat knotty problem on account of the relatively wide frequency band which it was required to pass while providing the maximum discrimination against interference. The width of the required band pointed very definitely toward the superheterodyne. This type of circuit is also very stable, permits of all the amplification that may be needed or that may be employed under ordinary noise conditions, and is very selective against interference immediately adjacent to the desired band. It is quite susceptible, however, to interference from components differing from the desired carrier frequency by an amount approximately equal to the intermediate frequency. If the interfering component lies in the neighborhood of the frequency of the oscillator, beats will be produced which may or may not pass the intermediate-frequency amplifier and the associated filters depending on their design. If the interfering component lies on the opposite side of the wanted carrier from the oscillator and differs from the former by the intermediate-frequency, it will be passed by the receiver, subject only to the attenuation due to the radio-frequency circuits (the input circuits tuned to the wanted carrier). This characteristic must be given careful consideration in the design of selective receivers of the superheterodyne type and has led to the introduction of carefully designed, loosely coupled, input circuits or an initial tuned radio-frequency stage for this purpose. Neither of these expedients were possibilities in the television receiver, however, because of the extraordinary width of the required transmission band. Recourse was had, therefore, to a triple detection arrangement. Speaking somewhat in the vernacular, the desired signal was "beat up" to 5000 kilocycles where it was passed through sharply tuned coupled circuits, then "beat down" to 120 kilocycles, amplified, filtered, and rectified, finally passing through a suitable low pass filter, audio amplifier, and output transformer to the television reproducing apparatus.

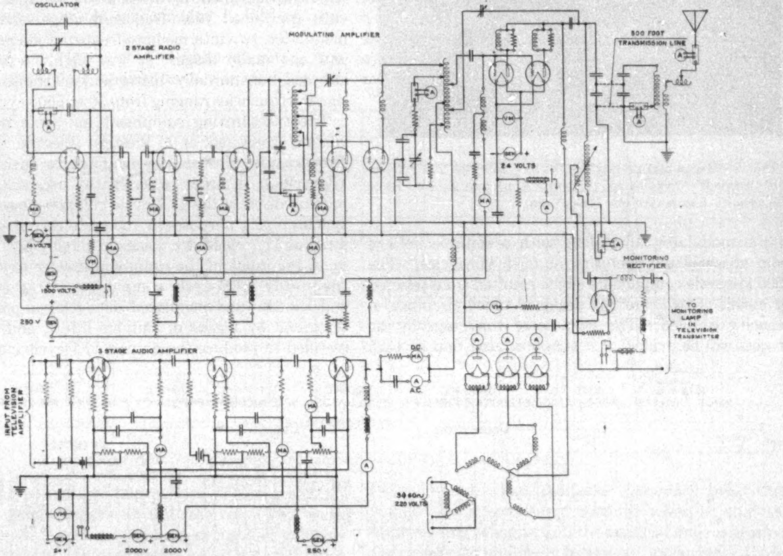

Fig. 4—Schematic of Radio Transmitter for Television Channel

The circuit arrangement is shown schematically in Fig. 6. Two tuned circuits with capacity coupling were connected to the input of the first detector or modulator. A relatively tight coupling was employed to produce the well-known double-peaked resonance curve capable of affording the required band width. The antenna was not tuned but was loosely coupled to the selective circuits by means of an adjustable capacity. The incoming radio signal was impressed upon the grid of the modulator tube along with a suitable voltage from an oscillator operating at 6575 kilocycles. The 5000 kilocycle components which resulted were selected by means of two carefully designed tuned circuits also capacity coupled. The purpose of this stage in the process will be evident if it is appreciated that at 1575 kilocycles ± 20 kilocycles requires a 2.6 per cent band while at 5000 kilocycles the same side frequencies require only a 0.8 per cent band. In the latter case, therefore, it is possible to employ materially sharper circuits without discriminating against the higher signal components. The 5000-kilocycle circuits connected to the grid of a second detector or modulator tube upon which suitable voltages from a 5120 kilocycle oscillator were impressed. The 120-kilocycle components in the output of this modulator were selected by means of a band-pass filter which worked into a two-stage intermediate-frequency amplifier. A second band-pass filter led to the third or final detector. A 20-kilocycle low-pass filter was employed in the plate circuit of the latter. This filter was designed for a low input impedance at 120-kilocycles in order to meet the necessary condition for efficient rectifier action and it also served as a coupling element for the audio stage which followed. A special output transformer with a permalloy core was provided to step down to the relatively low impedance of the line leading to the television apparatus proper.

FIG. 5—RADIO RECEIVING EQUIPMENT FOR THE TELEVISION AND SPEECH CHANNELS IN THE AUDITORIUM OF BELL TELEPHONE LABORATORIES, NEW YORK

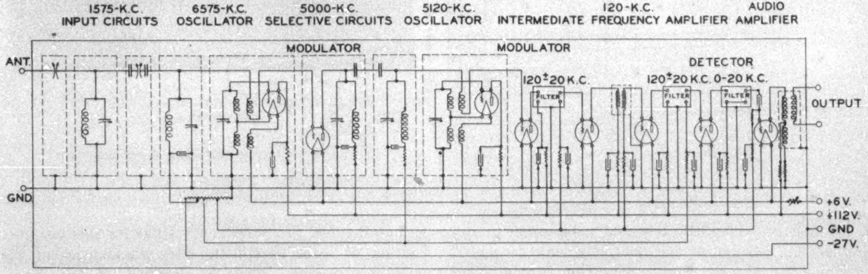

FIG. 6—SCHEMATIC OF TRIPLE DETECTION RECEIVER FOR TELEVISION CHANNEL

A superheterodyne receiver of more conventional design was employed for the speech receiver. The circuit arrangement involved a double-tuned input circuit, one tuned radio-frequency stage, oscillator and modulator, two intermediate-frequency stages, detector and one audio stage. It was highly selective and afforded substantially distortionless transmission for signal frequencies ranging from 50 to 5500 cycles.

The transmitting equipment for the synchronizing channel consisted of a Western Electric 6-A Radio Broadcasting Transmitter modified to operate at 185 kilocycles. In order to avoid the necessity of transmitting directly the 17.7-cycle component required for synchronizing purposes, a 760-cycle carrier was modulated at 17.7 cycles by means of a relay and impressed upon the input of the radio transmitter together with the steady 2125-cycle component. At the receiving end, the 2125 and modulated 760-cycle components were separated by means of suitable filters, and the latter rectified to produce the desired 17.7-cycle current.

TESTS OF THE SYSTEM

As soon as the various apparatus units could be made ready for service, a comprehensive series of transmission tests was undertaken. In order to determine the relative suitability of the 1575- and 1750-kilocycle bands disclosed by the preliminary survey, transmissions from Whippany at intervals throughout the day were arranged. Field strength measurements were taken at the receiving point employing apparatus of the type described by Messrs. Englund and Friis at the Spring Convention[2] and observations on the relative strength of the received signals were made by inserting a sensitive microammeter in the plate circuit of the third detector of the television receiver. These data indicated that the lower frequency band suffered considerably less attenuation and also afforded much more stable transmission. In spite of the comparatively short distance (approximately 22 miles) marked fading was experienced beginning with the sunset period and increasing in amplitude as the night advanced. The high frequency band proved to be particularly disadvantageous in this respect. It was decided, therefore, to fix upon the lower frequency band and to confine the demonstration to the afternoon when reasonably stable transmission conditions prevailed.

Following the choice of a definite operating frequency, a number of modifications was made in the transmitting antenna to improve its efficiency and increase the field strength at the receiver. This work finally resulted in a measured field strength of approximately 2500 microvolts per meter for an antenna input of 5 kilowatts.

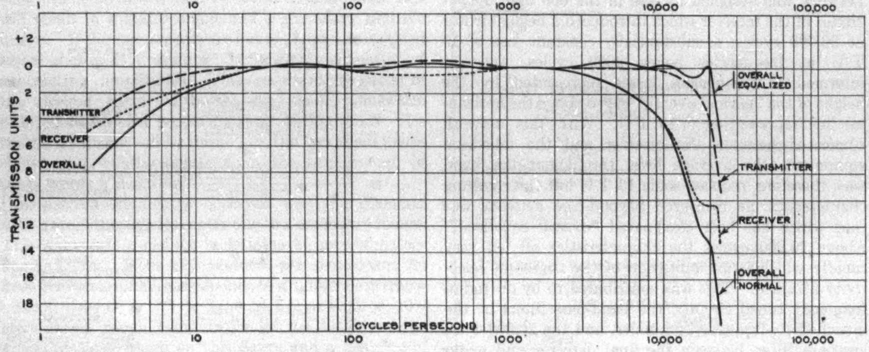

Fig. 7—Measured Characteristics of Television Channel

Further consideration of the available data on transmission and traffic conditions and the performance characteristics of the apparatus units involved lead to a choice of 1450 kilocycles for the speech channel. In spite of an antenna input of approximately 30 kilowatts, the initial tests at this frequency were very unsatisfactory due to inadequate field strength at the receiver which necessarily resulted in an abnormally high noise level. The height of the antenna was, therefore, increased from 60 to 100 feet by installing iron pipe topmasts. This change brought the field strength at the receiver to approximately the same value as that obtained for the television channel (2500 microvolts per meter) which was considered to be satisfactory for the purpose.

In order to insure that the reproduction of the picture might not suffer from serious discrimination against essential frequencies at some point in the radio system, very careful tests were made on the individual units and on the system as a whole.

The frequency characteristic of the transmitter was determined by connecting a vacuum tube oscillator producing a relatively pure wave to its input terminals through a suitable network involving a thermal milliammeter and an adjustable artificial line. A rectifier of known characteristics and a second thermal meter protected against radio-frequency currents by means of a low-pass filter were coupled to the output circuit of the water-cooled tubes. Employing a frequency of 1000 cycles, the input was adjusted to produce normal modulation and the readings of the input and output meters noted. The oscillator frequency was then changed by a convenient amount while holding the input reading constant and the artificial line readjusted, if necessary, to produce constant output current. Under these conditions, any change in the setting of the artificial line indicates an equal variation in the transmission efficiency of the transmitter which is evaluated by this method directly in Transmission Units (T U).

The characteristic of the receiver was determined in a similar manner. A low power transmitter of known characteristics was connected to it through a suitable attenuating network which, in so far as the receiver was concerned simulated the receiving antenna. The radio-frequency input to the receiver was adjusted to approximately the normal value and a series of measurements taken with variable audio-frequency inputs as indicated above.

The overall measurements were also based on a similar procedure impressing a constant input on the 600-ohm input terminals of the transmitter through a suitable artificial line and adjusting the latter to give a constant current into a 600-ohm load at the output of the receiver, taking necessary precautions, of course, to preclude overloading at any point in the system.

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[2] Methods for Measurements of Radio Field Strengths. C. R. Englund and H. T. Friis. See p. 492.

The experimental characteristic curves thus obtained are shown in Fig. 7, where the abscissae represent cycles per second and the ordinates departure from the 1000-cycle value in T U. As will be noted, at the lower frequencies exceptionally good performance was obtained, the overall characteristic being only 2 T U down (or deficient) at 10 cycles and only 6 T U down at 3 cycles. The results for the higher frequencies, however, were not so satisfactory, a loss of approximately 13 T U being observed at 20,000 cycles probably due to the tuned circuits in the receiver. Since modification of these circuits to obtain a flatter characteristic would have been difficult and would have occasioned a noteworthy sacrifice in selectivity, a compensation network was designed for use in the 600 ohm output circuit of the receiver which introduced a negligible loss at 20,000 cycles, a substantially constant loss of 13 T U at frequencies below 2000 cycles, and for intermediate frequencies losses represented by the height of the "normal overall" curve above the horizontal line representing – 13 T U. With this network connected between the receiver and the television equipment, the average level throughout the band was, therefore, reduced some 13 T U but the resulting characteristic as measured beyond the network was that which has been designated "overall equalized." Above 20,000 cycles the characteristics all fell very rapidly which is an indication of the degree of selectivity attained. This was contributed to by the radio-frequency tuned circuits, the band-pass filters in the intermediate-frequency amplifier, and the 20,000-cycle low-pass filter between the final detector and audio amplifier. The individual characteristics of the various filters were designed to be 60 T U down 20 kilocycles from the specified cut-off frequency.

Similar measurements were made upon the speech channel but a less thorough study was deemed sufficient in that case due to the existing background of experience.

EFFECTS OF FADING

With the system as outlined above, very satisfactory performance was obtained during the afternoon and early evening hours when reasonably stable transmission conditions were prevalent. Later at night, however, when marked fading became evident, some rather unexpected but easily explainable phenomena were observed which may be of sufficient interest to warrant brief mention.

When marked fading occurred, the normally clear reproduction was accompanied by "ghosts" or additional images which faded in and out in an erratic manner, sometimes appearing as positives and sometimes as negatives. The effect was most clearly observed when using one of the various types of test screens employed, a white card bearing a black diamond-shaped outline, approximately a square with its diagonals vertical and horizontal. With this simple type of pattern, it became evident that the secondary images were additional reproductions which were "out of frame" by a greater or less amount. In other words, each of these additional images consisted of a portion of two diamonds placed side by side with the corners just touching. Images "out of frame" along the vertical axis are frequently seen on the motion picture screen.

The explanation is fairly obvious. The present more or less generally accepted view of fading is that it is a manifestation of transmission along two or more paths, at least one of which is variable, producing a continually changing phase relationship between the components and a corresponding waxing and waning of the resultant signal. In the present case, the major image was probably produced by the so-called "ground wave." The secondary images probably resulted from components which were transmitted upward at a relatively sharp angle and reflected back to the receiving station from the Heaviside layer, the difference in framing being due to the longer time of transmission.

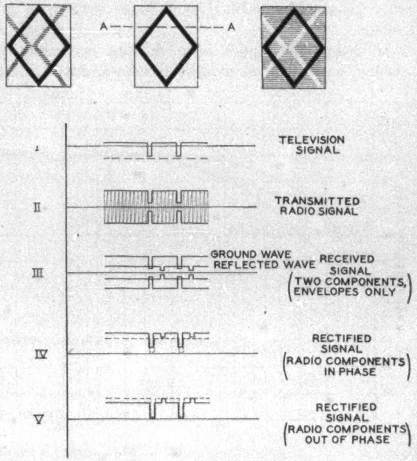
A
A
I
TELEVISION SIGNAL
II
TRANSMITTED RADIO SIGNAL
III
GROUND WAVE
REFLECTED WAVE
RECEIVED SIGNAL (TWO COMPONENTS, ENVELOPES ONLY)
IV
RECTIFIED SIGNAL (RADIO COMPONENTS IN PHASE)
V
RECTIFIED SIGNAL (RADIO COMPONENTS OUT OF PHASE)

FIG. 8—PRODUCTION OF POSITIVE AND NEGATIVE SECONDARY IMAGES DUE TO MULTI-PATH TRANSMISSION

The production of negative secondary images is a most interesting phase of the phenomena. This effect may be explained quite easily by means of a series of signal diagrams such as is shown in Fig. 8. If attention is confined to the interval during which scanning takes place along the line *AA*, it is evident that the television signal will have the form shown. Amplitudes above the dotted line indicate the current through the photoelectric cell. Since transformer-coupled amplifiers are employed in the television apparatus, however, the direct component is eliminated and the zero axis for the input to the radio transmitter is the solid line. Sketch II shows the modulated output of the radio transmitter. The received signal, shown in III, is assumed to consist of two components, the larger due to the "ground wave," and the smaller due to reflected energy from the Heaviside layer. The latter lags somewhat because of the greater length of the transmission path. The resultant of these two components will necessarily depend on the relative phase of the two carriers at the receiving point. Two cases are considered: when the components are exactly in phase, and when they are exactly out of phase. The effect at intermediate positions may be readily evaluated from these examples. With the components in phase, the detector output is proportional to their sum which is shown in IV. It is evident that this will result in a major image and a secondary positive image. If the components are out of phase, the rectified signal shown in *V* results. It is simply a matter of subtracting amplitudes. This resultant consists of the desired signal with the amplitude somewhat reduced which will produce a gray background. The secondary image will be formed by the two small peaks shown and will be lighter than the background, in other words a negative.

A pattern frequently observed was the diamond with a cross through its center due to a secondary image. This represents a change in framing of approximately one half line. With 17.7 pictures per second and 50 lines per picture, this corresponds to a difference in transmission time of $1/17.7 \times 1/50 \times 1/2$ or 5.65×10^{-4} seconds. A rough computation of the height of the reflecting layer based on this figure and a distance of 22 miles between transmitting and receiving stations gives 100 kilometers, which is substantially in agreement with determinations made by other methods.

TRANSMISSION OF VOICE AND IMAGE WITH A COMMON CARRIER FREQUENCY

Following the demonstration, a brief series of supplementary tests was arranged to obtain some appreciation on experimental grounds of the problems involved in transmitting both voice and image with a single radio transmitter. The system employed may be considered as the extension of carrier current technique to radio, but has been described in various other terms: "multiplex radio," "double modulation," "the Hammond system," etc. The output of a 30,000 cycle oscillator was modulated with the speech signal. The resulting carrier and sidebands were selected by means of a suitable filter passing frequency components ranging between 25,000 and 35,000 cycles and impressed on the input terminals of the radio transmitter along with the 10 to 20,000 cycle signal from the television apparatus. A suitable low-pass filter was employed in the line to the latter in order to preclude "crosstalk" due to 25,000-35,000 cycle energy working back into the final amplifier stages. The input to the radio transmitter thus consisted of a band extending from 10 to 20,000 cycles together with a 25,000 to 35,000 band, with a particularly strong component at 30,000 cycles representing the low-frequency carrier.

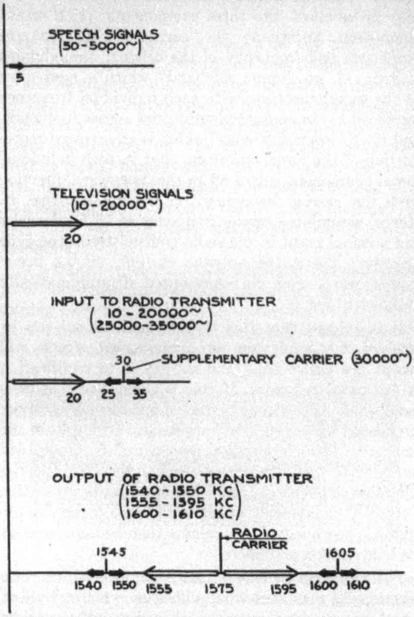

FIG. 9—DIAGRAMMATICAL REPRESENTATION OF FREQUENCY CONVERSIONS IN MULTIPLEX RADIO SYSTEM

In order that it might be capable of handling this wider band without discrimination, further modifications in the radio transmitter were required. In the case of some of the radio-frequency circuits, which were required to pass a 70,000-cycle band, it was found to be necessary to insert resistance to reduce the sharpness of resonance. On account of lack of time, it was not possible to obtain a complete series of characteristic curves for the transmitter under these conditions. Isolated measurements with a single-frequency input of 35,000 cycles indicated, however, that components of this order could be transmitted without serious loss and the subsequent performance of the system as a whole confirmed this conclusion.

It is well known that if a sinusoidal alternating current $i = I_0 \sin \omega t$ is modulated with a signal of frequency $f = \frac{\Phi}{2\pi}$ the resulting modulated current may be represented by the expression:

$$i = I_0 \sin \omega t + \frac{k\ I_0}{2} \sin(\omega + \Phi)t + \frac{k\,I_0}{2} \sin(\omega - \Phi)t$$

where k is a fraction indicative of the degree of modulation. In other words, a modulated current, or wave, may be resolved into three components: (1) a steady component, known as the "carrier," which has the amplitude and frequency of the original unmodulated current, (2) an "upper sideband" which is equivalent to the signal spectrum with each individual frequency increased by an amount equal to the carrier frequency, and (3) a "lower sideband" which is an inverted reproduction of the signal spectrum, that is, each individual signal component is laid off in the downward direction from the carrier frequency, or, subtracted from it. Hence, assuming a carrier frequency of 1575 kilocycles and a signal input to the radio transmitter of the type described above, the antenna current, or the transmitted wave, may be represented diagrammatically as shown in Fig. 9.

It is evident that this type of radio signal can be received by employing an arrangement which will accept the entire band and subject it to rectification in the usual manner. If this is done, the television signal and the 30,000 cycle supplementary carrier modulated with speech will appear at the output of the detector. Branch circuits with suitable filters will enable these two components to be separated and the television signal passed on to the reproducing apparatus. The other component must be rectified to derive the original speech signal, which may then be impressed on the loud speaker amplifiers.

The reception scheme actually employed during the experiments was somewhat different. The television signal was received separately by means of the triple detection set employed for the demonstration. The speech signal was received in a similar manner employing the set utilized for the speech channel during the demonstration. This latter receiver was tuned to 1545 kilocycles. That reception in this manner is feasible, is evident from the diagram. The 1540-1550 kilocycle zone contains two speech sidebands and a carrier of 1575 – 30 or 1545 kilocycles. It is quite possible, therefore, to demodulate in one step, instead of "beating" the various components against the main carrier (1575 kilocycles) to produce a 30-kilocycle supplementary carrier which must be rectified a second time to derive the speech signal. The 1600-1610 kilocycle band was ignored. The receivers were sufficiently selective that, with the 5-kilocycle interval which existed between the two bands, no noteworthy crosstalk was experienced.

The results obtained in this manner were not as satisfactory as those to be had with the other system described. This can be attributed to two factors, both concerned with the transmitting apparatus: (1) In order to transmit both signals with the same transmitter, that is, the same vacuum tubes, the individual current amplitudes had to be reduced to at least one-half, resulting in too weak a radio signal to clear the prevailing noise levels in New York, (2) In spite of the reduced amplitudes, a certain amount of inter-modulation was experienced in the transmitter which resulted in "crosstalk" between the channels. Notwithstanding these deficiencies, however, it was possible to recognize the speaker and to understand his remarks; but a short time ago, the performance would have been considered a very noteworthy achievement.

Experiments of this nature, although not new, are of particular interest where television is concerned, since, as Dr. Ives has indicated, the logical trend of development is toward a finer picture structure involving the transmission of much wider frequency bands, or what is more likely, the use of parallel scanning schemes and multi-channel transmission. The work, while necessarily somewhat cursory, may, therefore, be of value in affording an indication of the significance of multi-channel radio transmission in this connection. From a popular standpoint, these tests have been described as the transmission of both voice and image "on a single wavelength." To what extent this statement falls short of actually representing the facts in the case is obvious from Fig. 9. It will be seen that a wider frequency band is actually employed with this system than was required for two separate channels. Furthermore, this wider band is much less effectively utilized. Two bands are required for the voice channel in place of one. At the receiver, one of these bands was disregarded. To have received both would have required apparatus accepting twice the band width and the gain in signal would have been offset by the corresponding increase in noise level. For all useful purposes, therefore, the energy radiated in the form of the second band is wasted.

To proceed further with a discussion of multi-channel radio transmission is beyond the scope of the present paper. Whatever the system employed, however, one conclusion illustrated by these experiments may be pointed to with confidence: Television by radio requires a discrete and fairly wide frequency band. Hence the frequently predicted introduction of television as an adjunct to radio broadcasting without extensive changes in existing channel arrangements is extremely unlikely.

Joints in High-Voltage Multiple-Conductor Cable

BY THOS. F. PETERSON[1]

Associate, A. I. E. E.

Synopsis.—*In this paper the author has outlined the evolution of multiple conductor joints and explained in detail the development of a three-conductor, high-voltage joint for use with belted and metal sheathed cables or combinations of these.*

In the final design, spreaders and barrier tubes have been replaced by specially reinforced crotches and hand-wrapped insulation.

To date, all tests,—short time and accelerated life, a-c. and d-c.,—have produced failures in cable rather than in joints so constructed.

Many other claims of superiority over existing types are presented.

* * * * *

LITERATURE on high-voltage cable theory, design, testing, and operation is remarkably voluminous. This fact is especially noticeable when comparison is made with the meager data available on joints or splices and characteristics of jointed cable systems. Only recently have questions pertaining to the last mentioned been treated or emphasized in technical papers and discussions. It is quite pleasing however, to note their appearance in papers such as Del Mar's on the Effect of Internal Vacua, and others on the use of reservoirs and design of joints. This paper falls in the same category as the latter and constitutes a presentation of data on design, installation, and operation of joints on high-voltage systems with which the author has been intimately associated.

In its final analysis a joint may be considered as a reconstruction of cable brought into being because it is physically impossible to maintain continuity of cable from one end of a feeder to the other. Most efficient design would dictate using a joint somewhat stronger electrically than the cable with which it is made. Except in a few cases, this is theoretically possible, although often it would appear as though "brute strength" tactics were used in its accomplishment. Single conductor joints, those in metal sheathed and braided cable with immense amounts of insulation and those in multiple-conductor cable of monstrous size, typify these. The evolution of joints, with improvements gained partly from experience and more recently by the application of scientific principles of design, constitutes an extremely interesting subject for study.

When viewed in its entirety, one notices that the trend in joint design has followed rather closely that in cable design and quite justly so, since joint construction is, in a sense, cable reconstruction. When the old Edison tube was in use splices were made in enlarged coupling boxes, these being filled with hard compound as was the tube. Following this, various types of taped and rubber covered cable were developed. However, joints were made as before, except for the introduction of barrier tubes. Increase in operating voltage then gave rise to changes in impregnating compounds. At the same time difficulties with voids in hard compounds which were unavoidably produced during filling and with charring of paper, necessitated the use of such filling compounds as petrolatum and the introduction of hand wrapping, still retaining the barriers, however. In more recent developments based on knowledge of cable breathing, migration of compound into cable, stress distribution, etc., oil and reservoirs have been used at joints, barriers eliminated, crotches reinforced and, from one point at least, it would seem that joint construction has overtaken that of cable inasmuch as very little oil filled cable has as yet been placed in operation.

It is quite evident that in old designs, creepage paths were considered as the most important factors determining dielectric strength, this probably being derived from experience with joints on d-c. systems in which moisture, condensation, poor workmanship, impurities and the like contributed largely to failures. There seemed to have been no regard for stress distribution, dielectric constant relations of insulating materials and so we find barriers, tubes, porcelain, mica, paper, compounds, etc., in series with one another in various proportions and placed indiscriminately in the electric fields. These statements are not intended to belittle the importance of leakage paths but rather to encourage consideration of all phases of design.

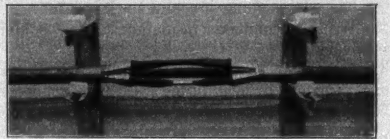

FIG. 1—METAL SHEATHED CABLE JOINT IN CONSTRUCTION

Quite general practise today consists in operating with alternating current and testing with direct current. Under alternating potentials at standard frequencies, the dielectric stress distribution is dependent on the configuration of electrodes and the constants of the dielectric circuits. On the other hand such distribution exists only at the moment of application of direct or steady potential differences. After this, redistribution takes place in the form of a slow transient with end conditions depending on the resistivities of paths. If then, total stress is sufficiently high, failure will occur along the path of minimum resistance. Though the

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[1] With the Brooklyn Edison Co., Inc.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

stress distribution under alternating current is determined primarily by dielectric constant relations, a component may always be determined acting in the direction of the leakage path and tending to produce this type of failure. It is obvious that the direct potentials with total voltage applied to path, are more severe. However, it is felt that defects brought to light by the comparatively high tests based on cable requirements are well worth removing and that the testing does not necessarily invite trouble. The importance of consideration of leakage paths as well as dielectric constant relations is thus brought out.

While the company with which the author is connected, was contemplating operation at 27 kv., many tests were made on multiple-conductor joints. Inasmuch as this was done under pressure of time no radical departures from standard types or accepted practises were made. Finally one, hand wrapped, petrolatum filled, with porcelain spreaders and mica barrier tubes was adopted. Results with this type were fairly satisfactory. After several years of operation, however, a rather high percentage of failures was experienced on test which, on examination, were almost all found to have started at connectors, then continuing along pencilling and conductor insulation or barrier tubes to ground. These were due to such causes as follows:

1. Copper dust produced by filing connectors.
2. Jagged pencilling of conductor insulation.
3. Wrinkled conductor insulation, (2 and 3 giving rise to voids).
4. Compound having been driven from mill insulation at pencilling by heat during sweating of connectors.
5. Migration of compound into cable, making most probable location of voids on contact surface between mill insulation and hand wrapping.
6. Sharp edges of solder, connectors, etc.

It is quite obvious why, with such causes as these conspiring to produce trouble, direct-current tests, at 95 kv., conductor to ground, and 135-kv., conductor to conductor, were so effective in weeding out defects.

The cures for these evils were soon found. Advances were made in cable manufacture (wrinkles being practically eliminated), pencilling was improved, one or two layers of varnished cambric tape was used over connectors, etc. These, with pressure filling, gave rise to a fairly satisfactory joint but, like "Champions and Records," this did not stand long, for tests at the laboratory under three-phase, alternating current revealed other weak places, namely, at the crotches and porcelain spreaders. Just about this time one company whose experience in this direction was very similar to ours began to experiment with wooden spreaders, the idea being that with a dielectric constant more nearly equal to that of impregnated paper better results would be obtained. Our progress was along different lines.

For some time manufacturers and those engaged in testing of cable had suffered a serious handicap in not being able to carry tests of three-conductor cable to satisfactory completion because of breakdowns which invariably occurred at the crotches. The author was confronted with and quite successfully solved this problem about 2½ years ago. Examinations of crotch failures of cable tested under oil indicated that there were three outstanding causes for these, namely:

1. Ruptured conductor insulation due to sharp bending.
2. Breakdown of film of oil and then paper at crotch.
3. Circumferential stresses on conductors.

The first was removed by fanning out legs in stages, that is, removing part of belt and bending legs, using edge of belt as a fulcrum.

The second did not yield so readily to the corrective measures applied. The attack was based on observations made during two tests devised to bring out the existance of this cause.

A roll of impregnated tape was used as a spreader in a crotch. After application of three-phase voltage, the roll was badly burnt due to breakdown of oil at its center. At another time a sample was prepared and the crotch placed in a glass jar of clear oil. Spark discharge across film of oil in crotch was very much in evidence.

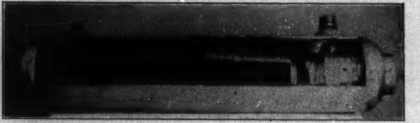

FIG. 2—CUT-AWAY OF JOINT IN BELTED CABLE

Joints of this type tested; 120-kv. three-phase, 10-65 hours.
Failures in cable; 130 kv. 15 hr. and 276 kv. instantaneous failure in cable.

The dielectric constant of oil is approximately 2.5 and that of impregnated paper 3.2 to 4.1. When insulations are in series in a dielectric circuit distribution of stress is in inverse proportion to dielectric constant. It is seen, therefore, why oil or compound was stressed beyond its rupturing strength, broke down, and the resultant heating, charring, etc., finally culminated in breakdown of adjacent paper[2].

Circumferential stresses on conductors manifested themselves by the appearance of tree formations at crotches through several layers of paper on the backs of conductors of certain types of cable, though these very rarely resulted in failure.

A rather simple solution for the above difficulties was arrived at after making some exhaustive tests using barriers, cotton, and the like jammed into crotches, short circuiting oil by wrapping legs with electrostatic shields, using wrapping of wick. Finally, after carefully bending conductors, each leg was built up conically with black varnished cambric tape starting deep in the crotch and extending out 5 inches, the maximum thickness being ⅜ in., 4 inches from belt. edge. (See Figs. 2 and 4.) The conductors were drawn together and a varnished cambric belt was applied at crotch to the level of the factory belt. Then the lead sheath of cable was extended, using lead foil, to the point of maximum thickness of varnished cambric. The lead foil and varnished cambric belt were punctured to allow oil to enter where normally fillers would be. With this type of construction, overstressed oil films at crotch were replaced with varnished cambric. The latter with its high dielectric constant (approximately 5) in comparison with impregnated paper (approximately 3) assumed very little of the total stress between conductors, but still took a sufficient per cent to relieve the paper at crotch and make the stress in it considerably lower than in the rest of the cable. The foil served to eliminate circumferential stresses by maintaining a zero potential surface at a distance outward from the center of the conductors equal to the distance inward to the actual neutral axis of the cable. Similar crotches were used in testing metal sheathed cables, except that after building up individual legs with varnished cambric the metallic tape of cable was extended and carried over each leg to the point of maximum thicknesses of varnished cambric (see accompanying illustrations).

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[2] See author's discussion of paper on a 132-kv. joint, by D. M. Simons, presented at A. I. E. E. Winter Convention, 1927.

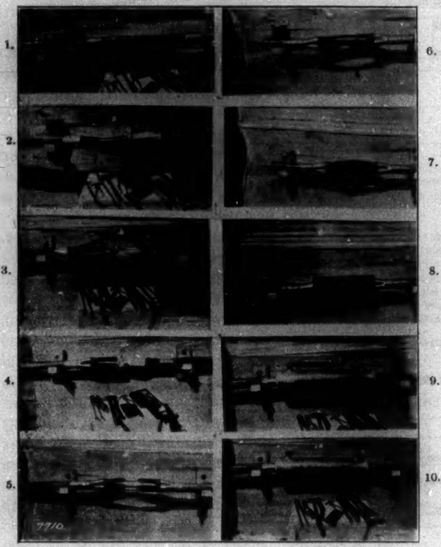


Fig. 3—Showing Stages of Construction

1. Cable butted
2. Fanning conductors
3. Conductors covered with one layer of tape
4. Crotch built up metal foil applied
5. Connectors sweated
6. Hand wrap
7. Hand wrap
8. Barrel slipped into place
9. Pressure filling
10. Caps on—completed joint

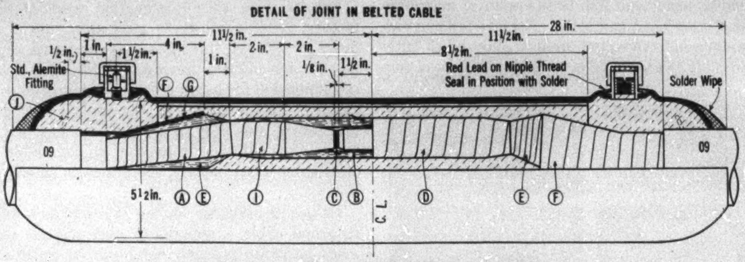
Results with this construction were highly satisfactory, yielding failures in cable after runs of 35 to 50 hours at 120 kv., three-phase as compared with 3 to 8 hour crotch failures previously obtained and so it was recommended for use in crotches of joints on our 27 kv. system. In the latter role, the applied varnished cambric served not only as a reinforcement, but also as pacer, thus eliminating another objectionable features of existing joint, namely, the porcelain spreader. (See Fig. 4.)

Before its adoption for use in connection with metal sheathed cable, comparative high-voltage tests were made on joints in which metal foil was ended at crotch, *i. e.*, cut off and tied down, carried across connectors, over hand-wrapped insulation, etc. In all these, the decided superiority of the built up construction was shown. Joints with metal tape carried across the hand-wrapped insulation failed after about twenty minutes at 120 kv., while those with tape ended at crotch were but a little better. In the first, failures occurred across stepped and conical penciling. This type of joint would seem unsatisfactory at first sight (despite the fact that some manufacturers stand by it because radial stresses are maintained throughout) since filling compound is of no avail in taking up stress and the total voltage is applied to a comparatively weak path between pencil and hand wrapped insulation. Joints made in manner described, that is, using built up crotches, have withstood tests at 120 kv. for 10 to 60 hours without failure. In all cases rupture has occurred in cable. This is quite favorable, when compared with the results obtained on standard Conducell joints which have averaged 3 to 10 hours at 120 kv., three-phase.

Hand wrapping of joints with tape has always been a long and tedious operation in which the personal equation of the worker is a most important item.

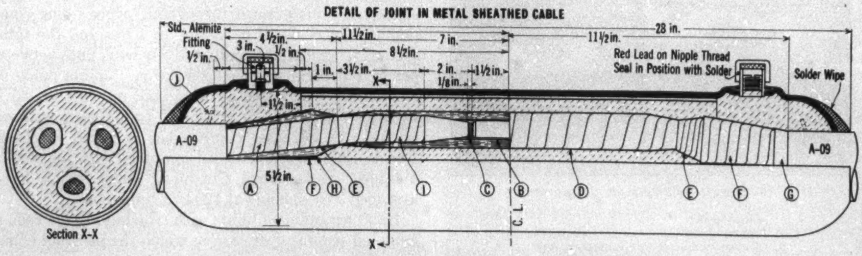
Prior to making the aforementioned studies and tests, an attempt was made to reduce manual labor involved in joint construction by substituting barrier tubes and spacers made of bakelite and the like, for the hand wrapping. Results of voltage tests were very discouraging since failures occurred at 120 kv. after from one-half to one hour, the higher values being obtained with the thinner tubes. Careful examination and investigation indicated that such devices which had high dielectric constants and were of such shape as to place small thicknesses of compound in series with them thus greatly overstressing the latter, were of no use and should be avoided. Barrier tubes were accordingly eliminated from final design (see Figs. 4 and 5) and some additional paper hand wrap was applied over connectors, a very reasonable change, considering the facts that the paper serves as the necessary barrier in compound and makes it possible to retain comparatively long paths through compound thus increasing strength of joint, without over-stressing component parts.

Confirmatory test results similar to those recorded have recently been obtained by another of the metropolitan companies which has adopted this type of joint for metal-sheath cable. In addition to the above alternating-current tests, run of five hours duration at 95-kv., direct potential differences, conductor to ground, have been made without the slightest indication of any developing failure. These data, together with very satisfactory operating experience serve as evidence for claims of superiority of this joint.


DETAIL OF JOINT IN BELTED CABLE
28 in.
11½ in.
11½ in.
½ in.
1 in.
1½ in.
4 in.
1 in.
2 in.
2 in.
8½ in.
1/8 in.
1½ in.
Std., Alemite Fitting
Red Lead on Nipple Thread Seal in Position with Solder
Solder Wipe
Ⓙ
Ⓕ
Ⓖ
09
09
5½ in.
Ⓐ Ⓔ Ⓘ Ⓒ Ⓑ Ⓓ Ⓔ Ⓕ
C L

A. One layer, 3/4-in., black varnished cambric on each conductor.
B. One layer, 3/4-in., black varnished cambric on each connector.
C. Varnished cambric strips 1/8 in. to 3/4 in. wide to level of connector diameter.
D. Paper tape
E. 3/4-in. black varnished cambric conical wound as shown.
F. One layer 5/8 in. lead foil around all three conductors.
G. Black varnished cambric around all three conductor.
H. Varnished cambric 5/16 in. thick 4 in. from belt.
I. Remove v. c. tape 2 in. from edge of pencil, also one layer of conductor insulation before applying paper tape.
J. Bell to be beat into original position after crotch is finished.

Fig. 4—Specifications For 3-Conductor, 27,000-Volt, 09 Cable Joint 27-3


DETAIL OF JOINT IN METAL SHEATHED CABLE
Std., Alemite Fitting
½ in.
3 in.
4½ in.
½ in.
11½ in.
7 in.
8½ in.
28 in.
11½ in.
1 in.
3½ in.
2 in.
1/8 in.
1½ in.
1½ in.
X
Red Lead on Nipple Thread Seal in Position with Solder
Solder Wipe
Ⓙ
A-09
A-09
5½ in.
Ⓐ Ⓕ Ⓗ Ⓔ Ⓘ Ⓒ Ⓑ Ⓓ Ⓔ Ⓕ Ⓖ
C L
X
Section X-X

A. One layer, 3/4-in. black varnished cambric on each conductor.
B. One layer 3/4-in. black varnished cambric on each connector.
C. Varnished cambric strips 1/8 in. to 3/4 in. wide to level of connector diameter.
D. Paper tape.
E. 3/4 in. black varnished cambric conical wound as shown.
F. One layer, 5/8-in. lead foil on each conductor from highest point to bottom of crotch and in contact with copper foil on conductors.
G. 5/8 in. lead foil two layers around all three conductors and in contact with outer metallic tape of cable.
H. Varnished cambric 5/16 in. thick 4 in. from bell.
I. Remove v. c. tape 3 1/2 in. from edge of pencil; also one layer of conductor insulation before applying paper tape.
J. Bell to be beat into original position after crotch is finished.

Fig. 5—Specifications For 3-Conductor, 27,000-Volt, A-09 Cable Joint, 27-3

Joints installed in practise do not differ materially from those described. Conductors are carefully fanned out and covered with one layer of tape to exclude foreign matter, crotches are built up, connectors are sweated, insulation pencilled conically (approximately 2 in. long) and sandpapered. Then gaps between insulation and connectors are built up with thin strips of varnished cambric and one layer is applied over connector. One layer of mill insulation is removed from conductors and finally each leg is built up with hand-wrapped tape. An insulating barrel is used inside of lead sleeve, merely for mechanical protection and joint is either pressure filled with petrolatum, or oil filled and equipped with a collapsible reservoir or the like. Though this joint is somewhat more difficult to make than the usual type and depends quite largely on the ability of the splicer, little difficulty is experienced in the field, when men are properly trained.

In the foregoing we have been concerned more particularly with electrical rather than mechanical characteristics of joints. The latter, however, are of immense importance. Absolute cleanliness should be exercised. Wipes, plugs, and material must be such as to insure against introduction of moisture. Though voids are not likely to exist at time of filling, they may develop in practise, hence fittings, location in manholes, etc., must be arranged to facilitate periodic inspection, refilling of joints, or installation of reservoirs. These can best be dealt with as individual problems.

Summarizing, then, it may be said that a joint has been developed having high leakage resistance, good stress distribution and on which our test results, both alternating-current and direct-current, and operating experiences, are quite satisfactory. (Note: No studies have been made of dielectric loss because to date nothing in our experience has indicated the need for consideration of same.) Furthermore, barriers and spreaders have been eliminated and a zero potential surface has been established around the crotch which gradually extends out toward sleeve, thus tending to maintain radial stresses throughout joint. In addition, due to removal of tubes, phasing out of cables is more easily accomplished since conductors in joint do not have to be run parallel to cable. Instead of having two bends in the conductors one at each end of the joint, and then a straight run through cells or tubes as is necessary in the barrier type joint, the lay or twist may be continued through the joint. Last, and perhaps most important, is the universality of the joint. Not only can it be used with belted cable and metal sheathed cable, round or sector conductor, but also with combinations of these.

It is not intended that these statements be construed to mean that perfection in joint design has been reached. The author expects to attempt improvements from time to time and profit by criticism which will undoubtedly accompany its more general adoption.

Discussion

D. W. Roper: I am not going to subscribe to the implications in the paper that this type of construction is the only one that will result in successful high-voltage joints, but I can subscribe to the statement that single-conductor cable joints which are made up in essentially the same manner as the joints on each of the three conductors in Mr. Peterson's three-conductor joints, are very successful. In Chicago we have about 750 single-conductor joints made up very much the same as the individual conductors in Mr. Peterson's metal-sheathed cable joint; that is, they have the reinforcement of the insulation, the extension of the metal sheath to the point of maximum reinforcement, and then a tapering of the reinforcement as well as a tapering of the conductor insulation.

Mr. Peterson mentions the careful selection of materials, absolute cleanliness, and careful workmanship. In order to secure the latter two features in the single-conductor joints which we made, we started a school for the training of our cable splicers, using for the purpose a full-sized cable. We had each one go through all of the details process of making up a joint, or several joints if necessary, until he became proficient in the mechanical execution of the several processes.

To make sure that a man was proficient, after he had made up each joint, it was cut apart, and examined, and the imperfections in his workmanship were pointed out to him. He was required to repeat the process as often as was necessary,—generally only two or three times—until he had eliminated those imperfections in his workmanship. After that was done, we had him make up a joint at a different location where a high-voltage test transformer was available, and in order to secure economy, we had three joints made up in series putting high-voltage potheads on the ends of the cable and applying the high-voltage test required by the specifications to be applied to the cable. In each case, the failures occurred in the cable and not in the joints.

Since then, we have made up 750 joints on the 66-kv. cable and placed them in service, and have applied the usual high-voltage test to the completed lines before placing them in service. There have been no failures whatever on any of the joints, either during the construction training, or on the testing of them after the completion of the lines or in service. We have had a total of about 400 joint-years of service on this type of joint. So I think, as Mr. Peterson said, we are rapidly approaching the condition where joints are stronger than the cable.

E. D. Eby: In this problem of splicing high-tension cables, it is gratifying to note in Mr. Peterson's work an honest attempt to make use of scientific principles of design.

His adoption of the tapered reinforcement of conductor insulation adjacent to the lead sheath with the reinforcement overlaid with metal tape unquestionably constitutes one of the main factors in the success of his design. I advocated this construction to the Cleveland Electric Illuminating Company in 1923, when the 66,000-volt single-conductor cable system was first being developed. I think this was the first occasion on which this construction was proposed and described. How effective it has proved in the solution of cable-joint problems is evidenced by the fact that practically all of the higher-voltage joints have been made in this way and with a success which has been unparalleled.

A feature of which Mr. Peterson has not taken advantage, but which has worked out to complete satisfaction in General Electric designs, is that of stepped conductor insulation adjacent to the connector. Mr. Peterson found trouble with both stepped and penciled insulation in some of his designs but this was probably due to the use of inflexible paper tape. With a flexible tape such a bias-cut varnished cambric, there is a much better bond between the conductor insulation and the hand-wrapped insulation. The stepped surface, however, offers decided advantage over the penciled surface. An exact number of tapes can be removed at each step so that the remaining insulation is of a known and uniform thickness. Accidental cutting into the remaining insulation, as sometimes happens with a penciled surface, is thus wholly eliminated. The tearing of the tapes against a fine steel wire looped around the conductor leaves a spongy edge which fills well with the flushing compound and forms an elastic cushion under the hand applied tape. The uniformity of the remaining insulation is particularly evident in the case of sector-shaped conductors where uniform penciling becomes even more difficult than with round conductors.

While, as Mr. Peterson remarks, it is true that the hand-wrapping of joints with tape has always been a long and tedious operation, it is also true that with the more scientific designs now available, the amount of taping to be done has been so much reduced and the kind of tape to be applied so much improved that hand-taped joints are not only wholly practicable from the installation standpoint, but are proving entirely reliable in service. Even machine wrapping of the tape does not offer sufficient advantage to warrant the use of machines except for paper tape. Furthermore, the introduction of the specially processed varnished-cambric tape with low dielectric losses has removed the one advantage of paper tape which formerly existed so that now a hand-taped varnished-cambric joint stands second to none, both with regard to ease of installation and its reliability.

While barrier tubes in three-conductor joints have proved disappointing in some designs, they are not an objectionable feature when properly shaped and located with respect to the conductors. The writer has developed a design for 33-kv. sector cable in which the barriers consist of sections cut lengthwise from standard Kerkolite cylinders, two such sections surrounding each conductor within an outer cylinder and holding it rigidly in definite relation to the other conductors and to the outer casing. The construction prevents displacement of the conductors within the casing which might occur either from short-circuit stresses or from mechanical movement of the cable.

In Mr. Peterson's joint for belted cable, he has found it desirable to enclose each conductor with an individual reinforcement of tape where it emerges from the belt. With no barriers to support the cables at the center of the joint, this serves a good purpose, mechanically. Electrically, however, it is not necessary, since a reinforcement enclosing all three conductors and overlaid with metal tape will accomplish the desired results. Sample joints tested by the writer have shown no weaknesses in the crotch.

Three-conductor cable of the belted type for 20,000 volts and above will probably give place sooner or later to cable of the shielded type so that future interest in joint design for three-conductor cables will center largely in the latter. It is natural that with each separate conductor enclosed in a metal tape under the lead sheath, a similar construction should prevail at the joint. Apparently Mr. Peterson did not have much success with such joints in which the tape was carried all the way across the hand-wrapped insulation. Here again perhaps the difficulty was with the kind of tape and a penciled rather than a stepped surface. At any rate, the writer has had no difficulty in making a satisfactory joint of this kind. Such a design for 33-kv. shielded cable is shown in the accompanying Fig. 1. The simplicity of this joint is at once apparent. The fact that each conductor is wholly enclosed within the grounded metal tape eliminates the transition made in Mr. Peterson's design from the equivalent of three single-conductor cables under the lead sheath to the equivalent of a three-conductor unshielded cable in the center of the joint.

That the totally shielded form of joint for three-conductor shielded cables is entirely practicable is further evidenced by the complete success of a 66,000-volt single-conductor joint of equivalent design. A further advantage in a totally shielded joint is the absence of immediate danger which results from an incomplete filling of the joint casing with oil or compound. In the open construction with oil between the conductors under stress, the absence of oil would be expected to develop trouble rapidly. In the totally shielded construction, providing moisture was excluded, an incomplete filling of the casing would not be serious.

Such descriptive papers as this will do more than anything else toward harmonizing ideas and unifying practise. There are certain fundamental principles in the design, installation, and operation of cable joints which this publicity will gradually help to establish. The standardization of materials and methods in this class of work is just as desirable as in other classes of equipment.

D. M. Simons: I should like to make a few remarks, particularly on the joints for Type-H or metal-sheathed cable. We have been making Type-H cable since 1914 (in commercial quantities since 1919), and have given the matter considerable thought. I approve of Mr. Peterson's solution, but I should like to point out that there is another solution.

I believe everyone agrees that one of the great advantages of Type-H cable is that all the so-called "crotch" failures can be avoided, since the metal can be carried into the joint until there is sufficient separation between the insulated conductors to avoid trouble. Apparently, however, there are two schools of thought on Type-H joints, differing in whether or not the metal foil should be carried completely across the joint.

We have made both kinds of joint, ourselves, having made those similar to the kind Mr. Peterson has described, with one refinement in addition; namely we used a large lead wire at the edge of the metal in order to reduce somewhat further the stresses there. The other form, which we have generally preferred, is to carry the metal shields across the joint. If this is done, then, theoretically, the joint has all the advantages of Type-H cable. Furthermore, and within the range of voltages where experience has not dictated the advantage (from the standpoint of the cable itself) of using soft or fluid jointing compounds with collapsible reservoirs, the carrying of the metal across the joint removes all stress from the compound, and there is no danger even if voids should exist. There is also of course no necessity for using reservoirs or otherwise maintaining the compound. Finally, even if a soft compound or fluid oil is used for the sake of the cable, there is always a possibility that the joint may be partially emptied of oil due to the improper functioning of reservoirs or other causes; and in this case, there will be a great advantage if all the jointing compound has been shielded from stress by metal wrappings.

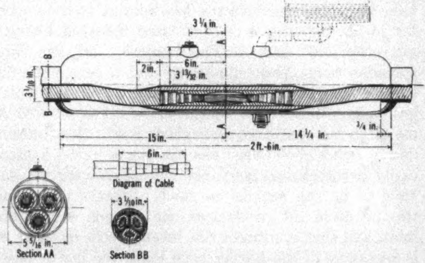


FIG. 1—33-KV. 3-CONDUCTOR JOINT FOR SHIELDED CABLE

I do not hold a brief for either type, but wish to emphasize that there are two types available, both of which have been eminently successful in service. I believe in general that Mr. Peterson's type of joint, in which the foil is not carried across the joint, would probably tend to give better laboratory tests than joints in which the foil is carried across, because the stresses are lower. From the practical standpoint, however, I feel sure that the metal foil should always be continuous through the joint, if hard or fairly hard joint-filling compounds are used, and that in case of fluid compounds with reservoirs, both types of construction should be carefully considered before a decision is made.

R. G. Hooke: At about the time Mr. Peterson started his development on high-tension cable joints, we were faced with a similar problem. Inasmuch as our solution of it is quite different from his, it seems that some of the details of the study which we have made and the results obtained may be of interest.

We first procured samples of several different types of three-conductor splices being used by various power companies. These were tested in the laboratory and at the same time careful analytical studies were made of the stresses which occurred in each. Mr. Peterson emphasizes the fact that in the past there seems to have been no regard for stress distribution or dielectric-constant relations of insulating materials and this was most certainly true in the majority of the specimens which we obtained. It would be impossible to agree too emphatically with the author's conclusion as to the harmful effects of porcelain spreaders, unreinforced crotches, and carelessly designed barrier tubes with high dielectric constants. By analytical comparisons, it is very easy to see the undesirability of such features.

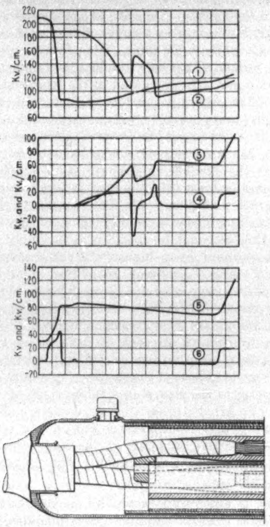


FIG. 2—VOLTAGE STRESSES IN AN OLDER TYPE OF CABLE JOINT

CURVE 1—MAXIMUM STRESS TOWARD SHEATH IN FACTORY INSULATION ALONG THE CORNERS OF THE SECTOR,—KV./CM.
CURVE 2—MAXIMUM STRESS BETWEEN PHASES IN FACTORY INSULATION ALONG APEX OF THE SECTOR—KV./CM.
CURVE 3—MAXIMUM VOLTAGE FROM SURFACE OF THE FACTORY INSULATION TO AN INSTANTANEOUS NEUTRAL PLANE BETWEEN CONDUCTORS—KV.
CURVE 4—AXIAL STRESS ALONG SURFACE OF FACTORY INSULATION BETWEEN PHASES—KV./CM.
CURVE 5—MAXIMUM VOLTAGE ON THE SURFACE OF FACTORY INSULATION TO THE NEAREST POINT ON THE SHEATH—KV.
CURVE 6—MAXIMUM AXIAL STRESS ALONG THE SURFACE OF FACTORY INSULATION NEAREST THE SHEATH—KV./CM.
NOTE: THE ORDINATE OF CURVE 4 IS AT ALL POINTS PROPORTIONAL TO THE SLOPE OF CURVE 3, AND THE SAME RELATIONSHIP EXISTS BETWEEN CURVES 6 AND 5

In our work, we have developed certain methods, more or less empirical, for quick calculation of numerous instantaneous stresses in different parts of the joints and we find it very helpful to represent some of these by curves showing progressive changes in the conditions at different points in the splices, proceeding axially from one end toward the connector in the center. Illustrations of two of these charts will make their use clear.

The first illustration, Fig. 2, shows a joint in which conditions were particularly bad. The very rapid decrease in the maximum gradient to ground, as shown by Curve 1 at the termination of the cable sheath, is an undesirable feature. Also, the "bump" in Curve 2, caused by the presence of the porcelain spreader between conductors, is an obvious point of weakness.

The next illustration, Fig. 3, indicates a very considerable improvement. In this case, the major irregularities are due to the abrupt terminating of a static shield around the three conductors. Elimination of these discontinuities is obviously simple and curves result which are perfectly smooth except for conditions at the steps.

The irregularities on stepped or penciled insulation as shown in these figures require some explanation. The calculations are made along the surface of the original factory paper and therefore, at these points, abrupt increases of voltage necessarily occur due to the decrease in thickness of the factory paper and the consequent nearer approach to the conductor surface. These changes in potential cause the gradients shown, which are really in a radial rather than an axial direction; but they are plotted as part of the same curve as the axial stresses because they occur between adjacent surfaces of insulating material. Comparative effects of different numbers of steps or of different length pencils can readily be studied from these curves.

Of course the figures indicated on these charts are qualificative rather than quantitative, and of interest only in so far as they can be used for graphical indication of the differences between various designs of joints. However, when compared with the careful observations of conditions in the joints after the application on test of high potentials for periods of time up to several days, it was found that in a great majority of cases the weak spots were entirely determinable from analytic study made in advance of tests.

I mention these studies because I think that they emphasize the very great value of a scientific approach to the problem of cable jointing. In fact, so far as the electric field is concerned, splices are very much more complicated and require very much more careful mathematical analysis than does the cable itself wherein the assumptions of a homogeneous insulating medium can be made without appreciable error.

As a result of these studies a joint was determined upon in which the objective was to introduce as few foreign materials as possible. In other words, the more nearly we could make a splice an exact continuation of the cable, the better were we satisfied. With this in view, we concentrated our work upon joints insulated with hand-wrapped paper, the tape being carried well into the crotches at each end and cable compound being applied liberally during the operation. After the conductors have been insulated, the central filler space between the three conductors is filled with paper fillers, compound, and jute. We have found it desirable to fill completely the inside crotches of the joint with paper fillers saved from the outside of the cable after the belt has been removed, it even being possible occasionally to thrust one or two fillers back for an appreciable distance into the cable itself. The outer spaces between the conductors and under the belt are also filled with jute and compound. Jute has been used since it is not feasible to obtain paper satisfactory for this purpose.

At first our procedure was to also fill the spaces between the connectors and the factory insulation with jute. Recently, however, we have adopted the use of wrappings of narrow varnished-cambric tape. In this connection, I am rather surprised at Mr. Peterson's practise of carrying the varnished cambric over the lower edge of his penciled factory insulation. He points out in numerous places the undesirability of having thin layers of low s. i. c. material in series electrically with rather heavier layers of high dielectric-constant insulation. It would seem from this consideration that he might experience an overstressing of the edge of his penciled paper, where it is under the varnished cambric, this being also immediately next to the conductor and therefore at a point where the gradients are at a maximum.

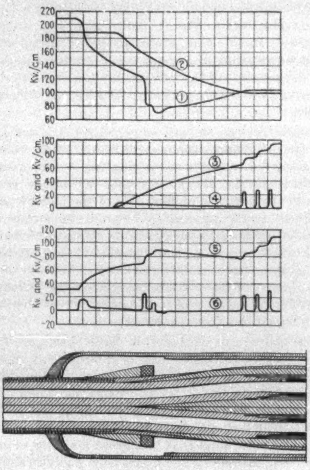


FIG. 3—VOLTAGE STRESSES IN A CABLE JOINT OF RECENT DESIGN

THE EXPLANATION OF THE CURVES IN THIS CASE IS EXACTLY THE SAME AS FOR FIG. 2

We have found that our workmen were much more successful in making steps than they were in cutting a pencil. For this reason, therefore, although the pencil gives the best theoretical stress distribution, we have adopted three steps on each conductor. In making the joint, the varnished cambric at the end of the conductor is not allowed to overlap any of the paper insulation. It is merely used as a filler, its superiority over jute being mainly that it is a higher grade material and less likely to contain foreign particles or moisture.

A paper belt is applied tightly over the three conductors with the fillers in place and over this is used a copper screen which carries the ground-potential surface completely across the splice and is held snugly against the paper belt by a wrapping of heavy wicking. The joint is filled under pressure with a light oil or grease and expansible reservoirs are used. This splice as nearly as possible, is a continuation of the cable and depends entirely for its strength upon impregnated, homogeneous, fibrous insulation.

Perhaps the most important difference between our splice and Mr. Peterson's is our use of hand-wrapped insulating material completely across the joint, whereas in the central part of his splice he depends to a large extent upon the insulating value of the compound. The same feature is brought out in Mr. Peterson's remarks about joints on conductor-sheath cable; that he does not believe that it is desirable to carry the metal sheath of the conductors across the joint. He states that if the metal tape is carried across, the "joint would seem at first sight unsatisfactory * * * since filling compound is of no avail in taking up stress and the total voltage is applied to the comparatively weak path between pencil and hand-wrapped insulation." To us, this is by no means an obvious conclusion. In the first place, over the steps, the zero-potential surface in our joints, whether of the conductor-sheath type with the metal carried across the joint or of the belted type with the copper screen outside of the belt insulation, is very nearly identical with the lead sleeve of the joint. In other words, there is very small clearance between the lead sleeve and the built-up insulation about the splice. Therefore, unless Mr. Peterson uses a larger diameter of sleeve than ours, the leakage path in his joint, either from the connectors to the ground surface or between conductors, is practically the same as it is in ours. The reduction in the length of this path due to the use of the screen about the conductors is exceedingly small. Secondly, there is one very important point in dielectric circuits which it seems to me Mr. Peterson does not sufficiently emphasize. It has been proved conclusively by a number of authorities that the breakdown strength of oil in volts per mil is very much greater for a thin film than it is if the material is not broken up into such films. Mr. Peterson accomplishes this division of the oil into films in his reinforcement of the crotch with varnished cambric, although he speaks of it as simply a replacement of the compound by a higher-strength material. It is actually both of these. However, the filling in of the space which he leaves free for compound between layers of fibrous insulation, in the middle of his splice between which would be distributed liberal amounts of oil or grease, should result in a definite increase in the breakdown strength of the joint at this point. As a matter of fact, some rather hurried and incomplete tests which we have made indicated that a direct puncture can occur from the connector to the sleeve on a splice made up as nearly in accordance with Mr. Peterson's design, as we are able to produce it, whereas, on numerous tests, no fault of this type or any other has ever occurred on joints of the kind which I have described and which we have adopted. The failures are invariably in the cable.

There are one or two questions which I should like to ask the author. He states that the cambric belt is punctured to allow oil to enter the cable where the fillers would normally be. Inasmuch as he does not carry the belt completely across the joint, we wonder that this puncture should be considered necessary. It would seem as if the compound would fill all of the spaces under the belt by entering through the empty filler spaces. Possibly his real objective is to provide an outlet for air bubbles which might collect under this belt insulation. Even this, however, would seem hardly necessary unless one end of the joint was appreciably higher than the other when it was being filled. The use of the static shield outside of the belt reduces to a safe value axial and circumferential stresses which might otherwise be harmful. We question, however, whether or not this shield *eliminates* the circumferential stresses in belted cable as is claimed. It certainly does maintain a zero-potential surface at a distance outward from conductors which is nearly equal to the distance from each conductor inward to the cable axis. This axis, however, for purposes of stress determinations, is not a zero-potential line. True, its *average* potential is zero, but at any instant its potential may not be zero. For example, assuming the three conductors to be carrying three-phase potential, if at a particular moment, the voltage of one conductor is zero, the voltages of the other two conductors will be opposite in sign and equal to 86 per cent of the crest value of the wave. Certainly, the geometric center of the cable would not then be at zero potential and for the moment, circumferential stresses about the conductors would exist. Determination of the magnitude of these stresses in a joint on sector-conductor cable using different types of splicing materials is exceedingly difficult. The fact seems to be that Mr. Peterson, by keeping his ground-potential surface nearer (than in the older joints) to the conductors, as each conductor is gradually moved away from the other two, has reduced the circumferential gradients to values which do not cause trouble; but these stresses are reduced only by virtue of the gradual transition from cable diameter to splice diameter, which is accomplished by the shield, the rate of transition being determined by the rate of separation of the conductors from each other.

We are inclined to question the author's reference to Conducell joints. We believe that the design of the Conducell barrier is excellent, if barriers are to be used at all and we should like to know whether or not Mr. Peterson tried reinforcing the crotches of the Conducell joints in the same way that he does his standard splice. We should expect this to improve greatly his test results on these joints.

In conclusion, I wish to say that the work which Mr. Peterson has done is extremely valuable. His joint is not difficult to make and it possesses a large number of very praiseworthy features. To me, one of the most interesting of these is the idea of a design which permits application of most of the hand-wrapped insulating material before the sweating of the connectors. With the ends of the conductors open, it should be very much easier to apply the insulation and also accomplish a considerable saving in time.

F. A. Brownell: We have tested a number of the author's cable joints between metal-sheath cables and between metal-sheath cable and belted cable, and have had one failure in the metal-sheath cable joint.

In each of the metal-sheath joints tested, evidence of overstressing was indicated by carbonized petrolatum at the end of the metal shield over the varnished-cambric cone and evidence of ionization at the lower edge of the cones.

In the joints where metal-sheath cable was joined to belted cable we found some evidence of overstressing at the surface of the conductors. This type of joint appears to be the best that has been offered for this type of construction.

We have been using the idea of the varnished-cambric cones for the past three years in making end-bells for testing cable in the laboratory, the only difference being that we apply two half sections of lead foil over the formed cone. We find that these sections can be applied in less time and eliminate more voids than the wrapping of narrow widths of foil.

Our test data on joints where the metal tape has been carried entirely across the hand-wrapped insulation are not in agreement with the author's. We have made numerous tests on this type of joint in the laboratory and have never broken down a joint nor opened one that has shown any indications of having been overstressed. In one test, failure occurred in the cable after 24 hours of testing at 125 kv. Another test at the same potential lasted for 20 hrs. with the failure in the cable. These tests do not compare with the author's test of 20 min. at 120 kv. Of the hundreds in service we have very recently had two failures.

To compare a joint for belted cable, we have tested and are now using the nearest thing to cable reconstruction that we believe is possible,—an all-paper joint with jute fillers, gauze stocking for shielding, and filled with the same oil as is used in the cable. This joint was designed a few years ago by Phillip Torchio, of the New York Edison Company, and is known as the Metropolitan joint. We have never broken down one of these joints on test. In one case we held 132 kv. for 25 hrs. when the cable failed. No evidence of overstressing was found in the joint.

We do not think it necessary to maintain the lay of conductors through the joint and we believe that another step in the art will be accomplished when phasing-out in manholes is eliminated.

J. F. Fairman: During the past five years an energetic program of cable and insulation research has been in progress in the Brooklyn Edison Company. The results, though by no means final or complete, have been very gratifying as reflected in the marked improvement in the performance of cable in operation. This improved performance is due both to better manufacture and to refinements in handling, jointing, testing, and maintaining cable by the operating company.

Mr. Peterson has discussed one of the more important contributions to improved operation made by the operating company in describing the evolution of our joint for multiple-conductor, high-voltage cable. I believe it is pertinent at this time to mention briefly another very important feature in this progress,—the installation of oil reservoirs on these joints.

In 1922 in the early stages of the redesign of the transmission system of the Brooklyn Edison Company, it was determined to make 27,000 volts the basic transmission pressure for future developments. Before placing this system in operation however, a great deal of consideration was given to the various factors of cable design and method of jointing and testing cables. Three fundamental factors essential to good performance of a cable system were brought out in this analysis: (1) Thorough initial impregnation; (2) a means of assuring against the formation of voids after installation; and (3) the prevention of entrance of moisture. Good cable could be had at the factory and careful handling gave reasonable assurance against oil leakage and the entrance of air and moisture, but after a cable is in operation with the inevitable cycles of heating and cooling, there is migration and absorption of the compound which, although very slow, produces voids. Vacua of considerable magnitude have been found in cable, and any breaks in the lead sheath or imperfections in the wipe at a joint will allow air and moisture to be drawn into the cable or the joint.

As a result of this analysis, the Brooklyn Edison Company, at the initial installation of 27,000-volt cable in 1922, adopted the policy of installing oil reservoirs on the cable system in the hope of insuring a positive internal pressure above atmospheric pressure throughout the cable system at all times. It is obvious that with such internal pressure and a supply of oil for filling in voids, a break in the sheath would result in bleeding rather than the entrance of air or moisture. Loss of oil in this way could be detected by inspection of the reservoirs.

Such reservoirs were first installed in the stations at transformer and switch potheads. It was found that the migration of oil throughout the cable was so slow that these reservoirs on the potheads were not sufficient to maintain a positive pressure throughout any great length of cable. The next step was to install such reservoirs on a few feeders at the cable joints in the manholes, and the results obtained were so satisfactory that this practise is being extended gradually to the whole 27,000-volt cable system.

Two examples will serve to illustrate the results obtained. One feeder of old cable which had had a fairly good record as to failures, was taken out of service during a period of rearrangement. On testing it before putting it back into service, a number of failures both in cable and in joints, as well as the dry condition of the paper in the sections examined after failure, led us to make an experiment on this feeder. Oil-filled reservoirs were connected to each joint and it was put through periodic heating and cooling cycles for one month at low voltage and 150 per cent rated current. During this process 27 gallons of oil were absorbed, which is approximately equal to 1 per cent of the original compound in the cable. Then the feeder was tested. It went back into service without a failure in the cable, and has been operating steadily ever since. Another old feeder gave so much trouble that the operating voltage had to be reduced to 13,000 volts. Each joint was then equipped with a reservoir filled with oil and operation at 13,000 volts was continued for six months. Following this, it was tested for 27,000-volt operation and has given very satisfactory service at this voltage for over a year. To date 72 gallons of oil, which is approximately 2.6 per cent of the original compound, have been absorbed by this feeder from 120 reservoirs.

W. F. Davidson: Mr. Peterson has called attention to the tests which were made to determine the merits of joints in which metal foil was carried across the connectors and those in which it was cut off at some distance back from the connection. This suggests a point which seems to have received too little attention in connection with cable and joint specifications.

The significance of this was not appreciated until after considerable experimental work had been done. In the early stages it became evident that in order to secure prompt results it was necessary to use cable of superior quality so that the failures would occur in the joint rather than in the cable. However, when field construction work was started a new factor became noticeable.

The ideal condition in a joint or terminal, or for that matter in the cable itself, is to secure strictly radial stresses in the insulation which is under stress. But it is not always possible to do this in joints and terminals. Consequently, we are faced with the necessity of meeting more or less severe longitudinal stress. In their ability to withstand such stress, various cables show extremes of performance. I think it quite safe to say that the range of values obtained under these conditions is far greater than the range obtained with respect to radial stress. As an example, during some recent test on several samples of cable of the metal-foil type, it was found that with an end prepared in the manner described by Mr. Peterson, one sample could be operated for barely 30 hrs. at 120 kv. between conductors before failure occurred and even when it was possible to get failure of the cable under the lead, the ends almost invariably showed signs of severe longitudinal stress. In contrast to this, another sample of cable prepared in the same way and having the same insulation thickness, has operated for over 640 hrs. at 120 kv. without showing the slightest signs of stress.

Mr. DelMar has called attention to surface-tension effects and these probably explain some of the differences just noted. We might expect therefore to reduce troubles of this nature by making sure that the compounds used to fill the joint or terminal were of exactly the same character as that used in the cable itself so as to make a minimum surface tension along the boundary between the cable insulation and the surrounding medium. However, there are obvious and serious practical objections to such a procedure and it seems absolutely necessary to find some other solution to the difficulty.

Summarizing, I wish to urge the need for "jointable" cable. That means not only cable which can stand up under the bending and working essential to making a joint or attaching a terminal, but cable so constructed and impregnated that it has longitudinal as well as radial dielectric strength.

Herman Halperin: About two years ago tests were made in Chicago with the crotch of a 33-kv., three-conductor (sector) cable placed vertically in clear oil in a glass jar with a copper screen at ground potential inside the jar. The observations corroborate those made by Mr. Peterson in that spark discharges were observed across the film of oil in the crotch; but before this occurred, streamers were seen along the tape edges just above the end of the belt insulation. Some of these streamers ran around the conductors. It was found that such discharges could persist for about an hour at 120 kv., three-phase, without leaving visible signs of deterioration in the insulation, and before discharges across the oil in the crotch were noticed. It was also observed that bubbles of occluded air or other gases, which were seen clinging to the insulation previous to the application of voltage, would become dislodged upon the application of the voltage and flow upwards.

In the last page of Mr. Peterson's paper, it is stated that the joints were filled with petrolatum or oil. Experience in Chicago with petrolatum in three-conductor, 33-kv. joints has been unfavorable. About three years ago some 75 three-conductor 33-kv. joints filled with petrolatum at atmospheric pressure were installed. During the following winter several failures occurred in the crotch of the joints. The petrolatum was full of small voids or air spaces the size of a pinhead and had pulled away from the factory insulation of the conductors, causing void spaces around the conductor insulation in the crotch. Apparently the petrolatum contracted on cooling and pulled away from the mill insulation, and the maximum operating temperatures of about 20 deg. cent. were insufficient to melt the petrolatum which had a melting point of 30 deg. cent. Therefore it appears that if a hard or semi-hard compound is used its properties of adhesion to the cable and cohesion should be carefully chosen.

When these joints were rebuilt, they were filled with a thin oil (switch oil) and they gave entirely satisfactory service for the seven months when they operated at 33 kv., although in rebuilding the crotches were not reinforced.

Some experiments were recently started to reinforce the crotches on 500,000 and 650,000-cir. mil, three-conductor, 13-kv. joints. These cables have 9/64-in. insulation around each conductor, which is less than half the insulation on the cable used by Mr. Peterson, and the conductors are considerably larger and stiffer than his conductors. These joints are to be filled with an asphaltic compound that has a Saybolt viscosity of about 750 sec. at 100 deg. cent., but to eliminate the variable introduced by the differences in filling obtained with the semi-hard compound used, oil was employed in these experiments.

The first joints were made with varnished-cambric tape applied around the conductors in the crotch in a fashion similar to that used by Mr. Peterson except that the foil was not used and, in addition, varnished cambric was applied around the three conductors. These joints were tested at 4.5 times the rated voltage for six hours, after which 10 per cent geometric increases were made every hour. The test results have been evaluated for the equivalent voltage for six hours. The joint with no reinforcement failed at nine hours, which test was equivalent to 69 kv. for six hours, using the 7th-root relation that has been developed for breakdown voltage—time characteristic of impregnated paper insulation. The corresponding voltage for the joint with the reinforcement was 75 kv., an increase of 10 per cent, but still failures were obtained in the joint. The failures were found in the crotch region at the end of the belt.

Later, insulation was applied to the conductors only and this was done either before or after the connectors had been sweated. In the few tests that have been made, the joint with the reinforcement applied before the copper sweating has been found to be the stronger. The equivalent voltage for this joint for six hours is about 90 kv., which is 30 per cent better than that found for a joint with ordinary construction.

The fillers were left extending 3 or 4 in. into the joint, and after insulating, a couple of turns of twine were wrapped around the three conductors to hold the fillers in place. Cutting fillers, especially the central one, close to the end of the belt is liable to result in damage to the conductor insulation.

In the discussion of Mr. Simons' paper in the Winter Convention, Mr. Oesterreicher presented* some data on the beneficial effects of flared-out shields applied at the cable lead; and I am wondering whether Mr. Peterson has used anything of that nature on three-conductor joints.

The experience of the Commonwealth Edison Company is in thorough accord with the statement at the end of the paper, that such a joint (or any high-voltage joint) "depends quite largely on the ability of the splicers," and that "little difficulty is experienced in the field when men are properly trained."

A. H. Kehoe: Unanimous agreement will not be had to the statement that a decided improvement has been made over types of joints previously used.

It seems desirable to emphasize that only operating results in service are conclusive and that high over-voltage test methods may be wrongly interpreted. My experience with joint operation is that failures due to poor workmanship, several types of which are mentioned by the author cannot be ignored. One of the most common causes of failure in operation apparently has been omitted; that is, "leaky wipes." Such workmanship defects are now the principal cause for joint failures but, as stated by the author, compound-filling arrangements have reduced the number of these failures.

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*See p. 245.

It does not seem wise to adopt a type of joint which requires very superior workmanship to make it successful, as compared to one in which ordinary careful workmanship will eliminate failure in service. This is likely to take place when high-voltage test results are used exclusively as a criterion for successful joint operation. They assume that factors of safety for all joint elements should have the same value, which is not required, particularly where the factory-formed type of joint insulation is one of the elements. The advantages of factory-formed insulation (the author mentioning Conducell, a well-known joint of this type) are evident in statistics of operation, and testing methods which give stress values beyond the breakdown values of certain of the elements do not demonstrate what the operating results will be if such values are never encountered.

Mention is made of the experiments with wooden spreaders. I can report on this that removal of all spreaders has resulted in a satisfactory mechanical joint and has eliminated an unnecessary element.

The author refers to confirmatory tests for metal-sheath cable which appear to be those made by the company with which the writer is associated. For joints on the metal-sheath type of cable, we have used a joint similar to that described in the paper. At present we know of no better way to make such joints, although possibly some improvement will be forthcoming which will reduce the workmanship hazard now ever present with the hand wrapping of three-conductor joints.

For the belted type of cable we doubt if operating results will show improvements over the factory-formed type, and so are continuing to use the latter. Experience has shown that both the time of making joints and their cost will increase by adopting the type suggested.

T. F. Peterson: Although practically all features of the joint described, together with the bases for design, have received favorable mention by one or another of those contributing discussion, there seems to be no general concurrence of thought or opinion. Possibly this is due to the fact that many have felt the call to defend joint designs with which they have had experience and there has been agreement of thought in just such measure as their designs are similar to the one in question.

Mr. Roper's report on the successful operation of single-conductor joints made up similar to the individual conductors of the joint described is very interesting and gratifying. His description of splicer's schools is quite timely and should serve to impress the importance of systematic training of men in the art of joint construction.

Mr. Eby has brought to our attention his early use of reinforcement of the insulation at the edge of lead sheath of single-conductor cable, overlaid with metal tape. While this means of relieving stress is highly commendable, it is felt that the use of varnished cambric (obtaining advantages of high strength and dielectric constant) instead of impregnated wick or cord, together with its use on three-conductor cable, is an advance. He feels that building up individual conductors of three-conductor belted cable is unessential electrically and cites his experience. This is not in accord with the results presented in the paper by me or in the discussion by Mr. Halperin. I dare say that the use of built-up crotches of test sections in many laboratories, including the Electrical Testing Laboratories, is based on some such observation as the last mentioned.

Undoubtedly the combination of low-loss varnished-cambric tape and stepped insulation described by Mr. Eby is a very good one and might supplant the use of paper tape and conical pencil. However, difficulties with the latter are not so great as intimated. The work is rather quickly and quite accurately done, using a sharp knife and sand paper. The resulting surface is rough and when insulated presents no abrupt changes or finite discontinuities. Low-loss varnished cambric is a comparatively new development, and although it possesses many advantages, its use may not become general until there is more than one source of supply. This is the situation which dictates certain practises despite technical data pointing to the contrary.

The use of barriers for electrical reasons will be discussed later. At this time suffice it to say that in the joint described they are not needed for mechanical stiffening. Short-circuit tests of 20,000 amperes for several cycles have resulted in no appreciable movement of conductors.

Whether the tape of metal-sheath cable should be continued through the joint or not is a matter of concern to many. The reasons for maintenance of it are variously stated; for example:

1. Since the cable is so made—the joint should be.
2. Abrupt discontinuities and transitions are eliminated.
3. Voids may exist or the joint may drain without harm.

The first has no particular justification except in so far as there are incidental advantages which may accrue. The second is accomplished fairly well in the joint offered. As for the third, if voids in compound are expected, by all means short-circuit them. However, in the case of joints which may drain,—that is, lose their oil,—since these will probably fail eventually due to water, drying out or the like, this joint offers very little in favor of carrying the tape through. On the other hand, if conductors can be satisfactorily insulated with tape only, it would seem quite possible to cut down on hand wrapping when oil is brought into use. At least, the entire burden will not be placed on hand wrapping (with its uncertain personal element) but some part will be sustained by the oil.

Mr. Hooke has presented some very interesting data and ideas on joint design, stress distribution, and dielectric circuit theory. The joint which he describes seems to be identical with the so-called Metropolitan type inasmuch as both are largely re-builds of cable. As such, considerable dependence is placed on the hand-made elements of the joint and as Mr. Kehoe points out, this is not always very desirable. In the early work in Brooklyn an attempt was made to eliminate hand operations entirely by insulating conductors with oil and barriers. When this failed, joints such as described were evolved. In these, the human element or personal equation is quite important so far as the hand wrapping is concerned. However, the oil serves as a second line of defense and renders inconsistencies in this less important.

Though Mr. Hooke has found a use for the Brooklyn Edison Company idea of thin strips of varnished cambric between connector and penciled mill insulation, he criticises the over-lapping of the latter with the tape and quotes my statements on the undesirability of having short paths of low specific inductive capacity material in series with long paths of high specific inductive capacity material. He has apparently overlooked the fact that in determination of stress distribution or values of gradients, the entire path from electrode to electrode must be considered to determine flux densities. These, multiplied by 1/s. i. c., give gradients. Obviously, a little varnished cambric (high s. i. c.) over a short path of mill insulation but having in series large amounts of oil, paper, etc. (low s. i. c.) cannot greatly alter the stress in the penciled insulation.

He also questions the use of a conical shield over the crotches of three-conductor belted cable, in eliminating circumferential stress. The basis for his argument is an attempted proof that the cable axis is not at zero potential. Given homogeneity and symmetry of position this must be the case when three-phase voltage is applied. I am at a loss to understand the reasoning in the case cited. When one conductor is at zero potential and the others at plus and minus 86 per cent of maximum potential, the locus of the zero-potential points is a plane half-way between the two conductors and perpendicular to their line of centers. Certainly the geometric axis of the cable falls on this.

Barriers in oil may be considered to produce beneficial results in two ways:

1. Where the free path in oil is broken up so as to reduce the distance in which ionization by collision may take place thus greatly increasing gradient for breakdown; for example, paper and impregnating oil.
2. Where paths are long and breakdown is due to lining up of ions—water, impurities, etc.—barriers prevent this action and so increase the over-all strength.

There remains a range from approximately ¼ in. to about 2 in. in which the use of barriers is questionable. Their s. i. c. is usually high and since their thickness may be no small part of the total path of electric flux, the incidental shifting of stress in oil may more than overcome the advantage of barrier action. In view of this and since operating experience indicated that they were very ineffective in preventing failure where moisture or poor workmanship were present, barriers were eliminated entirely from the joint described. The breaking up of the oil space into very short paths, as is done in the Public Service joint by the use of filling materials, is quite permissible, although one might question whether this done under unfavorable conditions will furnish much of an improvement over the use of good oil.

I also doubt if many will subscribe to Mr. Brownell's idea on the elimination of phasing-out in manholes, although it would seem that this is greatly to be desired in view of the introduction of isolated-phase construction, installation of networks, etc. At any rate, does it not seem advantageous to phase-out in joints (rarely ever more than one in any one location) rather than subject one cable end at the pothead to twisting each time there is need for change?

Mr. Davidson's remarks are a fitting supplement to a paper on joints. Regardless of the type of cable or joint, the introduction of longitudinal stresses is inevitable at the pothead if at no other place. I join with him in a plea for a cable better able to withstand these stresses.

In response to the queries of Mr. Halperin I might say that due primarily to migration, considerable difficulty had been experienced with petrolatum-filled joints, but periodic refilling (at six-month intervals) almost entirely eliminated this trouble. As for the use of flared metal shields at crotches—several of these carefully made of spun brass were tried but the results were no improvement on those obtained with metal tape laid on the built-up conical structure.

The Use of High-Frequency Currents for Control

BY C. A. BODDIE[1]

Associate, A. I. E. E.

THE rapid development of radio has given rise to a parallel development in the art of remote control. Remote and supervisory control is being applied to an ever increasing variety of problems. This control is now commonly effected by the use of special wires connecting the apparatus under control with the point from which control is exercised.

Wires suitable for control purposes are often difficult to obtain. If they are supported for any considerable distance on the same towers as the power line, induction from the power line may seriously reduce their value unless special measures are adopted. If the wires are carried on separate poles on a separate right of way, the cost becomes a formidable item. The other alternative is to lease the necessary circuits from the telephone company. In this case, the rentals are always so high as to be a serious burden on the whole project.

The expense of obtaining and difficulty in operation of special wire circuits have directed attention to the possibilities in the use of alternating currents of moderate or high frequency for control purposes. The object has been to utilize the existing power conductors as a control circuit. This has been accomplished by superimposing on the live power circuit a frequency sufficiently different from the power frequency to permit its being easily separated from the power frequency by suitable tuned circuits. Although this current flows in the power system together with the power current, it is independent of it. It may therefore be used for control purposes.

The application of alternating current to control problems opens up many new fields. The development of apparatus is already quite well advanced. Some of the equipment has been in commercial service for over two and one-half years and has given good account of itself. The applications already developed provide for the control of large main line oil circuit breakers, substation apparatus, and street lights. It is expected that this type of control will be used quite generally for all classes of control now requiring special circuits, where these circuits introduce a serious burden on the project as a whole.

The development has been carried out along two rather distinct lines. The apparatus may be classed according to the frequencies employed as

1. Medium-frequency systems,
2. High-frequency systems.

The medium-frequency system employs frequencies of the order of 500 cycles. This control frequency is so low that it passes through transformers just like power frequencies. The control frequency is generated by a motor-generator set and is fed into the circuit usually by means of condensers. The special advantage of this system is that the line losses are low on account of the moderate frequency used. It is possible to transmit sufficient energy to the receiving devices to directly actuate a relay magnet. This relay is tuned to the control frequency and therefore responds only to this frequency. The relay is simple and sturdy and requires no vacuum tube amplifier. The system is very flexible and is well adapted to quite a variety of applications.

The high-frequency system employs frequencies of the order of 50,000 cycles. Frequencies of this order are most readily produced by vacuum tubes. The energy employed in the high-frequency system is much less than that required in the medium-frequency system. Vacuum tube amplifiers are necessary to amplify the control frequency at the receiving point in order to get sufficient energy to operate a relay satisfactorily. On account of its requiring vacuum tubes, the receiving equipment is much more bulky and more complex than the corresponding equipment of the medium-frequency system. Currents of this frequency do not pass readily through transformers. Its application is therefore limited mainly to operation over high voltage power lines, but it is well adapted to this class of service.

HIGH-FREQUENCY SYSTEM INSTALLED AT TIPTON, INDIANA

Sporadic attempts at control utilizing radio or high-frequency currents over wires date back over a period of 10 or 15 years. These attempts assumed more of a spectacular than of a practical trend. Perhaps the first definite attempt to control the apparatus of a power system by high-frequency currents was made just previous to the opening of the Dresser Power Station of the Central Indiana Power Co. There was then in operation a high-frequency system controlling two 66-kv. circuit breakers at Tipton, Indiana. The control point was Kokomo, Indiana, some 20 miles distant. The 66-kv. line loops through the substation at Tipton. An oil circuit breaker is installed in each branch of the line as it connects to the high-tension bus at Tipton. This is indicated in Fig. 1.

The local power plant which previously supplied the town of Tipton was shut down shortly after the transmission line was built between Indianapolis and Kokomo and power was supplied from the line. The substation at Tipton is about a mile out of town. An operator is not maintained at this point. The object of installing the circuit breakers and the control system was to insure continuous service to the town from either of two power sources, namely, Indianapolis and Kokomo. In case of line trouble on either side of Tipton, the section in trouble could be cut clear by operating the proper breaker at Tipton and the town supplied with power from the remaining section of line. The absence of an operator at Tipton necessitated the installation of the control system.

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[1] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

At the time it became apparent that supervisory control of the Tipton breakers would be desirable, an efficient high-frequency telephone system was already in regular operation over the Indianapolis-Kokomo lines. It was decided to use the transmitter then installed at Kokomo and to install one of the standard receivers and calling selectors at Tipton for the operation of the Tipton circuit breakers. A special frequency was selected so that there could be no interference between the telephone system and the control system. This special frequency could be readily produced since the Kokomo transmitter was already provided with a wave-change switch as a regular part of the equipment, whereby its frequency could be changed from the normal frequency used in telephone service to any other desired frequency. The operation of changing frequencies was accomplished by the operation of the automatic calling dial.

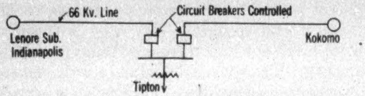

FIG. 1—SCHEMATIC DIAGRAM OF CIRCUIT BREAKER INSTALLATION AT TIPTON, INDIANA

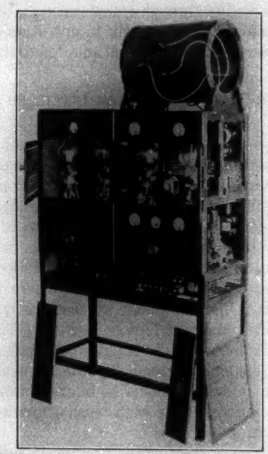

FIG. 2—TYPE OF TRANSMITTER INSTALLED AT KOKOMO

This transmitter is rated at 250 watts and is used for both control and telephone communication

HETERODYNE RECEPTION

The receiver was of the usual coupled circuit type as shown in Fig. 3. The receiver circuits employed were exactly the same as used regularly in the telephone calling system. This utilizes the well-known heterodyne method invented by Fessenden for radio telegraph reception. Because of its superior efficiency and a remarkable ability to ride through serious radio static, it soon displaced all other methods for radio telegraph reception. It was for these same reasons that it was selected as the basis of the calling system for power line telephone communication. The heterodyne method was of course retained for the application to supervisory control.

In the heterodyne system, the incoming high-frequency signal is combined with a frequency generated locally. In this application, it is customary to adjust the locally generated frequency to within about 1000 cycles of the incoming signal frequency. This difference of frequency gives rise to a third frequency equal to the difference between the two main frequencies. This third frequency is commonly called the beat frequency. The second or locally generated frequency is in this case produced by maintaining the receiving detector in a state of continuous oscillation. The beat frequency of 1000 cycles is amplified by a two-step amplifier, the second step of which is adjustable and the output used to operate a polarized relay. This is accomplished by connecting two vacuum tubes in parallel as shown in Fig. 4 and supplying their plate circuit through a polarized relay having a suitable winding. The tubes are provided with a grid leak and condenser just as in the case of an ordinary radio detector. When the amplified 1000-cycle beat frequency is applied to the grids of the relay tubes, a negative charge is built upon the grid condenser in the usual way which gives the grids a large negative bias. This greatly reduces the plate current drawn through the winding of the polarized relay and allows the tension of a spring to close the relay contacts.

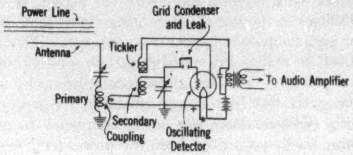

FIG. 3—SCHEMATIC OF RECEIVER USED FOR HIGH FREQUENCY CONTROL

The sector is kept in a state of continuous oscillation and heterodynes the incoming signal

When the control frequency is put on the power line by the Kokomo transmitter, a beat is produced between the oscillating detector and the incoming frequency which, as described above, causes the relay contacts to close. When the flow of current from the Kokomo transmitter is interrupted, the relay contacts automatically open. The calling dial of the sending transmitter is arranged to interrupt the flow of high-frequency current so as to produce a series of impulses. At each interruption, the contacts of the control relay close and thus give a corresponding series of impulses to the selector which it controls.

SELECTOR

The type of selector used is that commonly employed in automatic telephone systems and now so widely used for supervisory control. It consists essentially of an electromagnet commonly called a motor magnet which drives a contact arm over a bank of contacts by means of a ratchet and pawl, as shown in Fig. 5. At each impulse of the radio or control relay, the motor magnet advances its contact arm one step. By a combination of fast and slow relays associated with the motor magnet, the circuit through the contact arm is held open until the proper code sequence is received. After the final pause in the series of code impulses, a slow relay drops out and completes the circuit to perform the desired operation if the correct code sequency has been received. The relay combination is such that during the advance of the selector contact arm a pause must be made at two predetermined points and there must be no interruption, in each group of impulses between the pauses. The total number of impulses must also add up to a predetermined total. Unless all of these conditions are fulfilled, the operating circuit cannot be closed. This interlocking combination is more elaborate than that commonly used in the calling system of standard power line telephone equipment.

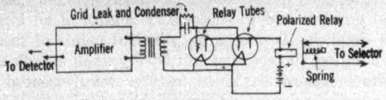

FIG. 4—SCHEMATIC SHOWING METHOD OF OPERATING POLARIZED RELAY

Two vacuum tubes are used in parallel to ensure ample mechanical pull at contacts

The installation at Tipton is perhaps the oldest practical installation of supervisory control using high-frequency currents transmitted over a power line. In its initial stage it involved merely the application of a standard power line telephone system to the service of controlling oil service breakers. It was but a short step from this to the full system with answer-back applying all the well-known functions of modern supervisory control.

ANSWER-BACK WITH INDICATING LAMPS

In order to provide for an answer-back signal, a small transmitting set was installed at Tipton and an additional receiver was added to the equipment at Kokomo. Controllers similar to those used for controlling oil circuit breakers replaced the automatic calling dial of the desk telephone set. These were mounted on a small panel and provided with the usual red and green indicating lamps. Automatic impulse senders were provided at Kokomo for sending the proper code impulses. The code sent out by the impulse sender was determined by the oil switch controller. Thus, to operate one of the Tipton breakers, the Kokomo operator was required to perform only the usual function of operating the controller of a standard oil circuit breaker. This started the automatic impulse sender which sent out a code of impulses corresponding to the particular controller operation performed.

When the code impulses were received at Tipton, the proper control circuits were completed and the desired breaker operation effected. At the completion of any circuit breaker operation a small transmitter was automatically started by means of a similar automatic impulse sender and a code was sent back to Kokomo corresponding to the breaker operation which had occurred. These impulses being received at Kokomo on selectors caused the proper indicating lamps to show on the Kokomo control board. The equipment was also arranged so that in case of doubt the operator could always check the position of the breakers.

IMPROVEMENTS IN MECHANICAL DESIGN

The mechanical arrangement and form of mounting used in the Tipton installation has been changed in later designs. The receiving equipment, instead of being built as a number of independent units and mounted on a table, has been changed to the arrangement shown in Fig. 6A-6B. The equipment is all mounted on panels which are accessible front and rear. The unit construction is still retained. The top panel carries the entire high-frequency equipment. A second panel carries all vacuum tubes and associated apparatus. The third panel carries the rectifier supplying the plate current to the vacuum tube system and the lower panel carries all relay and selective equipment together with the terminal board.

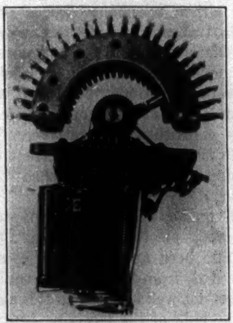

FIG. 5—VIEW OF SELECTOR SHOWING MOTOR MAGNET AND BANK OF CONTACTS

For work over short stretches of power line or over sections of badly exposed telephone line, a small unit has been developed. This is a complete transmitter and receiver and also carries with it sufficient selective equipment for some simple applications. The unit is adapted to either telephone service of supervisory control. The transmitter tubes are shown at the top. A master oscillator and four 7½-watt tubes are mounted so that they may be tied all four in parallel as oscillators for control work, or two may be used as oscillators and two for modulators for telephone service. The lower three tubes constitute the receiver. The unit is supplied with 500 volts of direct current from a dynamotor running on current furnished by a 24-volt storage battery.

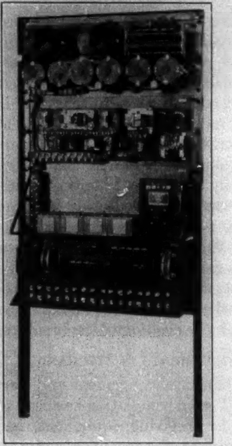

FIG. 6A—FRONT AND REAR VIEW OF PANEL TYPE RECEIVER SHOWING SELECTOR AND TERMINAL BOARD

The high-frequency system is well adapted for control using high voltage power conductors as a circuit. The system has also been applied to the control of series street lights fed from pole type regulating transformers. It is not well adapted to this class of service because of the bulk of the receiving equipment which must be hung on a pole and more or less exposed to the weather. It is not suitable to the control of multiple street lights This is again owing to the bulk and cost of the receiving equipment and to the fact that frequencies of the order of 50,000 cycles do not readily pass through transformers. The system is thus limited to service on high-tension lines.

MEDIUM-FREQUENCY SYSTEM

As early as 1901, Mr. Rhodes of the New York Edison Co. proposed to turn multiple street lights on and off by superimposing a 500-cycle control current on the power circuits. The early work did not show much promise and the project was dropped for some years. About four years ago the project was reopened and promising results obtained from preliminary work on the overhead system at Yonkers, New York. Subsequent development was carried out on the underground system of the Fordham substation of the New York Edison Co.

While the system was developed primarily for the control of multiple street lights, it has been found applicable to a wide variety of control problems. It is being applied to the control of street lights both series and multiple and to supervisory control of all kinds. Its simplicity is one of its principal points of merit. In addition, its receiving unit is very small in bulk and quite inexpensive. These features were essential to its success in the field of street light control.

The moderate-frequency system may possibly be best understood by discussing its application to street light control. Fig. 7 is a schematic diagram of a substation showing the method of energizing a single feeder. Feeders may be energized one at a time as in the schematic diagram, or in groups, or the entire bus may be energized according to the method of operation preferred. The control frequency is produced by the generator G. This is a rotating machine of a standard type, driven by a two-speed induction motor whose synchronous speeds are 1200 and 1800 rev. per min. This gives control frequencies of 440 and 660 cycles at synchronism. The motor is especially designed for low slip which is approximately 1½ per cent. Condensers of the oil-filled type similar to those regularly used for power factor correction are employed to couple the

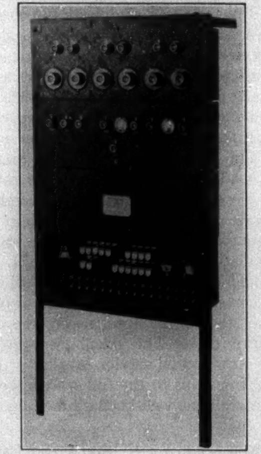

FIG. 6B—FRONT AND REAR VIEW OF PANEL TYPE RECEIVER SHOWING SELECTOR AND TERMINAL BOARD

generator to the line. Inductance coils are provided to tune the circuit as a whole. The power circuit presents a low impedance when viewed from the generator terminals. Hence a coupling transformer of suitable ratio is interposed between the generator and the tuned circuit to enable the generator to deliver its full output into the power system.

METHOD OF ENERGIZING FEEDER

When energizing a single feeder, it is preferable to connect on to the feeder just beyond the feeder regulator and reactor, as indicated in Fig. 7. This will be apparent when it is observed that current delivered by the generator through the tuned circuit to the feeder has two paths in which to flow. It may flow out along the feeder through the numerous distributing transformers and it may also flow back into the station bus and through the large station transformers. If the generator is connected beyond the reactor and feeder regulator, the impedance of these two elements is interposed in the path of the current flowing back through the station bus and less generator current is required to energize the feeder.

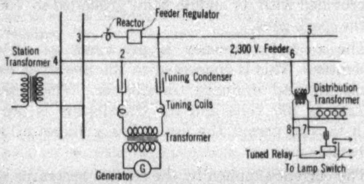

FIG. 7—SCHEMATIC DIAGRAM SHOWING METHOD OF ENERGIZING FEEDER FOR MEDIUM FREQUENCY CONTROL

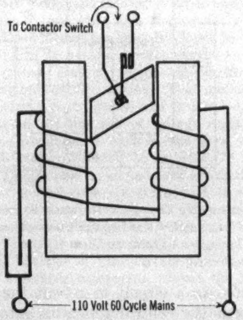

FIG. 8—SCHEMATIC DIAGRAM OF RESONANT RELAY

The control currents flow along the conductor just as though the power currents were not present. Sufficient current is fed into the system to establish a control frequency potential of approximately 100 volts at the outgoing terminals of the feeder. This control frequency potential acts throughout the whole length of the feeder in the same manner as the power frequency but it is quite independent of it. The various distribution transformers supplied by the feeder, step this control voltage down in the same ratio as they do the power voltages. Thus, with 100 volts of control frequency on the high side of a 2200-volt distribution transformer, five volts are delivered on the 110-volt side. It is this voltage which is available for the operation of the control relays.

At any point on the system where control is desired, a control relay is located as indicated in Fig. 7 and connected to the 110-volt side of a distribution transformer. It consists of a simple U-shaped magnet acting on a balanced armature as shown more clearly in schematic diagram, Fig. 8. A contact is mounted on the armature shaft and arranged to close when the relay is energized. A condenser is placed in series with the relay winding. The inductance of the winding is designed so that the inductive reactance of the relay is exactly equal to the capacity reactance of the condenser at the frequency the relay is intended to operate on. The total reactance of the relay circuit including its condenser is therefore zero, and the control current passing through the relay is governed by Ohm's law, thus:

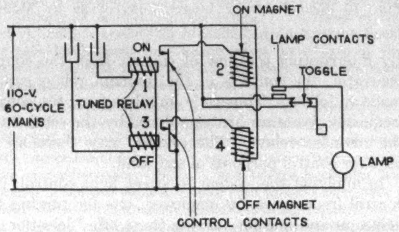

FIG. 9—SCHEMATIC DIAGRAM OF STREET LIGHT CONTROL UNIT USING RESONANT RELAYS

$$I = \frac{E}{R}$$

where E is the value of the control voltage impressed across the relay circuit, R is the effective resistance of the relay circuit, and I is the current flowing through the relay winding. When a circuit is adjusted so that its reactance is zero, it is said to be in tune or in resonance for this particular frequency.

APPLICATION TO STREET LIGHT CONTROL

In its application to street light control, it is clear that the lower the effective resistance of the relay circuit, the more energy will be available for its operation. This is owing to the low impedance of the supply circuits; the principal difficulty encountered in the development of the relay was in keeping its losses sufficiently low. By careful selection of materials and proper proportioning of the magnetic circuit, these losses were cut well under the values necessary for commercial operation. It was the successful development of these low loss relays which made it possible to draw sufficient energy from the power system to operate a pair of contacts by the direct magnetic pull of the control current itself.

It will be apparent from the foregoing that when the feeder is excited by the control generator, all relays tuned to the control frequency will close their contacts.

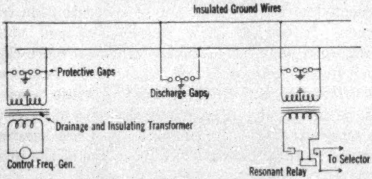

Fig. 10—Series Street Lights Controlled by Medium Frequency System

By interrupting the flow of current from the control generator, the contacts of the resonant relays can be made to follow. Thus code impulses from the control frequency generator are reproduced by the contacts of the resonant relays. The system may therefore be used for control purposes of any kind.

In applying the system to street light control, two control frequencies are employed, one for turning the lights on and one for turning them off. Selection by using different frequencies is preferred in this case because of its simplicity. This requires two relays at each control point, one resonant to each control frequency. The contacts of the resonant relays are not required to carry the lamp current, but merely to throw a toggle switch which is provided with heavier contacts to carry the lamp current. Fig. 9 shows a schematic diagram of the two resonant relays and toggle switch as used in street light control. These elements are assembled in a weather proof case. The whole control unit is small enough to permit its being installed on the base of most ornamental street light posts.

In its application to the control of series street lights, the control unit governs the position of an oil switch in the primary side of a pole type regulating transformer as in Fig. 10. In this case, the feeder potential is usually 2300 volts and a potential transformer is necessary to supply 110 volts to the control unit. This transformer furnishes power frequency energy to operate the oil switch and control frequency energy to operate the resonant relays.

Application to Supervisory Control

Supervisory control by means of moderate-frequency currents and resonant relays has recently been applied to a rather new problem. This is to provide for the control of numerous sectionalizing switches and some oil circuit breakers on a long 110-kv. line. The circuit over which the control system operates is rather novel. This circuit is obtained by insulating the ground wires ordinarily provided on a long high-tension line. The protective feature of the ground wires is not sacrificed appreciably owing to the installation of spark gaps at frequent intervals which provide a discharge path to ground. The sectionalizing switches to be controlled are installed at 15-mile intervals. At each point where a sectionalizing switch is located, a drainage transformer is provided with its middle point grounded in addition to the spark-gaps.

The control frequency is produced by a ½-kw. generator. This is connected to the line by a step-up insulating and drainage transformer. The generator voltage is 100 volts and the transformer is provided with taps to permit the use of line voltages of 300 to 500 volts.

The current supplied by the control generator is controlled by a standard system of supervisory control. The output of the generator is thus broken up into impulses and these impulses are received by resonant relays at the points where switches are to be controlled. The line voltage of 500 volts is stepped down to 10 volts by the insulating drainage transformer at each sectionalizing switch. This voltage is used to operate the resonant relays. The contacts of the resonant relays are connected in a circuit to repeat the impulses rereceived from the control generator into the selector system.

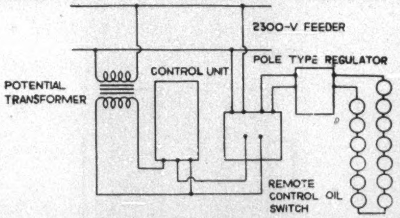

Fig. 11—Schematic Diagram Showing System of Medium Frequency Control Using Circuit Consisting of Insulated Ground Wires

The selector system employs a combination of fast and slow relays together with associated motor magnets and contact banks as ordinarily used in automatic telephony and now adopted as standard practise for supervisory control. The complete supervisory system and resonant control relays are mounted on a switchboard section one panel wide. The type of line construction and the location of the spark-gaps and drainage transformers is shown in Fig. 11.

This application of medium-frequency alternating currents to the control of sectionalizing switches is likely to find wide application on long transmission lines where conditions will not justify a double-circuit line. By this means it is expected to reduce very greatly the interruptions to service arising from the use of a single-circuit line. The system may also have important applications on double-circuit lines and assist materially in the solution of control problems of all kinds.

Discussion

Chester Lichtenberg: One thing that is disturbing to the designers, as well as the operators, is the high cost of the equipment required for carrier-current supervisory control. At present the cost of wires is large; however, the terminal apparatus required for radio supervisory equipment is also large. It is therefore necessary to make a quite careful analysis of the situation before a reasonable conclusion can be drawn as to whether or not it is cheaper to use supervisory equipment employing wires or supervisory equipment using carrier current. From some studies which we have made, the wires seem more economical for distances up to about 25 mi., beyond that distance however, the carrier-current design becomes feasible.

The ambition of radio engineers seems to be to put supervisory equipment on high-tension transmission system. This is a splendid idea. It should be recollected however, that the supervisory equipment is most essential during times of stress, and such times will almost always coincide with high-tension transmission-circuit interruptions. Following this idea further, it is gratifying to note that the Alabama Power Company instead of using its high-tension transmission system, uses insulated ground wires. This follows in general the practise of the Chicago, South Shore, and South Bend Railway Company which uses a single wire for the control and indication of eight automatic substations.

L. H. Junken: (communicated after adjournment) A system of street-light control using high frequencies superimposed upon the distribution feeders has been developed by the General Electric Company in its general development of high-frequency control. This system uses a vacuum-tube oscillator for the generation of high-frequency power. This oscillator has an output of approximately 100 watts and is coupled to the distribution feeders on the line side of voltage regulators at the substation. The oscillator is built in the form of a switchboard panel and is approximately 33 in. wide, 76 in. high and 15 in. deep. The coupling capacitors used are small due to the small amount of control energy required and to the high frequency used which is about 40 kilocycles. The coupling capacitors are permanently installed on each feeder over which control is to be sent. The oscillator can then be switched from feeder to feeder on the low side of the coupling capacitors by means of low-voltage switching equipment. Since the frequency is high, the possibility of telephone interference is avoided and the presence of voltage regulators between the oscillator and the station bus prevents any appreciable amount of energy from flowing toward the bus and insures a maximum amount of energy flowing toward the feeder. This is a very economical arrangement, because it uses very small amounts of control energy.

The receivers are located along the feeders where pole-type constant-current transformers are to be controlled. The receivers are coupled to the feeder through small coupling capacitors which can be mounted on the crossarm in a way similar to that used for mounting lightning arresters. The receivers use a single vacuum tube for the detection of the high-frequency control current. This tube is operated at reduced rating and has a life of approximately one year. All of the power for the filament, grid, and plate potentials of the vacuum tube is taken from the secondary lines, no batteries or rectifier being required. The whole receiver is enclosed in a weather-proof sheet-steel box arranged for mounting on a pole. The receiving tube operates a d-c. relay in its plate circuit and this relay in turn operates a time-selector relay which controls the position of the switch in the primary of the constant-current transformer. The time-selector relay is so arranged that it closes the lighting circuit on a short impulse of control energy and opens the circuit on a longer impulse of control energy. The receivers are built to receive a suitable band of frequencies and require no tuning or other adjustments after their installation.

This system of control has been installed in service for street lighting in Schenectady and Rochester, N. Y., also Bayonne, N. J., for about two years and has given very satisfactory performance.

A. H. Kehoe: (communicated after adjournment) While the author mentions the control of multiple street lighting by high-frequency relays, the section of the paper covering applications of street-lighting control deals primarily with series lighting circuits. This is probably due to the fact that the multiple type of street lamp has not been extensively used in the past except in a few large cities. As there are economical advantages to be gained by using this type, provided a satisfactory method of control exists, it should be emphasized that the principal disadvantage of the multiple type of lighting in the past has been the difficulty of control. In the future, comparisons of the series vs. multiple systems for street-lighting applications should take this development into consideration, as it extends the field where multiple street lighting will prove to be the more economical of the two types of system.

H. M. Trueblood: (communicated after adjournment) Mr. Boddie's paper and other recent ones along similar lines serve to call our attention to the extent of the range of application of the carrier-frequency art. It was only about ten years ago when carrier-frequency telephone and telegraph circuits first appeared in the Bell System. Since then we have seen, first, a rapid development of carrier frequencies in commercial telephone and telegraph circuits; second, the use of carrier frequencies superposed on power transmission lines for the private communication services of power companies; and we now have the announcement of working applications of carrier frequencies for control purposes, a field which Mr. Boddie expects will be extensively widened.

In making note of these achievements we do so with the satisfaction any electrical engineer must feel in the furthering of the applications of electricity to the useful arts. We ought, however, to realize that the development and application of carrier frequencies along these three lines and others which may later appear, need to be carried out with proper regard to the possibilities of interference. In the development of commercial carrier-frequency telephone and telegraph circuits, where different systems are carried on the same pole line, engineers of the Bell Telephone System have been made fully aware of the difficulties of the interference problem, which appears here as a cross-induction problem more formidable than at voice frequencies. Methods which have been used to deal with it have included special attention to circuit balance, the suppression of the carrier wave itself, the use of a single side band, the selection of appropriate power levels, and a system of frequency allocations within the frequency range employed. No other methods than these appear to be open to us for dealing with the problem of induction in commercial communication circuits from carrier-frequency circuits as applied for control or communication purposes in connection with power lines, and the most fundamental of these methods at present is undoubtedly frequency separation. In fact, unless carrier-frequency circuits for communication or control purposes in connection with power-circuit operation can be made to approach commercial communication circuits as regards balance and energy levels, I know of no means of avoiding interference in situations of close exposure other than frequency separation.

There is no serious situation at present as regards interference in commercial carrier-frequency communication channels from power lines, or any of the carrier-frequency applications used in connection with power-system operation, so far as I know. I believe that those of us who are working in carrier-frequency development in its various aspects should see that this fortunate condition continues. It is of interest in this connection to refer to the work of one of the project committees organized under the Joint Development and Research Subcommittee of the N. E. L. A. and Bell Telephone System which has been charged with, and is actively pursuing, a study of the problem of interference in carrier-frequency channels.

C. A. Boddie: I fully agree with Mr. Lichtenberg when he states that the application of high-frequency currents for control purposes is at present limited considerably by the high cost of the equipment. It is only on relatively long distances that there is a sufficient saving to justify the use of high-frequency currents so far as first cost is concerned. A good deal of work is being done in an effort to reduce these costs and it is hoped that a considerable improvement can be effected. But it is not the matter of first cost alone which is the chief consideration and stimulus to the use of high-frequency currents. It is rather the need of a mechanically more sturdy and more stable circuit than is attainable with telephone-line construction. It is this mechanical strength which can withstand sleet and storm and flood conditions inherent in power-line construction that offers the chief inducement to the use of high frequencies on such a line for communication and control.

The type of control described by Mr. Junken using relatively high frequencies generated by vacuum tubes is referred to in the paper. This type as stated is not suited to the control of multiple street lights, because such high frequencies will not pass through transformers and because of the size and cost of the individual receiving devices, each of which requires a vacuum tube and accessories. The system has a legitimate field of application for the remote control of regulating transformers for series lighting, since for this application relatively few receiving devices are required and the size and cost per unit is not a serious objection. The control currents are not required to pass through transformers as in the case of multiple lighting, but are taken off the high-voltage feeder by means of special coupling condensers.

I am glad Mr. Kehoe has emphasized the fact that the development of the system of control by the use of medium frequencies puts a new complexion on the problem of multiple vs. series street lighting. As he says the principal obstacle to the use of the multiple system has been the difficulty of control. In large cities the multiple system of distribution reaches everywhere. To supply street lights by the series system means a system within a system; that is, "duplication." The economic advantages of the multiple system which is universal for everything except street lighting may now be secured. The medium-frequency system of control was developed with special reference to the problem of controlling multiple street lights. Although as brought out in the paper it can be applied to the control of series lighting, its principal field of application is to multiple lighting. Perhaps this was not brought out in the paper as clearly as it should be.

With reference to Mr. Trueblood's discussion on the subject of interference, I might state that so far as street-light control is concerned he will have little to fear. The development has not overlooked the question of interference with existing telephone lines. In the first place, the system is worked out so that the control frequency is applied to the line only twice a day and this is only for a period of several seconds. One of these operating periods comes at a time when interference if there should be any, would be of little consequence. In the second place, the control currents are confined entirely to the metallic conductors of the power system and are not permitted to return by way of ground. In the several installations which have been made there is no known interference with the telephone system, even that in the substation itself being immune.

Electromagnetic Waves Guided by Parallel Wires
With Particular Reference to the Effect of the Earth

BY S. A. LEVIN*
Associate, A. I. E. E.

Synopsis.—*A theory of the propagation of electromagnetic waves guided by a system of parallel wires is developed with particular reference to the effect of the earth, and is simplified so that it is identical in form with the elementary theory.*

Important general properties of a system of parallel conductors and their application to problems of propagation in power or communication circuits, or in a system of both types of circuits, derived by Professor Pleijel from the elementary theory, are mentioned together with several remarks of practical importance. The simplified theory leads to the same conclusions and, consequently, to the same applications.

The simplified theory gives promise of successful application to such problems of propagation in transmission systems which heretofore as a rule have not been formulated with sufficient precision.

INTRODUCTION

FOR the propagation of electromagnetic waves along any number of parallel wires of any configuration, mainly the elementary theory[1,2,3] has been employed†. Usually the earth is considered as very remote or as a perfect conductor. Rigorous theories have been developed in special cases for conductors in free space, as well as with proper regard to the influence of the earth. Earlier literature is found in reference 4; some recent work is mentioned in references 5 to 17, inclusive. A theory of propagation on a system of parallel wires, with particular reference to the effect of the earth, is developed in the theoretical part of this paper. To make this theory practically applicable, certain assumptions are made in the discussion. The simplified theory is identical in form with the elementary theory.

In a paper based upon the elementary theory Professor Pleijel[18] has derived general properties of a system of parallel wires and pointed out their application to several problems connected with the propagation of electromagnetic waves along power or communication circuits, or systems of both. The section of the present paper dealing with applications of the theory shows that these properties and their applications follow also from the simplified theory mentioned above and, in addition, contains remarks of practical interest.

It is believed that the simplified theory will permit application to many of those problems regarding electromagnetic wave propagation in transmission systems which previously were often not formulated with a sufficient degree of accuracy.

THEORY

Consider v wires of circular cross-section parallel to the z-axis, (see Fig. 1). Let the x z-plane separate the two media A, the air, and B, the earth. The conductors may be located in one or both of these media. Some may be of copper, some of iron, or any other material. All the conductors in the earth must be insulated. In the air, some or all of them may be covered with insulation. Some conductors may be arranged so as to correspond to a cable, but some of the following considerations do not strictly apply to such an arrangement. The conductors, the media A and B, and the insulating materials are supposed to be homogeneous and isotropic.

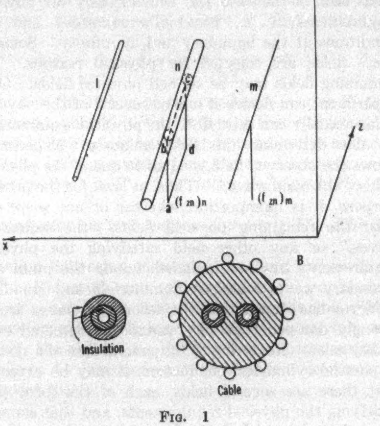

FIG. 1

The separations between the conductors, and their distances from the surface of the earth, are supposed to be so large compared to the radii of the wires that the electromagnetic field inside each wire is symmetrical about the axis of the wire. The insulation of the wires is supposed to be so thin that it can be neglected. The fundamental assumption is that at a given frequency f

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[1] See references at end of article.

†See reference 21. This paper, however, was not known to the author when the present theory was developed.

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the electromagnetic state* of the system can be represented, except at the ends, by a number of traveling waves. Only one of them with the propagation constant Γ will be considered, unless otherwise specified. The components of the corresponding electric and magnetic field vectors, F and H, respectively, are the real parts of the complex expressions, obtained by multiplying

$$F_x(x, y),\ F_y(x, y),\ F_z(x, y),\ H_x(x, y),\ H_y(x, y),\ H_z(x, y)$$

by the exponential factor $e^{-\Gamma z + j\omega t}$ where t is the time, $\omega = 2\pi f$ and $j = \sqrt{-1}$. The factors F_x, etc., are represented by different analytical expressions inside each wire, in the air, and in the earth. They satisfy the boundary conditions at the surfaces of the wires and at the surface of the earth.

Regarding the justification of the assumption of a field with the components

$$F_z\,(x, y)\, e^{-\Gamma z + j\omega t} \quad . \quad . \quad . \qquad (\mathbf{A})$$

the following may be noted: Considering first the simple case of a single conductor of circular cross-section in free space, it has been shown by Sommerfeld[22] and Hondros[23] (see also references 24, 25, and 26 for other important contributions) that there are several fields each of the form (**A**) which satisfy the physical requirements; *i. e.,* Maxwell's equations and the conditions at the boundary and at infinity. Some of these fields are rejected for physical reasons. The remaining fields may be termed physical fields. Only experiment can decide if one or several of the physical fields actually can exist since the physical requirements do not determine the field uniquely. Experiment shows the observed field is equal to one of the physical fields, "the main wave." Thus, at least for the present purpose, it is unimportant whether or not there can exist the remaining physical fields, "the secondary waves," or any other field satisfying the physical requirements but not included among the main and secondary waves found by Sommerfeld and Hondros. Hondros has shown that the secondary waves are so strongly damped that they cannot be detected even if they exist. Reverting to the general case of a system of parallel cylindrical conductors, it may be expected that there are several fields, each of the form (**A**), satisfying the physical requirements, and that some of these fields are physical fields. Experiment may be expected to show that the observed field is equal to a field which is the resultant of certain of the physical fields, "the main waves." It may not be important to attempt a proof of these expectations if a number of fields are found which approximately satisfy the physical requirements and adequately account for the phenomena observed.

The z-component of the electric current in the wire n equals $\int \gamma_n F_z\, d s$ multiplied by $e^{-\Gamma z + j\omega t}$. Let

$$\int \gamma_n F_z\, d s = I_n \qquad (1)$$

Hereafter the exponential factor $e^{-\Gamma z + j\omega t}$ will be omitted for the purpose of simplification. The conductivity of the wire is γ_n. The integral is extended over the cross-section of the wire, and its value, I_n, is independent of the location of the cross-section along the wire. The electromagnetic field everywhere outside the wires, *i. e.,* in the air and in the earth, would be the same as the field that would be produced there by currents I_n concentrated at the axes of the wires, if each wire were removed and replaced by the medium surrounding it. Consider an element of the concentrated current I_n. It produces an electromagnetic field that satisfies Maxwell's equations at all points in space and also satisfies the boundary conditions at the surface of the earth.[19] The same is true for the resultant field produced by any combination of elements. Let F_n, H_n be the vectors of the electromagnetic field produced outside the wires by all the elements of the current I_n alone. The components of these vectors are $F_{xn}\,(x, y)$, etc., multiplied by the exponential factor. The component H_{zn} can be neglected.† This permits of the definition of single-valued electric voltages and magnetic fluxes. Maxwell's equations are (elm. c. g. s. units),

$$\left.\begin{aligned} \operatorname{curl} H_n &= 4\,\pi\,\gamma\, F_n + K\,\frac{\partial F_n}{\partial t} \\[2ex] \operatorname{curl} F_n &= -\,\mu\,\frac{\partial H_n}{\partial t} \end{aligned}\right\} \qquad (2)$$

where the constants

γ = conductivity
K = dielectric constant
μ = permeability

have different values in the air and the earth. It follows from these equations and $H_{zn} = 0$ that

$$\left.\begin{aligned} F_{xn} &= \frac{\Gamma}{4\,\pi\,\gamma + j\,\omega\,K}\,\frac{1}{\alpha}\,\frac{\partial F_{zn}}{\partial x} \\ F_{yn} &= \frac{\Gamma}{4\,\pi\,\gamma + j\,\omega\,K}\,\frac{1}{\alpha}\,\frac{\partial F_{zn}}{\partial y} \\ H_{xn} &= -\,\frac{1}{\alpha}\,\frac{\partial F_{zn}}{\partial y} \\ H_{yn} &= \frac{1}{\alpha}\,\frac{\partial F_{zn}}{\partial x} \end{aligned}\right\} \qquad (3)$$

where

$$\frac{\partial^2 F_{zn}}{\partial x^2} + \frac{\partial^2 F_{zn}}{\partial y^2} = (4\,\pi\,j\,\omega\,\mu\,\gamma - \omega^2\,\mu\,K - \Gamma^2)\,F_{zn}$$

and (4)

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*Due to sinusoidal electromotive forces, of the frequency f, at the conductor terminals when the steady state is attained.

†See, for instance, reference 21.

$$\alpha = \frac{4\,\pi\,j\,\omega\,\mu\,\gamma - \omega^2\,\mu\,K - \Gamma^2}{4\,\pi\,\gamma + j\,\omega\,K} \qquad (5)$$

The field is proportional[19] to I_n.

Let

$$F_{zn} = f_{zn}\,(x, y)\,I_n \qquad (6)$$

The function f_{zn} satisfies equation (4) and consequently contains Γ^2. It is represented by different expressions in the air and in the earth, but it is continuous at the surface of the earth. For a given wire n the same expressions apply for all waves. Thus, two waves on the same wire with different values of Γ^2 give different values of f_{zn} only because the Γ^2 values are different. This will be expressed by saying that f_{zn} depends upon Γ^2. The value of f_{zn} at the $x\,z$-plane vertically below (or above) wires n and m is $(f_{zn})_n$, and $(f_{zn})_m$, respectively, see Fig. 1.

The z-component of the total electric field at the surface of wire n is

$$z_n\,I_n \qquad (7)$$

where z_n is the internal impedance of the wire and is practically independent of Γ^2.

The magnetic flux per unit length through the loop $a\,b\,c\,d$, see Fig. 1, due to I_n, equals

$$\int_0^{h_n} \mu\,H_{xn}\,d\,y = \int_0^{h_n} - \frac{\mu}{\alpha}\,\frac{\partial f_{zn}}{\partial y}\,I_n\,d\,y = (\phi_n)_n\,I_n \qquad (8)$$

where $h_n = a\,b$ is the distance from the surface of the wire to the $x\,z$-plane, counted positive when the wire is above the $x\,z$-plane. The flux per unit length through the loop due to I_m is

$$\int_0^{h_n} \mu\,H_{xm}\,d\,y = \int_0^{h_n} - \frac{\mu}{\alpha}\,\frac{\partial f_{zm}}{\partial y}\,I_m\,d\,y = (\phi_m)_n\,I_m \qquad (9)$$

Both $(\phi_n)_n$ and $(\phi_m)_n$ depend upon Γ^2.

The voltage to ground of wire n equals,

$$\int_{h_n}^{0} F_y\,d\,y = \int_{h_n}^{0} \frac{\Gamma}{4\,\pi\,\gamma + j\,\omega\,K}\,\frac{1}{\alpha} \sum_{n=1}^{v} \left(\frac{\partial f_{zn}}{\partial y}\,I_n \right) d\,y$$
$$= V_n \qquad (10)$$

The usual application of the second equation of (2) gives

$$\Gamma\,V_n = Z_{nn}\,I_n + \sum_{m \neq n} Z_{mn}\,I_m \qquad (11)$$

where

$$\left. \begin{aligned} Z_{nn} &= z_n - (f_{zn})_n + j\,\omega\,(\phi_n)_n = R_{nn} + j\,\omega\,L_{nn} \\ Z_{mn} &= -\,(f_{zm})_n + j\,\omega\,(\phi_m)_n = R_{mn} + j\,\omega\,L_{mn} \end{aligned} \right\} \quad (12)$$

All terms in (12) depend upon Γ^2. Z_{nn} is the self-impedance of wire n. Z_{mn} is the mutual impedance between wires m and n when m carries the current. $R_{nn}\,(L_{nn})$ is the self-resistance (inductance). R_{mn} (L_{mn}) is the mutual resistance (inductance) when m carries the current. All impedances are per unit length.

If (V) and (I) are the vector voltage and current, respectively, as they are understood in ordinary a-c. theory, it follows from equation (11) that

$$-\frac{\partial\,(V_n)}{\partial\,z} = Z_{nn}\,(I_n) + \sum_{m \neq n} Z_{mn}\,(I_m) = \sum_{m=1}^{v} Z_{mn}\,(I_m) \qquad (13)$$

Briefly, this 'is explained as follows. Note that all quantities in equation (11) are complex quantities. Put $V_n = V_{n1} + j\,V_{n2}$, etc. Multiply equation (11) by $e^{-\Gamma z + j\omega t}$, calculate the real parts of the expressions on both sides of the equality sign and put them equal to each other. The equation thus obtained will contain two expressions on the left side of the equality sign, each containing a factor one of which is the time-derivative of the other. A similar remark applies to the expression to the right of this sign. It is seen that the equation gives a relation between the instantaneous voltage and current values which are the real parts of $V_n\,e^{-\Gamma z + j\omega t}$, etc., so that the vector expression on the right in equation (13) equals the vector $\Gamma\,(V_n)$. Finally, the derivative with respect to z of the instantaneous voltage shows that the vector relation

$$\Gamma\,(V_n) = -\frac{\partial\,(V_n)}{\partial\,z} \text{ is true.}$$

From the usual application of the fact that the divergence of the sum of the conduction and displacement current is equal to zero, it follows that

$$\Gamma\,I_n = 2\,\pi\,r \left(\gamma + \frac{j\,\omega\,K}{4\,\pi} \right) F'$$

where r is the radius of the conductor n and F' the component, normal to the surface of the conductor, of the total electric field immediately outside this surface. The total leakage current from the conductor per unit length is $2\,\pi\,r\,\gamma\,F'$. The total electric charge on the conductor per unit length is $2\,\pi\,r\,\dfrac{K}{4\,\pi}\,F'$. The intersection between a plane $z =$ constant, the surfaces of the conductors, and the surface of the earth consists of a number of circles and a straight line. The voltage on each circle is constant. If, as an approximation, the voltage on the line is assumed constant, it follows by comparison with familiar electrostatic methods* that

$$\Gamma\,I_n = \sum_{m=1}^{v} \beta_{mn}\,V_m$$

where $\beta_{mn} = \beta_{nm}$ and both are complex coefficients independent of Γ^2. Consequently

$$-\frac{\partial\,(I_n)}{\partial\,z} = \sum \beta_{mn}\,(V_m) \qquad (14)$$

Equation (14) can also be written

$$-\frac{\partial\,(I_n)}{\partial\,z} = (A_{nn} + j\,\omega\,C_{nn})\,(V_n) +$$

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*Neglecting[7], in the dielectric, $\Gamma\,F_z$ in div. $F = 0$.

$$\sum_{m \neq n} (A_{mn} + j\,\omega\, C_{mn})\,[\,(V_n) - (V_m)\,] \qquad \textbf{(15)}$$

where the coefficients A and C are the leakages and capacities per unit length, respectively.

If

$$Z_{mn} = Z_{nm} \qquad \textbf{(16)}$$

then

$$\left.\begin{aligned} R_{mn} &= R_{nm} \\ L_{mn} &= L_{nm} \end{aligned}\right\} \qquad \textbf{(17)}$$

Since

$$\beta_{mn} = \beta_{nm} \qquad \textbf{(18)}$$

thus

$$\left.\begin{aligned} A_{mn} &= A_{nm} \\ C_{mn} &= C_{nm} \end{aligned}\right\} \qquad \textbf{(19)}$$

The general validity of equation (**16**) has not been investigated here.[15] In the following case the equation holds. In Fig. 1 let $n = 2$. The flux per unit length through the loop $a\,b\,c\,d\,a$ due to the current $I_1 = 1$ is

$$(\phi_1)_2 = \int_0^{h_2} - \frac{\mu}{\alpha}\,\frac{\partial f_{z1}}{\partial y}\,d\,y$$

The voltage to ground at wire 2 due to $I_1 = 1$ is

$$V = \int_{h_2}^{0} \frac{\Gamma}{4\,\pi\,\gamma + j\,\omega\,K}\;\frac{1}{\alpha}\;\frac{\partial f_{z1}}{\partial\,y}\,d\,y$$

Let $f_{z1}\,(h_2)$ represent the value of the function at the surface of wire 2. The second equation of (**2**) gives

$$f_{z1}\,(h_2) - (f_{z1})_2 = \frac{\Gamma^2}{(4\,\pi\,\gamma + j\,\omega K)\mu}\,(\phi_1)_2 - j\;\omega\;(\phi_1)_2$$

If $\Gamma = 0$, then

$$f_{z1}\,(h_2) = -\,Z_{12}$$

i. e., the z-component of the electric field at the wire 2 due to $I_1 = 1$ equals the negative value of the mutual impedance. The z-component of the electric field at the wire 1 due to $I_2 = 1$ equals

$$f_{z2}\,(h_1) = -\,Z_{21}$$

But[14] when $\Gamma = 0$

$$f_{z1}\,(h_2) = f_{z2}\,(h_1),$$

This relation is also true[20] when $\Gamma \neq 0$. Thus, when $\Gamma = 0$, equation (**16**) holds irrespective of the location of the wires.

It can easily be shown that all previous considerations can be extended to the case where the insulation of the wires has finite thickness and the space below the $x\,z$-plane is occupied by layers of different homogeneous and isotropic materials, provided no conductor is close to the surface separating two such layers. For instance, immediately below the $x\,z$-plane there may be a layer of water followed by a layer of earth, and so forth.

DISCUSSION OF THEORY

Consider equations (**13**) and (**14**) or (**15**) for a given wire n. Write down all equations (**13**), one for each Γ, also all equations (**14**). If the corresponding coefficients in equations (**13**) and the corresponding coefficients in equations (**14**) were all equal, the addition of equations (**13**) and the addition of equations (**14**) show that these equations apply to the total values of the voltages and currents. The coefficients in (**12**) are, however, unequal for all waves with unequal values of Γ^2. The equations do not apply strictly to the total values of the voltages and currents, except in the case when only two waves exist with the propagation constants Γ and $-\,\Gamma$. For most practical purposes it will be necessary to make an approximation.

The approximation mentioned consists in the use of equations (**13**), (**14**), and (**15**) in which the vectors denote total values, and equation (**16**), it being understood that it is desirable to determine experimentally the limitations of such a procedure. The equations are then identical in form with the equations of the elementary theory. The present theory represents an advance, at least in so far as the effect of the finite conductivity of the earth manifests itself in the coefficients. The theory gives a physical picture of the phenomena occuring in the conductors and in space. It also points out the approximations involved in the elementary theory.

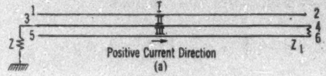

FIG. 2A

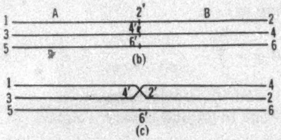

FIG. 2B, 2C

SOME APPLICATIONS OF THE THEORY

Let 1, 2, 3, 4, etc., denote the ends of wires, I, II, III, etc., see Fig. 2A. The total vector voltage to ground and the current at 1 will be denoted by v_1 and i_1, respectively, and so on. It follows[18] from equations (**13**) and (**15**) that

$$\left.\begin{aligned} v_1 &= b_{11}\,i_1 + b_{12}\,i_2 + b_{13}\,i_3 + b_{14}\,i_4 + \;.\;.\;. \\ -\,v_2 &= b_{21}\,i_1 + b_{22}\,i_2 + b_{23}\,i_3 + b_{24}\,i_4 + \;.\;.\;. \\ v_3 &= b_{31}\,i_1 + b_{32}\,i_2 + b_{33}\,i_3 + b_{34}\,i_4 + \;.\;.\;. \\ -\,v_4 &= b_{41}\,i_1 + b_{42}\,i_2 + b_{43}\,i_3 + b_{44}\,i_4 + \;.\;.\;. \end{aligned}\right\} \qquad \textbf{(20)}$$

etc.

where the coefficients $b_{nm} = b_{mn}$ are composed of the coefficients in equations (**13**) and (**15**) in a complicated way. Since the line is uniform throughout its length, it follows[18] that

$$\left.\begin{array}{lll} b_{11} = b_{22} & b_{31} = b_{42} & \\ b_{13} = b_{24} & b_{32} = b_{41} & \text{etc.} \\ b_{14} = b_{23} & b_{33} = b_{44} & \end{array}\right\} \qquad (\mathbf{21})$$

Let A and B be two sections, see Fig. 2B, such that equations (**20**) and (**21**) apply to each of them but with constants b differing on account of changes in the earth conductivity, the geometry of the system, and so forth. The changes in the geometry may be due to transpositions, see Fig. 2C, different spacings, different distances from the ground, etc. There will be a relation (**20**) between the quantities at 1, 3, 5 and at $2'$, $4'$, $6'$. A similar relation will hold between the quantities at $2'$, $4'$, $6'$ and at 2, 4, 6. Eliminating the quantities at $2'$, $4'$, $6'$ gives equation (**20**) where $b_{nm} = b_{mn}$, but equation (**21**) no longer holds. If several sections are added in this way, the result is always equation (**20**) with $b_{nm} = b_{mn}$.

Let A, Fig. 2D, be a system, composed of one or several sections. At one end of the system are impedances Z_1 and Z_2 (*e. g.*, loading coils); they may also have a mutual impedance Z_{12} (*e. g.*, booster transformer). One wire may be grounded through an impedance Z (*e. g.*, ground wire). Between the quantities at $2''$, $4''$, $6''$ and at 2, 4, 6, a relation (**20**) holds, which together with

$$\left.\begin{array}{l} v_2' = v_2'' \\[4pt] i_2' = i_2'' + \dfrac{v_2'}{Z} \\[8pt] i_4' = i_4'' \text{ and } i_6' = i_6'' \\ v_4' = v_4'' + Z_1\, i_4' + Z_{12}\, i_6' \\ v_6' = v_6'' + Z_2\, i_6' + Z_{12}\, i_4' \end{array}\right\}$$

permits the elimination of the quantities at $2''$, $4''$, $6''$. The result is equation (**20**) between the quantities at $2'$, $4'$, $6'$ and at 2, 4, 6 where now some of the coefficients b will contain Z_1, Z_2, Z, etc., but still remain such that $b_{mn} = b_{nm}$. If the system A, including its equipment of terminal impedances Z_1, Z_2, Z, etc., is connected to another system B, see Fig. 2D, the result is still equation (**20**).

If a transformer is connected as shown in Fig. 2E, it is easy to show that there is not a sufficient number of equations to permit the elimination of the quantities at $2'$, $4'$. Equation (**20**) does not apply in such a case.

If necessary, divide a system into sections so that equation (**20**) applies to each section. The terminal conditions of such a section will now be considered. The wires may be interconnected at the ends, ground connections may also be used. For instance, the point 3, Fig. 2A may be grounded through an impedance Z. Then

$$v_3 = -\, i_3\,(Z + R)$$

where R is the ground resistance of the grounding device. This resistance is included to correct, at least approximately, the conditions at the ends of the system where the considerations of the paper do not strictly apply. This resistance R also should be included in Z in Fig. 2D. If the points 4 and 6, see Fig. 2A, are connected by an impedance Z_1, then

$$i_4 = -\, i_6,\; i_4\, Z_1 = v_4 - v_6$$

and so forth. In this connection the following may be noted, see Fig. 2A. The wires I, II, III, may continue to the right of the points 2, 4, 6. The system to the right is then the termination of wires I, II, III at these points.

It is possible to determine the coefficients b in equation (**20**) by experiment.[18] These equations are of fundamental importance and, together with their consequences, permit of important applications.[18] At first sight the great number of coefficients to be determined would seem to hinder applications, except when the number of wires is small. Fortunately this is not the case. It is possible, for instance, in a lead containing a large number of wires, to study the currents and the voltages to ground at the ends of one wire, or the currents and the voltages between two wires at the ends of a loop of these two wires, by measuring only a few coefficients. This has been shown in a few cases,[18] which correspond, for instance, to the propagation of voltage and current along a single telephone wire, or a loop of two wires, from the end A, see Fig. 2F, of an exposure between power and communication circuits to a point B, where the currents and the voltages may be of interest. There are also other cases when a study can be made by measuring relatively few coefficients, as can easily be shown by methods similar to those employed in the paper referred to.[18] Thus, for instance, the voltages and the currents induced in a single wire or a loop of two wires in a lead can be studied by comparatively few measurements.

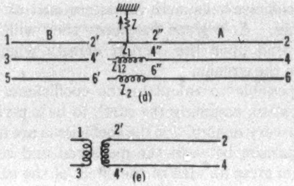

Fig. 2D, 2E

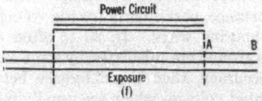

Fig. 2F

In conclusion, some remarks will be made regarding the application of equation (**20**) to problems of inductive coordination between power and communication circuits.

It is customary and useful to distinguish between uniphase and balanced currents and voltages on a power system and consider the induction on a telephone system due to them separately. It is seen that the coefficients b are applicable both to uniphase and balanced induction. Once the uniphase and balanced components are known in magnitude and phase at the ends, the inductive effects can be found by using the same coefficients b. It is not necessary to use separate coefficients for balanced and uniphase induction. This facilitates experimental work and analysis considerably.

The impedances of loading coils, booster transformers, etc., appear in the coefficients b, as seen above. The effect of the line proper can be determined, if all these devices are short-circuited. If the coefficients are then determined with these devices their influence can be found.

The coefficients have to be determined at different frequencies. At a given frequency they will, presumably, depend upon the current density when iron is present in the circuits.

It is possible to calculate the coefficients b for a simple system, assuming the earth to be a perfect conductor, or very remote. If the coefficients are measured the comparison between the measured and calculated coefficients gives an idea of the effect of the earth.

The irregularities in the geometry of the system, in the earth's conductivity, and so on, appear in the coefficients b, determined experimentally. This is of very great importance, particularly for the voltages between two neighbouring wires. If, as is often done, their voltages to ground are calculated by some method, even of great accuracy, then the difference between these two calculated voltages which are usually fairly large is not equal to the actual voltage between the wires, usually fairly small, since even small errors due to irregularities of the system in the voltages to ground give a large error in their difference.

CONCLUSIONS

A theory of propagation of electromagnetic waves guided by parallel wires with regard to the effect of the earth has been developed. The theory shows how the finite conductivity of the earth enters into the problem of propagation. It also gives an idea of the physical phenomena involved. The approximations necessary for the development of a simplified theory suitable for engineering applications have been pointed out. Some applications of the simplified theory to transmission systems for power and communication purposes, suggested by Professor Pleijel, have been indicated, together with remarks of practical interest.

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Discussion

L. C. Peterson: (communicated after adjournment) In connection with some tests planned to determine the inductive coupling at the higher frequencies between different wires of a system of parallel wires, a theoretical study was made in June and July of 1926, based on a paper by Professor Pleijel entitled "Current and Voltage Relations in a System of Parallel Conductors" and given as reference No. 18 in Mr. Levin's paper. In his paper, Professor Pleijel gives a set of linear equations connecting the terminal currents and voltages of a multi-wire line. In the underlying transmission theory, the ground is assumed to have infinite conductivity. Mr. Levin states that he has demonstrated the validity of Pleijel's equations, as regards their form, when the finite conductivity of the ground is taken into account. Such a demonstration is quite unnecessary; indeed, the fact that the equations maintain the same form irrespective of any assumption regarding the ground conductivity, or any approximation introduced in the transmission theory, follows immediately from the general properties of an n-terminal network. The coefficients in Pleijel's equations (given as eq. (**20**) in Levin's paper) uniquely specify the system and hold for all possible terminal connections of the line wires. They offer, therefore, a means of determining the induction in systems of parallel conductors. In a limited number of cases, these coefficients may be calculated from the dimensions of the system. However, calculations being rather laborious, it is advantageous to resort to the experimental determinations of the coefficients.

In a system of n conductors, $n\ (2\,n + 1)$ independent coefficients will be necessary to define the system completely but if the system is electrically symmetrical around its midpoint, this number of independent coefficients is reduced to $n\ (n + 1)$. For instance, an unsymmetrical three-phase system is determined by $3\ (6 + 1) = 21$ coefficients and a symmetrical three-phase system by $3\ (3 + 1) = 12$ coefficients. It is obvious that in the general case the determination of $n\ (2\,n + 1)$ coefficients will be required and hence, $n\ (2\,n + 1)$ measurements.

Methods for experimental determination of the coefficients b in eq. (**20**) of Mr. Levin's paper have been worked out. It can readily be shown that all the coefficients are obtainable from open- and short-circuit impedance measurements on the individual wires or on combinations of these wires.

S. A. Levin: The validity of Pleijel's equations for any finite value of earth conductivity, eq. (**20**), can be shown as done in the paper of reference (18) provided eqs. (**13**) and (**15**) are demonstrated for finite conductivity. According to Mr. Peterson, the validity of Pleijel's equations for any finite value of earth conductivity follows from the theory of an n-terminal network. It seems to me that the application of this theory to a system of parallel conductors also requires that eqs. (**13**) and (**15**) are demonstrated for finite conductivity. This is the only demonstration attempted in my paper. I cannot see, therefore, that it contains any unnecessary demonstration whether Pleijel's equations are obtained one way or the other.

Present Status of the International Electrical Units*

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Member, A. I. E. E.

Synopsis.—*This paper summarizes the present legal standing and practical usage of the international electrical units, particularly as maintained in the United States. The international agreement reached in 1910 was provisional and requires some revision. Further investigations of the international standards, as well as of the absolute units, are urgently needed to put the system on a satisfactory basis for work of high precision.*

Legal authority to deal with electrical units has now been given the International Committee on Weights and Measures. This provides a permanent working organization through which international agreements can be reached and can be made effective throughout the world.

When the committee takes up the question of electrical units for formal international adoption, it will have to decide whether to maintain as nearly as is practicable the values accepted at present or to revise them so as to accord with the fundamental c. g. s. system. With regard to primary standards, it will have to choose between the mercury ohm and silver voltameter, on the one hand, as against direct determination of the units by methods based on mechanical dimensions.

The Bureau of Standards has under way several investigations planned to give a better technical basis for final decisions on these questions. It is desirable also that they be discussed by those interested in making precise and accurate electrical measurements in order that all the advantages and disadvantages of the changes proposed may be given adequate consideration.

INTRODUCTION

While many systems of electrical units have been proposed, the leaders in electrical science and engineering since the time of Weber have almost invariably adhered to the principle that fundamental electrical measurements should be based on the mechanical effects of electricity, and thus be made concordant with measurements in other fields of science and engineering. The metric system has also been generally accepted as the basis for the electrical units.

Even though this general principle is accepted, there are many sets of alternatives between which a choice must be made. For example, one may start with the mechanical forces between electric charges at rest, or on the other hand, first consideration may be given to the magnetic effects which are of so much greater importance in connection with electric currents. In other words, the basis of the system of units may be either electrostatic or electromagnetic effects.

In fact, systems of both kinds are used, and each has special advantages for particular cases. The greater importance of electromagnetic relations in the practical use of electricity, and the facility with which precise measurements of electric current can be made by the use of magnetic effects, have combined to give the electromagnetic system a predominating position. It is nevertheless worth noting that developments of recent years, especially in high voltage work and in electronics, have made electrostatic effects more prominent than they formerly were. It has been established beyond reasonable doubt that all material is composed of constellations of electric charges. The numerical values of these elementary charges have been determined with precision. In numerous devices they already serve us, and the future importance of their direct uses no one can foretell. In this great and growing field of science and practise, essential values are naturally determined in absolute electrostatic units.

It would be beside the point to discuss here the various combinations of units which have been proposed for the purpose of simplifying the numerical relations between quantities and thus making computations easier. These proposed systems are treated at some length in Bureau of Standards Circular No. 60, "Electric Units and Standards,"[1] and Scientific Paper No. 292, "International System of Electric and Magnetic Units."[2] In brief, the view set forth in those publications is that in the "practical" electromagnetic system, as modified by the adoption of the present international units, there has been developed a set of units more generally satisfactory than any of the systems proposed on a theoretical basis. Consequently, it is concluded that there is no good reason to incur the confusion which would result from an attempt to change the units now ordinarily used.

During the last 10 years there has been no agitation for radical changes in the present system, and certainly no such proposals would now be received with favor. No one would seriously propose to do away with the ohm, the ampere, or any of the important units derived from them. It is, however, an open question whether the values of these units as now accepted should not be adjusted to make them accord more closely with the general system of measurements. Furthermore, we may well inquire whether the methods of determining these values have not reached such a stage of perfection that the old expedients for maintaining constancy of the units can be safely discarded. If any such changes of units or of fundamental standards are to be made, they must, of course, be thoroughly considered in advance in order that the advantages and disadvantages resulting from the change may be fully weighed. The purpose of this paper is therefore to set forth the present status of the units and to ask for discussion of the changes which might logically be made if found expedient.

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*Approved by the Director of the Bureau of Standards, Department of Commerce.

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Legal Basis of the Units in the United States

The legal basis for the electrical units used in the United States is still the Act of July 12, 1894. This accepted the international ohm, ampere, volt, coulomb, farad, joule, watt, and henry, as adopted at the International Electrical Congress held at Chicago in 1893, and incorporated definitions paraphrasing without essential changes the resolutions adopted by that Congress. The definitions given in the Act for the ohm, ampere, and volt were as follows:

> 1. The unit of resistance shall be what is known as the international ohm, which is substantially equal to one thousand million units of resistance of the c. g. s. system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 g. in mass, of a constant cross-sectional area, and of the length of 106.3 cm.
>
> 2. The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the c. g. s. system of electromagnetic units, and is the practical equivalent of the unvarying current, which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of 0.001118 of a gram per second.
>
> 3. The unit of electromotive force shall be what is known as the international volt, which is the electromotive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of an international ampere, and is practically equivalent to $\frac{1000}{1434}$ of the electromotive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of 15 deg. cent., and prepared in the manner described in the standard specifications.

It will be seen that these definitions do not draw a sharp distinction between the basic c. g. s. units and those defined in terms of concrete standards. If taken literally, the law is inconsistent with regard to the relations between the two sets of units. The exact value of the ohm is to be that obtained from the mercury column, the absolute unit being mentioned merely as a substantial equivalent, while in the case of the ampere this condition is reversed.

Section 2 of the same act provided,

> That it shall be the duty of the National Academy of Sciences to prescribe and publish, as soon as possible after the passage of this Act, such specifications of detail as shall be necessary for the practical application of the definitions of the ampere and volt hereinbefore given, and such specifications shall be the standard specifications herein mentioned.

This section has become a dead letter, since it was necessary to depart from the National Academy specifications in order to obtain consistent results and to obtain international agreement. Except in this detail, however, it has been possible to follow the terms of the law literally and still to put into effect the international agreements which have been reached. The differences between the absolute and the accepted international unit of current have been negligible, and the terms in which the Clark cell were referred to were not such as to require its use, so that there has been no legal obstacle to the adoption of the Weston normal cell as a substitute for the Clark.

The Present Units

The values of the units now accepted for practical use throughout the world were established, in principle, by the International Conference on Electrical Units and Standards, held in London in 1908. This conference made a clear distinction, so far as definitions are concerned, between the absolute units and those which were called international. The Conference used the term "fundamental" for the units here called absolute, that is, those derived from the basic units of length, mass, and time. With reference to these units, the following resolution was adopted:

> The Conference agrees that as heretofore the magnitudes of the fundamental electrical units shall be determined on the electromagnetic system of measurement with reference to the centimetre as the unit of length, the gramme as the unit of mass, and the second as the unit of time.
>
> These fundamental units are:
>
> 1. The *Ohm*, the unit of electric resistance which has the value of 1,000,000,000 (10^9) in terms of the centimetre and second,
>
> 2. The *Ampere*, the unit of electric current, which has the value of one-tenth (0.1) in terms of the centimetre, gramme, and second,
>
> 3. The *Volt*, the unit of electromotive force which has the value of 100,000,000 (10^8) in terms of the centimetre, gramme, and second,
>
> 4. The *Watt*, the unit of power which has the value of 10,000,000 in terms of the centimetre, the gramme, and the second.

As a system of units representing the above and sufficiently near to them to be adopted for the purpose of electrical measurements and as a basis for legislation, the conference recommended the adoption of the international ohm, ampere, volt, and watt, defined as follows:

> The *International Ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of a length of 106.300 centimetres. (The procedure to be followed in setting up mercury ohms was prescribed in detail in specifications attached to the resolutions.)
>
> The *International Ampere* is the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with the Specification II attached to these Resolutions, deposits silver at the rate of 0.00111800 of a gramme per second.
>
> The *International Volt* is the electrical pressure which, when steadily applied to a conductor the resistance of which is one international ohm, will produce a current of one international ampere.
>
> The *International Watt* is the energy expended per second by an unvarying electric current of one international ampere under an electric pressure of one international volt.

It is perhaps worth noting that the units of mass and length included in these definitions occur in the description of the apparatus by which the units are to be established, and are not involved in the real definitions of the units themselves. That is, these dimensions could be stated in any other mechanical units desired without affecting the value of the electrical units; the latter are essentially defined in terms of the properties of mercury and of silver without any reference to systems of measurement. They therefore become independent, fundamental units which, joined with the centimeter and the second, constitute the basis of a complete system from which even mechanical units including the gram might be derived.

While these definitions were thus made in form quite independent of the absolute electrical units defined in terms of the centimeter, gram, and second, the intention of the conference was to make the units adopted represent very closely the value of the absolute units. In fact, one reason advanced for choosing the ampere, instead of the volt, as a fundamental unit was the possibility of direct determination of the absolute value of the ampere by several methods independently of any assumed value for the ohm or other electrical units.

The definition adopted for the ampere carried the value to six significant figures although it was recognized that the specifications for the voltameter were incomplete and therefore indefinite. An International Committee on Electrical Units and Standards was created to complete the work of the conference and to carry it on until another conference should be convened. Among other duties, this committee was to secure agreement on precise specifications for the voltameter and to establish a standard value for the electromotive force of the average Weston normal cell consistent with the international ohm and the ampere as defined by these specifications. The accomplishment of an important part of this task was made possible by the generous support of four American societies, the American Institute of Electrical Engineers, Association of Edison Illuminating Companies, Illuminating Engineering Society, and National Electric Light Association. These societies contributed funds to bring representatives of Great Britain, France, and Germany to America for joint experiments with the Bureau of Standards upon the silver voltameter and the standard cell. The Technical Committee thus created worked at Washington for nearly two months in 1910. It did not agree on formal specifications for the voltameter, but it did bring the experimental results with different types of voltameters nearly enough into accord so that the value of 1.0183 at 20 deg. cent. for the Weston normal (saturated) cell was established and accepted internationally.

VALUES OF THE OHM

When the Technical Committee met in 1910, it had at hand wire standards calibrated in terms of the mercury ohms at the German Physikalisch-Technische Reichsanstalt and the British National Physical Laboratory. The two values for the ohm were found to differ by only one part in 100,000, and the mean value was accepted by the Technical Committee in the following resolution:

> The committee decides to choose, for the present and until there are other mercury ohms prepared, as the value of the international ohm, to be recommended to all countries for general use, the mean of the values of the units realized at the Physikalisch-Technische Reichsanstalt and at the National Physical Laboratory. Although the international ohm as defined by the London Conference has not yet been strictly realized, the committee believes that its value has been attained in two laboratories independently with a good degree of precision, and that future work is not likely to change it by more than two or three parts in 100,000.

It will be seen that this acceptance of a mean value to be called the International Ohm was really provisional. The International Committee never succeeded in completing the full and formal establishment of the unit, and no machinery was provided for the distribution and maintenance of a common unit. The work of the committee did, however, show that the resistance standards of the several national laboratories were already in fairly good agreement. Since the standards were thus reasonably concordant, in general each laboratory considered it best to maintain the continuity of its own values rather than to make small changes in advance of a complete international acceptance of a precise value.

In fact, a mercury ohm determination completed at the Bureau of Standards a few years later[3] gave a value differing from the international ohm accepted in 1910 by 25 parts in a million. This was considered to be a check within the limits of certainty of the mercury ohm, and the values assigned to the Bureau's wire standards were not changed. Since 1915 no mercury ohm determinations have been made at the Bureau. The unit has been preserved by sets of wire standards whose relative values have been found to be very stable. The unit has been maintained on the assumption that the mean resistance of a group of 10 one-ohm manganin wire standards remains constant. The ten standards included in the reference group are, however, not always the same. Intercomparisons with a considerably larger group are made from time to time, and those standards which have apparently been most stable are chosen for the reference group which is the custodian of the unit until the next intercomparison is made. Since 1910, 17 different standards have thus at various times been included in the reference group.

Comparisons of the ohm as maintained in different countries have been made only irregularly, but the results have indicated that all the national laboratories have remained in satisfactory agreement. The differences found have seldom been greater than two parts in 100,000, which is about the limit of accuracy obtainable in the establishment of the international ohm.

In recent years an entirely unexpected complication has arisen which makes the old definition of the international ohm indefinite. This is the discovery that mercury is not a simple element but includes several isotopes. If completely separated, these would presumably differ in density by as much as 3 per cent, although they have the same resistivity on the basis of volume. This difficulty is not a serious one, however, since mercury from many sources as ordinarily produced has been shown to have the same density within a few parts in a million. At any rate, the difficulty can be met by specifying the density of the mercury to be used in the ohm tubes or by prescribing the cross-section instead of the mass of the mercury column.

Although the mercury ohm affords a means, however, of checking the value of the unit to a few parts in 100,000, determinations of the absolute value of the international ohm have shown that this unit differs by a considerable amount from the basic unit with which it was intended to be practically equivalent. Two very careful determinations have been made since 1910 by entirely different methods, one at the National Physical Laboratory,[4] with an apparatus of the Lorenz disk type, the other at the Reichsanstalt[5] by comparison of resistance standards with the calculated self-inductance of a coil. The two results agree within one part in 100,000. This close agreement must be considered as partly accidental, since the relative values of the reference standards used in the two laboratories could not be depended upon to such a high degree of accuracy. Expressed in terms of the length of a column of mercury of the same cross-section as the international standard, these two determinations indicated that the absolute ohm would be represented by a column 106.245 or 106.246 cm. long instead of 106.300. It appears certain, therefore, that the absolute ohm is smaller than the international ohm by about five parts in 10,000, and incidentally that determinations of the absolute ohm can be made with the same degree of precision as the international ohm can be established and checked with mercury ohm tubes.

VALUES OF THE AMPERE

The present international ampere represents the value obtained by the Technical Committee of 1910 as an average of the results found with several different types of voltameter. Since current is transitory, the average result was recorded and is concretely expressed by the value assigned to the Weston normal cell. The international volt being the potential drop in the accepted international ohm with a current flowing which deposited 1.11800 mg. per sec. in the *average* voltameter, the committee found that the average cell had a voltage of 1.0183, and this has since been used as the standard value. Standard cells used in conjunction with resistance standards calibrated in international ohms presumably reproduce ampere values as then obtained.

The voltameters operated by the Bureau of Standards at that time would have established an ampere larger than the average by three parts in 100,000. That is, in these voltameters the deposits of silver were smaller, probably because they were more nearly pure silver. As a result of researches carried out over a period of several years[6] following the London Conference, the precision of the voltameter as a measuring instrument was materially improved, but since these improvements in procedure reduced the deposits they would increase the value of the ampere as measured by the silver deposited. Seven series of measurements made in five different countries since 1910 have shown an average deviation of only one part in 100,000 from their mean, but the mean is three parts in 100,000 different from the 1910 value. That is, these voltameter measurements would have made the standard cell value 1.01827 instead of 1.0183.

As has been remarked above, the ultimate value of the international ampere as defined by the silver voltameter is as yet indefinite because no precise specifications for the operation of the voltameter have been adopted. The Bureau of Standards has proposed specifications which are believed to assure the highest attainable degree of purity in the deposit. In voltameters used according to these specifications, the absolute ampere as measured by the Bureau's current balance was found to deposit 1.11804 or 11.1805 mg. per sec., of which about 0.004 per cent was foreign matter included with the silver. Consequently, according to these measurements, if the international ampere were based on *pure silver* deposited, it would be identical with the absolute unit within the limits of accuracy of these measurements.

This allowance for inclusions was not made, however, when the accepted value for the standard cell was established in 1910. Since it has to be added to the difference of three parts in 100,000 mentioned above, the "international ampere" then set up was smaller than the absolute by seven or eight parts in 100,000, according to the measurements made at the Bureau of Standards. Taking into account also measurements at the British National Physical Laboratory[7] and at the University of Groningen, Holland[8], it has been estimated[1] that the best value for the international ampere of 1910 was 0.99991 absolute ampere, and this conversion factor has been commonly used.

There are several distinctly different methods by which the absolute ampere can be found. Since the 1908 London Conference, good determinations have been made by the tangent galvanometer[8], the electrodynamometer[9], and several forms of current balance [7,10]. It is not possible, in most cases, to compare the results exactly because there have been no common standards of sufficient accuracy to preserve and express the values obtained by these experiments. The differences have been a few parts in 100,000, and it is a question how much of these differences is due to the errors in establishing the absolute ampere and how much to variations between the voltameters (or the standard cells and resistance coils) used to represent the international ampere. Certainly it is possible to establish the absolute value within two or three parts in 100,000 with a single instrument like the current balance. If several laboratories were to set up different types of absolute instruments and systematically compare the results, it should be possible to establish and maintain the ampere to one part in 100,000.

VALUES OF THE VOLT AND OTHER UNITS

Although the ampere is the second fundamental unit adopted, the unit actually maintained for practical measurements is the volt, as represented in the standard cell. Since 1910 the Bureau of Standards has maintained this unit by groups of reference cells in a manner closely analogous to that described above for the ohm. With a few substitutions of stable cells for some which showed a relative decrease in electromotive force, it is believed that these reference groups of selected cells have remained substantially constant for many years, some of them since 1906. A few new cells have been made recently, and these have agreed with the older groups within one part in 100,000. Results of comparisons with other laboratories in this country and abroad have also generally supported the belief that the Bureau's cells have maintained their values. During the last year, however, differences of several parts in 100,000 have arisen between the Bureau's measurements and those of the National Physical Laboratory, and no complete explanation for them has yet been found.

The absolute value of the international volt and of other international units is, of course, dependent on that of the ohm and of the ampere. On the basis of the estimates given above for the two fundamental units as established in 1910, the various international units have the following values[1]:

1 international ohm = 1.00052 absolute ohm
1 international ampere = 0.99991 absolute ampere
1 international volt = 1.00043 absolute volt
1 international watt = 1.00034 absolute watt
1 international joule = 1.00034 absolute joule
1 international coulomb = 0.99991 absolute coulomb
1 international farad = 0.99948 absolute farad
1 international henry = 1.00052 absolute henry
1 international gilbert = 0.99991 absolute gilbert
1 international maxwell = 1.00043 absolute maxwell

FUTURE UNITS AND STANDARDS

There are few applications in which a change of one-twentieth of one per cent (the maximum discrepancy existing between the two sets of units) would now be of any practical importance. The demands of industry for precise measurements have grown with surprising speed, however, and if the discrepancy is ever to be removed, it would be well to perform the operation before the change does become significant in industry. For those laboratories which carry on work of high precision, the changes involved in going over to the absolute units would undoubtedly be troublesome for a time, particularly because so much apparatus is precisely adjusted to values in the international units. The easier course at present would be to retain the old units, making such minor adjustments as may be found necessary for better international agreement, and to establish accurately the necessary conversion factors for those who must transfer from electromagnetic quantities to electrostatic and to heat or other mechanical units. When one considers, however, that this probably means laying up trouble in increasing amounts for decades to come in order to avoid some temporary inconvenience, it would appear that the logical course is to adopt the absolute units in the near future rather than to make merely minor adjustments in the international units.

Before the absolute units could be thus adopted for practical use, it would be necessary to have more laboratories set up apparatus to give these units and to find whether these newer determinations agreed satisfactorily with those which have been mentioned above. If such apparatus is set up in the several national laboratories and gives concordant results with a certainty as great as that of the mercury ohm and silver voltameter, the need for these inconvenient custodians of the units will have vanished. This will be true even if the present international units should be continued in use.

In the enactment of legal definitions, the concreteness of the standards representing the present international units offers some advantage, but in those countries having national laboratories there should be no serious difficulties in securing the legal recognition of units defined in terms of the centimeter, gram, and second, to be established and maintained by the national laboratory in cooperation with the International Committee on Weights and Measures. Other countries could define the units similarly and provide for obtaining copies of standards through the International Committee.

Although this Committee has never heretofore dealt with electrical standards, it was empowered to do so by an amendment to the international convention on weights and measures which was ratified by the United States in 1923. In accordance with its enlarged authority, the Committee is now inaugurating a series of systematic intercomparisons between the national standardizing laboratories, and it will consider eventually what steps shall be taken to coordinate more effectively electrical measurements as made in different countries.

Whatever course is followed, it is obvious that there is urgent need for comprehensive experimental studies, including the reexamination and further development of the fundamental standards whereby the units are established and maintained. This, of course, is primarily a problem for the national standardizing laboratories. In general, however, such laboratories in recent years have been pressed with problems of more immediate and obvious industrial usefulness, or their means available for fundamental scientific work have been otherwise curtailed.

PRESENT WORK AT THE BUREAU OF STANDARDS

In view of the long period over which the units have been maintained at the Bureau by means of secondary standards, coils, and cells, it would be desirable to check their values by new mercury-ohm and silver-voltameter determinations. It has been believed, nevertheless, that a revival of the researches on fundamental units is still more important, and that if successfully carried out, these researches would in effect give also as accurate a check on the international units as could be got by the use of the mercury column and the voltameter. Since it has not been practicable to take up work on both types of standards, attention has been given first to the development of apparatus for establishing the fundamental units. Of these units, the ohm has been given priority because the Bureau has never made any determinations of its value, and very few accurate determinations have ever been made. Two independent schemes for accomplishing this purpose are being developed. Both will use stationary inductance coils designed for calculation of the inductance from dimensions. One method, devised some years ago by Dr. Frank Wenner, will use mutual inductances so arranged that the electromotive force induced in the secondaries can be balanced against the potential drop in a resistance of which the value is to be determined. The other procedure is planned to make use of the experience of the Inductance Laboratory under the charge of Dr. H. L. Curtis. It will consist essentially of the comparison, by a-c. bridge methods, of a self-inductance of calculated value with the resistance to be measured. In each project, it is desired to carry the results to an accuracy approaching one part in 100,000, and this requires a theoretical and experimental study of many details which are neglected in ordinary measurements. Fair progress is being made, but it is impossible to predict when final results will be obtained.

For absolute measurements of current, the balance used by Dr. E. B. Rosa[10] and his associates has been reassembled and is being studied for possible improvements. It is hoped that this will soon give results which, in conjunction with the wire ohm standards, will serve to check the Bureau's standard cell values.

Whatever primary standards are used, the cells always serve as one of the essential secondary standards, and studies of their behavior are being made. This work, as well as the maintenance of the reference group of cells, is under the direction of Mr. G. W. Vinal.

CONCLUSION

This summary of the present situation in regard to the electrical units and standards has been offered because the Bureau of Standards has a large responsibility for their maintenance and improvement which in recent years it has not been able to meet to its own satisfaction. The kind of work necessary to give the accuracy desired in the basic standards can not be done hastily, and it will require several years of work on the part of all the national laboratories to provide a good technical basis for a final decision on the type of standards and on the units to be adopted for international use. Detailed results of the several investigations mentioned will be published as they become available. In the light of these results and those of similar work abroad, the International Committee on Weights and Measures will have to reach a decision. In the meantime, it is desired that those whose interests will be affected by this decision study the situation and make known their views as to the course which should be chosen.

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Methods Used in Investigating Corona Loss
By Means of the Cathode Ray Oscillograph

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Associate, A. I. E. E. — Non-member

Synopsis.—*Methods of employing the cathode ray oscillograph for investigation of corona loss are described. By deflecting the ray of cathode particles (electrons) in one direction within the tube by a voltage proportional to the applied voltage, and in a transverse direction by a voltage proportional to the current, a closed figure representative of the loss, is thrown upon the screen. Methods of obtaining photographic records of these figures and of calculating, therefrom, accurate values of the power expended are given. The instrument used is well adapted for this work. Power measurements of 0.1 watt can be measured with an accuracy of 1 per cent.*

From the volt-ampere cyclograms the characteristics of the positive and negative loss on the a-c. wave are readily observed. The instantaneous voltage at which the loss starts and the instantaneous values of the combined corona and capacity currents can be accurately determined.

Measurements of the corona starting point and loss on various conductors check the laws of corona established by Mr. Peek in 1910. The formulas of "critical disruptive gradients" and "visual disruptive gradients" were closely checked. The loss was found to follow a quadratic above the visual critical corona voltage, e_v. For cables and roughened conductors the excess loss below e_v due to surface irregularities approximately follows the probability law. For smooth, polished conductors the loss suddenly jumps from zero to a definite value at e_v and then follows the quadratic.

The practical effect of the condition of the conductor surface is forcibly brought out by the following data measured on a 336,400-cir. mil. cable at 63.5 in. spacing to neutral plane:

Voltage between lines (three phase)	Sixty cycle loss in kw. per mile of conductor		
	Smooth	Rough	Mutilated
258 kv. (eff.)	49	....	....
220 " "	18	29.4	....
205 " "	0	17.0	38.0
180 " "	0	0	....
132 " "	0	0	4.7

INTRODUCTION

THIS paper describes in detail the methods used in an investigation of corona loss with the low-voltage cathode-ray oscillograph. The work, which has been under way for several years in the High Voltage Engineering Laboratory at Pittsfield, Mass., is a continuation of the investigation of corona started by Mr. F. W. Peek, Jr. in 1910.[2] It is hoped that this detailed description of the methods employed in using the cathode ray oscillograph and making the laboratory measurements will be of assistance to other investigators. A discussion of the results and the conclusions are given in Mr. Peek's paper, *Law of Corona—IV*.[3]

In studying the corona discharge it is highly desirable to make use of an instrument having no power-factor limitations. It is also desirable that the instrument be capable of accurate indications when the current and power involved are small and the voltage high. The low-voltage cathode ray oscillograph is such an instrument.

THE CATHODE RAY OSCILLOGRAPH

This instrument depends for its operation upon a jet, or ray, of cathode particles (electrons) moving at high velocity within an evacuated tube and impinging upon a fluorescent screen. The electrons are discharged from an incandescent filament (the hot cathode). Under the action of a strong electric field, the electrons are rapidly accelerated toward the anode. A small hole in the anode permits certain of the electrons to pass along the axis of the tube in a narrow beam. This beam, or ray, of cathode particles, which subsequently passes between two pairs of metallic plates upon which voltages can be impressed, can be deflected from its normal course by transverse electrostatic or electromagnetic fields, or by a combination of such fields, and thereby made to trace definite figures on the fluorescent screen which becomes luminous at the point at which the electrons strike. These figures are representative of the electrical phenomena taking place within the circuit being investigated, and can be accurately interpreted when the various tube and circuit constants are known. The figures are accurately traced irrespective of the frequency of voltage or current producing the deflections of the electron beam, for this beam has practically no mass and hence no natural period, at least within the limit of the higher radio frequencies.[4]

Photographic Records. Although not strongly luminous, the figure traced on the screen of the tube can be photographed. A sharp image can be obtained by using a lens but the length of exposure required is very great (3 min. or longer). Consequently this method is not desirable when a large number of records is to be taken.

An accurate but somewhat blurred image can be obtained with a comparatively short exposure by operating the tube in a darkened room or box and placing the emulsion side of a "super speed" photographic film directly in contact with the end of the tube. The exposure required is about $^1/_{10}$ to $^1/_4$ sec. for a

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[1] Both of the General Electric Co., Pittsfield, Mass.

[2] F. W. Peek, Jr., *Law of Corona and Dielectric Strength of Air, I*, TRANS. A. I. E. E., Vol. XXX, pp. 1889-1965.

[3] F. W. Peek, Jr., *Law of Corona and Dielectric Strength of Air, IV*.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 21-24, 1927.

[4] For a more detailed discussion of this device, the reader is referred to an article by J. B. Johnson, *Bell System Technical Journal*, November, 1922, p. 142.

straight line image, and about ½ sec. for other figures. The exposures are made by bringing the filament of the tube up to the proper temperature and then closing the anode circuit for the required length of time.

Tube Calibration. The construction of the tube is such that the deflection of the cathode beam as shown on the screen of the tube is approximately proportional to the maximum voltage difference between the deflector plates. Photographs were taken of the individual deflections, horizontal and vertical, for various known sinusoidal plate voltages and different anode potentials. These films were measured and curves plotted showing the relationship between deflection, horizontal and vertical, and deflector-plate voltage. See Fig. 1.

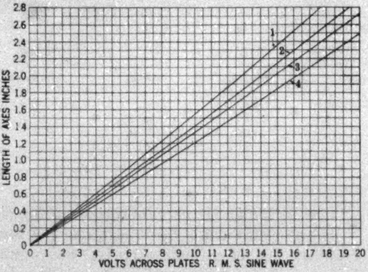


Fig. 1a—Calibration Curves for the Cathode Ray Oscillograph

1—Vertical deflection—anode potential 241.5 volts
2—Horizontal deflection—anode potential 241.5 volts
3—Vertical deflection—anode potential 315.0 volts
4—Horizontal deflection—anode potential 315.0 volts

It was thought that possibly the calibration might vary over different parts of the screen, but simultaneous exposures of the axes, producing straight diagonal lines across the screen, and photographs of the individual axes displaced as far from the center as practical, indicated that there was little or no change in the calibration.

Corona Loss Measurements

Volt—Ampere Cyclograms. The most satisfactory arrangement for studying corona losses by means of the cathode ray tube has been found in this investigation to be that shown diagrammatically in Fig. 2. A specimen of the conductor to be studied was suspended vertically at a uniform spacing from a broad, vertical metal plate having a length of about 20 ft. This plate was composed of three sections of equal width. The end sections, each five ft. in length, were grounded and the middle section, which was insulated from the other two, was grounded through the variable non-inductive resistance, R_3. The function of the end sections was to intercept the corona discharges from the ends of the conductor and leave a uniform section exposed to the active plate.

Although this ground plate, which served as a part of the neutral plane, was 5 ft. wide, it was rather narrow relative to some of the spacings used in the tests. In the near vicinity of the set-up there were several other grounded objects (principally a metal wall) which were, in effect, extensions of the ground plate. Consequently the active section of the plate was only a part of the practically infinite neutral plane, and therefore intercepted only a fraction of the flux emanating from the conductor. The relative amount of flux intercepted became less as the spacing was increased. The object of this work, however, was to study the critical voltage, the mechanism of the discharge, and the relative rather than the absolute value of the losses. For this purpose the set-up was found to be entirely satisfactory.

The step-up and potential transformers were identical, and were rated 10 kv-a., 25-cycle, 200/100,000-volt. These transformers were capable of delivering 150,000 volts intermittently without injury and, when operated at 60 cycles, without high flux density in the core. Voltage control was obtained by means of variable resistances in series with the low-voltage winding of the step-up transformer. Resistances are very flexible, and give practically no wave distortion when used in connection with a load of constant impedance, as in these tests. A constant-impedance load, consisting of high-voltage condensers of a total capacity of about 0.00125 μf., was shunted across the high-voltage winding. This was a load several times as great as that imposed by the exciting kv-a. of the transformers plus the maximum corona loss obtained. The function of this capacitance was to smooth out the voltage wave impressed on the conductor. It accomplished this by supplying the magnetizing current of the transformers and by providing a heavy sinusoidal current to minimize the disturbing influence of the non-sinusoidal corona currents.

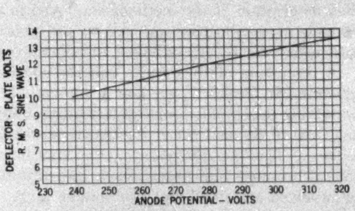


Fig. 1b—Curve Showing The Relation Between Deflector-Plate Voltage to Produce a Vertical Deflection of 1.8 in. (Pos. + Neg.) and Anode Potential of Tube

A resistance potentiometer, consisting of two non-inductive variable resistances, R_1 and R_2, was shunted across the low-voltage winding of the potential transformer and provided a means of obtaining a voltage of the desired value across the deflector plates of the cathode ray tube. The voltage thus obtained was for any particular setting of R_1 and R_2 proportional to and in phase with the voltage impressed on the corona conductor.[5]

The voltage drop across R_3 was proportional to and in phase with the current flowing to ground from the middle section of the test conductor and was impressed across the remaining pair of deflector plates.

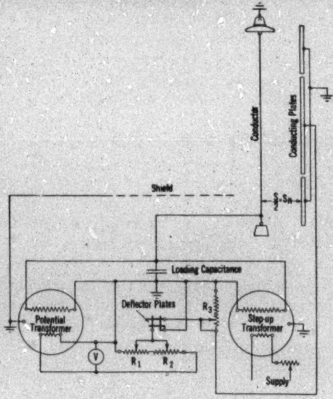
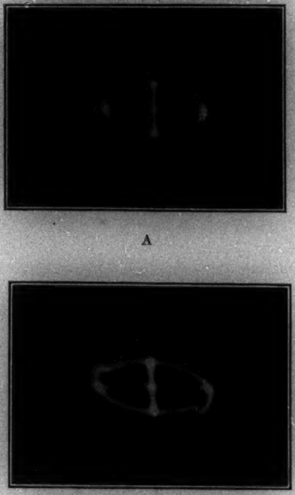


FIG. 2—DIAGRAM OF CONNECTIONS FOR A STUDY OF CORONA BY MEANS OF THE CATHODE RAY OSCILLOGRAPH

Since the corona phenomena are re-occurring, the figures traced on the screen of the oscillograph tube under the above conditions are stationary and are accurately indicative of the instantaneous relationships between voltage and current. When no corona exists, both the current and voltage waves are sinusoidal and 90 degrees out of phase as dry air is practically a perfect dielectric. The figure traced on the screen is then a pure ellipse. With the connections as shown in Fig. 2 the abscissas of the ellipse are proportional to voltage and the ordinates proportional to current. The cyclogram in Fig. 3A illustrates this condition. The individual axes were obtained by grounding one pair of deflector plates at a time and thus obtaining each separate deflection independent of the other. When corona is present on the test conductor the voltage remains sinusoidal but the current wave is badly distorted. An individual discharge takes place during each half-cycle of voltage, near the crest of the wave if the impressed voltage is only a few per cent above the critical disruptive value. These discharges appeared on the screen of the tube as an irregular "hump" on each half of the figure. The circuit employed was such that the lower right hand hump represented the discharge during the time that the conductor was negative. The upper left hand hump represented the positive discharge. The cyclogram in Fig. 3B illustrates this condition for a voltage slightly in excess of the visual critical value.

A permanent record of the figure traced by the electrons on the screen of the tube was most easily obtained, as noted above, by placing a "super speed" photographic film in contact with the end of the tube. The film was first placed in position in the dark box containing the tube. The tube was then moved ahead by means of an adjusting screw and forced into contact with the film. The filament of the tube was heated to the proper temperature, the required current having been determined previously by means of the filament ammeter and visual inspection of the figure on the screen. The desired voltage was then impressed on the test wire. One pair of deflector plates was grounded and the anode circuit closed for approximately ¼ sec. The first pair of plates was then connected in circuit and the other pair grounded, and another ¼-sec. exposure made. Both pairs of plates were then connected in circuit and an exposure of about ½ sec. was made. In this way a record of the figure



A

B

FIG. 3—VOLT-AMPERE CYCLOGRAMS

A—Below corona starting voltage
B—Just above corona starting voltage
Conductor; No. 00, polished
Spacing; 63.5 in. to neutral
Voltage; 91.0 kv. (eff.) to neutral

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[5] The phase angle and ratio errors of a high-voltage transformer when operating at low core densities on no load as a step down transformer are very slight and, for this investigation, can be neglected.

and both axes was obtained. The film was developed in the usual manner.

In order that the voltage drop across R_3 be proportional to and in phase with the current flowing to ground from the middle section of the ground plate, it was essential that there be relatively no capacitance or inductance in this part of the circuit. Since the ground capacity of the active section of the ground plate was approximately 4.9×10^{-4} μ f. and was, in effect, shunted across R_3, the value of this resistance, which was non-inductive, was kept less than 100,000 ohms. Consequently the maximum phase-angle error introduced was less than one degree.

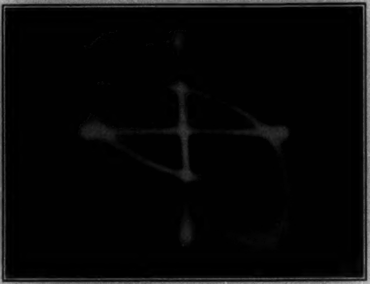


FIG. 4—VOLT-AMPERE CYCLOGRAM OF HEAVY CORONA

Conductor; No. 00 polished conductor
Spacing; 21.5 in. to neutral
Voltage; 150 kv. (eff.) to neutral

A grounded network was placed above the bushings of the transformers to shield out stray fields which otherwise would have reached the effective section of the ground plate.

Most of the work was done at 60 cycles but some measurements were made at 420 cycles. Practically the same test circuit was used in both cases. At normal frequency, even with the loading capacitance removed, the system drew a leading current, but at 420 cycles the impedance of the step-up transformer was so increased and the impedance of the high-side load so decreased that the system drew a lagging current. It was necessary, then, in order to reduce the current drawn from the supply lines to a minimum, to remove the extra capacitance from the high side and shunt a large capacitance across the low side of the step-up transformer.

The sources of power for all of the tests, both 60 and 420 cycles, were sine-wave alternators of large capacity.

Analysis of Cyclograms. Since the abscissas of the cyclograms are proportional to instantaneous voltage, and the ordinates proportional to instantaneous current, if the wave shape of one of the two quantities is known the other can be plotted with time as abscissa and the unknown quantity as ordinate. Fig. 5 illustrates a case in which the horizontal deflection was produced by a known sinusoidal voltage (a voltage proportional to and in phase with the voltage of the test conductor) and hence the current wave, in both magnitude and phase, was readily derived, as shown, from the cyclogram. The original cyclogram from which Fig. 5 was taken is given in Fig. 4.

As well as giving an indication of the wave form of the current in the corona discharge, the cyclograms also provide an accurate means of determining the instantaneous voltages, positive and negative, at which corona forms on the conductor. In Fig. 6, E_i (pos.) and E_i (neg.) represent these voltages.

Since the discharges fade out very gradually, the cyclograms give no definite indications of the exact stopping points. It is evident, however, that the discharges do not persist far beyond the crests of the voltage waves. As brought out later, the stroboscope verified this point.

The maximum charging current of the conductor due merely to the normal conductor capacitance and the maximum *rate of change* of voltage for that particular maximum voltage is represented (approximately) by I_c (max).

The maximum currents occurring during the discharge are represented by I_{max} (pos.) and I_{max} (neg.).

The rotation of the cathode beam in generating this general type of cyclogram is, as viewed normally, counter-clockwise.

STROBOSCOPIC OBSERVATIONS

In order to obtain an approximate check on some of the discharge characteristics disclosed by the cathode ray tube, a stroboscope, or synchronous shutter, was used to permit visual inspection, in a darkened room, of the corona discharge from a polished conductor at any point on the voltage wave. These observations checked the instantaneous disruptive voltages very closely, and indicated definitely that the discharges terminate shortly after the passing of the voltage crests. The corona "sparks" are most intense when they first appear. The illumination then gradually fades out and

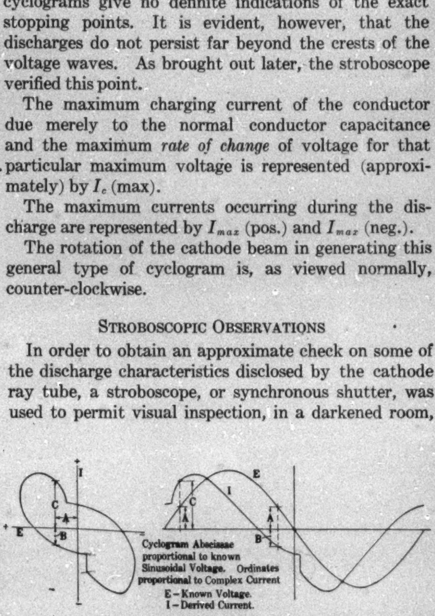

+
I
C
A
E
B
-
E
I
C
A
A
B
Cyclogram Abscissae proportional to known Sinusoidal Voltage. Ordinates proportional to Complex Current
E – Known Voltage.
I – Derived Current.

FIG. 5—GRAPHICAL ANALYSIS OF THE VOLT-AMPERE CYCLOGRAM SHOWN IN FIG. 4

Cyclogram abscissas proportional to known sinusoidal voltage. Ordinates proportional to complex current
E—Known voltage
I—Derived current

disappears at a point 10 to 25 deg. beyond the voltage crest.

POWER CALCULATIONS

When taken by means of the circuit shown in Fig. 2 the cyclograms obtained are volt—ampere curves, the horizontal deflections being practically proportional to and in phase with the sinusoidal line voltage to ground, and the vertical deflections practically proportional to and in phase with the complex current flowing between the conductor and ground plate.[6] The average power represented by the cyclogram can then be calculated by the method outlined below.

The figure is divided into a convenient number of equal-time sections, and the product of the mean abscissa and the estimated mean effective ordinate is obtained for each section. The sign of each of these products is determined algebraically from the sign of the factors, the abscissas being positive to the left of the vertical axis and the ordinates positive above the horizontal axis. The algebraic mean of these products is then proportional to the average power expenditure, and when multipled by the product of average volts per unit horizontal deflection and average amperes per unit vertical deflection is expressed in watts. This process is rather involved but, since the current waves are quite complex, it is the only accurate method of calculating the power represented by the figures.

The current and voltage factors are obtained from the constants of the test circuit, the tube calibration curves, and the line voltage. The voltage factor is equal to the maximum line voltage divided by the maximum horizontal deflection (positive or negative) in units. The current factor is equal to the deflector-plate volts (maximum) per unit vertical deflection (obtained from the tube calibration curves) divided by the value of R_3 in ohms. The calculation of the unit current constant was simplified by the use of a curve plotted for a given set of conditions and made applicable to other conditions by means of a simple formula.

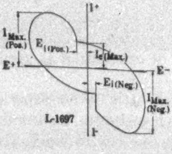


FIG. 6—CORONA DISCHARGE CHARACTERISTICS AS INDICATED BY THE VOLT-AMPERE CYCLOGRAM

E_i Proportional to instantaneous disruptive voltage
I_{max} Proportional to maximum current
$I_{c(max)}$ Approximately proportional to maximum conductor charging current

It has been found that sufficient accuracy is usually obtained by dividing the cyclograms into 20 equal-time sections. Each section then represents 18 degrees of the cycle. The instantaneous voltage at the mid point of each section is very nearly the average voltage for that section, and the corresponding average instantaneous current can be estimated with sufficient accuracy. Horizontal and vertical deflection readings are then taken at the mid point of each 18-degree section, or, beginning at a zero point on the voltage

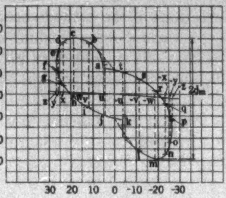


FIG. 7—METHOD OF COMPUTING POWER EXPENDITURE REPRESENTED BY THE VOLT-AMPERE CYCLOGRAM

LEGEND

$D_m = Z = -(-Z) = 26.3$
$D_1 = U = -(-U) = 4.1$
$D_2 = V = -(-V) = 11.9$
$D_3 = W = -(-W) = 18.6$
$D_4 = X = -(-X) = 23.4$
$D_5 = Y = -(-Y) = 26.0$
$d_1 = a = 13.6$
$d_2 = b = 22.9$
$d_3 = c = 24.5$
$d_4 = d = 22.5$
$d_5 = e = 18.3$
$d_5' = f = 8.8$
$d_4' = g = 3.0$
$d_3' = h = -2.0$
$d_2' = i = -6.4$
$d_1' = j = -9.8$
$d_{1a}' = k = -14.0$
$d_{2a}' = l = -25.0$
$d_{3a}' = m = -27.9$
$d_{4a}' = n = -25.8$
$d_{5a}' = o = -20.2$
$d_{5a} = p \doteq -9.0$
$d_{4a} = q = -2.8$
$d_{3a} = r = +2.8$
$d_{2a} = s = 7.2$
$d_{1a} = t = 10.0$

$$P = \frac{E_u\, I_u}{4\,N}\left[D_1\,(d_1 + d_1' - d_{1a} - d_{1a}') + D_2\,(d_2 + d_2' - d_{2a} - d_{2a}') + \ldots\right.$$
$$\left.\ldots + D_N\,(d_N + d_N' - d_{Na} - d_{Na}')\right] \text{ watts}$$

$N = 5$

$$D_1 = D_m \sin \frac{90}{2\,N} = D_m \sin 9^\circ = 26.3 \times 0.156 = 4.10$$

$$D_2 = D_m \sin \frac{3 \times 90}{2\,N} = D_m \sin 27^\circ = 26.3 \times 0.454 = 11.9$$

$$D_3 = D_m \sin \frac{5 \times 90}{2\,N} = D_m \sin 45^\circ = 26.3 \times 0.707 = 18.6$$

$$D_4 = D_m \sin \frac{7 \times 90}{2\,N} = D_m \sin 63^\circ = 26.3 \times 0.891 = 23.4$$

$$D_5 = D_m \sin \frac{9 \times 90}{2\,N} = D_m \sin 81^\circ = 26.3 \times 0.988 = 26.0$$

D_x	$+d_x$	$+d_x'$	$-d_{xa}$	$-d_{xa}'$	
(D_1 = 4.1)	(13.6	−9.8	−10.0	+14.0)	= 4.1 × 7.8 = 32.0
(D_2 = 11.9)	(22.9	−6.4	− 7.2	+25.0)	= 11.9 × 34.3 = 409.0
(D_3 = 18.6)	(24.5	−2.0	− 2.8	+27.9)	= 18.6 × 47.6 = 886.0
(D_4 = 23.4)	(22.5	+3.0	+ 2.8	+25.8)	= 23.4 × 54.1 = 1270.0
(D_5 = 26.0)	(18.3	+8.8	+ 9.0	+20.2)	= 26.0 × 56.3 = 1400.0
					Sum = 4058.0

$$\frac{\text{Sum}}{4\,N} = \frac{4058}{4 \times 5} = 202.9$$

$$E_u = \frac{E_{max}}{D_m} = \frac{150. \times 1.414 \times 10^3}{26.3} = 8060 \text{ volts per unit.}$$

$$I_u = I_u' \times \frac{R_3'}{R_3} \times \frac{E_{\text{A.P.}}}{E_{\text{A.P.}}} = \frac{8.996 \times 10^3 \times 12.48 \times 10^{-4}}{5040 \times 12.9}$$
$$= 0.000173 \text{ amp. per unit.}$$

$$P = E_u\, I_u \times 202.9 = 8060. \times 0.000173 \times 202.9 = 282.5 \text{ watts.}$$

---

[6] See tube calibration curves (Fig. 1). Deflections are not strictly proportional to deflector-plate voltage.

wave, at 9 deg., 27 deg., 45 deg., 63 deg., 81 deg., 99 deg., etc. The impressed voltage being sinusoidal, the instantaneous voltage corresponding to each of the above points is $E_{max} \sin \theta$, where θ is 9 deg., 27 deg., 45 deg., 63 deg., 81 deg., 99 deg., etc., as above.

Since the deflections of the cathode beam are practically proportional to deflector-plate voltage, the horizontal axis of the cyclogram, Fig. 8, is then divided at $u, -u; v, -v; w, -w; x, -x; y, -y$, according to the equation:

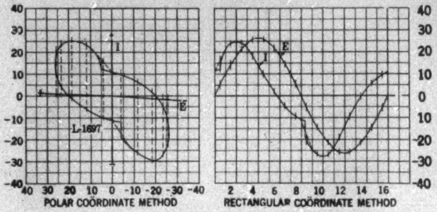


Fig. 8—Derivation of Power from a Corona Cyclogram

Voltage to neutral = 212 kv. max.
Spacing to neutral = 21.5 in.
Conductor: 365 in. diam. polished

POLAR COORDINATE METHOD

+ Half	− Half
(26.3 sin 9° = 4.1) (13.6 − 9.8	+ 14.0 − 10.0) = 4.1 × 7.8 = 32.0
(26.3 sin 27° = 11.9) (22.9 − 6.4	+ 25.0 − 7.2) = 11.9 × 34.3 = 408.
(26.3 sin 45° = 18.6) (24.5 − 2.0	+ 27.9 − 2.8) = 18.6 × 47.6 = 885.
(26.3 sin 63° = 23.4) (22.5 + 3.0	+ 25.8 + 2.8) = 23.4 × 54.1 = 1265.
26.3 sin 81° = 25.9) (18.3 + 8.8	+ 20.2 + 9.0) = 25.9 × 56.3 = 1459.
	4049.

$$\text{Average unit product} = \frac{4049}{20} = 202.4$$

Volts per unit = 8050
Amperes per unit = 0.0001735
Power = 8050 × 0.0001735 × 202.4 = 283 watts
% difference between two methods = 1.06%

RECTANGULAR COORDINATE METHOD

Sect.	*E*	*I*	*P*	Sect.	*E*	*I*	*P*
1	5.1 ×	14.8 =	75.5	9	− 5.1 ×	− 18.0 =	91.8
2	14.6 ×	24.0 =	350.	10	− 14.6 ×	− 26.9 =	393.
3	21.8 ×	24.0 =	524.	11	− 21.8 ×	− 27.2 =	593.
4	26.0 ×	18.0 =	468.	12	− 26.0 ×	− 19.6 =	510.
5	26.0 ×	10.0 =	260.	13	− 26.0 ×	− 9.5 =	247.
6	21.8 ×	2.3 =	50.1	14	− 21.8 ×	0 =	0.
7	14.6 ×	− 4.7 =	− 68.6	15	− 14.6 ×	+ 5.5 =	− 80.3
8	5.1 ×	− 9.8 =	− 50.0	16	− 5.1 ×	+ 9.8 =	− 50.0
			1509.0				1704.5

Average unit product = (1509.0 + 1704.5) ÷ 16 = 200.8
Volts per unit = 8050. Amperes per unit = 0.0001735
Power = 8050 × 0.0001735 × 200.8 = 280 watts

$$D_\theta = D_m \sin \theta$$

where D_θ = Horizontal deflection, positive or negative, at angle, θ, as above.

D_m = Maximum positive or negative horizontal deflection.

Ordinates of the figure corresponding to the abscissas $u, -u; v, -v$; etc., are taken as the points of intersection of the figure with vertical lines drawn through these abscissas, for all sections except those containing an abrupt break in the rate of change of the ordinate, or those having such a nature that the ordinate obtained in this manner is obviously not the approximate mean value for the section. An accurate estimate of the mean ordinate of a section having an irregular boundary is obtained by joining the limits of the given section by a straight line and taking the average ordinate of the straight line and the curve at the given abscissa as the value for use in the subsequent calculations. In this illustrative example (Fig. 7), a is the ordinate taken to correspond to abscissa u (dotted line joins limits of section), and c to correspond to abscissa w.

When obtained by means of the tube in use at present, the horizontal axis is not exactly a straight line, nor is either half of it perpendicular to the vertical axis. It is most convenient, therefore, when tracing the figure from the photographs on transparent coordinate paper, to place the vertical axis in coincidence with the axis of ordinates, and the horizontal axis in its true position, the origin being the intersection point of the vertical and horizontal axes. In measuring the ordinates it is necessary to take the displacement of the axis of abscissas into account.

Since vertical deflections above the horizontal axis represent current in the positive direction, and horizontal deflections to the left of the vertical axis represent voltage in the positive direction, the first quadrant represents negative power, the second positive, third negative, and fourth positive.

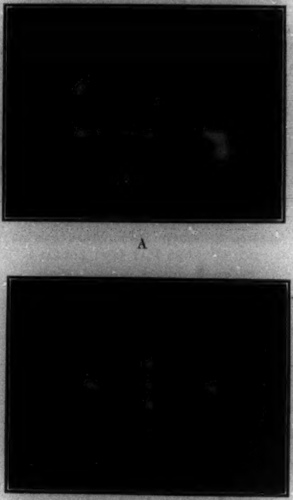
A complete example of the power loss calculations on a typical cyclogram is given in Fig. 7. Space is not available for including the derivation of the formula used, but it is simple and was developed in order to expedite the calculation of numerous cyclograms.

This method of calculating power expenditure is, in effect, the same as that of transcribing the figures to rectangular coordinates and integrating the power wave in the customary manner. Fig. 8 gives an example of the two methods of calculation as applied to the same cyclogram.

To one who had little or no experience in making these calculations it will seem, on account of the blurred figure with which one has to work, that the accuracy would be poor, but such is not the case. After a small amount of practise one can estimate the centers of the lines very accurately, and it has been found that different individuals working independently and at different times will obtain results that check within one per cent. Power values as low as 0.1 watt can be measured with this same accuracy.

A slight error is introduced in the results obtained by means of the methods illustrated in Figs. 7 and 8 by the deviation of the cathode-beam deflection from a linear function of the deflector-plate voltage. This error is, for all practical purposes, inappreciable and the error in abscissas practically compensates for that in ordinates. Under some circumstances, however, it may be desirable to make more exact calculations and a brief outline of this method follows:

The abscissas of the mid-point divisions on the horizontal axis of the cyclogram are located by determining, from the proper calibration curve, the deflector-plate voltage necessary to produce the maximum abscissa of the figure; multiplying this value by the sines of the mid-point angles; and determining, from the calibration curves, the deflections corresponding to these products. At these new abscissas, the average ordinates are read and each one is multiplied by the ratio of average deflector-plate volts per unit required to produce that ordinate to average volts per unit to produce the maximum ordinate. The current and voltage factors are determined from the maximum deflections as before.


A

B

Fig. 9—Cyclograms of Heavy Corona

A—Volt—coulomb cyclogram
B—Volt—ampere cyclogram
Conductor: 0.015 in. diam. smooth
Spacing: 7.5 in. to neutral
Voltage: 15.0 kv. (eff.) to neutral

It should be noted that in obtaining the various products of abscissas and ordinates the value of abscissa required is, for instance, maximum deflection (max. abscissa) times sin 9 deg., etc., and not the corrected abscissa from which the corresponding current is obtained, and the ordinate is the corrected value as given above.

These corrections, the effects of which are very slight, were not made in the loss data given in this paper.

Volt-Coulomb Cyclograms. The corona phenomena can be studied from a somewhat different angle if the resistance, R_3, in Fig. 2 is replaced by a capacitance C_3, or by a parallel combination of capacitance and high resistance C_3'. The ordinates of the cyclograms obtained are then proportional to the instantaneous charges induced on the ground plate by the voltage on the conductor.

If pure capacitance is used, the figure will drift over the screen of the tube due to the accumulation of a negative charge on the deflector-plate connected to C_3. Since in the case of 60-cycle corona on conductors at relatively close spacings, the positive and negative discharges are of unequal magnitudes, C_3 will accumulate a positive or an additional negative charge which will cause further drifting of the figure. In the case of a small conductor (0.015 in. diam.) at a fairly close spacing (7.5 in. to neutral) it was found that at about 15.0 kv. to neutral the positive excess was just sufficient to compensate for the negative charge acquired from the electron stream within the tube. Consequently the figure would remain stationary near the center of the screen and could be photographed when the line and anode voltages were properly manipulated.

When a high resistance is shunted across C_3 to provide a leakage path for the excess positive or negative component, the figure remains stationary at all voltages, and can be photographed in the usual manner.

Fig. 9A is a volt-coulomb cyclogram of the discharge characteristics of a 15-mil (0.015 in. diam.) conductor spaced 7.5 in. to neutral and operating at 15.0 kv. to neutral. This figure was generated in the clockwise direction. The approximately parallel top and bottom sides of the figure represent the 60-cycle charge (affected somewhat by the motion of the space charge surrounding the conductor) induced on the ground plate by the normal conductor capacitance. The slightly curved end sections represent the excess charge induced during a part of each half cycle by the periodically increased conductor capacitance. This increased capacitance exists only during the discharges and hence disappears near the crests of the voltage waves. Consequently the excess charges can not be returned to the circuit and are lost.

TABLE I

Conductor:—0.365 in. diam., polished.
Spacing:—63.5 in. to neutral.
Length:—10.0 ft.
Temp.: = 20 deg. cent. Bar. 28.77 in. hg. $\delta = 0.977$
Tube anode potential = 290.0 volts.
$E_c = 87.0$ kv. eff.

Kv. (eff.) to neutral	R_3 ohms	Power loss (watts)	
		Meas.	Calc.*
87.0	11,067	3.51	2.65
91.0	11,067	7.45	7.22
100.0	11,067	11.8	12.6
115.0	11,067	25.3	25.1
130.0	11,067	43.2	41.8
149.5	11,067	69.7	70.0

*Values calculated by means of Peek's formula (large wires) multiplied by 0.374 ($m_0 = 1.00$)

This type of cyclogram has been employed in other corona investigations, notably those of Dr. Ryan, and provides a convenient means of studying the phenomena from the standpoint of the charge and discharge of "space condensers," but is not so convenient as the first type for power calculation purposes.[7]

A volt-ampere cyclogram corresponding to Fig. 9A is given in Fig. 9B.

POWER LOSS DATA

By means of the first method described above (*i. e.* the volt—ampere cyclograms), loss measurements were made on several solid conductors of various sizes under different conditions of spacing and surface regularity, and also on a large, concentric-strand cable. Typical data are given in Tables I and II and on the accompanying curves.

Fig. 10 gives the 60-cycle loss characteristic of a very small, smooth wire (0.015 in. in diameter) at a rather close spacing (7.5 in. to "neutral"), and at voltages up to the arc-over value. It will be noted that the curve of square root of power against line voltage is very nearly a straight line for a considerable distance above the point marked e_v indicating that in this region the loss followed a quadratic law quite closely. Near the upper end this curve turns up rapidly, indicating that the loss increased at a higher rate just preceding arc-over. Dirt and oxide on this conductor had practically no effect on the loss above e_v. The effective diameter of the conductor was probably sufficiently increased by the foreign matter to compensate for the irregular surface.

Figs. 11 and 12 give the 60-cycle loss characteristics of a large, solid conductor (0.365 in. diameter), both polished and rough, at two spacings (63.5 in. and 21.5 in. to neutral). The curve for the roughened conductor in Fig. 11 was plotted from the data given in Table I, and the curve for the polished conductor in Fig. 12 was plotted from the data given in Table II. No loss whatsoever could be detected from the polished conductor until the visual critical voltage was reached. At this point it suddenly jumped to a definite value and then, with further increase of voltage, followed a quadratic law very closely. It will be noted that for this conductor at the closer spacing two critical voltages are indicated, one for increasing voltage and a different one for decreasing voltage. This characteristic seemed quite definite in the case of large polished conductors at small spacings, and was probably an effect of the space charge created by the uniform corona

TABLE II

Conductor 0.365 in. diameter polished.
Spacing:—21.5 in. to neutral.
Length:—10.0 ft.
Temp. = 18 deg. cent. Bar. = 28.47 in. hg., $\delta = 0.99$
Tube anode potential = 290.0 volts.
E_c: Increasing voltage = 77.0 kv. (eff.); Decreasing voltage = 75.0 kv. (eff.).

Kv. (eff.) to neutral	R_3 (Ohms)	Power loss (watts)	
		Meas.	Calc.*
75.0	5040	8.57	15.1
85.0	5040	35.4	31.5
90.0	5040	42.3	42.0
100.0	5040	70.3	67.5
115.0	5040	118.0	117.
130.0	5040	177.	180.
150.0	5040	283.	285.

*Values calculated by means of Peek's formula (large wires) multiplied by 0.73. ($m_d = 1.00$)

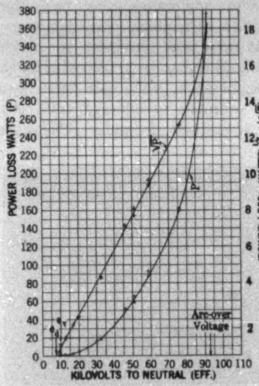


FIG. 10—OBSERVED CORONA LOSS, MEASURED BY MEANS OF THE CATHODE RAY OSCILLOGRAPH

Conductor: 0.015 in. diameter smooth
Spacing: 7.5 in. to neutral
Length: 10.0 ft.
Frequency—60 cycles

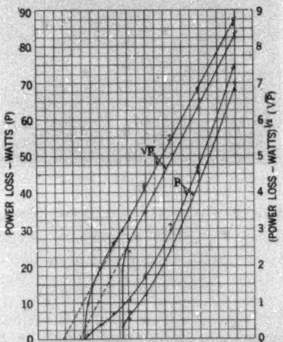


FIG. 11—OBSERVED CORONA LOSS, MEASURED BY MEANS OF THE CATHODE RAY OSCILLOGRAPH

Conductor: 0.365. in. diameter
° smooth surface
+ rough surface
Spacing to neutral: 63.5 in.
Length: 10.0 ft.
Frequency—60 cycles

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[7] Ryan and Henline, *The Hysteresis Character of Corona Formation*, A. I. E. E. TRANSACTIONS, Vol. XLIII, p. 1118.

TABLE III

Conductor (diam. in.)	Condition	Spacing to neutral (in.)	e_d (Kv. eff.)		m_d (Actual)	e_v (Kv. eff.)		e_c (kv.)	δ
			Observed	Calc. (for $m_d = 1$)		Observed	Calc. (for $m_v = 1$)		
0.015	Smooth	63.5	7.2	11.7	0.615	13.0	12.4	11.0	0.99
0.015	"	7.5	6.5	9.12	0.710	10.0	9.65	9.0	0.99
0.064	"	7.0	16.3	14.8	1.10	21.0	21.4	20.0	1.00
0.204	"	63.5	41.5	40.0	1.07	62.0	61.5	59.0	0.99
0.204	"	21.5	34.2	33.9	1.04	53.0	52.1	51.4	0.99
0.365	Polished	63.5	62.7	62.7	1.00	89.0	90.5	87.0	0.977
0.365	"	21.5	50.5	52.6	0.961	76.	76.0	76.0	0.99
0.365	Rough	63.5	54.0	62.7	0.86	87.0	90.5	66.0	0.975
0.365	"	21.5	44.5	52.6	0.848	77.0	76.0	60.8	0.99
336,400 C. M. A. C. S. R.	Smooth	21.5	77.0	90.4	0.852	125.	119.	100.5	0.963
"	"	63.5	95.0	113.	0.842	147.	149.	119.	0.975
"	Mutilated	63.5	....	113.	....	....	149.	63.	0.962
"	"Weathered" (Rough)	63.5	....	113.	....	....	149.	105.	0.964

e_d Disruptive critical voltage in Peek's formula. ($= e_o$, practically, for conductors of diameter greater than 0.15 in.)
m_d and m_v Irregularity factors.
e_v Visual critical voltage in Peek's formula. Observed value is that at which loss becomes quadratic.
e_c Actual corona starting voltage
δ Atmospheric correction factor.

envelope. The roughened conductor gave a loss at much lower voltages than the polished, and a greater loss at the higher voltages. In this case the loss apparently followed no definite law, and was very sensitive to accidental surface conditions until well above the starting voltage when the curve became a quadratic. The excess loss above the quadratic due to surface irregularities gave a typical probability curve.

Loss measurements were made on a 336,400-cir. mil A. C. S. R. (aluminum conductor steel reinforced) cable with the surface in three different conditions, namely, smooth, rough, and mutilated. A smooth, clean cable spaced 63.5 in. to neutral gave no loss below 119.0 kv. to neutral and closely followed the quadratic, for $m_0 = 0.85$, above 125 kv. This value of m_0 was about the average obtained with this conductor at the 21.5-in. and 63.5-in spacings. See Table III. In the case of the roughened conductor the outside strands were scratched, as if the cable had been dragged over rough ground, and were covered with spots of dirt and oxide. The loss started at a lower voltage (89 per cent of the former starting value) and was higher over the entire range. The outside strands of the mutilated conductor were in a condition such as would be produced by dragging the cable over hard

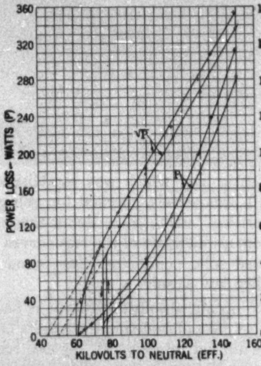

FIG. 12—OBSERVED CORONA LOSS, MEASURED BY MEANS OF THE CATHODE RAY OSCILLOGRAPH

Conductor: 0.365 in. diameter
• smooth surface
+ rough surface
Spacing to neutral: 21.5 in.
Length: 10.0 ft.
Frequency—60 cycles

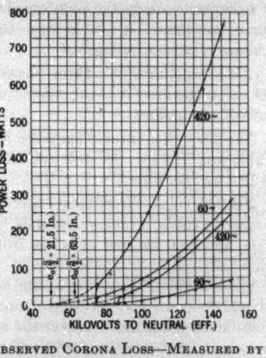

FIG. 13—OBSERVED CORONA LOSS—MEASURED BY MEANS OF THE CATHODE RAY OSCILLOGRAPH

Comparison of losses at 420 cycles and 60 cycles
Conductor: 0.365 in. diam., polished surface

Spacing to neutral: $\left(\frac{S}{2}\right)$; • = 21.5 in.

△ = 63.5 in.

Length: 10.0 ft.

sharp rocks. The loss started at a very low voltage (53 per cent of the starting value for the smooth conductor) and was of greater magnitude than in the other two cases.

It was found that the curve obtained by subtracting the quadratic from the loss curve of the mutilated conductor was a typical probability curve. Extrapolation of the curves for the three conditions beyond 150 kv. (up to which voltage the measurements were taken) showed them to come approximately together at 185 kv.

The loss from the smooth cable had much the same characteristics as that from the roughened solid conductor (Fig. 13). The surface irregularities, in the former case due to stranding and in the latter case due to burrs and scratches, caused the loss to begin below the "visual critical voltage," e_v, (for a smooth conductor of the same diameter) but not so low as the critical disruptive voltage, e_0. The quadratic seemed to apply when the true critical voltage, e_v, was approached. As was mentioned in connection with the solid conductors, the loss from the cables at voltages below e_v was very sensitive to changes in surface condition. When the strands were badly scratched the loss started at a relatively low voltage, far below the value of e_0 for the smooth cable, and was of considerable magnitude at voltages too low to cause any discharge from the undamaged conductor.

It is of interest to study the data obtained with the above three specimens of cable and the following conclusions are of particular interest:

1. At e_v, corresponding to a three-phase line voltage of 258,000 volts, the loss from the smooth cable was 49 kw. per mile of conductor.
2. At 220,000 volts, three-phase, the loss from the smooth cable was 18.0 kw. per mile and from the rough cable was 29.4 kw. per mile.
3. At 205,000 volts, three-phase, there was no loss from the smooth cable, 17.0 kw. per mile loss from the rough cable, and 38.0 kw. per mile loss from the mutilated cable.
4. At 132,000 volts, three-phase, the loss from the mutilated cable was 4.7 kw. per mile of conductor.
5. At sea level and in fair weather the smooth cable would operate at 205,000 volts, and the rough cable at 180,000 volts, three-phase, without corona loss. Corona would still be present on the mutilated cable at 130,000 volts, three phase.

From the above data it is evident that a great deal of care should be exercised in stringing the lines in order to keep them as free as possible from burrs and scratches. It is also evident that small changes in the surface condition of the conductor produce large changes in the magnitude of the loss near the critical voltage. The deviation of the loss curve from a quadratic below e_v seems to follow the probability law and, since the loss below e_v is determined by the size, projection, and number of irregular "spots" on the conductor, it is reasonable that this law *should* apply, at least approximately.

It seems that the best general practise is not to attempt to design the line for a given amount of corona loss, but to so choose the conductor and spacing that the dielectric strength of air (*i. e.*, 30 kv. per cm. max. under standard atmospheric conditions) multiplied by the irregularity factor of that conductor is not exceeded at the conductor surface for the highest operating voltages. Recent developments in the manufacture of cables having large diameters relative to their effective cross sectional areas make it economically possible to design and construct corona-free lines.

The irregularity factor is determined by actual measurement on a specimen of the given conductor or a similar conductor which has been installed and well weathered under the particular conditions involved.

Radio interference, and interference with successful carrier-current communication on the line itself are gaining importance as factors making the elimination of corona an important matter.

Loss at Higher Frequency. Fig. 13 gives the loss characteristics of a large, solid conductor, polished, at 420 cycles in comparison with the same at 60 cycles. The ratio of losses at the closer spacing was about 3 and at the greater spacing was about 3.5. The conduction component of the loss was apparently considerably greater at the small than at the large spacing and probably accounted for the difference in ratios. These curves are quadratics with the same critical voltages as the 60-cycle curves on the same conductor.

COMPARISON WITH RESULTS OF PREVIOUS INVESTIGATIONS

The quadratic law, which was established by Mr. Peek in 1910, was closely checked for voltages above the "visual critical" value (e_v). Below this voltage the deviation from a quadratic of the loss curves for cables and roughened solid conductors was found to follow the probability law closely.

In order to obtain absolute values it is necessary to correct the loss data obtained in these tests because the ground plate did not intercept all the electrostatic flux emanating from the conductor. The correction varies with the spacing. For any given spacing, however, the percentage of flux intercepted by the plate is approximately constant regardless of the voltage.

The correction is readily obtained by comparing the calculated charging current of the conductor with the measured values. In this manner it was found that at a spacing of 63.5 in. to neutral about 44 per cent of the total current was measured, and at a spacing of 21.5 in. to neutral about 78 per cent was measured. The uncorrected measured loss at the greater spacing was about 38 per cent of the value calculated by means of Peek's formula, and at the smaller spacing was about 70 per cent of the calculated value.

The formulas for critical disruptive gradients and particularly the visual critical gradients as determined by Mr. Peek's early work were closely checked.

The disruptive critical voltage, e_d, which is the voltage at which the quadratic curve starts; the visual critical voltage, e_v, and the actual corona starting voltage, e_c, for a number of conductors are tabulated in Table III. Both the calculated and observed values of the first two quantities are recorded. The calculated values of e_d are from Peek's[8] disruptive critical formulae for $m_d = 1$. The observed values of e_d are the voltages at which the projected curves of square root of power against voltage intersects the voltage axis. The quantity m_d (actual) is the ratio of observed e_d to calculated e_d.

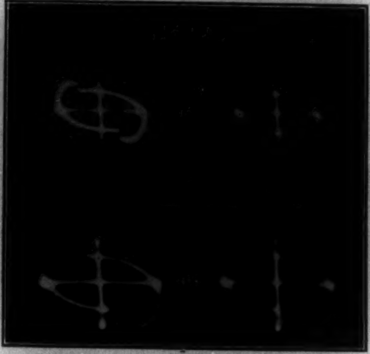

FIG. 14—COMPARATIVE CYCLOGRAMS SHOWING THE INFLUENCE OF CONDUCTOR SURFACE CONDITION UPON CORONA DISCHARGE CHARACTERISTICS.

Conductor: solid, 0.365 in. diameter
Spacing to neutral: 63.5 in.

The observed values of e_v are the voltages at which the losses began to obey a quadratic law. The calculated values were obtained from Peek's visual critical formulae for $m_v = 1$.

Over the entire range of conductors tested, a remarkably close agreement exists between the observed and calculated values of e_v. There is also a fairly close agreement between the observed and calculated values of e_d. It will be noted that in the case of the roughened solid conductors the loss actually started at voltages well above the e_d of the same conductor in a polished condition. The same may be said regarding the roughened cable. For the mutilated cable, however, e_c is lower than e_d ($e_d = e_0$ in the case of large conductors such as this). This condition is largely accounted for by the presence on the cable of numerous sharp projections which produced flux concentrations and consequently, local discharges at relatively low voltages.

CHARACTERISTICS OF THE CORONA DISCHARGE

Typical cyclograms showing the discharge characteristics under different conditions are given in Figs. 3B, 4 and 14. It will be noted that for polished conductors (Figs. 4, 14 left side) the individual discharges, particularly the negative, form very suddenly, and the current waves contain abrupt distortions. In the case of roughened conductors (Fig. 14 right side) and cables the individual discharges form much more gradually and the current wave distortions are much less abrupt. Fig. 14 illustrates the influence of surface conditions upon the magnitude and character of the discharge.

The decrease of instantaneous disruptive voltage with increasing line voltage is shown in Fig. 14.

SUMMARY

The low-voltage cathode ray oscillograph is well adapted to investigations of phenomena involving small currents of complex wave shape. This instrument was used in a study of the volt—ampere characteristics of the corona discharge. The circuit employed was such that the figures traced on the screen of the tube were volt-ampere cyclograms. These cyclograms were photographed by placing the film directly against the end of the tube and the power expenditure represented was calculated by a method which was in effect the same as that of transcribing the polar figures to rectangular coordinates and integrating the power wave in the usual manner. The results were converted to watts by means of the tube calibration data which were obtained by impressing definite voltages across the deflector plates and measuring the resulting deflections.

The cyclograms give accurate indications of the structural nature and characteristics of the corona discharge.

Some of these characteristics were checked visually by means of the stroboscope.

The following conclusions were drawn regarding the power loss and starting voltage:

1. Above the "visual critical voltage," e_v, as given in Mr. Peek's early work, the loss-voltage relation is a quadratic.

2. For polished solid conductors the loss suddenly jumps from zero to a definite value at e_v and then follows a quadratic.

3. For cables and roughened solid conductors the excess loss above or below that given by the quadratic law approximately follows the probability law below

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[8] See "Dielectric Phenomena in High Voltage Engineering," by F. W. Peek, Jr., pages 117-152.

e_v. This excess loss may be either positive or negative, or both.

4. The critical disruptive gradients and particularly the visual critical gradients as determined in Mr. Peek's early work were closely checked.

5. The loss near the starting voltage is greatly affected by the regularity and condition of the conductor surface.

The characteristics of the discharge are briefly as follows:

1. For polished solid conductors the individual discharges, particularly the negative, start suddenly and are accompanied by heavy rushes of current.

2. For roughened solid conductors or cables the individual discharges start gradually and the current waves contain no sharp breaks.

3. The instantaneous disruptive voltages decrease with increasing line voltage.

4. The individual discharges stop near the crests of the voltage waves.

Acknowledgment is made of the assistance of Mr. T. M. Hotchkiss, and others of the staff of the High Voltage Engineering Laboratory.

Discussion

For discussion of this paper see page 1020.

The Law of Corona and the Dielectric Strength of Air—IV

The Mechanism of Corona Formation and Loss

BY F. W. PEEK[1]

Fellow, A. I. E. E.

***Synopsis.**—The mechanisms of corona and corona loss have been studied with the cathode-ray oscillograph. High voltage power of the order of 0.1 watt can be measured with an accuracy of 1 per cent with this instrument. The measurements show that the loss follows the quadratic law above the visual critical voltage.*

On polished wires there is no loss until the visual critical voltage is reached. The loss then starts quite suddenly and takes a finite value on the quadratic curve. On cables and imperfect conductors there is a loss below the visual critical voltage on brushes at local "rough" spots. The loss due to these irregularities can be represented by the probability law. This is quite in accord with former work.

In practise it is important not to mutilate the conductors in stringing. The really important factor in design is the irregularity factor, m_o, for weathered conductors. No line should be operated with a corona loss under fair weather conditions. It is not necessary from the economic standpoint since large diameters can be obtained with special types of conductors.

The visual critical corona voltage can be calculated with great accuracy. As the applied a-c. voltage is increased above the visual critical value, the instantaneous critical voltage becomes lower and lower until finally corona starts at the zero point of the wave. This occurs when the applied voltage is twice the visual critical voltage. At still higher voltage, corona starts below zero or on the falling wave. The effect is as if the instantaneous critical voltage is reduced by an amount approximately equal to the excess of the applied voltage above the visual critical voltage. Thus when the excess is equal to the visual critical voltage the instantaneous voltage is zero. This occurs when the applied voltage is twice the visual critical voltage. The reason for this is clearly shown as well as many other interesting facts.

Artificial corona was readily produced with all of the characteristics of real corona after the mechanism was determined.

The quadratic law seems to be the rational expression for the loss.

Details of measurements are given in the supplemental paper on measurements by Starr and Lloyd, "Methods Used in an Investigation of Corona Loss by Means of the Cathode-Ray Oscillograph."

INTRODUCTION

THIS paper records the continuation of an investigation started in 1910, first reported to the Institute in 1911[2] and from time to time thereafter as seemed desirable.

At this time the results of a study with the cathode ray oscillograph are given. This instrument, not available in a very practical form when the investigation was started, has now made it possible to obtain a very good picture of the mechanism of corona formation and loss as well as to measure small losses. The instrument used was of the Western Electric hot cathode type. Probably the first work on corona with the cathode ray oscillograph was done by Prof. H. J. Ryan who has added considerably to our knowledge of this subject[3].

RÉSUME AND CONCLUSIONS

The cathode ray oscillograph used in the investigation described in this paper offers a means of studying the mechanism of corona loss as well as a means of measuring accurately small low power factor losses.

This investigation clearly shows the mechanism of corona loss. It was found that the corona loss follows the quadratic law above the visual critical voltage. Below this voltage there is no loss on polished conductors but loss may occur on an imperfect conductor due to local corona at surface irregularities caused by abrasions, dirt, etc. A weathered cable has the characteristic of a dirty wire and may approximate the quadratic down to the disruptive critical voltage. The operating voltage should be below the disruptive critical voltage for the weathered conductor. The losses due to chance irregularities near the critical voltage are closely approximated by the probability law. This quite confirms the results of the former investigations.

The measured visual critical corona voltages check the calculated values very closely.

The cathode ray oscillograms show quite clearly how the corona loss occurs. As the applied voltage is increased above the visual critical voltage, corona starts at a lower and lower instantaneous value of voltage on the a-c. wave. The instantaneous value of starting voltage on the a-c. wave is decreased by an amount approximately equal to the excess of the applied voltage above the visual critical voltage. For instance, when the applied voltage is twice the visual critical voltage, corona starts at the zero point of the voltage wave on each half-cycle. Thus the excess of the applied voltage above e_v is

$$2e_v - e_v = e_v$$

The critical voltage is then decreased by this amount or reduced to

$$e_v - e_v = 0$$

The reason for this is quite evident. After corona starts, a tube of corona surrounds the conductor and

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[1] Consulting Engineer, General Electric Co., Pittsfield, Mass.

[2] Peek, *Law of Corona and Dielectric Strength of Air—I*, TRANSACTIONS, A. I. E. E., 1911, Vol. 30, pp. 1889-1988.

[3] Ryan and Henline, *Hysteresis Character of Corona Formation*, TRANSACTIONS A. I. E. E., 1924, Vol. 33, p. 1118.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

is charged through the "corona arcs" up to the maximum of the wave when the arcs go out or corona stops. This corona tube or "space" charge increases quite suddenly to a finite diameter at the start and then more gradually as the maximum of the wave is approached. This charge caused by the excess voltage returns towards the conductor with the falling wave and adds to the charge caused by the applied voltage on the next half-cycle. When the sum of these charges is sufficient to cause the breakdown gradient, corona starts at an instantaneous voltage less than the visual critical voltage. With the start of corona there is a sudden rush of current. When twice the visual critical voltage is applied, the excess voltage is equal to the critical voltage. The charge due to this excess voltage is then sufficient to cause corona without any additional charge. Corona thus starts on the following half-cycles on the zero of the wave as shown above. If the applied voltage is further increased, corona starts below the zero of the wave or on the falling voltage.

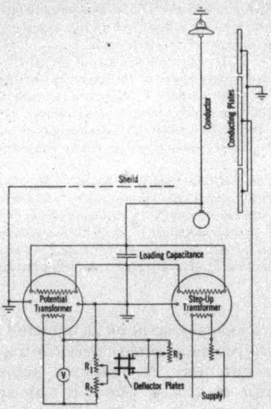

FIG. 1—ARRANGEMENT OF VERTICAL PLATE AND CONDUCTOR FOR A STUDY OF CORONA BY MEANS OF THE CATHODE RAY OSCILLOGRAPH

Corona characteristics can be produced artificially by means of condensers. For example, take two condensers and place a gap in series with a resistance across one of them. If voltage is applied and gradually increased, capacity current flows until the gap breaks down. There is a sudden rush of current. The spark, which represents the corona, continues to the maximum of the voltage wave when it stops. This leaves an excess charge on one condenser which adds to the charge caused by the line voltage on the next half-cycle. The gap breaks down at lower and lower instantaneous voltage as the applied voltage is increased and becomes zero when the applied voltage is twice the initial starting voltage as in the case of corona.

It is shown that the loss is caused by the charging of the space condenser through the "resistance" of the corona arc or by the motion of the charge through the field and that the quadratic is the rational form for the loss to take.

The formation of corona current is much faster on the negative than on the positive wave.

The results are of practical as well as theoretical importance. For instance, Fig. 7 shows very decidedly the importance of using care in stringing conductors.

METHODS OF TESTS

The cathode ray oscillograph was connected as shown in Fig. 1. With this arrangement the field across one pair of plates is proportional to the voltage while that across the other pair is proportional to the current. When voltage is applied across a capacity load, an ellipse is described, as shown in Fig. 2, the abscissas being proportional to the instantaneous voltages and the ordinates to the instantaneous current. These figures are recorded photographically. The voltage deflections to the left are positive while those to the right are negative. Positive currents cause deflections above the horizontal axis and negative currents below the axis. If the voltage wave is known, the voltage and current waves can be plotted as shown in Fig. 8. Power is easily obtained by measurements of the instantaneous voltage and current and by simple calculations as follows:

Each figure is divided into an integral number of equal-time sections and the product of the mean ordinate by the mean abscissa is obtained for each section. The average of all such products is then multiplied by the circuit calibration constants, volts and amperes per unit deflection of the cathode beam. The result is expressed in watts. This is equivalent to transcribing the figures to rectangular coordinates and then integrating the power wave in the usual manner. Power of the order of 0.1 watts can be measured with an accuracy of about 1.0 per cent.

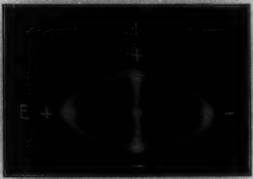

FIG. 2—CYCLOGRAM OF CAPACITY CURRENT BEFORE THE START OF CORONA

The general arrangement for corona measurements is shown in Fig. 1. The measurements were usually made between a single wire and a plane. Precautions were taken to eliminate the end effect by making the measurements on about 10 ft. of wire in the central part of the plane. Precautions were also taken to guard against stray fields and to prevent phase angle displacement errors.

Since they are very completely covered in the supplemental paper, it is not necessary to go into further details here regarding the measurements.[4]

The assistance of Messrs. W. L. Lloyd, E. C. Starr, T. M. Hotchkiss, and other members of the High Voltage Engineering Laboratory Staff in carrying on this investigation, is acknowledged. Mr. Starr's work in making the measurements was especially valuable.

Early Work and Laws for Determining Loss and Critical or Starting Voltage

The following brief statement of laws established in the first paper of the series should be of assistance in comparing the earlier work with the present.

The several laws of corona are as follows:

The *visual critical voltage* or the voltage at which corona first starts:

$$e_v = g_v\, r \log_e \frac{s}{r} \text{ kv. to neutral (crest)}$$

$$g_v = 30\,\delta \left(1 + \frac{0.3}{\sqrt{\delta\, r}}\right) \text{ kv. per cm. (crest)}$$

$$\delta = \frac{3.92\, b}{273 + t}$$

Where

r = radius in cm.,
s = spacing in cm.,
t = temp. deg. cent.,
b = barometric pressure cm.,

The power loss is:

$$p = \frac{241}{\delta} (f + 25) \sqrt{r/s}\, (e - e_0)^2\, 10^{-5} \text{ kw. per km. of one conductor}$$

$$e_0 = g_0\, m_0\, r\, \delta \log_e \frac{s}{r} \text{ effective kv. to neutral}$$

$$g_0 = 21.1 \qquad m_0 = \text{irregularity factor}$$

For small conductors:

$$e_d = g_0\, \delta \left(1 + \frac{0.30}{\sqrt{\delta r}} \left(\frac{1}{1 + 230\, r^2}\right)\right) r \log_e \frac{s}{r}$$

This has been referred to as the *quadratic law* and states that the loss increases as the square of the excess voltage above the disruptive critical voltage, e_0. The quadratic law obtains when a plot between $\sqrt{p}$ and e gives a straight line. The *disruptive critical voltage*, e_0, is lower than e_v and, in fact, corresponds to a gradient of $g_0 = 30$ kv. per cm. or the strength of air in a uniform field. In the early work it was stated that with a polished conductor, no loss would be expected below the visual critical voltage e_v. It was further stated that the loss should then start quite suddenly and follow the quadratic law. For rough conductors the loss should be expected below e_v due to the surface irregularities. This should follow because abrasions, dirt, and other chance irregularities cause high local stress and thus local corona and loss below e_v. Since local corona is caused by chance conditions, it is difficult to predetermine. It was shown that this loss followed the probability law.

Corona Loss Measurements By The Cathode Ray Oscillograph

Power Loss and the Quadratic Law. In Fig. 3A, measurements made by the cathode-ray oscillograph are plotted between loss p, and voltage e, for a small wire. The $\sqrt{p}$ is plotted with e in Fig. 3B where

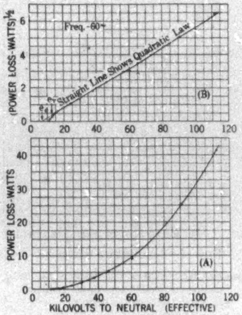

FIG. 3—OBSERVED CORONA LOSS

Measured by means of the cathode ray oscillograph
Conductor: 0.0382 cm. diam. smooth
Spacing: 161 cm. to neutral
Length: 305. cm.

TABLE I

CORONA LOSS ON SMALL CONDUCTOR

Conductor—0.0382-cm. diameter, smooth
Spacing —161 cm. to neutral
Length —305 cm.
Freq. —60~

Kv. (eff.) to neutral	Effective current (milliamperes)		Power loss (watts)
	Disruptive interval	Entire cycle	
11.6			0.0102
13.0	0.0392	0.0405	0.123
14.4			0.246
16.4	0.0749	0.0630	0.382
19.6	0.105	0.0868	0.588
24.0	0.137	0.115	1.11
30.0	0.181	0.157	1.90
38.0	0.249	0.218	3.36
46.4	0.322	0.296	5.53
60.5	0.461	0.400	9.14
91.0	0.740	0.669	25.4
112.0	0.939	0.860	41.5

Correction Factor—Divide current by 0.43
Divide power by 0.40

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[4] Lloyd and Starr, *Corona Loss Measurements by Means of the Cathode Ray Oscillograph*, p. 997.

the resulting straight line is the test of the quadratic. The loss starts suddenly at e_v and follows the straight line. The extension of this line cuts the axis at e_0 or e_d. Between e_0 and e_v there is very little loss for this relatively smooth wire. This curve is a check on the quadratic law for voltages up to ten times the starting voltage.

Fig. 4 shows that a larger polished wire also follows the quadratic law above e_v, or that

$$p = k\,(e - e_0)^2.$$

The Disruptive Critical Voltage and Visual Critical Voltage. The disruptive critical voltage e_d or e_0 is due to a gradient of 30 kv. per cm. at the conductor and is the voltage used in the quadratic, for large conductors; e_d is used for small conductors. The gradient g_0 corresponds to the strength of air. Quoting from the first paper, "With perfect conductors loss does not start at the voltage e_0, at which the disruptive gradient is reached at the conductor surface, but only after the disruptive strength of air has been exceeded over an appreciable distance from the conductor, that is, at a higher voltage e_v. With such conductors there would be no loss until e_v was reached. The loss would then suddenly take nearly the definite value calculated for this applied voltage by the equation."[3] With the usual imperfect conductors there is loss below e_v due to tufts of visual corona at local irregularities. Any appreciable corona loss is thus always accompanied by visual corona. For a dirty or mutilated wire, loss occurs at local brushes below the true e_v for the wire. These local losses below or near the critical voltage are difficult to predetermine because they are caused by chance conditions. This is generally not of great practical importance as it is desirable to operate below e_0. The loss on new wires decreases after operation as the irregularities are burned off. The local irregularity loss then becomes insignificant compared to those caused by dew, frost, rain, etc.

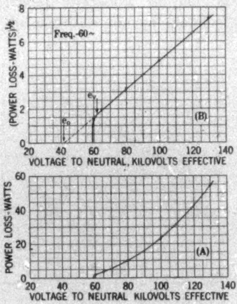

FIG. 4—OBSERVED CORONA LOSS

Measured by means of the cathode ray oscillograph
Conductor: 0.520 cm. diam. polished
Spacing: 161. cm. to neutral
Length: 305. cm.

TABLE II

CORONA LOSS ON MEDIUM CONDUCTOR

Conductor—0.520-cm. diameter, polished
Spacing —161 cm. to neutral
Length —305 cm.
Freq. —60~

Kv. (eff.) to neutral	Effective current (milliamperes)		Power loss (watts)
	Disruptive interval	Entire cycle	
60.0			1.80
65.0			3.81
80.0			10.3
100.0			23.3
130.0			54.3

Correction Factor—Divide power by 0.33

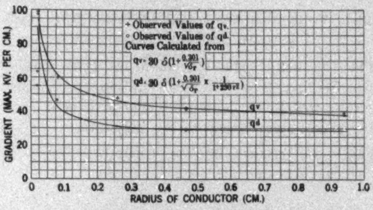

FIG. 5—CRITICAL DISRUPTIVE GRADIENT

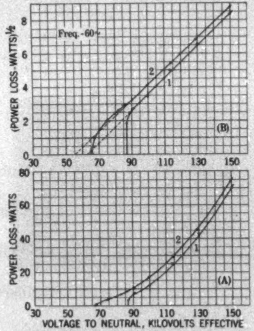

FIG. 6—OBSERVED CORONA LOSS

Measured by means of the cathode ray oscillograph
Conductor: 0.928 cm. diam. smooth (1)
mutilated (2)
Spacing: 161. cm. to neutral
Length: 305. cm.

Values of g_v and g_d or g_0 determined by the cathode ray oscillograph are the plotted points in Fig. 5. The drawn curves are the calculated values. The check is very good.

Loss Near the Critical Voltage and the Effect of the Condition of the Conductor Surface. In Fig. 6, loss curves are plotted for a rough and for a highly polished wire both directly between p and e and between $\sqrt{p}$ and e. The curve between the $\sqrt{p}$ and e shows that the loss follows the quadratic law. For the polished wire there is no loss until e_v is reached. At that voltage the loss suddenly takes a definite value on the quadratic curve. With the rough wire there is loss below e_v due to local brushes at irregularities. The loss on the rough wire is in excess of the loss on the smooth wire up to about twice e_v. As determined in the early work, this excess loss follows the probability curve

$$p = q\, \epsilon^{-h(e_0 - e)^2}$$

TABLE III

CORONA LOSS ON LARGE CONDUCTOR

Conductor—0.928-cm. diameter
Spacing —161 cm. to neutral
Length —305 cm.
Freq. —60~

Kv. (eff.) to neutral	Effective current (milliamperes) Disruptive interval	Effective current (milliamperes) Entire cycle	Power loss (watts)
	Polished surface		
87.0	0.192	0.361	3.51
91.0	0.330	0.404	7.45
100.0	0.508	0.485	11.8
115.0	0.695	0.622	25.3
130.0	0.915	0.789	43.2
149.5	1.18	1.01	69.7
	Roughened surface		
67.3			0.995
74.8			3.86
82.3			6.95
86.7			8.72
90.7			10.5
99.4			16.9
114.3			29.7
129.0			46.0
149.3			74.0

Correction Factor—Divide power by 0.35—Current by 0.44

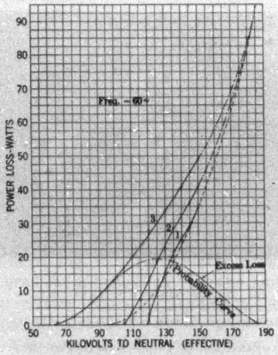

FIG. 7—OBSERVED CORONA LOSS

Measured by means of the cathode ray oscillograph
Conductor: A. C. S. R. Cable
Aluminum—30 strands, 336,400 cm.
Steel—7 strands
Diam. 0.741 in.
Length: 10.0 ft.
Spacing to neutral: 63.5 in.
1 Clean and smooth
2 Rough
3 Badly mutilated

Fig. 7 shows loss curves near e_0 for smooth, rough, and mutilated conductors. The excess loss for the mutilated conductor is shown by the dotted line. The ratio between e_0 for the rough and polished wires gives the irregularity factor m_0. After operation or weathering, the loss at e_0 becomes quite small.

The measurements by the cathode ray oscillograph are thus quite in agreement with laws formulated in the former work as follows:[5]

a. At the visual critical voltage and above, corona loss follows the quadratic law over a wide voltage range, or

$$p = k\,(e - e_0)^2$$

b. There is no loss below e_v for polished wires.

c. For roughened wires, there is a loss below e_v at spots due to local corona. The critical voltage, e_0, is then decreased by a factor m_0. This irregularity loss follows the probability curve. For the ordinary weathered cable, the loss is generally not far from the quadratic down to e_0.

TABLE IV

CORONA LOSS ON CABLE

Conductor—A. C. S. R. cable, 1.88-cm. diameter
Spacing —161 cm. to neutral
Length —305 cm.

Kv. (eff.) to neutral	Power loss (watts) Smooth	Power loss (watts) Rough	Power loss (watts) Mutilated
65.0			0.648
75.0			2.34
85.0			6.53
95.0			11.3
105.0			16.5
110.0		3.69	
115.0		7.44	
120.0	3.06	13.1	27.8
125.0	9.20	18.7	
130.0	15.3	22.5	33.6
135.0	21.6	28.5	
140.0	25.4	32.5	40.6
145.0	28.8	36.2	
150.0	34.6	41.0	48.8

d. e_0, e_d, and e_v check former measurements.

The loss as affected by various factors will be more critically studied later in connection with the cyclograms. In the curves in Figs. 3, 4, and 6, the loss is not reduced to the equivalent of parallel wires. This can be done by a correction factor. The loss is somewhat less than that for parallel wires due to the fact that the ground plate from which current was taken was not of infinite extent. Part of the flux went to this plate and part to the equivalent of an infinite plane. The loss may be corrected so that it corresponds to one of two parallel wires of the given length by multiplying by the proper factor.

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[5] (a) Peek, *Law of Corona and Dielectric Strength of Air*, TRANS. OF THE A. I. E. E., Vol. 30, pp. 1889-1988. (b) Peek, "Dielectric Phenomena in High Voltage Engineering," McGraw-Hill.

THE MECHANISM OF CORONA OR HOW CORONA FORMS

The Critical Voltage or Breakdown. The cathode ray oscillograph gives a very good picture of the mechanism of corona formation from instant to instant during the a-c. wave. Some exceedingly interesting facts have been observed. For instance, as the applied voltage is increased above the visual critical voltage, e_v, corona starts at a lower and lower instantaneous voltage on the wave. The instantaneous value of the starting voltage on the a-c. wave actually becomes zero when the applied voltage is approximately $2e$ and finally crosses zero at higher applied voltages.

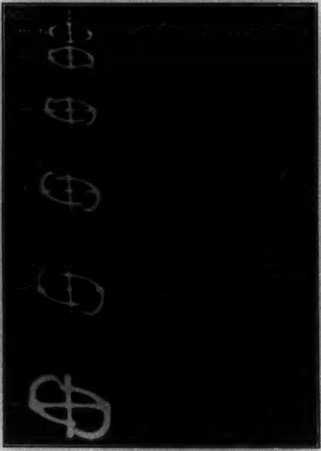
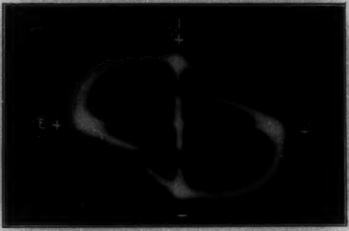


FIG. 8—VARIATION OF INSTANTANEOUS CORONA STARTING VOLTAGE WITH APPLIED VOLTAGE

Applied Voltage (kv.)		Visual critical voltage (kv.) (Max.)	Instantaneous critical voltage (e_i) (kv.)	
Eff.	Max.		Pos.	Neg.
87.0	123.	122.6	105.	101.
91.0	129.	"	102.	93.3
100.0	141.	"	90.4	81.2
113.5	161.	"	76.5	66.7
130.0	184.	"	60.4	49.1
149.5	212.	"	42.8	30.1

The visual critical corona voltage or the a-c. voltage at which corona starts when a low voltage is applied and gradually increased, is very sharp and definite and can be determined with accuracy. The formula for calculating this voltage is given above and seems to check well with the present tests. When the voltage is increased above the visual voltage and then reduced, the corona stops at the same voltage at which it started.



A

(a) Applied voltage e = 65.6 kv. max. to neutral:
Visual critical voltage e_v = 18.1;
Instantaneous critical voltage e_i = − 26 kv.

B

(b) Applied voltage, e = 212 kv. max.;
Visual critical voltage e_v = 18.1;
Instantaneous critical voltage e_i = − 142 kv.

FIG. 9—INSTANTANEOUS CORONA STARTING BELOW ZERO OR ON THE FALLING WAVE FOR APPLIED VOLTAGES FAR IN EXCESS OF THE VISUAL CRITICAL VOLTAGE, e_v

Spacing conductor to plane 161 cm. Diameter of conductor 0.371 cm. Frequency 60 cycles. Scale smaller in (b) than (a). Compare with Fig. 8

Loss occurs when the voltage is above the critical voltage. A study of the instantaneous corona at voltages in excess of the critical voltage is thus important to an understanding of the mechanism of corona loss. Fig. 8 shows a set of cyclograms taken on a wire. The top figure was made just at the start of corona at e_v, and represents capacity current with a slight corona hump at the maximum of the voltage. The ordinates give current, while the abscissas give voltage. Following the X axis to the left, it will be noted that a sudden increase of current or hump starts just before the maximum voltage is reached, and at an instantaneous voltage not far from e_v. At the maximum voltage, the corona hump disappears. The curve is then approximately the capacity ellipse, altered somewhat by the motion of the space charge, until corona starts again on the next half-cycle. The polar diagrams have been translated to the usual rectangular co-ordinates at the right. The corona hump is well shown in these figures. It will be noted that the corona starts at a lower and lower instantaneous voltage as the applied voltage is increased until in the last figure the instantaneous starting voltage on the a-c. wave e_i is almost zero. At a still higher voltage, e_i passes through zero or becomes negative, as in Fig. 9. Referring to Fig. 11A, when the voltage is gradually increased, corona starts at the maximum of the wave when the visual critical voltage e_v is reached. This voltage is very definite. When a higher voltage, e, is applied as in Fig. 11B, corona starts during the first half-cycle at the instantaneous voltage e_v. The charge on the conductor is then q_v or the flux is ψ_v. e_v is the voltage necessary to produce the flux or the charge required to cause the breakdown gradient g_v. In this particular example, assume that the wire at the start is negative. After corona begins, the positive ions are attracted to it and discharged. The repelled ions become in effect a charged cylinder, of varying diameter, surrounding the conductor. The stress between the conductor and this charged cylinder, or "space charge," now remains more or less constant with increasing voltage or just high enough to maintain the corona charging arcs; see Fig. 15. The stress is limited by the breakdown gradient of the air, g_0. Corona continues to form and to increase the outer cylinder until the maximum voltage is reached or slightly passed, when it stops. At this instant, the stress between the wire and the corona cylinder is just below the breakdown value. With decreasing voltage, the stress between the wire and the space charge decreases and somewhere on the descending wave becomes zero. This leaves for the instant the total flux on the space charge. The conductor and the space charge are then at the same potential. This occurs at $e - e_v$ on the descending wave. If there were no drop in the corona, this condition would occur at the maximum of the wave. With reversal of the voltage, the space charge, in effect, adds to the charge on the conductor. When the sum of these two charges becomes, in effect, equal to q_v, so as to cause a gradient g_v near the conductor surface, breakdown occurs. Then, if the charge due to the voltage is q_i, and that due to the space charge is q_s, breakdown occurs when

$$q_i + q_s = q_v$$

or in terms of flux

$$\psi_i + \psi_s = \psi_v$$

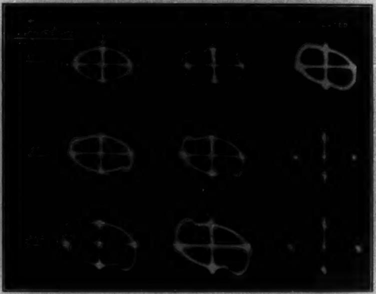


Fig. 10—Comparative Cyclograms Showing the Influence of Conductor Surface Condition upon Corona Discharge Characteristics

Conductor: A. C. S. R. Cable. 0.741 in. diam.
336,400 cm. cross-section area
Spacing: 63.5 in. to neutral

In Fig. 11B, the voltage has been increased above e_v. It is seen that corona starts at the reduced instantaneous voltage e_i on the second or + half-cycle. This is the voltage necessary to produce the flux ψ_i. The effect is as if the charge q_s produced by the excess in the voltage above e_v on the first half-cycle were added directly to the charge on the conductor in the next half-cycle so that corona starts when the sum of these becomes q_v. In other words, corona starts at the instantaneous voltage e_v on the first half-cycle that the voltage is applied. On the next and following half-cycle it starts on the wave at a lower instantaneous voltage, e_i. Fig. 12 shows this graphically.

In Fig. 12A, flux just sufficient to start corona at voltage e_v is shown leaving the conductor. Corona "arcs" are about to form, suddenly increasing the voltage across increased capacity and causing the sudden increase in current shown in Fig. 9. Fig. 12B shows the conditions at the maximum of the voltage wave. The outer cylinder has been charged through the corona "arcs." The stress at the conductor is still sufficient to maintain corona and there is considerable flux from the corona cylinder. The space charge cylinder has been fully charged and the "arcs" die out as the voltage starts to fall. Fig. 12C shows the point on the decreasing wave where all the flux is on the space charge and there is no voltage between the conductor and the space charge. In Fig. 12D, corona is starting on the second half-cycle at the reduced voltage e_i. The same flux is attached to the conductor as caused by the voltage e_v, in Fig. 12A. The flux is caused partly by ei and partly by the space charge. It is thus not necessary for e_i to be as great as e_v to cause the breakdown gradient.

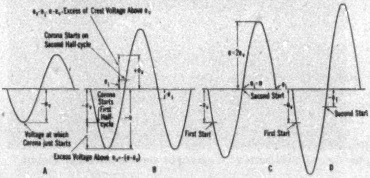

$e_v - e_i = e - e_v$ = Excess of Crest Voltage Above e_v
Corona Starts on Second Half-cycle
e_i
$+e_v$
Corona Starts First Half-cycle
$-e_v$
$-e$
e_i
Voltage at which Corona just Starts
Excess Voltage Above $e_v = -(e - e_v)$
$e = 2e_v$
$e_i = 0$
Second Start
First Start
e_i
$-e_i$
Second Start
First Start
A
B
C
D

Fig. 11—Mechanism of Corona

The reduction in instantaneous critical voltages is approximately equal to the excess of the applied voltage above e_v. Thus, if e is the applied voltage,

$$e - e_v \cong e_v - e_i$$

or

$$e_i = 2e_v - e.$$

This equation states that the instantaneous starting voltage, e_i, is zero when the applied voltage is $2e_v$, or that when the space charge is in effect q_v, no additional charge is needed to start corona. Instantaneous voltages, e_i, are plotted with applied voltages in Fig. 13. Note that e_i is zero at approximately $2e_v$.

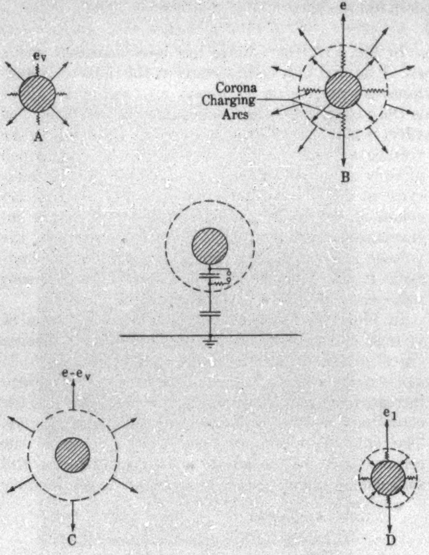


Fig. 12—Mechanism of Corona

A. Just at start of corona. Flux from conductor only

B. Just before corona stops at crest of voltage wave. Flux from conductor and space charge

C. After corona stops. Flux from space charge only

D. Just at second start of corona. Flux from space charge adding to conductor charge

Center figures. Schematic diagram of corona discharge circuit

If the above rule held over a wide range of voltage, it would be found that

$$e + e_i = 2e_v = \text{constant}$$

As a matter of fact, as would be expected, the tests show that $e + e_i$ is not constant, but approximately so, near the critical voltage. Actually, the effect is as if the total space charge were not effective in reducing the critical voltage, but that

$$e_i = e_v - (e - e_v)\,a$$

where a is a leakage factor and is less than unity. Fig. 14 shows this for several different wires. If the total charge were effective, the curves would be parallel to the x axis. At 420 cycles, as would be expected, the leakage is not appreciable.

It is stated above that the stress at the conductor after corona starts does not increase with increasing voltage but remains at a value just sufficient to keep the air broken down and to supply the outer cylinder or space charge. This is illustrated by the test curves in Fig. 15. The voltage was measured between the conductor and small wires placed at different points in space. It will be noted that the voltage between these points increases directly with increasing voltage until the critical voltage is reached, after which the voltage remains more or less constant.

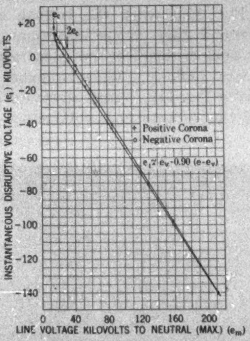


Fig. 13—Variation of Instantaneous Voltage of Corona Formation with Line Voltage

Conductor: Solid-smooth surface. 0.015 in. diameter
Spacing to neutral: 63.5 in.

Artificial Corona. The mechanism of the corona breakdown can be illustrated with condensers. For the sake of simplicity, take two condensers as in Fig. 16. Shunt one of these with a gap set to spark at voltage e_1 and of such characteristics that it never short-circuits c_1. This gap thus has a valve action sparking when a voltage e_1 is impressed across it but not short-circuiting c_1. On the first half-cycle, the spark, which represents

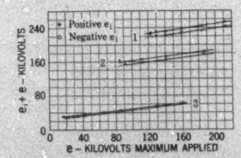


Fig. 14—Variation of $e_i + e$ with Line Voltage

e_i = Instantaneous disruptive corona voltage
e = Maximum line voltage to neutral
Conductor:
1 0.464 cm. radius polished
2 0.260 cm. radius polished
3 0.019 cm. radius smooth
Spacing: 161. cm. to neutral

the corona, starts at the instantaneous line voltage e_v, because e_v is the total applied voltage that causes the breakdown voltage e_1 on c_1. At that instant the voltage across c_2 is $e_v - e_1$. With increasing line voltage, the voltage across c_1 does not rise above e_1 because of the arc. The excess voltage is placed across c_2. When the maximum of the wave is reached, the current stops and the arc goes out. At a voltage $e - e_v$ on the falling quarter of the wave, the total voltage is on c_2. Finally, on the + half of the wave, the voltage across the gap again becomes e_1 and the spark again starts. This occurs at the instantaneous line voltage e_i. e_i is lower than the instantaneous voltage e_v at which corona started on the first half-wave, by the amount that e exceeded e_v because an excess charge, $(e - e_v)\,c$, on c_1, adds to the charge due to the line voltage, or

$$e - e_v = e_v - e_i$$
$$e_i = 2e_v - e$$

This is the relation arrived at for corona on wires and, as in the case of corona, the spark starts at zero instantaneous line voltage when the applied voltage is twice the critical voltage. The above holds whatever the relation between the capacities of the condensers, or whether one gap is used or several gaps are used to break down successively. The actual corona loss relation would be more nearly approached with a successive breakdown of gaps with increasing voltage.

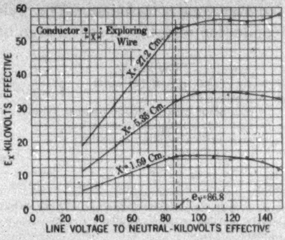
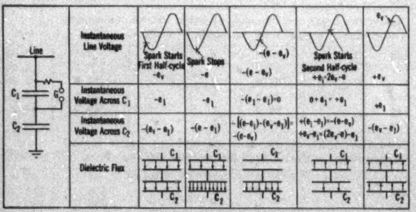


Fig. 15—Exploration of Static Field Surrounding a Conductor

X = Distance from conductor center to equipotential surface
E_x = Voltage across space X
Conductor: solid copper, polished
0.927 cm. diameter
Spacing: 161. cm. to neutral
Critical disruptive corona voltage—86.8 kv.



Instantaneous Line Voltage	Spark Starts First Half-cycle $-e_v$	Spark Stops $-e$	$-(e - e_v)$ $-(e - e_v)$	Spark Starts Second Half-cycle $+e_i = 2e_v - e$	e_v $+e_v$
Instantaneous Voltage Across C_1	$-e_1$	$-e_1$	$-(e_1 - e_1) = 0$	$0 + e_1 = +e_1$	$+e_1$
Instantaneous Voltage Across C_2	$-(e_v - e_1)$	$-(e - e_1)$	$-[(e-e_1)-(e_v-e_1)] = -(e-e_v)$	$+(e_1-e_1) = -(e-e_v)$ $+e_v-e_1 = (2e_v-e)-e_1$	$-(e_v - e_1)$
Dielectric Flux	C_1 C_2	C_1 C_2	C_1 C_2	C_1 C_2	C_1 C_2

Fig. 16—Operation of Single-Gap, Fixed-Capacitance Artificial Corona Circuit

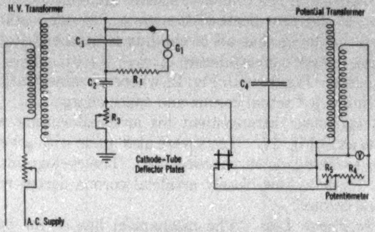


Fig. 17—Single-Gap Circuit for Producing and Studying Artificial Corona

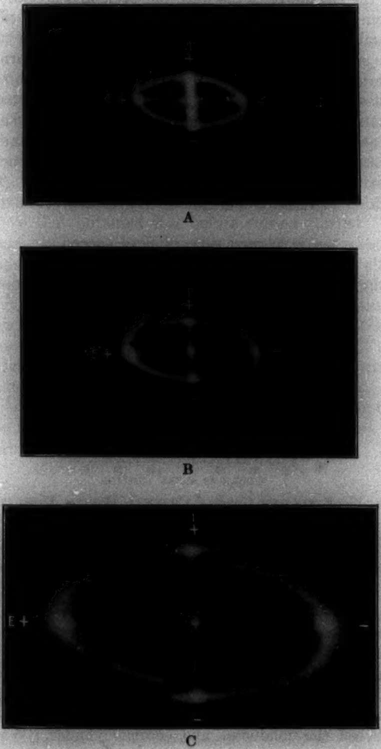


Fig. 18—Artificial Corona

A—Just above critical voltage
B—Considerably above critical voltage
C—Over twice critical voltage—corona starting "below zero"

A similar voltage relation holds if the gap is permitted to short-circuit the condenser except that the gap must then be set at double voltage. The voltage relations are followed out for several half-cycles in Fig. 16. The arrows on the condensers show the relation of the fluxes at different parts of the wave. In one instance it will be noted that all of the flux is attached to the "space charge" or condenser c_2.

Compare Fig. 16 with Fig. 12, where the same relations are shown for actual corona and space charge.

A condenser arrangement for artificial corona was made as in Fig. 17. Tests were also made with several gaps breaking down successively. The cyclograms in Fig. 18 show how closely artificial corona agrees with actual corona.

The Power Loss. The mechanism has so far been described up to the point where the charge of the ionized space of the first half-cycle in effect adds to the charge on the conductor of the next half-cycle and breakdown occurs when the sum of these two charges is in effect equal to q_v, or causes a stress g_v. In this particular case, the start was made with the wire (−). The returning charge was then also (−). Following the new break, this was cancelled by a (+) charge while a (+) charge was repelled from the conductor. The space charge cylinder moves out with increasing voltage and is charged through the corona arcs until the wave reaches maximum. The arcs then die out or corona stops. The sudden increase of current at the critical voltage is caused by a sudden increase of voltage across the increased capacity. Part of the energy in the "space charge" field is returned to the circuit and part of the energy is not returned or is returned at the wrong part of the wave and is lost. When the conductors are far apart, the main part of the loss occurs in the space between the conductor and corona cylinder. Where the conductors are close together, or the voltage is approaching the spark-over voltage, a conduction loss is also caused by ions migrating to the opposite conductor. The measurements show that over a very wide range of conditions the loss follows the quadratic law

$$p = k_1(e - e_0)^2$$

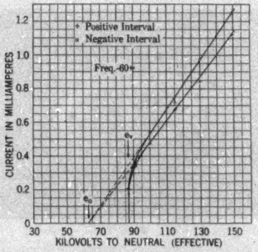


FIG. 19—EFFECTIVE CORONA CURRENT DURING DISRUPTIVE INTERVAL

Conductor: 0.927 cm. diameter, polished
Spacing to neutral: 161. cm.
Length: 305. cm.

An examination of the mechanism makes this appear the rational form for the loss to take. This seems so for the following reasons: The space charge is proportional to $(e - e_0)\,C$. Energy is required to move this charge through the field or through the voltage from the conductor to the corona cylinder. The voltage between the conductor and the space charge or corona

TABLE V

$P = 4\,f\,C\,(e - e_0)^2 \times 10^6$ watts
f = frequency
c = conductor capacitance to neutral, farads
e = max. line voltage to neutral, kv.
e_0 = max. critical voltage to neutral, kv.

e (Kv.)	e_0 (Kv.)	$e - e_0$	f	C ($F. \times 10^{-11}$	P (watts)*	
					Meas.	Calc.
Conductor: 0.0381-cm. diameter. spacing: 161 cm. to neutral C (corrected) = 0.747×10^{-11} farads. f = 60. e_0 = 9.90 s/r = 16,900						
16.4	9.90	6.5	60	0.747	0.014	0.076
18.4	"	8.5	"	"	0.100	0.130
20.4	"	10.5	"	"	0.246	0.198
23.2	"	13.3	"	"	0.382	0.318
27.7	"	17.8	"	"	0.588	0.570
33.9	"	24.0	"	"	1.11	1.04
42.5	"	32.6	"	"	1.90	1.92
53.8	"	43.9	"	"	3.36	3.45
65.7	"	55.8	"	"	5.53	5.60
85.6	"	75.7	"	"	10.2	10.3
128.8	"	118.9	"	"	25.4	25.4
158.0	"	148.1	"	"	41.5	39.5
Conductor: 0.519-cm. diameter, spacing: 161 cm. to neutral C (corrected) = 1.06×10^{-11} farads. f = 60. e_0 = 59.0 s/r = 1240.						
85.0	59.0	26.0	60	1.06	1.80	1.72
92.0	"	33.	"	"	3.81	2.78
113.	"	54.	"	"	10.3	7.42
141.	"	82.	"	"	23.3	17.2
184.	"	125.	"	"	54.3	39.8
Conductor: 0.927-cm. diameter spacing: 161 cm. to neutral C (corrected) = 1.13×10^{-11} farads. f = 60. e_0 = 89.6. s/r = 695.						
123.	89.6	33.4	60	1.13	3.51	3.04
129.	"	39.4	"	"	7.45	4.23
141.	"	51.4	"	"	11.8	7.20
160.	"	70.4	"	"	25.3	13.5
184.	"	94.4	"	"	43.2	24.2
212.	"	122.4	"	"	69.7	40.8
Conductor: 0.927-cm. diameter spacing: 54.6 cm. to neutral C (corrected) = 2.44×10^{-11} farads. f = 420. e_0 = 69.4. s/r = 236.						
117.	69.4	47.6	420	2.44	84.4	93.0
132.	"	62.6	"	"	165.	161.
147.	"	77.6	"	"	250.	247.
170.	"	100.6	"	"	415.	415.
191.	"	121.6	"	"	587.	601.
206.	"	136.6	"	"	764.	767.

*In making the loss calculations, the values of C in the formula were corrected to correspond to the actual capacitances of the conductor to the active section of the ground plate. The factors were determined by comparing the calculated and measured charging currents.

The measured values are the actual values computed from the cyclograms.

* * * * * * *

The check between the measured values of loss and the corresponding values calculated by the above simple formula is very good when s/r is large or when the frequency is high. When s/r is small or when the frequency is low, it is necessary to introduce a factor which is a function of the capacitance and also a factor to take care of ionic leakage as in the quadratic law. Thus in the tables above, for the lower values of S/r with 60 cycles, the difference between the measured and calculated watts is considerable. The corrective factor has not been applied to any of the above tables.

cylinder is proportional to $(e - e_0)$ and, in fact, appears to equal $(e - e_0)$ for the higher frequencies and very large spacings. The energy may be considered as being lost in the resistance of the corona arcs charging and discharging the corona cylinder. The loss is thus

$$w = (e - e_0)\ (e - e_0)\ k\, C = (e - e_0)^2\, k\, C$$

or the power

$$p = 4\, f\, C\, k\, (e - e_0)^2$$

The above relation checks the measured values for large spacings at 60 cycles and for all spacings at 420 cycles. When the frequency is low or when the spacing is small, ions must pass from conductor to conductor; see Table V. This is equivalent to a leakage loss or loss in a resistance intermittently placed from conductor to conductor. Then

$$p = k_2\, (f + a)\ C\ \ (e - e_0)^2$$

wherein the factors a and k are an integral part of the equation and were originally determined empirically in the quadratic law. The part of the energy in the space charge that is returned to the circuit at the proper part of the wave is not lost but gives the extra capacity effect of corona.

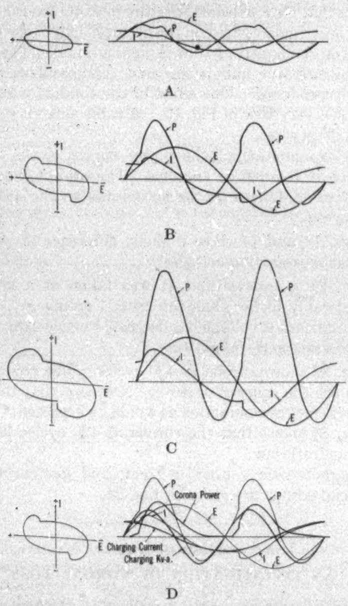


FIG. 20—VOLTAGE, CURRENT, AND POWER WAVES AS DERIVED FROM CORONA CYCLOGRAMS

E = Voltage I = Current P = Power
Conductor: 0.927 cm. diameter, polished
Spacing: 161. cm. to neutral
A—87.0 kv. (eff.) to neutral
B—113.5 kv. (eff.) to neutral
C—149.5 kv. (eff.) to neutral
D—113.5 kv. (eff.) to neutral

There is no loss at e_0 because breakdown does not take place until the voltage e_v is reached. There is then a sudden break over a finite distance from the conductor and the loss falls on the quadratic curve with e_0 as the disruptive critical voltage. The stress between the conductor and the corona cylinder is not reduced to zero when corona starts but has a value approximately equal to g_0 or g_d at the conductor and decreases outward to the corona cylinder along approximately the same curve that obtained just before corona started. Beyond the corona cylinder, the average stress must be higher or the curve flatter to maintain the voltage proportional to e_0 across the portion that is not ionized.

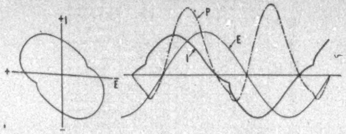


FIG. 21—WAVES OF VOLTAGE, CURRENT, AND POWER—VOLTAGE FAR IN EXCESS OF CRITICAL VALUE AS DERIVED FROM CORONA CYCLOGRAMS

E = Voltage I = Current P = Power
Conductor: 0.0382 cm. diameter, smooth
Spacing: 161. cm. to neutral
Voltage: 112.0 kv. (eff.) to neutral: e_v = 12.8 kv.
e_i = 9.5 kv.

It might be expected at first glance that the disruptive critical voltage should be e_v rather than e_0, since loss does not start on polished conductors until e_v is reached. A more critical examination, however, shows that following the initial break controlled by g_v, the strength of air becomes g_0. Thus, although g_v controls the start of the loss, after the initial break occurs and corona extends out, the controlling gradient is g_0. This follows up to the maximum of the voltage wave when corona stops. g_v is required to cause the next start, etc.

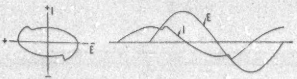


FIG. 22—WAVES OF VOLTAGE AND CURRENT AT 420 CYCLES AS DERIVED FROM A CORONA CYCLOGRAM

Voltage to neutral: 147. kv. max.
Spacing to neutral = 161. cm.
Conductor: 0.927 cm. diameter, polished

It is interesting that when the applied voltage is zero, all of the energy is on the space charge and is

$$(e - e_v)^2\ \frac{C\, C_2}{2\, C_1}$$

The voltage, current, and power relations are shown very well for a polished conductor 0.365 in. in diameter at different voltages, in Fig. 20. The relations are shown in rectangular coordinates transcribed from the measured cyclogram at the left of each figure. Fig. 20A was taken near the critical voltage. The (+) and (−) power waves are practically equal and correspond to the charging kilovolt-amperes except for the excess (+) hump caused by the loss. In Fig. 20B, the voltage has been greatly increased and the instantaneous starting voltage e_i has decreased. There is now a considerable excess (+) power, but without the decided hump. In Fig. 20C, the voltage has been increased to almost 2 e_v. e_i has approached zero. The power wave is now smooth.

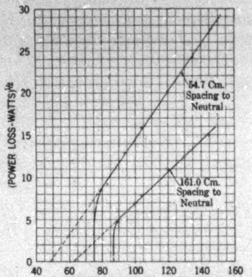


FIG. 23—OBSERVED CORONA LOSS AT 420 CYCLES MEASURED BY MEANS OF THE CATHODE-RAY OSCILLOGRAPH

Conductor: 0.927 cm. diameter, polished surface
Length: 305. cm.

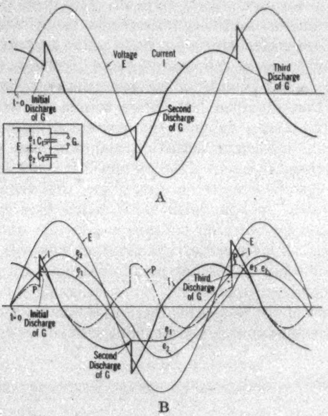


FIG. 24—ARTIFICIAL CORONA

A—Voltage and current waves
B—Voltage, current, and power waves
Gap *G* assumed to limit
Voltage of C_1 to e_1'

$$C_1 = C_2 \quad e_1' = \frac{E_m}{2.5}$$

P = Power input to circuit

Fig. 20D is the same as 20B with the addition of the charging kilovolt-ampere curve and the difference between this curve and the total power curve. This is the approximate corona loss curve.

An examination of the current curves in Fig. 20 shows that for a polished wire the instantaneous corona start is extremely rapid on the negative half of the wave, while on the positive half it is more gradual. The start on the negative half is so rapid that oscillations are readily produced. The effect of the conductor surface is shown very well in Fig. 10. A cable acts very much like a dirty wire.

An examination of the power curves shows a difference for (−) and (+) corona. There is not, however, any great difference in the actual loss on the different half-cycles.

Figs. 13 and 14 show a slight difference in the instantaneous starting voltages.

Fig. 21 is interesting. It was taken at a voltage considerably higher than the critical voltage, e_v. The instantaneous e_i voltage has decreased below zero or the *corona starts on the falling wave.*

Fig. 22 shows corona at 420 cycles. The characteristics are the same as at 60 cycles except that there is less evidence of conduction as would be expected.

Fig. 23 shows that the corona at 420 cycles follows the quadratic law.

Characteristic voltage, current, and loss curves for artificial corona are given in Fig. 24.

Discussion

AN INVESTIGATION OF CORONA LOSS

(STARR AND LLOYD)

LAW OF CORONA AND DIELECTRIC STRENGTH

(PEEK)

DETROIT, MICH., JUNE 24, 1927.

V. Karapetoff: The diagram of connections used by the authors permits of obtaining an oscillogram of the total current, consisting of a reactive charging component and an in-phase energy component. In some cases it may be of interest to obtain a picture of the latter component alone. Perhaps this could be accomplished by some differential arrangement; that is, by using a condenser which takes the same reactive charging current as the conductor under test but has practically no corona loss. The high-voltage side of the step-up transformer would then be wound for a double voltage, with the middle point grounded; the added condenser would be connected to the other terminal of the transformer, and the resistance R_3 would be connected to the neutral point. Only the difference of the currents taken by the conductor under test and the perfect condenser would flow through R_3.

A Mechanical Transcriber. With an increasing use of the cathode ray oscillograph in practical work, the problem of rapidly transcribing records with a sine-law axis of abscissas to those with

Cartesian coordinates becomes one of considerable importance. Where hundreds of similar records have to be transcribed, the tedious point-by-point method of transfer of individual ordinates can hardly be considered satisfactory. A mechanical transcriber is a device consisting of kinematic connections with a stylus at one end and a pencil at the other. By going with the stylus over the outline of an oscillogram on a photographic film, the pencil is made to draw on a strip of paper the corresponding curve in rectangular coordinates.

In order to enable those interested to build a mechanical transscriber, the underlying principle is shown in Fig. 1 herewith. The film containing the record is fastened to the platen 2-2, which has a slot, 3. A pin, 4, attached to the crank, 5, can slide in the slot. When the crank is turned by means of the handle, 6, the platen and the oscillogram perform a harmonic motion along the X-axis. The same crank, through the friction wheel, 7, drives the table 8-8, to the left. A strip of paper is fastened to this table, and the pencil, 15, draws a curve in Cartesian coordinates. It will be seen that by means of this mechanism the sine-law abscissas are wiped out, and abscissas proportional to time are substituted.

To transfer ordinates of the oscillogram from the platen to the table, the carriage 9-9, with a crossbar, 10, is used. The carriage can be moved up and down on the rails 12 and 13. The pencil, 15, is attached to the carriage by means of the bracket 14. The stylus, 11, is fastened to the crossbar, 10.

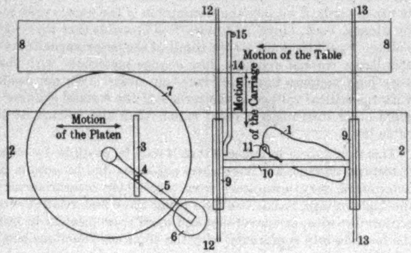


Fig. 1

To operate the transcriber, the crank is turned slowly and at the same time the carriage is moved up and down in such a way that the stylus always remains on the curve. The pencil then traces the transformed curve. When many oscillograms are to be transcribed, one person should turn the crank and another operate the carriage.

Determination of Power Loss by Weighing. When power loss is to be evaluated regularly from a large number of oscillograms, the point-by-point method described in the paper may prove to be too tedious. It is necessary to measure a large number of ordinates, multiply each by the sine of an angle, and add the products, keeping the positive and the negative quantities separate. An automatic method of weighing may prove useful in such cases, especially since it enables us to use a much larger number of ordinates with a comparatively small additional required amount of time. The principle of the method is shown in Figs. 2 and 3. For the sake of illustration, we shall assume, with the authors, that it is sufficient to divide each cycle into 20 equal intervals of time, Δt, each corresponding to 18 electrical degrees. Replacing integration approximately by summation, the average power, P, per cycle may be written in the form

$$P = (1/T) \sum_{t=o}^{t=T} i_m \, e_m \, \Delta t$$

where T is the duration of a cycle, and i_m and e_m are the average values of current and voltage during the small interval Δt. In Fig. 2, the curve, $a\ b\ c\ d\ f\ h$, represents a loop obtained experimentally, with sine-law abscissas. The circle, N is drawn with a radius equal to the amplitude of the voltage, $E = O\,g = O\,k$. The circle is divided into 40 equal parts, corresponding to 9 deg. each. Thus, the arc p corresponds to the interval Δt, and r is the middle point of this interval. $O\,s$ is the mean voltage e_m during this interval, and $s\,v$ is the corresponding mean current, i_m. Similarly, the ordinates x, y, etc., mark the middle points of the consecutive intervals Δt. The abscissa $O\,s = E \sin 9$ deg., the next abscissa $O\,z = E \sin 27$ deg., etc. Thus, eq. (1) becomes

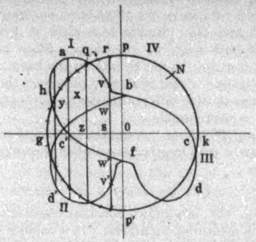


Fig. 2

$$P = (\Delta t/T)\ E\ [i_{m_1} \sin 9° + i_{m_2} \sin 27° + \text{etc.}] \qquad (2)$$

Instead of scaling off the ordinates and multiplying them by the sines of the corresponding angles, the simple balance beam, shown in Fig. 3, may be used. It is provided with notches at distances from the center proportional to sin 9 deg., sin 27 deg., etc. The beam is supported in the usual way on a knife edge at the center, has a pan P at one end and a counterweight, Q, at the other end, to balance the pan. Some strip or wire of uniform weight per unit length is provided, so that lengths can be measured by weighing. Let a piece of length, $s\,v$, be cut off and placed in the first notch to the left of A. The turning moment exerted upon the beam will be proportional to i_{m_1} sin 9 deg., that is, proportional to the first term within the brackets in eq. (2).

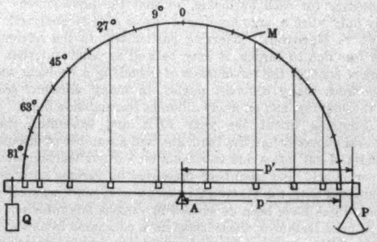


Fig. 3

Corresponding strips are placed in the remaining notches. The ordinates in the quadrants I and III of the loop give positive values of torque (counter-clockwise), and those in quadrants II and IV contribute to the negative torque. Having placed all the strips, the beam is balanced by putting some strip in the pan P. Let the length of this strip be l. If the pan were placed at the distance p from A, corresponding to the radius of the semicircle M, the length I would represent the value of the expression in the brackets. If the pan is at a distance p^1, the corrected value of I, reduced to the arm length p, is $I\ (p^1/p)$. All the quantities in eq. (2) being known, the power P may be readily computed.

The number of strips may be reduced to one-half by re-drawing the parts of the loop in the quadrants III and IV in quadrants I and II, as shown by the curve $b\ c^1\ d^1\ f$. This can be easily done by folding the film along $p\ p^1$. In this case, instead of cutting a piece of strip equal to $s\ v$, and later one corresponding to $s\ v^1$, a piece of length $v\ v^1$ can be cut off at once, covering both positive quadrants I and III. Similarly, a piece of length $w\ w^1$ will take care of both negative quadrants. Since both strips act on equal lever arms, the net results on the beam is the same as if piece of lengths $v\ w^1$ and $w^1\ v^1$ were placed on the positive side. This procedure of using different pieces on the beam may be used, if it appears to be preferable.

Other Uses of Weighing. A similar method of computation by weighing may be used in other cases when the unknown quantity is of the form

$$Q = a\,x + b\,y + c\,z + \text{etc.} \qquad (3)$$

wherein a, b, c, etc., are known constant coefficients, and x, y, z, etc., are some measured quantities. In a balance beam, notches are then made at distances a, b, c, etc. (positive or negative) from the knife-edge support, and strips of paper or metal, or wires of lengths x, y, z, etc., are placed at these lever arms, keeping in mind the sign of each product. The length of wire or strip necessary to balance the beam, multiplied by its distance from the center, will give the value of Q in eq. (3). The following examples may be cited:

(a) In cost estimates, where a, b, c, are unit costs and x, y, z, are quantities of materials,

(b) In computing the voltage drop along a line with branch loads.

The factors a, b, c, may then represent lengths and x, y, z, load currents.[1]

(c) When analyzing an irregular wave into its harmonics, equidistant ordinates are multiplied by the sines of certain angles.[2] By providing a balance somewhat like in Fig. 2, and another with cosine notches, the amplitudes of harmonics can be quickly and accurately determined. Two balance beams are necessary for each harmonic, so that the arrangement would pay only when a large number of waves is to be analyzed.

C. F. Harding: Referring particularly to the reference to the loss due to corona, it may be well to point out that these papers confirm the importance of attacking a problem such as this from many different angles, in many different sections of the country, and on many different transmission lines.

Beginning about the year 1912, and extending through several papers before the Institute, and a number of discussions, results of net losses due to corona on various transmission and experimental lines have been presented by various authors, and compared with the theoretical values, *i. e.*, the empirical equations which have been developed in various laboratories.

In most instances, those losses have compared favorably with the results calculated from the formulas which have been developed by Mr. Peek, and which have been repeated in this paper. That is particularly true for the losses at relatively low voltages, such as those up to and slightly exceeding the critical voltage between wires. If the net loss due to corona calculated by Peek's formula be plotted in kw. as ordinates, and the kilovolts between cables or between one cable and neutral be expressed as abscissas, a parabola results as indicated in the paper. If the actual net loss due to corona be plotted for an experimental line or for a transmission line operating under the same conditions, *i. e.*, the same frequency, density factor, size cables, spacing, etc., this curve checks very closely at low voltages with the quadratic curve. In many instances the test curve was found to be slightly higher in the lower range of voltage, but had considerably lower values of power loss throughout the higher ranges of voltage.

In other words, if we assume that the net corona loss in kw. is equal to some constant multiplied by the square of the difference between impressed voltage and critical voltage, the quadratic law of Peek's formula results. Actual tests show a curve of smaller loss and less slope at the higher voltages; at least on the lines at Purdue University. This would seem to indicate that either this exponent (2) is a little high, or that the critical voltage E_o varies with the voltage E impressed upon the line. Possibly both conditions exist simultaneously. Actual values for the exponent—call this N—have been of the order of 1.55 to 1.7 but always less than the 2 of the quadratic law of Peek.

In the paper presented by the speaker in 1924 at the Pasadena convention, the result of the possible increase of this E_o, *i. e.*, the disruptive voltage of air, because of the formation of the coronal cylinder of greater capacitance, at these higher values of voltage, was suggested. If we consider that this relatively large conducting cylinder of ionized air which has been termed the coronal cylinder does exist about the cable at extra high voltages, in other words, if we take the assumption of the papers presented by Messrs. Peek, Lloyd, and Starr, it is probable that the critical voltage E_o at this point, as the result of the larger capacitance of this larger coronal cylinder, does change materially with these very high voltages impressed between cables. We are dealing with the critical voltage at the surface of the coronal cylinder or some other intermediate point rather than the voltage at the cable itself.

This would seem to indicate that it may be possible, by means of tests on transmission lines where net losses due to corona are determined very accurately from wattmeter measurement in the high-voltage circuit itself, to bring these test relations into conformity with, or conversely, and more appropriately to bring the formula into conformity with the empirical functions established by such tests.

The important feature of this, it seems to me, is the fact that in practically all these tests, made over a long series of years, including a large number of spacings of cables ranging from 18 ft. to 38 ft., with sizes of cables from No. 0 up to 300,000 cir. mils stranded, that the Peek formula is a very conservative one, and that the corona loss actually is considerably less, at these higher voltages, than would be indicated by the calculations. We are on the safe side, therefore, if we design transmission lines in accordance with that formula.

The statement is made, however, in the papers under discussion that the line should not be operated at or above the critical voltage at which corona forms; *i. e.*, under dry-weather conditions. Designing for that voltage means that such a line will operate in many cases with considerable corona formation upon it, as many have noted substations and lines in more or less continuous operation with corona formation. It is important, therefore, to determine both by tests in laboratory and on operating lines just what this difference is between the two curves at the voltages in excess of the critical voltage.

W. F. Davidson: Mr. Lloyd and Mr. Starr described the method they have been using with the cathode ray oscillograph for studying corona formation. A short time ago I tried a similar tube in studying the distortion of current through a cable sample, with the idea of determining whether there was corona formation.

We used basically somewhat the ideas suggested by Professor

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[1] V. Karapetoff, Engineering Mathematics (Wileys), Part V, Chapter II. In the early days of electrical engineering, a German firm used to market a simple balance on which one could "hang amperes" at proper distances and balance them by a weight in the pan. The weights were stamped directly in square millimeters, to indicate the required cross-section of line conductor.

[2] V. Karapetoff, Experimental Electrical Engineering, Vol. II, Third Edition, p. 577.

Karapetoff of eliminating the straight capacity component of the charging current through the samples.

The supply transformer in this case was grounded at one end; the sample *S* connected as indicated in the accompanying Fig. 4 and brought down through a small resistance to ground, the no-loss air condenser *C* connected in parallel through a corresponding resistance, in fact, two resistances. The cathode ray is connected with one plate so as to take the drop $V_s - V_c$ giving a deflection proportional to watt-component current through the sample in the test, now, by controlling the total resistance *R* we had the possibility of shifting the base position of the current through the air condenser. That was sometimes of value and of help.

The other component was obtained in two different ways, in the first work, much as Mr. Lloyd has described, by taking the potential applied to the transformer. We found considerable difficulty in using our particular tube, however, because neither axis was straight; that is, the pattern instead of being straight, gave two rather distorted curves.

In order to overcome that difficulty, we made a calibration in the direction of abscissa, corresponding to these deflections, then we arranged our time curve. These were controlled by a phase shifter *P*, that was supplied from the same primary as the transformer. The cyclogram would then appear in various forms, and we would shift the thing around, changing the phase around, changing the phase shifter until the particular point we were interested in was on the central axis and then we scaled this point.

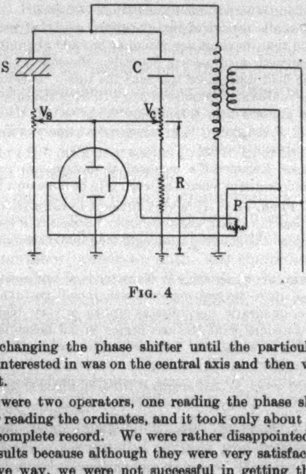

Fig. 4

There were two operators, one reading the phase shifts and the other reading the ordinates, and it took only about a minute to get a complete record. We were rather disappointed in some of the results because although they were very satisfactory in a qualitative way, we were not successful in getting them in a quantitative way. Perhaps we could do better with a tube that did not have such a badly distorted pattern.

J. Ryan: (communicated after adjournment) The results obtained by the authors of these corona papers and their corresponding conclusions will help greatly to establish a better understanding of the nature of corona. During the past four years by independent effort we have arrived at substantially the same results and corresponding conclusions. Our studies during the past year were mostly concerned with the mobility and make-up of the space charge and the manner in which its portions having constant and alternating polarities function.

Three methods have been used successfully for the cyclic study of corona by means of the cathode ray tube. In the first of these the ray-pointer is deflected electromagnetically.[3,4] In the second the deflections are produced electrostatically so as to yield quadrature ray-pointer deflections proportional to the voltage and to the rate of increase or decrease of the current in instantaneous relation. Thus the ray-pointer develops areas proportional to the time-integral of the volt-amperes or energy.[5] The third has been used by Mr. Peek and his co-workers. In it the ray-pointer is also actuated electrostatically and yields deflections proportional to simultaneous values of voltage and current. The first and third methods produce volt-ampere cyclograms and the second energy cyclograms.

The first avoids the necessity of using electrostatic deflectors mounted within the cathode ray tubes but requires a great variety of deflector current coils and potential-circuit resistors, some of which are expensive. Electrostatic deflectors mounted within the cathode ray tube are used in the second and third methods. Externally mounted electrostatic deflectors are not satisfactory because of the bound charges that assemble on the inner walls of the tube opposite the deflectors. We have used the three methods and recommend the use of the second and third. The choice depends upon the character of work to be done. Occasionally both are required in the same undertaking. Because a condenser must be used in the current circuit of the energy cyclograph it cannot be used when more than a mere trace of rectified current is present. Such current when small enough may be successfully bypassed with a "leak" of the proper value. Again when the observation of sudden changes of current or e. m. f. are to be made the energy cyclograph being a volt-ampere-time integrator, is less helpful and the electrostatically operated voltage-current cyclograph should then be used.

Following the recognition of the presence of the space charge that surrounds a conductor in corona it was found that the corona loss equals the energy that is dissipated in the space-charge condenser at unity power factor subjected to the voltage in excess of the critical voltage, *i. e.*, voltage above the lowest voltage that will sustain a fixed brush pattern.[6] Thereafter the capacitance and position of the space-charge condenser were studied with the energy cyclograph, using cylindrical barriers around the conductor to fix the radius within which the space charge is confined.[7] These studies were then followed by others to determine the central radial position of the alternating portion of the space charge by means of the potential exploring wire.[8]

During the past year my co-workers, J. C. Carroll and J. T. Lusignan, Jr., have studied the cyclic character of the ion-content of the space charge. One of the many purposes of the study was to add to the background of knowledge required to improve interpretation of the observations made with the potential exploration wire and to understand the function of the constant-polarity portion of the space-charge function in alternating corona formation. The corresponding results and conclusions are being embodied in a paper soon to be presented to the Institute.[9]

It has long since been known that the values of positive and negative corona-forming voltages differ. Dependent upon the particular conditions the difference varies from zero to a considerable fraction of either polarity of the critical voltage. The rectified portion of the space charge in alternating corona is caused by this difference. For example if the critical voltage is

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[3] *The Cathode Ray Alternating-Current Wave Indicator*, H. J. Ryan, A. I. E. E. Trans., Vol. XXII, 1903, p. 539.

[4] *The Conductivity of the Atmosphere at High Voltages*, H. J. Ryan, A. I. E. E. Trans., Vol. XXIII, 1904, p. 101.

[5] *A Power Diagram Indicator for High-Tension Circuits*, H. J. Ryan, A. I. E. E. Trans., Vol. XXX, 1911, p. 1089.

[6] *The Hysteresis Character of Corona Formation*, H. J. Ryan and H. H. Henline, A. I. E. E. Trans., Vol. XLIII, 1924, p. 1118.

[7] *On the Nature of Corona Loss*, C. T. Hesselmeyer and J. K. Kostko, A. I. E. E. Trans., Vol. XLIV, 1925, p. 1016.

[8] *The Space Charge that Surrounds a Conductor in Corona at 60 Cycles*, J. S. Carroll and H. J. Ryan, A. I. E. E. Jl., November, 1926, p. 1136.

[9] *The Space Charge That Surrounds a Conductor in Corona*, A. I. E. E. Pacific Coast Convention, Del Monte, Calif., September 14, 1927

greater when negative than when positive, the magnitudes of the corresponding space charges will have to be in reciprocal relation. When the conductor requires a higher negative than positive corona-forming voltage there will be formed and maintained the larger space charge from the positive to the negative voltage crests; and correspondingly the smaller space charge from the negative to the positive voltage crests. This gives rise to the phenomenon of "rectification" that is often witnessed in the study of corona. The portion of the space charge having constant polarity will increase the strength of the field attached to the conductor as the voltage approaches one crest and will diminish it as the voltage approaches the other crest. Thus the differences of the positive and negative critical fields and their corresponding initial voltages from their averages are compensated for by the presence of the constant-polarity portion of the space charge. It follows, therefore, that the initial value of the cyclic critical voltage must be as much below the value of the crest voltage as it is above the value of the subsequent critical voltage that starts corona formation in the succeeding cycles and that the positive and negative critical voltages must be equal. From the nature of things here set forth power-transmission conductors operated in corona will display constant polarity high potential phenomena to a limited extent and the direct currents thus set up in ionic mobility from the conductors through the air and over the insulators to earth are not likely to develop effects of much practical importance.

The model of corona given by Mr. Peek is helpful for gaining an understanding of the essential features of corona. The theory of this model as developed in our studies was given by Messrs. Hesselmeyer and Kostko in connection with Fig. 14 of their paper on the Nature of Corona.[8] Guided by this theory we made up in the summer of 1926 a model such as described by Mr. Peek, using a pair of Leyden jars with a spark-gap in parallel with one of them. It produced artificial corona cyclograms quite like cyclogram No. 4 in Fig. 6 of the Hesselmeyer-Kostko paper. It gave the oblique straight sided parallelograms that theory called for. Of course it did not repeat the minor departures that are witnessed in actual corona wherein the space charges are mobile. See for example the group of cyclograms in Fig. 10 of the same paper taken from a single-phase line mounted in the open to which corona forming voltage was applied.

All in this group conform liberally but not exactly to the oblique parallelogram wherein the slopes of the sides and ends determine the values of the capacitances of the conductors and space charges, and wherein the departures from the straight sided oblique parallelograms are measures of the effects due to causes that are not included, necessarily so, in the incomplete theory, such as conduction (thermal dissociation) and ionic mobility actions that Mr. Peek properly sums up in his paper by the phrase "corona arcs."

As a result of these studies we are convinced that Mr. Peek is right in his conclusion that the power-loss-voltage relation in full corona formation is a rational quadratic because of the space charge condenser. He is also right in recommending that care be used to avoid mechanical injury to the surface of the conductor. One should note, however, that losses due to local corona are greatly dependent upon the uniformity of the physical-chemical regularity. Greatest regularity is required to produce minimum local corona loss, since the maintenance of a smooth, polished conductor is not feasible. We have found this to be so for local corona loss when the mechanical irregularities or roughness of the surface of the transmission conductor have been carefully avoided. The local corona losses are likewise dependent to important extents upon the humidity in the air and the crest and effective values of the voltages. Because of the prominent part that the space charge condenser takes in corona formation one must expect the cyclic loss to be dependent upon the crest value of the voltage; again because the space charge is mobile its containing condensers do not have fixed value, and one must expect the cyclic loss to be dependent also upon the effective value of the voltage.

The local corona studies here referred to were made by our graduate students Messrs. Drodjin and Wiedeman and will be embodied in a paper that they will offer for publication at an early date. Local coronas are simply scattered brushes. In the local corona loss-voltage relation the numbers and positions of the brushes are large factors, which in turn depend upon the smaller factors just specified all of which in the full corona loss-voltage relation are of little or no importance. Local corona is an instability complex and much systematic study under all relevant conditions will have to be promulgated to know what high-voltage conductor diameters, and insulator hardware forms to adopt to meet all requirements.

E. C. Starr: Professor Karapetoff and Mr. Davidson have spoken of neutralizing the charging component of the current wave. We considered that, inasmuch as the portion of the figure that is said to represent charging current is not, strictly speaking, only charging current as based upon the capacitance of the wire itself, but charging current based upon the capacitance of the wire as affected by the motion of the space charge, we thought it best to leave it all in to show the whole wave. That suggestion, however, would allow us to show only what the corona does and under certain conditions would be useful.

Mr. Davidson spoke of his tube giving axes that were crooked. We found that the condition would be brought about if the source of the magnetic field used to neutralize the effects of the earth's magnetic field was placed too near the tube. It is necessary to place the source at a distance of 3 or 4 ft. from the tube in order to obtain a uniform field throughout the tube and thus eliminate distortion of the axes. It is possible that this was the source of Mr. Davidson's trouble.

Professor Karapetoff's mechanical devices are very clever, and should be useful where the work is to be done on a large scale.

F. W. Peek, Jr.: It is very difficult to estimate corona loss on practical lines with great accuracy because it is very difficult to determine the conditions along a long line; there are so many variables.

If a wire about an inch in diameter is taken and the loss is measured up to several million volts, it will be found that, at first, the quadratic law obtains up to a very high voltage. Finally, at some point the loss begins to fall below the original quadratic. It eventually follows another quadratic. The new curve, if extended, will be found to cut the axis at a new critical corona voltage. At the point where the deviation occurs, the appearance of the corona discharge changes. Great "cartwheels" form around the conductors. These cartwheels separate, act as shields, and change the whole electrostatic field. I do not think it is a practical condition because this deviation from the quadratic occurs at many times the operating voltage for the conductors. It is amazing, in fact, that any law is followed at these high voltages.

It seems to me that operation should be below the critical voltage for fair weather. It is practically impossible to eliminate all loss in wet weather. It is thus important to determine the irregularity factors for various types and sizes of weathered conductors. This offers a good field for investigation.

A Precision Measurement of Puncture Voltage

BY V. BUSH* and P. H. MOON*
Fellow, A. I. E. E. Associate, A. I. E.E.

Synopsis.—*A description is given of an apparatus for the automatic determination of the dielectric strength of thin sheet insulation. The machine makes about 1000 breakdown tests in a day. Almost no attention is required, thus reducing the human element to a minimum.*

The results of over 100,000 punctures are given. An investigation of the effect of temperature indicates that none of these punctures occurred according to the thermal theories of breakdown. Tests to determine the effect of electrode area are also described. It is shown that statistical theory can be applied to the results with some success. The standard deviation of the breakdown values is a measure of the inherent variability of the material, and thus gives promise of value in the specification of insulation.

* * * * *

THE pyroelectric theory of insulation breakdown, introduced in 1922 by K. W. Wagner[1] and independently by Steinmetz and Hayden[2], gave a great impetus to the study of the failure of solid dielectrics. Several investigators, particularly Dreyfus[3], Kármán[4], and Rogowski[5], have added to the theory, while others such as Mündel[17], Rochow[18] and Gabler[19], have attempted experimental verification.

The experimental results to date are, however, meager and unsatisfactory, and constitute a serious barrier in the way of further progress. One reason is the difficulty of getting consistent values of puncture voltage. Breakdown measurements are inherently more erratic than either resistivity or dielectric loss measurements. Puncture depends evidently only on the strength of the weakest path through the dielectric, while the measurement of other quantities is more or less an integration process giving an average of all the elementary paths through the insulator. The only solution of the difficulty seems to be to take the average of a great number of readings.

It was our purpose to build an automatic machine to make these breakdown tests in large numbers under accurately controlled conditions and with the minimum of manual labor. Such an outfit was completed at the Massachusetts Institute of Technology in the fall of 1925†. In the school year of 1925-26, 130 runs were made with a total of over 100,000 readings.

The apparatus will test any thin sheet material obtainable in long strips. A piece 1¼ in. wide and about 80 ft. long is sufficient for a day's run of 1000 punctures. The outfit consists essentially of a pair of electrodes, an insulation feed mechanism, and suitable means of automatically raising the voltage and recording the value at which breakdown occurs. The feed mechanism operates after each puncture, bringing a fresh length of insulation between the electrodes. A 4000-volt, d-c. generator is used for the tests, the voltage being raised at a constant rate by a motor-driven rheostat in the field circuit. The value at which puncture occurs is recorded by an ordinary graphic voltmeter.

That accurate results are obtainable is shown by the results of 16,000 punctures made on thin condenser paper, Fig. 12. The probable error of the daily average is 3.6 volts. Therefore, it may be said that with this material under the condition of the tests, *the daily average is correct to ± 3.6 volts, or less than 1 per cent.* An investigation of the sources of error has led us to believe that nearly all the variation from hour to hour and from day to day is caused by variations in the paper. The accuracy of the results depends, of course, on the number of readings averaged for each point, and upon the variability of the material. With a more uniform material than paper, still more accurate results could be obtained with the same number of readings.

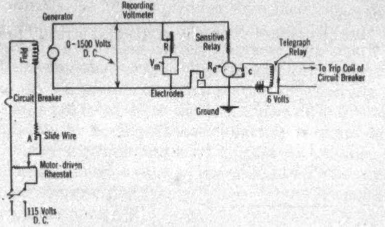

FIG. 1—SIMPLIFIED DIAGRAM OF CONNECTIONS

APPARATUS

Diagram of Connections. A simplified diagram of connections is shown in Fig. 1. All tests are made with direct current furnished by a 4000-volt generator. The field of this machine is separately excited from the 115-volt mains, a motor-driven rheostat being used to vary the voltage automatically. This rheostat is driven by a small synchronous motor, and twice a minute raises the generator voltage at a uniform rate from zero to a value somewhat above the breakdown point of the insulation.

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1. For all numbered references, see Bibliography.

†The authors wish to express their appreciation of the work of Mr. L. M. Dawes of the Elec. Engg. Dept., under whose direction most of the apparatus was constructed. Thanks are also due to the General Electric Co. for its help in the early stages of the work.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

This voltage is applied to the material between the electrodes. When puncture occurs, the rush of current operates the relay, *R e*, which trips the breaker in the field circuit of the generator. Across the line is also connected the recording voltmeter, *V m*. Every time the voltage rises, this voltmeter draws a line across its chart. The pointer continues to move slowly across the chart until breakdown occurs, when it falls back to zero due to the opening of the circuit breaker. The length of line is thus a measure of the breakdown voltage.

FIG. 2—INSTRUMENT BOARD

Just before the motor-driven rheostat is ready to start raising the voltage again, contacts on its rotor close and operate the feed mechanism. This pulls a fresh piece of insulation under the electrodes. At the same time, the circuit breaker closing coil is energized. The voltage is then built up as before. The cycle of operations is thus as follows: Voltage is raised slowly, puncture occurs tripping circuit breaker, feed mechanism operates, circuit breaker closes, voltage starts rising again. The cycle is repeated indefinitely.

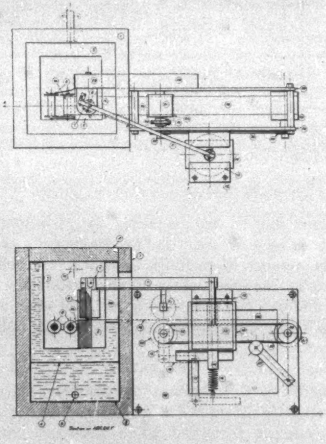

FIG. 3—ELECTRODE MECHANISM

Fig. 2 shows some of the apparatus. On the left of the board is the recording voltmeter. At the right is the sensitive relay. A series protective resistance of about 50,000 ohms is seen directly below the relay. The large box in the right foreground is called the constant humidity box.

Electrode and Feed Mechanisms. In the constant humidity box are the electrodes and the feed mechanism. As shown in Fig. 3, the lower electrode 5 is a copper block 2 in. sq. The upper electrode 7 is a copper rod ⅞ in. in diameter which slides up and down in the brass housing 6. The material to be tested, in the form of a long strip about 1¼ in. wide on the spools 8, passes between the electrodes and is wound up on the drum 23 after it has been punctured.

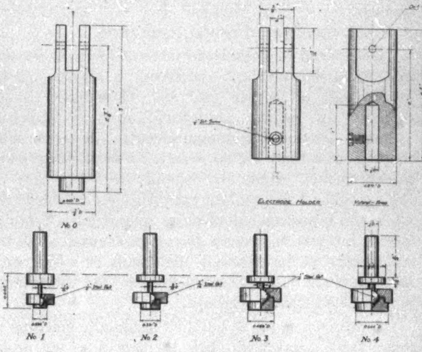

FIG. 4—ELECTRODES

After puncture occurs, the solenoid 13 is energized, lifting the plunger 12, and thus raising arm 11. This raises the electrode 7 slightly from the insulation and allows the strip to move. The movement of 12 also operates the lever 17 which moves the rolls 21 and 22 and brings a fresh portion of insulation between the electrodes. The plunger 12 then falls and the electrodes are again pressed against the insulating material with considerable pressure due to the weight of 11 and 12.

The electrodes are surrounded by the copper jacket 3. This is kept filled with oil which is circulated by means of a small motor-driven pump connected by the pipes 9 and 10. An electric heater in the piping system allows the temperature of the oil to be accurately controlled. For low temperatures a cooling coil immersed in an ice bath is used. A copper-advance thermocouple soldered to the lower electrode makes possible the accurate determination of the temperature. Wires from this thermocouple are run through the walls of the constant humidity box and are connected to a potentiometer.

Electrodes. The same lower electrode was used in all the tests. The various upper electrodes are shown in Fig. 4. No. *O* was the one used in the earlier tests. It consists of a solid rod of copper, the end being turned as shown. The electrode face was ground to an accurate fit with the lower electrode, using fine carborundum and oil. In this way, the electrodes made contact with the insulation throughout the area of the face.

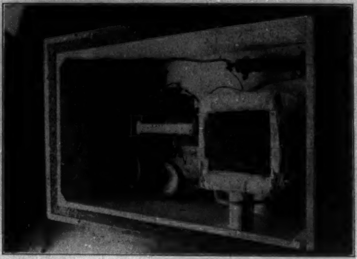

Fig. 5—View Looking Down into Constant Humidity Box

To make absolutely sure that this was the case, the other electrodes were made. These electrodes were fitted with ball-and-socket joints so that they would press evenly on the insulation throughout the face.

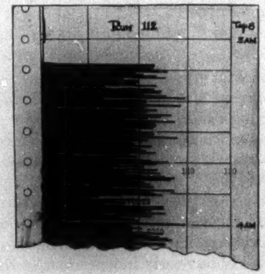

Fig. 6—Sample Voltmeter Chart

Range of scale = 325-700 volts using voltmeter tap No. 8

Electrodes 1, 2, 3, and 4 are used with the same holder. This is shown in the upper part of the figure. The faces were ground flat by using a piece of plate glass and some fine grinding compound. In all cases, the edges were rounded to a radius of about 0.001 in.

Control of Humidity. Fig. 5 is a view looking down into the constant humidity box, the cover being removed. The oil jacket surrounding the electrodes is shown in the lower part of the figure. The small fan in the upper part circulates the air over plates of phosphorus pentoxide in order to keep the humidity in the box at a very low figure. The sensitive hair hygrometer in the box indicates the per cent humidity.

PROCEDURE

All the tests were made on General Electric condenser tissue 0.0005 in. thick. This was a good grade of rag stock, and was not varnished, oiled, or otherwise treated. This material was used, not because of any special interest in its properties, but because it seemed to offer a cheap and fairly good material which could be obtained in long strips. All tests were made with paper from a single roll.

Twenty-four hours before starting a run, the paper for that run was wound from the roll to a small spool. This was done in a special winding machine in order to avoid all wrinkles. This eighty feet of paper on the spool was then baked for eight hours in a vacuum at 72 deg. cent. to remove moisture. The spool was then transferred to the constant humidity box, the cover bolted down, and the fan run all night to insure a constant low humidity during the run which was started the next morning. In this way, one run could be made every day, the paper being baked the previous day and transferred to the box each evening.

From 800 to 1000 punctures were made in a day. The resulting voltmeter chart consisted of a large number of lines ruled by the voltmeter, the length of each line representing the voltage at which one puncture occurred. A portion of such a chart is shown in Fig. 6. The lines are ruled 40 to the inch. The daily average is obtained by adding all these puncture voltages on a calculating machine and dividing by the number of readings. Hourly averages are also sometimes used, each consisting of the average of the 100 to 120 readings taken in one hour.

RESULTS

Preliminary Tests; Effect of Moisture. Preparatory to the regular tests, some runs were made to determine a good way of conditioning the paper. With such a hygroscopic material, considerable trouble was anticipated in eliminating all traces of moisture. There seemed to be no practical way of obtaining the actual moisture content of the paper under operating conditions. Our aim was therefore to develop a procedure which would produce consistent results, and to adhere strictly to this procedure in all runs.

Three runs made with the constant humidity box open to the air were as follows:

Run	No. readings	Temp. deg. cent.	Per cent humidity	Voltage (corrected to 20 deg. cent.)
15	239	19	5.8	588.6
16	222	21	8.7	571.0
17	1059	21	22.2	561.3

All values of voltage are referred to 20 deg. cent. by using a correction of − 1 volt per deg. cent. A considerable drop in voltage will be noted, due to increased humidity. As has been previously noted, the probable error of the daily average is about 3.6 volts. Three significant figures are therefore permissible. The fractional parts of a volt are probably meaningless and are retained merely to conform with the other runs of this group, some of which varied from each other by less than one volt.

The next investigation was to find how long the paper would have to be kept in the constant humidity box to reach a uniform condition of dryness. Several spools of paper were put in the box, phosphorus pentoxide was used, and the cover was bolted down. Under these conditions:

Run	No. readings	Temp. deg. cent.	Per cent humidity	Hours drying	Voltage (corrected to 20 deg. cent.)
18	1007	25	0.2	24	589.1
19	897	22	0.2	48	590.0
20	869	23	0.2	120	592.6

The results of these runs differ from one another by less than the probable error, and can thus be considered identical. Comparison of the hourly averages, however, shows a decrease in puncture voltage of 30 or 40 volts from the first readings of a run to the last. This indicates that the inner layers of paper on the spool were incompletely dried.

The next step was to bake the paper in an electric oven at 103 deg. cent. The results were:

Run	No. readings	Temp. deg. cent	Per cent humid.	Hrs. baked at 103 deg. cent.	Voltage (cor. to 20 deg. cent.)	Volts dev. from aver.
22	1064	23.6	0.2	8	614.6	−0.8
23	1040	26.1	0.2	17	616.4	+1.0
24	971	25.7	1.2	8	615.2	−0.2
					Av. = 615.4	

The hourly averages in these runs showed no downward trend with time, indicating uniform baking throughout the spool. The standard deviation of the individual readings was 46.0 volts, which is in agreement with later tests. The probable error of the daily average predicted from the σ of the individual readings is 1.0 volt. The agreement among the daily averages is thus even better than would be predicted from the σ of the individual readings, neglecting all long-time variations in the paper. Therefore, the small daily variations in the above runs are evidently due to some fortuitous combination of circumstances which cannot be expected to obtain generally. In fact, all the later runs show a greater probable error of the daily average than predicted from the single readings, due undoubtedly to long-time variations in the quality of the paper.*

This method of baking the paper was used in all tests up to Run 50. A change was then made to vacuum baking, which was used in all subsequent runs. It will be noted from Fig. 15 that the tests made with the two methods gave the same results. That there was very little moisture in the paper is also indicated by the fact that there is no break in the $E - \theta$ curve* near 100 deg. Our conclusions, then, are that the effect of moisture on the breakdown voltage is not as great as might be expected, and that there seems to be no difficulty in getting the paper dry enough to insure uniformity from this aspect.

Effect of Temperature. It seems reasonable to expect the thermal theories to apply in a great number of cases of breakdown. It seems equally likely, however, that the theories will fail in other cases such as sudden application of voltage, very low ambient temperatures, or extremely thin materials. In such cases there is little possibility of progressive heating of some path through the insulation, and breakdown must be of a different nature. There appear to be, then, at least two mechanisms of breakdown† which we will call "thermal" breakdown and "disruptive" breakdown. The question then arises as to the limits of applicability of each theory. This is a subject for experimental investigation.

If puncture voltage be plotted against temperature, the thermal theories predict an exponential curve *A*, Fig. 7. Purely disruptive punctures, on the other hand, would probably be nearly independent of the temperature, as in the curve *B*. It seems likely, therefore, that if a curve were obtained for any material over a wide enough range of temperatures, thermal breakdowns

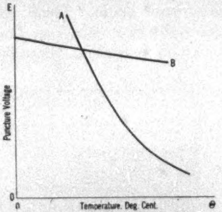

FIG. 7—PROBABLE SHAPES OF $E - \theta$ CURVES

A—Predicted by thermal theories of breakdown
B—Probable shape for distruptive breakdown

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*See "Precision of Results."

*Fig. 8.

†Hayden and Steinmetz, Bibliography No. 8, discuss several possible mechanisms of breakdown. In this connection, Rogowski says: "The opinion that breakdown is caused solely by heating must be abandoned in the case of thin plates, low temperatures, and sudden applications of voltage. On the other hand it may be correct for thick plates, high initial temperatures, and sufficiently long voltage applications; though a decisive verdict is not possible at present due to lack of suitable experimental material."

would occur in part of the range and disruptive breakdowns in the remainder. A combination of A and B would result. Such a curve would prove the existence of both kinds of punctures, and might lead to some criterion by which the transition point could be predetermined.

The results obtained are shown in Fig. 8. The temperature was varied from − 2 deg. to + 130 deg. cent., as wide a range as could be conveniently obtained. Each point represents a daily average of from 800 to 1000 readings. It will be noted that a rise in temperature of 100 deg. cent. causes a diminution in puncture voltage of only 15 per cent.

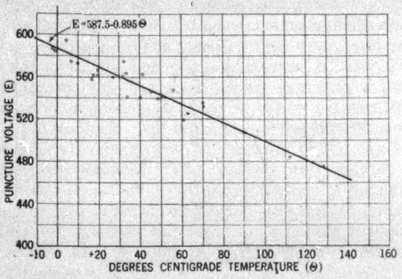

FIG. 8—BREAKDOWN VOLTAGE AS A FUNCTION OF TEMPERATURE

Runs 103-130
Electrode No. 3
Electrode pressure = 210 lb. per in.^{2}
Relay protective resistance = 30 × 10^3 to 56 × 10^3 ohms
Voltage raised at 1800 volts per min.
G. E. condenser paper baked in vacuum for 8 hr. at 72 deg. cent. before each test
Each point represents the average of from 500-1000 readings

The curve appears to be of the form B rather than A.[*] Further analysis given in Appendix A leads to the same conclusion; namely *none of the punctures obtained in these tests occurred according to the thermal theories of breakdown.* This is probably due to the very thin material used and the excellent heat-transmitting properties of the electrodes.[‡] Further tests are now in progress with thicker materials with which it is hoped to obtain curves of both the A and B types.

Effect of Electrode Area. Considerable work has been done in the past regarding the effect of electrode size. In 1913, Mr. F. M. Farmer[20] presented the results of tests on varnished cambric and hard rubber with electrodes of various sizes. Mr. Milnor[44] showed that, if we consider the insulation to be non-homogeneous, each elementary area between the electrodes having its individual value of dielectric strength, and if we assume further that these values are distributed according to the normal form of probability curve and that puncture occurs at the weakest point, then the breakdown voltage should be

$$E = C_1 - C_2 \log A$$

A is the area of the electrodes and C_1 and C_2 are constants. This is known as the "weak-spot" theory. Farmer's curves were of this form. The experiments of Gewecke and Krukowski[21] also seem to support the weak-spot theory. Further work by G. Y. Fong[23] and Kennelly and Wiseman,[24] however, have cast some doubt on the validity of the theory, and nothing really conclusive seems to have been decided.

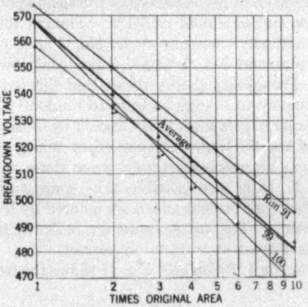

FIG. 9—BREAKDOWN VOLTAGE AS A FUNCTION OF ELECTRODE AREA

Electrode No. 1

In connection with an investigation of the various factors which might have caused errors in the $E - \theta$ curve, some runs were made to determine the effect of electrode area. While the evidence is not conclusive, it seems to support the weak-spot theory.

If the weak-spot theory is correct, the breakdown voltage obtained with large electrodes of area $n\,A$ should be the same as the lowest value obtained in n consecutive tests made with electrodes of area A. In other words, puncture should occur at a certain voltage corresponding to the weakest spot in a given area, regardless of whether this area is tested all at one time by the use of large electrodes, or is tested one section at a time with small electrodes. Thus if a large number of tests is available, using one size of electrodes, the puncture voltage for electrodes of twice this area should be obtainable in the following way: Consider the results in groups of two, discard the higher value in each case, and take a new average of the remaining figures. Similarly, the breakdown voltage for three times the original area should be obtainable by dividing the readings into groups of three and averaging the lowest values of each group.

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[*] In accordance with the Joffe theory of breakdown which has recently appeared, our curve is a measure of the number of free ions existing in the material at various temperatures.

[‡] See also Appendix B. Since our paper was written, some very valuable work by Inge, Semenoff, and Walther has been published[52]. $E - \theta$ curves are obtained showing a decided break where the mechanism of breakdown changed from the disruptive to the thermal type.

This was done in the case of Electrode No. 1 as shown in Fig. 9. The area of this electrode was taken as unity for convenience. It will be noted that the result is very accurately of the form

$$E = C_1 - C_2 \log A$$

as was predicted by Milnor. Our tests, therefore, support his proof. His assumption of a normal distribution of puncture voltages (an assumption which at that time had not experimental verification) is also verified by our tests.*

Similar analyses were made with electrodes of twice and four times the area. The results† are shown in Fig. 10. To be a perfect check of the weak-spot theory, the three curves should coincide. Though all curves have practically the same slope, No. 2 is about 2 per cent higher and No. 1 about 2 per cent lower than No. 3. Probability theory shows that this is too large a discrepancy to be due entirely to random variations in the paper. It is most likely caused by a "seasoning" action of the electrode faces, an effect which unfortunately was not discovered until after these runs were finished.‡ It is also possible that some thermal or other effects are present which are ignored by the weak-spot theory. The tests, however, indicate that the theory is at least approximately correct.

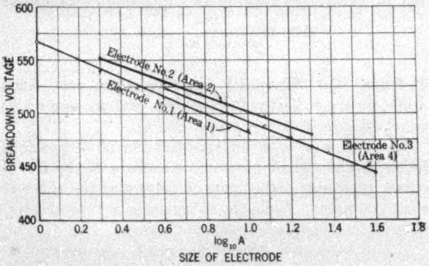

FIG. 10—BREAKDOWN VOLTAGE AS A FUNCTION OF ELECTRODE AREA

Average of 8808 readings
Electrodes Nos. 1, 2, 3
Electrode pressure = 210 – 850 lb. per in.^{2}
Relay protective resistance = 56 × 10^3 ohms
Voltage raised at 1800 volts per min.
G. E. condenser paper baked in vacuum for 8 hrs. at 72 deg. cent. before each test

The Standard Deviation. Fig. 11 shows the frequency distribution for a day's run. This was obtained by counting the number of readings falling in each 10-volt interval. The smooth curve is the probability curve having the equation—,

$$y = \frac{1}{\sigma\sqrt{2\pi}}\, \epsilon^{-\frac{x^2}{2\sigma^2}}$$

It will be noted that the points of our frequency distribution fall fairly well on this curve, showing that the distribution is of the normal form such as is obtained for the variations of any quantity governed purely by chance.§ Distributions obtained on other runs are very similar to the one shown. The highest point of the curve is, of course, at the arithmetic average of all the readings, and thus is the most probable voltage of breakdown.

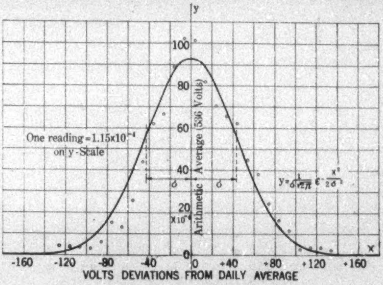

FIG. 11—TYPICAL PROBABILITY CURVE

Run 108
σ = 43.1 volts
$\frac{1}{\sigma\sqrt{2\pi}} = 92.6 \times 10^4$

The constant σ in the above equation is called the *standard deviation* or *dispersion*. It is a very important constant in statistical work, since it completely specifies the distribution, and thus takes the place of a mass of data. The standard deviation was obtained from the puncture voltage readings by the use of the equation

$$\sigma = \sqrt{\frac{\Sigma\ (\delta)^2}{n}}$$

where $\Sigma\ (\delta)^2$ is the sum of the squares of the deviations from the mean and n is the number of readings. In this case, $\sigma = 43.1$ volts. A more uniform and homogeneous material would have a more peaked curve and therefore a smaller value of sigma. A more variable material would have a larger sigma. Thus the constant σ is a measure of the inherent variability of the material.*

It is believed that the standard deviation will be of practical importance in future insulation engineering. Sigma is a constant of the material, just as important in its way as puncture voltage. The puncture voltage indicates the average value at which breakdown occurs; the standard deviation tells how much variation there will be from sample to sample. By knowing both these constants, the insulation designer can put his work on a much firmer foundation than would otherwise be possible. The constant should also be of use in the specification of insulating materials. The variability of the material bought by large cable or condenser manufacturers, for instance, is of the utmost importance to them. By specifying σ and keeping it up to standard by tests made on samples from each batch received, the manufacturer would eliminate one of the variables which tend to produce a non-uniform product.

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*See Fig. 11.

†All tests were made at a temperature of 48 deg. cent., humidity of 0.4 to 4 per cent, and a voltage rise of 1800 volts per min.

‡See "Effect of Polishing Electrodes."

§See, for instance, Bibliography, 44-50.

*It also includes of course, the accidental errors of measurement.

Precision of Results. As previously noted, Fig. 12 shows the results of 16,000 punctures made under the same conditions. The temperature was kept at 40 deg. cent., the humidity was between 0.2 and 0.4 per cent, and the electrode No. 0 was used throughout. Each point represents an hourly average of between 100 and 120 readings. The ordinates are the deviations of the hourly averages from the average of the day in which they occurred. The abscissas are merely consecutive hours of run. With the exception of the first run, the data do not appear to have any decided trend, either up or down, but seem to be purely random variations such as would be expected from variations in the paper itself.

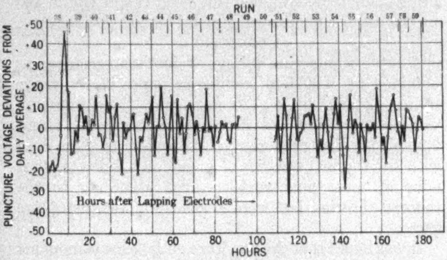

Fig. 12—Voltage Deviations, Runs 38-60

(Hourly averages minus daily average)

That the variations are random and follow the normal law is shown by Fig. 13. The points give the frequency distribution of these hourly averages. The smooth curve is the theoretical probability curve. The points fit the curve about as well as could be expected with such a small number of data, only 150 points. The next curve, Fig. 14, shows the deviations of the daily averages from the average of the eighteen runs from No. 39 to 59, inclusive.*

A statistical analysis was made of all the data, Appendix C. The actual values of σ are as follows: For the individual reading, 40 volts; for the hourly average, 9.19 volts; for the daily average, 5.3 volts. Comparison of the hourly and daily sigmas with the corresponding values predicted from the σ of the individual readings by probability theory, shows that in both cases the actual values are too high. The discrepancy of the daily value is, however, not much greater than that for the hourly sigma. This would seem to indicate that the unavoidable changes in humidity and baking (variations which did not occur from hour to hour) were not responsible for the variations in the daily average. It is thought that both hourly and daily variations are due principally to inherent variations in the paper itself.

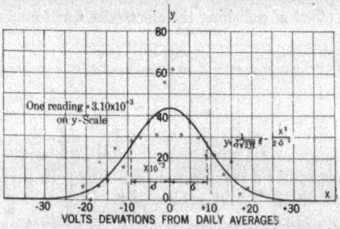

Fig. 13—Probability Curve of Hourly Averages, Runs 39-60

$\sigma = 9.19$ volts

$$\frac{1}{\sigma\sqrt{2\pi}} = 0.0434$$

Appendix C also shows that the probable error of the daily average is 3.6 volts, and thus the daily average is correct to less than one per cent.† Analysis of the runs used in obtaining the $E - \theta$ curve, Fig. 8, shows a similar situation. The probable error of the daily average in this case is 4.7 volts, giving again an error of less than 1 per cent. Thus the accuracy obtained by the machine is evidently much greater than could have

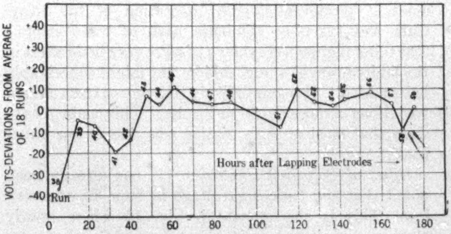

Fig. 14—Deviations of Daily Averages from Total Average of 18 Runs

All readings corrected to 30 deg. cent.

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*Run 38 was not used because of its low average. This is explained in the next section. Runs 49 and 50 were made, but the hourly averages were not computed.

†This, of course, ignores any constant errors which may have been present in all the tests.

been reached by the usual methods without an undue expenditure of time and labor.

A list was made of all possible sources of error. These included variations in the temperature and time of baking, variations in humidity, electrode size and pressure, rate of voltage rise, value of protective resistance, time which elapsed after polishing the electrodes, voltmeter calibration, and errors in temperature measurement. Nearly all of these factors were investigated over a wider range than would be likely to occur accidentally.

The effect of polishing the electrodes was found to be very considerable, as will be shown later. Too high a protective resistance was also found to have great effect, since it produced sluggish operation of the relay. Care was taken to eliminate both of these errors.‡ The other factors were shown to have very little effect in the limited range through which they might conceivably have varied. It seems unlikely that any of them caused variations of more than 1 or 2 volts. This again goes to show that most of the variation was due to the paper itself.

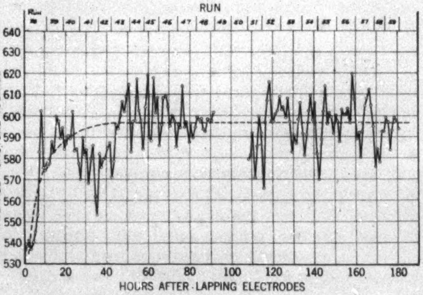

FIG. 15—HOURLY AVERAGES, RUNS 38-60

All readings corrected to 30 deg. cent.

Dotted curve indicates the upward trend of voltage due to seasoning of the electrodes

Effect of Polishing Electrodes. It has already been noted that the puncture voltage in Run 38 was very low. This is also shown in Fig. 15 where hourly averages are plotted against hours' use of the electrodes after polishing.§ Here the low puncture voltages obtained after polishing are very evident. To make sure that this was not a freak happening, hourly averages were plotted for a number of other runs, and in each case the time of polishing the electrodes was clearly shown by a set of readings lower than those obtained at any other time.

Brass and copper electrodes are commonly used for breakdown tests. It is probable that this phenomenon has been present in most tests, though it appears not to have been noticed before. Evidently if tests are made on the very steep part of the curve, the results will be much more inconsistent than would otherwise be the case. In our tests, it was necessary to discard at least 1000 readings after the electrodes were polished.

The cause of this trouble is not known. Current at breakdown was limited by the protective resistance to such a low value that there was no pitting of the electrodes nor was there any apparent roughening of the surface. One possible explanation is that the puncture produces an insulating coating on the electrode surfaces. Examination of the electrodes shows that each puncture causes a small black mark on the electrodes. These spots distribute themselves fairly evenly over the entire electrode surface. After a day's use, the faces reach a state of more or less uniform blackening, which is not altered much by subsequent runs. If each spot introduces a high resistance in series with the paper below it, puncture will tend to occur at places where there are no spots. This will have the effect of decreasing the effective surface, and will increase the puncture voltage accordingly. After a large number of punctures, the blackening effect will reach a nearly constant value, with like effect on the puncture voltage. Tests are now being made to determine if this effect can be eliminated by the use of a nonoxidizing metal for the electrodes.

CONCLUSIONS

A curve of puncture voltage against temperature was obtained for 0.0005-in. condenser paper in the range from − 2 deg. to + 130 deg. cent. The puncture voltage was found to decrease uniformly at about 0.9 volt per deg. cent. rise. Comparison of these data with the thermal theories of breakdown indicates that none of these punctures occurred according to the thermal theories, but were evidently of the disruptive type.

It was found that the puncture voltage measurements were distributed according to the normal law. A set of 8800 readings with different sized electrodes appears to support the weak-spot theory, though the evidence is hardly conclusive.

The standard deviation of the readings was found to be a constant of the material, independent of the electrode area and only slightly affected by temperature and number of readings. It is believed that this constant will be of considerable use in the specification of insulation, since it is a measure of the variability of the material.

It was found that the inherent variability of the paper was not confined to causing variations from reading to reading, but also caused long-time variations from hour to hour and from day to day. This reduces the accuracy of the daily averages below the values predicted by the theory of probability.

An analysis of the results indicates that the variations from hour to hour and from day to day are largely due to variations in the paper itself. An investigation was made of the various possible sources of error, and it appears that no one of them could have caused an error of more than one or two volts. The probable error of the results is shown to be less than 1 per cent. This precision is evidently considerably higher than could have been obtained in the ordinary way without a great expenditure of time and labor.

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‡Except in the investigation of electrode area, where the effect of polishing the electrodes was probably of importance.

§The gap at Runs 49 and 50 is due to the fact that hourly averages were not computed for these runs.

Copper and brass electrodes are commonly used for puncture tests. Our experiments, however, indicate that with such electrodes there is a "seasoning" effect. This produces a marked change in puncture voltage unless the electrodes are cleaned after each puncture or are allowed to reach a steady state before readings are taken. Such a condition was obtained only after about 1000 readings.

Appendix A

THERMAL THEORY OF THE $E - \theta$ CURVE

According to the thermal theories of breakdown, puncture occurs when

$$\frac{E^2}{\rho} = C, \text{ a constant*}$$

where E = voltage gradient at breakdown,

ρ = resistivity of the weakest path at the temperature of the surroundings, θ.

Physically, this means that for a condition of instability to obtain, it is necessary that the rate of heat generation per unit volume (E^2/ρ) shall reach a certain critical value (C). This value is a constant for a given material. It depends upon the thermal conductivity, the temperature coefficient, and also, in Rogowski's theory, upon the thickness.

If we assume that the resistivity is an exponential function of the temperature

$$\rho = \rho_0 \, \epsilon^{-\gamma\theta}.$$

then it is evident that

$$E = \sqrt{C \rho_0} \, \epsilon^{-\frac{\gamma}{2}\theta}$$

Thus the γ obtained from resistivity measurements will determine the change of puncture voltage with temperature.

Unfortunately, no resistivity measurements have yet been obtained on the paper used in these experiments. According to the work of J. E. Shrader[30], however, untreated fish paper has a temperature coefficient of about 0.07. Assuming this value for γ, we find that the puncture voltage at 100 deg. cent. should be

$$E_{100} = 587.5 \, \epsilon^{-\frac{0.07}{2}(100)} = 18 \text{ volts}$$

instead of the 498 volts actually obtained.

If, instead of a straight line, we use an exponential to approximate the data of Fig. 8, we find

$$E = 587.5 \, \epsilon^{-0.0016\theta}$$

The value of γ is thus 0.0032. That such a value is very improbable is shown by the following table:

Authority	Material	γ
J. E. Shrader[30] . . .	Untreated fish paper, vacuum dried	0.07
J. E. Shrader[30] . . .	Untreated fullerboard, vacuum dried	0.07
J. E. Shrader[30] . . .	Paraffined fish paper	0.104
H. H. Poole[29]	Mica	0.051
H. H. Poole[29]	Glass	0.109
E. Mündel[17]	Glass	0.084
Landolt-Bornstein[6]	Glass	0.120
Landolt-Bornstein[5]	Hard rubber	0.023
Landolt-Bornstein[5]	Porcelain	0.084

Appendix B

Some work of E. Mündel[17] has recently come to our attention, and a set of his curves has been replotted in Fig. 16. The material used in the tests was glass 0.4 mm. thick. Two curves, A and B, have been added to Mündel's results. Curve A is an exponential with ($\gamma/2 = 0.0171$). The straight line B falls 15 per cent with 100 deg. cent. increase in temperature just as in Fig. 8.

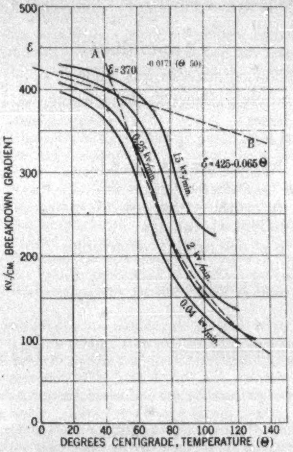

FIG. —16 PUNCTURE OF GLASS 0.4 MILLIMETERS IN THICKNESS

(From Mundel—*Archiv. f. El.* 15, 1925, p. 338)

These curves are a possible explanation of Mündel's results. The part of his curves from 50 deg. upward is evidently caused by thermal breakdowns, while the lower temperature punctures were probably disruptive.

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*According to Wagner, loc. cit., formula 2,

$$\frac{E^2}{\rho} = \frac{k}{\gamma \, \epsilon}$$

k = thermal conductivity,

γ = temperature coefficient of resistivity,

ϵ = 2.71828 . . .

This is based on the assumption that heat flow is radial from a thread into the surrounding insulating material.

Rogowski assumed that heat flowed only into the electrodes, and obtained the result, Rogowski, loc. cit., Formula 11,

$$\frac{E^2}{\rho} = \frac{8\,k}{\gamma \, D^2}$$

where D is the thickness of the insulation. The only difference is in the constant C.

It will be noted that all the curves appear to approach the slope of B as the temperature is decreased, while at the higher temperatures they approximate exponentials about as closely as the usual curves for resistivity do.

The value ($\gamma = 0.034$) from curve A seems to be a little low for glass; see table in Appendix A; but the discrepancy may be due to the fact that the resistivity of most insulating materials is a function of voltage as well as temperature, a fact which is neglected in the thermal theories.

Mündel's results, therefore, indicate the existence of both mechanisms of breakdown and show that the transition point for this particular case is about 50 deg. cent.

Thermal breakdowns are probably obtained in Mündel's tests and not in ours because of the difference in thickness of the two materials, his glass being about 30 times the thickness of our paper. All of our tests are undoubtedly on the straight line far to the left of his curves.

It is evident that for practical purposes, nothing could please the insulation expert more than to be sure that his insulation would operate on curve B rather than A, since its breakdown strength would then decrease so little with temperature. With a more thorough knowledge of insulation, the future engineer may be able to avoid thermal breakdowns altogether.

Appendix C

The following analysis was made:

Total readings, Runs 39-59, inclusive, = 16,505
Total hours . . . . = 154
Total days . . . . = 19

Av. hrs. per day = 8.11
Av. rdgs. per day = 869
Av. rdgs. per hr. = 107

From individual readings:

σ of one reading = 40 volts

$$\sigma \text{ of hourly aver.} = \frac{40}{\sqrt{107}} = 3.86 \text{ volts}$$

$$\sigma \text{ of daily aver.} = \frac{40}{\sqrt{869}} = 1.35 \text{ volts}$$

$$\sigma \text{ of total} = \frac{40}{\sqrt{16505}} = 0.31 \text{ volts}$$

From hourly averages:

σ of hourly aver. . . . . = 9.19 volts

$$\sigma \text{ of daily aver.} \ \frac{9.19}{\sqrt{8.11}} = 3.22 \text{ volts}$$

$$\sigma \text{ of total} \ \frac{9.19}{\sqrt{154}} \quad . \quad . \quad = 0.74 \text{ volts}$$

From daily averages:

σ of daily aver. . . . . = 5.30 volts

$$\sigma \text{ of total} = \frac{5.30}{\sqrt{19}} \quad . \quad . \quad = 1.22 \text{ volts}$$

Probable error of daily aver. = $5.30 \times 0.6745 = 3.6$ volts
Probable error of total = $1.22 \times 0.6745 = 0.82$.

These results can be tabulated as follows:

STANDARD DEVIATION

	Individ. rdg.	Hourly	Daily	Total	Daily σ — σ by Ind. rdgs.
By indiv. rdgs.	40_v	3.86	1.35	0.31	1.0
By hourly aver.		9.19	3.22	0.74	2.4
By daily aver.			5.30	1.22	3.9

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Discussion

W. W. Shaver: Mr. Moon stated that his results indicated that the breakdown was not a pyroelectric effect. I should like to mention some experiments which we have been carrying out on Pyrex and porcelain insulators which seem to warrant a similar conclusion.

These consisted in developing partial punctures; that is, carrying a puncture test almost to the point of breakdown, and then breaking the circuit and examining the partial puncture in the material. We have some photographs showing what might be described as "petrified lightning" in the breakdown in the Pyrex insulators. The discharge very much resembled a lightning flash permanently impressed in the glass. In the porcelain insulators, of course, we could not observe this in the same way, but by staining the material, we have found means to show that breakdown occurs in a similar manner. When examined under the microscope, the appearance of these partial punctures indicated that the breakdown was not due to a pyroelectric effect, but more probably to some type of ionization in the material.

F. M. Clark: In Pittsfield, we have worked for several years trying to devise a theory for insulation failure. We may not have made any more progress than other electrical laboratories, but we have reached some definite conclusions with regard to dielectric behavior. One conclusion is that the electrical breakdown of air-impregnated insulation is certainly not a breakdown phenomenon which we can explain on the basis of the pyroelectric theory. That checks, I believe, with the conclusions drawn by the authors. Air-impregnated insulation appears to give a breakdown which is a function of the chemical behavior and dielectric strength of air.

On the fifth page of their paper, the authors discuss the effect of large and small electrodes. They apparently reach the conclusion that the difference accompanying the use of large and small electrodes is a weak-spot difference. From such familiarity as I have with the paper they are using, I do not see how they can reach any other conclusion.

If the authors desire to get further evidence of the presence of weak spots, my suggestion would be to take a head set and locate those areas with low-voltage direct current.

In our work, we have extended the investigation to cover a number of laminations of this same type of paper as well as thick material, such as pressboard. We still find that the small electrodes give higher values than the large ones. This and other related observations make it almost conclusive to us that there is some effect present other than that which we can attribute to weak spots mechanically formed during the manufacture of the paper.

I was interested in the results with polished brass electrodes. We found the same thing. However, we have been led to believe that in all probability the low breakdown which is obtained for such thin materials when tested with new electrodes is due to the fact that the machinist has not polished the electrode surface sufficiently. With low-voltage direct current, weak-spot tests invariably show a larger number per sq. ft. with a new set of electrodes than with an old set. With frequent use, this difference between the electrodes tends to disappear. We have about concluded that the difference is largely due to a surface condition of the electrode which is mechanical and not chemical in any way.

Herman Halperin: In the body of the paper, the authors carefully point out the fact that the paper is only one-half a mil thick and for this reason thermal factors are favorable to disruptive breakdown.

Thermal breakdown. The equation for thermal breakdown is

$$E^2/\rho = C \text{ (Appendix A)}$$

Now, this equation does not contain a term for t (= time) since it is tacitly assumed that voltage is raised at a slow rate.

and the equation is the boundary or limiting value of the stable or steady-state condition. It takes a few hours to reach a steady condition. However, in Bush's tests, 100 to 120 readings were taken per hour. This is at the rate of two per minute, and assuming that half of the time is consumed in relay operation, feeding the tape, etc., the voltage must have risen at least 600 volts in 15 sec., or 40 volts per sec. This is far from a steady-state condition and it may well be that a more critical study of the thermal theory including the time and thermal storage coefficient of the paper would yield results consistent with the data.

At the end of Appendix B, they have this sentence: "With a more thorough knowledge of insulation, the future engineer may be able to avoid thermal breakdowns altogether."

That is, to my mind, a very nice hope, but I don't believe the authors have presented enough data to warrant that possibility.

Now, in Chicago, during the past few years, we have made over 300 tests at various times and durations on impregnated paper insulation in underground cables. We have found that about 75 per cent of the failures occurred in the region of the cable where the sheath temperature was higher than the adjacent sheath temperature. During the time the voltage was on the cable, our testing men went along the cable and felt it with their hands, and whenever they discovered a point a little hotter than the normal cable temperature, they placed the thermometer on the sheath and in that way we got the data as to the location of the so-called hot spots.

Furthermore we have found that a larger percentage of the failures was at hot spots when we had the thicker insulations, and so with real thin insulation, there may be more dissipation of the heat longitudinally so that it does not perhaps manifest itself in these hot spots.

Variation of thickness. If the failure is "disruptive" and follows "weak-spot" theory then running 1, 2, 3, . . or n paper layers through together, the dielectric strength should increase faster than the thickness of the layers. I should expect the equations to be of the form

$$E = E_1 (n + C \log n)$$

where E is strength of n layers and E_1 is the strength of one layer (as already determined). In other words, the weak spots would not line up and two papers would be far stronger than two times one paper.

However, on the other hand, if the thermal law were large or predominant, the strength could not be greater than

$$E = N\ E_1$$

and probably would follow the law

$$E = \sqrt{N}\ E_1\, n^{(1-x)}$$

If the failure were a complex combination of thermal and disruptive effects, (as is very probable), then the dielectric strength of several layers could follow the law

$$E = \phi\ (n)\ E_1$$

where $\phi\ (n)$ could be greater or less than unity for n = small number (1, 2, or 3, etc.). This may explain why different investigators may have found

$$E = n\ E_1$$

$$E = (n)^{3/4}\ E_1$$

$$E = \sqrt{n}\ E_1, \text{ etc,}$$

Now, Bush and Moon are in an excellent position to throw some light on this dispute by making identical tests with $n = 2$, $n = 3$, etc., up to the limit of the generator, 4000 volts (using wider strips, if necessary). Perhaps they could go as high as $n = 5$. Of course, they would have to be very careful about the pressure, since I am assuming thickness $t = n\ t_1$.

Variation of material. The tests described are on dry paper, unimpregnated. Without great complication, I believe it entirely feasible to surround the electrodes with an oil bath and have the paper run through this bath—(several zig-zag passes around several rolls under oil). The oil should be continuously renewed with degasified oil—the oil to flow in a closed cycle from the bath, into a vacuum chamber, pumped out and through a filter back into the bath. A surface float would prevent much gas from being absorbed at the surface during the short time the oil remains in the bath. Then the paper (being so thin), would quickly saturate with oil by capillary action, and small air bubbles trapped in the fibers would be quickly absorbed by the degasified oil. The test would then be a true test of oil-impregnated paper.

Again, several thicknesses n should be tried up to the limits of the apparatus. They should be spaced apart during impregnation and combined under the electrode only at the last minute.

Also it might be feasible and very valuable to adapt the apparatus to test oil films. All that is necessary is to have a continual flow of oil in the bath and a strip (or strips) of paper with perforated windows moved one at a time under the electrode. These "windows" would serve the double purpose of making the oil stagnant during test and positively sweeping it out clean for the next test.

Correlation. Then, having the breakdown strength of paper, oil, and impregnated paper separately, all with variations of thickness and temperature, the probability is high that much more will be known regarding the dielectric strength of "solid" oil-impregnated paper insulation. For the effects of ionization a separate analysis would be necessary but much simplified by exact knowledge of the "solid" insulation in series and parallel with the ionizing voids.

Wm. A. DelMar: Messrs. Bush and Moon have made an important contribution to the study of dielectrics in calling our attention to the importance of a large number of tests of dielectric strength, in order to obtain significant averages. Workers on dielectrics are indebted to them for developing an apparatus for putting this principle into effect.

The actual results obtained in their tests, however, are of doubtful value, due to the nature of the dielectric which they have chosen. In the first place, unimpregnated paper is merely air baffled by a tangle of fibers, and is not at all representative of a solid dielectric. Secondly, the use of samples as thin as ½ mil., especially in a material of this character, was bound to result in the way they have shown. It was not necessary to make tests in order to discover the relation shown in Fig. 11 of the paper. I therefore feel that they have contributed little or nothing towards settling the disputed explanation of the lowering of dielectric strength with increase of area.

The idea illustrated by their Fig. 7 may prove to be an important one for researchers on dielectrics if the slopes of the Curves A and B are really as different as shown. That such is the case is indicated by their analysis of the work of Mündel.

R. H. Marvin: (communicated after adjournment) This paper is of great interest on account of the ingenious methods used and the accuracy and completeness of the data obtained. However, the dielectric used in these tests,—untreated paper,—raises some question as to their significance.

If a piece of paper is examined with a microscope, it is seen to consist of great numbers of interlaced fibers. Such a structure indicates air spaces between the fibers, which is in accordance with its well-known porosity. The question then arises whether a puncture test on such a material gives a true puncture of the paper substance, or merely a breaking down of the air in the interstices. This is best checked by a comparison with the breakdown value of an equal air-gap.

Some excellent data on the strength of small air-gaps are given in, "The Sparking Distance Between Plates for Small Distances," Robert F. Earhart. *Philosophical Magazine.* 6th Series. Vol. I. 1901. Pp. 147-159. From Fig. 4 of this article the following values for various gaps and pressures are obtained:

Gap		Direct-Current Breakdown Voltage		
		Pressure, centimeters of mercury		
Millimeters	Inches	40	76	152
0.01	0.00039	340	385	490
0.02	0.00079	400	460	700
0.03	0.00118	460	530	900

The temperature is given as room temperature, and will be taken as 20 deg. cent. Since we may assume that changes in density caused by either temperature or pressure have the same effect, these data permit determining the breakdown at different temperatures as well.

Fig. 8 of the paper under discussion gives the breakdown strength of untreated paper 0.0005 in. (0.0127 mm.) thick at temperatures of from 0 deg. to 140 deg. cent. The curve is a straight line giving a puncture voltage of 586 at deg. cent., and 464 at 140 deg. cent. From the data given by Earhart let us determine these values on the assumption that the paper acts merely as a spacer for an air-gap. If the relative air density is taken as 1 at 20 deg. cent., then it will be 1.073 at 0 deg. cent., and 0.710 at 140 deg. cent. Plotting the values in the table and taking the values for a 0.0005-in. gap gives the following:

Pressure, centimeters	Relative air density	Breakdown voltage
40	0.527	354
76	1.000	406
152	2.000	547

Plotting the breakdown voltage against relative air density, we obtain the breakdown voltage at 0 deg. cent. (relative air density 1.073) as 415 volts, and at 140 deg. cent. (relative air density 0.710) as 373 volts. Thus the value for air at 0 deg. cent is 70.8 per cent of the value for paper, and the value at 140 deg. cent is 80.4 per cent. Both values for air are therefore a little low, but by nearly the same percentage.

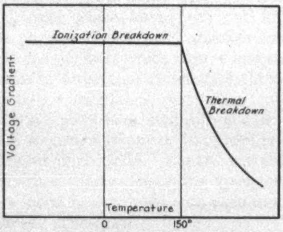

Fig. 1—Breakdown Gradient of Glass

Thickness = 1 mm.

From Joffe, Kurchatoff, and Sinelnikoff, *Jour. of Math. and Physics*, April 1927

There is also the possibility that the path of the spark among the paper fibers is not straight, but winding. This would increase the breakdown voltage of the air path, and bring the calculated values for air more nearly in accordance with the observed values for paper.

While it must be admitted that the data presented are not sufficient to prove that the untreated paper acted simply as an air-gap, still the relation is sufficiently striking to appear to deserve further study.

P. H. Moon: I was much interested in the suggestions made by Mr. Halperin. There is, of course, an immense field for further work along these lines; and our work, so far, can hardly be considered as more than a preliminary.

I agree with Mr. Del Mar that the material used was not just paper but was "air-impregnated" paper. I do not see, however, that this invalidates our results. If the breakdown were purely a breakdown of air, then—making due allowance for variations in the machine—I should still expect the results to be much more uniform than they were. The reason we used air-impregnated rather than oil-impregnated or varnished paper was simply to keep the number of variables as small as possible. In any measurement of insulation, the number of variables is large at best. If oiled or varnished paper had been used, additional "unknowns," connected with the kind and condition of the impregnating substance, would have been introduced.

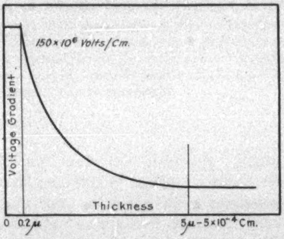

Fig. 2—Breakdown Gradient of Very Thin Specimens of Glass

In the paper by Del Mar, Davidson, and Marvin,[2] the conflicting results obtained by different investigators have been pointed out. These discrepancies are undoubtedly due to the use of different materials by the various investigators. However, I think that the differences are partly due to the fact that some of these breakdowns occurred according to the thermal theories and some did not; that is, two investigators might test samples of the same material and might get entirely discordant results due to difference in thickness or temperature. One might get thermal breakdowns and the other might get ionization breakdowns.

That thickness and temperature have a vital effect on the mechanism of breakdown is shown by some recent work by a group of Russian physicists. I am referring to Joffé, Inge, Semenoff, Walther, and some others. These investigators have obtained a very good check on Rogowski's thermal theory with glass and with rock salt, provided the thicknesses were of the order of a millimeter or greater. In this case, if voltage gradient is plotted against temperature, the curve is very low near the melting point of the material and rises exponentially as the temperature is decreased (see Fig. 1 herewith). This is just as we would expect from Rogowski's theory. But the curve does not continue to go up indefinitely. At some temperature—150 deg. for glass—it makes a sudden break and becomes horizontal. Their tests have shown that it remains horizontal out to liquid-air temperatures.

Joffé has shown that the exponential part of the curve is due to thermal breakdowns, while the horizontal part is caused by ionization. In the latter part, he has shown that the breakdown gradient is

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1. *Electric Strength of Solid and Liquid Dielectrics,* see p. 1049.

$$E = \frac{1}{u}\sqrt{\frac{2\,e\,P}{m}}$$

where u is the mobility of the ions, P is the ionization potential, and e and m are the charge and mass of the ion. This would indicate that the gradient at breakdown is independent of the thickness and temperature, provided the mobility stays constant.

It might seem that the horizontal part of the curve represented the maximum gradient obtainable, but some further work has shown that this is not the case. If gradient is plotted against thickness of the material (Fig. 2 herewith), for sheets less than 5 μ in thickness, the gradient goes up inversely as the thickness and rises to very high values. This again is supposed to be due to ionization by collision, but in this case the thickness is so small that there are insufficient ions to cause breakdown at the previous value (horizontal part of curve), and thus the gradient is higher than it would be for thicker materials.

Finally, a place is reached at about 0.2 μ, where the gradient is again a constant independent of thickness or temperature and having the immense value of 150 million volts per cm. Breakdown here is probably due to actual rupture of the molecules of the substance.

So, to summarize, these investigators have shown that there are at least three distinct mechanisms of breakdown, and that these different mechanisms apply in different cases.

The Electrical Resistivity of Insulating Materials[†]

BY HARVEY L. CURTIS[*]

Fellow, A. I. E. E.

Synopsis.—*A brief review is given of the literature on conduction through insulators. Every dielectric has a definite resistivity when the potential gradient is below a certain value, different for each substance. If the potential gradient is continually increased, a point is reached where an increase in voltage does not affect the current. This is called the saturation current. With still greater potential gradients, a point is reached where the current increases rapidly and breakdown soon results. All these phenomena of conduction are explainable as the movement of ions in an electric field.*

The resistivity of a dielectric depends on the number of ions in unit volume, on the charge on each ion, and on their mobility (velocity under unit potential gradient). The saturation current depends on the charge on each ion and on their rate of production. Breakdown is preceded by ionization by collision, which is determined by the ionization potential of the substance and the length of path of an ion.

The number of ions normally present in a dielectric depends not only upon the rate of producing them but also upon their rate of recombination. The rate of recombination is a constant of the material, but the rate of production may depend either upon outside agencies or inside forces. In gases, ions are generally produced by outside agencies, the important ones being rays from radioactive materials, X-rays and ultra X-rays. In liquids and solids, ions may be produced not only by the external agencies of rays and radiation, but also by the inside forces of solution and dissociation.

The ionic theory of conduction has been sufficiently developed to explain all the observed facts in the case of gases. Modifications and extensions are necessary, however, before all the experimental data on solids and liquids can be interpreted.

I. INTRODUCTION

An insulating material is defined as a material which either does not conduct an electric current or conducts it to a very limited extent only. There is, however, no definite lower limit of conductivity which sharply separates insulators from conductors. In fact, some substance on being subjected to a change in temperature will gradually and continually change from a state which would certainly be called conducting to one which, with equal certainty, would be called insulating. Since there is no sharp dividing line between insulators and conductors, it seems desirable to summarize, briefly, our knowledge of the process of electric conduction before attempting to apply it to the particular case of insulators. This paper will attempt to correlate our knowledge of the conductivity of insulators with the theories which have been put forward to explain the passage of electricity through conductors, and to point out the facts which require an extension of these theories in order that conduction through insulators may be explained.

Electricity is always associated with matter. Whenever there is a current of electricity, there is always a current of material particles. Electric currents can be divided into three classes depending upon the mass of the material particle associated with each fundamental electric charge. These are

1. Electronic conduction
2. Ionic conduction
3. Colloidal conduction.

In electronic conduction the moving electricity is associated only with the electrons. These are extremely small amounts of matter to which the elementary negative charge of electricity is inseparably bound. The most familiar example of electronic conduction is conduction through metals, although many interesting and important applications are found in vacuum tubes. Since electrons are all of the same mass and since the same current and hence the same number of electrons must pass every part of the circuit, there is no measurable transfer of matter in electronic conduction.

In ionic conduction the moving electricity is associated with a subdivision of a molecule, or in some cases, with such a subdivision to which a few molecules are attached. The most familiar example is conduction through electrolytes, although conduction through gases, through fused salts, and even through a number of solid substances is of this type. In this case there is always a transfer of matter, which can be detected at any surface of discontinuity in the materials which form the electric circuit.

In colloidal conduction, the electric current is carried by small masses of matter which are suspended in some inert medium. The electrical precipitation of smoke and dust by an electric current is an application of colloidal conduction.

While the three kinds of conduction are quite distinct, yet any two or even all three may take place simultaneously. As an example, it sometimes happens that when a solvent ionizes, one ion will consist of a single electron, while the positive ion will consist of all the rest of the molecule. Then when a difference of potential is applied, there is both electronic and ionic conduction. A complicated example of this is the Nernst filament at high temperatures, where both ions and electrons participate in carrying the electric current.

While the fundamental distinction between electronic and ionic conduction is the amount of matter carried by the current, yet there are other properties which are

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[*] Physicist, Bureau of Standards.

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Approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

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more or less characteristic. The most important of these is the temperature coefficient of resistance. In metals, this is generally positive, while in electrolytes it is negative. Hence attempts are often made to determine whether a substance conducts electronically or ionically by measuring the temperature coefficient of resistance. At best, this is a very indirect method, and results so obtained require additional confirmation.

The phenomena connected with conduction through insulators depends on the state of the material; that is, whether it is a gas, a liquid, or a solid. The process is more complicated in liquids than in gases, and still more so in solids. Only in the case of gases has a fairly complete explanation of the experimental phenomena been deduced. Hence, in this paper there will be given a brief review of the facts and theories concerning conduction through gases; following this, a digest of some of the more important facts concerning conduction through liquids and solids including a correlation of these facts with the phenomena in gases.

II. Conduction Through Gases

The experimental facts concerning conduction through gases and the theoretical explanation of these facts have been treated in several different books.[1-2-3]

In all cases the authors correlate the experimental facts with the ionic theory. The following is a very brief résumé of this theory as applied to gases:

Consider two plane electrodes which are the opposite sides of a cubical box having a volume of one cu. cm. The other sides of the box are perfect insulators. This box is filled with air at atmospheric pressure. If a very small potential difference is applied to the plates for a short time, the current which flows is proportional to the applied potential difference. Hence, under these conditions, the air will obey Ohm's law. The resistance in the assumed case will be about 3×10^{15} ohms. If the potential difference between the plates is increased, the current will increase more slowly than the voltage. When the potential difference is in the neighborhood of 100 volts, the current has become about 10^{-18} amperes. This is called the saturation current, since it does not increase with a further increase in the potential difference. When, however, the potential difference is of the order of 30,000 volts, there is a very rapid change of current with voltage, leading up to the breakdown of the dielectric.

The facts as outlined above can readily be shown by the curve in Fig. 1. It is not feasible, however, to draw to a definite scale such a curve for air at atmospheric pressure, partly because there is considerable variation in the air and partly because if drawn to scale many of the interesting features would be entirely masked. For instance the section of the curve representing the saturation current would extend 300 times as far as the first part of the curve. When drawn to scale, the resistance at zero voltage can be determined from the tangent to the curve at the origin.

The explanation of the above phenomena is very satisfactorily given by supposing that there is some agency which is continually causing molecules of air to separate into positive and negative ions. These ions usually consist of an atom of either oxygen or nitrogen having a free positive or negative charge equal to the elemental electric charge. In atmospheric air under ordinary conditions the rate of ionization is about four or five ions per cu. cm. per sec. Since in a gas the molecules and ions are in violent agitation, the ions are likely to collide with each other. Whenever there is a collision between two ions having opposite charges, they may unite to form a molecule. When the number of molecules which are reunited each second is equal to the number ionized, a stationary condition has been reached. In normal air, there are always present about 700 or 800 ions per cu. cm. When a voltage is applied to the electrodes of the cubic box, the positive ions are attracted towards the negative electrode while the negative ions are attracted towards the positive electrode. On account of numerous collisions with molecules, however, the actual velocity of an ion towards the electrodes is only about one cm. per second if the potential difference of the electrodes of the box is one volt. Hence, if the voltage is very small, the conduction current will not, in a short time, appreciably alter the number or distribution of the electrons in the box. As the voltage is increased, however, the velocity of the ions also is increased. A point is soon reached where an ion is carried from the field almost as soon as formed. Now as the number of ions produced depends solely upon some external agency, an increase in voltage does not increase the current, since the current depends on the number of ions which reaches the electrodes. The saturation current is, then, the current which flows between two electrodes when the potential is sufficiently high so that practically every ion which is formed is carried to the electrodes without combining with another ion. Although the velocity

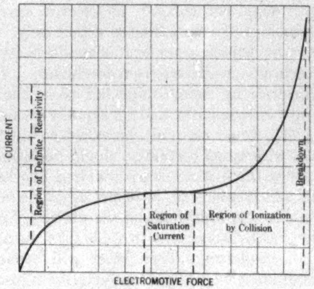

Fig. 1—The Current—Voltage Characteristics of a Gas

[1-2-3] For references see Bibliography.

of the ions increases with increasing voltage, this only decreases the time required for the ions to travel from the place where they are formed to the electrodes, without changing the number which arrives every second.

When the voltage gradient becomes sufficiently high, the velocity of the ions becomes so great that, on colliding with a molecule of gas, the molecule will be broken into ions. Those ions, of which the negative may be a free electron, will then start towards the electrodes and may, in turn, ionize other molecules. Hence, the one ion which was produced by an external agency may be the cause of several ions reaching the electrodes. Therefore, as soon as the voltage applied to the terminals of the cubical box is so high that ions may have a velocity sufficient to produce ionization by collision with a molecule, the current increases rapidly with increasing voltage. This is shown in the latter part of the curve of Fig. 1. It leads rapidly to breakdown, a phenomenon not discussed in this paper.

There are a number of external agencies which will cause a gas to ionize. The best known are X-rays, the α, β, and γ, radiations from radioactive substances, ultra-violet light, and the ever-present radiations of unknown origin which might be called ultra X-rays. The ultra-violet light differs from the other agents in that it generally produces ionization only as it is absorbed by one of the electrodes. In order to simplify the discussion, this type of ionization will not be considered. The activity of the other ionizing agents can be determined from the number of ions produced in a gas, per cu. cm. per sec. As already indicated, the ever-present ultra X-rays produce some four or five ions per cu. cm. per second throughout the habitable volume of the earth's atmosphere. By sinking a small vessel filled with air some 50 or 60 ft. below the surface of a lake, Millikan and his co-workers[6] were able to protect this air from the effect of the ultra X-rays. Some ions were produced in this air, however, which could easily be explained by supposing that the metal of which the containing vessel was constructed bore minute traces of some radioactive substance. While man has never had a mass of gas in which ions were not produced, yet it is possible that under normal condition all ionization of gases is caused by some outside agency.

By using X-rays or rays from radioactive materials, the rate of production of ions can be made as great as a thousand or more per cu. cm. per sec. These recombine rapidly, however, so that no appreciable part of the 10^{19} molecules in a cubic centimeter of air is ever ionized at one time.

In order to develop equations from which the current through a gas can be determined from the applied voltage, the following constants must be known:

Let

q = number of ions produced per cu. cm. per sec. by the ionizing agent.

e = elemental electric charge (1.591×10^{-19} coulombs.)

k_+ = mobility of the positive ions; *i. e.*, the velocity in cm. per sec. when the potential gradient is one volt per cm.

k_- = mobility of the negative ions.

α = recombination coefficient.

δ = ionizing potential; *i. e.*, the potential difference through which an electron must be allowed to accelerate in order to acquire enough energy to ionize a molecule.

If n is the number of positive ions per cu. cm. of gas at a time t, there will also be n negative ions. Now the rate of increase of ions must equal the number of ions produced minus the number of recombinations. But since the chance of a given positive ion colliding with a negative ion is in direct proportion to the number of negative ions, the number of recombinations is proportional to the square of the number of ions. Hence

$$\frac{d\,n}{d\,t} = q - \alpha\, n^2 \qquad (1)$$

This equation holds when there is no applied electromotive force. If this condition persists for some time, an equilibrium condition is reached, when $\frac{d\,n}{d\,t} = 0$. Then

$$n = \sqrt{\frac{q}{\alpha}} \qquad (2)$$

It is this condition which exists in air under normal conditions.

With a very low potential difference between the electrodes, the number of ions which reaches them depends on the total number of ions in the field and on the velocity of the ions. Hence the conductivity λ is given by the equation

$$\lambda = n\,e\,(k_+ + k_-) = e\,(k_+ + k_-)\sqrt{\frac{q}{\alpha}} \qquad (3)$$

The resistivity ρ is the reciprocal of λ. Hence the resistivity of a gas which has been exposed to an ionizing agent for a sufficiently long time for a stable condition to be reached depends on three constants of the gas; namely, the velocity of the positive ion, that of the negative ion, and the recombination coefficient.

The values of these constants for a few of the common gases are given in Table I. While the mobilities are quite different in different gases, yet there is relatively little difference between the positive and negative ions in the same gas. Also the recombination coefficient varies but little for the more permanent gases.

The saturation current S depends only upon the rate of ionization and the volume V of gas between the electrodes. It is represented by the equation

$$S = e\,q\,V \qquad (4)$$

TABLE I
IONIZATION CONSTANTS OF GASES

Gas	Mobility of positive ion k_+ $\frac{cm.}{sec.} \Big/ \frac{volt}{cm.}$	Mobility of negative ion k_- $\frac{cm.}{sec.} \Big/ \frac{volt}{cm.}$	Recombination coefficient α
Hydrogen	6.	7.5	1.4×10^{-6}
Helium	5.	6.	....
Oxygen	1.3	1.8	1.6×10^{-6}
Nitrogen	1.3	1.8	1.6×10^{-6}
Carbon dioxide	0.8	0.9	1.6×10^{-6}
Carbon monoxide	1.1	1.1	0.9×10^{-6}

This does not contain any constant of the gas, but depends entirely upon the activity of the ionizing agent and the volume of the gas from which the ions can be captured by the applied electromotive force.

The ions, in their passage towards an electrode, collide at frequent intervals with a molecule. If, after a collision, the velocity of the ion should be zero, then at the next collision the energy which this ion has acquired is $e \overline{V} l$, where $\overline{V}$ is the potential gradient and l the distance it has traveled. To understand what effect this collision will have on the molecule struck, it is desirable to consider the analogous case of an electron colliding with an atom.

Experiments on gases at low pressures have shown that when an electron collides with an atom, the atom will be ionized only if the kinetic energy of the electron is greater than a certain critical value. The electron will normally obtain its kinetic energy by accelerating in an electrostatic field. Starting from rest, the kinetic energy of the electron at any instant is $e V$, where e is the charge upon the electron and V is the potential difference between the starting point and the point under consideration. The value of V, which is just sufficient to give to the electron enough energy to ionize an atom, is called the ionizing potential. Values of the ionizing potential are known for the atoms of most gases.

Likewise a molecule will be ionized by collision with an ion or electron, only if the velocity of the latter is sufficiently high. Since the velocity at impact depends not only upon the potential gradient but also upon the length of path between collisions, the voltage at which ionization by collision will begin increases almost in direct ratio with the pressure.

Our knowledge of conduction through gases at normal pressure, when the applied voltage approaches that which produces breakdown, is very incomplete. There is sufficient evidence to show that the rapid increase of current is the result of ionization by collision. It is not as yet possible, however, to formulate the laws under which it acts.

III. CONDUCTION THROUGH LIQUID DIELECTRICS

The current is carried through most liquid dielectrics by ions. Hence most investigators have attempted to explain conduction through such substances by applying to them the same laws as hold for gases. The discussion of conduction in liquid dielectrics, therefore, centers around the production of ions, their mobility, and recombination.

Ions in a liquid dielectric may result from ionization of the dielectric itself, as is generally the case with a gas, or they may come from the ionization of substances dissolved in the liquid. The same external agencies that cause ionization of a gas will produce ionization in a liquid. In addition to these external agencies, the liquid may ionize spontaneously and ionization is often produced by some physical or chemical action between the liquid and a substance dissolved in it. The first is called ionization by dissociation; the second ionization by solution. As very minute amounts of a dissolved substance may produce a relatively large number of ions, impurities which can be removed only with difficulty often produce more ions in a liquid than is produced by a relatively intense external ionizing agent.

There is some evidence that all liquids are ionized by ever present ultra X-rays. In hexane the number of ions produced per cu. cm. per sec. is nearly fifty times as great as in air[25]. In most liquids the effect is masked by the effects of other ionizing agents.

X-rays and the rays from radioactive substances produce ionization in all liquids that have been examined. The effectiveness of a given ray is different in different liquids. For example, Jaffé[34] found that for hexane the ratio of ionization compared to air is about 1/1000 for α-rays but about 1/10 for β-rays. Also Greinacher[27] found that the number of ions produced by a given beam of α-rays was twice as large in paraffin oil as in petroleum ether.

The rate of recombination of ions in liquids is much less rapid than in gases. The few values of the recombination coefficient that have been determined[26] lie between 10^{-10} and 10^{-13}, or of the order of one millionth of that of gases.

The mobility of ions in a liquid is much less than in a gas. Values for a few liquids are given in Table II.

These values are of the same order of magnitude as the mobilities of ions in electrolytic solutions. There is a large increase in mobility with temperature, as is also the case with electrolytes.

No values of the ionization potential of a liquid molecule are available.

TABLE II
IONIZATION CONSTANTS OF LIQUIDS

Liquid	Average mobility of positive and negative ions—k $\frac{cm.}{sec.} \Big/ \frac{volt}{cm.}$	Recombination coefficient	Authority
Petroleum ether	500×10^{-6}	35×10^{-12}	26
Vaseline oil	5×10^{-6}	4×10^{-12}	26
Toluol	0.2×10^{-6}	....	17
Carbon tetrachloride	240×10^{-6}	....	33
Paraffin (60 deg.)	1600×10^{-6}	....	49
Electrolytic solutions in water	$\left.\begin{matrix}3000\\ \text{to } 300\end{matrix}\right\} \times 10^{-6}$	....	17

According to most investigators, the current which flows between two electrodes immersed in a liquid and maintained at a constant potential difference is determined by the differential equation

$$\frac{d\,n}{d\,t} = q - \alpha\, n^2 - s \tag{5}$$

where

n = the total number of ions at time t

q = the rate of production of ions

α = the coefficient of recombination

s = the rate at which ions reach the electrode—proportional to the current

If the ions are uniformly distributed through the liquid

$$s = \beta\, n \tag{6}$$

where β is a constant which among other things involves the mobility of the ions. Substituting this in (5) and separating the variables

$$\frac{d\,n}{q - \beta\, n - \alpha\, n^2} = d\,t \tag{7}$$

Integrating

$$\operatorname{logn} \frac{2\,\alpha\, n + \beta + \sqrt{4\,\alpha\, q + \beta^2}}{2\,\alpha\, n + \beta - \sqrt{4\,\alpha\, q + \beta^2}} = t\,\sqrt{4\,\alpha\, q + \beta^2 + C} \tag{8}$$

This shows that n is an exponential function of t. It follows that s, the current between the electrodes, decreases as the time during which the electromotive force is applied increases. This same solution holds in the case of gases, but in them, the mobility is so great that equilibrium is reached in less time than that required for the measuring instrument to reach a steady condition. With liquids, however, the mobility is much less and the decrease of the current with time can be readily observed. Most observers have made readings one minute after the application of the electromotive force. During this time, the current will in most cases become stationary.

The conductivity λ of a liquid is given in terms of ionic constants by an equation similar to that for a gas, namely:

$$\lambda = 2\, n\, e\, k \tag{9}$$

where

λ = the conductivity

e = the charge on an ion

n = the number of ions per cubic centimeter in the stationary state

k = the average mobility of the positive and negative ions.

As the mobility of ions in liquids is from a thousand to a million times less than the mobility in gases, the conductivity for a given concentration of ions is proportionally less or the resistivity proportionally greater. The number of ions, however, is generally so much greater in liquids than in gases that the resistivity of many liquids is less than that of gases under similar conditions.

As the potential gradient in a liquid is increased, the current approaches saturation in the same way as in a gas. Difficulties, however, are frequently encountered in the experimental determination of the saturation current of a liquid. As an example, when the number of ions is small, it is probable that most of them will be caused by the solution of traces of impurities. Before the current becomes stationary, many of the ions of the impurities will go to the electrodes, thus removing from the liquid the cause of the ionization. Hence, before the saturation current can be reached, the fundamental character of the liquid has changed.

As a second example, when the liquid contains a large number of ions which have been produced by any cause whatever, an ion concentration occurs near the electrodes before the current becomes stationary. The potential gradient in the region of this concentration is reduced so that the gradient near the electrodes is less than at a point midway between them. On account of this reduced gradient, the velocity of the ions towards the electrodes is reduced and the chance for recombination increased. These two examples show that the experimental difficulties of determining the saturation current in a liquid are much greater than in the case of a gas.

Another difficulty in connection with the determination of the saturation current in a liquid is that ionization by collision may start before the potential gradient is sufficiently high to produce saturation. When this is the case, no true saturation current occurs. This phase of conduction in liquids, however, has received very little systematic investigation.

Much larger potential gradients can be applied to liquids without appreciably affecting the conductivity than can be applied to gases. Thus, in most liquids, the conductivity obeys Ohm's law if the potential gradient is less than three or four hundred volts per cm., whereas in a gas it must be less than one volt per centimeter. Where higher potentials are applied the current does not increase as rapidly as the potential. For example Jaffe[23] found for several liquids that if the potential gradient were between 500 and 3000 volts per cm.

$$I = a + c\,E \tag{10}$$

where a and c are experimental constants.

The resistivity of most liquids decreases with increasing temperature. This is largely caused[16] by a change in mobility, as the number of ions is nearly independent of temperature. Attempts to correlate[11] the resistivity of a liquid with its viscosity, however, have been unsuccessful.

The above explanation of the passage of electricity through liquid dielectrics is based on the assumption that the ionic laws of gases can be applied to liquids. Some investigators are of the opinion that these laws will require some modification before a complete explanation can be given. Von Schweidler[17] has proposed two modifications; the first suggests that equation (**5**), which gives the rate of increase of ions, should be

$$\frac{d\,n}{d\,t} = f\,(n) - s \qquad \textbf{(11)}$$

where $f\,(n)$ is an unknown function. The reason for this suggestion is that the solution of equation (**5**) shows the current to be proportional to the square of the conductivity, while experimentally the current is found to be proportional to the conductivity with a fractional exponent. The second suggestion is that, in many liquids, several different kinds of ions exist, each having a different mobility. When an electromotive force is applied, the kinds of ions having the higher mobilities reach the electrodes more rapidly than those with the low mobilities. As a result, the average mobility of the ions which are left decreases as the time of application of the electromotive force increases. This agrees with the results of tests on several liquids.

IV. Conduction Through Solid Dielectrics

Conduction in solid dielectrics is difficult to distinguish from the phenomena of absorption and residual charge. In this respect, solids are entirely different from liquids and gases in which these phenomena are seldom, if ever, present. Also, electrons play a more important part in conduction through solids than liquids or gases. Ionic conduction, however, is much more common than electronic. Hence in the discussion it will be considered that conduction in solids is caused by ions, the electron being considered the smallest possible ion.

Although the external agencies which cause ionization in solids are the same as in liquids or gases,—namely, electromagnetic radiations and rays from radioactive substances,—the effectiveness of the different agencies is quite different. The α and β rays from radioactive substances ionize some, and perhaps all, solid dielectrics. Also it seems probable that for any given wave length of electromagnetic radiation some solid can be found which will be ionized by it. Moreover there is a number of solids which are ionized by a very wide range of wavelengths. Some illustrations of these facts are cited in the following.

There are few data concerning the ionization of solids by the ever-present ultra X-rays. It may, however, be because of these rays that no substance is known which has infinite resistivity. Sulphur, paraffin, and quartz are ionized by both λ-rays and X-rays. The ionization of selenium by X-rays is so definite that some roentgenologists have proposed using this material to measure dosage. Quartz is ionized by ultra-violet radiation. The visible part of the spectrum produces ionization in a number of substances, the most familiar being sulphur and selenium. In fact, one form of selenium is ionized not only by all of the visible spectrum, but also by the infra red rays.

In addition to the external agencies which produce ionization, substances dissolved in a dielectric may produce it. Familiar examples are water dissolved in rubber and alcohol dissolved in shellac. The influence of minute quantities of a dissolved substance is often very marked. Hence, there is often great difficulty in ascertaining the true resistivity of a substance since it is exceedingly difficult to remove the last traces of an impurity. Some dielectrics ionize spontaneously. There is some interaction between molecules so that ions are always present. With such dielectrics an increase in temperature frequently increases the number of ions in a very marked degree. Glass is an example of such a dielectric.

In general, the mobility of ions in a solid are less even than in a liquid. The values given in Table III however, are not entirely characteristic. In case of light-sensitive selenium, the negative ions are single electrons, the mobility of which is probably quite high, while that of the positive ion is negligibly small in comparison. In this case the conduction is electronic.

TABLE III
IONIC CONSTANTS OF SOLIDS

Substance	Average mobility of ions $\frac{\text{cm.}}{\text{sec.}} \Big/ \frac{\text{volts}}{\text{cm.}}$	Recombination coefficient	Authority
Paraffin............	0.14×10^{-6}	....	49

On the other hand the mobility of the positive ions is sometimes high relative to that of the negative. This is the case with some kinds of glass. In such glasses the positive ion can be replaced by other positive ions which are carried into the glass by the current. Many similar cases are known. These examples show that in solids, the difference in mobility of the positive and negative ions may be so great that the conductivity is entirely dependent on the ions having the highest mobility, and not, as is usually the case, in liquids and gases, on the average mobility of the two kinds.

The mobility of the ions in a substance frequently increases rapidly with the temperature. In crystalline quartz[67] the mobility at 100 deg. cent. is about 200 times that at 0 deg., while in calcite it is more than 10,000 times as much at the higher temperature. In rock-salt, on the other hand, there is little if any change in mobility with temperature.

There are few quantitative data on the coefficient of recombination of ions in solids. There is, however, a considerable amount of qualitative information. For instance, it is known that in light-sensitive selenium, practically all the ions are recombined in two or three seconds after removal to the dark. The recombination in crystalline quartz is much slower. At room temperature several months or even years are required before a piece of ionized quartz will reach an equilibrium condition. These examples doubtless represent extremes in regard to recombination.

There are practically no data concerning the ionization potential of the molecules of a solid. There are, however, sufficient data to show that ionization by collision occurs in solids although it is often difficult to separate it from other phenomena which are present when high voltages are applied to a solid dielectric.

It is probable that the law of ionic equilibrium given in equations (**1**) and (**5**) will require modification even in the case of homogeneous solids. At present, however, it furnishes the only accepted basis from which to discuss the observational data. On this basis materials should have a definite resistivity when the applied voltage is low and the time of application long, and should reach, or at least approach, a saturation curve when the voltage is sufficiently increased. Both conditions have been observed.

To conform with this law, conduction current in a solid must decrease with time according to an exponential law, as has been shown to be the case for a liquid. The measurement of the conduction current in solids, however, is complicated by the fact that the absorption current also decreases with the time in much the same way. There is no known method of distinguishing between these currents. In some materials, however, the conduction current is so great that the absorption current can be neglected. Moreover, in all materials, the absorption current tends towards zero, while the conduction current tends towards a fixed value, provided the ionization remains constant. The ionization in solids is generally due to conditions within the material. Often these conditions are disturbed by the passage of the current, so that the current at infinite time does not represent the true conduction current. It follows that no entirely satisfactory procedure can be established. The commercial practise of reading the current at the end of one minute after the potential has been applied appears to be as good a one as any that can be suggested.

The saturation current in solids has occasionally been observed. An interesting case is reported by Jaffe[67], who found that the current in crystalline quartz was independent of the applied potential if the potential gradient was between 10,000 and 50,000 volts per centimeter, and that a steady condition was reached in three seconds after the application of the voltage.

The change of resistivity with temperature depends both on the change in the number of ions and on the change in their mobility. In some solids the temperature coefficient is small and positive, but in most solids it is large and negative. Rasch and Hinrichsen[44] found that, in glass, the change in resistance R could be represented by the formula

$$\log R = \frac{A}{T} + B \qquad (\mathbf{12})$$

where T is the absolute temperature and A and B are constants. This same law has been found to fit a number of substances. As the dissociation of gases into ions (a subject not treated in this paper) obeys the same law, the conclusion is often drawn that the cause of the negative temperature coefficient in such solids is the dissociation of molecules into ions caused by the increase in temperature. This law, however, holds with some solids in which the change in resistance with temperature is known to be caused—at least in part—by the increase in mobility of the ions. Hence the dissociation coefficient can be determined from the temperature coefficient only when there is no change of mobility with temperature.

The above deductions concerning homogeneous dielectrics apply to the individual components of a heterogeneous dielectric. The most interesting form of such dielectrics is one composed of layers perpendicular to the direction of current flow. A consideration of this type leads immediately to the question of absorption, a subject which has been treated in another paper* of this series.

V. CONDUCTION OVER THE SURFACE OF SOLID INSULATORS

Gases or vapors are often condensed to a liquid form on the surface of a solid with which they are in contact. If the angle of contact between the solid and liquid is greater than 90 deg., the liquid collects in droplets; but if the angle is less than 90 deg., the liquid spreads, in a film, over the surface of the solid. In one case, the presence of the liquid has little effect on the insulation of electrodes fastened to the solid, but in the other, the current which flows from one electrode to another *through the solid* is often much less than the current which flows through the condensed liquid.

The condensation of water vapor from the air on solid surfaces is a familiar example of surface condensation. With waxes, the condensed water forms into droplets, but with most materials, it spreads in a film over the surface. If the film is formed on the clean surface of an insoluble material, like quartz or amber, the water is very pure. Since pure water is a poor conductor, the film may not carry an appreciable current. The slightest trace of a soluble salt however, will so greatly increase the conductivity of a film that the current through it may be much larger than that through the solid itself. Many solids, such as glass and porcelain, are sufficiently soluble so that films deposited on them are always conducting. The presence of delequiscent salts very greatly increases the amount of condensed water. In all cases, the amount of condensed water increases as a rather large *power* of the relative humidity.

Surface films play an important part in conductivity through porous materials, such as unglazed porcelain, marble, and slate. The pores extend deep into the material. If a specimen is exposed to a humid atmosphere, the pores become lined with a film of water, which

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*J. B. Whitehead: *Dielectric Absorption and Theories of Dielectric Behavior*, A. I. E. E. TRANS., Vol. 45, 1926, Mar., p. 102.

may have a conductivity many thousands of times greater than that of the material itself. The result is a decrease in the volume resistivity of the material. Evershed considers that conduction phenomena in fibrous materials is associated with surface films in the pores of the material. However, he makes the further assumption that the thickness of the film decreases as the applied voltage is increased. This explains the increase of resistance with increasing voltage which is shown by such materials.

VI. CONCLUSION

This paper includes only a small part of the facts which are known concerning conduction through insulators. There is every reason to believe that all the known facts can be explained by the ionic theory, the main tenets of which are herein set forth. The details of this theory are sufficiently developed to explain practically all the experimental facts in connection with the passage of electricity through gases. The same cannot be said in regard to liquids and solids. Here the phenomena are more complex and the experimental data more meager. A complete explanation must await further investigation.

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Discussion

F. M. Clark: With regard to Dr. Curtis' paper, I do want to emphasize the type of conduction which he calls colloidal conduction. I think that this is a type which we will have to recognize and consider in more detail than we have. When you consider that almost all of our insulations are colloidal in nature if not true colloids, you can see the significance of that type of conduction. I recognize that colloidal conduction is similar in many respects to electrolytic conduction. However, it owes its origin to larger charged masses and may occur in mediums where electrolytic conduction would be least expected.

Dr. Curtis discusses the conduction over the surface of solid insulators. The effect of water-vapor condensation is cited and its importance discussed. Although the author illustrates the effect on inorganic materials such as quartz and porcelain, it should not be overlooked that conductivity in fibrous materials appears to be closely related to the cases given. Our researches at Pittsfield have not been entirely completed, but we have already obtained considerable evidence that the quantity of water contained in an oil-treated paper, while important, is not the sole factor determining the a-c. and d-c. conductivity. It appears that the effect produced depends very largely on whether the water vapor present is absorbed on the surface of the paper fibers or whether it be held within the fiber wall.

Electric Strength of Solid and Liquid Dielectrics*

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Synopsis.—*This paper was prepared by a subcommittee of the Committee on Electrical Insulation of the Division of Engineering and Industrial Research of the National Research Council. The general purpose of the committee is to foster research on dielectrics and its initial activities have been confined to the preparation of a series of summaries of the published literature on this subject.*

The subcommittee which presents this paper is the second one to report, having been preceded by a paper on dielectric absorption and theories of dielectric behavior, by Chairman J. B. Whitehead, which was published by the A. I. E. E. in 1926.

The second paper under the auspices of the committee was inspired by the first one, and was an original exposition of Clerk Maxwell's theory of the double layer dielectric, by Professor Murnaghan, published by the A. I. E. E. in 1927.

The present paper, like the first, is a summary of existing literature which, it is hoped, will afford a starting point for original research in many directions. It is hoped that the discussion by the Institute will bring out obscure phenomena and new interpretations of the data reviewed. The report starts with a consideration of the general subject of instability in electrical circuits, and an explanation of instability in the case of dielectrics, in terms of a stress and strain characteristic. This is followed by a discussion of distribution of stress and strain in non-uniform fields, and their relation to breakdown. The reversible and non-reversible phenomena of dielectric failure are considered, the former in relation to the electron theory and the latter in relation to the pyroelectric theory, and the bearing of both upon the time-voltage relation is indicated.

The latter part of the report is devoted to the relation of breakdown voltage to various factors, such as insulation thickness, insulation area, the electrode form, heterogeneity, temperature, rate of voltage, variation, pressure, etc. Final conclusions are given which summarize in a few words the present state of the art, as it may be judged from published data.

GENERAL

FARADAY seems to have been the first to use the term "dielectric" to distinguish certain classes of materials by their electrical properties and he defined dielectrics as all bodies whose insulating value is such that when they are placed between two conductors at different potentials the electromotive force acting on them does not immediately distribute their electricity so as to reduce the potential to a constant value. It was not until considerably later, however, that we find recognition of the fact that if the potential difference across a dielectric is sufficiently increased, failure will occur; or, to state the matter in another way, the current will increase with a decrease of voltage and conditions will be unstable. The stress at which this unstable condition begins is called the dielectric strength of the material, but it depends on many factors, as will be discussed later in this paper, and is by no means an exact physical constant.

At the present time, it is proposed to discuss the literature in regard to the dielectric strength of solids and liquids. It should be recognized that liquids occupy a ground between solids and gases and may be treated, with equal reason, in connection with the behavior of gases or with the behavior of solids. In some respects they will behave more like one than the other but in general there is but little to choose between one grouping or the other. The methods for examining solids and liquids are in general different and this will introduce several questions which will need answer before we can arrive at definite conclusions.

An examination of the literature discloses the fact that a large number of solid dielectrics has been investigated by several workers but the range of liquids examined is far more restricted. This is not surprising, however, when it is recognized that the majority of our commercial insulating materials are solids and comprise such varied materials as glass, fiber, wood, moulded compounds, mica, sulphur, paraffin, rubber, paper, and porcelain, while in the list of liquids we find only a few oils, chiefly distilled from petroleum crudes, and usually classified under the general type of transformer oil. Many liquids, such as benzine, kerosene, toluol, xylol, and carbon tetrachloride, are good dielectrics but seem to have found little or no commercial use and hence not to have gained the interest of various investigators.

INSTABILITY

It is helpful to consider dielectric failure as the phenomenon of instability which exists when the voltage across the dielectric has been increased to a certain critical value beyond which the current increases more or less in unpredictable ways while the voltage tends to fall. Well-known examples of such cases are associated with arcs without ballast resistors, overloaded reactors, transformers, and series wound generators.

There is always difficulty in assigning credit for the first presentation of a line of thought, for when a number of men are working simultaneously on the same problem they are inevitably led into similar trains of thought. Perhaps the first clear statement connecting the failure of dielectrics with the general phenomenon of instability is that of Steinmetz[1] in 1923 when he said that "the view is gaining ground that the mechanism of dielectric failure is a phenomenon of the instability of the so-called constants of the material."

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*Report to the Division of Electrical Engineering of the National Research Council.

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[1] For all numbered references, see Bibliography.

STRESS-STRAIN CHARACTERISTIC

It was, however, a research by K. W. Wagner[2], initiated in 1914, interrupted by the war, and completed in 1919, which gave the first quantitative data on which to base a satisfactory theory. This physicist was the first to ascertain the volt-ampere characteristic of a solid dielectric, and he showed that the characteristic curve consists of a straight line corresponding to Ohm's Law, a curve which becomes vertical, *i. e.*, parallel to the current axis, and finally slopes backward, corresponding to a rising current with falling voltage, as shown in Fig. 1.

It is not difficult to obtain the first two parts experimentally, but the third part corresponds with a condition of unstable equilibrium and is normally passed at such an immense speed that it cannot be observed unless special precautions are taken.

Wagner succeeded in plotting the entire curve by using an electrode of such high resistance that the circuit, as a whole, had stable characteristics, *i. e.*, current rising with voltage, and thus stabilized the dielectric in the same way that a ballast resistance is used to stabilize an arc. Current and voltage readings were first taken with the electrodes in contact with one another, and then with the dielectric between. The difference in voltage for the same current, in the two cases, was the voltage across the dielectric corresponding to this current. The general form of these characteristics was verified in 1921 by J. C. Schrader[3] who used a hot cathode rectifier in series with the dielectric, to restrain the current.

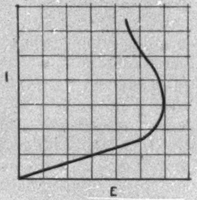

FIG. 1

In the same year, H. H. Poole[4], using the same method as Schrader, found that the conductivity of certain dielectrics rises with the potential gradient, the logarithm of the conductivity being nearly a linear function of the gradient.

In 1925, H. Gabler[5], also using a thermionic valve for ballast, obtained experimental curves of the same general form.

In all these tests, direct current was used. Corresponding data for alternating current have not yet been obtained, indicating a fruitful field for research.

No work appears to have been done on liquids in an effort to determine the volt-ampere characteristic although Gunther-Schulze[6] made determinations of dielectric conductivity over a considerable range of stresses and found a tendency for the current to decrease with increase of voltage. This may be explained by consideration of the fact that the voltage was gradually increased and this gave additional time for impurities, which apparently constitute a considerable number of carriers, to be swept out of the field.

The shape of the curve for solids has been explained by Wagner on the basis of the negative temperature coefficient of resistivity characteristic of all dielectrics and by Peaslee as due to ionic migration. These theories are explained at greater length, below.

One of the consequences of the form of volt-ampere characteristic is that an unequally stressed dielectric, by virtue of the different current densities in its parts, may have in one place a rising current density and in another a falling current density, with increase of voltage. This was expressed by W. D. A. Peaslee[7], in 1922, by saying that a dielectric, if unequally stressed, may have a positive value of $\frac{dI}{dE}$ in one part, and a negative value in another, but failure will not occur until the total $\frac{dI}{dE}$ becomes infinite. An important part of Peaslee's contribution is the interpretation of Wagner's volt-ampere curve in terms of voltage and current density.

DIELECTRIC OVERSTRESSED WITHOUT INJURY

That part of the dielectric which has $\frac{dI}{dE}$ negative is overstressed, that is, it would fail but for the stabilizing action of the remainder of the insulation, which acts as a ballast resistor.

Wagner, working simultaneously along similar lines, described an experiment on a dielectric which had been unequally stressed for a short time so that one part was stressed above the point where $\frac{dI}{dE} = \infty$, and he showed that this over-stressed part, when subsequently tested alone, had lost none of its dielectric strength.

STRESSES IN A DIELECTRIC WITH NON-UNIFORM FIELD

The next step toward putting the behavior of dielectrics upon a quantitative basis was taken by P. L. Hoover[8] who showed that the characteristic curve could be expressed in terms of potential gradient and current density and that therefore if the current density be plotted for the various parts of an unequally stressed dielectric, the corresponding potential gradients and values of $\frac{dI}{dE}$ can be plotted from the characteristic curve and the behavior of the dielectric deduced therefrom. In other words, if the current densities at various points be computed from the resistivity and geometrical configuration, the corresponding stresses may be derived from the characteristic curve.

Thus, in a dielectric between co-axial cylinders, *i. e.*, a single-conductor cable, if the same current be assumed to flow in all parts of the circuit, the current density must decrease radially, and may therefore be represented by an hyperbola, such as *A* in Fig.2. Referring now to the volt-ampere characteristic, Fig. 1, as each value of current density corresponds with a definite potential gradient, a curve representing the potential gradient at all radii may be plotted, as shown by the curve *B* in Fig. 2.

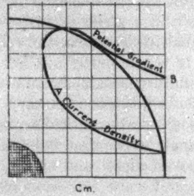

Fig. 2

If the current density is increased, the maximum of curve *B* moves outward. In doing so, the area to the left of the maximum increases, while that to the right decreases. The curve is not symmetrical; hence there is a certain current at which the area enclosed by the curve is a maximum. But this area is the total voltage between cylinders. Hence there is a value of the total voltage *E* such that $\frac{dI}{dE} = \infty$. This will be the breakdown voltage.

Applied to solids, this theory gives results which accord better with the facts than the well-known theory published almost simultaneously in 1901 by M. O'Gorman[9] and E. Jona[10], according to which the unequally stressed dielectric between co-axial cylinders will begin to fail at the inner surface, the failure spreading outward from that point until rupture occurs. Important modifications of this theory were published by W. I. Middleton, C. L. Dawes, and C. L. Davis[11] in 1922, and R. J. Wiseman[12] in 1923.

Considering this theory in its original form, if certain assumptions are made, it may be shown that

$$g = \frac{E}{x \log_e \frac{R}{r}}$$

where

g is the electric stress at any point distant x from the axis of the cylinders,

R is the inner radius of the outer cylinder,

r is the outer radius of the inner cylinder,

E is the potential difference between cylinders.

By this formula the maximum stress is always at the inner conductor surface, *i. e.*, with $x = r$, and the minimum stress at the outer conductor surface, *i. e.*, with $x = R$.

The assumptions which are made in deriving the above formula are that the dielectric has uniform resistivity and specific capacity in all directions, and that the resistivity and specific capacity do not alter with stress.

The first two assumptions are often unjustifiable, and the third is certainly wrong for most materials at stresses approaching failure. Since it is at these high stresses that the formula finds its principal application, it is evidently basically unsound, except for a few materials which may have characteristic curves without any maximum point.

So flagrant were the observed departures from the consequences of the logarithmic formula that in 1920, F. Fernie[13] suggested that failure of insulation in a single-conductor cable occurs when a certain value of the minimum stress is attained. This theory was ably criticized in 1922 by D. M. Simons[14] who showed that experimental data are as consistent with failure taken to occur at constant average stress as at constant minimum stress. It is now fairly evident that the maximum stress determines failure, but that this stress cannot be calculated by the ordinary formulas which neglect the peculiar shape of the volt-ampere characteristic at stresses approaching failure.

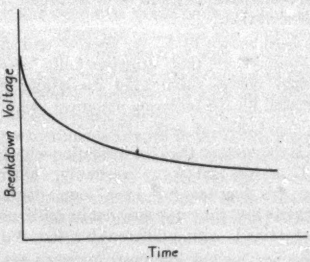

Fig. 3

Hendricks[15] mentions the results of tests on transformer oil using concentric cylindrical electrodes of different sizes, and finds that the breakdown strength as calculated on the maximum stress theory is not constant. On the other hand, Peek[16], as a result of similar tests, comes to the conclusion that there is a definite maximum stress, 36 kv. per cm., which is the value at which an increase in conductivity results from ionization by collision.

Another factor which may be of considerable importance in some cases and that seems to have been generally overlooked in the computation of stresses, although it is made the basis for many explanations of the phenomenon of absorption, is the combined influence of resistivity and specific inductive capacity on distribution of stress. That this may be an important influence is indicated by the different values of dielectric strength obtained with direct potentials and alternating potentials of different wave form and frequency, as discussed later in this report.

TIME—VOLTAGE RELATION

Thus far we have dealt exclusively with so called momentary stresses, *i. e.*, those which are not maintained sufficiently long to produce chemical deterioration of the insulation.

The relation between the breakdown voltage of a dielectric and the time of application of the voltage is of the character shown in Fig. 3, *i. e.*, the breakdown voltage for short periods is high and falls off rapidly with increasing time, whereas that for long periods is comparatively low and changes little with the time. This was clearly shown in 1907 by H. W. Tobey[17].

The general form of the curve suggests two influences at work, one corresponding to instantaneous action and the other to prolonged action, the intervening periods corresponding to a combination of the two. This idea is discussed at greater length under "Irreversible Deterioration."

It has been shown by F. W. Peek[16-18] and also by V. M. Montsinger[19] that experimentally determined time-voltage curves using solid insulations may be approximated by the following empirical formula:

$$E = A + B \,.\, T^{-4}$$

where E is the voltage that the insulation will stand for a time T, and A and B are constants. No rational basis for this formula has been found, and recent work suggests that the curve may not be continuous, but may have a break, with a different law for each side of the break.

Data presented by Farmer[20] as a result of tests on oil impregnated paper insulated cable may be expressed by the equation derived by Montsinger, except that the exponent of the time term is 7 instead of 4. There seem to be clear limitations as to the range of stress over which this expression may be applied, however, and it evidently has but limited use; certainly there is no ground for assuming that it may be applied to materials other than the specific ones investigated.

Another aspect of the time-voltage relation was indicated by C. P. Steinmetz[1] in 1923, namely, the differing time-lag of various dielectrics. If two insulating materials are in parallel, the one which will fail under stress will depend upon the time of application of the voltage and upon the time-lag of electric strength of the two materials. Thus, under lightning conditions, transformer bushings may flash over without injury to the oil insulation, whereas on test, the oil insulation may puncture far below the voltage at which the bushings flash over.

Air has no appreciable time-lag, but a sphere-gap with high resistance in series has an appreciable time-lag, due to the time required to charge the capacity of the spheres over the resistance.

True liquids do not seem to be influenced by the time of application of the voltage although there is a time-lag for rapidly applied voltages and Peek[21] has found this lag to obey almost the same laws as he established for gases. Dieterle[22], in a series of tests with semi-fluid materials intended to determine the relations between the magnitude of continuously applied stress and the time to produce failure, obtained results similar to those already noted for solids although they are too few to permit establishing a mathematical expression.

Yet another angle to the time-voltage relation has to do with the effect of direct and alternating potentials and the influence of wave form and frequency in the latter case. These will be discussed in more detail later on in this report, but it should be pointed out that at the present time these factors have to do with the short-time part of the curve.

PYROELECTRIC THEORY

Miles Walker[23], in 1912, was probably the first to call serious attention to the dielectric loss-temperature relation as a factor in dielectric failure. He was followed in 1917 by A. F. Bangs and H. C. Louis, and W. S. Clark and G. B. Shanklin[24], and in 1922 by D. W. Roper[25], all of whom dealt with average losses in relation to mass heating of cable dielectrics. At about this time C. P. Steinmetz and J. R. Hayden contributed ideas as to the general application of this theory and coined the name "pyroelectric theory."

K. W. Wagner[2] in 1922 went a step further by taking into account the well-known unevenness of dielectrics and considering the effects of a filament of higher dielectric loss or, as he expressed it, lower resistivity than the surrounding mass. He laid particular stress on the well-known fact that dielectrics, unlike metallic conductors, possess a negative temperature coefficient of resistivity; *i. e.*, their resistivity decreases with rising temperature. Hence, if a spot or filament of comparatively low resistivity exists in a dielectric, the extra $I^2 R$ loss which occurs in it will tend to raise its temperature, which in turn by decreasing the resistivity will increase the current and temperature, and so on accumulatively until failure occurs due to burning.

This theory was ingeniously developed by Wagner as a general theory of dielectric failure by assigning an empirical equation to the temperature-resistivity relation, and assuming arbitrarily a certain diameter for the hot filament. The quantitative results obtained were later found to depend very largely upon the value assumed for this diameter. Morover, work by L. Dreyfus[26], H. Rochow[27], W. Rogowski[28], Von Karman[29], H. Gabler[5], and others, has shown that the theory, as developed by Wagner, is untenable, both from a theoretical standpoint and because it yields numerical data which are widely at variance with facts.

Wagner's theory considers only the effects of filaments of low resistivity. C. P. Steinmetz, in 1923, called attention to the effects of particles of high specific capacity, as follows:

> "A particle of higher specific capacity than the surrounding material will concentrate the lines of electric force toward itself, creating points or edges of excessive flux density at the 'poles' of the particle. The insulation would char at these points or edges, and the shape of the product of chemical decomposition would tend toward the form of a conducting needle, with excessive voltage gradients at its ends, gradually piercing the dielectric until final puncture occurs between the terminals. Thus in a laminated insulation, consisting of very many layers, a foreign particle in one of the layers, though originally forming only an insignificant part of the *total* thickness of the dielectric, may gradually but cumulatively, in the course of time, pierce and destroy the insulation by its electrostatic cutting edges, the average voltage gradients within the dielectric being still very low compared with the tested 'dielectric strength' of the material."

Thus unevenness of specific capacity, as well as of resistivity, is a factor in pyroelectric failure. Perhaps there are other factors which explain the phenomenon noted in 1910 by H. Osborne[30], namely, that when a solid dielectric is over-stressed, it is not disrupted uniformly, but is affected as if it had been pricked by a number of needle points.

The failure of Wagner's theory in a quantitative sense should not be interpreted as discrediting the general pyroelectric theory, as cumulative heating is undoubtedly a factor in many cases of dielectric failure. It is not that the theory must be discarded but that it must be amplified and made part of a more general and complete theory.

There seems to have been no attempt to apply the pyroelectric theory to liquids and indeed it is hard to see how this could be done since the existence of convection currents would tend to largely mask any of the detail phenomena assumed for solids.

IRREVERSIBLE DETERIORATION

All solid dielectrics on which published data are available show some kind of permanent deterioration when exposed to stresses over a certain critical value. Thus, it was found by F. M. Clark[31] in 1925, that prolonged stresses permanently injure oil-impregnated paper insulation and that the injury is cumulative, even if the stresses be applied intermittently. Other experimenters have verified this conclusion and extended the rule to a wide range of materials. Clark also verified Wagner's observation that *momentary* overstressing produces no appreciable permanent injury.

The nature of the injury depends upon the material, and may be any of the following:

A. Direct effect of stresses,
    a. Polymerization and condensation,
    b. Redistribution of component materials according to specific capacity,
    c. Change in surface tensions between component parts,
B. Indirect effect of stresses,
    a. Thermal effects such as carbonization,
    b. Chemical combinations such as oxidation,
    c. Electrolytic effects.

An example of polymerization is the formation of the solid material provisionally known as "X", in mineral oils.

Redistribution of component materials according to specific capacity is exemplified by particles of cellulose in oil.

Change in surface tensions is one of the important factors in the failure of oil impregnated cables. In such composite insulations composed partly of solid and partly of liquid materials, an air or vacuous pocket may serve as a center of disturbance, and the liquid material, as observed by Dunsheath[32] in 1926, will be driven away from the pocket, thereby enlarging the latter. Such oil free spots would ultimately link together, and ionization would be established, leading to charring in dendritic or tree-like patterns. The projection of oil from places of high stress to those of low stress, in this case, is apparently due to an increase of surface tension between air and oil as compared to that between paper and oil.*

Many experimenters, such as A. P. M. Fleming and F. Johnston[33], in 1911, have found that where air is present and the voltage is high enough to create discharges, cellulose materials have their structure disintegrated; oils and gums are converted into various products. Some of these actions are thermal and others are chemical reactions, but all are cumulative and irreversible.

Little is known about electrolytic effects but they are under investigation in the case of glass.

The redistribution of moisture by endosmose may be a factor of some importance, as indicated by the work of S. Evershed[34] in 1914. Endosmose is a motion of films of water along the walls of an insulator, due to the water being electropositive to practically all solid insulating materials, and being, therefore, drawn through the pores of the solid toward their electronegative ends.

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*It had been observed by several people [35] in 1924 that impregnated paper cable insulation fails as the result of surface discharges rather than punctures.

The alignment of threads of moisture has the effect of partially short-circuiting the dielectric.

At this time it seems opportune to reinterpret the time-voltage curve in terms of the Steinmetz, Dunsheath, and similar effects. It has been observed that the shape of this curve suggests two influences at work, one corresponding to an action which is reversible up to the point of breakdown and the other to a cumulative and irreversible action. Both of these actions have been observed and explained. It is therefore clear that for any finite period of application of voltage, these cumulative effects will cause progressive deterioration until the bulk of the dielectric has become worthless as a ballast resistor, and the remainder of the insulation is carrying such a current that the critical point, where $\frac{dI}{dE} = \infty$, is reached therein, and failure occurs practically instantaneously. Thus each point on the time-voltage curve represents two actions varying in degree with the time.*

ELECTRON THEORY

Thus far, our studies of dielectric behavior have led us to explain failures in terms of volt-ampere characteristic and chemical deterioration under stress. To be consistent with the spirit of the times, explanations of these actions must be sought in the electron theory.

Considerable half-hearted groping in this direction might be recorded, involving ideas of ionization by collision or indefinite displacement or mobilization of electrons, but no really definite results have been obtained. Indeed, it is hardly reasonable to expect any more until physicists have agreed upon an atomic structure which explains the simpler phenomena of chemistry and physics. Assuming some form of molecule having a considerable number of electrons that can be detached without causing chemical disturbance, P. L. Hoover[8] suggested the following theory:

> "Both the polarization and the conductive current increase with the potential gradient, or, in other words, the number of mobile ions increases with the degree of polarization. This suggests that the mobile ions may come from the molecules of the dielectric.
>
> If this is the case there must exist for every potential gradient a state of kinetic equilibrium between the free mobile ions and the molecules of the dielectric.
>
> As the potential gradient is increased, ultimately a gradient will be reached where the number of ions required to establish equilibrium will be so great that the molecular bonds will be destroyed and dynamic rupture of the insulation will take place. In other words, at high stresses, the molecular fields will be changed and there will be a force on the electrons tending to move them in the direction of the external field. Some of the electrons will drift from molecule to molecule, and thus a conduction current will flow. The electrons that do not drift along will be acted on by the same forces, however, so that the molecules will be distorted; that is, the dielectric will be polarized. The external field thus changes the equilibrium conditions and causes a general drift of electrons."

The form of the volt-ampere characteristic, which Wagner deduced from the negative temperature coefficient of resistivity, is explained by Hoover on the basis of the above conception.

> "For low gradients the number of ions drifting, that is, the magnitude of the conduction current, will be proportional to the gradient. At higher gradients, as the polarization of the molecules increases, the molecular bonds will become weaker and the number of electrons which are drifting will increase more rapidly than the gradient, and there will be an increase in the conductivity of the dielectric. At some critical gradient the molecular bonds will be entirely broken and rupture will take place."

It must be remembered, however, that all this is purely speculative and beyond our present-day knowledge. Here, then, is an excellent field for theoretical and experimental attack.*

Another form of the same general theory is suggested by Gunther-Schulze[6] as applicable to liquids. He discards the theory of ionization by collision as presented by Peek[21] on the ground that the high viscosity and low mean free path require extremely high potential gradients to produce ionizing velocities and these in turn mean localized field distortions far beyond values that are reasonable to expect. Starting from the data of Freise[36] and Koch[37] on the relation between pressure and dielectric strength, he notes the rather surprising qualitative agreement between the behavior of liquids and gases under pressure, and then suggests that slow moving ions of considerable mass leave vapor tracks in the liquid and that in these spaces, as well as in the spaces formed by occluded gases, ionization is produced by collision. He also suggests that there may be originally small vapor spaces, which serve equally well to permit higher localized velocities of sufficient value to produce ionization. In these spaces the mean free path and viscosity would be those corresponding to a vapor and not to a liquid so that the required field strength would not be excessively large. Schumann[38] presents a somewhat similar theory but calls attention to the need for obtaining further and more accurate information as to the mechanics of moving ions and other charged particles within liquids.

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*This conclusion is verified by the work of Mündel[70] as analyzed by Bush and Moon.[79]

*Since the preparation of this report an important contribution on this subject has been published by Joffe, Kurchatoff, and Sinelnikoff.[80]

RESEARCH TO BE UNDERTAKEN

It might seem to a superficial observer that the theory of dielectric failure is fairly well established and that only a little remains to be done to round it to completion. Such, however, is far from being the case, and the remainder of this report will be devoted to consideration of some of the great mass of inconsistent experimental data which need elucidation and present a fertile field for the laboratory worker.

BREAKDOWN VOLTAGE AND INSULATION THICKNESS

Consider the simple problem of the variation of breakdown voltage with insulation thickness.

In the case of solid dielectrics, two facts have been fairly well known for many years, namely, that over a considerable range, the relation between breakdown voltage and wall thickness is linear, but that very thick walls are weaker than the linear relation would allow.*

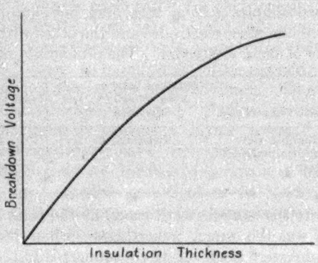

FIG. 4

This suggests a curve of the form shown in Fig. 4, which may be represented by an equation of the form

$$E = A\,T^n$$

where

E is the breakdown voltage

T is the thickness, and A and ^{n} constants.

Tests by F. M. Clark and V. M. Montsinger,[39] reported in 1925, indicate that the index depends upon the time of application of the stress, the following values being obtained for flat sheets of oil impregnated paper:

Time	Index n
Momentary	0.72
10 Days	0.56
51 Days	0.52
77 Days	0.52

This suggests an index of 0.5 for $T = \infty$.

The value of 0.72 for momentary stresses is not in agreement with most recorded data. Unpublished work by Montsinger indicates that the "momentary index" varies with the material, being unity in some cases and as low as 0.66 in others. Thus in 1912 B. S. Radcliffe,[40] working on porcelain, found a linear relation between 2 and 6 mm. This is confirmed by E. T. Montgomery.[41]

Work in 1925, by R. Dieterle,[22] accords with the following equation:

$$E = B + A\,t$$

using the same notation as above, B being another constant. The tests, however, were on mica sheets between 0.015 and 0.30 mm. thick, a very narrow range, so that the two equations are readily reconciled as shown in Fig. 5. Moreover, the test covered periods of application from momentary to 30 min., and Dieterle agrees with Clark and Montsinger in finding that the slope $A\,B$ decreases with increasing time of application of the stress.

H. Rochow[27] worked with flint glass from 0.032 to 0.262 mm. thick and obtained a linear relation like Dieterle, except that at thicknesses from 0.15 to 0.262 mm. the breakdown voltage seems to be higher than the linear law would require. This is at variance with the generally accepted view, as expressed quantitatively by Clark and Montsinger. F. W. Peek[42] confirms the view that with alternating voltages the breakdown voltage does not increase as rapidly as the thickness, but finds the linear law to hold with direct voltages.

In the field of liquid dielectrics we find little information and that bit is not consistent within itself nor does it agree with observations based on work with solids. Peek,[21] working with transformer oil and spherical electrodes, found a minimum value for the strength when expressed in terms of the computed stress at the surface of the sphere when the spacing was approximately five times the radius of the sphere. In terms of total volts across the gap the relations are in the form found by Dieterle although the value of the constants is different. On the other hand, his data obtained using flat disk electrodes cannot be reduced to any simple expression. In both of these cases geometrical symmetry has been ignored and it may partially explain results although Spath[43] after an extensive study, came to the conclusion that for reasonably well purified

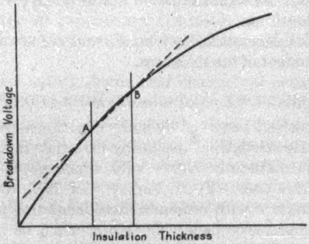

FIG. 5

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*As far back as 1902, F. J. Newbury observed that it was possible to increase the breakdown voltage of cables up to a certain point by increasing the thickness of insulation, but that beyond that point the cable would break down at almost the same voltage, irrespective of increased insulation thickness.[31]

oil the dielectric strength will increase with decrease in the separation between the electrode; with oil that is not well purified the results cannot be predicted. Wedmore[44] found that for needle points and parallel disk electrodes the total voltage was nearly independent of the gap, while Sorge found a decrease in average strength with an increase in spacing.

There is an excellent field for investigation in filling the gaps of our knowledge regarding the relation between breakdown voltage and insulation thickness, but the work must be done by trained physicists conversant with the various factors which must be controlled in order to eliminate everything but the two quantities under investigation. For instance, it is believed by some that the failure of the linear relation is due to the probability that thick material cannot be manufactured as perfectly as the thin.

The discussions which have centered about the pyroelectric theory of dielectric failure have placed much emphasis on the thickness relation. The following table shows the theoretical deductions which have been made from various aspects of the theory, Case I (b) having been developed by J. L. R. Hayden.[45]

Case	Insulation Homogeneous or Heterogeneous	Direction of escape of heat	Electrode Temperature	Relations between breakdown voltage (E) and thickness (T)	Index n in formula $E = A\ T^n$
I (a)	Homogeneous	Toward electrodes	Kept uniformly cool	Independent	0
I (b)	Homogeneous	Toward electrodes	Allowed to warm naturally	$E\ \alpha\ \sqrt{T}$	0.5
II	Weak filament along lines of stress	To surrounding insulation	(Immaterial)	$E\ \alpha\ T$	1.0

A modified form of this theory, suggested by Rogowski, which allows for changes in resistivity with voltage, as well as with temperature, gives

$$E\ \alpha\ T \text{ for thin insulation and } E\ \alpha\ \left(K_1 - \frac{K_2}{T}\right)$$

for thick insulation,[27] where K_1 and K_2 are constants independent of the thickness.

BREAKDOWN VOLTAGE AND INSULATION AREA

It has been observed by many experimenters that the dielectric strength of insulating materials in thin sheet form is materially higher with small electrodes than with large ones. F. M. Farmer,[46] in 1913, found that the variation with ordinary sheet insulating materials, such as prepared paper and cloth, may be 40 or 50 per cent between electrode diameters of 1/64th in. and 8 or 10 in. The usual explanation of this phenomenon had been that the probability of a weak spot increases with the area, but Mr. Farmer pointed out that if this were the case, the probable variation should be much greater for the small than for the big electrodes* and if this were so, the difference between the maximum and minimum breakdown voltages would differ materially for the large as compared with the small electrodes. The tests, however, indicated that there is substantially the same difference, thereby discrediting the theory of probability of weak spots.

This would have been fairly conclusive had not the work of Gewecke and Krukowski,[47] in 1914, renewed interest in the probability theory. These physicists made n times as many tests with small electrodes of area f as with large electrodes of area $n\,f$, took from each series of n successive tests with small areas, the lowest breakdown voltage, and found the mean of these minima to be equal to the mean of all the breakdown voltages, with large areas. This indicated that the probability of weak spots is proportional to the area regardless of whether that area is in one large unit or several small ones. These latter tests were repeated by Wagner and Stahl, who obtained the same results.[8]

The next step in this problem was taken in 1917 by A. E. Kennelly and R. J. Wiseman,[48] who made tests with two samples of equal total area, one equipped with a pair of one-piece electrodes and the other with a large number of small electrodes. The probability of a weak spot was the same in both cases, as the total area involved was the same; nevertheless, when the small electrodes were each separately connected to the source of supply through a high resistance, the breakdown voltage was equal to that obtained with each pair of small electrodes used separately.

This seemed to point to the existence of high frequency transients, surging from part of the insulation to another, as the cause of puncture. The breaking up of the electrodes and connection through high resistance would damp out these transients whereas they would flow freely through the solid electrodes. The difference between the breakdown voltages of large and small samples would be explained by the greater severity of high frequency transients with large (high capacity) samples.†

In 1907, W. LeRoy Emmet[49] suggested that dielectric loss protects insulation from destructive voltage surges, which are absorbed and harmlessly dissipated through the mess of insulation, instead of breaking down at some comparatively weak point.

These discrepancies leave another interesting field for physicists to clarify.

BREAKDOWN VOLTAGE AND ELECTRODE FORM AND MATERIAL

Somewhat akin to the questions of electrode area are those of their form, material, and surface. In the case of solids these details have had but little attention. There is somewhat extensive experimental work along these lines with liquids, however. In 1921 Wedmore[44] made tests to determine the best form of electrodes for testing commercial transformer oil. He came to the conclusion that the type has large influence on the uniformity of results but comparatively little influence on the average result obtained if the number of observations is sufficiently large to eliminate the experimental errors. Dieterle,[22] after using seven types of electrode, came to a similar conclusion. In 1924, Sorge[50] reported that over a considerable range the curvature radius of the electrode had but slight influence with any of several forms of liquid dielectrics. He found that the material from which the electrode was made had a noticeable effect on the majority of the values, silver being the best of eight materials tried and brass the poorest. Sorge and Engelhardt[51] both studied the influence of the condition of the electrode surface and found that it has a marked influence on uniformity of results and to some extent upon the average value obtained. Thoroughly cleaned electrodes will give smaller experimental variations and in general a higher average result. Similar observations have been reported before the American Society for Testing Materials and made the subject of a special investigation in dealing with revision of the test procedure for determining dielectric strength of transformer oil.

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*Because some small electrodes might cover no weak spots and others might cover one or more, whereas the large ones would be likely to cover at least one.

†Two reasons exist for supposing high frequency voltages to be especially destructive: first, the greater dielectric losses and second, the greater ionizing ability as illustrated in ozone formation.

BREAKDOWN VOLTAGE AND HETEROGENEITY OF DIELECTRIC

The relations between breakdown voltage and the degree of heterogeneity of the dielectric possibly covers the entire subject, as nothing whatever is known of the behavior of absolutely homogeneous solid insulation. The electrical continuity of such materials is almost invariably broken by minute quantities of air or moisture, and many of the most commonly used insulating materials are mixtures of substances having widely different specific capacities and resistivities.

James Clerk Maxwell showed that a dielectric composed of two substances differing in these two characteristics, will exhibit absorption phenomena which, as pointed out by F. Grunewald[52] in 1923, may have an important influence upon electric strength. Grunewald's thesis, briefly stated, is as follows:

> If a composite dielectric having two layers of different specific capacities be subjected to a condenser discharge, it will acquire different potential gradients in the two layers, the greater being in that of lower specific capacity. If now the dielectrics be discharged, the difference of potentials between the terminals will disappear, but there will be an internal potential gradient due to charges at the surface of separation.
>
> After an interval, this internal potential curve will straighten and establish a potential difference between electrodes, and the gradient in each layer will now be proportional to its conductivity instead of inversely as its specific capacity.
>
> If, now, a new condenser discharge occurs of such polarity that it will add its potential to that due to the residual charge, a severe stressing of the low capacity layer will result, especially if that layer has the higher resistivity.

This view was supported by experiments which showed a marked decrease of puncture voltage as the result of precharging.

Attempts by other physicists to duplicate these experiments have led to inconclusive results.

The presence of occluded air is a serious detriment to solid insulation, as shown by J. B. Shanklin and J. J. Matson[53] in 1919, as it leads to excessive local dielectric losses and to destructive chemical action under powerful electrical stresses. In the same year, F. Dubsky[54], reported experimental work on the electric strength of thin films of air and showed that the ionization stress is considerably higher than in thick films.

The stress in an air film is K times that in the contiguous solid dielectric, where k is the specific capacity of the latter. Hence knowing k and Dubsky's experimental data, it is possible to predicate the stress in the solid dielectric at which air ionization begins. Shanklin and Matson found this to be 19 kv. per cm. in the case of impregnated paper cable insulation. The detrimental effects of such ionization were described by J. B. Whitehead[55] in 1923.

It was shown in 1922 by Del Mar and Hanson[56] that impregnated paper insulation acts as oil insulation reinforced by baffles. When, however, the oil has been altered chemically as the result of exposure to ionized oxygen, the oil hardens and the insulation degenerates to a laminated type with air or vacuous spaces between layers.

By their nature, fluids lend themselves better than solids to studies of the effects of impurity and the literature contains reports of many such studies. Hendricks [15,57] in 1910 was among the first to report on the effect of moisture on the dielectric strength of transformer oil. He succeeded in deriving results which he reduced to the form of an equation covering small limits. Freise[36] made extensive studies and found that the amount of moisture was only one factor; the extent of diffusion was almost equally important. Hayden and Eddy[58] report the result of 3000 tests of the dielectric strength of transformer oil. They came to the conclusion that one of the principal factors influencing the uniformity of results is that of moisture. In their tests they used commercially dried oils. Schroter[59] arranged a test electrode so that the process of dielectric failure might be observed under a microscope and was able to establish clearly that breakdown was preceded by the movement of minute dust particles into the space between the electrodes. He found marked improvement in the dielectric strength of the oil as the moisture and dust were removed by successively more refined methods of treatment. Spath and Wedmore both report more irregular results with moist oil than with dry. Gyemant[60] has developed a mathematical theory to explain dielectric failure of liquids where moisture is present in small globules as the diffuse phase. This treatment is interesting but does not seem to have any great practical application.

BREAKDOWN VOLTAGE AND TEMPERATURE

If the failure of insulation is due to the heating resulting from dielectric energy loss, the breakdown voltage should drop with rising temperature, and failure should be preceded by temperature rise.

As with most experimental data relating to the failure of insulation, the facts are apparently inconsistent. Thus G. Weimer and C. T. Dun[61], experimenting with porcelain, reported a progressive drop of momentary electric strength from 25 deg. cent. to 325 deg. cent. at which latter temperature this material possessed no electric strength whatever. Similarly, W. S. Flight[62] stated in 1922 that composite materials like varnished cambric, mica products, impregnated paper, etc., all suffer a marked decrease in dielectric strength, ranging from 5 per cent to 72 per cent for a rise from 30 deg. cent. to 100 deg. cent., the voltage being increased by steps of one-minute duration.

Opposed to these results are the more recent and carefully obtained data published by H. Rochow[27] in 1925. This physicist used double concave specimens of Jena flint glass and made his tests over a range of 108 deg. cent., namely from 18 deg. to + 90 deg. The minimum thickness of insulation was 0.175 mm. and the rate of increase of voltage 340 volts per sec. The net result was a practically uniform electric strength over the whole range of temperatures.

Furthermore, Rochow found by an optical method based upon the change of refractive index with temperature, that no temperature rise, exceeding 10 deg. cent., occurred even when a voltage near the breakdown point was applied for 10 min. or longer until failure occurred. It was pointed out by Rogowski[28] in 1924, however, that according to Wagner's filament theory, a temperature rise of 5 deg. cent. should be sufficient to start accumulative heating in glass, so that Rochow's results are inconclusive. Here again is a field for investigation which has important aspects both from the practical and scientific points of view.

Results obtained with liquids in an effort to determine the influence of temperature are too varied to leave us with any idea as to their true significance. Sorge[50] found that xylol was almost uninfluenced by temperature between 20 and 80 deg. cent. while above 80 deg. there is a slight dropping off in dielectric strength. Hexane showed steadily decreasing values and an abrupt falling off after the temperature had risen above 69 deg. cent. Still another form of relation was obtained with transformer oil which showed slightly increasing values up to 50 deg. cent. and beyond this point a slight decrease.

BREAKDOWN VOLTAGE AND RATE OF VARIATION

It is difficult to distinguish experimentally between the effects of voltage and time as discussed under the heading, Time—Voltage Relations, and those having to do with effects of the rate of variation of voltage, for where the time of test is comparatively short the two influences tend to mask one another.

It has long been known that with solid dielectrics the electric strength increases with increasing rates of application of the voltage. This is clearly illustrated by the following data submitted by H. Rochow[27], a result of careful tests on Jena flint glass 0.167 cm. thick:

Volts per sec.	Kv. per cm.
26	2900
340	3810
650	4310

R. Dieterle,[22] experimenting with mica sheets from 0.015 to 0.030 mm. thick, and over a range of frequencies from 0 to 50,000 cycles, found the breakdown voltage both for momentary and 30-sec. applications, to be practically constant from 500 to 50,000 cycles. The average for all thicknesses was as follows, taking for both momentary and 30-min. periods a base of unity for 50,000 cycles:

	Relative Electric Strength	
Cycles	Momentary	30 Minutes
0 (d-c)	15.6	5.53
0.5	2.42	1.59
10	1.29	1.15
50	1.22	1.15
500	1.16	1.10
5000	1.12	1.09
50000	1.00	1.00

In general, it would appear that a dielectric will withstand a higher voltage if applied rapidly than if applied slowly.

As far back as 1908, A. S. Langsdorf[63] reported that at frequencies from 30 to 110 cycles, breakdown occurs after a definite number of repetitions of the electric stress, provided the applied voltage is above a certain critical value. It is interesting to note that this is consistent with the findings of F. M. Clark, regarding accumulative injury with intermittent voltage of long duration.[65]

Work by F. J. Vogel,[64] in 1924, on oil impregnated fullerboard, over a range of from 60 to 400 cycles, confirms Langsdorf in a general way, but indicates that the law of cumulative injury does not represent the facts with precision for such frequencies. Secondary actions are likely to complicate the results, especially if air is present, as ozone generation is almost proportional to the frequency.[65]

If these effects which are so prominent in solids are present in true liquids, the values seem to be so small in general as to have escaped detection. However, Dieterle[22] reports that with semi-fluid oils, he found a noticeable difference between the dielectric strength when the voltage was increased step by step in amounts of two-kv. each minute and that obtained when the increase of voltage was uniform and rapid so as to obtain failure in a few minutes. If the rate of application of voltage is increased to very high values, new phenomena are introduced. Peek[16] has investigated the behavior of commercial transformer oil when subjected to rapidly applied impulse potentials and finds that the dielectric strength under these conditions may be as much as four times that obtained with rapidly applied 60-cycle potentials. These tests indicate a clear time-lag although it is somewhat larger than that obtained for air and rather smaller than that obtained for solids under approximately similar conditions.

COMPARISON OF DIRECT AND ALTERNATING POTENTIALS

It has long been known that insulation will stand a much higher direct than alternating voltage. Thus, F. W. Peek,[42] in 1916, showed that the "d-c. breakdown voltages" of solid dielectrics in good condition are generally higher than the alternating crest voltages, especially when the time of application is long and the insulation thick. When appreciable amounts of moisture are present, the direct and alternating crest voltages are approximately the same.

J. Delon, W. S. Clark, and others have determined the ratio of direct to alternating voltages for oil impregnated paper, and in 1923, J. L. R. Hayden and W. H. Eddy[66] published a very comprehensive report on the subject. They showed that the ratio of direct to alternating 60-cycle crest voltage varies with temperature, insulation thickness, and rate of application of voltage.

The following figures for oil impregnated paper are typical:

Temperature 6 deg.	25	50	75	100
Ratio	2.04	1.64	1.61	1.53
Thickness, number of layers	2	4		
Ratio	1.86	1.65		
Rate of voltage rise, per cent per second	20	5	0.1	
Ratio	1.86	1.58	1.78	

There is abundant opportunity for research work in analyzing the changes of ratio and amplifying the scope generally, especially with relation to the time element.

It is well recognized that for air, the value of the direct potential and the crest value of the a-c. potential to produce failure are the same. We are not surprised, therefore, to find that investigations on liquids indicate a ratio of more nearly 1.00 or about the same value as for gases. However, it is a little surprising to find that Hayden and Eddy[68] report that for a Number Six Transil oil they obtained an average ratio of 0.975 for a two-mm. gap and 0.871 for four-mm. gaps and only slightly higher values, namely, 0.965 and 0.969 at two mm. and four mm. respectively. Using petrolatum, Sorge[50] reports what seems to be a more probable value as a result of tests on hexane when he gives 1.095 as the ratio between the direct potential and the crest value of the corresponding 60-cycle alternating potential. Using the same material and the same arrangement of test electrodes, the 50-cycle value was 1.36 times the d-c. value to produce failure, the time of breakdown being essentially constant and only a few minutes in either case. Draeger found similar results, but differences in test electrode and other details of procedure make it difficult to compare absolute values. He also reports that when alternating potentials are used, the "root-mean-square value" is of importance, as well as the crest value; the lower the r. m. s. value, the higher the crest value to cause breakdown. This is quite in line with results previously referred to in connection with the effects of impulse voltages on transformer oils.

BREAKDOWN VOLTAGE AND PRESSURE

In 1902, Perrine[78] noted the effect of rarefied gases in lowering the dielectric strength of insulation. Koch[37] reported in 1915 the results of elaborate tests on some fluid and semi-fluid insulating materials with the somewhat surprising conclusion that for pressures up to 20 atmospheres the breakdown strength may be expressed by the following formula in which p is the pressure in atmospheres:

$$\text{Breakdown strength (Kv.}_{eff}\text{/cm.)} = a + b\ .\ p$$

But there is a definite limiting maximum value for each liquid above which the dielectric strength is approximately constant. The factor a has values between 75 and 100 while b ranges from 5.0 to 13.0 and depends on the material. In some cases the limiting pressure was found to be as high as 50 atmospheres.

While this study was made the basis of the theoretical work of Gunther-Schulze previously referred to, very little significance seems to have attached to it until recently, although H. W. Fisher and R. W. Atkinson[67] obtained a patent in 1925, based upon a dielectric composed of a combination of compressed air and a solid. These investigators stated that pressure increases, in marked degree, the dielectric strength of insulation containing occluded air bubbles by reduction of internal corona effect.

L. Emanueli, about the same time, developed a special high-voltage cable in which provision was made for expansion and contraction of the oil. He also showed the effect of variation of air pressure upon dielectric losses.[68] In 1926, W. A. Del Mar[69] called attention to the consequences of the creation of voids by thermal contraction of oil in impregnated paper, pointing out that ionization will occur in ordinary high-tension cables unless the internal pressure is maintained above atmospheric.

ISOLATED PHENOMENA

A number of very curious phenomena has been noted which probably bear upon the ionization or electron theories. For instance, if a solid dielectric in series with air be subjected to an increasing alternating stress, at first a small positive charge is induced on the surface of the solid dielectric which, with higher stress, changes to an increasing negative charge until puncture takes place.[71]

Work on air suggests that the disruption of a dielectric may be principally the work of positive ions, the negative ions being of secondary importance. Perhaps this phenomenon is related to the preceding one.[72] Very high gas pressure can be created by an electric spark in a confined area, as shown by K. B. McEachron[73] in 1923, which may lead to the mechanical disruption of a solid dielectric in which sparking occurs due to air fissures.

If the duration of the applied voltage is short (say, 10^{-6} sec.), water disrupts like a dielectric and has several times the dielectric strength of air.[74] This suggests that pure water may be a dielectric differing from others, principally in its time-voltage function, the time required to produce ionization being very short, failure by conduction occurring if the time be any longer.

There is some evidence that ionization initiated by an over-voltage may persist at a lower voltage.

It was pointed out by W. Petersen that small funnel-shaped craters are likely to form in insulation from which ions may be shot into the surrounding medium.[75] Nelson Goodwin,[76] in 1925, told of an interesting experiment which suggests the ionic bombardment of Petersen:

> A plate of glass was tested, large enough to spark around at about 50,000 volts but which could not be punctured. A small patch of paraffin wax was then melted on the glass and allowed to cool. One of the electrodes, consisting of a wire 0.040 in. in diameter, was allowed to rest on top of the paraffin, end on. The other electrode under the glass was a disk about 1 in. in diameter. Under these conditions a potential of less than 30,000 volts was required to puncture glass and paraffin together. The absence of spark noise was very noticeable during the paraffin test in comparison to that when glass was tested alone.

The lowering in breakdown voltage might be accounted for by the introduction of ions sufficiently mobile to be accelerated to a velocity which will cause break-up of the glass molecules, or on the other hand by the stress concentration due to the specific capacity of the oil.

The test is an important one and the matter needs further investigation. The experiment has since been repeated successfully by others, using glass from photographic plates and drops of petrolatum, paraffin, and other oils. Water on glass, or oil on varnished cambric has no such action.

Perhaps connected with this is the remarkable increase of electric strength of oil when the electrodes have been very carefully cleaned. This has been previously mentioned under the heading, Dielectric Strength and Effects of Electrode Form and Material. When the electrodes are cleaned by washing off with carbon tetrachloride or benzol and are dried by passing them through the flame of a burner with a draft of warm dry air, the results are invariably much higher and more uniform than when the same oil is tested with electrodes prepared in the methods used for commercial testing.[77] In this latter method it is customary only to wipe the electrode and rinse out the test cup with a sample of oil free dry gasoline in order to remove fibers. However, there is another possible explanation than the one suggested and it has to do with the complete removal of small fibers such as would be accomplished by the action of the flame. Some work bearing on this phenomenon has been done, using x-rays and radium emanations on the dielectric but the results have generally been too indefinite to prevent forming the basis for any thorough-going theory or explanation.

Another peculiar condition was noted by Przibram in 1904 and quoted by Gunther-Schulze[6] after working with organic liquids. In general, it may be stated that first, in a homologous series the dielectric strength decreases with increasing molecular weights; second, the introduction of a halogen atom or the NH_2 group causes in benzol a considerable increase in dielectric strength; and third, oxygen-compounds seem to have a greater dielectric strength than the corresponding hydrocarbons.

These phenomena differ from those previously recorded in that they have not been correlated with other phenomena or theories.

CONCLUSION

Viewing the subject as a whole, while far from being on a satisfactory footing, either from the practical or theoretical standpoint, the developments of the last three years have given us a rough survey of the field with sufficient bench marks to enable us to visualize the general character of the intervening territory and to project our vision some distance beyond.

Research work can now be undertaken with an understanding of its significance in the general scheme of things, so that experimental work can generally be counted upon to have a broad as well as a specific significance.

This is a great advance over the empiricism of preceding years with its fragmentary and undigested data and crude theories.

Summarizing this report as a starting point for future research work, we may state our present position with regard to solid and liquid dielectrics as follows:

1. Dielectric failure is a phenomenon of circuit instability,

2. This instability results primarily from the form of the stress-current density curve, which has either an infinite or a negative slope in its upper ranges,

3. The explanation of this unstable characteristic is possibly to be found in the equilibrium conditions of free and combined electrons or other ions. In many cases these ions may be due to the presence of impurities in what we ordinarily consider to be the basic material,

4. A dielectric is not necessarily injured by operation at stresses corresponding to instability unless the entire circuit has unstable characteristics; hence this action is non-cumulative and reversible,

5. A dielectric under stress is subject to deterioration due to the energy of electric fields being destructively absorbed, in affecting chemical changes directly or by generating heat. This action is generally cumulative and irreversible,

6. In solids, practically every known instance of cumulative and irreversible deterioration seems to be traceable to the presence of gases or liquids although it is conceivable that this phenomenon might even occur in a pure solid dielectric. In the case of liquids, it does not seem to be entirely dependent upon the presence of impurities for there is considerable evidence of chemical changes of various types where hydrocarbons and similar liquids are used. No evidence is available as to the performance of liquids of simple molecular structure and consisting of one chemical element only, so that change is impossible,

7. The form of the time-dielectric strength curve is explained by both the reversible and the irreversible actions being at play, the former predominating for a short period and the latter for a long period,

8. There are indications that the cumulative deterioration in solids resulting from heat generated in the dielectric may be serious even for periods of less than a second in duration. In liquids no evidence of any kind having bearing on this point has been presented,

9. Attempts to frame a quantitative theory of dielectric failure in solids on the basis of heat generation and the negative temperature coefficient of resistivity which is characteristic of these dielectrics have not given quantitatively correct results,

10. The pyroelectric theory is not basically unsound but it will not be useful until it has been made part of a more general theory which must take many more factors into account,

11. The only theory approaching to general application to explain dielectric failure of liquids assumes the presence of gaseous or vapor spaces within the liquids so as to permit the development of velocities sufficient to cause ionization by collision and the consequent production of additional ions as evidenced by increase in current,

12. A theory to explain the failure of dielectric liquids due to the presence of moisture has only limited application and cannot receive a serious place in this report,

13. Future progress is dependent largely on obtaining reliable data by carefully controlled experiments on a wide range of materials and carefully noting the influence of gaseous, liquid, and solid impurities. These experiments should establish the relations between dielectric strength, thickness of dielectric, area of dielectric or form of electrodes, material of electrode, form of the voltage current waves, temperatures of dielectric and electrodes, mechanical pressure, and other variable factors,

14. Various isolated phenomena should be carefully studied. These are likely to prove more valuable leads than phenomena that are better understood.

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Discussion

F. M. Clark: I think if you read this paper carefully you will come to the conclusion that there is a considerable difference of opinion among "reliable experimenters." That is due, no doubt, to a fact which heretofore has been greatly ignored. Not until recently have the chemist and physicist come into agreement with regard to the structure of the atom. The chemist has had his own chemical atom, and the physicist his own physical atom. The real reason has been that the physicist has entirely neglected the chemist's problems and the chemist has been guilty of neglecting the problems confronting the physicist.

The chemist is compelled to recognize two entirely different types of material; that is, the organic and the inorganic. The molecular structure involved appears to be different for each of these classes.

The electrical engineer, however, disregards both the chemist and the physicist; he works upon "insulation" and pays no attention as to whether it is organic or inorganic material. I think if you go through the paper and pick out the discrepancies, you will find one man talking about inorganic matter,—glass or the like,—and the other man talking about oiled paper or other organic material.

Another point I want to bring out in the paper is the difficulty in handling empirical equations. For example, take the case of the time-voltage curve, described on the third and fourth pages. Mr. Del Mar points out that the index of the time factor obtained by Peek and Montsinger is -4. Mr. Farmer indicates that it ought to be -7 instead of -4. Now, we have found that you can change that index as you please, by merely changing the materials with which you are impregnating the paper, or using the same impregnating material and changing the solid. In using an empirical formula, therefore, one must be very careful to apply it only to those limited cases for which it has been already proved.

Mr. Del Mar asserts that the dielectric strength-thickness relation for oil-impregnated paper is 0.72. It ought to be stated as 0.72 for mineral-oil-impregnated linen paper. If you go to Kraft paper, it is something else. As a matter of fact, we have been able to change the strength-thickness relation at will merely by changing the type of material with which we are dealing. We have gone from a value of 1 to an index point of about 0.58, either by changing the solid or the oil. That is brought out in a paper to which Mr. Del Mar refers as unpublished. It was published by Mr. Montsinger and myself in the *General Electric Review* in 1925.

There is one other point. It is mentioned that there are two classes of breakdown. I should like to add another. The mechanical effects leading to electrical breakdown have been entirely neglected, except at the bottom of the column. I think they are extremely important, especially to the cable engineer and operator.

I can cite a case. In 1923 at Pittsfield in some of our apparatus which had seen field service, we found considerable wax formation. The problem of the cause of this wax formation was turned over to our laboratory. The chemists attacked the work, and in a short space of time came to rather definite conclusions regarding its origin,—conclusions which have been quite widely accepted since that time. They claimed the wax was nothing but a polymerizing product produced by the action of corona on the oil. They concluded that the corona was first, the wax being the result and not the cause. Nevertheless, we continued the work to determine the possible advantage occurring from the use of an oil which would not wax easily under corona bombardment. We found that if we took oil which did not wax easily, or if we used air impregnated paper, and subjected either of them to corona, we got a mechanical shock and the paper disappeared. They told us it was the oxidizing action of ozone on the paper. However, we could not find carbon dioxide after or during the experiment. We repeated the experiment in a hydrogen atmosphere. If we stopped the experiments halfway, "needle points" were noted, such as Osborn described some time ago. Eventually, with a continued test, the paper lost its structure and disappeared as a powder.

Can an oil be produced which will not wax under corona bombardment? This problem may not yet have been solved to the satisfaction of all. Perhaps such an oil may be eventually found, although its existence does not appear probable at present. However, suppose such an oil can be prepared, what will be the result? We shall then have to consult the paper chemist in order to obtain a paper which will not disintegrate under corona. In the end, we may reach the surprising viewpoint that wax formation in cables is not our worst enemy after all. It at least gives us a true indication of the condition of the cable before failure. I believe wax formation in cables will be well worth the trouble it has caused if it serves to focus attention on the fact that, to insure proper service, the cable must be not only properly treated at the factory but throughout its entire existence before and after installation must be handled in such a way as to prevent the formation of voids, gas pockets, and "dry" spots.

W. A. Del Mar: Mr. Clark's discussion clearly shows the necessity of attacking the insulation problem in a broad comprehensive way and of avoiding hasty generalizations. Since the preparation of our report, several papers on dielectric failure have appeared which, we are glad to note, support the main conclusions which we reached from the rather meager data previously available, such as the existence of two agencies of deterioration, neither of which is explained by the Wagner theory in its present forms, although a modified form of this theory may be expected to explain one of them.

W. F. Davidson: I think that we can only again emphasize the points which Mr. Clark has brought out; the need for clearly understanding that about which we are talking, and following the thing through with attention to every minute detail. So many times after the work is all done we find some essential point that might easily have been secured during the progress of the work still lacking. Somebody starts out to study the pyroelectric theory of material, and when he gets through, he finds that the thing has acted in a disruptive way, and that evidence bearing on the disruptive failure is quite useless because it has been collected in such a haphazard way.

Mercury Arc Rectifier Phenomena

BY D. PRINCE[1]

Fellow, A. I. E. E.

Synopsis.—*Peter Cooper Hewitt invented the mercury arc rectifier in 1902, so that it can hardly be considered a new development. More than a half dozen different manufacturers are producing mercury arc rectifiers of various types and sizes and have been producing such rectifiers for years, so that commercial development is not new, yet technical literature is astonishingly bare of treatments going to the fundamentals of rectifier behavior. Many articles appear, describing this and that installation. Descriptions of structural details are not wanting. For specific glass rectifiers, performance curves are available which give the relation between current and voltage at which failures occur under standard conditions. Even this information does not seem to be published for the iron tank rectifiers.*

An engineer wishing to familiarize himself with the quantitative relations underlying rectifier design has thus practically nothing to go on. We cannot assume from this that manufacturers the world over have proceeded blindly for nearly a quarter of a century, but if they do know what happens in a mercury arc rectifier they at least have not told the public.

The purpose of this paper is to present such information as is at present available to the author. This information does not include the knowledge of very important groups in the industry and would even seem to indicate that a very large aggregate capacity of rectifiers has been designed along incorrect lines. This offering is then made in a spirit of humility in the hope that those who know will point out wherein it is in error.

INTRODUCTION

FOR the purposes of this paper, structural details will be ignored as far as possible. That is, if a vacuum-tight vessel is secured, the performance of a rectifier of a given size and shape should not be altered whether the joints are sealed with mercury or rubber, or are of glass fused to metal. Erratic behavior due to bridging of insulators with mercury or oxidation residues merely hampers an investigation and is not a factor in fundamental design constants. Similarly, disturbances due to flying drops of mercury must be prevented, but this again should not be determining where size and shape are to be fitted to current and voltage ratings.

The function of a rectifier, as every one knows, is to convert alternating into direct current. This it does by an electrical action analogous to the familiar check valves used in pumps. A mercury arc rectifier consists of an evacuated vessel of glass or metal containing a mercury pool cathode and two or more anodes. A bright dancing spot on the mercury surface is the source of electrons which move toward any positively charged anodes. As long as the anodes are unable to give off electrons, conductivity in the other direction is normally nil. These phenomena are developed more in detail in subsequent sections.

THE CATHODE SPOT

The probable mechanism of the cathode spot is substantially as follows: Electrons are emitted from the spot and proceed into the space where they strike neutral vapor molecules and ionize them by removing an electron. The new electron joins the old in conducting the current. The remainder of the molecule has a net positive charge and is a positive ion. It is attracted to the cathode. As the positive ions approach the cathode, they produce a high space charge potential gradient which removes electrons from the relatively cold mercury surface. At the same time, the positive ions striking the surface heat it and cause a violent evolution of mercury vapor. As the ions are also mercury vapor, a pressure of mercury vapor is built up which enables the electrons to strike molecules after a very short travel. The entire process can thus take place very close to the mercury surface which enables a very few volts to produce a gradient of millions of volts per centimeter at the surface.

The cathode spot has been observed to have an area of 2.5×10^{-4} cm.$^2/A$. It moves about at a rate of 10 meters per sec. due to the vapor blast. It is relatively cool, not over 600 deg. cent. This point is demonstrated by noting its spectrum which is of the band type rather than the continuous spectrum radiated by a hot body. The vapor pressure is about 2.58 atmospheres, 1.8 atmospheres due to arriving positive ions and 0.78 to evaporating mercury. This pressure gives a mean free path of the order of 3.76×10^{-6} cm. near the surface. The cathode-drop is about nine volts so that the average gradient over one mean free path is about 2.5 million volts per cm. The removal of the electrons from a cold metal surface by high gradient is called the Schottky effect. Schottky calculated on theoretical grounds that about 40×10^{6} volts per cm. would be required for perfectly plane surfaces. For curvature of the order of molecular dimensions the average gradient might be reduced a great deal so that 2.5×10^{6} would seem a reasonable value. Excepting the spot temperature, most of the foregoing figures are due to Guntherschulze[2] who gives also a division of the lost energy. Mercury evaporated is 7.2×10^{-3} g. per sec. per ampere. Fifty-six per cent of the current at the cathode is carried by electrons, the rest by ions moving toward the cathode surface. At greater distances, almost all the current is carried by electrons on account of

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[1] Research Laboratory, General Electric Co., Schenectady, N. Y.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

[2] *Engineering Progress*, August, 1925; *Zeit fur Physik.* Band 11 Heft 2, 1922.

the slow movement of the heavy ions. An electron has a mass of 9×10^{-28} g. and carries a charge of 1.59×10^{-19} coulombs, while an ion carrying the same charge has a mass of 3.3×10^{-22} g. Their velocities are inversely proportional to the square roots of their masses, so that the electron velocity is about 600 times that of the mercury ion in the same field.

The ionization potential of mercury vapor is 10.4 volts; however, all this energy need not be imparted at one collision. A molecule may be activated by receiving 4.7 volts in which condition its free electrons are in an outer position from which they are more easily detached. The nine-volt cathode drop is presumably a weighted average of the various possibilities. 0.44 ampere, the positive portion of each cathode ampere, falling through nine volts, delivers 3.96 watts to the cathode. On entering the mercury, the ions are neutralized by conbination with electrons, and this releases further energy equal to 3.1 watts. The total energy is thus 7.06 watts per ampere. This energy is consumed as follows:

Consumption of energy by electrons leaving the mercury	2.20 watts
Evaporation of mercury in the cathode spot	2.20 watts
Conducted away by the mercury	2.68 watts
	7.08 watts

The heat conducted away by the mercury may evaporate mercury outside of the cathode spot or may be conducted to the outside of the rectifier. The electrons passing out through the nine-volt cathode drop acquire energy represented by $0.56 \times 9 = 5.04$ watts which is used in making ionizing collisions, but 3.1 watts is returned to the mercury by the positive ions. The difference, 1.94 watts, passes into the ionized space but must be added to the 7.08 to make up the total cathode drop. $7.08 + 1.94 = 9.02$ volts.

CONDUCTION IN SPACE

Once free of the cathode, the gas expands to a low pressure, which will be considered later. In this space the electrons travel in the general direction of the anodes but they are diverted by collisions with vapor not violent enough to ionize, and this adds heat to the vapor. Some electrons recombine with ions, giving off energy as radiation which is lost. Electrons and ions combine on the surface of the vessel giving up their energy as heat. These losses must be made up by a potential drop along the arc path. Data have been published giving this loss as 0.1 to 0.4 volts per cm. of arc length. It is thus quite variable and depends upon a variety of factors, such as temperature and geometry of the rectifier. In any given volume of ionized vapor there will tend to be equal numbers of electrons and ions at any instant. A surplus of either sign of charge will attract charges of the opposite sign until the balance is restored. Since for equal energy the electrons move 600 times as fast, the conducted or drift current will be carried by electrons and positive ions in the ratio of about 600 to one. The walls of the rectifier, if at space potential, would receive 600 times as many electrons as ions. They could therefore remain at space potential only if a large current were drawn from them. Actually the walls are usually allowed to charge themselves negatively until a balance is reached. This balance occurs when the walls are negative about five volts with respect to the space potential or about four volts positive with respect to the cathode. The cathode drop was thus long supposed to be the difference between the wall or sounding electrode potential and the cathode[3].

ANODES

The deflection of electrons and ions from the straight path by collisions gives rise to random velocities which may considerably exceed the velocity of drift toward the anode. Langmuir[3] has found that the density of electrons striking a test electrode is from 1½ to 4 times the drift current through the rectifier, depending upon the temperature. It is thus possible for an anode to collect all its current merely by picking up electrons which naturally strike it. These electrons give up their energy upon falling into the anode surface so that the anode is heated even though there is, strictly speaking, no anode drop.

Theoretically, a large enough anode might collect its entire current while at a potential negative with respect to the space. A metal tank rectifier employing the whole tank as an anode will show a drop as low as eight volts for considerable currents. The total drop is thus less than the cathode drop alone. Such an arrangement would be more or less impracticable for an operating rectifier but serves to show the nature of the anode loss.

Most practical rectifiers are arranged to have enough anode area so that the entire current is random current at ordinary loads and working temperatures. This is accomplished by making the anode area not less than ⅔ of the cross section of the arm in which it is located. The anode is then required to dissipate only the energy of recombination of the electrons; that is, the work function which is 3.7 volts for iron and 4.1 for graphite. This energy may be conducted away in anodes with cooled stems or radiated where small solid stems are employed.

If, due to insufficient anode area or a cold tank, the random current is too low, the anode must surround itself with an electric field to draw additional electrons in. This field accelerates the electrons which reach the anode at high velocity and cause excessive heating. It is not uncommon for the anodes in a cold tank suddenly loaded to become incandescent or melt, giving a failure before the apparatus has time to warm up.

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[3] Langmuir, *G. E. Review*, Nov., 1923, and July, Aug., Sept., Nov., and Dec., 1924.

TOTAL ARC-DROP

The total arc-drop from anode to cathode of a rectifier is easily investigated. Fig. 1* gives the arc-drop of a small glass tube under various conditions. The lower curves are taken with a tube having short straight anode arms. With natural air cooling, direct current was first collected by one anode and then by two in parallel using a resistance to divide the current. The curves are the same within the limits of experimental error. Also it is impossible to make two anodes divide current without some external means, for even though the drop increases with slow increase of current, it seems to drop with instantaneous increases.

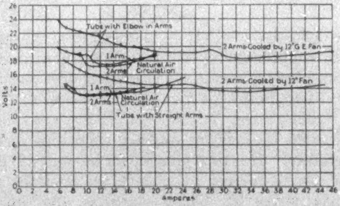

FIG. 1—ARC-DROP FOR 20-AMPERE GLASS TUBE

The upper curves are for a precisely similar tube with bent anode arms which are necessarily longer. The point of minimum drop is perhaps the best to take for comparison between different sizes. Table I gives an idea of the erratic connection between length of arc and total drop.

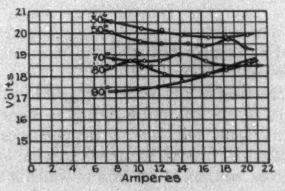

FIG. 2—EFFECT OF TEMPERATURE ON ARC-DROP OF 20-AMPERE GLASS TUBE

TABLE I

MINIMUM ARC-DROP FOR VARIOUS RECTIFIERS

Glass	Size	Minimum drop
	10 ampere	16
	20 "	17.1
	30 "	15
	50 "	15
	250 "	17.2
Steel	15 inch	14.1
	30 "	14.8

Fig. 1 also shows the effect of fan cooling the same two tubes. The point of minimum drop is moved to a considerably higher current but the minimum drop itself is larger. The temperature effect is clearly shown in Fig. 2. For small currents the drop is much lower at high temperatures.

The drop as a function of time is shown in Fig. 3. A rectifier was operated under oil at carefully controlled temperature and a record made of current and arc-drop. The arc-drop measurements at the same temperatures and currents obtained from d-c. measurements are shown dotted and check quite closely except at the transfer points where two anodes were exchanging current.

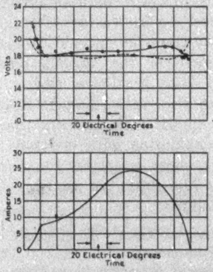

FIG. 3—INSTANTANEOUS ARC-DROP VARIATIONS

The effect of fan cooling in Fig. 1 suggests that even more current could be carried by more intensive cooling. Fig. 4 shows the effect of additional cooling on the same tube. Additional current possible with water cooling is accompanied by a considerable rise in arc drop. The connection between the increased minimum and tube shape is shown by comparing Fig. 4 with Fig. 5. The two tubes have almost the same external area but the one of larger cross section not only carries more current with natural cooling but allows an almost indefinite increase in current without much increase in drop under forced cooling. The sudden changes in drop attract attention, but we are not

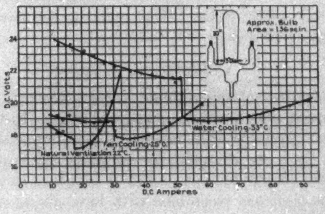

FIG. 4—EFFECT OF COOLING ON ARC-DROP OF 20-AMPERE GLASS TUBE

prepared to offer an explanation for them. It is associated with the anode conditions. When the drop is high, the whole anode is covered with glow. When it is low, the anode tip appears to receive most of the current.

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*Figs. 1, 3, 11, 12, 13, 14, 16, 18, 19, 20 are reproduced from "Mercury Arc Rectifiers and Their Circuits," by Prince and Vogdes, through the courtesy of the McGraw Hill Book Co.

When the vacuum is extremely poor, excessive arc-drops are obtained. The additional drop appears to be in the arc stream which contracts and becomes very hot, even melting glass or iron where it comes in contact at turns in the passage. The anode is also likely to be heated locally. A voltage breakdown or arc-back, however, will usually occur in operation long before this state is reached.

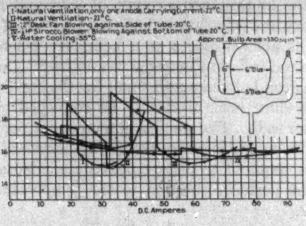

FIG. 5—ARC-DROP IN GLASS TUBE OF LARGE CROSS SECTION

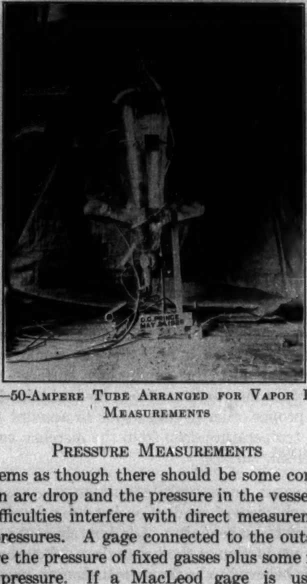

FIG. 6—50-AMPERE TUBE ARRANGED FOR VAPOR PRESSURE MEASUREMENTS

PRESSURE MEASUREMENTS

It seems as though there should be some connection between arc drop and the pressure in the vessel. Certain difficulties interfere with direct measurements of these pressures. A gage connected to the outside will measure the pressure of fixed gasses plus some mercury vapor pressure. If a MacLeod gage is used, the mercury vapor will practically all condense under compression. If the hot wire type of gage is used, connected by a small tube, the mercury may not all be condensed, but an indeterminate part will be condensed so that the reading is of doubtful value as far as indicating the total pressure in active zones in the rectifier. The hot wire gage cannot be introduced into the arc stream because recombination will then take place on its surface and the heat of recombination may more than offset the convection due to the gas on which the pressure indication depends. Similar difficulties prevent reliable measurements of temperature inside a rectifier. Investigators have thus been more or less in the dark as to the actual conditions in the arc stream and condensing chamber.

At the suggestion of Dr. Langmuir, a new method has been worked out for measuring pressures and temperatures within a mercury arc rectifier. Small copper thermometer wells are attached to the sides of a rectifier, as shown in Fig. 6. By heating and cooling these wells, the temperature is found at which the mercury just condenses or just evaporates from the inside surface. These temperatures are within one or two degrees of each other. At the balance temperature, the amount of mercury condensing must be equal to the amount of mercury evaporating. The mercury evaporating must evaporate in a saturated state, so that by consulting a saturation pressure temperature curve for mercury, the pressure of evaporating mercury is known. The pressure of the condensing mercury vapor can only be different from this due to superheat. If a calorimetric measurement were made on the thermometer well, the heat represented by superheat could be determined. To do this the temperature is measured at a point where there is a natural balance between evaporation and condensation. From information on the heat loss of surfaces, the energy passing through

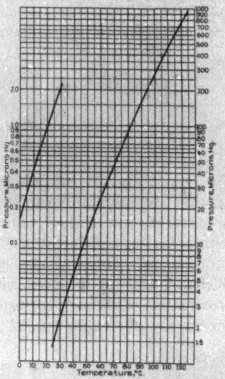

FIG. 7—TEMPERATURE AND PRESSURE OF SATURATED MERCURY VAPOR

the glass can be calculated. The superheat at this point is found to be about 20 deg. cent., so that at most points in the condensing chamber the degree of superheat is probably negligible. In the regions of high ionization density, the recombination on measuring surfaces would produce the same indication as superheat.

The relation between saturated mercury vapor pressure and temperature is given in Fig. 7., and the observed vapor pressures for a 20-ampere glass tube are given in Fig. 8 and Fig. 9. By comparison with Fig. 1, it appears that minimum arc-drop corresponds to about 0.1-mm. pressure for either natural or fan cooling, using the pressures obtained for the neighborhood of the anode arms. Other things being equal, the greatest efficiency will be obtained with a vapor pressure of approximately 0.1 mm. or corresponding to a temperature for saturated vapor of 82 deg. cent.

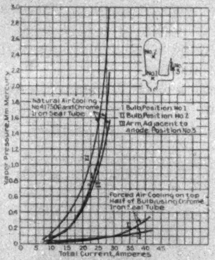

FIG. 8—VARIATION OF VAPOR PRESSURE WITH CURRENT IN GLASS RECTIFIER

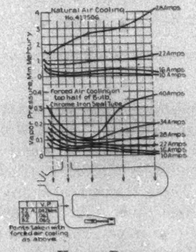

FIG. 9—VARIATION OF VAPOR PRESSURE WITH LOCATION IN GLASS RECTIFIER

Fig. 2 shows a slightly lower drop at 90 deg. cent. and a lower current. The tube had to be heated to secure this temperature at a current of six to eight amperes whereas the 90-deg. curve is higher than the 80-deg. curve under full load conditions.

Similar pressure measurements can be made on iron tank rectifiers by providing them with glass windows on which condensation and evaporation can be observed. The difficulty in this case is to secure the proper location for the windows but valuable information is likely to be secured in any case.

In running such pressure tests, it has been observed that the condensing chamber may run 20 deg. to 30 deg. cooler than the saturation temperature. Several obvious explanations for this phenomenon do not seem to apply. For instance, a temperature gradient might exist in iron or glass. But such a gradient is calculable from the known loss per unit of area and heat conductivity and calculations indicate fractions of a degree. There is always a loss at a boundary where heat is transferred from one material to another. The amount of this drop is known and small for the conditions employed, since heat need only be conveyed to a thermometer well. The amount of mercury vapor condensing and evaporating from the surface is several times the amount required to deliver heat by condensation and raise the parts in question to the temperature of the interior.

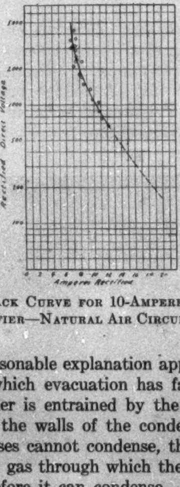

FIG. 10—ARC-BACK CURVE FOR 10-AMPERE 200-VOLT GLASS RECTIFIER—NATURAL AIR CIRCULATION

The most reasonable explanation appears to be that the fixed gas which evacuation has failed to remove from the rectifier is entrained by the mercury vapor and carried to the walls of the condensing chamber. Since these gasses cannot condense, they remain as a cushion of dead gas through which the mercury vapor must diffuse before it can condense. By assuming a reasonably small value for residual gas pressure and calculating how thick a film this gas would make if compressed against the condensing surfaces at the pressure of the mercury vapor, a value is obtained which is of the proper order of magnitude to account for the observed temperature drop from the mercury vapor to the condensing surface.

ARC-BACK

The mechanism of conduction has been developed in advance of the mechanism of failure since the failure is merely conduction at the wrong time. Before setting up what is believed to be the correct theory, some of the observed facts may be described.

OBSERVED ARC-BACK DATA

Fig. 10 is an arc-back curve of a standard 10-ampere, 200-volt mercury arc rectifier. Previous tests had been made of the dotted low voltage portion. The curve therefore represents actual test results up to 4300 volts. Other tests were made to 10,000 volts, but extraneous factors prevented consistent results. The method of making such tests is to connect three rectifiers, as shown in Fig. 11. Two are used for protective purposes and have auxiliary anodes from which a cathode spot is continuously maintained. The circuit to each anode of the tube under test is carried through a protective tube shunted by resistance. As long as rectification is practically perfect, there is little current in the shunting resistances and the full burden is borne by the tube under test. If a failure to rectify occurs, the reversal current flows through the shunt resistance which limits it to such a value that the rectifier under test is not instantly destroyed. The protective tubes are larger and better cooled than the tube under test and so do not give way. Such tests are most readily made on glass tubes because in the past they have been the only ones that could be depended upon to repeat themselves.

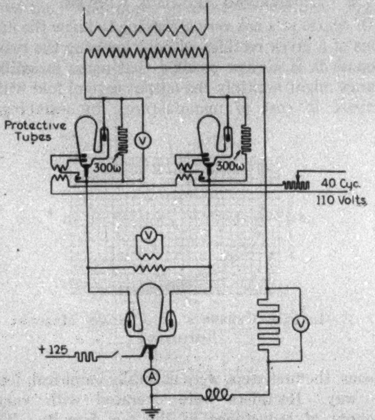

FIG. 11—ARRANGEMENT OF RECTIFIERS FOR ARC-BACK TESTS

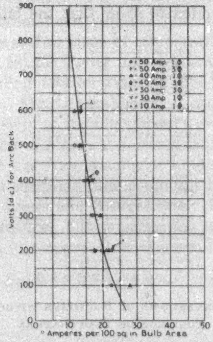

FIG. 12—COMPOSIT ARC-BACK FOR GLASS TUBES NATURAL AIR-COOLED

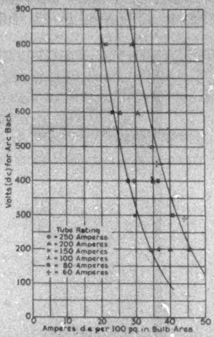

FIG. 13—COMPOSIT ARC-BACK CURVE FOR GLASS TUBES FAN-COOLED

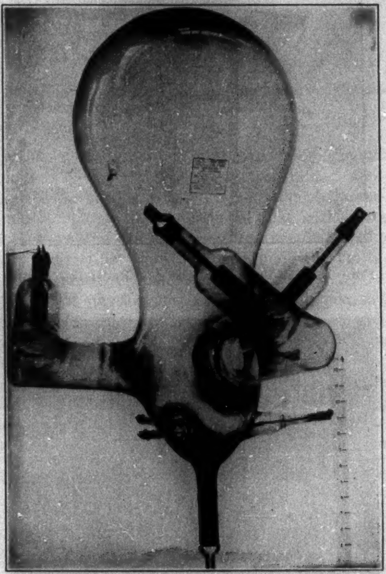

FIG. 14—250-AMPERE, THREE-PHASE 500- TO 600-VOLT MULTIPLE MERCURY ARC RECTIFIER TUBE

Fig. 12 gives the arc-back curve of a whole series of tubes of similar shape reduced to the performance per 100 sq. in. of bulb area. Fig. 13 gives similar data for tubes cooled by a fan. Two forms are represented. The left-hand curve is for tubes having a modified hour glass form, such as Fig. 14. The right-hand curve is for straight sided tubes such as Fig. 15.

As in the arc-drop measurements, a tube of greater section gives superior results. More intense cooling gives a further shift of the arc-back curve to the right.

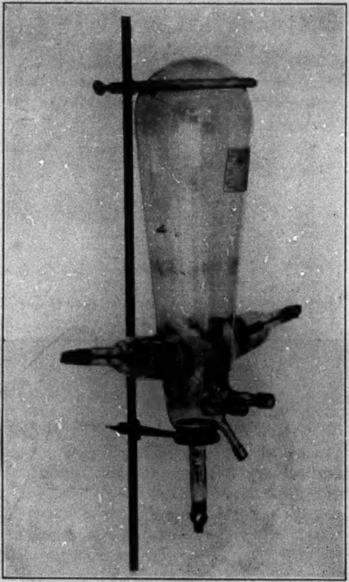

Fig. 15—150-Volt Mercury Arc Rectifier Tube

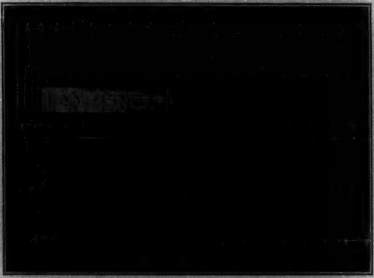

Fig. 16—Oscillogram of Arc-Back taking Place

Curves are not available because of the larger power required since the tubes were actually loaded up to the point where failure occurred and only 50 kw. were available for the tests.

Derived Arc-Back Curves

Of course it is not very satisfying to know the limitations of a given rectifier without knowing the reasons, because it is always possible that some insignificant change might multiply the output several fold with no increase in cost of manufacture. In searching for reasons, the first steps were naturally empirical, feeling the way. Rectifiers were operated with varying amounts of inductance in the a-c. circuits. These failed at a lower d-c. output voltage which corresponded to nearly the same voltage on the a-c. lines; that is, the peak inverse voltage was the same. Two tubes connected like the protective tubes in Fig. 11, but without the shunting resistance, carried more load than two tubes each rectifying both half waves. These bits of evidence tended to show that the failure occurred at the inverse voltage peak, but conclusive evidence of this was wanting until one tube came under test which arced so consistently that it could be caught in the act by an oscillograph. Fig. 16 is a record of actual arc-back taking place. It appears that the failure occurs at or near the voltage peak so that conditions at that time should yield the explanation for arc-back.

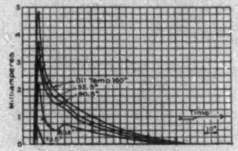

Fig. 17—Inverse Currents in a Glass Mercury Arc Rectifier

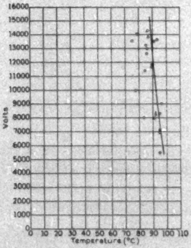

Fig. 18—Breakdown of Rectifier without Arc as Function of Temperature

Fig. 17 is a record of inverse currents at different points in the inverse half-cycle, and shows that at the negative inverse peak there is practically no ionization present, even at temperatures a good deal higher than those used in operating rectifiers. It should therefore be possible to test for breakdown in a tube not operating at all, provided the pressure conditions are duplicated, and have the results apply to an active rectifier. Fig. 18 shows the breakdown voltages for a considerable variety of tubes as functions of temperature. Fig. 19 produces Fig. 18 to higher temperatures. Fig. 20 gives corresponding data for a special tube having movable electrodes. From these figures it appears that a considerable variety of rectifiers break down at almost exactly the same voltage and temperature. Within these limits the electrode spacing is immaterial. The special movable electrode tube could not be exhausted so well as the others and it accordingly breaks down at lower temperatures, but as perfection is approached the points of failure approach a common curve.

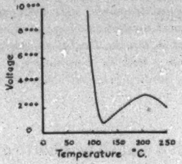

Fig. 19—Breakdown of Mercury Vapor

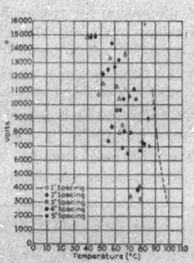

Fig. 20—Breakdown of Mercury Vapor for Variable Electrode Spacing

If these breakdown data are significant, it should be possible to check the breakdown curve by means of the current voltage arc-back curves reduced to pressure temperature abscissa by means of the pressure temperature measurements which have been described. Fig. 21 shows such a comparison. An envelope is formed by curves *A* and *B* which enclose the breakdown points obtained with no arc in the tube. The other curves are the actual observed arc-back curves for three sizes of tubes reduced to a pressure basis.

The envelope in Fig. 21 is rather a wide one but at least the elements of constructive design are present. The width of the envelope is a measure of vacuum technique. The relation between pressure and temperature of the cooling medium is a matter of thermodynamic technique. It is predicted that within five years there will be less mystery about the design of a mercury arc rectifier than now enshrouds the limitations of commutating machinery.

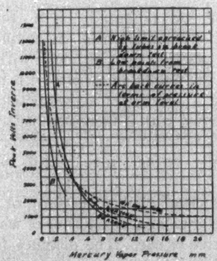

Fig. 21—Comparison of Breakdown and Arc-Back Curves for Mercury Arc Rectifier Tubes

Note: The physical mechanism of the breakdown between electrodes at low pressures has been roughly indicated by such investigators as J. J. Thompson, "Conduction of Electricity through Gases," and J. L. Townsend, "Electricity in Gases." A more detailed study of what happens when a cathode is formed on a conducting surface is being made by Dr. I. Langmuir and should be published in the near future.

Current Collection from an Overhead Contact System Applied to Railroad Operation

BY S. M. VIELE[1]

Non-member

Synopsis.—This paper discusses, from a non-mathematical standpoint, certain factors which should enter into the design of an overhead contact system on an electrified railroad. It emphasizes desirable features, mechanical and structural, and indicates practical limits which conditions will permit in the attainment of these features.

Tests made by the Pennsylvania Railroad with slow-motion photography are outlined. These tests were made to determine operating conditions of the catenary construction with two different types of supporting attachments, over both curved and tangent track. Deflections and oscillations were studied in order to bring out the most productive sources for future study and improvement of the design in respect both to elimination of wear and to maintenance of uniformly good current collecting qualities.

THE use of low potentials on a contact system permits the installation of the contact conductor in such a position that, although within relatively easy access to the public, in general, it is not an undue hazard to them. The use of higher potentials on such systems has carried with it increased hazard, which has necessitated the isolation of this conductor at distances which materially reduce the hazards of accidental contact.

Isolation has usually taken the form of an overhead contact conductor along which a shoe or wheel is carried by the locomotive or car. The distance from the rail to the contact wire varies through relatively wide limits due to the exigencies of the conditions to be met along practically all railroad rights-of-way. Overhead bridges, tunnels, and other obstructions require construction at heights which will permit little more than actual clearance for the rolling stock, whereas reduction of hazard to trainmen and the public generally requires a greater clearance than that usually obtainable through obstructed territory.

The height of a multiple-unit car, such as is used in our suburban service, to the crown of the roof is 13 ft., 0 in. (3.96 m.) The height of an electric locomotive of L 5 class is 13 ft., 5 in. (4.09 m.). These dimensions have resulted in our setting a minimum trolley wire height of 15 ft., 3 in. (4.65 m.) in completely electrified territory where steam locomotives are not permitted to move. This minimum trolley height is never used except where the conditions necessitate. It represents conditions in tunnels and is occasionally approximated at overhead bridges.

The installation of a contact system at such an elevation would make approach to the roof of cars or locomotives very dangerous and would not represent an operative condition if used, except in very limited stretches.

In case operation should require an employe on top of the rolling stock for minor repair work, the height of trolley should be 22 ft. (6.71 m.) as this is the minimum height with which it is at all feasible to make even minor repairs. This includes an allowance of about one ft. (0.30 m.) for the increased sag under hot weather conditions. Such a dimension for trolley height has been adopted for suburban territory. It is not an ample clearance but some work can be done if proper care is taken. At the same time, my own experience does not permit me to say that it is a position of mental comfort. Two feet more, or a total contact height of 24 ft. (7.31 m.) is, from a clearance standpoint, a very much better proposition. This additional two-ft. (0.61 m.) clearance above a minimum of 22 ft. (6.71 m.) is very hard to obtain and in some cases almost impossible in urban districts, owing to the requirements placed for city grades, adjacent property conditions, etc. In open country, where there are relatively few overhead bridges, the 24 ft. (7.31 m.) can be obtained and such a standard set.

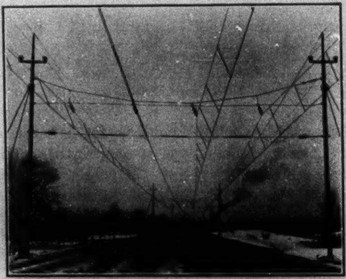

FIG. 1—VIEW OF STANDARD CONSTRUCTION ON CURVED TRACK

Steadying against side-sway on standard tangent track is accomplished in a similar manner

The distance from rail to contact wire will vary from about 15 ft. (4.57 m.) to approximately 25 ft. (7.62 m.). This variation requires that the device which carries the actual contact member and bridges the space between the locomotive or car roof and the contact wire shall be capable of operation at varying heights over a

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[1] Asst. Electrical Engineer, Pennsylvania Railroad.

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range of some 10 ft. (3.05 m.). Its operating range should permit of its minimum height position being about three in. (0.08 m.) below the minimum operating height position and it should be capable of extending some 6 in. to 12 in. (0.15 m. to 0.30 m.) beyond its highest operating position.

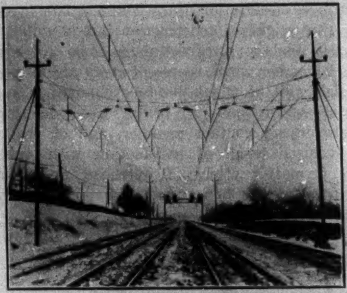

FIG. 2—VIEW OF SPECIAL CONSTRUCTION ON TANGENT TRACK

Steadying against side-sway on special curved track is accomplished in a similar manner

Pantographs, carrying collecting shoes, have been exclusively used by the Pennsylvania Railroad for this purpose, for high voltage service. We have two broad classifications for pantographs, depending upon the equipment to which they are to be applied, *i. e.*, locomotives and multiple-unit cars. The multiple-unit pantograph is a single shoe device of relatively light construction, whereas the locomotive pantograph is heavier and carries two shoes. The weights of these equipments are of the following order:

	Pounds		
	Gross	*"Live"	Shoe
Locomotive	1400 (635 kg.)	243 (110 kg.)	33 (15 kg.)
Multiple unit	710 (322 kg.)	100 (45 kg.)	14 (6.3 kg.)

*Shoe and extensible frame work

In both types of equipment, the "live" weight is carried by the journals of a rotative shaft at each end of the pantograph, the two shafts having connected thereto the bottom members of the pantograph movable framework, which members are held in supplementary angular relation with the base of the device by bell cranks and interconnecting links. In all operating positions, the weight of the "live" parts is eccentric to the shafts, which produces a torsional moment on the shafts which varies in amount throughout the entire operating range. This torsional moment is counterbalanced with helical springs, in tension, applied through chains operating over cam surfaces, the latter being attached to each end of each shaft.

The necessity for a small collapsed height of pantograph does not permit of a very large radius for the cams nor a large elongation of the springs and, consequently, the spring tensions are material. This spring tension is used to support the pantograph in all operating positions and to provide the force required for upward acceleration of the shoe and frame work, as well as to overcome the frictional resistance to upward movement.

Collection requires that the force of contact should be as near constant as is feasible and that the friction of vertical movement of pantograph should be relatively small. It becomes evident that this is not a simple problem when it is considered that the spring tension on a multiple-unit pantograph for operation on trolleys at 22-ft. (6.71 m.) elevation is 460 lb. (208.6 kg.) for the minimum operating height and 150 lb. (68.0 kg.) for the maximum operating height, in each of four springs, and that the weight of the moving members is of the order of 100 lb. (45.4 kg.), the latter being variably applied. A pantograph operated vertically, on a stationary block, exerts 18 lb. (8.2 kg.) of upward force under slow motion conditions, with a total variation of force of about 10 per cent. Downward motion produces approximately 10 per cent additional variation.

The above figures are based upon favorable conditions. If sleet accumulates on the framework, it does not take long until sufficient weight is added to overcome the 18 lb. (8.2 kg.) of upward force. Wind loads have affected the operation of some of the early designs to a sufficient extent to actually drop the mechanism.

It has been frequently asked why the upward pressure should not be increased so as to have a larger working margin. Two major limitations exist.

FIG. 3—ENLARGEMENT FROM SLOW-MOTION PICTURE

This shows the locomotive pantograh shoe parting contact at the trailing end of a wire splice

In case the sum of the pressures of all pantographs on a given span of contact wire approaches the weight of construction lying immediately adjacent to the contact plane, columnar instability of the overhead results. If the pressures are increased sufficiently, the contact wire will occasionally turn over, exposing the contact wire attachments to blows from the passing pantograph.

The wear on shoes and the contact wire itself is, for a non-lubricated contact, a function of the mechanical abrasive effect varying with pressure, and the pitting and burning effects caused by current flow.

The most economical pressure apparently varies with the current collected from a single shoe. As the current value decreases, lighter pressures can be advantageously used. This is fortunate, as otherwise it would be difficult to operate some four to six multiple-unit pantographs in a given span without increasing the weight of actual contact system or providing special stiffening attachments for the contact wire.

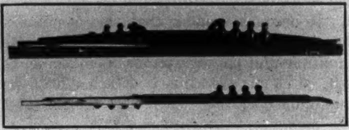

FIG. 4—CLOSE-UP OF BADLY WORN WIRE SPLICE

This picture shows the occurrence of wear at the approaching end and pitting action at the trailing end of the splice

The force required for downward acceleration of the "live" parts of the pantograph is the sum of the upward thrust of the shoe plus an increased reaction to cause downward acceleration. This sum must not reach a value which will permit the turning of the contact wire. It follows that the upward acceleration forces which are permissible in the pantograph are less, by an amount necessary to produce maximum downward acceleration, than those which will be sufficient, under any conditions, to cause overturning of the contact wire. It should be borne in mind that the downward accelerating forces vary with speed of traffic, gradients, changes of sag, etc., and a suitable margin must be left between these totaled upward pressures and the weight of the construction adjacent to the contact plane.

The pantograph has a natural period of vibration, in a vertical plane, if it is considered that it be pulled down at the same rates at which it will rise under the application of forces inherent in the design. This periodicity of vertical vibration is an inverse function of the mass involved in the "live" parts and the friction of operation of the pantograph mechanism, and a direct function of the actuating force (the latter limited by maximum permissible wire pressures). Commercial designs vary but are of the order of from 40 to 70 beats per min. The latter figure represents very careful attention to the entire pantograph design.

The more rapid oscillatory designs are of advantage in obtaining better collection on any given overhead design. At the same time, this periodicity can be and is improved by the secondary supporting of the shoe itself, by springs which produce pressures well above and below the forces applied through the main framework. With the reduced mass to be vibrated and with forces approximating those applied through the main framework, the shoe periodicity is much higher but, of course, of very low relative amplitude.

What is desired is a continuous contact with the trolley at as near a uniform pressure as is feasible. The nature of the overhead system involves major changes of elevation of the contact system, necessitating grades for transition between these different elevations; there are changes of elevation of the contact plane in a given span; the weight of construction is not uniform and cannot be made so; construction has to be utilized in the contact member which represents very abrupt changes in weight; all of which produce changes of contact pressure which show up in increased deterioration of the overhead contact member, and of the shoes. The eventual result of satisfaction with operation is not a matter solely of overhead design or of pantograph design but a result of coordinated effort in both designs.

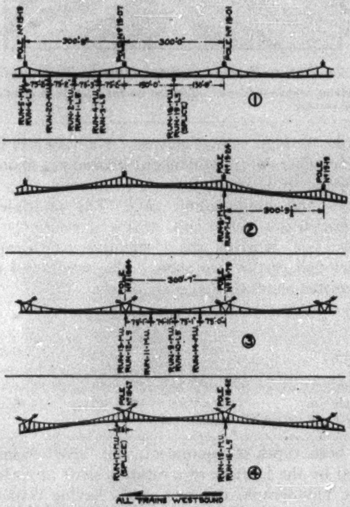

FIG. 5—LOCATION DIAGRAM OF SLOW-MOTION PICTURES

1. Standard tangent construction
2. Standard curve construction
3. Special tangent construction
4. Special curve construction

A live resilient pantograph operating on a poorly designed overhead system will necessarily not give as satisfactory results as would be produced with a better design of overhead. The same general statement may be made with reference to the design of the overhead system itself.

A pantograph shoe is limited in length (about four ft. (1.22 m.) transverse to the track), and if the overhead is not properly located over the track, or if the catenary is permitted to sway under wind conditions to a sufficient extent, the shoe passes to one side of the contact wire, rises to an elevation above the contact wire and practically always catches in some part of the overhead, with the result of damage to the pantograph sufficient to render it inoperative and occasionally materially damaging the overhead.

Reduction of occurrences of this nature on tangent track may be obtained by shortening the span length or by means of steadies applied at the support points. The latter is the more economical procedure. Spans may be installed with little trouble of this nature, of the order of 285 ft. (86.87 m.) in length, with proper steadies at support points, in locations exposed to wind. In protected localities, this value may be increased to about 325 ft. (99.06 m.). Without steadies, our experience would indicate that these figures should each be reduced about 100 ft. (30.48 m.).

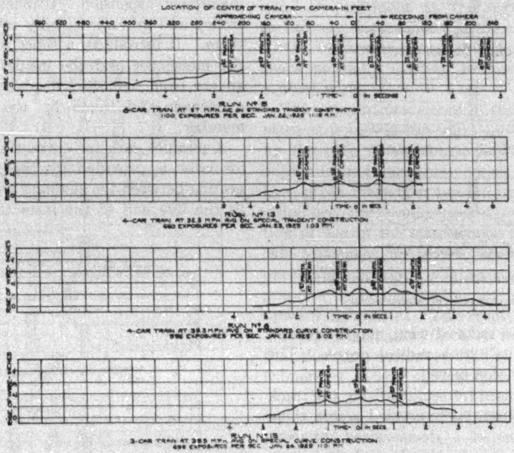

FIG. 6—UPWARD DEFLECTION OF CONTACT WIRE

Measured at support point of catenary span

On the assumption that the more economical construction should be used, that is, longer spans, one is immediately faced with the design of steady which will limit motion transverse to the tracks, allow longitudinal equalization of tensions and permit as great a vertical movement of the contact member at the support points as is possible. The above represents the mechanical requirements. Electrical requirements involve the maintenance of potential across the insulators and of suitable clearances. Maintenance requires easy access with suitable working clearances, prevention of burning of the vital members of construction in case of flashover of insulators, and in case of damage, the positioning of members such as to clear pantographs in as many cases as possible.

The trolley construction used in the Broad Street-Paoli district, carrying 11,000 volts single phase, consists of a ½-in. (12.7 mm.) steel messenger, from which is suspended, by 3/16-in. by 1-in. (4.75 mm. by 25.4 m m.) straps at 30-ft. (9.14 m.) intervals, a 1/0 round copper auxiliary wire, from which in turn is suspended, by clips at 15-ft. (4.57 m.) intervals, a 3/0 grooved bronze contact wire. The contact and auxiliary wires lie parallel to one another in a vertical plane. The connections between these two wires "break joints" with the connecting hangers lying between the messenger and the auxiliary wire. Span lengths vary from a maximum of 325 ft. (99.06 m.) to values necessitated by conditions with an average of about 300 ft. (91.44 m.).

The suspension of the catenaries consists of a three-unit cap and pin type insulator and assembling hardware, to which the messenger is attached. This represents a pendulum length of about 24 in. (0.61 m.) applicable to the messenger. The messenger has five ft. (1.52 m.) of sag in a 300-ft. (91.44 m.) span, which construction produces a total pendulum length at the support point of approximately 7ft., 6 in. (2.29 m.).

The original design did not call for the use of steadies except at a few locations. Operation, however, has necessitated their general installation in exposed locations.

The steady design adopted consisted of a horizontal strand installed between the catenary supporting poles at an elevation six in. (0.15 m.) above the contact wire. This strand was insulated from the poles by three-unit porcelain insulators adjacent to the poles, with a sectionalizing wood stick insulator between the auxiliary wires of adjoining tracks. Attachments were made by a slack two-way jumper from this strand to the auxiliary wire. This construction reduced what may be termed "pantograph derailments" to a radical degree. The disadvantages of this design were:

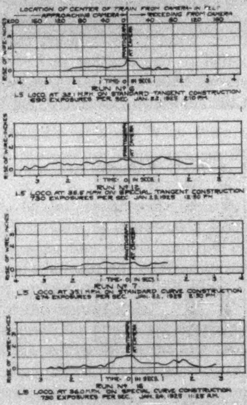

FIG. 7—UPWARD DEFLECTION OF CONTACT WIRE
Measured at support point of catenary span

a. The wood sticks used for sectionalizing the track trolleys do not have potential impressed across them except under conditions of a de-energized adjacent track, which condition usually develops their failure, if any, at the most inopportune time.

b. Small working clearances for certain repair operations.

c. Periodic cleaning, varnishing, and general overhauling of the wood sticks.

d. The impassability of one or more tracks in case of steady span failure, on account of portions of steady span construction hanging down below the elevation of the contact plane.

e. Small upward deflection of the contact wire at the support point.

In the effort to improve catenary support conditions, a new form of support was laid out which, in my opinion, reduces to a material degree the disadvantages of the previously described steady.

The standard form of back guyed poles, cross catenary, and body span member were used; however, instead of suspending an insulator string directly over the track for attachment of the messenger, a string was placed each side the center line of each track. Connected between these insulators is a short length of cable approximately eight ft. (2.44 m.) long, with sufficient sag in it that when drawn downward at its center, it forms the two legs of an obtuse triangle with its apex about 30 in. (0.76 m.) below the body member, this point being used for the attachment of the messenger. The ends of the insulators nearest the center line of the track have a second attachment with members running to a common point and attached to the auxiliary wire. In appearance, the construction forms a letter "V" with the auxiliary wire attached at the bottom apex and the upper ends of the legs of the "V" attached to sides of the obtuse triangle at the lower ends of the insulators. This construction fixes the position transversely of the messenger, and eliminates the pendulum length of the messenger suspension. It places all insulators between trolley and ground, thereby retaining potential across them. It introduces a short section of cable between the messenger and the insulators, and consequently, an arc from a flashed insulator is removed from the messenger and taken by the messenger supporting member.

A parallel path for current supply to an arc at the insulator is provided by the secondary hanger connecting between the auxiliary wire and a point adjacent to the live end of the insulator. This, we consider, will reduce the localized annealing of the messenger under flashed insulator conditions, as there is provided a shunt path for the current flowing from the trolley and auxiliary wires to the point of arc contact with the catenary supporting member.

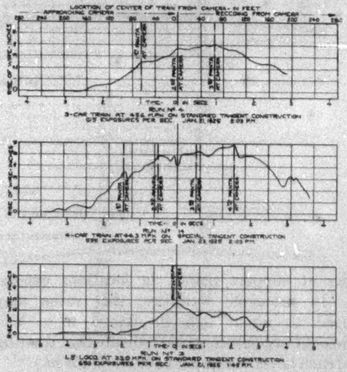

FIG. 8—UPWARD DEFLECTION OF CONTACT WIRE
Measured at entering quarter point of catenary span

The two members forming the lower "V" converging on the auxiliary wire leave the clearance between tracks as a maximum at the contact wire elevation, with the minimum clearance existing at the insulators, just below the body member.

Tests with this construction loaded horizontally and transversely of the tracks indicated greater displacement at the support point as compared with our usual form of horizontal strand steady. Displacement at the center of span with the two forms was within one per cent of one another. The total displacement of the special form of steady at the support point was approximately one-half of that obtained in the center of the span.

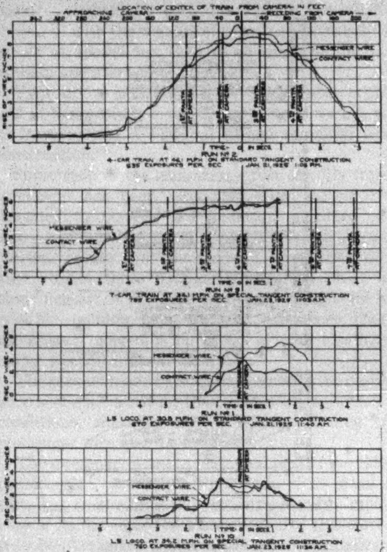

FIG. 9—UPWARD DEFLECTION OF CONTACT WIRE

Measured at center point of catenary span

In thinking of catenary construction, most engineers consider that contact is made with the overhead conductor in practically a uniform plane and that there is little or no upward displacement with pantograph passage. Even among engineers who have watched such things, the impression is that the deviation of the contact wire from a uniform plane is a matter of a few inches and that the wave of this deviation follows closely the position of the pantographs.

If we assume the messenger as carrying practically all of the weight of construction in a given span and that this weight is fairly uniformly distributed, we obtain an approximation of a catenary curve. The direction of the tension in the messenger makes an angle of varying magnitude with the upward thrust of the pantograph. At the point of maximum sag in the messenger, the tension in the messenger and the thrust of the pantograph are at right angles to one another; as the support points are approached, this angle decreases and is a minimum at the support point. The application of a vertical force applied at the center of sag, therefore, has a maximum effect producing upward displacement of the trolley wire, on account of the fact that there is no vertical component of tension at that point, but only horizontal tension, which is constant throughout the span; whereas at other points than the center of sag, a vertical component of tension exists, which, opposing the upward shoe pressure, reduces the upward displacement of the contact wire. There is little or no displacement of the construction as a whole at the support points such as occurs at the center of sag of the messenger. Displacement at the support point is almost exclusively displacement of the contact wire in relation to the messenger, in other words, a closing up of the space normally existing between the messenger and contact wires.

This upward movement of the catenary which takes place at points in the span is evidently brought about by the ease of equalization of messenger tension to adjoining and successive spans. The load supported by a given messenger tension being reduced by pantograph pressure, equilization of tension demands a reduction of sag in the span whose load is so reduced. If this statement is correct, then an increase of total pantograph pressure, whether applied by a single pantograph or by increasing the number, should increase the upward deflections. This result is confirmed by trials.

From what has so far been said, it is to be inferred that there is a major oscillation imposed upon the main framework of the pantograph, whose periodicity is a function of span length employed and an inverse function of the speed of the rolling stock.

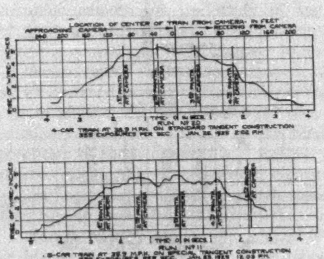

FIG. 10—UPWARD DEFLECTION OF CONTACT WIRE

Measured at trailing quarter point of catenary span

The oscillation of the contact wire travels ahead of the pantograph with various irregularities or harmonics imposed on it. Support points tend to become nodes of the wave. Changes of mass, usually splices, trolley wire intersections, or section breaks retard the propagation of these oscillations.

A traveling wave impinging upon a section of greater mass than that employed in the adjacent section is damped to a material degree. As the wave reaches the increased mass, the change of elevation of the contact plane does not take place as rapidly as it did in previous sections of the contact wire and there is produced, locally, a gradient at the approach to such mass of greater slope. This greater slope produces increased pressure between the shoe and the trolley wire which accelerates the shoe downward and the contact wire upward. The pressure produced between the shoe and the contact wire is such as to accelerate the two members in opposite directions at velocities sufficient that they over-travel and contact is momentarily lost.

Such loss of contact is usually caused by definitely establishing an out-of-phase relation between the oscillation of the shoe and the contact wire. There are two other causes of loss of contact occasionally present, however, one being the result of insufficient range of movement of the shoe supports and the other the inability of the shoe to follow in time the periodicity of oscillation of the contact wire.

Taking the more usual case of conditions existing at the time contact is lost, that is, an out-of-phase relation between the oscillation of the shoe and contact wire, the succeeding occurrences are that as the shoe and contact wire again approach one another, the phase relation of their oscillations is still materially displaced. Contact is made with a pressure increased above normal, due to the vertical velocities present, and excessive wear takes place in these areas.

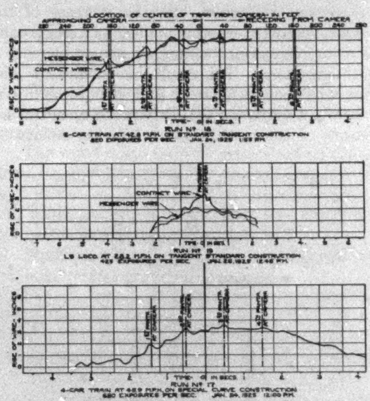

FIG. 11—UPWARD DEFLECTION OF CONTACT WIRE AT SPLICES

Runs 18 and 19—New splice located at center point of catenary span
Run 17—Old splice located 28-ft. beyond catenary support point

Such effects are well illustrated by a splice 24 in. (0.61 m.) long, weighing 3½ lb. (1.59 kg) after a number of years of service. On the entering end, the splice is smooth and bright with a well worn contour; near the middle it shows little deterioration of any nature; at the trailing end, there is practically no wear but a material reduction of section due to pitting and burning. On the wire one foot removed from the trailing end of the splice, there is severe wear and reduction of section.

In so far as splices alone are concerned, these conditions may be corrected, as experience with a splice weighing 20 ounces, (0.57 kg.) for use with 4/0 contact wire, over a period of more than a year, indicates an adequate design with a great improvement in reduction of hard spot effects.

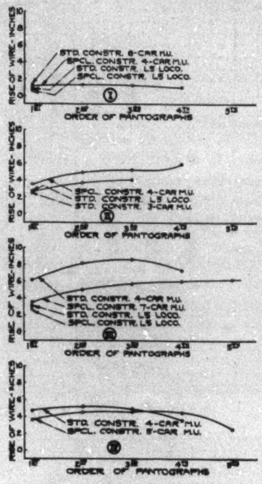

FIG. 12—RELATIVE UPWARD DEFLECTION OF CONTACT WIRE AT EACH PANTOGRAPH

I —Support point
II —Entering quarter point
III—Center point of span
IV—Trailing quarter point

The hard spot effects produced by the 3½ lb. (1.59 kg.) splice described above are typical in a general way of hard spots produced by other items or types of construction.

If wood section breaks, usually about six ft. (1.83 m.) long, are used in high speed territory, similar results may be expected, but of very much augmented degree.

The supporting attachments of the contact wire itself may produce similar or worse results. With the attachment of the contact wire directly to the messenger, each hanger becomes a point of increased mass, which, in amplitude of upward movement with pantograph passage, tends to lag behind the construction immediately preceding it. This action produces a hard spot of initially small importance but, as service progresses, the contact wire may be slowly deformed or kinked immediately ahead of the attachment, with the severity of the hard spot gradually increasing.

This result is the product of a number of intimately related conditions, such as size of contact member, shoe weight, and normal operating pressure, weight of hanger and its location in the span, the weight of messenger and, particularly, operating speeds.

Such a construction with a single contact wire will not operate satisfactorily with the pressure and weights of shoes and with the weights of construction usually used, except at speeds less than those employed in normal railroad operation.

Yard service is slow speed service and such construction at these speeds is satisfactory and is used in yards, as it represents a material saving in initial cost of construction.

Suspension of the contact member directly from the messenger has been attempted with hanger designs intended to correct the conditions outlined above. The use of loop hangers on systems in which the normal current values and short-circuit conditions do not approximate those necessarily involved on a major electrification, is not here criticized, though the writer is of the opinion, from our past experience, that the use of such hangers is not applicable to conditions of major electrifications.

The contact made by such hangers with the messenger wire is of such a nature that, under large current flow conditions, annealing and occasional burning of the messenger wire may be expected.

The messenger wire is a very vital part of the catenary construction, not only on the basis of its usual function, but on the basis of what is involved in repairs in case of its damage. The importance of this statement may be overlooked and it is wished to emphasize it by explaining that satisfactory current collection at the speeds used necessarily involves a very high degree of uniformity of construction; the higher the speed, the more exacting the requirements.

In case the messenger parts, from whatever cause, the unbalanced tension therein immediately tends to rip the construction to pieces, with the result of broken connections, distorted hangers, kinked contact wire, etc. These results, with decreasing severity, extend each way from the break. Repairs under such conditions are made in successive steps; the first thing is to get the overhead up out of the way of traffic quickly, supporting it in any way possible. Repairs of this nature are necessarily a makeshift and they require that traffic through the damaged area be at a reduced speed. Usually pantographs are lowered and equipment drifts through the damaged area. The successive steps are to position the messenger with the correct sag, to remove and replace temporary construction and to strengthen or replace bent hangers, wire, etc., and finally to position the various connections.

It may readily be realized that, if messenger tension in the damaged area is not lost, the amount of damage to the overhead system, with the various mishaps that occur, is very much less and, consequently, it is possible to restore the construction to a fully operative condition in very much less time.

The pantograph design and its operation on an overhead system bear a very intimate relation to the design of the overhead itself. Pantograph design is a specialized subject which has been localized in the hands of the manufacturers, whereas the design of the overhead has usually been in the hands of the owners of the property or their consulting engineers. While it has been appreciated to some degree that the design of the overhead is also a specialized matter, at the same time I do not feel that its importance in eventual satisfaction of operation is as fully appreciated as it should be.

An increase in the amount of available information on the subject of overhead construction is one of the necessities for eventual railroad electrification.

With this in mind, a study was undertaken some time ago by members of our organization, with the idea of determining and depicting, as far as possible, actual occurrences in the pantographs and in the overhead under operating conditions. This study included a comparison of the design of steady which we have been using for a number of years, with the special design described above.

A section of track was selected in the territory lying between Broad Street Station, Philadelphia, and Paoli, Penn., and a number of spans was equipped with the special support construction referred to. Deflections of the overhead system and movement of the shoes were recorded.

The apparatus for these tests included a high speed photographic camera for the study of the various movements which take place in a span of contact wire. Through the courtesy of W. H. Miner, Railroad Supplies, Chicago, such a camera was obtained, capable of taking, on standard moving picture film, photographs at the rate of more than 1000 pictures per sec., together with auxiliary equipment and a personnel familiar with the operation of the equipment and familiar with the possibilities of depicting occurrences at high speed.

The test apparatus consisted of a high speed camera handling film at speeds from 400 to 1100 exposures per sec., a storage battery for current supply to a shunt motor driving the camera and for use in the projectors, projectors for augmenting natural illumination, and a scale used as a background for the contact wire.

One of the center tracks of four was occupied by a tower car carrying the camera, projectors, and operators, a storage battery and work car, and a steam locomotive. The adjoining track towards the outside of the right-of-way was used as the test track. Outside the clearance lines of this latter track a scale was installed which was used as a determinative background against which the contact wire was to be silhouetted.

The camera and scale were supported at such a height that the camera lens, midpoint of the scale, and the contact wire under investigation were at the same elevation. In front of the scale and held to it by a string was a weight of about five lb., which weight was attached so that its center of gravity was at the zero of the scale. A tripping cord was arranged so that a pull on it would apply a horizontal force on the weight supporting string and break it, allowing the weight to fall.

The camera and scale were located at designated points in the several test spans and, as the multiple-unit train or locomotive approached, its position and approximate speed were signalled from a point ahead of the test equipment.

The camera mechanism was engaged with the running motor and the timing weight dropped. Some experimentation was necessary to obtain proper sequence and coordination with the passing equipment, but the arrangement finally worked out very well.

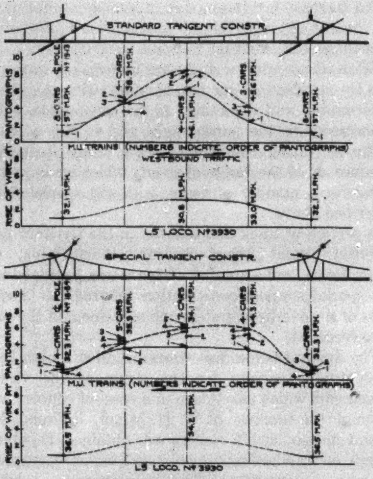

FIG. 13—RELATIVE UPWARD DEFLECTION OF CONTACT WIRE ALONG CATENARY SPAN

The scale was graduated in inch divisions, with each third inch marked.

The contact wire being located horizontally about midway between the camera lens and the scale, the scale reading is about double the actual wire movement. In working up the data afterwards, it was found that it was not possible to determine when the timing weight started to fall. It was necessary, therefore, to take the difference in theoretical time of fall from the zero to a point on the scale, 39 in. (0.99 m.) from zero, and the theoretical time of fall from zero to a point at a higher elevation, 12 in. (0.30 m.) from zero, as a determinant of the film exposure rate. The time of fall from the 12-in. (0.30 m.) point to the 39-in. (0.99 m.) point, with acceleration started at zero, is two-tenths of a sec.

The exact passage of the timing weight by these points was determined within a few exposures and the intervening pictures counted. This result multiplied by five gives the exposures per second. As the camera was driven by a shunt motor, the exposure rate per second for the few seconds involved was considered constant. Train speed was also considered constant over the test interval. Knowing the exposure rate and the distance between successive pantographs of a multiple-unit train, or the distance between marks on the contact wire, the speed of the pantograph passing the camera field is obtained from the number of exposures between pantograph passages, or from the passage of a single pantograph between the marks on the contact wire.

The distance ahead of the pantograph at which movement takes place was obtained in a similar manner, using the same data given above and the train or locomotive speed.

A number of determinations was made as follows: Deflections at the center, entering, and trailing quarter points of a span and points directly beneath the support points of the messenger. The above trials were obtained for multiple-unit equipment and a portion of them for locomotive equipment, both types of steady construction being photographed.

It was known that a splice in the contact wire caused a heavy flash with pantograph passage, which flash was reduced in intensity with service. A new splice was installed and photographed with current being collected with pantograph passage. The resulting arc fogged the film to such an extent that little may be told from these pictures. A locomotive shoe passing over the splice without current flow, however, tells a story of material departure of the shoes from the contact member. Several pictures were taken on curved construction, which show a much greater flexibility of construction at such locations than exists at similar locations in a span of tangent construction.

The exposures made of each test run were then consecutively numbered in 100 exposure intervals and passed through a projector a number of times until the observer was familiar with the film. Then the position of the contact wire in terms of the scale reading was observed at given intervals of the total exposure, with a notation of the exact picture from which the reading was taken. In cases where there existed some peculiar feature or where it was apparent that harmonics of readable magnitude existed, readings at much closer intervals were made.

These data, converted into distance ahead of the train, were then plotted, the ordinates representing rise of contact wire at the observation point in the span from its initial position, and the abscissas representing the position of the train with reference to the observation point at the instant of the occurrence of the position of contact wire indicated by the ordinate reading.

These results, shown graphically, are given in the several figures bearing proper title as to type of messenger support, position in the span of the observation point, type of rolling stock, number of multiple-unit cars, traffic speed, and exposure rate.

An analysis of these curves indicates clearly certain conclusions, a portion of which, for a specific design of overhead system, are quantitive. Mathematical conclusions, not included, indicate the basic phenomena to a sufficient extent that usable approximations may be made for construction embodying different weights, span lengths, and types of construction.

Such conclusions follow:

1. That a major oscillation is present in the contact member ahead of the pantographs and travels with the pantographs,

2. That this oscillation is of variable amplitude, depending on its location in the span,

3. That the position of any given point in the amplitude of oscillation has a variable spacing ahead of the pantograph,

4. The amplitude varies with the total pressure imposed by the pantographs in a given span,

5. The amplitude seems to be increased with the distribution through a number of pantographs of a given total pressure,

6. That when distribution of a given total pantograph pressure is made through several pantographs, the pantographs in the central portion of a train experience the greatest amplitude of movement,

7. That the leading pantograph does not have imposed on it as great an amplitude of movement as do all successive pantographs,

8. That the direction of traffic does not materially alter the symmetry of maximum upward deflection of the contact wire,

9. That the gradient to which a pantograph is subjected in passing under locations of depressed contact wire height is augmented by a material gradient imposed by the inherent deflections in the approaching spans and that such augmenting of gradient is of the order of the usually installed maximum gradient, one-half of one per cent, for such transition of elevation of the contact wire,

10. That harmonics of the major oscillation are present each side of the pantographs with relatively minor amplitudes and are apparently damped out at pantographs and do not cause trouble,

11. That the presence of material change in mass of the construction adjacent to the contact plane damps the oscillation and thereby causes relatively severe deterioration due to variation in pressure of contact shoes at such points,

12. That with a given increase of mass necessarily locally employed, shaping of the contact surface will decrease the deterioration at such points,

13. That support points practically represent nodes of the major oscillations, as the amplitude of oscillation is relatively small at such points,

14. That the amplitude of oscillation at the center of the messenger span is maximum with a reduction towards each messenger support point,

15. That the amplitude of movement at the center of the span is almost wholly a movement of the total catenary construction, that is, a movement of the messenger and its suspended construction,

16. That the amplitude of movement at support points is almost wholly a matter of change in spacing of the contact wire and its adjacent construction with relation to the messenger wire,

17. That improvement of hanger construction at points adjacent to the messenger support is possible and desirable,

18. That the use of so-called flexible hangers at the center of span is not necessary and their desirability is doubtful,

19. That flexible hangers adjacent to the support point may be advantageously used if of satisfactory design,

20. That harmonics in the contact wire at positions of change of mass in the construction adjacent to the contact plane and located remote from support points are of high periodicity and considerable amplitude,

21. That such oscillations cause similar oscillations of opposite phase in the contact shoes,

22. That departure of the shoes from the contact wire at such locations are usually the greatest in time and the greatest in amplitude,

23. That the reduction of changes of mass at points adjacent to the contact plane offers the greatest measure of improvement immediately available,

24. That further study of the construction from a trailing quarter point in a span through the support point to the entering quarter point of the succeeding span offers the next most promising step in improvement,

25. That improvement in the secondary supporting of shoes and the obtaining of a greater amplitude of movement of the shoe is probably desirable,

26. That further investigation and study of the tilting action of shoes is necessary and desirable, as there are indications that edge riding of shoes is at times present to a material degree.

Discussion

For discussion of this paper see page 1133.

Catenary Design for Overhead Contact Systems

BY H. F. BROWN[1]
Member, A. I. E. E.

***Synopsis.**—This paper is intended to outline methods which have been found useful in the design of overhead contact systems where catenary construction is employed, with especial reference to the single catenary construction installed in 1925 on the Danbury Branch of the New York, New Haven, and Hartford Railroad as an extension of its 11,000-volt, single-phase, a-c. electrified zone; and on the New York, Westchester, and Boston Railway on its extension from Larchmont to Harrison, N. Y.*

Part I deals with construction over straight or "tangent" track, and is a review of methods and formulas which, while not new, are necessary as an introduction to Part II, which not only includes tangent-chord construction, but deals at length with the design of the so-called inclined catenary and its adaptation to curved track.

THE fundamental requirements of an overhead contact system for high speed railway electrification are:

a. It must be parallel to the track centerline, or nearly so, within prescribed limits. When changes are necessary in the normal elevation above the track, the gradients must be such that the current collectors on the locomotives and cars will follow the contact wire without leaving it, and without excessive pressure due to inertia; that is, the relative grades must not be too abrupt,

b. It must support without great distortion its own weight together with superimposed vertical loads due to sleet and horizontal loads due to wind,

c. It must possess consistency in flexibility; that is, hard spots should not occur in a construction designed to possess a certain amount of yielding, nor should soft spots occur in a system possessing inherent rigidity,

d. It must transmit the power supply, and afford suitable contact area with the moving collecting device on the locomotive or car, at the required speed,

e. It must possess a high degree of reliability, and ease of maintenance.

The problems of design are closely allied to transmission line design as far as power transmission problems and the choice of mechanical design of supporting structures are concerned. Some of the formulas developed for sags and stresses in transmission lines may be applied in a general way to the calculation of the contact system supporting messenger, although since this messenger is to support a conductor which is to be practically level without sharp departures from the horizontal, there will be certain limitations and refinements in the design not met with in the ordinary transmission line calculations.

Many features, such as insulation, suspension methods, bracing on curves and for wind loads, tensioning devices, anchorage details, and sectionalizing details are also involved in the design, although outside the scope of the main subject.

Part I. Tangent Construction

For a simple system where a working conductor is supported by an elastic messenger directly above it, the well-known parabolic equations will apply for all span lengths generally encountered, since in general the sag is small compared with the span length.

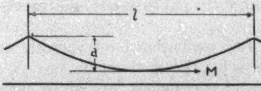

FIG. 1—SPAN, SAG, AND TENSION

In Fig. 1,

$$d = \frac{\gamma\, l^2}{8\, M} \tag{1}$$

$$L = l + \frac{8}{3}\,\frac{d^2}{l} \tag{2}$$

or

$$L = l + \frac{\gamma^2\, l^3}{24\, M^2} \tag{3}$$

where

d is the deflection, or sag, of the messenger,
γ is the supported weight per unit length of span,
l is the distance between supports, span length,
L is the actual length of the messenger,
M is the horizontal component of the tension in the messenger.

The errors made in using these formulas, instead of the more unwieldy equations of the catenary, are approximately 2 per cent in the case of (**1**), and 0.5 per cent in the case of (**2**) and (**3**) for spans where the sag is not greater than 2 per cent or 3 per cent of the span length, which errors are well within construction allowances.

Sags according to (**1**) may be most conveniently shown graphically on logarithmic cross-section paper, as shown in Fig. 2.

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HANGER DESIGN

If the hanger spacing is s and the minimum allowable vertical distance between the contact and the messenger is v_0 then the length of any hanger is

$$v_n = d_n + v_0 \quad \text{(see Fig. 3)}$$

where d_n is the sag of the messenger for a span length of $2\,n\,s$

or

$$v_n = \frac{\gamma\,(2\,n\,s)^2}{8\,M} + v_0 \qquad (4)$$

Such hangers are called "standard" hangers, in distinction from "special" hangers, calculated from above formula when v_0 is replaced by any greater distance.

For a catenary system with horizontal contact wire, and with the same length shortest hanger at the low-point in adjacent spans, the distance from the low-point to the support is the same for spans both sides of the support, if the messenger tension and unit weight (γ) are the same in both spans; see Fig. 4.

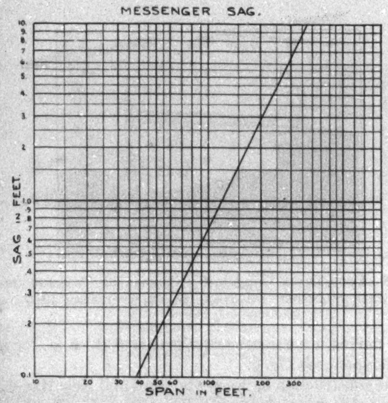

FIG. 2—MESSENGER SAG CURVE

The distance from low-point to low-point under these conditions has been called the "equipoise span" (l_E) since the weight either side of the support is the same; and the distance from the low-point to the support is designated the "equipoise half-span" $\frac{(l_E)}{2}$.

In Fig. 4, it is obvious that the weight supported by the structure is

$$W = \gamma\, l_E \qquad (5)$$

and the height of the supporting structure is

$$V = C + v_0 + d_E \pm I \qquad (6)$$

where

C is the height of contact wire above top of rail,

d_E is the sag for the equipoise span,

I is the height of the insulation, taken positive if of the suspension type, and negative if of the supporting type.

The formulas (**5**) and (**6**) are useful for the design of the supporting structures. It should be noted here that in calculating the horizontal loadings on the structures due to wind, these will be the unit wind loading on the catenary multiplied by half the sum of the adjacent spans, which may or may not be the same as the equipoise span.

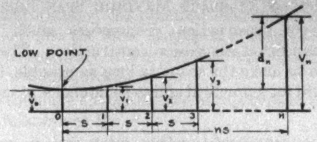

FIG. 3—"STANDARD" HANGERS

Spans having the same length of shortest hanger at the low-point will have the same hanger length at any given distance from the low-point. When the shortest hanger is "standard," all the other hangers in the equipoise span are standard, unless there is a change in grade in the contact wire.

If standard hangers can be used, the construction is greatly simplified, both as to cost of design, as well as manufacture and installation. Standard hangers will apply when the distances between supports are uniformly the same length, or when, if necessary to change the span length for any reason, length of any span is the sum of the lengths of each adjacent equipoise half-span. For example, if there is a series of, say, 300-ft. spans having standard hangers, and it becomes necessary to reduce the span length to, say, 200 ft. for several spans, to pass around a curve, if there is introduced between the last 300-ft. span and the first 200-ft. span a span having a length of 250 ft., then standard hangers can be applied to all the spans. Similarly, as in Fig. 5, successive span lengths of, say, 300, 250, 200, 150, 125, 200, 275, 300, may all have standard hangers, as each of these spans is the sum of the equipoise half-spans on either side. The equipoise spans for the above

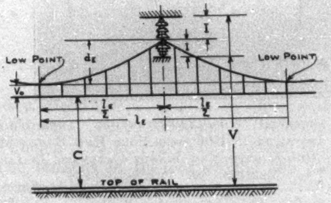

FIG. 4—EQUIPOISE SPAN, AND HEIGHT OF SUPPORT

would be: 300, 200, 200, 100, 150, 250, and 300, respectively.

This feature is worthy of emphasis, as a little extra study on the preliminary layout in location of supporting structures may save considerable calculations later if standard hangers are maintained.

CHOICE OF STANDARD MAXIMUM SPAN LENGTH

Catenary construction presupposes much longer spans than direct suspension construction. The trend has been to make the spans as long as possible, having due regard for a practical and sensible choice of sags and tensions for the material used for the messenger. Three hundred feet has been more or less standardized for catenary construction on tangents, and has worked out fairly well in some cases. The writer is of the opinion that this length of span is a little too great for "single catenary" construction, especially in exposed places, on account of the large possible displacement due to wind; and the large possible variation in the height of contact at the center of the span due to temperature changes. Further, when the track alinement consists of a large amount of curvature, with short tangent stretches between, there is not always sufficient length on the tangents to make a material reduction in the number of supporting structures by going to such long spans and there is also the danger of having unbalanced stresses on the structures at extreme temperature conditions in going from very long spans on tangent to short spans on curves.

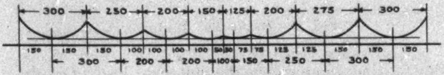

FIG. 5—ILLUSTRATING "STANDARD" HANGER APPLICATION AND EQUIPOISE SPANS

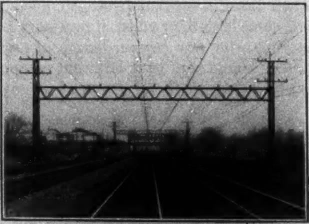

FIG. 6—ORIGINAL "WOODLAWN-STAMFORD" TYPE DOUBLE CATENARY INSTALLED IN 1906 OVER FOUR HEW HAVEN MAIN TRACKS AT RIGHT, WITH NEW SINGLE CATENARY INSTALLED 1925 OVER TWO "WESTCHESTER" TRACKS AT LEFT

Wind bracing may be necessary on exposed tangent stretches having 300-ft. spans, especially on the types of construction which have no lateral stiffness against wind loads.

For the above reasons, 250 ft. was adopted as the standard maximum span length on tangent track on the Danbury Branch. On the extension of the New York, Westchester, and Boston to Harrison, which parallels the New Haven main line, use was made of the New Haven right of way and structures. This line is mostly tangent, and the existing catenary supporting structures are spaced 300 ft. apart.

CHOICE OF MESSENGER MATERIAL AND SAG

The "New Haven" has on its electrified system more than 650 track miles of catenary construction, embracing several types, all of which have been fully described to the Institute at various times. The original construction, installed in 1906 between Woodlawn, N. Y., and Stamford, Conn., is "double catenary," having two 9/16-in. extra high strength steel messenger strands, each having a nominal vertical sag of 6 ft., 5 in. for a 300-ft. span; see Fig. 6.

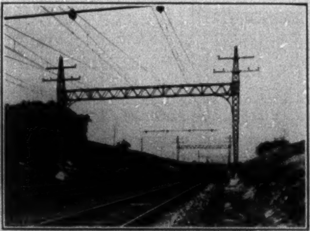

FIG. 7—FOUR-TRACK COMPOUND CATENARY ON TANGENT BETWEEN STAMFORD AND NEW HAVEN, INSTALLED 1913

The construction between Stamford and New Haven, and on the six-track Harlem River Branch, as well as on the four-track main line section of the N. Y., W., and B. Ry., is "compound catenary," with supports 300 ft. apart, from which main messengers, not alive, suspend the live single catenary. This has a normal span of 150 ft. The live catenary messengers are of extra high strength galvanized steel, 19-wire, ⅝-in.-diameter strand, having a sag which is equivalent to 5 ft., 0 in. in a 300-ft. span; see Fig. 7.

At certain points on this latter type of construction, the ⅝-in. messenger has suffered severe corrosion due to steam train traffic which is of necessity routed over part of the electrified zone. In replacing the steel messenger at these points with a high strength bronze strand, the factor of safety, which was five in the case of the steel strand to allow for the corrosion, could not economically be much greater than 2½, under assumed maximum loads, even though the messenger size were increased, on account of the increased weight. The lower factor of safety for bronze is allowable, because of the low corrosion loss; but it was a fact to be kept in mind that this sag and tension was right at the limit for bronze, for the contact wire weight supported.

In 1921, the New York, Westchester, and Boston started extending its New Rochelle Branch parallel to the New Haven main line. As stated above, the New Haven had four tracks electrified in this section, but the catenary bridges were designed for six tracks. The new "Westchester" tracks were electrified, using single catenary construction supported on the same bridges which supported the New Haven double catenary. This new catenary had the same nominal sag, 6 ft., 5 in., as the old double catenary for the 300-ft. spans, and a single 9/16-in. extra high strength steel messenger strand, galvanized, was used.

Steel was used because there is no steam traffic of any nature on this line, but the characteristics of this design are such that because of the larger sag, bronze can be economically substituted if the occasion ever warrants.

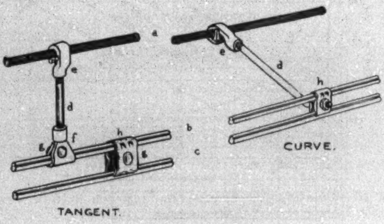

Fig. 8—Catenary Construction Details

a. 9/16-in. steel Messenger strand
b. 4/0 copper auxiliary trolley
c. 4/0 Phono contact wire
d. ½-in. steel hanger rod
e. Malleable iron messenger hook
f. Malleable iron auxiliary trolley clip
g. ½-in. steel carriage bolt
h. Pressed phono intermediate clip

This same design was selected, for these reasons, for the Danbury Branch.

Deterioration due to corrosion has an important bearing on the choice of messenger material. If steam traffic can be entirely eliminated by electrification, steel may be economically used with an expectancy of reasonably long life, if properly maintained by painting. Much of the original 9/16-in. messenger installed on the Woodlawn-Stamford section of the New Haven over 20 years ago, is in very good condition at the present time on this account. Maintenance of this kind on busy traffic routes is both difficult and expensive, however, and where the traffic is dense, therefore, the higher first cost of non-ferrous materials such as bronze, is justified on new work.

As against the excellent record of the Woodlawn-Stamford messenger, mention must be made of the steel messenger construction installed in 1912 in the large freight yards of the New York tidewater terminals, which have recently been entirely reconstructed with bronze messengers and hanger rods. The combination of the heavy salt fogs, with more or less steam traffic, produced conditions which were unusually severe.

On the Danbury Branch, however, because of the low traffic density, the practical elimination of steam operation, and the drier atmospheric conditions away from the salt water, it was felt that the higher cost of bronze was not justifiable.

The complete characteristics of this type of catenary are as follows: see Fig. 8

Messenger:	9/16-in.-diameter, 7-wire extra high strength steel strand, extra heavily galvanized.
Sag:	6 ft., 5 in. in 300-ft. span, with normal load at 60 deg. fahr.
Auxiliary trolley:	4/0 grooved hard drawn copper.
Contact wire:	4/0 grooved Phono bronze, 45 per cent conductivity.
Hanger rods:	½-in. round galvanized steel rods.
Hanger hardware:	Malleable iron castings, heavily galvanized.
Intermediate clips:	(Aux. to contact) pressed Phono bronze.

Unit Loadings

(Pounds per Foot)

Material	Vertical (Bare)	Vertical (½-in. Sleet)	Horizontal (Wind on Sleet)	Resultant (Wind and Sleet)
9/16-in. Mess.	0.668	1.329	1.042	1.69
4/0 Copper	0.641	1.241	0.973	1.575
4/0 Phono	0.641	1.241	0.973	1.575
Hangers and Clips	0.273	0.465	0.272	0.54
Total	2.223	4.276	3.260	5.380

Normal Tensions

At 60 deg. fahr.

9/16-in. Messenger	3900 lb.	(from equation (**1**))
4/0 Copper	1600 lb.	(from Fig. 9.)
4/0 Phono	1815 lb.	(from Fig. 9.)

The tensions in the copper and the phono were determined as follows:

The change (ΔL) in a given length (L) of any material having a coefficient of linear expansion (α) due to a change (Δt) in temperature, is

$$\Delta L = - L \alpha \Delta t \tag{7}$$

The change (ΔL) in a given length (L) of material having a modulus of elasticity (E), due to its elasticity, for a change in tension (ΔT) is

$$\Delta L = \frac{L \Delta T}{E a} \tag{8}$$

where a is the cross-sectional area.

If the change in tension is all due to a change of temperature, (**7**) will be equal to (**8**).

Then

$$-L\,\alpha\,\Delta\,T = \frac{L\,\Delta\,T}{E\,a}$$

or

$$\Delta\,T = -\,a\,E\,\alpha\,\Delta\,T \qquad (\mathbf{9})$$

For copper

α = 0.0000094 per deg. fahr.

E = 17,500,000 lb. per sq. in., solid wire.

For Phono

α = 0.0000093 per deg. fahr.

E = 18,140,000 lb. per sq. in., solid wire.

For 4/0

a = 0.1662 sq. in.

Substituting the above values in (**9**), and plotting for various values of t, and assuming a minimum of 100-lb. tension in the contact wire at the maximum assumed temperature (+ 120 deg. fahr.) gives the values shown in Fig. 9. It will be noted that these tensions are kept as low as possible on the theory that horizontal loads on the supporting structures on curves are thereby reduced to a minimum, and the maximum flexibility is provided for the contact wire. Although the curve of auxiliary wire tensions is shown as zero at the maximum temperature, it will always have some tension due to the weight of the contact wire which it supports.

MAXIMUM TENSIONS

The maximum stress in the messenger and trolley wires must be determined in order to insure that the material and sizes selected will have ample strength to withstand the maximum loads to be experienced. The maximum loading is assumed to be eight lb. of wind pressure on the projected area of all material when covered with a coating of sleet one-half-inch thick.

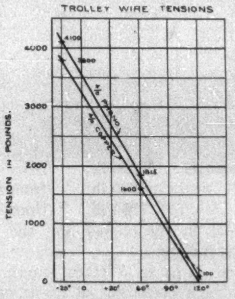

FIG. 9—TEMPERATURE-STRESS DIAGRAM FOR AUXILIARY AND CONTACT WIRES

The maximum stress in the auxiliary and the contact wires is readily found from Fig. 9. The messenger stresses are determined as follows:

From equation (**3**), for a given span length (l), the actual length (L) of the messenger may be plotted for various values of tensions (M) and unit loadings (γ); see Fig. 10.

γ_1 is for the bare messenger alone (0.668 lb.)

γ_2 is for the messenger with the two trolley wires but without hangers (1.95 lb.)

γ_3 is for the complete catenary, without wind or sleet loads (2.22 lb.)

γ_4 is for the complete catenary with wind and sleet loads included (5.38 lb.)

Point A on the normal load curve may be taken to represent the messenger length and tension at 60 deg. fahr., normal temperature.

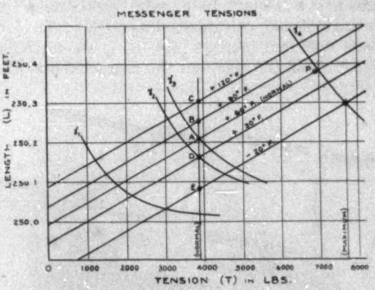

FIG. 10—TEMPERATURE-STRESS DIAGRAM FOR 9/16-IN. STEEL MESSENGER

From equation (**8**) may be calculated the amount the messenger would stretch for any increase in tension (assuming constant temperature), plotting this new length and tension at P.

For 9/16-in. steel strand,

a = 0.1992 sq. in.

E = 23,000,000 lb. per sq. in.

α = 0.0000064 per deg. fahr.

A straight line drawn through A and P will give any value of L for any value of T with constant temperature of 60 deg. fahr.

For any constant tension, the change in length due to a change in temperature may be found from equation (**7**), and points B, C, D, and E, using lengths as at A may be found thus for 90 deg., 120 deg., 30 deg., and − 20 deg. fahr., respectively. Lines through these points drawn parallel to $A\,P$ will indicate the variation in length due to a change in tension for that temperature.

The maximum stresses determined thus are:

9/16-in. Messenger. . 7700 lb.		at − 20 deg. fahr.
4/0 Copper. . . . . 3800 lb.		
4/0 Phono. . . . . 4100 lb.		

The tensions at which the bare messenger may be installed alone at various temperature conditions may be found from the intersections of the lines A, B, C, D, and E with the curve γ_1. If the stress is to be determined by the sag of the messenger, a curve may be made similar to Fig. 11, showing no-load sags for various spans and temperatures. Similar graphical data may be made for the sagging of the messenger with the load of the two trolley wires, but without the hangers, as is desirable in certain instances when these are all strung out together.

Fig. 12 shows the details of the tangent construction used on the Danbury Branch. The poles are 10-in., 49.5-lb. Bethlehem H sections, either 36 or 38 ft. in length. These are self-supporting, being bolted to the foundation channels with a sufficient number of one-in. bolts to develop the full strength of the section. Bracket arms are adjustable as to height, and are attached to the flanges of the poles by malleable iron clamp castings. This is a valuable feature of the design, as it is not necessary to specifically mark each pole for each location; fabrication is reduced; and if it ever becomes necessary to raise the grade of the track (by ballasting), or to change the number or dimensions of insulator units, these may be done without reducing the standard contact wire height, by raising the brackets, without additional work on the poles.

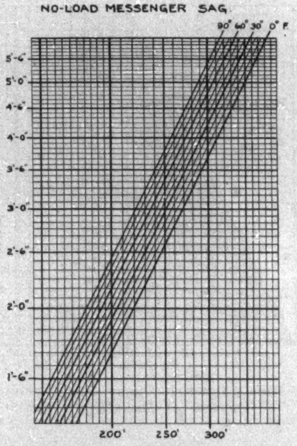

Fig. 11—No-Load Sags for 9/16-in. Messenger for Various Temperatures

Fig. 13 shows graphically the proper pole length and the bracket height for this type of construction, for various equipoise half-spans, calculated from formula (**6**) above.

Suspension insulators, consisting of three units, were used, instead of the pin type supporting the messenger above the bracket arm, used in previous installations, as the suspension type allows a greater insulation value, and takes care of the possibility of unbalanced stresses better. This makes a system having a greater possibility of deflection to transverse wind loads, and it

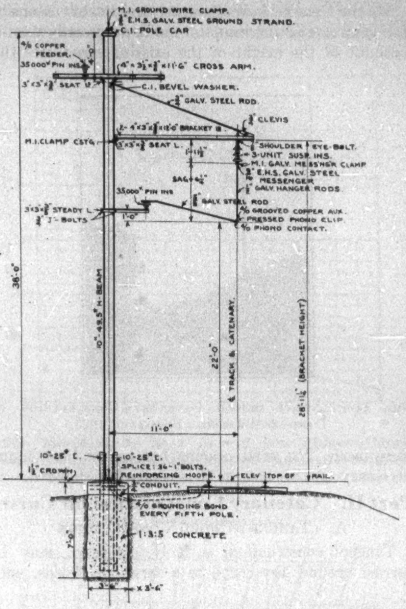
M.I. GROUND WIRE CLAMP.
3/8" E.H.S. GALV. STEEL GROUND STRAND.
C.I. POLE CAP.
4/0 COPPER FEEDER.
35000V PIN INS.
4"×3½"×⅜"×11'-6" CROSS ARM.
3"×3"×⅜" SEAT L.
C.I. BEVEL WASHER.
¾" GALV. STEEL ROD.
¾" CLEVIS
2-4"×3"×⅜"×12'-0" BRACKET L.
M.I. CLAMP CSTG.
3"×3"×⅜" SEAT L.
1'-11½"
SHOULDER EYE-BOLT.
3-UNIT SUSP. INS.
M.I. GALV. MESS'NG'R CLAMP
9/16" E.H.S. GALV. STEEL MESSENGER
SAG+4½"
½" GALV. HANGER RODS.
35,000V PIN INS
⅝" GALV. STEEL ROD
3"×3"×⅜" STEADY L.
¾" J-BOLTS
1'-0"
4/0 GROOVED COPPER AUX.
PRESSED PHONO CLIP.
4/0 PHONO CONTACT.
36'-0"
10"-49.5# H-BEAM
22'-0"
℄ TRACK & CATENARY.
28'-11½" (BRACKET HEIGHT)
11'-0"
10"-25# ⊏.
10"-25# ⊏.
SPLICE: 36-1" BOLTS.
REINFORCING HOOPS.
1½" CROWN
¾" CONDUIT.
ELEV. TOP OF RAIL.
4/0 GROUNDING BOND EVERY FIFTH POLE.
1:3:5 CONCRETE
7'-0"
3'-6" × 3'-6"

Fig. 12—Standard Tangent Bracket Pole for 250-Ft. Span

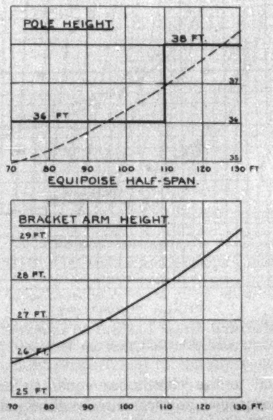

Fig. 13—Pole and Bracket Arm Heights

therefore seemed desirable to provide a steady rod or wind brace between the contact wire and the pole. This, with all other details, is shown in Fig. 14, which illustrates the finished appearance of the tangent construction on the Danbury Branch.

On the New York, Westchester, and Boston extension, pin type supporting insulators were necessarily used on account of the height of the existing structures that were used. No wind bracing has as yet been found necessary on this line, as it is somewhat shielded.

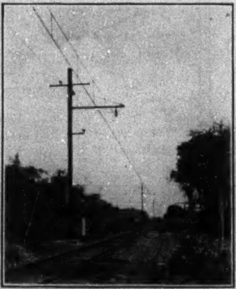

Fig. 14—Tangent Single Catenary Construction on Danbury Branch. Installed 1925

Part II. Catenary Construction on Curves

A. Tangent-Chord Construction

Tangent construction, suitably deflected, may be carried around any curve in a series of chords, with spans equal to the chords, or equal to the sum of several chords, using intermediate pull-off poles. Poles are preferably located on the outside of the curves.

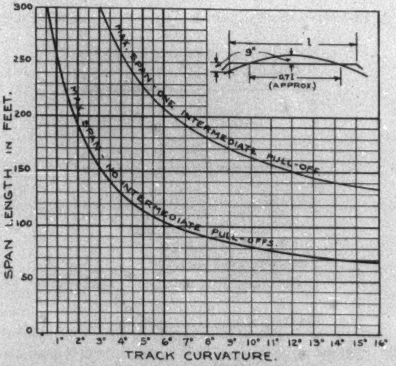

Fig. 15—Maximum Span Lengths—Tangent-Chord Construction on Circular Curves

The length of these chords will depend upon the degree of curvature and the maximum allowable departure (*q*) of the contact wire from the centerline of the pantograph collector. With a pantograph similar to that used on the New Haven equipment, the maximum allowable departure for design purposes is nine in. either side of the centerline. This means that a chord making a middle ordinate of 18 in. on any curve, is the maximum length of tangent catenary which can be used on that curve. Fig. 15 shows in graphic form the maximum chord length for tangent-chord construction, together with maximum span length for no intermediate pull-off poles, and for one intermediate pull-off pole, for a maximum middle ordinate of 1.5 ft., for various degrees of curvature. The contact wire is not, strictly speaking, a chord, but a secant to the curve, parallel to, and of the same length as, the chord, and distant from it an amount equal to half the middle ordinate, or nine in., (*q*). An interesting relationship, which is of value, is that this secant crosses the curve at points which are approximately 15 per cent of the chord length distant from the pull-off points, for all curves used in railroad work.

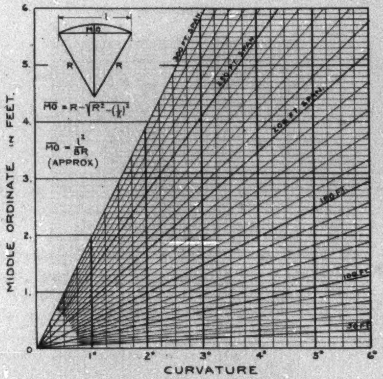

Fig. 16—Middle Ordinate Chart—For Various Spans and Degrees of Track Curvature

The middle ordinate, $\overline{MO}$, for any curve and chord is found from the expression

$$\overline{MO} = R - \sqrt{R^2 - \left(\frac{l}{2}\right)^2} \tag{10}$$

where *R* is the radius of the curve, and *l* is the chord length.

A convenient graphical representation is shown in Fig. 16. On curves and spans usually encountered in railroad work, the following parabolic expression may be substituted for (**10**):

$$\overline{MO} = \frac{l^2}{8R} \qquad (11)$$

which is more convenient, and accurate to within 0.5 per cent.

Since

$$R = \frac{5730 \text{ ft.}}{D} \quad \text{(approximately)} \qquad (12)$$

where D is the curvature in degrees, this may be written

$$\overline{MO} = 0.0000218\, l^2 D \qquad (13)$$

for all ordinary spans and curves.

In going from tangent to a curve, it is obvious that the limit to which the tangent can be extended beyond the point of curvature before a deflection becomes necessary, is where the "tangent offset" to the curve is equal to the allowable departure (q) from the centerline of the pantograph. In other words, the distance from the point of curvature to the first pull-off must not be greater than one-half of the chord whose $\overline{MO}$ is equal to q.

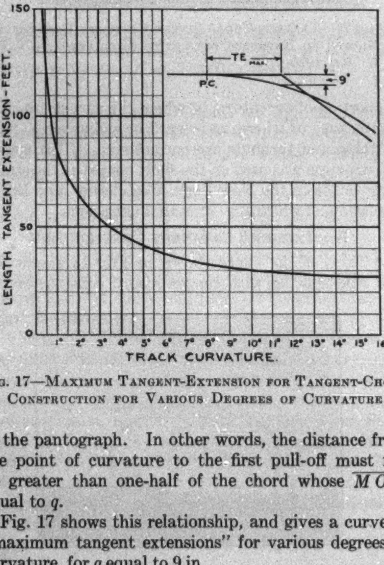

Fig. 17—Maximum Tangent-Extension for Tangent-Chord Construction for Various Degrees of Curvature

Fig. 17 shows this relationship, and gives a curve of "maximum tangent extensions" for various degrees of curvature, for q equal to 9 in.

On symmetrical reverse curves (having no tangent between) as on cross-overs and turnouts, or where main track is sometimes deflected for "island" station platforms, the wire must obviously be over the point of reverse curvature and should be secant to both curves so that all four maximum departures from the curves are equal; see Fig. 18.

It may be shown that the wire will cross the curve at points which are approximately 1/12 of the span length from the ends of the span, under these conditions. Fig. 18 also gives the maximum span which can be used on various degrees of reverse curvature, when q is equal to 9 in.

On tangent-chord construction, the contact wire will be offset from the pantograph centerline towards the outside of the curve an amount equal to q. The track, however, has the outside rail elevated above the level of the inside rail, which will tilt the centerline of the locomotive and pantograph towards the inside of the curve an amount proportional to the super-elevation of the outer rail and the contact wire height.

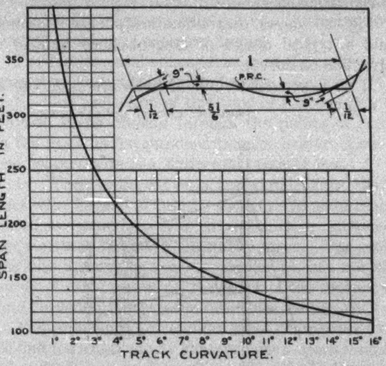

Fig. 18—Maximum Span Length for Tangent-Chord Construction on Reverse Curves of Various Degrees of Curvature

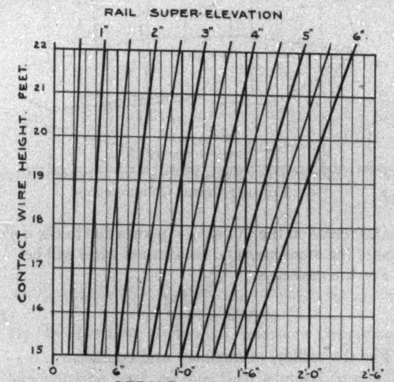

Fig. 19—Contact Wire Offset for Various Values of Rail Super-Elevation and Contact Wire Heights

Fig. 19 shows the offset z of the pantograph centerline from the track centerline, due to rail super-elevation, for various values of trolley height and super-elevation.

It is apparent that the true offset 0 of the insulator (and wire) from the track centerline, which is the base line for all construction measurements, will be

$$O = q - z \tag{14}$$

When O is positive, the wire is located outside the track centerline curve.

When O is negative, the wire is located inside the track centerline curve.

Fig. 20 shows diagrammatically this relationship, and a typical design of tangent-chord bracket construction on curves.

If no intermediate pull-off poles are used, the horizontal forces on the catenary supports due to the deflection of the messenger and the contact wires will be:

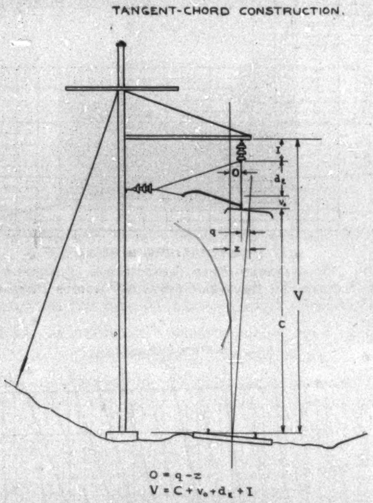

FIG. 20—ILLUSTRATING BRACKET-ARM AND INSULATOR OFFSET ON TANGENT-CHORD CONSTRUCTION

For the messenger,

$$P_M = \frac{l\,M}{R} \tag{15}$$

For the trolley, or auxiliary and contact,

$$P_T = \frac{l\,T}{R} \tag{16}$$

where P_T is the "curve-pull" in pounds and T is the sum of the tensions in the auxiliary and the contact wires.

The total curve pull on the structure will be

$$P = \frac{l\,(T + M)}{R} \tag{17}$$

in addition to wind loads. If intermediate pull-offs are used, l will be reduced to an amount equal to the chord length.

Tangent-chord construction has certain distinct advantages. The design details are the same on curves as on tangents, and the alinement of the wire is practically fixed, so far as temperature changes and wind loads are concerned. This type of construction is recommended for track alinements having a small proportion of curvature, or where the curves are short, and heavy; or where, as in yard or siding work, many switches and turnouts are encountered. This type of construction was used on the short curves at the station approaches on the New York, Westchester, and Boston extension, of which Fig. 21 is an illustration.

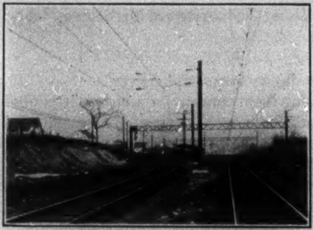

FIG. 21—TANGENT-CHORD SINGLE CATENARY CONSTRUCTION ON EIGHT-DEG. REVERSE CURVE AT STATION APPROACH, N. Y. B. & W. RY. 1925

B. INCLINED CATENARY CONSTRUCTION

A study of the curve shown in Fig. 15 shows that the pole spacing with tangent-chord construction decreases very rapidly for curves up to about five deg., and at a slower rate for curves above five deg. This means more supports, each with double the usual number of insulators required for tangent construction, on account of the pulloffs; which of course increase the cost.

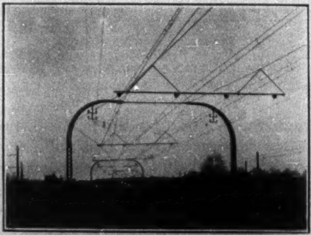

FIG. 22—THE ORIGINAL "INCLINED CATENARY" INSTALLATION. EXPERIMENTAL COMPOUND CATENARY CONSTRUCTION AT GLENBROOK, 1909

In 1909, the New Haven engineers developed the so-called "curved catenary," or what is more commonly now called "inclined catenary" (on account of the position assumed by the hangers) with the idea of eliminating to a large extent the necessity of pull-offs, and also increasing the span length, on curves of moderate degree of curvature.

The original installation was an experimental stretch of four-track construction one mile in length at the easterly end of the original double catenary construction, near Glenbrook, Conn.; see Fig. 22. This was compound catenary, with 300-ft. spans on both tangent and two-deg. curve. The live single catenary consisting of a ⅝-in. steel strand with a 4/0 copper auxiliary and 4/0 contact wire was supported from two main messengers, not alive, every 100 ft. Over the curved track, the contact wire followed the track alinement very closely, and the hangers were inclined at an angle which was the resultant of the weight of the trolley wires and the curve-pull reaction between these wires and the messenger.

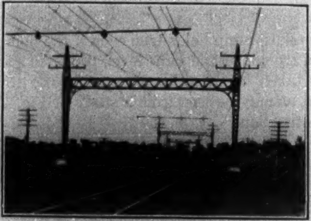

Fig. 23—Compound Catenary Construction Installed in 1913 Between Stamford and New Haven, Showing Inclined Catenary on Two-Deg. Curve

This type of construction seemed to present such distinct advantages over the double catenary, that it was adopted, in a form modified as to supporting structures and main messenger construction, for the electrification of the six-track Harlem River Branch, and the four-track New York, Westchester, and Boston, in 1911, and for the four-track extension to New Haven in 1913, which is shown in Fig. 23. The design of this type of construction has been very completely described by Mr. Sidney Withington, in the *Journal of the Franklin Institute*, December, 1914, Vol. CLXXVIII, No. 6.

On this type of compound catenary, the live catenary spans were fairly short, being but 150 ft. for the maximum condition. The calculation involving the inclined rods on curves was therefore comparatively simple, as the errors made in assuming the hangers had the same inclination throughout the span, were very small. These calculations were based on the theory that the contact wire had a parabolic shape, with the horizontal "sag" equal in value to the middle ordinate of the circular curve of the track for that span. The horizontal force or "curve-pull" was assumed to be uniform for each hanger rod. Both of these assumptions are fallacious, but do not introduce appreciable errors until it is attempted to apply this method of design to single catenary having much longer spans, and on sharper curves.

The following solution of this problem is not new; some of it is not original; but it has not, to the writer's knowledge, been set forth heretofore in a form which is available for engineers who may be interested in the general problem of catenary design.

The fundamental characteristics of the inclined catenary may be summarized as follows:

1. The projection of this type of construction on a vertical plane passing through the points of support is the same as for standard tangent construction, and the contact wire is a horizontal straight line.
2. When projected on a horizontal plane, the contact wire, instead of being a straight line as in tangent-chord construction, is a curve which is intended to approximate the track alinement curve. The messenger also appears as a curve, similar to, but not necessarily of same degree as, the shape of the contact wire, and curved in the opposite direction; see Fig. 24.

The problem is to combine these characteristics and to find the length of any hanger under various conditions of span length and track curvature. Before undertaking the solution, the factors entering into the problem must be resolved into their simplest forms. It is apparent that two opposing systems in static equilibrium are being dealt with and that some of the factors involved are the weights and tensions in each system. Therefore if either or both of the systems be composed of more than one element, the elements must be combined and replaced, for calculation purposes at least, by a single element whose tension and weight has the same effect on each system as the elements thus replaced.

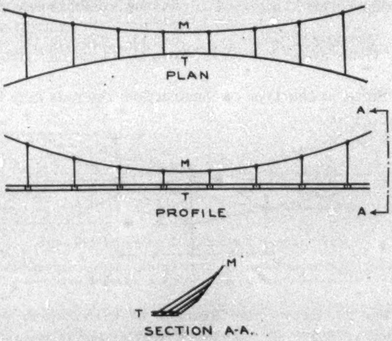

Fig. 24—Inclined Catenary. General Characteristics

The design used on the Danbury Branch, described in Part I, will be used to illustrate the theory throughout this discussion, although the general theory will apply to any other design. In the solution of this particular design, the two 4/0 wires must be replaced by a single element having a tension equal to the sum of the normal tensions in these two wires. Their weight must also be combined, and with it should be included the weight of the hardware used on the lower end of the rods, together with one-half of the weight of the rods themselves. This may be designated the "trolley system." To the messenger must be added the weight of the hardware on the upper end of the rods, as well as the other half of the weights of the rods. This is the "messenger system."

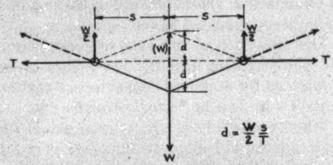

FIG. 25—ILLUSTRATING RELATION OF DISPLACEMENT TO STRESSES IN A SIMPLE SYMMETRICAL SYSTEM

The combined characteristics then are, for normal conditions:

	Tension	Unit weight
Messenger System	3900 Lb.	0.732 Lb. (Min.)
Trolley System	3415 Lb.	1.336 Lb. (Min.)

Since, in this type of construction, the rods may become quite long, it will make for closer accuracy to assume that the unit weight of each system is to be compensated for the increase in hanger weight from the center to the ends of the span. Therefore, the weights given above are based on short hanger lengths at the center of the span.

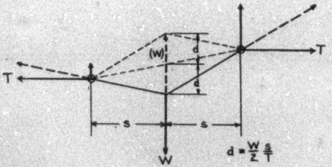

FIG. 26—ILLUSTRATING RELATION OF DISPLACEMENT TO STRESSES IN A SIMPLE ASYMMETRICAL SYSTEM

Relation Between Messenger and Trolley Curve. In Fig. 25, let W be a weight suspended from a flexible weightless strand at a point midway between two fixed supports which are on the same level, and at a distance $2\,s$ from each other.

Let d be the deflection or sag in the strand;

Let T be the horizontal component of the tension in the strand.

Then, by construction,

$$d : s :: \frac{W}{2} : T$$

or

$$d = \frac{W}{2}\,\frac{s}{T}$$

This relationship holds true even if the points of suspension are not on the same level, provided s and T are always taken horizontal (or perpendicular to W) and parallel, as may be seen from a study of Fig. 26, since, by construction,

$$2\,d : W :: s : T$$

or

$$d = \frac{W}{2}\,\frac{s}{T}$$

It is apparent that the above relationship will also apply to forces acting in a horizontal plane, where the weight W is replaced by a horizontal pull P; and the strand will indeed be "weightless" under these conditions.

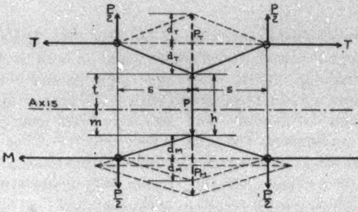

FIG. 27—TWO SIMPLE COMBINED OPPOSING SYSTEMS. ILLUSTRATING RELATION OF DISPLACEMENTS TO TENSIONS

The above expression will then become

$$d = \frac{P}{2}\,\frac{s}{T} \tag{18}$$

In a horizontal system just described, the pull P may be produced by a connection to another similar system having the same distance ($2\,s$) between its supports and an equal but opposite force P. The tension in the strands of each system may be the same, or they may be different, in which case the deflections will be different; for, in Fig. 27, let system 1 have a horizontal tension T in its strand, with a deflection d_T due to the pull P.

From (**18**):

$$d_T = \frac{P\,s}{2\,T}$$

Let System 2 have a horizontal tension M in its strand, with a deflection d_M due to the same pull P.

Then

$$d_M = \frac{P\,s}{2\,M}$$

and

$$2\,T\,d_T = P\,s = 2\,M\,d_M$$

whence

$$\frac{d_M}{d_T} = \frac{T}{M} \tag{19}$$

If, in Fig. 27, the distance between the systems at the tie through which P acts, is h, this length may be divided into two parts, t and m

$$h = t + m \tag{20}$$

such that

$$m : t :: d_M : d_T \text{ or } \frac{m}{t} = \frac{d_M}{d_T}$$

Substituting in (**19**),

$$\frac{m}{t} = \frac{T}{M} \tag{21}$$

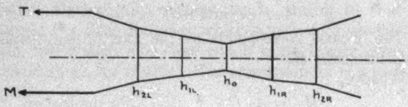

FIG. 28—MUTUAL REFLECTION OF SHAPE IN TWO OPPOSING SYSTEMS

then

$$m = t\,\frac{T}{M}$$

Substituting in (**20**),

$$h = t\left(1 + \frac{T}{M}\right) \tag{22}$$

If the two systems are balanced against each other by ties at regular intervals (s), it is apparent from the foregoing that the deflection in either system due to the pull at any tie, referred to its two adjacent tie points, will have the definite relationship expressed by (**19**), and that any tie can be divided into two parts which will have the relationship expressed by (**21**). It follows that a line drawn through each of these division points on every tie is a straight line perpendicular to each tie, since by construction the ties are all parallel. This line may be called the system axis.

The ties may be such that each system has an irregular shape as in Fig. 28, in which case some ties may have no tension (as h_{1L} in the figure), or some may have compression (as h_{1R} in the figure); or the ties may be so designed that the systems have a symmetrical shape about an axis through the shortest tie, perpendicular to the system axis, as shown in Fig. 29. A symmetrical system may be tangent to the other symmetrical system at the shortest tie, when this is equal to zero, or the two systems may even be crossed and reversed, as shown in Fig. 30.

In each and every case, the following relations obtain:

$$h = t + m \text{ for any tie.}$$

$$H = H_T + H_M$$

$$\frac{m}{t} = \frac{d_M}{d_T} = \frac{T}{M} = \frac{H_M}{H_T}$$

The general statement may therefore be made that the shape of the trolley system, referred to the system axis, will be a reflection of the shape of the messenger system, and vice versa; differing only in degree, which degree is expressed by the inverse ratio of their respective tensions.

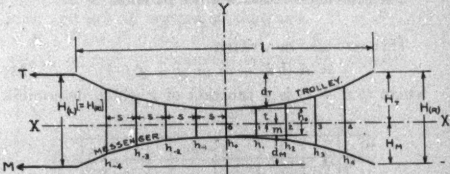

FIG. 29—INCLINED CATENARY CONSTRUCTION

Two combined opposing systems with symmetrical horizontal displacements about axis through shortest tie. (The *U*-shape.)

It is apparent from an inspection of the two symmetrical curves shown in Figs. 29 and 30, that they are both some function of the span length. If the system axis is represented by the X axis, and the perpendicular axis through the shortest tie is represented by the Y axis, then either of these curves may be expressed

$$y = f(x)$$

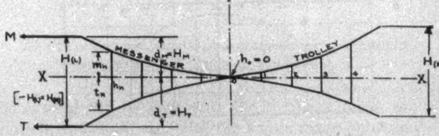

FIG. 30—INCLINED CATENARY CONSTRUCTION

Two combined opposing systems which are crossed and reversed, having similar horizontal displacements about axis through shortest tie. (The *S*-shape.)

Before these symmetrical shapes can be applied to the track curve, something of their nature must be known to determine their adaptibility and their limitations. One method of doing this would be to plot their shapes, as determined by use of their correct constants, for comparison with the shape of the track curve.

Track curves are, in general, circular, but the equation of the circle, and especially that of circular middle ordinates, (**10**), is rather unwieldly. If it can be assumed that the track curve is a parabola, the parabolic expression (**11**) for the middle ordinate is much more conveniently handled. The middle ordinate of a 300-ft. span on a four-deg. curve when calculated by (**10**) is 7.80 ft. and when calculated by (**11**) is 7.85 ft. The difference or error is only five parts in 780, or 0.64 per cent. As this is an extreme case, it is apparent that for the spans and curves involved, it is well within the allowable limits to assume that the track curve is a parabola instead of a circle.

The general expression for the parabola is

$$a\,y = b\,x^2 + c$$

This may also be written

$$y = A\,(b\,x^2 + c) \text{ or } y = A\,f\,(x) \qquad \textbf{(23)}$$

where $(b\,x^2 + c)$ is a function of x which determines the shape about the Y axis and A is a factor which determines the magnitude of the ordinates from the X axis. The function of x, $(b\,x^2 + c)$, may be called the "shape factor" and the factor A may be called the "multiplier."

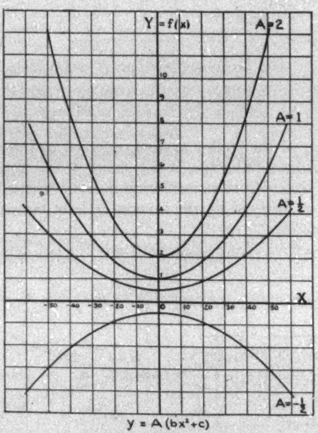

FIG. 31—ILLUSTRATING THE USE OF THE "MULTIPLIER" (A) ON THE "SHAPE FACTOR" $(b\,x^2 + c)$

As an illustration of their use, in Fig. 31, the parabola $a\,y = b\,x^2 + c$ is shown (plotted for certain values of b and c) for A equal to 1, 2, ½, and $-$ ½.

The shape in each case is the same, (parabola), and each curve has the same constants for b and c for a given value of x; but the curve where $A = 2$ has ordinates twice those of the curve $A = 1$, and similarly the latter curve has ordinates twice the value of those of the curve $A = ½$. The ordinates of the curve $A = -½$ are equal to those of the curve $A = ½$, but are on the opposite side of the X axis. If the ordinates of the two last mentioned curves are combined, and added geometrically, their result will be a new curve having the same "shape factor," but a new value of A, in this case, equal to 1.

Curve $A = 1$ has the following value of $f\,(x)$ for different values of x

$\pm x$	$f\,(x)$
0	1.0
10	1.2
20	1.8
30	2.8
40	4.2
50	6.0
etc.	etc.

Each of these ordinates, multiplied by any value of the "multiplier" A, will always give a curve having the same "shape factor" $f\,(x)$.

In Fig. 29, which is that of two symmetrically opposed horizontal systems, let the curve of the trolley system be

$$y = A_T\,F\,(x) \text{ and let } F\,(x) = U$$

The Y axis corresponds to the axis of horizontal displacements, H.

Whence

$$H_T = A_T\,U \qquad \text{for the trolley}$$

and

$$H_M = A_M\,U \qquad \text{for the messenger}$$

Then the distance between the trolley and the messenger is

$$H = A\,U \qquad \textbf{(24)}$$

where

$$A = A_T + A_M$$

and the multiplier A and the shape factor U are to be determined.

Similarly, in Fig. 30, which is also that of two symmetrically opposed systems, but which are crossed and reversed, let the curve of the trolley be

$$y = B_T\,F'\,(x) \text{ and let } F'\,(x) = S$$

Then, as above,

$$H_T = B_T\,S \qquad \text{for the trolley}$$

and

$$H_M = B_M\,S \qquad \text{for the messenger}$$

and the distance between the trolley and the messenger is

$$H = B\,S \qquad \textbf{(25)}$$

where

$$B = B_T + B_M$$

and the multiplier B and the shape factor S are to be determined.

In the symmetrical system shown in Fig. 29,

$$H_R = H_L = A\,U$$

Also in the symmetrical system shown in Fig. 30,

$$H_R = -H_L = B\,S$$

If the constants and variables, such as messenger and trolley tensions, weights, and materials which determine the system shape in Fig. 29 are the same for Fig. 30, and if H_R (Fig. 30) is equal to H_R (Fig. 29) for the same value of x, then, if the two shapes are added together algebraically, a new shape similar to that shown in Fig. 32 is obtained, which is, as before, similar about the system axis, but which is no longer symmetrical about the H axis. The expression for this new shape is

$$H = A\,U + B\,S \qquad \textbf{(26)}$$

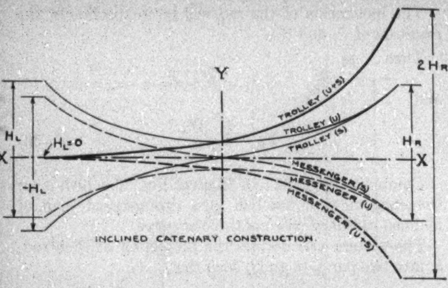

FIG. 32—THE COMBINED U AND S SHAPE, OR TRANSITION CURVE

It is clear that the shapes U and S may be combined with each other in any degree. If the factor B is zero, the shape S is not present; and the system has the shape shown in Fig. 29, which is for a symmetrical continuous curve. When A is zero, the shape U is not present, and the system has the symmetrical shape shown in Fig. 30, which is a reverse curve. When A and B are present to an equal degree, the system has the shape shown in Fig. 32, which is a transition curve of some kind.

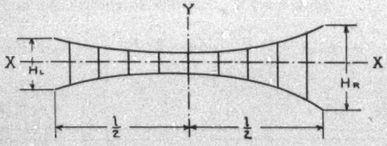

FIG. 33—INCLINED CATENARY WITH ASYMMETRICAL HORIZONTAL DISPLACEMENTS ABOUT LOW POINT

Conversely, it is also clear that any continuous curve, as in Fig. 33, which is not symmetrical with respect to the H axis, that is, where the distance between the trolley and the messenger is not the same, for the same distance each side of the H axis ($H_R \gtrless H_L$) such a shape partakes of both shapes U and S.

The expression (**26**) may therefore be said to be the general expression for any combined horizontal system.

If U and S can be determined for a given system whose variables are known, the shape of any span on that system may be determined by the expression (**26**), by determining the values of their multipliers, A and B.

The Determination of the U Shape. Fig. 34 is that of a funicular polygon, whose shape is assumed to be symmetrical about its normal axis H. The forces P_0, P_1, P_{-1}, P_2, P_{-2}, P_3, P_{-3}, etc., are assumed to be acting on a system whose horizontal component of tension (parallel to the horizontal axis X) is T; and these forces are assumed to occur at equal intervals (s) along the X axis, and are all normal to that axis. $P_1 = P_{-1}$, $P_2 = P_{-2}$, $P_3 = P_{-3}$, etc., and P_0, P_1, P_2, P_3, etc., may be equal, or may progressively increase or decrease. The system will be deflected, due to these forces, at points 0, 1, 2, 3, 4, etc., and the angles made between the sides of the polygon, and parallels to the X axis, are θ_1, θ_2, θ_3, θ_4, and θ_5 respectively. For convenience, the stress diagram is shown below in the figure.

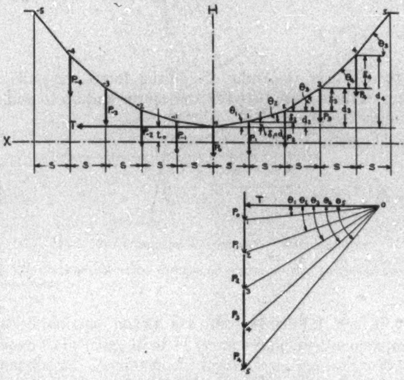

FIG. 34—THE FORCES AND DISPLACEMENTS IN A REGULAR SYMMETRICAL FUNICULAR POLYGON

Let t_0 be the distance from the polygon to the X axis at its nearest point (0); let δ_1, δ_2, δ_3, δ_4, etc., be the departure measured normal to the X axis, due to θ_1, θ_2, θ_3, θ_4, etc.; and let d_1, d_2, d_3, d_4, etc., be the departures from a line drawn parallel to the X axis through the nearest point, 0.

Then

$$\tan\theta_1 = \frac{\delta_1}{s} = \frac{P_0}{2\,T} \text{ (in the stress diagram)}$$

and

$$\delta_1 = \frac{s}{T}\,\frac{P_0}{2}$$

Similarly,

$$\tan\theta_2 = \frac{\delta_2}{s} = \frac{\dfrac{P_0}{2} + P_1}{T}$$

and

$$\delta_2 = \frac{s}{T}\left(\frac{P_0}{2} + P_1\right),$$

and in like manner,

$$\delta_3 = \frac{s}{T}\left(\frac{P_0}{2} + P_1 + P_2\right)$$

and

$$\delta_n = \frac{s}{T}\left(\frac{P_0}{2} + P_1 + P_2 + P_3 + \ldots + P_{n-1}\right) \tag{27}$$

$$= \frac{s}{T}\,\Sigma\, P_n$$

where $\Sigma\, P_n$ is the sum of all the forces, or pulls, acting on the system between the center and up to and including the point $n - 1$.

In like manner, for d

P_n is the horizontal pull, or displacing force, acting on the lower end of the rod r_n,

W_n is the weight, or restoring force acting on the lower end of the rod,

t_n and m_n are the distances from the trolley and the messenger, respectively, to the system axis,

r_n is the theoretical length of the rod, and is the geometrical sum of v_n and h_n.

The inclination of the rod will be in the line of the resultant of P_n and W_n.

Then

$$P_n : h_n : : W_n : v_n$$

and

$$P_n = h_n \frac{W_n}{v_n} \tag{30}$$

Combining Figs. 34 and 35 gives Fig. 36, which is an isometric view of one-half of a symmetrical span of inclined catenary on a continuous curve.

The system axis divides the projection of each rod, h, into two parts, m and t, such that

$$\frac{t}{m} = \frac{M}{T}$$

$$d_1 = \delta_1 \qquad = \frac{s}{T}\left(\frac{P_0}{2}\right)$$

$$d_2 = d_1 + \delta_2 = \delta_1 + \delta_2 \qquad = \frac{s}{T}\left(\frac{2\,P_0}{2} + P_1\right)$$

$$d_3 = d_2 + \delta_3 = \delta_1 + \delta_2 + \delta_3 \qquad = \frac{s}{T}\left(\frac{3\,P_0}{2} + 2\,P_1 + P_2\right)$$

$$d_4 = d_3 + \delta_4 = \delta_1 + \delta_2 + \delta_3 + \delta_4 \qquad = \frac{s}{T}\left(\frac{4\,P_0}{2} + 3\,P_1 + 2\,P_2 + P_3\right)$$

$$d_n = d_{n-1} + \delta_n = \delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n \qquad = \frac{s}{T}\left(\frac{n\,P_0}{2} + (n-1)\,P_1 + (n-2)\,P_2 + \ldots + P_{n-1}\right)$$

$$= \frac{s}{T}\,Q_n \tag{28}$$

where Q_n is the expression in the parenthesis.

The distance, or ordinate, of any point on the polygon from the X axis is

$$t_n = d_n + t_0 \tag{29}$$

The forces and the figure just discussed must now be combined with the shape and the forces of the tangent catenary.

For calculation purposes, each hanger is reduced to the elementary form shown in Fig. 35.

In this figure,

v_n is the projection of any rod r_n on a vertical plane (calculated from expression (**4**)),

h_n is the projection of that rod on a horizontal plane,

The known constants of the catenary used for illustration are:

M = 3900 lb.
T = 3415 lb.
s = 10 ft.
W_0 = 13.36 lb.
v_0 = 0.448 ft.

t_0 is assumed to be unity.

The derived constants are

$$\frac{T}{M} = 0.876 \text{ and } \frac{s}{T} = 0.00293$$

also

$$h_0 = 1.876 \qquad \text{(from } (\mathbf{22})\text{)}$$

$$\frac{W_0}{v_0} = 29.8 \text{ whence } P_0 = 55.9 \qquad \text{(from } (\mathbf{30})\text{)}$$

$$\Sigma P_0 = \frac{P_0}{2} = 27.95 = Q_0 \text{ and } \frac{s}{T} Q_0 = 0.0819 = d_1$$

(from (**28**))

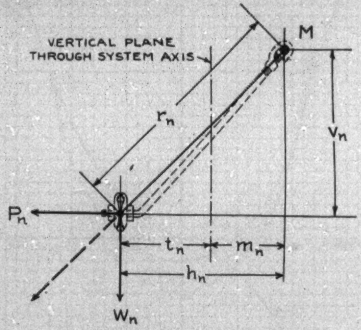

Fig. 35—Inclined Catenary Construction

Elements of hanger rods, contact, and messenger

Then, from (**29**), $t_1 = 1.0819$ which is the first point on the U curve. Continuing,

$$h_1 = 2.025 \qquad \text{(from } (\mathbf{22})\text{)}$$
$$v_1 = 0.476 \qquad \text{(from } (\mathbf{4})\text{)}$$
$$W_1 = 13.37$$

(Note: W will increase in accordance with a previous rough calculation of the weights of rods on an average curve (3 deg., using parabolic formula. The error made, leaving W constant, is very small.)

$$\frac{W_1}{v_1} = 28.1$$

$$P_1 = 56.9 \qquad \text{(from } (\mathbf{30})\text{)}$$
$$\Sigma P_1 = 84.85 \qquad \text{(from } (\mathbf{27})\text{)}$$
$$Q_1 = 112.8 \qquad \text{(from } (\mathbf{28})\text{)}$$
$$d_2 = 0.331 \qquad \text{(from } (\mathbf{28})\text{)}$$

and

$t_2 = 1.331$ from (**30**) which is the second point on the U curve.

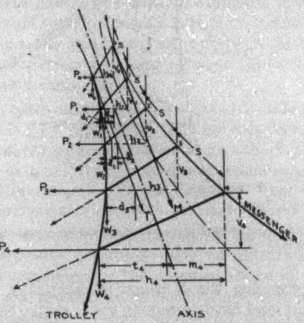

Fig. 36—Inclined Catenary Construction

The development of the U-shape for use on circular continuous curves. (See Table I)

In this manner, the calculations may be continued until enough points are determined for the longest span to be used, 16 or 17 being ample for the maximum condition. A convenient form for setting up the calculations is shown in Table I.

The points may then be plotted, with reference to an

TABLE I

	$\frac{l_E}{2}$	$\frac{S}{T}Q$	Shape Factor U $t_0 + d_n$	$t_n\left(1+\frac{T}{Sm}\right)$	From Tan. Constr.	See Note in Text.		$\frac{W}{v}h$	$\frac{P_0}{2}+P_1+P_2+P_3+\ldots+P_n$	$Q_n = \Sigma P_n + Q_{n-1}$
n	x	d	t	h	v	W	$\frac{W}{v}$	P	ΣP	Q
0	0	0.00	1.00	1.876	0.448	13.36	29.8	55.9	27.95	28
1	10	0.08	1.08	2.025	0.476	13.37	28.1	56.9	84.85	113
2	20	0.33	1.33	2.495	0.562	13.41	23.9	59.6	144.5	257
3	30	0.75	1.75	3.29	0.704	13.49	19.2	63.0	207.5	465
4	40	1.36	2.36	4.43	0.903	13.58	15.1	66.7	274	739
5	50	2.17	3.17	5.94	1.16	13.71	11.8	70.2	344	1083
6	60	3.18	4.18	7.83	1.472	13.88	9.44	73.9	418	1502
7	70	4.40	5.40	10.13	1.842	14.06	7.63	77.8	496	1998
8	80	5.85	6.85	12.84	2.27	14.28	6.3	80.9	477	2575
9	90	7.54	8.54	16.00	2.75	14.54	5.29	84.6	662	3236
10	100	9.46	10.46	19.6	3.29	14.80	4.5	88.2	750	3986
11	110	11.67	12.67	23.8	3.89	15.11	3.9	92.6	842	4828
12	120	14.15	15.15	28.4	4.55	15.45	3.4	96.6	939	5767
13	130	16.9	17.9	33.6	5.26	15.80	3.01	101.	1040	6807
14	140	19.93	20.93	39.2	6.04	16.25	2.69	105.3	1145	7952
15	150	23.3	24.3	45.6	6.87	16.75	2.44	111.2	1256	9208
16	160	27.0	28.0	52.5	7.76	17.3	2.23	117.	1373	10581
17	170	31.	32.	60.	8.70		2.1	123		

X and a *U* axis. It will be symmetrical about the *U* axis, and will always be positive, for any value, positive or negative, of *x*.

The Determination of the S Shape. The method used for determining the *S* shape is practically identical with that used for determining the *U* shape, as outlined above, with one or two exceptions. Fig. 37 shows an isometric diagram of half of a symmetrical span of inclined catenary on a reverse curve. It will be seen that the shortest rod in this diagram is not inclined; therefore $h_0 = t_0 + m_0 = 0$, and no assumptions can be made as to their value, to start the calculations, as in the case of the *U* curve.

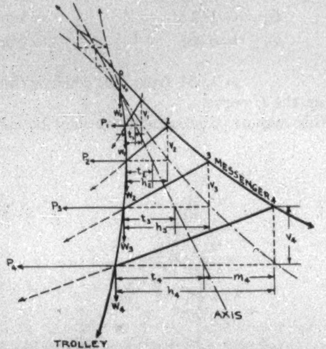

FIG. 37—INCLINED CATENARY CONSTRUCTION

The development of the *S*-shape for use on reverse curves and transitions. (See Table II)

A convenient value can, however, be assigned to the deflection at the next adjacent rod, with which assumption, and the same constants used in the calculations for the *U* curve, the calculations may be made, point by point, until the same number has been determined as for the *U* curve. These are shown tabulated in Table II.

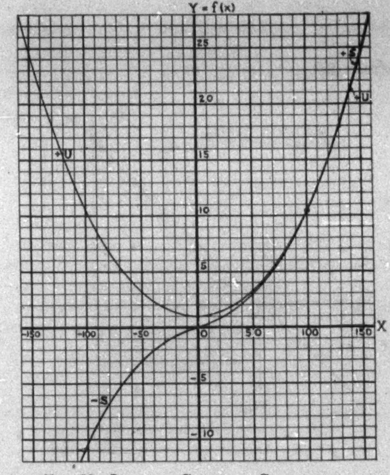

FIG. 38—INCLINED CATENARY CONSTRUCTION

The "Shape Factor" curves *U* and *S* plotted from Tables I and II for 5/16-in. catenary

TABLE II

	$\frac{l_E}{2}$	Shape Factor S ($d \times$ const.)	$\frac{S}{T}Q$	$t_0\left(1+\frac{T}{M}\right)$	From Tan. Constr.	See Note in Text.		$\frac{W}{v}h$	$P_0 + P_1 + P_2 + P_3 + \ldots + P_n$	$Q_n = Q_{n-1} + \Sigma P_n$
n	x	S	d (= t)	h	v	W	$\frac{W}{v}$	P	ΣP	Q
0	0	0.00	0.00	0.00	0.448	13.36	29.8	3.41[bc]	3.41[b]	3.41[b]
1	10	0.38	0.01[a]	0.019	0.476	13.37	28.1	0.526	3.94	7.35
2	20	0.82	0.022	0.040	0.562	13.41	23.9	0.965	4.90	12.25
3	30	1.38	0.036	0.067	0.704	13.49	19.2	1.292	6.19	18.44
4	40	2.08	0.054	0.101	0.903	13.58	15.05	1.526	7.72	26.16
5	50	2.95	0.077	0.144	1.160	13.71	11.82	1.70	9.42	35.58
6	60	4.00	0.104	0.195	1.472	13.88	9.44	1.844	11.26	46.84
7	70	5.28	0.137	0.258	1.842	14.06	7.63	1.965	13.23	60.07
8	80	6.77	0.176	0.331	2.268	14.28	6.3	2.085	15.31	75.38
9	90	8.5	0.221	0.414	2.753	14.54	5.29	2.19	17.50	92.88
10	100	10.46	0.272	0.51	3.293	14.80	4.5	2.30	19.80	112.08
11	110	12.7	0.331	0.62	3.888	15.11	3.89	2.41	22.21	134.89
12	120	15.2	0.395	0.741	4.548	15.45	3.4	2.52	24.73	159.62
13	130	18.0	0.468	0.876	5.258	15.8	3.01	2.64	27.37	186.99
14	140	21.1	0.548	1.028	6.038	16.25	2.69	2.765	30.13	217.12
15	150	24.45	0.636	1.193	6.868	16.75	2.44	2.915	33.05	250.17
16	160	28.2	0.733	1.374	7.758	17.3	2.24	3.08	36.13	286.3
17	170	32.25	0.839	1.572			2.09	3.29	39.42	325.7

NOTES: (a) Assumed, to start calculations.
(b) Calculating backwards from starting assumption.
(c) Due to component of pull of other half of span.

On this curve, $t_n = d_n$ and when calculated from the assumed value of d_1 their values may not be anything like the values of t in the U curve. When they are all determined, however, the value of d at any point (for example, at point $x = 100$) may be multiplied by a factor which will make it the same as the value of U for that point. Then if all the other values of d are multiplied by the same factor, the new curve will still have the S shape, but will be of a magnitude which can be conveniently compared with the U curve.

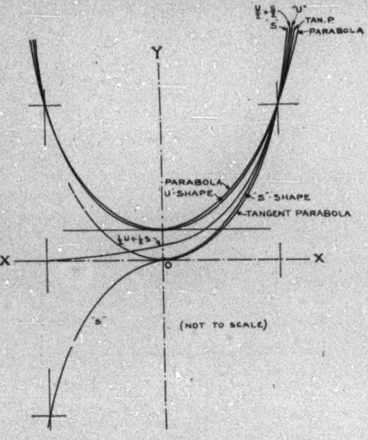

FIG. 39—INCLINED CATENARY

Comparison of U and S shapes with parabola. (See Table III)

The values of S are negative for negative values of x. Both curves are shown, plotted to scale for the determination of intermediate values, in Fig. 38.

COMPARISON OF THE U SHAPE AND THE S SHAPE WITH THE PARABOLA

Having determined the ordinates of the shapes U and S, they may now be compared with each other and with the parabola, which is the assumed shape of the track curve on ordinary curves and span lengths. If, by a proper choice of multipliers, these three curves are plotted for a given span, so that the U curve and the parabola are tangent to each other at the $f(x)$ or Y axis and all three curves have the same ordinate for $x = l/2$, it will be seen that the U curve lies outside the parabola from the point of tangency at the center until it crosses the parabola at the end of the span ($x = l/2$), and is inside thereafter.

The S curve passes through the origin, and, for positive values, lies closely outside of the U curve, crossing it and the parabola at the end of the span, and remaining inside of both curves thereafter; see Fig. 39. The amount of departure from the parabolic shape may best be seen from the tabulated values of the ordinates of the three curves, shown in Table III, calculated for equal values of all three curves for $x = 100$.

The U Shape and the Parabola. It will be noted that the maximum difference between the U shape and the parabola occurs at about $x = 60$ and that this difference is 0.23 units. If the value of the ordinate at $x = 100$ represents feet, then the maximum departure from the track shape is 0.23 ft., or less than three in. The curve having a middle ordinate of 10.46 ft. for a 200-ft. span is approximately 12 deg., which is a very extreme curve for this span. A four-deg. curve having one-third this ordinate for the same span would therefore have a maximum variation of not more than one inch between the shape of the track and the U shape for any point between the center of the span and the support.

For points outside the span ($x > l/2$) this difference increases very rapidly; so that for this reason, if for no other, the ordinates calculated for a short span should not be extrapolated and used on a longer span without careful investigation of the departure from the track curve.

It is clear, therefore, that the U curve (at least for the particular constants used for illustration) very closely approximates the assumed shape of the track curve, and presents no serious limitations on account of such departures as exist, at least for points within the span used for calculations.

The S Shape and the Parabola. It is obvious that the S shape is only used alone on reverse curves. These are rarely circular, but spiral, so that no accurate comparisons can be made. In general, however, it may be seen from the tabulated values in Table III, that for points near the end of the span, where the track begins to approach the parabolic or circular shape, the values are very nearly the same as for the U-shape. For comparison at the point of reverse curvature (the origin), it is necessary to shift the parabola tangent to the X axis. The values of the

TABLE III

x	Parabola $f(x)$	U Shape $F(x)$	S Shape $F'(x)$	Tangent Parabola	$\frac{1}{2}U + \frac{1}{2}S$
0	1.00	1.00	0.00	0.00	0.50
10	1.09	1.08	0.39	0.10	0.73
20	1.38	1.33	0.82	0.42	1.08
30	1.85	1.75	1.38	0.94	1.55
40	2.51	2.36	2.08	1.67	2.22
50	3.37	3.17	2.95	2.61	3.06
60	4.41	4.18	4.00	3.76	4.09
70	5.64	5.40	5.28	5.13	5.34
80	7.05	6.85	6.77	6.69	6.81
90	8.66	8.54	8.50	8.46	8.52
100	10.46	10.46	10.46	10.46	10.46
110	12.43	12.67	12.7	12.63	12.68
120	14.61	15.15	15.2	15.05	15.17
130	17.0	17.9	18.0	17.7	17.95
140	19.53	20.9	21.1	20.5	21.0
150	22.3	24.3	24.45	23.6	24.37
160	25.6	28.0	28.2	27.2	28.1

ordinates to the tangent parabola are also given in Table III, and from them it is seen that the maximum departure is for approximately $x = 30$. On a four-deg. curve, the indicated difference would be less than two inches.

If both shapes are present in equal amount, making the transition shape referred to in Fig. 32, it will be seen in like manner from the tabulated values of the ordinates of $U/2 + S/2$, also shown in Table III, that for the half of the span which lies on the curve, the variation from the tangent parabola is not more than 0.65, at approximately $x = 20$. Reduced to a four-deg. curve, this would not exceed three inches.

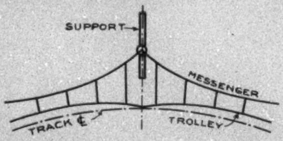

Fig. 40—Illustrating the Cusp at the Support when Contact Wire has Same Middle Ordinate as Track Curve, Inclined Catenary

This combination is where the greatest departure from the assumed track shape exists, and also the place where the greatest variation in track alinement may be expected, since most railroad curves are spiralled at the points of tangency and points of curvature. For these reasons it is sometimes well to make a set of field measurements of actual track departures from the tangent, on such spans, for comparison with the trolley curve shape, when designed.

The above differential values are, of course, true only for the catenary design used for illustration, but they may be taken as being representative; and in general, within the limits of reasonable application, these shapes do not, *within the span,* present serious limitations either used alone or in combination.

The most important limitation due to the shape of both the U curve and the S curve, occurs at the point of support or end of the span, where on account of the progressive increase in the curve pull with the span length, all the curves are sharper than the track curve, and cross it at this point, as indicated in Fig. 39. On long spans and short radius curves this will produce an irregularity, or cusp, between spans, as shown in an exaggerated manner in Fig. 40, which may spoil the appearance. Since this is due to the increase in curve pull, it is a function of the span length, and may be obviated by making the radius of the trolley curve at the point of support (or at the last hanger) equal to the radius of the track curve which is being fitted, instead of having the trolley curve tangent to and equal to the track curve at the center of the span.

The radius of the trolley curve (R_n) at the support may be determined by using the unit curve-pull, p_n, which is the value of P for any value of x in Table I or II, divided by the hanger spacing s, and substituting in the expression, derived from (**16**),

$$R_n = \frac{T}{p_n} \qquad \textbf{(31)}$$

The radius of the track curve (R_T) may be known, or may be found by substituting known values in the parabolic expression (**11**) for the middle ordinate

$$R_T = \frac{l^2}{8\,\overline{MO}} = \frac{l^2}{8\,d} \qquad \textbf{(32)}$$

where d is that of the U shape, taken from Table I.

The ratio

$$K = \frac{R_n}{R_T} \qquad \textbf{(33)}$$

will be constant for a given span length, and must enter into the calculation of the "multiplier" A in order to avoid the irregularity at the point of support. It represents the proportion of the track middle ordinate which must be taken to make the U shape have the same radius as the track, at the end of the span. This constant changes with the span length, and may be calculated from the U shape curve-pulls, and plotted for convenience, as shown in Fig. 41.

In a similar manner, it may be shown by a comparison of the S shape with the "tangent parabola," that these curves may be made parallel or tangent, of equal radius, at the point of support by using a similar constant J for a given span length, which is determined in a like manner from the curve-pulls calculated from Table II.

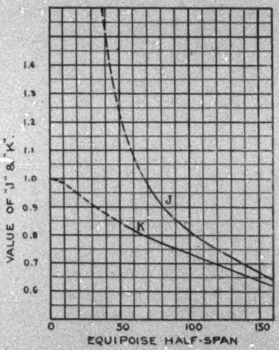

Fig. 41—Inclined Catenary Construction

Correction factors to be used with multipliers A and B of the shape factors U and S. They are the ratio $R\,n/R_T$

This constant

$$J = \frac{R_n}{R_T} \qquad \textbf{(34)}$$

is quite different from K because the rate of change of the curve-pull is different for the two shapes U and S. It changes as the span, and may be plotted with the corresponding values of K as shown in Fig. 41.

The necessity for using these constants to improve the shape at the point of support in reality limits the length of span on a given curve, since it is apparent that the middle ordinate (d_T) of the trolley curve should not be less than the track $\overline{MO}$ by an amount greater than the maximum allowable departure from the centerline of the pantograph. Allowing three in. either side for all other possible variations, the allowable departure due to this factor should not be greater than 12 in. total, or 6 in. either side of the pantograph centerline.

Then

$$\overline{MO} - d_T = 2\,q \tag{35}$$

The trolley curve will lie across the curve of the pantograph centerline as shown in Fig. 42.

The Determination of the Multiplier A for the U shape. Assume the symmetrical construction shown in Fig. 29 on a railroad curve of radius R, with a span length l having the low point in the center of the span. H is the horizontal distance between the trolley and the messenger at the point of support. By construction, the S shape is not involved,

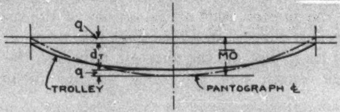

Fig. 42—Inclined Catenary Construction

Relative position of trolley curve and pantograph center-line when cusp at support is eliminated

whence,

$$H = A\,U$$

but

$$H = d_T + d_M + h_0 = d_T\left(1 + \frac{T}{M}\right) + h_0 \text{ from } (\mathbf{19})$$

Then

$$A\,U = d_T\left(1 + \frac{T}{M}\right) + h_0 \tag{36}$$

On the U curve, the ratio of any ordinate h_n to the shortest ordinate h_0 is

$$\frac{h_n}{h_0} = U \tag{37}$$

or

$$h_n = h_0\,U$$

If

$$h_n = H$$

then

$$H = h_0\,U$$

and

$$h_0 = A \tag{38}$$

In other words, on symmetrical curves involving the U shape only, the multiplier A is in reality the horizontal length of the shortest hanger at the center of the span.

Substituting (**38**) in (**36**),

$$h_0\,U = d_T\left(1 + \frac{T}{M}\right) + h_0$$

or

$$h_0\,(U - 1) = d_T\left(1 + \frac{T}{M}\right)$$

and

$$h_0 = \frac{d_T\left(1 + \frac{T}{M}\right)}{U - 1} = A \tag{39}$$

If d_T, the middle ordinate of the trolley curve *is the same value* as $\overline{MO}$, the middle ordinate of the track, then, using expression (**11**) for the $\overline{MO}$,

$$h_0 = \frac{\frac{l^2}{8\,R}\left(1 + \frac{T}{M}\right)}{U - 1} = A \tag{40}$$

But it has previously been shown that a proportion $\frac{R_n}{R_T}$ of $\overline{MO}$ (equal to $K\,\overline{MO}$) must be used, instead, to prevent a cusp at the support.

Then

$$h_0 = \frac{\frac{K\ l^2}{8\,R}\left(1 + \frac{T}{M}\right)}{U - l} = A \tag{41}$$

The Determination of the Multiplier B for the S Shape. Assume the symmetrical construction shown in Fig. 30 on a reverse track curve of radius R with a span length l, having the low point over the point of reverse curvature. H is the horizontal distance between the trolley and the messenger at the point of support. By construction, the U shape is not involved.

Therefore

$$H = B\,S$$

but

$$H = d_T + d_M = d_T\left(1 + \frac{T}{M}\right)$$

and

$$B = \frac{d_T\left(1 + \frac{T}{M}\right)}{S}$$

If d_T, the middle ordinate of the trolley curve, is regarded as part of a continuous curve having the same middle ordinate as the track curve, also regarded as continuous, then

$$B = \frac{\frac{l^2}{8R}\left(1 + \frac{T}{M}\right)}{S}$$

But it has previously been shown that to avoid the cusp at the support, a certain proportion J must be used instead,

whence

$$B = \frac{\frac{J\,l^2}{8R}\left(1 + \frac{T}{M}\right)}{S} \qquad \textbf{(42)}$$

The Application of the U Shape to Standard Hangers. Having determined the shape U and the value of the multiplier A in terms of a given span and curve, the horizontal lengths of all the rods may now be calculated.

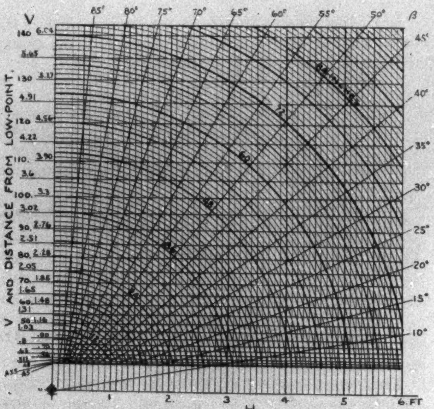

FIG. 43—HANGER LENGTH CHART. INCLINED CATENARY CONSTRUCTION

For example, to calculate standard hangers for a 250-ft. span on a 2-deg. curve:

$K = 0.685$, from Fig. 41,

$l = 250$,

$R = 2865, \quad T = 3415, \quad M = 3900,$

$U - 1$ (for $x = 125$) $= 15.5$, from Fig. 38 or Table I,

$\overline{MO} = 2.73, \quad K\,\overline{MO} = 1.87 = d_T,$

$\overline{MO} - d_T = 0.86$, which is within the allowable variation, 1 ft.

Substituting the above values in (**41**),

$$A = 0.236$$

Multiplying each value of U in Table I by this value of A will give the horizontal length of the rods at the corresponding distance x from the low-point. The value of $A\,U$ for $x = 125$ gives the horizontal separation between trolley and messenger at the support. In this case,

$$H = 16.50 \times 0.236 = 3.9 \text{ ft.}$$

The horizontal values of each rod thus determined may now be combined with their vertical projection, which is assumed to be the same as standard tangent rods. A chart similar to Fig. 43 saves a great deal of calculation, and gives at once the true length of the rod and its angle of inclination β to the horizontal.

If care has been taken in locating the supports so as to maintain "standard hangers" as outlined in Part I, at the same time maintaining as far as possible a fair degree of uniformity in the span lengths on curves of approximately the same radius, inclined hangers may be designed as outlined above for several different average degrees of curvature, using the maximum span for a given curve in each case.

On the Danbury Branch, hangers were designed in this manner for 1-deg., 2-deg., 3-deg., and 4-deg. curves having spans of 250 ft., 200 ft., 180 ft., and 190 ft., respectively. These were approximations when applied to shorter spans, or to intermediate curves, but care was taken that allowable limits were not exceeded. A further approximation was made in the design of these hangers in that after calculating the various values of h_0 for the shortest rods at the centers of the spans on the various curves, an adjustment was made in these values such that all the shortest rods had the same average length and the same average inclination. The design of a transition span was also included in this approximation, so that the resulting standard hangers could be used, within limits, for construction over most of the curves as well as the transitions. This did not make perfectly level wire, but the variations were less than the allowed variation due to temperature changes, and by this method a large amount of calculation was done away with. On curves above four deg. up to six deg., four-deg. rods were used, and the trolley was deflected at the supports by pull-offs, somewhat as on tangent-chord construction. Fig. 44 shows the appearance of the completed con-

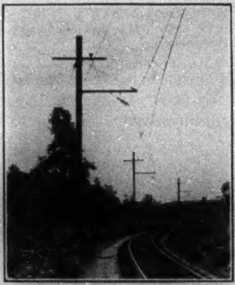

FIG. 44—INCLINED CATENARY ON FOUR-DEG. CURVE, DANBURY BRANCH, 1925

struction on curves where the rods fitted the track curve, and Fig. 45 illustrates the pulled-off construction on sharper curves. On one or two curves which were sharper than four deg., the rods were calculated to individually fit the track curve.

Computation of Special Spans, Involving both Shape Factors. On transition curves, reverse curves, or where there is a large difference between the horizontal separations of messenger and trolley at each end of the span, the values of these offsets H_R and H_L are usually known. These values may be substituted in the general expressions

$$H_R = A\,U + B\,S \qquad \text{at } x = n_1$$
$$H_L = A\,U + B\,S \qquad \text{at } x = -\,n_2$$

using the values of U and S given for $x = n_1$ in the case of H_R and those given for $x = -\,n_2$ in the case of H_L, and having proper regard for the signs of U and S. U is always positive for values of x either side of the low-point and S is positive for values of x to the right of the low-point, and negative for values of x to the left of the low-point.

The above equations may be solved simultaneously for A and B, and these, when determined with proper regard for the factors K and J and used with their respective shape factors, will give the horizontal length h_n of any rod, which may be combined with v by use of chart shown in Fig. 43, or otherwise.

All the above calculations are based on combining the horizontal lengths of the rods with "standard" tangent hanger values of v. If vertical lengths other than "standard" are used, the shape factors U and S calculated for standard hangers will not apply, but new values of the U shape and the S shape, based on the shortest "special" rod, must be calculated. This greatly increases the amount of calculation work, and for this reason, "special" hangers should be avoided on curve construction where possible.

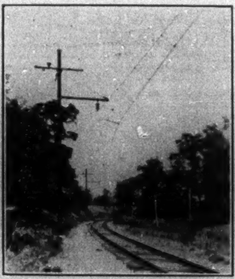

FIG. 45—INCLINED CATENARY ON FIVE-DEG. CURVE, DANBURY BRANCH, 1925

Showing four-deg. catenary pulled off at supports

The above methods and calculations as set forth are somewhat complex, but once developed for a given design, they may be easily applied to the solution of any standard span.

The Limitation of Inclined Construction. Some of the limitations of the inclined construction have already been touched upon, the most important being the necessity of using a factor in the calculation of the "multipliers" to avoid the cusp in the shape of the trolley wire at the point of support. The span limit is reached for a given curve, when the difference between the track middle ordinate and the trolley middle ordinate is greater than 1.0 ft.

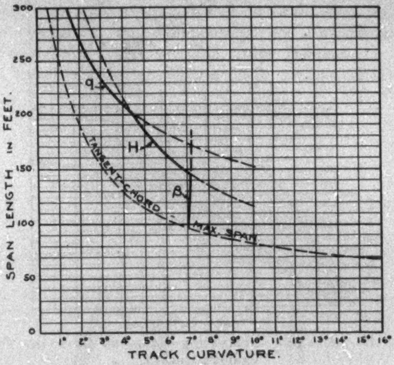

FIG. 46—MAXIMUM SPAN LIMITS FOR 9/16-IN. INCLINED CATENARY FOR VARIOUS CURVES

Another important limitation is the maximum permissible horizontal length of longest rod, or the offset at the support. Good practise indicates a limit for this of about six ft., except in special cases which cannot otherwise be avoided, to avoid construction which would extend over the adjacent parallel track which is usually a different electrical section.

A third important limitation is that due to the inclination of rods. When the angle of inclination to the horizontal is too small, the plane of the hanger rods approaches that of the pantograph, already inclined on curves on account of super-elevation of outer rail. Good practise indicates a limit to this inclination of approximately a horizontal value of three to a vertical value of one, except in special cases.

These limitations may be shown graphically for comparison with the limitations of tangent-chord construction, as shown in Fig. 46, where q is the span limitation due to the difference between track and trolley middle ordinates, H is the span limitation due to maximum allowable offset between trolley and messenger at the support, and β the span limitation due to hanger rod inclination. From this diagram it is seen that the practical application of the inclined catenary construction is limited to curves up to about seven deg. Above this point, tangent-chord construction is preferable.

Determination of Structure Heights. As in the case of tangent-chord construction, the structure height is given by formula (**6**)

$$V = C + v_0 + d_E \pm I$$

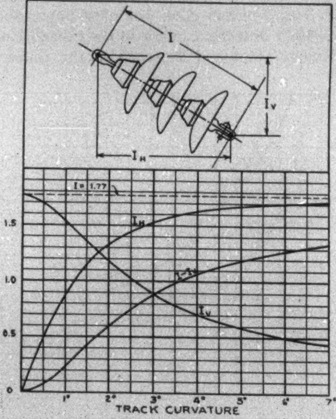

FIG. 47—INSULATOR INCLINATION—INCLINED CATENARY FOR VARIOUS CURVES

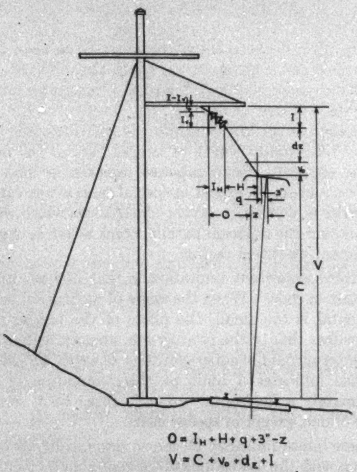

FIG. 48—ILLUSTRATING BRACKET-ARM HEIGHT AND INSULATOR OFFSET ON INCLINED CATENARY CONSTRUCTION

If suspension insulators are used, and this type is preferable on account of the horizontal stresses, they may assume an inclination which is the resultant of the weight supported and the curve-pull, W and P, respectively. If I_V and I_H are the vertical and horizontal projections of the inclined insulator assembly, then

$$\frac{I_V}{I_H} = \frac{W}{P} = \frac{\gamma l}{l\left(\frac{T+M}{R}\right)} = \frac{\gamma R}{T+M} = \frac{\gamma}{p} \qquad (43)$$

In other words, the inclination is independent of the span length, and is the ratio of the unit weight of the catenary to the unit curve-pull; and since γ, T, and M are constant, is directly proportional to the curvature. A set of curves giving the values of I_V, I_H, and $(I - I_V)$ is very useful in determining structure heights and insulator offsets. $I - I_V$ is useful to determine the

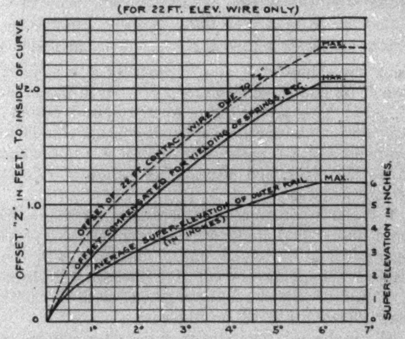

FIG. 49—AVERAGE SUPER-ELEVATION OF OUTER RAIL ON VARIOUS CURVES, WITH CORRESPONDING OFFSET z OF 22-FT CONTACT WIRE

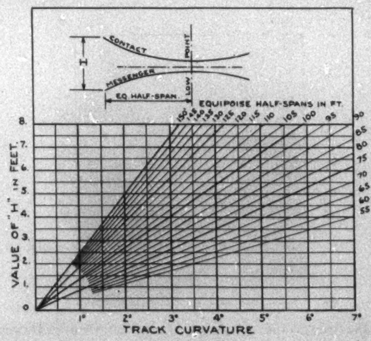

FIG. 50—HORIZONTAL DISPLACEMENT OF MESSENGER FROM CONTACT FOR INCLINED CATENARY AT SUPPORT, FOR VARIOUS SPANS AND CURVES

spacer length between the supporting structure and the top of the insulator when the same height structure is maintained with inclined construction as on tangent construction. Fig. 47 shows such curves.

Determination of Insulator Attachment Offsets form Track Centerline. The super-elevation of the outer rail will cause an offset z of the pantograph centerline from the track centerline, as in tangent-chord construction, towards the inside of the curve. A small amount, generally about three in., is also allowed for the yielding toward the outside of the curve of pantograph and locomotive springs, due to centrifugal action at running speeds.

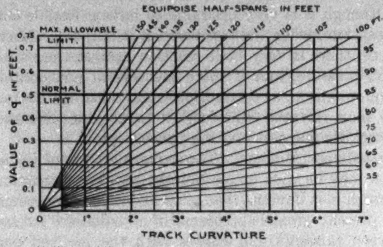

Fig. 51—Inclined Catenary

Maximum departure of U-shape of contact wire from shape of track at center of span on circular curves for radius of contact wire equal to radius of track curve under support; for various spans and curves

The messenger has a horizontal separation H from the trolley wire, toward the outside of the curve.

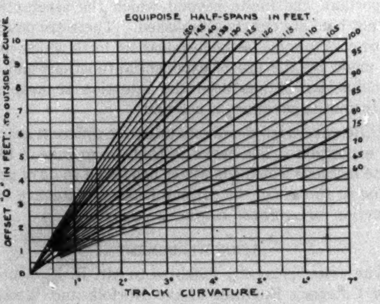

Fig. 52—Total Offset of Insulator Attachment for Inclined Catenary Construction, for Various Spans and Curves Under Normal Conditions

The insulator, if inclined, will increase the offset to the outside of the curve by the value of I_H.

If the curve of the wire is desired to follow the pantograph centerline curve as shown in Fig. 42, an additional offset to the outside of the curve of the value of q must be made.

Adding all these elements, the total offset from the track centerline is

$$O = I_H + H + q + 3'' - z \qquad (44)$$

diagrammatically shown in Fig. 48, which is typical of inclined catenary pole and bracket construction on curves.

It is very useful to have these various elements plotted for standard conditions on various curves. Fig. 49 shows values of z plotted for wire 22 ft. above rail, from known super-elevations for different degrees of curvature, and compensated for the allowance assumed for pantograph and spring yielding.

Values of H for standard spans for various curves are shown graphically in Fig. 50 and the values of q under similar conditions are shown in Fig. 51.

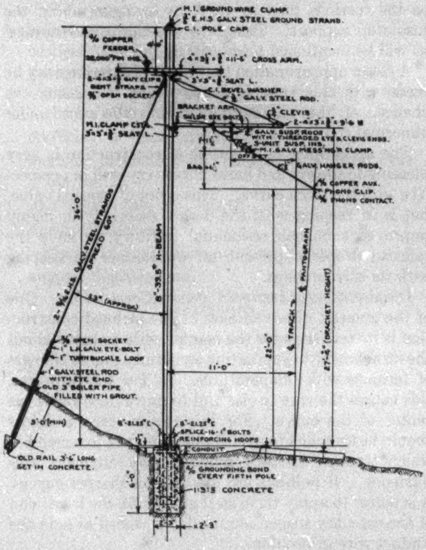
M.I. GROUND WIRE CLAMP.
⅜" E.H.S GALV. STEEL GROUND STRAND.
C.I. POLE CAP.
4/0 COPPER FEEDER.
4'-6"
4"×3½"×⅜"×11'-6" CROSS ARM.
2-4×3×⅜" GUY CLIPS
BENT STRAPS
⅝" OPEN SOCKET
3"×3"×⅜" SEAT L.
C.I. BEVEL WASHER.
¾" GALV. STEEL ROD.
BRACKET ARM
CLEVIS
M.I. CLAMP CSTG.
3×3"×⅜" SEAT L.
2-4×3×⅜"×9'-6" L
GALV. SUSP. RODS WITH THREADED EYE & CLEVIS ENDS.
3-UNIT SUSP. INS.
M.I. GALV. MESS'NGR CLAMP.
OFFSET
GALV. HANGER RODS.
4/0 COPPER AUX.
PHONO CLIP.
4/0 PHONO CONTACT.
36'-0"
2-⅜" E.H.S. GALV. STEEL STRANDS SPREAD 60°
8"-33.5# H-BEAM
℄ PANTOGRAPH
℄ TRACK
27'-2" (BRACKET HEIGHT)
22'-0"
23° (APPROX.)
⅝" OPEN SOCKET
1" L.H. GALV. EYE-BOLT
1" TURNBUCKLE LOOP
1" GALV. STEEL ROD WITH EYE END.
OLD 3" BOILER PIPE FILLED WITH GROUT.
11'-0"
3'-0" (MIN)
8"-21.25# C
SPLICE-16-1" BOLTS
REINFORCING HOOPS
CONDUIT
OLD RAIL 3'-6" LONG SET IN CONCRETE.
4/0 GROUNDING BOND EVERY FIFTH POLE
1:3:5 CONCRETE
6'-0"
2'-3" × 2'-3"

Fig. 53—Standard Bracket Pole on Curves up to Three Deg. for 200-ft. Span

All of these elements may then be combined and shown graphically as in Fig. 52.

Fig. 53 shows the details of the inclined construction used on the Danbury Branch. A comparison with Fig. 12 will show the similarity to the construction on tangents, except for the inclination of the catenary and the insulator.

Advantages of Inclined Catenary Construction. It is readily seen from Fig. 46 that longer spans are permissible on all curves with inclined construction than with tangent-chord construction. This decreases the cost of supporting structures.

The necessity for pull-offs is largely eliminated, none generally being required for curves up to four deg. This means fewer insulators, an important point where high voltage is used on the contact system.

Sharp deflections in the contact wire, which tend to cause sparking and ultimate wear and injury to the wire, are avoided.

The contact system has greater flexibility than with tangent-chord construction, and hard spots are practically eliminated.

The variation in tension in the trolley wires due to temperature changes will, in general, be less than on tangent-chord construction, since there are no fixed points and the wire can move in and out on the curve as the tensions tend to vary, by swinging about the insulator support. This is a possible disadvantage, as will be mentioned below.

A closer approximation to the track centerline may be made with this type of construction, decreasing the chances of the pantograph leaving the wire under abnormal conditions.

The appearance of this type of construction is very pleasing to the eye and harmonizes very well in perspective with the track curve. This is not of especial value, but is in keeping with the desire, especially in highly improved suburban residential territory, to have the railroad property present an appearance in keeping with its surroundings.

Disadvantages of Inclined Catenary Construction. One of the greatest disadvantages of the inclined construction is its tendency for the contact wire to move about the attachment to supporting structures due to changes of tensions with temperatures. At low temperatures, this causes the wire to rise and to move in towards the center of the curve. For this reason, the insulators should be attached to a fixed point and not to a member which through its own motion, may augment this tendency. If pull-offs are used, as on sharper curves, it is better to apply them so that they fix the lower end of the catenary suspension insulator as well as hold the contact wire in position.

On the short, sharp curves generally found in terminals, yards and sidings, and cross-overs, the inclined catenary is difficult to design, install, and maintain on account of the deflector and sectionalizing details, and should not be applied to such track.

The complexity of design is possibly another disadvantage, but this is largely academic. The fact remains, however, that this type of construction does require more time for study of conditions and for the proper design than does tangent-chord construction.

The cost of installation is possibly higher than the cost of tangent-chord construction, but this is usually compensated for by a saving in the cost of supporting structures.

Although on heavy curves, longer spans may be used than with tangent-chord construction, the extreme flatness of the inclined construction in such cases is generally a greater disadvantage from construction, operation, and maintenance points of view than the additional pull-offs required with tangent-chord construction. As an illustration, the offsets under such conditions may be so great that on multiple track construction, to replace a faulty insulator on one track, it might be necessary to de-energize an adjacent track before the work could be done with safety.

Although it is very desirable to have a flexible contact system, when the construction is flat, as on heavy curves, there is very little reaction against the pantograph pressure, and if there are several pantographs acting in unison, as on multiple-unit trains, the contact wire may rise and still further reduce the clearance between the hangers and the pantographs.

Installation Methods. The same methods of installation may be used as for tangent-chord construction, although it is obvious that until loaded, the messenger will present greater problems of unbalanced stresses between tangents and curves with the inclined construction. For this reason, the writer prefers the method of installation which allows the tensioning of the messenger with the trolley wires approximately in position and in tension, held to the messenger by a few temporary ties per span. This saves further lengthy calculations for unloaded messenger sags under no-load stress conditions which will vary considerably on various curves from the stress on tangent.

In this analysis certain assumptions may have been made which may not be strictly true; as, for instance, the rods are in reality radial to the curve, instead of being parallel to each other. The errors due to such assumptions are, for the most part, very slight; or if important, the limits beyond which the assumptions do not hold are clearly established. There are possible short cuts to some of the formulas set forth which could be made by the use of exponential functions and the calculus, but it has been the intention to keep the mathematics as simple as possible.

Different design details may require different graphical exposition. The curves shown are merely to illustrate methods which the writer has found to be of great value in the design of various types of catenary construction for overhead contact systems.

ACKNOWLEDGMENT

The writer is greatly indebted to Mr. R. P. Winton, now Catenary Engineer of the Norfolk and Western Ry., for many valuable ideas during the past few years, which have been used and incorporated in the development of the theory and methods used in this discussion of the inclined catenary; and to Mr. Sidney Withington for many helpful suggestions in the preparation and arrangement of this paper.

SYMBOLS USED

a area cross-section, wire or strand,

d deflection or sag of wire or strand,

f	function f of x,
h	horizontal projection of hanger rod length,
l	distance between supports; span length,
l_E	equipoise span,
$\frac{l_E}{2}$	equipoise half-span,
m	horizontal distance from messenger to axis,
p	unit curve pull,
q	allowable departure from pantograph centerline,
s	hanger spacing,
t	distance (horizontal) from trolley to axis; temperature in deg. fahr.,
v	vertical projection of hanger rod length,
x	distance along system axis from low point of span,
z	offset of trolley from track centerline due to super-elevation of outer rail,
A	the "multiplier" of the shape factor U,
B	the "multiplier" of the shape factor S,
C	height of contact wire above top of rail,
D	degree of curvature,
E	Young's modulus of elasticity,
F	function F of x,
H	horizontal distance between messenger and trolley at support,
I	length of insulator assembly,
I_V	projection on vertical plane of inclined insulator assembly.
I_H	projection on horizontal plane of inclined insulator assembly,
J	the value of the ratio R_n/R_T of the S curve,
K	the value of the ratio R_n/R_T of the U curve,
L	length of messenger for span length l,
M	horizontal component of messenger tension,
$\overline{MO}$	middle ordinate of a circular curve for a given span,
O	offset of insulator attachment from track centerline at support,
P	curve pull; horizontal force,
Q	the sum of a series,
R	the radius of a curve,
S	the function $F'(x)$; the shape of the reverse or S curve,
T	the horizontal component of trolley tension,
U	the function $F(x)$; the shape of the continuous or U curve,
V	height of supporting structure,
W	vertical force due to weight,
X	the system axis,
Y	the axis of $F(x)$, perpendicular to the system axis through the low-point,
α	coefficient of linear expansion,
β	angular inclination of hanger rods from the horizontal,
γ	unit weight (per unit length),
δ	linear displacement,
Δ	a change in value; increment or decrement,
θ	angular displacement.

Discussion

For discussion of this paper see page 1133.

Catenary Construction for Chicago Terminal
Electrification of Illinois Central Railroad

BY J. S. THORP[1]
Non-Member

***Synopsis.**—The object of this paper is to give briefly the procedure followed by the Illinois Central engineers in the layout, design, and erection of the catenary system. Brief reference also is made to the principal items of material entering into the construction and to methods of maintenance.*

* * * * *

THE Illinois Central decided upon the 1500-volt d-c. system of electrification with catenary construction for pantograph operation.

The electrified suburban trackage includes about:

4 route mi. of single main line track,
20 route mi. of double main line track,
1½ route mi. of 3 main line tracks,
8 route mi. of 4 main line tracks,
5 route mi. of 6 main line tracks, and about
20 track mi. of yards and sidings.

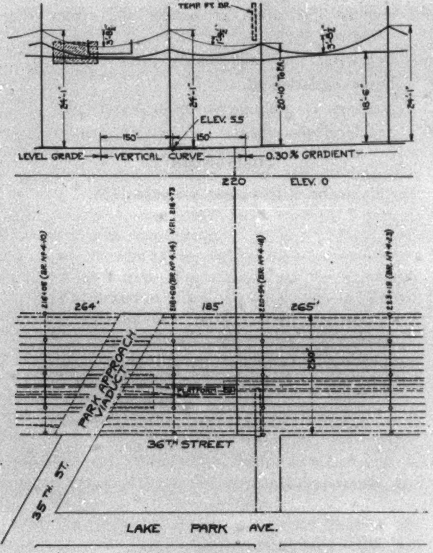
TEMP FT BR.
3'-8½"
1'-3½"
3'-8½"
24'-1"
24'-1"
20'-10" To F.R.
18'-6"
24'-1"
ELEV. 5.5
150'
150'
LEVEL GRADE
VERTICAL CURVE
0.30% GRADIENT
220
ELEV. 0
V.P.I. 218+73
264'
185'
265'
PLATFORM
PARK APPROACH VIADUCT
36TH STREET
35TH ST.
LAKE PARK AVE.

FIG. 1—TYPICAL TRACK PLAN

PLANS

Track plans were prepared showing the existing tracks and the proposed track arrangement to ultimately develop the right-of-way. Fig. 1 shows a typical section of the track plan in the vicinity of 36th Street.

The catenary structures were located tentatively on the track plans, adhering to the normal tangent spacing of 300 ft. as far as possible and reducing the spacing according to Fig. 2, where the span is wholly or partly on a curve, or the normal spacing maintained and pull-off-poles interposed as found desirable. These locations were given to the field engineer who first made a check to see if any shifting would be necessary due to physical obstructions not shown on the plans, and then when the structures were definitely located, made cross-sections of the entire right-of-way at each location. From the track plans and cross-sections, erection diagrams as shown in Fig. 3 were prepared and the catenary structures designed. The height of the structure is determined by the catenary profile which is drawn on the track plans.

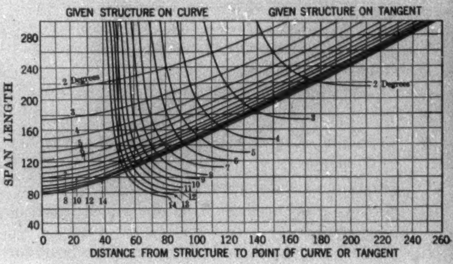

FIG. 2—SPACING CHART FOR STRUCTURES ON CURVES

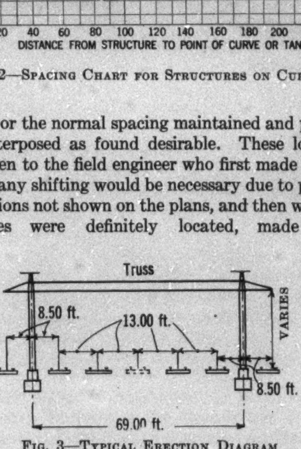
Truss
VARIES
8.50 ft.
13.00 ft.
8.50 ft.
69.00 ft.

FIG. 3—TYPICAL ERECTION DIAGRAM

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[1] Distribution Engineer, Illinois Central Railroad.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

CATENARY STRUCTURE FOUNDATIONS

The foundations of all permanent structures were concrete. Where the space between the ties was less than the across-track dimension of the foundation, "side-bearing" footings were used. These "side-bearing" footings were designed in accordance with the chart shown in Fig. 4.

Gravity type footings were installed where track shoreing was not necessary, and this type was designed by reference to the chart shown in Fig. 5. In addition to the determination of the bearing pressure under the footing, these foundations were checked against overturning, using a factor of 1½.

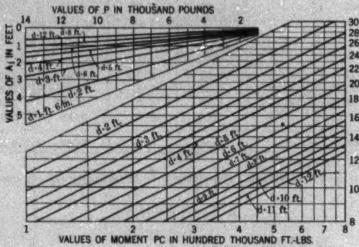
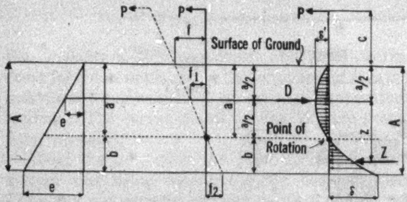


FIG. 4—CHART USED IN THE DESIGN OF SIDE-BEARING FOUNDATIONS

$$A = \frac{3\,P}{\delta\,d} + \sqrt{\frac{12\,P\,C}{\delta\,d}} = A_1 + A_2,\quad A_1 = \frac{3\,P}{\delta\,d}\quad A_2 = \sqrt{\frac{12\,P\,C}{\delta\,d}}$$

Earth pressure δ = 5000 lb./sq. ft.



NOMENCLATURE

a = distance from point of rotation to surface of ground.
b = distance from point of rotation to bottom of pier.
c = distance from surface of ground to point of application of overturning force.
P = overturning force on pier, or horizontal shear at surface of ground.
D = resisting force of earth above point of rotation
Z = resisting force of earth below point of rotation.
A = total depth of pier below surface of ground.
z = distance between centers of gravity of earth pressures above and below point of rotation.
δ' = maximum unit earth pressure above point of rotation.
δ = maximum unit earth pressure below point of rotation.
f = deflection of pier at surface of ground.
f_1 = deflection of pier at center of gravity of earth pressure above point of rotation.
f_2 = deflection of pier at bottom.
e = passive earth resistance at bottom of pier.
e' = passive earth resistance at center of gravity of earth pressure above point of rotation.
d = width or diameter of pier.

In staking out the foundations for construction, two stakes were set on each center line and grade was referenced from the top of an adjacent tie. The concrete was poured from a mixing plant mounted on flat cars and handled in a work train. An inspector supervised all operations of the concrete pouring outfit which placed as much as 140 cu. yd. per eight-hr. day where traffic conditions were favorable to the use of the track.

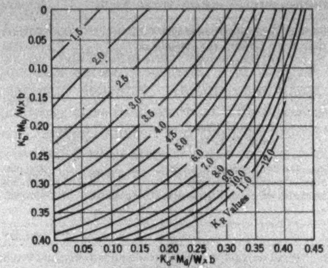
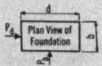


FIG. 5—CHART USED IN THE DESIGN OF GRAVITY FOUNDATIONS

CURVES GIVE VALUES OF MULTIPLIER K_R

Reduce moments to equivalent moments at base of pier.



M_d = moment due to force P_d, ft lbs.
M_b = moment due to force P_b, ft lbs.
W = total vertical load at base of pier, lb.
b = width of pier, ft.
d = length of pier, ft.

Moments in Two Directions.—Read K_R at intersection of K_b and K_d

Moments in One Direction.—Read K_R in the same manner as described above except that either K_d or K_b is zero.

$$\text{Maximum Toe Pressure} = \frac{W}{b\,d}\,K_R \text{ lb. per sq. ft.}$$

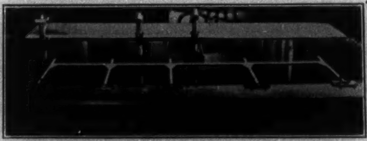


FIG. 6—BEGGS' APPARATUS AS APPLIED TO THE MODEL OF A 5-COLUMN STRUCTURE

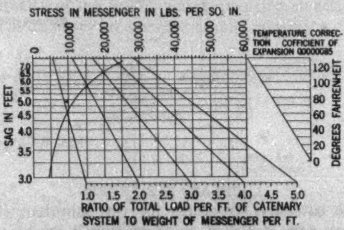


FIG. 7—MAIN MESSENGER DEFLECTION CURVE

TABLE I
LOADING OF CATENARY SYSTEM

Loadings of each Catenary System are shown in the following table:

Suburban catenary system 70th St.-North	Equiv. Conductivity cir. mils	Tension still air at 60 deg. fahr.	Tension at 0 deg. fahr. Ice and wind	Factor of safety	Dead weight	Dead weight with ice	Horizontal wind load. No ice 20 lb. wind	Horizontal wind load with ice 8 lb. wind
0.81-in. diameter composite messenger	370,000	7,750 lb.	12,350† lb.	2.55	1.51	2.32	1.35	1.21
0.512-in. diameter. Copper aux. messenger	200,000	800	1,555	5.82	0.61	1.25	0.85	1.01
Two 3/0, 80 per cent conductivity. Bronze trolley wire	268,900	4000	7,020	2.85	1.02	2.20	1.08*	1.43*
Hangers	..	..	..	..	0.11	0.22	0.14	0.13
Total	838,900	12,550	20,925	..	3.25	5.99	3.42	3.78
Suburban catenary system 70th St.-South, So. Chicago and B. I. R. Rs.								
0.81-in. diameter composite messenger	370,000	7,720 lb.	12,325 lb.†	2.55	1.51	2.32	1.35	1.21
0.375-in. diameter aux. messenger	105,500	800 lb.	1,555 lb.	3.55	0.32	0.87	0.63	0.92
Two 4/0 hard-drawn copper grooved trolley wires	423,200	4,000 lb.	7,020 lb.	2.2	1.28	2.51	1.21*	1.48*
Hangers	..	..	..	..	0.15	0.27	0.18	0.15
Total	898,700	12,520 lb.	20,900 lb.	..	3.26	5.97	3.37	3.76

NOTE. *Wind on double trolley wires figures at one and one-half times the wind on one wire.

†Wind on messenger and one-half wind on hangers assumed as acting at point of support of messenger, remaining wind on catenary system assumed as acting at steady wire.

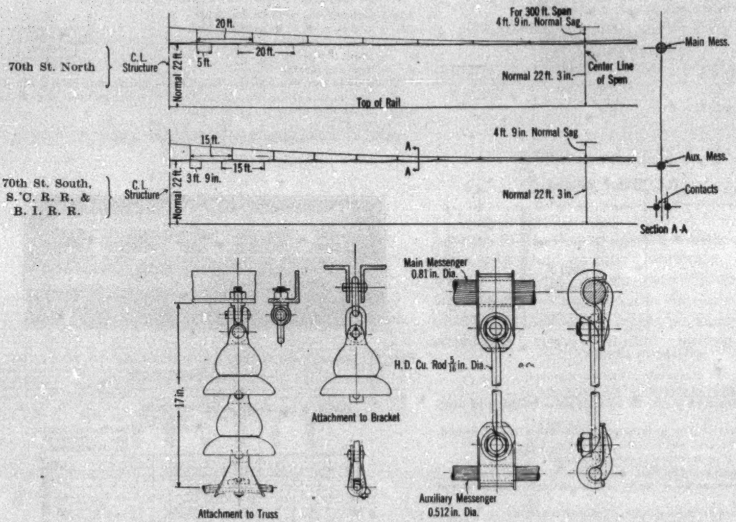

For 300 ft. Span
4 ft. 9 in. Normal Sag
20 ft.
20 ft.
5 ft.
Main Mess.
70th St. North
C.L. Structure
Normal 22 ft.
Normal 22 ft. 3 in.
Center Line of Span
Top of Rail
15 ft.
4 ft. 9 in. Normal Sag
A
A
Aux. Mess.
15 ft.
3 ft. 9 in.
70th St. South, S. C. R. R. & B. I. R. R.
C.L. Structure
Normal 22 ft.
Normal 22 ft. 3 in.
Contacts
Section A-A
Main Messenger 0.81 in. Dia.
H. D. Cu. Rod $\frac{5}{16}$ in. Dia.
17 in.
Attachment to Bracket
Attachment to Truss
Auxiliary Messenger 0.512 in. Dia.

FIG. 8—MAIN LINE CATENARY SYSTEM

CATENARY STRUCTURES

The catenary structures for the suburban electrification were designed so as to permit extensions to include the remainder of the right-of-way. In some instances this means an ultimate structure of 200 ft. or more in length, made up of several spans. A reference to Fig. 1 will show the suburban catenary structures in heavy line, proposed extension to the east for through passenger and freight tracks, and further extensions to the east and west, shown in dotted lines, to include the entire right-of-way which at this point has a width of 250 ft. This provision necessitated calculations of the stresses in the complete structure and a check of the stresses in the initial structure. To reduce the labor and time required for such troublesome calculations, it was decided to adopt the Beggs method for the mechanical determination of statically indeterminate stresses. This method proved very satisfactory and greatly facilitated the work. Fig. 6 shows an application of the Beggs apparatus to a five column structure.

The structures were erected, painted, and the base plates grouted by the railroad forces, very expeditiously and with little interruption to the regular traffic.

CATENARY SYSTEM

Table I shows the loadings of the catenary system and Fig. 7 shows a very convenient and easily workable chart for a determination of the tensions and sags in the messenger for any assumed condition of temperature or loading. The curves shown on Fig. 12 were prepared from this chart.[2]

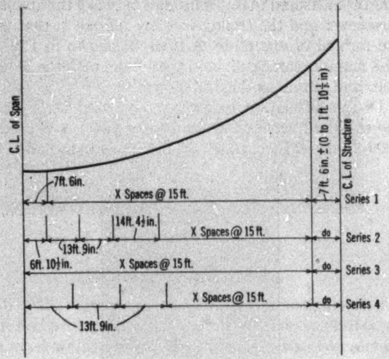

C. L. of Span
C. L. of Structure
7 ft. 6 in. ± (0 to 1 ft. 10½ in)
7 ft. 6 in.
X Spaces @ 15 ft.
Series 1
14 ft. 4½ in.
13 ft. 9 in.
X Spaces @ 15 ft.
do
Series 2
6 ft. 10½ in.
X Spaces @ 15 ft.
do
Series 3
X Spaces @ 15 ft.
do
Series 4
13 ft. 9 in.

FIG. 9—CATENARY HANGER CHART

About the time when the make-up of the catenary system was being considered, the General Electric Company was carrying out some tests of double trolley wire construction at Erie, Pennsylvania. Illinois Central representatives were invited to witness some of these tests and were favorably impressed with the practically arcless collection of heavy current at high speed, and shortly the decision was reached to adopt the double trolley system. At the same time it was decided not to use parallel feeders but to equip each track with a catenary system of sufficient current-carrying capacity to keep the voltage drop within the prescribed limit, when normally in parallel with other tracks at sub- and tie-stations.

After these decisions were made it was a simple matter to select the main and auxiliary messengers to make up an adequate catenary system. Fig. 8 shows the main line catenary assembly. On the main line 4/0 copper trolley wires were used south of 70th Street and on the South Chicago Railroad and the Blue Island Railroad. Bronze trolley wires were selected for use north of 70th Street where the traffic is most dense. To compensate for the lower conductivity of the 3/0 bronze as compared with the 4/0 copper trolley wires, the auxiliary messenger was increased in size. A 0.375-in. auxiliary was used with the 4/0 trolley and a 0.512-in. auxiliary with the 3/0 trolley.

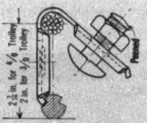

2 3/16 in. for 4/0 Trolley
2 in. for 3/0 Trolley
Peened

FIG. 10—TROLLEY WIRE CLIP

The main messenger cable is of composite construction made up of seven Copperweld wires forming the core around which are stranded 12 hard-drawn copper wires. This cable is 0.81 in. in diameter and has an ultimate strength of 31,500 lb., which allows a sag of 4 ft., 9 in. in a 300-ft. span at 60 deg. fahr., giving a factor of safety of 2½ under the maximum assumed loading of the system.

The auxiliary messenger is made up of 19 strands of hard-drawn copper and has a normal tension of 800 lb. which gives a little sag between hangers and increases the flexibility of the trolley wire supporting structure.

The catenary hangers are simple, consisting of bronze

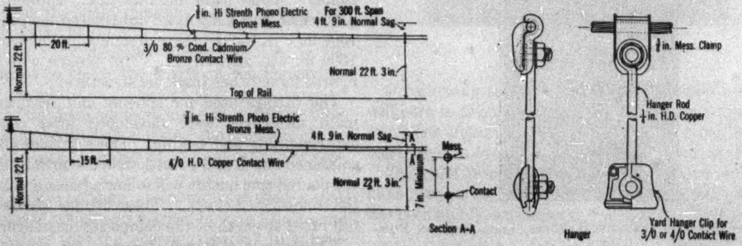

⅝ in. Hi Strenth Phono Electric Bronze Mess.
For 300 ft. Span
4 ft. 9 in. Normal Sag
20 ft.
3/0 80 % Cond. Cadmium Bronze Contact Wire
Normal 22 ft.
Normal 22 ft. 3 in.
Top of Rail
⅝ in. Hi Strenth Photo Electric Bronze Mess.
4 ft. 9 in. Normal Sag
A
A
15 ft.
4/0 H.D. Copper Contact Wire
Normal 22 ft.
Normal 22 ft. 3 in.
7 in. Minimum
Mess.
Contact
Section A-A
Hanger
⅝ in. Mess. Clamp
Hanger Rod
¼ in. H.D. Copper
Yard Hanger Clip for 3/0 or 4/0 Contact Wire

FIG. 11—CATENARY CONSTRUCTION FOR YARDS AND SIDINGS

clamps for main and auxiliary messenger connected with a 5/16-in. diameter hard-drawn copper rod with an eye in each end. To simplify the determination of the hanger lengths and to aid in the spacing of the hangers in the field, a diagram was prepared to divide the span lengths into four series. Fig. 9 shows the arrangement for the system having a normal hanger spacing of 15 ft. This diagram was laid out on a table-top to full vertical scale and to ¼ in. equaling one foot horizontal scale. The eye-to-eye lengths of the hangers were measured from points on the curve to a line representing the normal position of the eye of the hangers. In cases where the position, with reference to the messenger, of the trolley wires is other than normal, a straight edge was adjusted to represent this change and the hanger lengths measured to it. The clips between the auxiliary messenger and the trolley wire are unique in that they are formed of one piece of stock as shown in Fig. 10. The manufacturer delivered these clips with the T-head bolts and nuts assembled as shown.

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[2] The method of constructing this chart was published in the A. E. R. A. *Proceedings* for 1925.

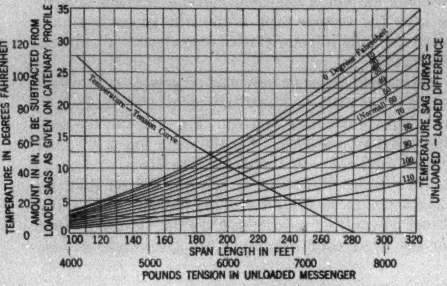

Fig. 12—Temperature—Sag Curve

Fig. 13—Hanger Board with Half Span of Hangers Cut to Length

Fig. 14—Complete Span of Hangers Assembled on Pipe

Double insulation to ground was used throughout, the standard suspension unit being two seven-in-disk, 8000-lb., M. & E., cap and pin insulators attached to the structure by means of either a galvanized eye-bolt-and-clevis or a double-link, depending upon the type of structure to which the attachment was made.

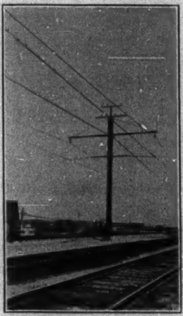

Fig. 15—Typical 2-Track T-Bracket Construction

The suspension or saddle clamp is U-shaped to which the messenger is clamped by means of one J-bolt and keeper. Tests made on this clamp showed that the messenger would slip at an unbalanced tension of about 1000 lb. In the structure design, a broken wire load of 1000 lb. was allowed for. This broken wire load was figured for only one track per structure.

The fittings used for splicing and terminating the main messenger were developed after considerable experimenting and testing. These fittings consist of a combined chuck and poured socket so arranged that the the poured zinc button will follow against and keep tight the cone-shaped chuck. These fittings will develop the full rated strength of the composite messenger cable.

The seven-in-disk insulators used for suspension were also used in steady- and pull-off-strand construction to insulate from structure and to insulate separate sections.

For terminating the main line catenary system a very sturdy type of double strain insulator was used, one set for the main messenger and one set for the two trolley wires and auxiliary messenger combined.

The yard construction is shown on Fig. 11. Only a single trolley wire of either 3/0 or 4/0 gage was used. All yard catenary was supported from steel structures except Weldon Yard where some cross-span construction was used.

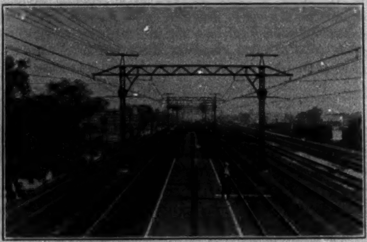

FIG. 16—TYPICAL 4-TRACK CONSTRUCTION

CATENARY CONSTRUCTION

All of the wires were strung and the remainder of the main line work practically completed from work trains. The yard work, except wire stringing, was completed from push-tower-car and ladders. The heavy equipment for construction consisted of reel cars with special shafts and brake rigging, tower cars with winch operated platforms adjustable from 15 ft., 6 in. to 19 ft., 6 in. above top of rail, and box cars fitted with work benches, shelves, and bins for tool cars.

The typical train for wire stringing consisted of a locomotive, reel car, tool car, and one or more tower cars. Only one main messenger was strung at a time, but the auxiliary messenger and the two trolley wires were strung together. The main messenger was tensioned by means of a sight rod and surveyor's level. The auxiliary messenger and trolley wires were tensioned with a dynamometer. These wires were all erected and tensioned in roller bearing rollers, the proper sag for the main messenger being determined from curves shown in Fig. 12.

The hanger chart described above was used in the material depot to check the length of the hangers as they were assembled on a piece of ½-in. pipe, three ft. long. After the hangers were assembled on the pipe, a linen tag was attached showing the location of the span and the series number of the hanger. See Figs. 13 and 14.

In erecting the hangers, the contractor used a steel tape, one side of which was marked in span lengths and the other marked with the recurring figures 1 to 4 inclusive, representing the four series of spacings. These figures were spaced as on the hanger board, but to full scale. This tape was stretched alongside the messenger, adjusted to the span, and the hangers were placed opposite the series figure.

MAINTENANCE

The maintenance of the catenary system is handled by the Maintenance of Way Department, using one 1-ton and one 1¼-ton motor trucks equipped with special bodies for hand tools, supplies, and ladders. These trucks are used for all light repair work as it is possible to approach the right-of-way in the electrified zone on paved or hard surface streets or roads. For heavy repair work two construction trains, each consisting of reel, tool, and tower car, are located at points on the terminal where a steam locomotive is readily available.

Soon after the start of the electric operation, trouble was experienced due to the pantographs striking the insulators which form part of the air-gap construction and either damaging the gap or so damaging the pantograph as to result in line trouble elsewhere. During the severe cold weather, some trouble was caused by failure of faulty fittings or by poor workmanship and to pantographs fouling the steady arms. The latter trouble was due to the fact that the trolley wires, at very low temperatures, lifted higher at the structures than was anticipated.

Main line construction was sectionalized by means of air-gaps, but wood section insulators were used in cross-overs where high speed is not attained. These insulators were originally equipped with two bronze gliders to give continuous feed. It was later found desirable, however, to install four gliders, two on each side of the wood stick to prevent the insulator rocking when the pantograph passed.

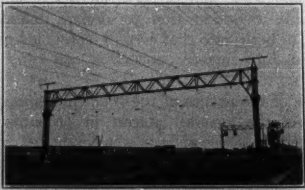

FIG. 17—TYPICAL 6-TRACK CONSTRUCTION

Occasionally the gliders of a wood section insulator will burn in two when a pantograph bridges a live and a grounded section, and on two occasions a trolley wire has burned in two at an air-gap due to the same cause.

The chief troubles were mentioned above and there have been minor failures but on the whole the operation has been highly satisfactory.

Discussion

For discussion of this paper see page 1133.

The Collection of Current From Overhead Contact Wires

BY R. E. WADE[1] Non-Member and J. J. LINEBAUGH[1] Associate, A. I. E. E.

Synopsis.—*Up to a few years ago, the generally accepted limitations for the amount of current which could be collected from an overhead distribution system were from 300 to 800 amperes in heavy interurban service and between 800 and 1000 amperes, with a maximum of 1500 in the case of the Chicago, Milwaukee, & St. Paul.*

It is not possible to determine the limits of current collection by theoretical calculations nor by the experience on any particular installation. The tests described and analyzed in this paper were demonstrated on a four-mile track using special overhead construction of the twin trolley type with observation towers at several points to enable observers to carefully inspect commutation between the collector and the trolley wire.

Tests were also made to determine the temperature rise which would be obtained as a result of delivering, for a period of five minutes, a current of 5200 amperes to a standing locomotive. Further tests were made to determine what damage would result, if any, should a pantograph leave the wire while delivering currents as high as 4000 or 5000 amperes. Test data are included and description of the several types of overhead construction used.

INTRODUCTION

THE problem of transferring current from overhead contact wires to moving cars and locomotives is one which cannot be solved by mathematics or laboratory tests. Neither can conclusions as to the limits of current collection be reached by experience on any particular installation.

There has been considerable discussion as to the amount of current that can be successfully collected from an overhead contact system. Except for standard railways, operating experience has been confined to the heavier types of interurban service, with current values of from 300 to 800 amperes, and heavy traction work such as the Chicago, Milwaukee, & St. Paul Railway, where normal current collected with single pantograph varies between 800 and 1000 amperes, with a maximum of about 1500.

Those who have made a study of the subject know that there is a large number of factors to be taken into account and that each of these factors is subject to considerable variation due to design, maintenance methods, or both; also that certain factors are essential for successful current collection on any given installation.

In view of the nature of the problem, as mentioned above, and the general interest in probable future requirements, a series of tests was conducted with the object of securing information as to the maximum amount of current that could be successfully collected from overhead contact wires with the conditions which are described later. These tests were made at Erie, Pennsylvania, on a section of track owned by the East Erie Commercial Railroad, and used by the General Electric Company for testing locomotives and cars. Following reconstruction of overhead contact system and completion of test runs, demonstrations of heavy current collection were given to engineers and railroad men on July 16, 17, and 18, and other dates, 1923.

These tests were planned and conducted in conjunction with the New York Central Railroad and the Cleveland Union Terminals Company.

It is thought that the importance of this subject, in connection with railway electrification work, justifies the compilation of a record covering the details of preliminary tests and investigations and the results as finally demonstrated.

NATURE OF TESTS

Due to the fact that the capacity of the substation supplying power was limited to 6000 kw., tests were made at 850 and 1500 volts, using the lower voltage for the higher currents. Comparative collection tests were made at 850 and 1500 volts under identical conditions which clearly indicated that the voltage of contact line makes no difference in the collection of current so long as the voltage is more than adequate to maintain any arc that might occur between contact wire and pantograph.

Tests were also made to approximate conditions existing under prolonged acceleration periods, and to determine the temperature rise in various members of the contact system and in the collector.

With each change in contact wire arrangement and suspension, tests were started with low current values and at low speed and gradually worked up to maximum allowable values. A number of duplicate runs was made at maximum values to check the final results.

TESTING EQUIPMENT

A. General. In order to make the tests contemplated, it was necessary to make certain changes in existing equipment and provide other equipment.

An attempt is made under the following headings to describe the various items of equipment provided and tried out, methods of making tests, the final type of overhead contact system selected for demonstration purposes, and the general conclusions reached following the tests and demonstrations.

B. Power Supply. Current was obtained from the substation used to supply power to the test track.

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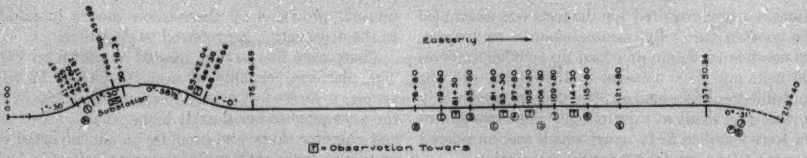
[1] Both of the General Electric Co., Schenectady, N. Y.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.


Easterly
0+00
1°-30'
41+93.67
42+25.00
43+12.61
45+
Feed Tap 49+69
50+78.34
1°-30'
Substation
0°-36½'
65+42.04
66+30
68+63.56
1°-0'
75+46.49
78+80
79+80
81+50
85+80
91+80
93+30
97+80
105+30
106+80
109+80
114+30
115+80
121+80
137+50.34
0°-31'
213+40
T = Observation Towers

Type of Catenary Construction					
Location	Messenger	Feeder Messenger	Number Contact Wires	Size Contact Wires	Suspension Contact Wires
① to ③	5/8" HS Steel	1,000,000 C M Copper	2	4/0 B & S Copper	Laced
③ to ④	5/8" HS Steel	150, 00 C M Copper	2	4/0 B & S Copper	Laced
④ to ⑦	5/8" HS Steel	150,000 C M Copper	2	4/0 B & S Copper	Laced

Type of Supporting Structure		
Location	Number of Structures	Type of Structure
Ⓐ to Ⓑ	13	Laced Column Bridges
Ⓑ to Ⓒ	5	Bethlehem "H" Column Bridges
Ⓒ to Ⓓ	5	Laced Channel Bracket Poles
Ⓓ to Ⓔ	5	10" Bethlehem "H" Bracket Poles
Ⓔ to Ⓕ	5	9" Bethlehem "H" Bracket Poles
Ⓖ	1	Laced Column Bridge

FIG. 1—SECTION OF TRACK USED FOR TESTS AND FINAL DATA ON CONSTRUCTION

The equipment consists of two synchronous motor-generator sets with switching arranged so that the generators could be used to supply any trolley potential from 600 to 3000 volts. One set with rating of 1000 kw. consists of two 500-kw., 1500/3000-volt generators direct-connected to a synchronous motor. The second unit is of similar construction consisting of two 750-kw., 1500/3000-volt generators. Full capacity could be obtained with series and parallel connections and lower voltages by adjusting the generator field rheostats.

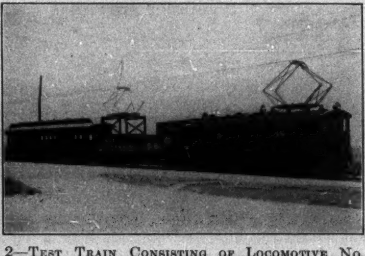


FIG. 2—TEST TRAIN CONSISTING OF LOCOMOTIVE NO. 18, GONDOLA

With loading rheostats and observation car

Both sets are designed to carry three times normal load for five min. or a total of 6000 kw. This substation is located approximately 0.9 mi. from the west end and four mi. from east end of the track. Power was supplied to the overhead line through a 1,000,000-cir. mil feeder shown in Fig. 15.

C. Test Train. A special test train was assembled consisting of a 110-ton gearless locomotive capable of operating at speeds up to 70 mi. per hr. with either 600 or 1500 volts. This locomotive was coupled to a special gondola car followed by a standard passenger car used as an observation car equipped with ammeter, speedometer, and telephone, Figs. 2 and 3. These cars were furnished by the New York Central Railroad. As the weight of this train was not sufficient to give the current desired, a sufficient number of iron grid rheostats with contactors and switches arranged to give the

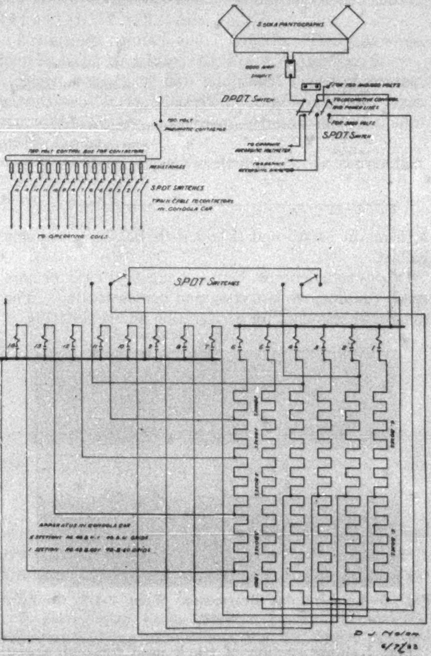

3 Sub A Pantographs
3000 Amp Shunt
DPDT Switch
For 750 and 1500 Volts
To Locomotive Control and Power Lines
For 3000 Volts
SPDT Switch
750 Volt Pneumatic Contactor
750 Volt Control Bus for Contactors
Resistances
SPDT Switches
Train Cable to Contactors in Gondola Car
To Operating Coils
To Graphic Recording Voltmeter
To Graphic Recording Ammeter
SPDT Switches
13
12
11
10
9
8
7
6
5
4
3
2
1
Apparatus in Gondola Car
D. J. Nolan

FIG. 3—WIRING DIAGRAM FOR CURRENT COLLECTION TESTS

Showing method of controlling current on test train consisting of Locomotive No. 18 and Gondola additional current required for the tests was assembled in the gondola car. By means of this equipment, it was possible to obtain any load up to 6000 amperes at 850 volts and 4500 amperes at 1500 volts. It was also possible to obtain load at 3000 volts although only a few runs were made at this voltage. The two pantographs were installed 57 ft. apart which was considered representative spacing for a two-unit locomotive.

D. Track. The track used for general testing is 4.15 mi. in length and is laid with 100-lb. rail in slag ballast.

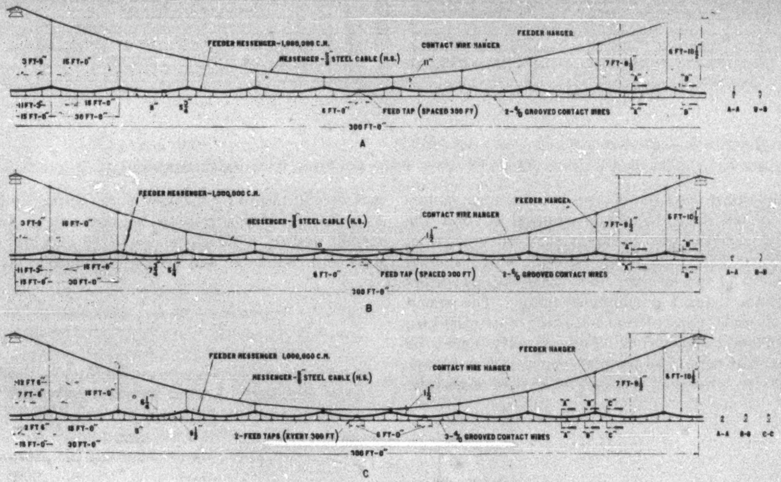

FEEDER MESSENGER—1,000,000 C.M.
MESSENGER—$\frac{5}{8}$" STEEL CABLE (H.S.)
CONTACT WIRE HANGER
FEEDER HANGER
3 FT-6"
15 FT-0"
7 FT-9$\frac{1}{2}$"
6 FT-10$\frac{1}{2}$"
FEED TAP (SPACED 300 FT)
2-4/0 GROOVED CONTACT WIRES
300 FT-0"
A
B
2-FEED TAPS (EVERY 300 FT)
3-4/0 GROOVED CONTACT WIRES
C
A-A
B-B
C-C

Fig. 4—Loop-Hanger Suspension

Of this total length, about two mi. were used for high speed running, while testing and demonstrating. The remainder was used for acceleration and retardation.

Fig. 5—Loop-Hanger Suspension With 2-4/0 Contact Wires

Fig. 1 shows section of track used for high speed running and current collection.

E. Observation Towers. It was decided to make the unusual provision of observation towers in addition to the observation car referred to elsewhere.

There were five towers, located as shown on Fig. 1. The platform, capable of accommodating 12 to 15 people, is located at a height which places the eyes of the average observer slightly higher than contact wires and collector shoes, and provides an unobstructed view of all the parts entering into current collection, when approaching, passing, and leaving. Towers were located as close to track as permitted by clearance requirements.

An added feature in this connection was that studies were made at night when the slightest spark between wire and shoe could be detected from these points of vantage.

F. Overhead Contact System. The existing contact system, within the limits selected for high speed and current collection, was not suitable for the purpose. This was constructed with more or less antiquated fittings and was not in first-class condition.

In order to supply current for contemplated tests, a 1,000,000-cir. mil feeder was installed from a point about 620 ft. west of substation tap to a point about 3580 ft. east of tap. The old 750,000-cir. mil feeder was used for extension from the latter point to the east end of high speed territory, a distance of 5400 ft.

1. General Design of Contact System. On account of the heavy current values contemplated, it was thought advisable to shorten and simplify the taps between feeder and contact wires as much as possible, and to reconstruct with compound catenary, suspending the feeder from the messenger and the contact wires from this feeder messenger. With the length of span

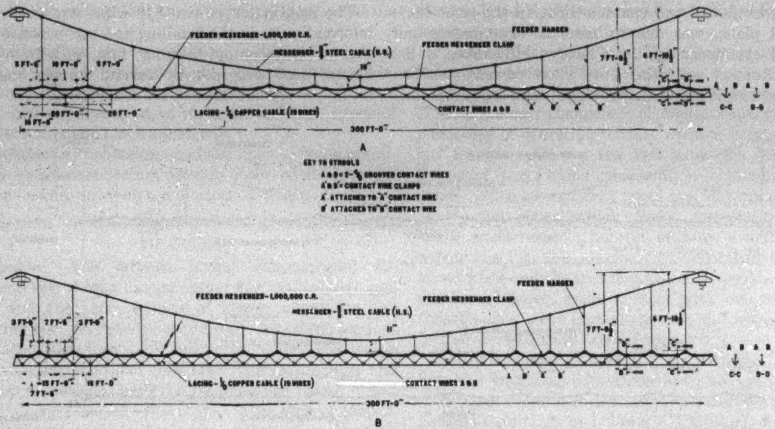

FEEDER MESSENGER-1,000,000 C.M.
MESSENGER-5/8" STEEL CABLE (H.S.)
FEEDER MESSENGER CLAMP
FEEDER HANGER
5 FT-0"
10 FT-0"
5 FT-0"
10"
7 FT-0 1/2"
6 FT-10 1/2"
LACING-1/2 COPPER CABLE (19 WIRES)
CONTACT WIRES A & B
20 FT-0"
20 FT-0"
10 FT-0"
300 FT-0"
C-C
D-D
A
KEY TO SYMBOLS
A & B = 2-4/0 GROOVED CONTACT WIRES
A' & B' = CONTACT WIRE CLAMPS
A' ATTACHED TO "A" CONTACT WIRE
B' ATTACHED TO "B" CONTACT WIRE
FEEDER MESSENGER-1,000,000 C.M.
MESSENGER-5/8" STEEL CABLE (H.S.)
FEEDER MESSENGER CLAMP
FEEDER HANGER
3 FT-9"
7 FT-6"
3 FT-9"
11"
7 FT-0 1/2"
6 FT-10 1/2"
LACING-1/2 COPPER CABLE (19 WIRES)
CONTACT WIRES A & B
15 FT-0"
15 FT-0"
7 FT-6"
300 FT-0"
C-C
D-D
B

Fig. 6—Laced Suspension

adopted, 300 ft., ⅝-in. high-strength steel cable was selected for the messenger.

The feeder messenger was suspended from the messenger by means of hangers made of No. 2 A. W. G. solid, hard-drawn copper, with bronze clamps for attachment to messenger and feeder messenger.

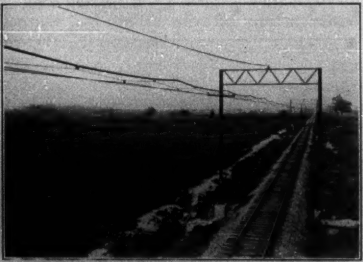


Fig. 7—Laced Suspension (Outside Rail for Testing Wide Gauge Locomotives)

2. Contact Wire Arrangements and Suspension. The first installation included several arrangements of contact wires and methods of suspension as described below, and referred to on Fig. 1. The table on this drawing gives the final arrangement.

a. Between points 1 and 2, 3600 ft. Two 4/0 wires, loop-hanger, suspension, (A), Fig. 4. Also Fig. 5.

b. Between points 2 and 3, 600 ft. Two 4/0 wires, laced suspension, (A) and (B), Fig. 6. Also Fig. 7.

c. Between points 3 and 4, 1200 ft. Two 6/0 wires, loop-hanger suspension, (B), Fig. 4.

d. Between points 4 and 5, 1200 ft. Three 4/0 wires, loop-hanger suspension, (C), Fig. 4. Also Fig. 8.

e. Between points 5 and 6, clamp suspension, 600 ft., (A) and (B), Fig. 9. Also Fig. 10.

f. Between points 6 and 7, 2400 ft. Two 4/0 wires, loop-hanger suspension, (A), Fig. 4. Also Fig. 5.

Short sections of two additional types of contact wire suspension were installed during preliminary tests.

g. Cable-hanger suspension, reproduction of photograph, Fig. 11.

h. Twin laced suspension with which the two contact wires were suspended by independent lacings, from symmetrical yokes, attached to feeder messenger, locating them about 3½ in. apart and in the same horizontal plane.

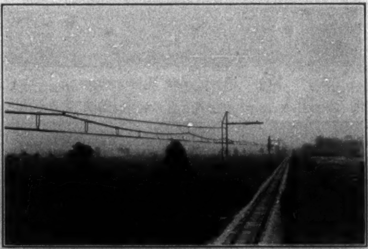


Fig. 8—Loop-Hanger Suspension With 2-4/0 Contact Wires

3. Feed Taps. Loop-hanger suspension, (a) and (f), required feed tap connections, Fig. 12, which were installed at the center of each span or 300 ft. apart. Feed tap cable was 4/0 copper with 19 wires.

4. Reasons for Several Contact Wire Arrangements.

The selection of two contact wires, in the same horizontal plane, was due to previous experiments and actual experience on the Chicago, Milwaukee, & St. Paul Railway and other lines where current collected was of considerable value. The main virtue of this arrangement, in addition to the increased contact surface, is that with alternate or staggered suspension of the two wires, the collector shoe is always in contact with two wires, and one wire is always without any additional weight, due to hangers or other fittings.

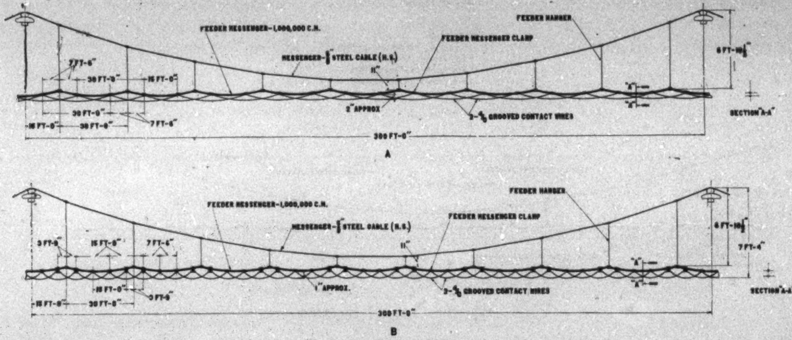


FIG. 9—CLAMP SUSPENSION

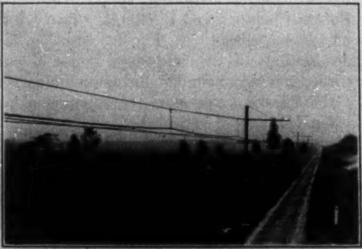


FIG. 10—CLAMP SUSPENSION

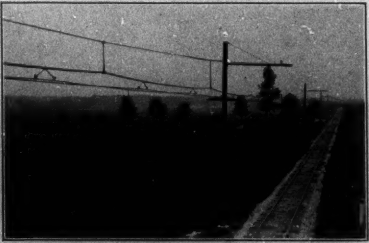


FIG. 11—CABLE-HANGER SUSPENSION

The installation of two 6/0 wires was made to get information as to the handling and performance of this size wire as a contact member: first, on account of its having been proposed by several foreign engineers; second, to try out this method of increasing cross-section should the two 4/0 wires show undesirable temperature rise; third, to study the effect of the additional weight in contact wires.

The installation of three 4/0 wires was made in line with possible need for more cross-section and more contact and to get experience in connection with their use, including the additional weight.

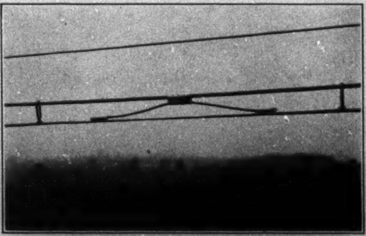


FIG. 12—FEED TAP WITH LOOP-HANGER SUSPENSION

METHODS OF MAKING TESTS AND BASIS FOR CONCLUSIONS

With the several types of contact wire arrangement and suspension in place, for any given set of test runs, the remaining fixed factor was line voltage. Those subject to change were speed, amount of current collected, number of pantographs, and pantograph pressure.

Observation tower construction and location permitted direct comparison between the performance of the several types of overhead construction with a certain line voltage and any desired combination of the variables.

Independent and simultaneous observations were made by four or five individuals from the same tower, and such observations made from several towers for each principal condition, so as to cover different types of construction and track alinement.

Satisfactory collection was fixed upon as being practically sparkless collection when so observed at night and when looking down on top of collector shoes.

RESULTS OF EXPERIMENTS WITH CONTACT WIRE SUSPENSION

General. The original layout contemplated the general use of loop hanger suspension (a), (c), (d), and (f), and the short section of clamp suspension (e). During the early stages of the test runs, short sections of laced suspension (b) and cable hanger suspension (g) were installed and compared with loop hanger and clamp suspension, with the results given below.

1. *Loop Hangers (a) and (f).* This type of suspension was selected for the larger portion of the work on account of flexibility and because it was being successfully used when currents of considerable magnitude were encountered, in regular service, though smaller in value than the heavy currents contemplated in this case.

With the general construction used for these tests, loop hangers, with feed taps spaced 300 ft. apart, provided the necessary conductivity between feeder messenger and contact wires for current values not in excess of 2500 amperes.

The Chicago, Milwaukee, & St. Paul Railway, using simple catenary construction, with loop hangers, steel messenger, and two 4/0 A. W. G. copper contact wires in the same horizontal plane, during an operating period of 11 years, and a total of 650 route mi., has had about three interruptions on account of messenger being burned at loop hangers.

These cases were all due to defective feed tap clamps at contact wire shunting the current through the steel messenger. General replacement of these clamps was made with an improved design and no further trouble of this kind has been experienced. Other roads equipped with the improved clamp have not experienced this trouble.

2. *Laced Suspension (b).* This method of suspension offered flexibility in line with that of loop hangers and the added feature of very frequent taps to feeder messenger and cross taps between contact wires. The first trial was made with two feeder hanger spacings, 30 ft. and 15 ft., with a minimum distance of six in. between top of contact wires and under side of feeder messenger. A few test runs indicated that the section with shorter hanger spacing gave better collection. Comparison with other types of suspension tested brought about the conclusion that the laced suspension provided the best conductivity between feeder and contact wires for the collection of the amount of current contemplated, and it was therefore adopted for further test runs and demonstrations.

3. *Clamp Suspension (e).* This design possessed the following features: Direct connection between contact wires and feeder messenger, the use of clamps instead of more expensive hangers or clamps and lacing, and the possibility of operating contact wires at comparatively low tension. After a few test runs this design was dismissed from further consideration for the purpose in hand. With two contact wires the lift was such that pantograph shoes struck feeder messenger clamps. While an increase in number of wires would tend to reduce the lift, the complication accompanying additional wires and the fact that one shoe would, with any number of wires, make contact with one wire only, throughout a great part of the line, did not encourage further investigation at that time.

4. *Cable Hanger Suspension (g).* About 150 ft. of line was equipped with these hangers, spaced 15 ft. apart, or 30 ft. on each contact wire. For test purposes, hangers were made of 1/0 B & S flexible copper strand, and the contact wire clamps used with laced suspension. Attachment to feeder messenger was made by copper wire wrapping.

This suspension approached laced construction in general principle and provided a manufactured unit, permitting attachment of clamps to strand by welding or other means supposedly preferable to clamped connection. This suspension showed no improvement over the simpler laced construction and the design was therefore eliminated.

5. *Twin Laced Suspension (h).* The trial of this suspension, incidental to collection trouble with laced suspension on a one-deg. curve, was made during the early stages of the tests. The design was never given serious consideration on account of inherent faults, including expense of two lacings, the special yokes for attachment to feeder messenger which would be expensive and difficult to maintain in any desired plane.

RESULTS OF EXPERIMENTS WITH CONTACT WIRE ARRANGEMENTS

Shortly before the completion of test runs and the demonstrations of current collection, it was decided that two 4/0 contact wires in the same horizontal plane with proper suspension and tension provided the necessary cross-section, contact surface, and weight for current collection covered by the tests.

While the experience gained with 6/0 contact wire was limited to a very small quantity, the condition of the wire, as installed, was never satisfactory owing to long kinks, presumably due to winding on reel. While this trouble may be avoided by using a reel with proper drum diameter, the general impression was that wire of this size should be passed through a wire straightener, mounted on tower car, as it is strung. As stated above, there seems to be no reason for adopting this special and undesirable size of wire.

While no particular trouble was encountered in connection with the three 4/0 contact wires, there was no indication of the additional wire being required, and its use would certainly complicate the contact system and introduce the undesirable requirement of maintaining approximately even tension in three wires instead of two in order to get the best results.

The following data apply to contact system finally used for tests and demonstrations:

Member	Material	Size	Wt. lin. ft. pounds
Messenger................	H. S. Steel	⅝ in.	0.8
Feeder Messenger.........	Copper	1,000,000 cir. mils	3.1
Feeder Messenger.........	Copper	750,000 " "	2.325
Lacing...................	Copper	105,000 " "	0.322
Contact wire 1...........	Copper	211,600 " "	0.64
Contact wire 2...........	Copper	211,600 " "	0.64

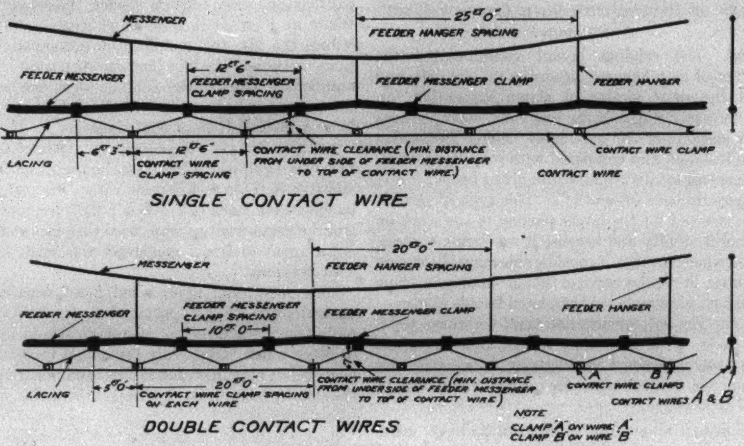

MESSENGER
25FT 0"
FEEDER HANGER SPACING
FEEDER MESSENGER
12FT 6"
FEEDER MESSENGER CLAMP SPACING
FEEDER MESSENGER CLAMP
FEEDER HANGER
LACING
6FT 3"
12FT 6"
CONTACT WIRE CLAMP SPACING
CONTACT WIRE CLEARANCE (MIN. DISTANCE FROM UNDER SIDE OF FEEDER MESSENGER TO TOP OF CONTACT WIRE.)
CONTACT WIRE
CONTACT WIRE CLAMP
SINGLE CONTACT WIRE
MESSENGER
20FT 0"
FEEDER HANGER SPACING
FEEDER MESSENGER
FEEDER MESSENGER CLAMP SPACING
10FT 0"
FEEDER MESSENGER CLAMP
FEEDER HANGER
LACING
5FT 0"
20FT 0"
CONTACT WIRE CLAMP SPACING ON EACH WIRE
CONTACT WIRE CLEARANCE (MIN. DISTANCE FROM UNDERSIDE OF FEEDER MESSENGER TO TOP OF CONTACT WIRE)
A
B
CONTACT WIRE CLAMPS
CONTACT WIRES A & B
NOTE
CLAMP "A" ON WIRE "A"
CLAMP "B" ON WIRE "B"
DOUBLE CONTACT WIRES

Fig. 13—Catenary Construction Laced Suspension for Contact Wires (Final Recommendations)

After completion of tests it was decided that the arrangement with laced suspension for single and double contact wires should be as shown on Fig. 13.

CONTACT SYSTEM ON CURVES

All curve work in the original installation was fitted with loop hanger suspension for contact wires, and both feeder and contact wire hangers were inclined, as shown in Fig. 14. This illustration also shows feeder connection at substation, consisting of one 1,000,000-cir. mil. cable, which supplied current to line for all tests including heat runs.

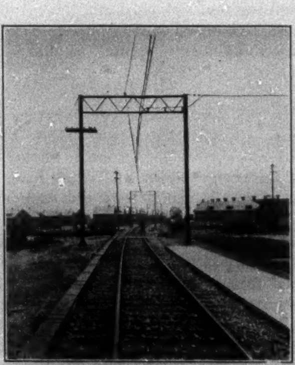


Fig. 14—Loop-Hanger Suspension on Curve

Showing inclined messenger construction

Current collection on this curve construction, as experimentally installed, in combination with other factors affecting current collection, was not satisfactory on account of arcing. Laced suspension with inclined messenger gave the same results. No further studies were made with inclined hangers but contact system was trimmed in chords with pull-offs located 150 ft. apart, as shown in Fig. 15.

The final combination of laced and chord construction gave practically sparkless current collection.

CONTACT WIRE TENSION

As originally installed, contact wires had a tension of 1000 lb. at 70 deg. fahr. Before tests were completed, this tension was considerably below that value due to seasonal increase in temperature and many changes made in line construction. When curve work was finally adjusted, late in June, tension was increased to from 1300 lb. to 1400 lb. at 70 deg. fahr. and with improved collection throughout.

COLLECTORS

The collector used throughout for test runs and demonstrations was the S-501-A slider trolley, with certain modifications mentioned later, Fig. 16.

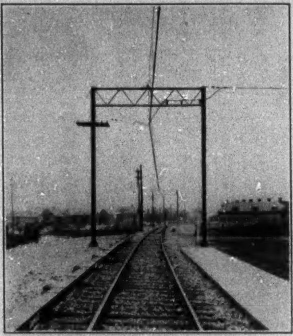


FIG. 15—LACED-TYPE SUSPENSION ON CURVE

Showing chord construction and feeder connection

The collector consists of two flexibly mounted, contact shoes on top of a jointed diamond or pantograph frame, the diamond or pantograph frame being constructed of Shelby tubing with malleable iron joint castings so hinged together that it can readily expand or contract to suit variations in height of contact wires.

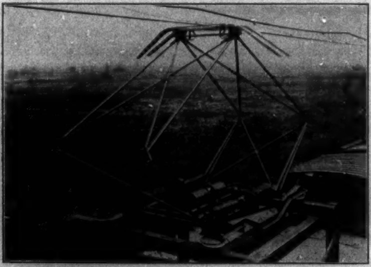


FIG. 16—S-501-A PANTOGRAPH USED FOR CURRENT COLLECTION TESTS

As mounted on wooden structure in the Gondola carrying the grids

The collector is expanded or raised to its operating position by means of four large coil springs attached to cams mounted on the same hinge shaft as the bottom part of the diamond frame. Two of the large springs are energized and permanently attached to the base of the collector for balancing approximately the dead load of the moving parts. The other two large springs are connected to air cylinders by means of which they can be energized or de-energized at will.

The small irregularities in the overhead are taken care of by the flexibility of the contact shoes. Each shoe is independently hinged on two spring supported cams which allow it to rise and lower two in. independent of the main or diamond frame. The contact shoes are composed of sheet steel pans with sheet steel horns attached to each end for picking up the siding wires. The wearing strips are of hard drawn copper, 3/16 in. thick and 1-3/16 in. wide, having one edge bent over slightly to prevent fouling, leaving approximately ¾ in. of width, flat surface. On each side of each pan is one long, renewable, wearing strip, and in the middle of each pan where most of the wear is concentrated between the outside strips, two short, renewable, wearing strips. This gives at the middle section of each shoe a line contact with the contact wires of approximately three in. or a total for both shoes of six in. The space between the renewable, wearing strips is filled with a lubricant for preventing a rapid deterioration of both strips and contact wire. Shoes are shown in plan in Fig. 17.

The standard collector has a rated continuous capacity of 1000 amperes, or 2000 amperes for two min., and is suitable for speeds up to 60 mi. per hr.

In order to bring the capacity up to the heavy current requirements, the collectors were fitted with additional shunts around each bearing and a flat copper strip along one side of both top and bottom frame arms. The additional weight imposed by this copper necessitated increasing capacity of balancing springs.

As far as principle of design goes, a standard collector was used and the only special precaution taken was to surface the wearing strips on an emery-covered face plate after assembly, which should be done in regular practise regardless of current values.

COLLECTOR PRESSURE

During testing period, collector pressures varying between 19 and 43 lb. were used. Reference to list of test runs shows that the pressure finally used for the collector and contact system dealt with was about 40 lb. This pressure was necessary on account of the increased moving weight due to copper shunts and strips.

CURRENT COLLECTION TESTS

The following list gives data on a group of representative test runs with both 850 and 1500 volts, using the type of contact system finally adopted. More than 350 test runs were made and records kept in connection with all runs.

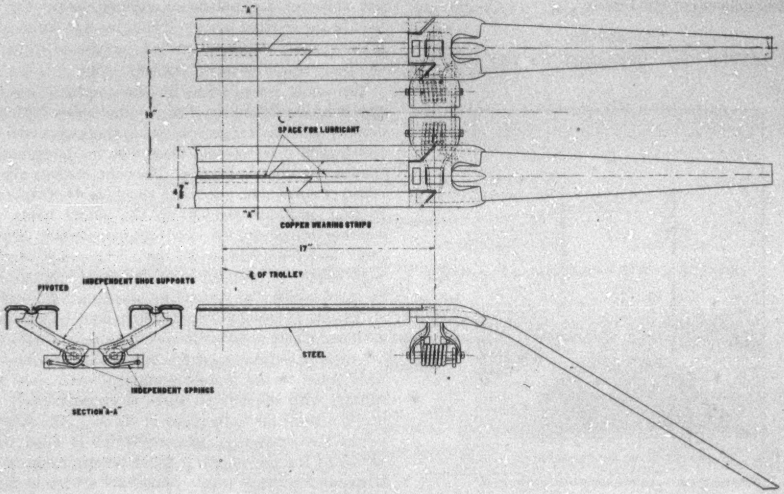

"A"
10"
SPACE FOR LUBRICANT
4 7/8"
"A"
COPPER WEARING STRIPS
17"
℄ OF TROLLEY
PIVOTED
INDEPENDENT SHOE SUPPORTS
STEEL
INDEPENDENT SPRINGS
SECTION "A-A"

FIG. 17—PANTOGRAPH SHOES

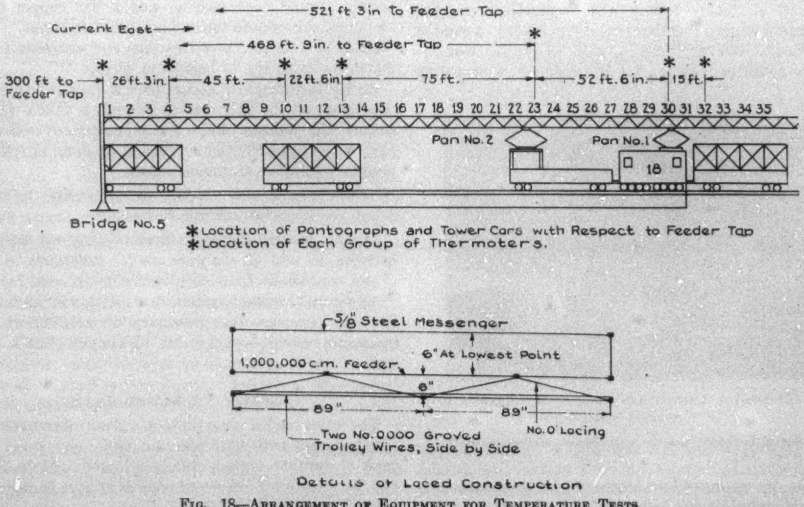

521 ft 3 in To Feeder Tap
Current East
468 ft. 9 in. to Feeder Tap
300 ft to Feeder Tap
26 ft. 3 in.
45 ft.
22 ft. 6 in.
75 ft.
52 ft. 6 in.
15 ft.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Pan No. 2
Pan No. 1
18
Bridge No. 5
* Location of Pantographs and Tower Cars with Respect to Feeder Tap
* Location of Each Group of Thermoters.
5/8" Steel Messenger
1,000,000 c.m. Feeder
6" At Lowest Point
6"
89"
89"
Two No. 0000 Groved Trolley Wires, Side by Side
No. 0 Lacing
Details of Laced Construction

FIG. 18—ARRANGEMENT OF EQUIPMENT FOR TEMPERATURE TESTS

TESTS AT 850 AND 1500 VOLTS

Test no.	Amps.	Speed mi.perhr.	Volts	No. of collectors used	Pressure lbs.	Date
125	5000	50	850	1	32.5	6–15–23
150	5000	50	850	1	30	6–19–23
160	6000	50	850	2	35 33	6–23–23
161	6000	50	850	2	35 33	6–23–23
162	6000	50	850	2	35 33	6–23–23
164	5100	50	850	2	35 33	6–23–23
172	5200	54	850	2	35 37	6–24–23
173	5200	54	850	2	35 37	6–24–23
224	5100	65	850	1	37.5	6–28–23
225	5000	60	850	1	37.5	6–28–23
227	5100	60	850	1	37.5	6–28–23
232	5200	50	850	2	39 40.5	7–10–23
240	5000	50	850	1	40.5	7–11–23
244	5000	55	850	1	40.5	7–12–23
249	5300	57	850	1	40.5	7–13–23
250	5400	56	850	2	40.5 42	7–13–23
254	5300	57	850	1	39.5	7–13–23
255	5300	58	850	1	39.5	7–13–23
256	5800	53	850	2	39.5 42	7–13–23
257	5800	55	850	2	39.5 42	7–13–23
278	5500	58	850	2	34.5	7–16–23
280	5400	58	850	1	34.5	7–16–23
96	4300	32	1500	2	32.5 34.5	6– 4–23
110	4000	52	1500	2	32.5 34.5	6–15–23
191	4200	48	1500	2	39 40.5	6–26–23
192	4000	48	1500	2	39 40.5	6–26–23
198	4500	40	1500	1	39	6–26–23
200	3000	60	1500	1	39	6–26–23
259	4500	46	1500	1	39.5	7–14–23
261	4150	48	1500	1	39.5	7–14–23
281	4500	50	1500	1	39.5	7–17–23
282	4600	52	1500	1	39.5	7–17–23

TEMPERATURE TESTS

Temperature tests were made to determine heating of the contact wire, feeder, messenger, pantograph shoe, etc., in order to definitely establish that it is possible to conduct heavy currents to a locomotive. These tests were made on the laced overhead trolley construction as shown in Fig. 7. The feeder in this section is a 1,000,000-cir. mil copper conductor acting as a messenger.

The general arrangement of the test train, location of pantographs, and overhead construction is shown in Fig. 18. The thermometers were placed on the contact wire, feeder, lacing, and messenger near lacings, 1, 4, 10, 13, 23, 30, and 32.

Test No. 1: This test was made with both pantographs raised and current maintained at approximately 5200 amperes for five min. Nine readings were taken at one-min. intervals, the first five being taken while current was flowing and the last four after the circuit was interrupted. A maximum temperature rise of 67 deg. cent. was obtained on the contact wire near pantograph 2, lacing 23.

Test No. 2: This test was made with one pantograph located near lacing 23 with approximately 5200 amperes for five min. Ten readings were taken, five while current was flowing and five after interruption of the circuit. A maximum temperature rise of 101 deg. cent. was obtained on the contact wire near the pantograph.

TEST NO. 1

5200 Amperes for Approximately Five Minutes, Two Pantographs.
Maximum Temperature Rise, Deg. Cent.

Thermometer location	at Pantograph		Distance from Pantograph No. 1				
	No. 1	No. 2	127.5 ft.	150 ft.	195 ft.	217½ ft.	15 ft.*
Feeder..........	4	28	27.2	30	27.5	27	1.0
Lacing..........	12	33	13.5	13.5	14.5	13	..
Contact wire...	35	67	31.0	28.4	29.0	21	4.2
Shoes..........	10	17	..	..	..	..	..

Temperature of air, 25 deg. cent.

TEST NO. 2

5200 Amperes for Approximately Five Minutes, One Pantograph.
Maximum Temperature Rise, Deg. Cent.

Thermometer location	at Pantograph		Distance from Pantograph No. 1				
	No. 1	No. 2	127.5 ft.	150 ft.	195 ft.	217½ ft.	15 ft.*
Feeder..........	29	40.5	31	30	28.5	28.0	115
Lacing..........	55	18.0	16.0	15.6	16.0	14	..
Contact wire...	101	36.0	28	29.2	30.5	23.5	27
Shoes..........	53	..	..	..			

Temperature of air, 25 deg. cent.
*Thermometer location on opposite side from source of power.

These tests do not reproduce exact operating conditions after a train has started but might apply while locomotive is standing still, attempting to start a train. The temperatures obtained are so low and the current and time used in the tests so high that they clearly indicate that no trouble would be experienced due to overheating of the distributing, contact, or collecting equipment.

BURN-OFF TESTS

These tests were made to determine the amount of burning which would be obtained if a pantograph should start to drop while carrying heavy currents and for some reason was checked and held in position a few inches from the contact wire. These tests were made with the locomotive standing still. While different amounts of current were passing through the contact wire and pantograph, the pantograph was released, stopped, and held. Potential of 1500 volts was held on the contact wire throughout the tests. Twenty tests of this kind were made, with current reaching a maximum of 4400 amperes, the maximum drop of the pantograph being 17 in. and the minimum one in. In no case was the contact wire or pantograph seriously damaged. A pantograph was also dropped without checking, with 5000 amperes, at 850 volts, without serious damage.

Several tests were made interrupting 4000 amperes at 1500 volts with locomotive control arranged so that if the pantograph should begin to drop due to low-air pressure, a pressure relay on the locomotive would open the main breaker and interrupt the circuit. With this system of control, there was no sparking at pantographs.

CONCLUSIONS

Contact System Design. Contact systems, in actual use, for heavy traction, vary in principle of design between direct suspension, with considerable weight concentrated at points of contact wire support, and catenary suspension, with practically no change in contact wire weight and freedom of vertical movement throughout. There is a corresponding variation in current transfer capacity.

It is desirable to provide a contact member of uniform flexibility and, as far as possible, of uniform weight throughout. The necessity for flexibility and uniformity in weight increases as current values increase.

It is important to avoid the use of fittings which obstruct in any way the contact between contact wire and pantograph shoes.

Contact Wire Lubrication. Experience gained by these tests and on lines equipped with the same or similar type of pantograph shows that a very thin film of lubricant should be maintained on under side of contact wire and that there is no difficulty in this connection provided pantograph shoes are properly lubricated.

It is interesting to note that contact resistance between wire and shoe is decreased when lubricant is used. This is probably due to elimination of chattering and actual glazing of contact wire and wearing strips of pantograph shoes.

Contact Wire Tension. Contact wire tension should be maintained at the highest value consistent with temperature conditions and other limiting features.

Current Collector Design. The efficiency of this device is affected by the following details:

Weight of moving parts and friction in bearings and joints which may interfere with its response to any change in contact wire height,

Design of contact shoe, including number and assembly of wearing strips, lubrication, and facilities for lubrication,

When two shoes are used, the degree of independence of movement and spring control,

The rigidity of pantograph frame in connection with side sway,

It is to be noted that a collector of standard design, with slight modifications to increase current carrying capacity, was used throughout these tests.

Shoe Pressure. The word "pressure" is intended to mean the pressure exerted at standard contact wire height with pantograph in motion or with friction practically eliminated. This value can be obtained by tying pantograph with shoe at standard contact wire height and reading pressure with spring balance while pantograph is shaken, thus approximating the condition while collecting.

It is desirable to emphasize the importance of pressure in connection with current collection, and, when more than one collector is used, the equalization of pressure.

Shoe pressure must be adapted to the overhead contact system design and should be maintained at the minimum value found practicable in each case.

Number of Collectors Used and their Spacing. These items determine the total upward pressure on contact wires and distribution of pressure, and certain combinations may disqualify a contact system suitable for use with a single collector or other combinations.

The number of collectors used must be determined by their design, current to be collected, and their minimum spacing, by overhead contact system design outside of locomotive design, and other considerations. It is desirable to make this distance a maximum.

Speed. The speed at which a collector is moved introduces the effects of wind pressure on pantograph frame and certain parts, such as shoe horns, the inertia of pantograph frame and shoes, and side whipping with bad track surface. It also places limits on grades in contact wire and changes in weight and flexibility of contact wire.

Effect of Voltage on Current Collection. Test runs were made with 750, 850, and 1500 volts. With all other conditions the same, no difference could be detected in quality of current collection with the three voltages mentioned.

General. Conservative evaluation of these tests and experience gained from operated lines indicate that 2000 amperes or more can be successfully collected, at any speed up to 60 or 70 mi. per hour, with one pantograph, and 4000 amperes with two pantographs.

These tests also demonstrate that it is practicable to design and construct an overhead contact and distribution system capable of delivering more than the amount of current required for train propulsion, with line potentials used to date for trunk line electrification.

The type of suspension, connecting messenger or feeder messenger and contact wire or wires, and its conductivity must be governed by the maximum current to be collected.

These tests and experience lead to the following approximate ratings for the types of construction given:

a. Steel messenger, two 4/0 copper contact wires, loop hangers, feed taps spaced 1000 ft., as used on Chicago, Milwaukee, & St. Paul Railway:

Normal current, 1000 amperes with a maximum of 1500.

b. Compound catenary with auxiliary feeder, messenger, loop hangers, and feed taps spaced 300 ft.

Normal current 2000 amperes with a maximum of 2700.

c. For higher current value, the frequencies of feeder taps must be increased in proportion to the current demand.

Discussion

For discussion of this paper see page 1133.

Railway Inclined Catenary Standardized Design

BY O. M. JORSTAD[1]

Associate, A. I. E. E.

Synopsis.*—A description is given of a new method of overhead contact design, the "ideal inclined catenary." This is based on an originally discovered tension and weight relation formula. A proof of the formula is given and other characteristics of the design are mathematically analyzed.*

A number of railroads now using inclined catenary is listed and data on the weights and tensions of their overhead constructions are given for comparison purposes with the "ideal."

The necessity of making a definite selection of a proper contact wire tension in any inclined catenary design is indicated and that this, together with the use of the design formula, leads to standardization of overhead systems is pointed out.

A RAILROAD track alinement is made up of a succession of tangents and curves. The overhead contact system in an electrification must be designed to follow the alinement so that the current collector of the car or locomotive will always make contact in a satisfactory manner.

Many types of overhead contact systems have been designed and applied on the many electrified railroads throughout the world. On tangent sections they are practically all similar in one respect, *i. e.*, their catenary hangers are vertical. The curve constructions, however, generally speaking, may be divided into two classes, one with hangers vertical as on tangent and the other with hangers inclined across the track. The vertical hanger type is called, by some, the polyhedral type and by others the chord type, as the catenary construction is pulled into a series of straight lines or chords over the track by pull-offs from a back-bone or pull-off posts. The inclined hanger type pulls the contact wire into a position over the curved track by inclining the hangers and thus causing them to function as combined pull-offs and hangers. Back-bones and pull-offs, except on the sharper curves, are usually omitted in the inclined catenary construction.

Chord construction is an adaptation of tangent construction to curves. Likewise, the more usual inclined construction has heretofore been the result of displacing the messenger of the correlated tangent construction laterally. In the United States, both types are in general use with inclined catenary the most common on main line electrifications. In other countries, however, the chord type has been the most favored. The following is a partial list of users of inclined catenary in this and other countries:

The New York, New Haven, and Hartford Railroad Co.

New York, Westchester, and Boston Ry.,

Boston and Maine Railroad.

Pennsylvania Railroad,

Norfolk and Western Ry.,

Virginian Ry.,

Detroit, Toledo, and Ironton Railroad.

Chicago, North Shore, and Milwaukee Railroad Co.,

Canadian National Rys.,

Lancaster, Morecambe, and Heysham Railroad in England,

Midi Railroad in France.

This list indicates that the inclined catenary is a practicable construction and that in every electrification of the immediate future it will in all probability come up for consideration. It also strongly indicates that an inclined catenary of some description will be a future standard overhead.

The good qualities of the inclined construction may be partially summarized as follows: It is artistic and makes a strong appeal to the esthetic sense. It is economical of material in that it employs two wire members to do the work of the usual three or four in the chord type. It has inherent automatic tension characteristics and above all it supplies a contact line that approaches most closely the ideal desired, *i. e.*, uniform flexibility. The greatest obstacle in the way of its more general use has been the comparative complexity of methods of design.

As stated above, the present forms of inclined catenary curve construction were developed from the correlated tangent construction and consequently acquired similar tensions and sags for similar lengths of spans. In the design of the tangent construction, there has been no fixed rule for determining the relative values of the various factors of design, tension, weight, sag, etc., of messenger and contact. The object sought was a maximum span with sags selected to keep the contact wire from being displaced by wind and leaving the collector. Such requirements resulted in a great variety of tangent catenary designs, each one depending on local and special conditions. The resulting related inclined catenary curve constructions were not entirely satisfactory from the designer's standpoint as there was difficulty in securing proper alinements but they were made operative and were usually a great improvement over previous chord constructions, particularly on multitrack sections.

There is now available, however, the discovered formula,

$$T_m/T_c = W_m/W_c \qquad (1)$$

which provides a basis for a simplified and precise design of inclined catenary. In the formula, W_c =

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[1] General Engineering Dept., Westinghouse Elec. & Mfg. Company, East Pittsburgh, Pa.

Presented at the Summer Convention of the A. I. E. E., Detroit, Mich., June 20-24, 1927.

weight of contact wire in lb. (kg.) per linear ft., T_c = tension of contact wire in lb., W_m = weight of messenger wire in lb. (kg.) per linear ft., and T_m = tension of messenger wire in lb. In this design, the contact wire is practically parallel to the center line of track and all hangers in any given curve are parallel, *i. e.*, they make the same angle with the vertical. Also it must be noted that it is the combination of the weight and tension relation indicated by the formula and the approximate parallelism of contact wire with center line of the curved track that provides what may be called an ideal inclined catenary. With one of the two conditions absent, an ideal inclined catenary is not secured.

It should also be noted that this ideal combination of conditions can be secured only at some one selected design temperature. A change in temperature tends to modify both of the combination factors in a construction, as T_c/T_m does not usually stay constant over a range of temperature and the contact wire distorts either vertically or horizontally with the least change in temperature. This variation of tension with temperature is, however, a characteristic of all non-automatic catenary constructions.

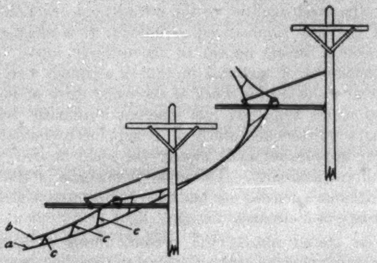

FIG. 1—SHOWING INCLINED CATENARY IN PERSPECTIVE

Also, any loss in weight of the contact wire after construction due to wear by collector causes a departure from the ideal design conditions. The loss in weight of contact is accompanied by a corresponding loss in tension and thus the formula is apparently satisfied. The loss in weight of contact wire, however, causes the tension of messenger to decrease due to its decreased load. This causes the messenger to rise, pulling up the contact wire at the middle of span with consequent distortion from its original position of parallelism to center line of track. Distortion due to loss of weight with wear, however, is common to all contact systems.

A demonstration of the truth of the formula may be made as follows. Fig. 1 is a sketch in perspective of the usual inclined catenary. Here *a* is the contact wire, *b* the messenger, and *c*, *c*, the inclined hangers. Fig. 2 is a plan view of an ideal inclined catenary span with zero length of shortest hangers. The contact wire *a* is a parabola and the curve *f*, the projection of the messenger on the horizontal plane of contact wire, is also a parabola.[2] The two parabolas are tangent at their vertices.

The ideal contact line or wire is defined as the wire under tension which lies in a horizontal plane and has the form of a parabola whose horizontal sag for a given span and curve is equal to the middle ordinate of the circular arc of the same span. For the usual spans and usual degrees of curvature there is practically no difference in the tensions or positions of the wire in the form of a parabola or an arc of a circle[3].

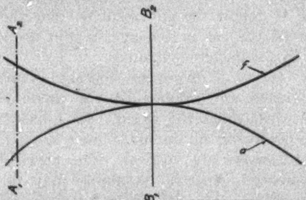

FIG. 2—PLAN VIEW OF IDEAL INCLINED CATENARY CONSTRUCTION

$A_1 - A_2$ Fig. 2 is a vertical plane through the messenger and contact parallel to the principal axis $B_1 B_2$ of the contact parabola. In Fig. 3 the triangle $g\ h\ i$ is a projection horizontally on the vertical plane $A_1 - A_2$ of Fig. 2 of the inclined catenary construction.

In Fig. 3:

$g\ h$ = horizontal sag of contact parabola,
$g\ i$ = hanger in the plane of projection,
$h\ i$ = sag of messenger.

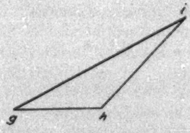

FIG. 3—PROJECTION HORIZONTALLY OF IDEAL INCLINED CATENARY CONSTRUCTION ON VERTICAL PLANE $A_1 A_2$ OF FIG. 2

Fig. 4 is a projection similar to Fig. 3 but with the following additional construction:

$g\ h$ is extended horizontally to o,
$i\ o$ is dropped vertically from i to o,
a vertical is constructed through g,
$i\ m$ is extended horizontally from i to meet vertical at m,
the diagonal $i\ h$ is extended to meet vertical at k.

In the triangle $m\ i\ g$ of Fig. 4, $i\ g$ has the slope of the hanger and if its length is assumed to represent the force on the hanger due to the dead load of contact wire and horizontal load caused by tension in contact wire then the latter two forces are represented respectively by the other two sides of the force triangle $m\,g$ and $m\,i$.

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[2] See Appendix I.

[3] See Appendix IV.

Likewise in triangle $m\,i\,k$ of Fig. 4, $i\,k$ has the slope of the messenger and as $m\,i$ represents the horizontal load on the messenger due to tension in contact, the other two sides of the triangle will represent the other two forces on the messenger, $m\,k$ the total dead load acting vertically and $i\,k$ the resultant load on messenger acting in the direction of slope of messenger.

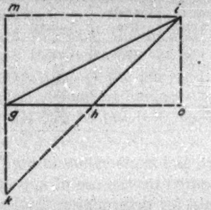

Fig. 4—Projection of Fig. 3 With Additional Construction Lines

Therefore

$m\,k = W_c + W_m$, weight of contact plus weight of messenger, and since $m\,g = W_c$ = weight of contact (2)

then

$$g\,k = W_m = \text{weight of messenger} \quad (3)$$

and

$$g\,k/m\,g = W_m/W_c \quad (4)$$

The two triangles in the Fig. 4, $g\,h\,k$ and $h\,o\,i$, are similar since their corresponding angles are equal.

Therefore

$$g\,h/h\,o = g\,k/i\,o \quad (5)$$

But

$$i\,o = m\,g, \text{ by construction} \quad (6)$$

Therefore

$$g\,h/h\,o = g\,k/m\,g \text{ from 5 and 6} \quad (7)$$

Also

$$g\,h/h\,o = T_m/T_c \text{ see Appendix III} \quad (8)$$

Therefore

$$T_m/T_c = W_m/W_c \text{ (axiomatic from (4), (7), and (8))} \quad (9)$$

It has thus been demonstrated that the ideal inclined catenary construction with zero length of shortest hanger is a true construction and that the relation of weights and tensions of members is as given in the formula.

The usual inclined catenary, however, has a shortest hanger of some length. If the contact line of such usual catenary is ideals,[4] it also has parallel hangers and the formula will also show the relation of its weights and tensions.

This can be demonstrated by considering Fig. 5. On Fig. 5, $g\,h\,i$ is the $g\,h\,i$ of Fig. 3; $g\,s\,p\,h$ is another figure showing the projection of the more usual inclined catenary with shortest hanger $h\,p$ of some length. The only difference between the ideal inclined catenary $g\,h\,i$ and the usual inclined catenary illustrated is in the length of hangers. The other factors in the two are identical, weight of contact, tension of contact, weight of messenger, and tension of messenger. Consequently, the usual inclined catenary with ideal contact line and parallel hangers will be in agreement with the formula. And conversely, if weights and tension are chosen in accordance with the formula, an ideal inclined catenary results if the contact line is made ideal.

In demonstrating the formula it has been assumed that the hangers were without weight. Practically, they will have some weight, this usually approximating five per cent of contact and messenger weight. This weight will have an effect on the actual slope and sag in practical construction and must be provided for. If the hanger weight is included by prorating it and including with contact and messenger weights, the ratio of weights and consequently of tensions in the formula is not changed so that if a basic contact tension is selected, the messenger tension will be fixed regardless of whether hanger weights are considered or not. Therefore, in preliminary determination or selection of messenger tension, hanger weights can be safely ignored.

In determining slopes of hangers and slopes of messenger and sag of messenger, however, the weights of hangers must be taken into consideration. Such consideration, however, will not destroy the value of the formula in practical applications as the distortion due to the hanger weights in the extreme case, *i. e.*, the case where the shortest hanger is assumed to have zero length, is practically negligible.

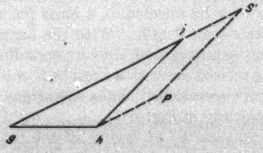

Fig. 5—Projection of Fig. 3 with Projection of Inclined Catenary with Lengthened Hangers

To assist in applying the formula in design work the nomogram in Fig. 6 is given. By its use the tensions for various messengers of different weights can be quickly and accurately determined. Likewise, the effect on messenger tension of changes in contact characteristics in the preliminary design can be readily determined. The index lines show that a four naught (106-sq. mm.) copper contact wire weighing 0.641 lb. per ft. (0.956 kg. per m.) tensioned to 2500 lb. (1135 kg.)

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[4] See previous definition.

requires that a messenger weighing 2.37 lb. per ft. (3.54 kg. per m.) have a tension of 9240 lb. (4200 kg.) in order to satisfy the design conditions for ideal inclined catenary.

On the assumption that tangent construction conditions are also satisfied by the indicated curve messenger tension—*i.e.*, that a practical span and sag results—the next step in the order of design is to find the length of hangers in the tangent span selected. Fig. 7 is a diagram method based on the formula and the supplementary truth that for every tangent hanger, a corresponding inclined hanger for each degree of curve can be selected.

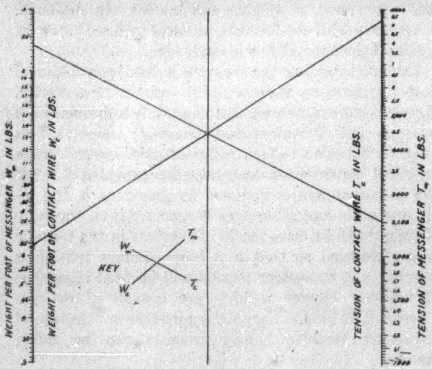

FIG. 6—NOMOGRAM OF FUNDAMENTAL FORMULA FOR DESIGN OF IDEAL INCLINED CATENARY $T_m/T_c = W_m/W_c$

The basic formula (**9**) is independent of span length or degree of curvature. Furthermore, the quantities in the formula are kept constant in all spans and curves in the usual practical alinement. Hence, any tangent span length may be selected as a basis for a group of curves of same span length. With the hangers of the basic tangent span selected, the corresponding hangers of any of the related curve spans can be found by simple mathematical operations. The operations are performed graphically in Fig. 7.

Fig. 7 is constructed by laying off the tangent half-span with the customary assumption that it is a parabola and that the contact line is horizontal. The tangent hangers of the half-span are drawn in to scale at desired hanger positions with the contact line shown the proper distance below the messenger. The slope lines of the inclined hangers for the various degrees of curvature are determined by laying off W_{ch}, the unit weight of contact with prorated hanger weight vertically and the radial load[5] per unit of contact horizontally and drawing in the straight lines through zero and such points. The radial load per foot of contact is T_c/R, where R is the radius in feet of the curve.[6] The length of the inclined hanger is found by projecting horizontally from the corresponding tangent hanger length to the proper slope line. The angle with the vertical is found with a protractor. A tabulation of corresponding hanger and angles is shown in Table I.

TABLE I
LENGTH AND ANGLE OF HANGERS

Alinement hanger no.	Tangent length in inches	1 Deg.		2 Deg.	
		Angle Deg.	Length	Angle Deg.	Length
1	9½	33½	12	52	16
6	12½	"	15	"	21
3	18½	"	22½	"	30½
4	27½	"	33	"	45
5	40	"	48½	"	66
6	54½	"	65½	"	89½

Both length and angle values in Fig. 7 can be readily checked if desired by the use of appropriate formulas.

The formula for determining the length of tangent hangers is based on commonly known characteristics of the parabola:

$$h = e + (4\,x^2\,s/L^2) \qquad (10)$$

where

h = hanger length,
e = length of shortest hanger at lowest point of sag,
x = horizontal distance of hanger from lowest point of sag,
s = sag of the half-span considered,
$L/2$ = length of the half-span considered.

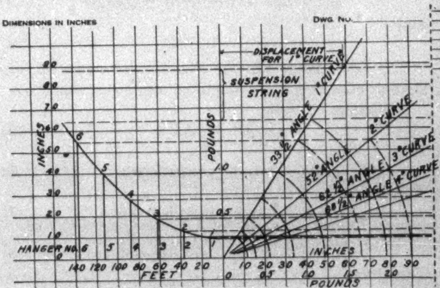

FIG. 7—DIAGRAM FOR GRAPHICALLY DETERMINING HANGER LENGTHS AND ANGLES

The corresponding inclined hanger length is determined by multiplying the tangent hanger length by multiplier as found in the following formula:

$$(T_c^2/R^2 + W_{ch}^2)^{1/2}/W_{ch} \qquad (11)$$

The angle of hanger with vertical is

$$\mathrm{Tan}^{-1}\, T_c/R\, W_{ch} \qquad (12)$$

The practically standard method of designating hangers is to give their angle of inclination from the vertical and their length from center line of messenger to center line of contact. Also, the practically standard term for designating the horizontal distance between center lines of contact and messenger is "the displacement."

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[5] See Appendix IV.

[6] See "Notes on Catenary Construction" by Sidney Withington, *Franklin Institute Journal*, Dec., 1914, p. 720.

The length of the shortest hanger is usually taken as 6 in. This length selection, however, depends on various conditions and the hanger may be made shorter or longer as circumstances may require. In Fig. 7 the shortest hanger is 9 in.

With the design following the formula, it can be demonstrated that the inclination of the suspension string is the same as that of the hangers in all cases, *i. e.*, with any length of shortest hanger.

The basic formula can be put into the form

$T_c/W_{ch} = (T_m + T_c)/(W_{mh} + W_{ch})$, a continued proportion **(13)**

Therefore

$T_c/W_{ch}\, R = (T_m + T_c)/(W_{mh} + W_{ch})\, R$, multiplying both sides of **(13)** by $1/R$ **(14)**

The formula for angle which the hanger makes with the vertical is previously given as

$$\tan^{-1} T_c/W_{ch}\, R \qquad \textbf{(12)}$$

The angle with vertical made by the suspension string is

$$\tan^{-1} (T_m + T_c)/(W_{mh} + W_{ch})\, R \qquad \textbf{(15)}$$

This is so since the suspension string takes a position in the direction of the resultant of the two forces acting on it, the vertical force due to dead load of span $(W_{mh} + W_{ch})\, L$ and the horizontal force $(T_m + T_c)\, L/R$. And hence the tangent of the vertical angle is

$$(\,(T_m + T_c)\, L/R)/(W_{mh} + W_{ch})\, L \qquad \textbf{(16)}$$

which upon simplification becomes

$$(T_m + T_c)/(W_{mh} + W_{ch})\, R \qquad \textbf{(17)}$$

Therefore the two angles are equal since their tangents are equal. (From **(12)**, **(17)**, and **(14)**.)

Consequently, the total displacement of the insulator support from center line of contact wire may be found by considering the suspension string as an extension of the theoretically longest hanger at the point of support; see Fig. 7. This is true only of the ideal inclined catenary.

With a theoretical "zero" length of shortest hanger, the displacement d of the messenger for a given contact and messenger and a given curve and span is a constant quantity and does not change with variation in the design contact tension and messenger tension.

Let

d = the displacement.

W_{ch} = unit weight of contact with hanger weight prorated

W_{mh} = unit weight of messenger with hanger weight prorated

Since W_{ch} and W_{mh} are assumed constant,

$$W_{ch}/W_{mh} \text{ is a constant} \qquad \textbf{(18)}$$

Since the design formula $W_{ch}/W_{mh} = T_c/T_m$ is assumed to be followed, then

$$T_c/T_m \text{ is also a constant} \qquad \textbf{(19)}$$

Likewise the various forms of the formula obtained by composition are constants.

$$(W_{mh} + W_{ch})/W_{ch} \qquad \textbf{(20)}$$

and

$$(T_m + T_c)/T_c \qquad \textbf{(21)}$$

are constants. With zero length of shortest hanger the length of longest hanger at support in tangent spans is the same as the vertical sag, *i. e.*,

$$S = (W_{ch} + W_{mh})\, L^2/8\, T_c \qquad \textbf{(22)}$$

The angle which the inclined hanger at support makes with the vertical will depend on the tension in the contact wire. The tangent of this angle is,

$8\, T_c\, d/(W_{ch} + W_{mh})\, L^2$ (displacement d over sag **(22)**) **(23)**

It is a variable.

Another expression for the value of tangent of hanger angle is

$$(T_m + T_c)/R\, (W_{ch} + W_{mh}) \qquad \textbf{(24)}$$

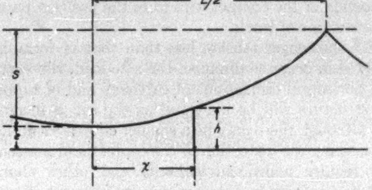

FIG. 8—DIAGRAM FOR TANGENT HANGER LENGTH FORMULA

which gives the slope of the supporting insulator string.

As the two angles are equal, their tangents are equal.

$$8\, T_c\, d/(W_{ch} + W_{mh})\, L^2 = (T_m + T_c)/R\, (W_{ch} + W_{mh}) \qquad \textbf{(25)}$$

and

$$8\, T_c\, R\, d = (T_m + T_c)\, L^2, \text{ simplifying } \textbf{(25)} \qquad \textbf{(26)}$$

and

$$d = L^2/8\, R \times (T_m + T_c)/T_c \qquad \textbf{(27)}$$

$L^2/8\, R$ = middle ordinate of curve for span of length L and radius R and is a constant for a given curve and span[7] **(28)**

Since $(T_m + T_c)/T_c$ is also a constant as per preceding **(21)**, then d is a constant as it is the product of two constants. If the shortest hanger is given some length, however, the displacement does not remain constant. Another disturbing factor in the total displacement is also introduced by the use of a suspension string of insulators for supporting the catenary.

Probably the most complicated part of the design of an inclined catenary is that covering the transition from the tangent to the body of the curve. This section of the overhead is somewhat similar to a spiraled section of the running track. With an inclined catenary designed according to formula, transition design is greatly simplified as the selection of hangers of suitably varying angle and length becomes a relatively simple process by means of the diagram of Fig. 7.

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[7] See Allen, R. R., "Curves and Earthwork," 1903 edition, p. 42.

Usually a simple design is economical and results in lowest cost for the duty performed. It is not proposed to demonstrate with mathematical exactness that the formula design of inclined catenary results in lowest over-all construction costs. The following analysis would indicate, however, that this is practically true.

Assuming equally good design of supporting structures, the combination of minimum loads consistent with minimum dimensions of catenary structure, both vertical and horizontal, should result in minimum supporting structures. If a messenger tension greater than that demanded by formula is selected by the designer, there will of course be a greater side load on the structures than if the formula were followed, since the side load varies directly as total wire tension. The dimensions may be reduced vertically somewhat but probably will not be reduced horizontally because of the distortion of the contact wire from the position parallel to center line of track.

If a messenger tension less than that of formula is selected in order to diminish the side load, the vertical and horizontal dimensions of catenary and of supporting structure will be increased as the sag is increased and although there may be a smaller effective side load, overcoming the distortion of the contact and messenger will require additional pull-offs and other changes such as auxiliary back-bones or more frequent supports with the probable net result of increased cost.

One of the greatest economic advantages of designing according to the formula is the resultant easy erection of the wires. All parts fit together naturally and there are minimum secondary stresses. The construction can be handled like a truss with members designed and connected properly, as compared to a truss in which members are not of proper length and in which the loads are carried eccentrically by the members. Cables are of course much more flexible than truss members and hence can be more easily pulled into connection with other members but nevertheless secondary stresses and distortion result with inexact design and must be provided for in the original design or as later experience dictates. With the inclined designed as per formula, the parts fit and come together easily so that the time and labor of erection and later maintenance become a minimum.

Design based on the formula also makes variations of the simple inclined catenary relatively easy. Double catenary, either with double contact or messenger or both, and staggered catenary on curve and tangent are examples of such variations. Furthermore, as is often desirable, if a stretch of chord construction is required, it can be readily introduced without complications between sections of inclined catenary.

The formula would appear to handicap design in that it selects but one messenger tension value and thus fixes the dimensions of the catenary on tangent as well as on curve after the other three factors, contact weight, contact tension, and messenger weight, are settled upon. There should be no objection to the limitations imposed if a reasonable construction results. If the resulting construction is not considered reasonble, it is still possible to change the other factors of design such as contact material and tension as well as messenger material and resulting tension in order to better satisfy tangent conditions.

The result of following this method of design will be the eventual selection of several contact weight and tension conditions as standards so that for similar service on comparable lines there will be available a suitable standard. Such a condition should enable sound comparisons to be readily made of operating conditions on such lines.

Assuming that a contact wire of a given weight and tension relation is installed as tentative standard practise, the inclined catenary construction has an inherent tendency to maintain this relation of weight and tension constant for a given temperature over a period of years, as the operating slope of hangers and position of contact wire over track depend on such relation being kept constant. The tension of the contact wire is a much more important design factor in any inclined catenary than in chord construction. It is in fact the basic or starting factor. In the chord type of construction the tension of the contact wire may vary over a wide range and not produce any noticeable effect on the shape of the construction. Two lines or sections of chord construction may appear exactly alike and yet have very different contact tensions and hence different collecting characteristics. This is not so of inclined catenary constructions. If they are otherwise alike, their contact tensions are also alike and consequently their collecting characteristics will be alike and they will stay alike with similar temperatures. True operating comparisons can then be made of such overheads under the same or differing services with the gradual result of continually improving the standards. The economic result of such a condition should be very satisfactory. It would only be a matter of time before the most economically designed and operated line or lines for certain conditions would be found.

It is interesting and valuable to use the formula as a check on already existing and operating inclined catenaries. Investigation shows that practically all lines heretofore installed depart more or less from this criterion, some more than fifty per cent. This might indicate one of two conditions. Either the formula application is not practically necessary and inclined catenary is very adaptable or present inclined catenary construction designs are subject to refinement.

Data showing the departure from the formula on certain important lines already installed are as follows:

Canadian National Railways. From data which appeared in the *Railway Age*, May 2, 1925, in article, "New Catenary Construction on the Canadian National," and elsewhere, the following values were secured:

$$W_m = 0.745 \text{ lb.}, \quad W_c = 0.641 \text{ lb.}, \quad T_c = 2400 \text{ lb.}, \quad T_m = 3600 \text{ lb.}$$

T_m as per formula should be 2790 lb. Hence, tension used is 30 per cent higher.

Pennsylvania Railroad. Philadelphia to Paoli Electrification. From data in *The Electric Journal*, February, 1916:

$$W_m = 0.510 \text{ lb.}, \quad W_c = 0.509 + 0.320 = 0.829 \text{ lb.}, \quad T_c = 3000 + 1000 = 4000 \text{ lb.}, \quad T_m = 3500 \text{ lb.}$$

T_m, according to formula, should be 2470 lb. Hence, tension used is 42 per cent higher.

New York, Westchester, and Boston Railway. Data in Sidney Withington's article, *Journal of Franklin Institute*, Dec., 1914, are as follows:

$$W_m = 0.810 \text{ lb.}, \quad W_c = 0.641 + 0.558 = 1.199 \text{ lb.}, \quad T_m = 4900 \text{ lb.}, \quad T_c = 3500 \text{ lb.}$$

T_m, according to formula, is 2370 lb. Hence, tension used is 107 per cent higher.

N. Y., N. H., & H. R. R. Danbury Branch. From page 313 of *Electric Railway Journal* for August 29, 1925:

$$W_m = 0.668 \text{ lb.}, \quad W_c = 0.641 + 0.641 = 1.282 \text{ lb.}, \quad T_c = 1600 + 1815 = 3415 \text{ lb.}, \quad T_m = 3900 \text{ lb.}$$

T_m, according to formula, should be 1785 lb. Hence, tension used is 118 per cent higher.

Midi Railway of France. From data on page 175 of *Le Génie Civil*, August 25, 1923, the following information is obtained:

$$W_m = 0.65 \text{ kg.}, \quad W_c = 0.89 + 0.89 = 1.78 \text{ kg.}, \quad T_c = 700 \text{ kg.} + 700 \text{ kg.} = 1400 \text{ kg.}, \quad T_m = 1350 \text{ kg.}$$

According to formula, T_m should be 510 kg. Hence, tension used is approximately 164 per cent higher.

A number of lines has been designed in accordance with the formula and several have been constructed and are in process of construction. Model spans have also been built. The results indicate that the formula is a very satisfactory and practical guide. The usual inclined catenary is a practical though somewhat empirical design. The design based on the discovered formula is proposed as an improvement as it is both practical and scientific.

The formula gives an exact statement of the relations of the various design factors involved. It is just as true as the equation of the parabola and to question a design based on it is comparable to questioning the use of the parabola formula in the design of the usual catenary.

The simplicity of the discovered formula indicates that the ideal inclined catenary construction with its two parabolas and connecting parallel hangers is a special shape which is closely allied to other geometrical shapes, the cone, the cylinder, and the sphere. And just as these are inherently best suited for certain mechanical purposes, the ideal inclined catenary shape is especially adapted for railway overhead construction on curves. It fits the track alinement perfectly.

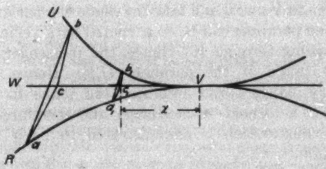

FIG. 9—SKETCH USED IN PROOF THAT TRACE OF ELEMENTARY HANGERS IS A PARABOLA

Appendix 1

Demonstration of proof that curve $U V$ is a parabola.

$R V$ = parabola of contact wire, by construction and definition,

$a b$, $a_1 b_1$ = parallel elementary hangers supporting contact wire,

$W V$ = intersection of horizontal plane of contact wire with inclined plane of messenger slope,

$U V$ = curve formed by elementary hangers $a b$, $a_1 b_1$, etc., piercing plane of messenger slope at points b, b_1, etc.

To prove curve $U V$ is a parabola:

Pass vertical planes through elementary hangers $a b$, and $a_1 b_1$ parallel to principal axis of parabola.

Triangles $a b c$ and $a_1 b_1 c_1$ are formed by the intersections of such parallel planes with horizontal plane of contact wire and inclined plane of messenger slope. Triangles $a b c$ and $a_1 b_1 c_1$ are similar since their corresponding sides are parallel **(29)**

$a b$ is parallel to $a_1 b_1$, by construction **(30)**

$b c$ is parallel to $b_1 c_1$, intersections of two parallel planes by a third plane **(31)**

$a c$ parallel to $a_1 c_1$, same reason as **(31)** **(32)**

$$a c / a_1 c_1 = b c / b_1 c_1 \quad \textbf{(33)}$$

but

$a c / a_1 c_1 = x^2 / x_1^2$, ordinate of parabola $R V$ **(34)**

Therefore,

$b\,c/b_1\,c_1 = x^2/x_1^2$, axiomatic (**33**) and (**34**) (**35**)

Therefore curve $U\,V$ is a parabola, since it has the properties of a parabola.

Therefore, the curve which the assumed elements trace in their intersection of the inclined plane of messenger slope is a parabola.

Likewise, the projection of the messenger on the horizontal plane of the contact wire is demonstrated to be a parabola.

Appendix II

Proof that $U\,V$ represents the true position of the inclined messenger.

Assume $a\,b$ is the position of longest hanger of some given span and that b is the point of attachment of hanger to messenger. The messenger with the calculated resultant load can be so tensioned as to pass through b and V and it will take the shape of a parabola.

Only one parabola can be constructed with vertex at V and passing through b. Hence, the messenger will take the shape of the parabola $U\,V$. Hence, the elementary hangers will intersect the plane in the parabola $U\,V$ formed by messenger and therefore the assumed construction is demonstrated to be a true construction.

Appendix III

Proof that tensions of contact and messenger vary inversely as their horizontal sags.

In Fig. 10,

$a\,b$ = length representing the sag of the inclined messenger in feet (**36**)

$a\,c$ = length representing the horizontal sag of projection of messenger on horizontal plane in feet (**37**)

$e\,a$ = length representing resultant load on messenger in pounds (**38**)

$d\,e$ = length representing dead load on messenger in pounds (**39**)

$d\,a$ = length representing horizontal load on messenger and also on contact wire in pounds (**40**)

$f\,a$ = length representing middle ordinate or horizontal sag of the contact parabola in feet (**41**)

$T_m = (e\,a)\,L^2/8\,(a\,b)$ parabola formula (**42**)

$T_c = (d\,a)\,L^2/8\,(f\,a)$ parabola formula (**43**)

$T_m/T_c = (e\,a)\,(f\,a)/(a\,b)\,(d\,a)$, dividing (**42**) by (**43**) (**44**)

$(d\,a)/(e\,a) = (a\,c)/(a\,b)$, similar triangles (**45**)

and

$(d\,a)/(a\,c) = (e\,a)/(a\,b)$, from (**45**) (**46**)

Therefore

$T_m/T_c = (d\,a)/(a\,c) \times (f\,a)/(d\,a)$, substituting in (**44**) (**47**)

Therefore

$T_m/T_c = (f\,a)/(a\,c)$, simplifying (**47**) (**48**)

Therefore the tensions are in inverse proportion to the horizontal sags.

Appendix IV

Proof that the radial load caused by a contact wire of a given tension and circular curve is practically the equivalent of the uniform horizontal load of a parabola with same tension at the middle of span and with sag equal to middle ordinate of arc of circular curve.

$T\,R = P$ = unit radial curve load[8] (**49**)

$m\,o$ = middle ordinate[9] $= L^2/8\,R$ (**28**)

The sag $S = m\,o$ by assumption (**50**)

$T = w\,L^2/8\,m\,o$ = parabolic catenary formula (**51**)

where w = unit load on horizontal parabolic catenary.

Prove $P = w$ (**52**)

$R = L^2/8\,m\,o$, from (**28**) (**53**)

$8\,T\,m\,o/L^2 = P$, substituting (**53**) in (**49**) (**54**)

and

$8\,T\,m\,o/L^2 = w$, from (**51**) (**55**)

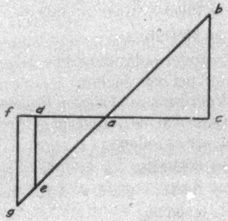

Fig. 10—Sketch Showing Projection of Ideal Inclined Catenary With Forces Acting On It

Therefore

$P = w$ axiomatic, from (**54**) and (**55**) (**56**)

Table of Letters or Symbols

W_c = weight of contact wire in pounds per linear foot,

T_c = tension of contact wire in pounds,

W_m = weight of messenger wire in pounds per linear foot,

T_m = tension of messenger wire in pounds,

a = contact wire in Fig. 1,

b = messenger wire in Fig. 1,

c = inclined hangers in Fig. 1,

d = displacement of messenger,

f = projection of messenger on horizontal plane, Fig. 2,

W_{ch} = unit weight of contact wire with prorated hanger weight in pounds per linear foot,

R = radius of curvature in feet,

h = hanger length,

e = length of shortest hanger at lowest point of sag,

x = horizontal distance of hanger from lowest point of sag,

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[8] Withington, *Franklin Institute Journal*, Dec., 1914, p. 720.

[9] Close approximate formula, Allen, "Railroad Curves and Earthwork," 1903 edition, p. 42.

s = sag of half-span considered,
L = length of span,
W_{mh} = unit weight of messenger wire with prorated hanger weight in pounds per linear foot.

Discussion

PAPERS ON OVERHEAD CONTACT SYSTEMS

(Viele, Brown, Thorp, Wade and Linebaugh, and Jorstad)
Detroit, Mich., June 24, 1927

H. F. Brown: Mr. Viele's paper emphasizes many points which are in line with similar experience on the New Haven system such as

1. The advantage of the auxiliary wire above the working conductor.
2. The necessity for proper steadying devices for the trolley, on both tangent and curve construction, especially against transverse wind loads.
3. The avoidance of long spans in exposed locations.
4. The advantage of locating sectionalizing and splicing devices at or near the supports.
5. The avoidance of hanger members which do not make a positive contact with both the messenger and the contact.
6. The importance of correlating the pantograph design with that of the overhead contact system, and the possibility and desirability of further refinement in the pantograph design.

In connection with the tests described, it would be of interest to know what effect the tensions of the auxiliary and the contact have on the amplitude of movement at the support point. Lower tensions, especially of the auxiliary, might offer an improvement. It would also be of interest to know the effect of the contact-wire tension on the speed at which the oscillations travel along the wire; also a comparison of the effect of the very light rods now being used on the newer installations, as compared with the columnar effects of the older, heavier types.

M. W. Manz: The amount of rise of the pantograph wire in any catenary span would be in proportion to the unloading or in proportion to the pantograph pressure. That can be laid out rather accurately with the force polygon, and if one follows the movement of the pantograph through a number of locations one gets an unloading diagram similar to the curve Mr. Viele has presented.

There is a rather peculiar thing about that type of diagram; the movement of the system is the largest at the center of the span. Has any thought been given to the possibility of increasing the weight of the system at the center of the span as opposed to increasing it near the supports which now happens due to the longer hangers? In other words, if you take the diagram and unload it at various points, you will find that an unloading at the support has much less effect on the system than at the center of the span. Has any consideration been given to the possibility of some correction, perhaps, for the unloading at the center as opposed to more unloading at the supports?

In connection with the long wave motion, which you have with a number of pantographs in a span, with a number of spans there is a rather interesting combination. Has any observation been made as to the extent of movement of the suspension insulator? When you unload a span, you reduce the tension. When you do that, the insulator will swing, giving an effect of balancing the tension back through the system. There might be another interesting point to consider for future investigations; can the loadings of the messenger be decreased as you approach the supports in such a way that the lift of the system is uniform?

G. I. Wright: We on the Illinois Central are considering the application of roller bearings to our pantographs—something that has not been done generally in this country, but I believe has been done abroad.

In both Mr. Viele's and Mr. Brown's papers, they mentioned a provision that they thought necessary to make for flashover of insulators—burning the construction and so on. I might say that due to the system we use and the design of it, we have had no flashover of insulators and practically no burning of wires. The only burning of wires we have had, except that caused by a direct stroke of lightning, has been the burning of the contact wire where the pantograph ran from a live section to a dead section on which there were a large number of trains standing. In leaving the gap, this caused an arc which was maintained, and burned the wires.

I might state that in order to prevent this burning we are installing dead-section signals which will stop a train from passing from a live to a dead section.

The fine results due to the elimination of burning and arcing are also largely due to the high-speed breaker protection we have on all feeders energizing the catenary.

In Mr. Jorstad's article, I thought possibly he gave a wrong picture of the popularity of the inclined catenary. I believe that both types have their use and their field, and that certainly the chord construction is better for a great many installations, particularly where there are few curves and considerable copper is required.

Mr. Jorstad listed eleven railroads which use inclined catenary, and stated that this indicated that this would be a future standard overhead. There are very many important railroads which use the chord construction.

Mr. Jorstad also said that in using inclined catenary only two wires were necessary whereas with chord construction three or four are used. I believe that is limited to short-span lifting-hanger construction, while two wires would not be feasible for long-span construction generally used for heavy electrifications.

Apparently, Mr. Jorstad's method of design greatly simplifies something that in the past has generally been considered very complicated, and such a simplification should be welcomed by all.

Norman Litchfield: As consulting engineers for a number of railroads, we have been closely connected with the development of the modern catenary, and a short review may therefore be of interest.

The first major installation of high-voltage catenary was on the N. Y., N. H. & H. R. R., in 1906, between Woodlawn, N. Y. and Stamford, Conn., the main line construction using two messengers with diagonal hangers forming a *B* with the trolley.

On the electrification of the Elkhorn grade of the N. & W. R. R., in 1912 we employed a single catenary for both main line and yards, the necessary stability of the trolley wire being obtained by a cross-span wire installed only at points exposed to winds. The catenary was supported by suspension insulators hung below the body strand, thus eliminating insulation in the latter between tracks. The same construction was used by us on the Paoli electrification of the P. R. R. and has since become practically a standard.

The original New Haven construction used a square, latticed column, bolted to a concrete foundation. To provide a lighter appearing structure, and to reduce first cost and maintenance, the N. & W. R. R. and the P. R. R. adopted tubular poles set in cored holes in concrete foundations, the poles being guyed to concrete anchors at points of curve pull.

On a further extension of the N. & W. R. R., rolled-steel H-columns were used for the first time, with structural bases bolted to the foundations.

For the Virginian Railway electrification in 1923, we used H-columns with a pre-cast concrete base, cast on the pole at a central manufacturing yard. These were set in holes sheathed with a section of corrugated steel culvert pipe, then wedged in place, and backfilled.

During the past year we have been engaged in the electrification of the Bay Ridge freight line owned by the New York Connecting and the Long Island Railroads, and have used plain H-columns set in culvert pipe, then wedged in place and the pipe filled with concrete.

To obtain a very light appearance and a clear view of signals, on the Paoli electrification, a cross catenary was used for the support of the messenger and trolley, this being its first general application to main-line work, although it had been used previously in the East Port Chester yard of the N. Y., N. H., & H. R. R.

Prior to the Virginian electrification, we had utilized galvanized steel wire and galvanized castings for the support of the messenger and trolley wire. On the Virginian, non-corrosive materials were used for the messenger and the trolley hangers (the two live elements above the trolley). For the Bay Ridge electrification, a further advance was made by the employment of non-corrosive hangers and wires throughout, including the cross-catenary.

Electrification is now under way of the P. R. R. Company's suburban service between Philadelphia and Wilmington which uses all non-corrosive materials. It is on a portion of this line that the interesting type of steady which is described in Mr. Viele's paper will be used.

At the time we undertook the Elkhorn grade electrification, we found the mathematics of the catenary system in an undeveloped state, the practise being to calculate the hangers only for those spans which followed around a uniform track curve, it being left to the construction forces to cut and fit the irregular spans at transitions, etc. On this line it became imperative to find some method of calculating the irregular spans also, and an original theorem was therefore developed by our engineers. This has since become generally used, and is described in Mr. Brown's paper. It should be remembered that while discussion of the mathematics is of value, nevertheless catenary design is largely empirical, requiring judgment and experience to produce an economical and smooth-running line.

H. S. Richmond: In abolishing the trolley pull-offs on high-grade and high-speed systems, by experience it was found necessary to meet the condition which demands that the trolley curve of one span shall be tangent to that of the adjacent span opposite the point of support. The trolley curve is not a true circle, but if the above condition is met, it forms a continuous and graceful curve, lying in a horizontal plane at the temperature for which the computations were made. We have deduced formulas by which spans are thus matched together or "balanced" both on regular curvature and on transitions.

The formula by which spans are "balanced" on a regular curve is

$$U = \frac{100008725\left(1 - \frac{T}{M}\right)\phi}{d\,\phi/d\,x}\,D\,S$$

and the maximum deviation Δ of the form of the trolley curve from the true circle is given by the formula:

$$\Delta = S\,D\left[\frac{S}{45900} - \frac{\phi - 1}{11470\,d\,\phi/d\,x}\right]$$

Here

S = the span in feet.
T = the combined trolley and auxiliary tension.
M = the messenger tension.
D = the track curvature in degrees.
U = the horizontal offset between messenger and trolley.
ϕ = the ordinate of the fundamental trolley curve referred to the system axis.

and

$d\,\phi/d\,x$ is the derivative of ϕ with respect to axial dimension, the particular values of the curve and its derivative at the point of support being used.

The deviation Δ, we have found to be a very small quantity and not of significance as regards conformity of trolley to pantograph center. In this we take exception to Mr. Brown's citation of this point as being an important limitation in the use of the inclined system. On the contrary, we find that the real limitations of this system for moderate curvatures lies in the length of the horizontal offset between contact wire and messenger at points of support and the temperature variation in vertical and lateral position of the contact wire.

J. C. Damon: Mr. Jorstad noted that in systems investigated, the tension used for the primary messenger of the catenary construction was considerably higher than his formula would give.

In the recent past, the tendency for catenary construction seems to have been to use very high-strength material for the messenger and to keep the sag as low as possible. Under these conditions, the stretch of the cable furnishes a large proportion of the excess length of the catenary curve over the straight-line distance between supports, and with a given uplift from the pantograph, the tendency for the catenary system to rise is consequently very great.

When the tension in the messenger per-pound-weight of the catenary system is reduced, the sag must be increased; but, in turn, there is less tendency for the catenary system to rise because of the pantograph uplift.

With the lower tension in the messenger cable, there is a greater variation of the height of contact wire due to temperature variation, but less variation due to the uplift of the pantograph.

Recently, there has been a tendency to put feeder capacity into the messenger, which generally increases the weight of the messenger and, in consequence, due to the pantograph, reduces the upward movement of the contact wire. By using a moderate tension and putting the necessary feeder capacity into the messenger, some systems get no appreciable wave traveling ahead of the pantograph. The Chicago, North Shore, & Milwaukee Railroad, which uses the heavy type of construction with feeder capacity in the messenger, is, of course, a low-voltage railroad and quite different from the Pennsylvania main line; but it has an entirely non-rust construction of very heavy messenger, which has given satisfaction at fairly high speeds. The Illinois Central Railroad is another system in which the feeder capacity has been put in the messenger and has operated in a very satisfactory manner.

Mr. Viele contradicted the statement in my verbal discussion and stated that the heavier messengers with lower-tension per pound did not reduce the wave which went ahead of the pantograph. We are not, however, as far apart as our necessarily brief statements in the very limited discussion permitted would make it appear.

If there were absolutely unrestricted longitudinal motion of the messenger, Mr. Viele's statement would be correct and my statement would be wrong. On the other hand, if no longitudinal motion of the messenger from one span to another were possible, Mr. Viele's statement would be wholly wrong and mine, correct.

In actual practise, Mr. Viele has developed an extremely flexible catenary support system which approaches somewhat, although not entirely, the conditions of his assumption. On the other hand, most of the existing catenary systems, and, I venture to predict, a great many of the future catenary systems, will have bridges with saddle insulators, or short strings of suspension insulators, and conditions will be more nearly those of a completely restricted than of an entirely unrestricted longitudinal motion—in which case my statements, which were relative only, will be correct.

A. G. Oehler: My question is essentially an elaboration of what has already been introduced by Mr. Manz. I should like to know how much the loop hanger does offset the increased rigidity at the points of support? How much is this variation in rigidity corrected by the use of a heavy contact wire, and is it or is it not desirable to use a flexible hanger to minimize this difficulty? Finally, is it necessary to worry about it?

K. T. Healy: It seems unfortunate that no place has been given in these papers to study the overhead distribution system costs. The future extension of electrification depends largely upon the abilities of the engineers and manufacturers to reduce the initial costs of the improvement, other things remaining equal. And a considerable part of this reduction must come in the distribution system, as this makes up from 25 to 35 per cent of the total cost of the electrification. Therefore, it is essential that the design of the overhead distribution system should, as a means of lowering costs, look not only to satisfactory operation but also to economical utilization of material and labor of construction.

As a general rule, economies of design are effected by a close study of the conditions and requirements at hand and an ample allowance for them, at the same time confining the design to these only. The main conditions affecting overhead design are: first, amount of conductivity to be provided; second, speed and method of collection of the current; and third, climatic conditions.

The requirements of conductivity immediately make a line of demarcation between high- and low-voltage systems, systems in level country and in mountainous country, and multiple-track and single-track systems. With the high-voltage systems, under all but mountainous conditions, ample conductivity can be secured in two conductors, so that there is no reason, so far as conductivity is concerned, for going into design with more conductors. With the low-voltage systems, more conductors are necessary with the consequent heavier loadings on supporting structures and greater amounts of steel necessary in the structures.

The second condition affecting overhead design—namely, speed of collection,—imposes on main-line, high-speed tracks, requirements decidedly different from those on yard and siding tracks. To a large degree collection is a function of the ability of the collector to keep in physical contact with the overhead in spite of variation in height of the overhead or varying hardness of the overhead. The ability to follow variations in height is dependent upon the velocity and lineal rate of variation in the height of the overhead with a given effective inertia of the collector. In yard operation the speeds are low and the currents low, so that greater lineal rates of variation in height are allowable and consistent with good collection and the overhead distribution does not necessarily have to be of the full catenary type. In Europe, notable strides have been made in decreasing the cost of yard electrification by taking advantage of this fact. The Swiss have their Renens yard arranged with a modified direct suspension, with 116-ft. and 165-ft. spans. This has given very satisfactory operating results.

The Paris and Orleans Railroad, on its 1500-volt electrification, has followed the same trend and has wired its yards in the Paris area with a direct suspended system with cross-span supports. Thus, many economies are possible by designing yard distribution systems, not for mainline track requirements, but for yard requirements of low-speed and less exacting requirements of uniformity in contact wire.

The second condition also has to do with the method of collection. Here again it may be well to emphasize Mr. Viele's remarks to the effect that coordinated effort in the design of both pantograph and overhead is necessary for satisfactory operation and that this coordination may be carried even further to affect economies in cost. Certainly one of the most important factors in the ability of a collector to work well is the inertia of its moving parts. Here, European pantograph design has taken a different course from ours on high-voltage systems by introducing a secondary bow for the shoe, swinging about its own central axis and held in position by small springs. This has reduced the effective inertia of the collector many times, because this small bow, weighing only a few pounds, is all that has to move to follow the small irregularities of the trolley height. It then requires much lower pressures to keep continuous contact on the trolley wire with consequent reduced wear on both wire and shoe. At the same time, the pantograph itself is made much lighter, using wire guys instead of pipe for side bracing and snugger joints with less friction at the points of support and other axes. This requires less pressure to operate and makes a smaller area for wind and ice loads to affect. The effects of hard spots in the contact system are of course greatly diminished because of the much lighter weight behind the impact of the shoe on the hard spot. Experience shows that a single shoe will collect 180 amperes perfectly at 55 mi. per hr. and 250 amperes at 27 mi. per hour, and that the shoes will run from 5600 mi. in Switzerland to 15,000 mi. in Germany. The usual practise is to run with two pantographs up, which nearly doubles the life of the shoes. The greasing of the shoes seems to be a mooted question; the Swiss, for instance, are in favor of it and the Germans are not.

Mention has been made of the need of coordinated effort in pantograph and catenary design, but the importance of this in relation to reducing both initial and operating costs cannot be stressed enough. With the low currents of normal high-voltage overhead distribution and the resulting possibilities of reducing shoe pressure, the possible simplification in catenary design is considerable.

In all the high-voltage European electrifications, this coordination has resulted in a great saving. Pantograph pressures have been kept down to 79 lb., using aluminum shoes with negligible wear on contact wires. As a consequence they have been able to use a much lighter and simpler catenary system with only a messenger and contact wire. In the past, to cut out the hard spots of the hangers, they have operated with an auxiliary or intermediate wire, but with the flexible hangers they use, they have found this wire unnecessary. In some cases the flexible hangers are made of strand so as to make a low-resistance, non-heating connection between messenger and contact. Particular care has been taken to avoid hard spots; splicing is rare, the contact wire being in lengths sufficient to reach from anchor point to anchor point; push-offs and other devices with compression members are not used; rigid deflectors are not used; and pull-offs are made very light.

The Swedish State Railroad catenary may be taken as an example of this type of catenary designed for weather conditions comparable to ours and with a conductivity of about 260,000 cir. mils. The costs of the materials at our prices would be about $528 per mile for 80 sq. mm. copper contact, and $316 per mile for 50 sq. mm. copper messenger, a total of $844.

A corresponding example of American practise may be a 4/0 phono contact, costing $792, a 4/0 copper wire, costing $686, auxiliary clips, costing $35, and a 9/16-in. steel messenger, costing $316, or a total of $1829 per mile, nearly $1000 more per mile than the other for a slightly greater conductivity of 310,000 cir. mils. The unit weights of the two systems are 0.81 lb. per ft., for the first and 1.95 lb. with the American, requiring a pole designed for roughly twice the catenary loading. This, of course, means an opportunity for the saving of considerable weight in the steel structures. The Swedish construction, using two 5-in. channels for a self-supporting single-track line, is comparable to the 10-in. H-section poles in use in this country for similar construction.

H. F. Brown: Mr. Thorp's paper among other things brings out the importance of the catenary profile in determining the structure heights and catenary details, especially where the hanger rods depart from standard conditions. This is a very valuable check on the calculations.

The unit weights and required conductivity of the overhead system described are impressive, but, of course, are required for the low voltage used. This is further reflected in the size of the supporting structures. I believe 14-in. H-beams were used for the typical two-track bracket poles illustrated in Fig. 15 of the paper. For the lighter type of catenary permitted by the higher voltage used on the New Haven, a 12-in. section is ample for similar two-track spans.

While Wade and Linebaugh show the feasibility of designing an overhead contact system which can transmit and deliver very large currents, and also the possibility of collecting large amounts by sliding collectors moving at high speeds, one cannot escape questioning the economics of a system which requires more than seven pounds of copper per foot of track, in the contact system alone.

If electrification is to be economically applied to steam railroads on any large scale, it must be done on a basis which gives the minimum new capital requirements. One of the largest items is the cost of the distribution system. The trend should, therefore, be towards lighter designs rather than heavier, indicative of higher voltages.

It is noted that the test track described contains no turnouts, heavy curves, or low highway bridges. With the high pantograph pressures required, the real test of current collection will come at points of special construction, such as deflectors, pull-offs on heavy curves, and the hard spots under low bridges; and while such difficulties might be overcome, they nevertheless increase at least in proportion to the catenary weight, and the amount of current to be collected. The wear on both the collector shoe and the contact wire must be greatly increased under such conditions, even with lubrication.

Mention was made of the use of a 6/0 conductor as being undesirable. Our experience seems to indicate that there is a real field of use for such a wire on busy yard leads and ladder tracks, where traffic is dense, and pantograph passages very frequent, and we have installed 6/0 wire in such locations to secure longer wear.

One word about lubrication. This is something to be desired but difficult to secure in actual practise, especially on a large system. If the lubricant is applied to the pantograph shoe at the start of a run, it is soon worn off in the first few miles, and the terminal-track trolleys are the chief beneficiaries. Tests made on the New Haven seem to indicate that the best shoe mileage and least wire wear is secured with a mild-steel shoe, and a pantograph pressure of about 18 lb.

Pantograph design is capable of being greatly refined. Lighter weights, lighter pressures, and lower inertias, especially of the collector shoe itself, seem to be the chief desiderate, all of which are inconsistent with heavy current collection.

Mr. Jorstad, in his paper, limits the shape of the trolley alinement to that of a true parabola, and shows that his formula, which gives the ratio of the weights of the two opposing systems (messenger and contact) equal to the ratio of their respective tensions, applies to the parabolic shape.

The use of this formula would therefore limit the designer in the choice of sag, or would fix the weights and sizes of the main members (messenger and contact) regardless of their economic choice.

As an illustration, assume that a 4/0 copper wire is the required contact member, with a normal tension of 2000 lb. The unit weight is 0.641 lb. and from the formula,

$$0.641\ \mathrm{T}_m = 2000\ W_m \text{ and } \mathrm{T}_m = 3120\ W_m$$

If we assume a 7/16-in. steel messenger, which past experience has shown to be of ample strength, properly sagged, for a 4/0 wire, its unit weight is 0.415 lb. and the tension, by the above formula will be 1300 lb.

The resulting catenary will weight approximately, including hangers, 1.15 lb. per ft., which would give a sag in a 300-ft. span of nearly 10 ft. This is obviously too great.

To decrease this sag to an economic value, the tension must be increased, which, according to the formula, must be accompanied by an increase in weight. To produce a 5-ft. sag, it would be necessary to go to a messenger having a unit weight of nearly three times the above, since the total weight is rapidly increasing. Assuming a unit messenger weight of 1.6 lb., the equivalent messenger tension would then be 5000 lb., and the resulting total weight of the catenary would be 2.3 lb. with a sag of 5.17 ft. This messenger would be approximately $^7/_8$ in. in diameter, which is obviously not economically applied, as the material is not required for the low tension used.

It is true that the trolley tension may be increased. The ultimate strength of the wire assumed is approximately 8000 lb. but it should not be stressed much more than $^2/_3$ of this amount on account of the low yield point of copper, and the danger of permanent stretch. This limits the maximum working tension to about 5300 lb., and sets the normal working tension at about 3000 lb.

Then, from the formula,

$$0.641\ T_m = 3000\ W_m \text{ or } T_m = 4700\ W_m$$

For a 5-ft. sag, the messenger must weigh approximately 0.671 lb. per ft. and the tension would then be 3140 lb.

This indicates a 9/16-in. messenger, which, although more nearly the economic size, is nevertheless larger than required, and is not working to its full capacity. Further, the horizontal loads on the structures on curves, due to the trolley tension, have now been increased 50 per cent, requiring this additional strength in such structures.

If, however, the characteristics of the design will permit the application of Mr. Jorstad's formula, it presents a very valuable method, and greatly simplifies the calculation, since the shape is parabolic. It may be mentioned here that the earlier installations on the New Haven, where the spans were short, were designed on the theory that the shape was parabolic.

Sidney Withington: (by letter) The design of pantographs in this country just at present, is, I believe, the least developed part of any of the electric equipment. We depend upon a collecting device capable of variation in height up to 10 ft., as mentioned by Mr. Viele. The inertia of such apparatus is very great and at high speeds, as pointed out by Mr. Viele, this is a serious problem and requires a very considerable pressure against the wire to avoid arcing. This, in turn, limits the design of the catenary system. The standard pantograph used abroad, which employs a trailing bow above the main pantograph, would seem to be considerably more logical. The trailing bow, being relatively light, follows the wire satisfactorily even at relatively low pressures, and the result is a far lighter catenary design with consequent economies both of construction and maintenance and without serious limitation in the amount of current which can successfully be collected. The advantages which would accrue from an improved design of pantograph in this country would repay, I believe, a very considerable amount of study.

Mr. Viele mentions spans of 325 ft. for the catenary system. It has been the experience on the New Haven that unless the location is pretty well protected from wind, some form of lateral support for long spans is necessary between bents in order to prevent trouble due to wind under maximum conditions. These supports may be obtained in the form of bridles or additional independent steady spans, or by shortening the main span.

The form of support suggested by Mr. Viele is of interest. A somewhat similar arrangement was installed on the New Haven in 1919. Other things being equal, it is of advantage to separate mechanically, as far as possible, each track from the others, in order that trouble which may occur may be localized. One advantage of the scheme mentioned by Mr. Viele is that the insulators are not directly over the track, and where steam locomotives are operated along with electric operation this is of considerable advantage.

Mr. Viele mentions hard spots and their damping affect on the oscillation which proceeds ahead of the pantograph. On the New Haven electrification, where wood section breaks were used at points of high speed an approach was designed consisting of a sheet-steel member of light gage about 6 ft. long, with the idea of damping the oscillations before they reached the relatively heavy section break. It was, however, found that this was not necessary, and its use has been discontinued. It is of course true that so far as possible, wood-stick section breaks are not used at points of high speed. Indeed, even splicing sleeves are eliminated so far as possible.

It may be of interest to note that on the New Haven electrification extensions installed in 1912 and 1913, the steel catenary supporting structures were designed to be self-supporting as units; that is, the corner connections between the truss and the posts were arranged to take the stress normal to the track due to wind, curve pull, etc., the anchor bolts at the base of the posts taking shear only. This is indicated in the shape of the posts, which taper down from the truss or corner connections.

This saved a considerable amount of concrete as compared with the original design wherein the posts were self-supporting and the concrete bases were obliged to take the entire overturning moment. There was some additional weight of steel and additional field labor, but the design resulted in considerable net saving.

In some of the supporting structures, especially those for six tracks, it was found advantageous to assume a point of contraflexure about 7 ft. above the top of the foundation. This resulted in some saving of weight of steel without much increase in the size of the foundation.

The magnitude of current collected from a trolley at 500, 1500, or 3000 volts is of course far greater, other things being equal, than that collected at 11,000 or 22,000 volts, and the problems are therefore of somewhat different nature when the lower voltages are considered, both from the point of view of current-carrying capacity in the catenary system itself, and what might be called the "commutation" at the point of contact.

The problem of sparkless collection of current is a function of the smoothness of the contact system, which in turn depends upon its uniformity of suspension—that is, the contact wire should be either free to move in a vertical plane easily at all points upon the passage of the pantograph collecting shoe, or should be relatively rigid at all points. Any change from soft to hard construction means sparking or arcing at high speeds. The real test of sparkless collection occurs at turnouts and at low bridges and other points where construction is limited by local conditions.

Mention is made of the freedom from burned messengers on the Chicago, Milwaukee, & St. Paul at loop hangers, which are employed on that system. The real test of efficiency of loop hangers would occur, I believe, only where the individual substation capacity feeding the system is relatively larger than is the case on the Milwaukee.

The design of pantograph shoe support with 4 in. of play between upper and lower position on the top of the pantograph is of much interest. It would seem that somewhat delicate adjustment would be necessary in order to maintain the supporting apparatus at the mid-position of its travel under normal conditions that it might be free to move up or down as roughness in the contact system required. Details of design to accomplish this would be of interest. Some data also would be of interest as to the mileage made by the "pans." The pressure of 40 lb. seems high when compared with that where less current-carrying capacity is required. A pressure of 40 lb. also means a necessarily heavy contact system, regardless of current-capacity requirements.

The economic limitations of any contact system, I believe, occur in the weight and expense of installation of the conductors and supporting structures rather than in the amount of current which can be successfully collected under conditions of high speed, and this should be considered in the design of an electrification installation. It is possible that the use of higher voltages, either alternating or direct current, would be justified in some instances by the saving of material in the contact structure.

S. M. Viele: Mr. Brown asked the effect of tension in the contact wire on vertical movement. We have not made any experiments on the basis of varying contact-wire tension. My opinion is it would not make any material difference.

Mr. Manz asked about the effects of unloading the span. Mr. Damon's remarks were along the same line. I do not nterpret upward deflections in the span on the basis of the possible impression that some have of that statement. If we assume a condition in which the span involved has a unit weight of 3 lb. per ft. and say a 300-ft. span, it means that the messenger is carrying roughly 900 lb. If we operate five pantographs on the span, it means that instead of carrying 900 lb., it carries 800 lb. This means that the sag in the span has to be reduced on account of the tension remaining practically constant. Therefore, the messenger sag will be decreased; the extra length of messenger will run back in the successive spans.

Now, if you change the contact tension, or if you change the loading of the span without change of pantograph pressures, it will not make a great deal of difference in the upward deflections with passage of the pantographs.

Mr. Oehler asked the question "What does it amount to?" Deterioration of contact wire has a very material bearing on how much it costs to operate such construction. Spliced contact wire means a deterioration of roughly five times that without splices. Whether we get a life of ten years out of it or 50 years is very material.

The question may also be asked "What are the relative effects of variation in height of contact wire?" Such effects are relatively small as compared with spliced effects. However, all variations of whatever nature which take place in the contact-wire height involve variations of pressure, both in enlarged and decreased pressures. Increased pressures mean increased wear; decreased pressures mean burning of the contact wire. Both result in increased maintenance cost. It is simply a question of the relative reduction of section which is produced by such variations in height, with the resulting cost. They have not been evaluated in money, though they have been evaluated in my mind as being worthy of study and the correction of value.

H. F. Brown: In answer to Mr. Litchfield I will simply state that the method of eliminating the cusp shown in Fig. 41 is admittedly an overcorrection. The importance of this correction is greater if the trolley tensions are lower. On the New Haven system, the tensions of the trolley and the auxiliary are lower, I think, than on many of the other systems mentioned here today. For that reason, the cusp effect is more important in their inclined design than it is where the trolley tensions are higher, and the method used in the paper is shown instead of the one referred to by Mr. Richmond because it was desirable to make this correction apply to all conditions involving even high temperatures as well as the normal temperatures. It is true that if one goes into the mathematics of the paper, the method suggested by Mr. Richmond is absolutely correct for normal conditions.

R. E. Wade and J. J. Linebaugh: Mr. Brown, in his written discussion with reference to tests on the collection of large currents from overhead wires as conducted at Erie, Pa. mentions the use of "more than 7 lb. of copper per ft. of track in the contact system alone."

The actual total weight of copper, installed per ft. of track for the tests involving the unusually large current values quoted, and for both feeder and contact, was 4.7 lb. maximum with 1,000,000 cir. mil feeder messenger and 3.92 lb. in the section with 750,000 cir. mil feeder messenger. With multitrack work and automatic cross ties, these weights would be reduced even for the lower voltages used.

While it is true that the recorded tests were conducted without deflectors, hard spots under bridges and pulloffs on heavy curves, temporary low bridges were later installed, and satisfactory collection demonstrated with a 1 per cent gradient in contact wires. Experience on lines in operation shows that with two contact wires, flexibly supported throughout, the additional wear at such points is negligible with pantograph pressures as high as 35 lb.

As regards lubrication, while it would of course be desirable to operate without this feature, experience has shown that even with comparatively infrequent service, satisfactory lubrication of contact wires can be maintained, although as stated by Mr. Brown, the contact wires within and in the vicinity of the large yards receive more lubrication, particularly in and near overhead switches. This also applies to curves due to wiping action.

While it is unquestionably true that generally speaking it is desirable to make refinements in pantograph designs as regards weights, lighter pressures, and lower inertias, it is commonly agreed that coordination in the design of both pantograph and overhead is necessary, and there is some question as to whether such coordination is not approached with the relatively higher pantograph pressures in combination with the weight of two wires in the same horizontal plane as compared with the very light pressures and correspondingly light overhead construction as used in some European installations.

Mr. Withington in his discussion very properly emphasizes the desirability of uniformity of suspension for contact wires, that is, the freedom of movement in a vertical plane at all points. By suggestion this of course includes uniformity in weight, which of course cannot be realized on account of the necessity for overhead switches and certain fittings attached to contact wires. From our observation, if the contact wires are free to move in a vertical plane at all points of suspension, the additional wear imposed on contact wire is reduced to a minimum.

As referred to in our paper, with regards to the freedom from burned messengers with loop hangers, we would suggest that in the case referred to, the current demand at locomotive rather than the substation capacity would be the determining factor.

As regards the flexibility of the pantograph shoe mounting, while this of course does not compare with the commonly used European design, either as to delicacy or range, there is no question but that the small amount of movement provided is a valuable asset, particularly with the flexible contact-wire suspension referred to elsewhere.

There is no question but that on some systems contact-wire splicing devices are responsible for considerable trouble in that the designs commonly used enclose the contact wire and offer an obstruction to the collector shoe, which becomes worse as wear on the splicing device increases. A splicing device which permits uninterrupted contact with the wire and is of the minimum weight consistent with the requirements, will do away with this trouble to a large extent. Such a device is available.

Calculation of Stray Load Losses

BY G. H. ROCKWOOD[1]

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INTRODUCTION

QUITE early in the history of the development of the alternating current generator it was recognized that there might be losses present under load conditions in addition to the no-load losses and the armature copper loss due to load current. Because no one had a satisfactory means of measuring this loss it was not included in the early definitions of conventional efficiency. As competition increased and as economic pressure demanded higher and higher efficiences the necessity of approximating this loss was apparent. Accordingly the A. I. E. E. rules were changed to include one-third of the loss under sustained polyphase short circuit.

This rule was short lived for, at the Winter Convention of 1913, papers were presented giving the results of carefully made input-output tests which tended to show that the entire loss on sustained polyphase short circuit should be taken as a measure of the stray load loss for the salient pole alternator. This rule has remained in force and in view of its general acceptance it is to the interest of designers of salient pole alternators to be able to predetermine this loss.

It is the purpose of this paper to present some general methods of attacking the problem. It is not intended that the results should be complete but they are presented simply as a starting point for the attack on this problem.

Proceeding on the principle that a formula cannot be correct unless it has the dimensions of the quantity which it represents or expresses, we may quite readily develop some of our familiar relations. As the complexity of the problems which we treat by this method increases, the difficulties encountered in its application also increase until a point is reached beyond which we cannot go without assistance.

In the case at hand, the calculation of stray load losses, this assistance may come in one of two forms; it may be experimental evidence as to the exponents of certain of the variables, or it may come in the form of assumptions as to the variables involved and the manner of their variation. It is evident, therefore, that the method is not rigorous but is one which will enable us to derive a formula from otherwise incomplete data.

The greatest difficulty encountered in the application of this, or any other, method to the calculation of losses in iron is the fact that we know the loss depends in some manner upon the flux density. But the flux density for any magnetizing force is a complex function of the permeability, a property of the iron which is not constant. Because we have no mathematical expression for the permeability of iron we must make certain assumptions in regard to the behavior of iron under strong magnetizing forces, such as are present in electrical machines. The two most commonly accepted assumptions in regard to permeability are those of constant permeability, or the "limiting value of the density" theory in which it is assumed that there is a maximum possible density. Which assumption will be the better for any given case depends upon the application, but, in general, the assumption of constant permeability is made.

The formulas derived in this manner will require the inclusion of empirical constants. These constants may be determined by statistical methods to be discussed later.

Instead of trying to develop a single formula for the short circuit core loss, it is desirable to separate the total loss into distinct parts and develop formulas for each of these parts separately.

The parts into which it is most conveniently divided are:

1. An eddy current loss in the armature copper.
2. A loss in the stator iron.
3. A pole face loss.
4. An end loss.
5. A loss in the amortisseur winding.

These component parts will be discussed in turn.

EDDY CURRENT LOSSES IN THE ARMATURE COPPER

Eddy current losses in the armature copper present the source of stray load loss usually recognized in design books. The method of calculating them has usually been to assume that the ohmic resistance of the armature was increased by an amount varying from 10 per cent to 50 per cent. The uselessness of such a method is evident.

In the early days of synchronous machine design when the existence of these losses was not recognized it was not at all uncommon to employ designs such that this loss constituted the major part of the total short circuit loss. It is only within the last five or six years that accurate and easily applicable methods of computing and minimizing this loss have been available. At the present time, however, this loss is rarely over 25 per cent of the total short circuit loss so we see the necessity of obtaining accurate knowledge of the factors on which the remainder of the stray load loss depends if improvements are to be made in the design of salient pole alternators.

There is extensive literature on the subject of eddy

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current losses in the armature copper and there will not be space here to review it. Reference should be made, however, to the work of Prof. W. V. Lyon as given in his A. I. E. E. paper in 1921. It might also be stated that the method given by Mr. I. H. Summers, and based on Prof. Lyon's work, at the 1927 Winter Convention of the A. I. E. E., was used in connection with the work on which this paper was based.

LOSS IN THE STATOR IRON

The flux wave form in the air gap of a salient pole alternator under load varies with the power factor at which the machine is operating. This condition renders it extremely difficult to determine the core losses under load. We are seeking, however, to express the core losses at a fixed power factor, and the power factor with which we are concerned is one which gives rise to very special relations between the main field flux and the armature reaction flux. The power factor which we are to consider is extremely close to zero in a lagging direction.

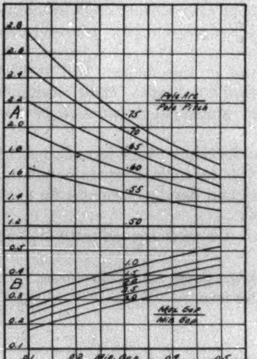


FIG. 1—THIRD HARMONIC OF THE NO-LOAD FLUX WAVE IN THE AIR GAP OF A SALIENT POLE SYNCHRONOUS MACHINE
THIRD HARMONIC = $A \times B - 0.7$

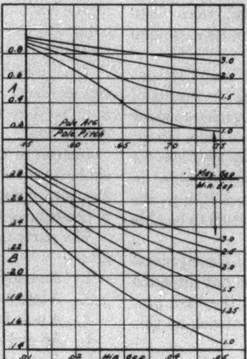


FIG. 2—THIRD HARMONIC OF THE AIR GAP FLUX WAVE WHEN A SALIENT POLE SYNCHRONOUS MACHINE IS EXCITED ONLY BY A SINE WAVE ARMATURE M. M. F. WHOSE AXIS COINCIDES WITH THE POLE CENTER
THIRD HARMONIC = $A \times B$

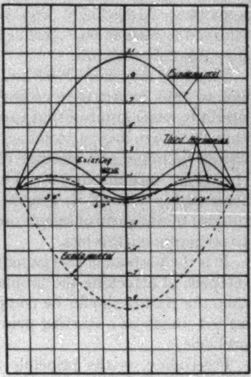


FIG. 3—GRAPHICAL DETERMINATION OF THE ZERO POWER FACTOR SHORT CIRCUIT AIR GAP FLUX WAVE

Since the flux densities under short circuit conditions are not ordinarily found in the usual course of design it will be necessary to determine them before the loss can be computed. It will be convenient to express them as a percentage of the open circuit densities.

If a salient pole alternator operates at zero power factor the axes of the main field flux and the armature reaction flux will coincide. If the power factor is leading the flux waves will add and if the power factor is lagging the net flux will be the difference of the two waves. The wave form of the two fluxes will not be the same, however, so the addition and subtraction mentioned above must be that of harmonics of the same order.

If we are to be able to calculate the flux wave existing in the air gap of a salient pole alternator operated on short circuit at such an excitation that rated current flows in the armature windings we must make certain assumptions. These assumptions are four in number and are given below:

1. That the air gap flux wave consists of four components and four only.
   - *a.* A fundamental due to the main field m. m. f.
   - *b.* A third harmonic due to the main field m. m. f.
   - *c.* A fundamental due to the armature reaction m. m. f.
   - *d.* A third harmonic due to the armature reaction m. m. f.
2. That the four components of the flux wave are determined by the value of the three ratios:
   - *a.* Maximum gap to minimum gap.
   - *b.* Pole arc to pole pitch.
   - *c.* Minimum gap to pole pitch.
3. That the armature reaction m. m. f. has a sinusoidal space distribution.
4. That the machine operates at zero power factor lagging.

At the winter convention of the A. I. E. E. in 1927, Mr. R. W. Wieseman presented a paper giving curves for the determination of the four components of the short circuit flux wave mentioned above in terms of the three ratios of assumption number two. (See Figs. 1 and 2.) The accuracy of these curves is discussed in the paper and no more need be said about them here.

It will be assumed that these curves give the actual amplitudes of the flux wave components.

Figure 3 shows a graphical combination of the four components to give the existing air gap flux wave and we shall now proceed to develop a method for doing this analytically. The upper half of the figure, in full lines, represents the flux components of the main field. For convenience this has been considered positive and the peak of the actual no-load flux wave, or main field flux wave, has been taken as unity. Since we have zero power factor in a lagging direction the flux armature reaction will subtract from that of the main field so the two dotted components representing this flux have been shown negative. In connection with this plotting it must be remembered that for a salient pole machine the armature reaction flux at zero power factor is always more peaked than a sine wave and so with the convention of plotting here adopted the third harmonic component will always be positive with respect to the main field. A glance at Fig. 1 will show that the third harmonic of main field flux may be either positive or negative. The existing flux wave in Fig. 3 is, of course, the sum of the four components shown in the figure.

Before any addition of components can be made we must find some common basis for their measurement since the curves of Mr. Wieseman's paper give the four components as a per cent of the total wave of either armature reaction or main field flux as the case may be. This common basis is found by expressing the m. m. f. applied under short circuit conditions as a percentage of the m. m. f. required for the air gap. This assumes that an ampere turn on the field is just as effective in producing flux as an ampere turn on the armature. We must determine another constant, however, because the value of the armature reaction as determined by the usual design formula, for three-phase $2.12\ N\ I\ K_p\ K_d$, is based on a time distribution instead of a space distribution. This constant may be found as follows:

I. Assume a winding having all turns concentrated in a single pair of slots.
II. Assume a full pitch winding.
III. Winding to have Z inductors/pole/phase.
IV. I = r. m. s. current/inductor—strictly sinusoidal.

Such a winding would give rise to a rectangular m. m. f. wave having a value of

$$\text{M. M. F.} = \sqrt{2}\, I\, Z/2.$$

The fourier series for such a wave would be

$$Y = 4/\pi\ (\sqrt{2} IZ/2)\ (\sin X + 1/3 \sin 3X + 1/5 \sin 5X + \ldots)$$

We shall now proceed to neglect all terms in this series except the first. In a three-phase winding the third harmonics will cancel, as will all the triplen harmonics, while the fifth, seventh, etc., will move relatively to the pole faces and so induce currents in the pole faces which tend to damp out the flux produced by these harmonics. Our usual formulas give us the height of the rectangular wave represented by the above fourier series. The ratio of the height of this wave to the maximum value of the fundamental of the series representing it is evidently $4/\pi = 1.27$ which is the constant we are seeking.

Making use of this constant the expression for the short circuit air gap density in terms of the open circuit air gap density becomes (for each component):

$$B_1 = B_{nL}\left[A_{1m}\left(X + \frac{A\ R}{F_g}\right) + \frac{1.27\ A\ R\ A_{1a}}{F_g}\right]$$

$$B_3 = B_{nL}\left[A_{3m}\left(X + \frac{A\ R}{F_g}\right) + \frac{1.27\ A\ R\ A_{3a}}{F_g}\right]$$

(The addition is algebraic. For nomenclature see the table of symbols.)

These then are the relations for determining the short circuit air gap density analytically. Similar relations between the amplitudes of the waves made it possible to construct Fig. 3.

The density distribution which has just been described will not obtain in the iron portions of the machine. We may, however, assume that it will give us an indication of the densities in the teeth, especially after the considerations which are now to be presented. If the armature be supplied with current, the field not being excited, the flux due to the armature current will follow a path somewhat as indicated in the figure below.

If the machine be short circuited and the field excited, so as to cause rated current to flow in the armature, there will be set up in the teeth and in the core immediately below the slot the same flux as before. In addition there will be a fundamental flux due to the main field m. m. f. By Lenz's law these two fluxes will be in opposition and, since they must be nearly equal, the difference being that necessary to compensate for armature IR drop and for end leakage drop, their resultant in the core will be negligible and we may neglect the loss in the core due to fundamental flux.

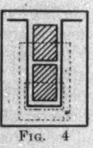


Fig. 4

The fundamental tooth density on short circuit is rarely more than 20 per cent of the open circuit tooth density and so causes not more than 4 per cent of the open circuit tooth loss. As the latter is only about ¼ of the total open circuit core loss we see that the corresponding short circuit core loss is negligible.

The third harmonic density in the armature core will produce a negligible loss, because of the fact that the third harmonic in the core has three times as many poles as does the fundamental, so we may neglect the loss due to this component of flux.

The only component which we need consider is the third harmonic in the teeth. This will be expressed by an equation similar to that given on page 2, except that B_{nL} will be replaced by the no-load tooth density. The loss will then be:

$$W_t = K_1 K_i K_f \frac{\text{volume of teeth}}{1000} w_3.$$

w_3 = loss per cubic inch in standard material at standard triple frequency corresponding to density B_3.

To calculate the triple frequency core loss in the armature teeth for the condition of sustained polyphase short circuit it is, therefore, necessary to compute the third harmonic tooth density as a percentage of the open circuit tooth density and to then obtain the loss from the usual laboratory loss curves.

SHORT CIRCUIT POLE FACE LOSS

Under sustained polyphase short circuit the current carried by each slot will give rise to an m. m. f. which will cause flux pulsations in the pole face. The magnitude of the loss produced will be a function of these tooth frequency flux pulsations. Since the path of this flux is largely in air we may assume, as a first approximation, that it is directly proportional to the m. m. f. producing it. This m. m. f. will be proportional to the current per slot, which, for any given machine, will be proportional to the ampere turns per inch of armature periphery. It so happens that the ampere conductors per π-inch of periphery are numerically equal to the ampere turns per inch of armature periphery at 77 per cent pitch. It will thus be convenient, and sufficiently accurate for any practical value of the pitch, to express the flux as proportional to the ampere conductors per π-inch of armature periphery.

To derive a formula for the short circuit pole face loss use has been made of a similar study of open circuit pole face losses. The method used was that of a dimensional analysis, starting with experimental evidence as to the best values of the exponents for certain of the variables. (See Appendix A.) As regards short circuit pole face loss, however, we have no experimental evidence on which to base a dimensional analysis so the following reasoning is proposed.

Under the assumption of constant permeability the exponent of stator slot pitch was found to be unity, a value lower than that indicated by the most reliable experimental evidence. The limiting value of the density theory gave rise to an exponent for the stator slot pitch of two, but it also involved the tooth frequency to the second power when experimental evidence and the conventional theory alike would indicate the one and one-half power. In both of these derivations for the open circuit condition the same basic assumptions were made. It was finally decided to use a form which was not dimensionally correct, but which involved the tooth frequency to the one and one-half power and the stator slot pitch to the second power, the other exponents being those in agreement with dimensionally correct formulas.

In formulating the short circuit pole face loss it seems desirable to use the formula developed for open circuit core loss, substituting a quantity which shall be proportional to the short circuit flux producing the pole face loss in place of the air gap density used in the open circuit form. It was something of a question as to whether or not the ampere conductors per slot should be used instead of the ampere turns per inch of periphery as outlined above. It will be noticed that the use of the ampere conductors per slot would necessitate involving the stator slot pitch to the fourth power. This exponent is so much higher than any given by experimental work or by the dimensional analysis that it seems better to use the ampere turns per π-inch of periphery as being proportional to the flux than to use the ampere conductors per slot.

The final form of the short circuit pole face loss formula will then be:

$$W_p = K_2 \, (16\, t) \left(\frac{H \times \tau_0}{100} \right)^2 \frac{P\,A}{1000} \left(\frac{V}{10\, g} \right)^{1.5}$$

END LOSS

Under sustained polyphase short circuit conditions the windings carrying current are linking the end fingers, clamping flanges and armature binding bands, thus inducing eddy currents in all of these portions of the machine. In addition there will be a considerable leakage flux from the end connections of the machine, some of which may get into the frame, end shields, and various other stationary portions. It is understood, of course, that this flux of the armature winding which is producing end loss is revolving at synchronous speed, in consequence of the polyphase nature of the winding which produces a revolving field.

If we are to be able to compute the loss we must first compute the leakage flux producing the loss. This may be done by multiplying the m. m. f. by the permeance of the leakage flux path. The m. m. f. will, of course, be the ampere turns of armature reaction, or some fraction thereof. In Appendix B is a dimensional derivation of a formula for end loss.

It now remains to find an expression for the permeance of the leakage flux path. If we assume that the adjacent masses of iron may be neglected we see, from Appendix C, that the permeance is of the form:

$$C = \mu \times \text{pole pitch} \times \text{coil pitch}.$$

Making the assumption of constant permeability and using the formula derived in Appendix B our end loss formula becomes:

$$W_e = K \times \text{pole pitch} \times \text{coil pitch}\; P\, V \sqrt{f}\, (A\, R)^2.$$

But the number of poles times the pole pitch is equal to gap diameter times a constant, whence:

$$W_e = K_3 \frac{D\, V \sqrt{f}}{1000} \left(\frac{A\, R}{10{,}000} \right)^2 \times \text{coil pitch}.$$

LOSS IN THE AMORTISSEUR WINDINGS

Thus far we have assumed that the armature reaction m. m. f. was sinusoidally distributed about the periphery. With a finite number of phases this can never be exactly true, but for the losses thus far considered the approximation has been close. There is, however, a loss due to the fact that the armature reaction m. m. f. is not sinusoidal. The wave form of the armature reaction m. m. f. will be determined by the pitch of the winding and by its arrangement in the slots. For our present purpose the effect of pitch alone will be considered. The arrangement of the winding will, in many cases, be more important but it is the present purpose to produce a formula which shall be comparatively simple to apply in a specific case rather than to obtain one which gives greater accuracy at the cost of greatly increased labor.

If the pitch is such that the wave form is not sinusoidal there will be certain of the higher harmonics which will move relatively to the pole faces. The triplen harmonics will cancel in the three-phase machine leaving all others to produce loss. Of these the fifth and seventh are the most important.

To determine the loss due to these flux harmonics it is necessary to find the current induced in the amortisseur winding by each of these harmonics and then to find the loss due to this current, taking into consideration the effective resistance of the rotor bars. Such a procedure is, of course, far too complicated for the case at hand and a simple approximation must be sought.

If we recognize the fact that, as far as this loss is concerned, the synchronous machine may be treated by an equivalent circuit exactly similar to that for the single-phase induction motor we may arrive at the desired approximation. By the use of this circuit we may deduce an expression for the loss in the bar windings of the rotor as some function of the stator coil pitch. (Appendix D.) To determine the form which this function will take we must remember that the flux harmonics due to the arrangement of the winding will also produce a leakage reactance drop in the phase belt. This reactance is easily calculable for the induction motor and we may simply use values taken from such a curve for the values of this function of coil pitch. This apparent mixing of the units of loss and reactance is not real since the units are proportional and we shall have to prefix this expression for the loss by a constant to be determined by statistical methods.

A dimensional check of the formula was also made and it was shown to be dimensionally correct. In its final form it is:

$$\text{Loss in bars} = K_4 \left(\frac{A\,R \times P}{100{,}000} \right)^2 \frac{L}{D} \times Q \times K_\rho.$$

Q is the function of coil pitch referred to above and K_ρ depends for its value upon the type of amortisseur winding.

DETERMINATION OF THE VALUES OF THE PROPORTIONALITY CONSTANTS

Each of the terms in the formula for short circuit core loss has been preceded by a coefficient. These coefficients have been included because the formulas as developed are not rigorous and they must be determined from considerations derived from the theory of probability. In determining their values we seek to find the most probable value for each one of the four coefficients. Statistical theory indicates that the most probable value of any observed quantity is that which makes the sum of the squares of the deviations of a set of observations, or a group of such sets, a minimum. In accordance with this theory the method of least squares may be applied for the determination of the values of the co-efficients.

For the purpose of determining these values the loss as given by the formula was computed for some 150 machines of various types and ratings. The expression for the loss including the unknown coefficients was then equated to the loss as determined by test and the method of least squares applied to find the separate values.

The method of least squares results in a number of "normal equations" equal to the number of unknowns. These equations are to be regarded as linear simultaneous equations and have to be solved. Unfortunately the coefficients of these "normal equations" are usually such as to require special methods of solution if proper values are to be obtained. Probably the simplest and best method for solving them is that due to Gauss and known as the method of Gauss. (See Whitaker & Robinson, "The Calculus of Observations.")

In applying the method of least squares in this particular investigation the computation of the eddy current loss in the armature copper should be assumed to be without error.

ACCURACY OF THE FORMULA

In using any formula such as this the designer of electrical machinery is interested not only in the error inherent in the formula but the probable error of the result. If the results of using this formula be plotted as a frequency distribution, *i. e.*, per cent error in each individual case against the number of times it occurs, the usual normal distribution or probability curve will result. The fact that we have such a frequency distribution is the thing of greatest interest to the designer. Associated with such a distribution are a number of means intended to give a reliable index of the accuracy of the result.

This normal distribution curve is completely defined by certain parameters, one of which is the so-called standard deviation, or σ. This value of σ happens to be one-half the distance between the points of inflection of the normal distribution curve. The standard deviation may be found from the following relation:

$$\sigma = \sqrt{\frac{\Sigma\ (x)^2}{N}}$$

where x is any deviation from the average and N is the total number of entries. This is, of course, simply the root mean square deviation.

The probable error is defined to be such that the chances are even whether the deviation exceeds it in absolute magnitude or is less than it. If this be denoted by the symbol Q we have:

$$Q = 0.67449\ \sigma.$$

The computation of the value σ requires considerable labor. Fortunately there is a constant relation between the value of σ and the mean deviation which is obtained by considering all deviations as positive and taking their average. Thus the probable error and the mean deviation differ only by a constant and we may use the mean deviation as a measure of the accuracy of the formula. The relation is, then:

$$Q = 0.8345 \text{ mean deviation.}$$

The usual manner of measuring the accuracy of such a formula is to form the ratio

$$\frac{\text{Test core loss}}{\text{Calculated core loss}}$$

for each application and to perform the above computations with this average. When dealing with stray load losses as determined by the sustained polyphase short circuit test, however, it is more convenient to express the error as a percentage of the copper loss at full load current. This is done for several reasons. First, it has been experimentally shown that the stray load loss, or short circuit loss, varies with the square of the current as does the copper loss and so it is natural to treat it in the same manner. Secondly, it has been the practise of designers from time immemorial to estimate this loss by assuming it to be a percentage of the copper loss at rated current. We shall, therefore, be measuring the accuracy of the formula in a unit which is already understood by those who are to use the formula. Finally it is desirable to minimize the testing errors which are greatest where the loss is but a small part of the copper loss.

Measuring the accuracy of the formula for its application to the above mentioned 150 machines the results were as follows:

Mean deviation.............	27.5 per cent
Average per cent error.......	10.9 per cent
Maximum per cent error	
Plus...................	41.2 per cent
Minus.................	41.5 per cent

The above per cent errors are expressed as a per cent of the full load copper loss.

The sources of error in the formula are chiefly due to the effort to apply a single formula, not in itself complete, to a large number of machines of different construction. A second source of error which is of considerable importance lies in the fact that the effect of the coil grouping has been omitted in computing the loss in the amortisseur windings.

Finally it must be pointed out that the accuracy of the experimental determination of this loss, thus far assumed to be without error, is not all that could be desired. In measuring the short circuit loss, as per the A. I. E. E. rules, it is necessary to deduct from the measured input not only the friction and windage but the armature copper loss as well. The ohmic resistance is to be used in this computation but it must, of course, be measured at a temperature corresponding to that obtaining in the armature windings when the loss was measured. This is not always known, so some temperature has to be assumed and consequently a quite appreciable error is introduced. Finally the loss we are seeking to determine is not capable of direct measurement but is a measured loss minus other more or less accurately determinable losses.

In this work the point most worthy of note is the recognition of stray load losses other than the armature eddy current losses. In recognizing these other losses and in formulating an expression for them a method has been advanced for determining the tooth densities of both fundamental and third harmonic components of the flux wave actually existing under zero power factor short circuit conditions.

Emphasis should be laid upon the fact that the dimensional theory and statistical methods have played the major role in all the work which has been here presented. The results cannot be claimed to be without error nor are they entirely novel, but it is believed that new light has been thrown on the subject of core losses in salient pole alternators and that the formulas of the type described here have wide application.

The writer wishes to express his indebtedness to Mr. P. L. Alger, for his constant interest in the work and for his many helpful suggestions.

Appendix A

The material of this appendix refers primarily to the problem of the calculation of open circuit pole face loss, the application of this work to the problem of pole face loss having been shown in the body of the paper.

An effort has been made to be guided by the work of previous investigators. In 1909, C. A. Adams gave the first noteworthy theoretical discussion of the subject in this country. He proposed the following:

$$W_p = \frac{3.3\ P\ A}{10^7}\ t\ B_0^2 \sqrt{\frac{V^3}{\mu\ \rho\ \tau_0}}$$

(B_0 dependent upon slot width/effective gap)

which it will be noted, involves tooth frequency and slot pitch to the 1.5 power.

In 1916, F. W. Carter proposed the following theoretical formula:

$$W_p = \frac{\pi}{24}\ t^2\ (s/g)^{-1.5} V^{1.5} B^2 \sqrt{\frac{1}{\mu\ \rho}}$$

All other theoretical formulas seem to be special cases of one or the other of the above, *i. e.*, applicable to special types of machines only.

Many special formulas based on experimental work have been proposed. The first of these was by Wall and

Smith in 1908. Their work applied only to a single machine and so does not include the lamination thickness. They gave:

$$W_p = K\,P\,A\,B^{2.1} f_t^{1.5} (s/g)^{3.5}.$$

In 1909 Adams gave the following based on experimental work:

For 0.06″ laminations:

$$W_p = \frac{4.62\,P\,A}{10^4}\left(\frac{B}{10^{-4}}\right)^{2.4}\left(\frac{V}{10}\right)^{1.55}(s/g)^{1.5}\frac{1}{\sqrt{\tau_0}}.$$

For 0.014″ laminations:

$$W_p = \frac{3.15\,P\,A}{10^4}\left(\frac{B}{10^{-4}}\right)^{2.3}\left(\frac{V}{10}\right)^{1.55}(s/g)^{1.22}\frac{1}{\sqrt{\tau_0}}.$$

Probably the most reliable experimental work is that of Spooner and Kinnard in 1924. They gave a separate formula for each thickness of lamination, but the exponents were practically the same for all thicknesses. Their formula was:

$$W_p = K\,B^{2.6} f_t^{1.6} (s/g)^{2.2}\,\tau_0^{1.3}\,P\,A.$$

Laminations	*Value of K*
0.0281″	.56
0.0625″	1.2
0.125″	2.6

The above table corresponds to including t with an exponent slightly greater than unity.

In this investigation it is proposed to develop a new formula by means of a dimensional analysis which shall make use of the results just summarized. It seems highly probable that the exponent of flux density should be two, the larger value given by the experimental work probably being due to the unavoidable inclusion of losses other than pole face loss, end loss, and tooth pulsation loss, for example.

In making a dimensional study we are confronted with the fact that we have no mathematical expression for permeability as a function of density. We may, however, assume constant permeability, *i. e.*, a straight line magnetization curve, or we may make use of the so-called limiting value of the density theory. This assumes an infinite permeability up to a certain point and zero permeability thereafter. A dimensional analysis by each method follows.

B = air gap density.
f_t = tooth frequency.
τ_0 = stator slot pitch.
t = lamination thickness.
μ = permeability of pole iron.
ρ = resistivity of pole iron.
B_{sat} = saturation value of density.

Constant permeability theory

$$\text{Watts/square inch} = K\,B^a f^c\,\tau_0{}^d\,t^e\,\mu^g\,\rho^h.$$

Dimensionally

$$M\,T^{-3} = L^{-\frac{a}{2}} M^{\frac{a}{2}}\,T^{-a}\,\mu\,T^{-c}\,L^d\,L^e\,\mu^g\,L^{2h}\,T^{-h}\,\mu^h.$$

$$\begin{aligned}
\text{Solving for}\quad L\,0 &= -\,a/2 + d + e + 2\,h\\
M\,1 &= a/2\\
T - 3 &= -\,a - c - h\\
\mu\,0 &= a/2 + h + g.
\end{aligned}$$

Whence $a = 2$ and:

$$\begin{aligned}
1 &= d + e + 2\,h\\
1 &= c + h\\
-1 &= g + h
\end{aligned}$$

Taking $c = 1.5$ from the work of Carter and Spooner
Using $e = 1$ from Spooner's work
$h = 1.5$
$g = -\,0.5$
$d = 1.$

Limiting value of the density theory

$$\text{Watts/square inch} = K B^2 f_t^c\,\tau_0{}^d\,t^e\,\rho^h\,B_{sat}^m.$$

Dimensionally

$$M T^{-3} = L^{-1} M\,\mu\,T^{-2}\,T^{-c}\,L^d\,L^e\,L^{2h}\,T^{-h}\,\mu^h\,L^{-\frac{m}{2}}\,M^{\frac{m}{2}}\,T^{-m}\,\mu^{\frac{m}{2}}$$

$$\begin{aligned}
\text{Solving for}\quad L\,0 &= -\,1 + 2\,h - m/2 + d + e\\
M\,1 &= 1 + m/2\\
T - 3 &= -\,2 - c - h - m\\
\mu\,0 &= h + m/2 + 1.
\end{aligned}$$

$$\begin{aligned}
\text{Whence}\quad m &= 0\\
h &= -\,1\\
c &= 2\\
d + e &= 3.
\end{aligned}$$

From Spooner and Kinnard's work:

$$\begin{aligned}
e &= 1\\
d &= 2.
\end{aligned}$$

The constant permeability theory gives rise to an exponent for τ_0 lower than that found by experiment, while the saturation value of the density theory gives rise to a value a little above the usual experimental values and an exponent of frequency which is too high, both by theory and experiment, for this relatively high frequency loss. It was, therefore, decided to use the exponents as given by the constant permeability method, except that the exponent of τ_0 would be taken as 2.0.

Shape constants such as s/g which enter as ratios cannot be handled by this method, so we shall have to include them with exponents derived from experimental work. The best value for the exponent of s/g seems to be 1.5.

As derived above the formula is in terms of watts per square inch of pole face area, so to get it in terms of kilowatts for the machine we must multiply by $P\,A/1000$ whence our final result will be:

$$W_p = K\,(16\,t)\left(\frac{B\,\tau_0}{100}\right)^2\frac{P\,A}{1000}\left(\frac{V}{10\,g}\right)^{1.5}.$$

Appendix B

Assuming that end loss is a function of the ampere turns of armature reaction (per pole), the permeance of the leakage flux path, the line frequency, the pole pitch, the permeability, and resistivity of the stator iron we may proceed as follows:

C = permeance of leakage flux path.
$A\,R$ = ampere turns of armature reaction per pole.
$p\,p$ = pole pitch.
P = number of poles.
f = line frequency.
V = peripheral velocity.

$$\left.\begin{array}{l}\rho = \text{resistivity of stator iron}\\ \mu = \text{permeability of stator iron}\end{array}\right\}\text{assumed constant.}$$

$$\text{Watts} = K\,C\,P\,f^n\,(A\,R)^2\,p\,p^m\,\mu^r\,\rho^s.$$

Dimensionally

$$L^2\,M\,T^{-3} = L\,\mu\,T^{-n}\,L\,M\,T^{-2}\,\mu^{-1}\,L^m\,\mu^r\,L^{2s}\,T^{-s}\,\mu^s.$$

$$\begin{array}{lrl}\text{Solving for } L & 2 &= 1 + 1 + m + 2\,s.\\ M & 1 &= 1\\ T & -3 &= -n - 2 - s\\ \mu & 0 &= 1 - 1 + r + s.\end{array}$$

Assuming $s = -0.5$, since for a high frequency loss ρ enters to the minus 1/2 power

$$\begin{aligned}2 &= 1 + 1 + m - 1\\ -3 &= -n - 2 + .5\\ 0 &= 1 - 1 + r - .5\end{aligned}$$

whence

$$\begin{aligned}m &= 1\\ n &= 1.5\\ r &= .5.\end{aligned}$$

But $V = 5\,f\,(p\,p)$.

So we may write:

$$W_e = K\,\frac{C\,P\,V\,\sqrt{f}}{1000}\left(\frac{A\,R}{10{,}000}\right)^2.$$

Appendix C

Since we are to neglect the effect of the adjacent masses of iron the permeability of the leakage flux paths will be that of air. The flux may then be considered as that due to an air core solenoid of length equal to the "effective length" of the end connections. Below is given a development of one end of the armature winding. Each turn of such a winding may be represented as below, one-half only being considered, since from symmetry we need only multiply by two to obtain final results.

If we consider this figure to represent one-half of a single turn we see that there will be four components of m. m. f. produced by the current. These components are.

$$F_1 = \frac{.4\,\pi}{4}\cos\alpha \qquad F_3 = \frac{.4\,\pi}{4}\sin\alpha$$

$$F_2 = \frac{.4\,\pi}{4}\cos\alpha \qquad F_4 = \frac{.4\,\pi}{4}\sin\alpha.$$

Evidently F_3 and F_4 will cancel at zero pitch, and in all cases their resultant will be small at low pitches, so as an approximation the net m. m. f. $= \dfrac{.4\,\pi}{2}\cos\alpha$.

If we now have a coil consisting of N turns, each carrying a current of I amperes, the net m. m. f. will be:

$$F = \frac{.4\,\pi}{2}\,N\,I\cos\alpha.$$

The flux will be (for both ends):

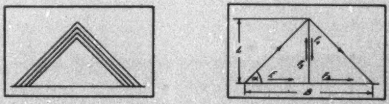
$$\phi = \frac{.4\,\pi\,N\,I\cos\alpha}{R}$$

$$R = \frac{L}{\mu\;A} = \frac{L}{\mu\,\pi\,r^2}\ \text{where } r = \text{radius of core}$$

$$\phi = \frac{.4\,\mu\,\pi^2\,r^2\,N\,I\cos\alpha}{L}.$$

If we had had a uniformly wound air core solenoid of the same diameter,

$$\phi' = \frac{.4\,\mu\,\pi^2\,r^2\,N\,I}{L'}.$$



A B

Fig. 5

Suppose the two solenoids were of the same length. Then $L = L'$ and $\phi/\phi' = \cos\alpha$, or the effective length of the end connections is:

$$L_{\text{eff}} = L\cos\alpha.$$

This effective length is, then, a measure of the permeance of the leakage flux path.

The value of α will be determined by the spacing between coils necessary to give the proper insulation for the voltage rating of the machine. Its value does not vary greatly, however, so we may assume it constant. The permeance may then be expressed as:

$$C = \mu \times \text{pole pitch} \times \text{coil pitch}.$$

Appendix D

In, so, far as the loss in the amortisseur windings is concerned a synchronous machine may be treated by an equivalent circuit exactly similar to that for the single-phase induction motor. The secondary circuit will be composed of two parts: the usual resistance and reactance where the resistance is expressed by $R_2/s = \infty$, since the machine is operating at synchronous speed, and a second portion which involves the reactance for the nth harmonic. In parallel with this secondary circuit will be the magnetizing circuit, having a reactance corresponding to that for the nth harmonic.

Below is given a diagram of the equivalent circuit which we are considering.

It will be noticed that the total current carried by the primary divides, part passing through the magnetizing reactance and the remainder through the secondary impedance. The division of this current will, of course, be inversely proportional to the impedances of the two parallel paths, but that current which passes through the magnetizing circuit produces no loss. This division of current might be expressed by setting the total current equal to $1 + X_m/X_2$. X_2 is the secondary, or rotor, reactance, which for open slots is approximately 10 per cent, for closed slots it may be as high as 40 per cent. Consequently this loss will be changed from what it would be if all the current passed through X_2 in the ratio of $(1 + X_m/X_2)^2$. Putting in the values which were given above this is:

$$\left.\begin{array}{l}(1/1.1)^2 \text{ for open slots} \\ (1/1.4)^2 \text{ for closed slots}\end{array}\right\} \left\{\begin{array}{l}\text{for important harmonics} \\ \text{such as fifth and seventh.}\end{array}\right.$$

Or, for our two-extreme cases the variation might be that from 0.8 to 0.5, a variation which may be neglected.

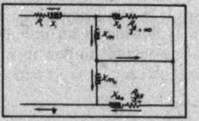


FIG. 6

Assuming that the loss is wholly an $I^2 R$ loss in the rotor bars we may derive the formula as follows:

For current of any given harmonic

$$k\, I^2 = k\, (A\, R)^2$$

$$A\, R = \frac{2.12\, N\, I\, K_p\, K_d}{P}$$

$$k = \frac{P^2\, (A\, R)^2}{(N\, K_p\, K_d)^2} \int (c\, p)\,.$$

The resistance of an induction motor secondary is given by:

$$R_2 = \left(\frac{N\, K_p\, K_d}{1000}\right)^2 \times \frac{\text{Bar length}}{\text{Total section}}.$$

(The resistance of the end rings is neglected since the number of poles is large for the harmonics.)

The total section of rotor bars may be replaced by the square inches of bar section per inch of periphery times the diameter D. The square inches of bar section may be taken as constant. In reality it will be slightly greater for large machines, due to the increased diameter of the bars, but, due to the high frequency of the currents, skin effect will increase the resistance of the bars, or tend to give them an apparently smaller section. Thus we may write:

$$R_2 = K_{pn}{}^2\, K_{dn}{}^2\, N^2\, \frac{L \times P}{c\, D}$$

and the loss will be

$$W = K\, (A\, R)^2\, \frac{L\, P_\rho}{c\, D} \int (c\, p)$$

or

$$W = K_4 \left(\frac{A\, R \times P}{100{,}000}\right)^2 \frac{L}{D} \times Q \times K_\rho.$$

Following is a dimensional check of the above.

In the above formula c has the dimensions of length, since it represents the square inches of bar section per inch of periphery.

Expressed in dimensional units:

$$L^2\, M\, T^{-3} = L\, M\, T^{-2}\, \mu^{-1}\, L\, L^2\, T^{-1}\, \mu\, L^{-1}\, L^{-1}.$$

Solving for (L) $\quad 2 = 1 + 1 + 2 - 1 - 1 = 2$
Solving for (T) $\quad -3 = -2 - 1 = -3$
Solving for (M) $\quad 1 = 1$
Solving for (μ) $\quad 0 = -1 + 1 = 0.$

Hence the formula is dimensionally correct.

TABLE OF SYMBOLS

A_{1m}	amplitude of fundamental of main field flux wave.
A_{3m}	amplitude of third harmonic of main field flux wave.
A_{1a}	amplitude of fundamental of armature reaction flux.
A_{3a}	amplitude of third harmonic of armature reaction flux.
X	armature leakage reactance expressed as a decimal.
B_{nk}	no-load air gap flux density.
B_3	short circuit third harmonic density.
$A\, R$	armature reaction ampere turns per pole.
F_g	air gap ampere turns per pole.
K_i	constant dependent upon quality of stator iron.
K_f	constant dependent upon frequency.
w_3	loss per cubic inch in standard iron at standard frequency at density B_3.
t	thickness of pole laminations.
H	armature reaction per π-inch.
τ_0	stator slot pitch.
P	number of poles.
A	area of air gap under one pole.
V	peripheral velocity in thousands of feet per minute.
g	effective air gap.
D	gap diameter in inches.
L	length of rotor stacking in inches.
Q	a factor dependent upon coil pitch.

Proximity Effect in a Seven-Strand Cable

BY J. E. L. TWEEDDALE[1]

Enrolled Student, A. I. E. E.[1]

THE calculation of the alternating current resistance ratio due to skin effect or proximity effect has been worked out for many shapes and combinations of conductors but often without recourse to experimental results. It is, accordingly, the purpose of this work to check calculations with tests for the type of calculation recently developed, covering the losses in several round wires connected in parallel, of which the arrangement is such that unequal currents flow in the different wires. The experimental results with which the calculated results are to be compared are those presented by A. E. Kennelly and H. A. Affel[2] in 1916 for seven-strand cables. Their work covered radio frequencies up to 100,000 cycles in rather small conductors. The derivations as presented here are applicable to all frequencies and sizes of conductors.

The method of attack is the same as that employed by Dr. H. B. Dwight of the Massachusetts Institute of Technology in the solution of the proximity effect in other arrangements of conductors. The writer wishes to acknowledge the aid and help of Dr. Dwight in the solution of this problem.

In a seven-strand conductor, inequalities of current exist in the separate wires of the conductor. If the return conductor is assumed to be at a distance such that its proximity effect is negligible then we have two unknown currents existing in the seven-strand conductor; *i. e.*, the outer concentric conductors will all carry equal currents I_1 and the center conductor will carry a current I_2 so that the total current $I = 6\,I_1 + I_2$.

The effect of spiraling of the wires has been neglected in this calculation. While the wires of a cable are spiraled, the test with which this calculation is to be compared, illustrated in Figs. 4 and 5, was made on seven straight, unspiraled round wires.

The method of attack is in the main as follows: The current density in a single isolated round wire is given by the following expression:

$$i_{r\theta} = \frac{I_1}{\pi\, a^2}\,\frac{j\,\alpha\,a}{2}\,\frac{J_0\,(j\,\alpha\,r)}{J_1\,(j\,\alpha\,a)} \text{ absamps./sq. cm.} \qquad (1)$$

where r and θ are the polar coordinates of any point in the section of the wire, where

I_1 = total current in the wire

a = radius of the wire

$\alpha = \sqrt{\dfrac{j\,\omega\,4\,\pi}{\sigma}}$

$\omega = 2\,\pi \times$ frequency

σ = resistivity of material of conductor

and $j = \sqrt{-1}$.

Absolute electromagnetic units are used throughout. The quantity $J_0\,(j\,\alpha\,r)$ is a Bessel function of the first kind, zero order and argument $j\,\alpha\,r$. This can be expressed as an algebraic series but is readily evaluated from tables in which $J_0\,(b\,j\,\sqrt{j}) = u_0 + j\,v_0$, where $j\,\alpha\,r = b\,j\,\sqrt{j}$. The above equation, (1), is based on the fact that the impedance drops at every section of the wire are equal.

The next point is to obtain the effect of the current in the other wires on the distribution of the current in the first wire. It has been shown by Manneback[3] that a current I_2 flowing in an infinitesimal wire will cause a circulating current to flow in a wire of radius a at an axial distance s whose current density will be

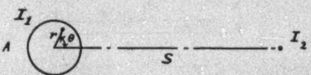


FIG. 1—ROUND WIRE AND INFINITESIMAL WIRE

$$i_{r\theta} = \frac{I_2}{\pi\ a^2}\,j\,\alpha\,a \sum_{n=1}^{n=\infty} \frac{a^n}{s^n}\,\frac{J_n\,(j\,\alpha\,r)}{J_{n-1}\,(j\,\alpha\,a)} \cos n\,\theta \qquad (2)$$

J_n represents a Bessel function of the first kind of order n.

The effect of the current distribution given by (2) is that the impedance drops at every section of the wire are equal. The total current in the wire, obtained by adding up expression (2) over the entire section of the wire, is zero. Since the above two conditions are satisfied, the current density given by (2) can be added to that given by (1) without changing the total current I_1. The necessary condition of equal impedance drops at every section of the wire is still met. The sum of (1) and (2) therefore gives the current distribution in a wire carrying current I_1, and with a concentrated current I_2 near it. Other expressions of the same form as (2) may be added for all the other concentrated alternating currents which may be near the wire.

Now, returning to the seven-strand cable we see that

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[2] "Skin-Effect Resistance Measurements at Radio Frequencies up to 100,000 Cycles per Second," by A. E. Kennelly and H. A. Affel. Proc. Inst. of Radio Engineers, May, 1916, and *Research Bulletin* No. 13, Mass. Institute of Technology.

Presented at the Regional Meeting of Dist. No. 1, Pittsfield, Mass., May 25-28, 1927.

[3] Equation (19), "An Integral Equation for Skin Effect in Parallel Conductors," by Charles Manneback. *Journal of Math. and Physics*, April, 1922, and *Research Bulletin* No. 30, Mass. Institute of Technology.

if we are considering wire *A*, the proximity effect of the other six conductors must be taken into account. The first step is to assume conductors *B, C, D, E, F*, and *G* as infinitesimal conductors. The resultant current density in wire *A* will be the sum of equation (1) plus the respective terms for the other conductors similar in form to equation (2).

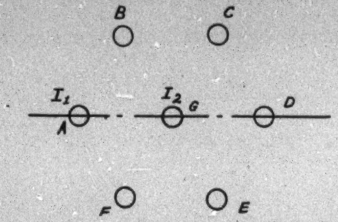

Fig. 2—Seven Wires Arranged in Cable Form

The resultant expressions will be, then, as follows:

$$i_{r\theta} = \frac{I_1}{\pi\, a^2} A_0 J_0 (j\, \alpha\, r) + \sum_{n=1}^{n=\infty} A_n J_n (j\, \alpha\, r) \cos n\, \theta \quad (3)$$

and

$$i_{u\gamma} = \frac{I_2}{\pi\, a^2} A_0 J_0 (j\, \alpha\, u) + \sum_{n=1}^{n=\infty} F_n J_n (j\, \alpha\, u) \cos n\, \gamma, \quad (4)$$

etc.,
where A_0, A_n, and F_n are coefficients which for this derivation are as follows:

$$A_0 = \frac{j\, \alpha\, a}{2\, J_1 (j\, \alpha\, a)}. \quad (5)$$

$$A_n = \frac{I_2}{\pi\, a^2}\, j\, \alpha\, a \frac{a^n}{s^n} \frac{1}{J_{n-1} (j\, \alpha\, a)} + \frac{I_1}{\pi\, a^2} \frac{a^n}{s^n} \frac{j\, \alpha\, a}{J_{n-1} (j\, \alpha\, a)} \Big[2 \cos \frac{n\, \pi}{3} + \frac{2}{(\sqrt{3})^n} \cos \frac{n\, \pi}{6} + \frac{1}{(2)^n} \Big]. \quad (6)$$

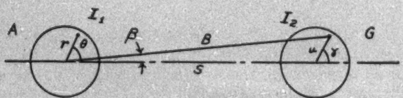

Fig. 3—Two Round Wires

$$F_n = \frac{I_1}{\pi\, a^2} \frac{a^n}{s^n} \frac{j\, \alpha\, a}{J_{n-1} (j\, \alpha\, a)} \Big[\cos n\, \pi + 2 \cos \frac{2\, n\, \pi}{3} + 2 \cos \frac{n\, \pi}{3} + 1 \Big]. \quad (7)$$

The expressions (3) and (4) are, therefore, the current densities due to uniform current densities in the others, and I_1 and I_2 in the wires themselves.

The next step is to find the current in each wire due to the A_0 and A_m or F_m currents in the others. For simplicity, let us consider just two of the wires as in Fig. 2, *A* and *G*, for the present, to show the method.

Let there be a current density in *A* in addition to the uniform current density

$$i_{r\theta} = -\frac{I_1}{\pi\, a^2} + \frac{I_1}{\pi\, a^2} A_0 J_0 (j\, \alpha\, r) + \sum_{m=1}^{m=\infty} A_m J_m (j\, \alpha\, r) \cos m\, \theta \quad (8)$$

and likewise for *G*

$$i_{u\gamma} = -\frac{I_2}{\pi\, a^2} + \frac{I_2}{\pi\, a^2} A_0 J_0 (j\, \alpha\, u) + \sum_{m=1}^{m=\infty} F_m J_m (j\, \alpha\, u) \cos m\, \gamma\, . \quad (9)$$

Each of these integrated over its section will be zero.

Then, the *n*th term of $i_{r\theta}$ due to $i_{u\gamma}\, u\, du\, d\gamma$ in *G* is as follows:

$$i_{u\gamma} \frac{j\, \alpha\, a}{\pi\, a^2} \frac{a^n}{B^n} \frac{J_n (j\, \alpha\, r)}{J_{n-1} (j\, \alpha\, a)} \cos n\, (\theta - \beta)\, u\, du\, d\gamma \quad (10)$$

where *B* and β are variables given by a series for each wire.

$$\frac{\cos n\, \beta}{B^n} = \frac{1}{s^n} \Big[1 + \sum_{k=1}^{k=\infty} \frac{\underline{/n+k-1}}{\underline{/n-1}\,\underline{/k}} \frac{u^k}{s^k} \cos k\, (\gamma - \pi) \Big] \quad (11)$$

and

$$\frac{\sin n\, \beta}{B^n} = \frac{1}{s^k} \sum_{k=1}^{k=\infty} \frac{\underline{/n+k-1}}{\underline{/n-1}\,\underline{/k}} \frac{u^k}{s^k} \sin k\, (\pi - \gamma)\, . \quad (12)$$

The proof of the two preceding expressions is given by H. L. Curtis.[4]

Substituting expression (9) in (10), expanding the difference of the angles, and substituting their respective values from (11) and (12), the *n*th term of $i_{r\theta}$

$$\Big[-\frac{I_2}{\pi\, a^2} + \frac{I_2}{\pi\, a^2} A_0 J_0 (j\, \alpha\, u) + \sum_{m=1}^{m=\infty} F_m J_m (j\, \alpha\, u) \cos m\, \gamma \Big]$$

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[4] H. L. Curtis, Scientific Paper No. 374 of the Bureau of Standards, Washington, D. C., 1920.

$$\left[\frac{j\,\alpha\,a}{\pi\,a^2}\,\frac{a^n}{s^n}\,\frac{J_n\,(j\,\alpha\,r)}{J_{n-1}\,(j\,\alpha\,a)}\right]\left\{\cos n\,\theta\left[1\right.\right.$$

$$\left.+\sum_{k=1}^{k=\infty}\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,\frac{u^k}{s^k}\cos k\,(\gamma-\pi)\right]$$

$$+\sin n\,\theta\left[\sum_{k=1}^{k=\infty}\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,\frac{u^k}{s^k}\right.$$

$$\left.\left.\sin k\,(\pi-\gamma)\right]\right\}u\,du\,d\gamma. \qquad (13)$$

The above expression is then integrated between the limits 0 to 2 π and 0 to a. The sine term will become zero upon integration and drop out, and likewise

$$\int_0^a\left[-\frac{I_2}{\pi\,a^2}+\frac{I_2}{\pi\,a^2}\,A_0\,J_0\,(j\,\alpha\,u)\right]u\,du=0.$$

Thus there is left the integral of the nth term of the terms in m which is

$$\sum_{m=1}^{m=\infty}\frac{a^{m+n}}{s^{m+n}}\,\frac{J_{m+1}\,(j\,\alpha\,a)}{J_{n-1}\,(j\,\alpha\,a)}\left[\frac{\underline{|n+m-1}}{\underline{|n-1}\,\underline{|m}}\right]$$

$$[F_m\cos m\,\pi]\,J_n\,(j\,\alpha\,r)\cos n\,\theta\,. \qquad (15)$$

Let B_n be the coefficient of $J_n\,(j\,\alpha\,r)\cos n\,\theta$.

Now let there be an additional current density in A as represented by the expression (15). The resulting current in A due to this additional current is given by

$$C_n\,J_n\,(j\,\alpha\,r)\cos n\,\theta \qquad (16)$$

where C_n is given by the formula for B_n except change A to B and F to G.

This process may be continued indefinitely, approaching the final limiting condition of the actual current density. The final current density in the two wires will be as follows:

$$i_{r\theta}=\frac{I_1}{\pi\,a^2}A_0\,J_0\,(j\,\alpha\,r)+\sum_{n=1}^{n=\infty}M_n\,J_n\,(j\,\alpha\,r)\cos n\,\theta \qquad (17)$$

and for conductor G

$$i_{u\gamma}=\frac{I_2}{\pi\,a^2}A_0\,J_0\,(j\,\alpha\,u)+\sum_{n=1}^{n=\infty}N_n\,J_n\,(j\,\alpha\,u)\cos n\,\gamma \qquad (18)$$

where $\quad M_n=A_n+B_n+C_n+\ .\ .\ . \qquad (19)$

and $\quad N_n=F_n+G_n+H_n+\ .\ .\ . \qquad (20)$

Thus far, only conductors A and G have been considered for simplicity. For the seven-strand problem, the effect of conductors B, C, D, E, and F may be determined by the introduction of their respective terms in the preliminary equations. The introduction of these terms will not affect the general form of the final equations for the current density as given in (17) and (18) but will affect the coefficients A_n, B_n, C_n, etc., and F_n, G_n, H_n, etc., which are dependent on the unknown currents I_1 and I_2 and on the geometry and dimensions of the circuit.

Thus, for the seven strand problem the coefficients are as follows:

$$A_0=\frac{j\,\alpha\,a}{2\,J_1\,(j\,\alpha\,a)}\,. \qquad (21)$$

$$A_n=\frac{I_2}{\pi\,a^2}\,\frac{a^n}{s^n}\,\frac{j\,\alpha\,a}{J_{n-1}\,(j\,\alpha\,a)}$$

$$+\frac{I_1}{\pi\,a^2}\,\frac{a^n}{s^n}\,\frac{j\,\alpha\,a}{J_{n-1}\,(j\,\alpha\,a)}\left[2\cos\frac{n\,\pi}{3}\right.$$

$$\left.+\frac{2}{(\sqrt{3})^n}\cos\frac{n\,\pi}{6}+\frac{1}{(2)^n}\right]. \qquad (22)$$

$$B_n=\sum_{m=1}^{m=\infty}\frac{a^{m+n}}{s^{m+n}}\,\frac{J_{m-1}\,(j\,\alpha\,a)}{J_{n-1}\,(j\,\alpha\,a)}\left[\frac{\underline{|n+m-1}}{\underline{|n-1}\,\underline{|m}}\right]\times$$

$$\left[F_m\cos m\,\pi+2\,A_m\cos\frac{\pi\,(m-n)}{3}\right.$$

$$\left.+\frac{2\,A_m}{(\sqrt{3})^{m+n}}\cos\frac{\pi\,(m-n)}{6}+\frac{A_m}{(2)^{m+n}}\right]. \qquad (23)$$

C_n is given by the same expression as B_n except change A to B and F to G, likewise for any subsequent terms.

$$F_n=\frac{I_1}{\pi\,a^2}\,\frac{a^n}{s^n}\,\frac{j\,\alpha\,a}{J_{n-1}\,(j\,\alpha\,a)}\left[\cos n\,\pi\right.$$

$$\left.+2\cos\frac{2\,n\,\pi}{3}+2\cos\frac{n\,\pi}{3}+1\right]. \qquad (24)$$

$$G_n=\sum_{m=1}^{m=\infty}A_m\,\frac{a^{m+n}}{s^{m+n}}\,\frac{J_{m+1}\,(j\,\alpha\,a)}{J_{n-1}\,(j\,\alpha\,a)}$$

$$\left[\cos n\,\pi+2\cos\frac{2\,n\,\pi}{3}+2\cos\frac{n\,\pi}{3}+1\right]$$

$$\frac{\underline{|n+m-1}}{\underline{|n-1}\,\underline{|m}}\,. \qquad (25)$$

H_n is given by the same expression as G_n except change A to B; likewise for any subsequent terms. For all terms except the sixth or multiples of the sixth, F_n, G_n, etc., will be zero.

As said before the expressions for the current density will be as given in (17) and (18) with the use of the above coefficients. The expressions for the current density in the other conductors, B, C, D, E, and F, will be similar to (17) except for the use of their respective polar coordinates.

There are now two unknown currents I_1 and I_2.

Their ratio may be determined, since the conductors of the cable are in parallel, by finding the voltage drop in each wire in terms of I_1 and I_2. Then, the voltage drops in any two parallel filaments are equal and can be equated, giving the ratio of I_1/I_2.

The simplest expression to derive is that for the drop in the central filament of the wire. This, then, is equal to the drop in any other filament. In wire A the resistance drop in the central filament is $i_0\,\sigma$ where i_0 is the current density at the center. Using equation (**17**) this is

$$\frac{I_1}{\pi\,a^2}\,A_0\,\sigma = \frac{I_1\,\sigma}{\pi\,a^2}\,\frac{j\,\alpha\,a}{2\,J_1\,(j\,\alpha\,a)} \tag{26}$$

since $J_0\,(0) = 1$ and $J_n\,(0) = 0$ where $n \neq 0$.

The reactive drop in the central filament due to the element of current $i_{r\theta}\,r\,d\,\theta\,d\,r$ is

$$j\,\omega\,2\,i_{r\theta}\left(\operatorname{logh}\frac{p}{r}\right)r\,d\,\theta\,d\,r \tag{27}$$

$$= j\,\omega\,2\,\frac{I_1}{\pi\,a^2}\,\frac{j\,\alpha\,a}{2}\,\frac{J_0\,(j\,\alpha\,r)}{J_1\,(j\,\alpha\,a)}\left(\operatorname{logh}\frac{p}{r}\right)r\,d\,\theta\,d\,r$$

$$+ j\,\omega\,2\sum_{n=1}^{n=\infty} M_n\,J_n\,(j\,\alpha\,r)\cos n\,\theta\left(\operatorname{logh}\frac{p}{r}\right)r\,d\,\theta\,d\,r. \tag{28}$$

This takes into account the flux up to a certain large distance p. The expression logh denotes the hyperbolic or natural logarithm. Equation (**28**) was obtained by the use of equation (**17**). To find the reactive drop due to the entire current I_1, the above expression is integrated from $\theta = 0$ to $2\,\pi$ and $r = 0$ to a. The integral of the second term of (**28**) will be zero since $\int_0^{2\pi} \cos n\theta\; d\theta = 0$. That of the first term integrating by parts, remembering that $\alpha^2 = \frac{j\,\omega\,4\,\pi}{\sigma}$ is

$$\frac{I_1\,\sigma}{\pi\,a^2}\,\frac{j\,\alpha\,a}{2}\,\frac{J_0\,(j\,\alpha\,a)}{J_1\,(j\,\alpha\,a)} + j\,\omega\,2\,I_1\operatorname{logh}\frac{p}{a}$$

$$-\frac{I_1\,\sigma}{\pi\,a^2}\,\frac{j\,\alpha\,a}{2\,J_1\,(j\,\alpha\,a)}\,. \tag{29}$$

Then the impedance drop at the center of the wire A due to I_1 is the sum of equations (**26**) and (**29**) and is

$$\frac{I_1\,\sigma}{\pi\,a^2}\,A_0\,J_0\,(j\,\alpha\,a) + j\,\omega\,2\,I_1\operatorname{logh}\frac{p}{a}\,. \tag{30}$$

The reactance drop at the same central filament in A due to currents expressed in the same form as in equations (**17**) and (**18**) in the other conductors may be added on.

Similarly, the impedance drop in the central filament of conductor G carrying current I_2 may be found. Since the two wires are in parallel, the impedance drops must be the same. The two equations may then be equated and the ratio of I_1 to I_2 determined.

For the case of the seven strand conductor the impedance drops in conductors A and G are as follows:

$$E_a = \frac{I_1\,\sigma}{\pi\,a^2}\,A_0\,J_0\,(j\,\alpha\,a) + j\,\omega\,2\,I_2\operatorname{logh}\frac{p}{s}$$

$$+ j\,\omega\,2\,I_1\operatorname{logh}\frac{p^6}{6\,a\,s^5}$$

$$+ j\,\omega\,2\sum_{n=1}^{n=\infty} N_n\,\pi\,a^2\cos n\,\pi\,\frac{a^n}{n\ s^n}\,\frac{J_{n+1}\,(j\,\alpha\,a)}{j\,\alpha\,a}$$

$$+ j\,\omega\,2\,\pi\,a^2\sum_{n=1}^{n=\infty} M_n\,\frac{J_{n+1}\,(j\,\alpha\,a)}{j\,\alpha\,a}\,\frac{a^n}{n\ s^n}\times$$

$$\left[2\cos\frac{n\,\pi}{3} + \frac{2}{(\sqrt{3})^n}\cos\frac{n\,\pi}{6} + \frac{1}{(2)^n}\right]. \tag{31}$$

and

$$E_g = \frac{I_2\,\sigma}{\pi\,a^2}\,A_0\,J_0\,(j\,\alpha\,a) + j\,\omega\,2\,I_2\operatorname{logh}\frac{p}{a}$$

$$+ j\,\omega\,12\,I_1\operatorname{logh}\frac{p}{s}$$

$$+ j\,\omega\,12\sum_{n=1}^{n=\infty} M_n\,\pi\,a^2\,\frac{a^n}{n\ s^n}\,\frac{J_{n+1}\,(j\,\alpha\,a)}{j\,\alpha\,a}\,. \tag{32}$$

It is to be noted again that M_n and N_n are expressions involving the unknown currents I_1 and I_2 as well as the geometry of the circuit.

For the determination of the watts loss and the resistance ratio, the process is as follows:

Let $$e_{r\theta} = \sigma\,i_{r\theta}\,. \tag{33}$$

Then by equation (**10**) of J. R. Carson's paper,[5]

$$j\,\mu\,\omega\,H_{r\theta} = \frac{\partial}{\partial\,r}\,e_{r\theta} \tag{34}$$

where μ is the permeability and, for this case, equal to 1. $H_{r\theta}$ is the tangential component of the magnetic force due to the currents in the wires.

Then

$$j\,\omega\,H_{r\theta} = \frac{I_1\,\sigma}{\pi\,a^2}\,A_0\,j\,\alpha\,J_0{}'\,(j\,\alpha\,r)$$

$$+ \sum_{n=1}^{n=\infty} \sigma\,M_n\,j\,\alpha\,J_n{}'\,(j\,\alpha\,r)\cos n\,\theta. \tag{35}$$

By equation (**18**) of Carson's paper, the true energy

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5. "Wave Propagation over Parallel Wires: The Proximity Effect," by J. R. Carson, *Philosophical Mag.*, April, 1921, page 607.

transferred to or from one centimeter of wire through its surface, according to Poynting's theory, is equal to the resistance loss and is

$$\hat{I}\, I\, R' = \text{real part of } \frac{a}{4\,\pi} \int_{\theta=0}^{\theta=2\pi} \hat{e}_{a\theta}\, H_{a\theta}\, d\,\theta. \qquad (36)$$

This it is noticed involves only values at the surface of the wire, so the integration is not complicated. The term $\hat{e}_{a\theta}$ is the conjugate of $e_{a\theta}$, taken at the surface, and is obtained by replacing j by $-j$.

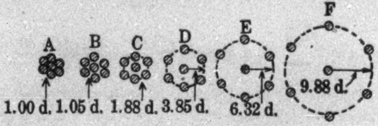

Fig. 4—Wire Spacing. (Kennelly and Affel, Reference 1)

Then the loss in conductor A is

$$\frac{b}{2}\left[\hat{I}_1\, I_1\, \frac{\sigma}{\pi\, a^2}\, \frac{(u_0\, v_0' - u_0'\, v_0)}{(u_0'^2 + v_0'^2)}\right]$$

$$+ \sum_{n=1}^{n=\infty} \frac{b\,\sigma^2}{4\,\omega}\, \hat{M}_n\, M_n\, (u_n\, v_n' - u_n'\, v_n) \qquad (37)$$

and the loss in conductor G is

$$\frac{b}{2}\left[\hat{I}_2\, I_2\, \frac{\sigma}{\pi\, a^2}\, \frac{(u_0\, v_0' - u_0'\, v_0)}{(u_0'^2 + v_0'^2)}\right]$$

$$+ \sum_{n=1}^{n=\infty} \frac{b\,\sigma^2}{4\,\omega}\, \hat{N}_n\, N_n\, (u_n\, v_n' - u_n'\, v_n)\,. \qquad (38)$$

It is to be noticed that

$$\hat{M}_n\, M_n = |M_n|^2 \text{ and } \hat{I}_1\, I_1 = |\, I_1\,|^2, \text{ etc.}$$

The loss in any one wire at zero frequency is given

$$= \frac{I^2}{49}\, \frac{\sigma}{\pi\, a^2}\,. \qquad (39)$$

Then by determination of the loss by equations (37) and (38) and the loss for the same conditions at zero frequency by (39) the ratio of losses will give the resistance ratio for conductors A and G and thence the average resistance ratio of the cable may be determined.

Results

The expressions having been derived, two calculations were made using tables of $J_n\,(b\,j\,\sqrt{j})$ as calculated by H. B. Dwight.[6] A value of $b = 2$ was chosen which for the size of wire corresponds to a frequency of 88,300 cycles/second. Calculations for two spacings, $s/a = 2$ and $s/a = 3.76$ corresponding to curves A and C of Fig. 5, were made.

The calculated results as compared with the corresponding results of Fig. 5 for these spacings are as follows:

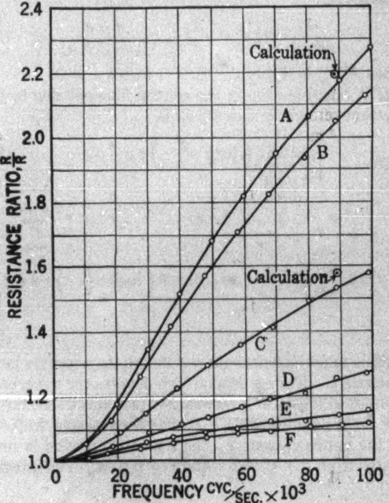

Fig. 5—Effect of Strand Spacing on the Skin Effect of a Stranded Conductor. (Kennelly and Affel, Reference 1)

	$s/a = 2$	$s/a = 3.76$
Calculated..............	2.19	1.58
Figure 5 (test)............	2.15	1.53

The resistance ratio for an equivalent single round conductor for the same conditions is 2.1457.

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[6] *Proximity Effect in Wires and Thin Tubes*, by H. B. Dwight, Trans. A. I. E. E., 1923, page 858.

A. I. E. E. Outstanding Features of the Past Year

President's Address

BY CUMMINGS C. CHESNEY

THE fact that the provision of the Constitution of the American Institute of Electrical Engineers which assigns the address of the President to the close rather than to the beginning of his term, seems to me to indicate a desire on the part of the Institute to have this address include in some measure, the ideas of the retiring President, acquired during his term on matters of general concern to the Institute, and to review those problems of procedure, organization and policy, which may assist the Institute to grow and develop in a way which is healthy and for the best interests of its members, for the engineering profession as a whole, for the good of the electrical industry and the communities which we represent. It seems fitting therefore at this time to bring to your attention several outstanding features of the year just past.

It has been my privilege as your President to have visited many of the more remote Sections of the Institute, for instance Salt Lake City, Los Angeles, San Francisco, Portland, Seattle, Spokane, and all of the Sections in Canada, together with many of the Sections located in the eastern cities and in the central West. I had in mind visiting all of those Sections, which, on account of their geographical location, have either never been visited by a national officer, or have been visited very infrequently. I am sure that it will be gratifying to all to learn of the increasing interest and activity in Institute affairs all over the country which it was my pleasure to find; and also to learn of the greater appreciation of the value of the Institute professionally and practically; but I have also noted many times in meeting with the officers of these Sections and discussing their problems, what an important part of our Institute activities these Sections constitute and how much of the future growth and vitalizing power of the Institute is dependent upon their success.

The Institute and its Sections can well be likened to a hydroelectric power system, where the Sections may be represented as so many small rivers reaching out in all directions, bringing their supply of power through the main artery to the central power house which converts their combined energy into a total useful effort.

At the regional meetings held by the Institute in various Sections of the country, out of a total membership of 19,000, several hundred national members may attend. For instance, at the regional meeting held at Madison, Wis., 180 were registered; at Niagara Falls 580 were registered; at New York 700; at Kansas City 225; at Bethlehem, Pa., 400. On the other hand, at the Sections meetings held during the year thousands of members get together and carry on in the many sections the real development work of the Institute. There are 95 Student Branches, which held 842 meetings last year, at which 42,650 members were registered. There are 52 of the regular Sections which held 432 meetings last year, at which 60,708 were registered; in other words there were over 100,000 members total of local Sections and Branches that attended meetings last year.

It is through the regular meetings of Sections that the young engineer makes his first contacts and receives, in many instances, the inspiration that influences the character of his life's work.

What the meetings of the Institute really mean to the members has been ably pointed out by President Scott in 1902, at the time of his presidency, when the Institute was developing at a rapid pace. A committee was formed on local Sections at this time with the authority to make arrangements for the holding of local meetings. There had been a few Sections operating at the time but active steps were then taken to develop broadly the Section idea. As Professor Scott so ably said, "In a profession, whose interests are so diversified and extended, workers should be brought together; they should have a common meeting place where discoveries may be announced, inventions discussed, engineering schemes criticized, and new undertakings presented and discussed. Here the student and professor, the investigator, the manufacturer, the operator and consulting engineer may meet upon common ground. The professor who regards lightly the work of the designing or construction engineer may find that his own cherished formulas are derived from rules and contain the constants, which the practical man has determined for himself. Association leads to mutual understanding, it curbs eccentricity and one-sided development, promoting symmetrical advancement."

During the year, I have been especially interested in watching the growth and accomplishments of all Sections of the Institute, and as might be expected, have been particularly interested in the growth and development of the Pittsfield Section, my home Section. This Section, the largest in the Institute, has a combined national and local membership of 1000 members, and it is not uncommon to have at the meetings more than 800 in attendance. This Section is not only a common meeting ground for all ambitious young engineers in the community, but at the same time it represents a definite part of the city and community life. A large number of people interested in the general meetings purchase a membership ticket and attend the popular

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Presented at the Summer Convention of A. I. E. E., Detroit, Mich., June 20-24, 1927.

meetings at which important investigators, scientists, and explorers give talks and demonstrations covering their experiences. The Institute Section is the leading dignified engineering group in the community and, being interested in the community's growth, all the leading citizens feel honor-bound to belong to it. The local papers report the meetings in an elaborate way, sometimes half a page being given to a single meeting. This publicity spreads the work of the Institute beyond the city limits so that the effectiveness of the Institute as an educational and social influence is far-reaching.

In addition to the large general meetings, discussion or round-table technical meetings are held, at which the more ambitious young engineer meets on a common ground, and can engage in oral discussion, with the senior engineers of the community upon subjects of vital interest to him. Such discussion develops self-confidence, attracts attention to the younger engineer, creates the desire on his part to present papers at regional meetings, and affords an outlet for his latent energy, his enthusiasm, and his creative ability.

As he gains in experience he is placed on committees and given an opportunity to do organization work. After he has served for several years in a minor capacity, further responsibilities are added and the extent to which he continues his efforts in the interest of his Section aids his progress in securing a higher office as a reward. Simultaneously with such progress he becomes identified with the national organization, taking an active part on the main committees.

This opportunity for self-expression and growth afforded the younger engineers through the local Section is vital, for here he learns one of the fundamentals of life—that progress comes only through constant effort. The methods, which have been followed by the Pittsfield Section, develop the activities of the Section and have resulted directly in the growth and development of the individual member; they have been followed to some extent by other Sections, and while the Pittsfield idea is worth copying there are other similar successful experiences in other Sections of the Institute that are equally entitled to thorough study and consideration by the national officers.

I believe there is need for outlining a more definite policy for Section activities based on the opportunities for training and growth afforded the younger members. Such a policy would include more specific plans for lectures and round-table discussions than have heretofore been the practise, and gives a chance for development of organizing and managerial ability. It should, of course, include the opportunity to prepare papers, to engage in oral discussions, and to take a more intimate part in community work.

An engineer, because of his education, is accustomed to analyze conditions and arrive at basic truths, and if his special abilities which invite confidence could be adapted to our community problems it would have a very direct bearing on the future progress and efficiency of our industry and our country.

The Speaker Bureau idea came into effect this year, in a limited way. Through this bureau it is expected the Sections may more readily get talented speakers and lecturers. The idea is sound and should become more and more valuable each year and should be given special consideration in the future.

The Committee on Public Relations, by formulating a definite plan of procedure based, of course, on the experience of the older sections, can greatly aid in extending more rapidly the influence in their several communities of the younger or more newly organized sections.

For the purpose of the study of the section idea and the ready exchange of ideas between them, and for the purpose of extending the sections' local and national influence, our new Assistant Secretary, Mr. Henline, was added to the national executive staff last January, and while he has not authorized me to speak for him, coming from the Golden West, I know his progressive spirit makes him ever ready to render any assistance within his power to any section. May I also at this point commend the work of Prof. Harold Smith during the past several years as Chairman of the Sections Committee. The Institute is under a deep debt of gratitude to him. We cannot too much emphasize the fact that without the help of the Sections, through their vigorous and helpful growth the usefulness and future stability and the influence of the Institute will be seriously handicapped; whereas with the constructive and vitalizing work which the Sections are able to contribute to the Institute's affairs there will be every reason to expect that the past effectiveness of the Institute will continue indefinitely.

During the past year, your executive officers and your Board of Directors have given more than usual attention to the subject of Electrical Standardization. It is a subject that has had the particular attention of practically every Board of Directors since the appointment of the first Committee of Standardization by the Institute in 1898.

There has been intense interest in standardization through the older engineering organizations and also through the newer trade associations, both national and international, which has brought forth recent statements from prominent writers, such as, "Standardization is the outstanding note of this present century. It ramifies to the remotest details of our industrial regime. Its trends are highly significant. They tap all sources of scientific knowledge and affect every phase of design, production, and utilization." From another author—"Standardization is a new and outstanding influence in modern industry. It is based on an economic conception of utility, and its trends and ramifications affect every aspect of design, production, and utilization."

These are excellent and general statements of fact, but submit no reason for this recent great activity in this line, which is now so generally recognized. It is, in my opinion, largely the direct outcome of the scarcity of labor since the great war and the passage of our new immigration law, and the laudible desire to maintain and extend the high standard of living which, in this country, we have enjoyed for the past quarter of a century, and during which tens of millions of people have attained standards of comfort and of culture far higher than those of any other country in the world, and immensely in excess of anything hitherto known in the world's history.

All this argues for the maintenance and increase of the present earnings of the worker and at the same time requires the lowering of the cost of production. Standardization—which permits more readily repetitive methods of production—stimulates the invention of machines to do more rapid, more accurate, and more skilled work. It stimulates the increased use of conveyors and other mechanical means for reducing the amount of labor required for handling and transportation, all of which makes for the increase of the productivity of the individual and thus directly for the increase of the national wealth.

Standardization and mass production contribute to decreased cost, not only through the economies effected in the manufacture of the product but also in the economies effected—

1. In calculations and designs.
2. In the preparation of drawings and specifications.
3. In making propositions in response to requests from customers.
4. In selling costs.

(1) Economies are effected in design largely through savings in time of engineers by the elimination of odd types and designs, thus freeing the engineers for other work. Standardization of circuit voltage and periodicity and of permissible limits of variation of these in service, permit the manufacturer to reduce the number of varieties of machines. Furthermore, by standardized working limits, such as dielectric strength and temperature rise and other characteristics such as regulations, stalling load, starting torque, etc., the engineer can more quickly complete the engineering work on a given design by reason of his knowledge of the results which are usually obtained by working to a single standard and by the familiarity and facility he has attained through the working out of many similar designs. If he has different limits to work to in different cases obviously he must employ more variables in his calculations.

As an example consider that an engineer has been accustomed to designing a given kind of electrical machine for a high potential insulation test of a given severity and that suddenly he must design a similar machine of the same rating but for a higher insulation test. He must employ more space for insulation (granting that a better kind of insulation cannot be obtained) and this will leave him less space for iron and copper. Immediately his whole design must be changed.

(2) The standardization of material and parts and the reduction of number of varieties, leads to less and simpler drawings and specifications so that a given staff of engineers and draftsmen can deal with a much larger volume of business.

(3) By the standardization of certain requirements, the buyer and seller become accustomed to specifying machines on these bases; useless or relatively unimportant tests are less likely to be demanded by the buyer; printed specification forms may be provided which simplify the labor of making up a specification and knowledge and familiarity with similar cases (based on the same standards) enable an estimate of cost to be made more quickly and easily. These, and many other considerations which will suggest themselves to any one investigating the matter, serve to reduce the cost of the preparation of specifications and making of tenders.

(4) Standardization makes cataloging possible. The greater the degree of standardization, the greater is the simplicity and the usefulness of the catalogue. Information which can be brought to the customer through the medium of the catalogue and handbook requires less effort on the part of the sales force; or, conversely, a given sales force can deal with a larger volume of business. A salesman to be fully informed needs to carry less in his head, consequently he can handle more work in one special subject or a greater number of special subjects than would otherwise be the case.

With the less variety of sizes of a product, the less is the value of the stock which has to be carried by various distributors and products.

Thus, costs, associated with engineers, draftsmen, salesmen, and some components of overhead, are, with modern mass production, materially decreased by standardization. This is in addition to the decrease in the strictly production costs. All together these combine to increase the growth and influence of the electrical industry which is primarily our concern.

The first standards for electrical machinery generally followed by the American electrical industry were those prepared by the original Institute committee, and adopted by the Institute in June 1899. A review of the proceedings of each successive Committee of Standardization since that date indicates that all of these committees were fully aware of the flux and changes that were taking place in industry, and while these committees consisted entirely of engineers—who, by nature and training, loved law and order and who might be supposed, on that account, to be ultra-conservative and possibly timid—they were, however, endowed with the spirit of progress which collectively turned their hopes and aspirations to the future; they saw the world of industry not as a still tableau but as a moving picture and in consequence the accomplishments of each year were progressively better than the year before.

They recognized that no standard could be final—since science was continually advancing and more effective equipment was steadily being introduced into industry—but they also recognized that changes must not be made so frequently as to unduly disturb the industry and only when a serviceable gain justified the change.

These committees to date have taken the initiative in the formulation of all electrical standards of America, and their work has been recognized as being authoritative throughout the entire world. Their procedure and their resulting standards during this period of more than a quarter of a century have been acceptable to the manufacturing and consuming interests, as well as to the general public. The industry has learned to value and to depend upon the A. I. E. E. Standards in commercial transactions covering matters of interest to all sections of the electrical industry. There has been no attempt to dictate to the industry but standards on any particular line have been introduced only when it is clear that all interested agree that the step is wise and desirable.

In the Institute Standards Committee, or in its subcommittees, the manufacturer and purchaser and the general interests came together and developed the required standards in a way which has been generally satisfactory in the past to all the interested sections of the industry. The electrical standards so issued always have been identified with the name of the Institute. It is well known that the Institute as an organization has no interest other than one of public service, which duty the Institute has always performed at its own expense. The Institute in performing this service, although voluntary, has assumed obligations during the past 28 years to the electrical industry and the public which would make it now embarrassing, if not impossible, to discontinue the present practise or lessen its responsibilities until a more simple and direct method has been devised and demonstrated.

For fear that some who have not had the opportunity to study in an intimate way the subject of Standards and Standardization, may not understand what the terms mean—I quote from my February 7th address:

"In this country and in Great Britain the term 'standardization' has grown to mean, in the minds of engineers, not only a simplification in the number of types and sizes and the securing of interchangeability, but also the laying down of performance rules or codes for all types of apparatus, including measuring instruments, prime movers, generators, transformers, and motors. Thus broadly the term 'Standard' in addition to being a measure of quality of standards of comparison, means a common unified practise, method, or dimension, which it is to the interest of industry and the community to adopt Back of any policy of standardization is primarily the purpose to furnish the public a better article or to render it a better service at a lower cost."

"In 1916 the need for a National Clearing House for engineering standards became apparent, in order to prevent duplication in standardization work and in promulgation of conflicting standards. To formulate a method of cooperation, a special joint committee—made up of representatives from the American Institute of Electrical Engineers, American Institute of Mining Engineers, American Society of Civil Engineers, American Society of Mechanical Engineers, and American Society for Testing of Materials—held its first meeting January 17, 1917. The result of this meeting and subsequent meetings was the organization of the American Engineering Standards Committee. This Committee initially consisted of representatives of these five institutions. Shortly after its organization, government representatives were admitted; and in 1919 the Constitution was broadened to permit the representation from other national bodies. The Committee now includes representatives from seven departments of the Federal Government, nine national engineering societies, and nineteen national industrial associations.

"The American Engineering Standards Committee, as at present organized, is a coordinating committee, and not a standards-making body. All standards are to be formulated and published by the respective societies, making the standard a function of great value and scope to industry. This intention is clearly expressed in the Constitution, and the American Engineering Standards Committee is primarily an administrative and policy-forming committee.

"As stated by its Secretary, perhaps the most important accomplishment of the American Engineering Standards Committee has been the actual launching of the work, setting up machinery, and securing the official cooperation of some three hundred national organizations, that is the fundamental job of breaking ground. Ninety-seven standards have been approved for the engineering and building trade—ten have to do with the electrical industry. In my opinion this is a very excellent record of accomplishments to date. The Secretary also states that everyone who has examined the work before the American Engineering Standards Committee, agrees that the whole movement of making American standards is being seriously crippled by the lack of adequate financial support. The total annual budget is $58,000. The American Institute of Electrical Engineers as such contributes to the American Engineering Standards Committee $1500 annually. Due to legal restrictions the government departments are unable to pay dues, and a special provision is made exempting them from such payments.

"The Secretary also expresses the opinion that inertia, lack of interest and understanding of the standardization method as a whole, and of its economic relations to their business on the part of executives and industrial groups, has been one of the greatest difficulties encountered in the successful accomplishment of the committee's work. It is now proposed, by the process of amendment, to make a material and fundamental change in the Constitution of the American Engineering Standards Committee This can only be done by the unanimous consent, notwithstanding the general provision in the Constitution, providing for its amendment by a lesser vote. Such provisions only apply to incidental amendments made to carry out the purpose of the organization, and not to fundamental changes—any amendement that aims to convert the American Engineering Standards Committee into a standards-making body, or into a body that would interfere in the autonomy, in standardization work, of the representative societies would be unconstitutional.

"On February 9, 1926, during Dr. Pupin's administration, the Board of Directors authorized a brief statement of its policy to its representatives on committees, or on joint bodies, dealing with the formulation of standards;

1. To continue to develop, publish, and maintain in the name of the Institute electrical standards as it has done for the past 25 years.

2. That in doing this work the Institute will continue as it has in the past to avail itself to the fullest degree of the assistance of others—both individuals and organizations—with a view to serving the interests of all who may be properly concerned in the work.

3. That Standards, after having been developed by the Institute in accordance with 1 and 2, and adopted by the Board of Directors as Institute Standards, will be presented to the American Engineering Standards Committee for approval by it as American Standards when, in the opinion of the Institute, such a step is proper.

4. That such presentation to the American Engineering Standards Committee for their consideration for approval as American Standard will be done in full conformity with the Constitution, By-laws, and Rules of Procedure of the American Engineering Standards Committee, which Committee the Institute was instrumental in initiating and has continued to and does now endorse and support.

5. That when and if Standards of the A. I. E. E. have been further advanced to the stage of being designated as 'Approved as American Standard by the American Engineering Standards Committee,' they shall continue to be printed as standards of the A. I. E. E. with a statement of approval by the American Engineering Standards Committee added to the title page of each particular standard."

This statement I understand to mean that the American Institute of Electrical Engineers is in sympathy with the American Engineering Standards Committee as it is now organized, but that any changes affecting the fundamental character of the committee may not be acceptable to it.

Something over a year ago, a movement was undertaken to form an International Standards Association, the proposed organization to have a national committee in each country. In America, the national committee was to be the American Engineering Standards Committee. As it has been stated heretofore, its (A. E. S. C.) principal constitutional object is to supervise standardization work, but it is expressly stated in its constitution that it shall not formulate standards. There is, however, a clause in its constitution which states that one of the objects of the American Engineering Standards Committee shall be "to act as the authoritative channel of cooperation in international engineering standardization."

The Institute has subscribed to the American Engineering Standards Committee constitution. It is also a member of the United States National Committee of the International Electrotechnical Commission, which has been and is at present the body through which the electrical industry of America is conducting its international standardization work. The Institute is thus faced with a conflict of obligations. The most reasonable course is for it to go before the American Engineering Standards Committee with a frank statement of the case, and ask the American Engineering Standards Committee for its support for the course which the Institute considers to be in the best interest of the electrical industry of this country, at the present time. Under the procedure of the International Electrotechnical Commission much of value has been accomplished in establishing international electrical engineering standards. This field alone is a very large one, and it would appear unwise to abandon the present successful plan for the untried plan of an International Standards Association which does not appear to be based on such sound fundamental principles. The success of the International Electrotechnical Commission has been so considerable that it could well be duplicated in other engineering fields such as mechanical, civil, and mining fields, etc. If this were done, then we should, in addition to the International Electrotechnical Commission have an International Mechanical Commission, an International Civil Commission, etc. At some future time it might become desirable to tie these international organizations into an international technical commission, but it would at this time be premature to try to decide whether this last step would be desirable and when it should be undertaken.

It would appear that if the electrical industry is a unit in desiring to continue the International Electrotechnical Commission, and in believing that its interests would be seriously endangered by going over to a new plan, representations to this effect should be made in proper form and on the proper occasion to the American Engineering Standards Committee. While the International Electrotechnical Commission recognizes that its field is electrical, and that the desired international accomplishments in that field alone constitute an enormous task, it is also recognized that its 20 years of experience constitute a considerable asset. The International Electrotechnical Commission has never taken advantage of this asset in a selfish way. In cases where in other fields of engineering, it should be desired to make use of the International Electrotechnical Commission's organization and experience, either temporarily or permanently, for work in other lines, the International Electrotechnical Commission is prepared, as in certain cases in the past, to offer its facilities and to adapt them to include the added work. In such cases the International Electrotechnical Commission could arrange with the organization in whose field the work belongs for additional representatives from that organization's membership. If at a later time the coöperating organization, through the establishment of its own international organization, or from any other reason for change of policy, should decide to discontinue the arrangement, this without doubt should and would meet with the hearty agreement of the International Electrotechnical Commission.

Three open problems on standardization thus confront the Institute and their importance justifies a prompt solution but not a hasty one.

First: The internal routine to be followed through its Committee on Standards for the handling of standards and standardization should be revised. This has to do primarily with the A. I. E. E. Standards.

Second: The present and future status of the American Engineering Standards Committee should be determined.

Third: The relation of the American Institute of Electrical Engineers to the United States National Committee of the International Electrotechnical Commission.

These I repeat are important and vital problems and need, for the best interest of the Institute and the industry, a prompt solution.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1927

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Forty-third Annual Report, for the fiscal year ending April 30, 1927. A general balance sheet showing the condition of the Institute's finances on April 30, 1927, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held seven meetings during the year; six in New York and one at White Sulphur Springs, W. Va. The Executive Committee acted upon various matters during intervals between Board Meetings.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees, and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

President's Visits.—President Chesney has attended the three National Conventions of the Institute held during the year and in addition visited many of the Sections. The following is a list of places visited: White Sulphur Springs, W. Va., (Summer Convention); Salt Lake City, Utah, (Pacific Coast Convention); New York, N. Y. (Winter Convention); Niagara Falls, N. Y., (Regional meeting); New York, N. Y., (Regional meeting); Los Angeles, Cal.; San Francisco, Cal.; Portland, Ore.; Seattle, Wash.; Spokane, Wash.; Vancouver, B. C.; Regina, Saskatchewan; Pittsfield, Mass.; Sharon, Pa.; Syracuse, N. Y.; Springfield, Mass.; Bridgeport, Conn.; Chicago, Ill.; Ithaca, N. Y.; Toronto, Ont.

Other meetings that President Chesney is scheduled to attend will be held in Cleveland, Ohio; Ft. Wayne, Ind.; Pittsfield, Mass. (Regional meeting); and Detroit, Mich., (Summer Convention).

Meetings.—Three national conventions of the Institute were held during the year, namely, the Summer, Pacific Coast, and Winter. The annual business meeting was held in May. Regional meetings under the auspices of the geographical districts were held in Madison, Wis., District No. 5; Niagara Falls, N. Y., District No. 1; New York, N. Y., District No. 3; Kansas City, Mo., District No. 7; Bethlehem, Pa., District No. 2. A regional meeting is also scheduled for Pittsfield, Mass., District No. 1, in May.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 21, 1926. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1926, was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1926.

Following the business meeting a talk was given on "A High-Power Laboratory for Testing Oil Circuit Breakers and other Apparatus" by W. R. Woodward of the Westinghouse Elec. & Mfg. Co. This part of the meeting was held under the auspices of the New York Section. Attendance about 600.

Summer Convention.—The Forty-second Annual Summer Convention was held at White Sulphur Springs, W. Va., June 21-25, 1926. Four technical sessions were held at which thirteen papers were presented. In addition two special sessions were devoted to the presentation of technical committee reports. The annual conference of the Sections Committee was held on Monday, June 21; forty-three Sections were represented. The entertainment program included golf and tennis tournaments, sightseeing, and special features. Attendance, about 350 members and guests.

Pacific Coast Convention.—The Fifteenth Pacific Coast Convention was held at Salt Lake City, Utah, September 6 to 9, 1926. Five technical sessions were held at which nineteen papers were presented. Attendance about 250.

Winter Convention.—The Fifteenth Winter Convention was held in New York, N. Y., February 7-11, 1927. Thirty-five technical papers were presented at eight sessions. Numerous inspection trips, a smoker, and a dinner-dance were also scheduled. Attendance was over 1900.

Regional Meetings.—During the year five Regional meetings were held. The first in District No. 5 at Madison, Wis., May 6 and 7, 1926, at which eight papers were presented during three sessions, attendance about 180; second, at Niagara Falls, N. Y. in District No. 1, May 26-28, 1926, twenty-three papers were presented at five sessions, attendance over 580; third, New York, N. Y., District No. 3, November 11 and 12, 1926, eleven papers were presented at three sessions, in addition the program also included inspection trips, a lecture, and a demonstration of the Vitaphone, registration over 700; fourth, Kansas City, Mo., District No. 7, March 17-18, 1927, twelve papers were presented at four sessions; student conference, inspection trips, attendance approximately 225; fifth, Bethlehem. Pa., District No. 2, April 21, 22, and 23, 1927, ten papers were presented at four sessions, inspection trips, lecture, attendance about 400.

Sections.—The increased activity of the Sections mentioned last year has been well-maintained. The cooperation and affiliation of Sections with Sections of other national engineering societies and with local engineering societies have remained prominent subjects of discussion. Definite advances have been made in the development of plans to bring about closer contact between engineers and the general public. The growing interest in this subject is well illustrated by the report of the 1926 Sections Committee Conferences, and by the records of several Sections which have planned some of their meetings to attract non-engineers. The increasing number and success of the Regional Meetings (see listing under Regional Meetings) are additional evidences of Section activity. The Sections have given greater attention to cooperation with neighboring Student Branches, with very good results. Four Sections have sponsored Student Conventions, and a considerable number have held joint meetings with Branches. Several have devoted regular meetings to Student programs, and in other cases joint programs have been arranged. During the past year, a Section was organized at Louisville, Kentucky.

Student Activities.—During the past year, provisions were made for the organization in each District of a Committee on Student Activities consisting of the Vice-President, District Secretary, and the Counselors of all Branches in the District, and for the holding, under its auspices, of an annual conference on Student Activities in order that the activities of all Branches in the District might be more effectively coordinated. Such conferences have been held by the Committees of seven Districts, and have resulted in much helpful discussion of all phases of Student activities. A considerable portion of the increase in Student Branch activity may be ascribed to these conferences and to the greater efforts of the Sections and the Counselors of the Branches to establish closer relations between Sections and Branches. Very successful Student Conventions, sponsored by Sections, have been held in four Districts, with approximately twenty Branches participating and an aggregate attendance of about 800. A separate department devoted to Student activities was inaugurated in the JOURNAL in January 1927. Upon recommendations of the Committee on Student Branches, the Board of Directors approved a statement of "Suggested By-Laws for Branches," and adopted a By-Law (59A) providing for the affiliation of established student engineering societies with the Institute. New Branches were authorized during the year at Louisiana State University, University of New Hampshire, Princeton University, Municipal University of Akron, Newark College of Engineering, University of Santa Clara, Mississippi Agricultural and Mechanical College, Duke University, and Northwestern University.

SECTION AND BRANCH STATISTICS

	For Fiscal Year Ending			
	April 30 1921	April 30 1923	April 30 1925	April 30 1927
SECTIONS				
Number of Sections.	42	46	49	52
Number of Section meetings held.....	303	344	386	431
Total Attendance....	37,823	46,672	49,029	60,708
BRANCHES				
Number of Branches.	65	68	82	95
Number of Branch meetings held.....	443	503	548	842
Total Attendance....	21,629	26,893	27,603	42,650

Meetings and Papers Committee.—During the past year, the Meetings and Papers Committee has followed practically the same policy as followed during the last three years in arranging technical programs for the national conventions and regional meetings.

Three national conventions have been held and five regional meetings. At all of the conventions and meetings there was a total attendance of over 4500; 144 technical papers were presented. Detailed information on these meetings is given in the accompanying tabulation.

One of the most note-worthy developments of the year was the growth in the number of regional meetings and the increased interest taken in them. These meetings seem to be fulfilling their purpose in a very desirable manner. They allow the attendance of a large number who could not attend a national convention of the Institute. Also, they allow the presentation in a particular territory of papers on subjects which are of special interest to members in the territory.

Most of the papers presented at the regional meetings are of as high quality as those presented at national conventions. The committee, however, has followed the policy of encouraging the regional committees to include in their programs certain papers which are of particular interest locally, although they might not be selected for a national convention.

In addition to arranging for programs for the meetings held during the past fiscal year, the programs also for three future meetings have been practically fixed. For the regional meeting at Pittsfield, Massachusetts, May 25-28, 1927, and the Summer Convention in Detroit, June 20-24, 1927, the programs are complete. For the Pacific Coast Convention in Del Monte, Sept. 13-16, 1927, due very largely to the foresighted efforts of the Pacific Coast Convention committee, that program also has been rather definitely arranged.

Publication Committee.—No change in the general arrangements governing Institute publications has been made by this year's committee as the policies adopted several years ago, after many months of careful consideration, seem to have fulfilled the requirements of the membership very completely. This is attested by the fact that the JOURNAL has become generally recognized, both at home and abroad, as the leading publication in its class.

The general scheme of publication of Institute papers is (1) publication in pamphlet form in full, these copies being available for use at the meeting where presented; (2) publication either in full or abridged, according to length, in the JOURNAL; (3) publication in full in the annual TRANSACTIONS. A few papers of merely transitory interest have been omitted from the TRANSACTIONS each year. These papers amount to less than six per cent of the total number printed and they are referred to in the TRANSACTIONS INDEX.

The TRANSACTIONS for 1926 will be completed early next Fall and will contain about 1350 pages, or 150 pages more than the previous volume. Six-hundred and eight pages have been printed to date, and 300 pages more are ready to print as soon as the current work for coming conventions permits.

Standards.—The Standards Committee has continued actively the revision of the Institute standards. There are now 23 standards available in the revised sectional or pamphlet form, several of which have themselves been revised and reprinted. The ease with which revisions can be made shows the wisdom of the change in form for the standards, adopted several years ago. Specially designed loose leaf binders can be supplied at a nominal price.

The Spanish translation of the standards has gone forward with gratifying speed. The translation of 17 of the standards has been completed and 11 have been published by the Bureau of Foreign and Domestic Commerce. By July 1 it is expected that all will be issued. The Spanish edition is printed in the same style and in the same size as the English edition. The translation has been received with interest by engineers in Spanish-speaking countries of South America. The Institute is very much indebted to the Bureau of Foreign and Domestic Commerce for the excellent manner in which the Spanish text has been published, and for the close cooperation that exists between the Bureau and the Standards Committee. American electrical manufacturers will no doubt distribute this Spanish edition widely in Spanish-speaking countries.

Six standards have been approved as American Standard by the American Engineering Standards Committee and others are before the Committee for consideration. The Institute is sole sponsor for 9 projects and joint sponsor for 9 more. These sponsorships and joint sponsorships are handled largely through the Standards Committee. Representatives on 26 additional Sectional Committees for which the Institute is not sponsor have also been appointed.

There has been during the year an increasing cooperation between the Standards Committee and the technical committees of the Institute. The chairman of each technical committee, or a member of the committee designated by the chairman as a "contact officer" is a member of the Standards Committee. Several standards have been formulated by the technical committees and accepted by the Standards Committee and in another case a subcommittee of a technical committee has been made the "working committee" of the Standards Committee on a project.

The Standards Committee has maintained close contact with the United States National Committee of the International Electrotechnical Commission, and at practically every meeting has received full reports on the work of the U. S. National Committee. This has been a very useful coordination of international problems in electrical standardization with the regular work of the committee.

U. S. National Committee of the I. E. C.—The scope of the work of the U. S. National Committee of the International Electrotechnical Commission has increased during the past year in several respects. One important increase in responsibility results from the adoption by the I. E. C. of a policy of decentralization of the secretarial work under which policy, various National Committees are asked to assume the Secretariat for one or more subjects. The U. S. N. C. has accepted the responsibility of the Secretariats on Nomenclature, on Prime Movers, and on the Rating of Rivers. This work involves stimulating and guiding the work of the International Advisory Committees on these subjects in the interim between meetings of the I. E. C.

A study of the problem of Rating of Rivers is a subject recently undertaken by the I. E. C. and illustrates the broadening scope of work of the Commission. Its importance is illustrated by the fact that the methods now used in different countries for rating the flow of rivers from the standpoint of their usefulness in the production of power, do not permit of comparative results.

Through its twelve groups of advisors the U. S. N. C. is kept closely in contact with the standardizing work of the various American organizations bearing on the subjects under consideration by the I. E. C., and has cooperated with these various organizations in the preparation of material to be presented as recommendations from America at the next meeting of the I. E. C. This meeting is to be held in September at Bellagio and Rome.

U. S. National Committee, International Commission on Illumination.—The United States Committee of the Commission is composed of representatives of various organizations interested in illumination, the Institute having three representatives on the committee. Following the plenary session in Geneva, July 1924, the general work of the committee has been proceeding. Measures are now being taken to prepare for the next session which probably will be held in the United States in 1928

Committee on Safety Codes.—The Committee on

Safety Codes investigates all matters relating to the formulation of rules for the protection of persons and property against fire, accidents, and other hazards in connection with electrical installations and equipments and to confer with similar committees of other bodies.

A subcommittee on procedure was instructed to include in its recommended procedure that the main Committee should make nominations to the President of representatives suitable for appointment on national safety code committees; that members of the A. I. E. E. now or hereafter, on safety code work, should submit drafts of their work to the Committee for review; that attention be given to having safety properly developed at the Institute meetings and in the Institute publications; that authors having papers where safety is involved be asked to emphasize that phase; that so far as possible helpful cooperation be set up with other engineering organizations.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the Annual Convention and printed in the JOURNAL.

Membership.—During the past year the National Membership Committee has carried on the usual activities for the purpose of obtaining the desired increase in membership. This work has been conducted, as heretofore, mainly through the organization of the various Membership Committees of the Institute Sections and a report of progress of the National Committee prepared on April 30, 1927 indicates that 1951 applications for membership have been received during the year.

The following table gives the number of new members added to the roll during the year and also the number of members whose affiliations have terminated through resignation, death, or delinquency in dues. The revised membership total as of May 1, 1927 is 18,344.

	Honorary	Fellow	Member	Associate	Total
Membership April 30, 1926.	4	625	2,623	14.906	18,158
Additions:					
Transferred...		33	307		
New Members Qualified...		2	100	1,635	
Reinstated....			7	71	
Total.........	4	660	3,037	16.612	20,312
Deductions:					
Died.........		12	15	68	
Resigned......		2	21	543	
Transferred...			25	315	
Dropped......		2	45	921	
Membership, April 30, 1927.	4	644	2,931	14,765	18,344

Net increase in Membership during the year........................186

Deaths.—The following deaths have occurred during the year.

Fellows: H. Fleetwood Albright, Adolphe A. Dion, Edward B. Ellicott, Dudley Farrand, Carl Hering, Alexander E. Keith, Edward N. Lake, Thomas D. Lockwood, James W. McCrosky, Charles E. Scribner, Ernest Thurnauer, C. Griffith Young.

Members: William Y. Avery, LeRoy Clark, Vahram Y. Davoud, Washington Devereux, George E. Luke, Leo Lustig, Shiras Morris, Russell Robb, Otto L. J. Schier, George D. Shepardson, Jesse Merrick Smith, Oberlin Smith, Charles C. Stutz, Victor H. Todd, C. Reginald Van Trump.

Associates: Charles H. Bedell, John I. Beggs, Albert P. Boeri, Alva A. Bonney, Anson W. Burchard, Paul Busher, Lawrence W. Cady, Charles A. Chapman, Edward L. Clark, Charles A. Coffin, Charles S. Cook, Richard J. Crandall, George S. Davis, Howard S. Davis, Samuel O. Edmonds, Olin J. Emmons, Louis M. Finkelstein, John J. Flather, William H. Forde, Frank E. Goodnow, Frank W. Graham, Walter Halfin, Arthur R. Henry, Albert J. Hoch, Harold O. Holte, Charles W. Holtzer, Everett D. Hunter, Charles R. Huntley, Arthur R. Jealous, Arthur Keller, Francis M. Kenny, William Korff, John V. Lannon, Don C. G. Linnell, Leo B. Masten, Charles C. Mathis, Joseph B. McCall, Henry K. McIntyre, Frederick McKeever, Walter R. McLeod, Richard N. Olds, Willis E. Osborne, George H. Pearson, Virgil Poston, John C. Potter, Lawrence W. Powers, Joseph H. Procter, William A. Rankin, Isidor M. Reguenga, Clarence E. Reid, Antonine R. Rivet, Frederick W. Roth, Earl A Schaefer, Rudolf Schmolck, Charles W. Shaifer, Frank C. Smallpiece, Leland L. Summers, Clark A. Sutton, Svatopluk Sychra, Hazen G. Tyler, Benjamin T. Viall, Edwin M. Walker, John M. Walshe, James C. Webster, Edgar M. Wilkins, Thomas W. Wilkinson, Fred W. Wilson, Ray D. Wilson.

Board of Examiners.—The Board of Examiners during the year held eleven meetings, averaging about three and one-quarter hours. It considered and referred to the Board of Directors a total of 4080 applications for admission or transfer to the higher grades.

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate......	1729	
Not recommended......................	16	1745
Recommended for grade of Member.......	93	
Not recommended for admission to this grade..............................	41	134
Recommended for grade of Fellow........	2	
Not recommended for admission to this grade..............................	5	7
Recommended for enrolment as Students..		1834

APPLICATIONS FOR TRANSFER

Recommended for grade of Member.......	293	
Not recommended for transfer to this grade	32	325
Recommended for grade of Fellow.........	30	
Not recommended for transfer to this grade	5	35
Total number of applications considered...		4080

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute three scholarships in electrical engineering. In consequence, the Institute is now authorized to award a scholarship each year so that there may be one man in each class holding a scholarship on the nomination of the Institute; these scholarships will continue until further notice. Each scholarship pays $350 toward the annual tuition, and reappointment for completion of course is conditioned upon the maintenance of good standing. The award in 1926 was made to E. R. Riethmiller of the University of Michigan.

Institute Prizes.—A report of a special committee on prizes and regulations regarding them, was received and adopted by the Board, February 9, 1926. The committee suggested the yearly award of four National Prizes of $100 each and a suitable certificate, as follows: (1) Best Paper Prize, (2) First Paper Prize, (3) Best Regional Paper Prize, (4) Best Branch Paper Prize.

The following Regional Prizes may also be awarded yearly in each of the ten Geographical Districts, each prize to consist of $25 and a suitable certificate: (1) Best Paper Prize, (2) First Paper Prize, (3) Best Branch Paper Prize. For complete details and conditions of award, see the February 1927 JOURNAL.

The following National Prizes were awarded during the past year, under the conditions in force prior to the adoption of the above-mentioned regulations:

First Paper Prize for the Year 1925. To R. W. Wieseman, Schenectady, N. Y., for his paper, "A Two-Speed Salient-Pole Synchronous Motor."

Transmission Prize for the Year 1925. To J. H. Cox and J. W. Legg, East Pittsburgh, Pa., for their paper, "The Klydonograph and its Application to Surge Investigation."

Various Regional Prizes have been awarded as announced from time to time in the issues of the monthly JOURNAL.

The National awards for papers presented during the year 1926 have been made and the prizes will be presented at the annual Summer Convention, in Detroit, in June, as follows:

Best Paper Prize for the Year 1926. To F. M. Farmer, New York, for his paper, "Tests of Paper-Insulated High-Tension Cable."

First Paper Prize for the Year 1926. To Othmar K. Marti, Camden, N. J., for his paper, "Steel Enclosed Power Rectifiers."

Best Regional Paper Prize for the Year 1926. To F. M. Farmer, New York, for his paper, "Tests of Paper-Insulated High-Tension Cable."

Best Branch Paper Prize for the Year 1926. To R. A. Schaefer, of Marquette University Branch, for his paper, "A Study of Transverse Armature Reaction in Synchronous Machines by Means of a Second Machine with an Adjustable Stator."

Edison Medal.—The Edison Medal awarded last year to Dr. Harris J. Ryan, Stanford University, California, "for his contributions to the science and the art of high-tension transmission of power," was presented to Dr. Ryan at the Pacific Coast Convention, Salt Lake City, Utah, on September 8, 1926.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers, awarded the 23rd medal to Elmer Ambrose Sperry of New York, "for the development of the gyro-compass and the application of the gyroscope to the stabilization of ships and aeroplanes." The medal was presented at New York, at the annual meeting of the American Society of Mechanical Engineers, December 7, 1926.

Lamme Gold Medal.—A bequest was made by the late B. G. Lamme, to cover the cost of an annual award by the Institute of a gold medal, to a member who has shown meritorious achievement in the development of electrical apparatus. The conditions governing the award of the Lamme Gold Medal are now being prepared by the Committee on Award of Institute Prizes.

Commission of Washington Award.—The Washington Award for 1926 was voted to John Watson Alvord, Consulting Engineer, Chicago, Ill.

The award is made annually "to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good," by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E., and the A. I. E. E.

Employment Service.—The employment service which the Institute has maintained for many years is now conducted as a cooperative bureau in conjunction with a similar service maintained by the National Societies of Civil, Mining, and Mechanical engineers under the title, "Engineering Societies Employment Service." In addition to the main office in the Engineering Societies Building, New York, offices have been opened in Chicago in cooperation with the Western Society of Engineers and in San Francisco under similar arrangements with the California Section, American Chemical Society, and Engineers Club of San Francisco. Arrangements have been completed to open a branch office in Denver, Colorado in cooperation with the Colorado Engineering Society. It is hoped to continue this development from year to year as conditions warrant. The service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past it consists principally in acting as a medium for bringing together the employer and the employee. In addition to the publication of the "Employment Service Bulletin" in the monthly JOURNALS, weekly subscription bulletins are issued for employers and those seeking positions.

American Engineering Council.—This organization is the instrument of the constituent engineering societies through which the engineering profession may contribute toward the solution of the social, economic, and political problems of the day.

The annual meeting of the American Engineering Council was held in Washington, D. C., in January 1927. Meetings of the Administrative Board have been held during the year. The activities of Council have covered an extensive field, the following being a partial summary: study of accidents and production; forest conservation; represented on National Board for Jurisdictional Awards; has endorsed the development of the Patent office and endeavored to secure a new building; Public Works Bill written to conform with recommendations of Councils' committee; report of Committee on Radio Broadcasting presented to Federal Radio Commission; preliminary study of stream pollution; has Committee on Street Signs, Signals, and Markings with local committees in over one hundred cities; etc.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechancial, and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1927 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining, and Mechancial Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc. A lending department is also maintained.

A copy of the annual report of the Engineering Societies Library covering the calender year 1926, may be obtained by applying to Institute headquarters.

Engineering Foundation.—Engineering Foundation is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." The fund has been generously increased through the gifts of Edward D. Adams and others, and also through bequest under the will of the late Henry R. Towne. It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a Department of United Engineering Society.

The Foundation has made appropriations for various research projects and has cooperated in others.

The annual report of the Foundation is available in printed form.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has accepted sponsorship and appointed representatives upon a number of new Sectional Committees of American Engineering Standards Committee. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report follows:

Respectfully submitted for the Board of Directors.

F. L. HUTCHINSON,

National Secretary.

New York, May 20, 1927

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
OFFICES IN THE PRINCIPAL CITIES OF
THE UNITED STATES OF AMERICA
AND IN
LONDON, PARIS, BERLIN, SHANGHAI,
MONTREAL, HAVANA, MEXICO CITY

37 WEST 39TH STREET
NEW YORK

May 16, 1927.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have made a general audit of your accounts for the year ended April 30, 1927, and submit the following exhibits and schedule:

Exhibit
- A—Balance Sheet, April 30, 1927.
  - Schedule
    - 1—Reserve Capital Fund—Securities.
- B—Summary of Income and Profit and Loss for the Year Ended April 30, 1927.

WE HEREBY CERTIFY that in our opinion Exhibits A and B correctly set forth the financial condition of the Institute at April 30, 1927, and the results of its operations for the year ended that date.

Yours truly,
HASKINS & SELLS

Exhibit A.

Balance Sheet, April 30, 1927

ASSETS

Real Estate:			
One-fourth interest in United Engineering Society's Land, Building, and Building Equipment, 25 to 33 West 39th Street (Depreciation carried on books of United Engineering Society)			$493,352.60
Equipment:			
Library—Volumes and Fixtures		$40,607.84	
Works of Art, Paintings, etc		3,001.35	
Office Furniture and Fixtures	$20,917.93		
Less Reserve for Depreciation (including $5,556.81 funded)	10,445.97	10,471.96	
Total Equipment			54,081.15
Working Assets:			
"Transactions" etc		$ 3,821.50	
Paper and Cover Paper		4,254.65	
Badges		1,504.30	
Total Working Assets			9,580.45
Current Assets:			
Cash		$24,991.26	
Accounts Receivable:			
Members—For Dues		25,919.93	
Advertisers		1,986.25	
Miscellaneous		1,584.91	
Accrued Interest on Investments		1,444.13	
Total Current Assets			55,926.48
Funds:			
Reserve Capital Fund:			
Securities—Schedule 1	$105,520.63		
Cash	613.07	$106,133.70	
Life Membership Fund:			
Cash	$ 5,056.03		
Chicago, Burlington & Quincy Railroad Company 4% Bonds, 1958, Registered, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	9,958.11	
International Electrical Congress of St. Louis Library Fund:			
Cash	$ 853.64		
New York City 4½% Corporate Stock, 1957, par Value $2,000.00	2,204.05		
New York Telephone Company 4½% Bond, 1939, Registered, Par Value $1,000.00	878.75		
Accrued Interest	67.50	4,003.94	
Mailloux Fund:			
New York Telephone Company 4½% Bond, 1939, Registered, Par Value $1,000.00	$ 1,000.00		
Accrued Interest	22.50	1,022.50	
Midwinter Convention Fund—Cash		106.53	
Lamme Medal Fund:			
Cash	$ 1,034.14		
Baltimore and Ohio Railroad Co. 6% Refunding and General Mortgage Series C Bond, 1995, Par Value $4,000.00	4,330.00		
Accrued Interest	16.67	5,380.81	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 544.31		
Cleveland Union Terminals Co. 5% Sinking Fund Series B Gold Bonds, 1973, Registered, Par Value $5,000.00	5,012.50	5,556.81	
Total Funds			132,162.40
Total			$745,103.08

LIABILITIES

Current Liabilities:		
Accounts Payable	$12,227.44	
Dues Received in Advance	2,842.19	
Entrance Fees and Dues Advanced by Applicants for Membership	633.85	
Subscriptions for "Transactions" received in Advance	38.00	
Total Current Liabilities		$ 15,741.48
Fund Reserves (Not Including Depreciation Reserves):		
Reserve Capital Fund	$106,133.70	
Life Membership Fund	9,958.11	
International Electrical Congress of St. Louis—Library Fund	4,003.94	
Mailloux Fund	1,022.50	
Midwinter Convention Fund	106.53	
Lamme Medal Fund	5,380.81	
Total Fund Reserves (Not Including Depreciation Reserves)		$126,605.59
Surplus, Per Exhibit "B"		602,756.01
Total		$745,103.08

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT & LOSS

FOR THE YEAR ENDED APRIL 30, 1927

EXHIBIT B.

INCOME:			
Dues		*$224,565.02	
Students' Dues		10,424.50	
Entrance Fees		17,215.50	
Transfer Fees		1,740.00	
Advertising		73,980.78	
Journal Subscriptions		9,088.83	
"Transactions" Subscriptions		8,887.00	
Miscellaneous Sales		10,195.49	
Badges Sold	$ 4,724.00		
Less Cost	3,114.35	1,609.65	
Interest on Securities in Reserve Fund		5,119.17	
Interest on Bank Balances		1,320.67	
Total			$364,146.61
EXPENSES:			
Publications:			
Journal	$103,370.22		
"Transactions"	24,531.76		
Year Book	7,623.60		
Miscellaneous	4,933.69	$140,459.27	
Institute Meetings		21,832.44	
Administrative Expenses		54,664.83	
Sections		30,443.70	
Membership		8,502.34	
Standards		4,750.53	
Finance		290.18	
Headquarters		1,586.76	
Code Committee		60.00	
Technical Committees		85.14	
Edison Medal		178.36	
Engineering Education		2,056.47	
Geographical Districts:			
Traveling Expense:			
Executive Committees	$ 2,424.86		
Vice-Presidents	372.02		
Speaker's Bureau	65.69		
Best Paper Prize	176.50		
First Paper Prize	201.50	3,240.57	
Branches:			
Traveling Expense—Counsellors	$ 3,746.36		
Salaries	583.25		
Stationery and Printing	525.65		
Best Paper Prizes	175.00	5,030.26	
American Engineering Standards Committee		1,500.00	
International Electrotechnical Commission		782.28	
United States National Committee of International Commission on Illumination		300.00	
President's Appropriation		955.66	
Board of Directors—Mileage		3,085.20	
National Nominating Committee, Mileage		1,172.86	
Institute Representatives—Traveling Expenses		246.91	
Honorary Secretary		4,000.00	
John Fritz Medal Award		375.38	
Engineering Societies Library—Maintenance		8,000.00	
United Engineering Society Assessment		4,860.00	
American Engineering Council		17,451.50	
Engineering Societies Employment Service		1,635.00	
International Annual Tables		100.00	
Best Paper Prizes—1926		187.50	
First Paper Prizes—1926		187.50	
Total			318,020.64
NET INCOME—(FORWARD)			$ 46,125,97

NET INCOME—FORWARD		$46,125.97
PROFIT AND LOSS CREDITS:		
Adjustment of Inventories—Library Volumes and Fixtures	$ 62.25	
Increase in Equity of United Engineering Society's Land, Building, and Equipment	1,710.24	1,772.40
GROSS SURPLUS FOR THE YEAR		$ 47,898.46
PROFIT AND LOSS CHARGES:		
Adjustment of Inventory—"Transactions"	$ 1,288.00	
Uncollectible Dues Written Off	8,259.91	
Furniture and Fittings Scrapped—Loss	327.14	
Provision for Depreciation of Furniture and Fixtures	799.93	
Total		10,674.98
SURPLUS FOR THE YEAR		$ 37,223.48
SURPLUS, MAY 1, 1926	$588,967.53	
Less Transferred to Capital Fund Reserve in Accordance with Resolution of Board of Directors	23,435.00	565,532.53
SURPLUS, APRIL 30, 1927		$602,756.01

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

RESERVE CAPITAL FUND—SECURITIES

APRIL 30, 1927

EXHIBIT A.
SCHEDULE NO. 1.

	Par Value	Book Value
The American Telephone and Telegraph Company 5% Gold Debentures Sinking Fund, Due 1960, Registered	$ 15,000.00	$ 14,625.00
Consolidated Gas Company of New York 5½% Gold Debentures, Due 1945, Registered	5,000.00	5,187.50
Pacific Gas & Electric Company 5½% First and Refunding Mortgage Gold Bonds, Due 1952, Registered	5,000.00	5,137.50
Chicago, Burlington & Quincy Railroad Company 5% First and Refunding Mortgage Series "A" Gold Bond, Due 1971, Registered	1,000.00	1,010.00
The New York Central Railroad Company 5% Refunding and Improvement Mortgage, Series "C" Bonds, Due 2013, Registered	6,000.00	5,742.50
Southern Railway Company 5% First Consolidated Mortgage Gold Bond, due 1994, Registered	1,000.00	980.00
Great Northern Railroad Company 5½% General Mortgage Series "B" Gold Bonds, Due 1952, Registered	10,000.00	9,847.50
The Detroit Edison Company 6% First and Refunding Mortgage, Series "B" Gold Bonds, Due 1940, Registered	5,000.00	5,178.13
The Western Electric Company 5% Bonds, Due April 1, 1944	10,000.00	9,818.75
Baltimore and Ohio Railroad Company 4½% Convertible Gold Bonds, Due 1933, Registered	10,000.00	9,287.50
St. Louis, San Francisco Railway Company 5% Prior Lien Mortgage, Series "B" Bonds, Due 1950, Registered	6,000.00	5,497.50
American Smelting and Refining Company 5% First Mortgage 30-Year Gold Bonds, Due 1947, Registered	9,000.00	9,085.00
Florida East Coast Railway Company 5% First and Refunding Mortgage Series "A" Gold Bonds, Due 1974, Registered	5,000.00	4,918.75
Public Service Corporation of New Jersey 5½% Gold Bonds, Due 1956	8,000.00	7,952.50
Chicago, Terre Haute & Southeastern Railroad Company 5% First and Refunding Mortgage Gold Bonds, Due 1960	3,000.00	2,846.25
Commonwealth Power Corporation 6% Sinking Fund 25-Year Bonds, Due 1947	3,000.00	3,150.00
United States Rubber Company 5% First and Refunding Mortgage Series "A" Bonds, Due 1947	2,000.00	1,915.00
American Smelting and Refining Company 6% First Mortgage Bonds, Due 1947	3,000.00	3,241.25
Total	$107,000.00	$105,520.63

Officers A. I. E. E. 1926-1927

PRESIDENT

(Term expires July 31, 1927)
C. C. CHESNEY

JUNIOR PAST PRESIDENTS

(Term expires July 31, 1927)	(Term expires July 31, 1928)
FARLEY OSGOOD	M. I. PUPIN

VICE-PRESIDENTS

(Terms expire July 31, 1927)	(Terms expire July 31, 1928)
P. M. DOWNING (District No. 8)	H. M. HOBART (District No. 1)
HERBERT S. SANDS (District No.6)	B. G. JAMIESON (District No. 5)
W. E. MITCHELL (District No. 4)	GEORGE L. KNIGHT (District No. 3)
ARTHUR G. PIERCE(District No. 2)	H. H. SCHOOLFIELD (District No. 9)
W. P. DOBSON (District No. 10)	A. E. BETTIS (District No. 7)

MANAGERS

(Terms expire July 31, 1927)	(Terms expire July 31, 1929)
W. M. McCONAHEY	M. M. FOWLER
W. K. VANDERPOEL	H. A. KIDDER
H. P. CHARLESWORTH	E. C. STONE
(Terms expire July 31, 1928)	(Terms expire July 31, 1930)
JOHN B. WHITEHEAD	I. E. MOULTROP
J. M. BRYANT	H. C. DON CARLOS
E. B. MERRIAM	F. J. CHESTERMAN

NATIONAL TREASURER	**NATIONAL SECRETARY**
(Terms expire July 31, 1927)	
GEORGE A. HAMILTON	F. L. HUTCHINSON

HONORARY SECRETARY	**GENERAL COUNSEL**
RALPH W. POPE	PARKER & AARON 30 Broad Street, New York

PAST PRESIDENTS—1884-1926

*Norvin Green, 1884-5-6.
*Franklin L. Pope, 1886-7.
*T. Commerford Martin, 1887-8.
Edward Weston, 1888-9.
Elihu Thomson, 1889-90.
*William A. Anthony, 1890-91.
*Alexander Graham Bell, 1891-2.
Frank Julian Sprague, 1892-3.
*Edwin J. Houston, 1893-4-5.
*Louis Duncan, 1895-6-7.
*Francis Bacon Crocker, 1897-8.
A. E. Kennelly, 1898-1900.
*Carl Hering, 1900-1.
*Charles P. Steinmetz, 1901-2.
Charles F. Scott, 1902-3.
Bion J. Arnold, 1903-4.
John W. Lieb, 1904-5.
*Schuyler Skaats Wheeler, 1905-6.
*Samuel Sheldon, 1906-7.
*Henry G. Stott, 1907-8.
Louis A. Ferguson, 1908-9.
Lewis B. Stillwell, 1909-10.
Dugald C. Jackson, 1910-11.
Gano Dunn, 1911-12.
Ralph D. Mershon, 1912-13.
C. O. Mailloux, 1913-14.
Paul M. Lincoln, 1914-15.
John J. Carty, 1915-16.
H. W. Buck, 1916-17.
E. W. Rice, Jr., 1917-18.
Comfort A. Adams, 1918-19.
Calvert Townley, 1919-20.
A. W. Berresford, 1920-21.
William McClellan, 1921-22.
Frank B. Jewett, 1922-23.
Harris J. Ryan, 1923-4.
Farley Osgood, 1924-25.
M. I. Pupin, 1925-26.

*Deceased.

LOCAL HONORARY SECRETARIES

T. J. Fleming, Calle B. Mitre 519, Buenos Aires, Argentina, S. A.
H. W. Flashman, Aus. Westinghouse Elec. Co. Ltd., Cathcart House, 11 Castlereagh St., Sydney, N. S. W., Australia.
Carroll M. Mauseau, Caixa Postal No. 571, Rio de Janeiro, Brazil, S. A.
Charles le Maistre, 28 Victoria St., London, S. W. 1, England.
A. S. Garfield, 45 Bd. Beausejour, Paris XVI, France.
F. W. Willis, Tata Power Companies, Bombay House, Bombay, India.
Guido Semenza, 39 Via Monte Napoleone, Milan, Italy.
P. H. Powell, Canterbury College, Christchurch, New Zealand.
Axel F. Anstrom, 42a Grefturegatan, Stockholm, Sweden.
W. Elsdon-Dew, P. O. Box 4563, Johannesburg, Transvaal, Africa.

A. I. E. E. Committees

GENERAL STANDING COMMITTEES

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C. C. Chesney, Chairman, General Electric Co., Pittsfield, Mass.
H. P. Charlesworth, G. A. Hamilton, G. L. Knight,
W. P. Dobson, H. A. Kidder, I. E. Moultrop.

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H. P. Charlesworth, G. L. Knight.

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E. H. Hubert, Secretary, 33 W. 39th St., New York, N. Y.
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J. E. Macdonald, F. D. Newbury, C. E. Skinner.
F. W. Peek, Jr.,
Chairman of Committee on Coordination of Institute Activities, *ex-officio.*
Chairmen of technical committees, *ex-officio.*

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F. L. Hutchinson, Donald McNicol, I. E. Moultrop.
E. B. Meyer,

COMMITTEE ON COORDINATION OF INSTITUTE ACTIVITIES

Farley Osgood, Chairman, 31 Nassau Street, New York, N. Y.
F. L. Hutchinson, E. B. Meyer, W. I. Slichter,
H. A. Kidder, L. F. Morehouse, Harold B. Smith.

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H. W. Drake, S. P. Grace, N. L. Pollard,
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F. M. Farmer, A. E. Waller,

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C. E. Magnusson,
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F. L. Hutchinson, H. A. Kidder.

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R. F. Schuchardt,

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F. B. Jewett, William McClellan, Harris J. Ryan.

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H. E. Farrer, Secretary, 33 W. 39th St., New York
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A. M. MacCutcheon, F. L. Rhodes, W. I. Slichter,
F. D. Newbury, L. T. Robinson, R. H. Tapscott.

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Chairmen of delegations on other standardizing bodies.
President of U. S. National Committee of I. E. C.

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Appointed by the President for term of five years.

(Terms expire July 31, 1927)
Gano Dunn, Chairman, F. A. Scheffler, W. R. Whitney.
(Terms expire July 31, 1928)
C. C. Chesney, Robert A. Millikan, M. I. Pupin.
(Terms expire July 31, 1929)
N. A. Carle, W. C. L. Eglin, John W. Lieb.
(Terms expire July 31, 1930)
George Gibbs, Samuel Insull, Ralph D. Mershon.
(Terms expire July 31, 1931)
John W. Howell, L. F. Morehouse, David B. Rushmore.

Elected by the Board of Directors from its own membership for term of two years.

(Terms expire July 31, 1927)
W. P. Dobson, Farley Osgood, A. G. Pierce.
(Terms expire July 31, 1928)
B. G. Jamieson, H. A. Kidder, G. L. Knight.

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F. L. Hutchinson, National Secretary.

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L. F. Morehouse, John B. Whitehead.

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J. C. Forsyth,
R. C. Fryer,
D. H. Gage,
H. B. Gear,
P. J. Howe,
L. C. Ilsley,
M. G. Lloyd,
Ernest Lunn,
Wills Maclachlan,
J. C. Martin,
J. E. Moore,
R. W. E. Moore,
Farley Osgood,
George Quinan,
Joseph Sachs,
H. R. Sargent,
W. H. Sawyer,
M. L. Sindeband,
Paul Spencer,
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Gano Dunn,
L. E. Imlay,
E. W. Rice, Jr.

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H. M. Hobart,
H. A. Kidder,
E. C. Stone,
J. B. Whitehead.

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Randolph H. Nexsen,
George K. Thompson.

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J. L. Clarke,
Charles E. Davies,
H. W. Drake,
Major P. W. Evans,
R. D. Evans,
E. H. Everit,
L. F. Fuller,
D. H. Gage,
S. P. Grace,
P. J. Howe,
F. H. Kroger,
Ray H. Manson,
R. D. Parker,
H. S. Phelps,
Lt. Commdr. B. B. Ralston,
F. A. Raymond,
Chester W. Rice,
J. K. Roosevelt,
H. A. Shepard,
J. F. Skirrow,
E. B. Tuttle,
K. L. Wilkinson,
F. A. Wolff,
C. A. Wright.

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R. E. Doherty, Vice-Chairman, Schenectady, N. Y.
C. A. Adams,
Edward Bennett,
Nelson J. Darling,
W. C. Eglin,
J. P. Jackson,
John Mills,
H. H. Norris,
Harold Pender,
W. L. Robb,
Harold B. Smith,
R. W. Sorensen,
J. B. Whitehead,
W. R. Whitney,
W. E. Wickenden.

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J. C. Parker, Vice-Chairman, Brooklyn, N. Y.
E. B. Pacton, Secretary, General Electric Co., Schenectady, N. Y.
C. A. Adams,
P. L. Alger,
B. F. Bailey,
B. L. Barns,
B. A. Behrend,
C. O. Bergstrom,
A. C. Bunker,
James Burke,
W. M. Dann,
L. L. Elden,
G. Faccioli,
C. J. Fechheimer,
W. J. Foster,
G. H. Garcelon,
C. M. Gilt,
Harold Goodwin,
J. L. Hamilton,
W. E. Haseltine,
R. A. Hentz,
C. F. Hirshfeld,
B. G. Jamieson,
J. A. Johnson,
V. Karapetoff,
A. H. Kehoe,
P. M. Lincoln,
A. M. MacCutcheon,
O. K. Marti,
V. M. Montsinger,
C. A. Mudge,
F. D. Newbury,
L. C. Nichols,
N. L. Poliard,
C. H. Sanderson,
R. F. Schuchardt,
A. Still,
E. C. Stone,
R. H. Tapscott,
R. B. Williamson,
H. L. Zabriskie.

ELECTRIC WELDING

J. C. Lincoln, Chairman, Coit Road and Kirby Avenue, Cleveland, Ohio
C. A. Adams,
P. P. Alexander,
C. W. Bates,
Ernest Bauer,
A. M. Candy,
Alexander Churchward,
O. H. Eschholz,
F. M. Farmer,
H. M. Hobart,
C. J. Holslag,
C. L. Ipsen,
Ernest Lunn,
J. W. Owens,
William Spraragen,
H. W. Tobey,
Ernest Wanamaker.

ELECTROCHEMISTRY AND ELECTROMETALLURGY

G. W. Vinal, Chairman, Bureau of Standards, Washington, D. C.
Lawrence Addicks,
A. N. Anderson,
T. C. Atchison,
Farley G. Clark,
Safford K. Colby,
F. A. J. Fitzgerald,
Walter E. Holland,
F. A. Lidbury,
C. G. Schluederberg,
Magnus Unger,
J. B. Whitehead,
J. L. Woodbridge,
J. L. McK. Yardley.

ELECTROPHYSICS

V. Karapetoff, Chairman, Cornell University, Ithaca, N. Y.
Carl Kinsley, Secretary, 22 Lynwood Road, Scarsdale, N. Y.
Oliver E. Buckley,
V. Bush,
J. F. H. Douglas,
Charles Fortescue,
Carl Kinsley,
W. B. Kouwenhoven,
K. B. McEachron,
R. A. Millikan,
J. H. Morecroft,
Chester W. Rice,
J. Slepian,
Harold B. Smith,
Irving B. Smith,
J. B. Whitehead.

INSTRUMENTS AND MEASUREMENTS

A. E. Knowlton, Chairman, Dunham Laboratory, Yale University, New Haven, Conn.
E. D. Doyle, Vice-Chairman, New York, N. Y.
O. J. Bliss,
Perry A. Borden,
W. M. Bradshaw,
H. B. Brooks,
J. R. Craighead,
Melville Eastham,
W. M. Goodwin, Jr.,
C. F. Hanson,
W. B. Kouwenhoven,
P. M. Lincoln,
W. M. McConahey,
Wm. J. Mowbray,
H. A. Perkins,
R. T. Pierce,
L. T. Robinson,
E. J. Rutan,
B. W. St. Clair,
G. A. Sawin,
I. B. Smith,
H. M. Turner,
Roy Wilkins.

APPLICATIONS TO IRON AND STEEL PRODUCTION

A. G. Pierce, Chairman, 1329 Guardian Building, Cleveland, Ohio
A. C. Bunker,
A. C. Cummins,
F. B. Crosby,
J. H. Hall,
S. L. Henderson,
R. H. Keil,
W. C. Kennedy,
A. G. Place,
F. O. Schnure,
J. W. Speer,
G. E. Stoltz,
T. S. Towle,
J. D. Wright.

PRODUCTION AND APPLICATION OF LIGHT

P. S. Millar, Chairman, Electrical Testing Laboratories, 80th St. & East End Ave., New York
W. T. Blackwell,
J. M. Bryant,
J. R. Cravath,
W. T. Dempsey,
William Esty,
G. C. Hall,
L. A. Hawkins,
H. H. Higbie,
C. L. Kinsloe,
A. S. McAllister,
George S. Merrill,
F. H. Murphy,
F. A. Rogers,
B. E. Shackelford,
C. J. Stahl,
G. H. Stickney.

APPLICATIONS TO MARINE WORK

G. A. Pierce, Chairman, Wm. Cramp & Sons S. & E. B. Co., Philadelphia, Pa.
R. A. Beekman, Vice-Chairman, Schenectady, N. Y.
Commdr. C. S. Gillette,
E. M. Glasgow,
H. F. Harvey, Jr.,
Wm. Hetherington, Jr.,
H. L. Hibbard,
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